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Professional Paper 189—E

THE FORCE REQUIRED TO MOVE PARTICLES ON A STREAM BED

BY

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THE FORCE REQUIRED TO MOVE PARTICLES ON A STREAM BED

By William W. Rubey

ABSTRACT

The movement of particles on a stream bed has been explained in several ways—by the impact or momentum of water against the particle, by frictional drag upon its surface, and by differences in pressure at its top and bottom caused by the gradient of velocities. The familiar textbook law, that the weight of the largest particles moved by a stream varies as the sixth power of the velocity, is based upon the impact theory. Frictional drag is the basis of the so-called law of "critical tractive force", which river engineers prefer because it gives the maximum size of moving particles in terms of the readily measurable quantities, depth of water and slope of a stream. The "hydraulic lift", due to differences in pressure above and below, has the support of several physicists.

G. K. Gilbert's laboratory experiments afford data for a test of these rival theories. Properly qualified by using "bed" velocities near the particle instead of mean velocities of the entire stream, the "sixth-power law" seems valid for coarse sand and gravel but not for fine sand and silt, which require much higher velocities to start movement than are indicated by this law. The evidence suggests that the smaller particles are protected by a laminar film of low velocity; but further data are needed to show what forces finally cause movement of these smaller grains.

The equations based on laboratory data give reasonable estimates of the maximum size of pebbles moved by certain large natural streams.

INTRODUCTION

In the study of sedimentary rocks and of the geomorphic work of ancient streams it seems a matter of first importance to understand as thoroughly as possible the principles that govern the erosion and transportation of rock materials by moving water. Some of these principles apparently are understood sufficiently for purposes of geologic interpretation; others, equally fundamental, are subject to considerable differences of opinion. Several alternative theories of the force required to move particles on a stream bed have long been in the literature, and each has its adherents today. It seems worth while to compare some of these theories with observational data. The more complex and difficult problem of the bottom currents that move coarse sand and keep rock surfaces bare at places on the sea floor, which has recently attracted attention, will probably not be solved till after that of movement on stream beds is better understood.

The so-called "sixth-power law",[1] announced more than a century ago and since then rephrased in several forms, states that the weight or volume of the largest particles that can be moved by a stream varies as the sixth power of the stream velocity. In the development of geologic thought the formulation of this relationship has served effectively to focus attention upon the essential fact that the transporting and erosive power of a stream increases tremendously with increase of velocity. But beyond this, the classic "law" has been of very little practical use, because it refers not to the surface or the maximum or the average velocity of a stream, any one of which can be measured readily enough, but to the velocity very near a stream bed, and "bed" velocities are almost impossible to measure accurately.

An alternative theory that has been found useful for estimating the sizes of particles actually moved by running water is in wide use today by river engineers and laboratory workers. According to this alternative law of "critical tractive force", force d'entrainement, or current drag, as it is variously called,[2] the diameter of the largest particle moved by a stream varies as the depth of water times slope of stream.

A third theory is that the dislodgment of particles from a stream bed depends not upon the velocity nor the drag force but upon the lift induced by the velocity gradient or the rate of shear between adjacent fluid filaments.[3]

Actually the three theories are not so completely different as they may at first appear. Their predicted results are in rough qualitative agreement, because of the fact that the square of the mean velocity of a stream and also the velocity gradient near the stream bed tend to vary approximately as the depth times slope. Yet this is far from saying that the three theories are identical or can be used interchangeably to explain the familiar facts of stream behavior. The magnitude of

[1] Leslie, John, Elements of natural philosophy, pp. 426–427, 1829; cited by Gilbert, G. K., The transportation of debris by running water: U. S. Geol. Survey Prof. Paper 86, p. 16, 1914. Hopkins, William, On the transport of erratic blocks: Cambridge Philos. Soc. Trans., vol. 8, pp. 225–233, 1844. Shelford, W., On rivers flowing into tideless seas, illustrated by the River Tiber: Inst. Civil Eng. Proc., vol. 82, p. 25–26, (discussion by Wilfred Airy), 1885. Law, Henry, idem, pp. 29–31.

[2] Du Buat-Nançay, L. G., Principes d'hydraulique, vol. 1, pp. 98–105, vol. 2, pp. 91–98, Paris, 1786. Du Boys, P., Le Rhone et les rivières à lit affouillable: Annales des ponts et chaussées, 5th ser., vol. 18, pp. 150–155, 1879. Winkel, Richard, The limits of transferability of experimental results and model similarity in river hydraulic experiments, in Hydraulic laboratory practice, pp. 57–58, 1929. Marzolo, Francesco, Some considerations regarding hydraulic models: Idem, p. 755. Kramer, Hans, Modellgeschiebe und Schleppkraft: Mitt. preuss. Versuch. Wasserbau und Schiffbau, Heft 9, pp. 33–38, 1932; Sand mixtures and sand movement in fluvial models: Am. Soc. Civil Eng. Trans., vol. 100, pp. 798–838 (with discussion, pp. 839–878), 1935.

[3] Hooker, E. H., The suspension of solids in flowing water: Am. Soc. Civil Eng. Trans., vol. 36, pp. 246–251, 264, 1896. Jeffreys, Harold, On the transport of sediment by streams: Cambridge Philos. Soc. Proc., vol. 25, pp. 272–276, 1929. Havelock, T. H., The vertical force on a cylinder submerged in a uniform stream: Royal Soc. London Proc., ser. A, vol. 122, pp. 387–393, 1929. Richardson, E. G., The transport of silt by a stream: Philos. Mag., ser. 7, vol. 17, pp. 769–783, 1934. Hjulström, Filip, Studies of the morphological activity of rivers as illustrated by the River Fyris: Geol. Inst. Upsala Bull., vol. 25, pp. 267–270, 1935.

the forces basic to the three theories differs greatly, and so also does their degree of dependence upon other factors in stream regimen that are known to vary widely. Hence a series of observations that extends over a wide range of velocities, depths, slopes, and grain sizes may reasonably be expected to indicate the conditions under which certain theories fit the facts better than others.

Ideally, in order to afford a thoroughly satisfactory test, a series of observations on competent velocity or "critical tractive force" should be sufficiently comprehensive to represent adequately the endless variety and complexity of natural streams. Practically, however, that is out of the question, because data of such comprehensiveness are not now available and may never be. Numerous observations have been made, either in the laboratory or under essentially laboratory conditions, of the competent velocity or "critical tractive force" required to move particles of different sizes; but very few quantitative data of this sort have been gathered from natural streams. Still, if the objective is an analysis or the better understanding of some particular process rather than a synthesis of the combined effects of many different processes, the data from the laboratory may be more useful than data from natural streams, because under controlled conditions some at least of the many extraneous and confusing factors of nature are thereby eliminated or held constant. The behavior in natural streams, of course, remains the final criterion by which conclusions founded on the relatively simple conditions of the laboratory must ultimately be tested.

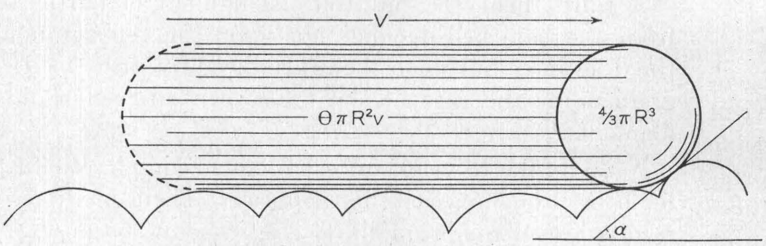

FIGURE 16.—Impact of moving water against particle on a stream bed. Schematic diagram to illustrate volumes of particle and of water cylinder which causes movement. (See text.)

Of the available data, the results of the comprehensive laboratory investigations by G. K. Gilbert [4] seem particularly worthy of critical study. Gilbert's data apply to very closely sorted sands and gravels, thus simplifying interpretation and facilitating direct comparisons, which are exceedingly difficult for poorly sorted materials. Furthermore, the sizes used by him ranged all the way from diameters greater than 5 millimeters down into the critical field of diameters less than ⅛ millimeter. It is true that Gilbert's experiments were chiefly measurements of stream capacity, or the quantity of debris transported, but numerous measurements were also made of stream competence, or the size of debris moved. Furthermore, his experiments on competence would have been much more significant had the investigation included still smaller debris. Nevertheless, no other set of data known to me compares with Gilbert's as a basis for at least preliminary testing of some of the conflicting theories of the force that moves particles on a stream bed.

The results of this inquiry have not been as clear-cut as was hoped, yet the need for certain additional data stands out distinctly, and some conclusions of general interest are definitely indicated. If this analysis of the problem should stimulate or provoke additional experimentation or a more critical examination of the available data, it will have accomplished one of its principal purposes.

The substance of this paper was presented orally before the Geological Society of Washington [5] April 10, 1935. As a result of generous discussion at that time and suggestions from many of my colleagues of the United States Geological Survey since then, the paper has been modified materially and expanded in its scope. I am particularly grateful to James Gilluly, W. H. Bradley, P. G. Nutting, R. W. Davenport, and C. H. Pierce for their helpful criticism.

PHYSICAL BASIS OF ALTERNATIVE THEORIES

The "sixth-power law."—The essentials of the theory that underlies various statements of the familiar "sixth-power law" may be given rather briefly.

Movement of a particle on a stream bed starts when the component of the force of water tending to lift the particle up a slope angle, α, becomes equal to the oppositely directed component of weight of the particle. The dynamic pressure of the water or the moving force is taken as equal to the momentum or the mass times velocity of the column of water that strikes the particle in a unit of time. This mass of moving water in any unit of time is the product of its density (ρ_F), its velocity (v), and its area of cross section. The column of water that strikes a spherical particle has a cross-sectional area of $\theta\pi R^2$, where θ is an empirical coefficient discussed below and R is the radius of the particle. The force of the water is thus represented by $(\rho_F \cdot v \cdot \theta\pi R^2)\ v$ (see fig. 16), and the component of this force acting up slope angle α is $(\rho_F \cdot v \cdot \theta \cdot \pi R^2)\ v \cos \alpha$.

The weight or the mass times acceleration of a spherical particle immersed in fluid is $\frac{4}{3}\pi R^3(\rho_S - \rho_F)\ g$, where ρ_S is the density of the particle and g is the acceleration due to gravity. The component of this weight that must be overcome by the force of moving water is given approximately [6] by the sine of the angle α up which the particle must be lifted. When this angle is very small—that is, when the particle rests on a smooth flat surface—the force required to start movement is obviously very small.

[4] Gilbert, G. K., The transportation of debris by running water; U. S. Geol. Survey Prof. Paper 86, 1914.

[5] Rubey, W. W., The force required to move particles on a stream bed: Washington Acad. Sci. Jour., vol. 25, pp. 571–572 [abstract], 1935.

[6] Approximately because the general slope of the stream bed is relatively small and therefore neglected.

When the component of current force is less than the component of particle weight, there is no movement of the particle. When the component of current force exceeds the component of particle weight, the particle moves, and the dynamics of the problem is then complicated by the velocity acquired by the particle and by the moment of force which acts upon it. But when the component of current force exactly equals the component of particle weight—that is, when the particle balances on the verge of moving—the physics of the problem is much simpler. When these two forces are equal,

$$(\rho_F \cdot v \cdot \theta\pi R^2)v \cdot \cos\alpha = \tfrac{4}{3}\pi R^3(\rho_S - \rho_F)g \cdot \sin\alpha$$

or

$$(\rho_F \cdot v \cdot \theta\pi R^2)v = \tfrac{4}{3}\pi R^3(\rho_S - \rho_F)g \cdot \tan\alpha$$

so that

$$R = \tfrac{3}{4} \cdot \frac{\theta}{\tan\alpha} \cdot \frac{\rho_F}{\rho_S - \rho_F} \cdot \frac{v^2}{g} \text{ ------------ } (1)$$

If g is taken in c. g. s. units, then R is measured in cm and v in cm/sec.

The empirical coefficient θ requires some explanation. Its value depends partly upon the proportion of the particle that is exposed to the current and partly upon the proportion of the force of the directly impinging water that is actually expended on the particle. For example, if a sphere is shielded by adjacent particles so that only one-half of its cross-sectional area is exposed to the current and if only one-fourth of the force of the impinging current [7] is expended on the sphere, then $\theta = \tfrac{1}{2} \times \tfrac{1}{4} = \tfrac{1}{8}$. A priori deduction will not give reliable estimates of the value of this coefficient,[8] but, for a given degree of sorting and a given shape of particle, the coefficient probably remains statistically constant and thus can be determined empirically.

The angle α, up which the particle must be lifted, depends upon the shape and the relative size of adjacent particles and thus it also is probably statistically constant for a given degree of sorting and a given shape of particles.

For water of known density flowing over a sand mixture of given roundness, sorting, and density, θ, α, ρ_F, ρ_S, and g may all be taken as statistically constant. Hence it follows from equation 1 that, other things being equal, the radius of particles barely moved by a stream varies as the square of the current velocity. This becomes the familiar "sixth-power law" when both sides of the equation are cubed. Thus

$$R^3 \propto v^6$$

that is, the weight or volume of the largest particles moved varies as the sixth power of the velocity of the stream. On page 129 a sharper definition is attempted of just which particular velocity of the stream is meant. The coefficient $\left[\tfrac{3}{4} \cdot \frac{\theta}{\tan\alpha}\right]$, which depends partly upon the shapes and relative sizes of the different particles making up the stream bed and partly upon other factors, is discussed more fully on pages 134–135.

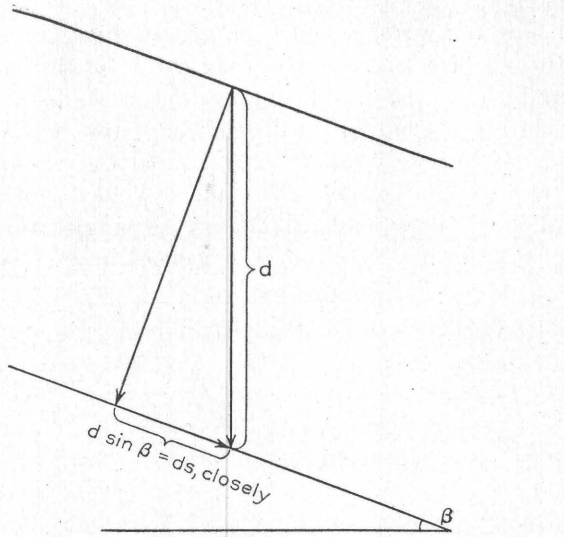

FIGURE 17.—In the "critical tractive force" theory, the drag on unit areas of a stream bed is taken as proportional to the depth-slope product. (See text.)

