

Some Aspects of the Shape of River Meanders

GEOLOGICAL SURVEY PROFESSIONAL PAPER 282-E



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By RALPH A. BAGNOLD

PHYSIOGRAPHIC AND HYDRAULIC STUDIES OF RIVERS

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SYMBOLS

A	cross-sectional area of flow
a	a numerical coefficient
b	channel width
C_s	a coefficient of wall friction
c	a coefficient related to head loss
d	diameter of pipe
e	overall resistance coefficient
F	force
F	Froude number
g	acceleration of gravity
H	a drawdown in head in a pipe
j	a numerical coefficient
k	a numerical coefficient
n	a numerical coefficient
r_m	mean radius of curvature of channel
R	mean radius of curvature of pipe
s	slope of water surface
u	velocity
v	tangential velocity
τ	distributed stress
ρ	mass density of water
θ	angle subtended by a curve, being the angle between radii at beginning and end of curve

PHYSIOGRAPHIC AND HYDRAULIC STUDIES OF RIVERS

SOME ASPECTS OF THE SHAPE OF RIVER MEANDERS

By RALPH A. BAGNOLD

ABSTRACT

From consideration of the probable nature of flow resistance in curved channels, a simple dynamical model is proposed to relate resistance to a criterion of bend curvature applicable both to closed pipes and to open channels. The theory indicates that resistance should fall to a minimum when the radius of channel curvature bears a certain critical ratio to the channel width; and this critical ratio should have approximately the same value for both closed and open channels, irrespectively of scale or of boundary roughness.

Resistances of pipe bends are known to fall to a sharply defined minimum when the curvature ratio, mean radius to diameter, is between 2 and 3. Recently measured flow resistances of sinuous open channels disclose that the same minimum occurs at approximately the same curvature ratio.

The theory therefore appears to go some way in explaining the mechanism tending to restrict the bends of rivers of all sizes to a curvature ratio between 2 and 3.

ENERGY DISSIPATION IN A CURVED CHANNEL

When water flows through a curved circular pipe of diameter d and mean radius of curvature R a transverse flow is created (fig. 81). Such a transverse flow is inevitable so long as the pipe walls exert a frictional drag on the tangential flow, for the radial acceleration gives rise to a calculable excess of fluid pressure on the outside of the bend and a corresponding pressure deficit in the inside. Throughout the main body of the flow the radial pressure gradient is balanced by the gradient of the centrifugal pressure. But it must remain unbalanced over the periphery, owing to the effect of boundary friction in reducing the flow velocity and therefore the peripheral pressure gradient in the radial direction. Hence water must flow radially inwards over the periphery, down the radial pressure gradient. And its place is taken by an outward flow across the center-line of the cross section.

The effect of the transverse flow is to reduce the internal shear due to the distortion of the flow round the bend. For if the transverse flow were prevented, say by the insertion of frictionless fins projecting in-

wards towards the central axis of the pipe, the maintenance of unaccelerated flow would require a uniform shear between the outside water and the inside water

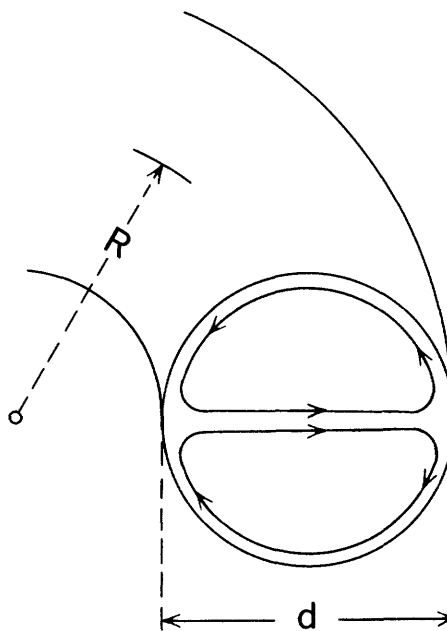


FIGURE 81.—Idealized diagram of transverse flow in a pipe bend. R is mean radius of pipe curvature, and d is diameter of pipe.

as in figure 82. The rate of shear would be $\frac{v}{R + \frac{d}{2}}$,

where v is the tangential velocity, R is the radius of curvature, and d is the pipe diameter.

The effect of the transverse flow may be compared to the effect of the lay in a rope which makes it flexible by greatly reducing the internal shear occasioned by bending it.

If the path of a peripheral water particle is regarded as a helix whose projected cross section is semicircular, the internal distortion over any given central angle

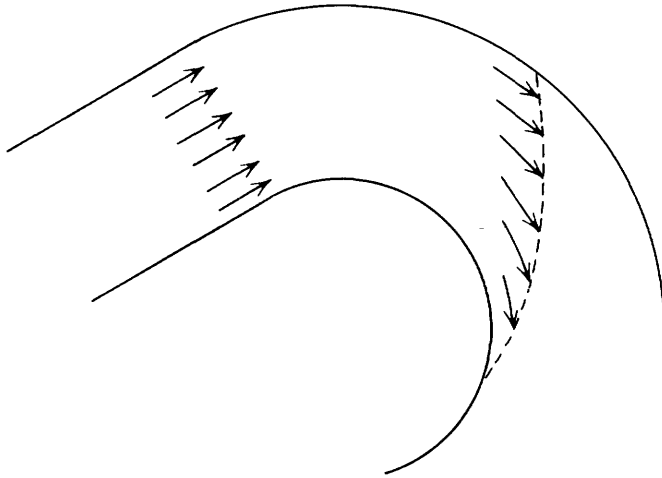


FIGURE 82.—Idealized diagram of shearing motion between water filaments in a pipe bend if transverse flow were prevented.

subtended by the arc of the channel bend may be measured as the difference in path length between that of the helix and that of its core arc. For any given ratio R/d this difference has a minimum value for a certain pitch of the helix. That is, a certain optimum ratio exists between the transverse and the tangential velocities.

