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GRAPHIC AND MECHANICAL COMPUTATION OF THICKNESS OF STRATA AND DISTANCE TO A STRATUM.

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INTRODUCTION.

Two problems that constantly confront the stratigraphic and structural geologist are the computation of the thickness of a geologic section and the computation of distance to a stratum from some designated point at the surface when the position of the outcrop of that stratum is known. The solution of each of these problems is divisible into three parts—a geometric solution, a trigonometric generalization, and simplified methods of computation. It is the purpose of the present paper to consider these three phases of each of the two problems above mentioned.

Analyses of these two problems, so essentially a part of the geologist's work, have doubtless been previously made, but it is odd that so little has been published on this subject, and particularly significant that most of the published material has been of recent origin. The obvious inference is that we are approaching a period in the development of geologic science when accurate data will be considered more and more essential to correct stratigraphic interpretation; and the recent interest shown in these and related problems is an index of the general appreciation of this fact by geologists. In other words, geology is changing progressively from a qualitative to a quantitative science, and older methods are giving way to newer ones more adapted to present needs.

The only fault that may be found with the material so far published on this subject lies in its incompleteness. In some of the published papers the writers have not worked out general formulas but have confined themselves to the consideration of special cases, the solution of which, though useful, is not of universal application. In other articles, in which universal solutions have been evolved, the treatment is not well balanced because the above-mentioned three phases of each of the two main problems have not been considered adequately. Thus, a geometric solution is of interest, but if that alone comes within the scope of the article its value will be impaired because no formula is deduced, and the geologist will have to repeat the solution for every individual set of data. The trigonometric solution is of much more value, but it will not be used by many workers because it requires mathematical computation. It is very desirable that graphic or mechanical methods be employed in the solution of all geologic formulas, first because in using such methods no knowledge of trigonometry is required, second because of the saving in time they permit, and third because the resulting solutions are well within the limits of accuracy imposed by the nature of geologic observations.

The principal publications known to the writer in which the problems of thickness of strata and depth to a stratum are considered are as follows:

Hayes, C. W., Handbook of field geology, 1909.

Roe, J. W., Application of descriptive geometry to mining problems: Am. Inst. Min. Eng. Trans., vol. 41, pp. 512–533, 1911.

Smith, W. S. T., Some graphic methods for the solution of geologic problems: Econ. Geology, vol. 9, No. 1, 1914.

Palmer, H. S., Nomographic solutions of certain stratigraphic measurements: Econ. Geology, vol. 11, No. 1, 1916.

Palmer, H. S., New graphic method for determining the depth and thickness of strata and projection of dip: U. S. Geol. Survey Prof. Paper 120, pp. 123–128, 1919.

In Hayes's Handbook trigonometric formulas are derived, but only that special case is considered where the field traverse is made perpendicular to the strike of the beds. Both Roe and Smith have made descriptive geometric solutions, but neither derives formulas therefrom. In his first article Palmer has derived the general formula for the calculation of thickness of strata and developed three-variable alinement charts for its graphic solution. In his second article he has developed three-variable alinement charts for the solution of both thickness and depth of beds, but only in the plane perpendicular to the strike of the formation. The present paper is devoted to four topics, as follows:

1. The graphic and numerical solution of the problem of thickness of strata and the construction of a five-variable alinement chart for the graphic solution of the general formula.

2. The graphic and numerical solution of the problem of distance to a stratum, and the construction of a five-variable alinement chart for the graphic solution of the general formula for depth to a stratum.

3. The construction of a chart for the graphic solution of a right triangle, to be used in conjunction with the two charts above mentioned.

4. The construction of a trigonometric computer for the graphic solution of all trigonometric formulas that may be used in geologic field work.

THICKNESS OF STRATA.

OUTLINE.

It is required to find the thickness of geologic strata lying between two known points, when the following data are given:

1. The horizontal and vertical location of two points, which may be considered the beginning and end points of a traverse.

2. The azimuth angle between the strike of the rocks and a line joining the two points.

3. The dip of the rocks.

In connection with No. 1, any two of the following measurements will suffice: (*a*) Angle of slope between the two stations, (*b*) difference in elevation between the two stations, (*c*) slope distance between the two stations, (*d*) horizontal distance between the two stations. Therefore four sets of data are given, and these together with the answer (thickness of strata) will necessarily produce a trigonometric equation of five variables.

GEOMETRIC CONSTRUCTION.

In the first publication by Palmer, previously mentioned, the general formula for this problem is derived. A different solution using descriptive geometry, from which the formula is derived, is here used. It is well known that two cases requiring this formula exist—(1) where the dip of the beds and the slope of the hillside are in opposite directions, which is the more usual condition; and (2) where the dip of the beds and the angle of slope of the hillside are in the same direction. The solution for the first of these cases is here given.

In figure 2, let AB be a horizontal reference plane which passes through the station S_1. Let s be the slope distance between the two stations S_1 and S_2 (traversed distance), h the horizontal distance between the two stations, e the difference in elevation between the two stations, and σ the angle of slope of the hillside. Let α be the azimuth angle of the traverse, or angle between the direction of traverse and the strike of the formation, and let δ be the angle of dip of the rocks. It will be assumed that s and σ are given. By revolving the right triangle S_1S_2O from its vertical position downward 90° on OS as an axis into the plane of reference, e and h may also be measured.

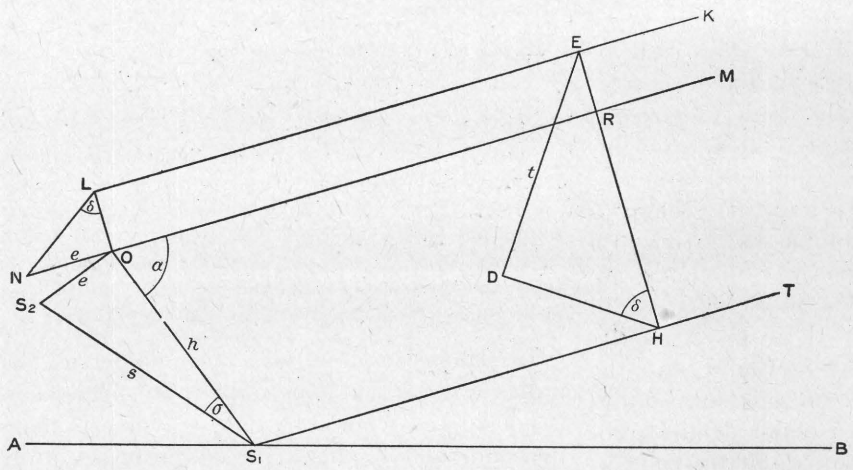


FIGURE 2.—Geometric representation of the thickness of a stratum when the dip of the stratum and the relative positions of a point on the upper surface of the stratum and another on the lower surface are given.

Let S_1T be the strike of the beds. It will also be the trace of the base of the stratum to be measured upon the reference plane. Through O, the horizontal projection of S_2 upon the reference plane, draw OM parallel to S_1T. Lay off $ON = OS_2 = e$. Then, e and δ being known, the right triangle NOL, which has been revolved downward 90° on OL as an axis into the plane of reference, may be measured, thus determining the distance OL. The line LK drawn parallel to S_1T is the trace of the top of the stratum upon the reference plane. Draw a line connecting and perpendicular to LK and S_1T. Such a line, EH, is the distance between the traces of the outcrops of the base and top of the stratum, upon the horizontal plane. When δ and EH are known the right triangle HDE may be revolved 90° upward on EH as an axis into the plane of reference and the thickness of the stratum (DE or t) may be measured.

TRIGONOMETRIC FORMULA.

The trigonometric solution from this construction is as follows:

$$t = (\mathrm{HR} + \mathrm{RE}) \sin \delta$$

$$\mathrm{HR} = h \sin \alpha \quad \text{and} \quad h = s \cos \sigma$$

$$\therefore \mathrm{HR} = s \sin \alpha \cos \sigma$$

$$\mathrm{RE} = \frac{e}{\tan \delta} \quad \text{and} \quad e = s \sin \sigma$$

$$\therefore \mathrm{RE} = \frac{s \sin \sigma}{\tan \delta}$$

Hence

$$t = \left(s \sin \alpha \cos \sigma + \frac{s \sin \sigma}{\tan \delta} \right) \sin \delta$$

$$t = s\ (\sin \alpha \sin \delta \cos \sigma + \cos \delta \sin \sigma)$$

When the dip of the beds and the slope of the hillside are in the same direction, the + in the formula changes to −. The general formula therefore is

$$t = s\ (\sin \alpha \sin \delta \cos \sigma \pm \cos \delta \sin \sigma) \quad \text{---} (1)$$

GRAPHIC REPRESENTATION OF THE FORMULA.

GENERAL PRINCIPLE.

Computations may be performed numerically, graphically, mechanically, or by a combination of methods. Numerical solutions of formulas require the use of logarithmic tables and are avoided when possible chiefly because they require too much time. It is highly desirable to represent formulas graphically or to compute them by some mechanical device based on a graphic representation of the functions involved. If some one formula is used a great deal, it should preferably be represented graphically, thus saving much time in computation and reducing greatly the liability of errors in the result. If a variety of formulas are being used, it will perhaps be found more convenient to perform the computations by means of some universal computing machine, such as a slide rule.

