# Uncertainty Analysis of Index-Velocity Meters and Discharge Computations at the Chicago Sanitary and Ship Canal near Lemont, Illinois

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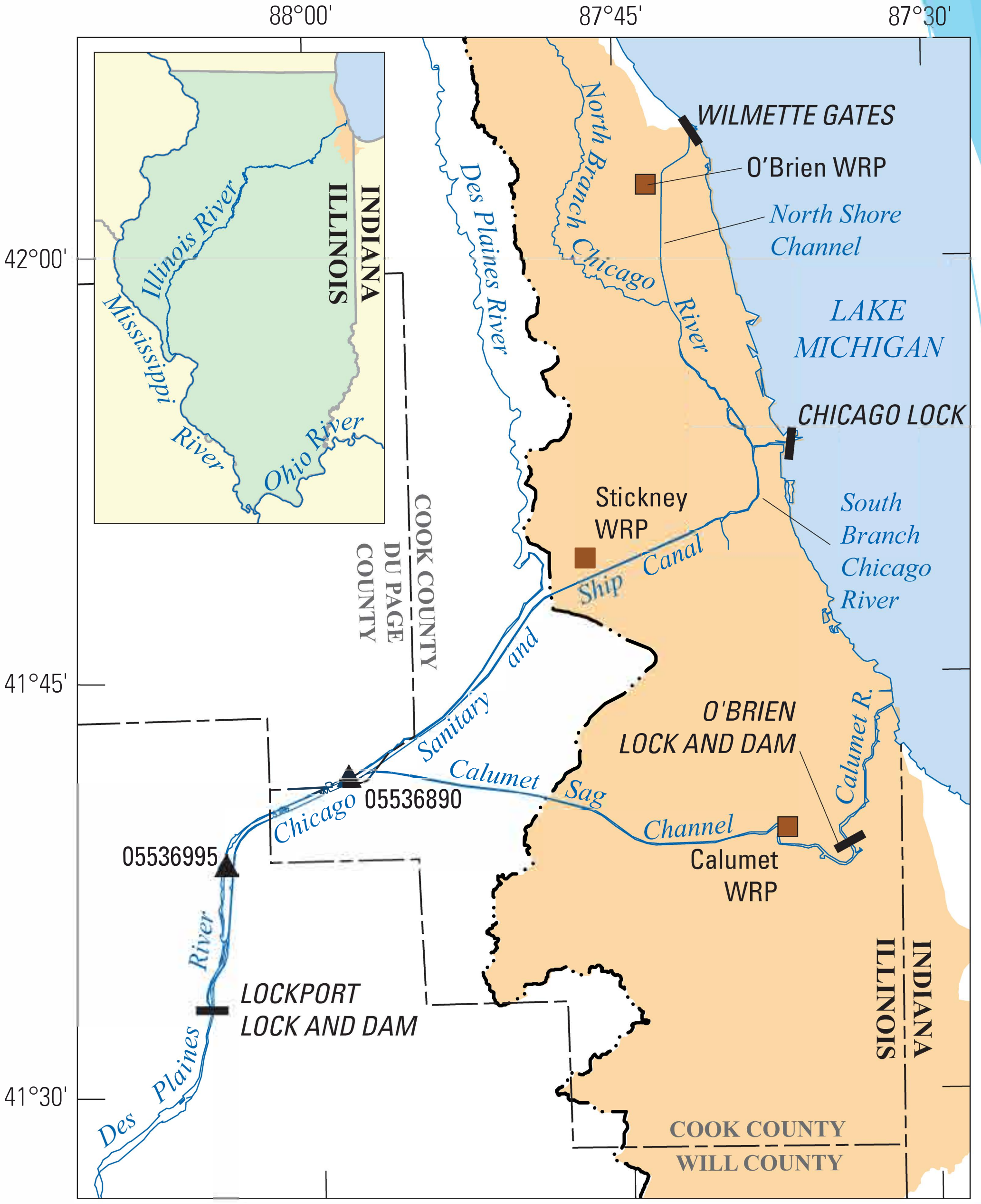
# Lake Michigan Diversion and its monitoring

- Series of canals built from 1836 to 1922 connecting from Lake Michigan (Great Lakes Basin) to Illinois River (Mississippi River Basin).
- Primary canal is Chicago Sanitary and Ship Canal, completed in 1900.
- Reverses flow of Chicago River and diverts water from Lake Michigan.
- Diversion by State of Illinois from Great Lakes is limited by U.S. Supreme Court decree to a long-term mean of 3,200 cubic feet per second ($ft^3/s$).
- U.S. Army Corps of Engineers is charged with monitoring the diversion: Lake Michigan Diversion Accounting (LMDA) program.

# Chicago Sanitary and Ship Canal and related features of the Chicago Area Waterway System




Base from U.S. Geological Survey 1:100,000-scale digital data, 2014, Albers Equal-Area Conic projection
Standard parallels 33° and 45°, central meridian 89°
North American Datum of 1927

**EXPLANATION**

[WRP, Water Reclamation Plant]

- Diverted Lake Michigan watershed
- Divide between Lake Michigan Basin and Mississippi River Basin before Lake Michigan diversion
- 05536995 ▲ U.S. Geological Survey streamgage and identifier

U.S. Geological Survey streamgages shown on the map are as follows: Chicago Sanitary and Ship Canal at Romeoville, Illinois (station 05536995), and Chicago Sanitary and Ship Canal near Lemont, Illinois (station 05536890).

# Streamgaging for LMDA and estimation of the uncertainty of computed discharge based on it

- U.S. Geological Survey (USGS) has operated one or more streamgages on the Chicago Sanitary and Ship Canal for the LMDA program since 1984.
- Current primary streamgage for LMDA is “Chicago Sanitary and Ship Canal near Lemont, Illinois”, USGS streamgage 05536890 (hereafter, the “Lemont streamgage”) .
- Prior LMDA discharge computation uncertainty work focused on a statistical approach, using properties of the index-velocity rating regression (Over and others, 2004; Duncker and others, 2006).
- Present (2021) effort continues that approach but also investigates incorporation of measurement uncertainty in stage, index velocity, and discharge.

# Lemont streamgage


Gage house and instrumentation (Modified from Jackson (2018). Photograph by Clayton Bosch, U.S. Geological Survey.)

On the Chicago Sanitary and Ship Canal looking downstream (southwest) toward streamgage during installation of uplooking ADCP (photograph by James Duncker, U.S. Geological Survey.)

# Lemont streamgage
# Plan v

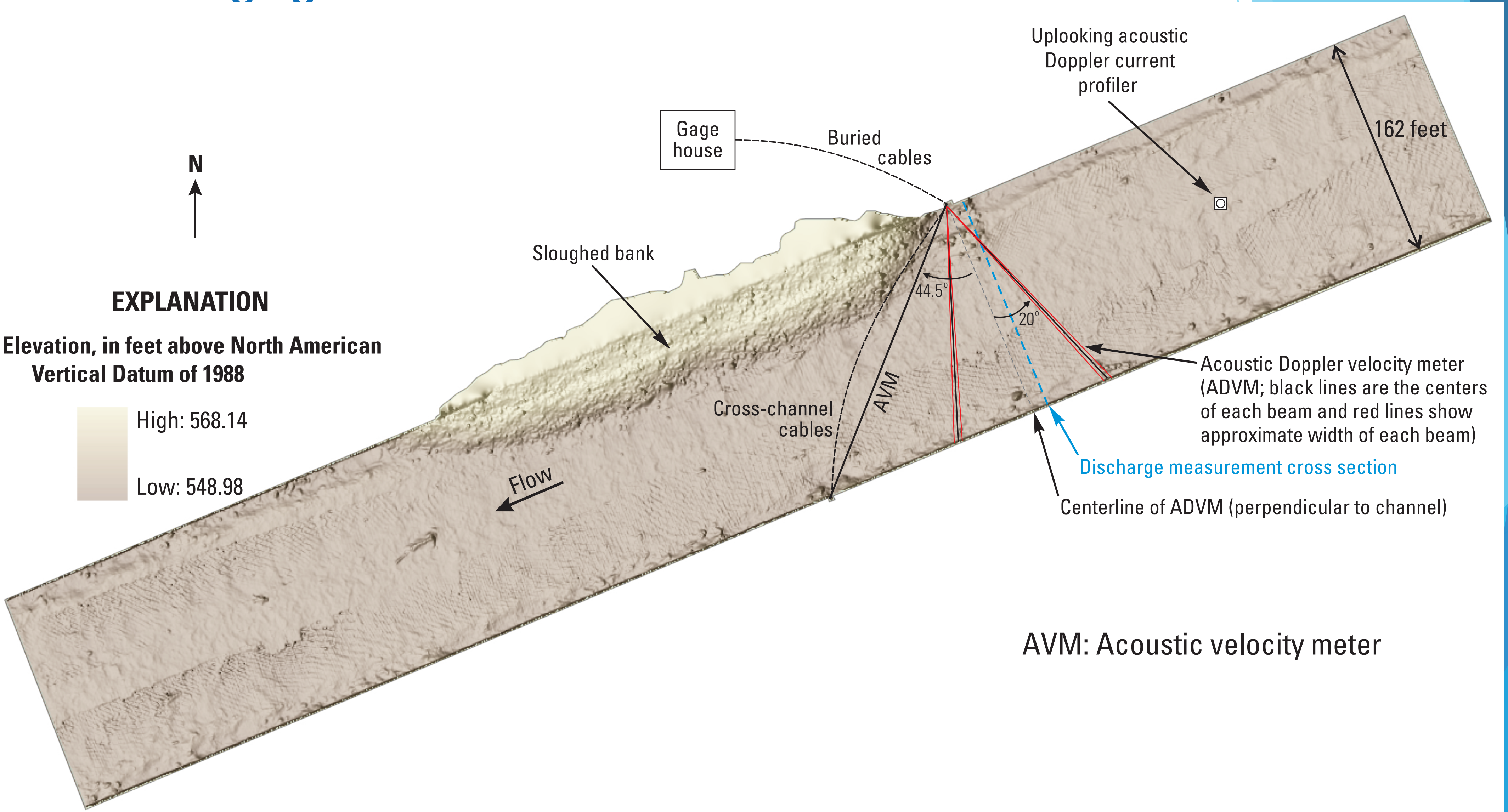
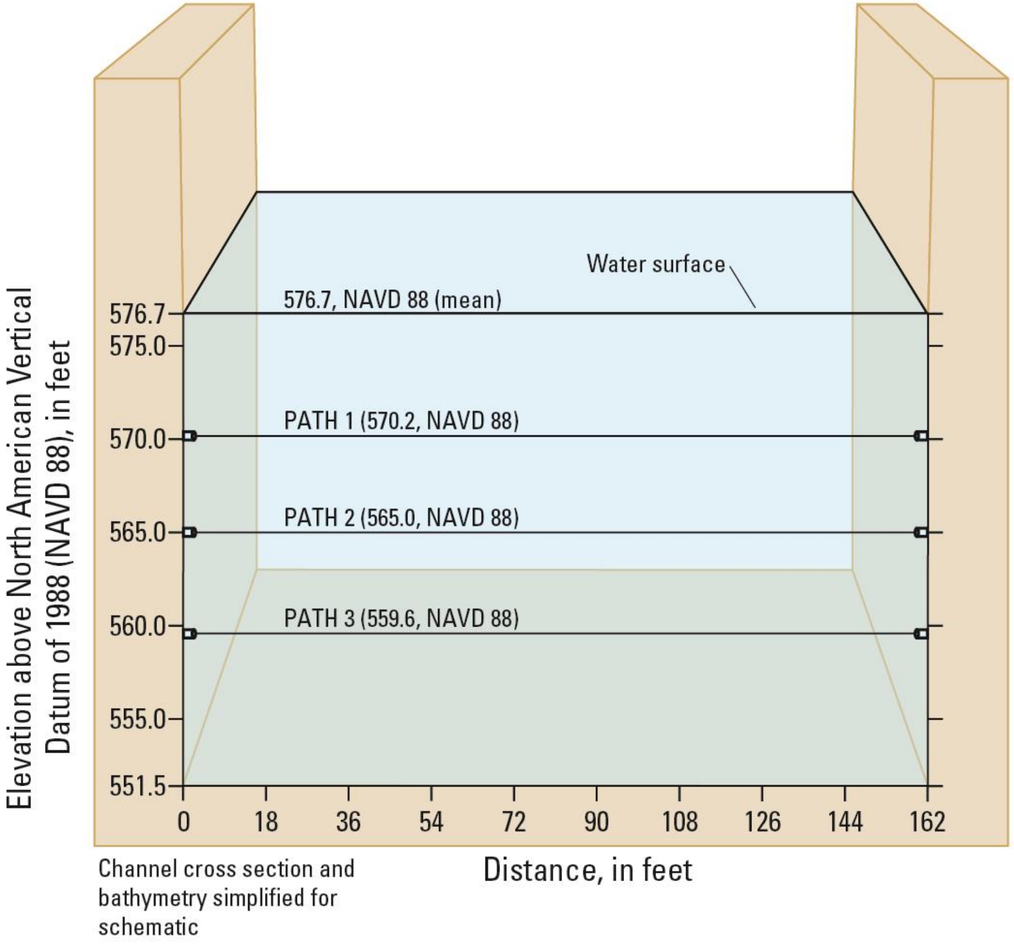



Figure is from Jackson and others (2012)