The "critical tractive force."—A column of water exerts a force or weight upon unit areas of a stream bed proportional to its density (ρ_F) and its depth (d). The down-slope component of this weight of water per unit area is taken to be the tractive force that tends to drag along downstream the particles that lie on a stream bed. Thus the unit tractive force is equal to $\rho_F \cdot g \cdot d \cdot \sin\beta = \rho_F \cdot g \cdot d \cdot \tan\beta \cdot \cos\beta = \rho_F \cdot g \cdot d \cdot s \cdot \cos\beta$, where β is the angle of the general stream slope and s is $\tan\beta$ or the fall per unit distance. (See fig. 17.) For slope angles less than about 10°, the term $\cos\beta$ is so nearly unity that, by comparison with the accuracy of other measurements, it may be neglected.

The surface area of a spherical particle on which this tractive force is exerted may be taken as $\psi\pi R^2$, where ψ is a coefficient that depends upon how nearly completely the particle is exposed. According to this theory, a particle will begin to move when the component of unit tractive force times the exposed area of the particle equals the component of particle weight:

$$\rho_F \cdot g \cdot d \cdot s \cdot \cos\alpha \cdot \psi\pi R^2 = \tfrac{4}{3}\pi R^3(\rho_S - \rho_F)g \cdot \sin\alpha$$

or

$$\rho_F \cdot g \cdot d \cdot s \cdot \psi\pi R^2 = \tfrac{4}{3}\pi R^3(\rho_S - \rho_F)g \cdot \tan\alpha$$

so that

$$R = \tfrac{3}{4} \cdot \frac{\psi}{\tan\alpha} \cdot \frac{\rho_F}{\rho_S - \rho_F} \cdot d \cdot s \text{ ---------------- } (2)$$

Inasmuch as ψ, α, ρ_F, and ρ_S may all be taken as statistically constant for a particular sand mixture, it follows that the radius of the largest particles moved varies as the product depth times slope.

Here, as in the "sixth-power law", the coefficient $\left[\tfrac{3}{4} \cdot \frac{\psi}{\tan\alpha}\right]$ depends upon the shapes and relative sizes of the different particles making up the stream bed; and therefore, for any given degree of roundness and of sorting, the term should have a characteristic value.

[7] For a discussion that indicates something of the uncertainties in this estimate, see Rubey, W. W., Settling velocities of gravel, sand, and silt particles: Am. Jour. Sci., 5th ser., vol. 25, pp. 327–329, 1933.

[8] Thoulet, M. J., Expériences relatives à la vitesse des courants d'eau ou d'air susceptibles de maintenir en suspension des grains minéraux: Annales des mines, 8th ser., vol. 5, pp. 523–524, pl. 16, fig. 14, 1884.

"Hydraulic lift" caused by velocity gradient.—It is well known that the velocity of flow is not the same at all points in a vertical cross section of a stream. The point of maximum velocity commonly lies some distance below the water surface, and from this point or points the velocity decreases toward the bed, the side walls, and the surface, the change per unit distance being relatively gradual until the bed and walls are approached closely, and there the velocity gradient becomes very steep (fig. 18). The mean velocity of a stream (defined more explicitly below) is simply the average of all the many different velocities in a cross section.

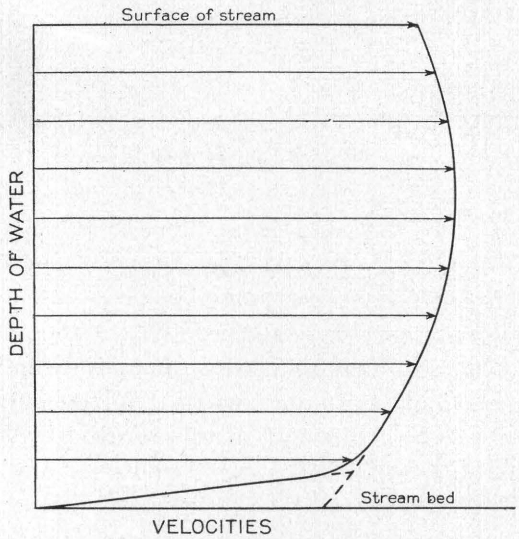

FIGURE 18.—Typical vertical velocity curve of a stream. Steep velocity gradient near stream bed and parabolic distribution of velocities above.

A particle resting on the bed of a stream lies in the zone where the velocity gradient is steepest. At the base of the particle the velocity is zero; at its top the velocity may be very high. Because of the diminution of pressure with increase of velocity, a pressure difference is set up which tends to lift the particle.[9] If this lifting force exceeds the particle weight, the particle is lifted off the stream bed and carried up to a position in the stream where the velocity gradient is gentler.

Several writers have attempted to evaluate the effects of this force on bodies of various geometric forms, but the problem is complex, and no simple solution directly applicable to debris particles appears yet to have been found.

GILBERT'S EXPERIMENTS ON COMPETENCE

Gilbert's apparatus and methods of laboratory procedure are fully described in Professional Paper 86.[10] The results of all his experiments on competence [11] are given in table 1, recalculated from feet to centimeters and accompanied by other pertinent data derived by computation from the fundamental observations. Computations were carried to the limit of reading on a 20-inch slide rule but are here given to only three significant figures. The various columns in the table require some explanation.

Grade of debris.—The materials used were moderately well-rounded river sand and fine gravel (mean density, 2.69) that had been closely sized by sieving. Photographs[12] of each size separate, or grade, show the degree of rounding. The maxima and minima radii given in table 1 are taken from the sieve openings recorded in unpublished notes of Gilbert and his assistant, E. C. Murphy, in the files of the United States Geological Survey. The mean radius given in the table is simply the geometric mean of the maximum and minimum radii. For well-sorted aggregates the geometric mean appears to be a simpler and more directly comparable average than the relatively complex number averages given by Gilbert,[13] which depend largely upon particle shapes and are greatly influenced by minor quantities of smaller particles.[14]

Discharge, width, and depth.—The quantity or volume of water passing through any cross section of a stream in a unit of time, such as 1 second, is called the discharge. In a trough of rectangular cross section, such as Gilbert used, the discharge equals the product, width times depth times mean velocity of flow ($Q=wdv_M$). The discharges from a constant-level tank were measured with a calibrated valve.

The depths given in table 1 were gage measurements. A few depths were also determined by the method of profiles,[15] but as the differences from gage measurements were slight they have been omitted here.

Slope of surface.—Most of the measurements of stream slope or gradient or fall per unit distance were made on the stream bed. For relatively steep slopes this measurement was sufficiently accurate; but for very gentle slopes it was found inaccurate, and consequently most of the gentler slopes were measured on the water surface instead, or on both water surface and bed surface. Where both slopes were measured, the two results have been averaged in preparing table 1. The surfaces measured in each experiment are indicated by the words "bed" and "water."

Hydraulic radius.—The hydraulic radius of a stream is defined as the area of cross section, wd, divided by the length of the wetted perimeter, P. In a wide, shallow stream in which the height of the channel side walls is negligible in comparison with the channel width, the hydraulic radius is essentially equal to the mean depth. In a trough of rectangular cross section the hydraulic radius $r=\frac{wd}{w+2d}$.

Mean velocity.—The values of mean velocity given in table 1 are calculated from the records of discharge,

[9] Jeffreys, Harold, op. cit., p. 272. Hjulström, Filip, op. cit., pp. 267–268.

[10] Gilbert, G. K., op. cit., pp. 17–26, 68–69.

[11] The much more numerous experiments on capacity are not given here.

[12] Gilbert, G. K., op. cit., pl. 2.

[13] Idem, p. 21.

[14] See Perrott, G. St. J., and Kinney, S. P., The meaning and measurement of average particle size: Am. Ceramic Soc. Jour., vol. 6, pp. 417–439, 1923, and Weigel, W. M., Size and character of grains of nonmetallic mineral fillers: U. S. Bur. Mines Tech. Paper 296, pp. 18–27, 1924.

[15] Gilbert, G. K., op. cit., pp. 25–26.

width, and depth by the defining equation $v_M=\frac{Q}{wd}$.

Turbulence criterion.—At low velocities and through small openings a fluid moves by a smooth gliding of filaments past one another that is called laminar, viscous, or stream-line flow. At higher velocities and through larger openings the motion of a fluid becomes irregular and distinctly eddying and is then called turbulent flow. Abundant investigation has shown that the transition from laminar to turbulent flow coincides with a change in the frictional resistance, which in laminar flow varies as the first power of the velocity and in turbulent flow varies essentially as the square of the velocity.[16] The flow through pipes and channels and past obstructions of various form changes rather abruptly from laminar to turbulent at some fairly definite value of the dimensionless Reynolds number $\frac{\rho_F v_M l}{\mu}$ (where ρ_F=density of the fluid; v_M=mean velocity; l=some linear dimension of the channel—for example, the pipe diameter; and μ=coefficient of viscosity). This critical value of the Reynolds number depends, of course, upon the particular linear dimension that is chosen; for similar geometric forms a corresponding dimension must be used.

The critical Reynolds number at which the flow changes from laminar to turbulent in open channels of rectangular cross section has been found [17] to be around 1,400 to 2,000, in which $l=r$, the hydraulic radius. This means that for water with a density of 1.0 and a viscosity coefficient of 0.01 (corresponding to a temperature of 20° C.), the flow would be laminar if the product, mean velocity times hydraulic radius, were less than about 14 to 20 and turbulent if this product exceeded 14 to 20.

It appears to be an open question whether or not sediment is ever transported during laminar flow. If debris transportation is caused solely by turbulence or "mass mixing",[18] then even a slight movement of debris would be evidence of turbulence, and, as some debris was moved during one of Gilbert's experiments in which the product $v_M r$ was only 9.0, the turbulence criterion should be less than 9 instead of 14 or 20. However, if other processes, such as rolling on the stream bed, saltation, or "hydraulic lift", are effective in debris movement,[19] it is possible that particles could be moved almost as readily by laminar flow as by turbulent flow. In fact, the "hydraulic lift" should be more effective in laminar flow, because the velocity gradient is steeper.[20] Until the possibility of transportation during laminar flow has been disproved by experimental studies, it appears unwise to accept transportation as an independent criterion of turbulence.

Whether or not transportation is possible during laminar flow, the flow of nearly all natural streams is turbulent; laminar flow of water in nature is thought to be virtually restricted to ground-water movement and the flow of thin surface sheets. For this reason, laboratory investigations of stream processes are usually designed to assure turbulent flow, either directly by observation of the movement of dyes or indirectly by recording for each experiment the Reynolds number or some comparable criterion of turbulence. Gilbert made no direct observations on turbulence, and, as he did not record temperatures of the water with which his experiments were made, exact values of Reynolds numbers cannot be calculated. However, it seems reasonable to take the product $v_M r$ as a turbulence criterion and to assume tentatively that a value between 14 and 20, say about 17, marks approximately the transition from laminar to turbulent flow. By this criterion the flow was turbulent in all but 6 of Gilbert's 105 experiments on competence.

"Bed" velocity, shearing stress at channel walls, and thickness of laminar film.—The significance of these three quantities and the methods by which they have here been estimated are discussed on pages 130, 132, and 133.

Movement of debris, symbol, and page reference.—The notes on movement of debris are quoted from Professional Paper 86.

The symbols assigned to each experiment in table 1 are intended to summarize the information on movement of debris. The hollow circle (○) signifies no movement of debris. The cross (×) indicates very few, few, or occasional grains moving or dunes forming. The solid circle (●) indicates some, several, or many grains moving.

The page reference gives the page in Professional Paper 86 from which the data for that experiment are taken.

[16] Stanton, T. E., Friction, pp. 24–47, London, 1923. Prandtl, L., in Ewald, P. P., Pöschl, T., and Prandtl, L., The physics of solids and fluids, pp. 269–283, 289–290, 297–302, 314–319, 1930. Dryden, H. L., Murnaghan, F. D., and Bateman, H., Hydrodynamics: Nat. Research Council Bull. 84, pp. 3–11, 20–24, 89–96, 335–339, 385–399, 1932.

[17] Allen, J., Stream-line and turbulent flow in open channels: Philos. Mag., 7th ser., vol. 17, pp. 1094–1096, 1934. Falkner, F. H., Studies of river-bed materials and their movement, with special reference to the lower Mississippi River: U. S. Waterways Exper. Sta. Paper 17, p. 53, 1935. See, however, Horton, R. E., Leach, H. R., and Van Vliet, R., Laminar sheet flow: Am. Geophys. Union Trans. 15th Ann. Meeting, pt. 2, pp. 393–404, 1934.

[18] Åkerblom, F., Recherches sur les courants les plus bas de l'atmosphère au-dessus de Paris: R. Soc. Sci. Upsaliensis, Nova Acta, ser. 4, vol. 2, no. 2, pp. 1–45, 1908. Taylor, G. I., Eddy motion in the atmosphere: Royal Soc. London Trans., ser. A, vol. 215, pp. 1–26, 1915. Schmidt, Wilhelm, Der Massenaustausch bei der ungeordneten Strömung in freier Luft und seine Folgen: K. Akad. Wiss. Wien, Math.-nat. Kl., Sitzungsber., pt. 2a, vol. 126, pp. 757–804, 1917; Der Massenaustausch in freier Luft und verwandte Erscheinungen: Probleme der kosmischen Physik, vol. 7, pp. 1–118, Hamburg, 1925. Leighly, J. B., Toward a theory of the morphologic significance of turbulence in the flow of water in streams: California Univ. Pubs. in Geography, vol. 6, pp. 1–22, 1932; Turbulence and the transportation of rock debris by streams: Geog. Rev., vol. 24, pp. 453–464, 1934. O'Brien, M. P., Review of the theory of turbulent flow and its relation to sediment transportation: Am. Geophys. Union Trans. 14th Ann. Meeting, pp. 487–491, 1933. Christiansen, J. E., Distribution of silt in open channels: Am. Geophys. Union Trans. 16th Ann. Meeting, pp. 478–485, 1935. Hjulström, Filip, op. cit., pp. 270–291.