The formula for the thickness of a stratum, as above given, is one that may be used repeatedly in certain kinds of stratigraphic work or only occasionally in other kinds but certainly is of use to every stratigraphic geologist. There are several objections to the numerical computation of this formula. First, too much time is required; second, the use of figures introduces a greater liability to error than a graphic computation; and third, the accuracy of the answer, if five-place logarithmic tables are used, is much greater than the character of the original data justifies. The matter of needless accuracy is often overlooked by geologists, with the result that meaningless figures and incongruous results are sometimes published. In general it is true that in formulas used by geologists in the field and office some one of the variables will depend on an observation of the strike or dip of rocks. The answer to the formula should obviously be no more accurate than the least accurate of the component variables. For example, it is very doubtful whether determinations of strike or dip can be made with an error of less than 1°. But even if 1° represents the maximum probable error in careful work, it must be remembered that geologic strata do not by any means have mathematically perfect surfaces. Therefore an additional possibility of error is introduced in the liability of the measured direction of strike or dip of surface to change within a comparatively short distance, thus vitiating a most carefully made measurement. Extreme accuracy in computation of geologic formulas is therefore neither needful nor desirable, and graphic methods should be used.

The graphic representation of a formula is commonly accomplished by means of Cartesian coordinates, but this system has serious drawbacks when equations of more than two variables are to be plotted. When equations of three variables must be represented, a system of curves must be drawn and an awkward interpellation used. For equations of more than three variables Cartesian coordinates are not suitable. The equation

$$t = s\ (\sin \alpha \sin \delta \cos \sigma \pm \cos \delta \sin \sigma)$$

comprises five variables, namely, t, s, α, δ, and σ. Equations of three variables are most easily represented by means of a nomograph or alinement chart, which in reality is a system of plotting by means of parallel coordinates. Three excellent treatments of this method of graphic analysis have been written, by D'Ocagne,[1] Lipka,[2] and Peddle,[3] and the reader is referred to their publications for an understanding of the theory of the alinement chart.

In plotting the above formula Palmer [4] used their three-variable nomograms, thus necessarily solving the formula by several independent operations. Thus sin δ and cos σ were multiplied in one operation, and the product multiplied by sin α in a second operation. Cos δ and sin σ were multiplied in a third operation. The products of the second and third operations were then added numerically by a fourth operation, and this sum multiplied by s in a fifth operation, to solve for t. Three charts were required for these operations, one to multiply sines by cosines, a second to multiply numbers by sines, and a third to multiply numbers by numbers. Moreover, as the nomographic solution with parallel scales, which is the one employed, is essentially a method of addition and subtraction, and as all the above-mentioned operations that were performed graphically involve multiplication, all the calibrated scales were necessarily logarithmic scales. A serious drawback exists in the use of logarithmic scales, because the accuracy of the reading is greater at one end of the scale than at the other, and this weakness is specially pronounced in logarithmic scales of the trigonometric functions. By the method here used, the solution of the equation

$$t = s\ (\sin\alpha \sin\delta \cos\sigma \pm \cos\delta \sin\sigma)$$

is effected by a compound operation, in which a single chart and natural instead of logarithmic functions are employed.

The above equation, containing five variariables, can not be plotted directly by any method in two dimensions known to the writer, but by separating it into two parts and equating each of these to some auxiliary variable, the equation may be readily charted. Thus the equation may be written in two parts as follows:

$$t' = \frac{t}{s} \qquad (2)$$

$$t' = \sin\alpha \sin\delta \cos\sigma \pm \cos\delta \sin\sigma \qquad (3)$$

where t' is the introduced auxiliary variable. Equation (2) is a problem in division or, when written $t = t'\ s$, a problem in multiplication and therefore can not be plotted with natural scales if an alinement chart with parallel scales is used. By employing a nomographic Z chart, however, natural scales may be employed in multiplication and division, and this is the method which has been used.

Equation (3), however, is well adapted to graphic representation by an alinement chart with parallel scales, as the primary operation to be performed is addition or subtraction, as indicated by the symbol $\pm$. This equation, however, presents a difficulty in that it expresses a relationship between four variables—that is, t', α, δ, and σ. If one of these variables could be regarded as a constant, the equation would be reduced to a three-variable type. The obvious solution consists in assigning to one variable a series of fixed values and computing the resulting curves for each particular value. Two variables will be plotted on two parallel scales, and a third variable, whose position is partly determined by the fixed value assigned to the fourth variable, will be plotted between the two parallel scales. For each fixed value assigned to the fourth variable a different curve of the third variable will be developed, and the composite result will be a series of curves expressing the third variable in terms of the fourth. These curves may be joined together by a set of auxiliary curves, drawn through points of equal value of the third variable, and a gridwork of intersecting curves will be formed which will express graphically the true relationship between the third and fourth variables.

In the practical application of this method the variable t' is assigned to one of the outer parallel scales, in the plotting of both equations (2) and (3). The same scale modulus is used, and as both solutions involve only natural functions, the scale of t' for each solution is the same, and a common support for the scale t' may be used. Three parallel supports are therefore used to plot the variables α, t', and t.

[1] D'Ocagne, Maurice, Traité de nomographie, Paris, Gauthier-Villars, 1899.

[2] Lipka, Joseph, Graphical and mechanical computation, New York, 1918.

[3] Peddle, J. B., The construction of graphical charts, 2d ed., New York, 1919.

[4] Palmer, H. S., Nomographic solutions of certain stratigraphic measurements: Econ. Geology, vol. 11, 1916.

The variables δ and σ are expressed in a gridwork of curves lying between α and t, and the variable s is plotted upon a diagonal line connecting opposite ends of the t' and t scales. As no numerical value of t' is required, the support of the t' scale is not calibrated. Thus in the operation of the chart a point upon the α scale representing some value of α is connected by a straight line with a point which represents given values of δ and σ in the gridwork of curves and produced to meet the uncalibrated scale t'. The intersected point is then connected by another straight line with a point on the diagonal line representing some value of s and projected to the t scale, the reading on which shows directly the thickness of the stratum, vein, or formation.

MATHEMATICAL ANALYSIS.

EQUATION (2).

The equation $t=t's$ may be written as

$$f_1(u)=f_2(v)\cdot f_3(w)$$

where $t=u$, $t'=v$, and $s=w$. In figure 3, let t' and t be plotted upon two parallel straight-line scales, oppositely directed. The diagonal line joining the zero ends of these two scales will be the locus of the scale z and will be considered to have a length of k. Draw any nomographic index line joining the t' and t scales and intersecting the z scale. In the diagram,

$$y : x :: k-z : z$$

$$x=\frac{z}{k-z}\,y$$

The above equation is evidently in the form $f_1(u)=f_2(v)\cdot f_3 w$. Therefore, assigning scale moduli of m_1 and m_2 respectively to $f_1(u)$ and $f_2(v)$, we may say that

$$x=m_1 f_1(u) \text{ and } y=m_2 f_2(v)$$

As t' and therefore t and s must be plotted as natural functions, in order to be coordinate with the chart of equation (3), the Z type of alinement chart is used. The method of analysis is that used by Lipka.[5] Hence the equation becomes

$$m_1 f_1(u)=\frac{z}{k-z}\cdot m_2 f_2(v)$$

or

$$f_1(u)=\frac{m_2 z}{m_1(k-z)}\cdot f_2(v)$$

[5] Lipka, Joseph, Graphical and mechanical computation, pp. 65–66, New York, 1918.

Therefore

$$\frac{m_2 z}{m_1(k-z)}=f_3(w)$$

and from the solution of this equation it is found that

$$z=k\frac{m_1 f_3(w)}{m_1 f_3(w)+m_2} \qquad (4)$$

From equation (4), by substituting the specified moduli and values of $f_3(w)$, a series of values of z can be computed, which will represent the calibration of the diagonal scale, or scale of s.

EQUATION (3).

Consider the positive form of equation (3) that is to be plotted:

$$t'=\sin\alpha\sin\delta\cos\sigma+\cos\delta\sin\sigma$$

or

$$t'-\sin\alpha(\sin\delta\cos\sigma)=\cos\delta\sin\sigma$$

If some definite value is assigned to σ, so that $\cos\sigma$ and $\sin\sigma$ become temporarily constants, the equation may be written

$$f_1(u)-f_2(v).\,f_3(w)=f_4(w)$$

where $t'=u$, $\alpha=v$, and $\delta=w$. In this form we have an equation of three variables, one of

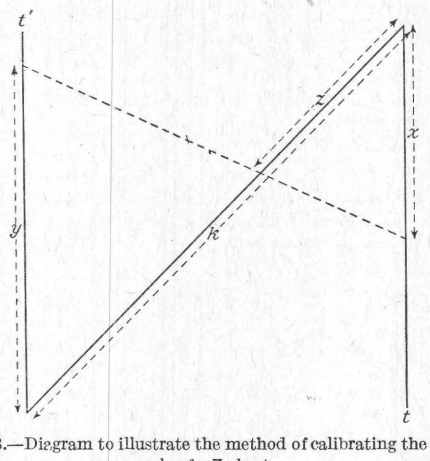


FIGURE 3.—Diagram to illustrate the method of calibrating the diagonal scale of a Z chart.

which (w) occurs on both sides of the equation as two different functions—that is, $f_3(w)$ and $f_4(w)$. Such an equation, when plotted as an alinement chart, will require two parallel straight-line scales and one curvilinear scale. The two parallel straight-line scales, representing the functions $f_1(u)$ and $f_2(v)$, may be drawn and calibrated in the ordinary manner used in building the simpler type of alinement chart, but the curvilinear scale representing the two functions of the variable w must either be projected graphically or computed by some system of coordinates. The latter procedure is here shown, the solution given by Lipka [6] being followed very closely.

