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Abbreviations

7Q	annual minimum 7-day streamflow
7Q2	annual minimum 7-day streamflow exceeded in 1 out of 2 years on average
7Q10	annual minimum 7-day streamflow exceeded in 9 out of 10 years on average
7Q20	annual minimum 7-day streamflow exceeded in 19 out of 20 years on average
LN2	two-parameter lognormal distribution
LN3	three-parameter lognormal distribution
LP3	log-Pearson type 3 distribution
RMSE	root mean-square error
USGS	U.S. Geological Survey

Methods for Computing 7Q2 and 7Q20 Low-Streamflow Statistics to Account for Possible Trends

By Luther Schalk,[1] Robert W. Dudley,[1] and Annalise G. Blum[2]

Abstract

Low-streamflow statistics, such as the annual minimum 7-day streamflow (which is the 7-day streamflow likely to be exceeded in 9 out of 10 years on average [7Q10]), that are computed by using the full historical streamflow record may not accurately represent current conditions at sites with statistically significant trends in low streamflow over time. Recent research suggests that using a contemporary subset of the historical streamflow record (specifically, the most recent 30 years) to compute an estimate of 7Q10 more accurately represents current streamflow conditions when a statistically significant trend in the streamflow record is present. This report presents the results of a Monte Carlo simulation experiment on artificial low-streamflow records, derived from the characteristics of streamflows at 174 U.S. Geological Survey streamgages, to test whether a similar approach is appropriate for the computation of the annual minimum 7-day streamflow exceeded in 1 out of 2 years on average (7Q2) and the annual minimum 7-day streamflow exceeded in 19 out of 20 years on average (7Q20). The results indicate that using the most recent 30-year subset of the low-streamflow record also may be the best approach when computing 7Q2 and 7Q20 at sites where a statistically significant trend in low streamflows is detected.

Introduction

Historical observations of streamflows are often used to estimate the frequency of occurrence of future streamflows in some quantifiable way. Under an assumption of stationarity, the frequency at which a particular streamflow occurred in the past is representative of its probable present and future frequency of occurrence, and it is taken for granted that all historical values should have equal weight in the computation of such streamflow statistics. Stationarity is commonly assumed when streamflow-frequency statistics are computed by using the full period of recorded streamflow; however, assuming stationarity when it does not exist may yield inaccurate results in the statistic (for example, see Yu, 2017, for peak-streamflow statistics).

The environmental drivers of nonstationarity at a site are many and complex. Researchers have sought to explain trends in low streamflows using interdependent hydrologic processes in copula models (Jiang and others, 2015; Ahn and Palmer, 2016), climatological variables demonstrated to be hydrologic predictors for particular regions (Liu and others, 2015), and antecedent sea-surface temperatures (Steinschneider and Brown, 2012). However, the precise influence that any one of these processes may exercise on change in hydrologic statistics is quantitatively uncertain, as has been demonstrated in studies of urbanization (Price, 2011; Allaire and others, 2015; Dudley and others, 2020). When some of the drivers of hydrologic change are dependent on others in complex ways, attributing hydrologic change—or even portions of it—to any single causal variable (Hirsch, 2011; Allaire and others, 2015) becomes difficult and presents a challenge to adequate modeling of the observed variability of low streamflows.

The presence of nonstationarity at a site calls into question exactly how much of the historical record is representative of present or future conditions and to what degree. For example, the use of low-streamflow statistics for regulatory purposes related to energy production, effluent dilution, drinking water, irrigation, or maintenance of minimum flow standards to support in-stream ecological health requires water-resource managers to know whether those statistics adequately represent current or future conditions pertinent to their application. Studies have indicated that low-streamflow statistics based on the recent past rather than the full record may more satisfactorily represent present or near-future values (Riggs, 1972; Gebert and others, 2016). Using data from 174 selected U.S. Geological Survey (USGS) streamgages in the Chesapeake Bay watershed, Blum and others (2019) sought to quantify which subset of the recent low-streamflow record was most suitable for computing what may be considered the most accurate or least biased low-streamflow statistics under nonstationary conditions for the 7Q10 low-streamflow statistic (the annual minimum 7-day streamflow exceeded in 9 out of 10 years on average). Practitioners in water-quality management, water-supply planning, industrial regulation, and agriculture commonly make decisions based on 7Q10 (for example, U.S. Environmental Protection Agency, 1986, 2018).

[1]U.S. Geological Survey.

[2]Johns Hopkins University.

Blum and others (2019) found that the optimal approach for computing 7Q10 was to use the most recent 30 years of the record when a statistically significant trend was detected and to use the full historical record when no significant trend was detected.

Other statistical measures of low streamflow are also of interest to hydrologists and decision makers. Performing such an analysis on other statistics similar to 7Q10 may yield greater insight into the results of the 7Q10 analysis by Blum and others (2019). This study uses the same data and experimental methodology as Blum and others (2019) to determine which subset of the recent low-streamflow record is most suitable for computing the most accurate or least biased estimates of alternate low-streamflow statistics: the annual minimum 7-day streamflow likely to be exceeded in 1 out of 2 years on average (7Q2) and the annual minimum 7-day streamflow likely to be exceeded in 19 out of 20 years on average (7Q20).

Data

The same data used by Blum and others (2019) were used in this study. For the 174 USGS streamgages in the Chesapeake Bay watershed in the mid-Atlantic region of the United States, daily streamflow values for the period of record ending in water year 2013 (October 1, 2012, to September 30, 2013) were obtained from the USGS National Water Information System (USGS, 2016). These sites represented a diversity of low-streamflow characteristics and geology, included sites that were affected by changes in regulation or land use (or both), and had from 56 to 75 years of data. Dudley and others (2019) provided (1) USGS streamgage identification numbers; (2) the beginning and ending years for the period of streamflow record tested; (3) Sen slope trends in the annual minimum 7-day streamflow for the period of record tested; (4) the *p*-values (significance) of the trends; and (5) the trend Sen slopes standardized by the standard deviations of the residual errors, defined as the difference between observations and the Sen slope lines (hereafter, “standardized trends”). The standardized trends varied from −0.033 to 0.029.

Reference should be made to Blum and others (2019) for an in-depth discussion of the source and derivative data. However, the definition of the standardized trend is repeated here because of its frequent use in this report. Dudley and others (2019) provided a nonparametric Sen slope computed for each of the 174 streamgages as the median slope of all slopes computed between every pair of points in that streamgage’s annual minimum 7-day flow (7Q) time series. The slope was then divided by the standard deviation of the residual error from the 7Q data to produce the standardized trend value for each streamgage. A positive standardized trend indicates a wetting trend (an increase in 7Q magnitude over time) at a streamgage, and a negative standardized trend indicates a drying trend (a decrease in 7Q magnitude over time).