[19] Gilbert, G. K., op. cit., pp. 26–34, 223–233. Richardson, E. G., op. cit., pp. 770, 777–779. Hjulström, Filip, op. cit., pp. 268–270.

[20] Prandtl, L., op. cit., p. 281. Hjulström, Filip, op. cit., p. 331.

TABLE 1.—*Gilbert's experiments on competence*

Grade of debris	Radius (cm): Minimum	Radius (cm): Maximum	Radius (cm): Mean	Discharge (Q) (cm³/sec)	Width (w) (cm)	Depth (d) (cm)	Slope of surface (s)		Hydraulic radius (r) (cm)	Mean velocity, (v_M) (cm/sec)	Turbulence criterion ($v_M r$)	Shearing stress at channel walls ($\rho F g r s$)	Calculated "bed" velocity at boundary of laminar film (v_B) (cm/sec)	Calculated average thickness of laminar film (δ) (cm)	Movement of debris	Symbol	Page reference, U. S. Geol. Survey Prof. Paper 86
B	0.0155	0.0200	0.0176	20,800	30.5	22.4	0.00012	Bed	9.07	30.5	276	1.07	13.7	0.128	Zero load collected. Dunes forming	×?	40
				10,300	30.5	14.9	.00029	Bed+water	7.54	22.6	170	2.12	13.9	.0655	A very few grains moving. Small dunes forming	×	69
				10,300	30.5	14.5	.00029	Bed+water	7.42	23.3	173	2.11	14.1	.0667	A very few grains moving. Some dunes forming	×	69
				10,300	30.5	14.5	.00037	Bed+water	7.42	23.3	173	2.69	15.0	.0556	Zero load collected. Dunes forming	×?	40
				10,300	30.5	13.0	.00045	Bed+water	7.00	26.0	182	3.09	16.3	.0528	Many grains moving. Dunes forming	●	69
C	.0200	.0290	.0241	10,300	30.5	13.7	.00055	Bed+water	7.22	24.5	177	3.89	19.3	.0496	A very few grains moving in a few places	×	69
				10,300	30.5	11.9	.00071	Bed+water	6.69	28.3	189	4.65	20.3	.0436	Many grains moving. Dunes forming	●	69
D	.0290	.0450	.0361	10,300	30.5	12.7	.00037	Water	6.94	26.5	184	2.51	15.2	.0604	Zero load collected. Dunes forming	×?	46
				10,300	30.5	11.9	.00045	Bed+water	6.70	28.2	189	2.95	16.3	.0552	A very few grains moving. No dunes forming	×	69
				10,300	30.5	11.4	.00077	Bed+water	6.50	29.7	193	4.91	18.9	.0386	Some small grains moving	●	69
				10,300	30.5	10.9	.00078	Bed+water	6.35	31.0	197	4.85	19.3	.0398	Several small grains moving. No dunes forming	●	69
				10,300	30.5	14.2	.00115	Bed+water	7.34	23.8	175	8.26	19.4	.0235	No grains moving	○	69
				10,300	30.5	12.7	.0018	Bed+water	6.94	26.5	184	12.2	22.6	.0184	An occasional grain moving	×	69
				20,800	30.5	22.6	.00175	Bed+water	9.11	30.1	274	15.6	25.9	.0166	Very few if any grains moving	×?	69
				20,800	30.5	20.1	.00295	Bed+water	8.66	34.0	208	25.0	30.9	.0123	Many grains moving. Dunes forming	●	69
E	.0450	.105	.0688	538	40.2	.427	.0100	Bed	.418	31.3	13.1	4.10	Laminar flow?		No grains moving	Not plotted	70
				538	40.2	.549	.0100	Bed	.534	24.4	13.0	5.23	Laminar flow?		do		70
				283	30.5	.457	.0200	Bed	.444	20.3	9.0	8.70	Laminar flow?		A few grains moving in a few places. Dunes forming		70
				283	20.1	.670	.0200	Bed	.628	21.0	13.2	12.3	Laminar flow?		A few grains moving in a few places		70
				538	40.2	.457	.0195	Bed	.447	29.3	13.1	8.54	Laminar flow?		Some grains moving; many moving as small channels are formed or surface becomes rough.		70
				538	30.5	.762	.0194	Bed	.726	23.2	16.8	13.8	Laminar flow?		Many grains moving. Dunes forming		70
				10,300	30.5	11.8	.000465	Bed+water	6.65	28.6	190	3.03	16.1	0.0531	No transportation	○	69
				1,100	30.5	1.61	.0050	Bed	1.46	22.5	32.8	7.15	16.4	.0230	No grains moving	○	70
				10,300	30.5	10.9	.00046	Bed+water	6.35	31.0	197	2.86	16.4	.0575	No transportation	○	69
				20,800	30.5	18.8	.00030	Bed+water	8.42	36.2	305	2.48	17.4	.0700	do	○	69
				10,300	30.5	10.9	.00065	Water	6.34	31.0	197	4.04	17.9	.0445	Zero load collected	○	48
				538	20.1	1.28	.0113	Bed	1.13	20.9	23.7	12.6	18.0	.0143	A few grains moving in a few places	×	70
				10,300	30.5	10.1	.00068	Bed+water	6.06	33.5	203	4.04	18.6	.0461	Very few grains moving in lower half of slope	×	69
				20,800	30.5	18.8	.0004	Water	8.42	36.2	305	3.30	18.7	.0565	Zero load collected. [Dunes]	×?	48
				20,800	30.5	17.1	.0007	Water	8.06	39.8	321	5.52	19.0	.0344	Trace collected. [Dunes]	●?	48
				1,640	30.5	1.83	.0050	Bed	1.63	29.5	48.2	8.00	19.5	.0243	A few grains moving in a few places	×	70
				1,100	40.2	.792	.0100	Bed	.762	34.7	26.4	7.47	19.9	.0267	Many grains moving	●	70
				1,100	40.2	.732	.0100	Bed	.706	37.5	26.5	6.92	20.3	.0293	Several grains moving. Dunes forming	●	70
				20,800	30.5	17.1	.00052	Bed+water	8.06	39.8	321	4.11	20.6	.0502	Some transportation in lower half of slope	●	69
				10,300	30.5	9.57	.00098	Bed+water	5.88	35.2	207	5.64	20.7	.0367	A few grains moving	×	69
				10,300	30.5	9.57	.0011	Water	5.88	35.2	207	6.34	21.3	.0337	Trace collected. Dunes forming	●?	48
				2,120	30.5	1.98	.0050	Bed	1.75	35.2	61.6	8.58	21.7	.0253	Several grains moving in several places	●	70
				538	20.1	.853	.0200	Bed	.786	31.3	24.6	15.4	22.8	.0148	Several grains moving in nearly all parts of trough	●	70
				2,630	30.5	2.13	.0050	Bed	1.87	40.5	75.8	9.17	23.8	.0259	Transportation	●	70
				1,100	20.1	1.52	.0113	Bed	1.32	36.0	47.7	14.7	24.8	.0169	Many grains moving. Surface becoming rough	●	70
F	[.135] [1]	.175	.154	2,120	20.1	2.16	.0105	Bed	1.78	48.8	86.9	18.3	29.7	.0162	A few grains moving in a few places	×	70
				1,100	20.1	1.31	.0251	Bed	1.16	42.0	48.6	28.5	30.0	.0105	An occasional grain moving. Several grains moving after a time.	×?	70
				2,630	20.1	2.80	.0105	Bed	2.19	46.7	102	22.6	30.9	.0137	Several grains moving in some places	●	70
				4,140	30.5	2.62	.0100	Bed	2.24	51.8	116	21.9	32.4	.0147	Few grains moving in some places	×	70
				4,650	30.5	2.83	.0100	Bed	2.39	53.8	128	23.4	33.7	.0144	Several grains moving	●	70
G	.175	.250	.209	10,300	30.5	7.80	.0031	Bed+water	5.16	43.2	223	15.7	28.0	.0179	No grains moving	○	69
				10,300	30.5	7.25	.00345	Bed+water	4.91	46.5	229	16.6	29.4	.0177	do	○	69
				10,300	30.5	6.86	.0048	Bed+water	4.73	49.2	233	22.2	32.4	.0146	A few grains moving at one place	×	69
				5,160	30.5	3.78	.0100	Bed	3.03	44.8	136	29.7	32.5	.0109	No grains moving	○	70
				1,640	20.1	1.58	.0250	Bed	1.37	51.6	70.5	33.5	34.4	.0103	do	○	70
				6,170	30.5	4.30	.0100	Bed	3.35	47.1	158	32.8	34.5	.0105	No (or very few) grains moving	○?	70
				2,120	20.1	2.13	.0205	Bed	1.76	49.5	87.2	35.4	34.7	.00980	No grains moving	○	70
				2,630	30.5	1.86	.0252	Bed	1.66	46.5	77.0	40.9	34.7	.00848	do	○	70
				3,140	30.5	1.80	.0190	Bed	1.61	57.4	92.4	30.0	35.6	.0119	do	○	70
				4,140	20.1	3.63	.0110	Bed	2.67	56.7	151	28.7	36.0	.0125	do	○	70
				7,700	40.2	3.51	.0105	Bed	2.99	54.6	163	30.7	36.2	.0118	do	○	70
				7,220	30.5	4.66	.0100	Bed	3.57	50.8	181	35.0	36.4	.0104	Very few grains moving	×	70
				2,630	20.1	2.68	.0205	Bed	2.12	48.8	103	42.5	36.4	.00856	do	×	70
				10,300	30.5	6.13	.00675	Bed+water	4.37	55.0	241	28.9	36.4	.0126	Very few if any grains moving	×?	69
				3,140	30.5	2.04	.0252	Bed	1.80	50.5	91.0	44.5	37.1	.00834	Very few grains moving in part of trough	×	70
				4,650	20.1	3.84	.0110	Bed	2.78	60.2	167	29.9	37.5	.0125	Very few grains moving	×	70
				4,140	30.5	2.44	.0190	Bed	2.10	55.7	117	39.1	38.1	.00972	do	×	70
				8,210	30.5	4.85	.0100	Bed	3.68	55.6	204	36.0	38.4	.0107	Many grains moving	●	70

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				6, 170	40. 2	2. 29	.0140	Bed	2. 05	67. 2	138	28. 1	38. 4	.0137	Very few grains moving near center of trough	×	70
				5, 160	20. 1	4. 24	.0110	Bed	2. 98	60. 5	180	32. 1	38. 5	.0120	A few grains moving near middle of trough	×	70
				5, 160	30. 5	2. 71	.0145	Bed	2. 30	62. 4	144	32. 7	38. 7	.0118	Very few small grains moving	×	70
				2, 120	20. 1	1. 65	.0250	Bed	1. 41	64. 2	90. 8	34. 6	38. 8	.0112	A few grains moving	×	70
				9, 260	40. 2	3. 81	.0105	Bed	3. 20	60. 4	193	32. 9	38. 8	.0118	Very few grains moving	×	70
				3, 630	30. 5	2. 26	.0252	Bed	1. 97	52. 8	104	48. 5	38. 9	.00802	A few grains moving except near head of trough	×	70
				4, 140	40. 2	1. 68	.0250	Bed	1. 55	61. 4	94. 9	37. 9	38. 9	.0103	Very few grains moving	×	70
				10, 300	30. 5	5. 61	.0080	Bed+water	4. 10	60. 2	247	32. 1	39. 0	.0121	An occasional grain moving	×	69
				6, 710	40. 2	2. 41	.0140	Bed	2. 15	69. 3	149	29. 5	39. 6	.0134	None moving except at middle of trough	×	70
				9, 770	40. 2	3. 90	.0105	Bed	3. 27	62. 3	203	33. 6	39. 7	.0118	A few grains moving	×	70
				3, 140	20. 1	2. 68	.0205	Bed	2. 12	58. 3	123	42. 5	39. 8	.00935	do	×	70
				20, 800	30. 5	11. 0	.0048	Bed+water	6. 38	62. 2	397	30. 0	39. 8	.0133	A few grains moving in places	×	69
				5, 660	20. 1	4. 36	.0110	Bed	3. 03	64. 6	196	32. 7	40. 0	.0122	A few grains moving	×	70
				2, 630	20. 1	2. 19	.0250	Bed	1. 80	59. 7	107	44. 1	40. 3	.00914	Many grains moving	●	70
				4, 650	40. 2	1. 86	.0250	Bed	1. 70	62. 1	106	41. 7	40. 4	.00968	Several grains moving	●	70
				7, 220	40. 2	2. 56	.0140	Bed	2. 27	70. 1	159	31. 2	40. 5	.0130	Very few moving except at middle of trough	×	70
				10, 300	40. 2	3. 96	.0105	Bed	3. 31	64. 5	214	34. 0	40. 5	.0119	Several grains moving	●	70
				6, 170	20. 1	4. 66	.0110	Bed	3. 18	65. 9	210	34. 3	40. 9	.0119	Several grains moving in lower half of trough	●	70
				7, 700	40. 2	2. 74	.0140	Bed	2. 41	69. 8	169	33. 1	41. 2	.0124	A few grains moving	×	70
				5, 160	30. 5	2. 74	.0190	Bed	2. 32	61. 7	143	43. 2	41. 3	.00955	Several grains moving	●	70
				7, 220	40. 2	2. 96	.0200	Bed	2. 58	60. 7	156	50. 5	41. 3	.00818	do	●	70
				5, 160	40. 2	2. 10	.0250	Bed	1. 90	60. 9	116	46. 6	41. 4	.00888	do	●	70
				6, 170	30. 5	2. 99	.0145	Bed	2. 50	67. 8	169	35. 5	41. 4	.0117	do	●	70
				6, 170	40. 2	2. 41	.0200	Bed	2. 15	63. 8	137	42. 1	41. 5	.00986	A few grains moving	×	70
				8, 210	40. 2	3. 02	.0140	Bed	2. 62	67. 6	177	36. 0	41. 6	.0116	Several moving in middle and lower parts; very few in upper.	●	70
				10, 300	30. 5	5. 27	.00965	Bed+water	3. 92	63. 9	251	37. 1	41. 6	.0112	Several grains moving in center half of slope	●	69
				6, 710	40. 2	2. 74	.0200	Bed	2. 41	60. 8	147	47. 3	42. 0	.00888	Several grains moving in middle part of trough	●	70
				4, 650	20. 1	3. 41	.0158	Bed	2. 55	67. 6	172	39. 5	42. 5	.0108	Some grains moving in lower half of trough	●	70
				4, 140	20. 1	3. 23	.0205	Bed	2. 44	63. 6	155	49. 1	43. 5	.00885	Several grains moving in middle of trough, many in lower part. Cutting of grade.	●	70
				7, 220	30. 5	3. 20	.0145	Bed	2. 64	74. 1	196	37. 6	44. 0	.0117	Many grains moving	●	70
				5, 160	20. 1	3. 17	.0158	Bed	2. 41	80. 8	195	37. 3	45. 7	.0122	Several grains moving in lower half of trough but very few in upper half.	●	70
				5, 660	20. 1	3. 17	.0158	Bed	2. 41	88. 8	214	37. 3	47. 9	.0128	Many grains moving in lower half but few in upper half	●	70
H	.250	[.370][1]	.304	5, 160	20. 1	4. 02	.0205	Bed	2. 87	63. 7	183	57. 7	44. 7	.00774	No grains moving	○	70
				7, 700	20. 1	4. 97	.0130	Bed	3. 33	77. 0	256	42. 4	45. 9	.0108	Occasionally one grain moving	×	70
				15, 400	20. 1	12. 2	.0110	Bed	5. 50	63. 1	347	59. 3	46. 4	.00782	Several grains moving in lower two-thirds of trough, and occasionally a grain in upper third.	●	70
				10, 300	20. 1	6. 46	.0110	Bed	3. 94	79. 0	311	42. 4	46. 9	.0111	Occasionally a grain moving	×	70
				8, 720	20. 1	5. 21	.0130	Bed	3. 43	83. 2	286	43. 8	48. 1	.0110	Some grains moving below middle; 1 to 3 grains in a cross section.	●	70
				6, 170	20. 1	4. 17	.0205	Bed	2. 95	73. 5	217	59. 2	48. 2	.00813	A few grains moving in lower three-fourths of trough	×	70
				12, 900	20. 1	7. 92	.0110	Bed	4. 43	80. 7	358	47. 8	49. 2	.0103	A few grains moving in lower half of trough	×	70
				9, 770	20. 1	5. 61	.0130	Bed	3. 60	86. 6	312	45. 9	49. 8	.0109	2 to 5 grains moving in a cross section	●	70
				7, 220	20. 1	4. 30	.0205	Bed	3. 01	83. 6	251	60. 4	51. 9	.00860	Many grains moving in lower three-fourths of trough	●	70
				18, 100	20. 1	10. 0	.0110	Bed	5. 02	89. 8	451	54. 1	53. 8	.00994	Several grains moving in middle third of trough; a few in upper and lower parts.	●	70

