Water-Loss Investigations: Lake Hefner Studies, Technical Report

GEOLOGICAL SURVEY PROFESSIONAL PAPER 269

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Douglas McKay, *Secretary*

GEOLOGICAL SURVEY

W. E. Wrather, *Director*

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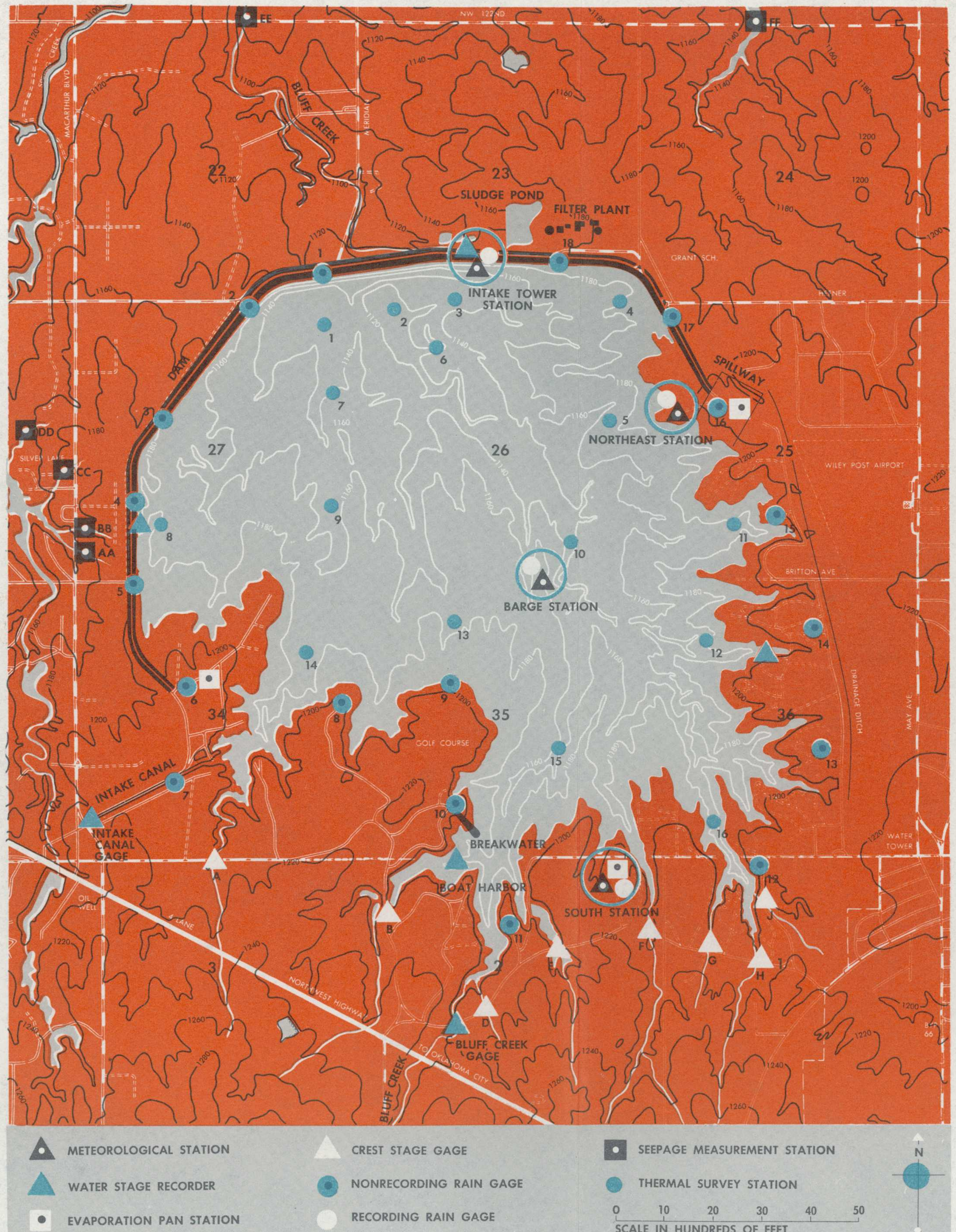
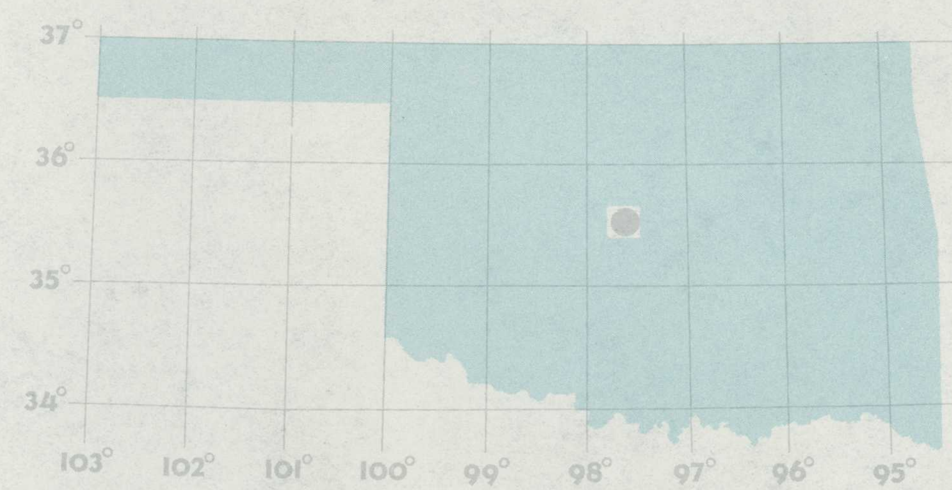


Map of Lake Hefner, Showing Equipment Installations, Adapted from Britton (SE) Quadrangle, U. S. Geological Survey, 1951.

Map of State of Oklahoma showing location of Lake Hefner.



FOREWORD

Water and its wise management and use are vital to the economic stability and growth of the western United States. Thus, every "lost river" must be explored to determine and, if possible, prevent water losses.

Evaporation is one of these "lost rivers." Loss through evaporation must be considered in the operation of any reservoir but, in the case of large reservoirs with extensive surface areas, it becomes a major factor in planning and management.

Methods of estimating evaporation which have proved adequate for small reservoirs may not suffice for a large reservoir in a region where water is scarce. A case in point is Lake Mead, the largest man-made body of water in the world, created by the construction of Hoover Dam on the lower Colorado River between Nevada and Arizona.

Lake Mead is the key reservoir for the distribution of lower Colorado River water to Arizona, southern California, and portions of Nevada, Utah, and New Mexico. Without water this area would be almost uninhabited, and the bulk of the supply comes from the lower Colorado. Every drop is precious; the demand far exceeds the supply.

For this reason, the Bureau of Reclamation is obligated to remove every possible source of error in its handling of the water in Lake Mead. The evaporation loss cannot be prevented, but accurate knowledge of evaporation permits more efficient use of the remaining water, increases the security of existing developments, and allows better planning of future developments.

To obtain the best knowledge of evaporation it was necessary to bring together experts in many fields. Ideas, theories, and methods for estimating evaporation have been proposed by engineers, meteorologists, hydrologists, oceanographers, physicists, and many others, but there has been no sound basis for agreement as to their relative merits. Furthermore, some of the theories and ideas required a precision of measurement of natural factors unattainable with existing instruments. By cooperative work involving agencies of the Department of the Navy, the Geological Survey and Bureau of Reclamation of the Department of the Interior, and the Weather Bureau of the Department of Commerce, it was possible to assemble the necessary assortment of skills.

Lake Mead presents a large and complex problem and, therefore, was undesirable as a "proving ground" for the various ideas and theories. A smaller, simpler reservoir was required for the first studies to test the procedures effectively. Lake Hefner was selected for this purpose and this report describes the work done there and the results obtained. The results were satisfactory and the work is being continued at Lake Mead.

The aim of the studies was not only the development of methods for use on large reservoirs, but also the development of new methods, or the improvement of existing methods, of potential use on all reservoirs, even the smallest, and an enlargement of knowledge of evaporation in general. The results as well as having a direct practical application, may be a major contribution toward an understanding of evaporation and related subjects.

Commissioner of Reclamation

PREFACE

This report was originally published in 1952, jointly by the Geological Survey and the Navy Electronics Laboratory, as Geological Survey Circular 229 and as N. E. L. Report 327; it was arranged and reproduced by the Navy Electronics Laboratory. The continuing demand for the report exhausted the first printing and made a reprinting necessary. In reprinting by the Geological Survey it was considered desirable to do so in a form that would permit wider distribution to reference libraries. The new format does not involve changes in the body of the report except for corrections indicated on the errata slip that accompanied the first printing and three additional corrections (the exponent in the denominator of the equation for *a*, p. 60, was changed from $1-n$ to $2-2n$, the Gatewood reference, p. 69, remains unchanged except for Lower Safford Valley, and the Richardson reference, p. 119, now reads American Society of Civil Engineers). Reprinting was accomplished by use of reproducible material generously provided by the Navy Electronics Laboratory.

Observational data, on which the report is based, are presented in a second report published as "Water-Loss Investigations: Lake Hefner Studies, Base Data Report," Geological Survey Professional Paper 270 and Navy Electronics Laboratory Report 328.

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STATEMENT OF PROBLEM

Develop an improved method or methods for the determination, and if possible the prediction from climatological and limnological data, of water losses by evaporation using mass-transfer and energy-budget theory. Develop improved techniques for converting evaporation-pan data to estimated lake evaporation. Test the proposed methods and techniques at Lake Hefner, Oklahoma, using evaporation computed from the water budget of the lake as the control.

CONCLUSIONS

1. The water-budget control met the requirement that errors in the water budget should not exceed 5 per cent of the monthly evaporation. In addition, 62 per cent of the determinations of daily evaporation were sufficiently accurate to provide a check against estimates of daily evaporation by mass-transfer and energy-budget theories.
2. Only two (O. G. Sutton's and Sverdrup's 1937 form) of the many mass-transfer equations tested proved adequate for predicting daily evaporation with sufficient accuracy to be generally useful, employing equipment now available.
3. A simple empirical equation was developed from the water-budget data, using wind speeds and vapor-pressure differences measured at Lake Hefner. An operational version of this equation proved satisfactory for computing daily evaporation at Lake Hefner using standard meteorological observations from a nearby weather station, plus the surface-water temperature of the lake.
4. The energy-budget equation proved satisfactory for computing evaporation for periods of 7 days or longer provided the water budget can be approximated closely enough to estimate advected energy. The primary limitation on the accuracy of the method, for short periods of time, is the evaluation of change in energy storage.
5. The Cummings Radiation Integrator (CRI) promises to provide a satisfactory substitute for the expensive radiation-measuring equipment used at Lake Hefner, thus simplifying the computation of evaporation by the energy-budget method.
6. Study of evaporation from four types of evaporation pans confirmed previous studies in showing that pan evaporation is generally higher than lake evaporation and that pan-to-lake coefficients have a pronounced seasonal variation. Annual pan-to-lake coefficients are reasonably consistent with previous values, but further studies of such coefficients under different conditions of climate, lake area, and lake depth will be required for more accurate interpretation of available evaporation-pan records in terms of lake evaporation.

RECOMMENDATIONS

1. Conduct further studies of the mass-transfer equations (Sutton's, Sverdrup's, and the new, simplified, empirical one), the energy-budget equation, and pan evaporation on a comparative basis at Lake Mead.
2. Test the proposed simplified method for obtaining continuous records of evaporation for any reservoir at Lake Mead and at a number of additional reservoirs with different conditions of terrain and climate.
3. Study the mass-transfer and energy-budget techniques as methods for estimating water losses by evapotranspiration.
4. Expand the present program for the collection of water-temperature data in lakes, reservoirs, and streams throughout the country and obtain wind data at the sites of selected existing and proposed reservoirs. These data, together with available climatological information, would assist in predicting evaporation prior to the establishment of a new reservoir. They would also be useful in interpreting existing records of pan evaporation in terms of lake evaporation.

INTRODUCTION

Net water loss by evaporation, and by transpiration from vegetation along water courses, in the 17 western states has been roughly estimated to be over 30 million acre-feet per year, more than twice the annual flow of the Colorado River. This would be enough water to irrigate 15,000 square miles of desert, an area nearly half that of the state of New York or Pennsylvania.

Over half this loss is by evaporation from reservoirs, lakes, and streams, approximately 15 million acre-feet being from reservoirs alone. Each new reservoir adds to this loss, yet reservoirs must be built to salvage water which otherwise would be lost and make it available at the time it is most needed.

There is no practicable known method for preventing evaporation losses, but it is possible to reduce them by proper planning of reservoir location and by establishing suitable operating procedures. Evaporation loss is only one of many factors involved in reservoir location and operation, but it is important that its magnitude be accurately determined so that it can be properly weighted against the other factors. Average yearly evaporation figures are not sufficient even if fairly accurately known; the monthly variation must be determined, especially when a chain of reservoirs of different areas and depths are operated as an integrated group, as is the case on the lower Colorado River.

Water losses by transpiration from phreatophytes (deep-rooted plants, mostly nonbeneficial, which use a great deal of water, for example, salt cedar) are estimated to be as great or greater than losses by evaporation. These losses may be greatly reduced by removal of the phreatophytes, but this procedure is uncertain and expensive. Again, accurate estimates of water losses by transpiration are needed to determine whether a removal program is worthwhile.

The Lake Hefner study represents an integrated attack on the water-loss problem. It comprised investigations of both old and relatively new methods of determining evaporation. It is an excellent example of a successful cooperative effort by specialists in different fields; the collaborating scientists and engineers were from five fields of science (geology, hydrology, meteorology, oceanography, and physics) and from five agencies in three departments of the federal government (Bureau of Reclamation and Geological Survey of the Interior Department, Bureau of Ships and Navy Electronics Laboratory of the Navy Department, and Weather Bureau of the Department of Commerce).

Historical Review

The Lake Hefner study was an outgrowth of the cooperative investigations at Lake Mead, in Arizona and Nevada, undertaken in late 1947, 1948, and early 1949 by representatives of the Departments named above. The Navy Electronics Laboratory contributed to the Lake Mead investigations by applying oceanographic techniques to the determination of seasonal patterns of circulation in the lake; the temperature data obtained were also used, with meteorological data, for estimating the thermal energy budget of the lake.

The latter study resulted in preliminary estimates of monthly evaporation. Discussion of the energy-budget technique by representatives of the agencies concerned indicated that a wider application of this approach might be worthwhile. The application of mass-transfer theory, which attempts to explain evaporation in terms of the factors affecting the transport of water vapor, to the evaporation problem was also discussed. Numerous investigators had studied the mass-transfer problem, but the practical instrumentation required for measuring the meteorological factors in the field had not been developed. The Navy Electronics Laboratory was in the process of developing instruments for measurement of meteorological factors affecting radio-wave propagation; these instruments appeared to be adaptable to the mass-transfer measurements.

At a joint conference on 13-15 December 1948 in Boulder City, Nevada, it was decided to investigate these relatively untested techniques, and a program was outlined. Subsequent conferences modified the details, but the basic program remained unchanged except for the addition of investigations by the Weather Bureau of evaporation from standard and experimental evaporation pans — a necessary part of any comprehensive study of methods for determining evaporation.

The Problem

At present there are four basic methods for determining evaporation from water bodies:

1. *The water-budget* method is the most direct; evaporation is determined by difference from measurements of inflow, outflow, and changes in storage.

Unfortunately this method is not generally practicable because errors in measuring inflow and outflow are often large compared to evaporation, and seepage and bank storage are uncertain and often unmeasurable terms.

2. *Evaporation pans* have been used for many years as an index to evaporation from lakes and reservoirs; over 4000 station-years of record have been collected at points throughout the country. In spite of many investigations, however, the general validity of "pan coefficients" (determined under specific experimental conditions) used for converting pan evaporation to lake evaporation has always been subject to question.

3. The thermal *energy-budget* method has been used for some time to estimate evaporation from the oceans and has been applied to restricted studies of evaporation from inland water bodies. It is based upon assessment of all the sources of incoming and outgoing thermal energy plus changes in energy storage, the difference being the energy utilized in evaporation. General application of this technique has awaited the development of adequate instruments.

4. Modern *mass-transfer* theory is the basis for the fourth method — measurement of the factors affecting the actual removal of water vapor from a lake by processes of turbulent diffusion and transport.

The last two methods had never been adequately tested at natural scale; for such a test an acuurate independent determination of evaporation by the water-budget method was essential. Tests of evaporation pans against independent measurements of evaporation from a large natural water body, with concurrent measurement of pertinent meteorological factors, were also inadequate.

Lake Mead was not suitable for the initial test; the water budget could not be determined with sufficient accuracy and its large size, irregular shape, and mountainous terrain introduced too many complicating factors. The first step in the program, therefore, was the search for a lake or reservoir where an accurate water budget could be established as a control. Requirements for testing the energy-budget and mass-transfer approaches introduced further restrictions, so that more than a hundred lakes and reservoirs in the West had to be considered to find one whose characteristics approached the stringent requirements.

After considering the advantages and disadvantages of all of these lakes and reservoirs, Lake Hefner, Oklahoma City, was chosen at a joint conference of the cooperating agencies in Boulder City on 17-19 October 1949. Installation of the equipment at Lake Hefner was completed in April 1950 and, after a preliminary study of meteorologic and hydrologic conditions, routine measurements began the latter part of April and continued through August 1951.

This volume reports the results of this investigation — the most comprehensive study of its type yet undertaken. Several million measurements were recorded, read, tabulated, and analyzed. These will be summarized and published in a subsequent volume (Volume II, Data Report) of this series. The results appear to have justified the effort; in addition to increasing the value of records obtained by traditional methods, the new techniques have proved both scientifically sound and operationally practicable.

PERSONNEL

The following list of personnel contributing to this project includes only those assigned by the collaborating agencies for appreciable periods of time; contributors not directly assigned are listed under "acknowledgments."

General Supervision

Bureau of Reclamation. J. R. Riter, Chief, Hydrology Branch, Project Planning Division, was in general charge of the work for this division of the Bureau, with technical direction of the work under the immediate supervision of W. U. Garstka. Washington representative of the Division was M. D. Dubrow, succeeded by W. D. Romig. C. P. Vetter, Chief, Office of River Control, Region III, was in charge for that office.

Geological Survey. R. W. Davenport, Chief, Technical Coordination Branch, Water Resources Division, was responsible for general supervision of the work of the Survey, with technical direction by W. B. Langbein, Chief, Research Section, Technical Coordination Branch. G. E. Harbeck, Jr., was responsible for the water-budget work and for field liaison. W. O. Smith acted as the Geological Survey representative during preliminary phases of the work.

Bureau of Ships. Throughout most of the project, G. B. Cummings, Chief Civilian Assistant, Sonar Branch, Electronics Division, was responsible for the Bureau's participation. His personal interest and unusual ability to coordinate diverse activities contributed greatly to the establishment and success of the project.

Navy Electronics Laboratory. R. Dana Russell, Senior Consultant (Geophysics), was in general charge of the project for the Laboratory, with L. J. Anderson responsible for the instrumentation, E. R. Anderson for the energy-budget approach, and J. J. Marciano for the mass-transfer work.

Weather Bureau. W. E. Hiatt, Chief, Hydrologic Services Division, was in general charge for the Bureau, succeeding the late Merrill Bernard, who with R. K. Linsley, now at Stanford University, was responsible for the initiation of the Bureau's phase of work.

Field Personnel

S. K. Jackson, District Engineer (SW), Water Resources Division, Geological Survey, served as local coordinator for the field work at Oklahoma City. The construction and operation of the stream-gaging and water-level installations at Lake Hefner, together with other observations required for the water-budget control, were directed locally by F. W. Kennon, Hydraulic Engineer, Geological Survey. He was assisted by H. O. Wires and, for part of the time, by G. E. Koberg. This group also constructed and operated the Cummings Radiation Integrator, mapped the shore of the lake, and provided horizontal control for the hydrographic survey. During part of the time they were also responsible for the maintenance and operation of one of the shore meteorological stations.

Installation, operation, and maintenance of the meteorological equipment at Lake Hefner were directed locally by LT C. C. McCall, USN, assisted by J. D. M. Freitas, MNC, USN, and H. E. Knudsen, MNC, USN, assigned to the project by the Bureau of Ships. This group was also responsible for taking the bathymetric data required for the hydrographic survey of the lake. In addition, they were responsible for the thermal surveys of the lake (observations of temperature-depth profiles at 16 stations distributed over the lake at intervals of 7 to 10 days, plus a daily profile at one station).

The field work of the Weather Bureau was coordinated by R. J. MacConnell, South Central Area Hydrologic Engineer. The observational program was under the direction of W. E. Maughan, Meteorologist in Charge, Oklahoma City Office. E. L. Smith served as observer until April 1951, when he was relieved by B. J. Stringer.

The success of the entire project was largely due to the conscientious and untiring efforts of these field personnel, who regularly obtained observations under a variety of weather conditions, frequently unfavorable to the operation of experimental equipment.

Office Personnel

Processing of the data for the mass-transfer and energy-budget studies required the maintenance of a staff of six to seven people at the Navy Electronics Laboratory. Though these individuals initially were Laboratory personnel, the Laboratory had to concentrate its effort on urgent defense work soon after the beginning of hostilities in Korea, and personnel thereafter were furnished by the Geological Survey.

Processing of the meteorological data was at first under the supervision of J. J. Marciano, Navy Electronics Laboratory, and later under G. E. Harbeck, Jr., Geological Survey, with G. E. Koberg, also of the Survey, in immediate charge of the work. Machine computations of the voluminous data (on about 200,000 IBM punch cards) were made by the

Institute of Numerical Analysis, National Bureau of Standards, under a contract with the Geological Survey. The data were analyzed by J. J. Marciano and G. E. Harbeck, Jr., assisted by G. E. Koberg.

Processing and analysis of the energy-budget data were under the supervision of E. R. Anderson, Navy Electronics Laboratory; O. E. Leppanen of the Geological Survey assisted with the statistical analyses.

Analyses for the Weather Bureau part of the program were performed in the Hydrologic Investigations Section (T. J. Nordensen, Chief), with W. E. Fox and E. S. Thompson responsible for most of the analytical work. Planning of the analyses and preparation of the Weather Bureau's section of the report were under the direct supervision of M. A. Kohler, Chief Research Hydrologist.

Final editing of the complete report and preparation for publication were done by the Technical Information Division of the Navy Electronics Laboratory. R. Dana Russell acted as editor for the collaborating agencies and prepared the sections for which no author is listed.

ACKNOWLEDGMENTS

In a large and complex cooperative effort such as the study reported here, it is almost impossible to acknowledge the assistance of all the people who contributed, directly or indirectly, to the successful completion of the work. A great many individuals in the field offices of the Bureau of Reclamation and the Geological Survey contributed data and assistance in the search for a suitable lake; their help has been acknowledged in the report on that phase of the study. It is hoped that most of the other major contributors not directly assigned to the work have been included below.

The active cooperation of officials of the Oklahoma City Water Department is gratefully acknowledged, particularly the assistance of M. B. Cunningham, Superintendent and Engineer, F. S. Taylor, then in charge of the Lake Hefner Filter Plant, S. K. Bean, Civil Engineer, and Frank Herrmann, Office Engineer.

Calibration of the Venturi meter at the Filter Plant was performed by D. M. Lancaster, R. H. Kuemmich, and R. B. Dexter of the Hydraulic Laboratory Section, Research and Geology Division, Bureau of Reclamation, Denver, Colorado. The description in this report of the technique and equipment used was taken from their unpublished report prepared upon completion of the calibration.

D. W. Pritchard, formerly of the Navy Electronics Laboratory and now Director, Chesapeake Bay Institute, The Johns Hopkins University, contributed to the preliminary planning and some of the theoretical analysis, and also made suggestions for the adaptation of the energy-budget technique to the Cummings Radiation Integrator. R. L. Denton, Navy Electronics Laboratory, designed the galvanometer amplifier used in the recording equipment for the meteorological measurements, and F. R. Bellaire, also of the Laboratory, was responsible for the construction of much of the meteorological instrumentation and for many of the basic instrumentation ideas. E. E. Gossard of the Laboratory assisted in the review and analysis of mass-transfer theories. Chemical analyses of the samples of water from Lake Hefner and Lake Mead used in the emissivity measurements were made by T. B. Dover and C. S. Howard of the Quality of Water Branch, Water Resources Division, Geological Survey.

Critical review and comments on the development of the energy-budget equation were contributed by Dr. H. U. Sverdrup, Director, Norsk Polarinstitutt, Oslo, Norway, and by Dr. R. B. Montgomery, Brown University, and on the mass-transfer approach by O. G. Sutton, Military College of Science, Shrivenham, England.

Rough drafts of this report have been critically reviewed by a number of people whose suggestions have been incorporated in the text without specific acknowledgment. Review and comments by J. T. Gier and R. V. Dunkle, University of California, E. C. Buffington, Navy Electronics Laboratory, and H. C. S. Thom, C. S. Gilman, F. D. White, L. P. Harrison, J. L. H. Paulhus, and T. H. MacDonald, Weather Bureau, have been particularly helpful.

General Description of Lake Hefner

by G. Earl Harbeck, Jr.*

BASIS FOR CHOICE OF LAKE HEFNER

The selection of a reservoir or lake suitable for a comprehensive study of the exchange of energy between the atmosphere and a water surface is not a simple matter. It was planned to determine evaporation by three methods: (1) water budget, (2) energy budget, and (3) mass transfer. Since the water budget was to be the control for evaluating the other two methods, it was imperative that the water budget be accurate. This was the paramount consideration in selecting the lake (Harbeck *et al.*, 1951).

The energy-budget and mass-transfer methods also imposed restrictions on the choice of the lake. It was desired, for example, to minimize the effects of mechanically induced turbulence on the wind structure, and for that reason canyon reservoirs were deemed unsatisfactory. A deep lake was desired to avoid the problem connected with the penetration of solar radiation to the lake bottom.

The requirements established for the lake were as follows, in order of importance:

1. Water Budget

a. The error in the monthly difference between total inflow and outflow, including both surface and subsurface flows, and allowing for changes in storage (including bank storage), must be less than 5 per cent of the best existing estimate of the mean monthly evaporation loss. Inflow and outflow preferably should be as small as possible during the period of high evaporation.

b. Infrequent short periods of storm inflow, during which the water budget cannot be determined with the accuracy required under 1a, can be tolerated.

c. Subsurface inflow and outflow must be negligible compared with evaporation, unless it is known that they can be measured accurately.

d. Substantial bank storage is undesirable.

e. Transpiration losses must be small.

f. An accurate area-capacity curve is needed, but if not available, a hydrographic survey can be made.

2. Size

a. The minimum desirable dimensions are 3 miles in width and 5 miles in length, or the area should not be less than 10 square miles if the water body is nearly circular or if the longer dimension is in the direction of the prevailing summer winds.

b. The maximum desirable size is 50 square miles if the lake is nearly circular, and otherwise 30 square miles.

3. Shape

a. A circular shape is ideal. A very irregular shore line is unsatisfactory; an irregular shore line downwind is not so objectionable as one upwind. A long, narrow lake is not satisfactory.

b. An unobstructed expanse of water (few or no islands) extending 5 miles in the direction of the prevailing summer winds is desirable.

4. Depth

a. At least 80 per cent of the lake should be more than 5 feet deep (and preferably more than 10 feet). A deep lake is desirable.

b. Playa lakes should be considered if they meet all requirements except depth.

5. Topographic Setting

a. Low relief is preferable; lakes in canyons are not satisfactory.

b. The drainage area preferably should be small.

6. Location and Climate

a. An arid region is preferable.

b. There should be fairly long periods of no rainfall during the season of high evaporation, but infrequent storms are not objectionable.

c. It is preferable, but not essential, that the lake remain unfrozen in the winter.

The search for a lake suitable for the study has already been discussed (Harbeck *et al.*, 1951). The Geological Survey district offices in the western states and the Bureau of Reclamation regional offices were furnished copies of the specifications and were asked for recommendations. Their comments were also solicited on the suitability of lakes that appeared, on the maps available, to approach the desired size and

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shape. Those that appeared to satisfy the requirements in most respects were visited, and field examinations were made.

The final choice of Lake Hefner near Oklahoma City involved no compromise as to the accuracy of the water budget, although in certain other respects, such as size and climate, it does not meet the specifications completely. These departures from the specifications were unimportant and, in practice, the smaller size actually proved advantageous.

It was believed that satisfactory water budgets could be obtained for certain other lakes, such as Big Sage Reservoir in northern California and Pyramid and Walker Lakes in western Nevada, but these lakes were rejected because Lake Hefner appeared to approach the ideal lake more closely in most other respects.

PHYSICAL AND CLIMATOLOGICAL CHARACTERISTICS

The Reservoir Area and The Dam

Lake Hefner is a water-supply reservoir, owned by the City of Oklahoma City. The lake is fairly regular in shape (see map, frontispiece) and is formed by a long horseshoe-shaped dam on Bluff Creek in the southeastern part of T. 13 N., R. 4 W., approximately 8 miles northwest of the center of the city. The topography surrounding Lake Hefner is flat to gently rolling. The dam itself is the only significant obstruction to smooth wind flow in the area. The land slopes generally toward the northwest, roughly 30 to 40 feet per mile. The recent hydrographic survey of the lake shows its capacity at full pool (elevation 1199 feet) to be 75,355 acre-feet and its surface area 2587 acres. During the period 26 April 1950 to 31 August 1951, however, the stage was maintained between 1190.8 and 1195.3 feet; contents ranged from about 55,900 to 66,100 acre-feet and surface areas from 2148 to 2386 acres.

Although Lake Hefner lies in the Cimarron River Basin, it is supplied principally by a canal from the North Canadian River. In this area the elevation of the North Canadian River is several hundred feet higher than that of the Cimarron River, and only a short diversion canal was needed to permit gravity flow of selected water (chemical quality being the criterion) from the North Canadian River into Lake Hefner. Under the general operational plan, diversions are infrequent and of only a few days' duration, but they may be of considerable magnitude since the canal capacity is 1500 cfs.

Natural inflow to Lake Hefner is of little consequence so far as reservoir replenishment is concerned. The natural drainage area above the lake is only about 30 per cent larger than the area of the lake at full pool, so that storm runoff into Lake Hefner is usually much less than rainfall on the lake surface.

Vegetation is sparse except along the water courses. The soil is generally thin, and native grass is the predominant cover. The lake shore is relatively clean. There are a few marshy areas north and west of the dam, supplied by seepage through the dam. The average size of the marshy areas is considerably less than an acre, and the water consumed by the vegetation is accounted for in the water budget.

The Lake Hefner area has been developed for recreational purposes by the City of Oklahoma City. On the south shore, concrete boat pens have been provided in the boat harbor and a golf course has been constructed. A good road encircles the lake.

The dam and dike are of rolled earth fill with a clay core. The maximum height of the dam from thalweg to the road on top of the embankment is approximately 105 feet, but the average height is considerably less. The total length of the dam and dike is 3½ miles. Flow through an ungated spillway at the east end of the dam would begin if the stage reached 1201 feet. Tile drains and catch basins were installed in the main part of the dam to collect seepage through the dam and rainfall on the downstream face. The water thus collected is drained to the Bluff Creek channel below the dam.

Climatology

The climate at Lake Hefner has been classed as subhumid by Thornthwaite (1931). For the purposes of the water-loss study, a site having an arid climate would have been preferred, but of all the requirements governing the selection of a site, this was considered to be of least importance. Theoretically, at least, it would be desirable to choose a site where there is no rainfall, inflow, or outflow, and where evaporation rates are high. Evaporation rates at Lake Hefner were considered to be satisfactorily high, and the only effect of rainfall was to decrease the number of days for which an accurate water budget could be obtained.

Normal annual rainfall at Oklahoma City is approximately 31 inches. During the last 12 months of the observation period, rainfall at Lake Hefner totalled 27.1 inches. Rainfall at Oklahoma City is usually greatest during the April-June period, as was observed in 1951. During the 4-month period May to August 1950, however, rainfall at Lake Hefner totalled approximately 18 inches, with abnormally high rainfall in May and July. The monthly variation in rainfall is illustrated in figure 1.

Wind speeds measured at the 8-meter level at the barge station averaged 11.5 knots* during the 16-month period of observation. The relative frequency of occurrence of various wind speeds is illustrated in figure 2. Wind speeds were in the range 4 to 14 knots 59 per cent of the time and were less than 4 knots only 8 per cent of the time. The modal wind speed was approximately 7 knots.

The monthly variation in wind speed at Lake Hefner is shown in figure 3. (Data obtained during May and June 1950, the instrument "shakedown" period, were not used.) Although the month-to-month variation is much more erratic than for other climatic factors, such as temperature, strong winds predominate during the winter and spring months; even in summer months, the winds can hardly be classed as light.

Wind directions at the barge were tabulated using a special code, as shown in figure 4, in which the figures in the sectors are the code numbers used. This code was designed to facilitate selection, by the computing machines, of data from the station considered to be most representative of the air upwind from the barge. The considerations underlying construction of the code were as follows: (1) normal wind-direction distribution, as shown by a wind rose furnished by the Oklahoma City Weather Bureau Office; (2) number and location of the shore measuring stations, each location being determined by terrain and other practical considerations as well as by dominant wind directions; (3) terrain and other obstructions affecting air flow; (4) the fact that the equipment recorded wind direction only by the eight cardinal and intercardinal points of the compass. The arbitrary code shown in figure 4 represents a compromise between these various factors for sectors 2, 3, and 4, the intent being that measurements at stations 2, 3, and 4 be representative of their respective sectors. The remaining area was arbitrarily divided into three equal parts (sectors 1, 5, and 6). It was later found that the winds at station 4 (the tower station) were affected by the dike, particularly below 16 meters, and that station 3 was more rep-

*The Navy anemometers used in this study registered wind speed in knots. A knot is one nautical mile, or 1.15 statute miles, per hour.

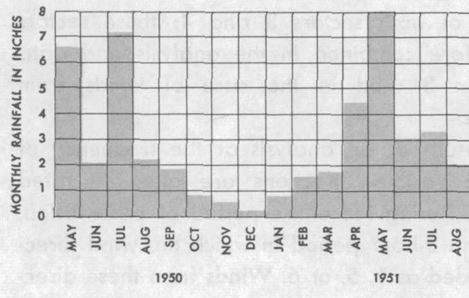

Figure 1. Monthly rainfall at Lake Hefner.

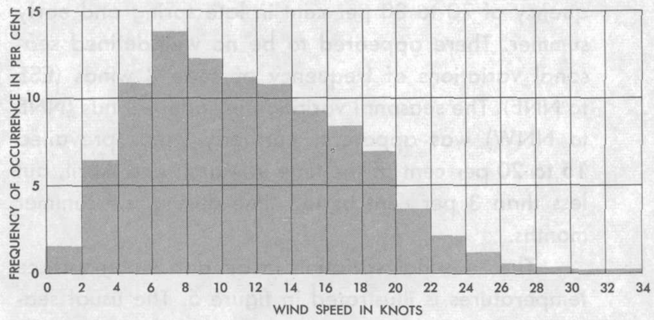

Figure 2. Frequency of occurrence of various wind speeds at Lake Hefner.

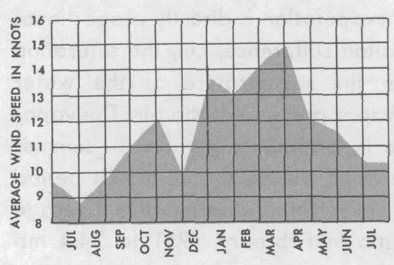

Figure 3. Monthly variation in wind speed at Lake Hefner.

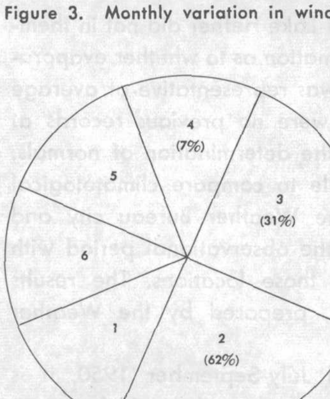

Figure 4. Frequency of occurrence of winds from various directions at Lake Hefner.

resentative of both sectors 3 and 4; these sectors were therefore combined in the analysis and data from station 3 used in the case of winds from both sectors.

The results of an analysis of the frequency of winds from various directions are also illustrated in figure 4. During the entire period of observation, there was no 3-hour period in which the wind direction was coded as 1, 5, or 6. Winds from these directions undoubtedly occurred for short periods of time, but sustained westerly winds were not observed. The code 2 winds (SSW to ESE) reached a maximum frequency of 70 to 80 per cent in late spring and early summer. There appeared to be no well-defined seasonal variations of frequency of code 3 winds (ESE to NNE). The seasonal variation in code 4 winds (NNE to NNW) was apparent; northerly winds prevailed 15 to 20 per cent of the time in March and April, but less than 3 per cent of the time during the summer months.

The seasonal variation in air and water-surface temperatures is illustrated in figure 5. The usual seasonal lag is readily apparent, particularly during the fall, winter, and spring.

The monthly variation in evaporativity as indicated by vapor pressures is illustrated in figure 6. For constant wind, evaporation is directly proportional to the so-called Dalton Difference, i.e., the saturation vapor pressure at the temperature of the water surface minus the vapor pressure of the air. The vapor pressure difference is, of course, greatest in summer and least in winter, and during the 16-month period of observation at Lake Hefner the monthly average ranged from 1.1 mb in February 1951 to 14.4 mb in August 1951.

The observations at Lake Hefner did not in themselves provide any information as to whether evaporation during this period was representative of average conditions, since there were no previous records at the lake available for the determination of normals. However, it was possible to compare climatological records obtained at the Weather Bureau city and airport stations during the observational period with long-term averages at those locations. The results are shown in figure 7, prepared by the Weather Bureau.

During the period July-September 1950, it is probable that evaporation from Lake Hefner was below normal; wind speeds, and vapor pressure differences as indicated by air and dew-point temperatures, were below normal. As might be expected, sunshine was also below normal.

During the period June-August 1951, wind speeds and air temperatures were near normal. Dew-point temperatures were above normal, however, and evaporation from Lake Hefner was probably slightly below normal during this period also.

Monthly precipitation at Oklahoma City was substantially above normal during May and July 1950, and May 1951, but was well below normal during the period September-December 1950.

For the entire 16-month period of observation, it is believed that evaporation may have been slightly below average because of the two below-normal summers but that the period, in general, was reasonably representative.

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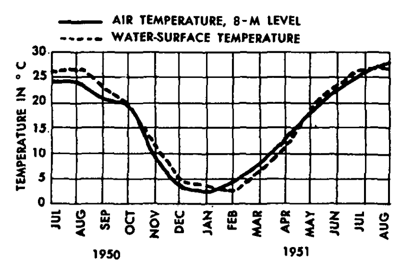

Figure 5. Average monthly air and water-surface temperatures at Lake Hefner.

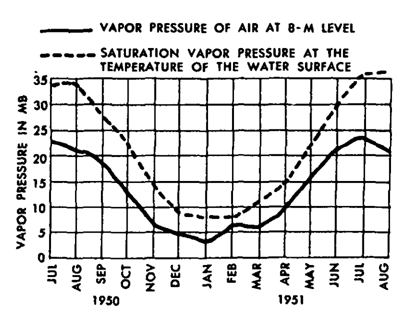

Figure 6. Average monthly vapor pressures and saturation vapor pressures at Lake Hefner.

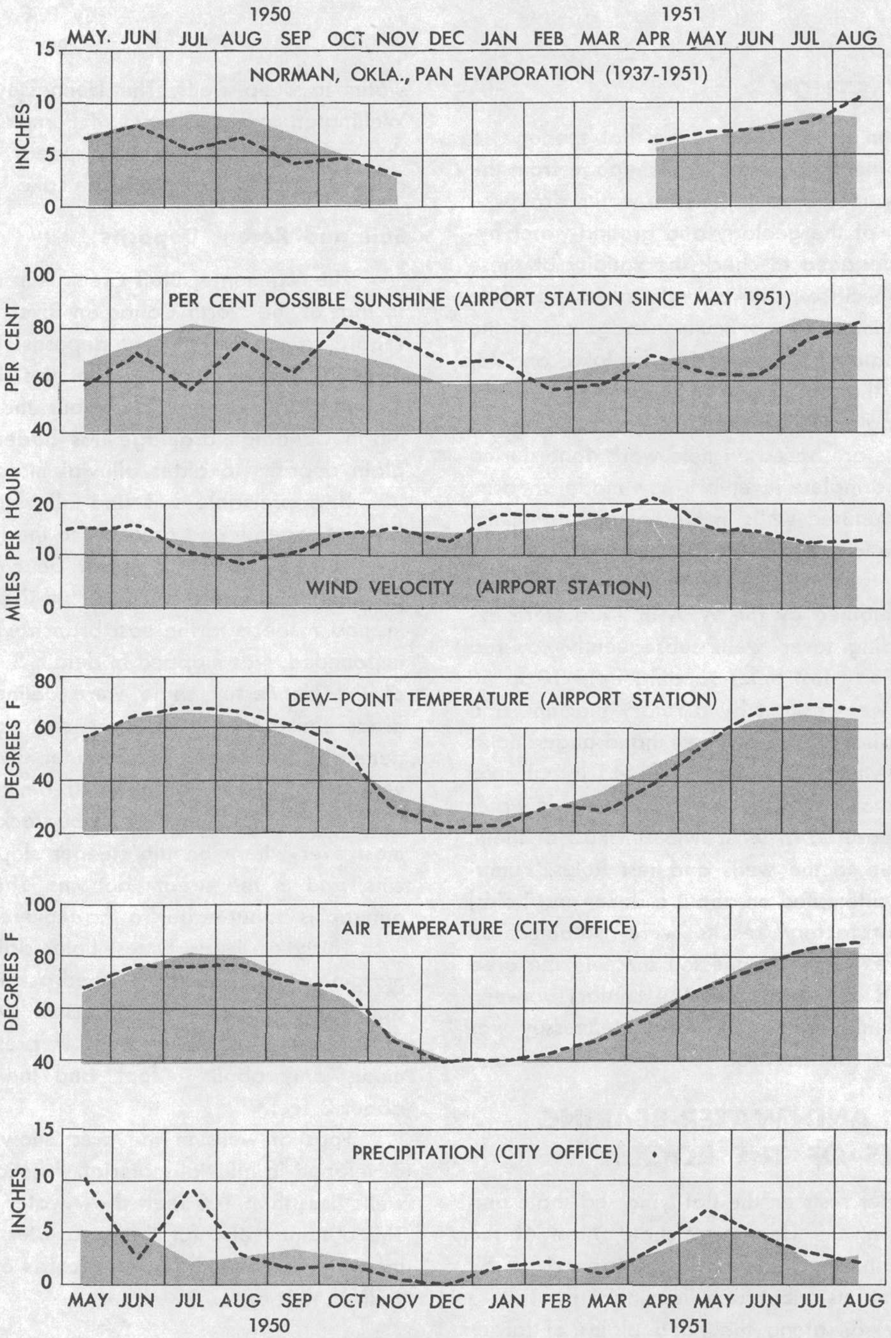

Figure 7. Comparison of meteorological data at Oklahoma City during period of Lake Hefner investigations with 1934-1951 averages.

Geology and Ground-Water Hydrology of the Lake Hefner Area

by P. E. Dennis*

SCOPE OF STUDY

It has been generally assumed that seepage to Lake Hefner is negligible, and that seepage from the lake is largely measured in surface-water drains.

The study of the geology and ground-water hydrology was designed to check the validity of these assumptions. Specifically, it was undertaken to provide the best (1) estimate of maximum seepage out of the lake, (2) estimate of seepage into the lake, and (3) evaluation of the present surface measurements of outward seepage through the dam.

The results are based on field work done during June 1950. A complete inventory was made of farm, domestic, and unused wells in the immediate vicinity of the lake; static water-level measurements were obtained in twenty-five of the wells. Records of wells in the area obtained by the WPA in 1936 were examined, including seven wells subsequently covered by the lake. Forty test holes ranging from 10 to 50 feet in depth were drilled by a rotary machine and logged from ditch samples. Three hand-auger holes were put down to the water table. In all, 68 points were obtained on the water surface and water levels were measured at least twice in most of them. Levels were run to the wells and test holes. Pumping tests were attempted on about a dozen test holes; reasonably satisfactory results were obtained on three. Exposures of the Hennessey shale in the area were examined and described. The contact between soil-covered Hennessey and exposed Hennessey was mapped in part of the area.

GEOLOGY AND WATER-BEARING PROPERTIES OF THE ROCKS

Lake Hefner rests on the flat-lying red shale and siltstone beds of the Hennessey shale. Most of the alluvial cover in this area has been stripped off by Bluff Creek and its tributaries, leaving only 1 to 3 feet of clayey soil along the flood plains of larger streams and on some of the broader divides. Older alluvial deposits are found only on the divide between the North Canadian and Cimarron Rivers. The Hennessey shale is more than 250 feet thick and is underlain by the Garber sandstone and Wellington formations, whose sandstone beds yield soft artesian water to deep wells. The Hennessey, Garber, and Wellington formations are of Permian age, and are the only formations which appear related to the ground-water hydrology of the Lake Hefner area.

Soil and Recent Deposits

The regimen of Bluff Creek is in marked contrast to that of the North Canadian River, especially with regard to soil and Recent deposits. The Bluff Creek drainage has stripped most of the soil and alluvial mantle from the bed rock, but the more sluggish North Canadian drainage has added Recent flood-plain deposits to older alluvial materials and soils.

If appreciable soil and alluvial deposits were present beneath and adjacent to the lake, their effect on the area hydrology might be considerable. Accordingly, the west branch of Bluff Creek, comparable in most respects to the east branch where the lake is impounded, was mapped in detail. Surface exposures of the Hennessey shale were delineated from the areas covered by soil. Some soil was found to be generally present on the broader divides and in the vegetated parts of the larger stream courses. This soil, however, is thin and clayey. Bedrock is exposed almost everywhere on the steeper slopes, along road cuts, and in the stream bottoms. The total area of outcrop is about equal to that covered by soil.

Thirty of the forty test holes drilled around the margin of the lake entered bedrock immediately; no appreciable amount of soil cover was found. The thickness of soil penetrated, if present, was commonly only about 1 foot, and the maximum was about 3 feet.

Logs of wells in the area show that none are developed in alluvial material. Practically all shallow wells (less than 100 feet) draw water from red shale. This obtained also for the area later covered by the lake, as indicated by WPA records of seven shallow wells in that area.

Pleistocene Terrace Deposits

Alluvium, consisting of gray-to-buff-colored sand, gravel, and clay, underlies the broad terrace-like divide separating the Bluff Creek and North Canadian River drainage basin. Jacobsen and Reed (1949) have described these materials as Pleistocene terrace deposits. Some beds consist of well-sorted sand and others of unsorted clay, sand, and gravel. Part of the

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terrace area is covered by windblown sand. Much precipitation is absorbed by these terrace materials, and sand beds near the base constitute an important aquifer in the Bethany and Warr-Acres areas.

The canal connecting the North Canadian River with Lake Hefner is cut through these terrace materials for 3 miles of its 5-mile length and acts as a drainage ditch when it is not being used to transport surface water. Drainage water entering the lake through the canal comes from this source.

Much of the low-flow surface water that enters the lake through the tributaries of Bluff Creek also originates in these terrace materials. All springs and seeps examined in the Lake Hefner drainage area issued at the contact between these alluvial materials and the underlying shale. Thus all ground water from the terrace materials that enters the lake can be measured in various channels.

The terrace materials probably recharge the upper part of the Hennessey shale also. Joints and other openings in the shale are small, however, and they accept recharge slowly. Where saturated terrace sand is in contact with the shale, a continuous supply of water is available; direct precipitation on the shale would largely run off. The rate of this recharge process was not determined, but it is probably slow; the recharge would affect only the shale that underlies or is very near the alluvial cover.

Hennessey Shale

The Hennessey shale controls seepage conditions in the vicinity of the lake. The formation consists of bright red interbedded silty shale and siltstone. Most of the more massive beds are siltstones, but a few beds of fine-grained sandstone also occur. The beds were revealed to be lenticular by tracing some of them along their strike. The lithology of the beds underlying the lake can be approximated from the composite section in the adjoining column compiled from descriptions of exposures in the lake area. Breaks in the section are indicated by spacing; the composite nature of this section may involve unavoidable minor duplications or omissions.

Another composite section compiled from logs of the test holes is similar to the above except that the massive siltstone and sandstone beds at the base of the outcrop section were not penetrated by the drill.

The weathered Hennessey is a stiff, plastic clay, and wetted cuttings from test holes showed no permeable openings. The shale exposed above the water table, however, had numerous joints some of which were probably produced by shrinkage on drying.

The regional dip of the beds in the Lake Hefner area is west at about 40 feet per mile. Brunton-compass measurements of outcrops show strikes ranging between N. and N. 20° E., and dips ranging between 1° and 7° W. A few minor folds and dips to the east and south were noted. Much of this surface structure has probably resulted from slumping and soil creep.

Composite Section of Hennessey Shale in the Vicinity of Lake Hefner

Description of Material	Thickness in Feet
Soil, shaly, reddish brown	2
Shale, bright red, fissile	4
Sandstone, red, fine-grained; lenses out east and west of section	2
Shale, with interbedded siltstone	3
Shale, red, fissile	2
Shale and siltstone, thin beds interbedded	3
Shale, red, fissile	7
Shale, red, silty	2
Siltstone, red, shaly	5
Shale, silty	3
Siltstone with intraformational conglomerate, shaly partings (seeps issue from this member)*	½
Siltstone	2
Shale, silty	3
Clay, red, shaly	5
Shale, red, silty	8
Shale and siltstone, thin beds interbedded	13
Shale, red, fissile	9
Shale with thin beds of siltstone	5
Shale, red, fissile	15
Shale, red, fissile	4
Siltstone, red	1/3
Shale, red, fissile	3
Shale, red, fissile	6
Shale and siltstone, interbedded	5
Sandstone, fine-grained, cross-bedded	1
Siltstone and fine-grained sandstone, mostly thin-bedded	13
Siltstone, shaly and with shale partings	8
Shale, fissile, mostly red but containing 1- to 3-inch layer of greenish-gray shale (seeps)†	5
Sandstone, fine-grained	1

* Seeps issue from shaly partings and from vertical joints in siltstone.

† Seeps issue from bedding-plane joints, or shears, in shale above underlying sandstone.

The Hennessey Shale as an Aquifer

General indications suggest that the Hennessey shale is extremely impermeable. However, in the Lake Hefner area the Hennessey is an aquifer and furnishes small quantities of water to wells. This is substantiated by the WPA well records, oral reports of drillers and farmers in the area, and the inventory of wells and test holes pumped during this investigation.

Wells north of the terrace materials can be classified in three groups. About 80 to 90 per cent are shallow and derive their water from red shale. The water from these is generally satisfactory in quality but in many places inadequate in quantity for a windmill. A few wells have been drilled to depths of 100 to 300 feet; these derive their water from sandstone or shale, or both. The water generally is highly mineralized and is used only for watering stock. The third type of well is 400 to 800 feet in depth and derives water from the Garber sandstone and Wellington formation. We are only concerned with the first group of wells.

One of the most interesting features of the shallow wells is that the water in them comes from shale and not from sandstone beds in the shale. Most of these wells supply small quantities required for household use, but not every drilling is successful. The water generally comes from depths of 10 to 20 feet, although such wells are usually drilled 40 to 70 feet deep to provide storage. If any particular permeable bed were the aquifer for these wells, one would expect the wells to become deeper westward because of the regional dip. Such is not the case. Also, in wells on higher ridges, water comes from shallower depths than would be expected if the aquifer were a nearly horizontal bed.

In all the forty test holes, water was encountered at depths of 2 to 25 feet. When pumped, the open test holes yielded a fraction of a gallon to 7 gallons a minute, being replenished from the surrounding shale. In none of the test holes was water obtained from siltstone or sandstone.

This evidence supports the conclusion that the near-surface part of the Hennessey shale in the Lake Hefner area is an aquifer of low yield. The water-bearing zone is generally within 10 to 20 feet of the surface and is not restricted to any particular beds, although the shale is more likely to be water-bearing than is the siltstone or sandstone. The interstices are joints and other fractures (possibly solution cavities, also) which do not persist at depth. It seems likely that the openings may be confined to the zone that has been above the water table during some part of Recent geologic time.

The Garber Sandstone and Wellington Formation

Sandstones of the Garber sandstone and Wellington formation constitute the most important aquifers in the Oklahoma City area (Jacobsen and Reed, 1949).

It is evident that the artesian heads in the wells in these aquifers are not sufficient to force water through the shales into Lake Hefner. According to Jacobsen and Reed (1949), the original static head probably was about 100 feet below the surface. Sufficient head probably never was present to make the formations a source of the shallow ground water.

GROUND-WATER HYDROLOGY

Adequacy of the Present Surface Measurements of Outward Seepage Through the Dam

The dam that contains Lake Hefner is of rolled-earth fill and is not completely impermeable. Drains have been provided around most of the embankment to channel and remove the outward seepage through the dam. Points at which measurements are presently being made on the drainage ditches are marked AA to FF on the piezometric map (fig. 8).

On the west side of the lake some seepage through the dam probably reaches the west branch of Bluff Creek without being caught in the measured drains. On the other hand, a part of the seepage measured at station FF comes from the east and is unrelated to seepage through the dam and deep seepage from the lake. However, the amount of such seepage from the east appears to be small.

All the measured drains have rich growths of water-consuming plants along their reaches, and evapotranspiration losses during summer months probably are very high. Therefore, only the early spring, winter, and late fall measurements are likely to be reliable. These were made and are reported in the section entitled "Water-Budget Control."

The Piezometric Map

Elevation points on the water surface as determined in 68 shallow wells and test holes around the lake are the basis for the piezometric map (fig. 8). When the data were being assembled, it was not

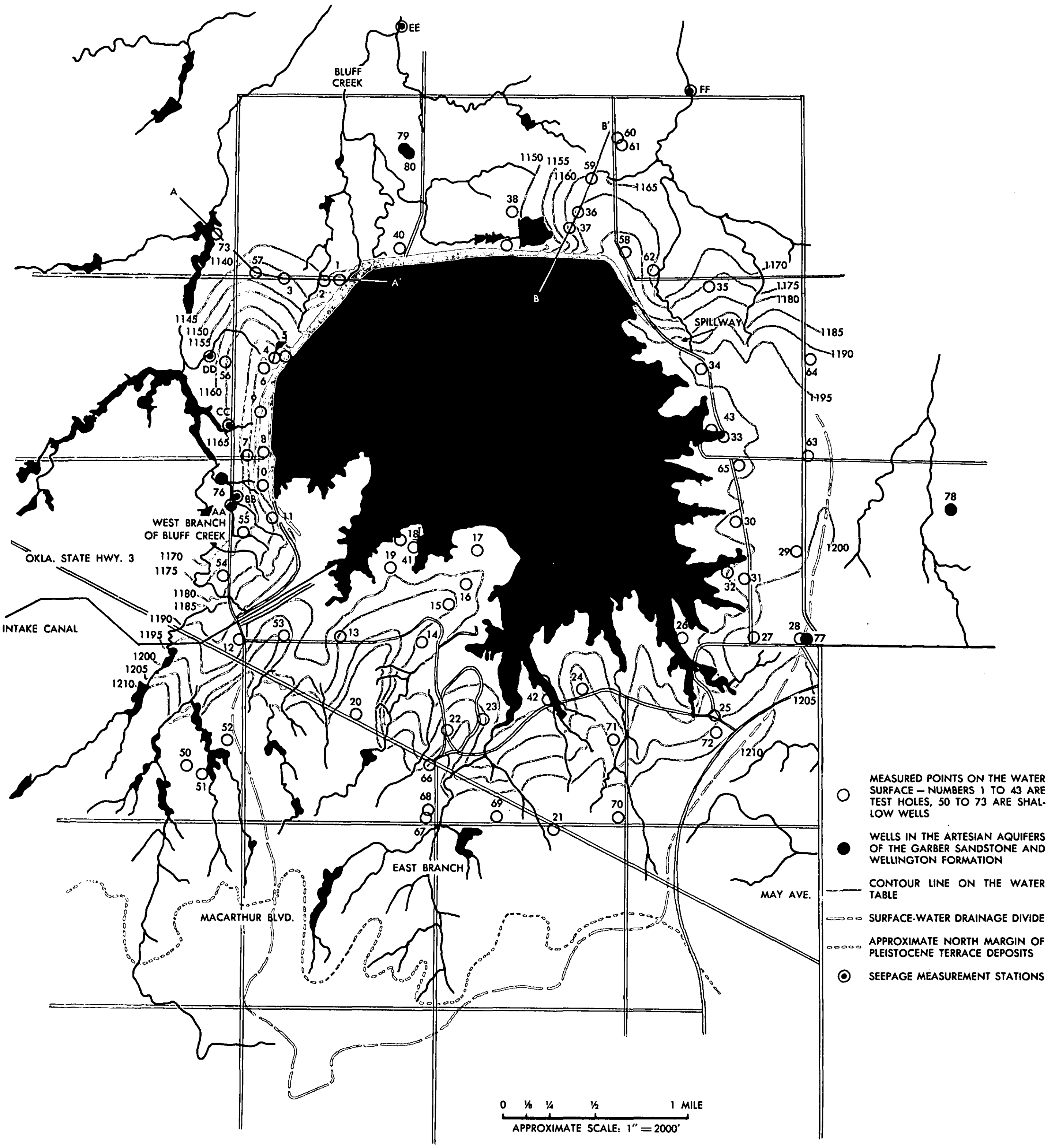

Figure 8. Piezometric map of Lake Hefner area showing water-table contours and other hydrologic features.

known that a simple water-table surface was present in the shale. On the contrary, a number of confined bodies of water, each with its own particular piezometric surface, was the condition to be expected in gently dipping, relatively impermeable beds. However, if the aquifer is a series of interconnected joints and other fractures in the upper part of the beds, then a water table typical of fracture openings might be expected.

Most elevations on the shallow-water surface, when plotted on the map, are compatible with water-table conditions. The highest water-surface elevations correspond with the points that are highest topographically and the water surface appears to be a subdued replica of the land surface. Only locally were artesian conditions or perched water-table conditions encountered. In test hole 21, water was encountered in shale below siltstone at a depth of 18 feet. When the hole was bailed below that depth, water could be heard spouting into it under considerable pressure. The water level in this hole is 6.44 feet below the land surface and at an elevation at least 10 feet higher than the water-table surface at that point. Thus small local artesian aquifers do occur in the shale. In the three hand-auger holes that were put down to the water-table, a thin perched water body was penetrated above several inches of dry shale before the main water body was encountered.

If the piezometric map is considered as a water-table map in fractured material, several characteristics are noted that appear to have resulted from the impounding of the lake. From the intake canal and eastward and northward around the south and east sides of the lake to the spillway the lake is effluent, the movement of the shallow ground water being toward the lake. The lake probably has modified the water table in this area very little, except possibly to decrease the gradient slightly. The gradient for about 2 miles at the south end of the lake is approximately 40 to 50 feet per mile. Along the east shore of the lake the gradient is much gentler and the ground-water divide is within half a mile of the lake.

North of the intake canal on the west and north sides of the lake to the spillway, the lake is losing water by seepage. The dike that contains the lake on the west side was built near the crest of the original drainage divide between the east and west branches of Bluff Creek. The water table has been raised and a steep gradient away from the lake has been created by recharge from the lake. The gradient westward and northwestward from the lake in this area is approximately 80 to 100 feet per mile. A trough in the water table is created by the drain at the northwest end of the dam, and a part of the ground water flowing northwestward along the ridge finds its way back into this drain. These relations are shown by section A-A′ in figure 9.

At the northeast end of the dam a major trench sloping westward is created by the drains at the base of the dam. Beyond the trench, water levels rise rapidly again. Whether they are higher than they were before the impounding of the lake is not known. Relations in this area are shown on section B-B′ (fig. 9). Too few points of control were obtained to permit completing the map at the northwest end of the lake.

Permeability of the Hennessey Shale

Examination of the Hennessey shale in outcrops and in well cuttings reveals a clayey, silty fissile red shale of apparently very low permeability. On the basis of such an examination only, one would conclude that the material is an ideal confining bed. The fact remains, however, that in its upper part it is sufficiently permeable to furnish water to most of the wells in the area. The specific capacities of several of these wells are of the order of 1 to 2 gallons per minute per foot of drawdown. Bailing of the test holes showed that none would yield as much as 40 gallons a minute and some only a fraction of a gallon per minute. However, one test hole was pumped at the rate of 7 gallons per minute for 1 hour with only 2 feet of drawdown. All these facts indicate that the upper part of the Hennessey shale is an aquifer of substantial importance, though of small capacity and low permeability.

Pumping tests were attempted on several test holes and farm wells. For test holes 1 and 26 and farm well 53, recovery curves were plotted from which transmissibilities were computed by the Theis nonequilibrium formula (Theis, 1935). Test hole 1, which is northwest of the lake and very close to the base of the dam and near the drain, gave a transmissibility of about 2500 gallons per day per foot. Test hole 26, which is at the southeast edge of the lake, showed a transmissibility of 125. These are thought to represent approximately the extremes of transmissibility of the shale. Farm well 53, southwest of the lake, showed a transmissibility of about 1600, which is thought to be considerably higher than average because the well has a reputation of being unusually "strong."

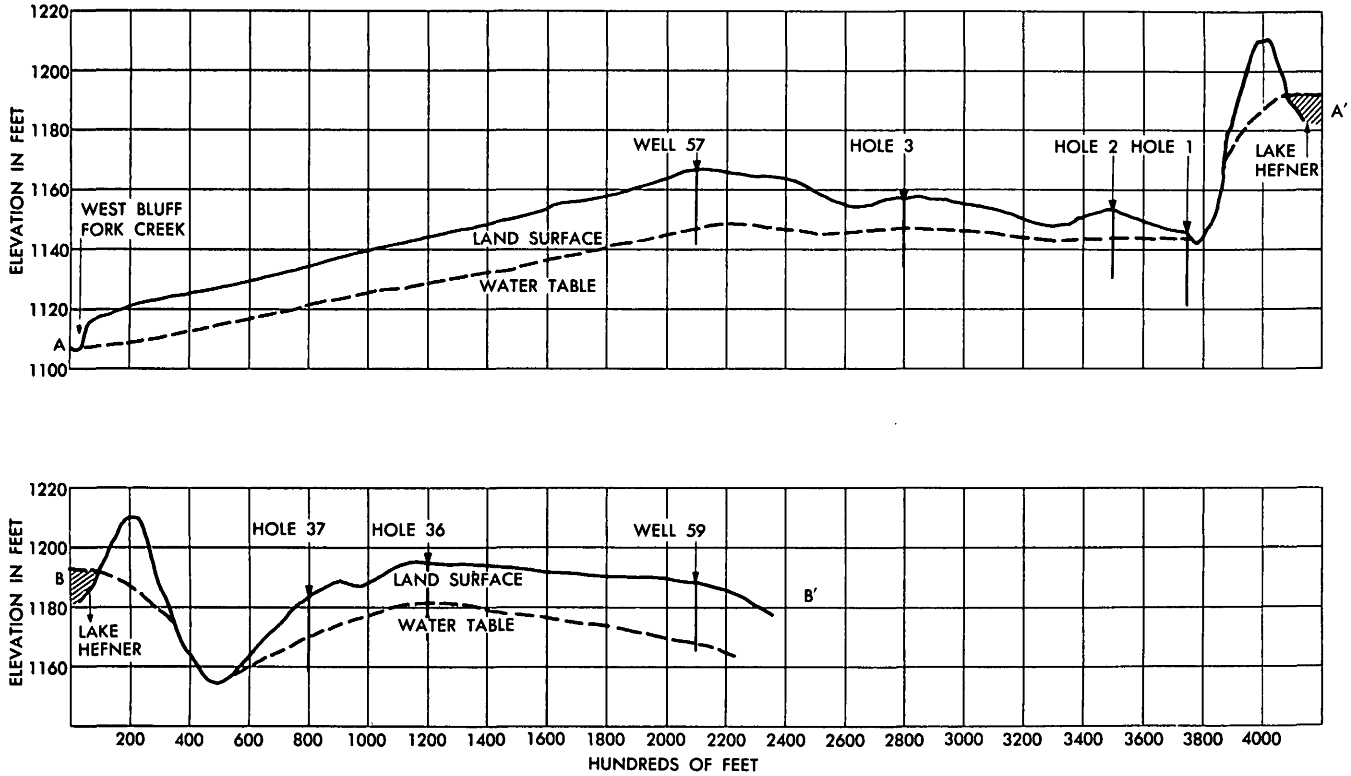

Figure 9. Cross-sections along lines shown in Figure 8.

Cross-Sectional Areas

The piezometric map (fig. 8) shows clearly that the lake is receiving ground water south of the intake canal and around the south and east sides of the lake to the spillway, and is losing water to the ground north of the intake canal around the west and north sides of the lake to the spillway. The effluent and influent stretches are approximately equal, each being about 4 miles in length.

The proper thicknesses to use in computing cross-sectional areas of effluent and influent seepage are difficult to determine. Evidence of several kinds already has been cited indicating that the chief movement of water in the Hennessey shale occurs near the surface, principally in joints that hardly could exist at depth. Where the water table is near the surface, as it is in much of the Lake Hefner area, most of the movement of water probably occurs within the upper 20 feet of saturated shale. Where the water table is deep, the thickness of the zone of active movement is likely to be even less.

Computation of Influent and Effluent Seepage

A large amount of detailed work probably would be required to make any reasonably accurate computation of the influent and effluent seepage at Lake Hefner. This is so chiefly because of the nature of the interstices that carry the water in the shale. In granular material a rather small number of samples may permit reasonably accurate determination of the average permeability, but in shale the average permeability is much more difficult to determine because the fracture openings may be extremely diverse in size and distribution. The following estimates are therefore to be considered as giving only the probable order of magnitude of the seepage losses from and gains to the lake.

The specific capacities of the better wells in the Hennessey shale are of the same order of magnitude (1 to 2 gallons per minute per foot of drawdown) as the specific capacities of the wells in the Garber sandstone, whose permeability has been determined to be 40 gallons per day per square foot. Assuming, on the basis of similar specific capacities of wells, that the Hennessey shale may have about the same permeability as the Garber sandstone, we have one of the three factors necessary for computing the seepage. The second factor, the water-table slope, as obtained from figure 8, is approximately 40 feet per mile for portions of the lake gaining water and 80 feet per mile for portions losing water. For the third factor, the cross-sectional area, we assume that most of the ground-water movement occurs in the upper 20 feet of saturated material. On the basis of these assumptions, it would appear that seepage to the lake might be of the order of 125,000 gallons per day, and that the seepage from the lake might be of the order of 250,000 gallons per day.

These figures are based on the assumptions that all the seepage moving toward the lake is intercepted by the lake and that none of the seepage moving out of the lake is intercepted by drains but continues on as "deep seepage." Neither of these assumptions is correct, and it is difficult to estimate what correction factor should be applied. Certainly some of the deep seepage continues beneath the lake. The fact that part of the deep seepage from the lake is intercepted by Bluff Creek and the drains at its north end is indicated by the large trenches they have made in the water table and by the visible seeps from the bed rock along their channels. Perhaps the figures given above should be reduced by half, or more, to take care of this factor.

The amount of seepage to the lake can be no greater than the amount of seepage originally contributed to Bluff Creek in the area now covered by the lake. Actually this seepage should be less, because the formation of the lake has decreased the gradient. Engineers and others who were acquainted with the area before and during the period of lake construction say that all the visible seeps and springs feeding this branch of Bluff Creek entered above the area subsequently covered by the lake, issuing chiefly at the base of the terrace sands.

From all the evidence available it therefore appears that the amount of unmeasured seepage into the lake is small, perhaps of the order of 50,000 to 100,000 gallons per day (0.15 to 0.3 acre-foot per day), and that the unmeasured deep seepage escaping from the lake is at least twice as much, perhaps of the order of 100,000 to 200,000 gallons per day (0.3 to 0.6 acre-foot per day).*

CONCLUSIONS ON GEOLOGY AND HYDROLOGY

The Hennessey shale underlying Lake Hefner is considered to be relatively impermeable. The net deep seepage loss from the reservoir is estimated to be 0.2 acre-foot per day. There is insufficient evidence to warrant any attempt to determine seasonal changes, if any, in influent and effluent seepage.

Measurements of shallow seepage losses through the dam and dike appear to be adequate if made during late fall, winter, and early spring, in order to minimize evapotranspiration losses.

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* Subsequent to the field examination and preparation of this section of the report, sporadic measurements of water levels in the forty test holes were made during the period June 1950 to September 1951. Unfortunately, many of the holes caved in or were otherwise destroyed during that time. Ten measurements were made at each of the following test holes: 1, 2, 5, 7, and 8, along the west side of the lake; 12, 20, and 21, along the south side; 26, and 33, near the east shore; and 38, 39, and 40, on the north side of the lake.

The magnitude of water-level fluctuation in the various wells varied greatly, but a general seasonal trend was evident. Water levels were generally lowest during the period February to April 1951, but rose rapidly thereafter in response to the heavy rains of May, and were at their highest in June 1951. Water levels at the end of the period were in general not more than 2 or 3 feet higher or lower than at the beginning of the period.

Since the seasonal trend in water levels in the area near the lake resulted from precipitation rather than changes in lake levels, it appeared reasonable to assume that water levels over the entire surrounding area followed a similar trend. There was insufficient evidence of any change in hydraulic gradients of sufficient magnitude to warrant recomputation of influent and effluent deep seepage.

The Water-Budget Control

by G. Earl Harbeck, Jr., and Frank W. Kennon*

INSTRUMENTATION AND METHODS

Since daily evaporation, as determined by the water budget, was the control for the entire project, it was imperative that every effort be made to measure each item of the budget to a precision consistent with its significance in the water budget and to evaluate the errors inherent in the measurements. Daily evaporation was computed using the familiar formula

$$E = I - O - S, \qquad (1)$$

in which E = evaporation,
S = change in reservoir contents,
I = inflow, and
O = outflow.

The acre-foot was used as the unit of volume in all water-budget computations. Daily evaporation was computed in acre-feet and then converted to equivalent depths in inches and centimeters.

Changes in Reservoir Contents

An accurate area-capacity curve was considered essential for the determination of daily changes in reservoir contents. An area-capacity curve had been prepared on the basis of a preconstruction transit-stadia survey by the consulting engineer employed by Oklahoma City to design the dam and reservoir. It is believed that some material was removed from the reservoir area and used in the construction of the dike, but the exact quantity was unknown. The amount of reservoir silting since the reservoir was constructed in 1944 was also unknown, although it was believed to be small. A resurvey of the reservoir was therefore considered necessary.

A plane-table survey was made of the shore areas between elevations 1192 and 1200 feet. A hydrographic survey of the lake was made, using a U. S. Navy plane-personnel boat equipped with a Bludworth NK-2 echo-sounder and outboard transceiver to obtain continuous bottom profiles along selected courses. A new area-capacity curve was developed on the basis of the resurvey. The curve based on the resurvey checks the original curve within 0.4 per cent at full pool, indicating that the net effects of reservoir silting and other changes have been small.

Since it was planned to determine daily changes in the reservoir contents by multiplying the area of the lake by the change in stage, accurate stage measurements were needed. The principal reference gage for all stage measurements was installed at the intake tower (see map). It was set to read the same as the City Water Department gage, which is also fastened to the intake tower, and which is graduated from 1180 to 1200 feet.**

Changes in stage were recorded on a Stevens type A-35 continuous recorder with a 10:12 gage-height ratio and a 4.8-inches-per-day time scale, placed in a wooden shelter over an 18-inch watertight corrugated-steel pipe well. The intake opening was made as small as practicable to eliminate surge in the well. Seiches were not eliminated, however, although it was possible that their amplitudes were damped. Standard stage-measurement procedures, as described by Corbett (1943), were employed.

The south lake gage was located in the small boat harbor at boat pen 498 (see map). Prior to 23 May 1950, the south lake gage was located at a site 1200 feet southwest, but this site was soon found to be unsuitable under certain wind conditions because of pile-up in the constricted boat harbor. The station equipment was the same as at the intake tower station.

The first two lake gages installed were placed at the north and south ends of Lake Hefner because of the overwhelming preponderance of northerly and southerly winds, but it was soon evident that additional gages were required to evaluate stage changes with sufficient accuracy.

Another lake gage was installed on 30 May 1950, on the east shore of Lake Hefner at the west end of the Oklahoma City Yacht Club dock (see map). The station equipment was like that at the intake tower and south gages except that the recorder gage-height scale was 1:6.

A fourth lake gage was installed on the southwest shore of the lake on 27 June 1950. Station equipment was similar to that at the other stations, except that a Stevens weekly type-F recorder with

* Hydraulic Engineers, Water Resources Division, U. S. Geological Survey.

** To convert to USC and GS datum, subtract 0.10 foot; based on levels from USC and GS traverse station No. 23-A-14 TT, elevation 1203.44 feet, datum of 1929.

a 1:2 gage-height scale was used. From 27 June until 10 July, the record was unusable because of faulty intake action. The intake was enlarged on 18 July, and the records obtained thereafter were good except during periods of strong northerly winds. The lake was shallow at this location and the bottom sloped so gently that considerable pile-up resulted from northerly winds. On 11 December 1950, the gage was moved to a new location on the west shore of the lake (see map), and no further trouble from pile-up occurred.

The intake tower gage was used as the reference gage. Daily comparisons of recorded stages during periods of light winds were made to detect possible changes in datum of the other gages, relative to the intake tower gage. Since the other three gages were used only to record daily changes in stage, it was not necessary that they be set to the same datum as the intake tower gage, but it was essential that any abrupt changes in datum be detected. For this purpose water-level comparisons were deemed much more reliable than precise leveling. The east and south gages proved to be quite stable. For the period 28 May 1950 to 31 August 1951, the net change in stage indicated by these two gages and the intake tower gage was as follows: intake tower gage, +0.005 foot; south gage, —0.002 foot; east gage, —0.006 foot. The west gage was not nearly as stable, however, mainly owing to the fact that there was no permanent structure to which it could be affixed. Corrections for changes in datum because of settlement were made as indicated by comparison with the intake tower gage during relatively calm periods.

To evaluate the effects of errors in recorded stage changes (needed for computation of daily changes in reservoir contents) the range in indicated values of daily change in stage proved useful, for it was deemed impracticable to compute the standard error of each daily mean change by the usual statistical methods. The variance in daily stage changes indicated by the various water-level recorders is a measure of the error in this mean. The range in indicated stage change is an easily determined measure of the variance. Tippett's mean values of the ratio of the range to the standard deviation for various sample sizes have been tabulated (Snedecor, 1946). The resultant error in the computed change in reservoir contents, and therefore also in evaporation, is shown in figure 10.

It has been suggested that the variance resulting from the nonagreement of indicated stage changes

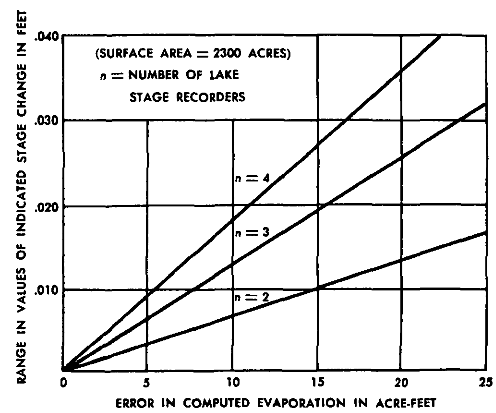

Figure 10. Error in computed evaporation resulting from inconsistencies in measurements of change in stage of Lake Hefner.

is not entirely error variance, but also includes systematic variance. A better estimate of the error variance may be obtained from

$$S^2 = \frac{1}{32}\left[(h_1 - h_2)^2 + (h_2 - h_3)^2 + (h_3 - h_4)^2 + (h_4 - h_1)^2\right] \qquad (2)$$

in which S = standard error of the mean, and h = indicated stage change.

The subscripts refer to the gages, which are numbered consecutively around the lake.

The standard error of the mean has been computed using equation (2) for 4 days selected at random. The standard error of the mean was also determined using the range as a measure of the standard deviation (see fig. 10), and the results were as follows:

Date	Standard Error of the Mean in Acre-Feet	
	From Equation (2)	From Figure 10
28 June 1950	21.2	22.8
31 July 1950	12.0	13.4
22 October 1950	30.1	31.2
25 May 1950	2.5	3.3

As expected, the standard error of the mean computed using equation (2) is consistently less than that obtained using figure 10, for the effect of systematic variance has been reduced if not eliminated.

