Summary of Calibration and Inversion Parameter Testing on the Yukon Flats AEM Data

B.J. Minsley
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Summary of Calibration Testing

1. Lines tested: L10020 (crosses Stinking Lake), L10140/1 (crosses Twelve Mile Lake), L10220 (contains multi-elevation test data).

2. Observations from preliminary inversions using the leveled, but uncalibrated data from Fugro:
   a. Models are reasonable, but with definite misfit biases, suggestive of calibration errors.
   b. Preliminary inversions were run with 'nominal' errors [10 10 10 20 40 50] ppm, which seem to be quite large relative to the data amplitudes. It is possible that areas with good RMSE only appear this way because of the large error bars being used.
   c. Very poor misfits are observed over conductive areas (lakes). Possible explanations are (a) amplification of calibration errors due to the conductive environment and (b) amplification of elevation errors over the conductive features.
3. The stochastic Markov Chain Monte Carlo (MCMC) algorithm was utilized to assess (a) calibration factors and (b) data errors using the multi-elevation data from L10220 at FID 7035.5. The calibration factors are found by sampling earth models and calibration factors (G,ϕ,B^I,B^Q) that make the predicted response match the measured data according to

$$d_{obs}^I + jd_{obs}^Q = Ge^{j\phi}\left(d^I\left(m\right) + jd^Q\left(m\right) + B^I + B^Q\right). \quad (1)$$

   a. In the first trial, fixed 'nominal' data errors are used and the calibration factors corresponding to the MCMC algorithm's most probable model are output. The data along all three test lines were corrected by applying the inverse of calibration relationship in equation 1 to the observed data, such as

$$d_{corr}^I + jd_{dcorr}^Q = \frac{1}{G}e^{-j\phi}\left(d_{obs}^I + jd_{obs}^Q\right) - B^I - B^Q \quad (2)$$

   Inversion of the calibrated data results in generally improved misfit, and specifically a reduction in the misfit bias for any given frequency. However, large data misfits remained over the (more conductive) lakes.

b. One hypothesis was that the fixed data errors are too large over resistive features where amplitudes are small, and too small over conductive features with large data amplitudes. To address this question, data errors were assessed as a free parameter along with conductivity models and calibration factors in the MCMC algorithm. A single unknown parameter, c, that quantifies error as a fraction of the data amplitude was solved for, such as

$$error = \chi \left| d_{obs}^{I,Q} \right| . \tag{3}$$

The posterior distribution of data errors ranged from approximately 0.02–0.14, with a peak near χ=0.06 (6 percent error). The calibration parameters associated with the most probable model are reported in **Error! Reference source not found.**.

Table 1. 'Most-probable' calibration parameters for the multi-elevation data with relative data errors

Frequency	Gain	Phase (degree)	Bias (in phase, parts per million)	Bias (quadrature, parts per million)
400	0.89	0.95	2.74	6.59
1800	1.13	1.39	2.00	-0.54
3300	1.06	2.28	9.54	-1.28
8200	0.86	-0.70	0.21	-12.97
40k	1.00	-0.08	-4.11	-17.09
140k	1.08	3.07	-35.43	-0.77

The updated calibration parameters were applied to the data, and all three lines were re-inverted using an assumption of relative data errors of 6 percent. This resulted in less over-fitting of the data in resistive areas, as well as better data fits over the more conductive lakes, suggesting that the relative error assumption is valid. A minimum absolute error value of 5 ppm was used for all frequencies, that is

$$EM1DFMerror = \max\left(.06 \cdot \left| d_{obs}^{I,Q} \right|, 5ppm\right) \tag{4}$$

c. One remaining problem area was over Stinking Lake (L10020), which appears to be much more conductive (<10 ohm-m) than the other lakes. After calibration and use of the relative errors, there was still a large data misfit over this lake. Two competing hypotheses were proposed for this: (a) the multi-elevation calibration was inadequate in this different hydrogeologic regime and (b) modest elevation errors over the lake could be amplified due to the low resistivity. Though it may be impossible to assess which is the case, we decided to pursue option (a).
The calibration parameters are non-unique; that is the values in **Error! Reference source not found.** are one of many sets of factors that can be applied to the data and result in an acceptable data fit. We therefore wanted to explore whether there was a set of calibration parameters in this distribution that would satisfy both the multi-elevation data as well as the Stinking Lake data.
To accomplish this, the MCMC algorithm was solved for the calibration parameters at a single location (and elevation) over Stinking Lake (FID 3552). Because only a single elevation is used, there is significant uncertainty in the distribution of calibration parameters. However, we observe that the distributions of calibration parameters for the multi-elevation dataset and the Stinking Lake

dataset overlap. That is, there may be a single set of parameters that satisfy both datasets.