[1] Sieve openings not recorded in manuscript notes but plotted in unpublished graphs.

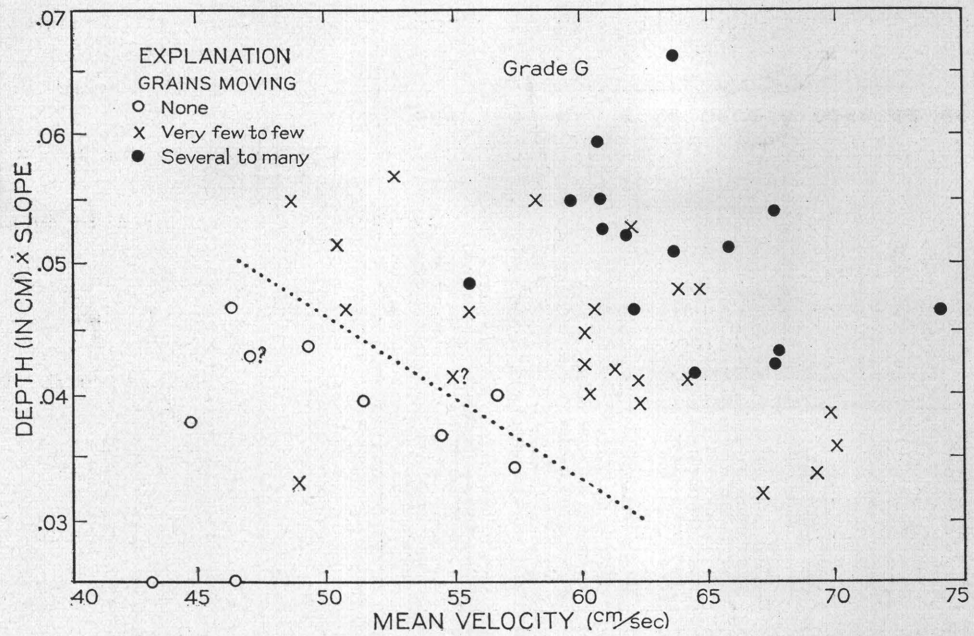

FIGURE 19.—Mean velocities, depth-slope products, and movement of debris of grade G. Oblique line separating points of no movement from those of slight movement indicates conditions at which movement of particles begins.

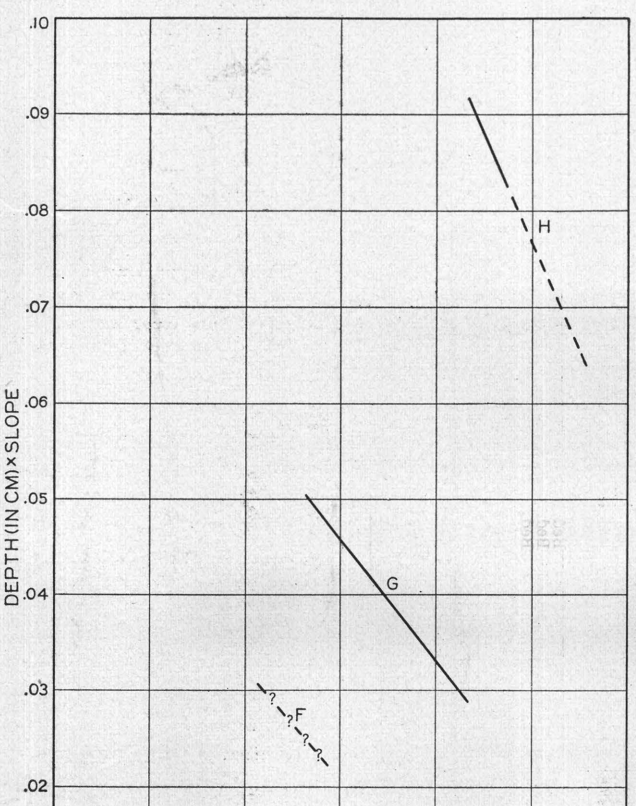

FIGURE 20.—Mean velocities and depth-slope products at which debris of grades B, D, E, F, G, and H begins to move on a stream bed. Oblique lines separate the experiments in which there was no movement of debris of the indicated grade size from experiments in which there was slight to much movement.

EFFECT OF MEAN VELOCITY AND DEPTH-SLOPE PRODUCT ON PARTICLE MOVEMENT

Probably the most direct way to illustrate the extent to which the "sixth-power" and the "critical tractive force" theories agree or fail to agree with observational data is to plot mean velocities against depth-slope products and to record thereon the observations on debris movement.

Figure 19 shows the data for grade G plotted in this manner. If the critical force required to start particle movement depends solely upon the mean velocity, the line separating the hollow circles (no movement) from the crosses and solid circles (slight and much movement) should be vertical. If the critical force depends solely upon the depth-slope product, the line should be horizontal. Actually the line is inclined, thus indicating that neither mean velocity nor depth-slope product alone accounts for the particle movement. At relatively low velocities the particles start moving if the depth-slope product is sufficiently great. At relatively high velocities the particles start moving at a much smaller depth-slope product.

Gilbert's observations on the other grades of debris were much less extensive than those on grade G. Nevertheless, the data are sufficient to show the approximate position and slope of the lines that separate the symbols indicating no movement from those indicating slight to much movement. These lines are plotted to the same scale in figure 20, the dashed and queried lines indicating those based on the fewest observations. Several facts are evident from this diagram. As would be expected, the larger particles require higher velocities or higher depth-slope products before movement starts. At a depth-slope product of 0.03, grade F particles (diameters 2.7 to 3.5 mm) start moving at a mean velocity of about 42 cm/sec.; whereas at the same depth-slope product grade G particles (diameters 3.5 to 5.0 mm) do not start moving until the mean velocity reaches 62 cm/sec. At a mean velocity of 48 cm/sec., grade F particles start moving at a depth-slope product of about 0.023, but at this same velocity grade G particles do not start moving until the depth-slope product reaches 0.048.

Another and perhaps more significant fact is shown clearly in figure 20. The lines that separate the symbols indicating no movement from those indicating slight to much movement slope more steeply as the size of particles increases. This means that the larger particles, such as grade H, start moving at a mean velocity that is relatively independent of the depth-slope product, and the smaller particles, grades B, D, and E, start moving at a depth-slope product which is relatively independent of the mean velocity. In other words, the "sixth-power law" appears to hold more nearly for gravel, and the theory of "critical tractive force" for finer sands.

The relationships brought out by figures 19 and 20 seem to show that neither the "sixth-power law" (at least as customarily stated in terms of an unspecified velocity) nor the theory of "critical tractive force" adequately fits the facts of observation. Some sort of compromise between the two theories—one that approximates the "sixth-power law" for larger particles and the theory of "critical tractive force" for smaller particles—would fit the observations better.

"BED" VELOCITY

The most obvious criticism that may be raised against the preceding test of alternative theories is that it compares particle movement with the mean velocity of the entire stream instead of with the velocity in the immediate vicinity of the particle on the stream bed. The criticism is well founded. Although many writers have assumed, some implicitly and others explicitly, that velocities near a stream bed are proportional to mean velocities of the entire stream, there appears to be no adequate basis for such an assumption. Other writers, recognizing this element of uncertainty, have sought to compare particle movement with velocities at the stream bed as determined either by actual measurement or by correction of surface or mean velocities.[21]

But this effort has been hampered by the difficulty of defining precisely what is meant by "bed" velocity. At the actual boundaries of a stream the velocity is zero. From the boundaries toward the center the velocity increases, very rapidly at first through a thin marginal layer and then more slowly toward the center (fig. 18). Just which particular velocity in such a continuous series of velocities is the one to be designated and measured as the "bed" velocity?

Investigation has shown that in the zone of turbulent flow that makes up the body of a stream the velocity gradient is relatively gentle and closely follows a parabola.[22] But in the thin marginal layer or film where the flow is dominantly laminar the velocity gradient is steep and approximately uniform.[23] At the boundary between these two zones there is a transitional rounding off of the velocity curve, which makes exact measurement difficult (fig. 18). Nevertheless, a distinct significance attaches to the marginal film of laminar flow with approximately uniform velocity gradient and to the velocity that marks the transitional boundary between this film and the interior zone of turbulent flow with parabolic distribution of velocities. Inasmuch as the average velocity at this boundary between the laminar and turbulent zones can be estimated [24] from other observational data on a stream, it is here tentatively adopted as the particular "bed" velocity which seems most useful for interpretations of particle movement.

A method of estimating this velocity at the boundary of the laminar film may be deduced from general considerations.

> A stream of water flowing down its bed expends an amount of energy that is measured by the quantity of water and the vertical distance through which it descends. If there were no friction of the water upon its channel, the velocity of the current would continually increase; but if, as is the usual case, there is no increase of velocity, then the whole of the energy is consumed in friction.[25]

In a unit of time, t, the energy generated by a stream in flowing downslope equals the mass of water, $\rho_F Qt$, times the acceleration due to gravity, g, times the vertical distance of fall, $v_M\ s \cos \beta\ t$. The energy spent in overcoming friction in the same unit of time is customarily [26] stated in terms of the mean velocity as the area of channel surface, Pv_Mt, times the mean frictional force per unit area, $k_M \rho_F v_M^2$, times the distance through which the total frictional force acts, v_Mt, where k_M is the coefficient of frictional resistance referred to the mean velocity and the other symbols have the same significance as before. For any short section of a channel where bends in the stream course and changes

[21] For compilations of earlier experimental results on competent mean and "bed" velocities see the following citations:

Penck, Albrecht, Morphologie der Erdoberfläche, vol. 1, pp. 283, 319–341, 1894. Grabau, A. W., Principles of stratigraphy, pp. 55–56, 247–251, 1913. Gilbert, G. K., op. cit., pp. 163–216, 1914. Schoklitsch, A., Über Schleppkraft und Geschiebebewegung, pp. 22–26, Leipzig and Berlin, Wilhelm Engelmann, 1914. Bucher, W. H., On ripples and related sedimentary surface forms and their paleogeographic interpretation: Am. Jour. Sci., 4th ser., vol. 47, p. 151, 1919. Twenhofel, W. H., and others, Treatise on sedimentation, pp. 31–32, 464, 1926. Fortier, S., and Scobey, F. C., Permissible canal velocities: Am. Soc. Civil Eng. Trans., vol. 89, pp. 942, 947, 951–953, 955, 1926. Hjulström, Filip, op. cit., p. 295, 1935.

[22] Stanton, T. E., Friction, pp. 28–31, London, 1923. Merriman, Mansfield, Treatise on hydraulics, 10th ed., pp. 321a–322, reprinted 1931.

[23] Prandtl, L., op. cit., pp. 275–276, 281–282. Dryden, H. L., Murnaghan, F. D., and Bateman, H., op. cit., pp. 252–253, 342–343, 346, 386.

[24] Jeffreys, Harold, The flow of water in an inclined channel of rectangular section: Philos. Mag., 6th ser., vol. 49, p. 795, 1925.

[25] Gilbert, G. K., Geology of the Henry Mountains, p. 106, U. S. Geog. and Geol. Survey Rocky Mtn. Region, 1877.