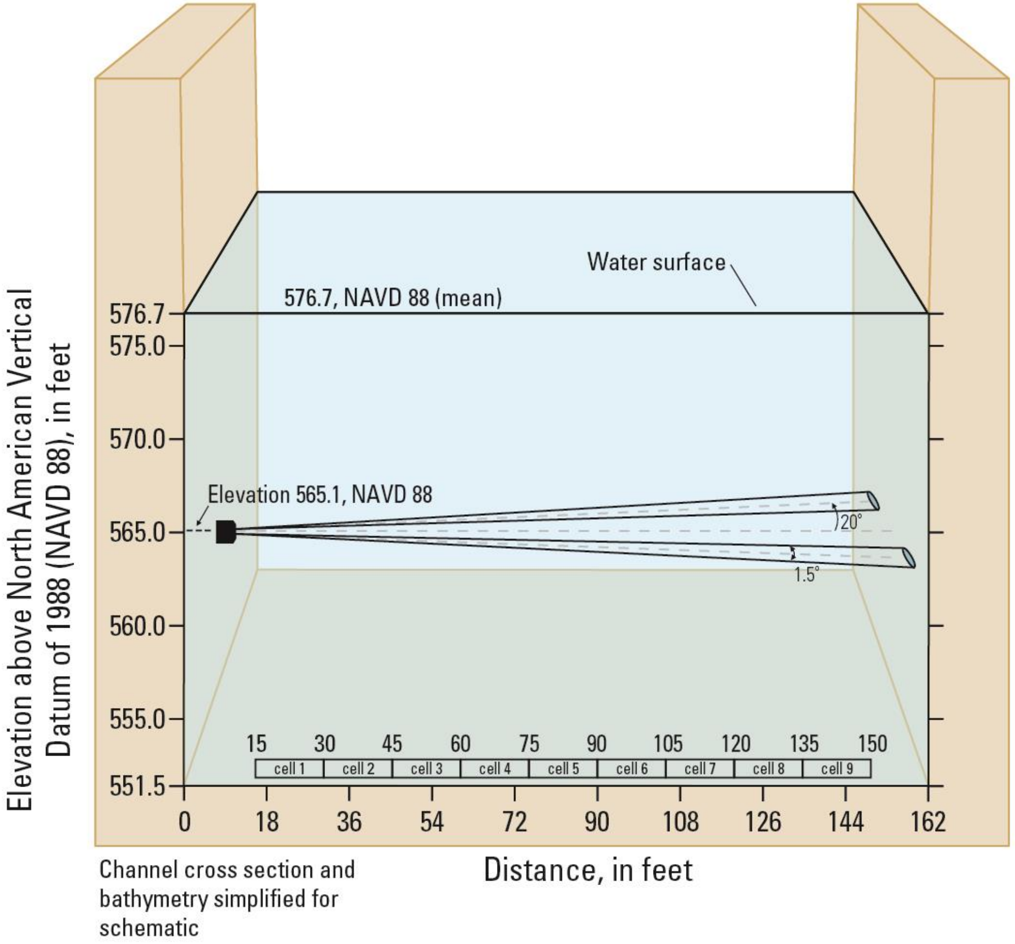
# Instrumentation

1. Stage: Paroscientific PS-2 pressure transducer with nitrogen tank and Conoflow gas-purge bubbler system.
2. AVM: 3-path Accusonic O.R.E. 7510 GS temperature-compensated time of travel velocity meter.
   a. Path length: 234.8 feet (ft), angle: 44.5 degrees
   b. Path stages: 18.4, 13.2, 7.8 ft
3. Horizontal ADVM (primary index velocity): TRDI Channel Master, 600 kHz
   a. 9 cells at 5 meters (16 ft) per cell beginning 2 meters (7 ft) from right (north) bank.
   b. Stage: 13.2 ft.
4. Discharge: ADCP (TRDI Rio Grande) measurements from moving boat (using tagline).
5. Uplooking ADCP: not used in this study.

# Deployment schematics of the AVM (left) and the ADVM (right) at the Lemont streamgage




Schematic of the channel cross-section showing the three paths of the AVM (looking upstream).




Schematic of the channel cross-section showing the ADVM beams (looking upstream).

Figures are from Jackson and others (2012).

# Approach to Uncertainty Estimation

Four steps:

1. Estimation of measurement uncertainty at continuous sensors by first-order second moment (FOSM) method using a type B approach: AVM, ADVM, and stage.
2. Estimation of uncertainty of ADCP calibration measurements (Qms).
3. Determination of AVM and ADVM-based index-velocity ratings (IVRs).
4. Computation of discharge and its uncertainty using IVRs.

# Discharge measurement dataset

- 155 ADCP discharge measurements (Qms) and corresponding ADVM and AVM velocities were processed using QRev (Mueller, 2016) for this study (Prater and others, 2021). Of these,
  - 140 have corresponding AVM velocities.
  - 130 have corresponding ADVM velocities.
  - 115 have both.
- Dataset properties include:
  - Date range: Jan. 12, 2005, to Oct. 23, 2013.
  - Measured discharge, range: –344 to 16,794 $ft^3/s$; median: 2,605 $ft^3/s$
  - Duration of measurement, range: 162 to 3,563 seconds; median: 831 seconds.
  - Number of transects, range: 1 to 18; median: 4.

# Step 1: Estimation of measurement uncertainty at continuous sensors

- Sensors: AVM, ADVM, pressure transducer
- General approach: “type B,” with computations by first-order second moment (FOSM) method.
- Software: QMSys (http://www.qsyst.com), implements FOSM and Monte Carlo methods.

# Error Propagation by the First-order Second Moment (FOSM) Method

Uncertainty in FOSM method is computed as the variance $\sigma_Y^2$, the 2nd moment, of *Y*, which is a function of some *X* variables:

$$Y = g(X_1, X_2, \ldots, X_n)$$

Using properties of variance and a Taylor series (first-order) expansion, variance of *Y*, is approximately:

$$\sigma_Y^2 \cong \sum_{i=1}^{n}\sum_{j=1}^{n} \left.\frac{\partial g}{\partial X_i}\right|_{\mu} \left.\frac{\partial g}{\partial X_j}\right|_{\mu} \sigma_{X_i,X_j}$$

If *X* variables are independent, then:

$$\sigma_Y^2 \cong \sum_{i=1}^{n}\left(\left.\frac{\partial g}{\partial X_i}\right|_{\mu}\right)^2 \sigma_{X_i}^2$$

For example, for *Q* = *AV*, one gets:

$$\sigma_Q^2 \approx \left(\left.\frac{\partial Q}{\partial V}\right|_{\mu}\right)^2 \sigma_V^2 + \left(\left.\frac{\partial Q}{\partial A}\right|_{\mu}\right)^2 \sigma_A^2 = A^2\sigma_V^2 + V^2\sigma_A^2$$

(adapted from Duncker and others, 2006).

# AVM uncertainty – Data reduction equation (DRE)

$$V_L = \frac{B}{2\cos(A)}\left(\frac{1}{T_{DC}} - \frac{1}{T_{CD}}\right)$$

where:

$V_L$ = "line velocity," velocity along acoustic path, used as index velocity,

$B$ = acoustic path length,

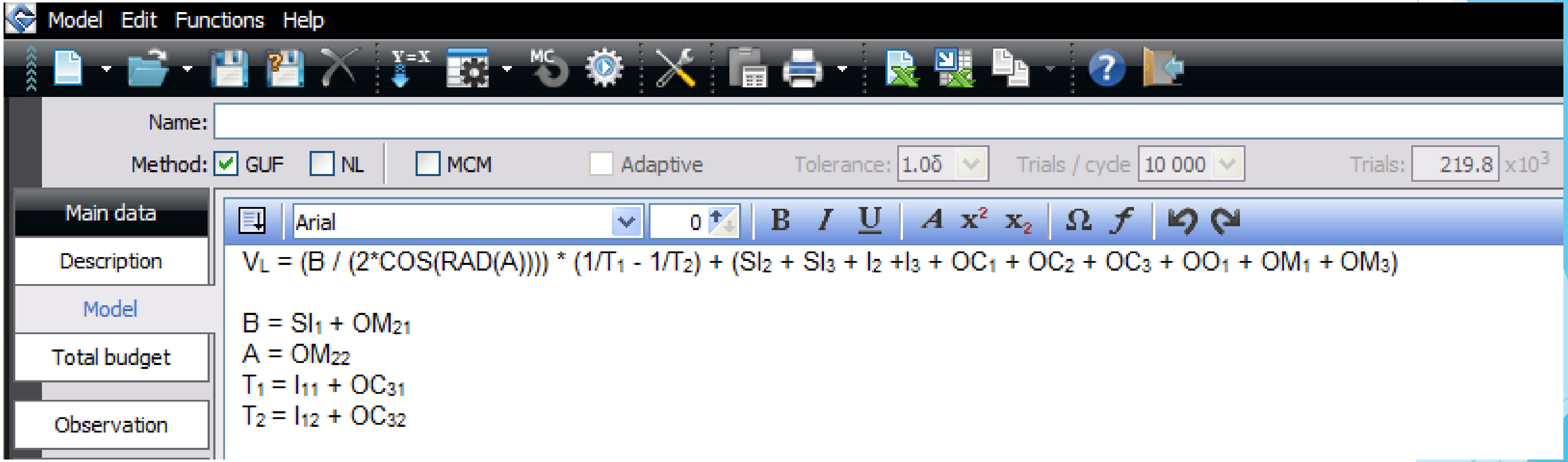
$A$ = angle between acoustic path and velocity, and

$T_{CD}$, $T_{DC}$ = travel times of acoustic signal between transmitter and receiver and back.

At the Lemont streamgage, the line velocities from three acoustic paths are averaged (with weighting when one or more is missing) to obtain a composite mean AVM velocity (Jackson and others, 2012).

Note that the DRE is nonlinear in all quantities except the acoustic path length $B$.