The 'best' set of calibration parameters was assessed by comparing the multi-elevation calibration parameters with the Stinking Lake calibration parameters. The set of calibration parameters from the multi-elevation data that were closest (in a minimum-norm sense) to the Stinking Lake data were chosen and are reported in **Error! Reference source not found.**.

Table 2. Calibration parameters for the multi-elevation data with relative data errors that are closest to the calibration parameters derived from the Stinking Lake data.

Frequency	Gain	Phase (degree)	Bias (in phase, parts per million)	Bias (quadrature, parts per million)
400	1.00	0.14	-2.56	-2.70
1800	0.95	0.05	-2.06	4.80
3300	0.98	0.04	4.25	7.45
8200	0.90	0.40	-4.81	-15.28
40k	0.99	0.25	-1.16	-29.20
140k	1.01	0.37	-24.51	3.43

These new calibration parameters were applied to all three lines of data, which were again inverted using 6 percent relative errors. The result was an acceptable level of misfit over Stinking Lake, as well as over the other portions of the lines. We therefore concluded that the set of calibration parameters in **Error! Reference source not found.** were optimal, in that they are consistent with both the multi-elevation data as well as the Stinking Lake data.

For the most part, the difference in models derived using the parameters in **Error! Reference source not found.** versus **Error! Reference source not found.** are modest. However, the overall trend is for models that exhibit lower resistivity at shallower depth using the parameters in **Error! Reference source not found.**. Without proper ground truth, it will be difficult to ever know if this is correct.

Summary of Inversion Parameter Selection

1. A comprehensive suite of tests was carried out regarding the optimum parameters for use in EM1DFM (Farquharson, 2000; Farquharson and others, 2003), which are summarized below:
    a. Model type: conductivity only.
    b. Starting conductivity model: A best-fitting half-space model is used for the starting model. No single starting model could be found that provided acceptable results over both resistive (thick sands/permafrost) and conductive (lakes) areas. Twenty-five layer models are used with increasing layer thickness (175 m to the top of the semi-infinite half-space) according to:

Table 3. EM1DFM layer thicknesses.

Layer #	Thickness (m)
1	1.20496
2	1.36646
3	1.54962
4	1.75732
5	1.99287
6	2.25998
7	2.5629
8	2.90642
9	3.29599
10	3.73777
11	4.23877
12	4.80691
13	5.45121
14	6.18187
15	7.01046
16	7.95012
17	9.01572
18	10.2242
19	11.5946
20	13.1486
21	14.911
22	16.9097
23	19.1762
24	21.7464
25	∞

c. Reference model: Three different reference models are used for the purpose of the DOI calculation. These are 5.6 ohm-m (conductivity = 0.1786), 28 ohm-m (conductivity = 0.0357), and 140 ohm-m (conductivity = 0.00714). The high- and low-resistivity models represent a factor of 5 above and below the 'central' model of 28 ohm-m. Higher resistivity reference models were tested (with central values up to 700 ohm-m), but it was found that these values were too high relative to typical 'basement' resistivities that were on the order of tens of ohm meters.

d. Inversion type: Fixed tradeoff (fixed β). A fixed value of $\beta = 3$ was determined after many fixed β trial runs, as well as runs using the GCV criteria. Though generally acceptable, the GCV too-frequently allowed extremely low β values ($\ll 1$) that introduced excessive structure and contrasts in the model; $\beta = 3$ was determined to be the lowest value that produced acceptable results over all three test lines.

e. Model norm components: acs = 0.01, acz = 1. Several values of the relative model norm were tested, with acs = 0.01 being an acceptable tradeoff between smooth and 'small' models.