[26] See, for example, Merriman, Mansfield, op. cit., pp. 216, 275; Stanton, T. E. op. cit., pp. 83–84; Prandtl, L., op. cit., pp. 297–299; Dryden, H. L., Murnaghan, F. D., and Bateman, H., op. cit., p. 438.

of mean velocity are relatively small,[27] these energies balance:

Energy supplied = Energy consumed

$$(\rho_F Qt)g(v_M s.\ \cos\ \beta t) = (Pv_M t)(k_M \rho_F v_M^2)(v_M t) \text{---} \quad (3)$$

Simplifying, to obtain the mean frictional force per unit area,

$$k_M \rho_F v_M^2 = \frac{\rho_F Qgv_M s.\ \cos\ \beta}{Pv_M^2} = \rho_F grs.\ \cos\ \beta \text{-----} \quad (4)$$

Furthermore, this frictional force per unit area is the shearing stress at the channel walls,[28] or, in other words, the velocity gradient, v_B/δ, in the laminar film multiplied by the coefficient of viscosity, μ. That is,

$$\mu \frac{v_B}{\delta} = \rho_F grs.\ \cos\ \beta \text{----------------} \quad (5)$$

where δ = thickness of laminar film and

v_B = "bed" velocity at boundary of laminar film.

Here, as above, the term cos β is nearly unity for slope angles less than 10°, and the shearing stress at the channel walls in all Gilbert's experiments is therefore measured closely by $\rho_F grs$. Values of this quantity given in table 1 were calculated by taking $\rho_F = 1.00$, and $g = 980$ cm/sec^2.

In the derivation of equation 3, the energy consumed in friction was stated in terms of the mean velocity. It may, however, by analogous reasoning be stated in terms of the "bed" velocity as the area of channel surface, $Pv_B t$, times the "bed" frictional force per unit area, $k_B \rho_F v_B^2$, times the distance through which the total frictional force acts, $v_B t$, where k_B is the analogous coefficient of frictional resistance referred to the "bed" velocity. Equating the energy supplied to the energy consumed, as above,

$$(\rho_F Qt)g(v_M s.\ \cos\ \beta t) = (Pv_B t)(k_B \rho_F v_B^2)(v_B t) \text{-------} \quad (6)$$

Simplifying, to determine the "bed" velocity, and for slope angles less than 10°,

$$v_B^4 = \frac{Qgv_M s.\ \cos\ \beta}{Pk_B} = \frac{g}{k_B} v_M^2 rs \text{------------} \quad (7)$$

Inasmuch as all terms except v_B and k_B are known for Gilbert's experiments, the "bed" velocities are determinable if k_B, the coefficient of "bed" resistance, can be evaluated.

By this theoretical derivation, v_B varies with both v_M and rs. It is significant to recall in this connection that the data plotted in figures 19 and 20 indicate empirically that particle movement depends in a somewhat similar manner upon mean velocity and the depth-slope product. That is, the empirical data seem to show that the larger particles, but probably not the smaller particles, start moving when some critical "bed" velocity v_B is reached.

From equation 7,

$$\sqrt{k_B} v_B^2 = \sqrt{g}\ v_M \sqrt{rs}$$

and thus it is evident that for any particular "bed" velocity

$$\sqrt{k_B} \propto v_M \sqrt{rs} \text{-----------------} (8)$$

COEFFICIENT OF FRICTIONAL RESISTANCE

The coefficient of "bed" resistance, k_B, is by definition analogous to the coefficient of mean resistance, k_M, of which numerous studies have been made to determine the controlling factors. The principal results of these studies may be summarized briefly.

At low values of the Reynolds number, the observed k_M agrees closely[29] with the theoretical k_M computed from formulas for laminar flow—that is, k_M varies inversely as the Reynolds number, $\frac{\rho_F v_M r}{\mu}$. At high values of the Reynolds number, under conditions of turbulent flow, k_M is more nearly constant. A great many observations on turbulent flow of water, oil, and air through smooth pipes of different sizes[30] indicate that

$$k_M \propto \left[\frac{\mu}{\rho_F v_M r}\right]^{0.35} + \text{constant}$$

However, under conditions of turbulent flow in rough pipes, k_M is essentially independent of the Reynolds number and depends almost solely on a roughness ratio (the mean height of irregularities in the pipe divided by the pipe radius). The relationship has been found to be approximately

$$k_M \propto \left(\frac{\epsilon}{r}\right)^{1/3}$$

where ϵ = mean height of irregularities. By substitution in equation 4 and putting cos $\beta = 1$, this relationship gives

$$v_M \propto \left[g\left(\frac{r}{\epsilon}\right)^{1/3} rs\right]^{1/2}$$

The latter proportionality is equivalent to the Manning formula (p. 138) for mean velocity widely used by river engineers,[31] in which a roughness factor, n, takes the place of $\frac{\epsilon^{1/6}}{g^{1/2}}$; that is to say, n varies as $r^{1/6}\left(\frac{k_M}{g}\right)^{1/2}$. In a later analysis of more extensive data on turbulent flow

[27] For a more complete statement of this derivation, see Rubey, W. W., Equilibrium conditions in debris-laden streams: Am. Geophys. Union Trans. 14th Ann. Meeting, pp. 497–502, 1933.

[28] Jeffreys, Harold, op. cit. (1925), p. 795. Dryden, H. L., Murnaghan, F. D., and Bateman, H., op. cit., p. 386. Lindquist, E. G. W., On velocity formulas for open channels and pipes: Ing. vetensk. akad. Handl., nr. 130, p. 43, 1934.

[29] Stanton, T. E., op. cit., pp. 52–53. Prandtl, L., op. cit., p. 300, 1930.

[30] Lees, C. H., On the flow of viscous fluids through smooth circular pipes, Royal Soc. London Proc., vol. 91, A, pp. 46–53, 1915. Stanton, T. E., op. cit., pp. 55–56. Prandtl, L., op. cit., pp. 299–300, 1930; Dryden, H. L., Murnaghan, F. D., and Bateman, H., op. cit., p. 337.

[31] Dryden, H. L., Murnaghan, F. D., and Bateman, H., op. cit., p. 441. Lindquist, E. G. W., op. cit., pp. 17–18, 41, 51–52, 56–57. Falkner, F. H., op. cit., pp. 25, 47–48.

in rough pipes Prandtl[32] derived a more generally applicable formula,

$$\frac{1}{\sqrt{k_M}}=\gamma \log \frac{r}{\epsilon}+\lambda \text{-------------} (9)$$

where γ and λ are constants. However, it should be noted that recent laboratory investigations [33] indicate that the roughness term in Manning's formula depends quite as much upon the condition of the bed as upon the dimensions of particles and stream channel. After particle movement is well started, ripples or dunes are formed, and the roughness factor, n, is thereby increased.[34]

In short, from previous studies of the factors that control frictional resistance, it would be expected that under conditions of turbulent flow in debris-floored open channels the roughness of the channel would be the dominant factor controlling the coefficient of resistance. For Gilbert's experiments on the force required to start particle movement, the expected change of channel roughness after the particles start moving would not be involved. For his experiments the channel roughness is probably to be measured by the ratio of particle to channel dimensions, R/r.

proportionality 8, $\sqrt{k_B}$ varies as $v_M\sqrt{rs}$. (*c*) For turbulent flow, k_B probably depends upon a roughness ratio, R/r. A plot of the three variables—$v_M\sqrt{rs}$, R/r, and the particle-movement data—affords a rigorous test of these supposed relationships.

After several preliminary attempts, it was found that the actual relationships are shown most clearly if log $\frac{R_{MEAN}}{r}$ is plotted against $\frac{1}{v_M\sqrt{rs}}$. Figure 21 shows the plot of the data for grade G. The equation of the straight line that separates the field of hollow circles (no movement) from the field of crosses and solid circles (slight to much movement) is

$$\frac{1}{v_M\sqrt{rs}}=-0.025 \log \frac{R}{r}+0.074$$

$$=0.025 \log \frac{r}{R}+0.074 \text{--for grade G}$$

Gilbert's observations on other sizes of debris were less complete than those on grade G, but his data are sufficient to indicate the approximate equations of the lines separating the symbols indicating no movement

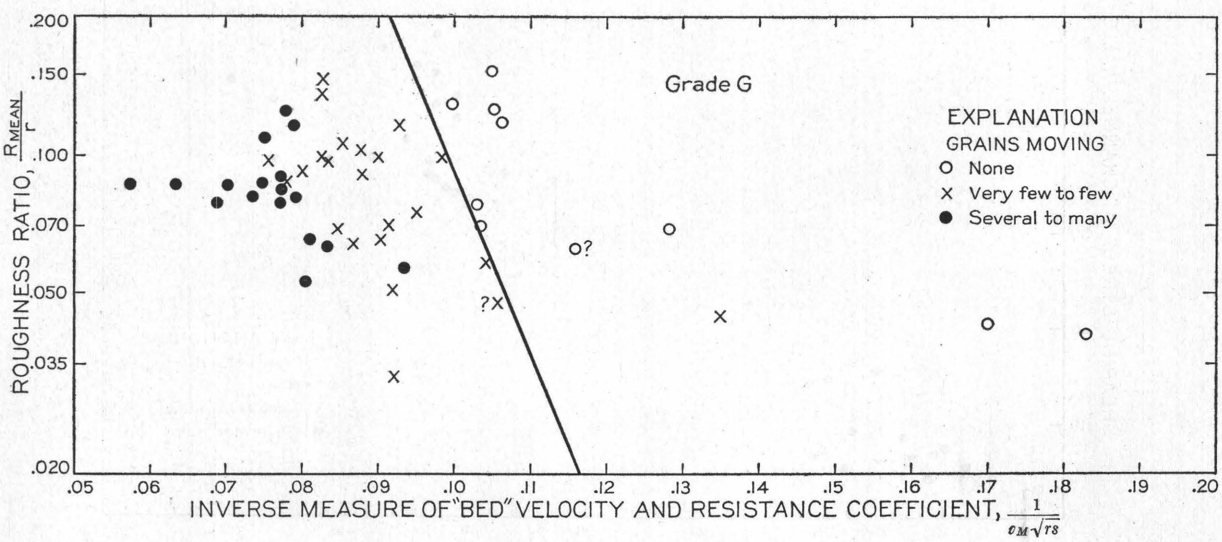

FIGURE 21.—Roughness ratios, inverse measures of "bed" velocity and coefficient of resistance, and movement of debris of grade G. Oblique line marks conditions at which movement of particles begins.

ESTIMATION OF "BED" VELOCITY AND THICKNESS OF LAMINAR FILM

Several conclusions thus far reached may now be assembled and tested. (*a*) The data plotted in figures 19 and 20 seem to indicate that the larger particles start moving when some critical "bed" velocity has been reached. (*b*) For a given "bed" velocity, according to

from those indicating slight to much movement for two other sizes.

$$\frac{1}{v_M\sqrt{rs}}=0.11 \log \frac{r}{R}+0.32 \text{----for grade E}$$

$$=0.016 \log \frac{r}{R}+0.048 \text{---for grade H}$$

It is noteworthy that the right-hand members of these equations have the same form as Prandtl's general formula (equation 9). The two constants in each equation decrease as the particles become larger, but the ratio between them is nearly the same for all three equations (2.91, 2.96, and 3.00).

[32] Prandtl, L., Neuere Ergebnisse der Turbulenzforschung: Ver. deutsch. Ing. Zeitschr., vol. 77, pp. 105–114, 1933. Lindquist, E. G. W., op. cit., pp. 40–42.

[33] Falkner, F. H., op. cit., pp. 47–48, pls. 24–38.

[34] This observation that the formation of ripples increases the roughness of a stream bed is of interest not because it bears closely on the present problem but because it seems to contradict the widely held theory that ripples in sand, like waves at the boundary between two fluids, decrease the frictional resistance to relative movement between two media.

Inasmuch as $\sqrt{k_B}$ is proportional to $v_M\sqrt{rs}$ (proportionality 8) for the particular "bed" velocity at which particles of a given size begin to move, it appears justifiable to conclude that

$$\frac{1}{\sqrt{k_B}} \propto 0.025 \log \frac{r}{R_{MEAN}} + 0.074$$

and, by substitution in equation 7,

$$v_B^2 \propto (0.025 \log \frac{r}{R_{MEAN}} + 0.074)\ \sqrt{g}\ v_M\ \sqrt{rs} \text{ -- } (10)$$

That is, the "bed" velocity increases with an increase of mean velocity, hydraulic radius, and slope and with a decrease of the roughness ratio (increase of channel smoothness). The relationship is such that the "bed" velocity is most sensitive to changes of the mean velocity and only very slightly sensitive to changes of the roughness ratio.

This relation may be tested further by some direct observations on velocity distribution made during a few of Gilbert's experiments.[35] The velocity gradients observed during flow over beds of paraffin, smooth board, and debris of grades A, C, D, and F have been fitted empirically to parabolic curves and the "bed" velocities then estimated from the individual curves. "Bed" velocities derived in this manner fit fairly well the equation

$$v_B^2 = \left[1.05 \log \frac{r}{R_{MEAN}} + 3.11\right] \sqrt{g} \cdot v_M \sqrt{rs} \text{ ------ } (11)$$

in which the bracketed term $\left[1.05 \log \frac{r}{R_{MEAN}} + 3.11\right]$ corresponds to $\frac{1}{\sqrt{k_B}}$ in equation 7. Needless to say, far more data than those just used would be required to establish the generality of such an equation, but it is here accepted provisionally as a basis for estimation.

It is of interest to remark that the increase in coefficient of resistance that might be anticipated (p. 131) after particle movement starts is not large enough to be perceptible in these data. Three of the velocity gradients shown in Gilbert's figure 82 were measured when loads of 38, 53, and 194 gm/sec of grade C debris were being transported through a trough 20.1 cm wide. Yet the "bed" velocities under these conditions of considerable transportation fit the same equation (11) as the "bed" velocities in the experiments where there was no movement of grades A, D, and F particles.