# AVM uncertainties: Expanded DRE as entered into QMSys



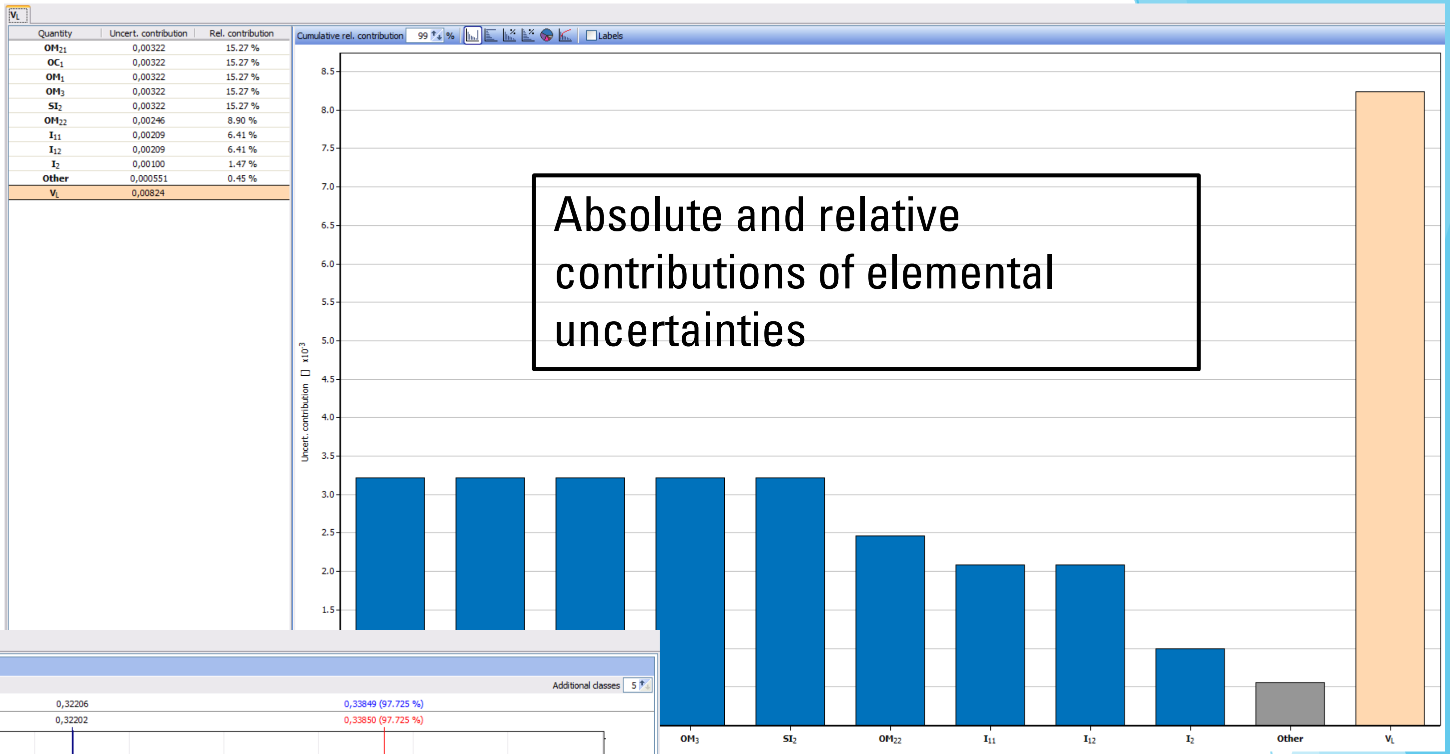

Part of QMSys input window

# Elemental uncertainties for AVM velocity at low flow*

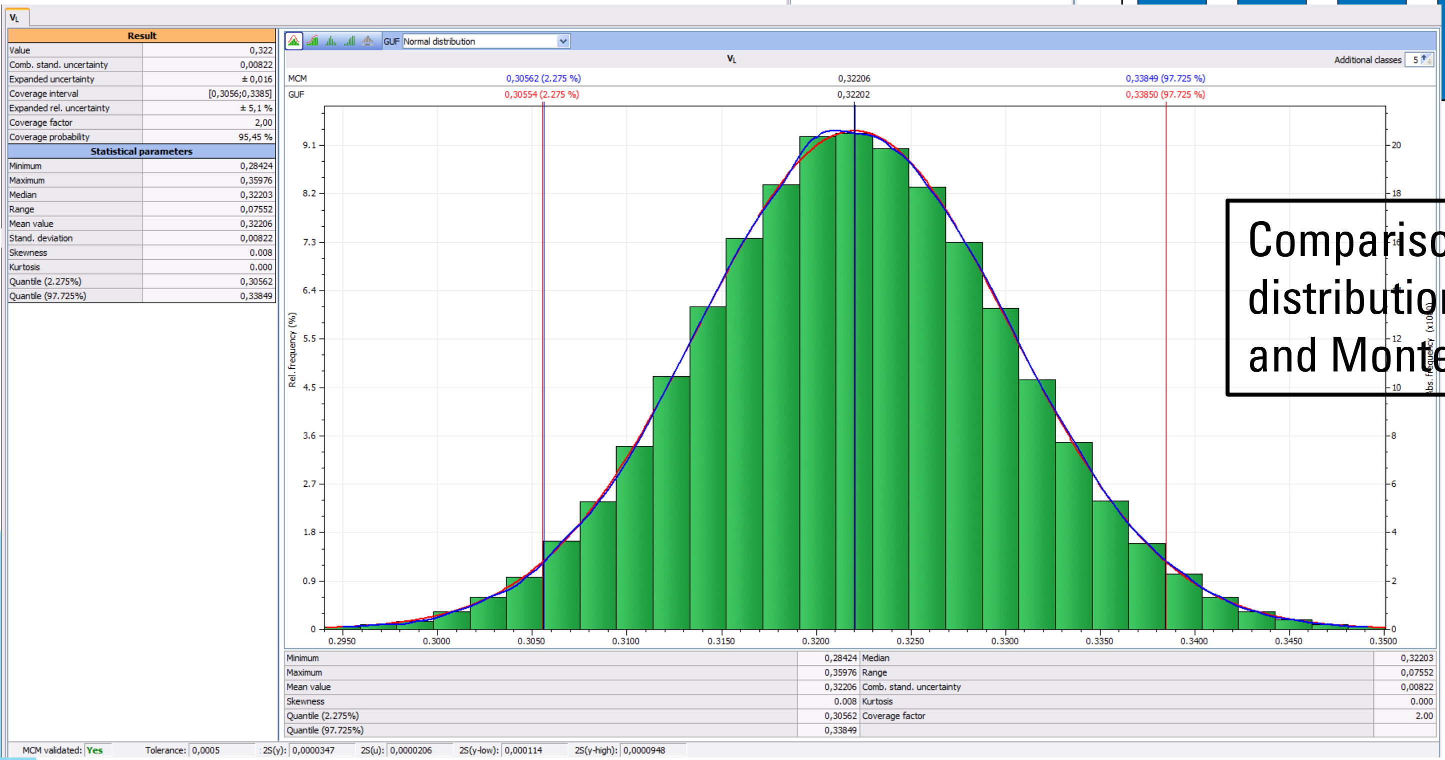
| Name | Notation | Error category | Absolute uncertainty | Contribution to total uncertainty (ft/s)** |
|---|---|---|---|---|
| Acoustic path length | $SI_1$ | Site and installation | 0.02% | 8e-7 |
| Changes in spatial flow distribution | $SI_2$ | Site and installation | 1% | 0.001258 |
| Acoustic path angle | $SI_3$ | Site and installation | 0.17% | 0.000036 |
| Travel time | $I_1$ | Instrument | 30 nsec | 0.001056 |
| Instrument resolution | $I_2$ | Instrument | 0.001 ft/s | 0.000121 |
| Sampling frequency | $OC_1$ | Operation - Configuration | 1% | 0.001258 |
| Sampling time | $OC_2$ | Operation – Configuration | 0 | 0 |
| Time synchronization | $OC_3$ | Operation – Configuration | Not applicable*** | 0 |
| Operational issues | $OO_1$ | Operation – Operator | 0 | 0 |
| Wind-induced shear | $OM_1$ | Operation – Measurement | 1% | 0.001258 |
| Acoustic path deflection | $OM_2$ | Operation – Measurement | 1% of *B*, *A* | 0.001992 |
| Temporary changes in flow distribution | $OM_3$ | Operation – Measurement | 1% | 0.001258 |
| Total | | | | 0.00824 (2.56% of *V*) |

*Low flow selected as discharge = 1,300 $ft^3/s$; velocity (*V*) = 0.322 ft/s; stage = 24.92 ft.
**Computed as the square root of the variance associated with this component.
***Not applicable to AVM velocity by itself but when used in rating curve development.

# AVM uncertainties: FOSM estimates from QMSys




Absolute and relative contributions of elemental uncertainties




Comparison of distributions of FOSM and Monte Carlo results

# ADVM uncertainty estimation: DREs

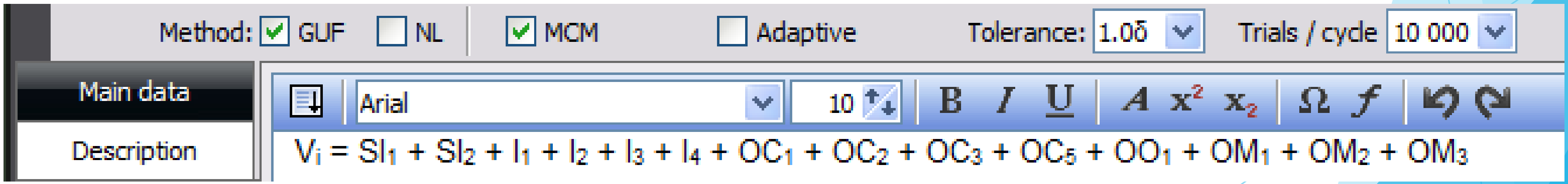
## Basic data reduction equation (DRE):

$$V_i = \frac{F_d}{2F_s} C$$

where:

- $V_i$ = Velocity along the acoustic path
- $F_d$ = Doppler shift of received frequency
- $F_s$ = Transducer transmit frequency
- $C$ = Speed of sound

## Expanded DRE as used for uncertainty estimation in QMSys:



# Elemental uncertainties for ADVM velocity at low flow*

| Name | Notation | Error category | Absolute uncertainty | Contribution to total uncertainty (ft/s)** |
|---|---|---|---|---|
| Beam orientation | $SI_1$ | Site and installation | 0 | 0 |
| Changes in spatial flow distribution | $SI_2$ | Site and installation | 2% | 0.003501 |
| Accuracy | $I_1$ | Instrument | 2 mm/s | 0.003634 |
| Resolution | $I_2$ | Instrument | 1 mm/s | 0.000908 |
| Factory settings | $I_3$ | Instrument | 1 mm/s | 0.000908 |
| Analytical methods | $I_4$ | Instrument | 1 mm/s | 0.000908 |
| Measurement volume setting | $OC_1$ | Operation - Configuration | 0 | 0 |
| Sampling frequency, sampling time | $OC_2$, $OC_3$ | Operation – Configuration | 0, 0 | 0, 0 |
| Data logger, compass | $OC_4$, $OC_5$ | Operation – Configuration | 0, 0 | 0, 0 |
| Salinity input | $OO_1$ | Operation – Operator | 0.463% | 0.000187 |
| Wind-induced shear | $OM_1$ | Operation – Measurement | 1% | 0.000875 |
| Acoustic path deflection | $OM_2$ | Operation – Measurement | Not determined | 0 |
| Temporary changes in flow distribution | $OM_3$ | Operation – Measurement | 1% | 0.000875 |
| Total | | | | 0.0118 (3.66% of $V$) |

*Low flow selected as discharge = 1,300 $ft^3/s$; velocity ($V$) = 0.322 ft/s; stage = 24.92 ft.

**Computed as the square root of the variance associated with this component.

# Predicted AVM and ADVM velocity uncertainties

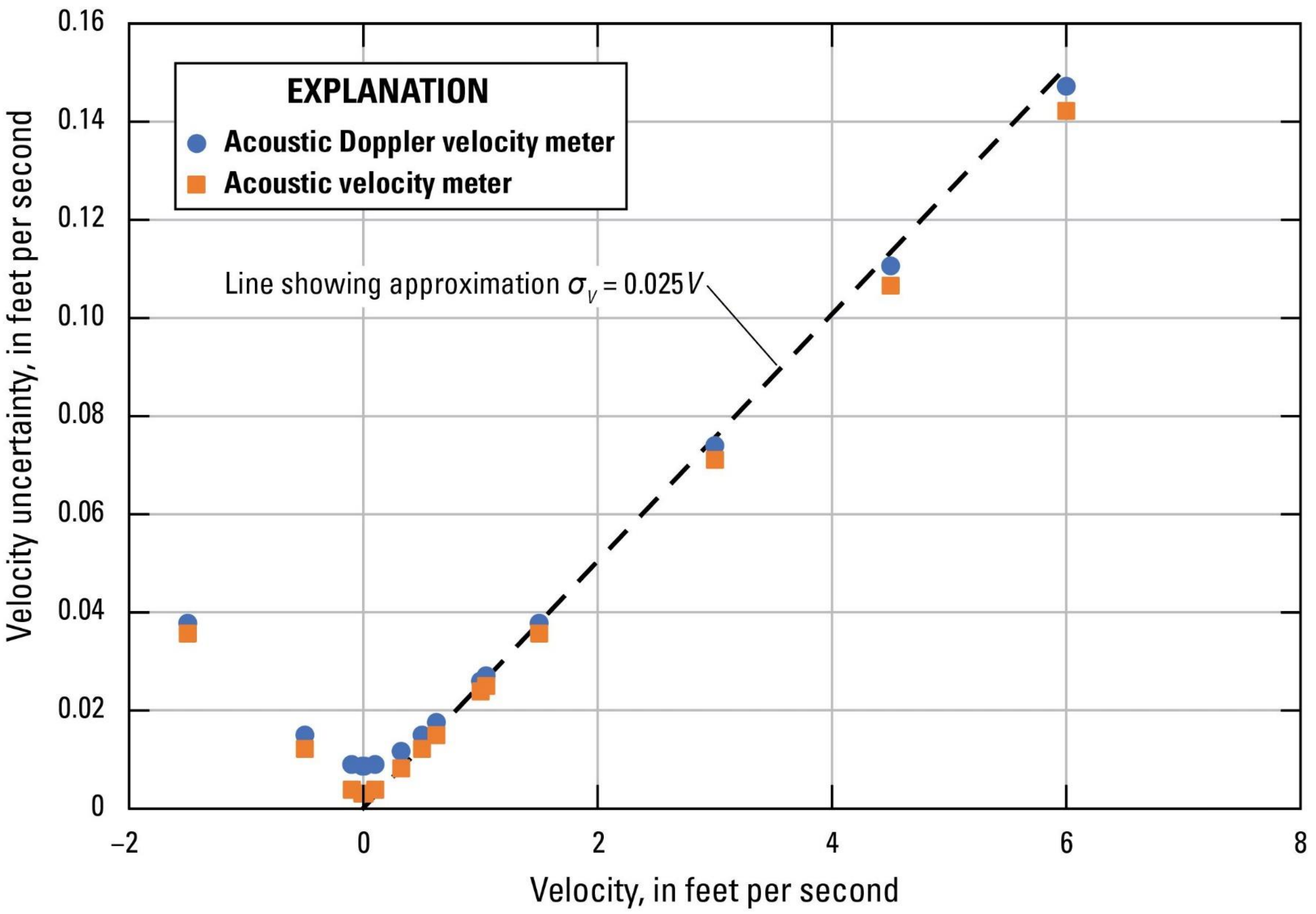
Notice:

- Strong dependence of total velocity uncertainty $\sigma_V$ on velocity $V$: $\sigma_V \approx 0.025V$.
- $\sigma_{AVM\_V} < \sigma_{ADVM\_V}$ esp. for small $V$
- At $V$=0, $\sigma_{AVM\_V} = 0.0031$ ft/s and $\sigma_{ADVM\_V} = 0.0087$ ft/s.
- Per current DREs, $\sigma_{-V} = \sigma_V$ (mirrored around $V = 0$).

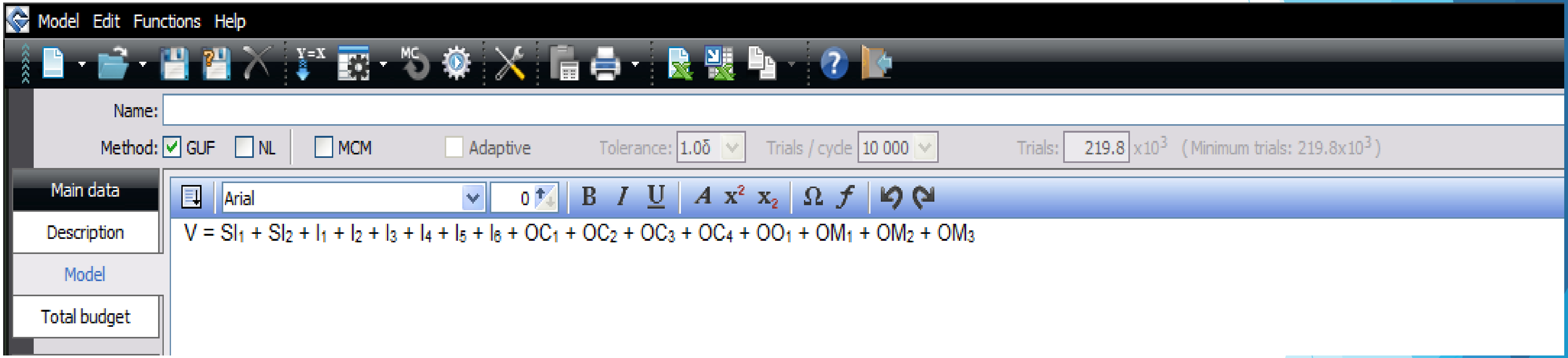
$\sigma_{AVM\_V}$: total uncertainty of AVM velocity.

$\sigma_{ADVM\_V}$: total uncertainty of ADVM velocity.

Note: The ADVM results shown here do not consider the effect of salinity.



