NEW GRAPHIC METHOD FOR DETERMINING THE DEPTH AND THICKNESS OF STRATA AND THE PROJECTION OF DIP.

By Harold S. Palmer.

INTRODUCTION.

Geologists, both in the field and in the office, frequently encounter trigonometric problems the solution of which, though simple enough, is somewhat laborious by the use of trigonometric and logarithmic tables. Charts, tables, and diagrams of various types for facilitating the computations have been published, and a new method may seem to be a superfluous addition to the literature. However, it is felt that the simplicity and accuracy of diagrams such as are presented in this paper warrant the addition. The alinement diagram is used to a very considerable extent by engineers, especially mechanical engineers. The principle of the alinement diagram is explained herein, but rigorous demonstration is not attempted.

Alinement diagrams are given for the following problems, which are numbered to agree with the numbering of the diagrams. The charts published herewith are suitable for use.

I. To find the depth to a stratum, given the dip of the stratum, the horizontal distance from the outcrop, and the difference in elevation between the outcrop and the point at which the depth is desired. (See fig. 15.) The formula is

$$D = H \tan A \pm E \text{..........} (1$$

in which D = depth to stratum, H = horizontal distance from the outcrop, A = dip, E = difference in elevation.

(Note.—Diagram I may be used not only for the problem above stated but also to obtain the difference in elevation between two points by omitting E and letting D equal the difference in elevation, H the horizontal distance, and A the vertical angle.)

II. To find the thickness of a stratum, given the dip of the stratum, the horizontal distance across its outcrop at right angles to the strike, and the difference in elevation of the upper and lower boundaries of the stratum. There are three cases, depending on the relation of the dip of the beds to the slope of the ground. (See fig. 16, *a*, *b*, and *c*.)

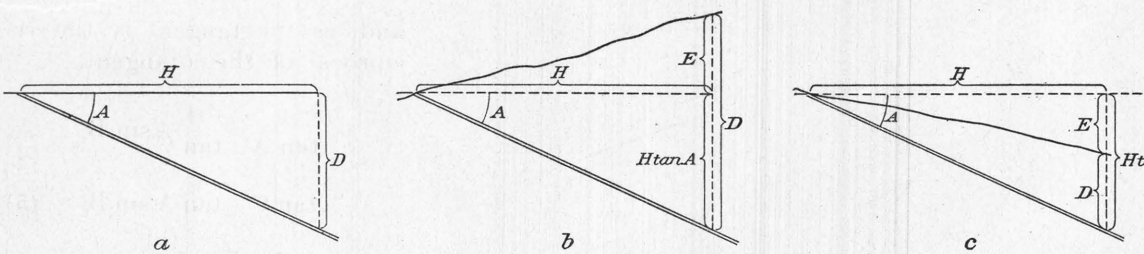


Figure 15.—Diagram showing three cases in computing depth.

Case 1, where the ground is level. By inspection of figure 16, *a*, it is seen that the formula is

$$T = H \sin A \text{............} (2)$$

in which T = thickness, H = horizontal distance from outcrop, and A = dip.

Case 2, where the ground slopes opposite to the dip. In figure 16, *b*,

$$T = m + l$$

but

$$m = H \sin A$$

Now

$$l = E \cos A$$

or, as

$$\cos X = \sin (90° - X)$$

$$l = E \sin (90° - A)$$

Therefore

$$T = H \sin A + E \sin (90° - A) \quad \text{-----} (3)$$

in which T, H, and A have the same values as before, and E is the difference in elevation.

Case 3, where the ground slopes with the dip. In figure 16, *c*, $T = m - l$; making the same substitutions as in case 2, we find

$$T = H \sin A - E \sin (90° - A) \quad \text{-----} (4)$$

in which T, H, A, and E have the same values as before.