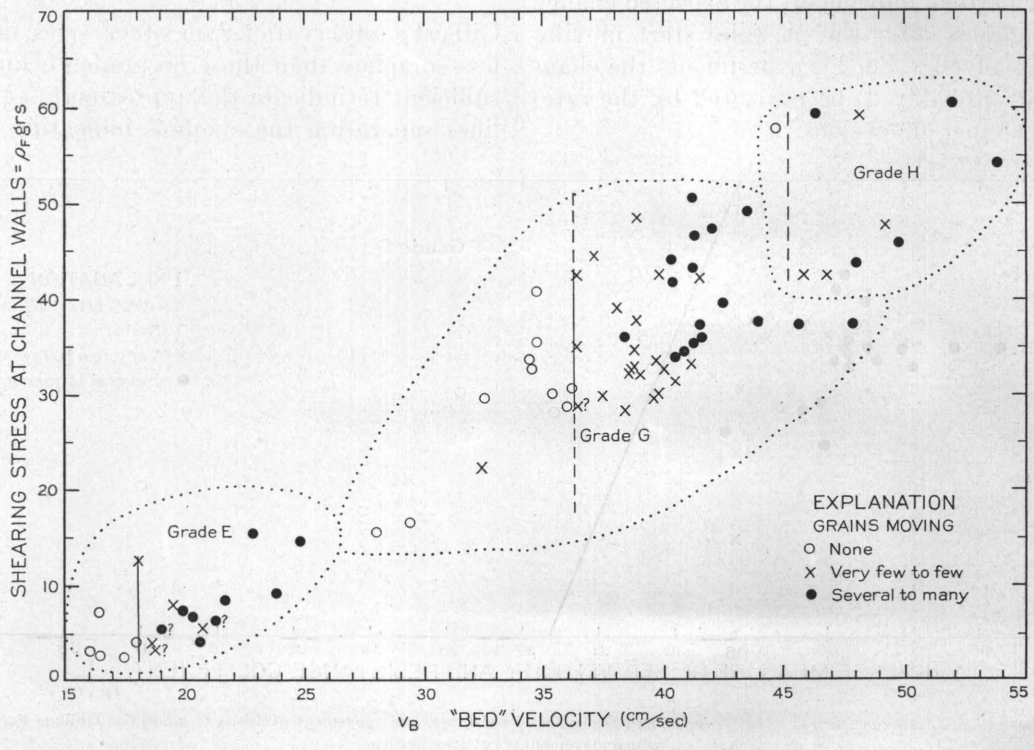

FIGURE 22.—"Bed" velocities, shearing stresses, and movement of debris of grades E, G, and H. Vertical lines mark "bed" velocities at which movement of particles begins.

Inasmuch as equation 11, which is based on measured velocity gradients above floors of grades A, C, D, and F debris, accords closely with the relationships found by plotting data on the movement of grades E, G, and H, and also with Prandtl's general formula, it is here accepted as a basis for calculating the probable "bed" velocities in all of Gilbert's experiments on competence. The results of these calculations are given in table 1.

If the "bed" velocity at the boundary of the laminar film and the velocity gradient within the laminar film

[35] Gilbert, G. K., op cit., pp. 245–246, figs. 78, 80, 82.

are known, an approximate average thickness of the laminar film, δ, can then be calculated from equation 5. This equation gives only an approximate thickness, because it assumes no rounding off of the velocity gradient at the transitional boundary between the zones of laminar and turbulent flow but instead a uniform gradient within the laminar film and an improbably abrupt change to turbulence. This calculation has been made for each of Gilbert's experiments on competence by assuming a coefficient of viscosity of 0.01 (equivalent to a temperature of 20° C.). The calculated average thicknesses are given in table 1. So far as known to me, no direct measurements have been made of the thickness of laminar films in water flowing over sandy beds.[36]

EFFECT OF "BED" VELOCITY AND SHEARING STRESS AT CHANNEL WALLS ON PARTICLE MOVEMENT

With these estimates of the "bed" velocity, v_B, and the shearing stress at the channel walls, $\rho_F grs$, for each of Gilbert's experiments, the data are available for a revision of figures 19 and 20 (the plots of mean velocity against depth-slope product and particle movement). Figure 22 shows the "bed" velocities, shearing stresses, and extent of movement of particles of grades E, G, and H. One difference between figures 20 and 22 stands out conspicuously. In figure 20 the lines that separate the symbols indicating no movement from those indicating slight to much movement slope obliquely down from left to right, thus indicating dependence upon both mean velocity and the depth-slope product. But in figure 22 the observations on grades E, G, and H seem to show that these lines are vertical or nearly so, thus indicating an independence of the shearing stress or the $r \cdot s$ product and a dependence upon "bed" velocity alone.

Unfortunately, Gilbert's experiments were not sufficiently numerous to establish or even to indicate clearly whether the lines that separate the fields of no movement from those of slight to much movement of the finer debris of grades B, C, and D are vertical or inclined. Additional data are needed on the conditions required to start movement of fine sand and silt. But so far as the available observations go, the force required to start movement of coarse sand and gravel seems to be independent of the depth-slope product (the "critical tractive force") and of the shearing stress at the channel walls (the "hydraulic lift") but to be closely dependent upon the "bed" velocity instead.

The data in table 1 that bear directly on the critical "bed" velocities of the different sizes of debris are summarized in table 2.

TABLE 2.—*Critical "bed" velocity for different grades of debris*

Grade	Radius (cm)		Critical "bed" velocity at which movement starts (cm/sec)
	Minimum	Maximum	
B	0.0155	0.0200	13.7−
D	0.0290	0.0450	15.2−
E	0.0450	0.105	17.9+ / 18.0−
F	[0.135]	0.175	29.7−
G	0.175	0.250	36.2+ / 36.4−
H	0.250	[0.370]	44.7+ / 45.9−

Gilbert noted that "when an experiment was begun with a velocity well below competence, and the velocity was gradually increased, the first movement detected would be the saltation of some small or light particle, and then the number of particles moving would gradually grow with the quickening of current."[37] In a few of his experiments, he recorded that only small

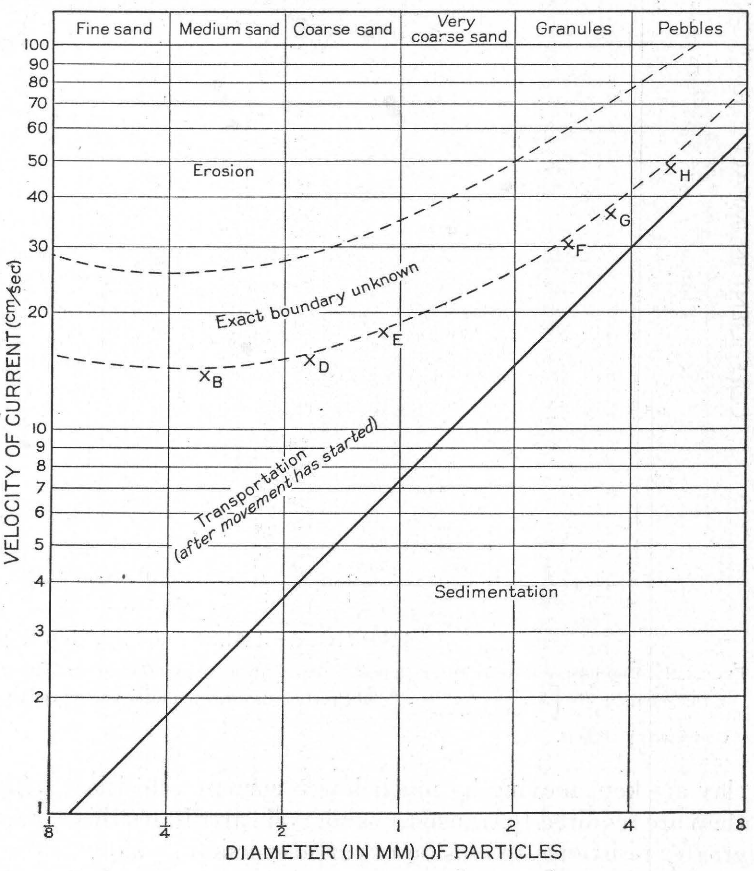

FIGURE 23.—Portion of Hjulström's diagram of the fields of erosion, transportation, and sedimentation for well-sorted sediment. The calculated "bed" velocities which start movement of Gilbert's debris of grades B, D, E, F, G, and H (shown by crosses) follow closely Hjulström's lower limit of the boundary between the fields of erosion and transportation.

[36] Since the above was written, my attention has been called to an entirely different method of calculating the thickness of the laminar film. In the notation of the present paper, Von Karman (Turbulence and skin friction; Jour. Aeronautical Sci., vol. 1, pp. 10, 19, 1934) finds that $\delta = 11.5 \frac{\mu}{\rho_F (grs)^{1/2}}$ approximately. This equation, applied to Gilbert's data, gives values of δ that range from the same to two times as thick as those calculated by the method of the present paper.

[37] Gilbert, G. K., op. cit., p. 69.

grains were being moved.[38] In the absence of information on differences in the specific gravity of individual grains, it is here assumed—somewhat arbitrarily, perhaps—that the first particles to be moved in Gilbert's experiments were the smallest ones in the mixture.

In figure 23 the critical "bed" velocities of each grade of debris are plotted on a portion of Hjulström's general diagram of the relations between current velocity, grain size, and the fields of erosion, transportation, and sedimentation.[39] This interesting compilation by Hjulström shows that higher velocities are required to start movement of silt and clay than of sand; but that, once movement has been started, the fine-grained silt and clay are kept moving by much lower current velocities than are required to transport sand. He attributes this greater resistance to erosion of particles less than about ⅓ mm in diameter to the "cohesion and adhesion" which unites smaller particles.[40] It is noteworthy that the six points from Gilbert's experiments follow closely the lower limit of Hjulström's boundary between erosion and transportation and especially that the points for grades B, D, and E confirm the flattening of the diameter-velocity curve. Gilbert's data, unfortunately, afford no information on the "bed" velocities required to start movement of particles smaller than grade B. The reversal of Hjulström's curve with smaller particles is based upon considerable laboratory and field evidence,[41] and later observations in the laboratory[42] seem to show the same reversal. However, even if this greater resistance of silt and clay is considered proved for clear water, it still remains questionable that the same would be true for water carrying sharp sand grains as tools of erosion. The very interesting corollaries in engineering, sedimentation, and physiography that may be drawn[43] from this supposed reversal of the curve make it highly desirable that additional studies be undertaken of the force required to erode silt and clay.

DEPARTURES FROM THE "SIXTH-POWER LAW" FOR SMALLER PARTICLES

The analysis of Gilbert's data thus far has tended to substantiate the "sixth-power law" for coarse sand and gravel but to indicate significant departures from this law for smaller particles. A further examination of Gilbert's data tells something about the nature of these departures.

As was pointed out in the discussion of the "sixth-power law", the coefficient $\left[\frac{3}{4}\cdot\frac{\theta}{\tan\alpha}\right]$ in equation 1 depends partly upon the shapes and relative sizes of the different particles making up a stream bed. Yet, even if the degree of roundness and of sorting of the particles were nearly the same throughout a group of debris samples such as Gilbert used, it seems likely that the value of this coefficient would still be influenced greatly by other factors, particularly by the relative dimensions of the particles and the laminar film. That is to say, the "bed" velocities at the upper boundary of the laminar film do not affect small particles lying on the stream bed or affect them only to the extent that the particles protrude through the laminar film into the mass of turbulently flowing water. If the particles lie entirely within the thickness of the laminar film, no definite relation between "bed" velocities and particle move-

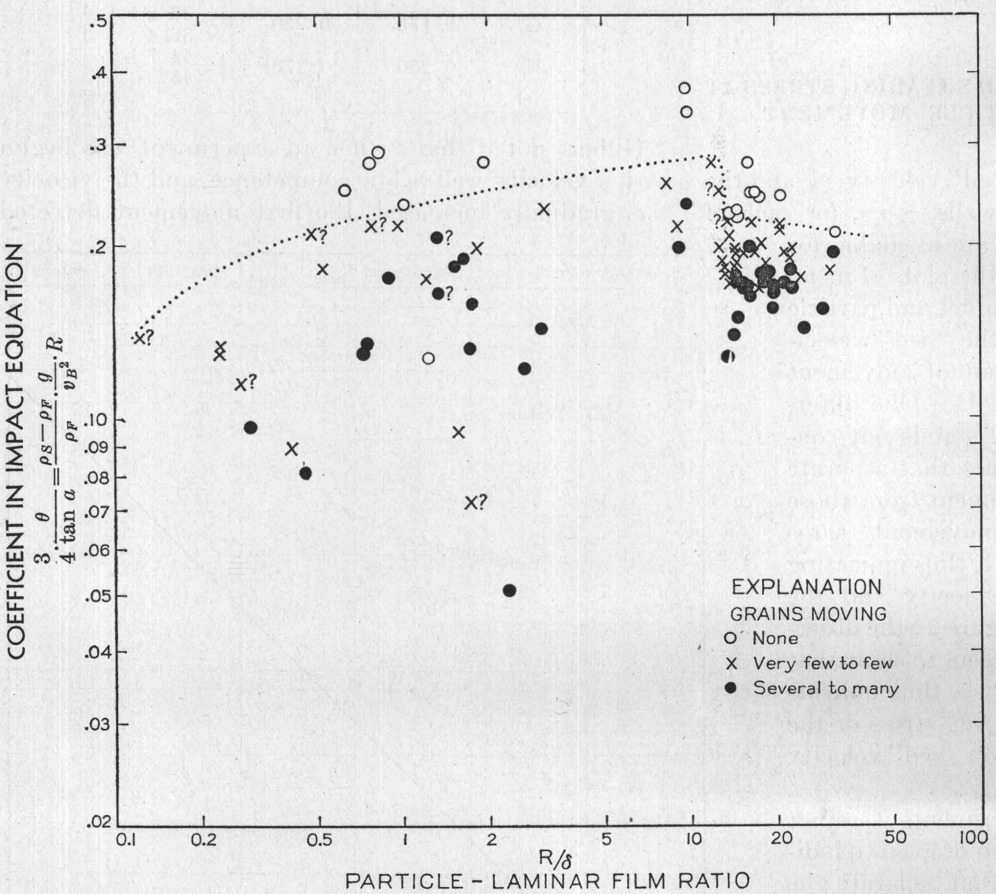

FIGURE 24.—Dependence of coefficient in impact equation upon the particle-laminar film ratio. The coefficient bracketed in the equation $R=\left[\frac{3}{4}\cdot\frac{\theta}{\tan\alpha}\right]\frac{\rho_F}{\rho_S-\rho_F}\cdot\frac{v_B^2}{g}$, the particle-laminar film ratio, and movement of debris of all grade-sizes investigated by Gilbert.

[38] Gilbert, G. K., op. cit., pp. 69, 70.