The standard error of the mean was determined only for use in classifying daily figures of water-budget evaporation as to their relative accuracy, as discussed in more detail below. The refinement made possible by using equation (2), instead of the much simpler method based on the range, was not believed warranted in view of the additional computational work involved. Every effort was made to devise an accuracy classification scheme that was completely objective, although necessarily somewhat qualitative because the time that could be allotted justifiably to this analysis was limited. It should be noted that the error introduced by using the short-cut method is on the conservative side; some daily evaporation figures might actually deserve a higher accuracy rating than they were given.

Errors in computed evaporation resulting from errors in measuring changes in stage are not cumulative and are therefore independent of the length of period. The relative effect of stage errors therefore decreases as the length of the period increases.

A study was made of the relation between length of period and the magnitude of errors resulting from nonagreement of gages. In order to eliminate bias in their selection, periods were chosen to begin and end on days when the number of lake stage recorders in operation changed. For example, in 1950 during the 15-day period 20 May to 3 June, only two recorders were operating. During the 24-day period 4 June to 27 June, three recorders were in operation, and during the 11-day period 28 June to 8 July, all four recorders were in operation. Then followed a 10-day period in which three recorders were in operation. Thus, in effect, the periods were chosen at random, and they varied in length from 1 to 112 days. The total change in stage for each recorder was computed for each of the 32 periods, and the error in evaporation determined from figure 10. It would have been possible to select periods for which the agreement among the various gages was considerably better, merely by having each period begin and end during a near-calm. The procedure adopted, however, eliminates any possible bias.

The standard error of the mean change in stage was computed for each of the 32 periods. The median value of the standard error was 0.0036 foot and, as expected, there was no discernible correlation with the length of period, since stage errors are not cumulative. Thus, for any length of period, the standard error of the indicated stage change is approximately 0.004 foot, which is equivalent to an error of 9.6 acre-feet in evaporation, assuming the area to be 2300 acres.

The effect of thermal expansion of water in the reservoir on the computed figures of water-budget evaporation was also studied. In most computations of this nature the effect of thermal expansion is ignored, possibly because of the difficulty in evaluating it correctly. It is generally inconsequential in shallow bodies of water.

If the datum to which stage measurements were referred remained constant, the error in evaporation computed by the water-budget method would depend only on the thermal expansion of the water in the lake. At Lake Hefner the basic reference point to which all stages were referred was a point on the walkway surrounding the intake tower a few feet above the water. Thus the problem resolved itself to the determination of the difference in thermal expansion between the water in the lake and the reinforced concrete intake tower. If, for a given temperature change, the expansion of the water was exactly the same as the expansion of the tower, there would be no errors in the water-budget evaporation figures.

The intake tower is a reinforced concrete structure, circular in cross-section. The bottom of its footing rests on the Hennessey shale at an elevation of 1142 feet, according to the construction drawing of the City Water Department. The top surface of the walkway is at an elevation of 1199 feet. Thus, the effective height of the tower for computing thermal expansion is 57 feet. The commonly accepted coefficient of thermal expansion for reinforced concrete is 0.0000117 per °C. The thermal expansion of the tower is, therefore, 0.000667 foot per °C temperature change. The thermal expansion of the water in the lake can be most easily determined from tables showing the volume of water at a given temperature relative to the volume at 0°C (Chemical Rubber Publishing Co., 1947).

A sample computation for the month of November 1950 is included to illustrate the relative magnitude of the various items. Based on thermal surveys of the lake, the average water temperature at the beginning of the month was 17.9 °C and at the end of the month 7.9 °C. The variation in the thermal expansion coefficient with temperature is not linear, but since Lake Hefner remained nearly isothermal at all seasons, little error was introduced by using an average temperature for the entire lake. Average content during the month was 62,232 acre-feet, and the average area 2298 acres. From the previously mentioned tables, the ratio of the volume at 17.9 °C to the volume at 0 °C is 1.001186 and the ratio of the volume at 7.9 °C to the volume at 0 °C is 0.999989. Therefore, the ratio of the volume at 17.9 °C to the volume at 7.9° is 1.001197. The contraction in volume during the month is $(1.001197 - 1) \times 62{,}232$ or 74.5 acre feet, which is equivalent to a depth of 0.389 inch. During the same period, the tower contracted $0.000667 \times 12 \times (17.9 - 7.9)$ or 0.080 inch. Thus, during November, the lake contracted 0.389 inch and the tower only 0.080 inch, a difference of 0.309 inch. Indicated water-budget evaporation during November was 6.298 inches, but since part of the apparent reduction in volume resulted from thermal contraction rather than evaporation, the indicated evaporation is too large, and the corrected volume is 5.989 inches. In this instance, the correction was approximately 5 per cent, but it would be proportionately greater for a spring month when evaporation is less.

Unfortunately, daily evaporation figures could not be readily adjusted because day-to-day variations in lake temperatures cannot be evaluated with sufficient accuracy. Thermal surveys of the lake were not made each day, so that no direct measurements of daily temperature changes are available. Average daily lake temperatures can be estimated on the basis of observed water-surface temperatures, but the standard error of estimate of approximately 0.6°C is unduly large in comparison with the usual daily change in lake temperature.

If the average lake temperatures on each of two successive days can be estimated with a standard error of 0.6°C, the standard error of the difference between the two estimated temperatures is

$$\sqrt{(0.6)^2 + (0.6)^2} \text{ or } 0.85°\text{C}.$$

If the lake temperature on the first day is assumed to be 20°C and the error in the indicated 24-hour temperature change is 0.85°C, the resultant error in the adjustment for thermal expansion would then be 0.13 cm, which is 32 per cent of the average daily evaporation during the period of observations. Under average conditions a week is the minimum period for which the error in the adjustment for thermal expansion can be assumed to be less than 5 per cent, if average lake temperatures are estimated from water-surface temperatures.

Although it was deemed impracticable to adjust daily evaporation figures for the effect of thermal expansion, evaporation figures for some longer periods could be corrected. Since the only requirement was that the average lake temperature at the beginning and end of the period be measured or estimated with sufficient accuracy, evaporation for periods determined by thermal surveys could be adjusted for thermal expansion, as could certain other periods for which it was believed that temperatures were known with sufficient accuracy.

Rainfall on the Lake Surface

Rainfall on the lake surface was frequently a major item in the Lake Hefner water budget. Rainfall of 0.05 inch or greater was recorded on 86 days during the 16-month period of observation. Nearly 60 per cent of the water withdrawn from Lake Hefner for municipal use during the entire period of observation was replaced by precipitation falling directly on the lake surface.

Tipping-bucket recording rain gages were located at each of the meteorological stations, including the barge, and 18 Weather Bureau nonrecording gages were located on the periphery of the lake (see map).

The recording gages operated without loss of record except for the barge station. It was apparent that the gage on the barge caught spray a few times. Also, the motion of the barge was sufficient to cause the rain-gage bucket to tip occasionally under certain wave conditions. Records for the nonrecording gages were complete except for two periods of a day or two when a gage was stolen.

Mean rainfall on the lake was taken as the simple average of the catches of the 22 gages. For a storm that extended over more than one calendar day, the amounts credited to each day were computed on the basis of the recording gage records by simple proportion.

For five selected storms a comparison was made of computed values of mean precipitation on the lake as given by (1) the simple average, (2) the Thiessen method, and (3) an isohyetal map. The results are as follows:

Date of Storm	Mean Rainfall in Inches: Average	Thiessen Method	Isohyetal Map
5 May 1950	0.073	0.070	0.066
4 July 1950	0.750	0.785	0.795
25 July 1950	1.077	1.013	1.012
9 May 1951	1.511	1.497	1.486
27 May 1951	1.849	1.864	1.895

It will be noted that results obtained using the Thiessen and isohyetal-map methods agree with each other better than with the arithmetic mean. This may in part result from the much greater weight given the record at the barge when computing Thiessen factors or preparing an isohyetal map. Since the other gages were more or less uniformly distributed around the periphery of the lake, the results would be practically the same for all methods if the records at the barge were not used. Because the record at the barge was less reliable than the others, owing to the possibility of mechanically induced bucket tipping or spray, it appeared advisable not to weight it more heavily than the others. Moreover, the preparation of an isohyetal map for each storm would have been an arduous task and would have been open to the criticism that the procedure is somewhat subjective. Another advantage of using the arithmetic mean is that the standard error of the mean is easily determined. The simple average was used as the mean rainfall on the lake in the water-budget computations.

Of considerable interest, because of its tremendous areal variability, is the storm of 24 July 1951, for which an isohyetal map was prepared (fig. 11). At the south station, where the recording gage showed the heaviest rainfall, precipitation began at 2132 and ended at 2244. The total catch was 1.76 inches, of which 1.48 inches fell between 2200 and 2230. At rain gage No. 11, 0.45 mile southwest of the south station, 1.99 inches fell. In contrast, the storm rainfall at gage No. 2, which is 2.40 miles northwest of the south station and directly across the lake from it, was only 0.02 inch. The average for the 22 gages was 0.534 inch. The standard error of the computed mean rainfall was 0.125 inch, or 23 per cent of the mean.

To study the areal variability of storm rainfall, the standard error of the mean and the coefficient of variation were computed for each storm. The standard error of the mean was computed by dividing the standard deviation of the individual rainfall amounts by the square root of the number of gages; it is thereby assumed that the systematic variance is negligible in comparison with the error variance. The results, which are illustrated in figure 12, indicate that rainfall during large storms tends to be more uniformly distributed than during light showers, as might intuitively be expected. The coefficient of variation is in general 10 per cent or less for storm rainfalls of 0.1 to 1 inch, and about 5 per cent or less for larger storms. The standard error of the mean rainfall for each storm, expressed in acre-feet, was used in determining the relative accuracy of computed water-budget evaporation.

Surface Inflow

Surface inflow to Lake Hefner results from precipitation on the natural drainage area above the lake and from diversion from the North Canadian River. Natural inflow usually is a small item in the water budget because of the relatively small size of the drainage basin.

The measurement of natural inflow to Lake Hefner was simplified to a considerable degree by the "dike" effect of the road encircling the lake, which served to confine runoff to relatively few channels where it could be measured. The total drainage area tributary to Lake Hefner above the encircling road is 2484 acres.

The principal stream, Bluff Creek, and its tributaries have cut back into a high terrace of the North Canadian River about a mile south of Lake Hefner. This terrace was once a flood plain of the North Canadian River, and is now being dissected by tributaries of both the North Canadian and Cimarron Rivers.

The deposits underlying the terrace contain relatively large quantities of water. Bluff Creek is cut into these water-bearing deposits and is therefore seldom dry, in contrast to other small streams in the area whose drainage basins are underlain by the Hennessey shale. Diurnal fluctuation in flow of Bluff Creek is caused by operation of a gravel plant upstream. A gaging station was established on 8 March 1950 at the bridge on the reservoir road (location C, see map).

A Stevens A-35 continuous recorder was housed in a wooden shelter over a 24-inch-diameter corrugated-steel pipe well fastened to the upstream left wing wall of the bridge (fig. 13). The zero of the reference gage was at elevation 1199.86 feet (based on an elevation of 1203.44 for U.S.C. & G.S. traverse station No. 23-A-14 TT, datum of 1929). The control for all stages was a sharp-edged steel 90° V-notch weir set in the center of a shallow concrete V weir 6 inches thick and 30 feet long at the upstream side of the bridge.

Records of stream flow were obtained using a standard stream gaging procedure described by Corbett (1943). The rating curve (fig. 14), was well defined between 0.2 and 40 cfs by 13 discharge measurements made by wading near the gage. Below a stage of 1 foot, the rating was based on a standard weir rating (U. S. Bureau of Reclamation, 1946). It was extended to a stage of 2.6 feet on the basis of known hydraulic characteristics of similar controls. The momentary peak discharge was 931 cfs on 27 May 1951. The peak rate of flow was maintained for only a very short period of time, as is shown by the fact that average discharge on that day was only 39.6 cfs. However, total inflow to Lake Hefner on this and certain other days was so great as to preclude

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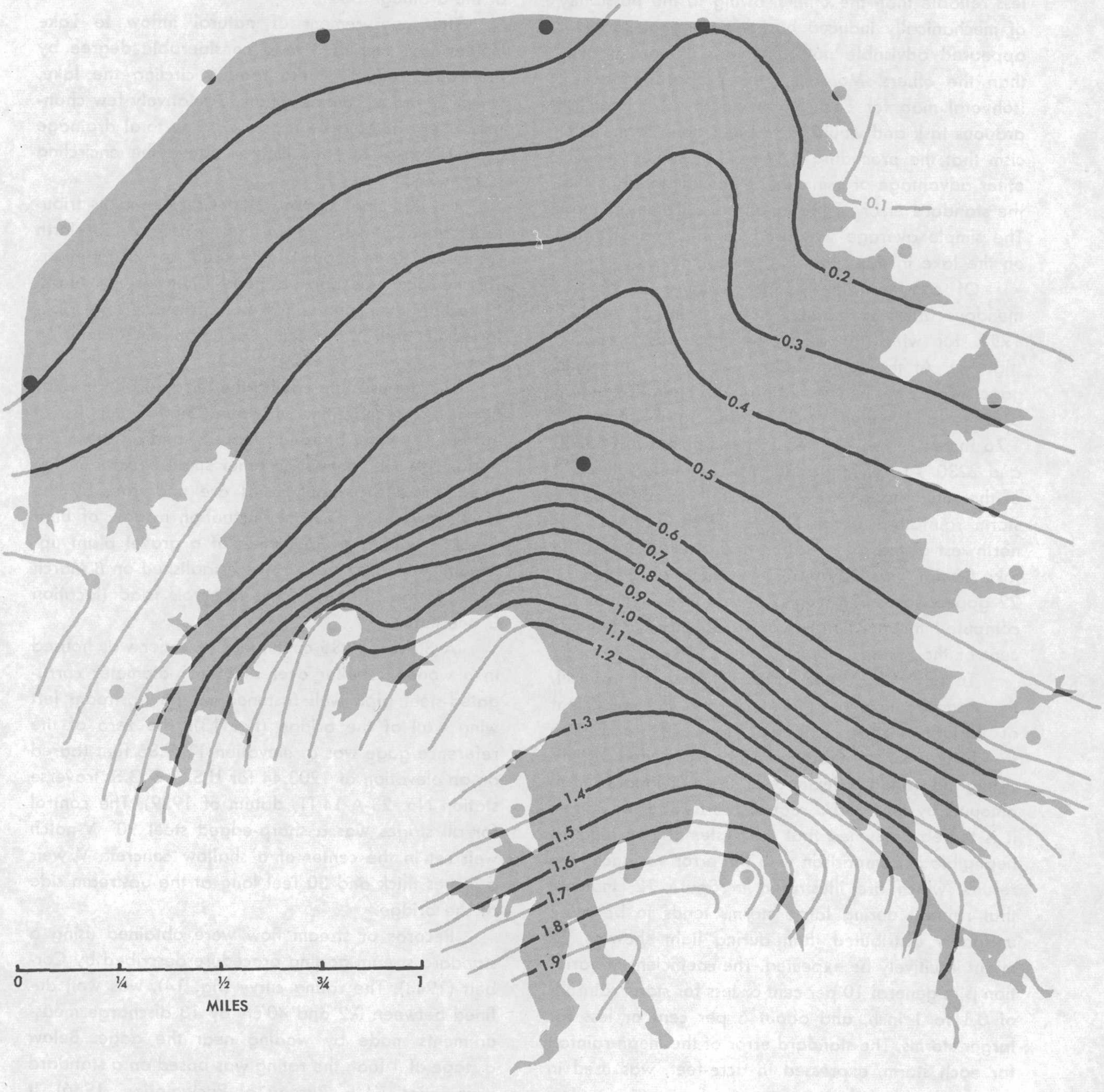

Figure 11. Isohyetal map for the storm of 24 July 1951 at Lake Hefner.

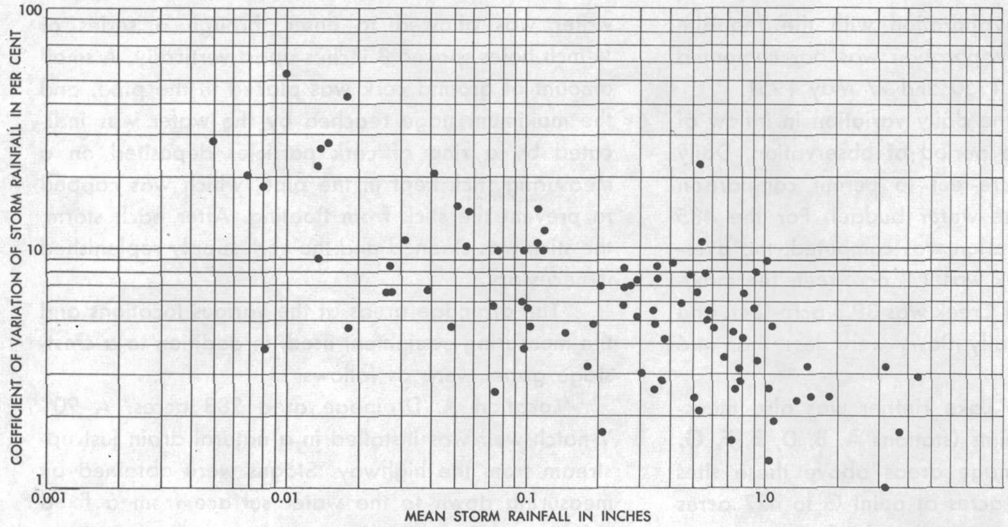

Figure 12. Variability of storm rainfall.

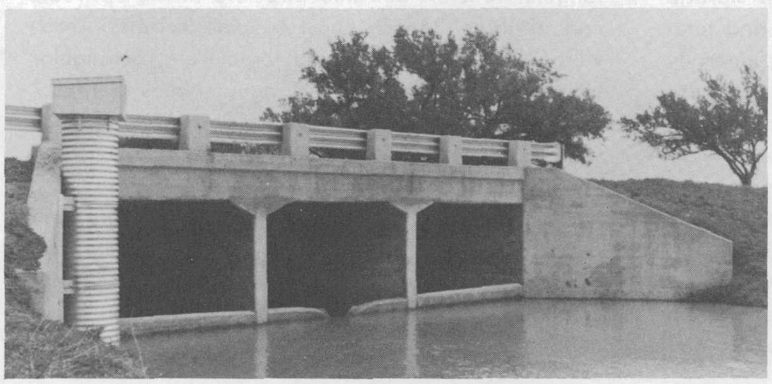

Figure 13. Pipe well and recording-gage shelter; concrete control with V-notch weir above Lake Hefner (location C).

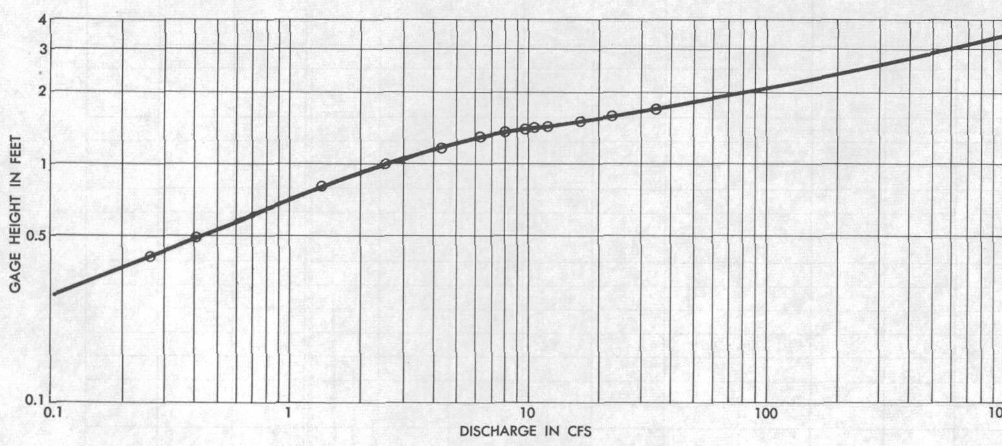

Figure 14. Rating curve for gaging station on Bluff Creek above Lake Hefner.

the computation of evaporation with the requisite accuracy; therefore, evaporation was not computed for 9, 13, 18, 19 May 1950 and 27 May 1951.

Figure 15 shows the daily variation in inflow of Bluff Creek during the period of observation. Daily flows are given in acre-feet to permit comparison with other items in the water budget. For the 485 days for which evaporation was computed, the average flow of Bluff Creek was 1.3 acre-feet; the maximum daily flow of Bluff Creek was 39.3 acre-feet, and 95 per cent of the daily flows were less than 4.6 acre-feet.

Natural inflow to Lake Hefner was also measured at eight other points (stations A, B, D, E, F, G, H, J, see map). Drainage areas above these sites range in size from 17 acres at point G to 427 acres at point F, and the total of all eight drainage areas is 1377 acres. Flow occurred at these gaging stations only as a result of storm rainfall. Farm ponds, constructed in upstream drains, have sufficient storage capacity to reduce substantially both peak and total flows. Except after severe storms, runoff generally occurred at only three or four of the stations because empty or partly empty farm ponds intercepted all surface runoff.

One or more weirs and a crest-stage gage (fig. 16) were installed at each of the eight locations. A continuous record of stage was not obtained. The crest-gage used at Lake Hefner consisted of a piece of 2-inch pipe mounted vertically just upstream from the weir or weirs. It was closed at the bottom and water was allowed to enter through a series of ¼-inch holes spaced 3 inches apart vertically. A small amount of ground cork was placed in the pipe, and the maximum stage reached by the water was indicated by a ring of cork particles deposited on a measuring stick kept in the pipe, which was capped to prevent the stick from floating. After each storm, the stick was cleaned and the cork supply replenished if necessary.

The drainage areas at the various locations and the measuring equipment used, in addition to a crest-stage gage, were as follows:

Location A. Drainage area 288 acres. A 90° V-notch weir was installed in a natural drain just upstream from the highway. Stages were obtained by measuring down to the water surface from a fixed reference point.

Location B. Drainage area 280 acres. Weirs were installed in six 18-by-24-inch openings in the rectangular-concrete-box inlet structure at the upstream end of three 36-inch concrete pipe culverts. Three were 90° V-notch weirs and three were rectangular weirs. The crests of the rectangular weirs were 0.7 foot higher than the point of zero flow of the V-notch weirs, so that flows of 27 cfs or less were carried entirely by the V-notch weirs.

Location D-J, incl. 90° V-notch weirs were concreted in place at the upstream ends of concrete pipe culverts. The drainage area and the number and diameter of pipe culverts at each location were as follows:

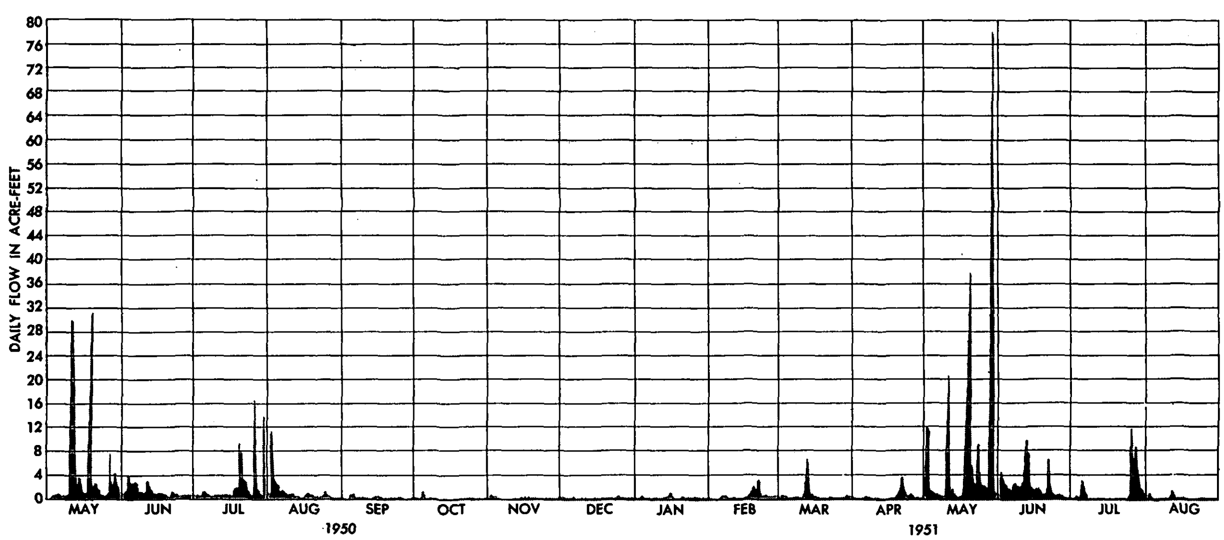

Figure 15. Hydrograph of daily flow of Bluff Creek above Lake Hefner.

Figure 16. Culvert, weirs, and crest-stage gage at location J.

Location	Drainage Area in Acres	Number of Culverts	Diameter of Culverts in Inches
D	35	1	36
E	74	1	36
F	427	3	60
G	17	1	36
H	96	2	36
J	160	4	36

Total flow at all eight gages, with an aggregate drainage area of 1377 acres, averaged 1.1 acre-feet per day, as compared with 1.3 acre-feet per day for Bluff Creek, which has a drainage area of 1037 acres. The difference in yield is caused by the fact that some of the Bluff Creek tributaries tap the water-bearing deposits of the Bethany terrace, while the other small streams do not.

The general procedure was to visit each of the gages after storms to obtain one or more observations of stage on the recession limb of the hydrograph. These observations, together with the crest stage, recorded rainfall intensities, and the Bluff Creek discharge hydrograph, were used to estimate the hydrograph for each gage at which flow occurred, as illustrated in figure 17. The Bluff Creek stage graph shows pronounced diurnal fluctuations, caused by operation of a gravel plant upstream from the gage, but this does not preclude its use as an indicator of the general shape of a flood hydrograph. During the storm of 2 June, precipitation at the south station began at 2149 and ceased at 2342; the total amount recorded was 0.44 inch. Flow at gage H on 1 June was 0.06 acre-foot as a result of the storm on 28 May, and gage H was read on the afternoons of 1 and 2 June to define the recession from the previous peak. The continuous record at Bluff Creek gage showed that Bluff Creek began to rise at about 2230 on 2 June and peaked at 0130 on 3 June. Since the drainage area at gage H is 96 acres as compared

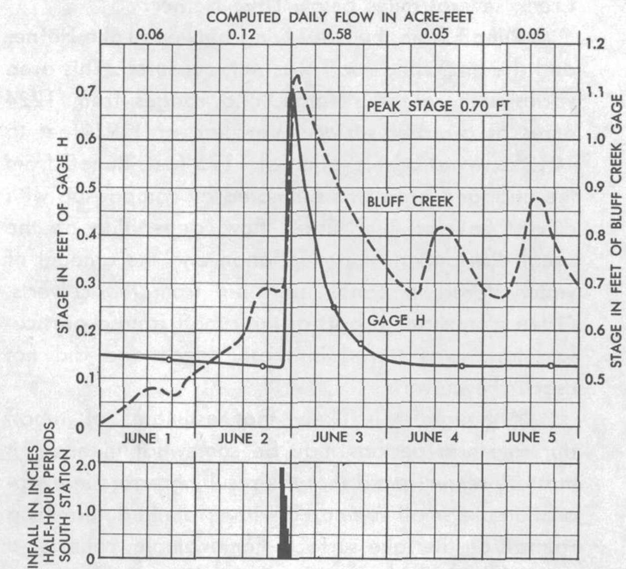

Figure 17. Estimated stage hydrograph at gage H resulting from storm of 2 June 1950.

with 1037 acres at the Bluff Creek gage, it is assumed that the peak at gage H occurred earlier than at Bluff Creek. The heaviest half-hour rainfall (0.19 inch) occurred from 2200 to 2230, which coincides with the beginning of the rise in Bluff Creek. It seems, therefore, that the rise started at gage H at approximately 2230 also, and reached its peak at midnight or shortly thereafter. The gage at H was visited on the morning of 3 June, the previous peak stage noted, and the stage at the time of the visit recorded. Stages were observed once daily, or oftener, thereafter until flow ceased. Flow on 3 June at gage H was computed to be 0.58 acre-foot.

It is believed that the storm discharge hydrograph at the various culvert gages was estimated with sufficient accuracy. Flow of Bluff Creek on 3 June was 4.17 acre-feet, and flow at the other culvert gages ranged from 0 to 0.95 acre-foot. Total inflow was 13.18 acre-feet.

Surface runoff from a small part of the area lying east of the Lake Hefner road in the southern part of Section 36 (see map) enters Lake Hefner through the culverts at location J, but flow from the remainder of Section 36 and from the south half of Section 25, that normally would drain into Lake Hefner, does not reach the lake. An intercepting drainage ditch along the east side of the road conducts surface runoff from this area in a northerly direction to the spillway, whence it eventually flows into Bluff Creek several miles below Lake Hefner.

Runoff from the area lying between Lake Hefner and the encircling road was not measured. This area varies with reservoir stage, and ranges from 1224 acres at a water surface elevation of 1191 feet to 1012 acres at an elevation of 1195 feet. Runoff from the ungaged area was estimated by comparison with culvert flow or Bluff Creek flow, depending on the areal distribution of precipitation and the amount of water stored in ponds upstream from the culverts. Often a substantial part of the runoff from a particular storm was stored in these ponds and did not reach the culverts.

Although it is likely that estimates of runoff during storm periods may be somewhat in error, it must be remembered that the resulting errors in evaporation are small compared with errors in determining rainfall on the lake surface. For example, consider a storm for which the average rainfall on the lake was computed to be 2 inches, or about 380 acre-feet on the average area of the lake. From figure 12 it is estimated that the coefficient of variation for a storm rainfall of 2 inches is 4 per cent, from which it follows that the standard error of the mean rainfall on the lake is 15 acre-feet. Storm runoff is rarely greater than 25 per cent of the rainfall, and is usually much less. Using the 25-per-cent figure, the runoff from the ungaged area of 1090 acres would be 23 acre-feet. An error of 20 per cent in the estimate of runoff from the ungaged area would result in an error of less than 5 acre-feet, only one-third of the error in mean rainfall. Moreover, it is considered that because of the wealth of hydrologic data on which to base estimates of runoff from the ungaged area, the error in these estimates is probably considerably less than 20 per cent.

Lake Hefner is replenished by infrequent diversions of water from the North Canadian River through the intake canal (see map). The rated capacity of the canal is 1500 cfs. Records of canal inflow were obtained at a site approximately half a mile above Lake Hefner at the point of emergence of an inverted siphon under the Northwest Highway. A short concrete transitional section just below the siphon mouth is terminated by a low concrete weir. The top of the weir is 12 inches above the paved approach section and is practically level for its entire length. It has a rounded downstream face. In order to measure flows with the desired accuracy, a 4-foot rectangular steel weir was installed near the left end of the concrete weir; its crest was set at 0.35 foot below the crest of the concrete weir.

Provisions were made for accurate measurement of low flows because of ground-water seepage into the intake canal. The canal cuts through the water-bearing deposits of the Bethany terrace, and most of the ground and surface water flow it collects is drained to a tributary that enters Bluff Creek several miles below the dam. Some of the perennial ground-water flow does reach Lake Hefner through the intake canal, however, and is occasionally augmented by natural runoff from an area of approximately 200 acres.

A Stevens type-F weekly water-stage recorder having a 1:2 gage-height ratio was housed in a small wooden shelter over a 12-inch corrugated-steel well fastened to the left wall of the concrete transitional section just upstream from the weir. The zero of the gage was at elevation 1200.96 feet, datum of 1929. Discharge measurements were made by wading or boat at a section approximately 700 feet below the gage.

The rating curve for the intake canal (fig. 18) is based on 34 discharge measurements and is well defined between 0 and 1500 cfs. Below a stage of 0.35 foot (the top of the rectangular weir) a standard weir rating was used. Current-meter and portable-V-notch-weir measurements at low stages check the theoretical rating fairly well. The concrete weir was rather insensitive because of its length; at discharges of from 500 to 1000 cfs, a change of 0.01 foot in stage results in a change of approximately 2 per cent in discharge. Twenty-two measurements were made at stages above 0.35 foot. Their average deviation from the rating without regard to sign was 2.7 per cent.

Water was diverted from the North Canadian River to Lake Hefner on ten occasions during the 16-month observation period. During two of these times natural inflow, including rainfall, was also of considerable magnitude, and no attempt was made

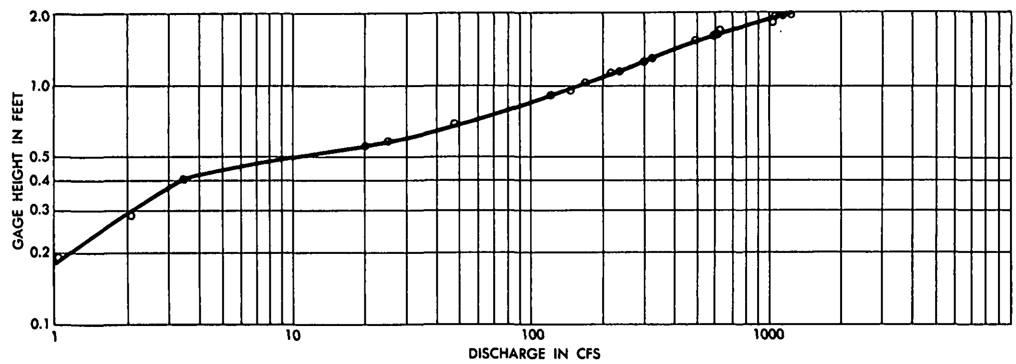

Figure 18. Rating curve for gaging station on Lake Hefner canal.

to compute evaporation. Diversions ranged in size from approximately 800 to 5500 acre-feet in periods of 2 to 4 days.

Despite the fact that the records of canal flow were considered to be of excellent accuracy by usual stream-gaging standards, the diversions were of such magnitude that even small percentage errors, when expressed in acre-feet, were large compared with evaporation. Computed daily evaporation during periods when diversions were being made is therefore of doubtful accuracy.

Excluding periods during which diversions were being made, the average daily canal inflow was approximately 1 acre-foot. Except during periods of reservoir replenishment by diversion from the North Canadian River, surface inflow was generally a minor item in the water budget. A statistical study to determine the standard error of each daily figure was not necessary since wide experience with gaging-station rating curves and stage hydrographs indicates the standard error of these operations to be 5 per cent. For the 485 days for which evaporation was computed, average inflow (excluding canal diversion) was 4.4 acre-feet per day as compared with average filter plant withdrawals of 30.6 acre-feet per day.

Withdrawals For Municipal Use

Filter plant withdrawals were measured by a Venturi meter in the 48-inch raw water line leading from the intake tower to the filter plant. Although the manufacturer's calibration of the Venturi meter was probably sufficiently accurate for the needs of the City Water Department, filter plant withdrawals were known to be the largest single outflow item in the water budget, and precise calibration of the Venturi meter was therefore deemed essential. The traditional formula for Venturi meter flow (King, 1939) is

$$Q = Cd_2^2 \sqrt{\frac{2g\,(h_1 - h_2)}{1 - (d_2/d_1)^4}} \tag{3}$$

where Q = discharge in cfs,
C = Venturi meter coefficient,
g = acceleration of gravity in feet per second per second,
d_1 = diameter of Venturi meter entrance in feet,
d_2 = diameter of Venturi meter throat in feet,
h_1 = head of water at Venturi meter entrance in feet of water, and
h_2 = head of water at Venturi meter throat in feet of water.

The salt-velocity method of calibration was used. The major items of equipment were a brine pressure tank, a pop valve, two sets of electrodes, an oscillograph, and a water-tube manometer. Brine was injected through the quick-acting pop valve into the raw water line at an unbalanced pressure of approximately 80 pounds per square inch. The pop valve and each set of electrodes were connected into separate oscillograph circuits so as to produce a deflection of the galvanometer when the brine decreased the resistance in the circuit. Each set of electrodes was connected to form one leg of a bridge in its respective circuit. This arrangement made the oscillograph galvanometers very sensitive to any unbalance of the bridge, caused by brine passing the electrodes, thus permitting the use of low-potential circuits, and also of small quantities of brine, thus minimizing density currents in the raw water line. The electrodes were placed in the 48-inch raw water line at distances of 100.08 and 211.10 feet downstream from the pop valve. The water-tube manometer was installed with one tube connected to a manifold at the Venturi meter throat and the other tube connected to a manifold at the upstream end of the meter to measure the differential head of the meter. The water-tube manometer remained in place during the entire period of observation at Lake Hefner and was the basic reference gage for the Venturi meter. Dimensions of the pipe and Venturi meter were measured to the nearest 0.001 foot.

Brine was injected into the 48-inch line by opening and closing a hand-operated quick-acting gate valve as rapidly as possible, which allowed approximately ½ gallon of brine to flow from the pressure tank through the pop valve. The oscillograph was started just before the brine was injected and remained in operation until the salt cloud had passed the second set of electrodes. The differential head at the Venturi meter was observed at 1-minute intervals during each test run. Readings of the totalizing Venturi meter recorder in the filter plant were made at frequent intervals during each run. Water temperatures were also recorded.

Twelve runs were made for rates of flow ranging from approximately 14 to 70 cfs. Values of C, the Venturi meter coefficient, were plotted against the differential head ($h_1 - h_2$) as shown in figure 19. A value of 0.988 was chosen as the upper limit on the basis of LeDoux calibration tests of many other Venturi meters of different sizes (King, 1939). Fewer observations were made at the lower end of the curve because of the infrequency of low flows. Results for two runs were discarded because the magnitude of the computed values of C were unreasonable. Data from another run were discarded because two injections of brine were made, and the salt clouds overlapped at the second set of electrodes.

Using values of C taken from figure 19, the curve shown in figure 20 was computed. The results of the individual runs are also shown for comparison. The maximum deviation of any run is 0.5 per cent.

The Venturi meter calibration curve is well defined throughout the range of 14 to 65 acre-feet per day. During the period of observation, 95 per cent of daily withdrawals were in the range 14.1 to 48.4 acre-feet per day. For the 485 days for which evaporation was computed, the average filter plant withdrawal was 30.6 acre-feet.

The minimum was 4.0 and the maximum 63.2 acre-feet per day. It should be noted, however, that even though the percentage error at extremely low flow may be high, the absolute error is small. Although the daily withdrawal rate was not generally constant, sustained low rates of flow were uncommon, and small daily flows generally occurred when the filter plant was shut down completely for part of a day.

Since it appeared impracticable to obtain a continuous record of differential head as measured by the sensitive water-tube manometer at the Venturi meter, it was decided to rely as much as possible on the record of raw water flow as indicated by the Bristol recorder in the filter plant, applying such adjustments as might be indicated by simultaneous readings of the water-tube manometer and the Bristol recorder. Generally the procedure was to record the total flow shown by the Bristol meter totalizator over a period of 10 minutes to an hour or more, the length of the period depending on the rate of flow. The water-tube manometer was read frequently during the period and the flow, as determined from figure 20, was compared with that indicated by the Bristol meter to determine the adjustment to be applied to the Bristol meter readings.

Unfortunately, the relation between flow as indicated by the Bristol meter and flow as indicated by the water-tube manometer did not remain constant over the entire period of operation. The Bristol meter was adjusted frequently by the City Water Department, and each time it was adjusted the relation changed. As far as is known, the Bristol meter was not adjusted during the period 25 April to 2 September 1950, but it was adjusted 21 times during the period 2 September 1950 to 31 August 1951. It was necessary, therefore, to make frequent comparisons between the flow as indicated by the Bristol meter

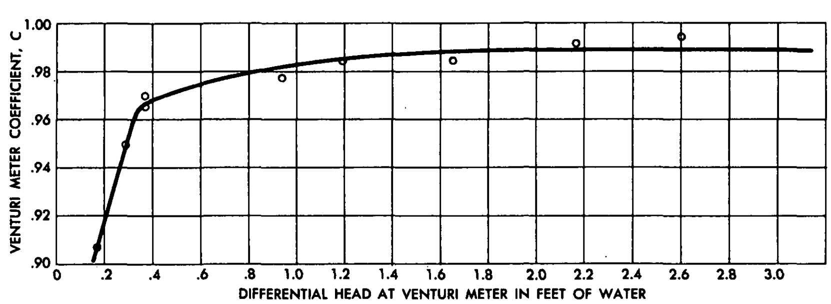

Figure 19. Relation between Venturi meter coefficient and differential head.

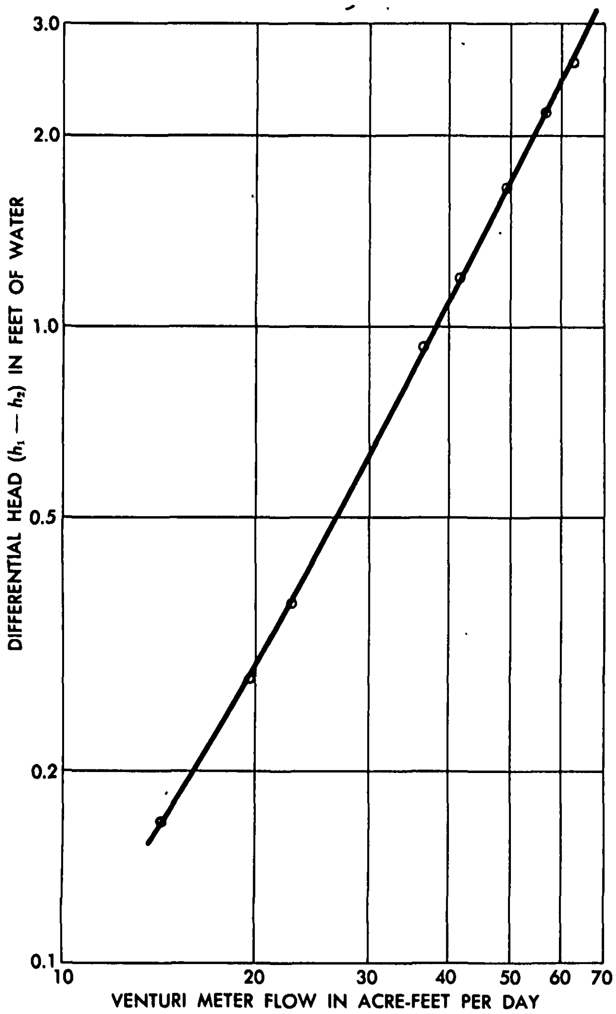

Figure 20. Venturi meter calibration curve.

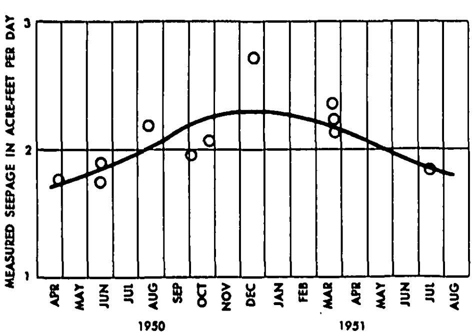

Figure 21. Effect of evapotranspiration losses on measured shallow seepage from Lake Hefner.

and the flow as indicated by the water-tube manometer. Altogether, 680 such comparisons were made. A correction curve was prepared for each period between Bristol meter adjustments. During the period June to August 1951, a test was made to verify the accuracy of the computed filter plant withdrawals. During most of this period the water-tube manometer was read each time the rate of flow was changed, and daily flow was computed from these readings. Daily flow was also computed using the correction curves previously described and the results were compared. The standard error of the difference in daily flow computed by the two methods during the three month period was 2 per cent of the daily mean flow. The difference between monthly flows computed by the two methods was less than 0.5 per cent.

As a further check of the accuracy of computed figures of filter plant withdrawals, a study was made of the amounts of water consumed in the plant and pumped to the city mains. The average daily flow computed from pumping records, with allowance for water consumed in the plant, averaged 1.6 per cent greater than that computed from the raw water Venturi meter records. It is believed that the major part of this difference results from error in estimating the quantities of water consumed in the filter plant. Although the water used to wash filters is metered, that used to blow sludge from the flocculators is not. Records of daily withdrawals computed from pumping records were used only during periods when records from the raw water meter were lacking or questionable, as follows: 7-9, 18, 19 June, 19-29 September, 22-25 November, 12-14 January, and 14-20 February.

For the purpose of classifying the accuracy of the daily water-budget figures, an estimate of the standard error of all figures of filter plant withdrawals was required. A detailed study made during the months of June, July, and August 1951 indicated that the standard error of computed withdrawals, based on the filter plant record, is 2 per cent, but this assumes no error in the relation between water-tube manometer readings and actual flow. This error is probably much less than 1 per cent, as indicated by the range in Venturi meter coefficients shown in figure 19. Also, records for a total of 30 days were based on pumping records rather than Venturi meter records, and their standard error is probably slightly greater than 2 per cent. Assuming that the errors follow the normal law, it is estimated that the standard error in computed daily figures of filter plant withdrawals is 3 per cent.

During the spring and summer of 1950, a golf course was established on the south shore of Lake Hefner, just west of the boat harbor. The greens and fairways were irrigated with water from the lake. Withdrawals were measured by a totalizing Sparling meter. During the entire period of observation the amount of water withdrawn from the lake for irrigation rarely exceeded 0.1 acre-foot per day and never exceeded 0.2 acre-foot per day, but was, nevertheless, taken into account in computing daily evaporation figures.

Shallow Seepage

Seepage into or out of Lake Hefner may be classified as shallow and deep seepage; these will be discussed separately. Shallow seepage includes inseepage from the thin soil mantle overlying the Hennessey shale on the south and east sides of the lake and outseepage through the dam and dike on the west and north sides of the lake, most of which is collected and drained to Bluff Creek below the dam. Deep seepage, on the other hand, is considered to be the flow in the near-surface part of the Hennessey shale, an aquifer of low transmissibility. The geologic examination of the Lake Hefner area indicated a net deep-seepage loss (see preceding section) of 0.2 acre-foot per day, and this figure was used in computing daily evaporation for the entire period.

Shallow seepage into Lake Hefner occurs along the east and south side shores where the terrain slopes gently toward the lake. The reddish-brown, shaly soil is quite thin, averaging perhaps 1 foot in thickness. The maximum thickness of soil penetrated in any of the holes drilled in connection with the geologic investigation was 3 feet, and in most instances there was no appreciable soil cover. The movement of water through the thin or nonexistent soil cover into Lake Hefner was, therefore, considered to be negligible.

The possibility of return flow from golf-course irrigation during the spring and summer of 1950 was considered. The quantity of water applied was generally less than 0.1 acre-foot per day and never exceeded 0.2 acre-foot per day. The amounts applied were absorbed by the soil and were less than the evaporative and transpirative requirements of the vegetative cover. Hence, it was considered improbable that any return flow from irrigation reached Lake Hefner at any time.

The earth dam and dike on the north and west sides of Lake Hefner is of rolled-earth fill with a clay core, and is not impermeable. A seepage-collection system was provided in the main part of the dam and drainage ditches were constructed to carry the collected seepage away from the dam. Seepage measurements were made during dry periods when there was no natural runoff, as evidenced by complete cessation of flow in nearby water courses known to be unaffected by seepage from Lake Hefner. Most measurements were made with a portable aluminum 90° V-notch weir, but a few were made with a Price pygmy current meter. The variation in lake stage throughout the project was only a few feet, so the seepage head remained practically constant.

At the time of the detailed geologic and hydrologic examination of Lake Hefner, made shortly after observations were begun, it was suggested that the computation of seepage inflow be based on measurements made during periods when evapotranspiration losses were at a minimum. Measurements were made throughout the period of operations at locations AA, BB, CC, DD, EE, and FF (see map). Seepage measurements during the winter were made on days of above-freezing temperature to eliminate the possibility of flow being arrested by ice formation. The effect of evapotranspiration losses in the seepage areas and in the collection channels between the dam and points of measurement is apparent (fig. 21). Evapotranspiration losses in summer were approximately 0.6 acre-foot per day. The size of the area covered by vegetation supplied from seepage was not determined.

At AA, BB, and CC, measurements were made of the collected flow from a number of small seeps that arise in the area between the dike and the north-south road just west of Lake Hefner. Most of the seepage flow at DD comes from a marshy area just west of the road in Section 28.

It was recognized that some seepage from Lake Hefner may reach the west branch of Bluff Creek without first appearing in any of the surface drains. On 28 January 1950, current-meter measurements were made to determine the increase in flow of the west branch of Bluff Creek owing to seepage from the west side of Lake Hefner. The reach selected was from a point approximately 1700 feet north of the south line of Section 33 to a point on the north line of Section 28. There was no flow in any of the natural drains entering the stream from the west, so the increase in flow in the reach was considered to be seepage from Lake Hefner. All ponds in the reach were full and spilling except Silver Lake. A temporary staff gage was installed on the shore of Silver

Lake and the change in contents was computed on the basis of the observed change in stage and the area of the lake. The difference between inflow and outflow, allowing for the change in contents of Silver Lake, was computed to be 0.10 acre-foot per day. Although admittedly not of high accuracy, this result is of the same order of magnitude as those obtained the next year. The average of five measurements of seeps AA-DD inclusive, during the late fall, winter, and early spring of 1950-51 was 0.22 acre-foot per day. Thus, it is probable that the amount of unmeasured seepage reaching the west branch of Bluff Creek in the area west of Lake Hefner is small.

All the seepage collected by the drains along the north and west sides of the dam is conducted into Bluff Creek. In addition, small seeps in Sections 22, 23, and 27, which are undoubtedly supplied from the reservoir, also drain into Bluff Creek. Several small uncontrolled farm ponds have been constructed to take advantage of the never-failing supply from these seeps. These remain full and overflow continuously. All seepage from the northwest side of the lake reaches Bluff Creek and was measured at EE.

The determination of seepage at EE was complicated to some extent by waste water from the filter plant, which is discharged to the sludge pond (see map). Normally the sludge pond is full, and overflows through an uncontrolled morning-glory spillway. It can be drawn down if desired, however. Measured flow at EE during the period January to August 1950 may have included some waste water from the filter plant. A 90° V-notch weir was installed at a point approximately 300 feet below the sludge pond (location JJ). When measurements were made at EE on 28 April, 16 June, 18 June, and 15 August 1950, the flow at JJ was also measured. The flow at JJ, as thus determined, included both the waste water from the filter plant and seepage from the reservoir. In order to separate the seepage flow from the waste-water flow, it was decided to draw down the sludge pond prior to making seepage measurements. The flow at JJ was then only seepage since the sludge pond was not spilling. The results of six measurements made at JJ with the sludge pond not spilling indicated that average seepage at JJ was 0.60 acre-foot per day. It was then possible to adjust the measurements made at EE during the period April to August 1950 for filter-plant waste water.

Seepage measured at FF included flow from several seeps in the west half of Section 24 and from several others in the area below the spillway in Section 25. It was obvious that these seeps were supplied from the reservoir, for their flow was perennial, in contrast to those farther east. Although there is a remote possibility that the flow at FF includes some seepage from sources other than the reservoir, no such flow was observed during dry periods and, if present at all, it must necessarily be small compared with the total flow at FF, which is itself an extremely small item in the water budget.

Five sets of seepage measurements were made during the period 25 October 1950 to 26 March 1951. The results were as follows:

Date	Daily Flow in Acre-Feet			
	AA-DD, inc.	EE	FF	Total
25 Oct 1950	0.24	1.60	0.23	2.07
19 Dec 1950	.27	2.10	.35	2.72
21 Mar 1951	.20	1.79	.38	2.37
21 Mar 1951	.20	1.57	.38	2.15
26 Mar 1951	.19	1.66	.35	2.20
Average	0.22	1.74	0.34	2.30

Total seepage during periods of minimum evapotranspiration losses averaged 2.3 acre-feet per day, as shown in the above table. This figure was used in computing daily evaporation throughout the period of observation at Lake Hefner.

RESULTS

Daily evaporation from Lake Hefner was computed for all but 8 days of the 493-day period of observations. On those 8 days, inflow volumes were so large as to preclude computation of evaporation with reasonable accuracy.

A weekly reporting form was designed to facilitate orderly tabulation of the various items comprising the water budget. Sample computations for the week ending 9 September 1950 are shown in figure 22. Daily values of the various items are believed to be reasonably representative of average conditions. Daily inflow was small except on Monday, 4 September, and on that day rainfall on the lake surface was the largest single item in the water budget. Surface inflow resulting from that storm, it will be noted, was relatively small. Withdrawals for municipal use were the major outflow item, ranging from 20.7 to 33.8 acre-feet per day. Indicated daily stage changes were in reasonably good agreement throughout the week; the maximum daily difference between any two recorders was 0.010 foot on 4 September,

WATER BUDGET FOR WEEK ENDING 12 P. M. SATURDAY, 9 SEP 1950

	SUNDAY	MONDAY	TUESDAY	WEDN'SD'Y	THURSDAY	FRIDAY	SATURDAY
EVAPORATION							
Intake tower stage change (ft)	—0.021	+0.059	—0.031	—0.035	—0.026	—0.036	—0.025
Boat harbor stage change (ft)	— .024	+ .059	— .029	— .038	— .025	— .036	— .029
East shore stage change (ft)	— .027	+ .055	— .030	— .035	— .020	— .030	— .028
West shore stage change (ft)	— .028	+ .065	— .036	— .034	— .027	— .035	— .030
Avg. stage change (ft)	— .025	+ .060	— .032	— .036	— .024	— .034	— .028
Avg. area during day (acres)	2365	2365	2367	2365	2363	2362	2360
24-hour change in contents	—59.1	+141.9	—75.7	—85.1	—56.7	—80.3	—66.1
Inflow	6.7	165.7	4.6	1.9	1.7	1.4	1.4
Outflow	23.2	28.5	27.2	30.1	34.0	36.3	36.3
Evaporation (acre-feet)	42.6	—4.7	53.1	56.9	24.4	45.4	31.2
Evaporation (inches)	.216	— .024	.269	.289	.124	.231	.159
Evaporation (cm)	.549	— .061	.684	.733	.315	.586	.403
Avg. contents during day	65,170	65,180	65,250	65,170	65,100	65,030	64,960
Mean lake stage	1194.886	1194.890	1194.920	1194.886	1194.856	1194.826	1194.794
INFLOW							
Rainfall on lake	5.12	160.23	0	0	0	0	0
Intake canal	1.45	1.86	1.59	1.23	1.23	1.23	1.23
Runoff at A	0	0	0	0	0	0	0
Runoff at B	0	.30	.36	.08	.02	0	0
Runoff at C	.12	1.21	.99	.36	.28	.22	.22
Runoff at D	0	0	0	0	0	0	0
Runoff at E	0	0	0	0	0	0	0
Runoff at F	0	.81	.61	.06	.04	0	0
Runoff at G	0	.02	0	0	0	0	0
Runoff at H	0	.26	.22	0	0	0	0
Runoff at J	0	0	0	0	0	0	0
Ungaged area	0	1.0	.8	.2	.1	0	0
Total inflow	6.69	165.69	4.57	1.93	1.67	1.45	1.45
OUTFLOW							
Seepage:							
AA + BB + CC + DD	0.22	0.22	0.22	0.22	0.22	0.22	0.22
EE	1.74	1.74	1.74	1.74	1.74	1.74	1.74
FF	.34	.34	.34	.34	.34	.34	.34
Deep seepage	.2	.2	.2	.2	.2	.2	.2
Total seepage	2.5	2.5	2.5	2.5	2.5	2.5	2.5
Irrigation pumpage	0	0	0	0	0	0	0
Venturi-meter flow	20.7	26.0	24.7	27.6	31.5	33.8	33.8
Total outflow	23.2	28.5	27.2	30.1	34.0	36.3	36.3
RAINFALL ON LAKE							
Recorder no. 1 (inches)	0.01	1.02					
Recorder no. 2 (inches)	.01	1.26					
Recorder no. 3 (inches)	.02	1.08					
Recorder no. 4 (inches)	.02	.74					
Non-recording gage no. 1	.02	.56					
Non-recording gage no. 2	.02	.44					
Non-recording gage no. 3	.04	.44					
Non-recording gage no. 4	.04	.48					
Non-recording gage no. 5	.03	.56					
Non-recording gage no. 6	.03	.56					
Non-recording gage no. 7	.03	.75					
Non-recording gage no. 8	.03	.79					
Non-recording gage no. 9	.02	.99					
Non-recording gage no. 10	.02	.96					
Non-recording gage no. 11	.03	1.09					
Non-recording gage no. 12	.03	.72					
Non-recording gage no. 13	.03	.67					
Non-recording gage no. 14	.03	1.08					
Non-recording gage no. 15	.03	1.07					
Non-recording gage no. 16	.03	.99					
Non-recording gage no. 17	.03	.88					
Non-recording gage no. 18	.02	.75					
Sum	.57	17.88					
Average	.026	.813					
Area of lake (acres)	2365	2365					
Rainfall (acre-feet)	5.12	160.23	0	0	0	0	0

Figure 22. Water budget computation for the week ending 9 September 1950.

when the east-shore gage indicated a rise of 0.055 foot and the west-shore gage a rise of 0.065 foot.

A study of the sample data presented in figure 22 would lead to the intuitive conclusion that daily evaporation figures are probably not of equal accuracy. For example, computed evaporation on 4 September is probably not as accurate as for other days in the week because of the large volume of inflow and because of the somewhat larger than usual variation in indicated stage changes on that day.

It was, therefore, considered desirable to classify the computed daily values of evaporation as to accuracy. The approximate magnitudes of the errors in measuring each item in the water budget have already been discussed. The error due to nonagreement of indicated stage changes was determined for each day using the range as a measure of the standard error. The error resulting from areal variability of rainfall was taken to be the standard error of the mean of the catches in the 22 rain gages. The standard error of measuring filter plant withdrawals was taken as 3 per cent, and the standard error of other inflow and outflow items as 5 per cent. The individual errors were then combined in the usual manner by adding the individual variances to obtain the total variance.

When Lake Hefner was selected as the site for the study, the accuracy criterion was that the error in mean monthly evaporation computed from the water budget should not exceed 5 per cent. Average monthly evaporation was originally estimated to be approximately 1190 acre-feet (Harbeck, 1951). On this basis, the error in monthly evaporation should not exceed 60 acre-feet. Assuming the standard errors in daily evaporation to be normally distributed, the standard error in daily evaporation equivalent to a monthly error of 60 acre-feet would be $60/\sqrt{30}$, or 10.9 acre-feet. The following limits for the various accuracy classes of daily evaporation were therefore selected, using 10 acre-feet as the reference level.

Class	Standard Error of Computed Daily Evaporation in Acre-feet
A	0 — 4.9
B	5 — 9.9
C	10 — 19.9
D	20 and over

Each daily figure of evaporation was classified on this basis. The eight days for which evaporation was not computed because of extremely large inflow volumes were included in class D. The distribution was as follows:

Class	Number of Daily Figures of Evaporation in Indicated Class
A	142
B	165
C	108
D	78
	Total 493

Daily evaporation figures for 62 per cent, or 307 days, were classified as either A or B. It should be remembered that this scheme is not infallible; errors may be larger or smaller than indicated. Comparisons between daily water-budget evaporation, and evaporation computed by one or another of the various equations, indicate that the accuracy classification scheme is satisfactory for selecting the most reliable figures of daily evaporation. The fact that 62 per cent of the daily figures were classed as A or B indicates that the original requirement as to accuracy of the water budget was met.

The system of accuracy classification was also used to indicate the relative accuracy of water-budget evaporation for selected periods of any number of days. Class limits for such periods were computed so as to be statistically consistent with the class limits for daily evaporation. In general, it was found that the accuracy classification improved as the length of the period increased. This might be expected since, as previously mentioned, the stage measurement error was independent of the length of period and its relative effect therefore decreased as the length of period increased.

Monthly figures of evaporation from Lake Hefner as determined by the water-budget method, including adjustments for thermal expansion, are shown in table 1. Daily figures will be published in Volume II (Data Report) of this series.

Of particular interest in table 1 is the variation in monthly evaporation. The maximum monthly evaporation of 8.828 inches occurred in August 1951, and the minimum of —0.638 inch in February 1951. Evaporation for the year September 1950 to August 1951 totalled 54.30 inches. Daily evaporation rates of 0.5 inch or greater were frequently observed. Although generally small in amount, condensation occurred on 56 out of 483 days, most of which were in winter or spring, with 15 days in February alone. The frequency of occurrence of condensation was surprisingly large, but not inconsistent with what might have been predicted from a study of the meteorological data.

TABLE 1. Monthly Evaporation from Lake Hefner, Oklahoma.

Month	Indicated Monthly Evaporation (inches)	Adjustment for Thermal Expansion (inches)	Monthly Evaporation Adjusted for Thermal Expansion (inches)	(cm)
May 1950	3.457*	+0.283	3.740	9.50
Jun	6.059	+ .292	6.351	16.13
Jul	6.519	— .021	6.498	16.50
Aug	7.410	— .020	7.390	18.77
Sep	5.839	— .196	5.643	14.33
Oct	6.741	— .228	6.513	16.54
Nov	6.298	— .309	5.989	15.21
Dec	2.848	— .006	2.842	7.22
Jan 1951	2.448	+ .018	2.466	6.26
Feb	—0.617	— .021	—0.638	—1.62
Mar	3.369	+ .021	3.390	8.61
Apr	2.533	+ .197	2.730	6.93
May	3.319†	+ .170	3.489	8.86
Jun	5.502	+ .370	5.872	14.91
Jul	7.029	+ .148	7.177	18.23
Aug	8.896	— .068	8.828	22.42

* Estimated from 24-day total of 2.677 inches
† Estimated from 30-day total of 3.212 inches

CONCLUSIONS ON THE WATER BUDGET

Although it is theoretically possible to use the water-budget method for the determination of evaporation from any lake or reservoir, it is usually impracticable to do so because of the effects of errors in measuring the various items. Evaporation as determined by this method is a residual, and therefore may be subject to considerable error if it is small relative to other items. At Lake Hefner, however, evaporation was a major item in the daily water budget; during the 16-month period of observation, outflow was only 10 per cent greater than evaporation. Inflow was generally much less because of the sporadic nature of reservoir replenishment.

The requirement that the error in monthly evaporation computed from the water budget should not exceed 5 per cent was met. Daily figures are, of course, subject to somewhat larger percentage errors, particularly during the spring and fall months when the effect of thermal expansion of the water in the reservoir results in a more or less systematic, but unavoidable, error in the daily figures.

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Instrumentation for Mass-Transfer and Energy-Budget Studies

by Lloyd J. Anderson*

To provide the necessary meteorological data for computing lake evaporation by the mass-transfer and energy-budget methods, it was necessary to develop special equipment in some cases and to modify standard equipment in others to meet the particular requirements of the problem.

The meteorological variables to be measured were: air temperature, humidity, and wind speed, at 2, 4, 8, and 16 meters above the lake surface; lake surface temperature; wind direction; and rainfall. These were measured and recorded at each of four stations. In addition, it was necessary to measure, at one station, the incoming solar and terrestrial radiation and the solar energy reflected from the lake surface.

GENERAL DESCRIPTION OF EQUIPMENT

In the design of the equipment system, four factors were considered essential:

1. The data must be recorded as accurately as possible.

2. The equipment must maintain its calibration over extended periods of time.

3. It must operate unattended for as long periods as possible.

4. It must use a minimum of electrical power.

Wind speed and direction were measured, respectively, with standard 3-cup Robinson-type contact anemometers and a conventional wind vane with contacts at cardinal compass points. Rainfall was measured with a tipping-bucket rain gage. The only adaptation needed for this equipment was the provision of a low-power recording system. An operational-type Esterline-Angus recorder with a spring-wound paper feed was used in conjunction with a capacitor discharge circuit for operating the recorder relays.

Temperatures and humidities were measured with wet and dry thermocouples. In this way all elements, including the lake temperature element, could be made up having identical calibrations and, using No. 30 wires for the wet elements, the minimum ventilation required for attaining the true wet-bulb depression was reduced to 0.5 mph. This eliminated the need for forced ventilation, since out of doors the wind speed is rarely below this minimum.

To use thermocouples, as well as thermopile radiation-measuring equipment, it was necessary to develop a low-power amplifier which could amplify voltages a thousandfold in order to drive the 1-mil Esterline-Angus recorder and which would maintain its amplification factor constant within a fraction of 1 per cent, regardless of normal changes in battery voltages and tube characteristics. The amplifier used for this purpose was a negative-feedback galvanometer type and fulfilled the requirements with a power consumption of 4 watts. It could thus operate for periods of two to three weeks on a set of batteries small enough to be easily transportable to the station sites. Further details on the amplifier, as well as on the thermocouple psychrometer, may be obtained by referring to Anderson, Anderson, and Marciano (1950), Bellaire and Anderson (1951), and Denton (1951).

Careful comparison was made out of doors between the wet and dry thermocouple traces and calibrated mercury thermometer readings. In this comparison, the thermocouples were mounted in a radiation shield designed to protect them from solar radiation and were ventilated by the wind, which varied from 4 knots to nearly calm. The thermometers were shaded and ventilated at 8 knots by a blower. A plot of thermometer readings taken at 10-second intervals showed agreement with the thermocouple trace within +0.1°C over the 30-minute test period, except during a period of flat calm, when thermocouple readings rose as much as 0.5°C above the thermometer values.

As a result of the above test, and careful spot checks in the field, it is believed that wet- and dry-bulb temperatures were obtained with errors of ±0.1°C. Although the Esterline-Angus Company does not guarantee its recorders to better than 2 per cent of full scale, or 1.0°C in our case, this accuracy was considerably improved by calibrating all recorders against a standard milliammeter and adjusting them so that all were within ±0.1°C at five points over the scale. By loading the pens as lightly as practicable, it was possible to reproduce the same trace for a given temperature within ±0.1° when approached from above or below.

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Numerous attempts to calibrate the cup anemometers resulted in closer agreement between individual units than with the standard. Since the anemometers were to be used in the field, they were calibrated out of doors against a hot-wire anemometer. The rapid response of the hot wire and the slow response of the cup anemometers made it difficult to compare results. Because of the complicated hot-wire traces obtained, and the logarithmic response to wind, it was not possible to planimeter the trace for accurate average winds. Installing the anemometers on a jeep and driving up- and down-wind also gave inconclusive results because of variable ambient winds, even under conditions as calm as can be expected.

The results of these tests led to the conclusion that, whatever the calibration errors, all units were consistent to ± 0.5 knot for winds above 3 knots. Since the anemometers were maintained in good condition and the cups were new, it is reasonable to assume that the absolute calibration was within the limits given by the manufacturer.

The radiation measurements consisted of incoming and lake-reflected solar flux measured with an Epply pyrheliometer, and total incoming radiation measured with a Gier and Dunkle flat-plate radiometer.

The pyrheliometer is a flat circular plate mounted horizontally inside a lime-glass bulb. The plate is divided into a central white spot, a black ring, and an outer white ring. A 10-junction thermopile measures the temperature difference between the black and white areas, which is proportional to the radiation flux penetrating the glass bulb.

The radiometer consists of a flat 2-inch square plate, mounted horizontally in the blast of a small blower. The plate is a sandwich with a blackened aluminum upper surface and a polished aluminum lower surface; between is a thermopile measuring the vertical temperature gradient across an insulating sheet forming the center layer of the sandwich. The thermopile voltage is thus proportional to the heat flow down through the plate, which in turn is proportional to received energy at the blackened surface after deduction of the black-body radiation. To obtain the latter correction, a separate thermocouple is used to measure the black-surface temperature. The function of the blower blast is to eliminate the effect of the wind on the calibration coefficient of the device.

Pyrheliometer and radiometer voltages were recorded using an amplifier and recorder system similar to that used for temperature and humidity.

For obtaining temperature-structure data within the lake itself, a Western Electric thermistor bead was used. It was lowered from a boat on a conducting cable, and water temperatures were recorded on a 1-mil Esterline-Angus recorder in the boat. Temperature changes of the bead caused its resistance to change, and this in turn produced an off-balance voltage in a Wheatstone-bridge circuit. This voltage was amplified and recorded as lake temperature. Calibration showed the instrument capable of an accuracy of $\pm 0.1°C$, when corrected by bucket checks.

After reconnaissance of the Lake Hefner area and a study of the wind rose data, it was decided to place the four meteorological stations as indicated on the map. The primary station, which included the radiation equipment for the energy-budget studies, was installed on a barge anchored near the center of the lake. The most important land station was located on the southeast shore of the lake, and served to measure the properties of the air before reaching the lake surface. The prevailing winds, particularly during the summer, were such that this location was representative of the up-wind air most of the time. During the winter the wind direction is more variable and, in order to utilize northerly winds, a station was located on the northeast shore of the lake to act as the up-wind station for winds between north and east. The fourth station was installed on the intake tower on the north shore of the lake, about 50 yards south of the dike. This station was to collect down-wind data for southerly winds. It was not felt necessary to provide for westerly winds, since these occurred only a very small percentage of the time.

A more detailed discussion of the development of the equipment described above, and the details of its construction, are given in Anderson, Anderson, and Marciano (1950).

MASS-TRANSFER STUDIES

Initial Measurements

Before the routine evaporation data could be collected, it was necessary to evaluate certain parameters such as the variation of wave height with wind speed and fetch, as well as the effect of stability on the wind profiles. The requirements for this study included the measurement of very small differences between winds and temperatures at 8 and 2 meters above the lake surface and of wave height as a function of distance down-wind. The basic equipment for these purposes has been described by Anderson, Anderson, and Marciano (1950). It was installed at all stations except the northeast.

The wind equipment consisted of two hot-wire anemometers connected on opposite sides of a Wheatstone bridge. Due to the logarithmic response of the hot wires to wind speed, the off-balance bridge voltage was a measure of the ratio of the winds at 2 and 8 meters. It was found that this ratio could be measured to ± 1 per cent, which was far better than using two conventional cup anemometers.

The temperature-difference equipment utilized a 5-junction thermopile installed in the radiation shields at 2 and 8 meters. The accuracy of measurement was $\pm 0.02°C$. The wind ratios and temperature differences, as well as the lake temperature, were alternately recorded through the same amplifier and Esterline-Angus recorder system, using the 20-point programming switch.

The wave-height equipment consisted of three vertically-floating staff gages marked with alternate 6-inch-wide black and white bands. One gage was anchored near the first barge location, the second near the middle of the lake, and the third near the north shore. In addition to visual and motion-picture observations of the staff gages from shore, using a 10-inch reflecting telescope, close-up observations of wave height were made from a small boat. The three methods yielded essentially identical data. Six sets of such measurements were made, covering wind speeds up to 30 miles an hour.

During the first three-week period, the barge was located about midway between the south station and the center of the lake, and for the remaining three weeks the barge was relocated at its final position near the center of the lake. The major equipment modification found necessary was damping of the wind-ratio recorder to obtain a more readable trace. The time constant was increased from 2 to 6 seconds, resulting in a trace from which average wind ratios ($\overline{u_8/u_2}$) could be much more easily obtained. The 5-junction thermopiles, used to measure small temperature differences between 2 and 8 meters, performed as expected and no modifications were necessary.

Routine Measurements

After the initial measurements, the installation of the equipment for routine operation was completed at all four stations, and routine measurements were begun, on 15 April 1950. The equipment, as designed and installed at the model station in San Diego, was found to perform satisfactorily. A few modifications were needed, most of which resulted from maintenance problems and which will be discussed under that heading. The photographs in figures 23 through 26 show over-all views of the four stations. The south station (fig. 24) was located about 100 yards from the shore of the lake, in order to be above the high-water level. The northeast station (fig. 25) was mounted on a platform supported by 4-foot pilings for the same purpose. To mount the equipment on the intake tower, it was necessary to replace the original conical roof by the flat one shown in figure 26, and to hang the lower-level instruments on 10-foot booms in order to avoid, as far as possible, the influence of the tower itself. A provision was made at this station for raising or lowering the instrument levels to follow the lake stage. At this particular station commercial power was available, and the recording equipment could be located inside the tower. Battery power was required for the other three stations. An additional complication was the need for providing aircraft warning lights for the northeast and barge stations. This required additional batteries and clock switches for turning off the lights during the daytime. Barge lights were mounted in such a way that they had no effect on the radiation measurements.

The lake-temperature thermocouples were the only elements requiring appreciable modification for satisfactory operation. It was soon found that a galvanic voltage was being generated between the thermocouple and float assembly (fig. 27) and the steel anchoring cable. During wet weather, partial shorting by exposed wet surfaces caused this voltage to appear across the amplifier terminals. To eliminate this effect, the thermocouple junction was electrically insulated from the lake by encasing it in a transparent plastic rod. It was considered that no loss of accuracy would result by this procedure, since any solar energy absorbed by the plastic would be quickly carried away by the lake water. The constant wave motion prevented any pile-up of warm water at the junction. Since it was necessary to measure the lake temperature as close to the surface as possible, the junctions were adjusted to float about ¼ inch below the surface. The lake-surface temperature thus measured is an average of the top ½ inch of water under quiet conditions and probably the top 6 inches of water and lower 3 inches of air under rougher conditions. Under virtually all conditions except those when still warm air overlies a flat still water surface,

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the wave action tends to remove thermal gradients in the top few inches of water. Thus the temperatures as recorded should be representative of the interface. The temperatures will be even more representative of the interface as the wind increases and wave mixing becomes stronger, in which case one would also expect thermal equilibrium between the air and water within the first few inches above and below the interface. Since the wind was above 4 knots 92 per cent of the time at Lake Hefner, it is believed that steep thermal gradients in the top fraction of an inch occurred very rarely.

ENERGY-BUDGET STUDIES

Radiation Measurements

Figure 23, showing the barge station, gives an over-all picture of the radiation equipment installation. The flat-plate radiometer and the Eppley pyrheliometer are on top of the central mast. The two downward-looking Eppleys for measuring reflected solar energy are mounted on 4-foot arms near the tops of the two shorter masts. To avoid radiation pickup from the barge, two downward-looking Eppleys were used, one covering the south half of the lake and the other covering the north half. The bulbs were painted to prevent pickup from overlapping sections of the lake and from the barge. The outputs of both Eppleys were added in series and recorded as a single voltage; this was possible because the calibration constants of the two instruments were essentially equal. The instrument shelter for the recording equipment is the smaller one nearest the central mast. Figure 28 shows a close-up of the southernmost mast with a downward Eppley shown near the top. Figure 29 shows the interior of the instrument shelter, with the amplifier and recorder on the top shelf and the batteries and accessory equipment on the bottom shelf.

The flat-plate radiometer installed on top of the central mast performed well under all conditions. It was found during analysis of the data, however, that the long-wave radiation, which was constant during the night, began to decrease at sunrise and reached a minimum of about half the nighttime value at midday. It began rising again during the late afternoon and by sunset was up to the nighttime value. Since this quantity was derived by subtracting the incoming solar radiation, as measured by the Eppley, from the total incoming radiation, as measured by the flat-plate radiometer, there are several possible

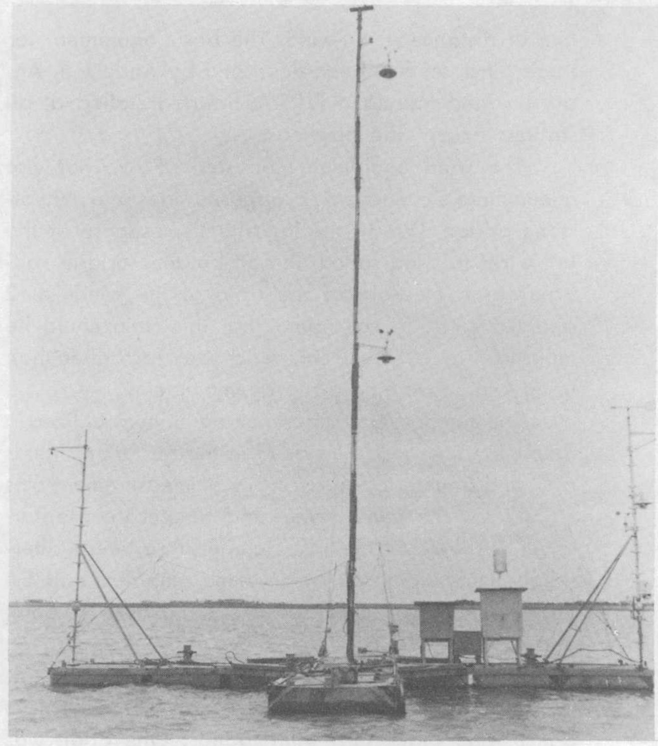

Figure 23. barge station.