# Water-level (stage) uncertainty estimation: DRE




Part of QMSys input window

# Elemental uncertainties for water level (stage)

| Name | Notation | Error category | Absolute uncertainty | Contribution to total uncertainty (ft)* |
|---|---|---|---|---|
| Datum errors | $SI_1$ | Site and installation | 0 | 0 |
| Local disturbances | $SI_2$ | Site and installation | 0.02 ft | 0.014728 |
| Accuracy | $I_1$ | Instrument | 0.02% | 0.001325 |
| Resolution | $I_2$ | Instrument | 0.01 ft | 0.003682 |
| Calibration, Data transmission | $I_3$, $I_4$ | Instrument | 0, 0 | 0, 0 |
| Timing, Data logger | $I_5$, $I_6$ | Instrument | 0, 0 | 0, 0 |
| Gage offset | $OC_1$ | Operation - Configuration | 0.01 ft | 0.003682 |
| Recorder, Data retrieval | $OC_2$, $OC_3$ | Operation – Configuration | 0, 0 | 0, 0 |
| Periodic stage correction | $OC_4$ | Operation – Configuration | 0.01 ft | 0.003682 |
| Operational issues | $OO_1$ | Operation - Operator | 0 | 0 |
| Hydraulically induced | $OM_1$ | Operation - Measurement | 0 | 0 |
| Temperature effects | $OM_2$ | Operation – Measurement | 0 | 0 |
| Temporary flow disturbances | $OM_3$ | Operation - Measurement | 0 | 0 |
| Total | | | | 0.0271 ft |

*Computed as the square root of the variance associated with this component.

# Step 2: Estimation of discharge (ADCP) uncertainties

- No generally accepted detailed method of ADCP uncertainty estimation yet exists.
- Used approximate estimates from QRev (Mueller, 2016).
- QRev uncertainty estimates consider the following:

  1. Random uncertainty: coefficient of variation (CV) = standard deviation / mean of transect discharges
  2. Invalid data uncertainty
  3. Top/bottom extrapolation uncertainties
  4. Edge estimation uncertainty
  5. Moving bed test uncertainty (when bottom tracking used)
  6. Systematic uncertainty: 1.5 percent (one-standard deviation estimate)

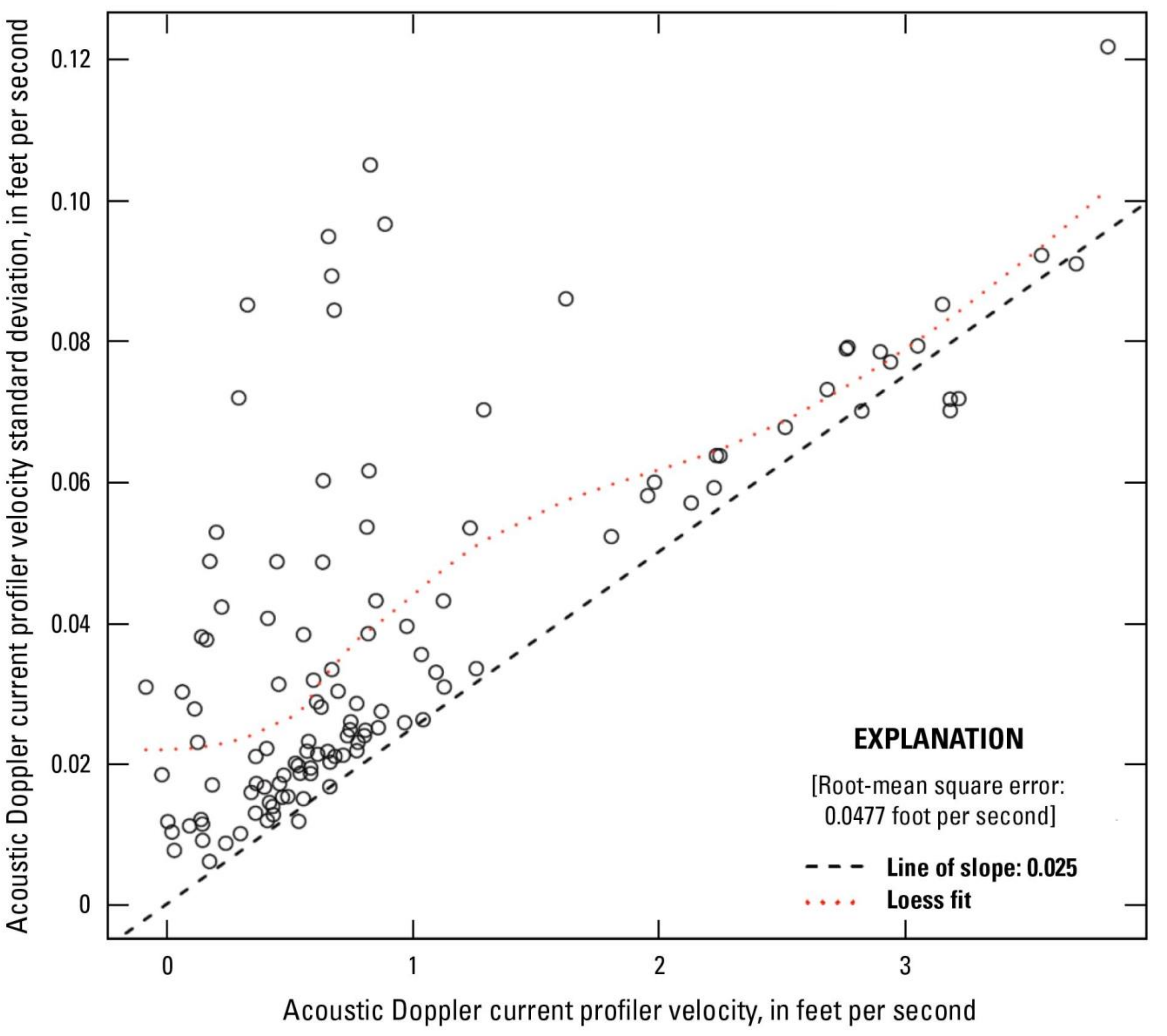
Note: Any such single Qm uncertainty estimate does not address correlation among Qms, which would affect the index-velocity rating and its uncertainty.

# Results of QRev estimation of ADCP uncertainties

| Uncertainty source | Computation (in terms of standard uncertainties in percent) | Uncertainty statistics for all 155 measurements considered in this study, expressed as standard uncertainties in percent | | |
|---|---|---|---|---|
| | | Minimum | Median | Maximum |
| Random uncertainty | Proportional to the CV of the transect discharges | 0.2 | 3.3 | 243 |
| Invalid data uncertainty | 10 percent of the percentage discharge for invalid cells and ensembles | 0 | 0 | 3.75 |
| Edge uncertainty | 15 percent of the percentage discharge in the edges | 0.45 | 1.75 | 7.9 |
| Top/bottom extrapolation uncertainty | Based on variation among results of different extrapolation methods | 0.05 | 0.2 | 9.45 |
| Moving-bed test uncertainty | If bottom track used: 0.5 if no moving bed, 0.75 if moving bed is present, 1.5 percent if moving-bed test not performed; otherwise 0. | 0 | 1.5 | 1.5 |
| Systematic uncertainty | Fixed at 1.5 percent | 1.5 | 1.5 | 1.5 |
| Total uncertainty | | 2.2 | 4.3 | 243 |

# Estimated ADCP uncertainties

- Plot shows common Qms with ADCP *V*s computed using ADVM stage-area rating.
- Minimum "base" uncertainty estimate of ~0.025 *V* (~same as ADVM, AVM):
  - Primary error sources: systematic, edges, moving bed.
- High "outliers":
  - common for $V < 2$ ft/s
  - Primary sources: random (CV), extrapolation.
- Root-Mean Square Error (RMSE) of ADCP *V*s, RMSE($V_{ADCP}$) = 0.0477 ft/s
  - RMSE($V_{ADVM}$) = 0.0356 ft/s
  - RMSE($V_{AVM}$) = 0.0345 ft/s



# Step 3: Determination of Index-Velocity Ratings

# Index-velocity ratings

- Per standard USGS practice (Levesque and Oberg, 2012), index-velocity ratings (IVRs) are developed by fitting a linear regression line with velocities from discharge measurements,

  $$V_{Qm} = Qm / Area_{index},$$

  where

  *Qm* is a discharge measurement and

  $Area_{index}$ is the cross-sectional area from the stage-area rating associated with the index-velocity meter,

  as the predictand (y-axis variable) to velocities from the index-velocity meter, $V_{index}$.
- A straight-line fit, that is,

  $$V_{Qm} = a + bV_{index}$$

  is used unless a pattern such as curvature is seen in the regression residuals.
- Each of the two index-velocity meters at the Lemont streamgage considered in this study has its own stage-area rating curve and its own set of IVRs.
- In this study, only straight-line fits were deemed necessary.

# Stage-Area ratings

- At the Lemont streamgage, there are two index-velocity meters; each has a stage-area rating curve to be used for determining $Area_{index}$:

  AVM $Area_{index} = 179.403 \times Stage - 638.05$

  ADVM $Area_{index} = 164.6766 \times Stage - 71.0209.$

- The differences between these stage-area rating curves arise from the AVM paths traversing the part of the measurement site that includes the sloughed bank, whereas the ADVM is located upstream (west) from this part of the channel (see measurement site plan, slide 6).

# Assumptions of ordinary least squares (OLS) regression and their implications

Table taken from Helsel and others (2020), p. 228.

**Table 9.2.** Assumptions necessary for the purposes to which ordinary least squares (OLS) regression is applied.

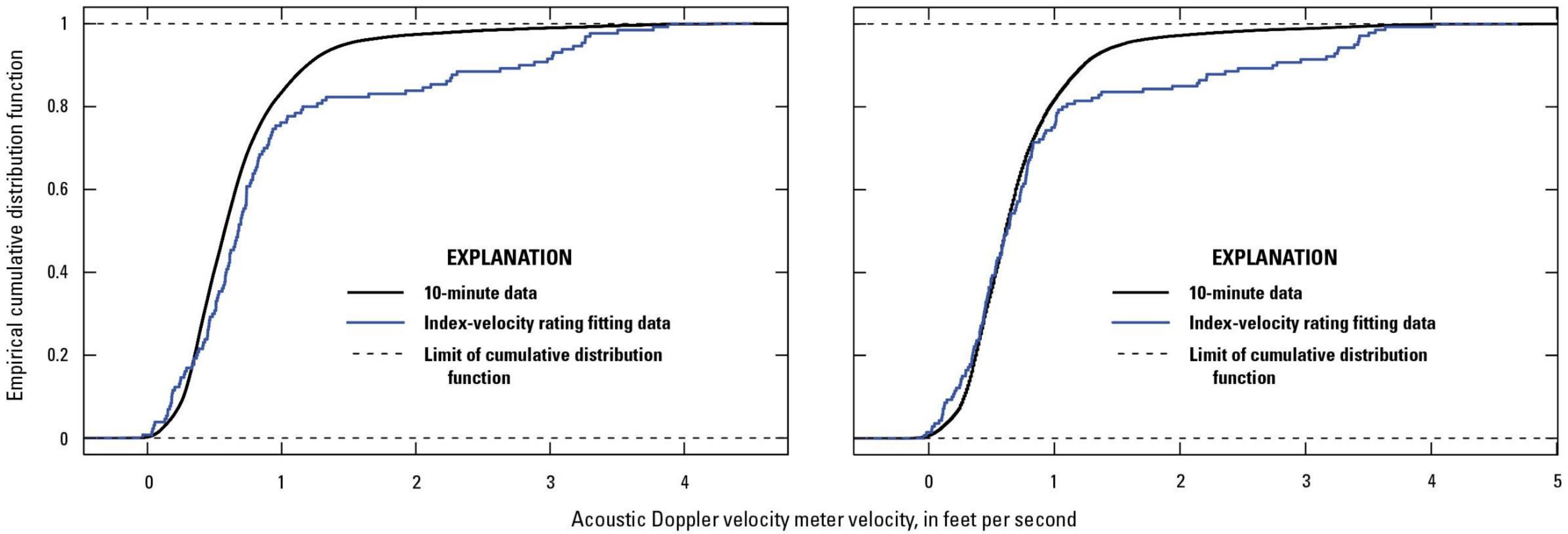
[X, the assumption is required for that purpose; -, assumption is not required]

| Assumption | Purpose | | | |
|---|---|---|---|---|
| | **Predict *y* given *x*** | **Predict *y* and a variance for the prediction** | **Obtain best linear unbiased estimator of *y*** | **Test hypotheses, estimate confidence or prediction intervals** |
| Model form is correct: *y* is linearly related to *x*. | X | X | X | X |
| Data used to fit the model are representative of data of interest. | X | X | X | X |
| Variance of the residuals is constant (homoscedastic). It does not depend on *x* or on anything else such as time. | - | X | X | X |
| The residuals are independent of *x*. | - | - | X | X |
| The residuals are normally distributed. | - | - | - | X |

[*y*, response variable; *x*, predictor variable]

# Empirical distributions of ADVM and AVM velocity data

| Data type | Number of data points | Mean | Mini-mum | 1% quantile | 10% quantile | Median | 90% quantile | 99% quantile | Maxi-mum |
|---|---|---|---|---|---|---|---|---|---|
| ADVM velocities used for IVRs, ft/s | 130 | 0.978 | −0.036 | 0.034 | 0.183 | 0.68 | 2.78 | 3.69 | 3.88 |
| ADVM prediction velocities, ft/s | 464,077 | 0.685 | −0.28 | 0.061 | 0.269 | 0.57 | 1.19 | 3.00 | 4.58 |
| AVM velocities used for IVRs, ft/s | 140 | 0.947 | −0.037 | −0.003 | 0.195 | 0.62 | 2.74 | 3.60 | 4.03 |
| AVM prediction velocities, ft/s | 509,996 | 0.719 | −0.390 | 0.030 | 0.280 | 0.61 | 1.23 | 3.11 | 4.80 |



# Regression methods considered

| Regression method | Are errors specified? | Are errors on the x (index velocity) axis considered? | Is correlation of ADCP errors considered? | Software packages used |
|---|---|---|---|---|
| Ordinary least squares (OLS) | No | No | No | R (lm function) |
| Weighted least squares (WLS) | No | No | No | R (lm function) |
| Weighted least squares (WLS) | Yes | No | No | R (lm) and TS28037[a] |
| Gauss-Markov regression (GMR) (also called generalized least squares: GLS) | Yes | No | Yes[b] | TS28037 |
| Generalized distance regression (GDR) (also called errors-in-variables regression: EIV) | Yes | Yes | No | TS28037 |
| Generalized Gauss-Markov regression (GGMR)[c] | Yes | Yes | Yes | TS28037 |

[a]TS28037: Software to Support ISO/TS 28037:2010(E) (National Physical Laboratory, 2010).