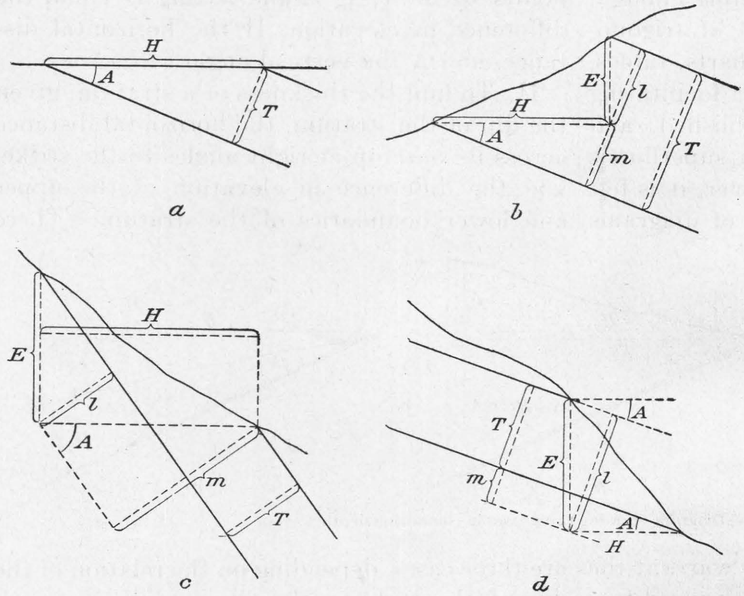


FIGURE 16.—Diagram showing three cases in computing thickness.

The above formula is strictly true only where the dip of the beds is greater than the slope of the ground. (See fig. 16, *d*.) Where the dip is less than the slope the formula becomes

$$T = E \sin (90° - A) - H \sin A \quad \text{-----} (4a)$$

The difference is algebraic and not arithmetic, so that the formula is practically workable for either subcase.

III. Projection of dips—that is, to find the slope of the trace of an inclined plane upon a vertical plane oblique to the strike. Consider figure 17, *a*, which is a plan or map; at A is an inclined plane the strike and dip of which are shown in the conventional way, and along S–P is a vertical plane upon which this dip is to be projected. B is the angle between the strike and the section plane. In figure 17, *b*, A is a dip whose projection on SPQR, the plane of a supposed structure section, is desired. C is the desired projection of A. B is the angle between the plane of the section and the strike of the beds, and angle PFS is a right angle. The plane PFS is horizontal and the lines PQ, FG, and SR are normal to it and of equal length, so that the triangle QGR is also horizontal and is equal to PFS.