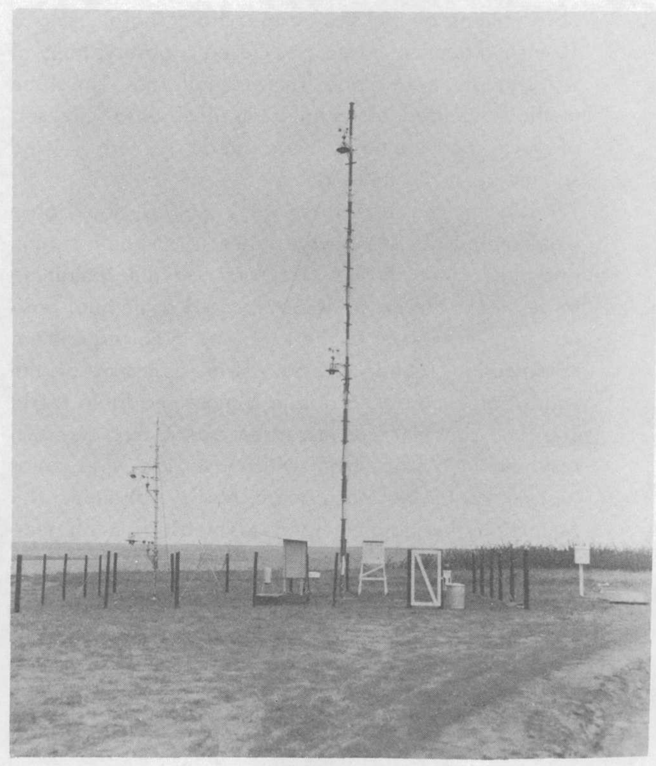

Figure 24. South station.

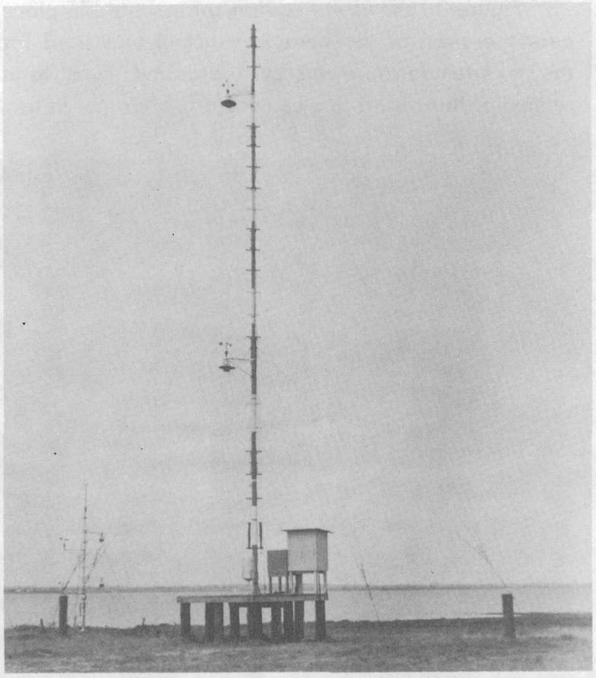

Figure 25. Northeast station.

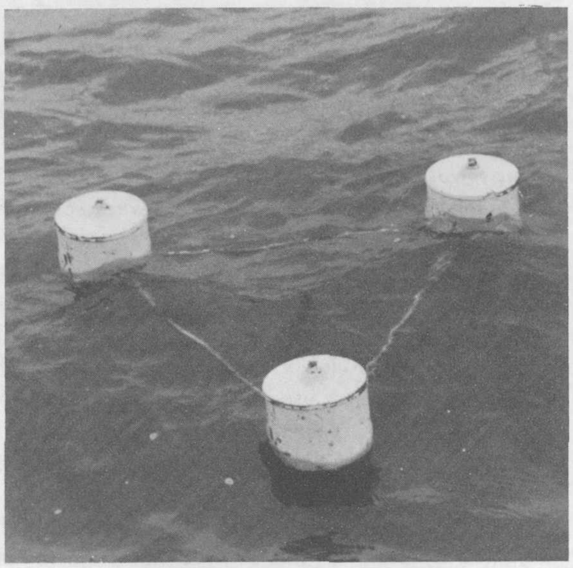

Figure 27. Lake thermocouple float at barge.

Figure 26. Intake tower station.

Figure 28. Lower level instruments on barge station.

causes for this apparent daytime decrease. It does not seem reasonable that it should be a real effect. Either the Eppley calibration was too high or the flat-plate calibration too low or, more probably, the reflection coefficient of the flat-plate for visible energy was not constant with angle of incidence. To check this point, the Eppley readings were compared with those of nearby Weather Bureau installations at Oklahoma City and subsequently at Las Vegas, Nevada. Comparison showed that the Oklahoma City values were 2.6 per cent higher and the Las Vegas values 2 per cent lower than the readings of the Eppley pyrheliometer used in these studies. Since the probable error of calibration is of the order of ± 2 per cent, it seems evident that the instrument in question lies about in the middle of this range.

Accurate terrestrial radiation values might be obtained by using a glass-covered flat plate for obtaining solar-energy values. If the covered and uncovered flat plates were connected in series with opposite polarity, the resulting recorded value should approximate the terrestrial energy flux. Such a scheme may be an oversimplification of the problem, but should be investigated.

To prevent interaction between the radiation recorder and the temperature recorder, it was necessary to use separate ice baths for the reference junctions of the flat-plate temperature thermocouple and the other thermocouples. When a common ice bath was used, interaction was noticed between the two circuits during rainy weather, presumably caused by partial shorting via the wet surfaces of the mast and barge. Similar shorting difficulties were encountered when attempting to use a common storage battery for the flat-place blower motor and the filament supply of the amplifier. Both difficulties were eliminated by suitable isolation.

The flat-plate blower was driven by a small 6-volt Haydon motor which performed satisfactorily for 6 months of almost continuous operation. This performance far exceeded expectation. The motor was replaced during a general overhaul of the equipment but was still in good operating condition.

The three Eppley pyrheliometers performed quite satisfactorily for the entire 16 months of operation and are now in use at Lake Mead.

Temperature Profile Recorder (TPR)

This equipment was developed at NEL to obtain more accurate profiles of temperature within the lake. It was described in detail by Anderson and Burke (1951).

Figure 30 shows the equipment mounted in place near the stern of the boat in which it was used for the lake-temperature surveys described in a later section of this report. It was covered, when not in use,

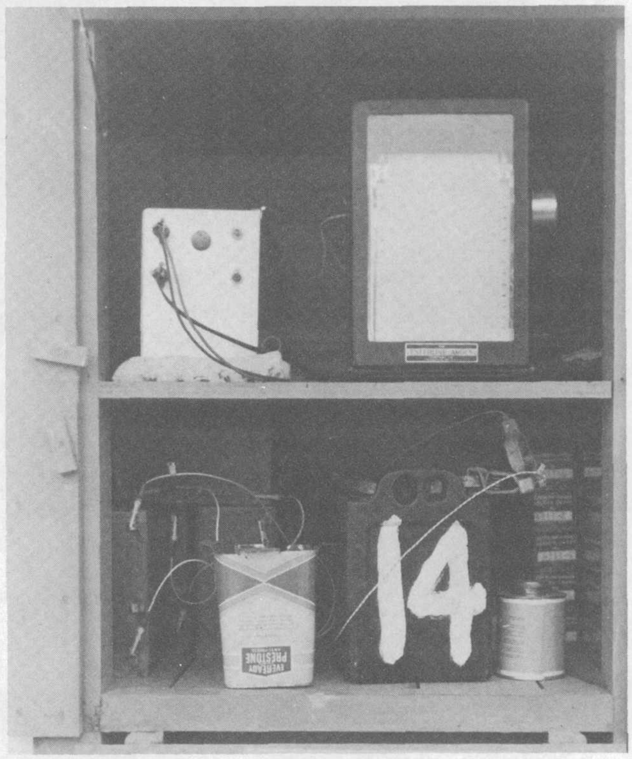

Figure 29. Radiation instrument shelter on barge.

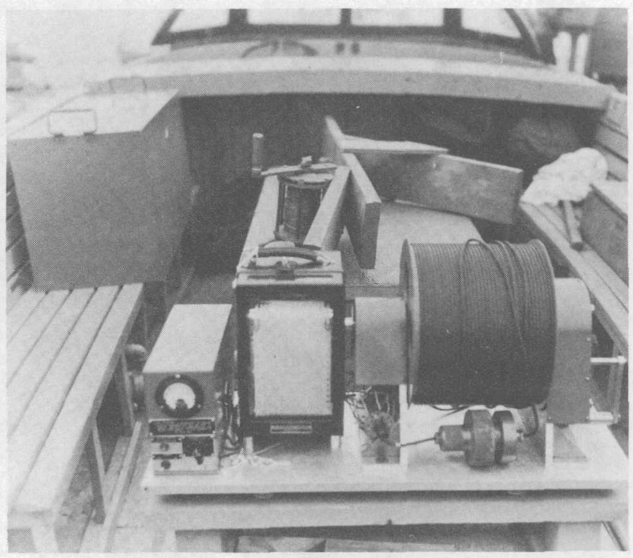

Figure 30. TPR installation on boat.

by the metal box shown in the left background. Amplifier power was supplied by the boat storage battery. Performance was satisfactory except for the last 2 months of the Lake Hefner studies, when the vibrator became faulty. Similar difficulties had occurred on a few previous occasions, but were remedied by replacing the vibrator. In June 1951, however, all the vibrators became faulty and the equipment was sent to NEL for overhaul. It was also recalibrated at this time and, after 14 months of operation, the thermistor beads were found unchanged in calibration. Tests showed that the entire amplifier had maintained its calibration over this length of time. The only component which affected the calibration of the equipment was the vibrator. Substituting various vibrator units shifted the entire temperature scale up or down a few divisions, because of the different noise levels inherent in the individual units. Correction for this shift, however, brought all scales back to the original calibration. In practice this effect offers no difficulty, since frequent checks were made by comparing the TPR readings with bucket temperatures as measured with a calibrated thermometer.

OVER-ALL PERFORMANCE AND MAINTENANCE

Accuracy Checks

To maintain the accuracy required for evaporation computations, it was necessary to make periodic checks of the equipment in the field. The temperature and humidity data were checked weekly at all stations by means of a sling psychrometer. The observed wet- and dry-bulb readings were entered on the recorder traces and compared with recorded values during the data analysis. The sling-psychrometer values were occasionally several tenths of a degree higher than the recorded values, although the wet-bulb depression always agreed within 0.1°C. It is believed that insufficient shielding of the psychrometer from the sun is responsible for these minor discrepancies.

No suitable method was found for checking anemometer accuracy in the field. All units were replaced at intervals of 30 days or less. The units which had been in use were completely overhauled, the overhaul including removal of the shaft and bearings, washing out of the lubricant, and filling with fresh lubricant. During this time the contact points were also checked, and the shaft friction was checked by the following procedure. The anemometer body with cups attached was arranged with its axis horizontal, and a small weight was placed on one of the cups which was held at the same level as the axis. The cup was released, and the weight caused it to rotate. At the bottom of the arc the weight dropped off, and the cup continued to rotate until eventually stopped by shaft friction. It was found that anemometer shafts in good condition made about nine revolutions before stopping. Any units which did not meet this criterion were relubricated.

The reference bath for the nine thermocouples at each station was contained in a 1-gallon Dewar flask. Since all temperatures were referred to the bath temperature, it was essential that 0°C be maintained in the bath at all times. A mercury thermometer was permanently mounted in the flask and checked at 2-day intervals. No appreciable discrepancies were noted.

Usable Data Yield

Table 2* shows the percentage of occurrence during each month of operation of the commonly occuring difficulties in collection of data. Some of these faults are minor, but others result in unusable data. A gradual decrease in the occurrence of most of these difficulties is evident throughout the summer and fall months of 1950. During the winter, however, wet bulbs dried out very frequently, because of freezing of the reservoirs. This effect generally occurred several hours after the air temperature dropped below 0°C, and disappeared abruptly at all levels about 2 hours after the air temperature rose above 0°C. On several occasions the temperature dropped a few degrees below freezing without the wicks drying out, because presumably, the water in the reservoirs did not freeze and thus continued to feed water to the wicks. By March, the occurrence of frozen reservoirs had decreased to only 11 per cent and no further difficulty was encountered after 21 March.

Missing anemometer pips occurred frequently during the first five months. This defect was eliminated by periodic checking of the contacts during anemometer overhaul. Fortunately, this defect was not serious, because the pips were missing in such a way that they could be inserted in the traces during analysis of the data.

Figure 31 shows the yield of usable data during the operating period. The radiation equipment, after a rather poor start, settled down to an average yield of 24 usable days per month during the last year of operation. Since the barge meteorological data were crucial for all evaporation computations, the yield

* See page 45.

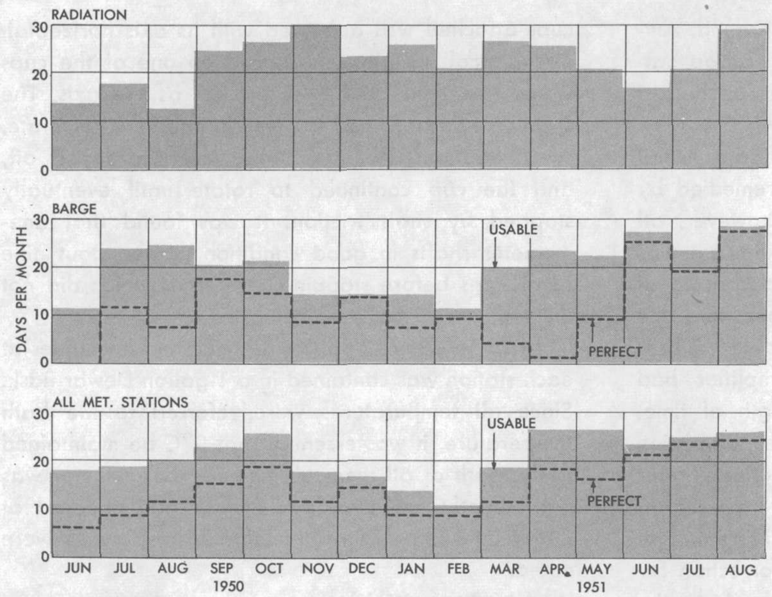

Figure 31. Data yield (days per month) of meteorological and radiation equipment.

of this station is shown separately. The average was 20 usable days per month, with 14 days per month of perfect data. The bottom row of the figure shows the average for all four meteorological stations, 21 days per month of usable data and 16 days of perfect data.

To qualify a day as "usable," there could be no more than two 3-hour periods containing unusable data. Both the temperature and wind recorders had to produce usable data simultaneously at all four levels (2, 4, 8, and 16 meters). The "perfect" days, of course, were those in which no data-recording failures occurred during the entire 24-hour period.

Maintenance Problems

The above data yields were the result of careful and conscientious attention to maintenance detail by the resident maintenance crews. After the initial installation by the equipment designers, it was the duty of the maintenance crews to solve the everyday problems that arose, and to solve them quickly to keep the data loss as small as possible. Table 2 and figure 31 give a graphic record of the maintenance crews' performance.

As mentioned previously, all the equipment except that at the intake tower was battery-operated. Originally, the 180-volt power required by the thermocouple amplifiers was supplied by dry batteries, and 6-volt power by storage batteries. Because of the persistent wind at Lake Hefner, it was possible to install a wind-driven propeller-type battery charger at each station. This greatly reduced the labor of transferring storage batteries and made it practicable to use a vibrator-type power supply, operating from a storage battery, to supply the 180-volt amplifier. In this way all the electrical requirements of the stations were fulfilled by the wind chargers. The cost of the charger and vibrator units was saved in a few months of operation by eliminating the expense of dry batteries. The procedure used with a wind charger was to connect it to one set of batteries at the station while a second set was in use on the equipment. It was not possible to connect the wind charger to the batteries while they were in use, because this would have necessitated connecting all storage batteries together, which was not permissible in the amplifier circuit.

The thermocouple amplifier, which represents the heart of each station, performed quite satisfactorily with periodic maintenance checks and the replacement of tubes and exciter lamps. Table 2 shows that the temperature trace was off scale a fair percentage of the time during the first 4 months of operation. This was due to sticking of the galvanometer coil against its stops whenever it happened to go off scale. The problem was solved by modifying the stops so that the coil form contacted only a fine point on the stop, thus virtually eliminating the cohesive force that seemed responsible for the sticking. Very little difficulty was experienced after making this modification, in early September.

A routine maintenance check on the input impedance of the amplifier was found necessary to prevent inaccuracies resulting from too much current being drawn from the external thermocouple circuits. It was accomplished by substituting a fixed 500-ohm resistor in place of the thermocouple circuit and adjusting the mechanical zero of the galvanometer until zero input voltage was indicated on the recorder. Such readjustment was found to be required about once a month. In figures 29 and 32, the amplifiers are shown resting on a pad of foam rubber. This was necessary to eliminate high-frequency vibration of the galvanometer suspension. Such vibration was particularly troublesome at the northeast station (fig. 25), where the entire platform would be set in vibration by winds above 15 miles per hour. With a foam-rubber pad under the amplifier the trace remained unperturbed until the wind exceeded 30 miles per hour.

The Esterline-Angus recorders used to record all the data were clock-driven to conserve electrical power. They performed satisfactorily except during conditions of high humidity in the summer. Table 2 shows the increase in clock stoppage during these periods. The difficulty was partially eliminated by installing small light bulbs to dry the inside of the recorder case. The bulbs were powered by worn-out amplifier batteries.

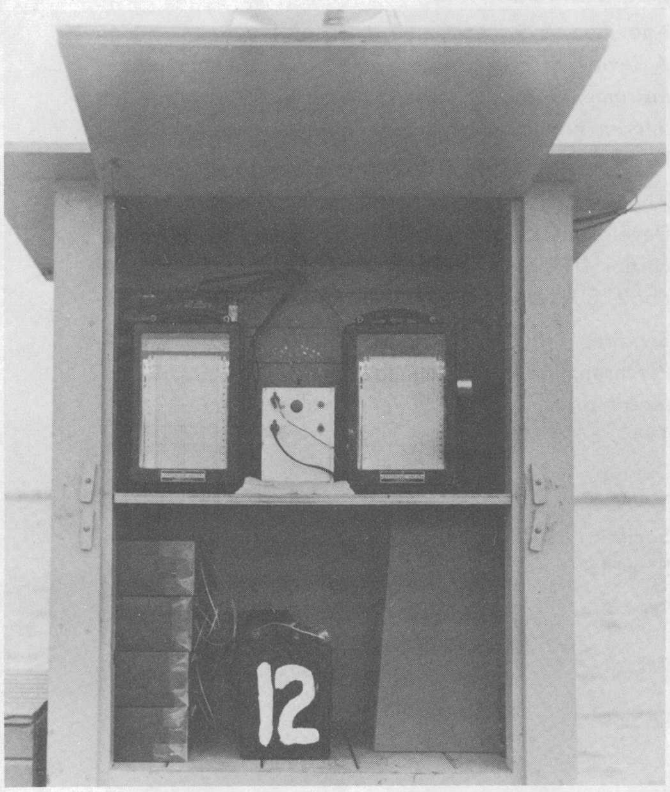

Figure 32. Meteorological instrument shelter on barge.

A 20-point switch was used to present, in turn, each of the nine thermocouple voltages to the amplifier. In this way all temperatures could be recorded on a single tape. The switch was driven by an auxiliary shaft on the recorder and made one revolution per hour. Since it was desirable to record each temperature twice an hour, each thermocouple was connected to two contacts. The two extra contacts were used to short the amplifier at half-hour intervals. This served to identify the beginning and end of each sequence and also to indicate the 0°C reading to which all temperatures were referred. For the first four months, faulty switch operation occurred about 8 per cent of the time, caused by accumulation of dust and oxidized metal on the wiper contact, such that when one contact was broken, the next was not immediately made. Thus the amplifier was open-circuited for a few seconds, resulting in an off-scale trace. Because of its short duration, this did not affect the data except when the galvanometer coil stuck off-scale, as mentioned above. Routine polishing of the wiper and stationary contacts with crocus cloth at weekly intervals eliminated this difficulty. A light film of instrument oil seemed to reduce further the occurrence by retarding oxidation of the metallic surfaces.

The reference baths for the thermocouples required refilling with ice at weekly intervals during the summer and monthly intervals during the winter. Several of the Dewar flasks imploded during the Lake Hefner observations, in spite of each being mounted inside a wooden case filled with rock wool. Even very careful handling did not prevent such occurrences. Perhaps smaller flasks refilled at more frequent intervals would have suffered fewer casualties, but the convenience of the larger size was a very desirable feature in field operation.

The wet-bulb reservoirs required filling at 2- to 4-day periods, depending primarily upon the wind speed. After the first 2 months, no appreciable data were lost because of dry reservoirs. As mentioned previously, frozen wet bulbs during the winter caused considerable difficulty, but this was unavoidable and represents the major disadvantage of the psychrometric technique. It was believed from the beginning, however, that the advantages of the technique far outweighed the disadvantages and that heavy loss of data during the winter months was less undesirable than the disadvantages of other methods of measuring humidity.

Each time the reservoirs were filled, the accumulated dust was removed from the wet thermocouple wick. This served to extend the life of the wick as well as to insure attaining of true wet-bulb temperatures. The copper wire used on one side of the junctions gradually corroded, and several wires had to be replaced toward the end of the operating period. The four spare thermocouple assemblies enabled this replacement to be accomplished with virtually no loss of data. It was found necessary to install insect screens around the psychrometer radiation shields to prevent wasps from building nests in the air passages. These screens also served to reduce wind speeds in the passages and to reduce excessive evaporation from the wicks during high winds.

Figure 27 shows a lake thermocouple float. The thermocouple is barely discernible inside the triangle. As mentioned previously, it was found necessary to insulate the junction of this thermocouple from the lake water by encasing it in a lucite rod. The thermocouple cable, from the plastic rod to the instrument shelter, was also encased in heavy-wall rubber hose, both to prevent electrical leakage and to protect the cable from mechanical abrasion. Table 2 shows that lake-temperature difficulties were essentially eliminated by this treatment.

The radiation equipment required little maintenance except for occasional dusting of the pyrheliometer bulbs and radiometer plate. The latter was given a coat of flat black enamel when necessary to preserve its absorption properties. The enamel used was the same as used by the manufacturer.

The Temperature Profile Recorder required little maintenance except for occasional polishing of the slip-ring system used for connecting the cable on the drum to the amplifier. Accumulation of dust in the system resulted in an erratic trace. Repolishing of the slip rings with crocus cloth eliminated the difficulty for several weeks.

Figure 33 shows a further maintenance problem occasionally encountered during the winter. The view shown is of the same mast as shown in figure 28. It was necessary on this occasion to chip away a great deal of ice to keep the barge floating level. Pictures such as this make one doubly appreciative of the efforts of the maintenance crew.

CONCLUSIONS ON INSTRUMENTATION

The equipment developed for the Lake Hefner studies proved to be reliable and practicable. The stations had to be visited at 2- to 4-day intervals for filling wet-bulb reservoirs, and at weekly intervals for other maintenance requirements. At windy locations, all power needs were met by wind chargers and storage batteries.

The thermocouple psychrometer was found suitable for obtaining accurate humidity data in regions where freezing temperatures are not prevalent for extended periods. The radiation recording system, comprising the Eppley pyrheliometer, flat-plate radiometer, galvanometer amplifier, and Esterline-Angus recorder, performed satisfactorily. Indicated daytime terrestrial radiation values were too low, possibly due to variable reflection of solar radiation by the flat-plate surface.

The Temperature Profile Recorder performed well and yielded much more accurate profiles than those provided by the conventional bathythermograph. When surface checks were made against a calibrated thermometer the profiles were accurate to 0.1°C.

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Denton, R. L., "A Thermocouple Voltage Amplifier for Temperature Recording," *American Meteorological Society, Bulletin,* vol. 32, no. 6, June 1951, pp. 214-216.

TABLE 2. Per Cent Occurrence of Instrument Faults, Meteorological Data, Lake Hefner Studies

	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Trace Off-scale	12	14	15	16	3	3	2	4	5	0	0	0	1	3	4	2
All Temp. Data Unusable	7	3	3	0	0	2	5	2	4	8	5	1	3	2	4	0
Lake Temp. Unusable	5	15	5	0	1	0	3	0	3	0	0	0	0	0	0	0
Dry Wet Bulb	3	3	1	0	0	3	17	23	28	27	11	0	0	0	0	0
Faulty Switch	8	7	6	10	1	2	0	0	0	0	0	0	0	0	0	0
Clock Stopped	1	2	2	2	3	2	1	0	0	4	0	0	0	0	0	0
Dry Pens	0	0	2	1	1	0	0	3	5	0	2	0	0	0	0	0
Anemo. Pips Missing	48	56	15	26	13	4	6	0	0	0	0	0	0	0	0	0

Figure 33. Icing on barge.

Mass-Transfer Studies

by J. J. Marciano* and G. Earl Harbeck, Jr.**

The foundation used for obtaining practical results from mass-transfer theory was NEL Report 159 (Anderson, Anderson, and Marciano, 1950). That report contains a detailed review of theory, based on the discontinuous- and continuous-mixing concepts applied to mass transfer in the boundary layer. As a product of the review, a selection was made of a series of evaporation equations judged suitable for testing the two concepts. Although both concepts had some experimental support, they had not been tested *to the extent or with the precise controls contem*plated in the Lake Hefner program.

Several important questions were left unanswered by Anderson, Anderson, and Marciano. First was the question of the existence of a critical wind speed for air-water boundary-layer processes. The answer is tied closely to another question, namely, the value of the roughness parameter of the lake surface. Evaporation equations are strongly influenced by answers to these two questions. A third unknown was the degree of coupling between wind and temperature fields in the boundary layer; this problem involves a search for a general wind equation for the boundary layer. Finally, it was necessary to ascertain whether theory could predict the height of wind-generated waves on a relatively small lake.

In the following pages, these problems are again discussed in connection with the theory of the boundary layer. Evaporation equations are again reviewed, with the addition of the case of evaporation in zero wind. The remaining discussion presents the results of tests of several evaporation equations against a reliable control.

THEORY

Structure of the Boundary Layer

Few concepts have had more influence on aerodynamics and fluid mechanics than Prandtl's description of the boundary layer in 1904. The theory of the boundary layer forms the basis for treating evaporation as a mass-transfer problem.

Consider a fluid flowing with constant velocity U past a fixed solid boundary. The velocity will increase from zero at the boundary and reach the value U a small distance away. Tangential stresses are set up, retarding the flow at distances farther and farther from the boundary, so that the width of the layer of retarded fluid will increase downstream. By definition, the boundary layer is that layer of fluid retarded by the boundary. Figure 34 illustrates the process involved.

We can thus divide the fluid into two portions:

1. The relatively thin boundary layer where the velocity gradient is large and, consequently, where shearing stresses cannot be neglected.

2. The region outside the boundary layer where the velocity gradient is small and shearing stresses negligible. In the second region the flow closely resembles that of the classical nonviscous fluid, with the motion determined by pressure forces and the flow described with sufficient accuracy by Euler's classical equation. Our main concern is the boundary layer, particularly the velocity distribution within it.

In nature, the boundary layer is formed as shown in figure 34. If the air has been flowing for some distance over a reasonably uniform surface, the thickness of the layer will be essentially constant with respect to downstream distance, changing height only insofar as the over-all degree of turbulence may warrant. If a new surface is suddenly encountered,

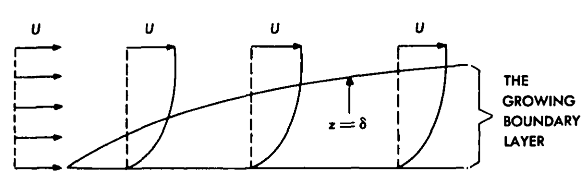

Figure 34. Growth of a boundary layer along one side of a flat plate.

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such as would occur at the edge of a lake, a new boundary layer will develop, its rate of growth depending on distance as well as the properties of the air and the new surface.

Scalar fields, such as temperature and vapor pressure, may also be thought of as having boundary layers, although it does not follow that the boundary layers for different properties are of equal thickness. Boundary-layer theory was first developed with momentum as the important property. Since then, the concept of "similarity" has been used to connect the momentum and heat boundary layers (Dryden, 1934; Goldstein, 1938). The same approach has been used to connect heat transfer and evaporation (Schmidt, 1929), and momentum transfer and evaporation (Millar, 1937). It is now generally accepted that turbulent transports of momentum and water vapor are the same, that is, their coefficients of eddy transport are interchangeable. Evaporation equations discussed later were developed with that principle in mind. The similarity of momentum and vapor transfer was shown in some excellent photographs by Albertson (1948) whose work confirmed that evaporation is a boundary-layer phenomenon.

The character of the flow and the nature of the boundary are important. In experimental work it is possible to set up conditions of laminar and turbulent flow, but in nature laminar flow is rarely observed. The present discussion is based on the postulate of turbulent flow at all times. The underlying surface may be "smooth" or "rough" in the aerodynamic sense. There exists at the boundary a very thin layer, of the order of a fraction of a centimeter in thickness, in which the velocity gradient is constant, that is, velocity is proportional to distance from the boundary. Such flow is laminar and the thin layer is called the "laminar layer." If the protuberances and irregularities of the surface project above the laminar layer, they affect the velocity distribution, resulting in turbulent flow over a rough surface. If the laminar layer completely encloses the roughness elements, the flow above is smooth.

Boundary-Layer Velocity Distributions

Considerable evidence exists that for turbulent flow without density gradients the velocity in a fully established boundary layer over a plane surface varies with the logarithm of height. Goldstein (1938, pp. 376-382) gave the following equations, based on the work of Prandtl and Schlichting:

$$\frac{u}{u_*} = C + 5.75 \log \frac{z}{\epsilon_s}. \qquad (4)$$

(a) Smooth flow: $C = 5.5 + 5.75 \log \frac{u_* \epsilon_s}{\nu}$;

$$\frac{u_* \epsilon_s}{\nu} < 7.08.$$

(b) Transitional: $C = 9.58;\ 7.08 \leqq \frac{u_* \epsilon_s}{\nu} \leqq 14.1.$

(c) Transitional: $C = 11.50 - 1.62 \log \frac{u_* \epsilon_s}{\nu}$;

$$14.1 \leqq \frac{u_* \epsilon_s}{\nu} \leqq 70.8.$$

(d) Rough flow: $C = 8.48;\ 70.8 \leqq \frac{u_* \epsilon_s}{\nu}.$

The variation of the roughness constant C is shown by figure 35*, taken from Schlichting (1949). In general, C must be determined experimentally. The values in equation (4) are for sand roughness of the type used by Nikuradse (1933). The actual roughness ϵ and the corresponding roughness constant C are related to the equivalent sand roughness ϵ_s, for rough flow, by

$$\frac{u}{u_*} = 8.48 + 5.75 \log \frac{z}{\epsilon_s} = C + 5.75 \log \frac{z}{\epsilon}. \qquad (5)$$

* All symbols and units used are explained in the Appendix.

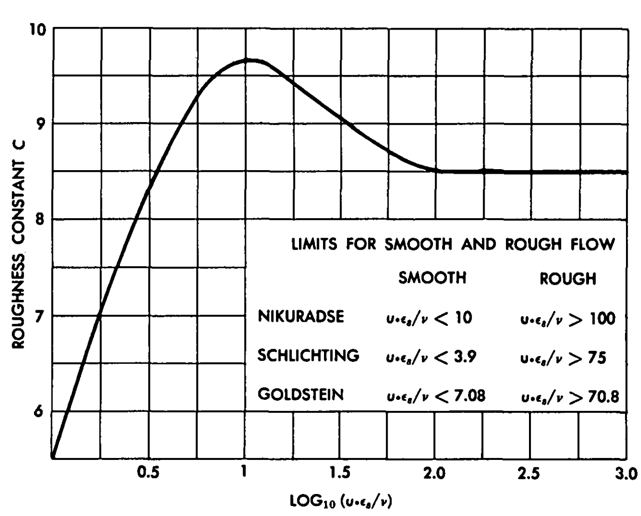

Figure 35. The roughness constant C as a function of $(u_* \epsilon_s/\nu)$.

In many cases, particularly for flow over water, the roughness ϵ cannot be precisely specified; such a specification could be made only if the roughness elements were of uniform height, shape, and spacing. Therefore, equation (4) is used, with the introduction of a "roughness parameter" z_0, which is related to ϵ_s by $z_0 = \epsilon_s/30$ (Nikuradse, 1933). Substitution for ϵ_s in equation (4) gives

$$\frac{u}{u_*} = 8.48 + 5.75 \log \frac{z}{30\, z_0} = 2.5 \ln \frac{z}{z_0}, \quad (6)$$

which is the equation generally quoted as applicable for rough flow in the atmospheric boundary layer. The use of 5.75 and 2.5 requires the use of $k_0 = 0.40$ for von Kármán's constant (in equation (6), $2.5 = 1/k_0$). Although various values of k_0 appear in the literature (Millar, 1937, Montgomery, 1943), the value chosen here is consistent with equation (4), the most generally accepted wind equation for the type of flow treated, and also with the assumption that the friction velocity u_* is constant with height in the boundary layer. This assumption is part of the discontinuous-mixing (mixing-length) approach to mass transfer. In reality u_* does vary slightly with height, but apparently the small errors so introduced are of equal and opposite magnitude to those introduced by the assumption of a linear mixing length (Montgomery, *ibid.* p. 94). Near a boundary, that is, within a boundary layer, any value of k_0 between 0.31 and 0.45 seems appropriate (Montgomery, *ibid.* figs. 1, 2). The value of 0.40 appears to be the most generally accepted value and was therefore used in the computations.

It is somewhat misleading, therefore, to say that equation (4) "applies." The wind generates waves on a lake surface, and the waves change in height, spacing, steepness, and speed downwind. Whether the waves or some other surface characteristic represent the roughness elements, it would be difficult to describe the entire surface by a single roughness ϵ. Therefore equation (6), involving the equivalent sand roughness ϵ_s, must be used for rough flow. Only under controlled (laboratory) conditions, could one use the right side of equation (5).

Figure 36 illustrates laminar, smooth, and rough flow, based on equation (4). In analyzing wind data from Lake Hefner, the criteria for the various types of flow are those given with equation (4). Using $\epsilon_s = 30\, z_0$, these become:

$$\text{Smooth flow: } \frac{u_*\, z_0}{\nu} < 0.236;$$

$$\text{Transitional: } 0.236 \leqq \frac{u_*\, z_0}{\nu} \leqq 2.36; \quad (7)$$

$$\text{Rough flow: } \frac{u_*\, z_0}{\nu} > 2.36.$$

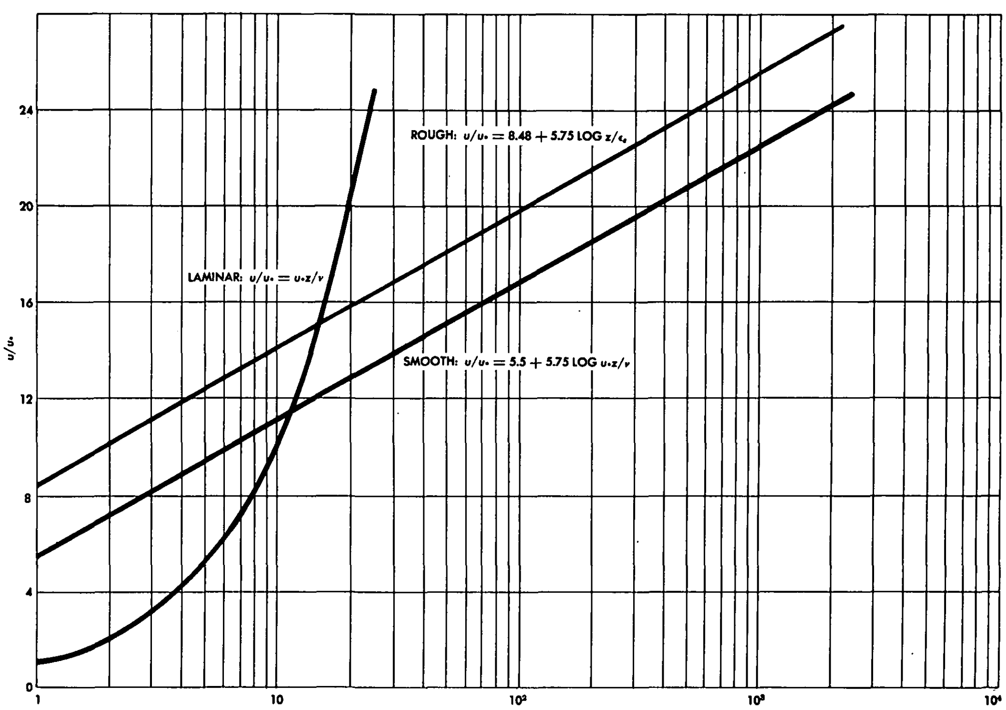

Figure 36. Laminar, smooth, and rough flow.

In using these criteria, z_0 is determined from equation (6). Calder (1949) quoted slightly different limits for the criteria, based on limits of 4 and 75 for the parameter $u_* \epsilon_8/\nu$. Lake Hefner wind data do not require a closer definition of these limits.

Although equation (4) is based on considerable experimental evidence, the expressions for smooth and rough flow have also been connected with the mixing-length theory of turbulent flow. Since the theory may have to be modified, wind measurements at Lake Hefner were analyzed in the light of this equation. It should be noted that equation (4) does not apply very near the boundary since it does not describe the laminar flow which exists there. According to evidence summarized by Goldstein (1938), equation (4) should not be used when $u_* z/\nu$ is less than 30. The lowest height at which wind measurements were made at Lake Hefner (2 meters) meets this requirement.

Analysis of some 23,000 individual wind profiles measured between 2 and 8 meters over the center of Lake Hefner yielded the averages of significant parameters given in table 3, based on 98 cases (3-hour averages) in which the lapse rate between the surface and 8 meters was essentially adiabatic.* Because of the limited number of adiabatic cases, an alternate procedure was used to check results. A multiple-regression analysis was made, using u_8/u_2, $(T_8 - T_0)$, and u_8 as the variables. It was then possible to use all the data and to separate the effects of temperature and wind, thus obtaining values of u_8/u_2 (and therefore also z_0) for adiabatic conditions.

Two somewhat unexpected conclusions can be drawn from table 3: (1) the lake surface is aerodynamically rough at all times; (2) the roughness parameter z_0 and the resistance coefficient $(u_*/u_8)^2$ increase with wind speed. Previous consideration of the same problem led to the suggestion, here discarded, that there is a critical smooth-to-rough velocity of the order of 7 meters per second at 8 meters, and that in the rough range z_0 and $(u_*/u_8)^2$ are independent of wind speed. Even if u_8 equals 100, for example, the profiles give a value of 5.5 for the friction velocity u_*, whereas with assumed smooth flow and the same u_8, u_* would assume the value 3.5. The friction velocity is a particularly reliable parameter since, for a given height range, it is uniquely determined by the wind profile without regard to any assumption of smooth or rough flow. Although the ratio u_8/u_2 was used to simplify the analysis, the profiles were also examined on semilog plots. When the measurements at 2, 4, and 8 meters were considered, no significant deviation from the logarithmic equation could be detected. It was therefore assumed that logarithmic profiles prevailed down to the heights z_0 given in table 3.

There is still some doubt about the character of the flow below a u_8 of about 4 meters per second, since light winds were infrequent at Lake Hefner. If smooth flow is assumed for light winds, and u_8/u_2 plotted against u_8, the curve for smooth flow based on equation (4a) meets the curve represented by the data in table 3 at a value of u_8 of approximately 20 cm per sec. Moreover, it will be seen later that even for very low wind speeds two evaporation equations based exclusively on rough flow give satisfactory results.

The roughness parameter of a natural water surface is a critical factor because the computed evaporation depends so much upon it. Values of z_0 for open water, generally the open ocean, have variously been quoted between 0.02 and 0.6 cm. Even if there were agreement as to value, it would not follow that the same value would apply to a small lake. Furthermore, there have been suggestions of a more or less abrupt transition between smooth and rough flow at a velocity of about 7 meters per second, measured at a height of about 8 meters.

A separate investigation of this question was carried out, but the necessary adiabatic conditions were so rare during the 2-week observation period in April 1951 that final results had to await the analysis described above. Although z_0 was found to vary from 0.55 to 1.15 cm, the results can be considered in fair agreement with the value of 0.6 cm given by Rossby (1935) for the open ocean under equilibrium conditions. That no transitional velocity existed was at first surprising, but was confirmed by application of every known criterion.

Another test was made to decide whether equilibrium conditions prevailed over the lake surface

* An adiabatic lapse rate would give $(T_8 - T_0) = -0.1°C$. Differences of −0.2, −0.1, and 0°C were considered adiabatic.

TABLE 3. Adiabatic Wind Profiles over Lake Hefner

u_8 (cm sec^{-1})	u_8/u_2	u_* (cm sec^{-1})	z_0 (cm)	ϵ_8 (cm)	$u_* z_0/\nu$	$(u_*/u_8)^2$	Surface
100	1.239	5.5	0.58	17.4	21.4	0.00306	Rough
200	1.241	11.3	.63	18.9	47.5	.00320	Rough
300	1.243	17.1	.66	19.8	75.2	.00325	Rough
400	1.245	22.8	.69	20.7	105.2	.00326	Rough
600	1.250	34.7	.77	23.1	178.2	.00335	Rough
1000	1.259	59.2	.94	28.2	371.0	.00350	Rough
1200	1.265	72.8	1.05	31.6	510.0	.00366	Rough
1500	1.269	91.9	1.15	34.6	704.0	.00375	Rough

which is always a wave-generating area. The Sverdrup-Munk (1947) wave-height theory was tested with the aid of instruments already described. If equilibrium conditions prevailed, then fetch rather than wind duration would control wave height. Such was found to be the case (fig. 37). The curve for measurements at Abbotts Lagoon, reported by Johnson (1950), is shown in the figure as further verification.

Although the values of z_0 quoted above are based only on measurements near the center of the lake, they have been considered applicable to the entire surface and used accordingly. There is certainly a possibility that z_0 varies downwind, since all the conventional surface characteristics (wave height, etc.) do so. On the other hand, one can not precisely define, with present knowledge, the roughness elements of a surface which is changing shape in a complex fashion downwind.

GROWTH OF THE BOUNDARY LAYER. Schlichting (1949) gives a derivation of an expression for the thickness of a growing boundary layer over a smooth plate. A similar analysis can be performed for a rough plate. The drag W of the water surface can be expressed as

$$W = \int_0^x \tau(x)\,dx = \int_0^{\delta(x)} \rho u\,(U - u)\,dz, \tag{8}$$

so that

$$dW/dx = \tau(x). \tag{9}$$

Equation (8) can be rearranged to give the drag in terms of the momentum integral J,

$$W = \rho U^2\,\delta(x)\,J = \rho U^2\,\delta(x)\int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\left(\frac{z}{\delta}\right). \tag{10}$$

Based on table 3, u can be replaced by (see equation (37), Anderson, Anderson, and Marciano, 1950)

$$u = qu_*\,(z/z_0)^a \tag{11}$$

where q and a vary slightly over the velocity range observed. In evaluating J we use

$$u/U = (z/\delta)^a \tag{12}$$

with the result that J is a slowly varying number equal to $a/[(a + 1)(2a + 1)]$. The drag is then

$$W = \frac{a}{(a + 1)(2a + 1)}\,\rho U^2\,\delta(x) \tag{13}$$

and, using equation (9),

$$\tau = \frac{dW}{dx} = \frac{a}{(a + 1)(2a + 1)}\,\rho U^2 \frac{d\delta}{dx}. \tag{14}$$

The shear stress τ is eual to ρu_*^2 so, using equation (11),

$$\tau = \frac{\rho U^2}{q^2\,(\delta/z_0)^{2a}}. \tag{15}$$

Now from equations (14) and (15),

$$\frac{a}{(a + 1)(2a + 1)}\frac{d\delta}{dx} = \frac{1}{q^2\,(\delta/z_0)^{2a}}, \tag{16}$$

and the solution of equation (16) is

$$\delta = \left[\frac{(a + 1)(1 + 2a)^2\,z_0^{2a}\,x}{aq^2}\right]^{\frac{1}{1+2a}}. \tag{17}$$

The thickness of the boundary layer at a downwind point representing the average value of x at Lake Hefner is given in table 4.

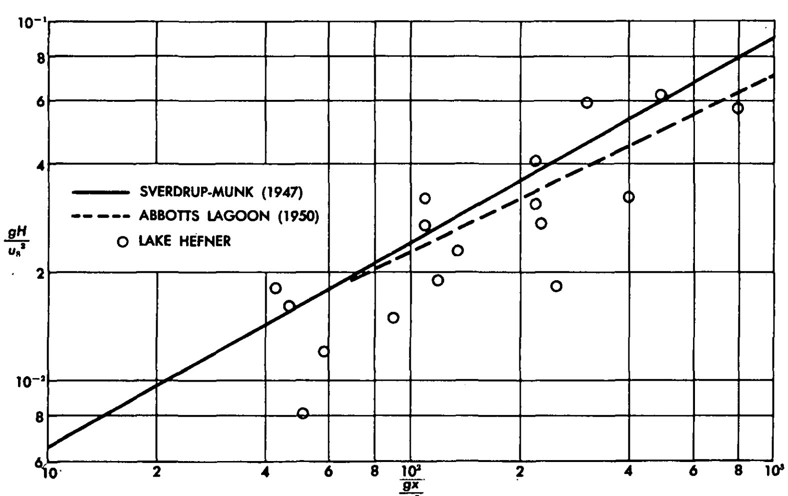

Figure 37. Wave height as a function of wind speed and fetch.

TABLE 4. Boundary-Layer Thickness at $x = 1750$ Meters.

u_8 (meters per second)	1	4	10	15
δ (meters)	41.0	42.9	43.2	47.4

These values of δ seem high, and in view of the uncertainty (Prandtl, 1934) as to boundary-layer thickness, they may be considered approximations only. The variation with height of various properties in the upper part of a boundary layer is so gradual that it is difficult to define the exact height at which the influence of the boundary ends. During the course of the measurements, upwind vapor-pressure profiles were compared occasionally with those over the middle of the lake. Little or no modification was detectable at 16 meters, the highest level of measurement. Considering the possibility that under stable conditions the boundary layer would be even thinner, it was decided to use the next lower level, 8 meters, as the upper limit of integration in the various computations.

In the development of "point" evaporation equations (for example, Sverdrup's equations, where $E = F$ times area) the upper limit of integration can be any height within the boundary layer, including the value $z = \delta$. For "area" evaporation equations (for example, Millar's or Sutton's), the upper limit of integration must be outside the boundary layer, with $z = \delta$ the lowest value permissible. The choice of $z = \delta = 8$ meters for both approaches, therefore, satisfies these requirements and insures that if the barge is properly located, the point solution will be merely a special case of the general vapor-blanket approach.

GENERAL WIND EQUATIONS. At the close of the first phase of the project, it was concluded that the search for a general wind equation was far from successful. Anderson, Anderson, and Marciano (1950) pointed out some of the objections to the Rossby-Montgomery (1935) approach. Holzman (1943) made an intuitive transformation of Rossby's and Montgomery's results, with little actual improvement. Sverdrup's (1936) wind equation removed some of the objections to the first approach, but retained much of its mathematical complexity. All these attempts at generalization were restricted to assessing the influence of stability, although Sverdrup (1938) later suggested that his results also applied to the unstable case.

The Rossby-Montgomery equation is

$$\frac{\partial u}{\partial z} = \frac{u_*}{k_0 z}\sqrt{1 + \sigma Ri}, \qquad (18)$$

while Holzman's transformation is

$$\frac{\partial u}{\partial z} = \frac{u_*}{k_0 z \sqrt{1 - \sigma Ri}}. \qquad (19)$$

The derivation requires that σ be constant with respect to stability. Sverdrup (1936) found that his measurements, mostly in the stable range, indicated $\sigma = 11$ and he later (1938) suggested that the same value applies for instability. Deacon (1949) argued that, with observed values of $Ri = -0.3$ and consistent wind profiles, $\sigma = 11$ would result in $(\sigma Ri) < 1$ and give imaginary values of $\delta u/\delta z$ when equation (18) was used. While $\sigma = 11$ is satisfactory in the stable range, the requirement $\sigma =$ constant thus casts doubt on Sverdrup's extension of his results to the unstable case. Insofar as equation (19) is concerned, Deacon's conclusion that $\sigma = 7$ has the same drawback — the equation would break down for $Ri > 1/7$, giving imaginary values of $\delta u/\delta z$ and, therefore, imaginary values of evaporation. To confuse the issue further, Pasquill (1949) concluded that the Rossby-Montgomery equation (18) provides "reasonable agreement with the results in stable conditions if $\sigma = 12$, while in unstable conditions the Holzman formula (19) gives corresponding agreement using the same value of σ." This complete and arbitrary change of basic equations with changing stability can hardly be considered satisfactory when one is seeking a general wind equation.

A series of unusually detailed wind and temperature profiles was obtained to explore this question further. Temperature differences were measured with a 5-junction thermocouple accurate to $\pm 0.02°C$. Wind profiles were obtained with hot-wire anemometers accurate to ± 2 per cent. After grouping the observations, it was found that, at the 4-meter level, the most stable group gave an average value of $Ri = 0.225$, while the most unstable gave $Ri = -1.7$. Neither $\sigma = 11$ nor $\sigma = 7$ would appear to be proper. In fact, it appears from the data that σ, which is the reciprocal of a proportionality constant originally used in the Rossby-Montgomery approach, must vary with height in order to prevent the two proposed wind equations from breaking down.

Figure 38 shows the values of σ based on these measurements, the notation across the top indicating the number of observations entering into each group average. Since one cannot use an equation to test

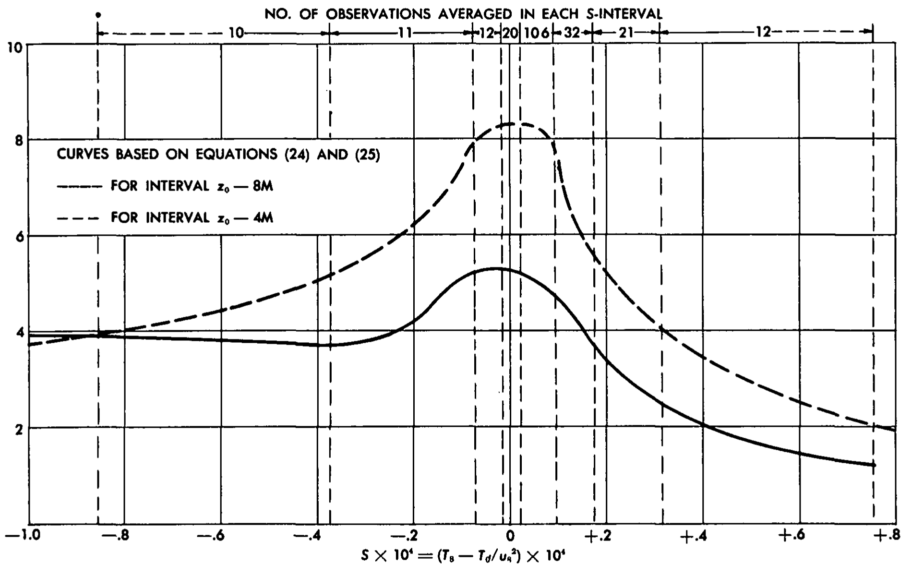

Figure 38. The stability parameter σ.

itself, use was made of Deacon's (1949) empirical wind equation:

$$\partial u/\partial z = az^{-\beta}. \qquad (20)$$

This is very similar to Sverdrup's (1936) approach, with the added advantage that it can be applied to instability as well. In equation (20)

$$\begin{aligned} &\beta > 1 \text{ for instability,} \\ &\beta = 1 \text{ for the adiabatic case, and} \\ &\beta < 1 \text{ for stability.} \end{aligned} \qquad (21)$$

Using the definition

$$Ri = \frac{g}{T}\,\frac{\partial T/\partial z}{(\partial u/\partial z)^2} \qquad (22)$$

with equation (20) and Deacon's (1949) approximation*

$$\partial T/\partial z = bz^{-\beta}, \qquad (23)$$

it can be shown that

$$Ri = \frac{g}{T}\,\frac{z_0{}^{1-\beta}\,z^{\beta}}{1-\beta}\left[(z/z_0)^{1-\beta} - 1\right]\left[\frac{T_z - T_0}{u_z^2}\right]. \qquad (24)$$

The constant a in equation (20) is determined from the boundary condition that, as the surface is approached, the effects of stability are suppressed, so that the wind approaches that given by the logarithmic law. This results in $a = u_*/k_0\, z_0{}^{1-\beta}$. The constant b in equation (23) is determined from the condition that as z approaches z_0, T approaches T_0. The latter is not exact but simplifies the computations and makes no essential difference in the results. Deacon's equation can be combined with equation (19) above to give (Anderson, Anderson, and Marciano, 1950, p. 36)

$$\sigma = \frac{1 - (z/z_0)^{2\,\beta-2}}{Ri}. \qquad (25)$$

When the value of Ri given by equation (24) is substituted in equation (25), σ is obtained and figure 38 is based on this analysis. The abscissa is based on the last term in equation (24), that is,

$$S = \frac{T_8 - T_0}{u_8{}^2}, \qquad (26)$$

in order to compare results with Deacon's. The relation between S, Ri, and β for Lake Hefner is shown in figure 39, and since this does not differ appreciably from Deacon's results for a much smaller roughness parameter, one must conclude that the relationship is unique and that β is a parameter governed only by the Richardson number. It should be noted that the great mass of the routine measurements at Lake Hefner over a 16-month period verify, in figure 39, conclusions based on the relatively few preliminary measurements. It can be concluded, therefore, that the curves shown in figure 38, which depend on the relationship between S, Ri, and β, would also be further substantiated by using additional data.

* The validity of the assumption that the index β is the same for wind and temperature was examined using Pasquill's (1949) measurements and found to be approximately correct. However, the Lake Hefner data do not substantiate this assumption since the wind profiles between 2 and 8 meters were essentially logarithmic regardless of stability. The validity of Deacon's assumption is therefore questionable, but it has been tentatively accepted in order to test the entire theory.

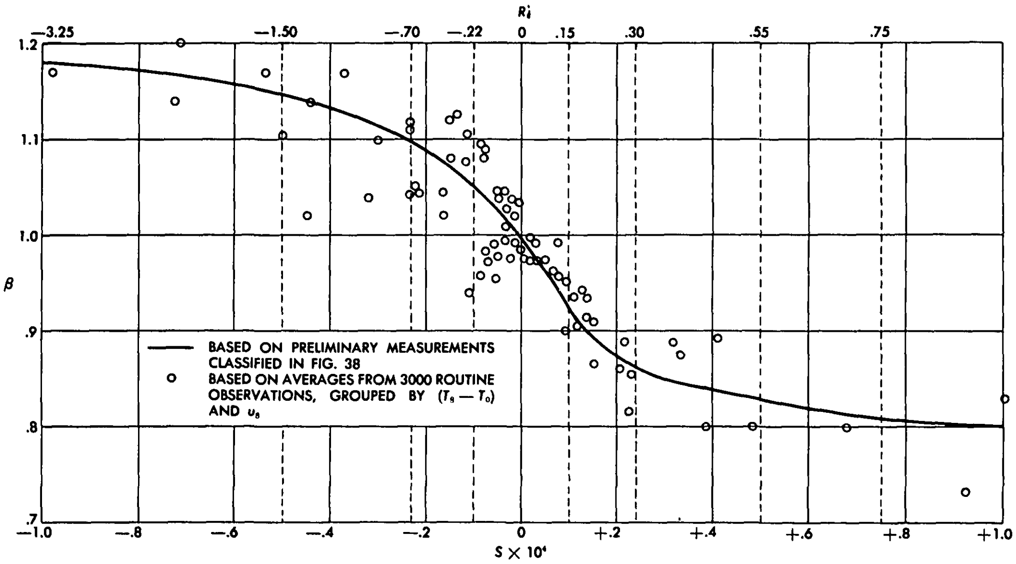

Figure 39. Relation between Richardson's number, β and S.

In summary, there is no basis for the assumption that σ is a constant. The measurements indicate that σ decreases markedly with height and, at any given height, varies with stability. Any choice of a single reasonable value for σ would cause equations (18) and (19) to break down numerically, entirely aside from the fact that the theoretical basis for σ would break down unless it were constant with respect to stability. The alternative, the use of the actual variation found in σ, would be only an arbitrary adjustment of an empirical number.

A final attempt was made to obtain workable results with equation (19). It was found that σ and Ri vary with height in such a way that their product is quasi-constant with height. An evaporation equation based on this finding produced such poor results in all comparisons that the approach was abandoned.

The possibilities of the above approaches apparently having been exhausted, it was decided to consider further Deacon's (1949) empirical wind equation, based on equation (20) above:

$$u = \frac{u_*}{k_0(1-\beta)}\left[(z/z_0)^{1-\beta} - 1\right]. \qquad (27)$$

The relation between S and β shown in figure 39 is based on this equation. With observed values of T_8, T_0 and u_8, β can be obtained from figure 39. This value of β, with the observed u_8 and z_0 already known, leads to a value for u_*, using equation (27). Then, with the usual assumption that u_* does not vary with height, and accepting Deacon's (1949) statement that β does not vary appreciably with height, equation (27) can be used to reconstruct the wind profile. It was found that this technique enabled measured wind profiles to be reproduced with reasonable accuracy. Doubts as to the validity of equation (27) are raised below but, even apart from these, it would not necessarily follow that a relationship suitable as an empirical wind equation would yield an evaporation equation which would successfully account for the effects of changing stability. Nevertheless, an evaporation equation based on equation (27) is developed and tested below.

There is an apparent paradox in connection with this analysis which, when resolved, leads to some interesting speculations. First, the measured wind profiles indicate no appreciable stability effects between the 2- and 8-meter levels. With respect to equation (27) and the associated analysis, this should lead to the conclusion that β does not vary appreciably from unity, the adiabatic case. On the other hand, large variations in stability S were observed, leading to large variations in β. Furthermore, the relationship between S and β is based on a large number of observations and verifies similar measurements (Deacon, 1949) over a land surface. Finally, it was found that the measured wind profiles could be reproduced with reasonable accuracy by the method described above. Considering equation (27) and the possible variation (that is, either variable or constant) with height of each of the three parameters k_0, u_* and β, which must appear together, there are eight possible combinations which could reconcile the apparently contradictory results. One of these possible combinations, the one actually used, has all three parameters constant with height. This combination leads to the conclusion, in order to reconcile the results, that the effects of stability are not discernible above 2 meters. On the other hand, it is well known that the local lapse rate ($\partial T/\partial z$) usually changes with height, so that a combination having β changing with height would also appear attractive. Therefore, the departure of β from unity should be increasingly large as the surface is approached, should decrease with height from the surface to 2 meters, and then remain essentially constant beyond 2 meters. This is in accord with common knowledge that the largest lapse rates, both stable and unstable, occur just above the surface.

In contrast to this model, Deacon (1949, p. 97) states that "it is found that the values of β obtained from the 4:1 m velocity ratios . . . are generally somewhat closer to unity than those giving the best fit to the whole velocity profile between 0.5 and 8 metres. It is considered that this is due to β not being exactly constant with height; the discrepancy would be explained if the departure of β from unity increased somewhat with height." Since the boundary conditions for equation (27) require β to approach 1 as z approaches 0, this requires that the departure of β from unity increase with height, which is exactly opposite to the model proposed above.

The contradiction between these models probably arises from the neglect of the slight, but nevertheless real, variation of u_* with height. The first model appears to violate the boundary condition imposed on β near the surface in deriving equation (27). However, from a physical point of view, it is preferable to relate β to the local temperature gradient ($\partial T/\partial z$), which the first model does, so again the inclusion of the real variation of u_* with height would probably act to reproduce the proper value of u.

These considerations lead to the belief that an evaporation equation based on equation (27) will probably be unsuccessful in assessing the effects of varying stability since equation (27), although useful as an empirical wind equation, cannot unambiguously couple the local wind shear $\partial u/\partial z$ to the local temperature gradient $\partial T/\partial z$.

A new approach to the turbulence and mass-transfer problems has recently been made by Lettau (1949). Lettau's boundary layer is not the same as that discussed here and is equivalent to the ordinary "friction layer." The top of Lettau's layer is the level at which the wind vector is first normal to the pressure-gradient force. Near the ground, Lettau uses a "surface layer," in which turbulence increases, and an upper layer in which turbulence decreases, with height, presumably reaching some residual value at the geostrophic level.

If we arbitrarily designate the 4-meter level as the top of Lettau's surface layer, in which the vapor flux is constant with height, then Lettau's evaporation equation would be

$$F = \frac{0.623\,\rho k_0\, u_4\,(e_0 - e_4)}{P\left(\ln \dfrac{400}{z_0}\right)^2 \left[1 + \dfrac{g}{T_m}\dfrac{\partial T/\partial z}{(\partial u/\partial z)_\theta{}^2}\right]^2}. \quad (28)$$

In $\partial T/\partial z$, T replaces θ (potential temperature); T_m is the mean temperature in the surface layer; and $(\partial u/\partial z)_\theta$ is the adiabatic wind shear. The main problem would be to evaluate the term $\partial T/\partial z$, which varies with height. Equation (23) may be a reasonable first approximation.

For the adiabatic case, the term in brackets in the denominator reduces to unity and the above equation becomes similar to Sverdrup's (1946), except for the smaller interval of integration. Although no results are presented below, it is obvious that Lettau's equation corrects Sverdrup's in the right direction. The ratio of Lettau's evaporation to Sverdrup's works out to

$$\frac{F_L}{F_S} = \frac{1.11\,(e_0 - e_4)}{(e_0 - e_8)}. \quad (29)$$

In order to give close agreement between F_L and observed values, the ratio of vapor-pressure differences in the above equation must have a value of about 0.5. The true ratio probably is close to that value, and if the exact height of Lettau's surface layer were known this point could be examined. But Lettau's surface layer is closely related to time changes in vapor flux ($\partial F/\partial t$), so it is likely that over relatively short periods the height of this layer would vary within such wide limits that the choice of one average value, as in the usual boundary-layer approach, would not be proper. While Lettau's theory is in several ways an improvement over existing approaches, this study was not designed to test his equations. Much more precise measurements and more elaborate calculations than those used here would be needed to verify his theory.

MASS-TRANSFER EQUATIONS

Two basic approaches were used in the development of the various evaporation equations tested. One group of equations is based on the concept of discontinuous mixing as developed in Prandtl's mixing-length theory. One of the equations tested is based on the concept of continuous mixing first expressed quantitatively by Taylor (1922). A physical and mathematical review of the two approaches has been given by Anderson, Anderson, and Marciano, (1950).

Discontinuous Mixing

EXISTING METHODS. The most general of the equations involving the discontinuous-mixing concept is that of Millar (1937). His approach is based on the growth of a "vapor blanket" or a "moisture boundary layer," analogous to the growth of the momentum boundary layer illustrated in figure 34. This approach will be considered in more detail, with only the case of rough flow treated.

Rearranging Millar's symbols, his parameter f, the "potential of vapor concentration," can be rewritten as

$$f = \frac{0.622\,\rho}{p}(e_0 - e_z). \qquad (30)$$

In the discussion below, f is regarded as a transferable property in the same sense that momentum is transferred by eddy processes.

Millar's approach is based on the skin friction of a flat plate as discussed by Prandtl (1934) and von Kármán (1934). The underlying principle is that friction between a fluid and the underlying surface is accompanied by a corresponding change of the momentum carried by the fluid.

Compare the momentum carried by a fluid through cross section (1) upstream from the edge of a plate and through cross section (2) at a distance x (fig. 40). The mass passing per unit time per unit area through (2) is $\rho u A$. This mass had momentum U per unit mass in (1) and u per unit mass passing through (2). The loss of momentum is then $\rho u\,(U - u)\,dA$, and the total loss of momentum in unit time is then $\int \rho u\,(U - u)\,dA$, the integral being taken over the whole of section (2). This is equal to T, the total frictional force* acting on the plate from the leading edge to x, that is,

$$\mathrm{T} = \int_0^{\infty} \rho u\,(U - u)\,dz \qquad (31)$$

for a plate of unit width. The value of $(U - u)$ is negligible for $z > \delta$, the height of the boundary layer. In fact, the height where this term becomes negligible is taken as the definition of the (momentum) boundary layer. Therefore,

$$\mathrm{T} \cong \int_0^{\delta(x)} \rho u\,(U - u)\,dz. \qquad (32)$$

The friction per unit area, or the shearing stress τ, is then $d\mathrm{T}/dx$. Therefore,

$$\tau = \frac{d\mathrm{T}}{dx} = \frac{d}{dx}\left[\int_0^{\delta(x)} \rho u\,(U - u)dz\right]. \qquad (33)$$

We now make the transition to the property f. In cross section (1) of figure 41 the evaporation per unit area per unit time is Uf_0 while in section (2) it is uf. If in section (2) the velocity is taken at $z = \delta$, then we can use u for both sections. Then the total mass of vapor evaporated per unit time, from the leading edge to the distance x, is

$$M(x) = \int_0^{\delta_w(x)} u\,(f_0 - f)\,dz. \qquad (34)$$

The evaporation per unit area at a point is dM/dx or

$$F = \frac{dM}{dx} = \frac{d}{dx}\left[\int_0^{\delta_w(x)} u\,(f_0 - f)\,dz\right]. \qquad (35)$$

Equation (34) is analogous to equation (32), and equation (35) to equation (33). The term in brackets in equation (33) is the momentum integral, while the corresponding term in equation (35) may be called the "vapor integral."

Millar points out that f should be measured at the top of the vapor blanket, $z = \delta_w$, while wind speed can be measured at any level. We here make the assumption that $\delta \cong \delta_w$ and measure both properties at the same level. It was pointed out above, in the discussion on the growth of the momentum boundary layer, that 8 meters has been chosen for this reference level. Values of δ_w will be calculated below for comparison with δ.

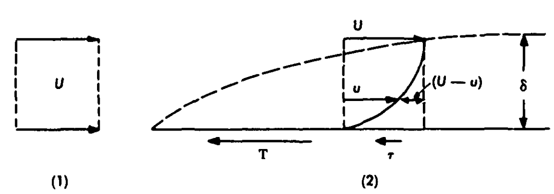

Figure 40. Momentum exchange in a boundary layer over a flat plate.

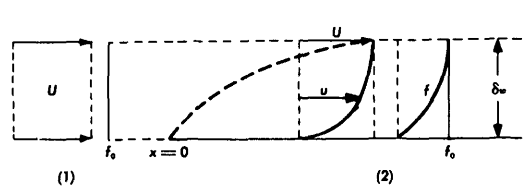

Figure 41. Analogy between momentum boundary layer and the vapor blanket, for $\delta = \delta_w$.

* T is the same as W of equation (8), a different symbol being used here to maintain Millar's notation.

It is necessary at this point to consider the several approaches made by Millar. For evaporation from "an infinitely extensive, free water surface without waves" (Millar, 1937, Section 7), he considers a smooth surface, including the laminar layer. For evaporation from "an infinitely extensive surface, roughened by waves" (Millar, 1937, Section 8), he uses a three-layer model (Anderson, Anderson, and Marciano, 1950, p. 27), accepting the Rossby (1936) distribution of shearing stress in the vicinity of the waves. In both these derivations there is no mention of a boundary layer or vapor blanket, although the wind equations used apply only in a boundary layer. Obviously, Millar was implying that δ becomes constant as x approaches infinity. Therefore, equilibrium having been established, it is necessary only to perform the various integrations up to the proper height. Evaporation per unit area per unit time would not be a function of x, and multiplication by an area would give total evaporation. These are essentially the assumptions involved in equations referred to in Anderson, Anderson, and Marciano (1950) as "point" equations. Sverdrup's evaporation equations, for example, are of this type and somewhat similar to Millar's. For moderate values of x, F is assumed to be computed at the particular value of x which yields a true mean for the entire surface.

On the other hand, the vapor-blanket analogy to the skin-friction plate was applied to "evaporation from finite pans" (Millar, 1937, Section 9), and the mathematical treatment indicates the assumption of a smooth water surface. However, Millar found that "the predicted evaporation was only about half as great as was actually observed. Many minute ripples were to be seen on the surface of the water during the experiments, which gave it a rapidly fluctuating, dimpled appearance." He accordingly changed an empirical constant, which he had originally derived from Nikuradse's experiments in a smooth-walled wind tunnel. It is not clear that the mere changing of one constant makes the entire derivation, based on smooth flow, applicable to rough flow. It must also be noted that Millar chose a value for this constant which "best agreed with the observations."

For evaporation from a lake of finite length, with the surface roughened by waves, it would be more appropriate to combine the vapor-blanket approach with the three-layer model used by Millar. It is desirable to determine first the accuracy of Millar's original approach.

Millar uses a general wind equation of the form*

$$\frac{u}{u_*} = \phi = \frac{1}{k_0} \ln \frac{\eta}{\beta}. \tag{36}$$

For a rough surface $\beta = 1$, and $\eta = z/z_0$. In equation (34) above, the variables are transformed so that $dz = z_0\, d\eta$. For the lower limit of integration, at $z = z_0$, $\eta = 1$. At the upper limit, $z = \delta_w$, $\eta = \eta_1 = \delta_w/z_0$.

Therefore equation (34) is rewritten, using Millar's equation (9b),

$$\begin{aligned} M(x) &= \int_\beta^{\eta_1} u_* \phi f_0 \left(1 - \frac{\phi}{\phi_1}\right) \frac{z_0}{\beta}\, d\eta \\ &= f_0 \left(\frac{u_* z_0}{\beta}\right) \int_\beta^{\eta_1} \left(\phi - \frac{\phi^2}{\phi_1}\right) d\eta \\ &= f_0 \left(\frac{u_* z_0}{\beta}\right) \Psi_r(\eta_1). \end{aligned} \tag{37}$$

This is analogous to Millar's equation (9d) which applies to a smooth surface. Since $\eta_1 = \eta_1(\delta_w) = \eta_1(x)$, equation (37) gives total evaporation from a strip of unit width and length x. By a similar procedure based on the transformation of variables, Millar's equation (9c) becomes, for the rough case,

$$\frac{\beta x}{z_0} = (Rx)_r = \mathrm{X}_r(\eta_1) = \int_\beta^{\eta_1} \frac{d\phi_1}{\phi_1} \int_\beta^{\eta_1} \phi^2\, d\eta. \tag{38}$$

The mean evaporation per unit area from a strip of length x is $M(x)/x$, or

$$F = f_0\, u_* (\Psi/\mathrm{X})_r. \tag{39}$$

This is of the same form as Millar's equation (9e), but with a different value of F arising from differences in u_* and (Ψ/X).

Millar describes the evaluation of Ψ and X, and the relation between X_r and $(\Psi/\mathrm{X})_r$ can be determined with the aid of tables given in Jahnke and Emde (1945). This relationship is shown in figure 42.

A corollary result of this analysis is figure 43, which shows the growth of the vapor blanket downwind over the lake. It will be seen, by comparing the values given in the table in figure 43 with those given in table 4, that $\delta \cong \delta_w$. This is gratifying, since it tends to verify one of the basic assumptions of the mass-transfer approach: complete similarity of eddy transport of momentum and water vapor in the atmospheric boundary layer.

* Here Millar's β is not to be confused with Deacon's, there being no connection.

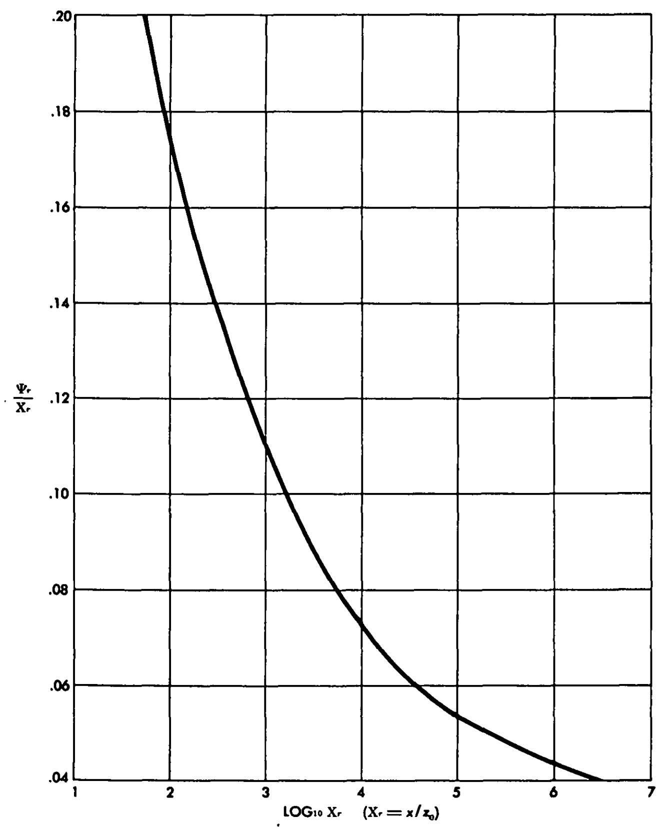

Figure 42. The relation between X_r and Ψ_r/X_r.

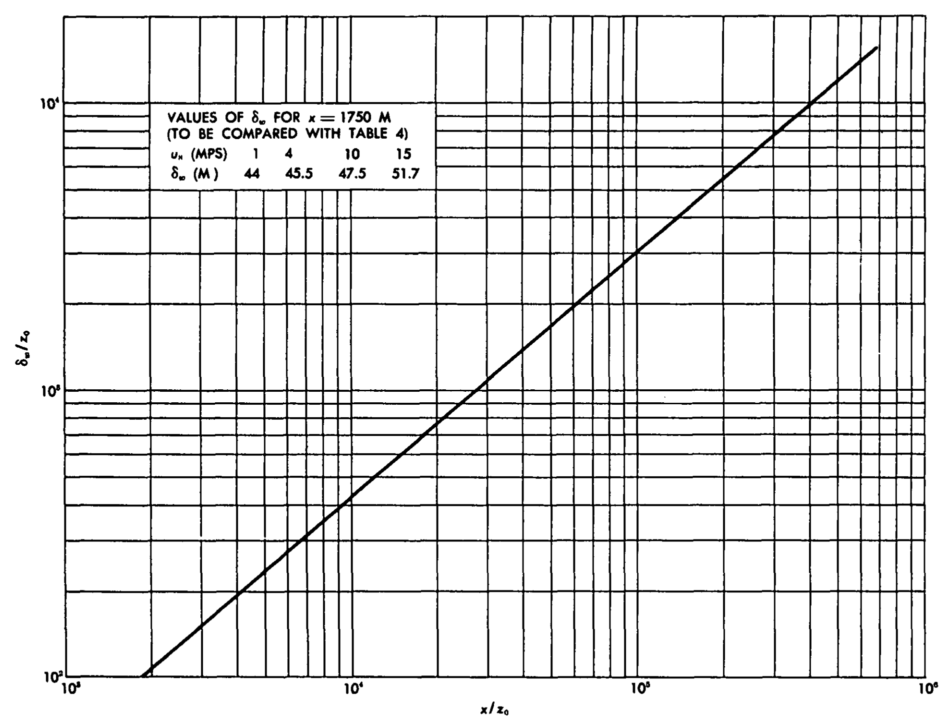

Figure 43. Growth of the vapor blanket downwind.

The above analysis can be used to develop evaporation equations by two methods. The exact method is to extend equation (37), which gives total evaporation from a strip of unit width and length x, to a circular area. Curve-fitting figure 42 with the relation $\Psi/X = 0.275\,(x/z_0)^{-0.135}$, equation (37) can be used with polar coordinates to give total evaporation from a circular area as

$$E = 0.275\, f_0\, u_*\, z_0^{0.135}\, r^{1.865} \int_0^{2\pi} (\cos\theta)^{1.865}\, d\theta. \quad (40)$$

After integration and simplification, with 8 meters as the reference level,

$$E = \frac{0.535\, \rho k_0\, u_8\, (e_0 - e_i)\, r^{1.865}}{P\,[\ln (800/z_0)]}. \quad (41)$$

A more direct but less accurate approach is simply to multiply equation (39) by area. But since $M(x)$ given by equation (37) is not linear in x, then $M(x)/x$ given by equation (39) is only an approximation to the true value of mean unit evaporation. However, if equation (39) is multiplied by area and use is made of the same curve-fit for figure 42, we obtain

$$E = \frac{0.475\, \rho k_0\, u_8\, (e_0 - e_i)\, r^{1.865}}{P\,[\ln (800/z_0)]}. \quad (42)$$

The more exact form, equation (41), gives values of E about 12 per cent greater than those obtained with equation (42). It will also be found, for Lake Hefner, that equation (41) gives values of E within 2 per cent of those given by Sverdrup's (1946) equation (44) below.