Methods

For this study, we followed Blum and others (2019) to estimate 7Q2 and 7Q20 statistics, to evaluate their accuracy and bias, and to identify the best approach for computing the statistics from full-record or recent-record subsets. The two-parameter lognormal probability distribution (LN2) was used to generate the synthetic 7Q records. This distribution was favored because of (1) its simplicity; (2) its relation to both the three-parameter lognormal distribution (LN3) and the log-Pearson type 3 distribution (LP3), both of which are widely used for this purpose; (3) its proven applicability for describing low-streamflow distributions in previous studies (Vogel and Kroll, 1989, 1990; Dingman and Lawlor, 1995; Modarres, 2008; Grandry and others, 2013; Jiang and others, 2015); and (4) the confirmation—through L-moment ratio diagrams—that it approximates the probability distribution of 7Q at gaged sites (Blum, 2017).

For each of the 174 standardized trends, a true 7Q2 value and a true 7Q20 value in the last year of record were computed by using a nonstationary LN2 quantile function (Vogel and others, 2011) and assuming a log-linear trend (eq. 1, reproduced from equation 1 in Blum and others, 2019). These values were the true values of the statistics as derived from the statistical models used to generate the synthetic records, as opposed to hypothetical “true” values of the statistics at a real-world streamgage, which—given nonstationarity and data limitations—cannot be known and were estimated instead.

$$7Qx_{true} = e^{\left(\mu_y+\beta(n-\bar{n})+z_x\sqrt{\sigma_y^2-\beta^2 s_n^2}\right)}, \quad (1)$$

where

- x_{true} is the true recurrence interval (2 for 7Q2, 20 for 7Q20);
- μ_y is the mean of the log-transformed 7Q record y, with y equal to ln(7Q);
- β is the magnitude of the standardized trend, assumed to be the true slope;
- n is the record length (75 years for all synthetic records);
- $\bar{n}$ is the mean year (38);
- z_x is a standard normal variable with an annual exceedance probability of $1/x$ (0.5 for 7Q2, 0.05 for 7Q20);
- σ_y is the standard deviation of y; and
- s_n^2 is the variance of record length, which has been derived for a nonrandom time variable so that $12s_n^2$ is equal to $n(n+1)$.

The mean μ_y was set to 0, and the standard deviation σ_y was set to 1. This equation assumes that a log-linear function of 7Q is representative; Blum and others (2019) found this assumption to fit well in the study area.

In addition to the true 7Q2 and 7Q20 values computed as above for each of the 174 standardized trends, a set of 10,000 synthetic 75-year annual series of 7Q values was simulated for each of the 174 standardized trends by using equation 2 (reproduced from equation 3 in Blum and others, 2019):

$$y_i = \mu_y + \beta(n_i - \bar{n}) + \varepsilon_i, \quad (2)$$

where

y_i is the log-transformed 7Q value in year i, with y equal to ln(7Q);

n_i is the year i from 1 to 75;

$\bar{n}$ is the mean year (38); and

ε_i is the residual error in year i, where ε_i has a normal distribution with a mean of 0 and a standard deviation of 1.

Estimating 7Q2 and 7Q20 Statistics

A nonparametric estimator is appropriate for the low-streamflow statistics 7Q2 and 7Q20 on a record length of 75 years because (1) the true distribution of 7Q would be unknown in practice and (2) record lengths are constrained to equal or exceed the return intervals of interest (Blum and others, 2019). For the return interval of 2 years (that is, 7Q2), statistics were estimated for eight subsets of the synthetic records, all exceeding the return interval: the most recent 3, 5, 10, 20, 30, 40, and 50 years of record and the full 75 years. For the 20-year return interval (that is, 7Q20), the subsets of the most recent 3, 5, and 10 years of record were excluded because they were all less than the return interval of the statistic being estimated. Therefore, 7Q20 was estimated for only five subsets of the synthetic records. Equation 3 (reproduced from equation 4 in Blum and others, 2019) was used for the 7Q2 and 7Q20 estimations:

$$7Qx_{est} = (1-\theta)\,q_i + \theta\,q_{i+1}, \quad (3)$$

where

x_{est} is the estimated recurrence interval (2 for 7Q2, 20 for 7Q20);

q_i is the value from the 7Q series with ranking i based on sorting the entire record from smallest (i equals 1) to largest (i equals record length);

i is also computed as the integer portion of $(n + 1)p$;

θ is the remainder after rounding down in the calculation of i, such that θ is equal to $[(n + 1)p - i]$;

n is the record length for the subset; and

p is the annual exceedance probability corresponding to recurrence interval x, such that p is equal to $1/x$.

Accuracy and Bias of 7Q2 and 7Q20 Estimates

The values produced by equation 3 are considered to be “estimated” 7Q2 and 7Q20 values, in contrast to the true values computed by using equation 1 for each of the 174 standardized trends. For each of the 174 standardized trends, there were 10,000 simulations, and 7Q2 was estimated from each synthetic record by using the full 75-year record and the most recent 3, 5, 10, 20, 30, 40, and 50 years of record (eight record subsets). This produced 13,920,000 estimates of 7Q2 (1,740,000 for each of the eight record subsets), each of which corresponds to the true 7Q2 value for the standardized trend upon which the synthetic record was established. Similarly, there were 8,700,000 estimates of 7Q20 (1,740,000 for each of the five record subsets used for 7Q20), each corresponding to a true 7Q20 value.

The error of each 7Q2 and 7Q20 estimate is simply the difference between the estimated 7Q2 or 7Q20 statistic resulting from each simulation and the corresponding true statistic. This per-point error was computed for all estimates, but the abundance of data points precludes producing plots with any usefulness or efficiency. Instead, the entire set of 1,740,000 error values for each record subset and each statistic (that is, 7Q2 and 7Q20) was divided into 100 bins of equal size, each containing 17,400 error values. The root mean-square error (RMSE) was computed for each bin for each of the shorter record subsets (for the 3, 5, 10, 20, 30, 40, and 50 most recent years for 7Q2; for the 20, 30, 40, and 50 most recent years for 7Q20) and compared to the same bin’s RMSE for the full 75-year record to produce an “improvement factor.” The improvement factor was computed as the ratio of the RMSE for the full record to the RMSE of the partial record (see equation 6 in Blum and others, 2019). Therefore, an improvement factor of 2 indicates that the RMSE of the full record is twice as large as that of the partial record; in other words, the 7Q2 (or 7Q20) computed from the partial record could be considered twice as accurate—on average—as that computed from the full record for the bin under consideration. An improvement factor of 1 indicates that the 7Q2 (or 7Q20) computed from the partial record is as accurate as that computed from the full record.