Now,

$$FS = PS \sin B$$

$$FS = FG \cot A$$

$$PS = PQ \cot C$$

Therefore

$$FG \cot A = PQ \cot C \sin B$$

but

$$FG = PQ$$

so

$$\cot A = \cot C \sin B$$

and, as the tangent is the reciprocal of the cotangent,

$$\frac{1}{\tan A} = \frac{1}{\tan C} \sin B$$

$$\tan C = \tan A \sin B \quad \text{---} (5)$$

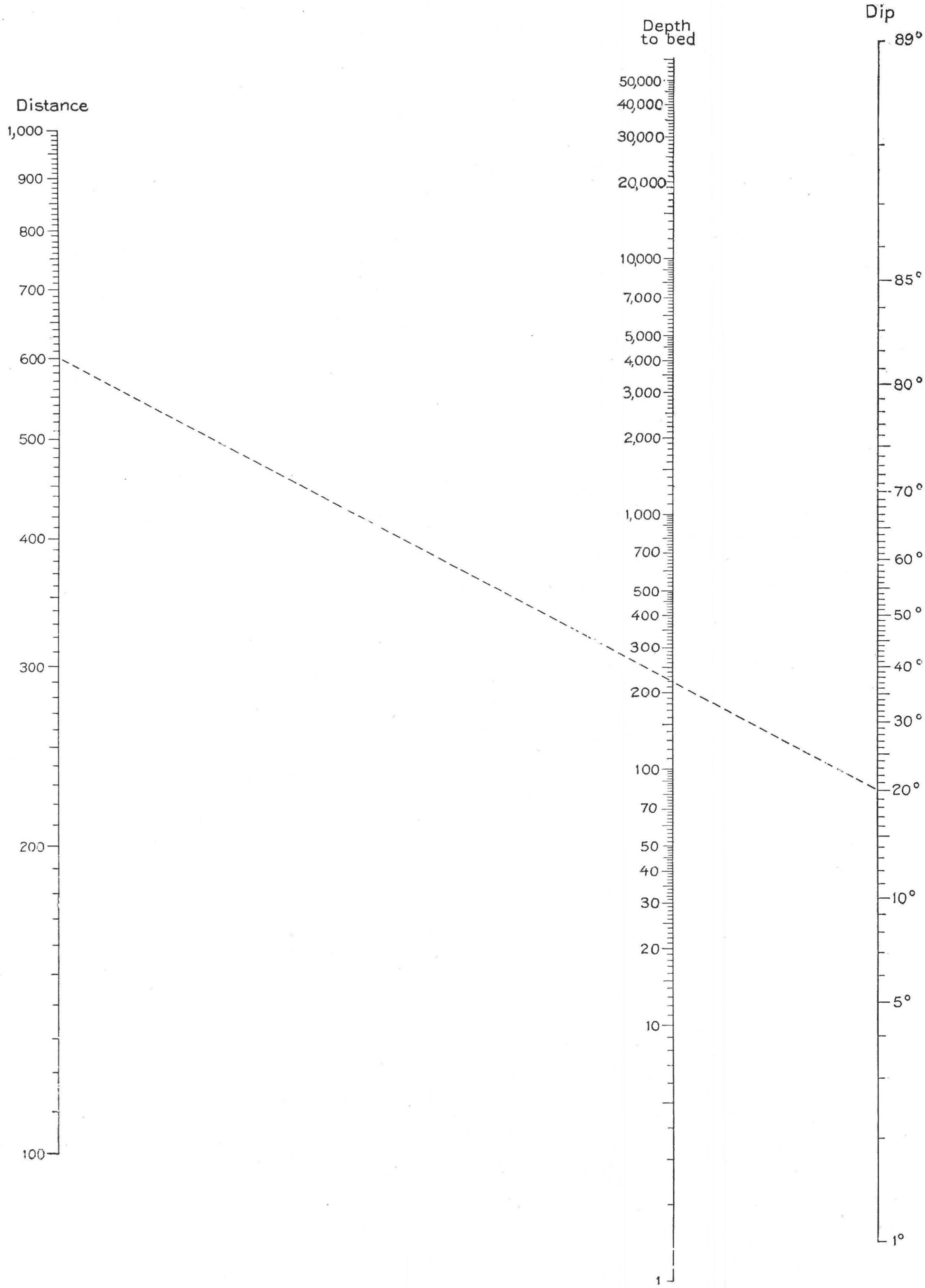
in which C = dip of the projected angle, A = dip of the bed, B = angle between strike and plane of section.

USE OF THE ALINEMENT DIAGRAMS.

The alinement diagram is worked in an extremely simple way. The points on the outer scales representing the given quantities are joined by a line, and the unknown quantity, or result, is read at the intersection with the middle scale. Interpolation between the values marked on the scales is effected just as on a slide rule. It is suggested that the edge of a draftsman's transparent triangle or a piece of slender black thread be used for projecting the cross line.

USE OF THE DEPTH DIAGRAM.

In using the depth diagram (Pl. XIV and problem I), find the point on the left-hand scale that represents the distance from the outcrop to the point at which the depth is desired, and join it to the point on the right-hand scale that



ALINEMENT DIAGRAM I, FOR COMPUTING DEPTH.

represents the dip of the beds. At the intersection of the line determined by these points with the middle scale the depth may be read. The dotted line on Plate XIV shows the solution when the distance is 600 feet and the dip 20°. The depth to the stratum is found to be 220 feet.

The above rule holds only if the surface is level. If the point at which the depth is to be determined is higher than the outcrop, the difference in their elevation is added to the depth as read by the above-described method. If the outcrop is the higher of the two points, the difference of elevation is subtracted from the reading.

The diagram may be used for the converse problem, namely, to find the dip of the bed when the depth and distance from the outcrop are given. In this case the given quantities are set on the middle and left scale, and the line joining them is produced till it cuts the right scale, at which the dip is read. Another converse problem arises when the distance from the outcrop is unknown but the dip and depth to the stratum are known. In this case known points on the right and middle scale are joined by a line the extension of which gives the result at its intersection with the left scale.

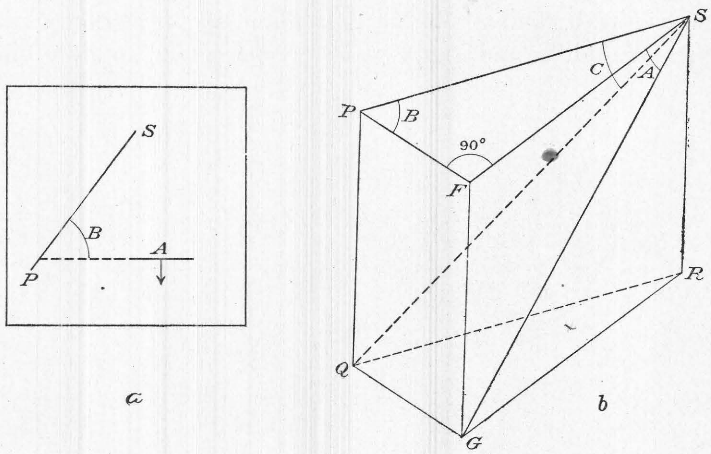


FIGURE 17.—Diagram for derivation of formula for projected dip.