[39] Hjulström, Filip, op. cit., p. 298.

[40] Idem, p. 299. See also Matthes, G. H., Floods and their economic importance: Am. Geophys. Union Trans. 15th Ann. Meeting, pp. 428–429, 1934, who attributes this resistance to compaction of the finer-grained deposits.

[41] Hjulström, Filip, op. cit., pp. 295, 299–300.

[42] Falkner, F. H., op. cit., pp. 32–33, 34–35, 60–86.

[43] Hjulström, Filip, op. cit., pp. 324–325.

ment is to be expected. It is true that the laminar film probably tends to cling to and follow over the surface of larger particles that protrude into the main current, but the force impinging against such protuberances is significantly greater than that which acts upon small particles lying entirely immersed in the laminar film.

It thus seems probable that the coefficient $\left[\frac{3}{4}\cdot\frac{\theta}{\tan\alpha}\right]$ should show some relationship to the ratio, radius of particle divided by thickness of laminar film.

In figure 24 the values of this coefficient, $\frac{3}{4}\cdot\frac{\theta}{\tan\alpha}=\frac{\rho_S-\rho_F}{\rho_F}\cdot\frac{g}{v_B{}^2}\cdot R$, are plotted logarithmically against the ratio R/δ (where R is the radius of the particles first moved) and the observations on debris movement of all grade-sizes recorded thereon. This diagram seems to show that the coefficient depends at least in part upon the value of the ratio R/δ. The calculated points are far from adequate to establish any precise relationship, but they indicate that, for ratios of 0.1 to 1.0, the coefficient increases rapidly from about 0.13 to 0.23; for ratios from 1.0 to 13.0, the coefficient increases more gradually from 0.23 to 0.28; at a ratio of 13, the coefficient appears to drop abruptly to 0.22; and then it decreases very gradually to 0.21 at a ratio of 31.[44] In other words, when the particles are relatively small compared with the thickness of laminar film, the force of the current is less efficient, so that "bed" velocities higher than those indicated by the "sixth-power law" are required to start movement; when the particle radii are from 1 to 13 times as great as the thickness of laminar film, the current is considerably more efficient; and when the particles are relatively large compared with the thickness of laminar film, the current is of intermediate efficiency.[45]

If "bed" velocity were the only factor in particle movement, the coefficient should be the same for all similarly shaped and sorted particles of whatever size and for all values of the ratio R/δ. The fact that the coefficient varies shows that "bed" velocity (at least, the one used here) is not the only factor in particle movement. Furthermore, the fact that the coefficient varies most with smaller values of the ratio R/δ suggests a possible explanation for the variation. The abnormally low values of the coefficient where the laminar film is thicker than the particle radius means that "bed" velocities higher than those indicated by the "sixth-power law" are required in order to start particle movement. This is what might be predicted either for surface drag or for hydraulic lift on particles entirely immersed in the laminar film; because, when thus immersed in a film of steep velocity gradient, the force acting upon a particle increases rapidly with the dimensions of the particle or the height to which it stands above the stream floor (fig. 25). Under these conditions of complete immersion in the laminar film,

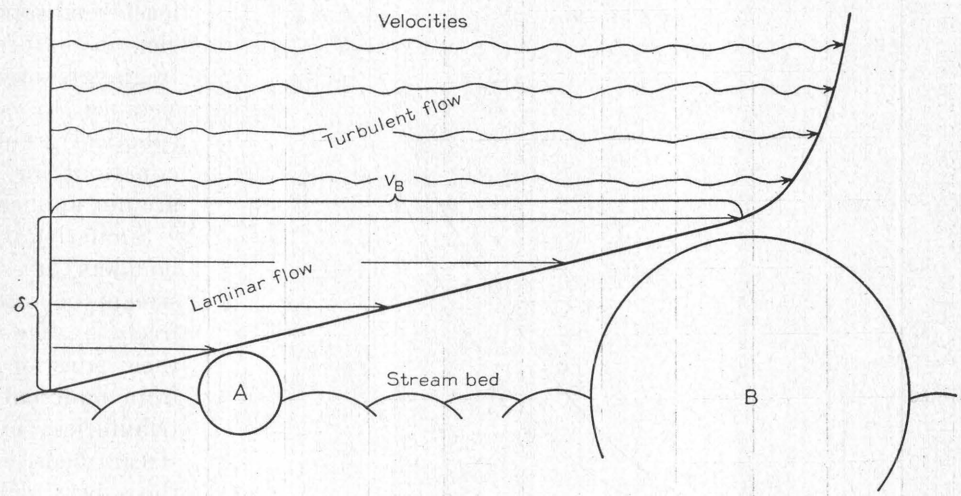

FIGURE 25.—Idealized velocity gradient very near a stream bed. Within the film of laminar flow the velocity increases greatly with distance from the bed. Thus the forces acting on particle B to push, drag, and lift it are much greater than those acting on particle A—perhaps enough greater to move particle B, despite its greater weight, without moving particle A.

the steepness of the velocity gradient rather than the "bed" velocity at the upper boundary of the laminar film would be the controlling factor in particle movement. Somewhat larger particles, protruding short distances into the mass of turbulently flowing water, would be acted upon by two or more forces—(1) the head-on impact of the current (the "sixth-power law") and (2) either the surface drag or the "hydraulic lift" within the laminar film or both. For still larger particles the head-on impact would become relatively more effective and the surface drag or "hydraulic lift" relatively less effective. For very large ratios of particle to laminar film the effects of the laminar film would thus become negligibly small and the coefficient $\left[\frac{3}{4}\cdot\frac{\theta}{\tan\alpha}\right]$ in the "sixth-power law" should remain constant.

[44] If θ is estimated at ⅓ (p. 123), the coefficient, 0.21, means that the angle α is approximately 24°. This may be compared with maximum, minimum, and average angles of 35°19′, 19°22′, and 22°03′ up which a sphere that rests upon three other mutually touching spheres of the same size would have to be lifted in order to dislodge it.

[45] If Von Karman's thickness of the laminar film is used (see footnote 36, p. 133), the relationship is almost exactly the same as that shown in figure 24, except that the abrupt decrease of the coefficient comes at a ratio of 8 instead of 13.

The apparent discontinuity in the coefficient curve shown in figure 24, where the particle radius is about 13 times as great as the thickness of the laminar film, suggests an analogy with the somewhat similar abrupt change in the coefficient of frictional resistance when the flow changes from laminar to turbulent.[46] The ratio at which this discontinuity appears may possibly represent a sort of Reynolds number, so to speak, at which the laminar film clinging to protuberances on the stream bed is broken up and torn away by the force of the main current.

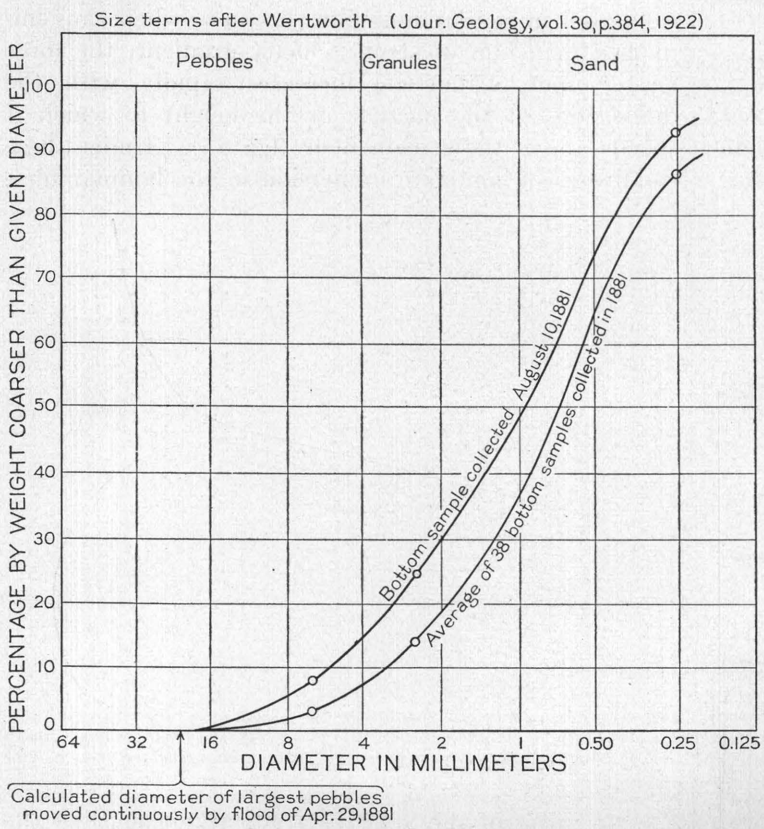

FIGURE 26.—Mechanical composition of samples collected in 1881 from the bed of the Mississippi River at Hannibal, Mo., compared with calculated diameter of largest pebbles continuously moved by the flood of April 29, 1881.

These departures from the "sixth-power law" for smaller particles, when $R/\delta<1$, need much additional study before they can be interpreted satisfactorily. Whatever their correct explanation, the variations in the value of the coefficient $\left[\frac{3}{4}\cdot\frac{\theta}{\tan\alpha}\right]$ are relatively small for larger values of the ratio, and when $R/\delta>13$ equation 1 may be rewritten:

$$R=0.21\frac{\rho_F}{\rho_S-\rho_F}\cdot\frac{v_B^2}{g}$$

By substitution from equation 11,

$$R=0.22\left[\log\frac{r}{R_{MEAN}}+2.96\right]\cdot\frac{\rho_F}{\rho_S-\rho_F}\cdot\frac{v_M\sqrt{rs}}{\sqrt{g}} \text{ -------- } (12)$$

POSSIBLE APPLICATION TO NATURAL STREAMS

The relationships considered and the equations proposed in this paper are based almost entirely upon laboratory evidence. It is a long jump from the effect of uniform discharge on uniform debris in small experimental troughs to that of variable discharge on natural aggregates of sediment in large stream channels. Nevertheless, it is the hope of understanding principles and finding relationships that may be applicable to full-scale natural processes that leads to laboratory investigation, and the temptation is strong to see how the equations derived here may work when applied to the immensely more complicated circumstances of nature.

As a matter of fact, when properly and sufficiently qualified, the conclusions indicated by Gilbert's experiments should be applicable, at least qualitatively, to the movement of particles in natural streams. Obviously, very flat fragments of rock offer a greatly different resistance to stream flow than approximately spherical grains of the same volume; therefore, equations for the movement of rounded grains are not applicable to very flat fragments.

Similarly, the large "erratic" pebbles and boulders present in minor quantities in many stream deposits may never have been carried freely by the streams. They probably reached their sites of deposition by rafting, by caving from undercut banks, by dumping from steep tributaries, by slow creeping on locally steep stream beds, etc.—processes quite different from those by which the bulk of sediment is moved. Until these "erratics" have been ground down to manageable dimensions by the blast of passing finer materials, they may perhaps be moved only very short distances on the stream bed.[47] Any relationships found for normal sediment transportation would certainly not apply for these "erratics."

Again, a very large part of the work done by natural streams is accomplished in the relatively brief periods of high floods. Consequently, we cannot expect to learn much about the average regimen of an ancient stream from a study of the land forms and sedimentary deposits it has left. At best we can only hope to learn something about the flood conditions of that stream.

And so on through a wide range of many other necessary qualifications, such as gentle slopes and turbulent flow, which must not be forgotten in any attempt to apply the laboratory results to natural streams. The real question is to learn whether or not the laboratory

[46] Stanton, T. E., op. cit., pp. 52-53. Prandtl, L., in Ewald, P. P., Pöschl, T., and Prandtl, L., The physics of solids and liquids, p. 300, 1930.

[47] Barrell, Joseph, Marine and terrestrial conglomerates: Geol. Soc. America Bull., vol. 36, pp. 330-331, 335-336, 338, 1925.

results apply, even approximately, to those natural streams in which conditions seem to be roughly analogous; and this necessarily implies a considerable knowledge of the conditions in the natural streams chosen for comparison.

I have made several calculations where sufficient information was available about a natural stream to afford a critical test. The examples given are typical of others, but they are chosen for illustration here because I happen to have some first-hand familiarity with the geology and physiography of the particular areas cited.

One of the largest floods of the Mississippi River at Hannibal, Mo., for which cross-section and velocity measurements are available, occurred April 29, 1881. The river was then 2,535 feet wide, its area of cross section was 52,498 square feet, its mean velocity 5.19 feet per second, and its discharge 272,609 cubic feet per second.[48] The exact slope of the river at the line of measurement is not known, but inasmuch as the gradient is fairly uniform in this part of the Mississippi, the average slope above and below Hannibal, a fall of 21 feet in the 42 miles from Quincy, Ill., to Louisiana, Mo.,[49] may be taken as representative.

Mechanical analyses of 38 samples dredged from the river bed at Hannibal in 1881 are also available.[50] The coarsest one of these samples was collected August 10 (fig. 26). The analyses of the 38 samples collected in 1881 agree fairly well with more detailed mechanical analyses of 11 samples collected near Hannibal in 1925 by Lugn.[51]

TABLE 3.—*Mechanical composition of samples from the bed of the Mississippi River at Hannibal, Mo., collected in 1881*

[Percentage of particles by diameter in fractions of an inch]

	Greater than 0.75 inch	0.75 to 0.25 inch	0.25 to 0.1 inch	0.1 to 0.01 inch	Less than 0.01 inch	Total
Coarsest sample	----	8	17	68	7	100
Average of 38 samples	----	3	11	72	14	100

Transposing these data from feet and inches to centimeters gives

$v_M = 158$ cm/sec.

Mean depth $= 631$ cm.

$r = 622$ cm.

$s = 0.000095$.

$R_{MEAN} = 0.07$ cm [the median diameter of average bottom sample, estimated from graph, fig. 26].

$R_{MAX} = 0.80$ to 1.0 cm [estimated from graphs of mechanical analyses, fig. 26].

Take

$\rho_F = 1.00$.

$\rho_S = 2.66$.

$g = 980$.

$\mu = 0.011$ (corresponding to a temperature of 15° C.).