This agreement between the rigorous solution, equation (41), and the point equation, equation (44), is important. The results verify the belief that if one is forced to use a point-evaporation equation, requiring an installation on the lake, such as a barge, the computation of total evaporation can be made with the equation $E = F$ times area if the equipment is properly located. The fact that neither equation (41) nor equation (44) gives satisfactory results is of no concern — the significant point is that when F is computed for the properly chosen downwind point, the simplified equation gives practically the same results as the rigorous solution if the same flow model is used in both cases. It should be noted that the relation between equations (41) and (44) remains fixed under these conditions. The constant 0.535 in equation (41) arises from the curve-fit for figure 42, and the values quoted above are applicable to Lake Hefner. For another lake a different portion of the curve would be used and the constant would change accordingly.

The most desirable location of an on-lake installation can be obtained from a graph such as figure 44, which shows the decrease of unit evaporation (F) downwind at Lake Hefner. The "best location" gives the true mean value of F. Anchoring problems dictated a location farther downwind but still close enough to the "best" location to give a reliable value of F. It will be seen that only for relatively small lakes would the location of an on-lake installation have a major effect on the computed evaporation.

It will be shown below that a laminar-turbulent two-layer model produces satisfactory results with a point-evaporation equation. It therefore follows that the same model would produce about the same results as the rigorous Millar approach described above. The latter requires no measurements on or above the lake except water-surface temperature. In support of this conclusion, it will also be shown below that a simplified area-type equation will give results comparable to those given by the best point-type equation.

Preliminary calculations were necessary to decide whether the use of a three-layer model, based on a discontinuity in u_* at the wave crests, would improve equation (41). Since equation (41) was found to give essentially the same results as Sverdrup's (1946) equation given below, and since this three-layer model is known to increase computed evaporation, as compared to Sverdrup's values, by about 10 per cent, attention was shifted to Sverdrup's equation. Only if Sverdrup's equation gave results which were too low, would any such modification of equation (41)

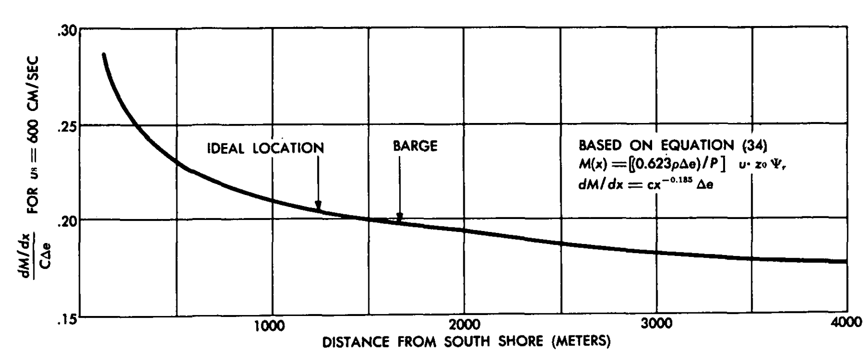

Figure 44. Variation in unit evaporation downwind.

be worthwhile. It turns out, as discussed later, that there is no reason for considering equation (41) further. The decision is made on strictly numerical grounds, but even the three-layer model (Anderson, Anderson, and Marciano, 1950, pp. 27-29) is open to question. It sems odd that a model having a smooth wind profile below the wave crests should yield more evaporation than one in which the rough flow extends to the surface.

Several of the equations reviewed during the course of the study can be considered special cases of general-boundary-layer or vapor-blanket approach. With these equations, one obtains vapor flux through unit area in unit time and multiplies by the surface area to obtain total evaporation. Most of these equations have already been reviewed in detail and are only given below because they were tested against the water-budget control. Only equation (48) is intended to be "general" with respect to effects of atmospheric stability; the others are limited to the adiabatic case.

Three of the point equations already reviewed were tested in the following form:

C. W. Thornthwaite and B. Holzman (1939)

$$E = \frac{0.623\ \rho k_0^2\ (u_8 - u_2)\ (e_2 - e_8)\ A}{P\ [\ln\ (800/200)]^2}. \qquad (43)$$

H. U. Sverdrup (1946)

$$E = \frac{0.623\ \rho k_0^2\ u_8\ (e_0 - e_8)\ A}{P\ [\ln\ (800/z_0)]^2}. \qquad (44)$$

R. Norris (1948)

$$E = \frac{0.623\ \rho k_0^2\ u_8\ (e_0 - e_8)\ A}{P\ [\ln\ (800/z_0)]\ [\ln\ 800/a_1\ z_0)]}; \qquad (45)$$

where $\ln a_1 = \frac{1.76\ u_{*8}}{u_*};\ \frac{u_{*8}}{u_*} = \frac{\ln\ (l/z_0)}{\ln\ (u_{*8}\ l/0.017)}.$

Anderson, Anderson, and Marciano, 1950 (equation (4), p. 64) quote an evaporation equation based on Deacon's wind equations (20), (21), and (27) above. This equation was not listed among those to be tested. Since then, in view of the unsatisfactory status of the problem of generalizing the logarithmic wind law, it appeared that this relatively new wind equation should be investigated. It has been shown above that Deacon's equation appears to be satisfactory as a wind equation. Given T_8, T_0, and u_8, with z_0 known, one can construct wind profiles agreeing very closely with those observed between 2 and 8 meters. It remains to be seen whether the parameter β successfully shows the effect of stability on evaporation. Although preliminary indications were that an evaporation equation based on these wind equations might not be satisfactory, the equation was developed and tested in the hope that more could be learned about the stability problem.

Point evaporation is

$$F = \frac{0.623\ \rho}{P} K \frac{\partial e}{\partial z}. \qquad (46)$$

Using equation (27), $K = \frac{u_*^2}{\partial u/\partial z}$ or

$$K = k_0\ u_*\ z_0\ (z/z_0)^{\beta}. \qquad (47)$$

Combining equations (46) and (47) and integrating as usual to the 8-meter reference level:

$$E = \frac{0.623\ \rho k_0^2\ u_8\ (1-\beta)^2\ (e_0 - e_8)\ A}{P\ [(800/z_0)^{1-\beta} - 1]^2}. \qquad (48)$$

For β approaching 1, the adiabatic case, equation (48) reduces to equation (44). For $\beta = 1$, equation (47) gives $K = k_0\ u_*\ z$ which, with equation (46), yields equation (44). Values computed from equation (48) compared with the water-budget values of E are discussed below.

Although specific comparisons are made in detail in another section, a general idea of the results is necessary at this point. Neither Millar's vapor-blanket approach, equation (41), nor any of the point equations (43), (44), (45), and (48) yielded satisfactory results. If some modification of theory or some correction of possible errors could not be made, then the entire discontinuous-mixing approach would have to be abandoned. But the approach has been so fruitful over the years that such a step would seem drastic. There is still the possibility that the flow models used are not complete. After some comparison of approaches it is found that all equations, except equation (43), ignore the laminar layer. Sverdrup (1946) recognized the laminar layer in formulating his expression for the eddy coefficient, but actually extended the turbulent layer to the surface in his integrations. The logical next step would be to consider a two-layer model.

Sverdrup (1937) investigated a laminar-turbulent two-layer model. This was not listed for testing because it was thought that his more recent equation would be more satisfactory. His earlier evaporation equation (Sverdrup, 1937, equation 14) is, in our notation*

* In an earlier paper Sverdrup (1936) developed essentially the same equation, but for its application discussed values of z_0 increasing with wind speed and having values ranging up to about 11 cm. These values are much too large and, in his 1937 paper, he stated that they are applicable only to rapidly changing conditions. For steady conditions he recommended $z_0 = 0.6$ cm, and this is the value used in equation (49).

$$F = \frac{0.623\,\rho k_0\,u_*\,(e_0 - e_8)}{p\left[\ln\dfrac{800 + z_0}{\delta_l + z_0} + \dfrac{k_0\,\delta_l\,u_*}{D}\right]}. \qquad (49)$$

Sverdrup defined the thickness of the laminar layer, for fully rough flow, as $\delta_l = 27.5\nu/u_*$, based on only a few measurements. This is close to the limiting value for laminar flow ($\delta_l = 30\nu/u_*$) proposed by von Kármán (1934) on the basis of extensive measurements. Equation (49) was tested using the definition $\delta_l = 30\nu/u_*$.

Continuous Mixing

The concept of continuous mixing, first advanced by Taylor (1922), has been applied by Sutton (1932, 1934, 1949) to the diffusion problem. The latest of Sutton's papers may be considered a generalization of his earlier work applying only to flow over smooth surfaces.

Sutton (1949) introduces the "macroviscosity" $N = u_*\,z_0$, presumed to play a part in exchange processes over rough surfaces similar to that of kinematic viscosity in smooth flow. For the adiabatic wind equation Sutton uses

$$\frac{u}{u_*} = \frac{1}{k_0}\ln\left[\frac{u_*\,z}{N + (\nu/9)}\right]. \qquad (50)$$

For smooth flow $z_0 = 0$, so $N = 0$ and equation (50) reduces to equation (4a). For rough flow, $N > \nu$ and equation (50) reduces to equation (6). As in his previous work, the more exact logarithmic wind equation is replaced by a power-law approximation in order to solve the conventional diffusion equations.

The macroviscosity also appears in Sutton's new form for the correlation coefficient R_ξ, where

$$R_\xi = \left[\frac{N + \nu}{N + \nu + w'^2\,\xi}\right]^n.$$

For smooth flow $N = 0$, and this equation reduces to the expression previously used (Sutton, 1934). The data selected by Sutton (1949) for illustration appear to verify the above equation as a satisfactory approximation.

The evaporation equation resulting from Sutton's latest work is similar to the one he developed for smooth flow. From the equations given by Sutton (1949), evaporation from a circular lake of radius r is

$$E = \frac{0.623\rho}{p}\,G'\,u^{\frac{2-n}{2+n}}\,r^{\frac{4+n}{2+n}}(e_0 - e_t) \qquad (51)$$

where

$$G' = \frac{\left(\dfrac{2+n}{2-n}\right)^{\frac{2-n}{2+n}}\dfrac{2+n}{2\pi}\sin\left(\dfrac{2\pi}{2+n}\right)\Gamma\left(\dfrac{2}{2+n}\right)a^{\frac{2}{2+n}}\,z^{\frac{n^2}{-4-n^2}}\,2^{\frac{2}{2+n}}\sqrt{\pi}\,\Gamma\left(\dfrac{3+n}{2+n}\right)}{\Gamma\left(\dfrac{8+3n}{4+2n}\right)}$$

and, in G', a is given by

$$a = \frac{(\pi k_0^2/2)^{1-n}\,(2-n)^{1-n}\,n^{1-n}\,z^{\frac{n^2-n}{2-n}}\,(\nu + u_*\,z_0)^n}{(1-n)\,(2-2n)^{2-2n}}.$$

This is to be compared with equations (24) through (27) in Anderson, Anderson, and Marciano (1950), to which the above would reduce if the surface were assumed to be smooth ($z_0 = 0$).

It should be noted that the n used by Sutton is connected to the α used in equation (11) above by the relation $\alpha = n/(2 - n)$. Over the range of velocities observed at Lake Hefner, α varies between 0.150 and 0.175, which is equivalent to a variation of 0.26 to 0.30 in n. Sutton indicates that typical values of n, presumably for a typical range of stabilities, are from 0.10 to 0.30. This illustrates one of the difficulties involved in using the power law (a variation of equation (12)) between two levels as an approximation to the logarithmic law. The variation of n quoted for Lake Hefner is for the observed adiabatic wind profiles, and it is presumed that the observed variation in z_0 is the cause of this variation. Approximately a twofold variation in z_0 at Lake Hefner is accompanied by a very small change in n. On the other hand, the threefold variation in n, which Sutton implies is typical, can be accounted for only by additional effects of stability. But one cannot be sure of the relative importance of stability and dynamic effects on the parameter n. From the evidence at hand, one could say that n is principally a stability parameter, increasing with increasing stability. However, Sutton uses the power law, apparently with no intention of accounting for effects of stability. Further, it can be calculated from the exact logarithmic law that n must also increase slightly with height to allow equation (51) to fit the entire profile closely. This can be illustrated by the data in Sutton's (1949) table II, where his power law closely represents the logarithmic profile only between 1 and 3 meters. This is perhaps the most serious drawback to a wind equation of this type. The power law is made approximately correct by fitting, and this can be done only for small height ranges. Unless the necessary variation of n with height is accounted for, a power law assumed to apply over a greater interval than 2 or 3 meters introduces a fictitiously large wind shear in the expressions for eddy diffusivity. This fictitiously large wind shear would prevail at higher levels, so one might say that for high winds (large evaporation) the computed evaporation would be too small. At low levels the power law would give a fictitiously small wind shear, so by similar reasoning, computed evaporation would be expected to be too large for low winds.

Another aspect of Sutton's equations has been mentioned in Anderson, Anderson, and Marciano (1950). Although originally based on Taylor's concept of continuous mixing, in the course of his derivations Sutton used results obtained from a mixing-length (discontinuous-mixing) theory. It would be proper to refer to the results as a method, rather than a theory.

Although these remarks have been offered to clarify the background to this approach, it is recognized that they give the impression that Sutton's evaporation equation might not give satisfactory results. Such was not the case, as will be shown below. In fact, Sutton's equation gave better results than any of the equations originally proposed for test.

EVAPORATION IN THE ABSENCE OF WIND. Many of the familiar empirical equations relating evaporation to meteorological parameters are of the general form

$$E = C_1 (1 + C_2 u)(e_0 - e_a), \qquad (52)$$

where C_1 and C_2 are experimental constants. This implies that evaporation takes place in the absence of wind. Equations of this type are generally obtained by a statistical analysis of data from evaporation-pan records.

Evaporation in the complete absence of wind occurs only by molecular diffusion, an extremely slow process. Point evaporation is given by

$$F = \rho D (\partial q/\partial z) \qquad (53)$$

which on integration and substitution yields

$$F = \frac{0.623 \rho D (e_0 - e_z)}{Pz}. \qquad (54)$$

For $z = 8$ meters,

$$F = 2.52 \times 10^{-6} (e_0 - e_z) \text{ cm/3 hours}. \qquad (55)$$

This serves to give a first approximation. Using boundary-layer concepts, one finds that as u approaches zero, the thickness of the laminar layer approaches infinity. Therefore, for z approaching infinity in equation (54), F approaches zero. Thus, in comparison with evaporation under turbulent conditions, evaporation by molecular diffusion is negligible, unless a strong temperature gradient exists so that free convection cannot be ignored.

If substantial temperature gradients do exist, evaporation into still air is somewhat greater than that given by equation (54). Although stating that the experimental verification of his theory had not yet been completed, Yamamoto (1950) gave an equation for evaporation into still air, which on substitution of the units and symbols used in this report, is as follows:

$$E = \frac{0.327\rho}{P}\left(\frac{D}{x}\right)\left[\frac{x^3 g \gamma_t (T_0 - T_z)}{\nu D}\right]^{0.25}(e_0 - e_z), \qquad (56)$$

in which γ_t is the coefficient of expansion of air. Using $x = 3.45 \times 10^5$ cm for Lake Hefner and $T_0 - T_z = 5°C$, equation (56) reduces to

$$E = 2.05 \times 10^{-4} (e_0 - e_z) \qquad (57)$$

where e_0 and e_z are in mb and E in cm/3 hours. Thus for a given vapor pressure difference, evaporation into still air is approximately the same as evaporation computed using equation (49) with $u = 0.32$ knot.

There is sufficient reason, therefore, for ignoring the case of zero wind in the experimental tests, aside from the fact that this case was never observed. Those interested in this aspect of the evaporation problem may refer to Hickox (1946), who presents the results of a series of excellent experiments.

COMPARISON OF COMPUTED AND MEASURED EVAPORATION

Data Processing and Computing

The computation of evaporation from Lake Hefner based on the mass-transfer theory required the processing of a tremendous number of data. Observations were begun on 26 April 1950 and were terminated on 31 August 1951, a period of 493 days. At each of the four meteorological stations, wet- and dry-bulb temperatures at the 2-, 4-, 8-, and 16-meter levels and water-surface temperatures were measured once each half hour and recorded on an Esterline-Angus milliammeter recorder. Each knot of wind movement at the same four levels, wind direction, and rainfall depths were recorded on an Esterline-Angus recorder. Excluding rainfall, a total of 14 items were recorded during each half-hour period at each station. For all four stations over the entire period of observations, the total number of items recorded was about 1,325,000. It was apparent that manual tabulation of the data and computation of meteorological parameters was impracticable, and the basic data were therefore placed on punch cards.

For each half-hour period the value of each of the items, as read from the trace, was entered directly on the recorder tape by one person and checked by another. The time of observation was taken as the half-hour at which the cycle of observations was completed. Temperatures were read to the nearest 0.1°C, and wind speed to tenths of a knot. The wind direction was taken to be that which was recorded most frequently during the half-hour cycle. For all stations except the barge, wind direction was coded to eight points of the compass, with code figure 1 representing NE and proceeding therefrom in a clockwise direction. For the barge station a different code was adopted, as described in the section "General Description of Lake Hefner" and shown in figure 4. For certain of the mass-transfer equations it was necessary to use the vapor pressure of the unmodified air, and it was belived that this would be best accomplished by using dry- and wet-bulb temperatures at the 16-meter level at the station upwind from the barge. If the wind at the barge station was coded as 2, for example, humidity data at the south station were used; if the wind was coded as 3, data at the northeast station were used, and for code 4, data at the intake tower station. Wind directions coded as 1, 5, or 6 were not observed over any 3-hour period during the studies.

Two sets of punch cards were prepared and corresponding cards checked by visual comparison. Thus it was possible to correct almost all the punching errors before the two sets of cards containing the half-hourly data were sent to the Institute for Numerical Analysis of the National Bureau of Standards. The two sets of cards were then compared by machine. Very few discrepancies were found as a result of the second comparison, and most of these could be resolved quickly. If there was any doubt, the observation in question was deleted before the half-hourly data were tabulated.

Three-hourly averages were computed for each item except wind direction. For the barge station the most frequent wind direction during the 3-hour period was taken as the mean and used in the computations. The results were punched on another set of cards and then tabulated.

The set of cards containing the 3-hour average data was then used for all subsequent computations. Although the evaporation equations tested generally appear quite complicated, it was found that they could be reduced to but a few variables and parameters. The parameters were given in graphical form. Since a digital rather than an analog type of computer was used, it was necessary to prepare a card deck for each graph. Evaporation was then computed for each 3-hour period for all the equations being tested. The results were tabulated and daily totals obtained.

Computed and Observed Evaporation

In considering the results of the mass-transfer studies, A and B water-budget days were used for all equations except Sverdrup's (1946) form, for which only A days were used.* The results are shown in figures 45 through 48.

Three different symbols are used in figures 45 through 48 to indicate atmospheric stability. The quantity ($T_8 - T_0$) was computed for each 3-hour period. If this difference was found to be positive for six or more of the eight 3-hour periods, the day was classified stable. If the difference was negative for six or more of the eight 3-hour periods, the day was classified as unstable. All other days were classified average.

The following values of physical constants were used in all computations:

$$
\begin{aligned}
k_0 &= 0.4, \\
p &= 1000 \text{ mb}, \\
\rho &= 1.2 \times 10^{-3} \text{ gm cm}^{-3}, \\
r &= 1.725 \times 10^{5} \text{ cm}, \\
\nu &= 0.15 \text{ cm}^2 \text{ sec}^{-1}, \\
D &= 0.25 \text{ cm}^2 \text{ sec}^{-1}.
\end{aligned}
$$

The Thornthwaite-Holzman and Sverdrup (1946) equations can be discussed together since they evolve from basically the same approach. Figures 45 and 46, however, show a striking difference. Sverdrup's

* When results were first assembled it was soon evident that Sverdrup's (1946) equation would give values of E which were consistently too large. The pattern having been definitely established, it was decided to use only A days so that attention could be gvien to more promising equations.

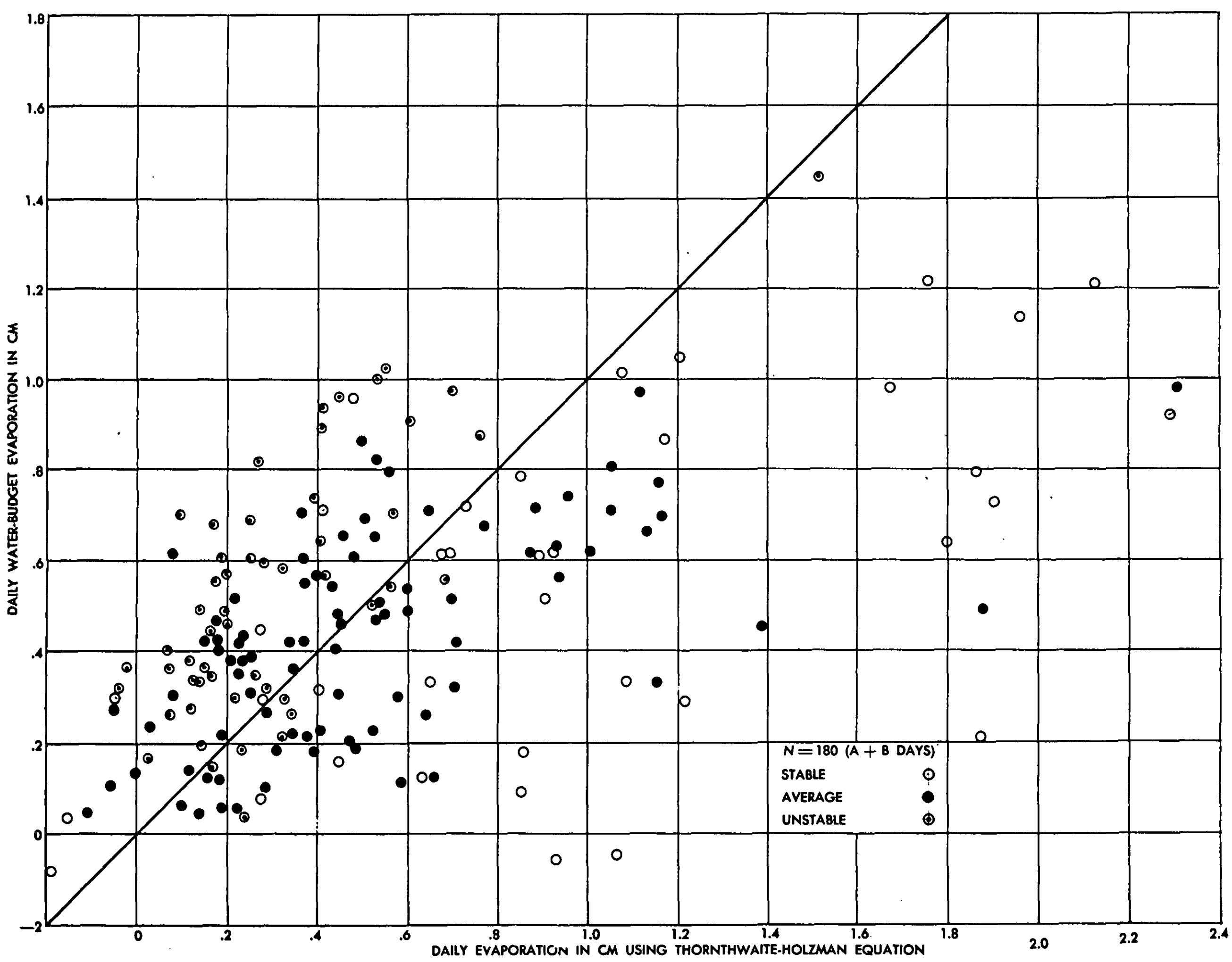

Figure 45. Experimental test of the Thornthwaite-Holzman evaporation equation.

(1946) equation (fig. 46), which essentially ignores the laminar layer in extending the turbulent layer to the very surface, yields results that are much too large. The discrepancy, furthermore, appears to be consistent, suggesting some fundamental inconsistency in the approach. It is possible that Sverdrup's (1946) equation gives high results because his limits of integration for the wind profile are from $z = z_0$ to $z = z$, but for the humidity profile they are $z = 0$ to $z = z$.

The Thornthwaite-Holzman results (fig. 45), while they are little better from the point of accuracy desired, at least show a random scatter about the perfect correlation line. The Thornthwaite-Holzman approach has in its favor the fact that it is much simpler than any of the others. The complicated aspects of flow near the surface are avoided by selecting two reference levels entirely within the turbulent layer. However, the problem arises of measuring small differences in wind speeds and vapor pressures, particularly the latter. Vapor-pressure differences between 2 and 8 meters could not be measured with sufficient accuracy with the equipment used at Lake Hefner. Moreover, so far as is known, equipment which will measure small vapor-pressure differences with the required accuracy and which is also suitable for unattended operation under field conditions is not yet available.

The results obtained with Sverdrup's (1937) equation are shown in figure 47. These are believed to be the best results obtainable with Sverdrup's two-layer model. If, for example, the "average" thickness of the laminar layer ($\delta_l = 11.5\nu/u_*$) is used, the agreement between measured and computed evaporation worsens. It should be remembered that the individual values shown in figure 47 are daily observations, and seldom in practice is a knowledge of daily evaporation necessary. As is true for the results shown for the other equations tested, the effect of atmospheric stability is apparently not significant, at least for daily figures of evaporation. It was considered impracticable to attempt to determine evaporation by the water-budget method for periods of less than 24 hours, so comparisons for shorter periods could not be made.

A graph illustrating the results obtained using Millar's vapor-blanket approach was not prepared because it was found that it would yield results closely approximating those obtained using the Sverdrup (1946) equation. In spite of the difference in methods, the two equations, when put in the form used for computing, were so nearly alike that it was considered unnecessary to use Millar's equation for detailed computations. Figure 46, which gives the results obtained from Sverdrup's (1946) equation, can be considered as showing the results to be expected from Millar's equation.

The same argument can be used with Norris' (1948) equation, based on a three-layer model examined by Millar (1937). Norris modified the Sverdrup (1946) approach to yield even more evaporation, and Sverdrup's (1946) equation has already been shown to give values that are much too large.

Holzman's (1943) evaporation equation, which contains the term $\ln(1 - \sigma Ri)$, must be discarded completely on the basis of figures 38 and 39. Some observed values of (σRi) cause this term to become imaginary and computed evaporation therefore also becomes imaginary. Moreover, temperatures at the reference levels would have to be measured with extreme precision, particularly when winds are light. For example, if $u_8 - u_2 = 50$ cm sec^{-1} and $T_8 - T_0 = 1°$C, computed evaporation is ten times as great at when $T_8 = T_0$.

Deacon's evaporation equation (48) was also tested for a brief period. It was quickly found that, although his wind equation is suitable as an empirical wind law, the evaporation equation derived therefrom breaks down. The effect of the stability parameter β is to over-correct at both ends of the stability range. For increasing stability the equation yields values which are much too low, while at the other end of the stability scale it yields values which are much too high.

Frost's (1946) evaporation equation was not tested. He assumed a form for the mixing length which causes that factor to depend greatly on the roughness of the surface, whereas it should be primarily a property of the air. This evaporation equation is otherwise similar in development to Sutton's and, in view of the above discrepancy, was dropped.

Sutton's equation gives essentially as good results (fig. 48) as Sverdrup's (1937) form. It has been pointed out above that Sutton's equation was evolved from concepts derived from two fundamentally opposed theories of turbulent mixing. Nevertheless, if one is seeking a satisfactory method, this equation must be considered. However, any modifications of Sutton's approach which would increase the computed values, such as discussed by Anderson, Anderson, and Marciano (1950, pp. 11-12) should not be considered.

Table 5 compares the average results that can be expected using the several equations discussed above. Water-budget evaporation is taken as unity.

TABLE 5. Ratios of Average Computed to Observed Evaporation.

Source	Equation Number in This Report	Evaporation
Water Budget (observed)	(1)	1.00
Sverdrup (1937)	(49)	1.12
Sverdrup (1946)	(44)	2.10
Millar (1937)	(41)	2.13
Norris (1948)	(45)	2.20
Sutton (1949)	(51)	1.07

It is evident that Sverdrup's (1937) equation and Sutton's equation are the only ones that give satisfatctory results. The standard errors of the computed figures of daily evaporation (about the line of best fit for the points shown in figures 47 and 48) are 0.109 for Sverdrup's (1937) equation and 0.118 for Sutton's. Assuming that the errors are random and normally distributed, the standard error of a computed figure of weekly evaporation for both equations may be expected to be about 10 per cent, and for monthly evaporation about 5 per cent. Because both equations give computed values that are too high, these computed values must be divided by the factors shown in table 5 to give average figures of evaporation in agreement with the water-budget data. These factors are not constant, but vary with wind speed. Computed values are particularly high at high wind speeds, especially with Sutton's equation. These relationships are shown in figure 49, where these two equations are compared with the empirical equation of best fit for the Lake Hefner data, computed from water-budget evaporation, wind speed, and vapor-pressure difference. This equation is

$$E = 6.25 \times 10^{-4}\, u_8\, (e_0 - e_8). \qquad (58)$$

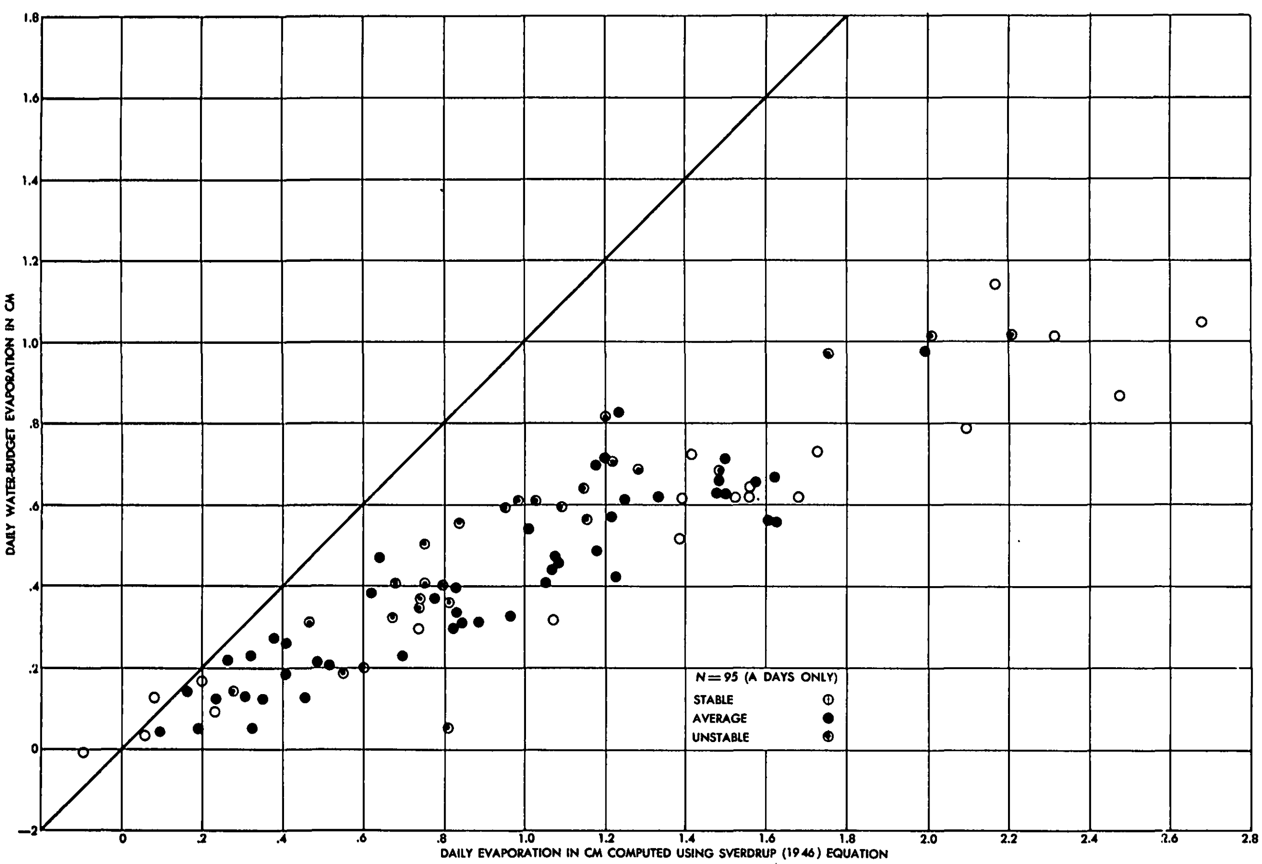

Figure 46. Experimental test of Sverdrup's (1946) evaporation equation.

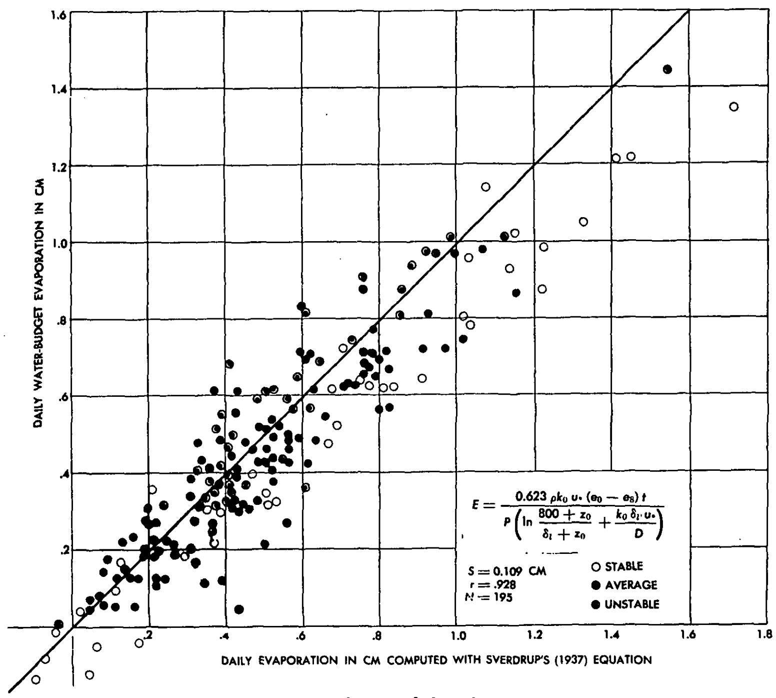

Figure 47. Experimental test of Sverdrup's (1937) evaporation equation.

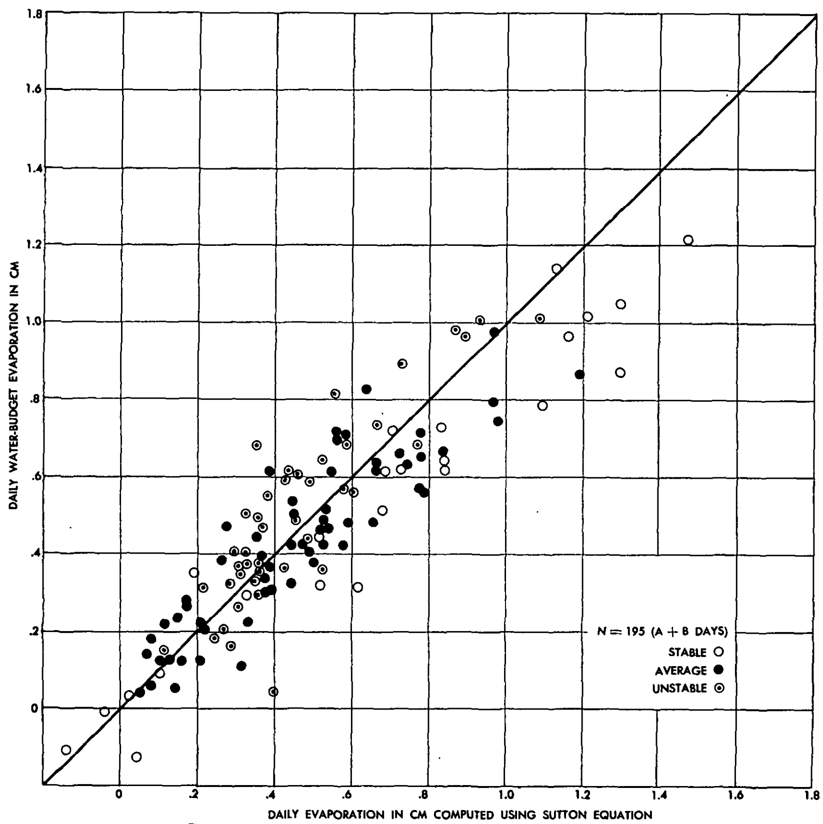

Figure 48. Experimental test of Sutton's evaporation equation.

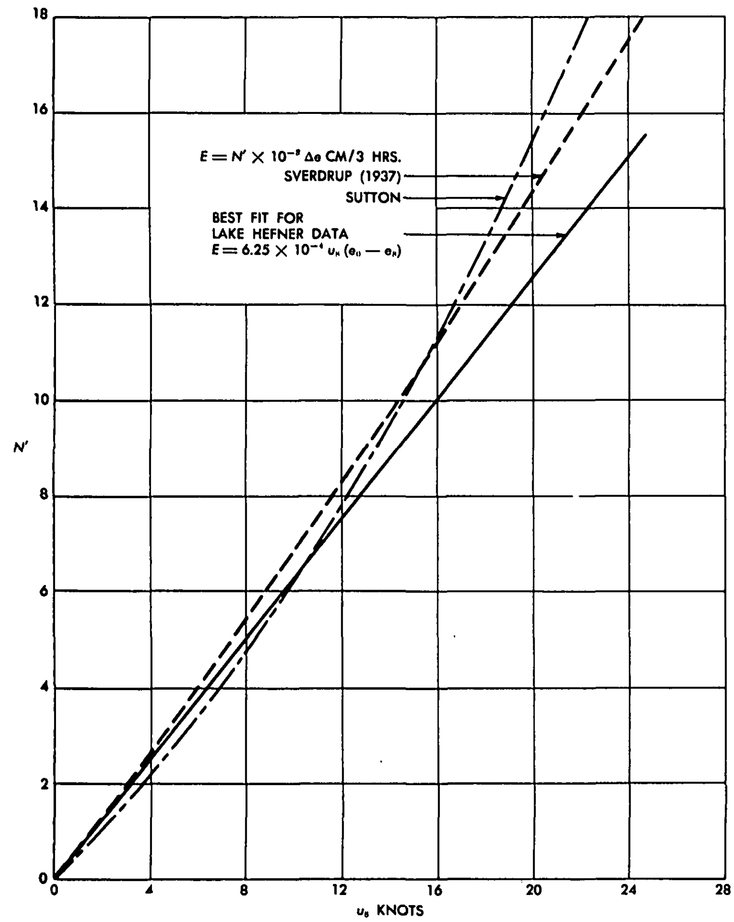

Figure 49. Comparison of Sverdrup's (1937) equation, Sutton's equation, and the Lake Hefner empirical equation.

The deviations of computed evaporation from the observed water-budget figures are believed to result from imperfections in the theoretical equations arising from our incomplete knowledge of boundary-layer wind structure, and are therefore not peculiar to Lake Hefner.

Notwithstanding the unsuccessful result of the search for a general wind equation, the most striking feature of the study is that evaporation can be accurately predicted by neglecting stability, that is, by using adiabatic equations exclusively. The principal effect of stability is to change the thickness of the boundary layer (vapor blanket). This effect can be taken into account by the proper choice of a reference level.

The effect of the size of lake on the growth of the boundary layer has already been discussed. For large lakes, the choice of the reference level for wind and humidity measurements can be approximated using equation 17, and can be determined more accurately by a few field measurements of the height to which modification extends since the only requirement is that the reference level be at or above the top of the boundary layer. Other evidence is available to demonstrate that the 8-meter level chosen for Lake Hefner was satisfactory and is also suitable for smaller lakes.

During the course of developing an empirical equation that could be used by the Oklahoma City Water Department for day-to-day computations of evaporation from Lake Hefner (Harbeck, 1952), a study was made using wind and humidity data observed at the Weather Bureau station at the Will Rogers Airport, located 13 miles south of Lake Hefner. Water-surface temperatures were taken as the average of the daily maximum and minimum recorded at the barge station. It was believed that this procedure gave a record equivalent to one that could have been obtained from a maximum-minimum thermometer floating in deep water. Using the airport wind and humidity data and the observed water-surface temperature, the following empirical equation for daily evaporation was obtained as best fitting the data: $E = 0.00177\ u\ (e_0 - e_a)$, in which E is in inches, u in miles per hour, and $(e_0 - e_a)$ in mb. Converting to cgs units, $E = 6.47 \times 10^{-4}\ u\ (e_0 - e_a)$ cm/3 hours, which is in close agreement with the empirical equation (58) shown in figure 49.

Because of this close agreement, it appears reasonable to assume that the thickness of the modified layer at the barge on Lake Hefner was less than 8 meters, thus substantiating the choice of this level. For smaller lakes the modified layer would be thinner, and the level selected for Lake Hefner could therefore be used.

CONCLUSIONS ON MASS-TRANSFER STUDIES AND APPLICATIONS

General

Although boundary-layer processes are still imperfectly understood, a complete and detailed knowledge of them does not appear necessary for the determination of evaporation for periods of a day or more.

It was concluded that the wind profile over most lakes between the 2- and 8-meter levels can be satisfactorily approximated using the logarithmic wind law. The effects of changes in atmospheric stability are not clearly understood, but appear to be of little practical significance so far as the determination of daily evaporation is concerned.

An evaporation equation based on Sverdrup's (1937) two-layer model gives results that are in better agreement with observations than those obtainable from any other equation based on the mixing-length theory. Sutton's equation gives about the same results. It is realized that all mass-transfer equations are, in a sense, partly empirical, for not until our understanding of boundary-layer processes is complete can a truly theoretical equation be developed. A rigorous wind law is still not available, for example, nor are the laws governing the variation of humidity with height. The empirical equation (58) developed from the water-budget data is, on the other hand, not entirely empirical, but embodies some of the principles of mass-transfer theory. Though further tests of this equation should be made, it is now believed possible, despite gaps in our knowledge, to compute daily evaporation from a water surface with reasonable accuracy, and in that respect one of the chief objectives of the study has been realized. It is therefore considered feasible to apply the results from Lake Hefner to the next phase of the study — the determination of evaporation from Lake Mead.

The use of meteorological data from a nearby Weather Bureau station offers tremendous advantages in economy of operation. If it can be demonstrated, for any particular reservoir, that meteorological data obtained at some distance from the site are reasonably representative of unmodified air conditions prevailing at the reservoir, the determination of evaporation for operational purposes becomes relatively simple. The only other physical measurement needed, the water-surface temperature, is not difficult to obtain.

Of considerable practical significance to those who may have occasion to use a mass-transfer equation is the fact that, for routine computations, these equations are not nearly so complicated as they appear to be at first glance. Although employing such mathematical devices as gamma functions and integral logarithms, for example, the equations can readily be simplified for routine use. Any of the equations can be reduced to a simple product of several variables, of which one or two may be obtained graphically with little loss in accuracy.

The use of the mass-transfer technique also offers considerable promise as a means of determining evapotranspiration losses. The magnitude of such losses has been approximated in the past by inflow-outflow determinations and other methods (Gatewood *et al.*, 1950), and by the use of the mass-transfer theory (Thornthwaite and Holzman, 1939). In the first of these studies, a water-budget control was provided, but the meteorological observations necessary for the computation of evapotranspiration losses using the mass-transfer theory were not obtained. In the second of these studies, good meteorological data were secured and evapotranspiration computed, but a water-budget control was lacking. Both of these omissions need to be made good for a conclusive test of the theory. Once the theory has been tested, including the determination of appropriate roughness parameters and wind and humidity profiles, the technique can be used with confidence for the determination of evapotranspiration losses from different types of vegetation under conditions of minimum, maximum, and optimum water supplies.

Specific Findings

THEORY AND BACKGROUND. The results of the mass-transfer studies, from the point of view of general boundary-layer theory, may be summarized as follows:

1. Wind speed near the center of Lake Hefner varied logarithmically with height between 2 and 8 meters, within the limits ascribable to instrumental error, regardless of stability variations.

2. The water surface was found to be aerodynamically rough at all times, with no evidence of a

critical wind speed. The resistance coefficient increased with wind speed, varying from 3.06×10^{-3} to 3.75×10^{-3}; the roughness parameter varied from 0.58 to 1.15 cm.

3. No completely satisfactory general wind equation could be found. For the empirical wind equations reviewed, it was found that if wind shear is proportional to the height raised to a power, that power must vary with Richardson's number and therefore with height.

PRACTICAL RESULTS. From the point of view of obtaining an evaporation equation suitable for field use, the findings can be summarized as follows:

1. Two of the theoretical equations, Sverdrup's (1937) form and Sutton's, give good results.

2. It is believed that the Thornthwaite-Holzman equation would give satisfactory results with proper instrumentation, but the instrument requirements are exacting.

3. All other theoretical evaporation equations based on existing models and methods were found to be unsatisfactory.

4. A simple empirical equation was developed from the water-budget evaporation data, using wind speed and vapor-pressure differences measured at Lake Hefner. An operational version of this equation uses standard data from a nearby U. S. Weather Bureau station and the water-surface temperature of the lake. No other measurement at the lake is required.

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Energy-Budget Studies

by Ernest R. Anderson*

INTRODUCTION

Since 1915, when Schmidt first attempted to utilize the (thermal) energy budget to obtain estimates of the annual evaporation from the oceans, applications of the energy budget have been restricted because of the difficulty of evaluating certain terms, such as reflected solar radiation, atmospheric radiation, reflected atmospheric radiation, long-wave radiation from the body of water, advected energy, energy conducted to or from the body of water as sensible heat, and the change of energy storage in the body of water.

Schmidt (1915), for instance, neglected the change in energy storage by computing evaporation from the oceans over a yearly interval. Richardson (1931) investigated evaporation from California lakes during time intervals when the change in air temperature was small and assumed a corresponding negligible change in energy storage. Sverdrup (1940) investigated the energy budget of two particular regions of the oceans: (1) a region off the Bay of Biscay which was without distinct currents so that advected energy could be considered negligible, and (2) a portion of the Kuroshio current where the advected energy was assumed constant throughout the year and was determined from the energy budget by first assuming that evaporation was negligible during the early summer. During recent years, Holzman (1941) and Penman (1947) both commented on the difficulty of evaluating atmospheric radiation, long-wave radiation from the body of water, and energy storage.

In addition, difficulty has been experienced in evaluating the conduction of sensible heat to or from the body of water. From the energy-budget equation it is possible to obtain the sum of energy conducted as sensible heat and energy utilized by evaporation. During the early use of the energy budget for obtaining evaporation rates, the ratio of these two terms was estimated to be around 0.1 or 0.2. Later investigations showed that the ratio varied considerably. Bowen (1926) theoretically examined the ratio and developed an expression for it in terms of easily measured quantities. Considerable controversy, however, still exists over the meaning and validity of Bowen's ratio (R).

Thus, past investigations have been confined to specific cases and large-scale features to minimize the effect of terms that could not be evaluated. However, the increasing attention on evaporation from lakes and reservoirs (Anderson, Anderson, and Marciano, 1950), on the cooling action of lakes as applied to industrial problems (Throne, 1951), and on the forecasting of thermal conditions in proposed reservoirs requires that methods be devised to evaluate each of the terms of the energy budget for any specified season, locale, and time interval.

The principal objective of the energy-budget investigation at Lake Hefner was to determine the utility of the energy budget as a method for computing evaporation from natural bodies of water. This made it necessary to improve our understanding of certain terms in the energy budget, for example, reflected solar radiation, change in energy storage, the Bowen ratio, and effective back radiation.

The Energy-Budget Equation

The energy budget for the ocean, and as applied in the past to lakes and storage reservoirs, is generally expressed as

$$Q_s - Q_r - Q_b - Q_h - Q_e + Q_{v'} = Q_\vartheta, \quad (59)$$

where Q_s is the solar radiation incident to the water surface, Q_r reflected solar radiation, Q_b the net energy lost by the body of water through the exchange of long-wave radiation between the atmosphere and the body of water, Q_h energy conducted from the body of water to the atmosphere as sensible heat, Q_e energy utilized for evaporation, $Q_{v'}$ net energy advected into the body of water, and Q_ϑ the increase in energy stored in the body of water. Conduction of energy through the bottom, heating due to chemical and biological processes, and the transformation of kinetic energy into thermal energy are generally neglected because of their small magnitude.

From equation (59) it follows that the evaporation, E, can be expressed as

$$E = \frac{Q_s - Q_r - Q_b + Q_{v'} - Q_\vartheta}{\rho L (1 + R)}, \quad (60)$$

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where L is the latent heat of vaporization and R, generally referred to as the Bowen ratio, is the ratio of energy conducted to or from the air as sensible heat to the energy lost through evaporation.

The ratio of conduction of sensible heat to evaporation as given by Bowen (1926) is

$$R = \frac{0.61P}{1000}\left(\frac{T_0 - T_a}{e_0 - e_a}\right), \tag{61}$$

where P is the atmospheric pressure in mb, T_0 is the water-surface temperature and T_a the air temperature in °C, and e_0 is the saturated vapor pressure at the surface-water temperature and e_a the vapor pressure of the air in mb.

Generally,

$$Q_b = Q_a - e\sigma T_0^4, \tag{62}$$

where Q_a is the atmospheric radiation (principally from water vapor, cloud droplets, carbon dioxide, and ozone), T_0 the water-surface temperature in °K*, σ the Stefan-Boltzmann constant, and e the emissivity of the water surface. The emissivity of water has variously been taken to be 0.900 to 1.000. In addition, the reflection of atmospheric radiation has generally been neglected.

Saur and Anderson (paper in preparation) discuss in detail the relations existing between advected energy, change in energy storage, and change in volume of a body of water. These relations will be briefly reviewed.

When considering the energy content of a substance it is necessary to refer the energy content to some arbitrarily selected base temperature. The energy content of a unit mass of a liquid substance may be written

$$c\,(T - T_b) + \text{constant}, \tag{63}$$

where T is the temperature in °C, the unknown constant depends on the base temperature T_b and the character of the substance, and c is the specific heat. Thus, in calculations of a change in energy content of a fixed mass of water, the constant and base temperature cancel out. However, if the mass of water is not fixed, the change in energy content is not independent of the base temperature and must be considered with the advected energy.

Let V_1 and V_2 be the volumes of the body of water at the beginning and end of the period under consideration, N and M the total volumes of inflow and outflow, and ρ_1, ρ_2, ρ_N, ρ_M the densities of the respective volumes of water; then, the equation of continuity is

$$\rho_1 V_1 - \rho_2 V_2 = \rho_M M - \rho_N N. \tag{64}$$

Let

$$\rho_N N = \rho_I n_I + \rho_p n_p + \ldots$$

and $$\rho_M M = \rho_0 m_0 + \rho_s m_s + \rho_e m_e + \ldots, \tag{65}$$

where $\rho_0 m_0$ is the mass of surface outflow, $\rho_s m_s$ the mass of out seepage, $\rho_e m_e$ the mass of evaporated water, $\rho_I n_I$ the mass of surface inflow, and $\rho_p n_p$ the mass of precipitated water. Then, using equations (64) and (65)

$$\rho_1 V_1 - \rho_2 V_2 = \rho_0 m_0 + \rho_s m_s + \rho_e m_e + \ldots - \rho_I n_I - \rho_p n_p - \ldots. \tag{66}$$

From equations (63) and (66) it follows that the quantity (Q_v-Q_s) can be expressed as

$$c\,[V_1 \rho_1 (T_1 - T_b) - V_2 \rho_2 (T_2 - T_b) - \rho_0 m_0 (T_0 - T_b) - \rho_s m_s (T_s - T_b) - \rho_e m_e (T_e - T_b) + \rho_I n_I (T_I - T_b) + \rho_p n_p (T_p - T_b) + \ldots - \ldots], \tag{67}$$

where T_1, T_2, T_I, T_0, T_p, T_s, and T_e are the temperatures of the respective volumes of water. Since equation (66) holds, it is clear that equation (67) is independent of the base temperature, and that the base temperature can be arbitrarily selected. In fact, T_b could be omitted from equation (67) but, as will be seen, it is convenient to retain it for the purpose of computation. Equation (67) further shows that those terms in the energy budget associated with a term in the water budget cannot be considered individually, but are independent of the computational method only when the sum is considered. The constant in equation (63), being a function of only the base temperature and the nature of the substance, cancels out as long as equation (66) holds. It is noted that, by neglecting differences in density (thermal expansion) and using an average value, little error is introduced and terms can be computed on the basis of volume. The results obtained from equation (67) are in units of energy per lake per interval of time. This result must be divided by the area of the lake to obtain calories per unit area per interval of time in order to be consistent with the units used to measure the radiative terms.

In addition to the energy lost because of the latent heat of vaporization, the mass of evaporated water represents an advected loss from the body of water, as indicated by equation (65), solely by the

* °K represents degrees Kelvin (the absolute centigrade scale).

loss of a mass of water at some temperature. Thus equation (59) should be written:

$$Q_e + Q_h = Q_s - Q_r - Q_b - Q_\vartheta + Q_v - Q_w, \tag{68}$$

where Q_w is the energy advected out of the body of water by the mass of evaporated water, and Q_v contains all other advected volumes. Dividing equation (68) by Q_e and rearranging:

$$Q_e (1 + R) + Q_w = Q_s - Q_r - Q_b - Q_\vartheta + Q_v. \tag{69}$$

Also,

$$Q_e = EL\rho_e \text{ and } Q_w = c\rho_e E (T_e - T_b), \tag{70}$$

where T_e is the temperature at which evaporation takes place — usually taken as the water-surface temperature. Substituting from equation (70) in equation (69):

$$E [\rho_e L (1 + R) + c\rho_e (T_e - T_b)] = Q_s - Q_r - Q_b - Q_\vartheta + Q_v \tag{71}$$

and

$$E = \frac{Q_s - Q_r - Q_b - Q_\vartheta + Q_v}{\rho_e [L (1 + R) + c (T_e - T_b)]}, \tag{72}$$

where

$$Q_b = Q_a - Q_{ar} - e\sigma T_0^4, \tag{73}$$

Q_{ar} being the amount of reflected atmospheric radiation. The term $(T_e - T_b)$ in the denominator of equation (72) is of the nature of a correction factor and will vary the evaporation about 5 per cent, depending upon the magnitude of the other terms in the equation. Obviously, the nearer the base temperature is to the evaporation temperature, the smaller will be the correction.

Intelligent selection of the base temperature can materially increase the accuracy of the computed evaporation obtained from equation (72). Referring to equation (67), generally it is difficult to evaluate n_p and m_s. Since the loss of energy through the volume of evaporated water is corrected for in equation (72), the base temperature should be selected near the temperature of the seepage and/or the precipitation temperature, since seepage and precipitation volumes are generally difficult to evaluate. By selecting the base temperature in this manner the effect of errors due to the errors in volume determination are *minimized*. In general, the base temperature should be selected as the best temperature estimate of the largest unknown advected volume.

If evaporation could be determined directly from the water-budget equation, there would be no necessity for using the energy budget. However, except in extremely well-controlled investigations, inaccuracies exist in the determination of the water budget which can be of the same order of magnitude as the evaporation. Since the latent heat of vaporization appears in the denominator of equation (72) for computing evaporation from the energy budget, a real advantage is obtained by use of the latter method. To utilize equation (72) it is necessary to have some knowledge of the water budget, but not to the degree of precision necessary to determine evaporation from the water budget.

Observational Program at Lake Hefner

The location of the instruments used for the energy-budget investigation is shown on the map.

Table 6, showing the number of days of each month for which complete radiation data were recorded, indicates in a general way the amount of data obtained. Considerable data covering all months and seasons were available for the study of energy exchange.

During October 1950, difficulty was experienced because the thermocouple ice jug rose above 0°C, with the result that the flat-plate temperature was in error during much of the month. An attempt was made to correct the flat-plate temperatures, but it

TABLE 6. Number of Days Per Month of Complete Radiation Record.

Month	Number of Days of Data	Per Cent of Total No. of Days
May 1950	9	29
Jun	11	37
Jul	11	35
Aug	13	42
Sep	20	67
Oct	27	87
Nov	30	100
Dec	22	71
Jan 1951	29	94
Feb	20	71
Mar	30	97
Apr	29	97
May	19	61
Jun	20	67
Jul	22	71
Aug	30	97
TOTAL	342	70

was difficult to judge how far back in time to carry the correction. This may have resulted in too high an effective back radiation and consequently too low an evaporation.

Ice formed on the lake during the month of February 1951, making it necessary to beach the barge from 9 February to 23 February. During this period all energy-budget parameters except the reflected solar radiation were measured at the beach position, on the south side of the lake. In addition, it was impossible to obtain vertical temperature profiles of the lake from 26 January to 26 February. Accurate data for evaluating the change of energy storage were therefore lacking during most of February.

In late June 1951, difficulty was encountered with the Temperature Profile Recorder (TPR), so that for the last two months of the observational program it was necessary to utilize a bathythermograph to obtain water-temperature data.

During months other than those mentioned above the instrumentation operated with only minor or local difficulties which influenced the observations for only short periods of time. For the months discussed above, the results, even the monthly average values, may have been affected by the instrumental difficulties.

Data Processing

The radiation data were recorded on an Esterline-Angus 1-mil recorder using a 15-point programming switch. The following elements were recorded: solar radiation, reflected solar radiation, flat-plate radiometer reading, and the temperature of the flat plate — plus a reference. This recording procedure resulted in a 4-minute record of each element spaced at 20-minute intervals.

The data were processed by constructing a continuous diurnal curve for each element and then reading off values of each element at half-hourly intervals. These 48 readings were averaged to obtain a daily average value, and then converted into a daily total. The totals were then utilized to establish an energy budget from which evaporation could be computed.

The TPR data were processed by reading and recording the mean water temperature by 1-meter intervals. From the mean temperatures and mean areas for each 1-meter-thick layer, the areas being obtained from the area-capacity table for the lake, the energy content in each layer was computed. The energy contents of the layers were then summed to obtain the total energy content, above an arbitrary base temperature, of the lake.

SOLAR RADIATION

The short-wave radiation (0.3 μ to approximately 4.0 μ) incident to the outside of the earth's atmosphere comes primarily from the sun. This radiation in passing through the earth's atmosphere undergoes depletion through scattering, reflection, and absorption by gases of the air, water vapor, clouds, and dust. As a result of these complex processes, the short-wave radiation arrives at the earth's surface partly as direct radiation and partly as diffuse radiation.

Two approaches are available for evaluating the solar radiation incident to the earth's surface: (1) indirect evaluation in terms of easily observable or measurable quantities, and (2) direct measurement by suitable instrumentation.

Knowing the amount of radiation incident to the outside of the earth's atmosphere, many investigators, for example, King (1913), Mosby (1936), Hewson (1943), Haurwitz (1948), Klein (1948), and Kennedy (1949), examined, both theoretically and empirically, the modification of solar radiation by the earth's atmosphere, with a view to obtaining an indirect method for evaluating this term. Because of the complexity of the processes involved, complete theoretical treatment was not possible; hence various assumptions, simplifications, and restrictions were necessary. For example, King (1913) examined theoretically the scattering of solar radiation by gaseous media and obtained complex expressions for the intensity of direct and diffuse solar radiation. He considered only the effect of molecules of the atmosphere and small particles, neglecting the effect of clouds, and assumed the earth was a smooth plane and the density of air a function of height only; he ignored reflection from the earth's surface and refraction by the atmosphere. Mosby (1936) developed the following empirical formula for computing the incoming radiation on a horizontal surface, in terms of the average altitude of the sun and the average cloudiness:

$$Q_s = k\,(1 - 0.071\,C)\,\bar{h} \text{ cal cm}^{-2}\text{ min}^{-1}, \quad (74)$$

where Q_s is the solar radiation, k a constant that is a function of latitude, C the average cloud cover in tenths of sky covered, and $\bar{h}$ the average altitude of the sun in degrees. This equation is not valid for sun altitudes greater than 60 degrees, but gives correct results if they are replaced by reduced altitudes. Hewson (1943) considered theoretically the effect of cloud thickness, water content, and drop size on the transmission of solar radiation. Haurwitz (1948) examined empirically the relation between total solar radiation reaching the earth's surface and the amount of cloudiness, cloud density, and cloud type. Klein (1948), utilizing the results of previous theoretical and empirical investigations, presented a method for computing direct and diffuse radiation received on the earth's surface. He considered the effects of terrain reflection and of depletion by dry air, water vapor, dust, and clouds. He summed up: "However, the computations described in succeeding sections of this report all involve approximations from empirical relations which are true only in the mean and subject to large deviations in individual cases. Therefore, they should be applied only to average values from climatological records taken over as long a period of time as possible." Kennedy (1949) presented a method for computing daily insolation. The method makes use of a pyrheliometer station, similar in latitude and altitude to the place where the radiation data are desired. Data are extrapolated from the station to the place in question. The method involves obtaining a solution to the following equation:

$$Q_s = I_0 a^m, \qquad (75)$$

where Q_s is the solar radiation, I_0 the solar radiation received on a horizontal surface at the exterior of the earth's atmosphere, a the atmospheric transmission coefficient, and m the solar air mass or the ratio of the length of the actual path of the solar beam to the path through zenith. Using known values of Q_s, I_0, and m at a pyrheliometer station, the daily atmospheric transmission coefficient a may be computed. The daily value is then plotted against cloudiness in tenths, and a smooth curve fitted to the points. From this plot the value of the atmospheric transmission coefficient for the unknown station may be obtained for different degrees of cloudiness. Utilizing the proper value of a and tables giving I_0 and m as functions of latitude and the declination of the sun, the solar radiation is computed from equation (75).

In this study, solar radiation was measured directly by means of an Eppley pyrheliometer. The observations are summarized in figure 50 in terms of average monthly values. The numerals indicate the number of complete days of solar-radiation observations obtained for each month. In addition, the maximum and minimum daily radiations during the month are shown. As expected, there is a roughly sinusoidal variation throughout the year with a variation from about 250 cal cm^{-2} day^{-1} in December to 625 cal cm^{-2} day^{-1} in June. A comparison of July 1950 with July 1951 indicates the variation that may occur from year to year for any given month. July 1950 had an average monthly value of about 175 (or about 25 per cent) lower than July 1951. According to the U. S. Weather Bureau, July 1950 was one of the coldest and rainiest on record and this is reflected in the low monthly radiation.

The monthly range in maximum and minimum varies from about 250 cal cm^{-2} day^{-1} in December to 450 cal cm^{-2} day^{-1} in June.

In past investigations, solar radiation has usually been evaluated by one of the indirect methods mentioned above. The direct solar measurements made at Lake Hefner over a period of 16 months provided an opportunity for evaluating some of these indirect methods. Two were selected for evaluation: (1) Mosby's formula, equation (74), which represents an empirical method designed to provide average monthly or annual values of solar radiation, and (2) Kennedy's method, which represents an extrapolation designed to provide daily solar-radiation values. These methods were selected because both have been widely used for oceanographic and limnologic studies.

The monthly solar radiation for Lake Hefner from July 1950 to August 1951 was computed from Mosby's formula. According to Mosby the constant k is a function of latitude, and for the latitude of Lake

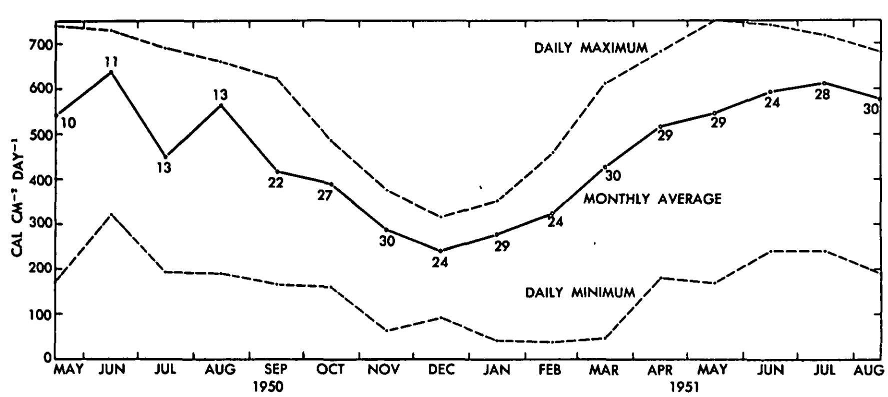

Figure 50. Average monthly solar radiation for Lake Hefner from May 1950 to August 1951.

Hefner its value is 0.024. The average cloud cover was computed from the hourly cloud cover as reported by the U. S. Weather Bureau station located 13 miles south of Lake Hefner at Will Rogers Airport. The average monthly sun altitude was computed by means of a method described by Bowditch (1939).

The results obtained are summarized in table 7. As indicated, the solar radiation values obtained from Mosby's formula are in general too low.

TABLE 7. Comparison of Solar Radiation Computed from Mosby's Formula and that Observed at Lake Hefner.

Month	Number Days Observation	Observed Q_{s0}	Computed Q_{sc}	Difference $Q_{sc} - Q_{s0}$	Per Cent Difference
		(cal cm^{-2} day^{-1})			
Jul 1950	13	448	383	—65	—15
Aug	13	565	550	—15	—3
Sep	22	417	339	—78	—19
Oct	27	390	367	—23	—6
Nov	30	287	226	—61	—21
Dec	24	233	184	—49	—21
Jan 1951	29	275	189	—86	—31
Feb	24	322	239	—83	—26
Mar	30	426	312	—114	—27
Apr	29	516	449	—67	—13
May	29	541	478	—66	—12
Jun	24	591	521	—70	—12
Jul	28	611	563	—48	—8
Aug	30	576	599	23	4

The mean difference is about 70 cal cm^{-2} day^{-1}, or about 15 per cent of the observed values. The values obtained from Mosby's formula are dependent to a great extent on the accuracy of the estimate of the amount of cloud cover. Since cloud cover is only estimated to tenths, the formula is rather inflexible and considerable difference in the results may be caused by varying the cloud cover by one-tenth. Obviously, if cloud cover were estimated with greater accuracy, say to twentieths or hundredths, greater flexibility would be attained. However, since cloud cover is generally observed from the ground and furthermore is visually estimated, the use of a scale in greater detail than a tenth is not justified. Under certain conditions it is difficult for observers to agree even within a tenth as to the amount of cloud cover.

The negative bias, indicated by table 7, can be removed by adjusting the constants in equation (74). The constant k and the cloud-cover coefficient were adjusted individually and simultaneously. Changing (1) k to 0.0276, or (2) the cloud-cover coefficient to 0.0532, or (3) the constant k to 0.025 and the cloud-cover coefficient to 0.059 removes the negative bias. The standard error of estimate, using the new constants, is ± 50, ± 38, and ± 41 cal cm^{-2} day^{-1}, respectively. These amount to approximately ± 10 per cent of the mean observed solar radiation. Hence, there is some improvement from the adjusted constants. The use of these constants, in preference to those originally established by Mosby, must be made with caution since they are probably only applicable to the Lake Hefner area.

The effect of the differences between computed and observed solar radiation on evaporation computations depends upon the magnitude of $(Q_e + Q_h)$ and R. For example, during August 1950, September 1950, and January 1951, the use of solar radiation computed from Mosby's formula, using the original constants, introduces differences of 6, 31, and 55 per cent, respectively, in computed evaporation.

On a mean annual basis, the observed solar radiation from 1 September 1950 to 1 September 1951 was 432 cal cm^{-2} day^{-1}, while that obtained from Mosby's formula was 362 cal cm^{-2} day^{-1} or approximately 15 per cent lower.

In summary, Mosby's formula leads to results approximately 15 to 20 per cent too low for monthly or yearly values. Solar radiation values obtained from Mosby's formula may therefore lead to large errors in evaporation computations, unless the values are substantiated by independent data; for other purposes, the values may be sufficiently accurate.

Kennedy's method of obtaining daily solar radiation consists basically in extrapolating measured data from a radiation station to the place in question, the two locations being similar in latitude and elevation. Since the U. S. Weather Bureau maintains about sixty radiation stations scattered throughout the United States, it is generally possible to find a radiation station that meets the latitude and elevation requirements. The radiation station at Fresno,* California, most nearly meets the requirements for Lake Hefner.

Solar radiation for Lake Hefner was computed for each of the 358 days for which a direct measurement of solar radiation was available. Plots of the difference between Kennedy's value and the observed value versus the observed value indicated that the difference was a function of cloud cover. The data were therefore examined in terms of cloud cover — clear, scattered, broken, or overcast. The results are presented in figure 51 in terms of the standard error of estimate and the standard error of estimate as a percentage of the mean observed solar radiation.

* The Oklahoma City data were not used because they would not have provided a test of Kennedy's method.

For clear skies the standard error of estimate is less than 10 cal cm^{-2} day^{-1}, or only about 2 per cent of the mean observed radiation. This error is acceptable for purposes of computing evaporation. However, as the cloud cover increases the standard error increases to about 115 cal cm^{-2} day^{-1}, or 25 and 50 per cent for broken and overcast skies, respectively. Errors of this magnitude cannot be tolerated in computing evaporation. The increase in error with cloud cover is related to the height of the cloud cover. Kennedy's method considers radiation only as a function of amount of cloud and not of cloud height. It is apparent, however, that the amount of solar radiation penetrating a high overcast will be greater than that penetrating a low overcast, all other conditions being the same; in other words the average daily atmospheric coefficient for high clouds is different from that for low clouds. This dependence was also demonstrated by Haurwitz (1948). Hence, to improve the accuracy for cloudy conditions, it is necessary to select a radiation station similar not only in latitude but also in types of clouds to the location in question. Alternatively, separate atmospheric transmission coefficients should perhaps be defined for each of the major cloud types — low, middle, and high.

In addition, solar radiation values were computed for time intervals greater than one day. The number of data did not permit a breakdown with respect to cloud cover or height. The results are presented in figure 52. The standard error of estimate decreases from a value of 85 cal cm^{-2} day^{-1} for daily periods to 45 cal cm^{-2} day^{-1} for a 10-day period, and then remains essentially constant with increasing length of time interval. Expressed as per cent of mean observed solar radiation, the error decreases from 20 for single days to about 12 for intervals greater than 5 days.

It thus appears that Kennedy's method gives excellent results for daily solar radiation with clear skies. For sky conditions other than clear, errors up to 50 per cent may occur when computing daily values. These errors may possibly be reduced by taking into account the characteristics of the cloud cover. For periods longer than 5 days, an error of approximately 10 per cent may be expected.

Summarizing, of several indirect methods for evaluating solar radiation, two were studied, namely, Mosby's, representative of long time intervals, and Kennedy's, representative of short time intervals. The results obtained were examined in detail and compared with direct observations made at Lake Hefner, Oklahoma. The conclusion was that these indirect methods of evaluating solar radiation will give the necessary accuracy for evaporation computations only in selected circumstances. For applications other than to evaporation, where accuracies of the order of 15 per cent are required, they are satisfactory. It is necessary to measure solar radiation directly to obtain the accuracy required for applying the energy budget to evaporation determinations.

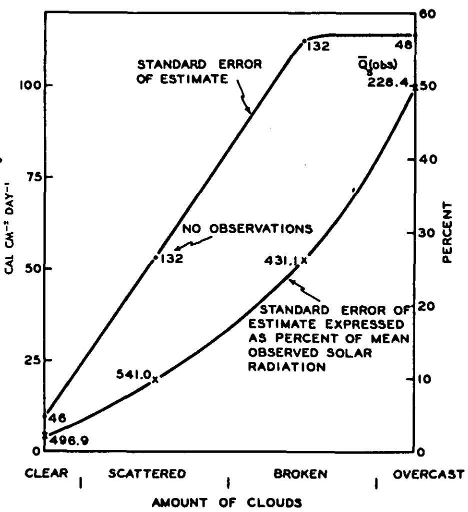

Figure 51. Standard error of estimate of solar radiation computed from Kennedy's method as a function of cloud cover.

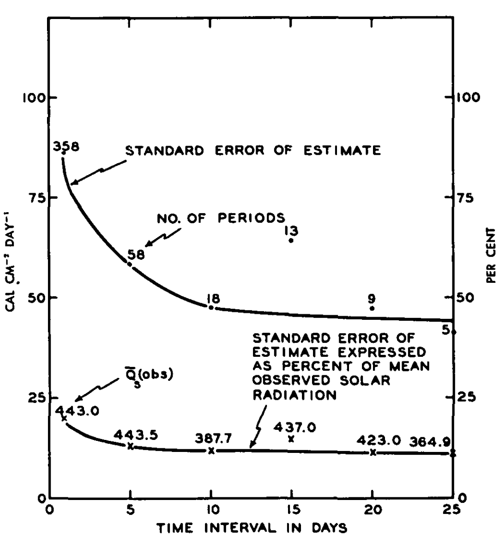

Figure 52. Standard error of estimate of solar radiation computed from Kennedy's method as a function of length of time interval.

REFLECTION OF SHORT-WAVE RADIATION

As stated by Neiburger (1948), a "knowledge of the reflectivity of the sea surface is of fundamental importance for heat-balance studies, both of the atmosphere and the oceans. In order to evaluate the amount of solar energy absorbed by the sea, it is necessary to know not only the amount incident on the surface, but to know also what portion of that is returned, unchanged, by reflection at the surface and by scattering from bubbles and suspended particles immediately below it."

The necessity for evaluating each term in the energy budget accurately when attempting to determine evaporation has been pointed out in the preceding section. For example, an error of 30 to 40* cal cm^{-2} day^{-1} in the evaluation of any term in the energy budget results in an error of 10 to 20 per cent in the computed evaporation.

The reflectivity of short-wave radiation by a natural water surface has been studied by relatively few investigators. Zenker (1888) made the first theoretical study of the subject, and Schmidt (1915) investigated the evaporation for the oceans as a whole by utilizing the energy-budget approach. In evaluating the incoming solar radiation, Schmidt discussed the theoretical reflectivity of both solar and sky radiation and obtained a theoretical value of 0.17 for the reflectivity of sky (diffuse) radiation.

Kimball and Hand (1930) made measurements from an airplane of reflectivity over Chesapeake Bay, Potomac River, and Patuxent River, at altitudes ranging from 10 to 1000 feet. They obtained values of reflectivity under an overcast sky of 0.06 to 0.10. For Chesapeake Bay, they reported a value of 0.097 for a hydrodynamically smooth surface and 0.034 to 0.049 when whitecaps were present, suggesting a decrease of reflectivity at higher wind speeds.

Powell and Clarke (1936) made 34 measurements of the reflectivity of the sea water at Buzzard's Bay in July and August 1935. Measurements were made for red and violet light, and a variety of winds, sky covers, and sun altitudes. They concluded that: (1) the solar radiation is reflected not only from the water surface but also from a stratum of relatively opaque water (bubbles and suspended material) just beneath the surface, and for a reflectivity of 0.09 cr less, one-half the radiation is returned by the water surface itself; (2) reflectivity increases slightly when the surface is hydrodynamically rough and in the presence of clouds; (3) reflectivity is not more than 0.09 for sun altitudes greater than 30 degrees in clear weather; (4) average reflectivity under an overcast sky (diffuse radiation) is 0.08.

Sverdrup (1942) synthesized Schmidt's theoretical values for reflectivity of solar radiation and Powell and Clarke's observed values for sky radiation, and obtained values of reflectivity of 0.25, 0.06, 0.03, and 0.03 for sun altitudes of 10, 30, 60, and 90 degrees, respectively.

Neiburger (1948), using a blimp, studied diffuse radiation off the California coast under a stratus overcast. He made 117 observations on 45 days during the summer of 1945. He concluded that: (1) the radiation was not completely diffuse; (2) the average reflectivity was 0.105, varying with cloud thickness and sun altitude; (3) for completely diffuse radiation, Schmidt's value of 0.17 would apply; (4) Powell and Clarke's value of 0.08 should be applied only for radiation from a high sun passing through relatively thin clouds; (5) no correlation exists between wind speed and reflectivity.