The mean bias was computed for the 7Q2 and 7Q20 estimation approaches by using the full record, using subsets of the most recent observation years, and using the most recent years of record with adaptive approaches (see next section) from statistical significance levels of $p<0.01$, $p<0.05$, and $p<0.1$. As with the computations for improvement factor, bias also was computed for each bin as the average over 17,400 points of the difference between the estimated 7Q2 or 7Q20 statistic and the corresponding true 7Q2 or 7Q20 value (see equation 7 in Blum and others, 2019).

Adaptive Approach

We also analyzed "adaptive" approaches for the record's subset selected as the best suited for computing 7Q2 and 7Q20 at trend-affected sites. The analyses described above are termed "nonadaptive" because the selected record subset was used for estimating the low-streamflow statistic at all sites, even sites where the full record outperformed the subset record in accuracy (that is, improvement factors less than 1, which was the case for sites with little to no trend). In contrast, the adaptive approach adapted to the presence of a trend at a site by using the full 75-year record where a trend was not detected and by using the chosen subset record where a trend was detected. We identified whether a trend existed according to the Mann-Kendall test with significance levels of $p<0.01$, $p<0.05$, and $p<0.1$ for each of the 1,740,000 synthetic 7Q datasets. For each of the significance levels, each of those synthetic datasets was categorized as either trend-affected or not trend-affected. For trend-affected datasets, the 7Q2 or 7Q20 statistic was computed by using the selected record subset; for datasets that were not trend-affected, the statistic was computed by using the full 75-year record.

Results

The range of improvement factors in the 7Q2 analysis (fig. 1) was found to be greater than those in the original 7Q10 analysis (Blum and others, 2019) or in the 7Q20 analysis (fig. 2). Improvement factors as a function of standardized trend had a roughly parabolic shape for all 7Q*x* analyses ($x = 2, 10, 20$), with a minimum at or near a standardized trend magnitude of 0 and maxima at or near the extreme standardized trend values (figs. 1 and 2; reference fig. 3 in Blum and others, 2019). All series also followed the general rule that the parabolic shape is shallowest for the longest record subsets and deepest for the shortest record subsets; that is, the longest record subsets had the greatest minimum improvement factors but the least maximum improvement factors. Inversely, the shortest record subsets had the most extreme minimum and maximum improvement factors of all record subsets.

The subsets of the most recent 20 and 30 years were the two most optimal options for estimating the 7Q2 statistic under the considered trend characteristics (fig. 1, appendix 1). The subsets of the most recent 10, 40, and 50 years each had narrow ranges of standardized trend magnitudes for which they were superior to other subsets. The subset of the most recent 20 years offered more improvement than the subset of the most recent 30 years for a wider range of standardized trend magnitudes, but those magnitudes were at the extremes of all those considered; the subset of the most recent 30 years was superior for more of the bins that correspond to the most commonly occurring standardized trends.

The subset of the most recent 30 years may be the most optimal option for estimating the 7Q20 statistic under the considered trend characteristics (fig. 2, appendix 1). This subset outperformed the subset of the most recent 20 years for almost all bins except those corresponding to the most extreme standardized trends. Compared with the subsets of the most recent 40 and 50 years, the subset of the most recent 30 years offered less improvement at trend magnitudes close to 0, but it surpassed their improvement factors as absolute trend magnitudes became greater than about 0.015.

Adaptive Approach

The subset of the most recent 30 years was chosen for improvement-factor analysis by using an adaptive-record-subset approach for comparison to the nonadaptive approaches for both the 7Q2 and 7Q20 statistics. The adaptive approach used the full record for computation at sites with no statistically significant trend present in low-streamflow magnitude and only the most recent 30 years of the record at sites where a statistically significant trend was detected. This approach combined the accuracy of the full-record analysis at sites with little to no trend with that of the subset-record analysis at sites with a greater trend so that there were few scenarios where the improvement factor was less than 1.

For datasets without statistically significant trends, the improvement factor was precisely 1 because the adaptive-subset 7Q*x* and full-record 7Q*x* were identical (figs. 3 and 4). Datasets with statistically significant trends, on the other hand, had improvement factors greater than 1 because the adaptive-subset 7Q*x* was equal to the nonadaptive-subset 7Q*x*, not to the full-record 7Q*x*.

Each of the three selected trends' significance levels for the adaptive approach ($p<0.01$, $p<0.05$, $p<0.1$) produced a curve that was flat, with an improvement factor of 1, at and near the point of no trend and that rose and met the nonadaptive data series as the improvement factor increased above 1. For all three analyses, more of the data points for the significance level of $p<0.1$ (inclusive of the most trends) matched those of the nonadaptive data series—and thus resulted in greater improvement factors for a given standardized trend magnitude—than data points for the other two significance levels, although the differences were minor. These results suggest that $p<0.1$ may be the best significance level to use when testing a site for the presence of a trend for the adaptive approach, although little accuracy would be lost by using a different significance level.