In figure 4, let α and t' be plotted on two parallel straight lines, as shown; and let δ be represented by some hypothetical curvilinear line. Let the zero point of each of the two parallel scales be connected by a base line, whose length is k; and let the two outer scales be so placed that this base line lies perpendicular to both. In this way a system of rectangular Cartesian coordinates will be assured. Take for an origin of such a coordinate system the intersection of k with the t' scale. Draw any nomographic index line connecting the α and t' scales and cutting the δ scale. From the intersection of the index line with δ, draw a line parallel to k to meet the α scale and another parallel to the α scale to meet k. From the intersection of the index line with the t' scale, draw a line parallel to k. In the diagram,

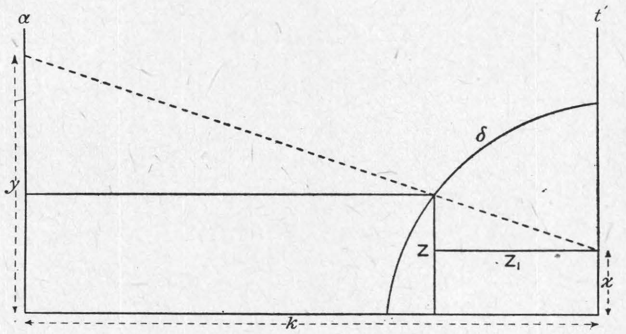


FIGURE 4.—Diagram to illustrate the method of determining the locus of the curvilinear scale in an alinement chart consisting of two parallel straight-line scales and a curvilinear scale.

$$y-z : z-x :: k-z_1 : z_1$$

$$z_1y-zz_1=kz-kx-zz_1+z_1x$$

$$kx-z_1x+z_1\,y=kz$$

$$(k-z_1)\;x-z_1y=kz$$

$$x+\frac{z_1}{k-z_1}y=\frac{kz}{k-z_1}$$

This equation is evidently in a form similar to the one to be plotted—that is,

$$f_1(u)-f_2(v)\,.\,f_3(w)=f_4(w)$$

Therefore, assigning scale moduli of m_1 and m_2 respectively to $f_1(u)$ and $f_2(v)$, we may say that

$$x=m_1f_1(u) \quad \text{and} \quad y=-m_2f_2(v)$$

[6] Lipka, Joseph, op. cit., pp. 106–107.

Then in order to satisfy the equation, it is necessary that

$$\frac{z_1}{k-z_1}=\frac{m_1f_3(w)}{m_2} \quad \text{and} \quad \frac{kz}{k-z_1}=m_1f_4(w)$$

Solving the first equation, we find that

$$z_1=\frac{km_1\,f_3\,(w)}{m_2+m_1f_3\,(w)} \text{-----------} (5)$$

And solving the second equation and substituting in it the value of z_1 frome quation (5), we find that

$$z=\frac{m_1m_2f_4\,(w)}{m_2+m_1\,f_3\,(w)} \text{-----------} (6)$$

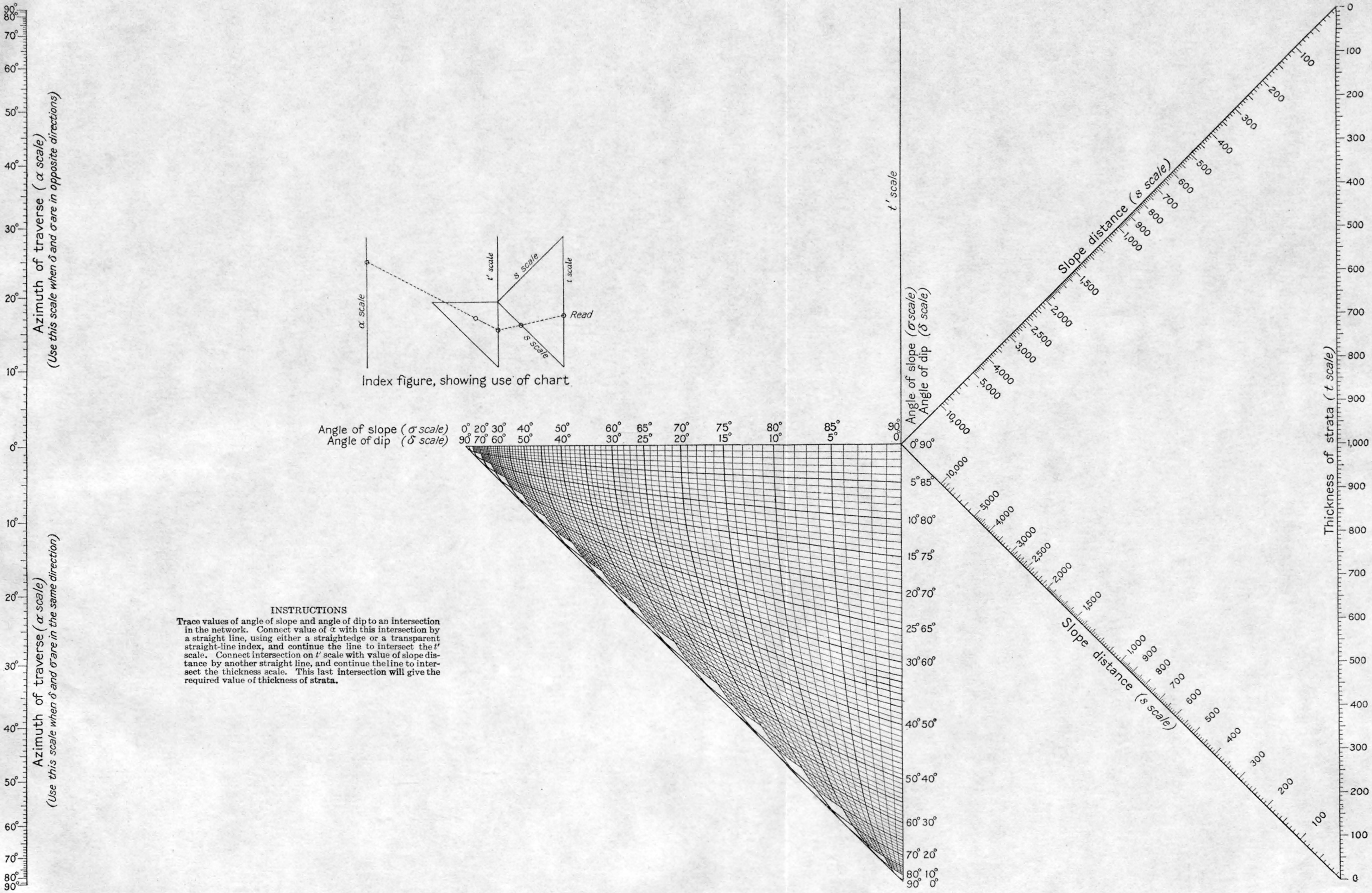
The values z and z_1 are the rectangular coordinates of any point on the curvilinear scale, representing a definite value of δ, measured from the intersection of k and the t' scale as an origin. The locus of the curvilinear scale δ can then be determined, for a series of assigned values of δ will give the coordinates of a series of points which may be joined together into a smooth curve.

To plot such a curve, however, a fixed value was assigned to the variable σ. Therefore for every assigned value of σ a new curve will result. In the preparation of the chart a series of such curves may be computed for a regular series of values of σ. If only a single curve were charted it would be calibrated in terms of δ, in a way similar to the parallel straight-line scales. But with a series of such curves the points on each curve that represent like values of δ are joined together, forming auxiliary intersecting curves that may be regarded as loci of definite values of δ. The original curves may then be regarded as loci of definite values of σ, and we shall have a series of intersecting curves representing the relationship between the variables δ and σ.

PREPARATION OF CHART.

The complete formula for the thickness of a stratum was charted by the methods here described. (See Pl. VI.) The details of the process have to do mainly with the selection of suitable scale moduli and the selection of such values for the variables s, α, δ, and σ that the resulting scales will have an adequate and balanced calibration.