As indicated by these studies, there is lack of agreement concerning the reflectivity of completely diffuse radiation and the effect of wind speed. In addition, so far as is known, no one has investigated the effect of variable amounts of high, middle, or low clouds on the total reflectivity.

The observations made at Lake Hefner provide the necessary data for examining the reflectivity of a natural water surface in considerable detail. In this section the theory of reflectivity of solar and sky radiation for an optically flat water surface under clear skies will be reviewed; reflectivity will be considered as a function of sun altitude, turbidity* of the atmosphere, hydrodynamic nature of the surface, and amount and height of cloud cover; and a method of determining reflectivity in terms of more easily measured parameters will be presented.

As previously mentioned, total incoming and reflected radiation was measured every 20 minutes from 1 May 1950 to 31 August 1951. Observations were made covering variable cloud conditions, winds from 0 to 40 knots, sun altitudes from 0 to 78 degrees, and several different types of air masses.

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* An approximate daily average value of the amount of reflected energy at Lake Hefner.

* Turbidity, unless otherwise noted, is used in this section to refer to the turbidity of the atmosphere when no clouds are present. As will be pointed out later, the type of air mass is related to the turbidity of the air, which in turn is related to the reflectivity.

Theoretical Reflectivity Under a Clear Sky

Short-wave radiation reaches any surface on the earth partly as direct radiation from the sun (solar radiation) and partly as scattered or reflected radiation from the sky (sky radiation). When considering the total reflectivity of a surface it is necessary to consider the reflectivity of each of these components. If a and b are the reflectivities of solar and sky radiation, and m and n are the respective fractions of the incoming radiation from the sun and sky, then the total reflectivity, R_t, of any horizontal surface may be expressed as

$$R_t = am + bn. \tag{76}$$

The reflectivity of both solar and sky radiation from an optically flat water surface under a clear sky can be examined by application of Fresnel's formula for the reflection of natural unpolarized light.* Theoretical expressions for the amount of short-wave radiation coming from the sun and sky were obtained by King (1913).

The theoretical examination of each term in equation (76) makes it possible to examine the nature of the total reflectivity of an optically flat water surface under a clear sky.

King applied the results of Rayleigh on the scattering of parallel radiation by molecules and small particles to radiation and absorption in the earth's atmosphere. He made the following simplifying assumptions: the earth is a smooth plane, the density of air is a function of height only, and reflection from the earth's surface and refraction by the earth's atmosphere may be neglected. In addition, he considered only the effects of molecules of the atmosphere and small particles, neglecting the effect of clouds.

By much mathematical manipulation he developed two equations (approximations of a complicated exact equation which is difficult to apply) for obtaining the total intensity of sky radiation from any direction. One is applicable to large sun altitudes and the other to small altitudes. These equations indicate that both solar and sky radiation are functions of the sun's altitude, the coefficient of attenuation of scattered radiation, and absorption. His attenuation coefficient, C, includes the effect of both molecules of gas and dust particles. Hence,

$$\text{solar radiation} = f_1\,(S_A,\, C), \text{ and} \tag{77}$$

$$\text{sky radiation} = f_2\,(S_A,\, C), \tag{78}$$

where S_A is the altitude of the sun.

Using mean coefficients of attenuation for Mt. Wilson, California, and Washington, D. C., King computed for these two stations the amount of energy coming from the sun and sky as a function of sun's altitude. His values are graphically presented in figure 53a. The sky radiation is given as a fraction of the total short-wave radiation in figure 53b. These

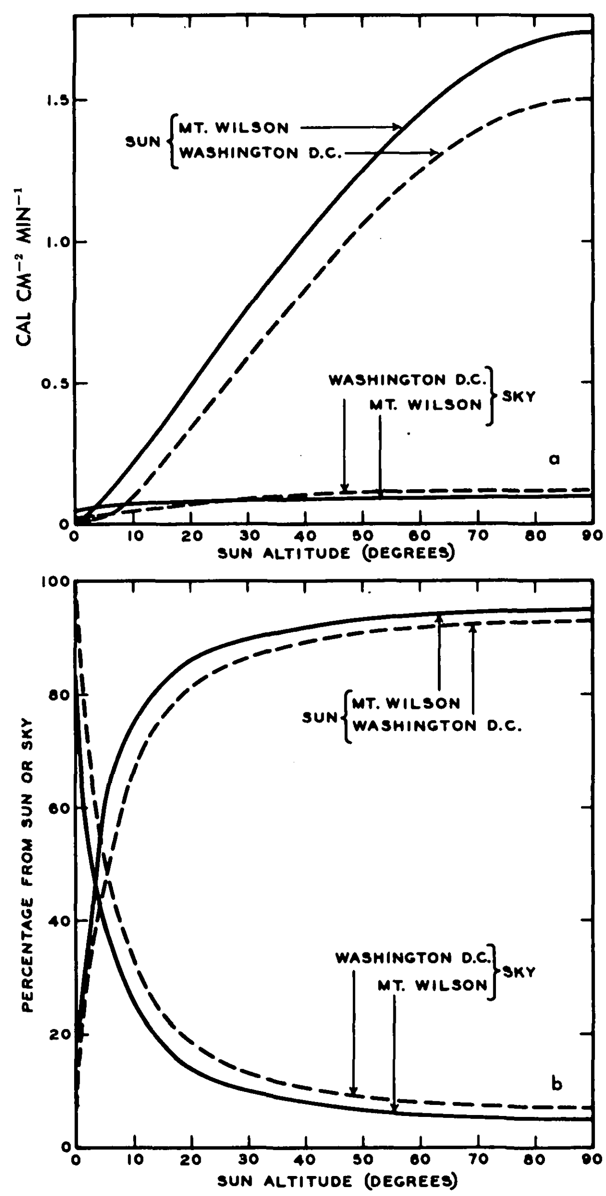


Figure 53. Theoretical radiation coming from the sun and sky for Mt. Wilson and Washington, D. C., according to King.

* A complete discussion and mathematical treatment of the Fresnel formula is given by Sears (1945) and Houstoun (1938).

data show the greater amount of sky radiation at Washington because of the increased scattering due to lower elevation of the station above sea level and the greater number of small particles in the atmosphere. The Mt. Wilson data probably reflect scattering caused primarily by molecules, while at Washington there is a greater particulate effect. In each place, the sky radiation is nearly independent of the altitude of the sun.

King's coefficient of attenuation is similar to Linke's (1922) turbidity factor. This factor may be written as

$$T = 1 + \frac{1}{a}(a_w w + a_d d), \tag{79}$$

where T is a mean turbidity factor, a a mean extinction coefficient due to molecular scattering, and a_w and a_d mean coefficients of extinction by water vapor (disregarding selective absorption) and dust, respectively.

Haurwitz (1934) demonstrated that the turbidity factor varies with respect to the air mass present. Table 8 presents the average turbidity factor obtained by Haurwitz for various types of American air masses. The effect of the variation in turbidity with air mass would be to vary the ratio of sky radiation to solar radiation. Thus, for modified Polar Atlantic air the relative amount of diffuse radiation would be greater than for Polar Continental air.

TABLE 8. Average Turbidity Factors for Various Types of Air Masses According to Haurwitz.

Air Mass	Turbidity Factor
Polar Continental	2.42
Modified Polar Pacific	2.65
Modified Polar Continental	2.81
Tropical Gulf plus Tropical Atlantic	3.49
Polar Atlantic plus Modified Polar Atlantic	3.61

As previously stated, the reflectivity of an optically flat water surface under a clear sky for direct solar radiation can be examined by considering Fresnel's formula for the reflectivity of natural unpolarized light. This formula is

$$a = \frac{1}{2}\left[\frac{\tan^2(90 - S_A - r)}{\tan^2(90 - S_A + r)} + \frac{\sin^2(90 - S_A - r)}{\sin^2(90 - S_A + r)}\right], \tag{80}$$

where a is the reflectivity of direct solar radiation, S_A the altitude of the sun above the horizon, r the angle of refraction, and $\sin(90 - S_A) = \mu \sin r$ where μ is the index of refraction for water relative to air. For sun alititude of 90 degrees, equation (80) reduces to

$$a = (\mu - 1)^2 (\mu + 1)^{-2}. \tag{81}$$

The index of refraction for pure water relative to air at 25°C is 1.33251 (Handbook of Chemistry and Physics, 1947) while for sea water of salinity 34.00 $^0/_{00}$* it is 1.33873 at 25°C (Sverdrup, *et al*, 1942).

Figure 54 shows the variation of reflectivity of direct sun radiation with altitude of the sun for pure water. In the case of sea water, which has a slightly higher index of refraction, the reflectivity is only 0.000 to 0.003 greater.

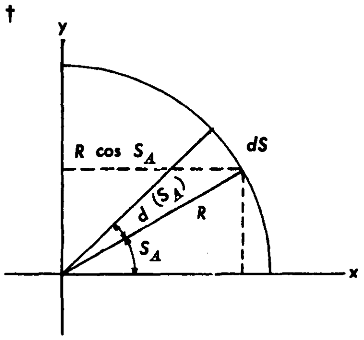
In the presence of thick clouds, assuming the atmosphere obeys Lambert's cosine law of diffuse radiation, the sky light is of equal intensity from all directions. Knowing the reflectivity for various sun altitudes, and assuming the light is completely diffused, then, as indicated by Schmidt (1915),

$$b = \int_0^{\pi/2} a(S_A) \cos S_A \, dS_A, \tag{82}$$

where $a(S_A)$ is the reflection of direct sun radiation as a function of sun altitude.† Equation (82) can be

* Parts per thousand, the standard method of expressing the salinity of sea water.

†



The area of a surface of revolution around the y-axis is $2\pi \int_0^{\pi R/2} x\,ds$ or in this case $2\pi R^2 \int_0^{\pi/2} \cos S_A \, dS_A$. The amount of reflected radiation is $2\pi R^2 \int_0^{\pi/2} a(S_A) \cos S_A \, dS_A$. The mean value of a function is $\int x\,ds / \int ds$. In this case the average reflectivity becomes

$$\frac{2\pi R^2 \int_0^{\pi/2} a(S_A) \cos S_A \, dS_A}{2\pi R^2 \int_0^{\pi/2} \cos S_A \, dS_A} = \int_0^{\pi/2} a(S_A) \cos S_A \, dS_A.$$

evaluated graphically if $x = \sin S_A$ and $y = a(S_A)$. Then the integral becomes $\int_0^1 y dx$. The area under the curve in figure 55 is then the total reflectivity of completely diffuse sky radiation for an optically flat water surface. This evaluation gives 0.173 for the reflectivity of sky radiation.

The total reflectivity of an optically flat water surface under a clear sky can be evaluated from equation (76) using values for a, b, m, and n previously discussed. Rewriting equation (76),

$$R_t = am + bn, \quad \text{where}$$

$$a = \frac{1}{2}\left[\frac{\tan^2(90 - S_A - r)}{\tan^2(90 - S_A + r)} + \frac{\sin^2(90 - S_A - r)}{\sin^2(90 - S_A + r)}\right], \quad (76a)$$

$$b = \int_0^{\pi/2} a(S_A) \cos S_A \, d S_A, \quad (76b)$$

$$m = \frac{f_1(S_A, C)}{f_1(S_A, C) + f_2(S_A, C)}, \text{ and} \quad (76c)$$

$$n = \frac{f_2(S_A, C)}{f_1(S_A, C) + f_2(S_A, C)}, \quad (76d)$$

we obtain

$$R_t = \frac{1}{2}\left[\frac{\tan^2(90 - S_A - r)}{\tan^2(90 - S_A + r)} + \frac{\sin^2(90 - S_A - r)}{\sin^2(90 - S_A + r)}\right] \times \frac{f_1(S_A, C)}{f_1(S_A, C) + f_2(S_A, C)} + \frac{f_2(S_A, C)}{f_1(S_A, C) + f_2(S_A, C)} \times \int_0^{\pi/2} a(S_A) \cos S_A \, d S_A. \quad (83)$$

From equation (83) it may be concluded that the total reflection from an optically flat water surface under a clear sky is a function of the altitude of the sun and the type of air mass (turbidity, excluding effect of clouds).

The effect of increasing turbidity (turbidity data from King, 1913) is indicated by figure 56, where equation (83) is applied to radiation data for Mt. Wilson and Washington. This shows that the effect of increasing turbidity at sun altitudes of less than 20 degrees is to decrease the reflectivity slightly, and at sun altitudes greater than 20 degrees to increase the reflectivity slightly. For different air masses and various amounts of cloud cover, the variations would be similar.

The total actual amount of reflected solar and sky radiation is

$$Q_r = R_t Q_s, \quad (84)$$

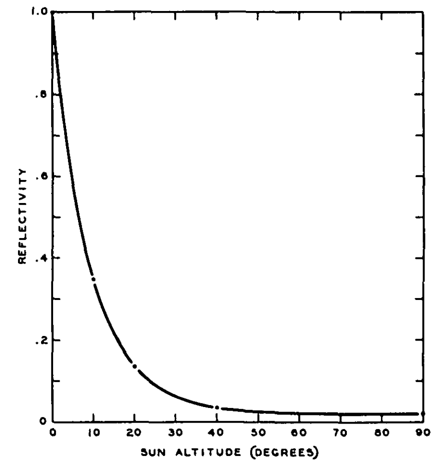


Figure 54. Variation of reflectivity of direct solar radiation with altitude of the sun for pure water as obtained from Fresnel's formula.

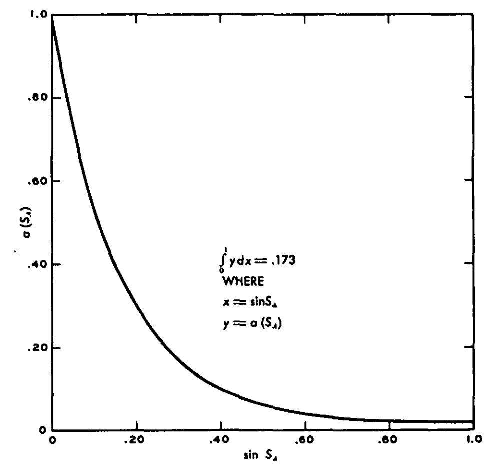


Figure 55. Theoretical total reflectivity for completely diffuse radiation.

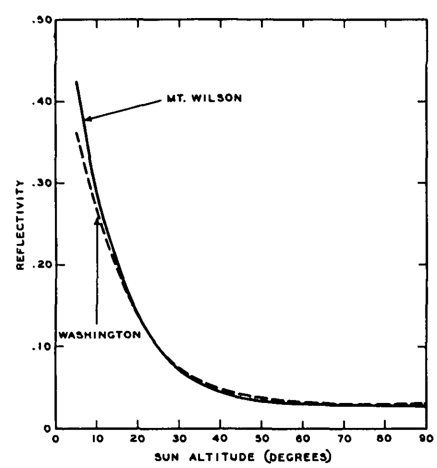


Figure 56. Effect of the variation of air-mass turbidity on the reflectivity of a natural water surface.

where Q_r is the total amount of reflected energy and Q_s the total incoming short-wave radiation. Using values of reflected energy computed from equation (83) and values of total incoming short-wave radiation computed from equations (77) and (78), one can examine the variation of the amount of reflected energy as a function of sun altitude and time.

If knowledge of the coefficient of attenuation for short-wave radiation were adequate, it would be possible to obtain a family of curves showing the amount of energy reflected from an optically flat water surface under a clear sky as a function of sun altitude. In addition it would be possible to present graphically the amount of reflected energy as a function of latitude and time. The amount of reflected energy could then be evaluated for any given location and day.

Since the data are inadequate, the above formulas have been applied to specific cases only. Figure 57 shows the variation of reflected energy as a function of sun altitude for Mt. Wilson and Washington (King, 1913). Mt. Wilson represents a low coefficient of attenuation and Washington a moderately high coefficient. The amount of reflected energy increases rapidly at low sun altitudes, reaching a maximum at 15 degrees for Mt. Wilson and then falling rapidly to virtually a constant value at 40 degrees. For Washington where the turbidity is greater, the amount of reflected energy is less at all sun altitudes, the maximum being at 25 degrees and less pronounced. As in the case of low turbidity, the reflected energy increases rapidly at low sun altitudes. The maximum effect of turbidity is for sun angles less than 30 degrees. From figure 57 it can be seen that the maximum amount of reflected energy for relatively transparent air is 0.085 cal cm^{-2} min^{-1} and for turbid air 0.06 cal cm^{-2} min^{-1}.

Figure 58 shows the theoretical variation of reflected energy for a given location (Lake Hefner, 35° 30′ N) as a function of season and time of day. The amount of reflected energy increases rapidly to a maximum at about two hours after sunrise; it then decreases, the amount of the decrease depending upon the season of the year, being greater during the summer and less during the winter. By the late afternoon, the reflected energy reaches an almost constant value. It then increases rapidly until two hours before sunset, and then decreases rapidly to zero shortly after sunset. A decrease in turbidity increases the reflected energy at all hours of the day but exerts its greatest influence just after sunrise and before sunset.

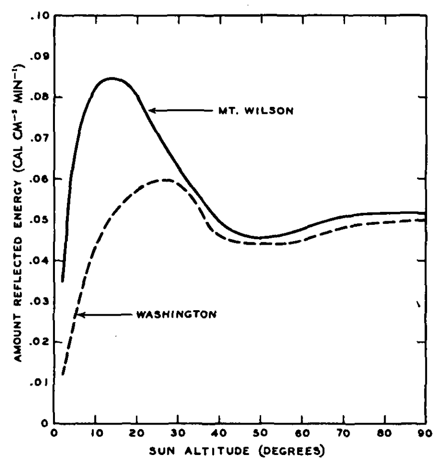


Figure 57. Theoretical variation of reflected energy as a function of sun altitude for Mt. Wilson and Washington, D. C.

At this stage, it becomes possible to speculate on the effects of cloud cover and air mass. If a uniformly dense cloud cover were introduced, it would effectively increase the turbidity of the atmosphere. This in turn would theoretically decrease the amount of reflected energy for all sun angles, latitudes, and times of day. Hence, for any given sun altitude, reflected energy would be maximal on a clear day and minimal on an overcast day. Variations in air mass should have a similar effect, and the more turbid the air mass, the smaller the amount of reflected energy for a given sun altitude.

As previously mentioned, Powell and Clarke (1936) concluded that the reflected radiation consists of radiation reflected both from the water surface and from bubbles and suspended matter just beneath the surface. For a total reflectivity of 0.09 cal cm^{-2} min^{-1} or less, only one-half is reflected by the actual surface. It might be postulated that, with increasing winds, the bubbles and suspended matter would increase, thereby increasing the total reflected energy.

Previous investigations have indicated that a water surface changes from hydrodynamically smooth to rough at some critical wind speed. For example, Munk (1947), using evidence based on observations of whitecaps, soaring characteristics of sea gulls, and variations in resistance and evaporation coefficients, concluded that the critical wind speed was 6 to 7 meters per second (approximately 12.5 knots). It may be speculated that, if other characteristics change at this critical wind speed, the reflectivity of the water surface would also change. Hence, the wind

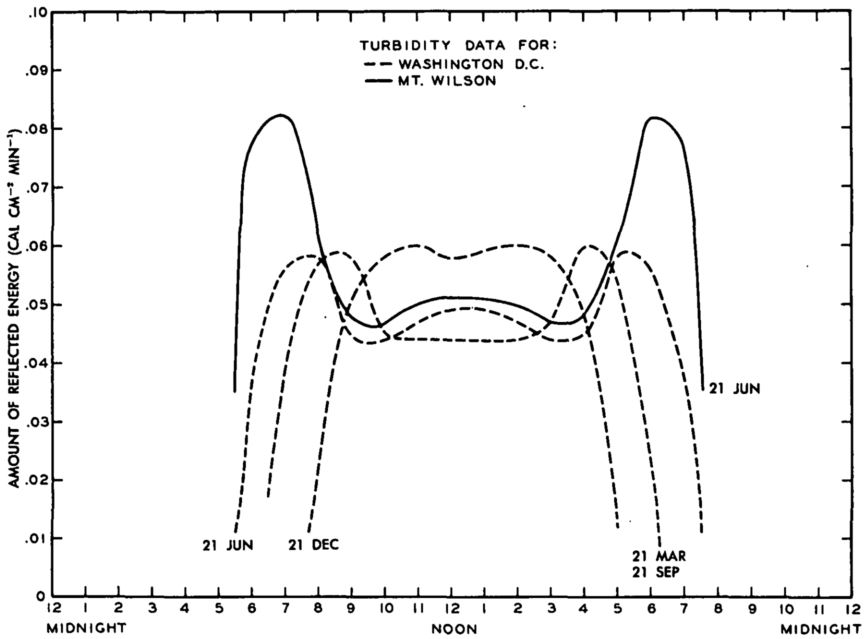


Figure 58. Theoretical variation of reflected energy at the latitude of Lake Hefner (35° 30′ N) as a function of season and time of day using turbidity data from Mt. Wilson and Washington, D. C.

in making the water surface hydrodynamically rough may also change its reflectivity characteristics.

Recapitulating, the above theoretical examination indicates that the reflectivity of an optically flat water surface is primarily a function of the altitude of the sun and secondarily a function of the turbidity of the atmosphere. The turbidity, in a general sense, is in turn a function of the type of air mass present. The introduction of clouds into the air mass increases the turbidity of the air and thus decreases the amount of reflected energy at all sun altitudes. Because radiation may also be reflected by bubbles and particulate material just under the surface, theoretical values for reflectivity should be lower than those actually observed. The effect of wind is to increase the suspended material and bubbles, thereby increasing the reflectivity. An abrupt change in reflectivity at wind speeds of 6 to 7 meters per second would confirm the concept of a critical wind speed at which a surface changes from hydrodynamically smooth to rough.

The actual energy reflected, for a given location, is greatest at sun altitudes less than 30 degrees, with a nearly constant amount reflected at greater sun altitudes. Therefore, on a given day, most energy is reflected just after sunrise and just before sunset. This characteristic is most pronounced during the summer and least during the winter.

Observations of Reflectivity Under a Clear Sky

At Lake Hefner, 1163 observations of reflectivity were made under a clear sky during the year 1 May 1950 to 30 April 1951. The observations were made under a variety of conditions: wind speeds of 0 to 30 knots; various combinations of air masses, and many examples of essentially unadulterated Tropical Gulf warm moist air and Polar Canadian cold dry air, the former being considerably more turbid than the latter; sun altitudes of 0 to 78 degrees. The resulting data offer an opportunity to examine in detail the reflectivity of a natural water surface under a clear sky.

Since the type of air mass present is indicative of the turbidity of the air, the effect of wind speed, when Tropical Gulf or Polar Canadian air only is present, will first be examined. A plot of wind speed versus reflectivity for a given, small sun-altitude range and a given air mass should indicate, by an abrupt change in reflectivity at some wind speed, any effect of a change in water surface from hydrodynamically smooth to hydrodynamically rough. Figure 59 presents the observations taken under these conditions. The number of observations is indicated by N, and S_A represents sun altitude. The data clearly indicate the absence of any wind effect or critical wind speed over the range of wind speeds shown, and the independence of wind speed and reflectivity. The greater scatter of individual observations at low sun altitudes, compared to high sun altitudes, is caused by the greater percentage error in measuring reflected solar and total solar radiation, due to their small magnitude, at the low sun altitudes. At sun altitudes greater than 30 degrees, the scatter is small.

Figure 60, a plot of the median reflectivity for 5-degree sun-altitude intervals, indicates the effect of turbidity, as defined by Polar Canadian and Tropical Gulf air masses, on reflectivity. Theoretically, an increase in turbidity will increase the reflectivity at high sun altitudes and decrease it at low sun altitudes. The Lake Hefner data do not show such a systematic variation. If it be assumed that the number of data are adequate to provide reliable median values of reflectivity, the lack of agreement between observation and theory may arise from the turbidity differential between these two air masses being too small to demonstrate the expected dependence. Hence, it may be concluded that reflectivity under clear skies is independent of air-mass turbidity, except for cases of extreme turbidity such as might occur in dust-laden or smoke-laden air.

Since the reflectivity is apparently independent of wind speed and air-mass turbidity, it can be a function only of the altitude of the sun. The relationship is shown in figure 61. The numerals indicate the number of observations at each altitude. Above sun altitudes of 55 degrees, the reflectivity is independent of the sun altitude and its value is approximately 0.05.

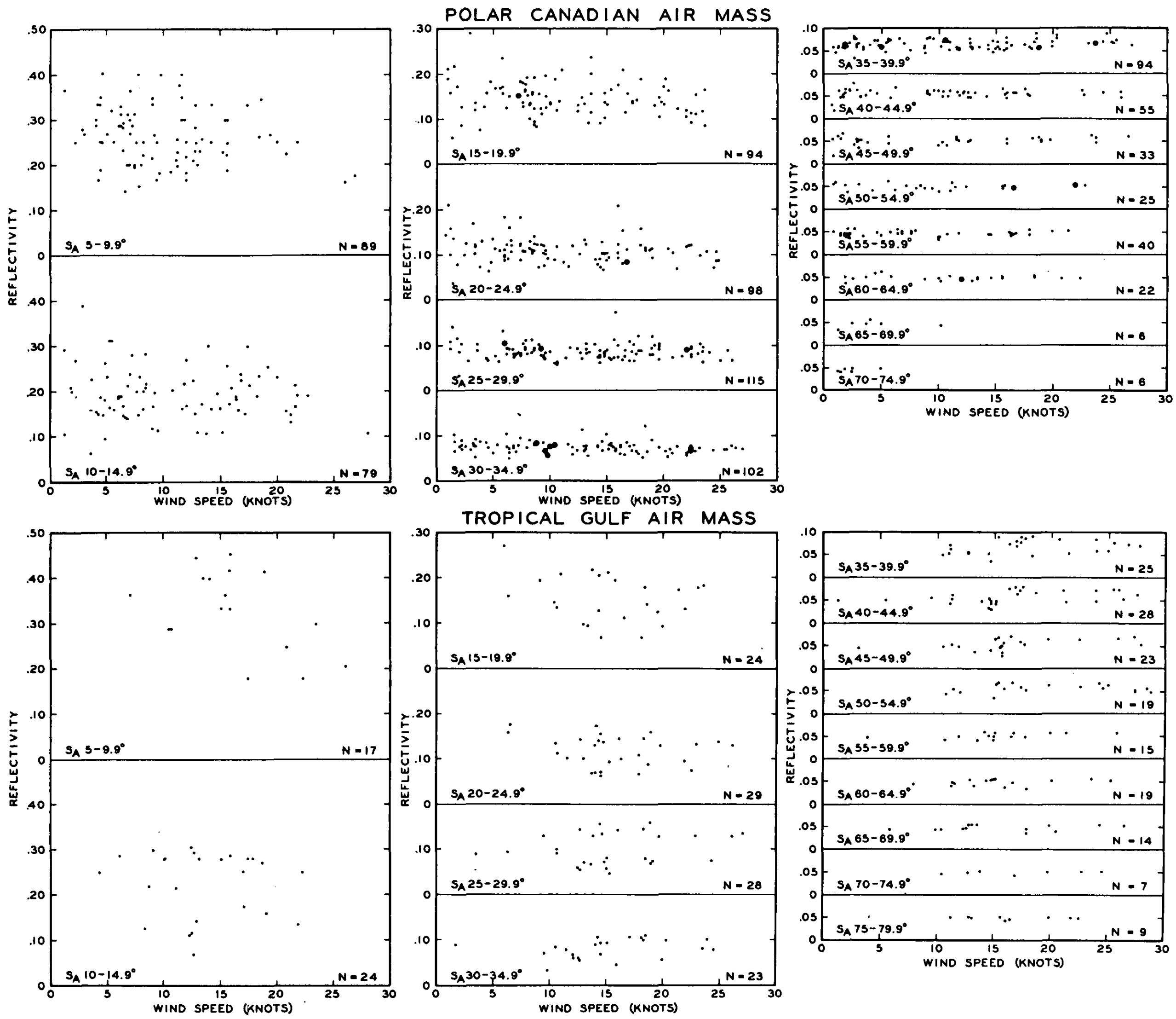


Figure 59. Observations on the reflectivity of a natural water surface under Polar Canadian and Tropical Gulf air masses as a function of the wind speed at the 8-meter level.

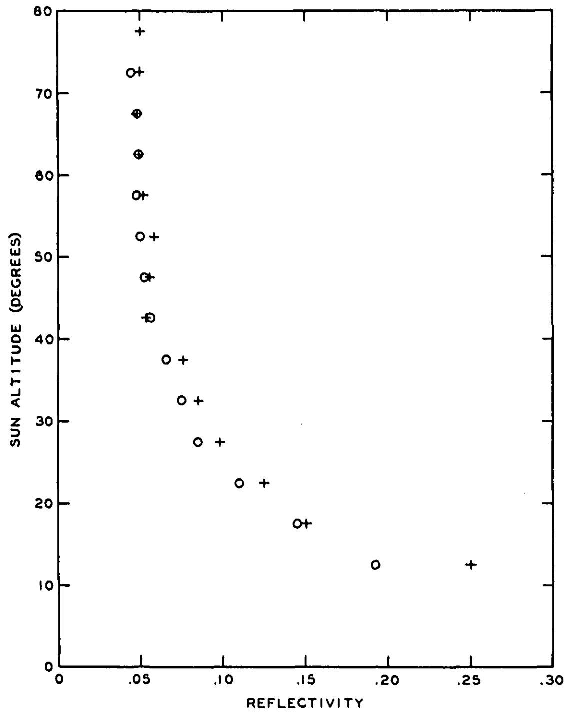


Figure 60. Median reflectivity of a water surface as a function of sun altitude for Polar Canadian (O) and Tropical Gulf (+) air masses under clear skies.

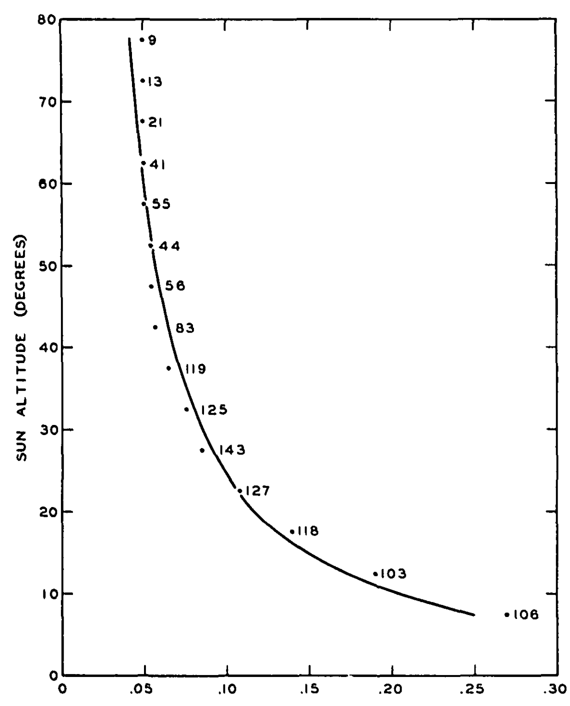


Figure 61. Reflectivity of a natural water surface under clear skies.

Comparison of these observations with the semitheoretical reflectivities of Sverdrup (quoted earlier) and those in figure 56 for Washington shows that for high sun altitudes the Lake Hefner observations give higher reflectivities and for low sun altitudes lower reflectivities. In general they agree within about ±0.02. Sverdrup's results would have been closer to those obtained at Lake Hefner if he had used Schmidt's value of 0.173 for reflectivity of diffuse radiation instead of Powell and Clarke's value of 0.08. The higher observed values of reflectivity obtained at high sun altitudes tend to support Powell and Clarke's contention that all reflection does not take place directly at the water surface but that about "one-half takes place from a stratum of relatively opaque water just beneath the surface." The semitheoretical results of Sverdrup and the Washington data do not include this component, while the Lake Hefner observations do include it.

The expression (equation 83) for the reflectivity of an optically flat water surface under a clear sky is complicated and suggests that the expression for the reflectivity of a natural water surface under a clear sky is also complicated. Nevertheless, the observations presented in figure 61 can be approximated by an equation of the form

$$R_t = aS_A{}^b, \qquad (85)$$

where R_t is the reflectivity, S_A the sun altitude, and a and b constants having values of 1.18 and —0.77 respectively. This equation is shown by a solid line in figure 61.

The reflected energy can be obtained from the reflectivity provided the total incoming solar radiation is known. Figure 58 shows the theoretical variation of the amount of reflected energy, from an optically flat surface, with season and time of day. Days without cloud cover were infrequent at Lake Hefner; hence very few observational data are available for comparison. On the occasional clear days, the reflected energy rose rapidly after sunrise, remained relatively constant throughout the day, and decreased rapidly just before sunset. The secondary features shown in figure 58 were unrecognizable. The maximum amount of reflected energy observed was 0.08 cal cm^{-2} min^{-1}, which agrees very well with the predicted maximum of 0.085 cal cm^{-2} min^{-1} (fig. 57).

Observations of Reflectivity Under a Cloudy Sky

Since for clear skies the reflectivity appears independent of the state of the water surface and atmospheric turbidity for Polar Canadian and Tropical Gulf air masses, it is safe to assume that under cloudy skies the same independence will exist. Enough Lake Hefner data are available to examine the reflectivity of a natural water surface under low clouds (50 to 6500 feet above the terrain) and high clouds (greater than 20,000 feet above the terrain) for three cloud amounts, scattered (1/10 to 5/10 sky coverage), broken (6/10 to 9/10 sky coverage), and overcast (10/10 sky coverage). Middle clouds (6500 to 20,000 feet above the terrain) occur infrequently at Lake Hefner, so not enough data are available to examine reflectivity under these conditions. The number of observations available for reflectivity study under low clouds was 696, distributed as follows: scattered clouds, 271; broken clouds, 183; and overcast clouds, 242. These observations were taken from 1 September 1950 to 31 August 1951.

Figure 62 illustrates the relation between reflectivity and sun altitude for different amounts of low clouds. Increased scattering of solar radiation with increasing amounts of low clouds is clearly demonstrated. As the amount of cloud increases, the effect of sun altitude on reflectivity decreases. The data can be closely approximated by empirical equations of the same form as for clear skies, giving the constants the following values:

constant	*low scattered*	*low broken*	*low overcast*
a	2.17	0.78	0.20
b	—0.96	—0.68	—0.30

These equations, using the proper constants, are plotted as solid lines on figure 62.

Clouds modify the reflectivity of a natural water surface the most under a low overcast. As shown in figure 62c, the reflectivity is almost independent of sun altitude, ranging from 0.05 at 80 degrees to 0.11 at 5 degrees. Although this range is small, it indicates that the solar radiation is not completely diffuse and that some direct solar radiation reaches the water surface. The average reflectivity over this range in sun altitude is about 0.085, which agrees well with Powell and Clarke's average reflectivity under an overcast sky of 0.08, and is slightly lower than the 0.105 obtained by Neiburger under a low stratus cloud cover. At low sun altitudes, when solar radiation is most diffused, the reflectivity approaches the theoretical value of 0.173 for completely diffused solar radiation. This indicates that complete diffusion of solar radiation is only attained in nature under a low overcast cloud cover at very low sun altitudes, and that the theoretical value of 0.173, rather than 0.08 as suggested by Powell and Clarke, is more correct. These results agree with Neiburger's conclusion, obtained from observations under California stratus.

The Lake Hefner observations under a low overcast give values of reflectivity at all sun altitudes that are slightly lower than those obtained by Neiburger under stratus. The difference can probably be accounted for in terms of the height of the cloud cover above the ground. In the case of Lake Hefner the observations were made for a period of a year under low clouds that varied in height from 50 to 6500 feet above the ground, with the median re-

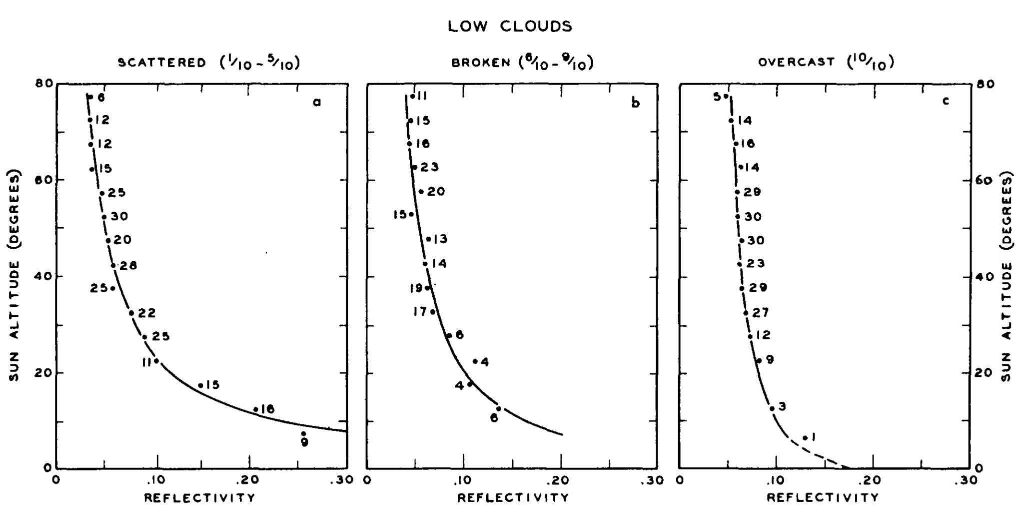


Figure 62. Reflectivity of natural water surface under low clouds.

flectivity values probably representing a median cloud height of about 3500 feet. Neiburger's observations were made under stratus, which has a fairly constant height of about 1500 feet. This indicates that, for a constant sun altitude, the lower the overcast the greater the reflectivity. Thus, for areas where the height of the low overcast averaged around 3500 feet, figure 62c would be useful in obtaining reflectivity but, if the cloud heights were consistently higher or lower, modification would be necessary.

The reflectivity of a water surface under variable amounts of high clouds is shown in figure 63. Again the data can be adequately represented by the empirical formula used for clear skies or low overcast, but the constants have the following values:

constant	*high scattered*	*high broken*	*high overcast*
a	2.20	1.14	0.51
b	—0.98	—0.68	—0.58

A comparison of the reflectivity under high clouds with that under low clouds indicates that for scattered-cloud conditions the reflectivities are similar — in fact, the two essentially coincide. In the case of broken and overcast skies, however, the reflectivity for high sun altitudes is greater under low clouds, and for low sun altitudes is smaller under low clouds. This indicates that the scattering or diffusing of solar radiation becomes greater as cloud height decreases or, what is probably more correct, the lower cloud height is associated with thicker clouds and hence greater diffusion of solar radiation.

A comparison of the reflectivity under a clear sky with that under a low overcast (the two extreme situations) shows that the reflectivity under low overcast clouds is slightly greater at high sun altitudes and less at low sun altitudes, the variation being due to the increased turbidity of the atmosphere because of the presence of water droplets in the air. The modification of the reflectivity, in these two extreme cases, is not great, showing that the effect of turbidity on reflectivity is small. Thus, it may be concluded that the reflectivity of a natural water surface is primarily a function of sun altitude and secondarily a function of atmospheric turbidity including the effect of clouds.

This discussion suggests that the determination of the amount of energy reflected from a water surface from empirical relationships, as represented by figures 61, 62, and 63, is possible. It is necessary to know only the sun altitude, type and amount of clouds, and the total incoming solar radiation. The type and amount of clouds can usually be obtained from Weather Bureau observations and the total incoming solar radiation measured at or near the site in question by a suitable pyrheliometer. This eliminates the necessity of attempting the difficult direct measurement of reflected solar energy. The amount of energy reflected from Lake Hefner on 20 days selected at random during the period 1 May 1950 to 31 August 1950 has been computed from the above relationships and compared in figure 64 with the observed reflected energy. On the average, the computed reflected energy is smaller than the observed by 0.004 cal cm^{-2} min^{-1} (5.8 cal cm^{-2} day^{-1}). Since the accuracy of the observation is 0.005 cal cm^{-2} min^{-1}, the agreement is satisfactory.

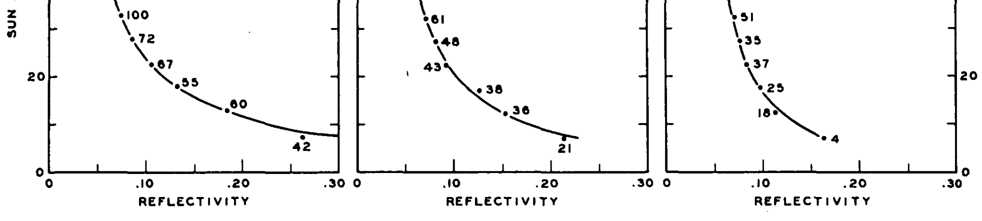


Figure 63. Reflectivity of a natural water surface under high clouds.

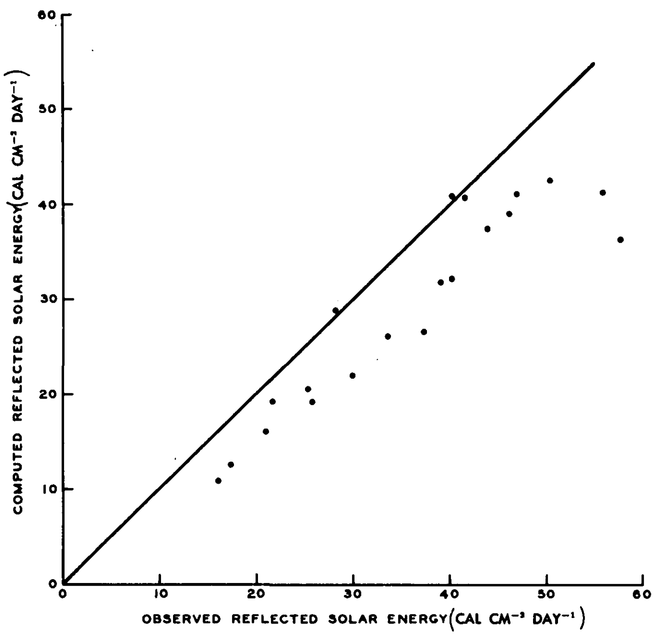


Figure 64. Comparison of reflected solar radiation observed at Lake Hefner with that computed from empirical equations.

Average Monthly Reflected Solar Radiation Observed at Lake Hefner

The average monthly reflected solar radiation and the maximum and minimum daily value for each month as recorded at Lake Hefner during the 16-month observational period are shown in figure 65. The reflected radiation was relatively constant, varying from 21 cal cm^{-2} day^{-1} in February 1951 to 48 cal cm^{-2} day^{-1} in June 1950, with an extreme daily minimum of less than 1 during February 1951 and an extreme daily maximum of 55 in June 1950. The high solar radiation during the summer is apparently offset by the greater reflectivity at the low sun angles during the winter.

Summarizing, the reflectivity of an optically flat water surface under clear skies is theoretically a function of sun altitude and atmospheric turbidity. The Lake Hefner observations indicate that under a clear sky the effect of wind and air-mass turbidity is negligible, and hence that reflectivity is primarily a function of sun altitude. In addition, the observations support previous conclusions reached by Powell and Clarke that the energy is reflected partly from the water surface and partly from a layer of opaque water just beneath the surface. Under cloudy skies the reflectivity is uniformly modified by an increase in scattering, caused by increased cloud cover and decreased height of cloud cover. Solar radiation is completely diffused only under conditions of low sun altitude and low overcast clouds, and then approaches the theoretical value.

The variation of observed reflectivity with sun altitude, for the various types and amounts of clouds, can be approximated by simple, empirical, hyperbolic relationships, which can be used to evaluate indirectly the reflected solar radiation. At Lake Hefner, the reflected radiation ranged from near zero to 55 cal cm^{-2} day^{-1}.

EFFECTIVE BACK RADIATION

The effective back radiation has been defined as the difference between the long-wave radiation leaving a body of water and that from the atmosphere being absorbed by the body of water. The process has often been referred to as the "nocturnal" radiation. The terminology is misleading, however, since the process takes place during the day as well as during the night. In addition, there is evidence that its magnitude is greater during the day.

A. Ångström (1915) presented an excellent review of effective back radiation and of instrumentation for measuring it. The reader is referred to his paper for information on the status of the study prior to 1915.

All the early instruments had disadvantages. Some were directional, that is, measurements had to be made at various zenith angles and then integrated over the hemisphere to obtain the total atmospheric radiation. All were uncompensated for convectional effects, and therefore were limited in use to calm or nearly calm conditions in order to obtain nonerratic readings. Most could be used only for night observations, since the daytime effects of short-wave solar radiation could not be eliminated. None were able to record continuously and automatically; it was necessary to take several individual measurements and then average to obtain a single representative reading. With the development of the Gier and Dunkle flat-plate radiometer in 1948, most of these disadvantages were overcome.

Since 1780, when the first primitive measurements of effective back radiation were made, countless observations have been recorded by investigators in all parts of the world. A great variety of instruments and techniques were utilized. It is not practicable to review all this work, but certain series of measurements are outstanding and deserving of attention. Examples are A. Ångström's (1915) measurements made at Bassour, Algeria, in 1912 and in California in 1913; Kimball's (1918) measurements made at Washington, D. C., and vicinity in 1914 and

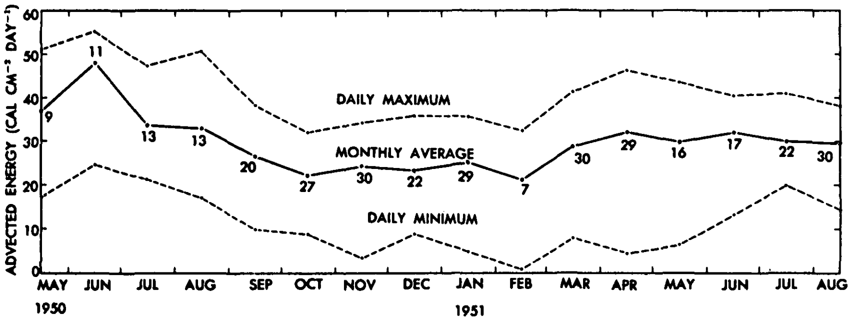


Figure 65. Average monthly reflected solar radiation for Lake Hefner from 1 May 1950 to 1 September 1951.

1915; Dines and Dines (1927) measurements at Benson, England, from 1922 to 1926; Robitzsch's (1926) measurements at Lindenberg in 1926; Asklöf's (1920) measurements in Sweden in 1918; Ramanathan and Desai's (1932) measurements at Poona, India, in 1930; Eckel's (1934) measurements made at Vienna in 1930 and 1931; Buttner's (1939) measurements made in Africa in 1938; Wexler's (1941) measurements made at Fairbanks, Alaska, and Fargo, North Dakota, from 1936 to 1938. These observations were made with various types of instruments during various months of the year.

From these observations several general conclusions regarding atmospheric radiation have been formulated, namely, variations of atmospheric radiation at low altitudes are due primarily to variations in atmospheric temperature and humidity; the atmospheric radiation increases as the moisture content of the air in the lower levels increases; the radiation during the daytime seems subject to the same laws that hold for the radiation during the night; the effect of clouds is variable, with low dense clouds considerably decreasing the effective back radiation and a high thin overcast having only a slight effect; there is always a positive radiation from earth to sky.

In most cases, effective back radiation has not been measured directly. Usually, the atmospheric long-wave radiation has been measured, and radiation from the body of water has been computed as black-body radiation utilizing Stefan-Boltzmann's law corrected by some emissivity coefficient, since water radiates as a gray body rather than a black body. Therefore, observations have usually been discussed in terms of atmospheric radiation, rather than effective back radiation.

The atmospheric radiation does not follow any simple law since it is a function of many variables, such as the distributions of moisture, temperature, ozone, carbon dioxide, and perhaps other atmospheric factors as yet unknown. It was early recognized that the total amount of moisture in the air was one of the primary variables. Since upper-air soundings are not always available, early observers attempted to correlate the incoming radiation with elements easily observed at the ground, such as air temperature and humidity near the place of observation. These efforts have led to the establishment of several empirical formulas for evaluating atmospheric radiation. These formulas agree only statistically with actual observations, since the scatter of any set of observations about the line of best fit is very great. The following four empirical formulas for computing radiation from a clear sky have been proposed, with the first two receiving the greatest attention:

$$\frac{Q_a}{\sigma T_a^4} = a - b\,\epsilon^{-\gamma e_a} \qquad \text{Ångström} \qquad (86a)$$

$$= a + b\sqrt{e_a} \qquad \text{Brunt} \qquad (86b)$$

$$= (a\,P - b\,e_a)/T_a \qquad \text{Robitzsch} \qquad (86c)$$

$$= a + b \log e_a, \qquad \text{Elsasser} \qquad (86d)$$

where Q_a is the atmospheric radiation, σ the Stefan-Boltzmann constant, T_a the absolute temperature of the air near the ground, ϵ the Naperian base, e_a the vapor pressure of the air near the ground, P the air pressure, and a, b, and γ empirical constants.

Over the usual atmospheric range of vapor pressures, 2 to 30 mb, and using the presently accepted values of the constants, the differences between the above formulas are small. For example the difference between Brunt's and Ångström's formulas, and between Brunt's and Elsasser's formulas, is only 10 per cent. This is not a significant difference, since the scatter of the observations is considerably greater. These formulas all attempt to relate atmospheric radiation to the local vapor pressure only, whereas it should be related to the total vapor content of the atmosphere, the carbon dioxide content, ozone content, and perhaps to other unknown factors. Therefore, these formulas can only be considered as empirical relationships which will only give average evaluations of atmospheric radiation from a clear sky over relatively long periods. This is brought out by the variations in the constants, as obtained from various series of measurements. For example, twelve different sets of observations fitted to Brunt's formula gave a value of a from 0.34 to 0.62 and of b from 0.029 to 0.082; and five different sets of observations fitted to Ångström's formula gave a from 0.71 to 0.81, b from 0.24 to 0.33, and γ from 0.04 to 0.074.

The above discussion is concerned only with the radiation from a clear sky. If clouds are present, the atmospheric radiation is increased greatly. Systematic observations under an overcast sky were made by Asklöf (1920) and Ångström (1929). They presented their results in the following form:

$$Q_{ac} = \sigma T_a^4 (1 - \lambda) + \lambda Q_a, \qquad (87)$$

where Q_{ac} is the atmospheric radiation from an overcast sky, T_a is the temperature of the air in °K, λ is an empirical coefficient which is a function of cloud height, and Q_a is the atmospheric radiation from a clear sky computed from equation (86a). The constant λ has been evaluated only for the Asklöf and Ångström observations.

Atmospheric radiation has been treated theoretically by Mügge and Möller (1932), Phillips (1940), Elsasser (1940), and more recently by Yamamoto (1950) who pointed out that theoretical investigations of atmospheric radiation are not in too good agreement with observation. This disagreement is probably due to inability of investigators to consider theoretically all the variables contributing to the atmospheric radiation because of the complex relationships of the variables.

The above discussion may be summarized as follows: At the present time it appears possible to evaluate from empirical relationships the atmospheric radiation received from a clear sky over a long time interval. For shorter time intervals, and for times when clouds are present, it is necessary to measure it directly.

The long-wave radiation from the body of water is evaluated from the Stefan-Boltzmann law for black-body radiation corrected by a suitable emissivity factor. A search of the literature revealed a rather meager knowledge of the emissivity of water: Dorsey (1940) gave a value of 0.985, Ångström (1920) 0.94, and Richardson (1931) 0.906. Variations of this magnitude in the emissivity factor are important in computing evaporation from the energy budget. For example, a change of emissivity from 0.985 to 0.94 will increase the evaporation by 30 per cent at an evaporation rate of 0.2 cm per day and by 20 per cent at an evaporation rate of 0.4 cm per day. It is evident that, for purposes of computing evaporation, a more precise evaluation of the emissivity of water is essential.

The remainder of this section is devoted to a discussion of the observations, pertinent to the long-wave radiation exchange, made at Lake Hefner. The characteristics of atmospheric radiation are examined with respect to the variation of the radiation as a function of local vapor pressure and the limitations placed upon the use of such relationships. In addition, the emissivity of a natural water surface is discussed.

Atmospheric Radiation

At Lake Hefner, the atmospheric radiation was measured directly during the night by means of a Gier and Dunkle flat-plate radiometer. During the day it was evaluated indirectly, using the radiometer for measuring both the solar and atmospheric energy irradiating the surface of the earth and an Eppley pyrheliometer for measuring only the solar radiation; the daily atmospheric radiation is the difference between the two measurements.

The diurnal variation of atmospheric radiation as obtained from this system of instruments was of an unexpected nature. On every one of 340 days of available measurements, the atmospheric radiation began to decrease at sunrise, reached a minimum at solar noon, increased until sunset, and then remained relatively constant until sunrise. The magnitude of the observed decrease is significant; for example, the nighttime value of atmospheric radiation might be 0.500 cal cm^{-2} min^{-1}, decreasing to 0.250 cal cm^{-2} min^{-1} at solar noon. The magnitude of the decrease was greatest under clear skies and least under a low overcast.

Intuitively one would expect, not a diurnal variation of this nature, but a relative constancy in the atmospheric radiation throughout the day, with perhaps a slight increase during the afternoon as the lower atmosphere warms up. Water vapor in the atmosphere is a fairly effective absorber of long-wave radiation, except for a transparent band from 8.5 μ to 11.0 μ. If the observed decrease in long-wave radiation is a true one, it would imply that some other component in the atmosphere radiates very strongly at the same wavelengths as the transparent band in the water-vapor spectrum and, furthermore, has a large diurnal fluctuation. Since this is improbable, the observed daytime values of atmospheric radiation appear incorrect, and the observed diurnal variation is therefore probably due to instrumental errors. (See "Instrumentation" section for description of instruments.)

Some possible instrumental errors are: (1) a high calibration constant for the Eppley pyrheliometer, (2) a low temperature reading on the flat plate, (3) a low calibration constant for the flat-plate radiometer, and (4) an imperfect flat-plate black body, with the reflectivity of the flat plate for the solar radiation dependent on the sun's altitude.

The U. S. Weather Bureau made solar radiation measurements at a site approximately 13 miles south of Lake Hefner. The Lake Hefner observations are approximately 2.5 per cent higher than those of the Weather Bureau. At the present time the same pyrheliometer is in service at Lake Mead. Comparison of the measurements being made at Lake Mead with those made by the Weather Bureau at Las Vegas, Nevada, 30 miles distant, indicates that the Lake Mead readings are about 1.5 per cent lower than those at Las Vegas. It is therefore concluded that these differences are within the limits of accuracy of the technique.

A low value for the temperature of the flat plate could be obtained if the reference thermocouple were not maintained at 0°C. Since ice in the reference ice jug was renewed at frequent intervals, any error from this source would be present only for short periods prior to the refilling of the jug, and could not account for the consistent observed variation in atmospheric radiation.

In the absence of a recalibration of the flat-plate radiometer, no direct evidence is available to indicate that the calibration was incorrect. However, nighttime recorded values agree satisfactorily with previous measurements made with other types of instruments.

The last possible instrumental error is concerned with the reflectivity of the flat plate for solar radiation. Dunkle *et al.* (1949) demonstrated in the laboratory that the flat plate is essentially independent of the altitude angle of an artificial light source. However, this may not be true when the flat plate is irradiated by solar energy, some of which is arriving as direct radiation and some as scattered. It has not been possible to investigate in detail this possible source of instrumental error.

It is tentatively concluded that the dependence of the flat-plate reflectivity on sun altitude is the primary cause of the observed daily decrease in atmospheric radiation, and that it is necessary to develop better instrumentation.

For this study, and in the absence of independent verification, it is assumed that the atmospheric radiation is relatively constant throughout the day and night, that the night values obtained from the Gier and Dunkle flat plate are correct, and that the day values are incorrect. An average daily value has therefore been obtained from the night measurements and this value assumed to apply throughout the 24-hour period.

Figure 66 indicates the magnitude of the atmospheric radiation as observed at Lake Hefner. The average monthly radiation (number indicates the number of days of data) and the highest and lowest daily value recorded during the month are shown. The radiation averages about 800 cal cm^{-2} day^{-1} during the summer and about 550 cal cm^{-2} day^{-1} during the winter. These values are approximately 200 higher than for comparable monthly average solar radiation. The greatest daily radiation occurred in August 1950, and amounted to 900; the minimum occurred in February 1951, and amounted to 375. These compare with maximum and minimum solar radiations of 750 and 40, respectively.

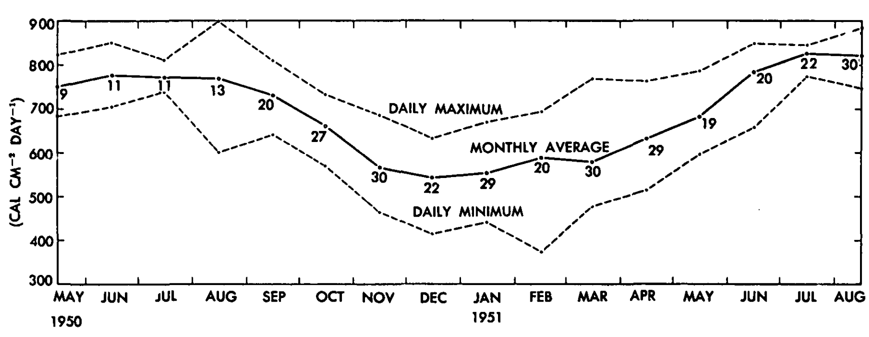


Figure 66. Average monthly atmospheric radiation for Lake Hefner from 1 May 1950 to 1 September 1951.

RADIATION FROM A CLEAR SKY

At Lake Hefner, for the period 1 September 1950 to 1 September 1951, 1893 observations of the nighttime atmospheric radiation were available for study. As previously indicated, the atmospheric radiation is primarily a function of the total amount of water vapor in the atmosphere. However, because of a lack of information concerning the total water-vapor content, most investigators have adopted the local vapor pressure as an index of the total water-vapor content. At the Will Rogers Airport, 13 miles south of Lake Hefner, two radiosondes per day were taken during the course of the Lake Hefner observations. It was originally hoped that these data might be used in connection with the study of atmospheric radiation at Lake Hefner. It was found, however, that two radiosondes per day were not enough to define the variation of the total moisture content with the accuracy needed when considering half-hourly radiation observations, and that accurate evaluation of total water vapor in the atmosphere could not be obtained, possibly due to the lack of accuracy or sensitivity of the humidity-sensing element and to the fact that the soundings did not consistently sample the entire atmospheric column. The use of total water vapor, as obtained from the two daily radiosondes, in place of the local vapor pressure, did not lead to better results. Hence, throughout this study, the vapor pressure at 2 meters has been used as the moisture variable. Atmospheric radiation is expressed as the ratio of the observed atmospheric radiation to the black-body radiation at the temperature of the air at 2 meters. Actually, the black-body radiation at the mean water-vapor temperature should be used, but the determination of this temperature is not feasible. As noted later, the use of the local vapor pressure and the local air temperature, under special circumstances, leads to anomalous results.

The 1893 observations were made over a vapor-pressure range of 2.9 to 30.3 mb. The data were edited to exclude observations made during periods of moderate and strong air-mass over-running. This editing made the local temperature and vapor-pressure observations more representative of the air column above the instrument. The ratio of atmospheric radiation to the black-body radiation of the atmosphere, defined by the 2-meter air temperature, was plotted against the 2-meter vapor pressure. To simplify plotting, these ratios were grouped by 0.5-mb intervals, and median values were obtained for each group. Median values were used rather than arithmetic means because the two did not differ markedly and the former were easier to compute. Figures 67 and 68 show the plots of these median ratios versus vapor pressure. The number indicates the number of observations, while the dashed lines include two-thirds of the individual observations within each 0.5-mb range in vapor pressure. In general there is a definite increase in the radiation ratio with increasing vapor pressure. However, at low vapor pressure (5 to 8 mb) the ratio is higher than expected, and also higher than most prior observations. These results are explainable in terms of the local meteorology. Observations at these vapor pressures were generally made during the winter, when a thin wedge of relatively dry Polar Canadian air was present at the ground with over-running of moist Tropical Gulf air aloft. Under these circumstances the total water vapor in the atmospheric column above the station was considerably higher than indicated by the local vapor pressure, thereby giving a higher atmospheric radiation than expected. In addition, the 2-meter air temperature was relatively lower than the mean water-vapor temperature.

Four empirical equations, plotted on figures 67 and 68, of the following form were fitted to these data:

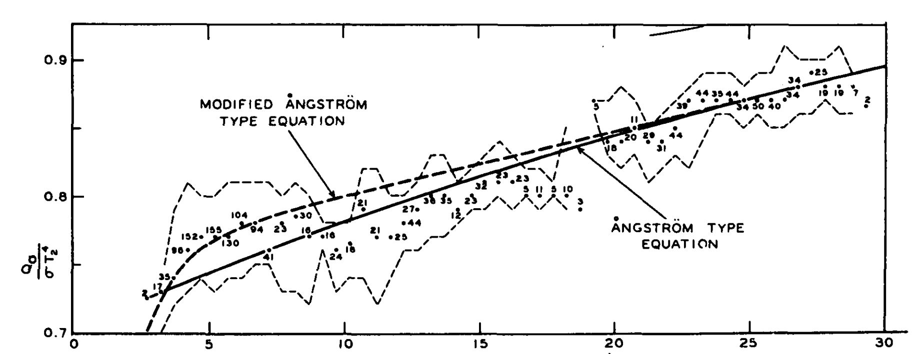


Figure 67. Plot of atmospheric radiation equations of the Ångström and modified Ångström type for the Lake Hefner clear sky observations.

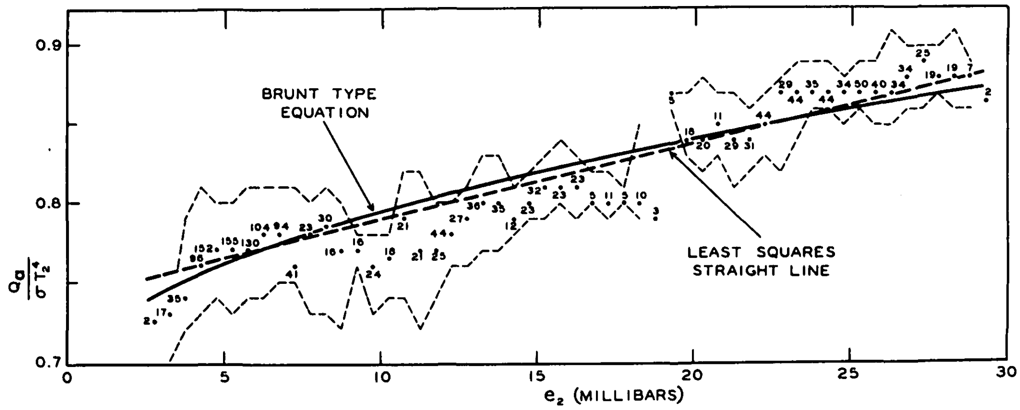


Figure 68. Plot of atmospheric radiation equations of the Brunt type and the least-square straight line for the Lake Hefner clear sky observations.

$$\frac{Q_a}{\sigma T_2^4} = a - b\epsilon^{-\gamma e_2}, \quad \text{Ångström}$$
$$= c + d\sqrt{e_2}, \quad \text{Brunt} \qquad (88)$$
$$= (f + ge_2) - h\epsilon^{\alpha e_2}, \quad \text{Modified Ångström}$$
$$\text{and} = i + je_2, \quad \text{Straight line}$$

where Q_a is the atmospheric radiation, σ the Stefan-Boltzmann constant, T_2 2-meter air temperature, e_2 the 2-meter vapor pressure, ϵ the Naperian base, and *a, b, c, d, f, g, h, i, j,* γ, *and* α are constants to be determined from the data. The modified Ångström type equation gives a better fit to the data at low vapor pressures. With the exception of one point at 18.75 mb, determined by only three readings, all four curves are within 5 per cent of the median points. In addition, the curves stay fairly well within the two-thirds band of actual radiation ratios.

The values of the constants are:

Constant	*Value*
a	1.107
b	0.405
γ	0.022
c	0.682
d	0.036
f	0.752
g	0.0048
h	0.542
α	0.781
i	0.740
j	0.0049

Various other determinations of the constants in Brunt's and Ångström's equations have been summarized by Elsasser (1942) and are listed for comparison in tables 9 and 10. The correlation coefficient indicates the correlation between the local vapor pressure and the observed radiation ratio, $Q_a/\sigma T_2^4$. In general, the radiation ratios obtained from the Lake Hefner data are higher than those obtained by others.

TABLE 9. Various Values of Constants for Brunt's Atmospheric-Radiation Equation.

Investigator	Place	c	d	Correlation Coefficient
Dines	England	0.53	0.065	0.97
Asklöf	Sweden	0.43	0.082	0.83
Ångström	Algeria	0.48	0.058	0.73
Ångström	California	0.50	0.032	0.30
Boutaric	France	0.60	0.042	—
Kimball	Washington, D. C.	0.44	0.061	0.29
Eckel	Austria	0.47	0.063	0.89
Raman	India	0.62	0.029	0.68
Anderson	Oklahoma	0.68	0.036	0.92

TABLE 10. Various Values of Constants for Ångström's Atmospheric-Radiation Equation.

Investigator	Place	a	b	γ
Ångström	Sweden	0.806	0.236	0.115
Kimball	Virginia	0.80	0.326	0.154
Eckel	Austria	0.71	0.24	0.163
Raman	India	0.79	0.273	0.112
Anderson	Oklahoma	1.107	0.405	0.022

There are also considerable variations between determinations. These variations are probably related to the implied hypothesis that previous investigators have made concerning the relation between the local vapor pressure and the total water-vapor content of the atmosphere. This relation is probably different for different locations, since the composition of the atmosphere differs from one place to another. If it were possible to express the moisture parameter in terms of total water-vapor content rather than local vapor pressure, the variation would undoubtedly decrease. Hence, it is concluded that the use of empirical relations are restricted to locations that have similar air-mass structures. To attempt to use them in other locations will lead to erroneous results and conclusions.

If the above relations are extrapolated to zero vapor pressure (index for an absolutely dry atmosphere), the radiation ratio should express the component of atmospheric radiation related to the other constituents in the atmosphere that contribute to atmospheric radiation, namely, carbon dioxide and ozone, provided there is no intercorrelation between water vapor and the other constituents. Yamamoto (1950) indicates that at a temperature of 280°K (7°C), the ratio for carbon dioxide is 0.146 and for ozone is 0.0086. The zero intercepts for the curves in figures 67 and 68 are considerably higher than these values. The radiating power of the atmosphere is high even for exceedingly small moisture contents, owing to the very high absorptivity of water vapor for some of the long waves. Hence, at very low vapor pressures, the ratio should decrease rapidly. Therefore, these empirical relations are only applicable over the range of vapor pressures considered, in this case from 3 to 30 mb.

The four empirical curves were checked against four sets of nighttime data not used in obtaining the curves themselves, and six sets of nighttime data used in obtaining the curves. The latter six sets were chosen arbitrarily, except that they were spread over the entire vapor-pressure range. The results are given in table 11. The empirical values for the four sets of data are from 16 per cent higher to 3 per cent lower,

TABLE 11.
Comparison of Computed and Observed Atmospheric-Radiation Ratios

Date	Observed	Ångström	Modified Ångström	Brunt	Straight Line	Maximum Difference (per cent)	Minimum Difference (per cent)
Days Not Included in Evaluating Constants							
13 Jun 1950	0.817	0.864	0.864	0.856	0.856	—5.8	—4.8
15 Jun	0.778	0.865	0.865	0.856	0.857	—11.2	—10.0
10 Aug	0.868	0.859	0.861	0.846	0.846	2.5	0.8
31 Aug	0.721	0.832	0.837	0.833	0.829	—16.1	—15.0
Days Included in Evaluating Constants							
10 Oct	0.802	0.804	0.817	0.812	0.806	—1.9	—0.2
4 Nov	0.699	0.748	0.771	0.765	0.767	—10.3	—7.0
19 Mar 1951	0.749	0.742	0.756	0.759	0.764	—2.0	0.9
2 May	0.728	0.789	0.806	0.801	0.795	—10.7	—8.4
7 July	0.854	0.862	0.863	0.854	0.855	—1.1	0.0
28 Aug	0.845	0.849	0.852	0.844	0.843	0.1	—0.8

and for the six sets of data from 10 per cent higher to 1 per cent lower than observed. Hence it might be hazarded that 10-per-cent accuracy might be expected from the empirical curves. Since there is little difference between the four empirical relations it seems reasonable to consider the straight line as the empirical expression for atmospheric radiation. Atmospheric radiation averages about 650 cal cm^{-2} day^{-1}; hence, a 10-per-cent error is too large for evaporation computations.

RADIATION FROM A CLOUDY SKY

Prior investigations of atmospheric radiation were concerned chiefly with the radiation occurring with a clear sky. The only investigations dealing with the effects of cloud cover are those of Asklöf (1920) and Ångström (1929). They treated the atmospheric radiation occurring under an overcast sky. The Lake Hefner observations include adequate data for considering the atmospheric radiation occurring when low, middle, and high clouds in variable amounts — scattered, broken, or overcast — are present.

These data have been examined in a manner similar to that used in treating the data concerning radiation from a clear sky. A linear relationship has been used since it was shown, for the case of clear skies, that such a relationship is as adequate as more complicated ones in empirically expressing the results over the vapor-pressure range generally encountered in nature.

Figure 69 summarizes the results, and gives atmospheric radiation as a function of cloud height, cloud amount, and local vapor pressure. *N* indicates the number of observations. In each case, the data have been treated in the same manner as that employed for clear skies. The heavy dashed lines indicate the least-squares line for each set of data, while the light dashed lines indicate the relationship for clear-sky condition. An inspection of this figure reveals a fairly consistent relation among the three variables. The radiation is least, and the effect of vapor pressure is greatest, for clear skies over the entire vapor-pressure range; the effect of vapor pressure decreases as the cloud amount increases. For any given amount of cloud cover, the radiation increases over the entire vapor-pressure range as the height of the clouds decreases. The modification of the radiation, from the clear-sky case, is least for high scattered clouds and greatest for low overcast. For low overcast, the radiation is independent of the vapor pressure. Since the average height of the low overcast is probably around 2500 to 3000 feet above

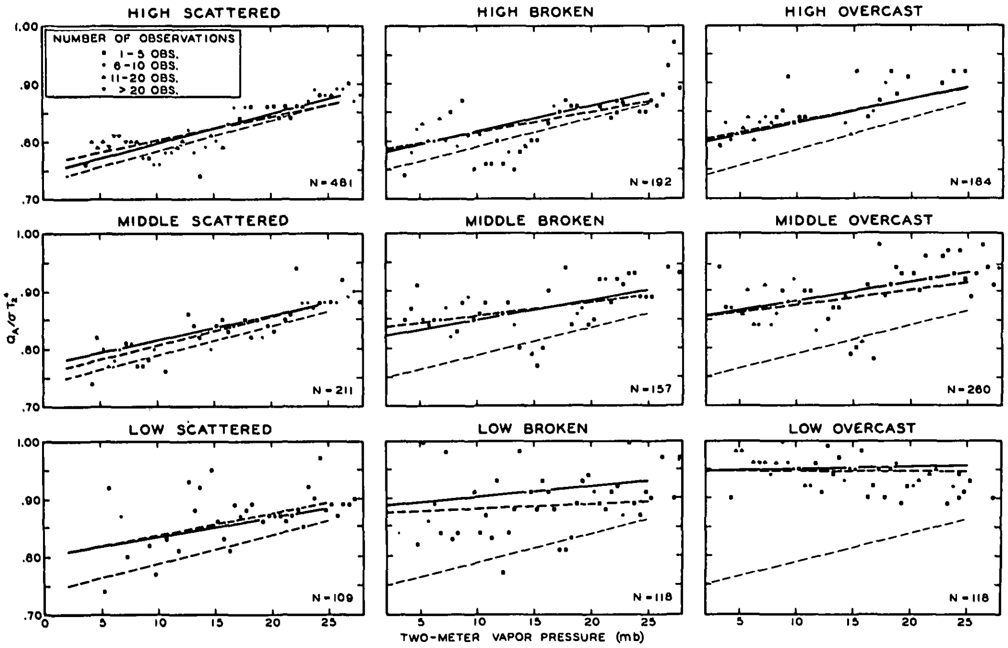


Figure 69. Atmospheric radiation as a function of cloud height, cloud amount, and vapor pressure.

the terrain, this indicates that for low overcast clouds below this altitude the radiation is independent of vapor pressure and height of clouds.

The data shown in figure 69 can be represented empirically by an equation of the following form:

$$Q_a/\sigma T_2^4 = a + be_a, \qquad (89)$$

where

$$a = 0.740 + 0.025\, C\epsilon^{-0.0584h},$$
$$b = 0.00490 - 0.00054\, C\epsilon^{-0.060h},$$

and $1600 \leqq h \leqq \infty$.

Radiation for cloud heights under 1600 feet must be considered equivalent to that occurring when the cloud height is 1600 feet, but actually the radiation would probably continue to increase as cloud height decreases. In the above equation e_a is the vapor pressure in mb, C the cloud amount in tenths of sky covered, ϵ is the Naperian base, and h the cloud height in thousands of feet. Equation (89), using appropriate cloud heights and amounts, is shown in figure 69 as a solid line. The agreement with the least-squares line is good.

Table 12 summarizes some comparisons of observed atmospheric radiation and that obtained from equation (89). These data indicate that accuracies around 5 to 10 per cent are possible using the empirical equation. Although the percentage errors are small, the actual error varies from 15 to 75 cal cm^{-2} day^{-1}, which is too large for purposes of computing evaporation.

Recapitulating, this analysis indicates that: (1) Empirical relationships between atmospheric radiation

TABLE 12. Comparison of Computed and Observed Atmospheric Radiation for Selected Days

Date	Atmospheric Radiation (cal cm^{-2} day^{-1}) Observed	Computed	Difference	Difference (per cent)	T_2 (°C)	e_2 (mb)	C (tenths)	h (1000's of feet*)
14 Jul 1950	699.8	743.0	43.2	5.8	22.8	18.4	3	37
6 Aug	905.8	829.4	—76.4	—8.4	24.5	26.4	7	14
14 Sep	803.5	822.2	17.7	2.2	18.5	19.5	10	1
24 Sep	733.0	710.1	—22.9	—3.2	20.6	14.1	3	32
23 Jan 1951	560.2	544.3	—15.9	—2.8	4.1	5.2	7	31
8 Apr	535.7	565.9	30.2	5.6	8.9	4.9	1	60

* For days with partly cloudy and clear skies, height is arbitrarily set at 100,000 feet for purposes of determining an average height for the day.

and local vapor pressure may be used if 10-per-cent accuracy is acceptable, provided the air-mass structure is similar to that of the area where the original observations were recorded. For other areas, with no consideration of the air masses involved, the accuracy of the relationships is more questionable. (2) A linear relationship between atmospheric radiation and local vapor pressure (over the vapor-pressure range generally observed in nature) is as adequate as the more complicated expressions of Brunt and Ångström. (3) These empirical determinations are inadequate for evaluating atmospheric radiation for purposes of evaporation determinations. (4) To obtain more accurate methods of determining atmospheric radiation, in terms of more easily evaluated parameters, it will be necessary to consider the total vapor content of the atmosphere as the moisture variable, rather than the local vapor pressure.

Radiation from the Body of Water

Long-wave radiation from a body of water can be computed from the Stefan-Boltzmann law for black-body radiation, corrected by a suitable emissivity factor for water, since water radiates as a gray body rather than a black body. It is necessary, for purposes of evaporation determination, to know the emissivity of the water surface to two significant figures. Since doubt existed as to the value of the emissivity of water, the Navy Electronics Laboratory contracted with the Physical Standards Laboratory, Institute of Engineering Research, University of California, for determinations of the emissivity of four water samples — distilled water, Lake Hefner water, Lake Mead water, and sea water. The salinities of the Lake Hefner, Lake Mead, and sea water were 347 ppm, 572 ppm, and 33.42 ‰ (parts per thousand), respectively. To determine the extremes of any possible variation of emissivity, the emissivity was determined over a range of water temperatures from 0 to 30°C and, in addition, for a sample of water contaminated with a thin oil film.*

The emissivity of a body of water can be evaluated by direct measurement or indirectly from a measurement of the reflectivity of the water surface.