USE OF THE THICKNESS DIAGRAM.

In using the thickness diagram (Pl. XV and problem II), find the point on the left scale that represents the width of the outcrop and join it to the point on the right scale that represents the dip of the beds. At the intersection of the line determined by these points with the middle scale the thickness may be read.

The above rule holds only if the surface is level. A correction must be added if the ground surface slopes in the opposite direction to the dip, and the same correction must be subtracted if the surface slope and the dip are in the same direction. This is the correction E sin (90° − A) in formulas (3) and (4). Subtract the dip from 90° and find the corresponding point on the right scale. Join it to the point on the left scale that represents the difference in elevation of the higher and lower boundaries of the outcrop, and at the intersection of the connecting line with the middle scale read the correction. The correction is added or subtracted as directed above.

In Plate XV the broken line shows the first step in solving the problem when the horizontal width of the outcrop is 600 feet and the dip is 20°. The thickness is read on the middle scale as 206 feet. The dashed line shows the correction to be introduced if the difference in elevation between the two edges of the outcrop is 50 feet. As there is no 50-foot graduation on the left scale, the decimal point is shifted to the right and the 500-foot graduation is joined by a line to the 70° mark on the left scale (90° − 20° = 70°). The reading is 470 feet, but the decimal point must be shifted back, so the true correction is 47 feet. Then if the bed dips "into the hill" the thickness is 206 + 47 = 253 feet; if it dips "with the hill" the thickness is 206 − 47 = 159 feet.

Like the depth diagram, the thickness diagram has converse uses, but they can be applied only when the ground surface is level. Thus if the width of the outcrop and thickness of the bed are known the dip can be computed, or if the dip and thickness are known the width of outcrop can be computed.

USE OF THE PROJECTION DIAGRAM.

In using the projection diagram (Pl. XVI and problem III) find the point on the left scale that represents the dip and the point on the right scale that represents the angle between the strike and the plane upon which the dip is to be projected. Join these points by a line, and at its intersection with the middle scale read the projected dip. As this problem involves only one case there are no corrections to be added or subtracted.

In Plate XVI the broken line shows the solution of the problem of projecting a dip of 43° upon a plane at an angle of 35° to the strike. The result, 28°, is read on the middle scale.

As with the other diagrams, the converse problems may be solved. Thus, if the trace of a bed along a vertical plane and the angle between the strike and the vertical plane are set on the middle and right scales, respectively, the dip of the plane may be read on the left scale. Again, if the dip of a bed and the slope of its trace on a vertical plane are known, and these points are set on the left and middle scales, respectively, the angle between the strike and the vertical plane may be read on the right scale.

PRINCIPLE OF THE ALINEMENT DIAGRAM.

The alinement diagram is a device by means of which certain arithmetic processes (addition, subtraction, multiplication, division, involution, and evolution) may be mechanically or graphically performed. It is similar in principle to the slide rule in that by means of it distances proportional to numbers or to logarithms are mechanically added, subtracted, multiplied, or divided, but this work is done by a different mechanical process. It differs further from the slide rule in that each diagram is good for only one formula, whereas the slide rule is applicable to many formulas.

Consider figure 18, *a*, in which *ab*, *cd*, and *ef* are parallel lines cut by two secants, *ace* and *bdf*. From *e* and *f* drop two lines, *eg* and *hf*, perpendicular to *ab*. By inspection of the diagram it is seen that

$$cd = \tfrac{1}{2}(ab + ef)$$

This equation is equivalent to saying that the space cut off on the middle line is equal to half the sum of the spaces cut on the outer lines.