Little need be said of the preparation of equation (2)—that is, $t=t's$. From the presence of the $\pm$ sign in the general formula it



INSTRUCTIONS

Trace values of angle of slope and angle of dip to an intersection in the network. Connect value of α with this intersection by a straight line, using either a straightedge or a transparent straight-line index, and continue the line to intersect the t' scale. Connect intersection on t' scale with value of slope distance by another straight line, and continue the line to intersect the thickness scale. This last intersection will give the required value of thickness of strata.

ALINEMENT CHART FOR GRAPHIC COMPUTATION OF THICKNESS OF STRATA.

results that both a positive and a negative scale for both t' and t are required. A scale modulus of 10 was adopted for the original drawing, calculation of the t scale being thus eliminated. The t' scale, though uncalibrated in the finished chart, was calibrated for purposes of projection in the actual work, and the calibration may be shown to be merely a natural sine scale. The scale modulus of 10 likewise eliminated calculation in the preparation of this scale as well as in the preparation of the α scale. The numbers 1, 2, 3, etc., might have been used on the t scale in keeping with slide-rule practice, instead of 100, 200, 300, etc. But as this chart is to be used solely to compute the thickness of geologic strata, it has seemed best to the writer to calibrate the scale in terms of the probable range of answers that will be obtained. The s scale has accordingly been numbered to accord with this convention.

In the plotting of equation (3)—that is, $t' = \sin \alpha \sin \delta \cos \sigma \pm \cos \delta \sin \sigma$—one point in particular requires explanation. For the positive form of the equation, only positive values of t' will result, but for the negative form of the equation both positive and negative values of t' will be obtained. Hence two nets of δ and σ curves would be required, one with positive and one with negative values; but only a single α scale, the positive one, would be necessary. To avoid drafting these two nets of δ and σ scales, both positive and negative α scales were drawn, and only one δ and σ network. In using the chart, therefore, the positive values of α are used for a solution of the normal or positive form of the general equation, and the negative values of α in solving the negative form of the general equation. This procedure is indicated on the chart.

As stated, the curves of δ and σ will ordinarily be calculated by some system of coordinates and joined together into smooth curves. In the case of this particular equation ($t' = \sin \alpha \sin \delta \cos \sigma \pm \cos \delta \sin \sigma$), however, the compensating form of the functions of δ and σ—that is, $\sin \delta \cos \sigma$ against $\cos \delta \sin \sigma$—results in a series of curves which are most easily prepared by a projective method. It is unnecessary to go into an analysis of the method, but a statement of the method used is given. It is stated above that a preliminary sine calibration was used on the t' scale. The positive and negative end points (90° positions) of the α scale being used as points of projection, two series of radiating lines were drawn to the points of sine calibration on the t' scale. The intersection of these two sets of radiating lines gave the loci of the required curves. Each of these curves is tangent to the base of the isosceles triangle that bounds the network, and each emerges to intersect both sides of this triangle. Each curve serves a double purpose, therefore—as a σ curve and as a curve of the complementary value of δ. As it is hard to trace several curves past a rather flat zone of tangency, the curves are doubly named, in order to avoid that necessity. Every curve cuts every other curve, and hence an intersecting point for values of δ and σ can always be found. Only one equivocal condition will be noticed, and that is where complementary values of δ and σ are given as field data. Under this condition, the same curve represents both values, and the point of tangency of the curve with the base of the isosceles triangle must be regarded as the point of intersection of a δ curve and a σ curve—that is, one limb of the curve will be regarded as a δ curve, and the other limb as a σ curve. The σ or angle of slope calibration was carried up to 90°, and this is open to criticism by field geologists, for hillsides of greater slope than 30° are rare. But the chart is also intended for measuring geologic sections in mines as well as in the open, and for this purpose the complete range from 0° to 90° for σ is required.

A small index chart showing five hypothetical points joined by a compound nomographic index line has been added as a guide to anyone using the chart.

USE OF CHART.

The use of the chart (Pl. VI) in obtaining the thickness of geologic strata is simple. At the left side of the chart is the α scale, on which are plotted the azimuth angles between the strike of the rocks and the line of traverse. This calibration comprises both a positive and a negative scale, the positive one starting at the middle of the line and extending upward and the negative one starting at the middle and extending downward. Use the upper scale where the angle of slope and angle of dip are in opposite directions and the lower scale where the angle of slope and angle of dip are in the same direction.

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Trace the two lines representing given values of angle of slope and angle of dip to an intersection in the δ–σ gridwork of curves. With a straight edge, or a transparent straight-line index, connect the point on the α scale with the δ–σ intersection, and the continuation of this line will give an intersection on the t' scale. Then connect the intersection on the t' scale with the point on the s scale which represents a given value of slope distance, and the continuation of this line gives an intersection on the t scale, which when read shows the thickness of the strata. It will be noticed that both the t' and t scales are divided into upper and lower parts, just as the α scale is. Also there are two s scales. When the first operation gives an intersection on the upper t' scale the second operation is performed likewise on the upper s and t scales; and conversely when the first operation gives an intersection on the lower t' scale the second operation is performed on the lower s and t scales.

The s and t scales are calibrated 100, 200, 300, etc., instead of 1, 2, 3, etc., because the answers will usually be of that magnitude. If desired, however, these calibrations may be regarded as 1, 2, 3, or 10, 20, 30, or 1,000, 2,000, 3,000, according to the use to which the chart is to be put, just as the ordinary slide-rule calibrations are used.

Another use to which the chart may be put, in addition to finding the thickness of strata, is the solution of equation (1) for any unknown quantity, if the other four are known. Thus, α, σ, s, and t may be known, and it is desired to find δ. A line connecting the t and s scales will intersect the t' scale. If this intersection is connected with the given point on the α scale, the resulting line will intersect the given σ line at a point which when read will show the required value of δ.

DISTANCE TO A STRATUM.

OUTLINE.

It is required to find the length of a tunnel, shaft, or drill hole from some selected point to some definite point on a stratum, when the following data are given:

1. The horizontal and vertical location of the starting point of the tunnel, shaft, or drill hole.
2. The horizontal and vertical location of a second point, which may lie anywhere on the surface of the stratum that is to be intersected.
3. The azimuth angle between the strike of the rocks and the line connecting these two stations.
4. The azimuth angle between the strike of the rocks and the direction of the tunnel, shaft, or drill hole.
5. The angle of dip of the rocks.
6. The angle of dip of the tunnel, shaft, or drill hole.

In connection with Nos. 1 and 2, which may be considered the beginning and end points of a traverse, any two of the following measurements will suffice: (*a*) Angle of slope between the two stations, (*b*) difference in elevation between the two stations, (*c*) slope distance between the two stations, (*d*) horizontal distance between the two stations. Therefore six sets of data are given, and these, together with the answer (the tunnel distance), will necessarily produce a trigonometric equation of seven variables.

This is the most general form of the problem of distance to a stratum. The problem usually considered by geologists, particularly in oil geology, and referred to as "depth to a stratum," is a special case of the more general problem, wherein the line joining the two points is vertical. In such a case the pitch is 90° and the line joining the two points has no horizontal azimuth angle. In other words, two variables are eliminated. The formula for the general problem will be developed, but for this paper only the formula for the special case—that is, depth to a stratum—will be charted.

GEOMETRIC CONSTRUCTION.

Let S_1 (fig. 5) be the starting point of the tunnel, shaft, or drill hole, and let S_2 be a point which is on the surface of the stratum that is to be intersected but is not in the horizontal plane through S_1. Let S_1 and S_2 be represented by their projections on the horizontal plane through S_1 and let S_2T be the strike of the stratum at S_2. The line S_1C, parallel to S_2T, is also the strike line, and AB is any reference line through S_1 in the horizontal plane. Also let h be the horizontal distance from S_1 to S_2, s the slope distance, e the difference of elevation, σ the vertical angle at S_1 between the horizontal plane and the station point S_2, and α the azimuth of the line joining S_1 and S_2 with reference to the strike line.

Pass a vertical plane through S_1 and S_2 and revolve this plane about the line joining the projections of S_1 and S_2 into the horizontal plane. The station S_2 will fall on S_2' and the right-angled triangle $S_1S_2S_2'$ will show in true proportions the quantities σ, s, h, and e. Pass a vertical auxiliary plane, perpendicular to the line of strike, through S_2. Its trace on the horizontal plane is the line P_1. Lay off S_2S_2'' equal to e and draw $S_2''L'$, making the angle $S_2L'S_2''$ equal to δ, the dip of the stratum at station S_2. On revolving the right-angled triangle $S_2S_2''L'$ 90° about the line S_2L' and then 90° about the vertical through S_2, the point L' will fall on L. Through L draw the line LM parallel to the strike line. The line LM is the line in which the dipping stratum intersects the horizontal plane through S_1, and the strike line S_2T is the line of intersection of the dipping plane with the plane through S_1C and the station S_2.