The direct method consists of heating the sample, and taking measurements of the surface temperature with a thermocouple and of the radiation from the surface with a directional radiometer. There are several objections to this method: (1) It requires a fairly large temperature difference between the sample and the surroundings. This involves cooling the surroundings, since the emissivity was desired over a water-temperature range of 0 to 30°C. (2) Since water is opaque to long-wave radiation, and since most of the emitted energy comes from a layer very close to the water surface, it is extremely difficult to obtain precise surface-temperature measurements. (3) An appreciable error in the direct emissivity measurement may arise because of the absorption of radiation by water vapor in the air.

The indirect method consists of measuring the reflectivity of the water surface, using calibrated energy sources of different temperatures. According to Kirchoff's law, when the source temperature is equal to the water-surface temperature, the emissivity is precisely equal to the absorptivity. In addition, the absorptivity is equal to one minus the reflectivity. Thus when the water-surface temperature is equal to the source temperature, the emissivity is equal to one minus the reflectivity. Therefore, determination of reflectivity as a function of source temperature and extrapolation of the resulting curve to the temperature of the sample enable an indirect determination of the emissivity of the water sample. The reflectivity is measured by a directional radiometer which is first calibrated against a calibrated energy source over the path lengths to be used in the experiment. The reflected energy is obtained by taking the difference between a measurement of the ambient energy (energy source shielded from the water sample) and a measurement made when the energy source is unshielded. Since the energy source is calibrated, the reflection is the ratio of the reflected energy to the total energy output of the calibrated energy source. The indirect method has several advantages: (1) The sample can be maintained at any convenient temperature, the only restriction being that the temperature of the environment remain constant. (2) Since the reflectivity of water is low, a large percentage error in the measurement of reflectivity results in a small error in the emissivity. (3) The problem of water-vapor absorption is avoided by calibrating the radiometer against the source used and over the same path lengths used in making the reflectivity measurement. The major disadvantage in this indirect method is drift in the measurements caused by changes in water-surface temperature. This source of error can be minimized by making the two required readings as rapidly as possible.

The Physical Standards Laboratory, therefore, used the indirect method of evaluating the emissivity. Two different energy sources were utilized. The high-

* This work was done under the supervision of R. V. Dunkle and J. T. Gier.

TABLE 13. Reflectivity of Various Water Samples as a Function of Source and Water Temperatures.

Distilled Water			Sea Water (33.42 $^0/_{00}$)		
Source Temp. (°F)	Water Temp. (°F)	Reflectivity	Source Temp. (°F)	Water Temp. (°F)	Reflectivity
4320	48	0.023	4320	51	0.022
	72	0.022		68	0.022
	86	0.022		86	0.022
4140	48	0.023	4140	51	0.023
	72.5	0.022		68	0.023
	87	0.022		86	0.023
3920	48	0.022	3920	51	0.023
	73	0.022		68	0.023
	86	0.023		86	0.023
315	43	0.030	303	46	0.032
313	71	0.031	316	71	0.030
316	86	0.030	320	86	0.031
			408	72	0.029
			420	86	0.030
			549	75	0.024
			557	75	0.026
			204	85	0.030
			203	62	0.030

Lake Mead Water (572 ppm)			Lake Hefner Water (347 ppm)		
Source Temp. (°F)	Water Temp. (°F)	Reflectivity	Source Temp. (°F)	Water Temp. (°F)	Reflectivity
4320	48	0.023	4320	44	0.022
	73	0.022		68	0.023
	86	0.021		86	0.022
4140	49	0.025	4140	44	0.023
	73	0.022		68	0.023
	86	0.022		86	0.023
3920	50	0.023	3920	86	0.023
	73	0.022		68	0.023
	86	0.022		86	0.022
311	46	0.029	307	47	0.032
307	73	0.033	315	72	0.033
312	86	0.030	317	86	0.031
315	90	0.033	408	76	0.030
405	69	0.026	411	86	0.029
549	72	0.027	551	73	0.028
551	86	0.024	556	86	0.026
405	67	0.026	212	69	0.032
212	65	0.031	207	85	0.030
221	85	0.030			
405	67	0.031			
406	85	0.031			

Water Analysis (ppm) Date 2 Oct 1951		Water Analysis (ppm) Date 27 Sep 1951	
Silica	11	Silica	2.4
Iron	.03	Iron	.00
Calcium	71	Calcium	46
Magnesium	27	Magnesium	17
Sodium	81	Sodium	53
Potassium	3.0	Potassium	5.8
Bicarbonate	137	Bicarbonate	170
Sulfate	244	Sulfate	75
Chloride	66	Chloride	60
Fluoride	.1	Fluoride	.3
Nitrate	1.0	Nitrate	.4

temperature source, a 1000-watt lamp, calibrated for total directional radiation and temperature as a function of source voltage, was used to produce source temperatures around 4000°F. The low-temperature source, a metallic surface coated with lamp black and heated electrically, produced source temperatures around 300°F.

Table 13 summarizes the individual reflectivity measurements. An inspection of the data in table 13 indicates that the effect of salinity and water-surface temperature is negligible. Since the data did not justify any other curve, a straight line was fitted to the plot of reflectivity versus source temperature. The equation for this line is

$$r = 0.0301 - 3.343 \times 10^{-6} T_s, \qquad (90)$$

where r is the reflectivity of the water and T_s the source temperature in °C.

The greatest deviations from the straight line of best fit for the high-temperature source were +0.002 and —0.001, with a mean deviation of 0.001. For the low-temperature source, the greatest deviations were —0.005 and +0.003, with a mean deviation of 0.002. Extrapolation of this line to temperatures (above freezing) normally encountered in natural bodies of water gives a reflectivity of 0.030. It is concluded that the emissivity of water is 0.970, with an estimated accuracy of ±0.005, and that it is independent of water temperature and dissolved solids.

A few data were collected on the reflectivity of tap water, muddy tap water, and tap water contaminated by an oil film. These data are summarized in table 14. As expected, the tap water is similar in reflectivity to the previously discussed samples. Sediment suspended in the tap water has no appreciable effect on the reflectivity. The contamination of the water surface by an oil film, however, modifies the reflectivity significantly, increasing it by about 50 per cent and decreasing the emissivity from 0.970 to 0.956. A modification of emissivity of this magnitude is significant when considering energy-budget evaporation computations, since it can introduce errors of

TABLE 14. Reflectivity of Contaminated Water Samples

Muddy Tap Water			Oil Film on Tap Water			Tap Water		
Source Temp. (°F)	Water Temp. (°F)	Reflectivity	Source Temp. (°F)	Water Temp. (°F)	Reflectivity	Source Temp. (°F)	Water Temp. (°F)	Reflectivity
208	64	0.027	216	67	0.045	138	66	0.028
			219	76	0.041			
			287	76	0.046			

10 to 15 per cent at evaporation rates of around 0.2 cm per day. It is speculated that for certain natural bodies of water, where considerable oil contamination may result from excessive plankton growth, domestic waste, or industrial waste, the evaporation rate may be considerably modified through the influence of the contamination on the emissivity.*

Because of time requirements it was necessary, in this study, to utilize a preliminary value of emissivity obtained prior to the completion of the above measurements. Preliminary data indicated an emissivity of about 0.967. This value was therefore used in computing evaporation. The evaporation figures were not recomputed because, over a range of water-surface temperatures of 7 to 32°C, the use of the emissivity value of 0.970 would increase the effective back radiation by only 2 cal cm^{-2} day^{-1}. The accuracy of the determination of emissivity is ±0.005, which amounts to an accuracy of about ±4 cal cm^{-2} day^{-1} in the effective back radiation.

The long-wave radiation from Lake Hefner is shown in figure 70. The average monthly, as well as the individual maximum and minimum daily values for each month, are indicated. The radiation averages around 900 cal cm^{-2} day^{-1} during the summer, and 680 cal cm^{-2} day^{-1} during the winter, with the highest daily recorded value being 930 and the lowest 660. This is the largest single term in the energy budget.

Reflected Atmospheric Radiation

In prior evaporation studies utilizing the energy budget, the reflected atmospheric energy has generally been considered negligible. In this section it will be demonstrated that the magnitude of this term is such that it must be considered.

In the preceding section it has been shown that the reflectivity of a water surface, for a source temperature ranging from 0 to 30°C, is 0.030. Since the atmosphere is radiating at approximately these temperatures, it follows that the reflectivity of a water surface for atmospheric radiation is also about 0.030.

The average monthly reflected atmospheric radiation, as well as the daily maximum and minimum for each month, is shown in figure 71. The reflected atmospheric energy is nearly constant throughout the year, averaging about 26 cal cm^{-2} day^{-1} during the summer and about 18 cal cm^{-2} day^{-1} during the winter, with a daily recorded maximum of 29 and a daily recorded minimum of 12. The reflected atmospheric radiation is, on the average, slightly less than the reflected solar radiation, and is large enough, if neglected, to introduce a definite error into evaporation determinations based on the energy budget.

In summary, the atmospheric-radiation observations at Lake Hefner were examined in an empirical manner similar to that employed by previous investigators. However, instead of considering atmospheric radiation only from a clear sky and as a function only of local vapor pressure, which was done in most previous studies, it was possible to take account of the effects of cloud amount and height. An empirical expression was derived relating atmospheric radiation to the local vapor pressure (used as the moisture-content index), cloud amount, and cloud height. This expression allows the computation of atmospheric radiation with an accuracy of about 10 per cent of the observed value, provided it is used in areas that have a relation between local vapor pressure and total atmospheric-water content similar to that at Lake Hefner.

In connection with long-wave radiation from the water surface, considerable work was done to determine the emissivity of water. It was shown that the emissivity of water is independent of its temperature and composition and is 0.970 ±0.005.

A consideration of reflected long-wave radiation indicated that this energy-budget term cannot be neglected.

ADVECTED ENERGY

In evaluating the energy advected into or out of a body of water, it is necessary to study the climatological, hydrological, and geographical characteristics of the particular location. These characteristics influence the choice of the most suitable method for measuring advected energy. It is not possible to establish a method that will apply in all cases, but it is possible to derive certain general principles, techniques, and cautions.

Advected energy is defined as the net energy gained or lost by a body of water through the ingress or egress of a volume of water. Advected volumes may result from surface inflow, rainfall, seepage, bank storage, controlled outflows, evaporation, and condensation. In some situations all these must be considered, while in other situations some may be negligible. The feasibility of applying the energy budget to the determination of evaporation at any

* Oil films, of course, also affect evaporation in other ways, but this problem was not studied.

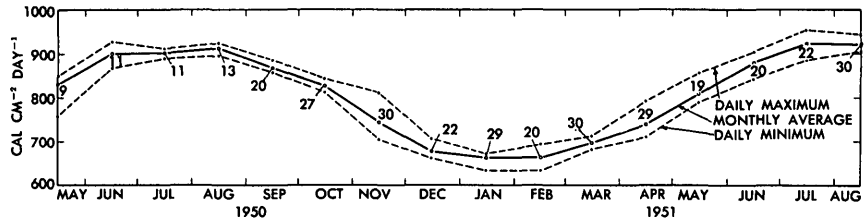


Figure 70. Long-wave radiation from Lake Hefner from 1 May 1950 to 1 September 1951.

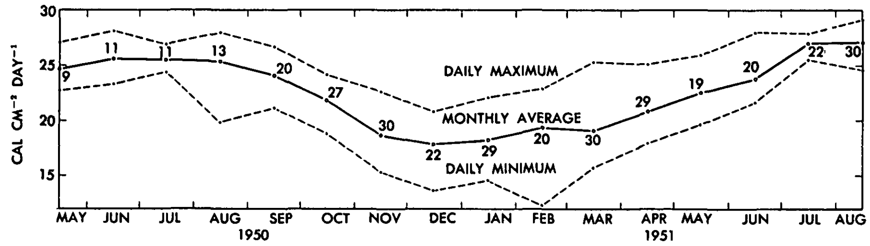


Figure 71. Reflected atmospheric radiation from Lake Hefner from 1 May 1950 to 1 September 1951.

lake will be determined largely by the ability to evaluate adequately the various parameters entering into the advected term. It is probable that, for certain situations, as in "run of the river" lakes or reservoirs, it may be impossible to utilize the energy budget because of difficulties encountered in connection with estimating advected energy.

Several aspects of advected energy have already been discussed in the beginning of this section, for example, the relationship between advected energy and change in energy storage for a lake of varying volume, the energy advected out of the lake by the volume of evaporated water, and the selection and use of a base temperature.

In the case of Lake Hefner, surface inflow, rainfall directly on the lake surface, seepage, and water withdrawals made by the city filter plant, are the primary advected volumes of water. The temperatures of these components were computed as follows: The temperature of the surface inflow was taken as the average of the maximum and minimum 2-meter air temperatures at the south station. Rainfall into the lake was assumed to be at the same temperature as the flat-plate radiometer at the time rainfall took place. Seepage temperatures were assumed to be the same as that of the deep bottom water and were obtained from vertical temperature profiles of the deep part of the lake. The temperature of city-filter-plant withdrawal was evaluated utilizing the periodic TPR cruise temperature profile taken at the intake tower and the day-to-day temperature profile as recorded in the filter plant; after March 1951 it was measured by a thermocouple placed in the intake tower at the withdrawal depth.

Due to the simplicity ot the geographic and hydrologic situation at Lake Hefner, it was possible to evaluate the various advected components with extreme accuracy. Table 15 summarizes information concerning the magnitude of the four advected components as well as the monthly average of advected energy. Surface inflow and seepage were extremely small, while filter-plant withdrawals were, on the average, equal to evaporation (average daily evaporation was approximately 30 acre-feet per day), and rainfall averaged about one-half the evaporation. Rain fell on 115 days out of 488, so on about 75 per cent of the days rainfall was zero.

TABLE 15. Monthly Averages of the Advected Water Volumes at Lake Hefner in Acre-feet per Day.

Month	Surface Inflow	Rainfall	Filter-Plant Withdrawal	Out-Seepage
May 1950	8.1	39.0	33.3	2.3
Jun	4.0	13.0	36.4	2.3
Jul	7.2	43.6	35.1	2.3
Aug	6.3	13.8	34.8	2.3
Sep	1.7	12.1	29.0	2.3
Oct	1.1	5.2	24.3	2.3
Nov	1.1	4.4	23.2	2.3
Dec	0.9	0.1	28.5	2.3
Jan 1951	0.9	4.2	25.2	2.3
Feb	2.1	9.6	26.3	2.3
Mar	2.2	9.3	25.8	2.3
Apr	3.8	25.9	26.6	2.3
May	15.0	37.3	29.0	2.3
Jun	8.3	20.0	34.7	2.3
Jul	3.5	17.1	41.4	2.3
Aug	0.5	11.7	40.7	2.3
	3.7	16.6	30.8	2.3

In evaluating the advected energy, it is necessary to select a base temperature for each time interval under consideration. Since, in the case of Lake Hefner, the magnitudes of the various advected components were both small and accurately measured, a constant base temperature of 0°C was selected. Figure 72 presents the daily values of advected energy. Figures at the top show the values for the corresponding days. In general, energy is advected out of the lake, and the magnitude is small, being about 15 cal cm^{-2} day^{-1} during the summer and approximately zero during the winter. It is evident that very little improvement could be effected by using a variable temperature base. Days on which the advected energy was positive were days when it rained and, generally, the heavier the rain the larger the advected term. On 18 May 1951 the advected energy amounted to 603.0 cal cm^{-2} day^{-1}. It is obvious that under these circumstances a more suitable temperature base than 0°C could be employed.

Anderson and Pritchard (1951) made a study of the energy advected into and out of Lake Mead. The geographic and hydrologic characteristics of Lake Mead are considerably more complex than at Lake Hefner, and a different base temperature was used.

The Lake Hefner and Lake Mead studies indicated that the evaluation of advected energy for any specific body of water is an individual problem, and extreme caution must be exercised in adopting procedures for the evaluation. It is possible that lack of adequate consideration of this term could lead to seriously erroneous evaporation results.

CHANGE IN ENERGY STORAGE

Early investigators, in attempting to utilize the energy budget, had difficulty in evaluating the storage of energy in a body of water over any specified time. It was necessary to select time intervals, or regions of the globe, where change in energy storage could be assumed negligible and therefore ignored. These assumptions were in many cases difficult to substantiate.

For example, Schmidt (1915) computed evaporation from the oceans over a yearly interval. Probably his assumption that energy storage was zero over a period of one year was valid, provided average meteorological conditions prevailed.

In 1931 Richardson discussed the application of the energy equation to the determination of evaporation from lakes and reservoirs. He stated, "Storage of energy . . . for a short period . . . , as determined from a study of data collected in California, is negligible." This may be true in certain California regions where the annual, seasonal, and daily range in air temperature may be small, and hence little energy stored in a body of water. However, since most areas experience a large air-temperature range, its truth is the exception rather than the rule. The application of the energy budget to evaporation determinations has, in the past, therefore been restricted to time intervals of a year or, in the case of shorter time intervals, to areas and times of small air-temperature range.

Through the development of instruments capable of obtaining vertical temperature profiles, and the development of the echo sounder for obtaining bottom depths, instruments are available for evaluating this energy-budget term. The following study of energy storage is based upon temperature data taken at Lake Hefner, utilizing primarily the TPR.

Methods of Evaluation

As indicated by equation (67), the change of energy storage is directly related to the advection of energy. This section is concerned with evaluating that part of equation (67), $V_1 (T_1 - T_b) - V_2 (T_2 - T_b)$, which represents the change in energy storage. The energy storage can be computed by a numerical integration method, provided vertical temperature profiles and an area-capacity table for the body of water are available. The time rate of change of energy storage is defined as the change of energy storage, Q_θ.

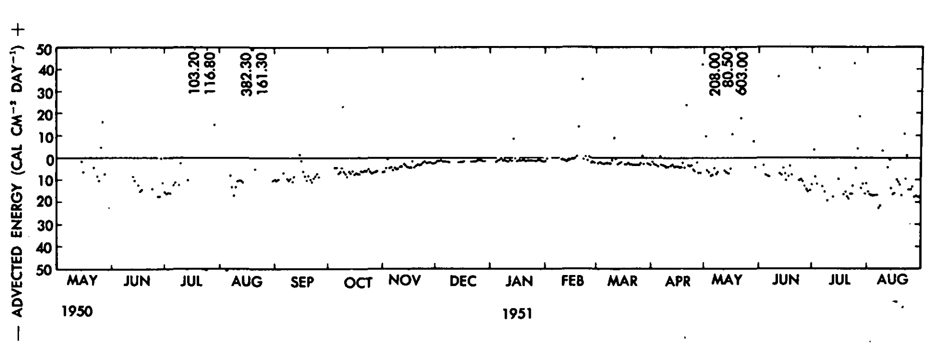


Figure 72. Daily advected energy for Lake Hefner from 1 May 1950 to 1 September 1951. Base temperature 0°C.

The energy storage for a column of water one square cm in area and extending from the surface to the bottom may be expressed as:

$$Q_{\vartheta(\text{column})} = \sum_{i=1}^{n} \rho c (\bar{T}_i - T_b) \Delta h_i + C, \qquad (91)$$

where $Q_{\vartheta(\text{column})}$ is the energy storage, ρ the density, c the specific heat, $\bar{T}_i$ the average temperature of a standard layer of thickness Δh, T_b the base temperature, n the number of standard layers, and C the amount of energy, above the base temperature, in a layer of nonstandard thickness extending from the bottom of the standard layer to the bottom of the body of water.

If the temperature structure of the body of water is uniform, then the energy storage above a base temperature is

$$Q_{\vartheta(\text{total})} = \sum_{i=1}^{n} \rho c \bar{A}_{\Delta h_i} (\bar{T}_i - T_b) \Delta h_i + A_c C, \quad (92)$$

where $A_{\Delta h_i}$ is the mean horizontal area of each standard layer and A_c the mean area of the nonstandard layer.

The change of energy storage over a given time interval is then the difference in the energy storage at the beginning of the time interval and the end (primed symbols), or

$$Q_{\vartheta} = \sum_{i=1}^{n} \rho c \bar{A}_{\Delta h_i} (\bar{T}_i - T_b) \Delta h_i - \sum_{i=1}^{n} \rho c \bar{A}_{\Delta h_i}{}' (T_i' - T_b) \Delta h_i + A_c C - A_c' C'. \qquad (93)$$

It should be reiterated that equation (93) is independent of the base temperature, provided the volume of the body of water remains constant. If the volume varies, the energy budget itself is independent of the base temperature, since it is necessary to consider both advected energy and stored energy in this case.

It is probable that in any body of water the regions of similar energy storage change are defined by the depth of water. That is, shallower water would probably experience a change of energy storage different from that in deeper water. Therefore the most accurate procedure for obtaining the energy storage is to (1) divide the body of water into several smaller regions representing different depths of water, (2) take several temperature profiles of each region and average them to obtain a representative temperature profile for the region, (3) compute the energy storage for each smaller volume from equation (92), and (4) sum the individual energy storages to obtain the total energy storage of the body of water. It is obvious that this procedure is time consuming and tedious, and hence would restrict the use of the energy-budget approach.

A second method of evaluating the energy storage is to (1) take many individual temperature profiles in all regions of the body of water, (2) average all the individual profiles regardless of depth of water, and (3) compute the energy storage from equation (92).

A third method is to take one profile in a deep section of the water and consider it representative of the entire body of water. This procedure, of course, assumes that the temperature structure of the body of water is uniform and, if sufficiently accurate, is desirable because it eliminates the necessity for making many temperature profiles and simplifies the computational procedure.

Energy Storage at Lake Hefner

The original observation program for Lake Hefner required thermal surveys at 10-day intervals at 16 stations located with respect to the depth of water, as indicated on the map. The program was later changed to complete thermal surveys every 7 days, with a daily profile being made at either station 6 or 2, depending upon weather conditions. Because of the high wind speeds experienced in this area, it was difficult to adhere rigorously to the program. During the 16 months of observations, 50 complete thermal surveys and 160 daily measurements at station 6 or 2 were made.

Prior to 1 March 1951, the observations were made at a time of day convenient for the observers, usually the forenoon but frequently the afternoon. As the analysis progressed, it was found that the time of day was important. If the observation were made in the morning at one time and in the afternoon at a later time, the afternoon heating of the upper layers of the water introduced an apparent change in energy storage. To eliminate this transient effect on energy-storage computations, after 1 March 1951 the observations were made about an hour before dawn when thermal conditions were most stable. The best times for the observations are obviously the beginning and end of the period under consideration. In this study evaporation was computed daily from midnight to midnight, but it was not practicable to make midnight observations.

From 1 May 1950 to 1 July 1951, the observations were made with the TPR. During July and August 1951, a 70- or 180-foot BT was employed, because of operational difficulties experienced with the TPR. Evaluations of energy storage based on observations made with the TPR are more accurate than those with the BT, due to the greater precision of the former instrument.

In addition to periodic information regarding vertical temperature structure, it is necessary to know the volume of water. Prior to the construction of the Lake Hefner dam, an area-capacity table had been prepared from a topographic survey. Although this table was probably accurate at the time it was prepared, no information was available on sedimentation that may have occurred or on the possibility that earth had been removed from the reservoir basin in the construction of the dam. Hence a bathymetric survey, using transit shore control, was made on 2-11 August 1950, with the upper 10 feet of the lake plane-tabled on 14-24 March 1950. The results of these surveys indicate that the reservoir capacity is approximately 0.4 per cent less than indicated by the preconstruction survey.

The energy storage for each cruise was computed, using equation (92), by each of the three methods outlined above: (1) averaging by depth, (2) averaging the 16 stations, and (3) using a single deep station. The change in energy storage, expressed in cal cm^{-2} day^{-1}, was then computed for the intervals between cruises, using equation (93). The results are shown graphically in figures 73a and 73b. The correlation coefficients are 0.99 and 0.98, respectively, while the standard errors of estimate are ± 16.8 and ± 31.3 cal cm^{-2} day^{-1}, respectively. Method (2) gives results close enough to those of method (1) to justify its use, but method (3) does not agree with method (1) as closely as desired. However, in this study it was not practicable to obtain complete 16-station data all the time, and hence it was necessary to make single-station evaluations.

The temperature measurements were accurate to ± 0.1°C. If the maximum error were present in a temperature profile in the same direction at all depths, it would introduce a maximum error of approximately 70 cal cm^{-2} over the time interval considered. Such an energy error is considerable over short time intervals, for example, of a few days, but over a period of 7 to 10 days is not great.

Figure 74 presents the energy storage for lake Hefner from 14 April 1950 to 31 August 1951. The numerals indicate the complete cruise numbers and the "dots" without numerals a single temperature profile. As indicated by equation (67), the energy storage is independent of advected energy only if the volume of the water remains constant. Figure 74 also shows the variation in the volume of Lake Hefner for the above time interval. The volume varied from a minimum of 56,000 acre-feet on 8 May 1950 to 65,000 acre-feet on 25 August 1950, with a mean volume of approximately 62,000 acre-feet, or during the 16 months of observation it varied by about 15 per cent of the mean volume.

From these data on energy storage, it is possible to examine the assumptions of various investigators concerning the magnitude of the change in energy storage over both long and short time intervals when volume changes are nil or small. Table 16 gives the variation of energy storage for monthly and yearly intervals. The variation over a yearly cycle is less than 10 cal cm^{-2} day^{-1}, while over the months of January and August, when the monthly variations would probably be the least, it is from 2 to 60 cal cm^{-2} day^{-1}. During months other than January

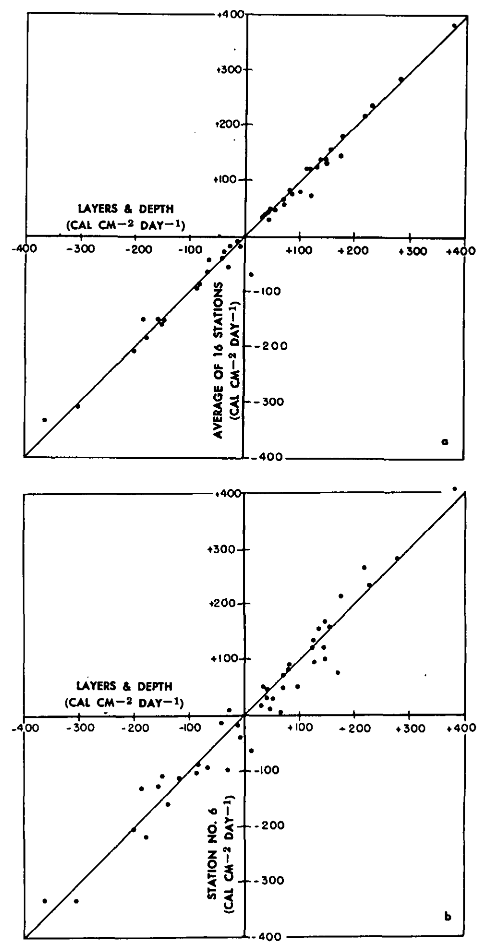


Figure 73. Comparison of energy storage computed by averaging by depth with average of 16 stations (a) and single station (b).

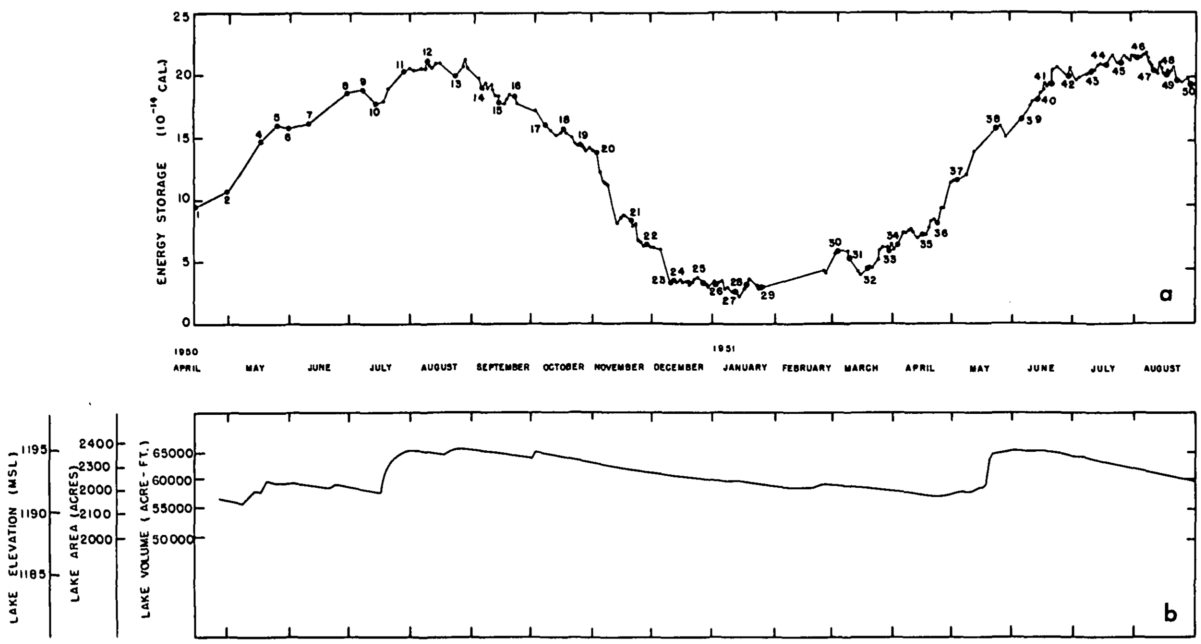


Figure 74. Energy storage (a) and variation in lake elevation, area, and volume (b) for Lake Hefner from 15 April 1950 to 1 September 1951.

TABLE 16. Change in Energy Storage Over Yearly and Monthly Intervals.

	Time Interval	Change in Storage (cal cm^{-2} day^{-1})	Change in Volume (per cent)
Yearly	15 May 1950-15 May 1951	+0.7	—8.5
Yearly	20 Jul 1950-20 Jul 1951	+7.8	+1.1
Monthly	1 Jan 1951-31 Jan 1951	—1.7	—2.0
Monthly	1 Aug 1950-31 Aug 1950	—13.5	0.0
Monthly	1 Aug 1951-31 Aug 1951	—58.4	—3.9

and August, the change in energy storage is considerable as shown by figure 74. In addition, the variation from year to year for the month of August is considerable, being about 15 and 60 cal cm^{-2} day^{-1} for August 1950 and August 1951, respectively. It may be concluded that the assumption of negligible change in energy storage is (1) valid over a yearly cycle provided the volume does not change, (2) is of dubious validity over monthly intervals during January and August, and (3) is definitely invalid for other months.

Table 17 indicates the change in energy storage over two short time intervals when the range in air and water-surface temperature was small and the volume constant. The change in energy storage in one case was negligible and the other considerable, making it impossible to draw general conclusions.

TABLE 17. Change in Energy Storage During Short Time Intervals When Range of Air and Surface-Water Temperature is Small and Lake Volume Constant

Time Interval	13 Dec 1950-16 Dec 1950 (4 days)	6 Aug 1950-12 Aug 1950 (7 days)
Average Daily Air-Temperature Range (°C)	2.9 to 4.9 (range 2.0)	24.5 to 27.5 (range 3.0)
Average Daily Water Temperature Range (°C)	4.3 to 4.7 (range 0.4)	25.8 to 27.4 (range 1.6)
Change in Energy Storage (cal cm^{-2} day^{-1})	+1.4	+46.5

On several occasions attempts were made to obtain 24-hour series of vertical temperature profiles taken at half-hour intervals. Although the change in energy storage is a residual term from several components, each varying in some manner during a 24-hour period, it was believed that perhaps some information on the diurnal variation of energy storage might be obtained. Only two such attempts were completely successful: one from 1300 CST, 16 November 1950 to 1130 CST, 17 November 1950; and another from 0730 CST, 21 March 1951 to 0730 CST, 22 March 1951. Figure 75 presents the results from these two series of observations. They fail to indicate any

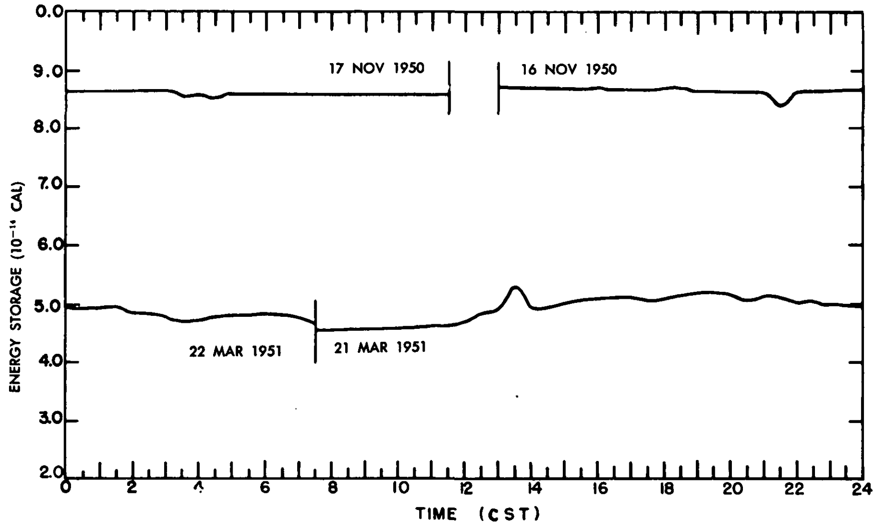


Figure 75. Diurnal variation of energy storage for Lake Hefner.

consistent diurnal trend — the November example indicates a possible linear decrease during the 24-hour period, while the March example indicates a possible sinusoidal variation, with a maximum energy storage during the late afternoon and a minimum during the early morning. Of more significance, however, is the abrupt decrease and increase in the energy storage observed at 2130 CST and 1330 CST on 16 November 1950 and 21 March 1951, respectively. These abrupt changes are apparently associated with the movement of cold and warm eddies of water through the area where the temperature profiles were taken. With the passage of the eddy, the energy storage returned to a more normal value. These abrupt changes are equivalent to approximately 350 cal cm^{-2}.

These examples indicate the possibility of introducing considerable error into the evaluation when single-station profiles are used since, under the above conditions, single stations did not yield profiles representative of the entire body of water. This error, however, compared to errors of other terms in the energy budget, decreases as the length of the time interval increases and, for longer time intervals, say a week or month, the technique may be used with confidence.

Summarizing, analysis of three methods of evaluating the change of energy storage indicates that the temperature structure of Lake Hefner is essentially vertically and horizontally uniform, suggesting that the change in energy storage can be evaluated from a single vertical temperature profile or at most from an average of several profiles taken in different depths of water. Because of limits to the accuracy of the temperature measurements, considerable error may be present when evaluating the change of energy storage over time intervals of a few days. For time intervals of the order of 7 to 10 days, however, the accuracy is probably adequate for evaporation determinations. Examination of the magnitude of energy-storage change substantiates that over annual cycles the change is negligible, but not over shorter time intervals, for example, a month or less.

RATIO OF ENERGY USED FOR CONDUCTION OF SENSIBLE HEAT TO THAT USED FOR EVAPORATION

From the energy budget, it is possible to evaluate the sum of the energy utilized by evaporative processes and that conducted to or from the body of water by the air as sensible heat. Since neither term can be evaluated directly, it is necessary to employ a ratio, R, the ratio of conducted heat to energy utilized by evaporation. Early workers, utilizing the energy equation for obtaining evaporation, have assumed various values for this ratio.

Bowen Ratio

In 1926 Bowen attempted theoretically to relate the value of R to easily measurable quantities. He concluded that the ratio could be expressed as follows:

$$R = c\left(\frac{T_0 - T_a}{e_0 - e_a}\right)\frac{P}{1000}, \qquad (94)$$

where T_0 and T_a are the temperatures of the air and water surface, e_0 and e_a the saturated vapor pressure at the temperature of the water surface and the vapor pressure of the air, respectively, and P the atmospheric pressure. The limiting values of c are 0.58 and 0.66, depending upon the state of the atmosphere.

Bowen concluded that c, under normal atmospheric conditions, was approximately 0.61.

The fundamental equations from which Bowen started apply to the transfer of heat and water vapor by processes of molecular diffusion. It might, therefore, be concluded that the computed Bowen ratio is valid only under conditions of laminar flow. However, as Bowen states, "it can be expected that heat losses by evaporation and diffusion and by conduction will follow the same laws and will be affected in the same way by convection." This implies that the ratio between the energy losses will be independent of the state of atmospheric turbulence.

In addition, Pritchard (unpublished notes) derived the same ratio as Bowen from mass-transfer concepts, using Sverdrup's (1946) relationships as a basis. He found that c varied from 0.57 for a smooth surface to 0.66 for a rough surface, which agrees with Bowen's limiting values. A similar analysis was made by Thom (1951) using the Thornthwaite-Holzman evaporation equation.

Sverdrup (1943) attempted an analysis of the general validity of the Bowen ratio. Recent work had suggested to him that it might be necessary to consider the effects of radiation on the transfer of heat and of spray upon evaporation. These effects might make the Bowen ratio dependent on turbulence, and it would then have to be modified.

Sverdrup first examined radiation for air in laminar motion or at rest. In a moist atmosphere, a flux of radiant energy is directed from regions of higher to regions of lower temperature. This flux is dependent upon the selective emission and absorption of long-wave radiation by water vapor, and is therefore not present in dry air. Its effect can be described by means of a coefficient which Brunt (1944) calls "radiative diffusivity." The radiative diffusivity has the same dimensions as the diffusion coefficient for heat energy and, when problems of diffusion of heat energy in moist air are considered, it must be added to the heat-energy diffusion coefficient. The ratio, *R*, in this form is greater than in Bowen's form, the increase depending upon the relative magnitudes of the diffusivity coefficients for heat energy and radiation.

Very little is known about the value of radiative diffusivity near a boundary surface. However, making certain primitive assumptions concerning radiative diffusivity, Sverdrup concluded that the ratio was about 1.30 times that obtained from equation (94). This conclusion was only semiquantitative. He also concluded that the application of Bowen's formula to atmospheric conditions breaks down because evaporation is increased at high wind speeds by spray. Montgomery (1940) suggested that evaporation from spray was of great importance and that, as a result, the total evaporation would not remain proportional to the wind speed as is nearly the case when transfer by eddy diffusivity only is considered, but would increase more rapidly at high wind speeds. If this is true, the Bowen formula gives values of the ratio that are too high at higher wind speeds. N. W. Cummings, in a discussion of Sverdrup's (1943) paper, indicated that in his opinion the presence of spray does not invalidate the Bowen ratio.

In summary, Sverdrup (1943) concluded that Bowen's formula gives only an approximate value, the closeness of the approximation depending upon the radiative diffusivity, turbulence near the surface, and the effect of spray upon evaporation.

The various developments of the ratio of heat losses by convection to those by evaporation depend fundamentally on the assumption that the eddy diffusivities for these two quantities are identical. There is evidence indicating that this assumption is not always valid. For example, Pasquill (1949) made an experimental study of the various factors involved in the turbulent transfer of water vapor and heat in the lowest layers of the atmosphere over open grasslands. From the data he evaluated the vertical components of the eddy diffusivities for water vapor and heat. He demonstrated, from evaluation of the turbulent heat flux using heat-balance considerations, that the eddy diffusivity for heat is reasonably equal to that for water vapor under stable conditions, but is substantially greater for unstable conditions, and may be twice as great.

Commenting on these observed differences in eddy diffusivities for water vapor and heat, Pasquill stated: "Although it is not proposed in this paper to deal in any detail with theories of turbulent mixing it may now be noted that, in qualitative implication at any rate, the present results are not entirely unexpected. In a recent treatment of convection near the ground Sutton (1948) has assembled evidence for the existence of a difference in heat and momentum transport. Furthermore, Priestley and Swinbank (1947) have recently put forward a modification of the classical theory of turbulent transport of heat, taking into account buoyancy effects. The latter treatment possesses implications as regards the transfer of physical quantities other than that of heat, and provides reasons for expecting different values of the eddy diffusivity to apply to the transport of different properties. As far as is known the present results provide the first direct experimental demonstration of the existence of this feature in the cases of matter and heat."

Although the eddy diffusivity for heat is approximately equivalent to that for water vapor under stable conditions, it may be twice as great for unstable conditions, and the computed evaporation rates 33 per cent too high. Hence Pasquill concluded that the methods described for computing evaporation from the energy budget may only be applied with confidence when the atmosphere is stable, and may be seriously in error under unstable conditions when evaporation is at a maximum.

Since considerable controversy exists over the validity and meaning of the Bowen ratio, an attempt was made with Lake Hefner data to compute Bowen ratios by observing all factors in equation (72) and comparing the results with those obtained from meteorological data. However, it was found that the various factors had not been measured with the requisite accuracy, and inconsistent results were obtained. It is suggested that a wind tunnel experiment be designed to investigate the validity of the Bowen ratio.

When its value is small, the Bowen ratio occurs in equation (72) as a corrective term. Hence, errors in its evalution are reflected as much smaller errors in evaporation. When the ratio approaches unity, it no longer is a small corrective term, and errors in its determination are of greater importance.

The remainder of this section covers a study, based on Lake Hefner data, of the implications of the Bowen ratio and its effect upon the evaluation of evaporation from the energy budget.

Bowen Ratios Observed at Lake Hefner

From the data collected at Lake Hefner, the Bowen ratio was computed for single days and also for periods varying in length up to 30 days. The ratio was computed from equation (94), utilizing a value of 0.61 for c, the surface water-temperature taken at the barge, and the barge 2-meter wet- and dry-bulb temperatures. Temperatures over the length of period under consideration were averaged. The ratios for daily, 5-, 10-, and 20-day periods are presented in figures 76 and 77. The most striking feature of these figures is the relatively small variation in the ratio, from an extreme of +0.5 to —0.5. The average variation is from +0.2 to —0.2 from May through October, but from October to May the daily variation is much greater, the greatest variation occurring during February. For the 16-month period, the extreme variation in the ratio was from —20.02 occurring the latter part of April to +31.50 in February. As the length of the period increases, the range becomes smaller, until for periods of 20 days the ratio only varies throughout the year from +0.25 to —0.25.

Bowen ratios of approximately —1.00, large positive ratios, and large negative ratios will next be examined with respect to their validity and their effect upon evaporation computations.

Referring to equation (71), ratios having a value of or close to —1.00 are of interest. Under these conditions the first term in the brackets on the left side of the equation assumes very small values (the second term is always small), with the result that small variations in $(Q_s - Q_r - Q_b - Q_\vartheta + Q_v)$ will vary the computed evaporation extremely. For the case when the ratio is equal to —1.00 and the base temperature equals the surface temperature, the quantity $[\rho_e L (1 + R) + c\rho_e (T_e - T_b)]$ is equal to zero. Then the right-hand side of equation (71) must also be equal to zero, and the evaporation is indeterminate. It must be pointed out that on the right-hand side of the equation the quantity $(Q_v - Q_\vartheta)$ is not independent of the base temperature, since the term Q_v does not include the energy advected out of the body of water by the mass of evaporated water. This latter advected mass of water is accounted for on the left-hand side of the equation by the quantity $c\rho_e (T_e - T_b)$. Hence, if the base temperature is changed in the situation discussed above, the quantity $[\rho_e L (1 + R) + c\rho_e (T_e - T_b)]$ will have a finite value. The quantity $(Q_s - Q_r - Q_b - Q_\vartheta + Q_v)$ may then assume any value depending upon the particular situation.

On a daily basis, there were 14 days when the ratio was between —0.70 and —1.30. In all 14 cases the lower layer of the atmosphere was stable, with an increase in temperature between 0 and 2 meters of 1.6 to 8.9°C. The vapor pressure in the lower 2 meters decreased with height from 1.1 to 4.9 mb. Unfortunately, only two of these days had water budgets classified as A. Pertinent data for these two days are summarized in table 18. In order that the computed evaporation equal the observed evaporation, $(Q_s - Q_r - Q_b - Q_\vartheta + Q_v)$ should have a value of 3 and —1 cal cm^{-2} day^{-1} on 22 December and 18 January, respectively. In other words, $(Q_s - Q_r - Q_b - Q_\vartheta + Q_v)$ would be in error by 35 and 185 cal cm^{-2} day^{-1}, respectively. The only term in the energy budget that could vary the amount indicated above is the change in energy storage. On a daily basis, errors of the above magnitude in evaluat-

TABLE 18. Pertinent Data for A Water-Budget Days When Bowen Ratio is Approximately —1.00.

Date	Bowen Ratio	$T_2 - T_0$ (°C)	Evaporation Water Budget (cm)	Evaporation Energy Budget (cm)	$e_2 - e_0$ (mb)	$Q_s - Q_r - Q_b - Q_\theta + Q_v$ (cal cm^{-2} day^{-1})	$L(1 + R) + T_w$
22 Dec 1950	−0.865	1.6	0.043	−0.376	−1.1	−32	85
18 Jan 1951	−1.109	5.1	0.015	−1.291	−2.7	184	−60

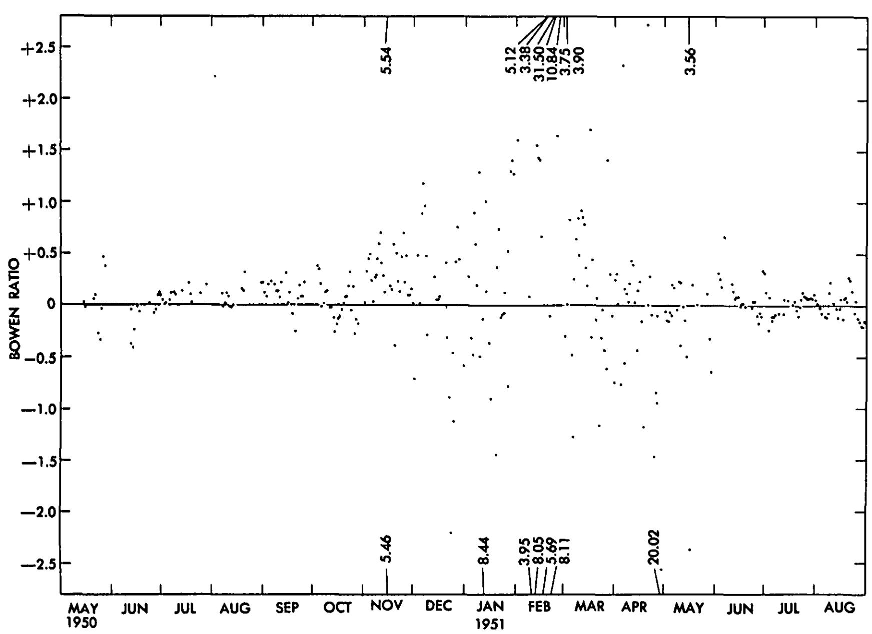


Figure 76. Daily Bowen ratios for Lake Hefner from 1 May 1950 to 1 September 1951.

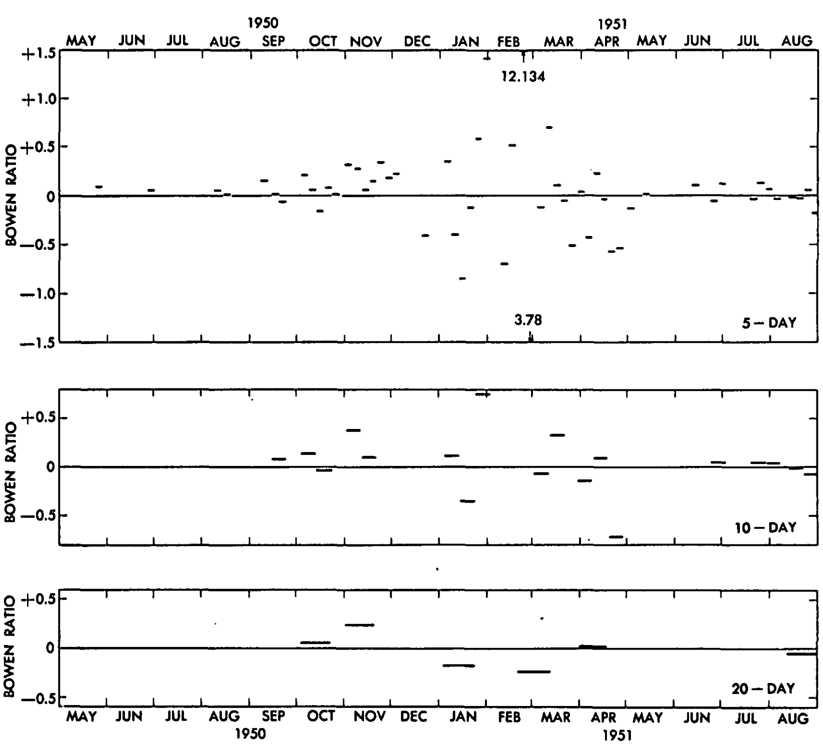


Figure 77. Bowen ratios for 5-, 10-, and 20-day periods for Lake Hefner from 1 May 1950 to 1 September 1951.

ing this term are possible, since temperature-depth measurements were not made at the beginning and end of the evaporation period. Greater accuracy in evaporation determinations will be obtained if the energy-storage measurements are made at the proper time intervals.

In addition, Bowen ratios around —1.00 occurred during eight periods varying in length from 4 to 16 days. Results similar to those above are indicated. Errors in evaluation of $(Q_s - Q_r - Q_b - Q_\vartheta + Q_v)$, *of a magnitude attributable to errors in energy-storage* évaluation, can account for the difference between computed and observed evaporation. One of these periods, 7 days in length, is of particular interest since complete 16-station thermal surveys were made both at the beginning and end of the period. Under these conditions it was possible to make the most accurate evaluation of the change in energy storage that the technique employed would permit. Pertinent data for this period follow:

Date	12 Jan to 18 Jan 1951
Bowen ratio	—1.126
Water-budget evaporation	—0.027 cm
Energy-budget evaporation	—0.123 cm
Temperature difference between 0 and 2 meters	3.2°C
Vapor pressure difference between 0 and 2 meters	—1.5 mb
$(Q_s - Q_r - Q_b - Q_\vartheta + Q_v)$	18 cal cm^{-2} day^{-1}
$L(1 + R) + T_w$	—150

If $(Q_s - Q_r - Q_b - Q_\vartheta + Q_v)$ were equal to 4 cal cm^{-2} day^{-1}, or 14 cal cm^{-2} day^{-1} smaller than observed, then the computed evaporation would agree exactly with the observed evaporation. Perhaps this difference is suggestive of the accuracy with which the change in energy storage can be evaluated.

Negative ratios greater than —3.00 occurred on 7 days, and positive ratios greater than +3.00 on 8 days. In addition, for 13 periods, varying from 2 to 8 days in length, ratios greater than ±3.00 were computed. These all occurred during the winter and spring when there was a southerly flow of warm moist air over a cold lake, with a resulting temperature inversion between 0 and 2 meters of 2.2 to 9.3°C, and a difference in vapor pressure between the air and air saturated at the temperature of the water of 0.1 to 1.0 mb. For large Bowen ratios the denominator of equation (72) becomes very large. Under these conditions, differences between computed and observed evaporations cannot be accounted for by errors in evaluating the change of energy storage, since large changes in $(Q_s - Q_r - Q_b - Q_\vartheta + Q_v)$ are necessary to vary the computed evaporation by an appreciable amount. However, small errors in evaluating the vapor-pressure difference in equation (94) become extremely important, since these small differences are reflected as a large change in the Bowen ratio.

From these observations it might be concluded that evaporation determinations made when the Bowen ratio approximates —1.00, or attains large positive or negative values, may be in considerable error. In the first case, errors in evaluating the change in energy storage are reflected as large changes in evaporation while, in the second, small errors in vapor-pressure difference cause the same effect. However, only 8.0 per cent of the time did the Bowen ratio attain the extreme values discussed above, and only during periods of low evaporation or condensation. During periods of high evaporation the ratio was small, and errors in its evaluation were reflected as smaller errors in evaporation due to its entering equation (72) as a corrective term. For periods greater than 10 days, the extreme values of the ratio never occurred. In a practical sense, errors in the Bowen ratio do not greatly affect the accuracy of the computed evaporation, because evaporation determinations for periods of under 10 days are rarely necessary or desired.

As previously stated, doubt has been cast upon the validity of using the Bowen ratio due to three assumptions: (1) neglect of the effect of radiative diffusivity, (2) equality of the coefficients of mixing for water vapor and heat over all ranges of stability, and (3) neglect of the effect of spray. As previously indicated, differences of opinion exist concerning the effect of spray on the Bowen ratio. No attempt will be made to resolve these differences. In the following discussion it is assumed that spray modifies the ratio, as suggested by Sverdrup and Montgomery.

Figure 78 has been prepared to indicate hypothetically what the effect would be on the computed evaporation provided these three effects were principally functions of wind speed and varied in some assumed fashion.

For example, assume, as Pasquill suggests, that the eddy diffusivity coefficients for water vapor and heat are equal under stable or neutrally stable conditions, and that the diffusivity for heat is greater than for water vapor when the atmosphere is unstable. Then, assuming that increasing wind speed is indicative of an increasingly unstable atmosphere, the presently computed evaporation rates for higher wind speeds would be too large, and it would be necessary to apply a negative correction to the computed evaporation as indicated in figure 78a.

Radiative diffusivity is assumed to be independent of wind speed. Hence, there would always be a negative correction to apply to the computed evaporation, since the neglect of radiative diffusivity gives a Bowen ratio that is too small. This is shown schematically in figure 78b.

Figure 78c indicates the possible error if spray is neglected. No error would occur at wind speeds lower than the speed necessary to produce whitecaps. When this speed is attained, water droplets are blown into the air. Evaporation would then occur from these water droplets before they fall back into the body of water. The computed evaporation would then be too small, and a positive correction would have to be made to the computed evaporation.

The composite of these three effects would be as indicated in figure 78d. At low wind speeds the computed evaporation would be too large and at high wind speeds it would be too small.

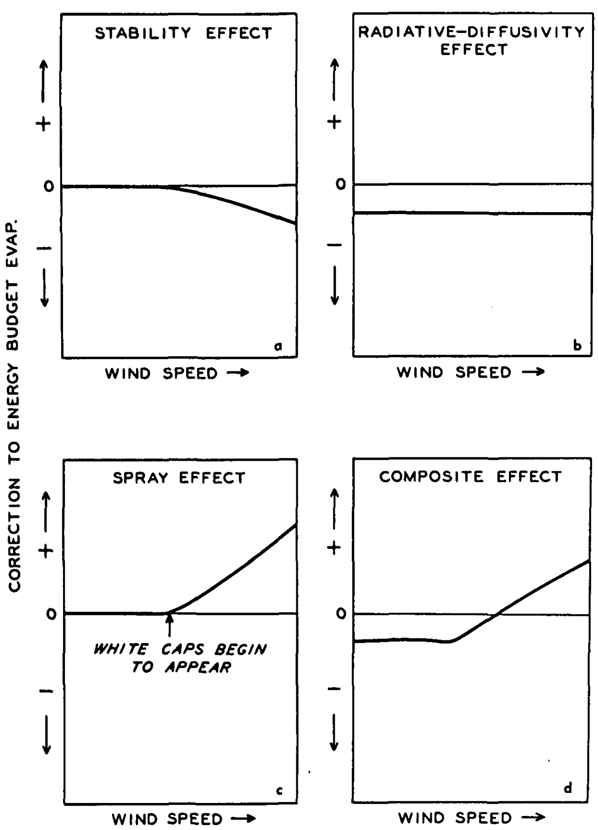


Figure 78. Composite hypothetical effect on evaporation of neglecting radiative diffusivity, variations in equality of diffusivity coefficients for water vapor and heat, and spray in computing Bowen ratios.

An examination of the Lake Hefner data should decide whether or not a consistent composite error, such as indicated above, is introduced into the evaporation results when using the Bowen ratio in its original form. The results of such an analysis are given in figure 79 for 142 evaporation periods of 7 or more days in length. For periods of the same length (that is, 7 days) there was no overlapping, but the longer periods necessarily included the shorter ones. The difference between these water-budget evaporations and energy-budget evaporations are plotted versus the average wind speed at 8 meters. The longer evaporation periods were selected in order to minimize errors in energy-storage evaluations. It is evident that the scatter is essentially random, and no relation, such as suggested above, is indicated.

Finally, the Lake Hefner data indicate that the assumptions made in the development of the Bowen ratio concerning spray, radiative diffusivity, and atmospheric stability do not introduce a consistent error into evaporation determinations. The Bowen ratio appears to be sufficiently accurate for computing energy utilized by evaporation for most conditions. In exceptional conditions, for example, when evaporation rates are small and the difference in vapor pressure of the atmosphere and that of air saturated at the surface-water temperature approaches instrumental accuracy, the Bowen ratio is inadequate. The latter conditions occur only 8 per cent of the time.

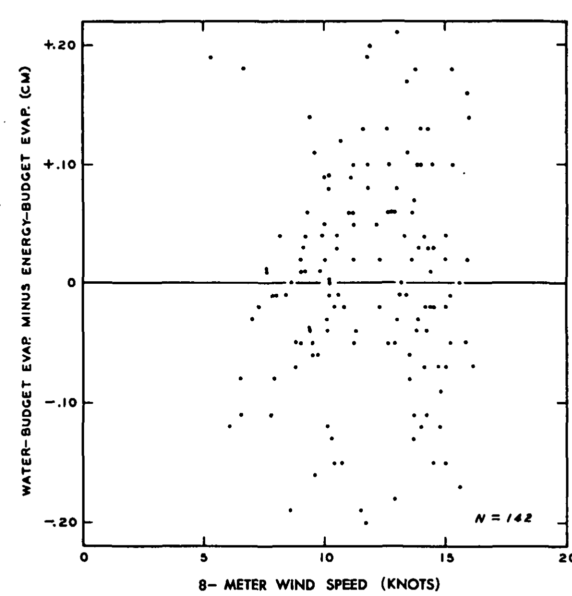


Figure 79. Relationship between wind speed and differences between water-budget evaporation and energy-budget evaporation.

ENERGY CONDUCTED AS SENSIBLE HEAT BY THE ATMOSPHERE

The heat carried away from a water surface through a unit area in unit time may be expressed as follows:

$$Q_h = -c_p A \left(\frac{dT}{dz} + \gamma\right), \tag{95}$$

where c_p is the specific heat of the air at constant pressure, A the vertical component of the eddy conductivity, dT/dz the temperature gradient of the air, and γ the adiabatic lapse rate. The term c_pA enters, instead of the molecular coefficient of heat conductivity of the air, because the air is nearly always in turbulent motion; the degree of turbulence is indicated by A, the eddy conductivity. To utilize equation (95), it is necessary to evaluate the eddy conductivity.

Observations were made at Lake Hefner of all quantities necessary to evaluate eddy conductivity. It was hoped to compute eddy conductivity from the data, and then find a method for obtaining it from more easily measured parameters, thus making it possible, in the future, to compute Q_h from equation (95). However, it was found that $(Q_e + Q_h)$, the evaporation, and/or dT/dz were not measured with enough precision to obtain consistent values of eddy conductivity. Hence, the attempt was abandoned.

It was possible to obtain a knowledge of the gross magnitude of the conducted heat through use of the Bowen ratio. From the preceding discussion of this ratio, it was concluded that satisfactory values of conducted heat could be obtained in all but the few situations where the Bowen ratio leads to erroneous results.

The average monthly conducted heat for the 16 months of observations is shown in figure 80. Most energy was conducted to the lake in April 1951, when it averaged 84.0 cal cm^{-2} day^{-1}; most was conducted away in November 1950, when it averaged 75.0 cal cm^{-2} day^{-1}. During most months, the conducted heat is a large enough term to be considered in the energy budget.

The ratio of conducted heat to energy utilized by evaporation varied, during the last 12 months of observations, from —0.324 in February 1951 to +0.252 in November 1950; the annual average ratio was —0.030. Hence, on a monthly basis the conducted heat at Lake Hefner must be considered, while on an annual basis it may be neglected.

ENERGY UTILIZED BY EVAPORATIVE PROCESSES

Evaporation was computed, by means of the energy budget, for periods of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20, 25, and 30 days, and for periods of variable length for which complete (16-station) thermal-survey data were available at both the beginning and the end of the period. Only periods where all terms in the energy budget were measured continually were used. No extrapolated data are included. The periods are non-overlapping and consecutive in time, and no attempt was made to exert judgment as to which days should be included in a particular period. The data were edited only to exclude periods where the evaluation of energy storage was obviously inadequate — for example, no storage determinations were made from 26 January 1951 to 26 February 1951.

Enough data for periods varying from 1 to 10 days in length and for periods coinciding with the thermal cruises were available for statistical treatment. For periods greater than 10 days, the data were too few to treat statistically.

The data for the above periods are plotted in figures 81 through 85. These figures clearly indicate the better agreement between observed and computed evaporation as the length of the time interval increases. In nearly every case the number of points above the 45-degree line equals the number below, while the average-energy-budget and the average-water-budget evaporations agree within —2.4 to +7.1 per cent. It may be concluded that the data are essentially randomly distributed about the line of perfect agreement.

Since the data were scattered normally about the 45-degree line, it was possible to compute standard errors of estimate; these are shown as dashed lines in figures 81 through 85. One standard error of estimate should contain, by definition, approximately two-thirds of all the cases. These standard errors of estimate, as well as the standard errors of estimate expressed as a percentage of the mean energy-budget evaporation, are shown as a function of the length of period in figure 86. The improvement in accuracy of the computations is marked as the length of period increases to 4 days, and thence more gradual, until for periods of 10 days the standard error of estimate is about ±12 to 15 per cent of the mean energy-budget evaporation. The curve appears to follow the expected square-root law reasonably well.

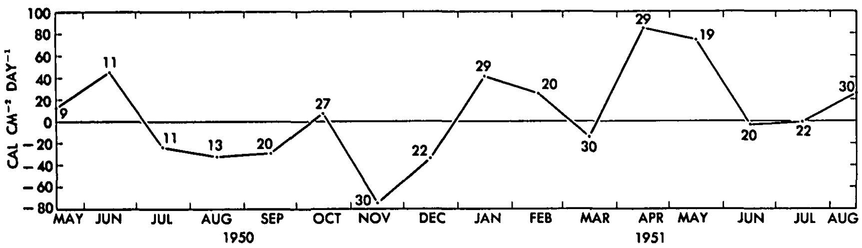


Figure 80. Average monthly amount of energy conducted to (+) or from (—) Lake Hefner from 1 May 1950 to 1 September 1951.

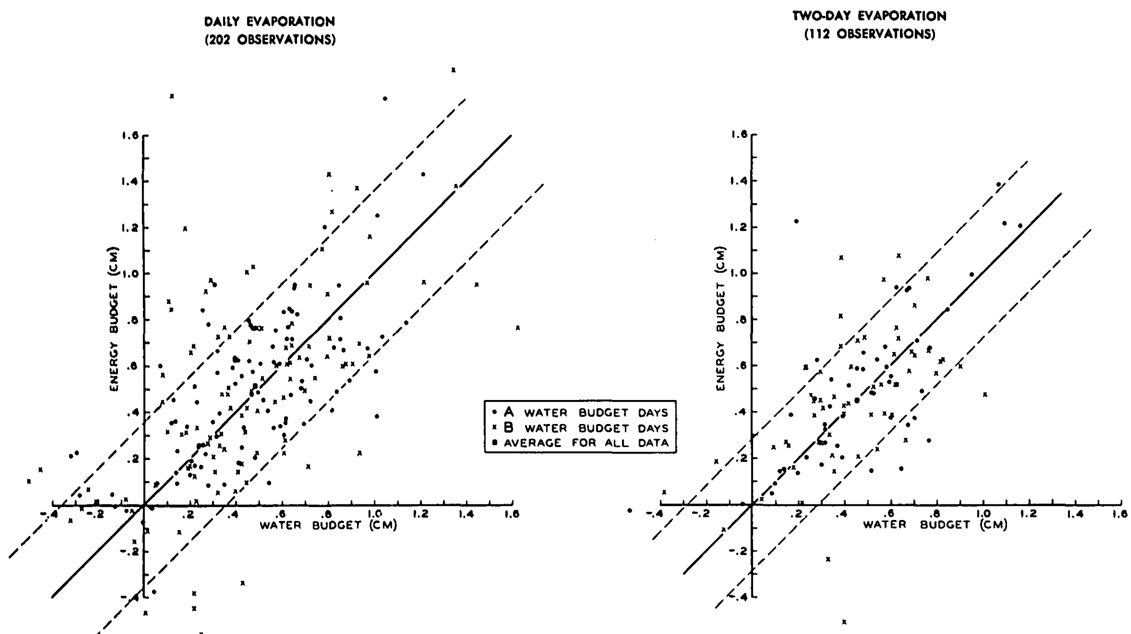


Figure 81. Comparison of daily and two-day evaporation from Lake Hefner. Dashed line represents one standard error of estimate.

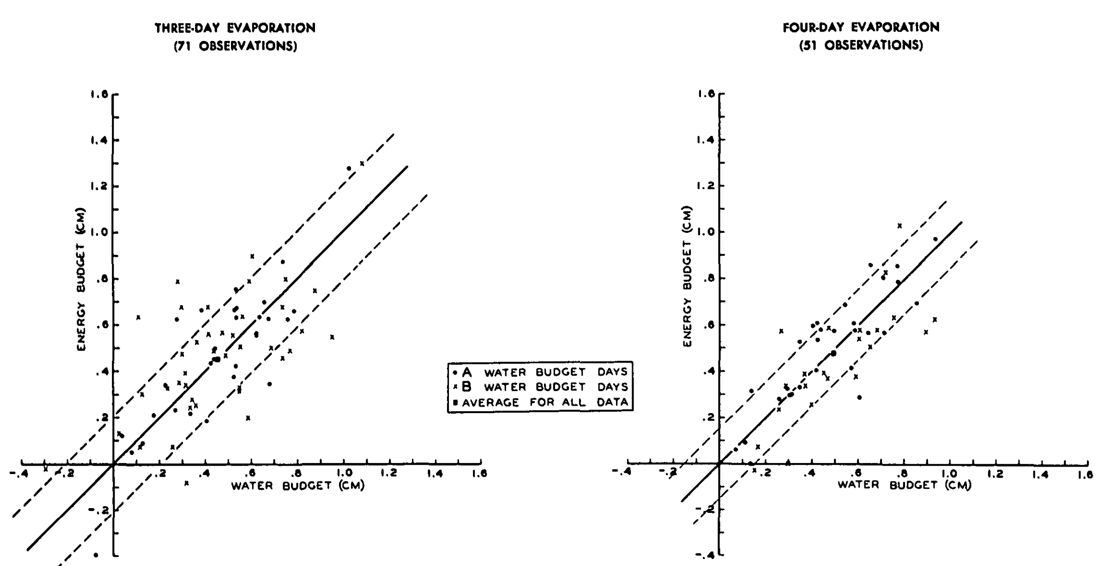


Figure 82. Comparison of three- and four-day evaporation from Lake Hefner. Dashed line represents one standard error of estimate.

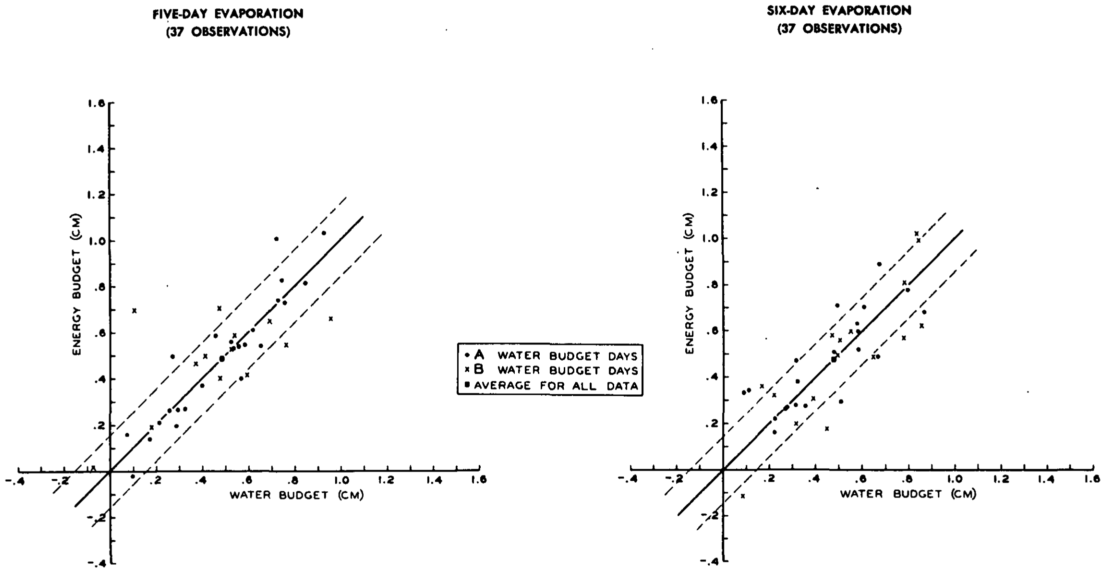


Figure 83. Comparison of five- and six-day evaporation from Lake Hefner. Dashed line represents one standard error of estimate.

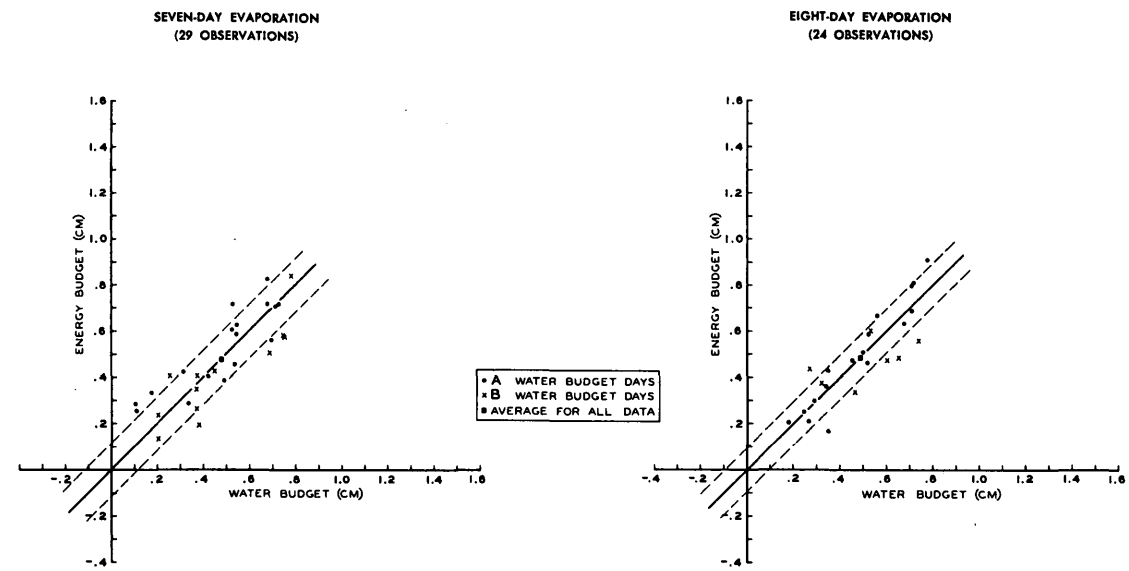


Figure 84. Comparison of seven- and eight-day evaporation from Lake Hefner. Dashed line represents one standard error of estimate.

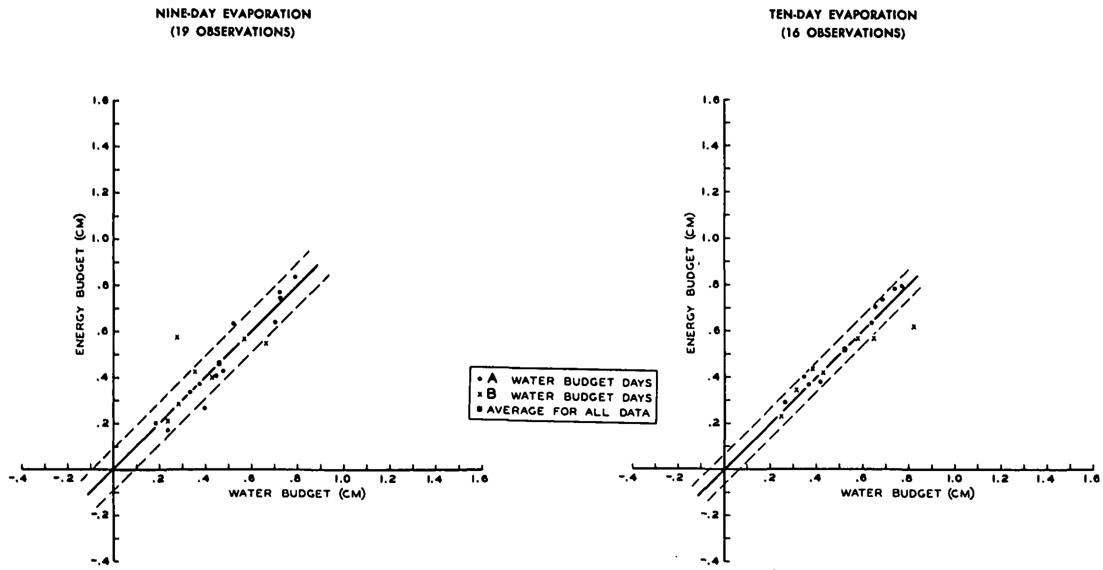


Figure 85. Comparison of nine- and ten-day evaporation from Lake Hefner. Dashed line represents one standard error of estimate.

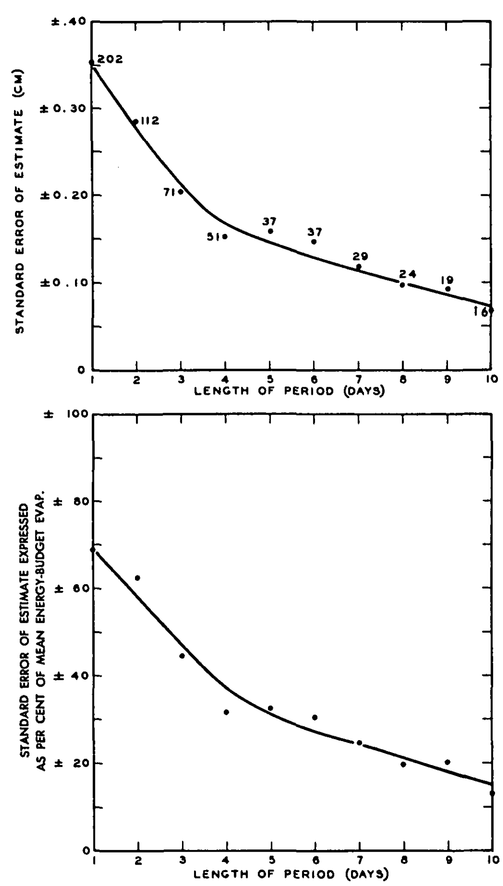


Figure 86. Accuracy of energy-budget evaporation determinations as a function of the length of computational period for Lake Hefner. Figure indicates the number of A and B periods included.

It must be remembered that these data include both A and B water-budget days, and that the control itself is subject to certain inaccuracies previously discussed.

The individual sets of data for periods longer than 10 days, which could not be treated statistically, are listed in table 19. With respect to percentages, the accuracies of these data vary considerably. On the whole, there does not appear to be any improvement in accuracy as the length of period increases beyond 10 days. This may be related to the fact, previously indicated, that the standard errors of estimate of computations of energy-storage changes from an average of all cruise stations or a single deep station and from the more detailed layer-depth method are ± 17 and ± 31 cal cm^{-2} day^{-1}, respectively. The available A and B data, for periods when evaporation was greater than 0.5 cm day^{-1}, indicate that an accuracy of ± 19 per cent is possible for periods longer than 10 days. For shorter periods the accuracy is inferior and decreases to about ± 70 per cent for single days.

A better agreement between observed and computed evaporation was obtained for A periods only. The errors in the water budget were then minimal, and the differences should be representative of the energy-budget variations alone. Figure 87 represents such an analysis. As expected, the standard error of estimate becomes less, having a value of ± 0.036 cm for a 10-day period; expressed as a percentage of the mean energy-budget evaporation, the error is ± 6.4.

At Lake Hefner there is a season of high evaporation and one of low evaporation, occurring during the summer and fall, June through November, and during the winter and spring, December through May, respectively. When considering accuracies in terms of percentages of the mean evaporation, the results differ for the two seasons. The results for the high-evaporation season are shown in figure 88. Comparing figure 88 with figure 87 shows that the standard error of estimate for the high-evaporation season is about the same as for all A period data, but the standard error of estimate expressed as a percentage of the mean energy-budget evaporation decreases. For periods of 10 days the standard error of estimate is approximately ± 5 per cent of the mean energy-budget evaporation. For the low-evaporation season, the data are insufficient for statistical treatment. However, they do indicate that the standard error of estimate would be about the same as for the high-evaporation season, but the percentage error considerably larger. For example, for single-day periods, the standard error of estimate was ± 72 per cent of the mean, whereas in the case of the high-evaporation season it was ± 47 per cent.