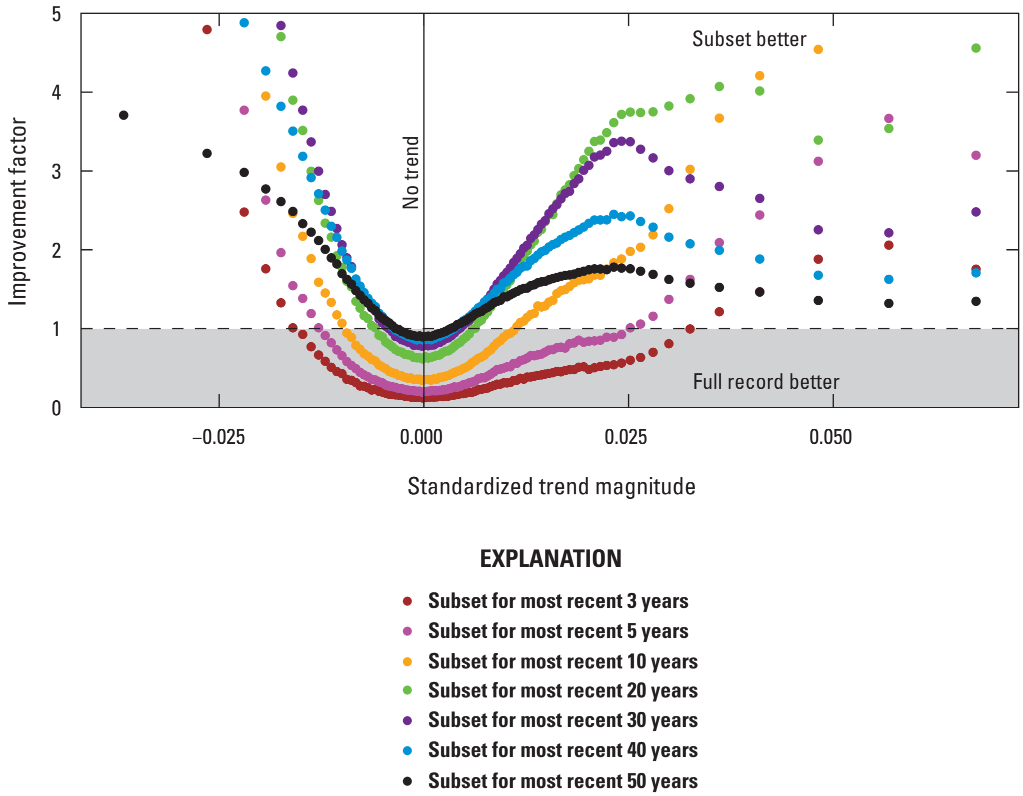

Figure 1. Accuracy of 7Q2 estimators based on subsets of the most recent 3, 5, 10, 20, 30, 40, and 50 years of record, relative to 7Q2 estimators calculated by using a full 75-year record. Improvement factor is defined as the ratio of the root mean-square error (RMSE) for 7Q2 estimated by using the full 75-year record relative to the RMSE for 7Q2 estimated by using the specified subset. Standardized trend magnitude refers to the nonparametric Sen slope standardized by residual errors relative to the Sen slope line. To focus on the large amount of data near the origin, where the differences among the seven record-subset series can be most clearly seen, bins with improvement factors greater than 5 are not shown. Above the y-axis limit, all series continue upward along a rough continuation of the same shape as the visible portions of their series. 7Q2, annual minimum 7-day streamflow exceeded in 1 out of 2 years on average.

Bias

In addition to the improvement factor, the bias of each 7Q2 and 7Q20 estimator compared to the true 7Q2 and 7Q20 can indicate which estimator is the most accurate. For the 7Q2 analysis, there were eleven 100-bin series for which the bins of average bias were plotted against the standardized trend magnitude: the nonadaptive subsets of the most recent 3, 5, 10, 20, 30, 40, and 50 years; the nonadaptive full 75-year record; and the adaptive series for the subset of the most recent 30 years with significance levels of $p<0.01$, $p<0.05$, and $p<0.1$ (fig. 5). For the 7Q20 analysis, there were eight 100-bin series: all the nonadaptive series from the 7Q2 analysis except the subsets of the most recent 3, 5, and 10 years; and the adaptive series for the subset of the most recent 30 years with significance levels of $p<0.01$, $p<0.05$, and $p<0.1$ (fig. 6).

The bias for the full record and all adaptive variations of record subsets, which were equivalent to the full record at trend magnitudes close to 0, always intersected the origin; that is, the mean bias was 0 for no-trend scenarios. All other record subsets had biases other than 0 at no standardized trend. Most, though, were near 0 for no standardized trend, with

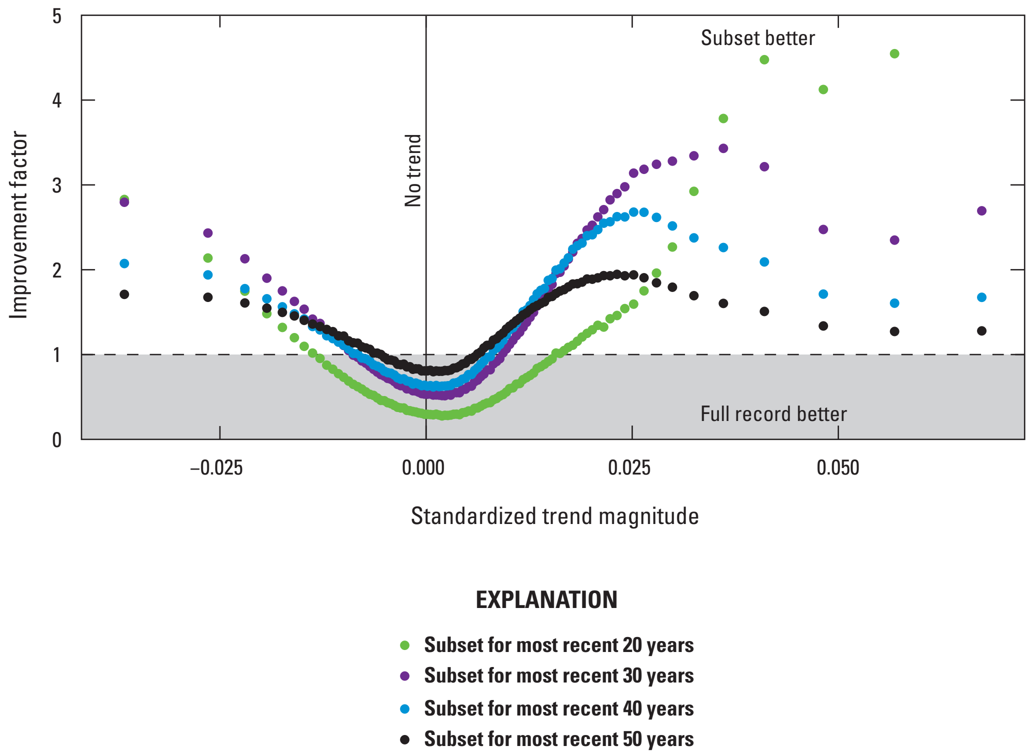

Figure 2. Accuracy of 7Q20 estimators based on subsets of the most recent 20, 30, 40, and 50 years of record, relative to 7Q20 estimators calculated by using a full 75-year record. Improvement factor is defined as the ratio of the root mean-square error (RMSE) for 7Q20 estimated by using the full 75-year record relative to the RMSE for 7Q20 estimated by using the specified subset. Standardized trend magnitude refers to the nonparametric Sen slope standardized by residual errors relative to the Sen slope line. 7Q20, annual minimum 7-day streamflow exceeded in 19 out of 20 years on average.

standardized trend magnitude and bias generally increasing together in absolute value. The full record was the least biased only at and very near a trend magnitude of 0; the shorter record subsets outperformed the full record at larger standardized trend magnitudes. The longer record subsets displayed greater absolute mean bias for almost all the bins that were considered. Bias was generally minimized across all trend magnitudes by using the shortest record subsets.