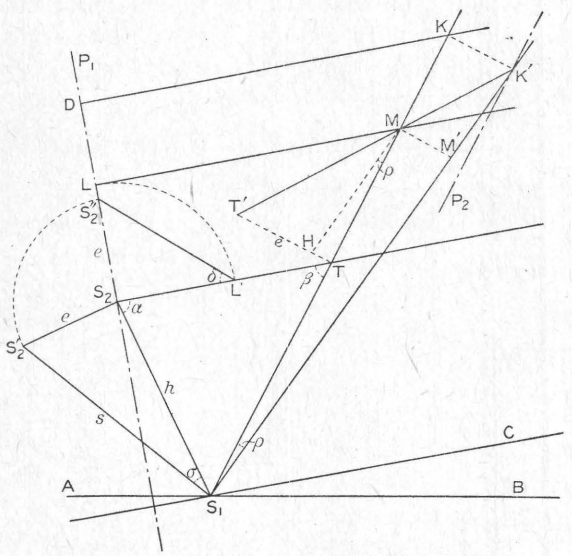


FIGURE 5.—Geometric representation of the distance to a stratum when the dip of the stratum, the position of a point on the stratum relative to the starting point of measurement, and the horizontal and vertical directions of the line of measurement are given.

Let S_1K be the projection of any sloping tunnel or drill hole which makes an angle ρ with the horizontal plane and has an azimuth β with reference to the strike line. Through S_1K pass a vertical plane. This plane will cut the strike line S_2T in point T and the line LM in point M. If we revolve this plane about S_1M into the horizontal plane, the point T, whose distance above the horizontal plane is e, will fall at T', M will be unmoved, and S_1K' will be the revolved position of the tunnel or drill hole. The angle ρ will be shown in true value. Draw the line $T'M$, cutting S_1K' in K'. The line $T'M$ is the revolved trace of the vertical auxiliary plane through S_1M and the dipping stratum through S_2, the point K' is the revoved position of the point in which S_1K pierces the inclined stratum, and M' is the revolved position of the point in which the vertical through M cuts the tunnel or drill hole. If the auxiliary plane is revolved back to its original position the projection of K' on the horizontal plane will be found at K, and line DK, drawn parallel to the strike line, is the projection, on the horizontal plane through S_1 of the line in which the dip plane is cut by plane P_2, the horizontal plane through K.

The distance S_1K' is the length of the tunnel required and is to be derived in terms of the known slope distance s and angles σ, δ, and ρ.

TRIGONOMETRIC FORMULA.

From the figure:

$$S_1K' = \frac{S_1K}{\cos\rho} = (S_1T + TM + MK)\frac{1}{\cos\rho}$$

$$S_1T = \frac{h\sin\alpha}{\sin\beta} = \frac{s\sin\alpha\cos\sigma}{\sin\beta}$$

$$TM = \frac{S_2L}{\sin\beta} = \frac{e}{\tan\delta}\cdot\frac{1}{\sin\beta} = \frac{s\sin\sigma}{\sin\beta\tan\delta}$$

$$MK = M'K'\cos\rho = \frac{MM'\cdot HM}{T'H}\cos\rho$$

$$\left.\begin{aligned} \text{but } MM' &= S_1M\tan\rho = (S_1T + TM)\tan\rho \\ &= \left(\frac{s\sin\alpha\cos\sigma}{\sin\beta} + \frac{s\sin\sigma}{\sin\beta\tan\delta}\right)\tan\rho \\ &= \frac{s\tan\rho}{\sin\beta\tan\delta}(\sin\alpha\tan\delta\cos\sigma + \sin\sigma) \end{aligned}\right\} \text{- - } (a)$$

$$HM = \frac{TM}{\cos\rho} = \frac{s\sin\sigma}{\sin\beta\tan\delta\cos\rho} \text{- - - - - - - } (b)$$

$$\left.\begin{aligned} T'H &= e - HT = s\sin\sigma - TM\tan\rho \\ &= s\sin\sigma - \frac{s\sin\sigma}{\sin\beta\tan\delta}\tan\rho \\ &= \frac{s\sin\sigma}{\sin\beta\tan\delta}(\sin\beta\tan\delta - \tan\rho) \end{aligned}\right\} \text{- - - - } (c)$$

Then by substitution from equations (a), (b), and (c),

$$MK = \frac{s\tan\rho}{\sin\beta\tan\delta}\cdot\frac{\sin\alpha\tan\delta\cos\sigma + \sin\sigma}{\sin\beta\tan\delta - \tan\rho}$$

From the values of S_1T, TM, and MK just found

$$\left.\begin{aligned} S_1T + TM &= \frac{s\sin\alpha\cos\sigma}{\sin\beta} + \frac{s\sin\sigma}{\sin\beta\tan\delta} \\ &= \frac{s}{\sin\beta\tan\delta}(\sin\alpha\tan\delta\cos\sigma + \sin\sigma) \end{aligned}\right\}$$

$$MK = \frac{s\tan\rho}{\sin\beta\tan\delta}\cdot\frac{\sin\alpha\tan\delta\cos\sigma + \sin\sigma}{\sin\beta\tan\delta - \tan\rho}$$

Adding these two last equations and factoring, we get

$$S_1K = s\,(\sin\alpha\tan\delta\cos\sigma + \sin\sigma)\frac{1}{\sin\beta\tan\delta - \tan\rho}$$

and as

$$S_1K' = \frac{S_1K}{\cos\rho}$$

Therefore

$$\left.\begin{array}{l} S_1K' = s\,(\sin\alpha\tan\delta\cos\sigma + \sin\sigma) \\ \cdot\dfrac{1}{\cos\rho\,(\sin\beta\tan\delta - \tan\rho)} \end{array}\right\}$$

By means of a similar though simpler construction it may be shown that the formula for "depth to a stratum" is $d = s\,(\sin\alpha\tan\delta\cos\sigma + \sin\sigma)$, in which the term $\sin\sigma$ is positive or negative as is determined by the value of σ. It therefore appears that the expression

$$\frac{1}{\cos\rho\,(\sin\beta\tan\delta - \tan\rho)}$$

in the equation for the value of d is a factor which must be applied where the hole is other than vertical. If the hole or shaft is vertical this factor reduces to unity.

In the construction above given, the angles σ and δ have been drawn in opposite directions, and also the angles σ and ρ in opposite directions. It has been found that four different formulas can result by the use of other constructions, and the composite formula covering all cases is as follows:

$$\left.\begin{array}{l} d = s\,(\sin\alpha\tan\delta\cos\sigma \pm \sin\sigma) \\ \cdot\dfrac{1}{\cos\rho\,(\sin\beta\tan\delta \pm \tan\rho)} \end{array}\right\} \text{----} (7)$$

The following rules govern the use of this composite formula:

Use $+\sin\sigma$, when σ and δ are in opposite directions.

Use $-\sin\sigma$, when σ and δ are in the same direction.

Use $+\tan\rho$, when σ and ρ are in opposite directions.

Use $-\tan\rho$, when σ and ρ are in the same direction.

GRAPHIC REPRESENTATION OF THE FORMULA.

As stated before, only that part of formula (7) which relates to the measurement of "depth to a stratum" will be plotted here. The whole formula could be plotted by the same methods, but in this paper other methods will be given for its solution.

Consider the formula $d = s\,(\sin\alpha\cos\sigma\tan\delta \pm \sin\sigma)$. This may be split into two formulas, like equation (1), and plotted by the same methods. Inserting an auxiliary variable t', we have

$$t' = \frac{d}{s} \text{-----------------} (8)$$

$$t' = \sin\alpha\cos\sigma\tan\delta \pm \sin\sigma \text{------} (9)$$

These two equations have been plotted exactly as equation (1) was plotted, and the result is shown on Plate VII. The two charts, Plates VI and VII, are analogous in every respect, and no further explanation is required.

USE OF CHART.

The depth to a stratum is computed from the chart (Pl. VII) exactly as the thickness of strata is computed from Plate VI. All directions are identical.

GRAPHIC SOLUTION OF RIGHT TRIANGLE.

OUTLINE.

In the consideration of the two preceding problems of thickness of strata and distance

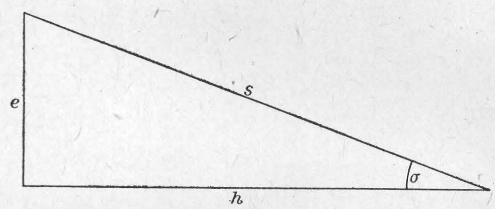


FIGURE 6.—Right-angled triangle showing the relations of slope distance, horizontal distance, difference of elevation, and vertical angle between two station points.

to a stratum, it has been assumed, in the right triangle determined by the two stations S_1 and S_2 and the plane of reference (see fig. 6) that the slope distance and angle of slope were given. It may be, however, that instead of s and σ any one of the five additional combinations is given as follows: σ and h, σ and e, s and h, s and e, or h and e. It is desired to derive graphically the values of s and σ when any of these combinations are given.