Now, if, as in figure 18, *b*, the outer lines are graduated evenly with equal divisions and the middle line is graduated with divisions half as

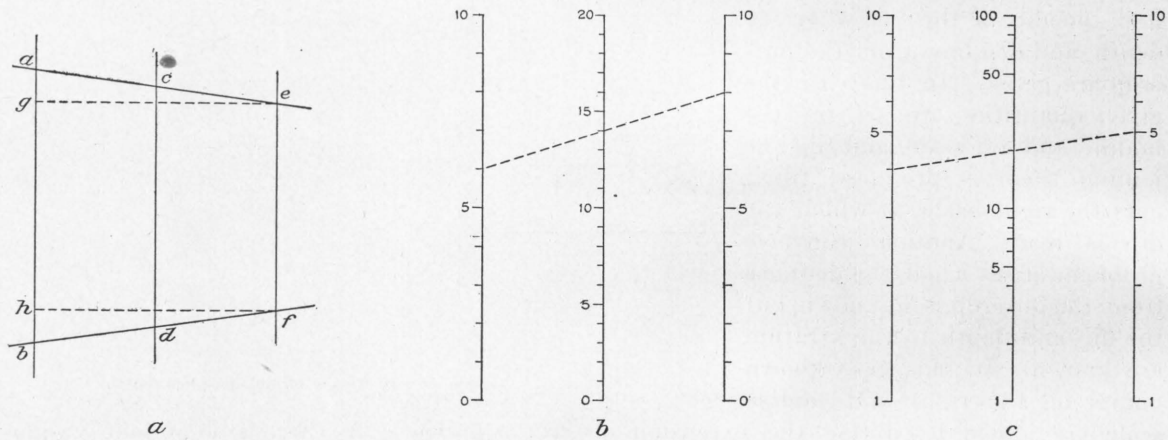


FIGURE 18.—Diagram showing three stages in derivation of principle of the alinement diagram.

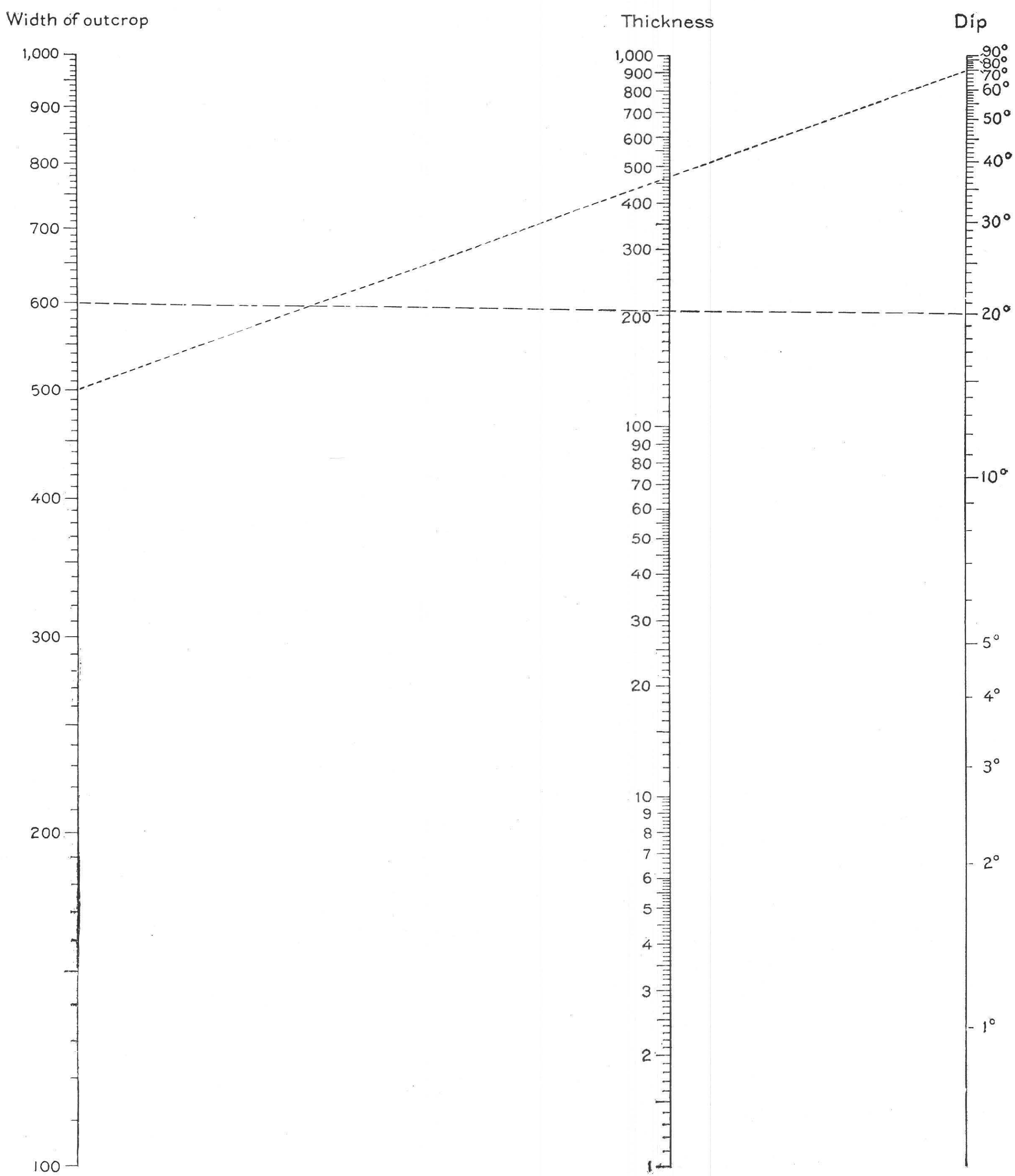
great, addition can be graphically performed. The bases of the three scales are put on the same level for convenience only. The secant line is taken so as to cut off six divisions on the left scale and eight on the right scale. It must, perforce, cut 14 divisions on the middle scale, and the addition of 6 and 8 is graphically accomplished.