Discussion

The threshold value of the standardized trend magnitude at which the 30-year-record subset became favorable to the full record (that is, the improvement factor became greater than 1) varied slightly among the three low-streamflow statistics (7Q2, 7Q10, 7Q20) and among the trends' significance levels ($p<0.01$, $p<0.05$, $p<0.1$). Generally, however, a standardized trend magnitude of 0.01 may be considered a reasonable threshold upon which to consider using only the most recent 30 years of record for computing low-streamflow statistics.

For all three statistics, the 30-year-record subset produced less bias than the full record when a significant trend was detected at any tested significance level. The biases of the 30-year-record subset and its three adaptive approaches (significance levels of $p<0.01$, $p<0.05$, $p<0.1$) were quite similar to each other for almost all the bins. Differences were observed only very close to a trend magnitude of 0. Of the three adaptive approaches, the significance level of $p<0.1$ appeared to be most favorable in this analysis, showing less

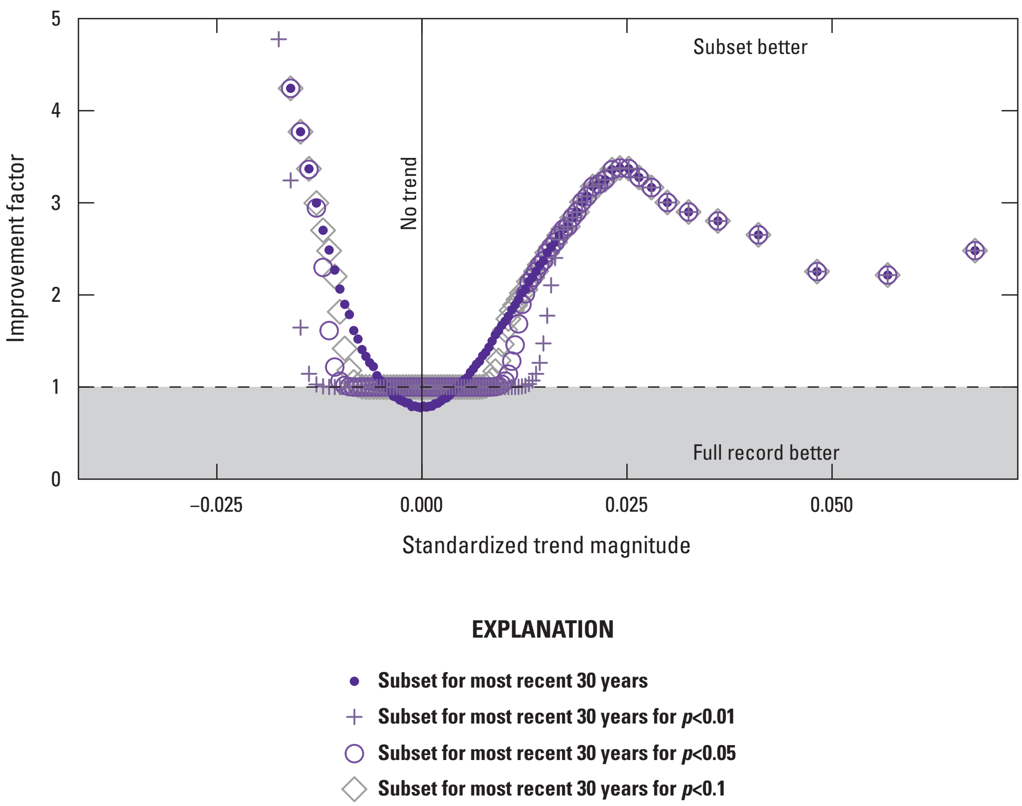

Figure 3. Accuracy of 7Q2 estimators based on adaptive approaches, where the subset of the most recent 30 years of record is only used when a statistically significant trend is detected at the $p<0.01$, $p<0.05$, and $p<0.1$ significance levels, relative to 7Q2 estimators calculated by using a full 75-year record. Improvement factor is defined as the ratio of the root mean-square error (RMSE) for 7Q2 estimated by using the full 75-year record relative to the RMSE for 7Q2 estimated by using the specified subset. Standardized trend magnitude refers to the nonparametric Sen slope standardized by residual errors relative to the Sen slope line. 7Q2, annual minimum 7-day streamflow exceeded in 1 out of 2 years on average.

absolute mean bias than the other adaptive approaches in bins in which they differed and matching the nonadaptive curve for the subset of the most recent 30 years more quickly than the other adaptive approaches as the trend magnitude increased.

The difficulty of choosing between the subsets for the most recent 20 years and the most recent 30 years for the 7Q20 analysis suggested that an adaptive approach with more than the two options proposed here (full 75-year record and a subset of the most recent 30 years) might provide the most accuracy for all the analyzed ranges of trend magnitudes, although the complexity of such an algorithm could make it unwieldy for many practitioners. To demonstrate, for all analyses (figs. 1 and 2, and see Blum and others, 2019), there was a range of standardized trend magnitudes for which each record subset provided the largest improvement factors. The longest record subsets provided the greatest improvement factors for trend magnitudes closest to 0, and the shortest record subsets provided the greatest improvement factors for trend magnitudes furthest from 0; the intervening record subsets provided the greatest improvement factors for certain ranges of trend magnitudes between the extremes. The exceptions to this general observation were the subsets of the most recent 3 and 5 years in the 7Q2 analysis, which did not provide the greatest improvement factors for any of the ranges in standardized trend magnitudes tested in this analysis. In a multiple-choice adaptive approach, a practitioner might (1) compute a site's standardized trend in low-streamflow magnitude at a significance level of $p<0.1$; (2) use appendix 1 to determine which of the analyzed subsets (or the full record) provides the greatest improvement factor at that magnitude; and (3)