[b]Two assumptions on correlations of the ADCP errors were tested: (1) that those measurements made in immediate succession have correlations of 0.5; and (2) that all the ADCP measurements are correlated with correlation = 0.1.

[c]Basic results with GGMR regression were computed but are not presented here to avoid unnecessary complexity.

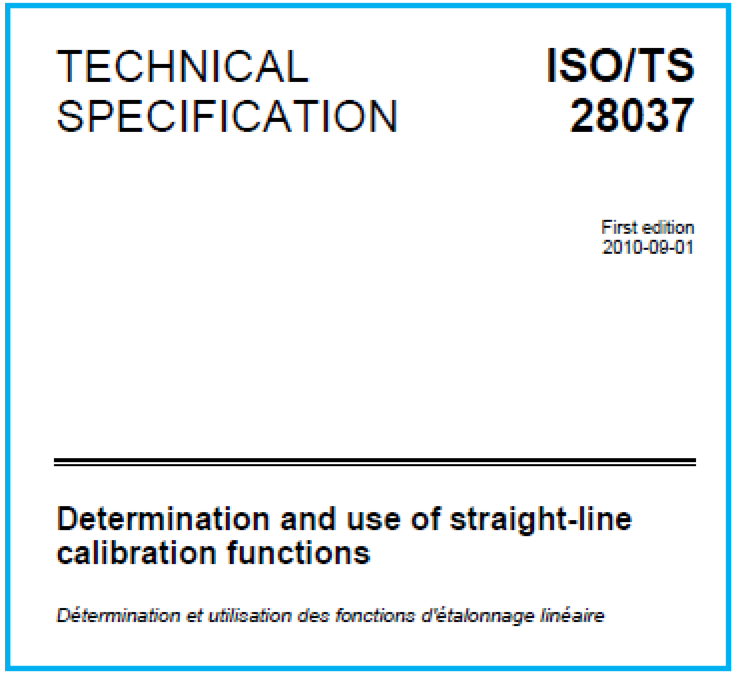
# Regression software used for fitting IVRs

Two packages:

1. Specified errors: MATLAB software built to implement ISO/TS 28037 (National Physical Laboratory, 2010).
2. Unspecified errors: General linear regression by lm function (R Core Team, 2019).

**Software to Support ISO/TS 28037:2010(E)**

This software supports the ISO Technical Specification (TS) *Determination and use of straight-line calibration functions.*

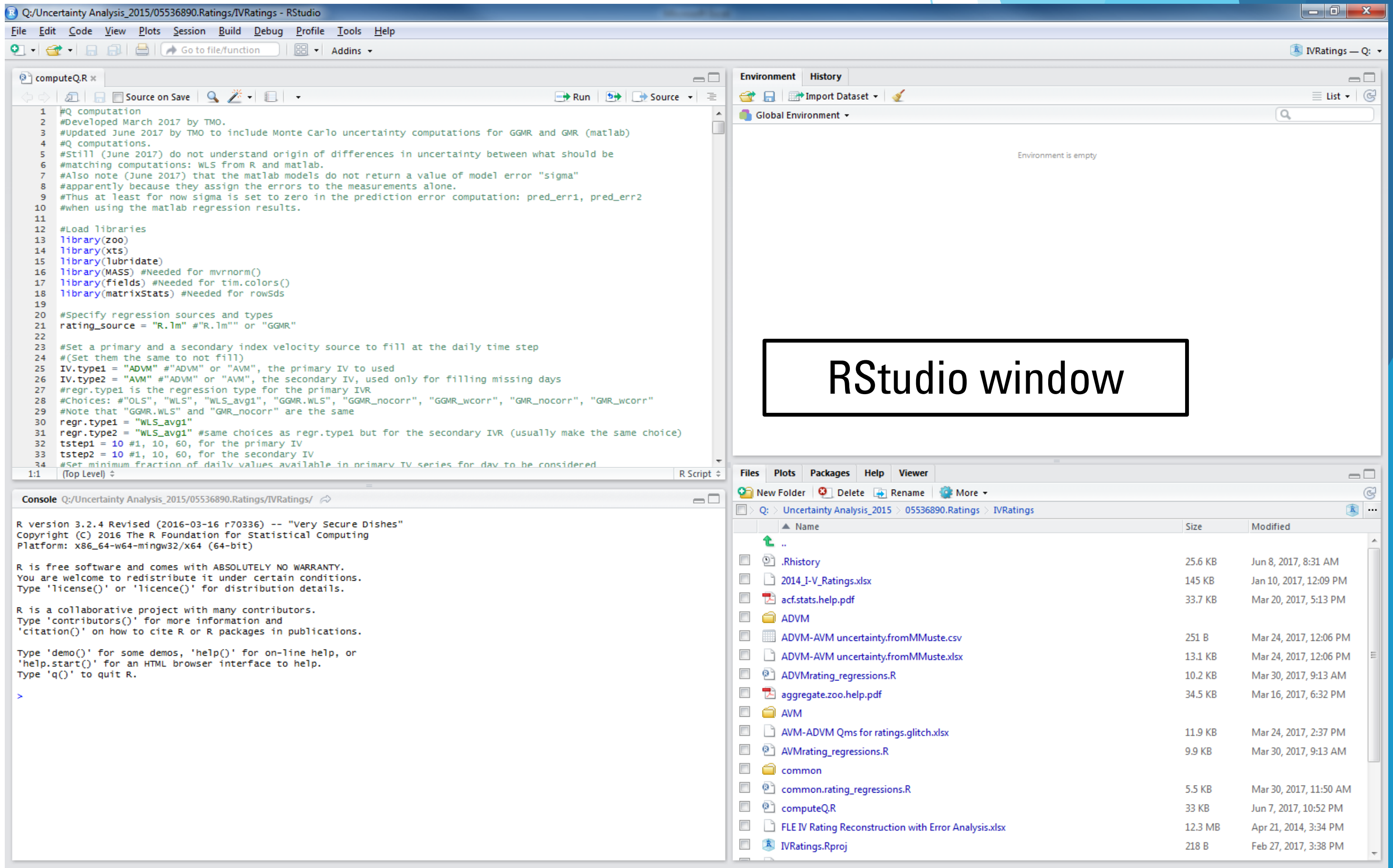

TECHNICAL SPECIFICATION

ISO/TS 28037

First edition 2010-09-01

**Determination and use of straight-line calibration functions**

*Détermination et utilisation des fonctions d'étalonnage linéaire*



# Chi-squared ($X^2$) tests of regression model fits

- *Without specified measurement uncertainties*, one typically uses a variety of diagnostics that mostly focus on the properties of the residuals (Helsel and others, 2020, chapter 9).
- *With specified measurement uncertainties*, the $X^2$ test (Press and others, 1992, p. 653–655), can be applied to test the model fit as follows:
  - If the specified measurement uncertainties are correct (that is, they are Gaussian with specified variances) and the (linear) model is appropriate, then

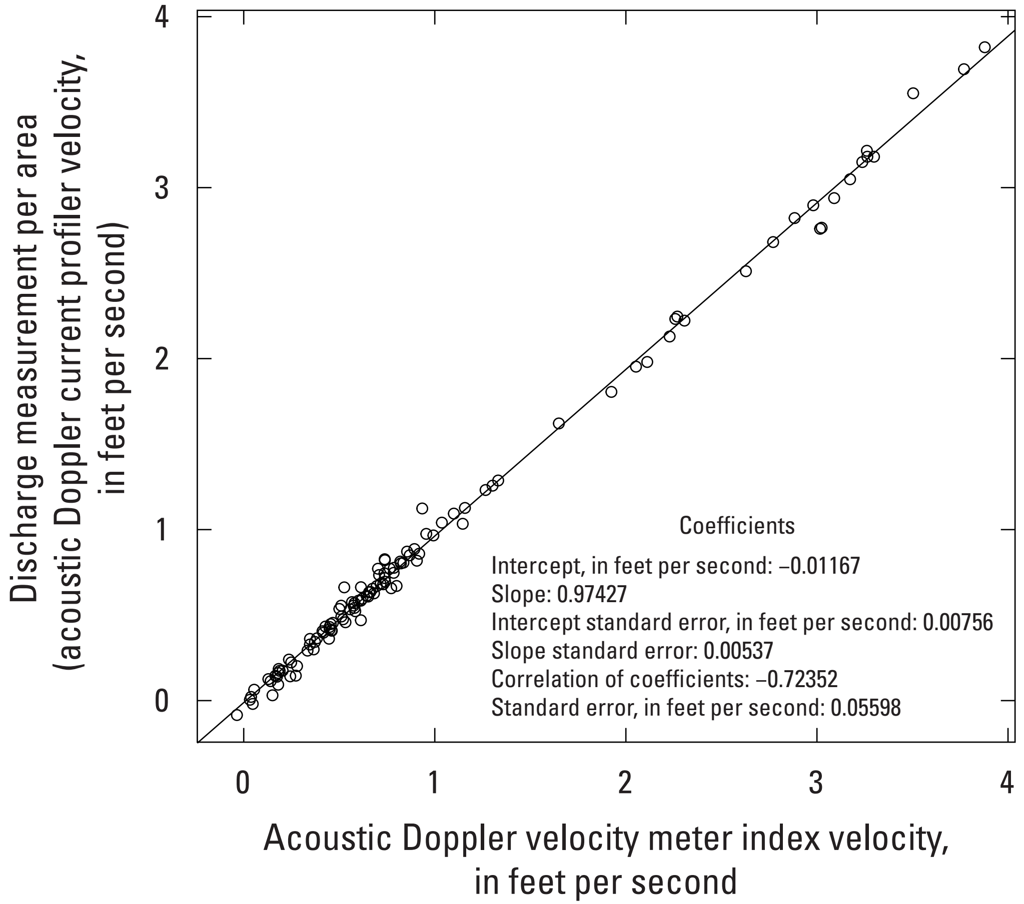
    $SSR_{norm}$ = sum of squared regression residuals normalized by uncertainties has a $X^2$ distribution with $n-p$ degrees of freedom, denoted $X^2_{n-p}$,

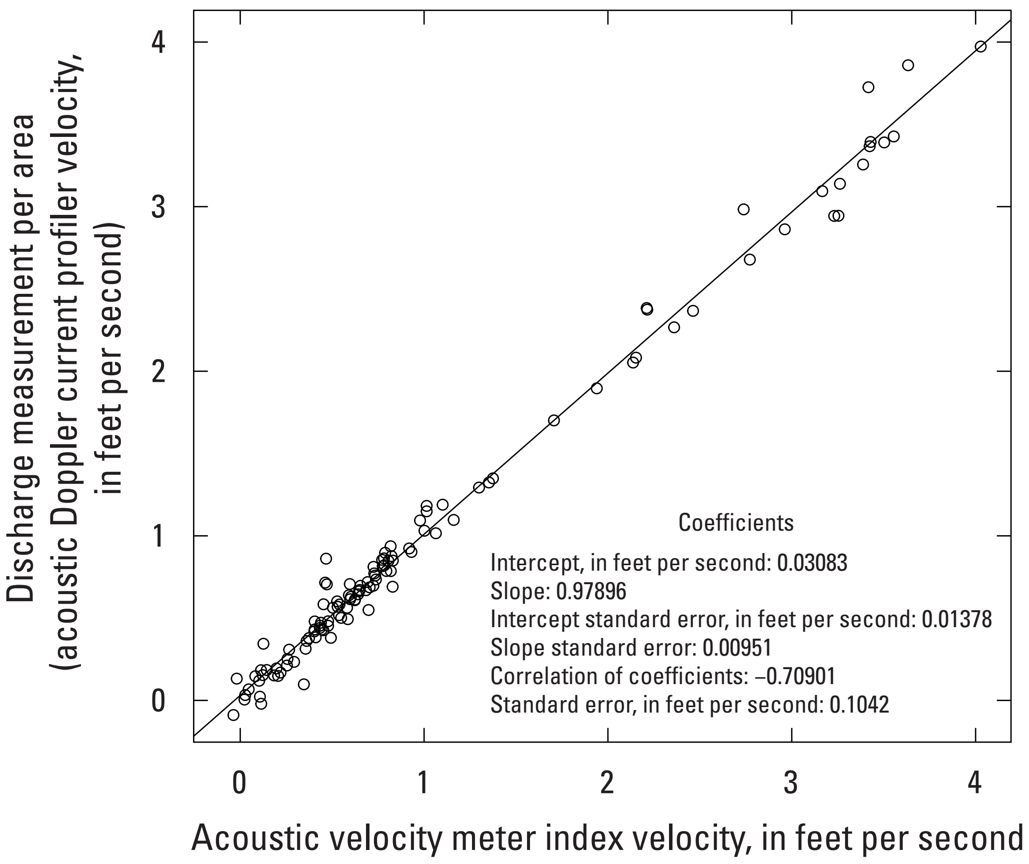
    where

    $n$ = number of data points used and
    $p$ = number of parameters fitted (here, 2).
  - $X^2_{n-p}$ has mean $n-p$ and variance $2(n-p)$ so a value of $SSR_{norm}$ near $n-p$ is expected.
  - If the test fails, meaning $p = \text{Prob}(X^2_{n-p} > SSR_{norm}) < \alpha$, for some selected significance level $\alpha <= 0.05$, then one or more of the following is true:
    - "The model is wrong" (here this would mean the IVR is nonlinear);
    - The measurement errors are larger than reported;
    - The errors are non-Gaussian.
  - Because of the strictness of the Gaussian errors criterion, it is "not uncommon" to accept models with $p$ as small as 0.001.