It will be remembered that logarithms are essentially exponents (to the base 10 in the common system), and that multiplication is accomplished by adding the logarithms. This is by the algebraic law

$$x^m x^n = x^{(m+n)} \text{ - - - - - - - - - - - - - -(6)}$$

which may be verbally stated as follows: The multiplication of powers of the same quantity is effected by using the sum of the exponents as the exponent for the product.

Now, if the markings of the scales are not equidistant as in figure 18, *b*, but are laid off as



ALINEMENT DIAGRAM II, FOR COMPUTING THICKNESS.

in figure 18, *c*, so that the distances from the base are proportional to the logarithms of numbers, we can in effect add logarithms. But this is equivalent to multiplying the numbers. In the figure the multiplication of 4 and 5 is shown, the result being 20. In figure 18, *c*, the distance on the outer scales proportional to a logarithmic difference equal to 1 is 2 inches; on the middle scale it is half as great, or 1 inch.

The scales may be graduated in proportion to logarithms of trigonometric or other functions as well as in proportion to logarithms of numbers. It is therefore possible to manipulate a variety of factors.

Sometimes it is desirable to compress a scale, as for example a log. sin. scale in which the divisions between 1° and 20° come relatively far apart. In the charts here presented (Pls. XIV–XVI) the scales for logarithms of numbers were drawn with a modulus[1] of 10 inches. A log. sin. scale with a modulus of 10 inches would have to be 17.5 inches long in order to range from 1° to 90°. A log. tan. scale would have to be 35.2 inches long to range from 1° to 89°. When the moduli of the outer scales are not equal, the modulus of the middle scale is determined by the following formula, in which M_3 = modulus of middle scale, M_1 = modulus of right scale, and M_2 = modulus of left scale:

$$M_3=\frac{M_1\,M_2}{M_1+M_2} \quad (7)$$

At the same time, the middle scale must be shifted toward that outer scale which has the smaller modulus. The position of the middle scale is determined by the following equation, in which M_1 and M_2 have the same values as before, D_1 = distance from middle scale to right scale, and D_2 = distance from middle scale to left scale.

$$\frac{D_1}{D_2}=\frac{M_1}{M_2} \quad (8)$$

The principles underlying this and the last equation may be deduced from figure 19. The principles of the diagram require that the middle scale must be so placed and have such a modulus that any two lines which intersect on one of the outer scales and cut off a distance on the middle scale equal to its modulus will also cut off a distance on the third scale equal to its modulus.

Let M_1, M_2, and M_3 be the moduli and D_1 and D_2 the distances of the outer scales from the middle scales. Assume the lettering as indicated.

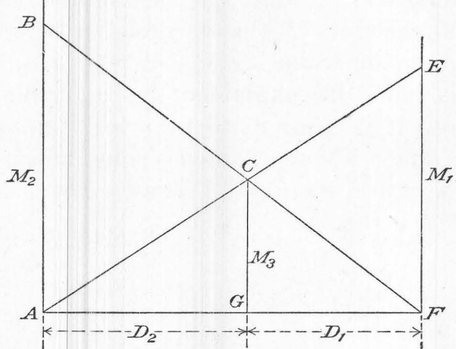


FIGURE 19.—Diagram for derivation of the interrelation of M_1, M_2, M_3, D_1, and D_2.