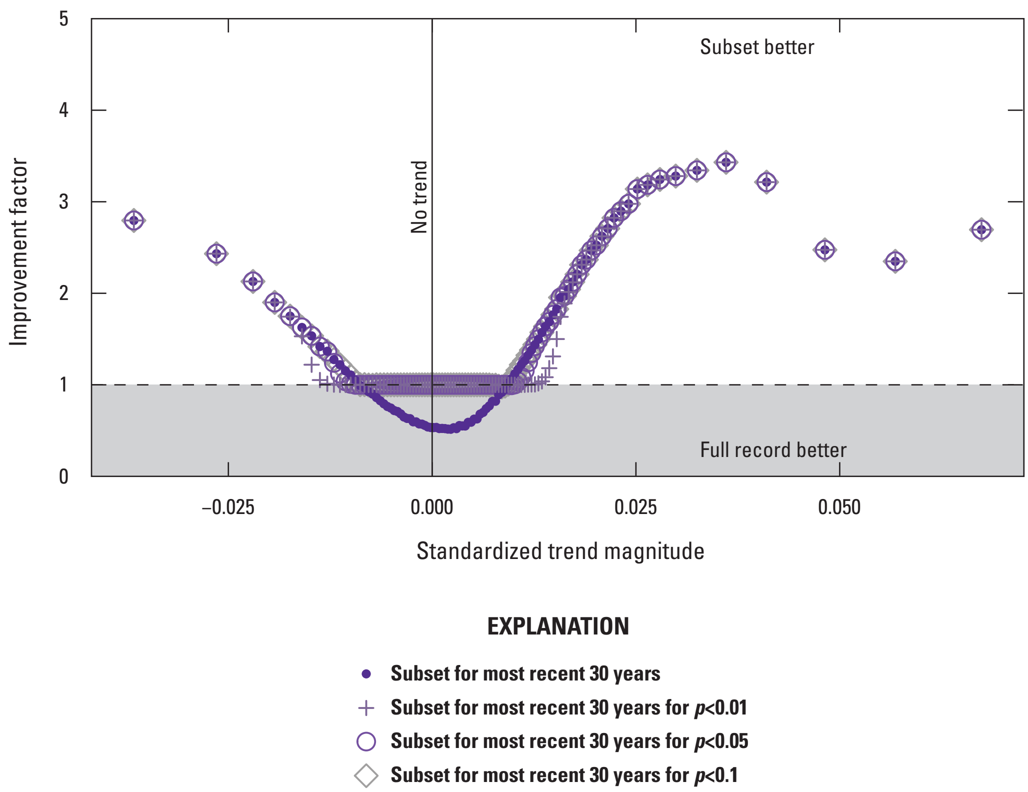

Figure 4. Accuracy of 7Q20 estimators based on adaptive approaches, where the subset of the most recent 30 years of record is only used when a statistically significant trend is detected at the $p<0.01$, $p<0.05$, and $p<0.1$ significance levels, relative to 7Q20 estimators calculated by using a full 75-year record. Improvement factor is defined as the ratio of the root mean-square error (RMSE) for 7Q20 estimated by using the full 75-year record relative to the RMSE for 7Q20 estimated by using the specified subset. Standardized trend magnitude refers to the nonparametric Sen slope standardized by residual errors relative to the Sen slope line. 7Q20, annual minimum 7-day streamflow exceeded in 19 out of 20 years on average.

proceed to compute that statistic using that subset. However, in most ranges of trend magnitude and for most bins analyzed, the differences in improvement factor between similar subsets are so slight that the simple two-choice adaptive approach recommended by Blum and others (2019) and this study can be expected to yield results nearly identical to those computed by using this marginally more accurate multiple-choice adaptive approach. Regardless, any practitioner should be sure to incorporate the most recent measures of low streamflow in both the standardized trend computation and in the subset used to compute the low-streamflow statistic.

The subset of the most recent 30 years appears to be the optimum length (or one of the optimum lengths) for 7Q2, 7Q10, and 7Q20, despite the difference in return interval (fig. 7). It is unclear from these analyses why the optimum subset length is not clearly dependent on the return period. It is important to consider that the standardized trends used in this study were derived from a set of gages limited in geographic scope to the mid-Atlantic United States, that only log-linear trends were explored, that the 7Q records are LN2 distributed, and that temporal correlation was not considered. Further analyses using information from other geographic regions, other trend types (such as nonlinear or reversing), correlation analysis, and perhaps other return periods could provide further insight into the relation between the return period and the optimum record subset length.