It is evident that it is difficult to assign a particular percentage error to the energy-budget-evaporation determinations unless the basis of comparison is clearly indicated. It may be concluded that comparisons with A water-budget periods indicate that:

TABLE 19. Comparison of Energy-Budget and Water-Budget Evaporation for Periods of 12 to 30 Days.

Length of Period	Classification	Water Budget (cm)	Energy Budget (cm)	Difference (cm)	Difference (per cent)
12 days	A	0.425	0.421	—0.006	—0.9
	B	0.614	0.578	—0.036	—5.9
	A	0.494	0.398	—0.096	—19.5
	B	0.773	0.654	—0.119	—15.4
	A	0.392	0.360	—0.032	—8.2
	A	0.361	0.261	—0.100	—27.6
	A	0.139	0.249	0.110	79.0
	B	0.108	0.257	0.149	138.0
	B	0.129	0.336	0.207	161.0
	A	0.379	0.493	0.114	30.1
	A	0.606	0.723	0.117	19.3
	A	0.640	0.650	0.010	1.6
	B	0.690	0.758	0.068	9.9
	A	0.699	0.565	—0.134	—19.2
14 days	A	0.623	0.445	—0.178	—28.6
	A	0.712	0.628	—0.084	—11.8
	A	0.368	0.325	—0.043	—11.7
	A	0.325	0.356	0.031	9.5
	A	0.377	0.378	0.001	0.3
	A	0.363	0.446	0.083	22.8
	A	0.634	0.667	0.033	5.2
	A	0.698	0.670	—0.028	—4.0
16 days	A	0.578	0.511	—0.067	—11.6
	B	0.609	0.554	—0.055	—9.0
	B	0.379	0.358	—0.021	—5.5
	A	0.314	0.328	0.014	4.5
	B	0.408	0.423	0.015	3.7
	A	0.190	0.327	0.137	72.0
	A	0.704	0.761	0.047	6.7
18 days	A	0.538	0.476	—0.062	—11.5
	A	0.604	0.509	—0.095	—15.7
	B	0.392	0.354	—0.038	—9.7
	B	0.431	0.447	0.016	3.7
	A	0.744	0.804	0.060	8.1
20 days	A	0.518	0.493	—0.025	—4.8
	A	0.751	0.801	0.050	6.7
25 days	A	0.551	0.482	—0.069	—12.5
	A	0.573	0.502	—0.071	—12.4
	B	0.382	0.447	0.065	17.0
30 days	A	0.518	0.482	—0.036	—7.0
	B	0.338	0.401	0.063	18.6

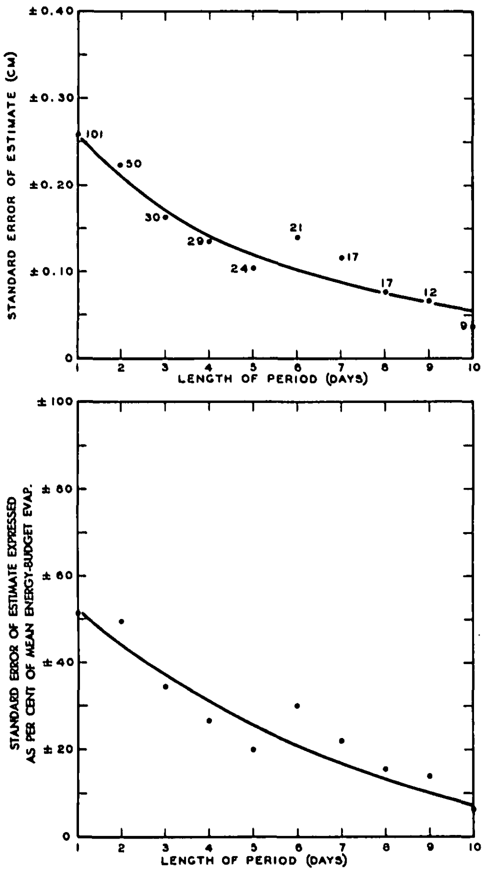


Figure 87. Accuracy of energy-budget evaporation (A periods only) determinations as a function of length of computational period. Figure indicates number of A periods included.

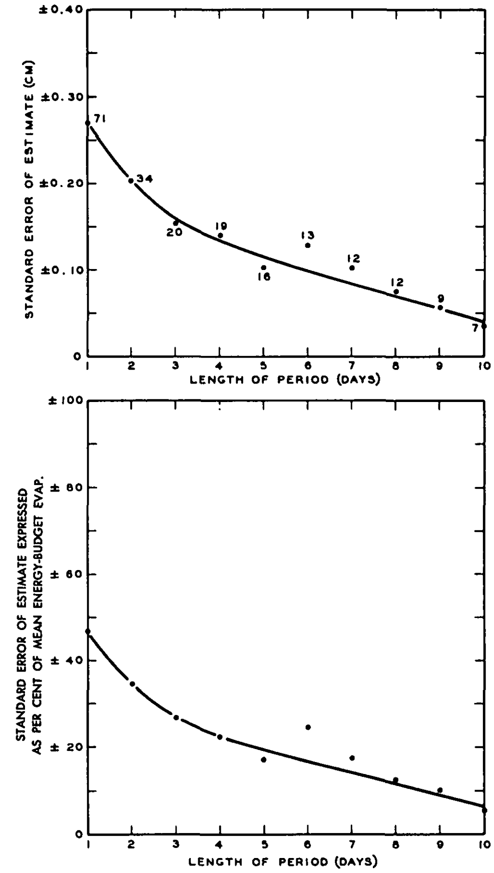


Figure 88. Accuracy of energy-budget evaporation (high-evaporation months and A periods only) determinations as a function of the length of computational period. Figure indicates the number of periods included.

(1) the standard error of estimate over a period of a year, a high-evaporation period, or a low-evaporation period is about 0.250 cm for daily periods, decreasing rapidly to about ±0.140 cm for 4-day periods, and then less rapidly to about ±0.050 cm for 10-day periods; (2) very little additional improvement occurs for periods longer than 10 days; (3) the standard error of estimate expressed as a percentage of the mean energy-budget evaporation varies, depending upon how the mean energy-budget evaporation is selected. For mean energy-budget evaporations obtained for a complete year of data, for the high-evaporation season, and for the low-evaporation season, the error for daily periods is about ±52, ±47, and ±72 per cent, respectively; while for 10-day periods it is about ±7, ±5, and ±40 (estimated) per cent respectively.

The above comparisons of the water-budget and energy-budget evaporations are characterized by a considerable scattering of the individual points, with the scattering decreasing as the length of the period increases. An inspection of the energy-budget equation, equation (72), indicates that errors in evaluating the Bowen ratio and/or the change in energy storage could introduce such scattering.

The Bowen ratio has already been discussed in considerable detail and the conclusion reached that the ratio is sufficiently accurate. However, during periods when the difference between vapor pressure of the air and that of air saturated at the water temperature is small, considerable error is introduced because of instrumental inaccuracies. Such conditions occurred rather infrequently and only during periods of low evaporation; hence, they could not be responsible for the above scattering.

By inference, then, the scattering can be explained in terms of the errors in the technique used in evaluating the change of energy storage. During the Lake Hefner observational program, attempts were made to obtain complete thermal surveys every 7 days, although the average time interval was probably closer to 9 days; single temperature soundings were made on as many other days as possible. The time of determinations rarely coincided with the evaporation periods, since the latter were selected chronologically. In general, any error in the change in energy storage would decrease as the number of days in the time interval increased, since the error is not cumulative. Hence, the agreement between the energy-budget and water-budget evaporations will improve as the length of the period increases. This improvement was observed.

To illustrate the effect of errors in determinations of change in energy storage, the energy-budget evaporation was computed for periods that coincided with the times of the thermal cruises, minimizing, of course, the error attributable to the change in energy storage. The periods varied from 7 to 19 days, the majority being 8 or 9 days long. It must be remembered that the errors inherent in the technique employed to evaluate this term, as previously pointed out, may introduce considerable error for periods shorter than 7 days. Hence, periods shorter than 7 days were eliminated from consideration. Data for nine A periods and four B periods were available, the pertinent data being given in table 20 and in figure 89. Of the A periods, seven occurred during the high-evaporation season and two during the low-evaporation season. The standard error of estimate for the nine A periods was ±0.044 cm or ±9.1 per cent of the mean energy-budget evaporation; for the seven A periods during the high-evaporation months, the standard error of estimate was ±0.028 cm or ±5.5 per cent of the mean energy-budget evaporation.

The significance of the above results as related to errors in the evaluation of the change in energy storage is obscured by the variation in the length of the periods. Of more interest are the results obtained during the five 8- and 9-day A periods. The standard error of estimate for these periods was ±0.023 cm or ±4.4 per cent of the mean energy-budget evaporation. In comparison, the standard error of estimate obtained for all 8- and 9-day periods was ±0.077 and ±0.066 cm, respectively, or ±15.5 and ±13.7 per cent of the mean energy-budget evaporation. These comparisons indicate a definite improvement related to the better evaluation of the change in energy storage.

It is concluded that evaluation of the change in energy storage is the primary limitation to the accuracy of evaporation determinations utilizing the energy-budget technique for periods shorter than a year, while the Bowen ratio is a secondary limitation of importance only during periods of low evaporation and in specific circumstances which occur infrequently. In addition, for periods of greater than 7 days, optimum accuracies of about ±0.030 cm or ±5.8 per cent of the mean evaporation can be expected during the high-evaporation months from the energy-budget technique.

Summarizing, comparisons of evaporation determinations at Lake Hefner by the energy-budget and water-budget methods result in the following major conclusions: (1) the energy-budget equation must be used with caution for periods of less than

TABLE 20. Pertinent Data for Evaporation Periods Coinciding with Thermal Cruises.

Date	Length (days)	Classification	Water Budget Evap. (cm)	Energy Budget Evap. (cm)	Difference (cm)	Per Cent
14-22 Sep 1950	9	A	0.372	0.372	0.000	0.0
8-17 Oct	10	A	0.533	0.493	—0.040	—7.5
17-25 Oct	9	A	0.384	0.396	0.012	3.1
2-20 Nov	19	A	0.468	0.496	—0.028	—6.0
3-12 Jan 1951	10	A	0.371	0.290	—0.081	—21.8
4-10 Mar	7	B	0.306	0.350	0.044	14.4
10-19 Mar	10	B	0.346	0.406	0.060	17.3
3-16 Apr	14	A	0.417	0.481	0.064	15.3
16-23 Apr	8	B	0.283	0.374	0.091	32.2
12-19 Jun	8	A	0.394	0.349	—0.045	—11.4
2-10 Aug	9	B	0.746	0.878	0.132	17.7
15-22 Aug	8	A	0.703	0.708	0.005	0.7
22-30 Aug	9	A	0.779	0.780	0.001	0.1

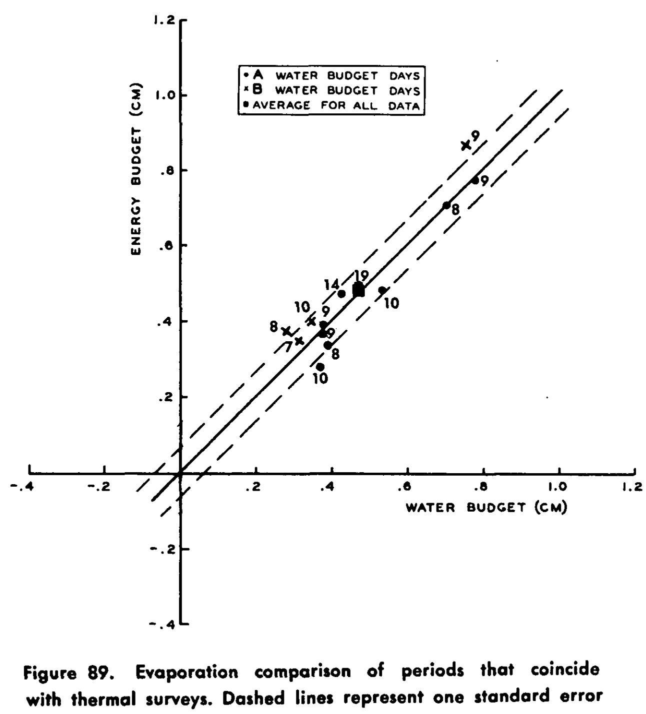


Figure 89. Evaporation comparison of periods that coincide with thermal surveys. Dashed lines represent one standard error of estimate.

7 days; (2) when applied to periods greater than 7 days, it will yield an accuracy approaching ±5.0 per cent of the mean energy-budget evaporation, providing all terms are evaluated with the utmost care, particularly the change in energy storage.

CONCLUSIONS FROM ENERGY-BUDGET STUDIES

1. The classical energy-budget equation for the oceans, and as applied in the past to lakes and storage reservoirs, must be modified to correct for the advected loss of the evaporated water. Neglect of this term will cause an error in the computed evaporation of the order of 5 per cent, depending upon the magnitude of the other terms in the equation.

2. For computing evaporation, indirect methods of evaluating solar radiation, such as those proposed by Mosby and Kennedy, are inadequate. It is necessary to evaluate solar radiation by direct measurement to obtain the requisite accuracy.

3. Theoretically, the reflectivity of an optically flat water surface under a clear sky, for solar radiation, is a function of sun altitude and atmospheric turbidity. In addition, previous evidence (Munk, 1947) indicated that wind speed may be a factor. The Lake Hefner observations showed that the effects of wind and air-mass turbidity are negligible, and hence that reflectivity is primarily a function of sun altitude.

4. Under cloudy skies, the reflectivity is uniformly modified by an increase in scattering, caused by an increasing amount and a decreasing height of the cloud cover.

5. Solar radiation is completely diffused only under conditions of low sun altitude and low overcast clouds. Under these conditions the theoretical reflectivity for completely diffuse radiation is approached.

6. Simple, empirical, hyperbolic relationships can be used to express the variation of reflectivity with sun altitude for any given cloud condition. These relationships can be used to evaluate indirectly the reflected solar radiation with enough accuracy for purposes of evaporation computation.

7. A linear function expresses the relation between atmospheric radiation and local vapor pressure (over the vapor-pressure range generally observed in nature) just as adequately as the more complicated expressions of Brunt and Ångström.

8. Empirical relations between atmospheric radiation and local vapor pressure may be used if 10-per-cent accuracy in computed atmospheric radiation is acceptable, provided they are used in areas of air-mass structure similar to that of the area where the original observations were recorded. If they are extended to other areas with no consideration of the air-masses involved, the accuracy of the result is questionable. For purposes of evaporation determination, empirical evaluations are inadequate for evaluating atmospheric radiation.

9. To obtain a general empirical method for evaluating atmospheric radiation it is necessary to consider the total vapor content of the atmosphere as the moisture variable, rather than the local vapor pressure (that is, as determined by standard observations).

10. The emissivity of a natural water surface is independent of water temperature and the composition of the water. The emissivity has been found to be 0.970 ± 0.005.

11. The evaluation of advected energy for any particular body of water is an individual problem. It is possible for certain situations, as in "run of the river" lakes and reservoirs, that inability to evaluate this term adequately will prohibit the use of the energy-budget approach to the determination of evaporation.

12. Errors in evaluating advected energy may be minimized through proper selection of the base temperature.

13. The Lake Hefner data indicate that the energy storage can be evaluated from an average of several profiles taken in different depths of water.

14. The Lake Hefner data indicate that the Bowen ratio is valid and gives consistent results except when the difference between the vapor pressure of the atmosphere and that of air saturated at the water-surface temperature is small.

15. No error seems to be introduced into evaporation computations by neglect of the effects of radiative diffusivity, stability of the air, and spray.

16. Because of limits in the accuracy of the temperature measurements, considerable error may occur in evaluating the change of energy storage over time intervals of a few days. However, for time intervals of the order of 7 to 10 days, the accuracy is probably adequate for evaporation determinations. Over annual cycles the change in energy storage is negligible.

17. The energy-budget equation, when applied to periods greater than 7 days, will result in a maximum accuracy approaching ±5 per cent of the mean energy-budget evaporation, providing all terms in the energy budget have been evaluated with the utmost accuracy, particularly change in energy storage.

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Cummings Radiation Integrator

by G. Earl Harbeck, Jr.*

Application of the energy-budget technique to the determination of evaporation from lakes and reservoirs was advocated for many years by the late N. W. Cummings, of San Bernardino Valley Junior College, San Bernardino, California. He suggested the use of a thermally insulated pan to measure certain net radiation items, and employed such a pan to compute evaporation from Bear Lake in Utah and Idaho in the summer of 1937 (Cummings, 1940). A water-budget control for this experiment was lacking, however, and Cummings could only compare his results with those obtained from a nearby Weather Bureau Class A pan.**

Shortly before Cummings' death, he outlined in a personal communication to the writer his suggested design of an insulated pan to be constructed and operated as a part of the Lake Hefner project. Cummings' proposed design was studied and the basic features incorporated in the instrument constructed at Lake Hefner.

THEORY

The energy-budget for a lake or reservoir is generally expressed as

$$Q_s - Q_r - Q_b - Q_h - Q_e + Q_v - Q_w = Q_\vartheta \quad (59)$$

in which Q_s = solar radiation incident to the water surface,

Q_r = reflected solar radiation,

Q_b = net energy lost by the body of water through the exchange of long-wave radiation between the atmosphere and body of water,

Q_h = energy conducted from the body of water by the atmosphere as sensible heat,

Q_e = energy utilized by evaporation,

Q_v = net energy advected into the body of water,

Q_w = energy advected by the evaporated water, and

Q_ϑ = increase in energy stored in the body of water.

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** See next section for discussion of Class A pans.

The effective back radiation, Q_b, is considered to be the net exchange of long-wave radiation between the atmosphere and the body of water, and for the purposes of this discussion can be separated into its various components as follows:

$$Q_b = Q_{bs} - Q_a + Q_{ar}, \quad (96)$$

in which Q_{bs} = long-wave radiation emitted by the body of water,

Q_a = incoming long-wave radiation from the atmosphere, and

Q_{ar} = reflected long-wave radiation.

Substituting in equation (59), we obtain

$$Q_s - Q_r + Q_a - Q_{ar} - Q_{bs} - Q_h - Q_e + Q_v - Q_w = Q_\vartheta. \quad (97)$$

Consider, then, a pan placed near the shore of the lake. Since the pan is in effect merely a small body of water, it will also have an energy budget which may be expressed as

$$Q_s' - Q_r' + Q_a' - Q_{ar}' - Q_{bs}' - Q_h' - Q_e' + Q_v' - Q_w' = Q_\vartheta' \quad (98)$$

in which the primed symbols refer to the pan.

The basic assumption is that the net sum of certain radiation terms is the same for the lake as for a pan on the shore of the lake, as follows:

$$Q_s' - Q_r' + Q_a' - Q_{ar}' = Q_s - Q_r + Q_a - Q_{ar}. \quad (99)$$

Over a short period of time, such as an hour, it is probably not true that long-wave and short-wave incoming radiations ($Q_s + Q_a$) are the same for the pan and the lake because of transient cloud effects, but for longer periods, such as a day or week, the assumption appears reasonable. Since the amount of reflected long-wave radiation, Q_{ar}, is dependent only on the emissivity, or rather the absorptivity, of the water, there appears to be no reason to question the validity of the assumption as far as this item is concerned, providing lake water is used in the pan and does not become contaminated with substances that might change its emissivity. The validity of the assumption that reflected solar radiation is the same for the lake as for the pan is not as obvious. In the discussion of reflectivity of a natural water surface (see preceding section entitled "Energy-Budget

Studies") it was concluded that reflectivity was independent of wind speed and, therefore, also of surface roughness. Therefore, the amount of solar energy reflected from the relatively smooth surface of the water in the pan should not be materially different from that reflected from the usually rougher lake surface. The possibility that some solar radiation may be reflected by the bottom of the pan has not been taken into account. However, it should be remembered that reflected solar radiation is a relatively small item compared with other items in the energy budget. During the period of observation at Lake Hefner, daily reflected solar radiation ranged between approximately 1 and 55 cal cm^{-2} day^{-1}, as compared with a range of approximately 50 to 750 cal cm^{-2} day^{-1} for incoming solar radiation.

Equation (98) may then be written as follows:

$$Q_s' - Q_r' + Q_a' - Q_{ar}' = Q_e' + Q_h' + Q_{bs}' - Q_v' + Q_\vartheta' + Q_w' \qquad (100)$$

Using the familiar relations $R' = Q_h'/Q_e'$, $Q_e' = \rho F' L'$, $Q_h' = \rho F' R' L'$, and $Q_w' = \rho c F (T_0 - T_b)$ and using a base temperature, T_b, of 0°C, equation (100) becomes

$$Q_s' - Q_r' + Q_a' - Q_{ar}' = \rho F' [L' (1 + R') + cT_0'] + Q_{bs}' - Q_v' + Q_\vartheta'. \qquad (101)$$

Combining equations (97), (99), and (101), and using the relations $Q_e = \rho FL$, $Q_h = \rho FRL$, $Q_w = \rho cF (T_0 - T_b)$, $T_b = 0°C$ and the close approximations $c = \rho = 1$,

$$F = \frac{F' [L' (1 + R') + T_0'] + (Q_{bs}' - Q_v' + Q_\vartheta') - (Q_{bs} - Q_v + Q_\vartheta)}{L (1 + R) + T_0}. \qquad (102)$$

INSTRUMENTS AND METHODS

The Cummings Radiation Integrator (CRI) was constructed as indicated in figure 90. Several features of its construction are worthy of note. The insulation was designed to limit the heat loss or gain by conduction to 3 per cent of the energy required to evaporate 0.1 inch of water per day, assuming a temperature gradient of 10°F through the insulation. Actually the average daily heat loss through the insulation did not exceed 0.5 per cent of the energy used for evaporation, principally because the observed temperature gradients were much less than had been assumed. A wooden platform covered with building paper was laid on a carefully leveled site. A block of Perlite concrete 24 inches thick was constructed on this platform as a base for the pan. It would have been preferable to use a more efficient insulating material for the base, such as rock wool, but it was obvious that the insulating effect of rock wool would be destroyed by the packing down resulting from the load of more than 1300 pounds of water alone. Perlite concrete appeared to be an acceptable compromise, for its structural strength was adequate and its thermal conductivity fairly low. After the base had been poured and the forms removed, it was covered with building paper, waterproofed with a bituminous coating, and then painted with aluminum paint. Because Perlite concrete is porous and its thermal efficiency seriously impaired by moisture, it was considered essential that the base block be placed above ground and waterproofed. When the pan was dismantled at the completion of the project, the Perlite block was dry, indicating that its insulation properties had been unimpaired.

The galvanized sheet-metal pan was 48 inches in diameter and 24 inches deep. It was surrounded by an 88-inch-diameter sheet-metal ring, and the 20-inch space between the pan and ring filled with rock wool. The rock wool was covered by an annular sheet-metal roof or rim which sloped toward the outside to shed rain. The roof was fastened to the outside ring, but was separated from the pan to prevent thermal conduction.

The rim projected inward over the pan for a distance of 2 inches, and was provided with a ½-inch lip to prevent driving rain from entering the air gap between the pan and the rim. The purpose of the overhanging rim was to maintain the area of the water surface exposed to solar radiation as nearly constant as possible. Obviously, for small sun angles, some of the incoming radiation was reflected from the side of the pan but, at least during the middle of the day when solar radiation was greatest, the water-surface area remained constant.

A fixed-point gage in a small stilling well was affixed to the side of the pan. The point of the gage was placed approximately 4 inches below the rim to allow for storm rainfall; overflow was to be guarded against to prevent wetting of the rock-wool insulation.

It was originally contemplated that the pan be serviced at intervals of approximately one week. The weekly schedule was adhered to for a time, but later it became evident that pan servicings should be timed to coincide with thermal surveys of the lake, which were made at somewhat irregular intervals because of weather conditions. When the pan was serviced, the water was stirred thoroughly with a stick and its

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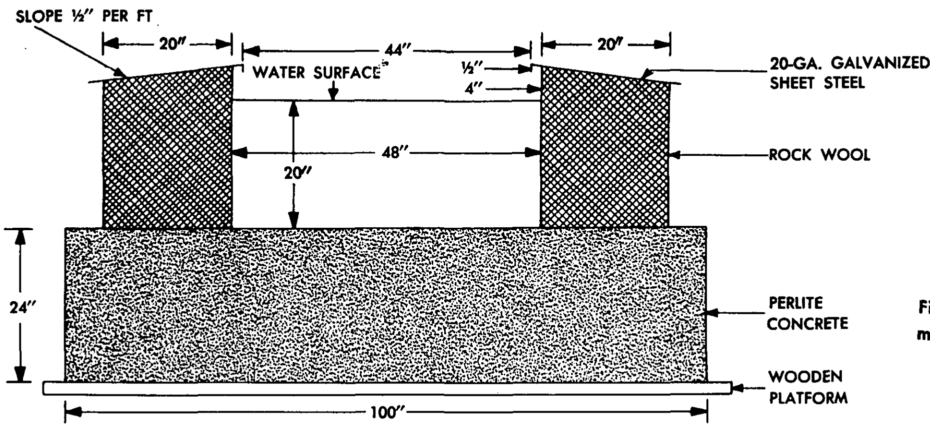

Figure 90. Cross-section of Cummings Radiation Integrator.

temperature then measured with a calibrated thermometer. Enough water was then added to bring the level to the point of the gage; both the quantity and temperature of water added were carefully measured. The water in the pan was stirred again and its temperature measured. On some occasions, it was necessary to remove storm rainfall to bring the water level to the point of the gage, but the procedure was similar.

Using the average temperature as found and the volume and temperature of the water added, the theoretical temperature of the full pan was computed and compared with the temperature measured after the pan had been filled and stirred. In no instance did the theoretical and observed temperatures differ by more than 0.3°C, and the difference was usually less than 0.1°C.

The CRI was located in the enclosure at the south meteorological station. It was placed in operation on 2 November 1950, and weekly observations were made until 30 November, when it was deemed advisable to remove the water from the pan to prevent possible damage from freezing. The pan was refilled on 7 March 1951 and observations were made until 31 August 1951. After a preliminary analysis of the data obtained in November, March, and April, it was evident that certain additional information was needed. Using equipment similar to that employed for temperature measurements at the meteorological station, thermocouples were installed to measure (1) the water-surface temperature in the pan, (2) the temperature of the underside of the overhanging rim, (3) the temperature of the soil under the pan, and (4) rainfall temperature. A reduced chart speed was used and each temperature was measured once each half hour.

The thermocouple for water-surface temperature was suspended from a small float in the center of the pan and was adjusted to remain just under the surface of the water. The rim thermocouple was placed against the underside of the steel rim but, because of the plastic insulation on the thermocouple junction, it was not in direct contact with the steel. The thermocouple for measuring soil temperature was buried ⅛ inch in the soil under the pan.

The instrument used for measuring rainfall temperature was essentially a small rain gage with a thermocouple in the storage receptacle. A plastic funnel having a maximum diameter of 4-13/16 inches and a throat diameter of 7/16 inch was used. A small plastic cup, ⅞ inch in diameter and ½ inch in height, was placed about 3 inches below the funnel. A thermocouple was placed in the cup. It is believed that during rainless periods the temperature indicated by the thermocouple would be, on a daily basis at least, not greatly different from the temperature of the ambient air. When precipitation occurred, however, it was reasoned that the funnel and cup would soon reach the temperature of the rain. The cup was purposely made very small so that it would soon fill and run over. It was placed 3 inches below the funnel in order that the falling water would splash in the cup, thus assuring thorough mixing. Even a rainfall of only 0.02 inch was sufficient to cause the cup to overflow, but little importance was attached to record temperatures of light rains because of the possibility that equilibrium had not been reached.

The analysis of the data was made using in general the methods described in the preceding section. The method used in computing certain items of the energy budget for the CRI are believed to be of interest, however.

Although evaporation takes place over the entire water surface of the CRI, the effective area for incoming radiation is only the area encompassed by the rim. The sum of the four radiation items in equation (100) (which is the basic equation for the energy budget of the CRI) is needed for the solution of equation (97). These must be computed in terms of the area encompassed by the rim, whereas each of the terms on the right side of equation (100), $Q_e' + Q_h' + Q_{bs}' - Q_v' + Q_\vartheta' + Q_w'$, must be evaluated on the basis of the entire water surface area. The sum of these last items must then be divided by the effective area for incoming radiation to determine the sum of $Q_s' - Q_r' + Q_a' - Q_{ar}'$ on a unit-area basis.

The computation of Q_{bs}', the outgoing long-wave radiation from the water surface in the CRI, was complicated by the effect of the overhanging rim. Temperature records obtained during the period May to August indicate that the rim temperature was generally 1° to 5°C higher than the water-surface temperature, and the interchange of energy between the water and the rim could not be neglected. The underside of the rim was painted black, using the same paint employed to blacken the flat-plate radiometer, and its long-wave emissivity was taken as 0.90. The preliminary value of 0.967 was used for emissivity of water. The amounts of energy being received, emitted, absorbed, and reflected by both the water and the rim were computed by successive approximations. In general, the result was a net gain for the water. The rim temperature was higher and its emissivity lower, but its reflectivity was also higher. The net Q_{bs}' for the entire pan was then computed as the difference between that being emitted by the water and that being received from the rim.

The computation of Q_v', the advected energy, was simple, since energy could be brought in only by rainfall or through the sides and bottom of the CRI. The volume of advected rainfall was determined by multiplying the depth of rainfall recorded at the south meteorological station by the area encompassed by the rim. The temperature of the rainfall was measured using the device described above. The results obtained for two selected storms are shown in figure 91. After the rainfall had cooled the small receiving cup, the indicated rainfall temperature remained fairly constant. Wet-bulb temperatures recorded at the 8-meter level are also shown in figure 91, and it is apparent that they agree closely with rainfall temperatures. Other investigators have previously obtained similar results so it appears unnecessary to discuss the matter at length. As far as the CRI is concerned, energy brought in by rainfall is usually a relatively small item in the energy budget, and rainfall-temperature measurements were made only to substantiate the generally accepted conclusion that rainfall and wet-bulb temperatures are approximately equal.

The heat exchange through the insulated sides and bottom of the pan was also a minor item. The thermal conductivity of the Perlite concrete was taken as the figure supplied by the manufacturer for the particular mix used, and the temperature gradient was taken as the difference between the water temperature and the temperature of the soil under the pan. Calculations of the heat exchange through the rock wool were made in similar fashion, the temperature gradient being taken as the difference between the water temperature and the dry-bulb temperature.

The change in energy storage in the CRI, Q_ϑ, was computed from volumes and temperatures measured each time the pan was serviced. Compared to a similar determination for Lake Hefner, it was a simple procedure.

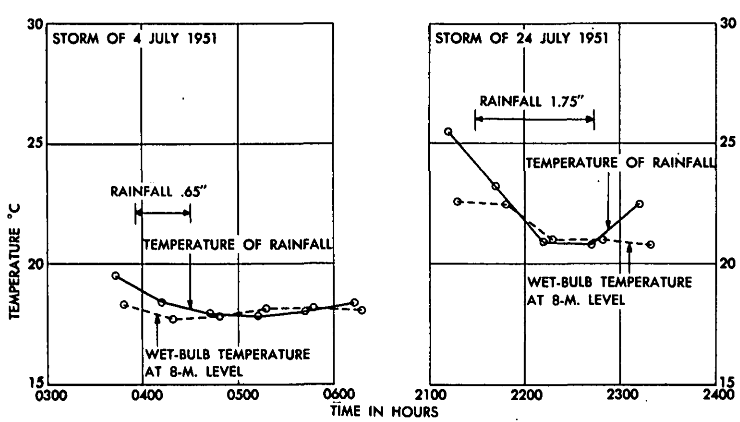

Figure 91. Comparison between rainfall and wet-bulb temperatures for selected storms.

The Bowen ratio for the lake, *R*, was computed using dry- and wet-bulb temperatures measured at the 2-meter level at the barge. Water-surface temperatures were also taken from records obtained at the barge. The Bowen ratio for the pan, *R'*, was computed using the same wet- and dry-bulb temperatures, and the temperature of the water surface in the pan. Wet-bulb, dry-bulb, and water-surface temperatures were averaged for each period. Vapor pressures were computed using these average temperatures. Although the relation between vapor pressure and temperature is not linear, for a week selected at random the vapor-pressure difference computed using average temperatures differed by only 2 per cent from the average of the vapor-pressure differences computed using the 3-hourly data.

Thermocouples for measuring water-surface and soil temperatures were installed in the CRI on 1 May 1951 and those for measuring rim and rainfall temperatures were installed 25 May 1951. Of these, the water-surface temperature measurement was by far the most important. The amount of long-wave radiation emitted by the water in the pan, which depends only on the water-surface temperature, is a major item in the energy budget for the pan. Its importance relative to that of other items was not recognized at the time observation began. An attempt was made to estimate water temperatures during the period before measurements were begun, but it was considered that the possible resultant error might be so large as to cause the results to be questionable. Rim temperatures during part of May were taken as equal to the air temperature at the 2-meter level at the south station.

RESULTS

Since the CRI was suggested as a possible replacement for the more expensive and complicated radiation instruments, it is necessary to show whether it measures the total radiation received, $Q_s - Q_r + Q_a - Q_{ar}$, with sufficient accuracy. Using all periods for which radiation data were sufficiently complete, the sum of the four radiation items was computed from the CRI records and from the radiation records. The results are shown in table 21. Although the general agreement is excellent, it should be remembered that a relatively small error in measuring radiation may affect computed evaporation considerably. For example, from the data in table 21, the standard error of measuring net radiation using the CRI is 36 cal cm^{-2} day^{-1}, which is only about 3 per cent of the mean. An error of this magnitude would result in an error of about 9 per cent in evaporation if it is assumed that the radiation instruments are without error.

TABLE 21. Comparison Between Average Net Incoming Radiation Measured by Cummings Radiation Integrator and by Pyrheliometers and Flat-plate Radiometer

Period	Net Incoming Radiation: Computed from CRI Records (cal cm^{-2} day^{-1})	Net Incoming Radiation: Measured Using Radiation Instruments (cal cm^{-2} day^{-1})
12-19 Jun	1310	1349
28 Jun-9 Jul	1296	1295
17-24 Jul	1434	1418
24 Jul-2 Aug	1281	1366
2-10 Aug	1357	1364
15-22 Aug	1282	1295
22-29 Aug	1306	1307
Weighted average	1321	1340

Because of possible large errors resulting from the lack of measurements of the temperature of the water surface in the pan, evaporation was not computed for periods in November, March, and April. An attempt was made to compute evaporation for each period between CRI servicings, which were made approximately once each week. After data for the first five months had been analyzed, it was discovered that CRI servicings should have been timed to coincide with thermal surveys of the lake. The change in energy storage, Q_θ was found to be of considerable importance particularly for short periods. The day-to-day change in storage in the lake was obviously erratic, and values interpolated between thermal surveys were therefore questionable. After this had been recognized, CRI servicings were made immediately after a successful thermal survey of the lake had been completed.

It was deemed advisable to group certain of the CRI servicing periods to obtain a smaller number of longer periods for which the change in energy storage in the lake was sufficiently well defined by thermal surveys. Commencing in June 1951, however, the coincident periods averaged slightly less than 8 days in length.

Evaporation was computed for 13 periods ranging in length from 5 to 20 days. The results are shown in table 22 and figure 92. For all periods, the mean daily evaporation as determined using the CRI

TABLE 22. Comparison Between Computed and Observed Evaporation from Lake Hefner for Periods of Various Length

Period	Length of Period (days)	Computed Evaporation (cm day^{-1})	Observed Evaporation (cm day^{-1})	Error (per cent)	Accuracy Classification
3-23 May	20	0.249	0.242	+2.9	D
23 May-5 Jun	13	.441	.415	+6.3	D
5-12 Jun	7	.242	.247	−1.0	A
12-19 Jun	7	.371	.371	0.0	A
19-28 Jun	9	.696	.566	+23.0	A
28 Jun-9 Jul	11	.593	.601	−1.3	A
9-17 Jul	8	.680	.531	+28.1	A
17-24 Jul	7	.835	.712	+17.3	A
24 Jul-2 Aug	9	.439	.516	−17.5	B
2-10 Aug	8	.911	.808	+12.7	B
10-15 Aug	5	.693	.631	+9.8	B
15-22 Aug	7	.733	.712	+2.9	B
22-29 Aug	7	.762	.766	−0.5	B
Weighted average		.544	.509	+6.9	

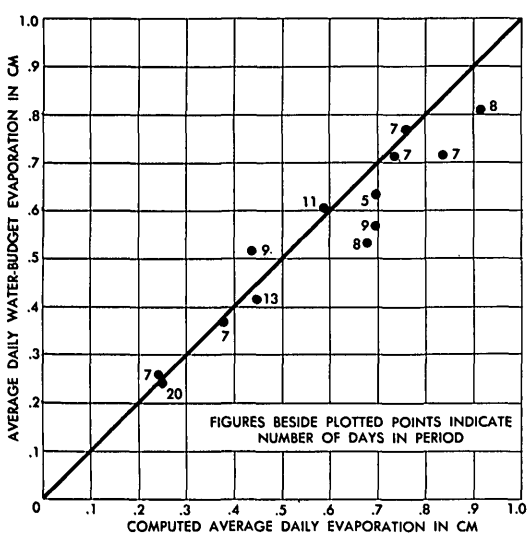

Figure 92. Comparison between computed and observed evaporation.

was 6.5 per cent greater than the corresponding water-budget evaporation, and the standard error of estimate was 0.72 cm, or 14.1 per cent of the mean. The computed results for periods of high evaporation appear to be consistently greater than those observed. No explanation can be given for this trend, if it be real, but it is suggested that it may result from incomplete knowledge of the interchange of energy between the water surface and the underside of the rim. The rim effect is not directly associated with the amount of evaporation, as might be inferred from figure 92, but rather with rim temperature, which in turn is correlated with solar radiation and thus with evaporation.

Similar computations were made using meteorological data obtained at the Weather Bureau station of the Will Rogers Airport, 13 miles south of Lake Hefner, for the determination of R and R'. There was no significant difference in the computed evaporation figures, which is not surprising, since T_a and e_a in the expression for the Bowen ratio are specified as the temperature and vapor pressure of the air before modification by the lake has occurred. It is believed that data obtained at the airport are representative of average conditions over periods of a week or longer. Moreover, this procedure indicated that it is not necessary to measure air temperature and humidity at the lake, which simplifies the instrumental requirements considerably. It should not be assumed that this procedure can be followed at any lake; but a series of sling psychrometer checks should be adequate to show whether this assumption is valid.

Table 23 illustrates the relative magnitudes of certain of the terms in equation (102). The differences between $L'(1 + R')$ and $L(1 + R)$ are highly correlated with the corresponding differences between T_0' and T_0 because the difference between R and R' depends largely on these two water-surface temperatures. The differences between $(Q_{bs}' - Q_v' + Q_\vartheta')$ and $(Q_{bs} - Q_v + Q_\vartheta)$ reflect to a large extent the change in energy storage in the lake. During the period 5-12 June, Q_ϑ was +235 cal cm^{-1} day^{-1}. If this had been neglected, computed evaporation for the period would have been 0.614 cm instead of 0.242 cm. An assumption that changes in energy storage in the lake are negligible cannot be justified, except possibly on an annual basis.

TABLE 23. Comparison Between Values of Indicated Radiation Items for Cummings Radiation Integrator and for Lake Hefner

Period	$L'(1 + R')$ (cal cm^{-1})	$L(1 + R)$ (cal cm^{-1})	T_0' (°C)	T_0 (°C)	$Q_{bs}' - Q_v' + Q_\vartheta'$ (cal cm^{-2} day^{-1})	$Q_{bs} - Q_v + Q_\vartheta$ (cal cm^{-2} day^{-1})
3-23 May	541	572	17.4	17.8	803	950
23 May-5 Jun	601	609	20.5	20.7	835	897
5-12 Jun	636	610	22.0	21.5	876	1076
12-19 Jun	617	609	24.2	24.0	876	1075
19-28 Jun	545	550	25.0	25.1	866	981
28 Jun-9 Jul	549	591	23.9	24.8	854	931
9-17 Jul	562	581	25.8	26.3	875	984
17-24 Jul	520	555	26.3	27.9	865	947
24 Jul-2 Aug	574	623	26.7	28.4	910	995
2-10 Aug	456	557	26.0	28.4	839	824
10-15 Aug	526	602	25.1	27.2	878	879
15-22 Aug	524	599	24.8	26.9	836	823
22-29 Aug	436	541	24.3	26.0	868	874

CONCLUSIONS ON CUMMINGS RADIATION INTEGRATOR

The conclusions presented here are based on only a relatively short period of observation. Further experimental work is desirable, and is believed warranted on the basis of the findings at Lake Hefner.

1. The Cummings Radiation Integrator offers considerable promise as an instrument for measuring certain net radiation items. For purposes of computing evaporation, using the energy-budget technique, the instrument appears to be a satisfactory substitute for the flat-plate radiometer and pyrheliometers. It should be emphasized that no empirical pan coefficient is involved if the CRI is used for this purpose. Only certain physical constants, whose values can be or have been established in the laboratory, are needed. It is estimated that the error in computed weekly evaporation during the summer months will ordinarily be 10 to 15 per cent or less, if measurements of energy storage in the lake are also made at weekly intervals; on a monthly basis, the error is estimated as 5 to 10 per cent. There is no apparent reason why the CRI cannot be operated in winter using a saline solution or some other liquid whose emissivity and other properties are known.

2. The problem of the interchange of energy between the water surface and the underside of the rim is as yet imperfectly understood. Elimination of the rim would be desirable if some means could be found of maintaining a completely filled pan or of satisfactorily accounting for the effect of a vertical rim. The rim effect could be minimized by using a larger pan.

3. The energy conducted to or from the CRI as sensible heat was determined by the use of the Bowen ratio. A comprehensive study of the validity of the Bowen ratio, particularly as used in connection with the CRI, would be of considerable value. The use of a wind tunnel for this study may be practical if the necessary physical measurements can be made with the requisite accuracy.

4. CRI or other radiation records obtained at a reservoir site prior to construction are desirable for making preconstruction estimates of evaporation using the energy-budget technique.

5. The use of the CRI in connection with studies of evaporation losses from vegetation should be investigated. No data are yet available as to the areal distribution of incoming long-wave radiation. Pyrheliometer records of incoming solar radiation at selected locations are available, but these are insufficient because long-wave radiation is excluded. Although the energy budget of a plant is admittedly more complex than that of a lake, the relation between total radiation received and evapotranspiration losses should be studied in relation to available moisture.

REFERENCE

Cummings, N. W., "The Evaporation-Energy Equations and their Practical Application," *American Geophysical Union, Transactions,* vol. 21, part 2, 1940, pp. 512-522.

Lake and Pan Evaporation

by Max A. Kohler*

BACKGROUND AND OBJECTIVES

The Weather Bureau has for many years collected evaporation data as observed in a standard 4-foot pan (commonly known as the Class A pan), and many additional stations equipped with pans of various types have been operated by other Federal, state, and local agencies. Admittedly, the data cannot be considered to represent evapotranspiration from adjacent land areas or evaporation from a nearby lake, but they do constitute an index to the evaporative power of the atmosphere, and thereby serve a real purpose in depicting geographical variations in climate.

Pan-evaporation data are of direct value in several fields, such as planning facilities for evapoative-cooling air conditioning, and have been widely used in an indirect manner for estimating evaporation from lakes and evapotranspiration. Estimates of lake evaporation are commonly derived by applying a "pan coefficient" to observed pan evaporation. This section is primarily devoted to a discussion of the Lake Hefner results relative to this latter application of pan data. Specifically, the objectives of the part of the Lake Hefner program devoted to pan studies were as follows:

1. Obtain further information on the relative evaporation from various types of pans.

2. Investigate the reliability of the pan-coefficient approach for estimating monthly and annual lake evaporation.

3. Derive a more reliable procedure for estimating lake evaporation from pan evaporation and related meteorological data normally collected by the Weather Bureau in its established observational programs.

To provide ready reference for comparative purposes, table 24 presents a summary of pan coefficients derived in numerous past studies. The investigator, geographical location, and reference publication are given in each case. Attention is called to the fact that in some cases a 12-foot sunken pan was assumed to be equivalent to a lake. Considering variations in climate and lake dimensions and the probable reliability of computed lake evaporation, the coefficients are reasonably consistent. Table 25, also presented for comparative purposes, summarizes results of simultaneous observations from various types of pans as ratios. The periods of record indicated in the footnotes of this table are only approximate. For example, "Apr-Oct 1940-45" would mean that observations during the 6-year period were, on the average, started by 1 April and discontinued at the end of October. Tables 24 and 25 contain all readily accessible comparative data; however, further search may reveal data which have not been included.

In the analyses in this chapter, particular attention has been directed to the Class A pan since it is the official Weather Bureau instrument. This is not to be construed as meaning that the other pans are not equally susceptible to such detailed analysis or that the Class A pan is believed to be the "best" of the group. Time was simply not available for complete analysis prior to publication of this report.

INSTRUMENTATION AND OPERATIONAL PROCEDURE

Three separate installations were mainatined around Lake Hefner to provide data for the evaporation-pan studies (see map). A plan view of each installation is shown in figure 93. The photographs in figure 94 show various aspects of the installations.

Observations at the pan stations consisted of pan evaporation, precipitation, pan-water temperature, humidity, air temperature, wind, and, at the south station only, soil moisture and temperature. Observations began on 24 April 1950, and terminated 31 August 1951, except that the BPI pan was not installed until 19 May 1950. Observations were initially made at about 1500 because of the heavy morning work load at the Oklahoma City Weather Bureau Office. Because of the inherent difficulties experienced in interpreting data from the maximum and minimum thermometers, and to terminate the day more nearly at the time of minimum evaporation, observation time was shifted to about 0900 on 11 June 1950.

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TABLE 24. Summary of Monthly and Annual Pan-to-Lake Coefficients

Investigator	Reference	Location	Years of Record	Basis of Coefficient	Class A	BPI (Sunken)	Colorado (Sunken)	Screened (Sunken)
Lippincott, J. B.	10	Lake Hodges, Calif.	1919-21	Jun-Oct[1]			0.96	
O'Neill, C. M.	12	Newell Reservoir, Canada	1919-25	Approx. May-Sep		0.95		
Rohwer, Carl	14	Ft. Collins, Colo. (85-ft reservoir)	1926-28	Apr-Nov[2]	0.70		0.79	
Sleight, R. B.	15	Denver, Colo. (12-ft pan, 3 ft deep)	1915-16[7]	Mean Annual[3]	0.67			
Sleight, R. B.	15	Denver, Colo. (12-ft pan, 3 ft deep)	1916	Jun-Oct		0.94		
White, W. N.	18	Milford Exp. Stn., Utah (pan, 12 ft x 3 ft)	1926-27	May-Oct	0.67			
Young, A. A.	21	Fullerton, Calif. (pan, 12 ft x 3 ft)	1936-39	Mean Annual	0.77	0.94	0.89	0.98
Young, A. A.	21	Lake Elsinore, Calif.	1939-41	Mean Annual	0.77			0.98
I. B. and W. C., United States and Mexico	24	Ft. McIntosh, Tex. (pan, 12 ft x 3 ft)	1950	Feb-Dec	0.72			0.89
Special Committee on[4] Irrigation Hydraulics	22	(recommendation of Subc. on Evap.)		Mean Annual	0.70		0.78	
Computed Values[5]	8, 23	Red Bluff Reservoir, Texas	1939-47	Mean Annual	0.68			
Computed Values[6]	8	Lake Okeechobee, Florida	1940-46	Mean Annual	0.81		0.98	
				Jan	0.77		0.87	
				Feb	0.69		0.83	
				Mar	0.73		0.89	
				Apr	0.84		1.00	
				May	0.82		0.97	
				Jun	0.85		1.08	
				Jul	0.91		1.19	
				Aug	0.91		1.20	
				Sep	0.85		1.05	
				Oct	0.76		0.84	
				Nov	0.71		0.77	
				Dec	0.83		0.90	
Rowher, Carl	14	Ft. Collins, Colo. (85-ft reservoir)	1927-28	Apr	0.60		0.75	
				May	0.63		0.76	
				Jun	0.69		0.77	
				Jul	0.69		0.76	
				Aug	0.71		0.75	
			1926-28	Sep	0.82		0.86	
				Oct	0.72		0.82	
				Nov	0.77		0.99	
Young, A. A.	21	Lake Elsinore, Calif.	1939-41	Jan	0.82			0.96
				Feb	0.63			0.77
				Mar	0.68			0.92
				Apr	0.66			0.87
				May	0.68			0.90
				Jun	0.77			0.99
				Jul	0.74			0.95
				Aug	0.78			1.02
				Sept	0.87			1.10
				Oct	0.93			1.12
				Nov	0.97			1.12
				Dec	0.95			1.03

[1] Fragmentary record; periods of data total 6 months; 2-inch chicken wire over pan.
[2] No data Apr-Aug 1926.
[3] Record broken for 2½ months in winter.
[4] Not an investigation, but recommendations based on study of all previous investigations.
[5] Computed from evaporation data on reservoirs and pans, respectively, from references (8) and (23); "Years of record" refers to pan data.
[6] From reference (8) and WB files; Class A pan at Belle Glade; Colorado (sunken) pan at Moore Haven.
[7] Nov-Nov.

TABLE 25. Summary of Pan-to-Pan Ratios.

Ratio = Pan X / Pan Y		PAN "X"			
		Class A	BPI (Sunken)	Colorado (Sunken)	Screened (Sunken)
PAN "Y"	Class A		0.77 Lake Kickapoo, Tex. (2) 0.83 Buchanan Dam, Tex. (3) 0.75 Denver, Colo. (4) 0.80 Balmorhea, Tex. (6) 0.78 Pardee Res., Cal. (8) 0.69 Ft. Assinniboine, Mont. (9) 0.78 Yuma Field St., Cal. (10) 0.82 Fullerton, Cal. (10) 0.68 Hays, Kans. (14) 0.77 Average	0.89 Ft. Collins, Colo. (5) 0.82 Belle Glade, Fla. (7) 0.84 Pardee Res., Cal. (8) 0.87 Fullerton, Cal. (10) 0.95 Henshaw Res., Cal. (13) 0.87 Average	0.91 Mansfield Dam, Tex. (1) 0.79 Lake Kickapoo, Tex. (2) 0.83 Buchanan Dam, Tex. (3) 0.73 Dryden, Tex. (3) 0.80 Ft. McIntosh, Tex. (3) 0.77 Yuma Fld. Sta., Cal. (10) 0.80 Fullerton, Cal. (10) 0.79 Lake Elsinore, Cal. (11) 0.75 San Jacinto, Cal. (12) 0.80 Average
	BPI (Sunken)	1.30 Lake Kickapoo, Tex. (2) 1.20 Buchanan Dam, Tex. (3) 1.33 Denver, Colo. (4) 1.25 Balmorhea, Tex. (6) 1.28 Pardee Res., Cal. (8) 1.45 Ft. Assinniboine, Mont. (9) 1.29 Yuma Fld. Sta., Cal. (10) 1.21 Fullerton, Cal. (10) 1.46 Hays, Kans. (14) 1.31 Average		1.09 San Pablo Res., Cal. (8) 1.03 Upper San Leandro Res., Cal. (8) 1.08 Pardee Res., Cal. (8) 1.05 Fullerton, Cal. (10) 1.06 Average	1.02 Lake Kickapoo, Tex. (2) 1.00 Buchanan Dam, Tex. (3) .98 Yuma Fld. St., Cal. (10) .97 Fullerton, Cal. (10) 0.99 Average
	Colorado (Sunken)	1.13 Ft. Collins, Colo. (5) 1.22 Belle Glade, Fla. (7) 1.18 Pardee Res., Cal. (8) 1.15 Fullerton, Cal. (10) 1.06 Henshaw Res., Cal. (13) 1.15 Average	0.92 San Pablo Res., Cal. (8) 0.97 Upper San Leandro Res., Cal. (8) 0.93 Pardee Res., Cal. (8) 0.95 Fullerton, Cal. (10) 0.94 Average		0.92 Fullerton, Cal. (10)
	Screened (Sunken)	1.10 Mansfield Dam, Tex. (1) 1.27 Lake Kickapoo, Tex. (2) 1.20 Buchanan Dam, Tex. (3) 1.36 Dryden, Tex. (3) 1.25 Ft. McIntosh, Tex. (3) 1.31 Yuma Fld. Sta., Cal. (10) 1.25 Fullerton, Cal. (10) 1.26 Lake Elsinore, Cal. (11) 1.34 San Jacinto, Cal. (12) 1.26 Average	0.98 Lake Kickapoo, Tex. (2) 1.00 Buchanan Dam, Tex. (3) 1.02 Yuma Fld. Sta., Cal. (10) 1.04 Fullerton, Cal. (10) 1.01 Average	1.09 Fullerton, Cal. (10)	

(1) Jan 1950-Dec 1951
(2) Apr-Oct 1950-51
(3) Jan-Dec 1950
(4) Jun-Oct 1916
(5) Sep 1926-Nov 1928
(6) Apr 1941-Dec 1948
(7) Jun 1941-Dec 1948
(8) Jan 1930-Dec 1944
(9) Apr-Sep 1949-51
(10) Jan 1937-Dec 1939
(11) Jul 1939-Dec 1943
(12) Jul 1939-May 1946
(13) Jan 1942-Dec 1943
(14) Apr-Sep 1937-1950

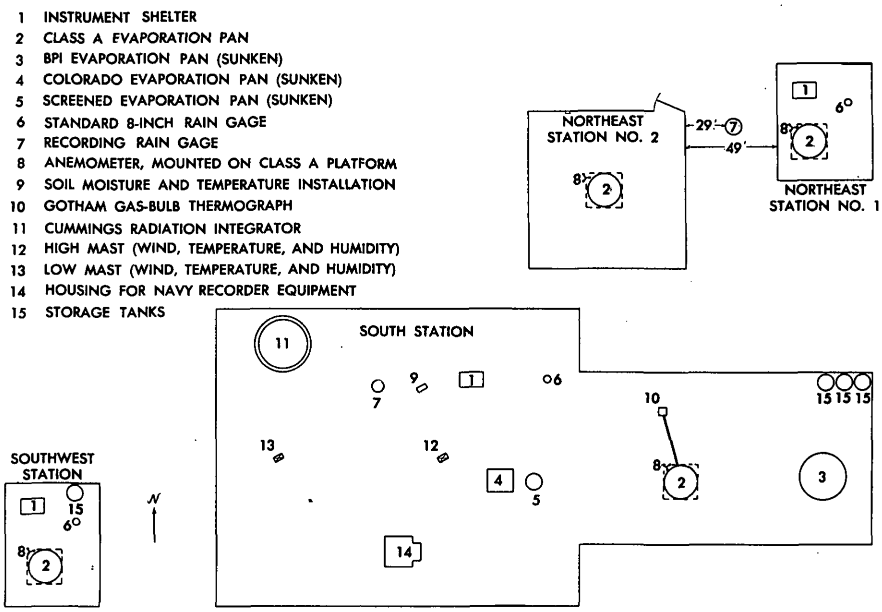


Figure 93. Plan view of pan installations at Lake Hefner.

Evaporation Pans

Class A pans were installed at each site to indicate areal variations, and additional pans of various types were installed at the south station for comparative purposes. Two Class A pans were maintained at the northeast site, one in a standard enclosure and the other more openly exposed, to test the effect of obstructions (shelter, rain gage, etc.) on observed evaporation.

The Class A pans were fabricated to standard specifications (4 ft in diameter and 10 in. deep) and mounted on standard wooden platforms. These pans were generally filled to within about 2 in. of the rim and refilled when the level had dropped approximately 1 in. The sunken screened pan was 2 ft in diameter and 3 ft deep, covered with ¼-in.-mesh screen; the Colorado sunken pan was 3 ft square and 18 in. deep; and the BPI pan was 6 ft in diameter and 2 ft deep. Each of the sunken pans was installed with the rim about 2 in. above the ground surface, and the water level was maintained at or slightly below ground level.

Water level in the pans was observed with hook gages calibrated to thousandths of an inch. Since the project was to be of limited duration, every effort was made to maintain observations as complete as possible, even during the winter. The pans were not emptied during cold weather as is customary. To avoid damage by freezing, small holes were drilled through the ice, thus relieving the pressure.

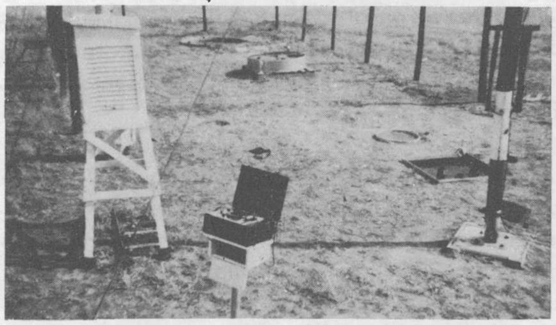
During September and early October 1950, windblown vegetation tended to collect on the windward side of the enclosures and in the sunken pans, particularly. Although an effort was made to keep the pans and fences free of vegetation, wind and pan evaporation were at times reduced somewhat. Visual observation and analysis of the data demonstrate that observed pan evaporation was occasionally in error due to blowing snow and to splash-out or spill-over during periods of heavy rain and high wind. This deficiency of the pan is discussed in considerable detail later in this section.

Water Temperatures

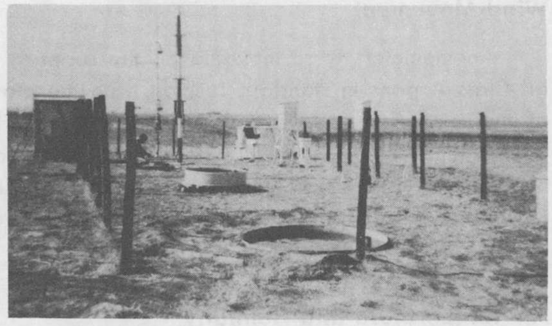
The Class A pan at the south station was equipped with a Gotham gas-bulb thermograph for recording surface-water temperature. The element was shielded from the sun and floated as near the surface as was feasible. It is believed the temperatures represent an average through about the top ¾-in. of depth. All other pans were equipped with Sixes maximum-minimum thermometers, floated at a depth of approximately ½-in. below the surface and shielded from the sun. The thermometers were removed from the pans during freezing temperatures to avoid damage.

Precipitation

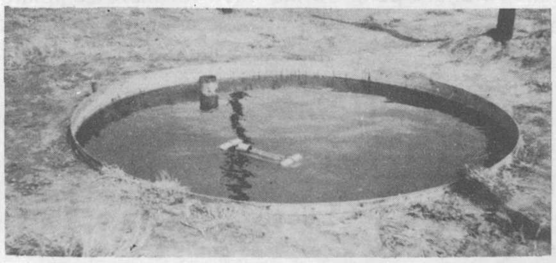
The network of precipitation gages utilized for the project is discussed in the section of this report on "Instrumentation." It will suffice to state here that standard 8-in. gages were placed at each of the three pan installations to assist in computing pan evaporation.



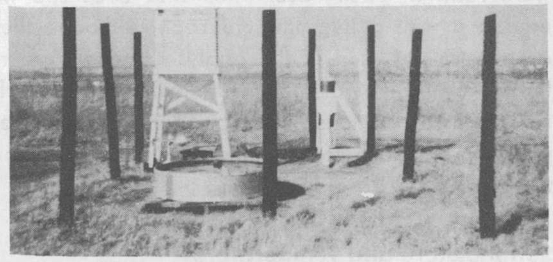
a. South station, looking east. Pans, in order of increasing distance from camera, are (1) Colorado, (2) screened, (3) Class A, and (4) Bureau of Plant Industry (BPI). Soil moisture-temperature shelter surmounted by Bouyoucos bridge in foreground.



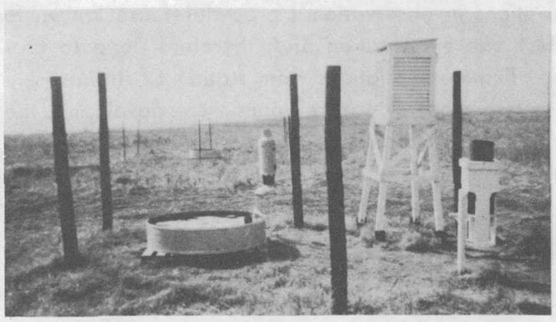
b. South station, looking west across BPI pan toward Class A pan.



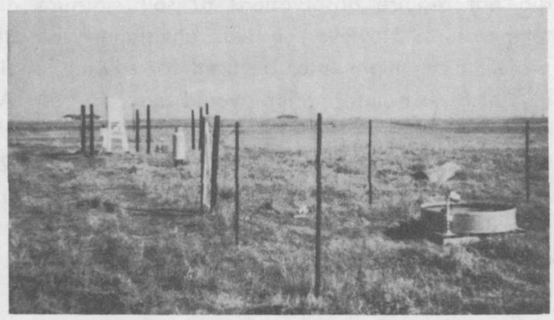
c. Showing float mounting of Sixes maximum-minimum thermometer in BPI pan.



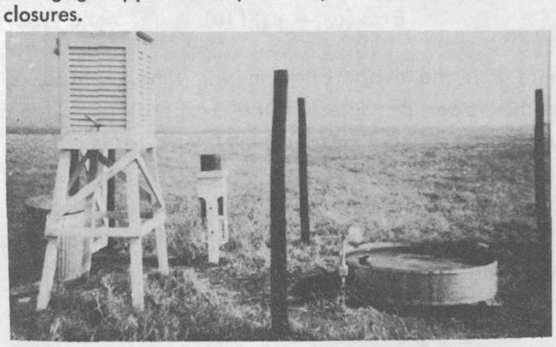
d. Northeast station no. 1, looking north.



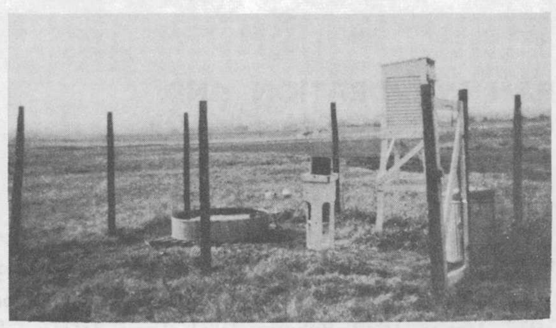
e. Northeast installation, looking west. Station no. 1 in foreground, station no. 2 in background, and recording rain gage approximately midway between the two enclosures.



f. Northeast installation, looking east. It will be noted that the pan at station no. 2 (foreground) is relatively unobstructed.



g. Southwest station, looking east.



h. Southwest station, looking west-southwest.

Figure 94. Photographs of pan installations at Lake Hefner.

Wind Movement

Anemometers were mounted on the supports of all Class A pans in standard fashion, with the plane of the cups 6 in. above the rim of the pans. The anemometers used were 3-cup totalizing (odometer) Bendix-Friez Model 349-N. Twenty-four-hour wind movement in statute miles was observed and recorded at each observation.

Air Temperature and Humidity

As is standard practice, maximum and minimum air temperatures were observed at each of the three sites. In addition, wet- and dry-bulb temperatures were read at each observation and each site was equipped with a hygrothermograph; although these data were not used in the analyses, since more reliable psychrometric data were available from the mass-transfer stations. These instruments were exposed in standard cotton region shelters (5 ft above the ground surface).

Soil Moisture and Temperature

The experimental work conducted at Lake Hefner did not require observations of soil moisture and temperature. However, it was envisioned that the project data might later be used for related studies in which such information would be required. Accordingly, thermistors and Bouyoucos blocks were installed at the south station at depths of 10, 20, 30, 50 and 100 cm. Satisfactory soil temperature records were obtained, but the soil-moisture data appear somewhat erratic, although the data have not been analyzed critically.

PAN EVAPORATION AND METEOROLOGICAL FACTORS

Numerous attempts have been made in the past to derive reliable relations between pan evaporation and meteorological factors (Bigelow, 1910; Horton, 1917; Meyer, 1942; Rohwer, 1931; Theis, 1931). Obvious purposes to be served by such relationships are:

1. Extrapolation of short-period and seasonal records.
2. Derivation of estimated data for meteorolgical stations at which pan observations are not made.
3. Tests of the reliability and representativeness of observed data.
4. Aids in studying lake-pan relations.

Over much of the country, pan observations are not made during the winter months because of subfreezing temperatures. This represents a serious deficiency in the data, particularly when studying pan-to-pan and pan-to-lake coefficients. Thus, a relation that could be used to estimate winter evaporation would be of practical value. Concerning estimates of evaporation at meteorological stations, continued and numerous requests for additional evaporation stations point up the need for more complete geographical coverage of evaporation data. Regarding the third purpose, it should be pointed out, that Weather Bureau evaporation stations are attended by cooperative observers and, under the circumstances, it is not always possible to select a truly representative site or to maintain observational procedures on a fully standard basis. Some observers, for instance, add water frequently and maintain proper water level, while others do not refill the pan until the water level has dropped far too low. Such variations in observational procedures are known to affect pan evaporation and, therefore, tend to produce fictitious variations from station to station.

Two types of pan relations were developed, the first being based on observed water-temperature data and the second utilizing an energy-balance approach.

Water-Temperature Approach

Most empirically derived equations for evaporation are similar in form to that set forth by Dalton well over a century ago, namely,

$$E = (e_0 - e_a)\, f(u), \tag{103}$$

where E is the evaporation in unit time, e_0 and e_a are the vapor pressures of the evaporating surface and the atmosphere, respectively, and $f(u)$ is a function of horizontal wind speed. Although some equations include a pressure term, variations in this factor have no significant effect on evaporation at a particular site. The form of $f(u)$ is usually assumed to be such that equation (103) becomes either

$$E = (e_0 - e_a)(a + bu) \tag{104}$$

or

$$E = c\,(e_0 - e_a)\, u^n, \tag{105}$$

where a, b, c and n are constants. Although these two equations appear quite different, they fit observed pan data almost equally well because of the limited range of the wind data. Equation (104) was fitted (least squares) to the pan data for the south station with the results shown in table 26 and figure 95. The derived constants (a and b) yield 24-hour evaporation in inches where u is the wind movement at the Class A pan in miles per day, and the atmospheric vapor pressure (e_a in inches of mercury) is observed at the 2-meter level (mass-transfer station). Values of e_a were obtained from mean 3-hourly wet- and dry-bulb temperatures and then averaged for the day. The vapor pressure of the water surface (e_0) was based on the mean 24-hour temperature (average of maximum and minimum). The derived constants are strictly applicable only when the vapor-pressure difference is computed as stated. The substitution of 6-hourly observations for 3-hourly in computing e_a should have little effect on the results. However, computation of the mean daily water temperature by any other method may produce a slight bias in computed evaporation.

Days with rain were excluded from the correlation because of the poor reliability of the evaporation data during such periods. The records prior to 12 June 1950 were excluded because of the change in time of observation, and periods when the pans were frozen could not be used because only accumulated evaporation was observed and water temperatures were not available. The fact that the Class A pan was occasionally frozen over when the sunken pans were ice-free accounts for the difference in number of observations used.

It will be noted from the table that the wind factor (b) displays but little variation from pan to pan as compared with the other constant (a). Water temperatures in the different pans, as indicated by values of $(e_0 - e_a)$, are consistent for the over-all periods used. However, with respect to comparative analysis of the data presented in table 26, it should be pointed out that the data used for the various pans were not altogether concurrent.

A pan relation based on water temperature is limited in application because water-temperature

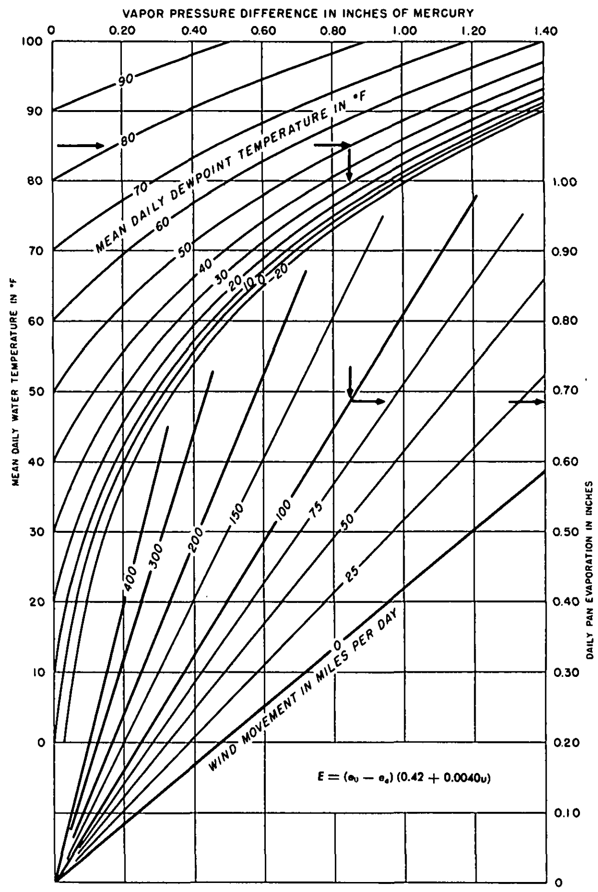


Figure 95. Relation for Class A pan at south station, Lake Hefner, using water temperature.

TABLE 26. Evaporation Relations $[E = (e_0 - e_a)(a + bu)]$ for Pans at South Station.

Pan Type	Days of Record	a	b	Correlation Index	Standard Error of Estimate: inches	Standard Error of Estimate: % of avg.	Daily Average*: Evap. (in.)	Daily Average*: $e_0 - e_a$ (in. Hg)	Daily Average*: Wind (mi/day)
Class A	266	0.417	0.0040	0.92	0.053	18	0.294	0.324	122.4
BPI	288	0.253	0.0040	0.91	0.040	19	0.211	0.295	125.0
Colorado	285	0.212	0.0044	0.89	0.050	22	0.229	0.320	126.0
Screened	292	0.104	0.0044	0.86	0.049	24	0.203	0.337	125.8

* For number of days shown in second column.

data are seldom available and, when this element is observed, pan evaporation is naturally observed as well. Nevertheless, such a relation is of value in pan-to-pan and pan-to-lake studies, as is demonstrated subsequently, and it is reasonably applicable for extended periods (e. g., monthly) if the air temperature is assumed to equal the water temperature in the pan (Linsley, *et al.*, 1949).

Energy-Balance Approach

Pan-water temperature obviously depends on a number of factors which affect the heat exchange between the sun, atmosphere, earth, and pan. From an energy approach, water temperature should correlate highly with air temperature, solar radiation, evaporation, and wind, all of which are readily observed. Thus, by assuming numerous functional relations, application of least-squares analysis should facilitate selection of a reasonably reliable regression. The required functional relation was known to be complex and of a form unsuited to least-squares analysis with water temperature as the dependent variable. Accordingly, resort was made to an approximate equation of energy balance to derive a suitable regression relation.

The concept of combining the aerodynamic and energy approaches for treatment of the evaporation problem was used by Jacobs (1942) in considering ocean evaporation, and by Penman (1948) for small water bodies. Basically, the two equations involving water temperature and evaporation are combined to eliminate the need for water-temperature observations. Assuming the change in heat storage of the water body to be negligible, Penman derived the equation

$$E = \frac{1}{\Delta + \gamma} (Q_n \Delta + \gamma E_a) \qquad (106)$$

where Δ is the slope of the saturation-vapor-pressure vs temperature curve (de_s/dT) at the air temperature T_a; E_a is the evaporation given by equation (104) or (105), assuming water temperature equal to air temperature; Q_n is the net radiant energy expressed in the same units as those of E; and γ is defined by the equation

$$R = \gamma \left(\frac{T_0 - T_a}{e_0 - e_a} \right) \qquad (107)$$

in which R is Bowen's dimensionless ratio (see section of report on "Energy-Budget Studies"). If evaporation and convective transfer of sensible heat are restricted to equivalent, identical surfaces, it can be shown that $\gamma = 0.010$ (P/P_0) inches of mercury per °F, where P and P_0 are actual and standard atmospheric pressures, respectively.

Penman used an equation in which

$$Q_n = f(Q_s, R_t, T_a, e_a, C) \qquad (108)$$

where Q_s is short-wave radiation from sun and sky, R_t is the reflection coefficient for the surface, and C is the cloud cover in tenths.

Equations (106) and (108), with the coefficients presented by Penman, are admittedly not strictly applicable to a Class A pan, but assuming the form of equation (106) to be applicable, pan data were correlated with meteorological elements by the coaxial, graphical technique (Linsley *et al.*, 1949) illustrated in figure 96. First assuming the standard value (0.01) of γ, curves were constructed in quadrants II and III (light, dashed lines) directly from equation (106) so that the entering axis of II represents $Q_n \Delta$ and the final axis of III is E. Since Δ is a function of T_a only, and this factor in conjunction with solar radiation Q_s constitutes a reasonably reliable index of Q_n, the data were plotted in quadrant I to define the relation between $Q_n \Delta$, Q_s, and T_a. That is, the chart sequence was entered in reverse order with E, T_a, and E_a [thus solving $Q_n \Delta = E\Delta + \gamma(E - E_a)$]; values of $Q_n \Delta$ were plotted against T_a in quadrant I; the points were labeled with values of Q_s; and a family of curves (light, dashed lines) was drawn to fit the plotted data. Values of E_a are those obtained from the inset relation in the upper left-hand quadrant which, incidentally, is the Class A relation of figure 95 with T_a substituted for T_0.

Since evaporation can occur only from the water surface while convective transfer of sensible heat takes place at the sides and bottom of the Class A pan as well, γ would be expected to have a value greater than that originally assumed (0.01). Accordingly, values of $Q_n \Delta$ were computed from quadrant I for each day and, using these data, the values of γ required to satisfy equation (106) were determined. The derived values of γ appeared to be independent of wind, averaging about 0.025. Curves in quadrants II and III were then revised to make $\gamma = 0.025$ (heavy, solid curves); data were again plotted in quadrant I; and the radiation curves were revised for the larger value of γ (heavy, solid curves). The final relation, based on data for 246 days without rainfall, has a correlation index of 0.96, and the standard error of estimate is 0.038 in. (13 per cent of the

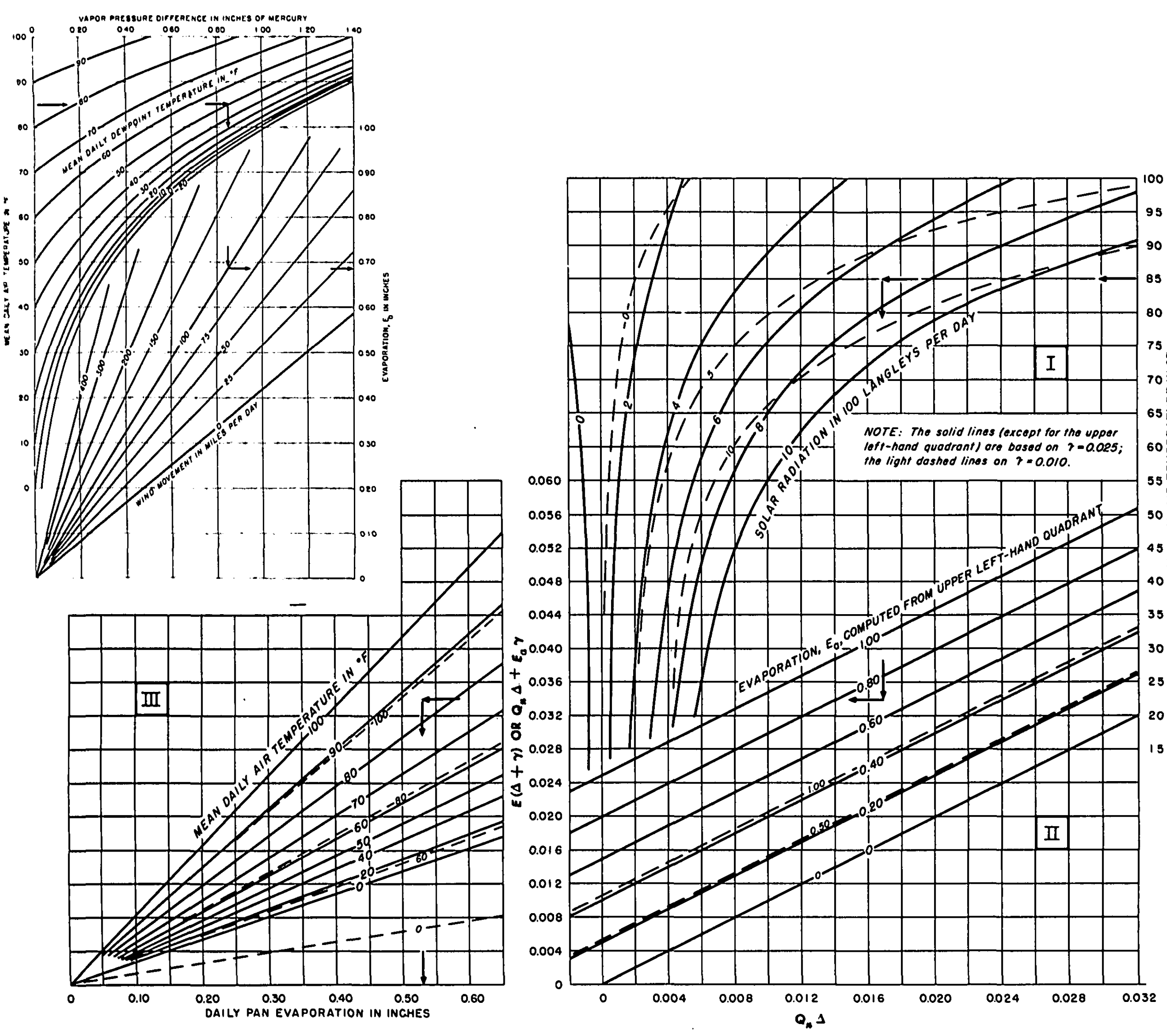


Figure 96. Relation for Class A pan at south station, Lake Hefner, using energy balance approach.

average evaporation for those days used in the analysis).