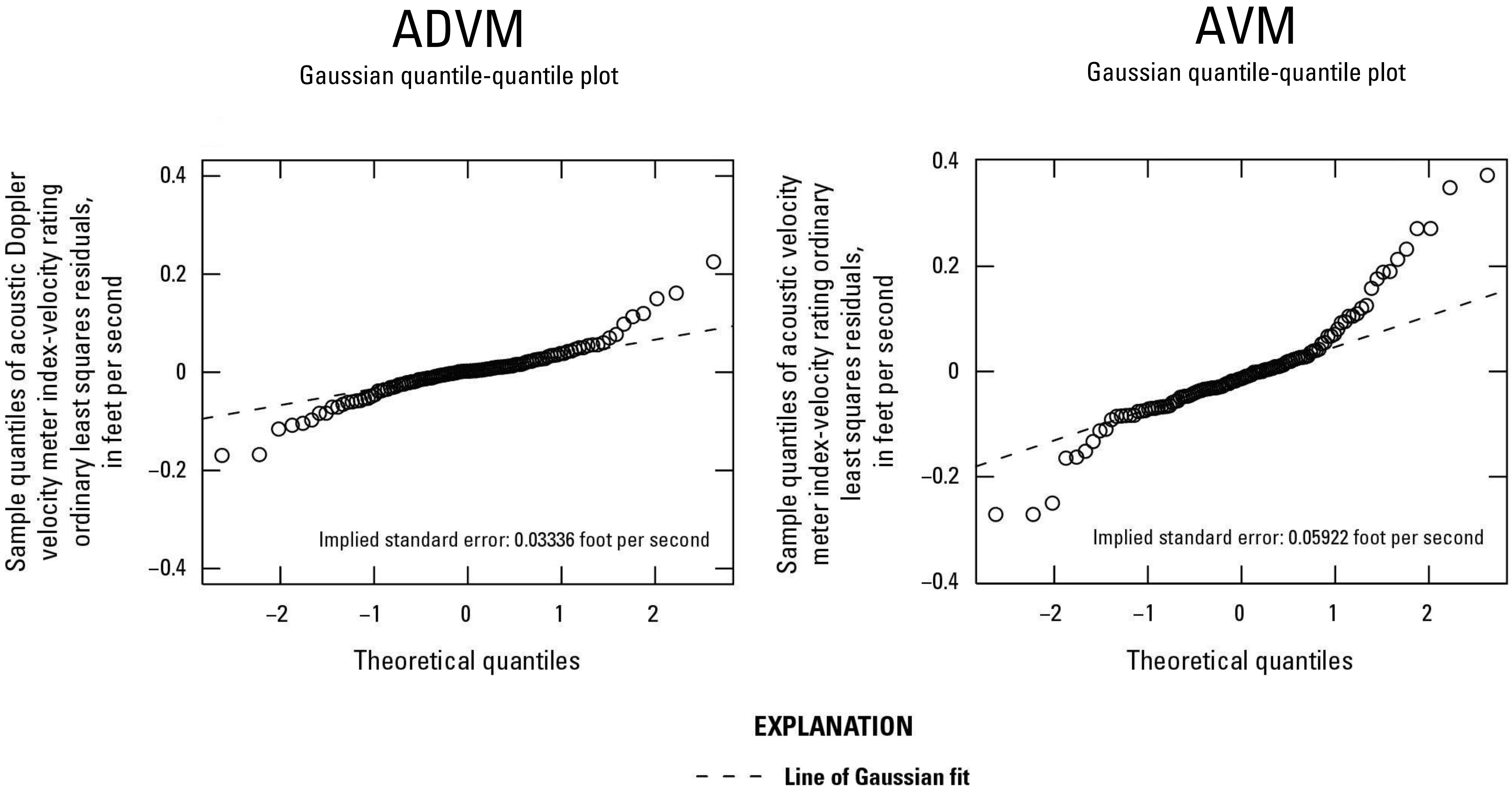
# Comparison of OLS ADVM and AVM Index Velocity Ratings (on common Qms)

SE(AVM IVR) = 0.104 ft/s > SE(ADVM IVR) = 0.056 ft/s



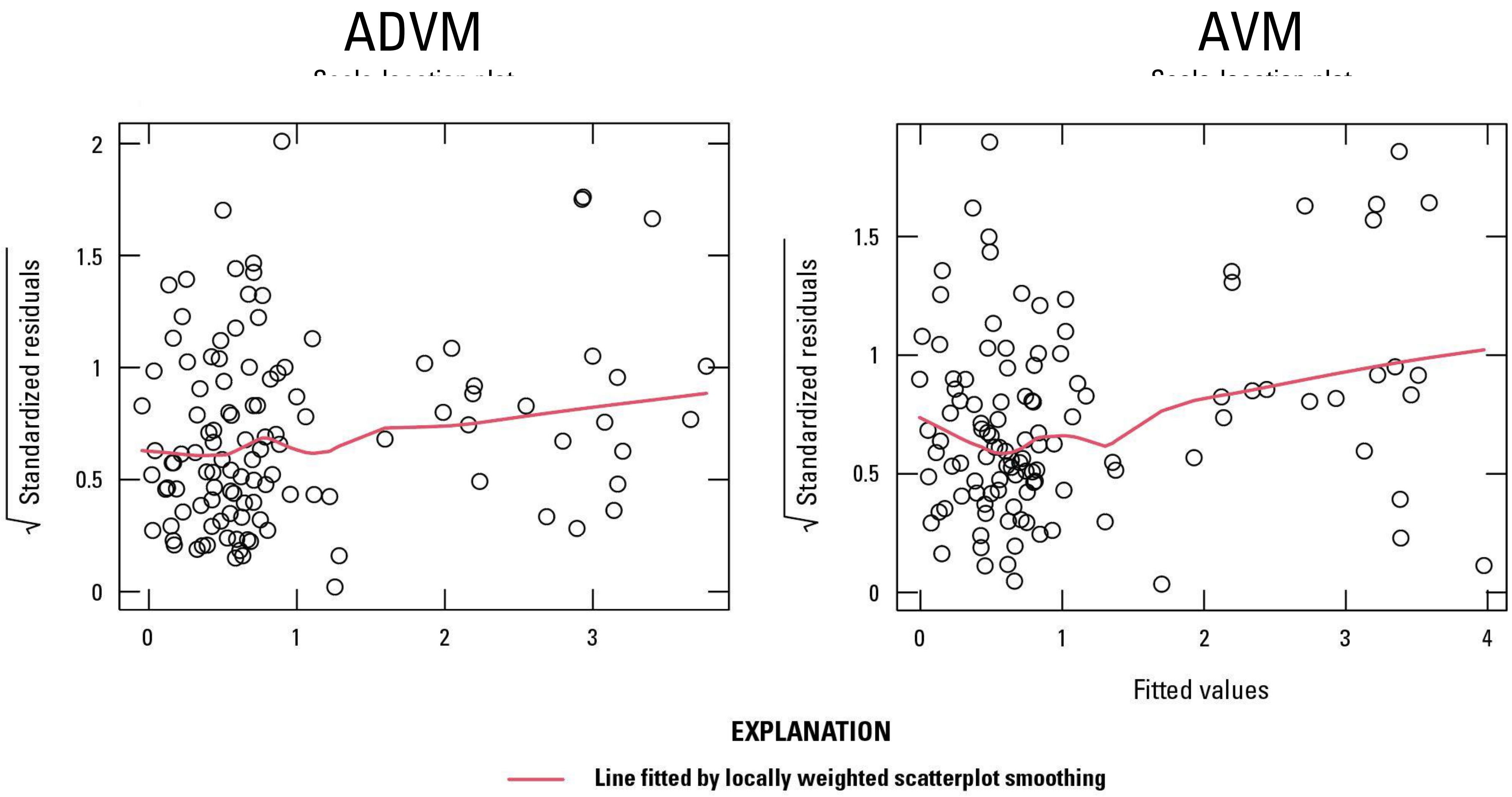



# Distributions of OLS IVR residuals (common Qms)




- Confirms AVM residuals > ADVM residuals
- Confirms presence of (non-Gaussian) outliers

# Heteroscedasticity of OLS IVR residuals (common Qms)

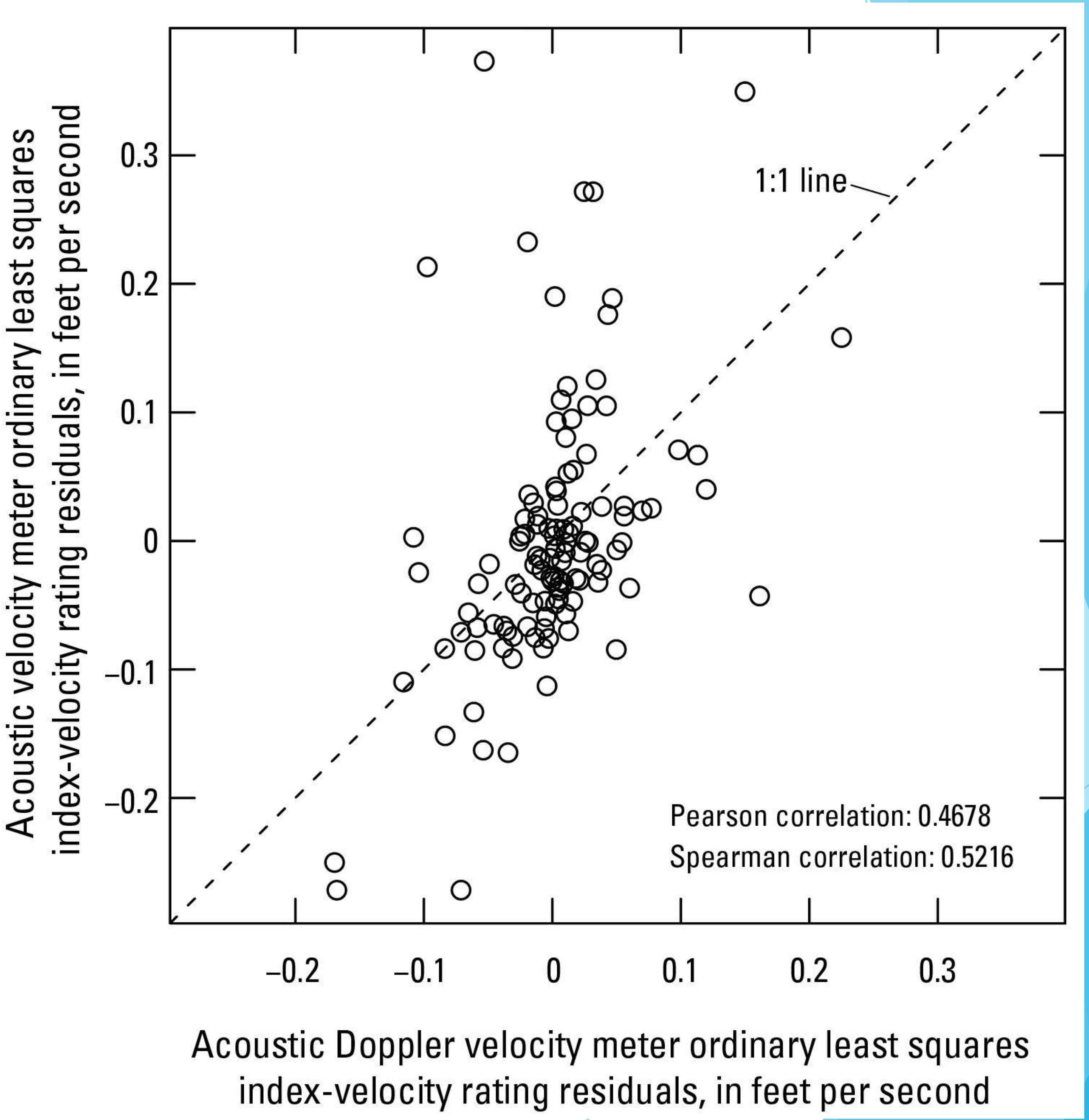



Results of statistical tests of heteroscedasticity (ADVM and AVM, respectively):

1. Non-constant variance test (Helsel and others, 2020, p. 244): $p = 0.006$ and 0.00001.
2. Kendall's tau correlation with fitted values (Helsel and others, 2020, section 8.4):
*tau* = 0.119 and 0.099; two-sided $p = 0.060$ and 0.117.

# AVM vs. ADVM OLS IVR residuals

Correlation of residuals is positive but not especially large, indicating ADCP errors are on the same order of magnitude as ADVM and AVM.



# Intercept, slope, and $X^2$ test statistics for ADVM-based IVRs for selected regression methods and error specifications

| Regression method | Are errors speci-fied? | Are IV errors consi-dered? | Correlation of ADCP errors | Intercept (ft/s) | Slope | Intercept SE (ft/s) | Slope SE | Correlation (Intercept, Slope) | $SSR_{norm}$ | Prob($X^2_{n-p}$ > $SSR_{norm}$) ($n-p$ = 128) |
|---|---|---|---|---|---|---|---|---|---|---|
| OLS | No | No | None | −0.0071 | 0.9740 | 0.0073 | 0.0054 | −0.727 | N/A | N/A |
| WLS | Yes | No | None | −0.0336 | 1.0017 | 0.0028 | 0.0049 | −0.747 | 743.3 | < 1e−16 |
| WLS | [a]Yes × 2 | No | None | −0.0336 | 1.0017 | 0.0056 | 0.0097 | −0.747 | 185.8 | 0.00064 |
| GMR (GLS) | Yes | No | [b]sel.corr. = 0.5 | −0.0383 | 1.0066 | 0.0032 | 0.0062 | −0.753 | 932.6 | < 1e−16 |
| GMR (GLS) | [a]Yes × 2 | No | [b]sel.corr. = 0.5 | −0.0383 | 1.0066 | 0.0064 | 0.0125 | −0.753 | 233.2 | 3.9e−8 |
| GMR (GLS) | Yes | No | [c]corr. = 0.1 | −0.0359 | 0.9937 | 0.0029 | 0.0061 | −0.246 | 822.0 | < 1e−16 |
| GMR (GLS) | [a]Yes × 2 | No | [c]corr. = 0.1 | −0.0359 | 0.9937 | 0.0058 | 0.0123 | −0.246 | 205.5 | 1.6e−5 |
| GDR (EIV) | Yes | Yes | None | −0.0349 | 1.0094 | 0.00395 | 0.00668 | −0.754 | 366.6 | < 1e−16 |
| GDR (EIV) | [a]Yes × 2 | Yes | None | −0.0349 | 1.0094 | 0.00790 | 0.01336 | −0.754 | 91.7 | 0.99 |

[IV, index velocity]

[a]"× 2" here means that the estimated standard errors, both ADCP and IV, were multiplied by 2.
[b]Errors of ADCP (y-axis) measurements in immediate succession were assigned correlations of 0.5.
[c]Errors of all ADCP (y-axis) measurements were assigned correlations of 0.1.