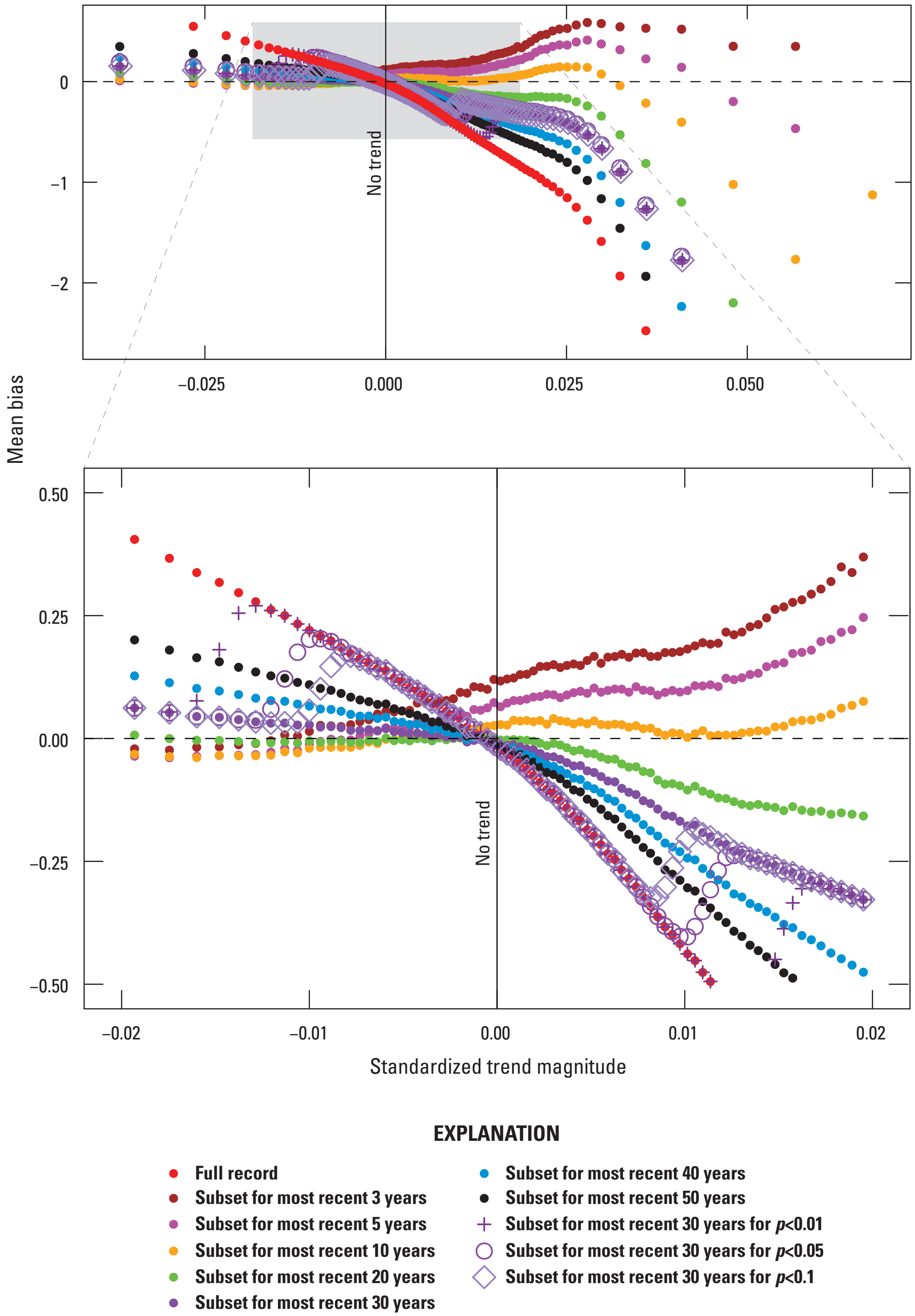

Figure 5. Mean bias of 7Q2 estimators calculated by using a full 75-year record, nonadaptive approaches (subsets of the most recent 3, 5, 10, 20, 30, 40, or 50 years), and adaptive approaches for the subset of the most recent 30 years. Standardized trend magnitude refers to the nonparametric Sen slope standardized by residual errors relative to the Sen slope line. 7Q2, annual minimum 7-day streamflow exceeded in 1 out of 2 years on average.

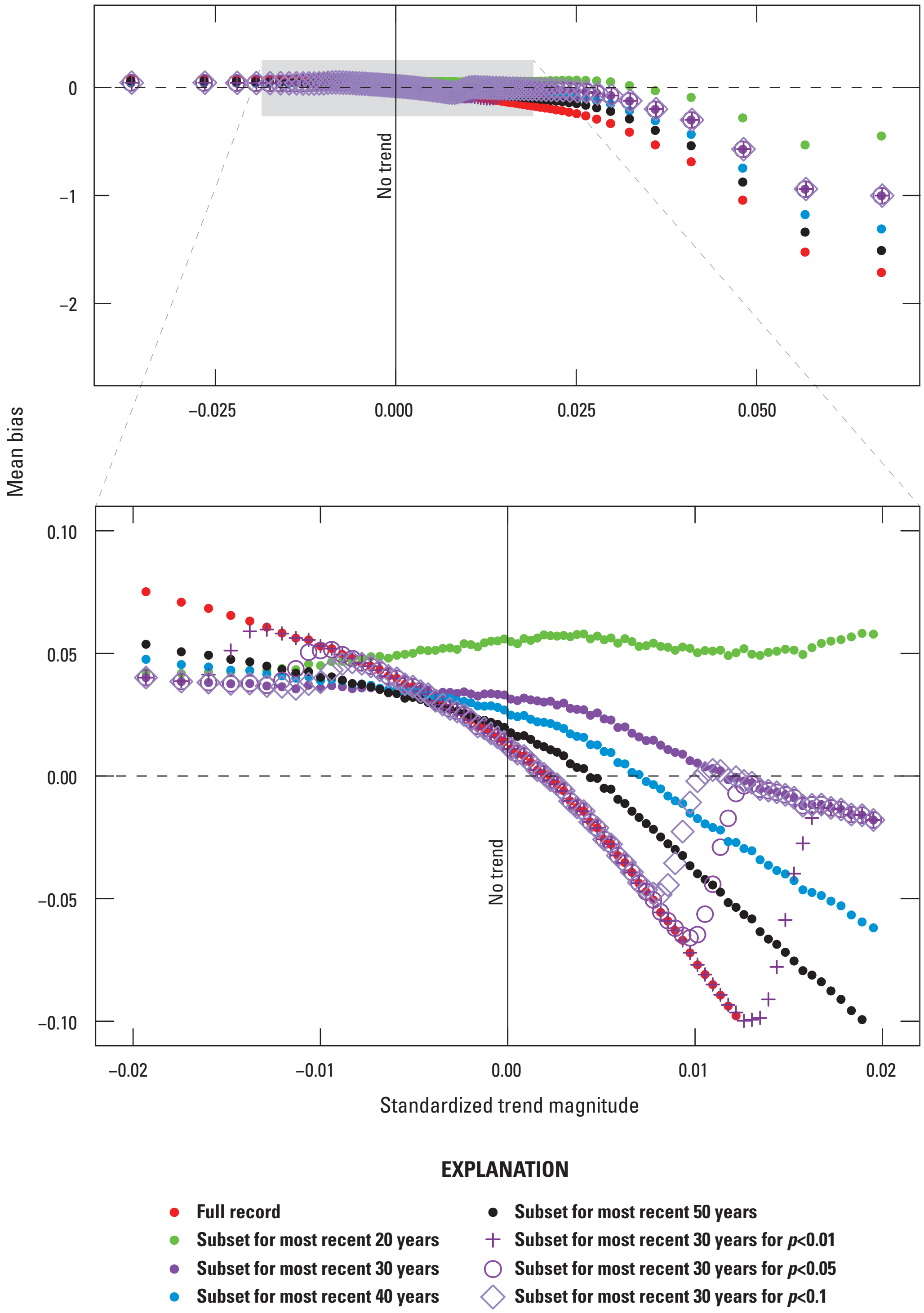

Figure 6. Mean bias of 7Q20 estimators calculated by using a full 75-year record, nonadaptive approaches (subsets of the most recent 20, 30, 40, or 50 years), and adaptive approaches for the subset of the most recent 30 years. Standardized trend magnitude refers to the nonparametric Sen slope standardized by residual errors relative to the Sen slope line. 7Q20, annual minimum 7-day streamflow exceeded in 19 out of 20 years on average.