Then, from equation 11

$$v_B = \left[1.05 \log \frac{r}{R_{MEAN}} + 3.11\right]^{\frac{1}{2}} g^{\frac{1}{4}} v_M^{\frac{1}{2}} (rs)^{\frac{1}{4}}$$

$$= \left[1.05 \log \frac{622}{0.07} + 3.11\right]^{\frac{1}{2}} (980)^{\frac{1}{4}} (158)^{\frac{1}{2}} (622 \times 0.000095)^{\frac{1}{4}}$$

$$= 2.69 \times 5.60 \times 12.57 \times 0.493 = 93 \text{ cm/sec.}$$

= the "bed" velocity.

From equation 5

$$\delta = \frac{\mu v_B}{\rho_F\, grs} = \frac{0.011 \times 93.4}{1.00 \times 980 \times 622 \times 0.000095}$$

$$= \frac{1.03}{57.9} = 0.018 \text{ cm}$$

= the average thickness of the laminar film.

Since $\frac{R}{\delta} = \frac{0.80 \text{ to } 1.0}{0.018} = 44$ to 56, which is greater than 13, then from equation 12

$$R_{MAX} = 0.22. \left[\log \frac{r}{R_{MEAN}} + 2.96\right] \cdot \frac{\rho_F}{\rho_S - \rho_F} \cdot \frac{v_M \sqrt{rs}}{\sqrt{g}}$$

$$= 0.22 \left[\log \frac{622}{0.07} + 2.96\right] \cdot \frac{1.00}{2.66 - 1.00} \cdot \frac{158 (622 \times 0.000095)^{\frac{1}{2}}}{(980)^{\frac{1}{2}}}$$

$$= 0.22 \times 6.91 \times 0.602 \times 1.227$$

$$= 1.1 \text{ cm,}$$

which is to be compared with the maximum radius of 0.80 to 1.0 cm estimated from the graph of the mechanical analyses, figure 26.

The data are much less complete for the other two examples, and so only the results of the calculations are given in table 4. The calculated radius of the largest particles moved by the Green River at Daniel, Wyo., agrees poorly with that estimated from direct observation. This lack of agreement may disclose fundamental weaknesses in the equations, or it may simply indicate that no very large flood has yet been measured at this point. In the Potomac and Mississippi examples, on the other hand, the calculations agree very closely with observations; and for all three rivers the equations give results that are at least of the correct order of magnitude.

It thus appears that the equations based on Gilbert's laboratory experiments give reasonably close estimates of the maximum size of particles transported by some large natural streams. Needless to say, the equations need far more extensive testing and considerable modification before they can be applied to streams generally. It appears certain, for example, that more detailed testing will show significant variations in the numerical coefficient 0.22, depending upon the degree of roundness and of sorting of the particles on a stream

[48] Results of discharge observations, Mississippi River and its tributaries and outlets, 1838–1923, p. 47, Mississippi River Comm., 1925.

[49] Gannett, Henry, Profiles of rivers in the United States: U. S. Geol. Survey Water-Supply Paper 44, p. 39, 1901.

[50] Vogel, H. D., Sediment investigations on the Mississippi River and its tributaries prior to 1930: U. S. Waterways Exper. Sta. Paper H, pp. 83–84, 1930.

[51] Lugn, A. L., Sedimentation in the Mississippi River between Davenport, Iowa, and Cairo, Ill.: Augustana Library Pub. 11, p. 49, 1927.

bed and upon the local curvature of the stream channel. Also the maximum size of particles moved must vary from point to point across a stream, depending upon differences in mean velocity, depth of water, slope of water surface, and local roughness of the bed. The approximate agreements thus far found must be largely the result of compensating errors. Yet this very probability of compensating errors may mean that equations of this type will ultimately be found useful for average stream conditions.

TABLE 4.—*Calculated "bed" velocity, thickness of laminar film, and size of largest particles moved at definite points in the Green, Potomac, and Mississippi Rivers during certain floods*

River	Locality	Largest measured flood [1]	Observed and estimated from observation						Calculations based on Gilbert's laboratory experiments		
			Discharge (cu. ft./sec.)	Slope	Hydraulic radius (cm)	Mean velocity (cm/sec.)	R_{MEAN} (cm)	R_{MAX} (cm)	v_B (cm / sec.)	δ (cm)	R_{MAX} (cm)
Green	Daniel, Wyo	June 16, 1918	8,750	0.0020	175	200	[2] 1.0	[3] 10	140	0.0045	2.7
Potomac	Chain Bridge, D. C	May 14, 1932	165,000	0.00087	412	340	[2] .10	[3] 5.0	210	.0066	5.7
Mississippi	Hannibal, Mo	Apr. 29, 1881	272,609	0.000095	622	158	[4] .07	[4] 0.80 to 1.0	93	.018	1.1

[1] Largest flood for which cross-section and velocity measurements are available. Data for the Green and Potomac Rivers from files of Geological Survey.
[2] Rough estimate.
[3] Estimate.
[4] Estimated from mechanical analyses.

In the form in which they have here been written, the equations based on Gilbert's experiments are of little direct use for interpreting the size and regimen of ancient streams. In order to estimate the dimensions of the largest particles moved by a stream, the mean velocity, the hydraulic radius, the slope, and either the mean grain size or the roughness ratio must be known. However, one of these determining quantities may be eliminated by using the Manning formula [52] for mean velocity in open channels,

$$v_M = \frac{1.486}{n} r^{2/3} s^{1/2} \text{ for ft.-sec. units}$$

or

$$v_M = \frac{4.642}{n} r^{2/3} s^{1/2} \text{ for cm-sec. units}$$

where the coefficient n (see also p. 130) is a roughness factor that may be taken from tables in engineering handbooks.[53]

With this simplification, something of significance may be learned about the probable conditions of flood flow of certain ancient streams. If the maximum and mean grain sizes of the deposits of an ancient stream are known and if the original undeformed slope of the stream can be determined from terrace remnants or from other evidence, estimates can be made of the mean velocity and the mean depth (or, more strictly, the hydraulic radius) of the stream at times of flood, and then, from a comparison with modern streams, inferences may be drawn about the probable flood run-off, climate, size and character of drainage basin, etc. But if the ancient deposits have been so deformed and eroded that the original stream slope cannot be ascertained, the problem appears indeterminate or, at best, determinate only within very broad limits. Under the most favorable circumstances the equations may be found useful for interpreting past conditions of streams whose relationships to the present topography have not been effaced—that is to say, for some Recent and Pleistocene and a few Tertiary stream deposits.

[52] However, it is fair to mention in this connection that Gilbert's experiments, on which these equations are based, fit the Manning formula very indifferently.

[53] Merriman, Mansfield, op. cit., pp. 288–288a.

STREAM COMPETENCE AND STREAM CAPACITY

It may not be out of place to close this paper with a comparison, in very general terms, of two entirely different aspects of the transporting power of streams that are sometimes confused.

According to Gilbert,[54] "A current flowing over debris of various sizes transports the finer but cannot move the coarser; the fineness of the debris it can barely move is the measure of its competence." A later definition [55] emphasizes an essential point: "By competency is meant the ability of a stream to transport in terms of dimensions of particles." The present paper is therefore a discussion of stream competence.

Gilbert [56] defined capacity as "the maximum load a stream can carry." Capacity is thus the amount of debris a stream can transport; it has nothing whatever to do with the maximum size of the particles transported. Elsewhere [57] I have proposed a method for evaluating stream capacity, and this method may be used to illustrate the distinction between capacity and competence. For laboratory streams flowing over transportable debris, it was found that approximately 2½ percent of the total energy of the stream was spent in transportation. For a few natural streams on which the necessary data were available, the proportion was found to be about 4 percent. Thus, in a general way,

$$L.v_S \propto \rho_F g Q v_M s, \text{ approx.}$$

[54] Gilbert, G. K., op. cit., p. 35.

[55] Twenhofel, W. H., and others, op. cit., p. 30.

[56] Gilbert, G. K., op. cit., p. 35.

[57] Rubey, W. W., Equilibrium conditions in debris-laden streams: Am. Geophys. Union Trans. 14th Ann. Meeting, pp. 497–505, 1933.

where L=load or mass of debris passing a cross section of the stream in a unit of time (measured in gm/sec.)

v_S=average settling velocity in quiet water of all the debris particles being transported (measured in cm/sec.)

and the other symbols follow the notation of the present paper. Dividing through by the wetted perimeter, this becomes

$$\frac{L}{P}\cdot v_S \propto \rho_F g v_M^2 rs, \text{ approx} \qquad (13)$$

Or, (the load per unit width of stream bed) × (the average settling velocity of the debris transported) varies approximately as the product v_M^2rs.

By equation 7,

$$v_B^4=\frac{g}{k_B}v_M^2rs$$

For the purpose of this general comparison of competence and capacity, variations in the coefficient of "bed" resistance, k_B, may be neglected as relatively minor second-order effects, and thus proportionality 13 will reduce to

$$\frac{L}{P}\cdot v_S \propto v_B^4, \text{ approx} \qquad (14)$$

That is, the load per unit width of a stream multiplied by the average settling velocity of the debris transported varies approximately as the fourth power of the "bed" velocity of the stream.

If the average settling velocity of debris transported should remain the same, then the weight of debris per unit width of stream carried past a given point would vary roughly as the fourth power of the "bed" velocity. But only under unusual circumstances would the average settling velocity remain the same in natural streams. A stream flowing in an alluvial channel of its own making has relatively easy access to unconsolidated debris of many sizes. As the "bed" velocity of such a stream is increased, the average settling velocity of debris transported does not remain the same but increases because larger and larger particles are picked up and added to the load.

Probably no natural stream ever carries exactly equal weights of all sizes of debris smaller than the largest particles it is able to move. Nevertheless, in a stream free to pick up larger and larger material as its "bed" velocity increases, the average settling velocity of all debris transported will be influenced greatly by the high settling velocity of the largest particles carried and hence if much of the larger material is picked up the average settling velocity will increase approximately in proportion to the settling velocity of these largest particles.

For small grains of silt and clay, to which Stokes' law of settling velocities is applicable, the principles governing the maximum size of particles transported are not yet known; and consequently no simplification of competence and capacity can be attempted here. But for larger grains of sand and gravel, to which the "sixth-power law" applies best, settling velocities vary closely as the square root of the particle radius.[58] That is, $v_S \propto \sqrt{R}$ for sand and gravel.

But according to the "sixth-power law,"

$$R^3 \propto v_B^6 \text{ or } R \propto v_B^2$$

So that

$$v_S \propto v_B, \text{ approx} \qquad (15)$$

That is to say, in a stream able to pick up considerable sand and gravel the average settling velocity of debris transported will tend to vary directly as the first power of the "bed" velocity of the stream.

Combining this relation with proportionality 14, we find that, because of the simultaneous effects upon both competence and capacity,

$$\frac{L}{P} \propto v_B^3, \text{ very approx} \qquad (16)$$

In a stream free to pick up much sand and gravel as its velocity is increased, the unit width load will vary roughly as the third power of the "bed" velocity.

The purpose of this generalized discussion has been to show that quantitative relationships of stream capacity are not to be confused with the "sixth-power law" of stream competence, which for comparison may be restated here as follows: The weight of the largest debris particles moved by a stream varies as the sixth power of the "bed" velocity.

SUMMARY AND CONCLUSIONS

1. The force that moves a particle on a stream bed has been explained in several ways—by the impact or momentum of the water which strikes against it, by the frictional drag upon its surface, and by differences in pressure induced by differences in current velocity at the top and bottom of the particle. The familiar textbook "law" that the weight of the largest particles moved by a stream varies as the sixth power of the velocity is based upon the impact theory. The theory of frictional drag is the basis of the so-called law of "critical tractive force," which river engineers prefer because it gives the maximum size of particles moved in terms of readily measurable quantities, the depth of water and the slope of a stream. The "hydraulic lift" caused by pressure differences is measured by the velocity gradient very near a stream bed.

2. G. K. Gilbert's laboratory experiments show that the force required to start particle movement depends not only on the mean velocity of a stream but also on the depth-slope product, the large particles being most

[58] Rubey, W. W., Settling velocities of gravel, sand, and silt particles: Am. Jour. Sci., 5th ser., vol. 25, pp. 327–335, 1933. Wadell, Hakon, The coefficient of resistance as a function of Reynolds number for solids of various shapes: Franklin Inst. Jour., vol. 217, pp. 467–475, 1934. Christiansen, J. E., Distribution of silt in open channels: Am. Geophys. Union Trans. 16th Ann. Meeting, pp. 480–481, 1935.

sensitive to changes of velocity and the smaller particles most sensitive to changes of the depth-slope product.

3. The velocity in the immediate vicinity of the particle on the stream bed is more significant for this inquiry than the mean velocity of the entire stream. The velocity at the boundary between the thin film of laminar flow on the stream bed and the main mass of turbulent water above it is here defined as the "bed" velocity and evaluated in terms of mean velocity, hydraulic radius, slope, and a coefficient of frictional resistance of the bed.

4. Experimental studies on the frictional resistance to flow in rough pipes suggest and Gilbert's data show that this coefficient of frictional resistance depends upon the channel roughness or the size of particles making up the stream bed. This relationship permits an estimate of the "bed" velocity in each of Gilbert's experiments.

5. From the shearing stress at the channel walls and the "bed" velocity, the thickness of the laminar film can then be estimated. With this information, a reexamination of Gilbert's results shows that movement of larger particles starts at some particular "bed" velocity and is independent of the velocity gradient or the depth-slope product. That is, the movement of coarse sand and gravel follows the "sixth-power law", but the smaller particles require much higher velocities than are indicated by this law.

6. A satisfactory theory of the force required to start movement of fine sand and silt must await additional observations. However, Gilbert's experiments suggest that the departures from the "sixth-power law" for smaller particles become large enough to be significant when the laminar film is thicker than the radius of the particle, thus indicating that frictional drag or "hydraulic lift," rather than impact, is the force that starts movement of small particles.

7. The equations based on laboratory data give reasonable estimates of the maximum size of pebbles moved by some large natural streams, but much information is required about a stream in order to make the necessary calculations. After more extensive testing and modification, equations of the type proposed may prove useful for interpreting the size and regimen of streams that laid down certain Recent, Pleistocene, and Tertiary deposits.

8. Although both size of particles and total load are aspects of the power of transportation of a stream, the two are not to be confused. The "sixth-power law" measures only the size of larger particles moved and has nothing to do with total load or amount of debris transported.

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