How well the relation will fit the data for other localities is not yet known, but data from eight first-order Weather Bureau stations equipped with Class A pans are being assembled for this purpose. Admittedly, γ is a function of pressure; and vapor pressure and cloud cover (equation 108) are neglected so far as Q_n is concerned. However, it is believed that these omissions will not materially affect the general applicability of the relation for low-level stations. In mountainous regions, γ will undoubtedly require adjustment for reduced pressure.

Figure 97, which is based on evaporation from the BPI pan, was derived in the same manner as figure 96. As would be expected, the value of γ for this pan is less than for the Class A pan (0.015 as compared to 0.025). It should be realized, however, that the value of γ, among other things, depends to some extent upon the type of material used in fabricating the pan because of the resulting effect on conduction.

The relation of figure 97, based on 288 days of record, has a correlation index of 0.86 and a standard error of estimate of 0.049 in. While the energy-balance approach yields a higher correlation for the

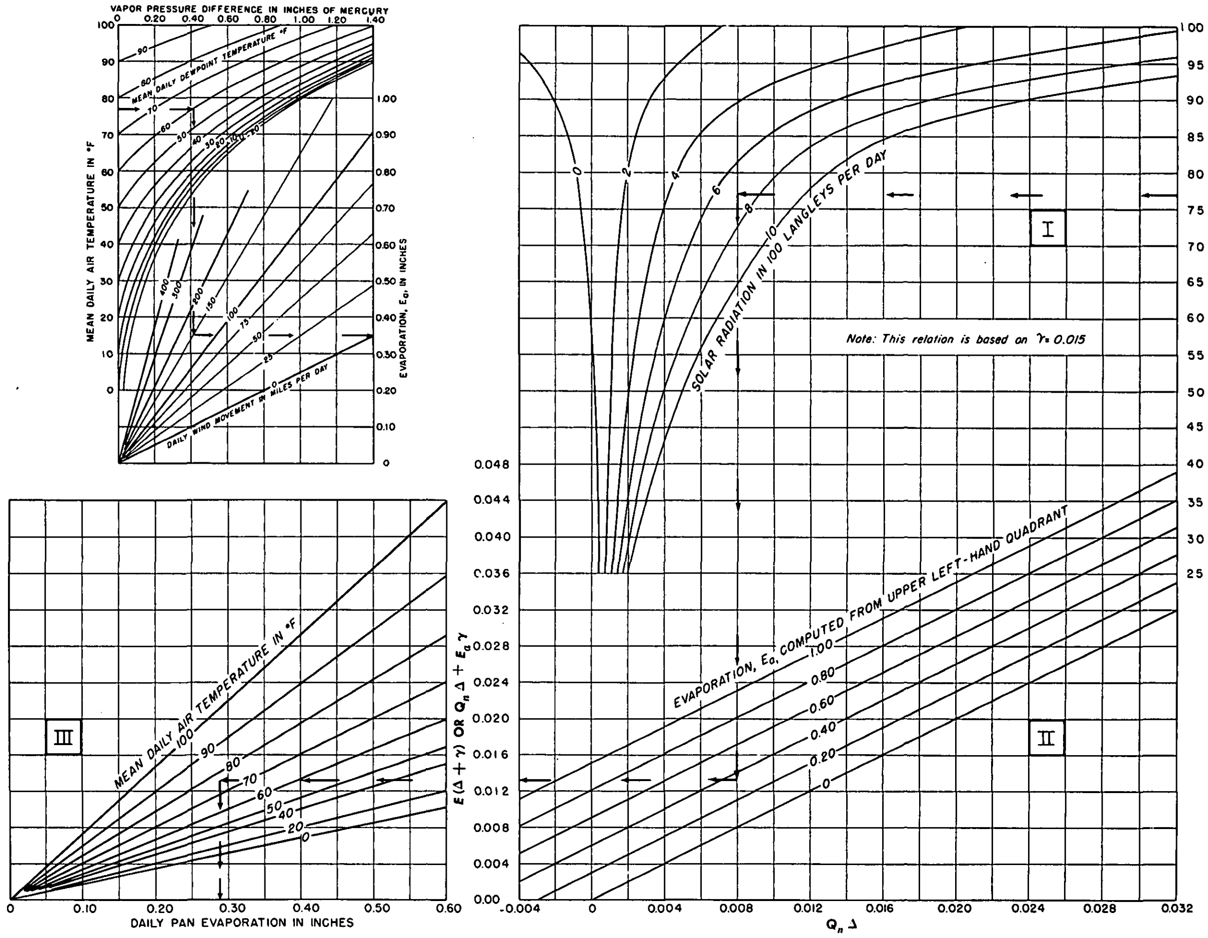


Figure 97. Relation for BPI pan at south station, Lake Hefner, using energy balance approach.

Class A pan than was obtained when using water temperature, the correlation index for the BPI pan dropped from 0.91 to 0.86. Realizing that the reduced correlation index was at least partially the result of heat storage in the relatively deeper, sunken BPI pan, the errors in computed daily evaporation were plotted against the changes in pan-water-surface temperature as shown in figure 98. The degree of correlation indicates that changes in heat storage cannot be assumed zero for the BPI pan when considering daily evaporation.

There is little doubt that similar relations could be developed for other types of pans. Moreover, it may be possible to apply a similar approach directly to a lake if some means can be found for extrapolating changes in heat storage in a fashion somewhat like that employed for water storage in streamflow routing.

LAKE EVAPORATION AND METEOROLOGICAL FACTORS

In the previous section it was shown that daily pan evaporation could be estimated from water temperature, vapor pressure of the air, and wind movement. It was visualized that if a similar relation could be developed for lake evaporation, the two could be compared for the purpose of analyzing the differences between pan and lake. Evaluation of similar equations derived by Rohwer (1931), Meyer (1942), Folse (1929), Hickox (1946), Horton (1917), and Sleight (1927) from less detailed information would also become possible.

With these objectives in mind, correlations were made with several combinations of data and assumed wind functions. The results of this analysis are presented in table 27.

The regression constants yield evaporation in inches per day with u in miles per day and vapor pressures in inches of mercury. Water temperature and 4-meter wind at the barge station were used in all six correlations, and dewpoint was taken at the 2-meter level for the barge or upwind station as indicated. Since the rate of evaporation at any instant is proportional to the corresponding instantaneous value of $(e_0 - e_a) \cdot f(u)$, best results should be expected when daily evaporation is related to the average value of $(e_0 - e_a) \cdot f(u)$ rather than daily averages of $(e_0 - e_a)$ and u. Accordingly, 3-hourly means of $(e_0 - e_a)$ and $(e_0 - e_a)\,u$ were averaged for the day and used in solving for regression constants. To determine the relative accuracy of the shorter method, the data used for line 5 of table 27 were reanalyzed using daily means of T_0 (to obtain e_0), e_a, and u. The resulting values of a and b were 0.064 and 0.00250, respectively, and the correlation index was 0.952. Thus, it is seen that the simpler approach is essentially as reliable, although the regression constants are slightly different. This conclusion cannot be considered necessarily of general application, however, since it is dependent upon the amplitude of diurnal fluctuations in $(e_0 - e_a)$ and u, as well as any difference in phase.

The selection of 2-meter dewpoint and 4-meter wind in the correlations of table 27 may appear somewhat arbitrary. It was believed that the 2-meter dewpoint (lowest level of observation) should constitute a reliable index to the vapor-pressure gradient near the water surface. Moreover, dewpoint observations taken in a standard Weather Bureau shelter (ground exposure) are at approximately the 2-meter level. The barge relations were developed first and, since it seemed possible that the barge itself might affect the wind observed at the 2-meter level, it appeared advisable to use the 4-meter wind. Actually, the levels selected are rather immaterial, since the resulting equations can readily be converted through analysis of the observations at different levels.

It will be noted that some correlations were made in which the coefficient a was assumed to be zero (lines 3 and 6 of the table), and others in which it was assumed to have a value other than zero. The latter are in the form of equation (104), while the former correspond to equation (105) with n equal to unity. Statistically, the derived values of a are not significantly different from zero and, therefore, may partially result from the tendency for imperfect correlation to reduce the slope of regression. Correlations using the same data as for line 2, assuming various values of n, and solving for corresponding values of c in equation (105), did not yield a correlation index greater than that obtained for equation (104). Thus, the Lake Hefner data provide little to support preference for either equation.

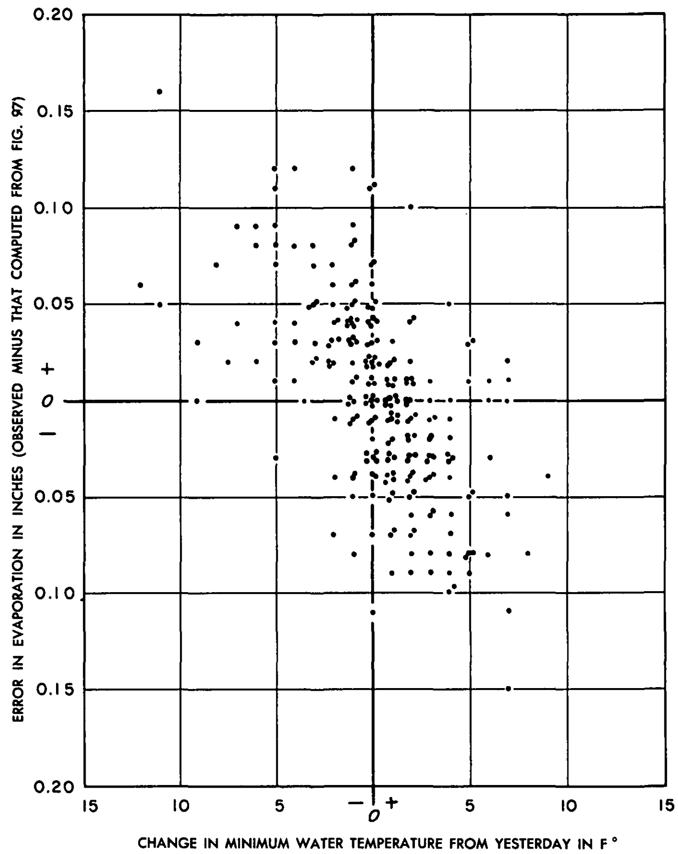


Figure 98. The effect of heat storage in the BPI pan.

By assuming equilibrium conditions and substituting reasonable values in the standard diffusion

TABLE 27. Equations for Lake Evaporation, $E = (e_0 - e_a)(a + bu)$

	Days of Record	Period of Record	Dewpoint Station	a	b	Correlation Index	Standard Error of Estimate: Inches	% Avg.
1	141	Apr 1950-Aug 1951	Barge	0.069	.00268	0.942	0.039	22
2	115	Sep 1950-Aug 1951	Barge	0.051	.00287	0.957	0.035	20
3	115	Sep 1950-Aug 1951	Barge	0.00*	.00304	0.956	0.035	20
4	141	Apr 1950-Aug 1951	Upwind	0.082	.00234	0.941	0.039	22
5	115	Sep 1950-Aug 1951	Upwind	0.068	.00246	0.954	0.036	21
6	115	Sep 1950-Aug 1951	Upwind	0.00*	.00270	0.952	0.037	22

* Assumed to be zero.

equation, it can be shown that evaporation should essentially cease when there is no wind (see section on "Mass-Transfer Studies"). Nevertheless, the possibility that a is significantly greater than zero cannot be dismissed by showing that evaporation from a large water body is insignificant under equilibrium conditions when $u = 0$. Other facts to be considered are:

1. The anemometers in use, although probably the best commercially available, have a starting speed of approximately 1.5 mph in laboratory tests. Thus, on days with light winds the true speed may be greater than observed — a systematic error which tends to increase the statistically derived value of a.

2. Because of insufficient ventilation, the true wet-bulb temperature is lower than the observed during relatively calm periods by an amount depending upon wind speed (see fig. 10 of reference 1 and section on "Instrumentation"). Since the error is negligible for wind speeds in excess of 0.5 mph, this effect is probably not appreciable, but would tend to increase the constant a.

3. During daylight hours when solar radiation is heating the water, evaporation into still air would occur primarily not by diffusion through a deep layer of air, but by natural convection created by (a) the temperature difference between the water and the air, and (b) the difference in the densities of the evaporated water vapor and the surrounding air.

It should be pointed out that the derived constant a is much larger for the pans (except the screened) than for the lake. This is to be expected because of the lip effect and because items (1) and (3) above become much more important in the case of the pans. With respect to item (1), the pan anemometer is at a lower level where the velocities are smaller, and with respect to item (3), some natural convection can occur directly over a pan without registering on the anemometer.

Referring again to table 27, it should be noted that the data are identical for correlations (1) and (2), and for correlations (4) and (5), except for the record periods. Analyses were first made using all days with A water-budget data. From examination of the resultant errors, it appeared that the records of surface-water temperature of the first few months were not as reliable as those of the later months and, accordingly, the regression equations were recomputed using only the last full year of data. The correlation indexes based on the last full year of record are significantly higher than those for the total period.

Correlations (4), (5), and (6) are equivalent to (1), (2), and (3), respectively, except that the dewpoint of the upwind station was substituted for that at the barge station. To the extent that evaporation at the barge can be considered to equal the average over the lake, the first three regressions represent point evaporation formulas. The three based on upwind dewpoint are in every sense geared to a lake the size of Hefner, and the regression constants may therefore require adjustment if the lake dimensions change appreciably.

Conclusions expressed in the literature vary considerably as to the effect of lake size, or downwind dimension, on the average rate of evaporation per unit area. Rohwer (1931) concluded that size has no appreciable effect for diameters in excess of 12 feet; Hickox (1946) derived an equation in which evaporation varies as the —0.25 power of diameter; Albertson, in a discussion of Hickox's paper, stresses the effect of assumed boundary conditions; and Sutton's equation (Anderson, Anderson, and Marciano, 1950) gives —0.11 power of diameter when the lapse rate is adiabatic. Data reported by Young (1947) on sunken pans follow —0.11 power for diameters of 2 to 12 feet, while for similar data collected by Sleight (1917) the power is more nearly —0.14. Figure 99 shows the data of Young, Sleight, and Rohwer, as well as several points for lakes. In those cases where 12-foot pan data were not available for comparison with lake data (such as the Hefner case), the points are based on data from other pans as explained in the notes.

Figure 99 indicates (within the range of data) that the evaporation from the relatively large lake is approximately equivalent to that from the 12-foot sunken pan. Moreover, examination of the figure will show that the only manner in which lake size could have any appreciable effect would require that evaporation from a 12-foot natural lake be considerably greater than that from the 12-foot pan. Since pans in general overestimate lake evaporation because of lip effects and heat conduction characteristics, it seems reasonable to conclude that lake size (within the range of interest) has no appreciable effect on the rate of evaporation. It should be noted, however, that this conclusion does not agree with fig. 44, based on Millar's (1937) theory.

If it is assumed that the rate of evaporation is essentially independent of lake size within the range of interest, the upwind regression for Lake Hefner may be reasonably reliable for other lakes providing representative observations of water temperature, 4-meter wind, and 2-meter dewpoint are available. Moreover, there is frequently a high degree of correlation of dewpoint and wind as observed at different sites within a limited area, so that a relatively small number of observations of these elements at the lake, when related to data from a nearby meteorological station, may be sufficient for estimating average annual or monthly evaporation. The reliability of such an approach can, of course, not be determined until verification data are available for numerous additional lakes. As a matter of interest, mean daily Lake Hefner observations were correlated with data for Will Rogers Airport (WBAS) with the following results:

4-meter barge wind = 0.82 WBAS wind;
2-meter upwind vapor pressure = 0.96 WBAS vapor pressure.

The correlation coefficient for the wind relation is 0.935, and that for vapor pressure is 0.994; the standard error of estimate for the wind relation is 1.8 mph, and for vapor pressure 0.025 in. of mercury. Both analyses were based on 479 days of record.

It will be noted that equation (58) in the "Mass Transfer" section is similar in form to the empirical equations shown in lines (3) and (6) of table 27. The question naturally arises as to how nearly these equations agree when reduced to the same units and levels of observation. To facilitate such comparison, the following relations were derived from approximately 40 days of data selected at random: $(e_0 - e_a)$ for 2-meter level $= 0.91$ $(e_0 - e_a)$ for 8-meter level, and

$$u_4 = 0.90\, u_8.$$

Using these equations and converting to proper units, the value of b in line (3) of table 27 becomes 6.5×10^{-4} compared to the value of 6.25×10^{-4} given in equation (58). This converted value of 6.5×10^{-4} is based on the last 12 months of record. Basing the computations on the full period of record ($b = 0.00292$) yields a converted value of 6.2×10^{-4}.

PAN VS LAKE EVAPORATION

The practice of applying a coefficient to pan evaporation to estimate that occurring from an adjacent lake is of long standing, even though the reliability of this approach has always been subject to question. Coefficients derived and recommended by various investigators are summarized in table 24, and results for the Lake Hefner experiment are presented in table 28.

There are several ways in which pan-to-lake coefficients can be computed for an experiment such as that at Lake Hefner, all of which merit consideration. The first approach is to use all observed data; second, to use only those data which are considered reliable; and third, to adjust all data which are believed to be in error.

If the errors in observed daily evaporation were randomly distributed, and if each weather event had an independent effect on evaporation, then all three approaches should give reliable and comparable results. However, this is not the case with respect to lake and pan evaporation. For example, the errors in pan evaporation brought about by water splashed out during periods of rain and high wind are always in the same direction. Similarly, during periods of heavy snowfall accompanied by high wind, the precipitation gage underestimates the amount of snowfall and this, in conjunction with the blowing snow trapped by the lake, results in an underestimate of lake evaporation as computed by the water budget. In the case of a pan, the effect of a particular weather event on evaporation is relatively independent of antecedent weather, largely because of its limited heat capacity. In the case of the lake, however, the evaporation occurring on one day is not independent of antecedent weather because the temperature of the lake is a function of the energy exchange over a considerable period of time. This carry-over effect results from the larger heat capacity of the lake. The large difference in heat capacities of the pan and lake produces marked variations in the daily pan coefficients during periods of changing weather. The elimination of questionable data would probably yield reliable annual coefficients except for the fact that the correlation between weather events and reliability of evaporation data produced biased results. In view of these facts, it would appear that pan coefficients should be based on continuous records adjusted where necessary for apparent discrepancies which would create a bias. Accordingly daily pan and lake evaporation were computed by applying the empirical relations described previously in this section. The computed values were substituted for observed data when the differences were large, provided the source of the errors could be accounted for.

Examination of the Lake Hefner pan evaporation data indicates that unreasonably high evaporation was frequently observed for the Class A pans during

TABLE 28. Summary of Monthly Pan and Lake Evaporation and Monthly and Annual Pan-to-Lake Coefficients. (Lake Hefner Experiment)

		Evaporation (inches)*									Pan Coefficients				
		Class A Pan									Class A Pan				
Mo.	Year	NE Sta. No. 1	NE Sta. No. 2	SW Sta.	S Sta.	Avg**	BPI	Colorado	Screened	Lake	S Sta.	Avg**	BPI	Colorado	Screened
May	1950	9.04	8.10	8.53	9.13	8.90	—	6.60	5.66	3.14†	0.34	0.35	—	0.48	0.55
Jun		10.72	10.28	10.24	10.98	10.65	8.38	8.56	7.08	6.35	0.58	0.60	0.76	0.74	0.90
Jul		7.77	7.61	7.52	7.54	7.61	5.64	5.93	5.00	5.43	0.72	0.71	0.96	0.92	1.09
Aug		8.42	8.05	8.18	8.41	8.34	6.62	6.65	5.19	6.83	0.81	0.82	1.03	1.03	1.32
Sep		6.13	5.97	6.11	5.96	6.07	4.76	4.81	4.32	5.65	0.95	0.93	1.19	1.17	1.31
Oct		7.32	7.11	7.00	7.32	7.21	5.38	5.94	5.51	6.51	0.89	0.90	1.21	1.10	1.18
Nov		4.53	4.36	4.49	4.56	4.53	3.83	4.16	4.59	5.99	1.31	1.32	1.56	1.44	1.31
Dec		2.56	2.45	2.77	2.70	2.68	2.08	2.21	2.81	2.84	1.05	1.06	1.37	1.29	1.01
Jan	1951	3.06	3.04	3.36	3.34	3.25	2.61	3.24	3.15	2.47	0.74	0.76	0.95	0.76	0.78
Feb		3.34	3.22	3.39	3.38	3.37	2.02	2.15	2.31	0.44	0.13	0.13	0.22	0.20	0.19
Mar		6.48	6.26	6.98	6.64	6.70	5.13	5.72	5.67	3.39	0.51	0.51	0.66	0.59	0.60
Apr		9.02	8.66	9.05	8.72	8.93	6.44	7.53	6.87	3.44	0.39	0.39	0.53	0.46	0.50
May		8.66	8.13	8.19	8.37	8.41	6.18	7.05	6.13	4.40‡	0.53	0.52	0.71	0.62	0.72
Jun		9.06	8.70	8.74	9.37	9.06	6.95	7.87	6.43	5.87	0.63	0.65	0.84	0.75	0.91
Jul		11.63	11.05	10.55	11.29	11.16	8.45	9.39	7.88	7.18	0.64	0.64	0.85	0.76	0.91
Aug		12.58	12.11	12.09	12.25	12.31	9.18	10.08	8.90	8.83	0.72	0.72	0.96	0.88	0.99
Annual Totals															
Jun-May					77.92	77.75	59.07	63.95	58.63	53.75	0.69	0.69	0.91	0.84	0.92
Jul-Jun					76.31	76.16	57.64	63.26	57.98	53.27	0.70	0.70	0.92	0.84	0.92
Aug-Jul					80.06	79.71	60.45	66.72	60.86	55.01	0.69	0.69	0.91	0.82	0.90
Sep-Aug					83.90	83.68	63.01	70.15	64.57	57.01	0.68	0.68	0.90	0.81	0.88
Annual Totals — Evaporation amounts and coefficients based on observed data, eliminating all days (approx. 14%) requiring adjustment for either pan or lake evaporation.															
Jun-May					68.92	68.50	51.66	56.23	51.58	47.95	0.70	0.70	0.93	0.85	0.93
Jul-Jun					65.64	65.30	49.11	54.17	50.10	46.82	0.71	0.72	0.95	0.86	0.93
Aug-Jul					70.55	70.09	52.49	58.23	53.47	48.96	0.69	0.70	0.93	0.84	0.92
Sep-Aug					74.21	73.82	54.90	61.33	56.78	50.34	0.68	0.68	0.92	0.82	0.89
Annual Totals — Evaporation amounts and coefficients based on observed water-budget and pan data for full period.															
Jun-May					80.59	81.01	58.81	64.36	58.40	52.67	0.65	0.65	0.90	0.82	0.90
Jul-Jun					79.48	79.76	57.74	64.00	57.52	52.19	0.66	0.65	0.90	0.82	0.91
Aug-Jul					83.61	83.46	60.86	67.73	60.29	52.87	0.63	0.63	0.87	0.78	0.88
Sep-Aug					87.65	87.76	63.58	71.20	64.03	54.31	0.62	0.62	0.85	0.76	0.85
Partial Yr. Totals — Evaporation amounts and coefficients based on adjusted water-budget and pan data.															
Jun-Oct					40.21	39.88	30.78	31.89	27.10	30.77	0.76	0.77	1.00	0.96	1.14
Nov-May					37.71	37.87	28.29	32.06	31.53	22.97	0.61	0.61	0.81	0.72	0.73
May-Oct					49.34	48.78	—	38.49	32.76	33.91	0.69	0.70	—	0.88	1.04

* Except as noted otherwise, the pan evaporation listed in this table has been corrected for splash-out, and the lake evaporation has been adjusted during periods of high inflow or blowing snow as explained in the text.

** Average of all Class A pans except NE No. 2, which was installed for comparative purposes.

† Records missing for 7 days. Monthly total obtained by adding computed values for missing days and adjusting "D" days adjacent to the missing days.

‡ Computed value used for one day of missing record. Further adjustments given in text.

periods of rain, particularly when accompanied by high winds. Raindrop splash and wave action, as well as pan overflow, are the probable causes. Accordingly, evaporation was computed for all pans from relations of the type shown in figure 95 for each storm period. The computed values were then compared with the observed data and substituted therefor when discrepancies were apparent. All monthly data in table 28 were adjusted in this manner.

The total observed and adjusted evaporations in inches at the south station for the 16-month period are as follows:

	Observed	Adjusted
Class A	124.48	119.96
BPI	83.92	83.66
Colorado	99.18	97.87
Screened	86.34	87.50

Records for the Class A pan required adjustment on 48 days; the adjustment was in excess of 0.20 in. on nine days; and for one exceptionally windy day when the pan was nearly full, the adjustment was 0.47. Apparently, about as much water (and silt) splashed into the sunken pans by raindrop impact as was splashed out. Since only the screened pan required an upward adjustment, it is possible that the screen reduced the splash-out of rain falling into the pan, but has essentially no effect on splash from the ground surface into the pan.

The computed lake evaporation was substituted for the observed only for those few periods when the reliability of the water budget could be questioned because of (1) heavy precipitation accompanied by considerable local runoff, (2) high inflow from the intake canal, and (3) heavy snowfall with high winds. Most of those water-budget days which were designated as C or D were so classfied because of the error in determining mean daily change in lake level brought about by variations in the wind pattern over the lake. Generally speaking, errors of this type are compensating and, therefore, need not be considered when evaluating monthly and annual evaporation. This is not necessarily true when such days precede or follow a period for which adjustments are made, since the compensating day (or days) may fall within the adjusted period. In each case, adjustments were made for periods between days classified as A or B. Following is a list of the periods for which adjustments were made and the reasons for the apparent errors:

Period	Remarks
7-14 May 1950	Missing water-budget data.
17-19 May 1950	Missing water-budget data.
17-21 July 1950	High inflow through intake canal.
24-26 July 1950	High inflow through intake canal.
18-20 August 1950	High inflow through intake canal.
12-16 February 1951	Snow accompanied by high wind and low temperature.
19-21 February 1951	High inflow through intake canal and heavy rainfall.
29 April-1 May 1951	Heavy rainfall — culvert weirs overtopped.
8-11 May 1951	Heavy rainfall and high inflow through intake canal.
17-22 May 1951	High inflow through intake canal and heavy local runoff.
27 May 1951	Missing water-budget data.

Comparison of the adjusted monthly lake evaporation values in table 28 with the observed water-budget amounts listed in table 1 shows substantial differences for individual months, although the 16-month totals agree reasonably well (78.76 in. for the adjusted data, and 78.30 in. for the observed water-budget data). The adjusted value of 0.44 in. for February 1951 is of particular interest, since the water budget shows a negative evaporation (condensation) of 0.638 in. for the month. Thus, the adjustments for two brief periods (12-16 and 19-21 February) account for a change of 1.08 in. in the month's total. Since the total evaporation for the remaining 20 days of the month computed from the lake relation is within 0.1 in. of the observed water-budget data, it is apparent that there is no pronounced bias during this season of the year.

All monthly data and coefficients in table 28 are based on adjusted pan and lake evaporation as described in the preceding paragraphs. The moving annual coefficients immediately below the monthly data and the seasonal coefficients at the bottom of the table are also based on adjusted pan and lake evaporation. The other two sets of annual coefficients, based on (1) only those days which did not require pan or lake adjustment, and (2) observed pan and lake data for the full period, are given in order that a comparison may be made of the three approaches. It will be noted that the first two sets of annual coefficients agree rather closely. However, this is not the case on a monthly basis; for example, the adjusted data give a June Class A coefficient of 0.65, whereas the selected data (23 days) give a value of 0.76.

Table 28 shows that the range in monthly coefficients is greater for all Lake Hefner pans than is the case for Lake Elsinore, Lake Okeechobee, and the 85-ft reservoir at Ft. Collins, Colo. This great range is undoubtedly caused partially by the sequence of weather during the Lake Hefner experiment. Nevertheless, there is little doubt that pan coefficients for Lake Hefner are normally much higher in late fall than in early spring because of the lag between pan (or air) and lake temperature. The limited data available for relatively few lakes indicate that the range is greatest for deep lakes in localities with large annual range in temperature.

Examination of tables 24 and 28 reveals variation in annual and warm-season coefficients of appreciable magnitude. The obvious question arises as to whether or not this variation is indicative of the reliability of the approach. There is, of course, the possibility that the variations can be explained and used to advantage. Among the possible causes of the indicated variations are:

1. Errors in lake evaporation (water budget).
2. Variations in observational practice at pan stations.
3. Variations from standard enclosure.
4. Variations in pan exposure, particularly with respect to the lake.
5. Differences in lake size and depth.
6. Geographical variations in climate.
7. Difference between part-year and annual coefficients.

Sufficient, reliable pan-vs-lake evaporation data should provide a means of evaluating these various factors, although in the case of climatic variations it may be necessary to resort to geographical differentiation. To be sure, little would be gained by the coefficient approach if the required water-budget data were sufficient of themselves to estimate evaporation reliably from all other existing and proposed lakes, but this is believed not to be the case. For example, if the Class A coefficient were assumed to be 0.70 for those lakes shown in tables 24 and 28, the extreme indicated error in mean annual lake evaporation would be about 12 per cent and, if this could be reduced by half through consideration of one or more of the factors listed, the coefficient approach would undoubtedly provide the required accuracy in estimating mean annual evaporation from existing and proposed lakes and reservoirs.

Discussing the seven items in the order listed, the accuracy of water-budget data from natural lakes is often in question because the magnitude of inflow and outflow seepage is not known. Although the water-budget data for Lake Hefner are exceptionally accurate, the data represented in table 24 were derived in a somewhat less rigorous manner. It is obvious, therefore, that a considerable portion of the variation in pan coefficients may be the result of incorrect lake evaporation. A detailed discussion of the effects of variations in observational practice at pan stations is not presented here, since they have been described in several previous publications (Hickox, 1946; Rohwer, 1931; Young, 1947; "Evaporation from Water Surfaces," 1934).

The effect of variations from standard exposure of the pan and related instruments is amply shown by the differences in evaporation and wind observed at the two northeast Lake Hefner stations. Quite the opposite of what might be expected, the site with least obstructions consistently recorded lower wind movement at pan height and, as a consequence, less evaporation by 4.3 per cent. Anemometers were interchanged to check possible instrumental deficiencies and observations made upwind from the enclosures showed no appreciable difference between the sites. Apparently, the obstructions at the site with standard layout create turbulence with a net transport of momentum downward to anemometer height.

Wind is probably the most important factor to be considered in selecting a pan site representative of a particular lake, since it has an appreciable effect upon pan evaporation, and it can be extremely variable over a relatively small area. If wind at the pan site is appreciably less than over the comparative lake, the true pan coefficient would necessarily be relatively high.

Figure 99 indicates that lake area has little effect on the rate of evaporation (depth per unit of time), although the data are admittedly insufficient to be conclusive. Depth of water, on the other hand, quite definitely influences the monthly coefficients — the greater the depth, the greater the range. Thus, part-year, or seasonal, coefficients must be related to lake depth. Although mean annual coefficients are not greatly affected by depth, it appears that they decrease as depth increases and as the annual air temperature range increases.

There is, of course, the possibility that the pan coefficients vary from region to region because of variations in climatic factors and this is frequently advanced as a reason why pan coefficients cannot be

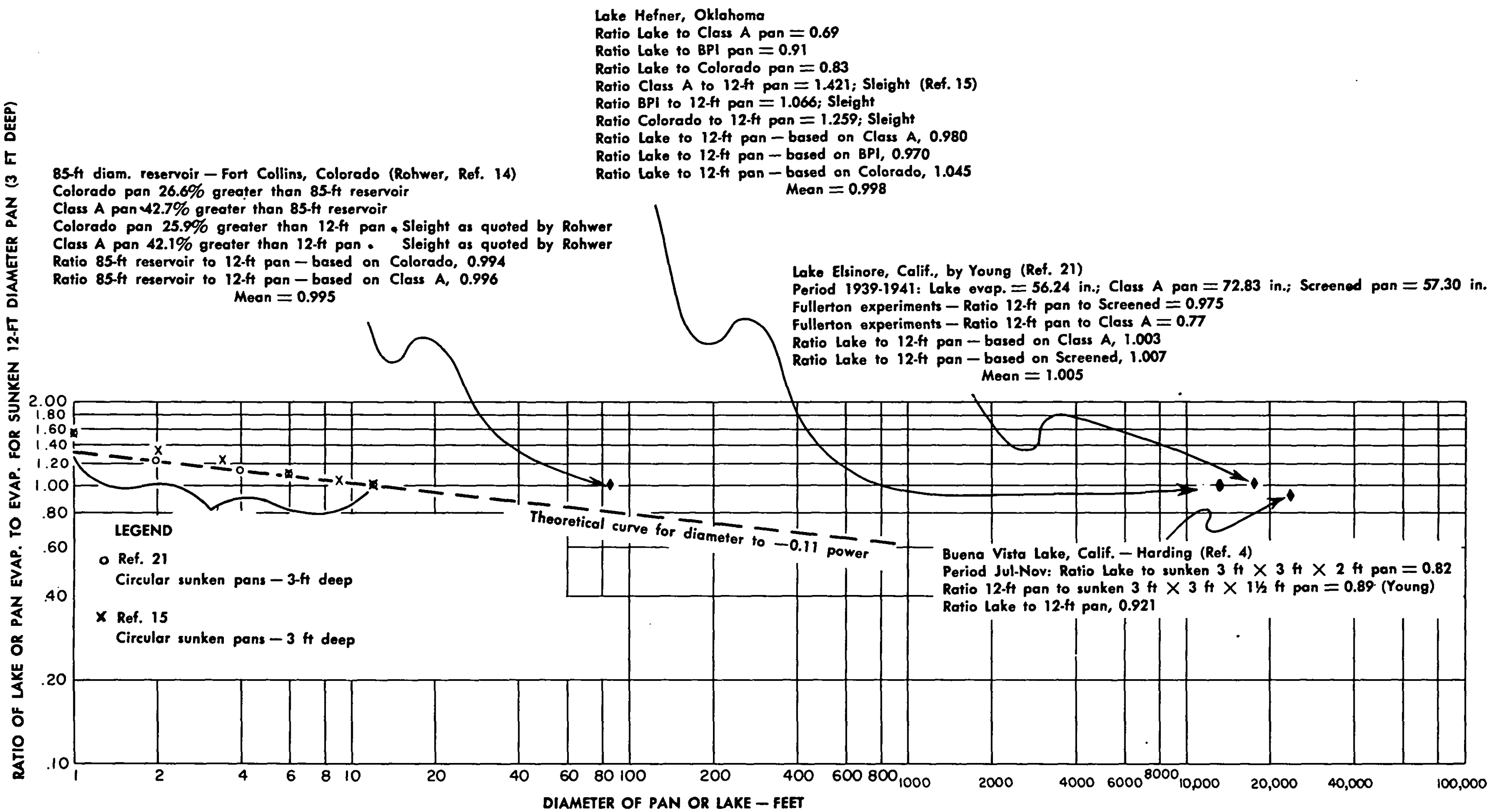


Figure 99. The effect of pan or lake size on rate of evaporation.

relied upon. If, however, the effect is not pronounced — and the rather minor variations observed in pan coefficients indicate this to be the case — rather limited pan-vs-lake data should provide the means for showing the variation geographically by isolines on a map.

Finally, it should be pointed out that coefficients based on part-year data are apt to be quite erratic because of the seasonal variation brought about by heat storage in the lake. That they should not be used as annual coefficients is demonstrated by the last three lines of table 28. Except for the close agreement of the annual and the May-October coefficient for the Class A pan, the part-year coefficients for all pans are considerably different from the annual values.

The preceding discussion has been directed toward evaluation of the pan-coefficient approach to estimating annual or mean annual lake evaporation. Observations demonstrate conclusively that the coefficients vary throughout the year and that the mean annual coefficient cannot be used to estimate monthly lake evaporation. To provide a ready basis for distributing annual evaporation computed from annual pan coefficients, the Class A equation in table 26 and the fifth equation in table 27 were combined as shown in figure 100. Since the wind data used in developing the two relations were different, it was necessary to relate wind at the pan to that at the barge (4-meter level). It will be seen that while the pan coefficient is a nonlinear function of wind, the effect is not great within the range of winds normally experienced, particularly for periods as long as a month. It should be pointed out that the relation of figure 100 is based on daily data and, because of the nonlinear effect of wind, application to monthly average data will result in an overestimate of the annual evaporation.

It is evident from table 28 that the coefficient for a particular month may vary considerably from year to year, even though the annual coefficient is reasonably stable. Variations in weather, coupled with the large heat capacity of the lake, can produce an appreciable seasonal shift in the monthly coefficients. Consequently, the monthly coefficients derived for the sixteen months of record at Lake Hefner may actually deviate considerably from the normal values. Moreover, even if normal coefficients were available for Lake Hefner, they would not necessarily be applicable to other lakes. To provide a means for distributing estimated annual evaporation, figure 101 was developed assuming that the seasonal march in coefficients is wholly the result of temperature lag between the pan and the lake. While figure 101 is quite similar to figure 100, the data plotted were computed on a mean monthly basis (evaporation, wind, water temperature, and dewpoint). It will be noted that the data display a moderate seasonal trend, but that the effect of wind is unimportant within the small range of speeds observed.

Figure 101 provides a means of estimating the monthly pan coefficients from observed pan and lake water-temperatures and dewpoint. Pan water-temperatures have not normally been observed at Class A evaporation stations, but can be computed by entering a relation of the type shown in figure 95 with observed pan evaporation, wind, and dewpoint. In most cases, it will be necessary to use dewpoint data from a nearby meteorological station. Lack of lake surface-water-temperature data constitutes the major obstacle to the application of figure 101. However, most approaches to the evaporation problem require surface-water-temperature data and, therefore, it would seem that a program for initiating nation-wide observations would be justified.

RELATIVE EVAPORATION FROM PANS OF VARIOUS TYPES

Because several types of evaporation pans are rather widely used in this country, numerous investigations have been conducted (Rohwer, 1931; Sleight, 1917; Young, 1947) to determine the relative evaporation from each. The results of many of these experiments are listed in table 25 in the form of ratios. In addition to ratios taken from references at the end of this chapter, values have been derived from published and unpublished data in Weather Bureau files.

It will be noted from the footnotes that some of the values are based on part-year data and, in most cases, the record periods are short. Considering these facts, the ratios are amazingly consistent — six determinations of BPI to Class A ratio in Texas and California average 0.797 with a maximum departure of 4 per cent. The three lowest values of 0.69, 0.75, and 0.77 are all based on part-year data, while all larger values are computed from full-year data. Whether the low ratios for Denver (0.75) and for Ft. Assinniboine (0.69) are entirely the result of climatic factors or, to some extent, of the use of part-year data cannot be determined with the limited records available.

Monthly and annual pan-to-pan ratios for the Lake Hefner observations are listed in table 29. It

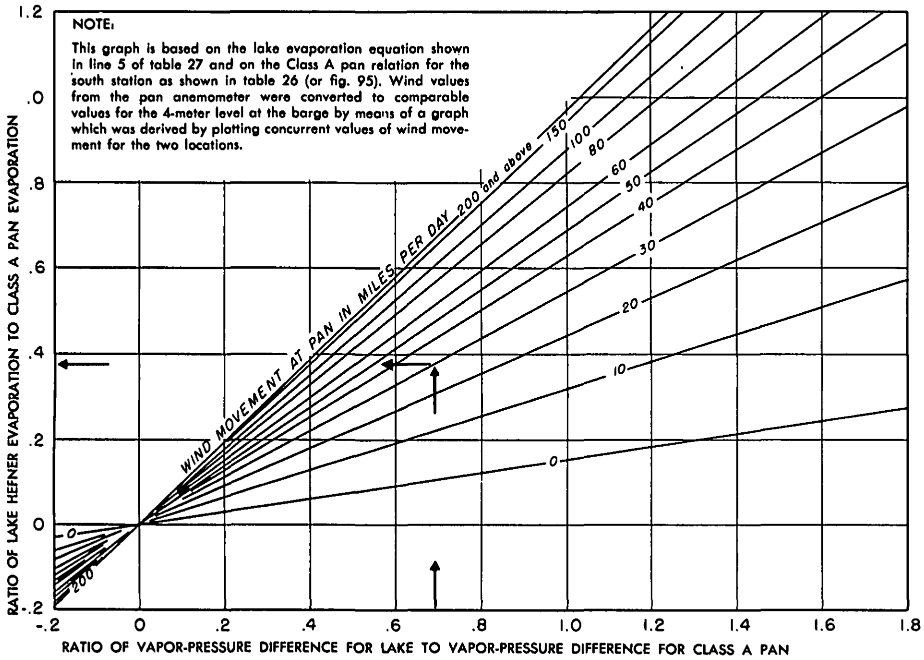


Figure 100. Relation of Class A pan coefficient to wind and relative vapor-pressure difference.

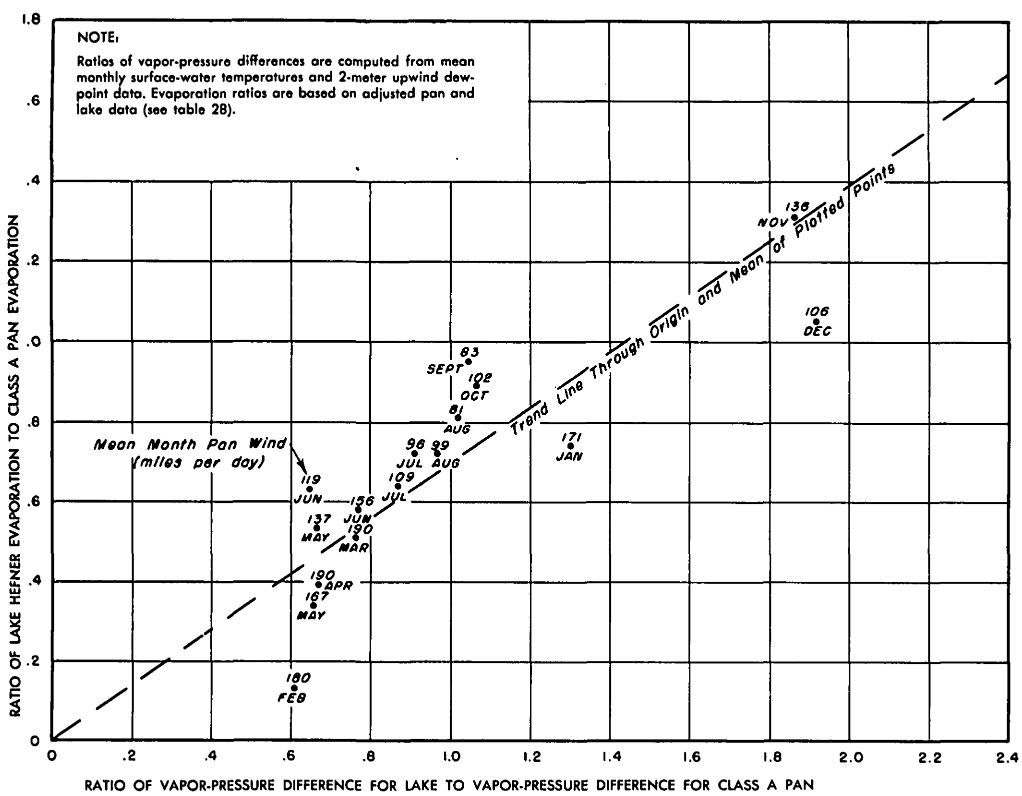


Figure 101. Plotting of monthly Class A pan coefficient versus ratio of vapor-pressure differences.

will be noted that two sets of ratios are listed; the first column in each block is based on data corrected for splash-out as described earlier in the section; the second column is based on incomplete data, eliminating those days when splash-out occurred. While the paired monthly values show minor discrepancies, the annual values are essentially equivalent.

Minor seasonal fluctuations may exist, but they are masked by other variations, except in the case of the screened pan. Ratios of the screened pan to each of the other pans show a definite seasonal march, with the maximum occurring in December and the minimum in June. Apparently, the screen affects the exchange of radiant energy (and convective transfer). It might be expected that the Class A pan with its vertical sides exposed to a maximum of winter solar radiation would display the opposite trend, but this is not borne out by the data for Lake Hefner.

Relations of the type shown in figures 96 and 97 should be of considerable value in studying the relative evaporation from different types of pans under specified conditions, particularly if they can be shown to be generally applicable. For example, the difference in γ (pages 134 and 135) for the Class A and the BPI pans apparently indicates that climatic variations in wind would result in corresponding variations in the pan-to-pan ratio. However, taking an air temperature of 80°F, a dewpoint of 50°F, and radiation of 500 cal cm^{-2}, figures 96 and 97 yield a Class A to BPI ratio of 1.48 for $u = 50$ miles per day, and 1.46 for $u = 200$ miles per day, indicating little wind effect on the pan ratio. A rather thorough analysis of this type is planned for the near future.

Comparing the Lake Hefner ratios of table 29 with the average ratios of table 25, it will be seen that the values are reasonably consistent. The greatest differences are in the ratios for the Class A to Colorado and Class A to screened. Much of the discrepancy in these values results from the determinations at Henshaw Reservoir and Mansfield Dam.

CONCLUSIONS ON LAKE AND PAN EVAPORATION

Before discussing the conclusions to be reached from the Lake Hefner observations and the analyses presented in this chapter, it seems appropriate to emphasize the "inconclusive" nature of a single evaporation experiment, at least with respect to the pan approach. The Lake Hefner experiment has undoubtedly provided the most reliable data available in an over-all sense, and has gone far in providing substantiating evidence. Nevertheless, additional similar experiments must be performed under other climatic regimes before the reliability of the empirical approaches described can be accurately determined. Nor can any conclusions be reached at this time as to the relative reliability of the several empirical approaches discussed. Possibly all should be considered if the problem at hand warrants.

The following is a summary of the more important conclusions drawn from the material in this section.

1. Measured pan evaporation is frequently greater than true pan evaporation because of splash-out and blow-out during periods of heavy rain and high winds.

2. Differences in wind movement and pan evaporation at the two northeast stations indicate that obstructions can increase wind at pan height and thus increase the rate of evaporation by a few per cent.

3. Comparative observations on the four types of pans at Lake Hefner agree quite closely with data obtained from other studies (tables 25 and 29).

4. Daily pan evaporation at Lake Hefner can be accurately estimated from air temperature, solar radiation, dewpoint, and wind using an energy-balance approach in conjunction with equation (104).

5. No appreciable seasonal variation was found in the pan-to-pan ratios for the Class A, BPI, and Colorado pans at Lake Hefner. The ratio of the screened pan to each of the other three, however, does display a minimum near June and a maximum in about December.

6. Annual pan coefficients based on Lake Hefner data are reasonably consistent with those derived from previous studies. The coefficients derived for Lake Hefner from adjusted pan and lake data are:

Class A 0.69	Colorado 0.83
BPI 0.91	Screened 0.91

To indicate the magnitude of the splash-out adjustment, the coefficients were computed using observed pan data and adjusted lake data; the results were as follows: the Class A coefficient became 0.66, the Colorado 0.82, the BPI 0.91, and the screened 0.91.

TABLE 29. Pan-to-Pan Ratios (Lake Hefner Experiment)

Ratio = Pan X / Pan Y	Pan "X": Class A (South Station)			BPI (Sunken)			Colorado (Sunken)			Screened (Sunken)		
Pan "Y": Class A (S. Station)				Jan	0.78	0.78	Jan	0.97	0.97	Jan	0.94	0.94
				Feb	0.60	0.56	Feb	0.64	0.63	Feb	0.68	0.64
				Mar	0.77	0.74	Mar	0.86	0.82	Mar	0.85	0.80
				Apr	0.74	0.77	Apr	0.86	0.87	Apr	0.79	0.79
				*May	0.74	0.72	*May	0.78	0.77	*May	0.67	0.66
				*Jun	0.75	0.74	*Jun	0.81	0.80	*Jun	0.66	0.66
				*Jul	0.75	0.74	*Jul	0.81	0.81	*Jul	0.68	0.68
				*Aug	0.76	0.75	*Aug	0.81	0.80	*Aug	0.68	0.67
				Sep	0.80	0.80	Sep	0.81	0.80	Sep	0.72	0.72
				Oct	0.73	0.73	Oct	0.81	0.81	Oct	0.75	0.75
				Nov	0.84	0.79	Nov	0.91	0.87	Nov	1.01	0.97
				Dec	0.77	0.77	Dec	0.82	0.82	Dec	1.04	1.04
				Ann.	0.75	0.74	Ann.	0.82	0.81	Ann.	0.75	0.74
Pan "Y": BPI (Sunken)	Jan	1.28	1.28				Jan	1.24	1.24	Jan	1.21	1.21
	Feb	1.67	1.79				Feb	1.06	1.13	Jan	1.14	1.15
	Mar	1.29	1.35				Mar	1.11	1.11	Mar	1.11	1.08
	Apr	1.35	1.36				Apr	1.17	1.19	Apr	1.07	1.08
	*May	1.35	1.39				*May	1.14	1.15	*May	0.99	1.00
	*Jun	1.33	1.35				*Jun	1.07	1.08	*Jun	0.88	0.89
	*Jul	1.34	1.35				*Jul	1.08	1.10	*Jul	0.91	0.93
	*Aug	1.31	1.33				*Aug	1.06	1.06	*Aug	0.89	0.89
	Sep	1.25	1.24				Sep	1.01	1.00	Sep	0.91	0.90
	Oct	1.36	1.36				Oct	1.10	1.10	Oct	1.02	1.02
	Nov	1.19	1.26				Nov	1.09	1.10	Nov	1.20	1.22
	Dec	1.30	1.30				Dec	1.06	1.06	Dec	1.35	1.35
	Ann.	1.33	1.35				Ann.	1.10	1.10	Ann.	1.01	1.01
Pan "Y": Colorado (Sunken)	Jan	1.03	1.03	Jan	0.81	0.81				Jan	0.97	0.97
	Feb	1.57	1.59	Feb	0.94	0.88				Feb	1.07	1.01
	Mar	1.16	1.22	Mar	0.90	0.90				Mar	0.99	0.97
	Apr	1.16	1.15	Apr	0.86	0.86				Apr	0.91	0.91
	*May	1.28	1.30	*May	0.88	0.87				*May	0.87	0.86
	*Jun	1.24	1.25	*Jun	0.93	0.93				*Jun	0.82	0.83
	*Jul	1.24	1.24	*Jul	0.93	0.91				*Jul	0.85	0.85
	*Aug	1.24	1.26	*Aug	0.94	0.95				*Aug	0.84	0.84
	Sep	1.24	1.24	Sep	0.99	1.00				Sep	0.90	0.90
	Oct	1.23	1.23	Oct	0.91	0.91				Oct	0.93	0.93
	Nov	1.10	1.15	Nov	0.92	0.91				Nov	1.10	1.11
	Dec	1.22	1.22	Dec	0.94	0.94				Dec	1.27	1.27
	Ann.	1.22	1.23	Ann.	0.91	0.91				Ann.	0.92	0.92
Pan "Y": Screened (Sunken)	Jan	1.06	1.06	Jan	0.83	0.83	Jan	1.03	1.03			
	Feb	1.46	1.56	Feb	0.87	0.87	Feb	0.93	0.99			
	Mar	1.17	1.25	Mar	0.90	0.93	Mar	1.01	1.03			
	Apr	1.27	1.26	Apr	0.94	0.93	Apr	1.10	1.10			
	*May	1.48	1.51	*May	1.01	1.00	*May	1.16	1.16			
	*Jun	1.51	1.51	*Jun	1.13	1.12	*Jun	1.22	1.21			
	*Jul	1.46	1.46	*Jul	1.09	1.08	*Jul	1.18	1.18			
	*Aug	1.47	1.49	*Aug	1.12	1.13	*Aug	1.19	1.19			
	Sep	1.38	1.39	Sep	1.10	1.12	Sep	1.11	1.12			
	Oct	1.33	1.33	Sep	0.98	0.98	Sep	1.08	1.08			
	Nov	0.99	1.03	Nov	0.83	0.82	Nov	0.91	0.90			
	Dec	0.96	0.96	Dec	0.74	0.74	Dec	0.79	0.79			
	Ann.	1.32	1.34	Ann.	0.99	0.99	Ann.	1.09	1.09			

Note: The first column in each block is based on data corrected for splash-out as described in the text; the second column is based on incomplete data, eliminating those days when splash-out occurred.

*Average of two years of data.

7. Monthly pan coefficients display a pronounced seasonal march which is principally the result of temperature lag between the lake and pan brought about by the difference in heat capacity of the two water bodies.

8. Annual lake evaporation can probably be estimated within 10-15 per cent (on the average) by applying an annual coefficient to pan evaporation, provided lake depth and climatic regime are taken into account in selecting the coefficient. Available data are admittedly insufficient to evaluate properly the effects of lake depth and climatic regime on pan coefficients. However, it is hoped that current Lake Mead observations, and possibly those from other anticipated projects, will shed some light on this phase of the problem.

9. Although equation (105) appears to be more in agreement with present theory, comparative analysis indicates that equation (104) fits the Lake Hefner data equally well.

10. Analyses presented apparently substantiate the conclusions of Rohwer and others that the rate of lake evaporation (depth per unit of time) is not greatly influenced by the area of the water surface in the range considered. This conclusion disagrees, however, with the material presented in the section on "Mass Transfer."

Although the Lake Hefner experiment has appreciably advanced our knowledge of evaporation, it is not yet possible to reach unalterable conclusions as to which of the pans tested provides the most reliable means of estimating lake evaporation. The physical characteristics of the BPI (sunken) pan seem to be most nearly representative of those of a natural body of water and, therefore, this pan merits high consideration on a theoretical basis. The Class A pan, on the other hand, is subject to convective and radiant heat transfer to and from the sides and bottom. One effect of this characteristic is to increase γ beyond Bowen's theoretical value, and even beyond that for the BPI pan, as shown in figures 96 and 97. This dissimilarity of the Class A pan is not necessarily serious, provided a coefficient appropriate to the given geophysical conditions is used. The relatively few determinations which have been made to date certainly indicate that climatic variations in the Class A pan coefficient are rather minor and, if all its operational advantages are viewed in the light of the many hundreds of station-years of record now available, any decision to replace the Class A pan in favor of another will merit considerable deliberation.

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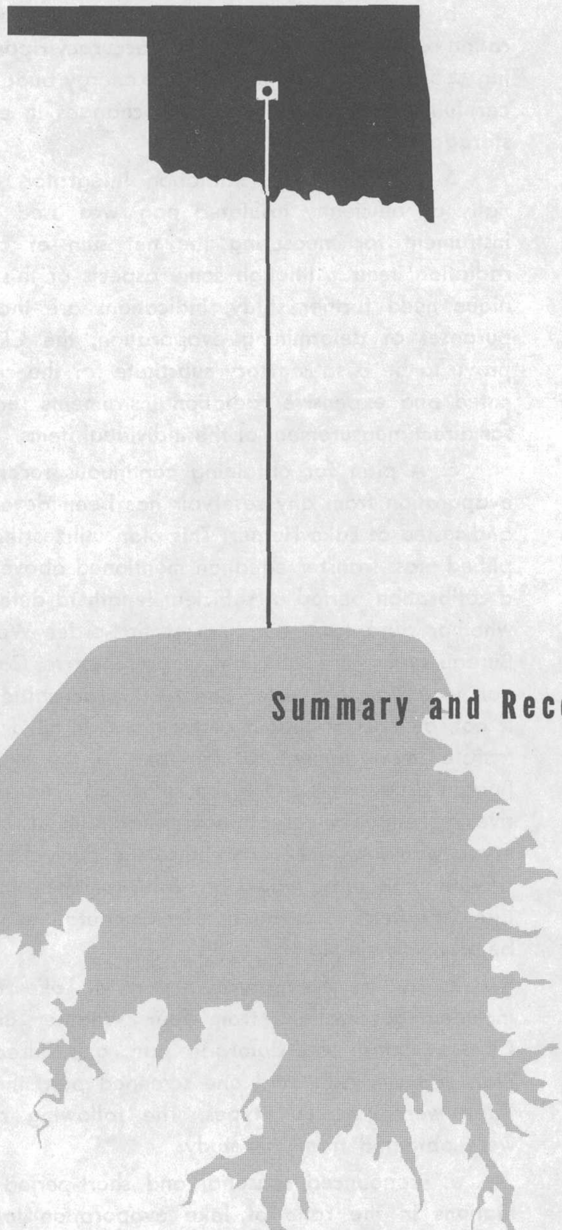

Summary and Recommendations-Lake Hefner Studies

SUMMARY OF RESULTS

1. Daily evaporation from Lake Hefner was determined by the water-budget method, that is, by measuring inflow, outflow, and changes in storage. Although this method is impracticable for most reservoirs, good results were obtained at Lake Hefner because: (1) natural surface inflow was generally negligible; (2) filter-plant withdrawals, the major outflow item, could be measured accurately; (3) changes in storage could be evaluated with sufficient accuracy; (4) deep seepage was almost negligible; (5) evaporation, although a residual by this method of determination, was a major item in the water budget, thus minimizing the effects of errors in measuring the other items. The water-budget control met the fundamental requirement that errors in the water budget not exceed 5 per cent of the monthly evaporation. An objective scheme of classifying daily water-budget evaporation figures as to their relative accuracy into four categories, A, B, C, and D, was used to select the most reliable daily figures for purposes of comparison with results obtained using the mass-transfer and energy-budget techniques. Sixty-two per cent of the daily figures were A or B, and only these values were used for the comparison.

2. The equipment system developed for measuring and recording the factors needed in the mass-transfer and energy-budget methods proved reliable and practicable. The average data yield during the 16-month operating period was 70 per cent, in spite of such difficulties as frozen wet-bulb reservoirs during the winter.

3. In the mass-transfer studies, both mixing-length and continuous-mixing concepts were reviewed in detail and the most promising evaporation equations derived therefrom were tested against the water-budget figures. The results of these studies were as follows:

a. No deviation from the logarithmic wind law could be detected between 2 and 8 meters, regardless of stability conditions, over periods of 3 hours or longer.

b. The lake surface was aerodynamically rough at all times, with no evidence of a critical wind speed for water-air boundary processes. The roughness length varied from 0.55 to 1.15 cm, increasing with wind speed. Taken as a whole, the evidence indicates that flow conditions very near the surface are critically important in predicting evaporation.

c. Of the mass-transfer equations tested, only O. G. Sutton's and Sverdrup's (1937) form proved satisfactory for use with available field instruments.

d. A new quasi-empirical equation was developed from the water-budget data, using wind speeds and vapor-pressure differences measured at Lake Hefner. A field version of this equation, using standard meteorological data from a Weather Bureau installation 13 miles from the lake and requiring only a measurement of the water-surface temperature at the lake, also gave good results. General application of this simplified technique depends upon whether the standard meteorological data are generally representative of the up-wind air approaching the lake.

4. The energy-budget equation was analyzed in detail to gain a more complete understanding of the physical processes involved, and an improved form of the equation was developed which makes it more generally useful. Evaporation determined with this equation, compared with results given by the water-budget control, showed that:

a. For periods of less than seven days, errors are likely to be excessive, primarily because changes in energy storage cannot be measured with sufficient accuracy.

b. For periods of seven days or more, evaporation can be determined with an accuracy approaching ± 5 per cent if all terms in the energy budget are carefully evaluated, particularly changes in energy storage.

5. The Cummings Radiation Integrator, essentially an efficiently insulated pan, was used as an instrument for measuring the net sum of certain radiation items. Although some aspects of this technique need further study, indications are that, for purposes of determining evaporation, the CRI will prove to be a satisfactory substitute for the complicated and expensive radiation instruments required for direct measurement of the individual items.

6. A plan for obtaining continuous records of evaporation from any reservoir has been developed and tested at Lake Hefner. This plan utilizes the simplified mass-transfer equation mentioned above, with a calibration period of sufficient length to determine whether data from the nearest first-order Weather Bureau station are sufficiently representative. Calibration would be by water budget if practicable and, if not, by energy-budget or by more detailed mass-transfer measurements at the lake. If the Weather Bureau data prove not to be sufficiently representative, a skeleton meteorological installation at the reservoir would be necessary. Since the only other observations required would be water-surface temperatures, the field observations and computations would be relatively simple.

7. Studies of evaporation pans at Lake Hefner included observations from four Weather Bureau Class A pans, one Colorado pan, one Bureau of Plant Industry pan, and one screened pan; the last three were sunken types. The following results were obtained from this study:

a. Pronounced seasonal and short-period fluctuations in the ratio of lake evaporation to that from all types of pans tested result principally from differences in surface-water temperature, augmented to some extent by other factors, such as variations in wind and the sun's altitude.

b. Annual coefficients between pan and Lake Hefner evaporation are as follows:

Class A pan	0.69
BPI pan	0.91
Colorado pan	0.83
Screened pan	0.91

While it is noted that the Class A pan coefficient is the lowest of the group, its range in monthly values was less (though not significantly so) than that of any other for the period of observation.

c. A high degree of correlation exists between pan coefficients and dewpoint, wind, and lake and pan temperatures. This relation explains much of the seasonal variation in pan coefficients.

d. Statistical relations were derived which show that daily pan evaporation at Lake Hefner can be reliably estimated from observations of surface-water temperature, dewpoint, and wind. Daily lake evaporation is also dependent on these factors, but may be appreciably affected by other factors as well, such as size and shape of lake, terrain in the vicinity, and possibly stability of the air. The true effects of these factors cannot be determined without additional observations at other lakes.

PRIMARY RECOMMENDATIONS

1. The techniques and methods tested at Lake Hefner should be given further tests at another lake with different terrain and climatic conditions. Although a water-budget control would be desirable for this additional test, it is considered that the results at Lake Hefner are sufficiently satisfactory and that enough has been learned of the physical processes involved to justify making the tests on a comparative basis without an accurate water budget. The following methods should be employed simultaneously and the results compared: (a) the mass-transfer method using Sutton's and Sverdrup's (1937) equations; (b) the quasi-empirical mass-transfer equation, including a comparison of measurements from the meteorological installation with data from the nearest first-order Weather Bureau station; (c) the energy-budget method; (d) the Cummings Radiation Integrator, comparing results with radiation measurements for the energy budget; (e) pan evaporation. In view of the importance of evaporation in the operation of reservoirs on the lower Colorado River, and in accordance with previous plans, these further tests are being conducted at Lake Mead.

2. The simplified field method for obtaining continuous records of evaporation described above (paragraph 6 of "Summary of Results") should be tried at a number of additional reservoirs under conditions of different terrain and climate.

3. Both the mass-transfer and energy-budget approaches appear promising as methods for determining water losses by evapotranspiration. It is recommended that these applications be further studied.

4. Both the mass-transfer and energy-budget techniques also offer promise for the prediction of evaporation from available climatological data, prior to the establishment of a lake or reservoir. The chief item lacking is information on both surface temperatures and energy storage in lakes of varied sizes and shapes and in different climates. Data on surface-water temperatures and wind movement are also needed for better interpretation of existing records of pan evaporation. It is therefore recommended that the present program for measuring water temperatures in lakes, reservoirs, and streams be considerably expanded, and that wind data be taken at selected lakes and reservoirs.

SECONDARY RECOMMENDATIONS

5. The experimental instrumentation for the mass-transfer and energy-budget studies at Lake Hefner proved very satisfactory and is recommended for use in other applications requiring measurements of temperature, humidity, wind, or radiation, provided that humidity measurements are not required during extended periods of below-freezing weather. For general application of mass-transfer or energy-budget techniques, however, further development should be undertaken with a view to producing commercial equipment requiring less servicing and maintenance.

6. Further research on mass-transfer processes is needed, with more refined measurements of wind, humidity, and temperature immediately above the water surface, especially just above, near, and below wave crests. Attention should also be focussed on evaluation of von Kármán's constant under field conditions.

7. Further research along the following lines is recommended to facilitate more general use of the energy-budget technique:

a. Additional observations on the reflectivity of a water surface under variable amounts of middle clouds.

b. Additional measurements of atmospheric radiation in conjunction with measurements of total atmospheric water-vapor content.

c. Development of better instruments and evaluation methods for measuring changes in energy storage.

d. Further studies of the Bowen ratio. Experiments in a suitable wind tunnel would probably yield useful results. The use of the Bowen ratio in connection with the Cummings Radiation Integrator might also be studied in this way.

Appendix: Symbols and Dimensions

The numbers in parentheses refer to the equations where the symbol first appears or where additional clarification may be obtained. There is some duplication of symbols because of the desire to preserve the notation used in the original papers.

Symbol	Dimensions	Description
a	LT^{-1}	A function of u_*, k_0, and z_0 (20).
a		Atmospheric transmission coefficient (75).
a		Reflectivity of solar radiation from a water surface (76).
a		Mean extinction coefficient due to molecular scattering (79).
a		Empirical constant (85, 86, 104).
a_d		Coefficient of extinction due to dust (79).
a_w		Coefficient of extinction due to water vapor (79).
a_1		A frictional parameter (45).
b		An empirical constant (23).
b		Reflectivity of sky radiation from a water surface (76).
b		Empirical constant (85, 86, 104).
c	$L^2T^{-2}\theta^{-1}$	Specific heat of water (63, 101).
c		Empirical constant (94, 105).
c_p	$L^2T^{-2}\theta^{-1}$	Specific heat of air at constant pressure (95).
d	L	Venturi-meter throat diameter (3).
e	$ML^{-1}T^{-2}$	Vapor pressure (numerical subscript indicates height in meters) (28).
e		Emissivity of a water surface (62).
e_a	$ML^{-1}T^{-2}$	Vapor pressure of the air (52).
e_i	$ML^{-1}T^{-2}$	Vapor pressure of unmodified air.
e_s	$ML^{-1}T^{-2}$	Saturation vapor pressure at T_a.
e_z	$ML^{-1}T^{-2}$	Vapor pressure at a height z (30).
e_0	$ML^{-1}T^{-2}$	Vapor pressure of saturated air at the temperature of the water surface (28).
f	ML^{-3}	Potential of vapor concentration (30).
f_0	ML^{-3}	Value of f outside the vapor blanket (34).
g	LT^{-2}	Acceleration of gravity (3).
h	L	Indicated stage change (2).
h	L	Venturi-meter head (3).
h	L	Cloud height (89).
$\bar{h}$		Average altitude of the sun in degrees (74).
k		Constant that is a function of latitude (74).
k_0		von Kármán's constant (18).
m		Solar air mass; the ratio of the length of the actual path of the solar beam to the path through zenith (75).
m		That part of the total incoming short-wave radiation that comes from the sun (76).
m_e	L^3	Volume of evaporated water (65).
m_o	L^3	Volume of surface outflow (65).
m_s	L^3	Volume of out-seepage (65).
n		An empirical constant (51).
n		That part of the total incoming short-wave radiation that comes from the sky (76).
n		Empirical coefficient or constant (105).

Symbol	Dimensions	Description
n_I	L^3	Volume of surface inflow (65).
n_p	L^3	Volume of precipitation (65).
q		Empirical constant (11); also specific humidity (53).
r	L	Radius of a circular evaporating surface (40).
r		Angle of refraction (80).
r		Reflectivity of a water surface (90).
u	LT^{-1}	Average wind speed in x-direction (numerical subscript indicates height in meters) (4).
u_z	LT^{-1}	Average wind speed at a height z (24).
u_*	LT^{-1}	Friction velocity, equal to $\sqrt{\tau/\rho}$ (4).
u_{*s}	LT^{-1}	Friction velocity over a smooth surface (45).
w'	LT^{-1}	Vertical component of fluctuating velocity
x	L	Distance along horizontal coordinate axis (8).
z	L	Distance along the vertical coordinate axis (4).
z_0	L	Roughness parameter (6).
A	L^2	Area of lake surface (43).
A	$ML^{-1}T^{-1}$	Vertical component of eddy conductivity (95).
A_c	L^2	Mean horizontal area of a nonstandard layer (92).
$\bar{A}_{\Delta h_i}$	L^2	Mean horizontal area of a standard layer (92).
C		Experimental constant (4).
C		Attenuation coefficient (77).
C	ML^2T^{-2}	The amount of energy in a nonstandard layer (91).
C		Average cloud cover in tenths of sky covered (74).
D	L^2T^{-1}	Molecular vapor diffusivity (49).
E	L^3T^{-1}	Volume of evaporated water in unit time (1).
E	L^3T^{-1}	Evaporation given by equation (104) or (105), assuming $e_0 = e_s$.
F	$ML^{-2}T^{-1}$	Evaporation per unit area (28).
F_L	$ML^{-2}T^{-1}$	Evaporation computed from Lettau's equation (29).
F_S	$ML^{-2}T^{-1}$	Evaporation computed from Sverdrup's equation (38), (29).
I	L^3T^{-1}	Volume of inflow in unit time (1).
I	L	Thickness of an intermediate layer (45).
I_0	MT^{-3}	Solar radiation received on a horizontal surface at the exterior of the earth's atmosphere (75).
J		Momentum integral (10).
K	L^2T^{-1}	Eddy diffusivity (46).
L	L^2T^{-2}	Latent heat of vaporization (60).
M	$ML^{-1}T^{-1}$	Total evaporation from a strip of unit width and length x (34).
M	L^3	Total volume of outflow (64).
N	L^2T^{-1}	The macroviscosity (50).
N	L^3	Total volume of inflow (64).
O	L^3T^{-1}	Volume of outflow in unit time (1).
P	$ML^{-1}T^{-2}$	Atmospheric pressure (28).
Q	L^3T^{-1}	Venturi-meter discharge (3).
Q_a	ML^2T^{-2}	Atmospheric radiation (long wave) (62).
Q_b	ML^2T^{-2}	The net energy lost by the body of water through the exchange of long-wave radiation between the atmosphere and the body of water (59).

Symbol	Dimensions	Description
Q_e	ML^2T^{-2}	Energy utilized by evaporation (59).
Q_h	ML^2T^{-2}	Energy conducted from the body of water by the atmosphere as sensible heat (59).
Q_n	ML^2T^{-2}	Net radiant energy defined by equation (108).
Q_r	ML^2T^{-2}	Reflected solar radiation (59).
Q_s	ML^2T^{-2}	Solar radiation incident to the water surface (59).
Q_v'	ML^2T^{-2}	Net energy advected into the body of water by all water volumes entering or leaving the body of water (59).
Q_v	ML^2T^{-2}	Net energy advected into the body of water by all volumes entering or leaving the body of water, except that volume leaving as evaporated water (68).
Q_{bs}	ML^2T^{-2}	Long-wave radiation emitted by the body of water (96).
Q_w	ML^2T^{-2}	Energy advected out of the body of water by the mass of evaporated water (68).
Q_θ	ML^2T^{-2}	The increase in energy stored in the body of water (59).
Q_{ao}	ML^2T^{-2}	Atmospheric radiation from an overcast sky (87).
Q_{ar}	ML^2T^{-2}	Reflected atmospheric radiation (73).
R		Bowen ratio or ratio of Q_h to Q_e (60).
R_ξ		Turbulent correlation coefficient
Ri		Richardson's Number (18).
R_t		Total reflectivity of solar and sky radiation from a water surface (76).
R_x		Ratio x/z_0 (38).
S	L^3T^{-1}	Change in reservoir contents in unit time (1).
S		Standard error of mean (2).
S	$T^2L^{-2}\theta$	Stability parameter (26).
S_A		Altitude of the sun, in degrees, above the horizon (77).
T	θ	Absolute temperature (22).
T	θ	Temperature (numerical subscript indicates height in meters) (24).
T		Mean turbidity factor (79).
T_a	θ	Temperature of the air (61).
T_a	θ	Absolute temperature of the air near the ground (86).
T_b	θ	Arbitrary base temperature (63).
T_e	θ	Average temperature of evaporated water (67).
T_I	θ	Average temperature of surface inflow (67).
$\bar{T}_t$	θ	Average temperature of a standard layer of water (91).
T_0	θ	Average temperature of surface outflow (67).
T_p	θ	Average temperature of precipitation (67).
T_s	θ	Average temperature of out-seepage (67).
T_s	θ	Source temperature (90).
T_z	θ	Temperature at a height z (24).
T_1	θ	Average temperature of a body of water at the beginning of a period (67).
T_2	θ	Average temperature of a body of water at the end of a period (67).
U	LT^{-1}	Horizontal wind speed outside the boundary layer (8).
V_1	L^3	Volume of a body of water at the beginning of a time interval (64).
V_2	L^3	Volume of a body of water at the end of a time interval (64).
W	LT^{-2}	Frictional drag of water surface (8).

Symbol	Dimensions	Description
α		An empirical constant (11).
β		An empirical constant (20).
β		An empirical number (36).
γ		A number defined by equation (107).
γ		Empirical constant (86).
γ	$L^{-1}\theta$	Adiabatic lapse rate (95).
δ	L	Thickness of the momentum boundary layer (10).
δ_e	L	Thickness of the laminar film (49).
δ_w	L	Thickness of the vapor blanket (34).
ϵ	L	Mean diameter of roughness elements (5).
ϵ		Naperian base (86).
ϵ_s	L	Equivalent sand roughness (4).
η		The ratio z/z_0 (36).
η_1		The value of η at $z=\delta_w$ (37).
λ		Empirical coefficient (87).
μ		Index of refraction for pure water relative to air (80).
ν	L^2T^{-1}	Kinematic viscosity of the air (4).
ξ	T	Time required for R_ξ to decrease from 1 to 0
ρ	ML^{-3}	Density of the air (8).
ρ	ML^{-3}	Density of water (60).
ρ_e	ML^{-3}	Average density of evaporated water (65).
ρ_0	ML^{-3}	Average density of surface outflow (65).
ρ_p	ML^{-3}	Average density of the precipitation (65).
ρ_s	ML^{-3}	Average density of out-seepage (65).
ρ_I	ML^{-3}	Average density of the surface inflow (65).
ρ_M	ML^{-3}	Average density of the volume of outflow (64).
ρ_N	ML^{-3}	Average density of the volume of inflow (64).
ρ_1	ML^{-3}	Average density of the body of water at the beginning of a time interval (64).
ρ_2	ML^{-3}	Average density of the body of water at the end of a time interval (64).
σ		An empirical number (18).
σ	$MT^{-3}\theta^{-4}$	Stefan-Boltzmann constant for black-body radiation (62).
τ	$ML^{-1}T^{-2}$	Eddy shearing stress (8).
ϕ		The ratio u/u_* (36).
ϕ_1		The value of ϕ at $z=\delta_w$ (37).
Γ		The gamma function (51).
Δ		Slope of saturation vapor pressure vs temperature curve (106).
T	MT^{-2}	Total frictional force (31).
X_r		The ratio x/z_0 (38).
Ψ_r		The integral defined in (37).

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