Then in the triangles ACG and AEF

$$\frac{M_3}{D_2}=\frac{M_1}{D_1+D_2}$$

and

$$M_3(D_1+D_2)=M_1D_2$$

Similarly in the triangles FCG and FBA

$$\frac{M_3}{D_1}=\frac{M_2}{D_1+D_2}$$

and

$$M_3(D_1+D_2)=M_2D_1$$

Therefore

$$M_2D_1=M_1D_2$$

and

$$\frac{D_1}{D_2}=\frac{M_1}{M_2} \quad \text{(q. e. d.)}$$

Taking this proportion by composition and transposing, we have

$$\frac{M_1}{M_1+M_2}=\frac{D_1}{D_1+D_2}$$

Now, in the triangles FCG and FBA

$$\frac{M_3}{M_2}=\frac{D_1}{D_1+D_2}$$

Substituting from the previous equation, we have

$$\frac{M_3}{M_2}=\frac{M_1}{M_1+M_2}$$

[1] The term "modulus" is used to mean the length on a scale proportional to a unit difference of logarithms. For example, the outer scales in figure 18 are said to have a modulus of 2 inches because the distance from the point representing 1 to the point representing 10 is 2 inches. The logarithm of 1 is 0.000 and the logarithm of 10 is 1.000, and the difference between these logarithms is unity.

and solving for M_3, we have

$$M_3 = \frac{M_1M_2}{M_1+M_2} \quad \text{(q. e. d.)}$$

The length of any scale may be determined by the following equation:

$$L = M\ (\log F_1 - \log F_2) \quad \text{--------(9)}$$

in which L = the length of the scale, M = its modulus, and F_1 and F_2 = the maximum and minimum values of the variable to be represented on the scale.

This may be explained by an example. Suppose it is desired to fit a log. tan. scale into a space about 12 inches long. Let $F_2 = \tan 1°$, and $F_1 = \tan 89°$. Then

$$\log F_2 = \log \tan 1° = 8.2419$$

$$\log F_1 = \log \tan 89° = 11.7581$$

then

$$\log F_1 - \log F_2 = 11.7581 - 8.2419 = 3.5162$$

and

$$L = M\ 3.5162$$

or

$$M = \frac{L}{3.5162}$$

Substitute 12 for L:

$$M = 12 \div 3.5162 = 3.41+$$

As an engineer's scale is used in drafting the scales and can be conveniently used only for moduli of 1, 1⅔, 2, 2.5, 3⅓, or 5 inches or these lengths multiplied by 10, we choose a modulus of 3⅓ inches, which is nearest to the inconvenient irrational decimal 3.41+. Substitute 3⅓ for M and we find L = 3⅓ × 3.5162 = 11.72 inches.

ACCURACY AND ADVANTAGES.