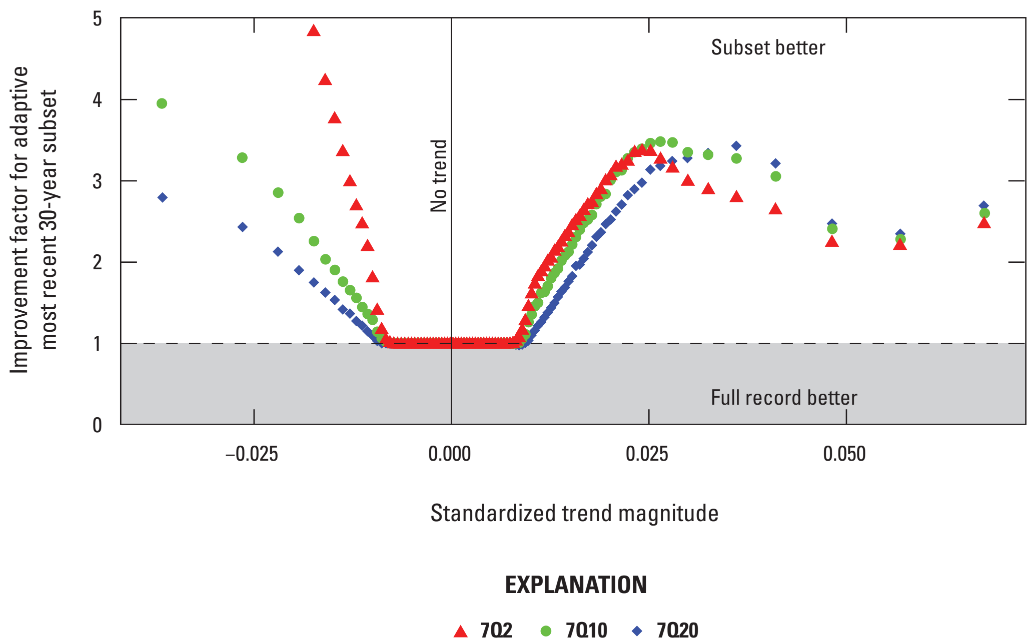

Figure 7. Accuracy of 7Q2, 7Q10, and 7Q20 estimators based on the adaptive subset of the most recent 30 years of record, relative to their respective estimators calculated by using a full 75-year record. Improvement factor is defined as the ratio of the root mean-square error (RMSE) for the statistic estimated by using the full 75-year record relative to the RMSE for the statistic estimated by using the specified subset. Standardized trend magnitude refers to the nonparametric Sen slope standardized by residual errors relative to the Sen slope line. 7Q2, annual minimum 7-day streamflow exceeded in 1 out of 2 years on average; 7Q10, annual minimum 7-day streamflow exceeded in 9 out of 10 years on average; 7Q20, annual minimum 7-day streamflow exceeded in 19 out of 20 years on average.

Summary

A previous study used Monte Carlo simulations of artificial low-streamflow records based on standardized trend magnitudes from 174 U.S. Geological Survey streamgages to identify an appropriate recent subset of a gage's low-streamflow record to use for computation of 7Q10—the annual minimum 7-day streamflow likely to be exceeded in 9 out of 10 years on average—when a significant trend in low streamflow was detected. The results of that study identified the most recent 30 years of the low-streamflow record as the best subset of the record for the computation of 7Q10 in such circumstances. Identical analyses conducted in this study for the annual minimum 7-day streamflow likely to be exceeded in 1 out of 2 years on average (7Q2) and 19 out of 20 years on average (7Q20) indicated that subsets of the most recent 30 years of low streamflow are also the appropriate subsets for computing these statistics when a trend is detected. For all three statistics, the subset of the most recent 30 years, in comparison with all other subsets analyzed and the full record, provided the best balance between a minimization of the error of the statistic estimated from the simulated records and a minimization of bias. The adaptive approach recommended by the previous study for 7Q10, in which the subset of the most recent 30 years was used only where significant trends were detected and the full record was used otherwise, was also supported by this study for computing 7Q2 and 7Q20. A significance level of $p<0.1$ for trend detection is recommended for the adaptive approach for all three statistics.

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Appendix 1. Tabulation of Highest Improvement Factor by Bin

For each of the 100 bins into which each analysis was split for plotting, the subset of the full record that produced the highest improvement factor was identified by tabulating results (table 1.1) from the improvement factor curves in figure 1 (7Q2), figure 2 (7Q20), and results from Blum and others (2019) (7Q10). Out of the 100 total bins for each subset, the table shows the count for which each subset produced the highest improvement factor and the range of standardized trend magnitudes for which each subset generally provided the highest improvement factor.

Table 1.1. Accuracy of 7Q2, 7Q10, and 7Q20 estimators: total maximum bin counts and trend ranges per subset of the most recent *n* years.

[7Q2, the annual minimum 7-day streamflow exceeded in 1 out of 2 years on average; 7Q10, the annual minimum 7-day streamflow exceeded in 9 out of 10 years on average; 7Q20, the annual minimum 7-day streamflow exceeded in 19 out of 20 years on average; *n*-year subset, analysis using a subset of *n* number of years of most recent 7-day low-streamflow record; Bin count as max, number of highest improvement factor bins for the given trend range in comparison to all other *n*-year subsets; Trend range as max, the range of standardized trend values over which the given *n*-year subset analysis yields the maximum improvement factor; improvement factor, ratio of root mean-square error for full record to root mean-square error of partial record; NA, not applicable]

n-year subset	7Q2		7Q10		7Q20	
	Bin count as max	Trend range as max	Bin count as max	Trend range as max	Bin count as max	Trend range as max
3	0	none	NA[1]	NA[1]	NA[1]	NA[1]
5	0	none	NA[1]	NA[1]	NA[1]	NA[1]
10	5	<−0.030, >0.040	1	none	NA[1]	NA[1]
20	22	−0.030 to −0.018, 0.016 to 0.030	7	<−0.022, >0.030	6	<−0.030, >0.035
30	32	−0.018 to −0.009, 0.007 to 0.016	30	−0.022 to −0.013, 0.013 to 0.030	22	−0.030 to −0.012, 0.018 to 0.035
40	16	−0.009 to −0.005, 0.005 to 0.007	20	−0.013 to −0.008, 0.008 to 0.013	14	0.012 to 0.018
50	25	−0.01	42	−0.016	58	−0.024

[1]Subset was not considered in analysis for this statistic because the period of record would be shorter than the return interval.

For more information about this report, contact:
Director, New England Water Science Center
U.S. Geological Survey
10 Bearfoot Road
Northborough, MA 01532
dc_nweng@usgs.gov
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