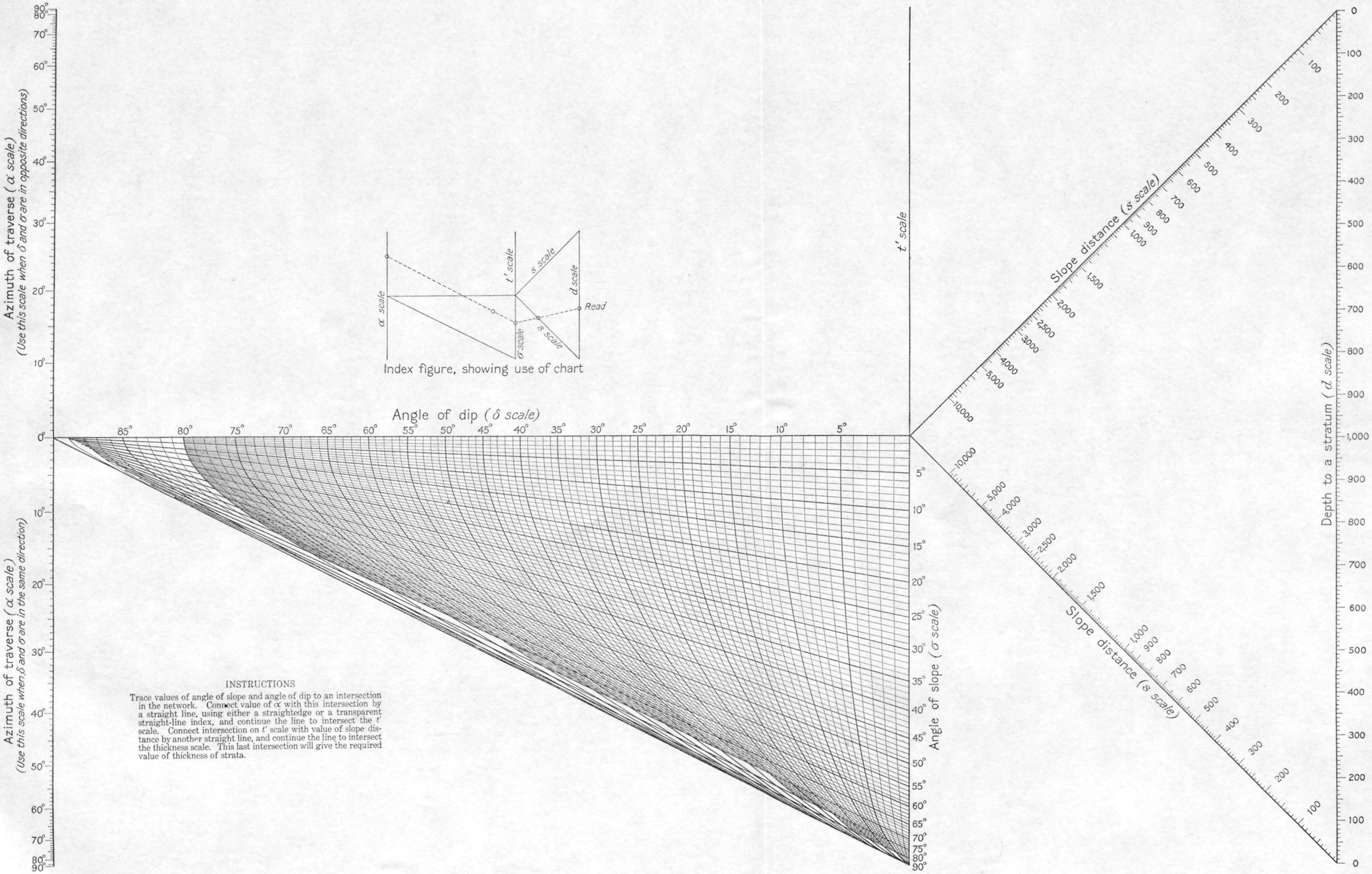
TRIGONOMETRIC FORMULAS.

It may be seen at a glance that the solution of this problem is essentially a graphic representation of the sine, cosine, and tangent conditions of a right triangle. The formulas involved are as follows:

$$e = s\sin\sigma \text{-------------} (10)$$

$$h = s\cos\sigma \text{-------------} (11)$$

$$e = h\tan\sigma \text{-------------} (12)$$



ALINEMENT CHART FOR GRAPHIC COMPUTATION OF DEPTH TO A STRATUM.

Each of these equations is in such form that it may be written $f_1(u)=f_2(v)\cdot f_3(w)$, and as shown before such equations are best plotted by means of the Z type of alinement chart.

PREPARATION OF CHART.

The method of plotting such equations has already been described, in connection with the

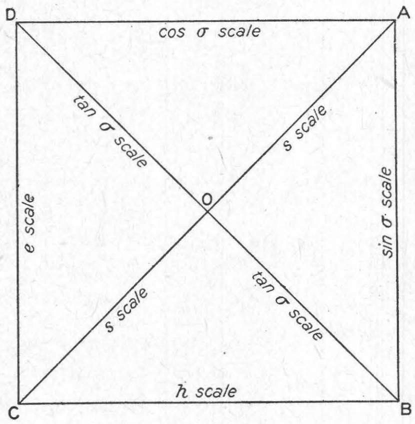


FIGURE 7.—Diagram illustrating the combination of three alinement charts for the solution of right-angled triangles.

plotting of equation (2). Two straight-line parallel scales, oppositely directed, are used to plot $f_1(u)$ and either of the other two variables, in this instance $f_3(w)$. Both of these are natural scales. A straight line joining the zero ends of these two scales carries the scale of the third variable, $f_2(v)$, and the calibrations are calculated by means of equation (4), as already explained.

In order to plot in one diagram all three of these equations, it is necessary to use one or more of the same scales in different solutions. Equations (10) and (11) have in common the variable s, wherefore it has seemed best to make a common scale of s, in the solution of these two formulas. In figure 7 the values of e are plotted on CD, the values of sin σ on AB, and the values of s on the diagonal joining the zero ends of these two scales. Two other lines, however, AD and CB, may be drawn to complete the square, and on these the values respectively of cos σ and h may be plotted. In this way a chart composed of five lines is provided for the plotting of e, h, s, sin σ, and cos σ.

The chart is now complete except for the solution of equation (12), which expresses the tangent condition. For this, the scales e and h, already plotted, are used in conjunction with a scale of tan σ which is plotted on the line BD. An index of two lines intersecting at right angles is used, such as EG and FH (figure 8). The method consists in passing the line EG through the given points on the e and h scales and then sliding the index along EG until the line FH passes through C. In this position the angle DCF$=\sigma$, and it is required to compute a calibration of tan σ on the line DB so that a reading indicated by the line HF on DB will give the required value of σ.

In figure 8

$$\mathrm{DC}:\mathrm{DS}:\sin(135^\circ-\sigma):\sin\sigma$$

$$\left.\begin{aligned}\mathrm{DS}&=\frac{\mathrm{DC}\sin\sigma}{\sin(135^\circ-\sigma)}=\frac{\mathrm{DC}\sin\sigma}{\sin(45^\circ+\sigma)}\\&=\frac{\mathrm{DC}}{\sin 45^\circ}\cdot\frac{\sin\sigma}{\sin\sigma+\cos\sigma}\end{aligned}\right\}$$

$$\mathrm{DS}=\mathrm{DB}\cdot\frac{\sin\sigma}{\sin\sigma+\cos\sigma}\text{------}(13)$$

Therefore the tan σ calibration, expressed in the diagram by successive values of DS, was

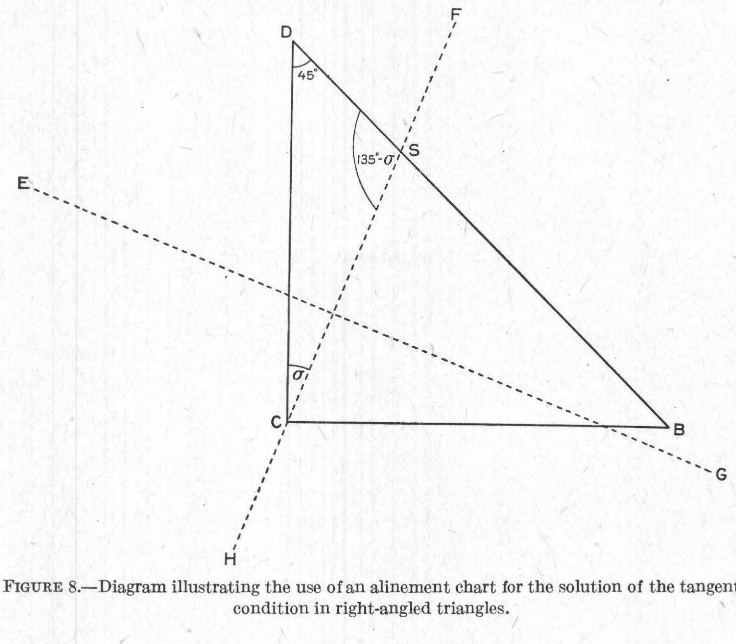


FIGURE 8.—Diagram illustrating the use of an alinement chart for the solution of the tangent condition in right-angled triangles.

computed by multiplying the length of the diagonal DB by the expression $\dfrac{\sin\sigma}{\sin\sigma+\cos\sigma}$.