# Intercept, slope, and $X^2$ test statistics for AVM-based IVRs for selected regression methods and error specifications

| Regression method | Are errors speci-fied? | Are IV errors consi-dered? | Correlation of ADCP errors | Intercept (ft/s) | Slope | Intercept SE (ft/s) | Slope SE | Correlation (Intercept, Slope) | $SSR_{norm}$ | Prob($X^2_{n-p}$ > $SSR_{norm}$) ($n-p$ = 138) |
|---|---|---|---|---|---|---|---|---|---|---|
| OLS | No | No | None | 0.0400 | 0.9768 | 0.0123 | 0.0092 | −0.707 | N/A | N/A |
| WLS | Yes | No | None | 0.0106 | 0.9899 | 0.0027 | 0.0048 | −0.733 | 1594 | < 1e−16 |
| WLS | [a]Yes × 3 | No | None | 0.0106 | 0.9899 | 0.0081 | 0.0143 | −0.733 | 177.2 | 0.014 |
| GMR (GLS) | Yes | No | [b]sel.corr. = 0.5 | 0.0014 | 0.9977 | 0.0031 | 0.0063 | −0.746 | 1748 | < 1e−16 |
| GMR (GLS) | [a]Yes × 3 | No | [b]sel.corr. = 0.5 | 0.0014 | 0.9977 | 0.0093 | 0.0188 | −0.746 | 194.2 | 0.0012 |
| GMR (GLS) | Yes | No | [c]corr. = 0.1 | 0.00076 | 0.9616 | 0.0029 | 0.0060 | −0.175 | 1721 | < 1e−16 |
| GMR (GLS) | [a]Yes × 3 | No | [c]corr. = 0.1 | 0.00076 | 0.9616 | 0.0088 | 0.0181 | −0.175 | 191.2 | 0.0019 |
| GDR (EIV) | Yes | Yes | None | 0.0069 | 1.0057 | 0.0032 | 0.0061 | −0.723 | 1168 | < 1e−16 |
| GDR (EIV) | [a]Yes × 3 | Yes | None | 0.0069 | 1.0057 | 0.0095 | 0.0182 | −0.723 | 129.8 | 0.68 |

[a]"× 3" here means that the estimated standard errors, both ADCP and IV, were multiplied by 3.
[b]Errors of ADCP (y-axis) measurements in immediate succession were assigned correlations of 0.5.
[c]Errors of all ADCP (y-axis) measurements were assigned correlations of 0.1.

# Prediction statistics for ADVM-based IVRs for selected regression methods and error specifications

| Regression method | Are errors specified? | Are IV errors considered? | Correlation of ADCP errors | Mean bias (predicted – observed) (ft/s) | Mean SE of prediction (ft/s) | Variance ratio (predicted to observed) |
|---|---|---|---|---|---|---|
| **OLS** | No | No | None | 0.00000 | 0.0572 | 0.996 |
| **WLS** | Yes | No | None | 0.00058 | 0.0627 | 1.054 |
| **WLS** | [a]Yes × 2 | No | None | 0.00058 | 0.0627 | 1.054 |
| **GMR (GLS)** | Yes | No | [b]sel.corr. = 0.5 | 0.00071 | 0.0649 | 1.064 |
| **GMR (GLS)** | [a]Yes × 2 | No | [b]sel.corr. = 0.5 | 0.00071 | 0.0649 | 1.064 |
| **GMR (GLS)** | Yes | No | [c]corr. = 0.1 | −0.00957 | 0.0608 | 1.037 |
| **GMR (GLS)** | [a]Yes × 2 | No | [c]corr. = 0.1 | −0.00957 | 0.0608 | 1.037 |
| **GDR (EIV)** | Yes | Yes | None | 0.00680 | 0.0664 | 1.070 |
| **GDR (EIV)** | [a]Yes × 2 | Yes | None | 0.00680 | 0.0664 | 1.070 |

[a]"× 2" here means that the estimated standard errors, both ADCP and IV, were multiplied by 2.
[b]Errors of ADCP (y-axis) measurements in immediate succession were assigned correlations of 0.5.
[c]Errors of all ADCP (y-axis) measurements were assigned correlations of 0.1.

# Prediction statistics for AVM-based IVRs for selected regression methods and error specifications

| Regression method | Are errors specified? | Are IV errors considered? | Correlation of ADCP errors | Mean bias (predicted – observed) (ft/s) | Mean SE of prediction (ft/s) | Variance ratio (predicted to observed) |
|---|---|---|---|---|---|---|
| OLS | No | No | None | 0.00000 | 0.1032 | 0.988 |
| WLS | Yes | No | None | −0.0169 | 0.1053 | 1.015 |
| WLS | [a]Yes × 3 | No | None | −0.0169 | 0.1053 | 1.015 |
| GMR (GLS) | Yes | No | [b]sel.corr. = 0.5 | −0.0188 | 0.1068 | 1.031 |
| GMR (GLS) | [a]Yes × 3 | No | [b]sel.corr. = 0.5 | −0.0188 | 0.1068 | 1.031 |
| GMR (GLS) | Yes | No | [c]corr. = 0.1 | −0.0536 | 0.1173 | 0.957 |
| GMR (GLS) | [a]Yes × 3 | No | [c]corr. = 0.1 | −0.0536 | 0.1173 | 0.957 |
| GDR (EIV) | Yes | Yes | None | −0.0057 | 0.1070 | 1.047 |
| GDR (EIV) | [a]Yes × 3 | Yes | None | −0.0057 | 0.1070 | 1.047 |

[a]"× 3" here means that the estimated standard errors, both ADCP and IV, were multiplied by 3.
[b]Errors of ADCP (y-axis) measurements in immediate succession were assigned correlations of 0.5.
[c]Errors of all ADCP (y-axis) measurements were assigned correlations of 0.1.

# Step 4: Computed Discharge Uncertainty

# Computation of discharge with an index-velocity rating

With an IVR $V_{Qm}(V_{index})$ and a stage-area rating $Area_{index}(h)$, where $h$ is stage, and continuous measurements of velocity $V_{index}(t)$ and stage $h$(t), where $t$ is time, discharge at time $t$, $Q(t)$, is computed by multiplying the rated velocity by the rated area given, that is:

$$Q(t) = V_{\mathrm{Qm}}(V_{index}(t)) \times Area_{index}(h(t)).$$

Because here $V_{Qm}(V_{index}) = a + b^{*} V_{index}$, where $a$ is intercept and $b$ is the slope of the IVR, the discharge computation relation can be written more specifically as:

$$Q(t) = [a + b^{*} V_{index}(t)] \times Area_{index}(h(t)).$$

# Sources of uncertainty in discharge computed with index-velocity ratings

Because the computed discharge is a product of rated velocity and rated area, likewise its uncertainty arises from those factors.

From the rated velocity, which is more properly written as

$$V_{Qm} = a + bV_{index} + \varepsilon,$$

where $\varepsilon$ is the IVR fitting error, there are two components:

1. The uncertainty of the IVR parameters $a$ and $b$, which are distributed as a joint Gaussian distribution with mean vector $[a,b]$ and covariance matrix parameterized by their individual SEs and their correlation (see slides 37 and 38 for values).

2. The uncertainty arising from $\varepsilon$, which may or may not be correlated in time, and whose uncertainty magnitude is computed from the regression residuals.

The error in $Area_{index}(h(t))$ is neglected here because, according to the argument of Duncker and others (2006, p. 28), the error in the stage-area rating is eliminated because the same stage-area rating is used in the fitting of the IVR and the computation of discharge.

# Effect of time scale of averaging on computed discharge uncertainty

At this similar streamgage, which was also on the Chicago Sanitary and Ship Canal and was the primary measurement location for LMDA discharge measurements before it was moved about 6 miles upstream to Lemont, and where discharge was computed with an IVR based on AVM velocities, at long time scales (monthly to annual), IVR regression parameter uncertainty was determined to dominate computed discharge uncertainty (Over and others, 2004).

USGS 05536995, Chicago Ship and Sanitary Canal at Romeoville, Illinois

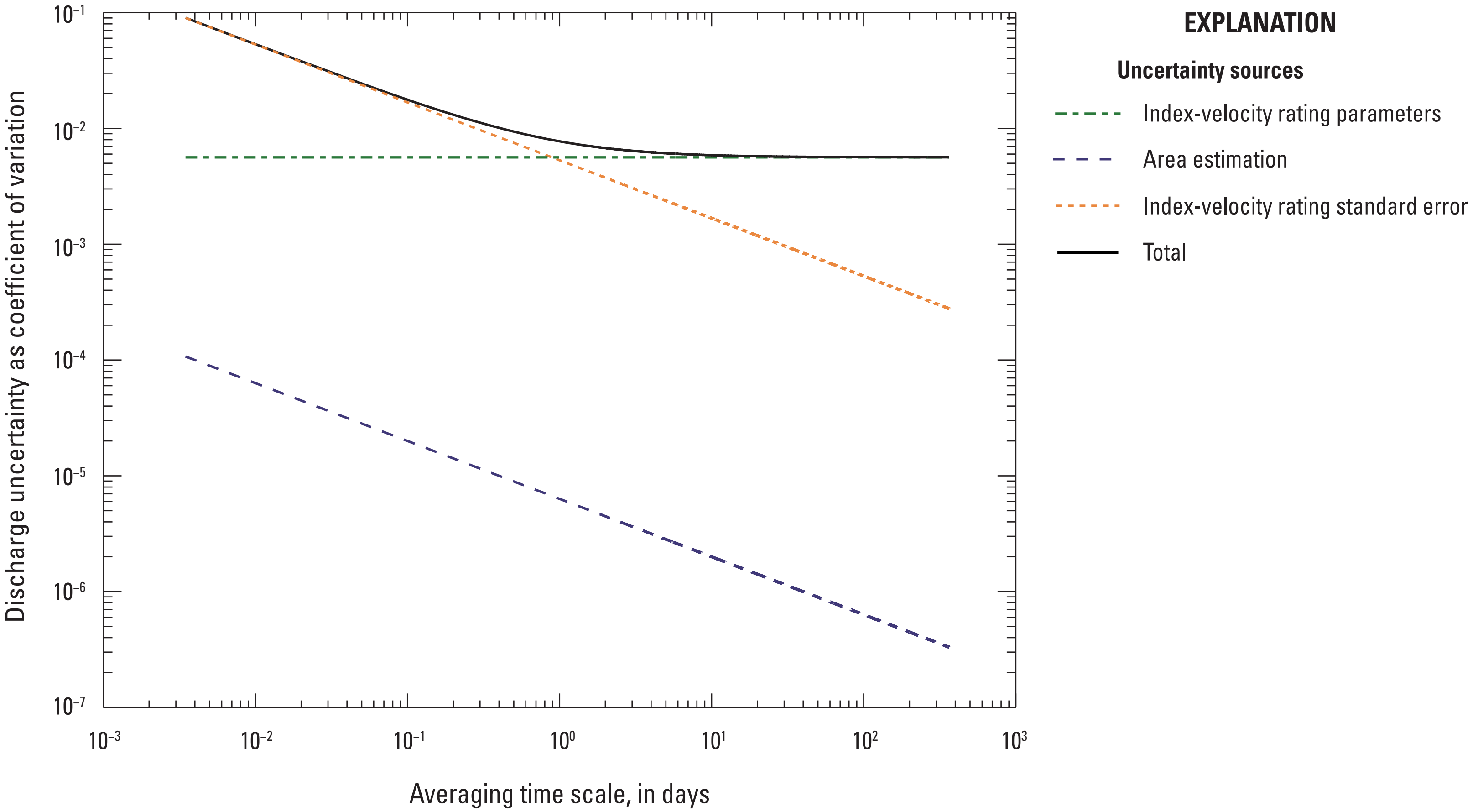



Figure is adapted from presentation by Over and others (2004).

# Computation of discharge and its uncertainty by Monte Carlo simulation

- For each index-velocity meter, the uncertainty in the IVR parameters $a$ and $b$ was simulated by sampling a vector $[a,b]$ from their joint Gaussian distribution 100 times.
- When using the OLS IVR, the IVR error term $\varepsilon$ was also simulated as an independent Gaussian time series $\varepsilon(t)$ with zero mean and standard deviations (stdevs) from the SEs of the OLS IVR regressions (ADVM: 0.0572 ft/s; AVM: 0.1032 ft/s).
- For each sample of $[a,b]$, discharge $Q(t)$ was computed by the standard method for streamgages with IVRs (slide 40) at a 10-minute time step.
- In each day, if at least half the 10-minute $Q(t)$ values were available, those $Q(t)$ values were averaged to compute a daily mean.
    - Otherwise, 10-minute $Q(t)$ values from the other index-velocity meter were used if at least half of those were available.
        - Otherwise, the day was left as missing.
- Finally, the daily values were averaged to obtain water-year-annual mean $Q(t)$ values.
- AVM data were available for most of WYs 2006–15 and ADVM data for most of WYs 2008–16, and therefore those years were simulated for each respective index-velocity meter.