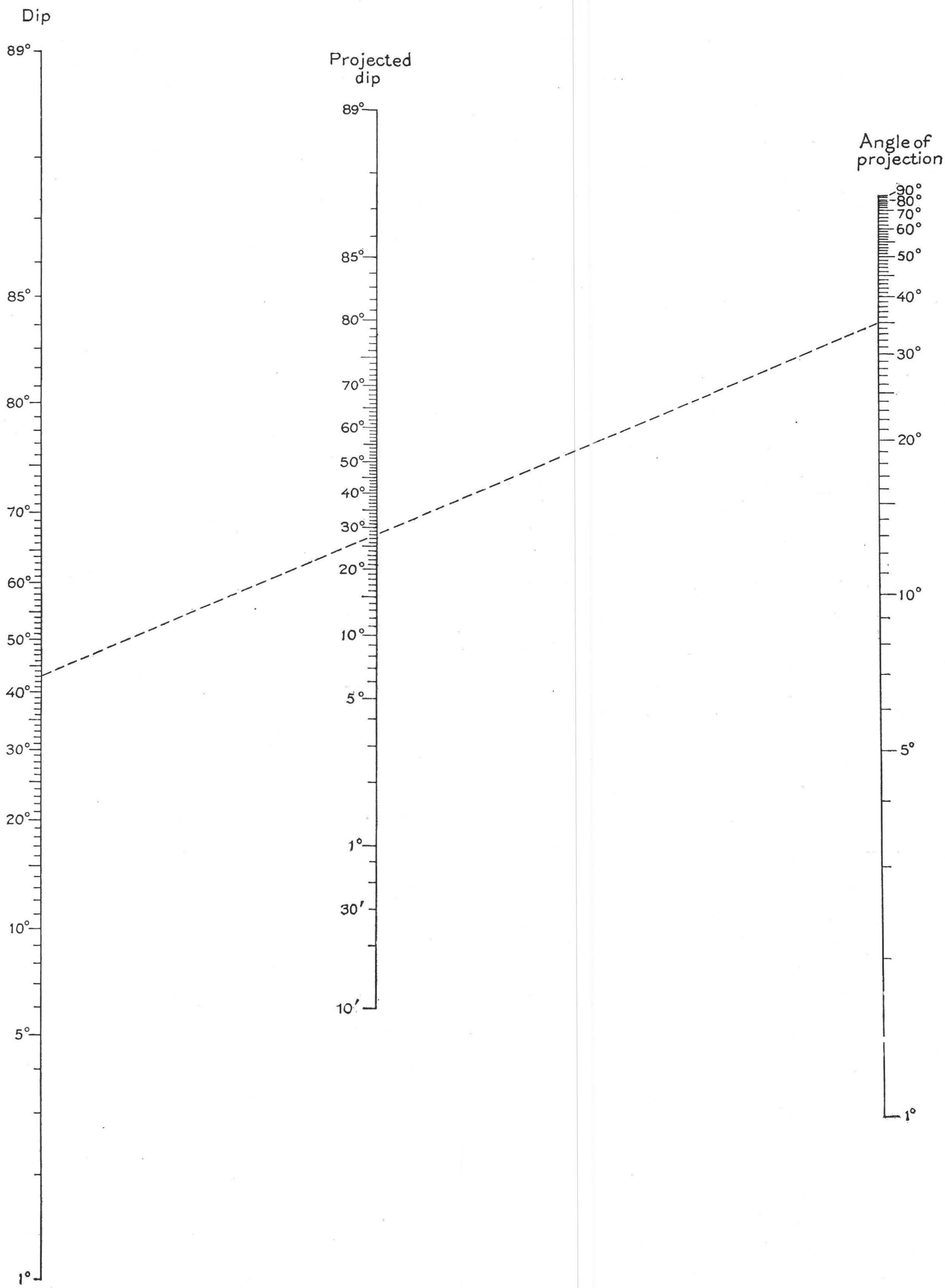
The point may be raised that the calculations made by these charts are not accurate, and in the strictest sense this is undoubtedly true. But there are two facts that should be borne in mind: (1) Measurements made by pacing or by a hand compass or clinometer are not accurate; (2) the surfaces with which the geologist deals are not true planes. Measurements of angles are seldom accurate within 1°, which is approximately equivalent to an error of 1 per cent, and pacing under the most favorable conditions can not be trusted closer than to 1 per cent. The undulations of fault or bedding planes will usually introduce an error of the same or greater magnitude. Therefore it is unsound logic to demand that the error in the calculations based upon such measurements be less than 1 per cent. If the chart is used with reasonable care, the error is somewhat less than 1 per cent and is therefore negligible. Moreover, the use of the chart gives less opportunity for the introduction of gross errors, such as come from a mistake in multiplication or in the addition of logarithms.

The charts, therefore, have the following advantages over the other methods for solving these equations:

1. They are more speedy. As a test of the speed of this method the writer used the first draft of the three charts for a series of test problems. The time required to solve the problems by means of the charts and by logarithms in the ordinary way was taken with a stop watch. It was found that the time with the charts ranged from one-sixth to one-fourth of that required by the logarithmic method. The test was fair to the logarithms, as the writer is reasonably proficient in their use.

2. The chances of gross errors are eliminated. In the test mentioned in the preceding paragraph the correctness of the answers was checked, and a mean error of about 0.2 per cent was found, with a maximum of 0.7 per cent. One gross error of about 40 per cent was made in one of the logarithmic solutions and was first detected by the check given by the chart solution.

3. It is unnecessary to keep the formula in mind, as it is incorporated in the construction of the chart.



ALINEMENT DIAGRAM III, FOR COMPUTING PROJECTED DIP.