The chart representing the complete solution of a right-angle triangle is shown in Plate VIII. It is of universal use in the graphic solution of right triangles, but it is here presented as an accessory chart, to be used in connection with the charts shown in Plates VI and VII. Three small index diagrams have been added as guides in the use of the chart. For the convenience of geologists, for whom this chart has been primarily constructed, the terms vertical distance, horizontal distance, slope distance, and angle of slope have been placed on the chart to prevent ambiguity in its use. Also as in the preceding charts, the calibrations are given as 100, 200, 300, etc., instead of 1, 2, 3, etc., for reasons stated on page 45.

USE OF CHART.

A straight edge or, better still, a piece of transparent or semifrosted celluloid with a black line ruled on the underside is required to solve equations (10) and (11). If, for example, σ and h are given, connect the σ value on AD (fig. 7) with the h value on BC, and the intersection on AC will give the value of s. Or if σ and s are given, connect the σ value on AD with the s value on AC, and the continuation of this line intersecting BC will give the value of h on BC. Similar solutions are used when e, s, and σ are involved. For equation (12) a piece of transparent or semifrosted celluloid will be required, on the underside of which are drawn two black lines intersecting at right angles. One of these lines is placed to pass through the given values of e and h, on CD and BC, respectively, and the other required to pass through the point C. Then the continuation of the line passing through C will show on DB the value of σ.

TRIGONOMETRIC COMPUTER.

OUTLINE.

Two good reasons exist for the use of a trigonometric computer. First, the geologist or surveyor will have numerous formulas to solve which, though essential, are not frequently used. It would be impracticable to have an alinement chart for every such formula, and it would be a laborious task to prepare so many such charts. Second, such charts, when reduced to a size which can be carried in the field, might not give sufficiently accurate results, particularly when the formulas are complex.

The alternative is some graphic computing device, which is accurate enough for general purposes and compact enough to be carried without difficulty in the field. The straight slide rule at once suggests itself as an instrument for this purpose, but it is open to two main objections—it is not of convenient shape to be easily and safely carried, and it is not easy to use for the solution of trigonometric formulas.

To fill this distinct want, the writer has designed a circular slide rule, which will not exceed five inches in diameter nor one twenty-fifth of an inch in thickness, which will be the equivalent in accuracy of a 12-inch straight slide rule, and which can easily be carried in a notebook, just as a protractor is carried. The principal practical advantages of this type of computer may be summarized thus:

1. It is compact and portable.
2. It enables all computations, including trigonometric computations, to be accomplished with the same ease and by exactly the same operations.
3. It possesses a continuous scale, so that it is never necessary to reset the instrument, as it is with the straight slide rule, because the answer may be off the scale.
4. Sufficient space is available through the use of concentric circles, or of a spiral, to plot the entire tangent scale, only half of which is plotted on the straight slide rule. This makes possible a direct setting to the tangents of angles between 45° and 90° and to the cotangents of angles between 0° and 45°, doing away with the necessity of computing these values from reciprocals, as in the straight slide rule.

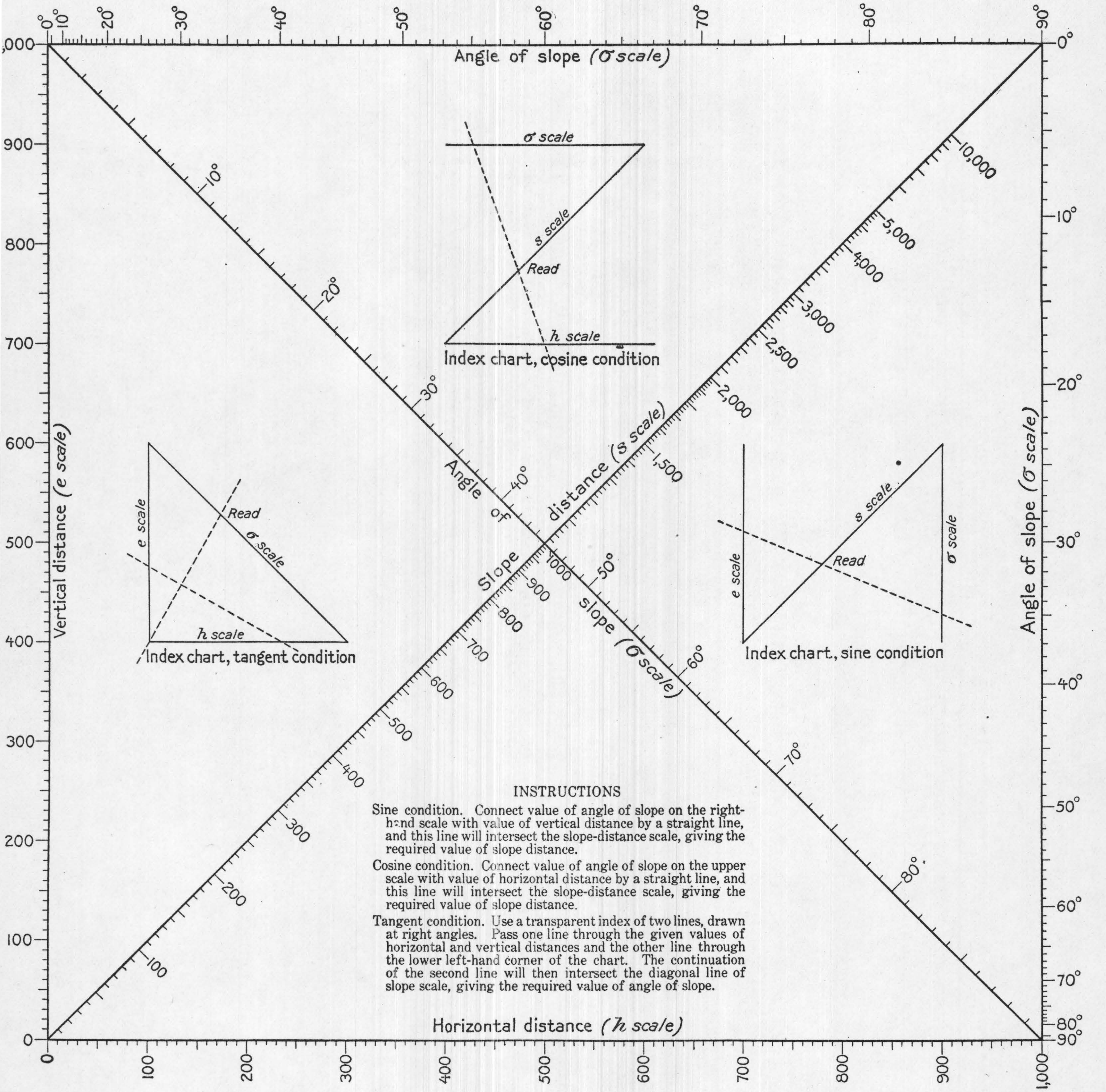
CONSTRUCTION OF COMPUTER.

A circular slide rule is constructed in exactly the same way as a straight slide rule, except that the calibration is computed and laid off in angular instead of linear magnitudes. In constructing a straight slide rule x inches long, for multiplication and division, which is to range from a scalar value of y at one end to a scalar value of z at the other end, the scale modulus (M) is expressed as follows:

$$M=\frac{x}{\log y-\log z}$$

The calibration is computed by multiplying the logarithms of each scalar value that will appear on the scale by M.

For a circular or spiral slide rule, consider a circle of indeterminate diameter depending on