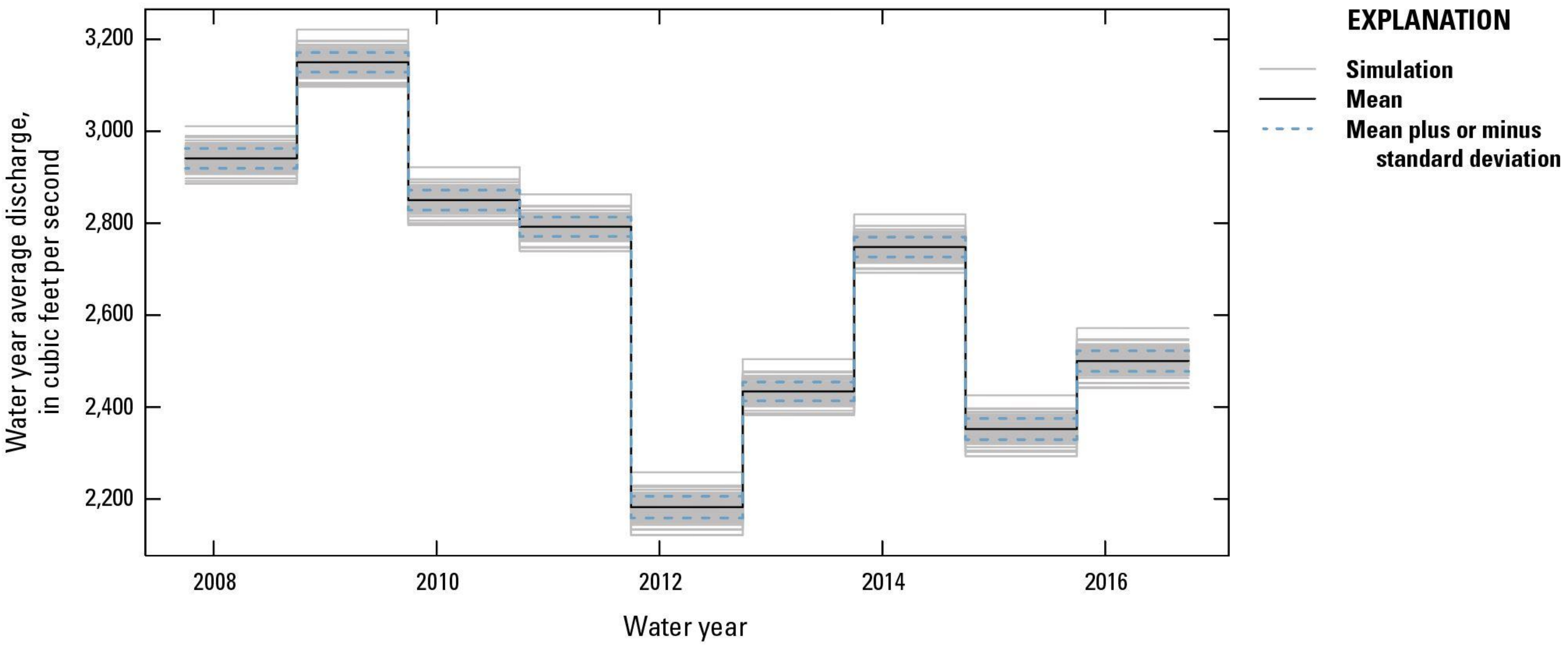
# Uncertainty statistics for annual mean (water years 2008–16) computed discharge for ADVM-based IVRs for selected regression methods and error specifications

| Regression method | Are errors specified? | Are IV errors considered? | Correlation of ADCP errors | Mean of annual means ± stdev(mean) ($ft^3/s$) | Mean of annual stdevs ($ft^3/s$) | Mean of daily CVs ± stdev(CVs) |
|---|---|---|---|---|---|---|
| OLS | No | No | None | 2,660 ± 2 | 21.2 | 0.595 ± 0.0007 |
| WLS | Yes | No | None | 2,625 ± 1 | 9.5 | 0.621 ± 0.0003 |
| WLS | [a]Yes × 2 | No | None | 2,625 ± 2 | 18.1 | 0.620 ± 0.0005 |
| GMR (GLS) | Yes | No | [b]sel. corr. = 0.5 | 2,622 ± 1 | 12.2 | 0.625 ± 0.0003 |
| GMR (GLS) | [a]Yes × 2 | No | [b]sel. corr. = 0.5 | 2,619 ± 2 | 18.0 | 0.626 ± 0.0006 |
| GMR (GLS) | Yes | No | [c]corr. = 0.1 | 2,593 ± 2 | 19.5 | 0.623 ± 0.0003 |
| GMR (GLS) | [a]Yes × 2 | No | [c]corr. = 0.1 | 2,596 ± 3 | 33.4 | 0.623 ± 0.0006 |
| GDR (EIV) | Yes | Yes | None | 2,638 ± 1 | 12.9 | 0.621 ± 0.0003 |
| GDR (EIV) | [a]Yes × 2 | Yes | None | 2,640 ± 2 | 22.2 | 0.623 ± 0.0008 |

[a]"× 2" here means that the estimated standard errors, both ADCP and IV, were multiplied by 2.
[b]Errors of ADCP (y-axis) measurements in immediate succession were assigned correlations of 0.5.
[c]Errors of all ADCP (y-axis) measurements were assigned correlations of 0.1.

Computed at a 10-minute time step with 100 Monte Carlo samples. On average 6.3 days per year were filled with AVM-based computed discharge, when available; on average 1 day per year remains missing.

# OLS regression parameter uncertainty-based Monte Carlo results for computed discharge using ADVM velocities



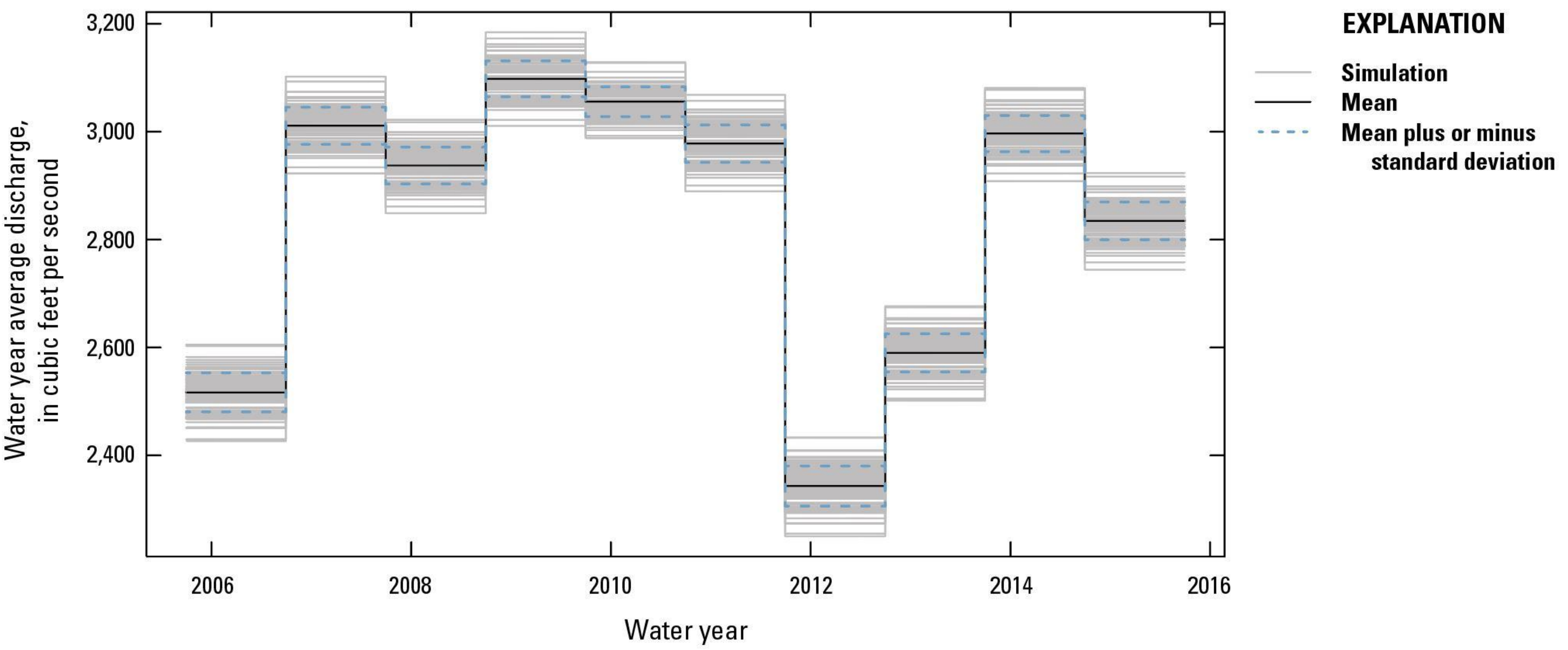
# Uncertainty statistics for annual mean (water years 2006–15) computed discharge for AVM-based IVRs for selected regression methods and error specifications

| Regression method | Are errors specified? | Are IV errors considered? | Correlation of ADCP errors | Mean of annual means ± stdev(mean) ($ft^3/s$) | Mean of annual stdevs ($ft^3/s$) | Mean of daily CVs ± stdev(CVs) |
|---|---|---|---|---|---|---|
| OLS | No | No | None | 2,834 ± 3 | 34.5 | 0.545 ± 0.0009 |
| WLS | Yes | No | None | 2,756 ± 1 | 8.9 | 0.568 ± 0.0002 |
| WLS | [a]Yes × 3 | No | None | 2,755 ± 3 | 28.7 | 0.566 ± 0.0006 |
| GMR (GLS) | Yes | No | [b]sel. corr. = 0.5 | 2,742 ± 1 | 12.1 | 0.575 ± 0.0002 |
| GMR (GLS) | [a]Yes × 3 | No | [b]sel. corr. = 0.5 | 2,750 ± 3 | 33.4 | 0.572 ± 0.0007 |
| GMR (GLS) | Yes | No | [c]corr. = 0.1 | 2,643 ± 2 | 18.2 | 0.575 ± 0.0002 |
| GMR (GLS) | [a]Yes × 3 | No | [c]corr. = 0.1 | 2,646 ± 6 | 57.6 | 0.575 ± 0.0007 |
| GDR (EIV) | Yes | Yes | None | 2,784 ± 1 | 11.9 | 0.570 ± 0.0003 |
| GDR (EIV) | [a]Yes × 3 | Yes | None | 2,786 ± 3 | 32.2 | 0.571 ± 0.0008 |

[a]"× 3" here means that the estimated standard errors, both ADCP and IV, were multiplied by 3.
[b]Errors of ADCP (y-axis) measurements in immediate succession were assigned correlations of 0.5.
[c]Errors of all ADCP (y-axis) measurements were assigned correlations of 0.1.

Computed at a 10-minute time step with 100 Monte Carlo samples. On average 9.1 days per year were filled with ADVM-based computed discharge, when available; on average 1.2 days per year remains unfilled.

# OLS regression parameter uncertainty-based Monte Carlo results for computed discharge using AVM velocities



# Summary I: Predicted measurement uncertainties

- Type B (non-empirical, scientific judgment-based) estimates for ADVM and AVM velocities and stage at the Lemont streamgage were developed using FOSM and checked using Monte Carlo simulation.
- Elemental uncertainties were taken as Gaussian, and predicted uncertainties were also Gaussian to a high degree of approximation.
- ADVM and AVM velocities were predicted to be similar and about 2.5 percent of velocity except near zero velocity, where AVM uncertainty was predicted as about 0.003 ft/s and ADVM uncertainty as about 0.009 ft/s.
- Stage uncertainty was estimated to be about 0.027 ft, independent of water level.
- Measured discharge Qm uncertainty was estimated using QRev (Mueller, 2016), which is part empirical and part judgment based.
- When converted to mean channel velocities, the predicted Qm uncertainties were estimated to be larger than the AVM and ADVM uncertainties, being at least 2.5 percent of velocity, but often much larger, especially for lower velocities.

# Summary II: Index-velocity ratings (IVRs) computed using ordinary least squares (OLS) regression

- Used a database of 155 ADCP discharge – index velocity pairs:
  - 155 total pairs, 140 with AVM velocity, 130 with ADVM velocity, 115 with both.
  - Distribution of velocities in database agree with prediction velocities except at moderately high values.
- OLS-fitted IVRs were found to be linear with high coefficient of determination ($R^2$) values (at least 0.988).
- Properties of OLS residuals:
  - Residual standard error of AVM regression (0.104 ft/s) were about twice that of ADVM (0.056).
  - Distributions of both sets of residuals had fat, non-Gaussian tails.
  - Both AVM and ADVM residuals showed modest growth with velocity.
  - AVM and ADVM residuals from common measurements were modestly positively correlated, indicating contributions from both index-velocity and ADCP errors.
- When repredicting fitting data, OLS predictions were unbiased, had the smallest prediction error, and had only a small variance reduction effect.

# Summary III: Index-velocity ratings computed using alternative regressions

- Alternative regression methods used allow:
  - Variation in uncertainties of regression data on y axis or both x and y axes.
  - Specification of measurement errors for which the predicted index-velocity and ADCP uncertainties were used.
- $X^2$ tests of residuals of alternative regressions indicate:
  - predicted measurement uncertainties are too small.
  - predicted measurement uncertainties need to be multiplied by about 2 (ADVM) or 3 (AVM).

# Summary IV: Computed discharge and its uncertainty

- Uncertainty sources other than uncertainty of IVR parameters assumed to be negligible at annual time scale.
- Used Monte Carlo simulation.
- Water years 2008–16 (ADVM) and 2006–15 (AVM) predicted.
- Properties of computed discharge using OLS regression:
  - Coefficients of variation (CVs) of 0.8 percent (ADVM) and 1.2 percent (AVM) of annual means.
  - Largest mean discharges.
  - Smallest variation of daily discharges.
- Properties of computed discharge using alternative regressions:
  - Similar CVs as OLS only when specified uncertainties were increased by 2 or 3.
  - Smaller mean discharges.
  - Larger variation at daily time scale.
- Concluded OLS-based results should be reliable if not exact.

# Summary V: Final Remarks

- Results indicate need to revisit predicted measurement uncertainties (magnitude, velocity dependence).
- Measurements with larger residuals could be examined individually regarding possible causes.
- Improved methods of ADCP uncertainty estimation may be or may soon be available.
- Main weakness of OLS regression probably is correspondence of fitting and prediction velocity distributions:
    - Could be addressed by subsampling or additional measurements.
- Better inference of growth in residuals with velocity would help IVR regressions and predicted measurement uncertainties.

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