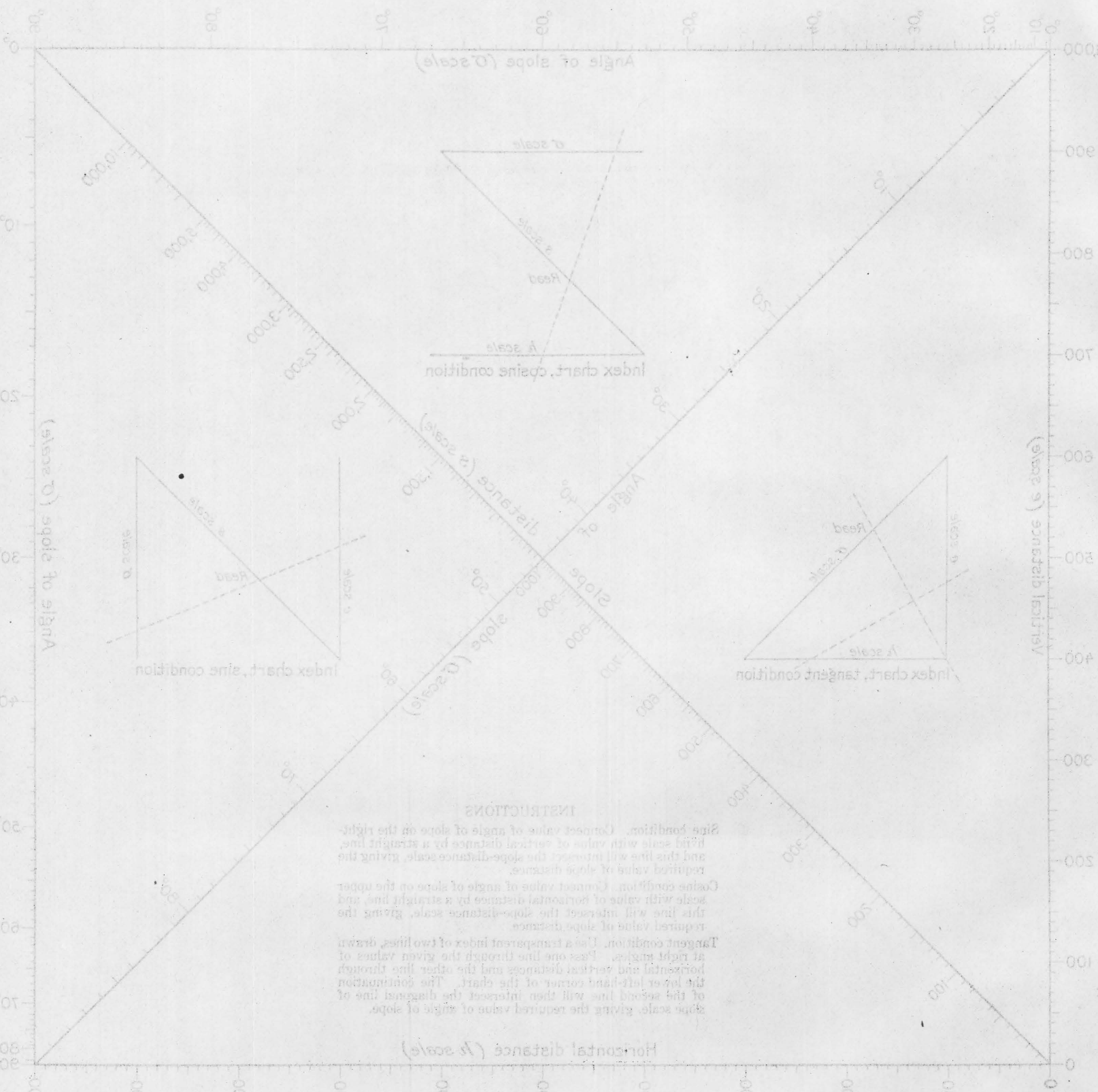
INSTRUCTIONS

Sine condition. Connect value of angle of slope on the right-hand scale with value of vertical distance by a straight line, and this line will intersect the slope-distance scale, giving the required value of slope distance.

Cosine condition. Connect value of angle of slope on the upper scale with value of horizontal distance by a straight line, and this line will intersect the slope-distance scale, giving the required value of slope distance.

Tangent condition. Use a transparent index of two lines, drawn at right angles. Pass one line through the given values of horizontal and vertical distances and the other line through the lower left-hand corner of the chart. The continuation of the second line will then intersect the diagonal line of slope scale, giving the required value of angle of slope.

ALINEMENT CHART FOR THE GRAPHIC SOLUTION OF RIGHT TRIANGLES.



INSTRUCTIONS

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Tangent condition. Use a transparent index of two lines, drawn at right angles. Pass one line through the given values of horizontal and vertical distances and the other line through the lower left-hand corner of the chart. The continuation of the second line will then intersect the diagonal line of slope scale, giving the required value of angle of slope.

ALINEMENT CHART FOR THE GRAPHIC SOLUTION OF RIGHT TRIANGLES.

the size desired for the finished product. Angular magnitudes are to be plotted, and as a circle is measurable in degrees, the same formula applies if x is considered to be the angular extent of the scale. If several concentric circles or a spiral of several turns is used to plot some one function, the circular scale modulus is expressed thus:

$$M=\frac{t\,360}{\log y-\log z}$$

where t is the number of concentric circles or the numbers of turns in the spiral.

The circular slide rule here considered was computed with a circular scale modulus of 180 instead of 360. The logarithmic range from 1 to 10 is 1, but the logarithmic range from sin 90° to sin 0° 45′ is almost 2, and it therefore requires twice as long a scale to plot the desired range of sines as to plot the usual numerical scale. If the numerical scale is plotted to a whole turn (360°), the sine range will require two turns, and if an answer is to be read off in sines, it will be ambiguous, as the index will give two possible values. To avoid this result an angular range of 180° was used for the numerical scale, which places the entire sine scale in one turn. The usual numerical calibration therefore takes but half of one turn, and to prevent the index from yielding an answer in the uncalibrated half of the number scale, the numerical range was doubled—that is, to read from 0.01 to 1, or from 0.1 to 10, as desired. Such a scale therefore takes a whole turn.

The tangent scale, if plotted with the same angular range as the sine scale, requires twice as long a scale as the sine scale, and in order to obtain this range the tangent scale is plotted in a two-turn spiral, the same circular scale modulus being used as before. As a result, any answer that is read off in tangents will theoretically be ambiguous, as the index gives two values, but practically the ambiguity is of no consequence, for the two values given by the index are so widely different that the operator, if he knows roughly the magnitude of the required answer, will be able to choose without difficulty the proper one.

The calibration of this computer is shown in figure 9. The outer circle is the number scale; the next circle inward is the sine scale; and the tangent scale is placed inside the sine scale in a two-turn spiral. This disk is mounted to turn upon an underlying support which extends outward a quarter of an inch or more and is equipped with two overlying indexes, made of transparent celluloid, which are attached to the center of the disk. One of these indexes turns freely, but the other is attached at the outside to the underlying support and is therefore immovable. It is intended that the freely moving index may be attached temporarily to the underlying support by pressure with the thumb and one finger.

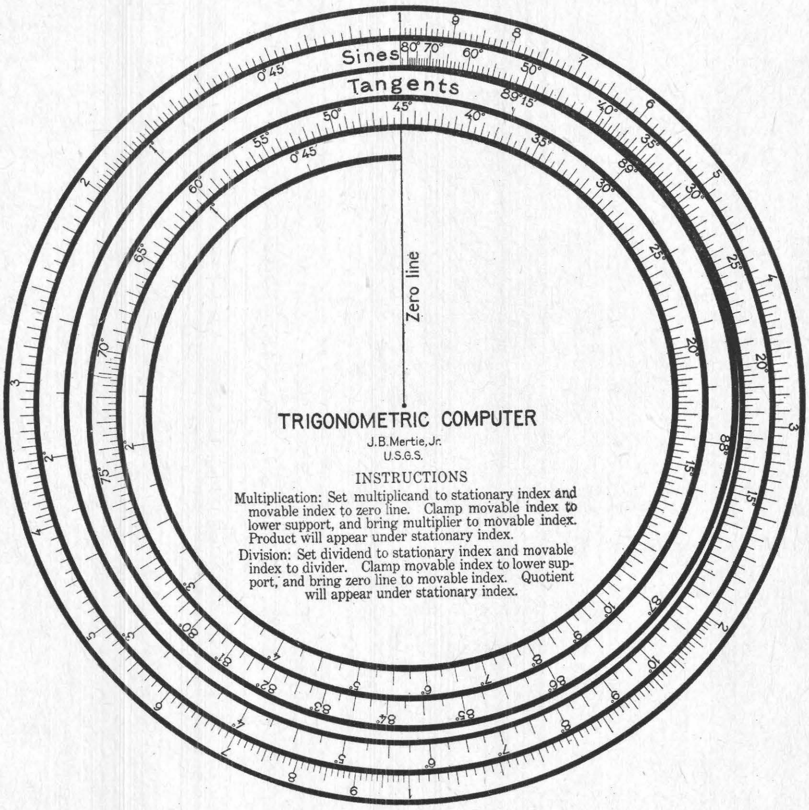

FIGURE 9.—Trigonometric computer for the solution of such problems as are readily solved with the 12-inch straight slide rule.

USE OF COMPUTER.

It is easy to remember how to manipulate this computer, because both in multiplication and division the start is made at the stationary index, and the answer is found at the same place. Thus, in multiplication the multiplicand is set under the stationary index; the movable index is set to the zero line of the scale; and then, the movable index being clamped to the underlying support with the thumb and finger, the multiplier is brought under the movable index; the product is then found under the stationary index. In division, the dividend is set under the stationary index, and the movable index is set to the divider and clamped; the zero of the scale is then brought to the movable index, and the quotient appears under the stationary index.

The computer also enables the operator to read natural sines and tangents to at least three digits, and by using complementary angular values he can read the natural cosines and cotangents. Secants and cosecants, though rarely used, may be obtained by taking the reciprocals respectively of cosines and sines.

As before stated, there is a twofold numerical range from 0.01 to 1 or from 0.1 to 10. In multiplying numbers by numbers, it is immaterial which of these scales is used; in fact, a multiplicand can be selected in one and a multiplier in another, and the product will be correct. In multiplying numbers by trigonometric functions, however, the true meanings of these two number scales must be utilized if the required answer is to be read as a trigonometric function. These two scales in reality represent any two number scales with a logarithmic range of 1, in which the calibrations of one are ten times the value of the calibrations in the other. This condition is not unique to this computer, being present in all duplex slide rules, but is mentioned here merely to prevent possible confusion in the use of the computer.

It is recommended that the computer be used in the field for all computations, thus saving the carrying of graphic charts or of a book of logarithm tables. The computer is in effect a graphic table of three-place logarithms arranged for general computations.