Let it now be assumed, on the general principle of least effort, that the transverse flow ultimately attains this optimum velocity ratio. An angular acceleration is required to create the transverse flow, and this may be supposed to occur within some finite initial length of the channel arc. So, let it also be assumed that the arc of the curved channel exceeds this initial length, as defined by a certain minimum subtended angle θ_0 .

Then if we neglect, as a first approximation, any residual energy dissipation due to the small continuing internal distortion after the steady state has obtained, the resistance to flow through the whole channel arc is measurable by the sum of: (a) the rate of energy expenditure required to create the transverse flow (assumed dissipated beyond the end of the channel bend); and (b) the rate of energy dissipation by wall friction.

The total resistance to flow, due to the channel bend, is thus the sum of two forces: (a) force F_a required to overcome the inertia-resistance to the creation of the transverse flow, and (b) a force F_b required to overcome wall friction.

F_a should be proportional to $A\rho v^2 d/R$ where A is the cross-sectional area of the flow $=\pi d^2/4$. F_b is equal to $\tau_s \pi d R \theta = C_s \rho v^2 4A/dR\theta$ where $C_s = \tau_s \rho v^2$, the coefficient of wall friction.

The total resisting force $F = F_a + F_b$ should therefore be

$$A\rho v^2 (ad/R) + 4C_s \theta R/d$$

where a is some unknown constant.

The overall resistance coefficient e should therefore be

$$e = ad/R + 4C_s \theta R/d \tag{1}$$

Comparing channel bends which subtend any constant central angle θ exceeding θ_0 , equation (1) can be written in the general form

$$e' = e/a = d/R + n^2 R/d \tag{2}$$

where n^2 is a constant (assuming as an approximation that C_s is constant, depending only on the wall roughness).

Equation (2) contains only two independent variables R/d and n , and e' has a minimum value $2n$ at $R/d = 1/n$.

The family of curves obtained by giving n a range of arbitrary values is shown in figure 83. It should be noted that the geometry (fig. 84D) imposes an absolute cutoff at $R/d = 0.5$, for the inner boundary then has a zero radius of curvature and becomes an abrupt angle. Hence the curves of figure 83 for $R/d < 0.5$ are imaginary only.

EFFECT OF BREAKAWAY AND EDDY FORMATION

Further, as R/d is reduced towards this limit the tacit assumption that the flow conforms to its solid

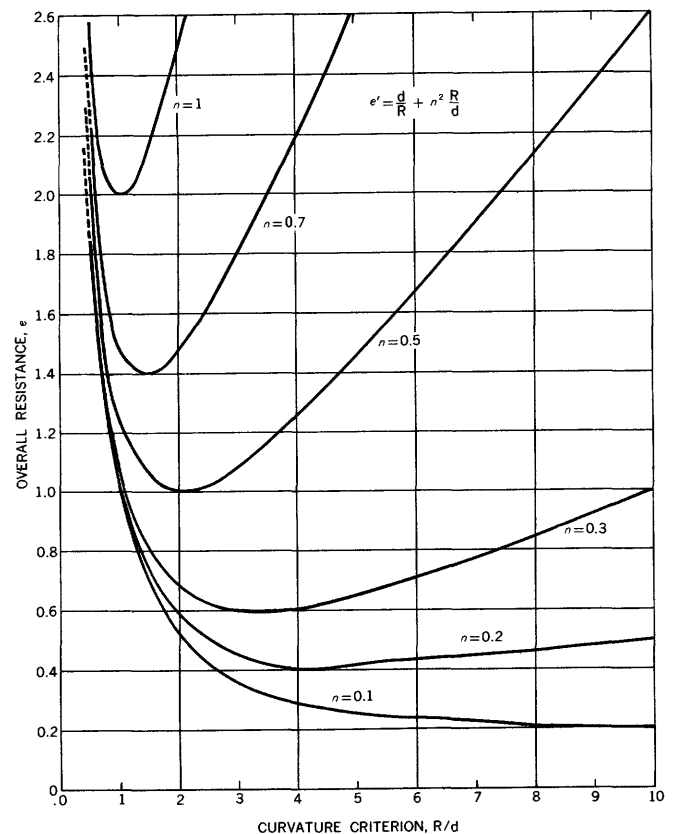


FIGURE 83.—Theoretical relation between overall resistance coefficient e' and curvature criterion, radius of curvature divided by pipe diameter, for different arbitrary wall resistances.

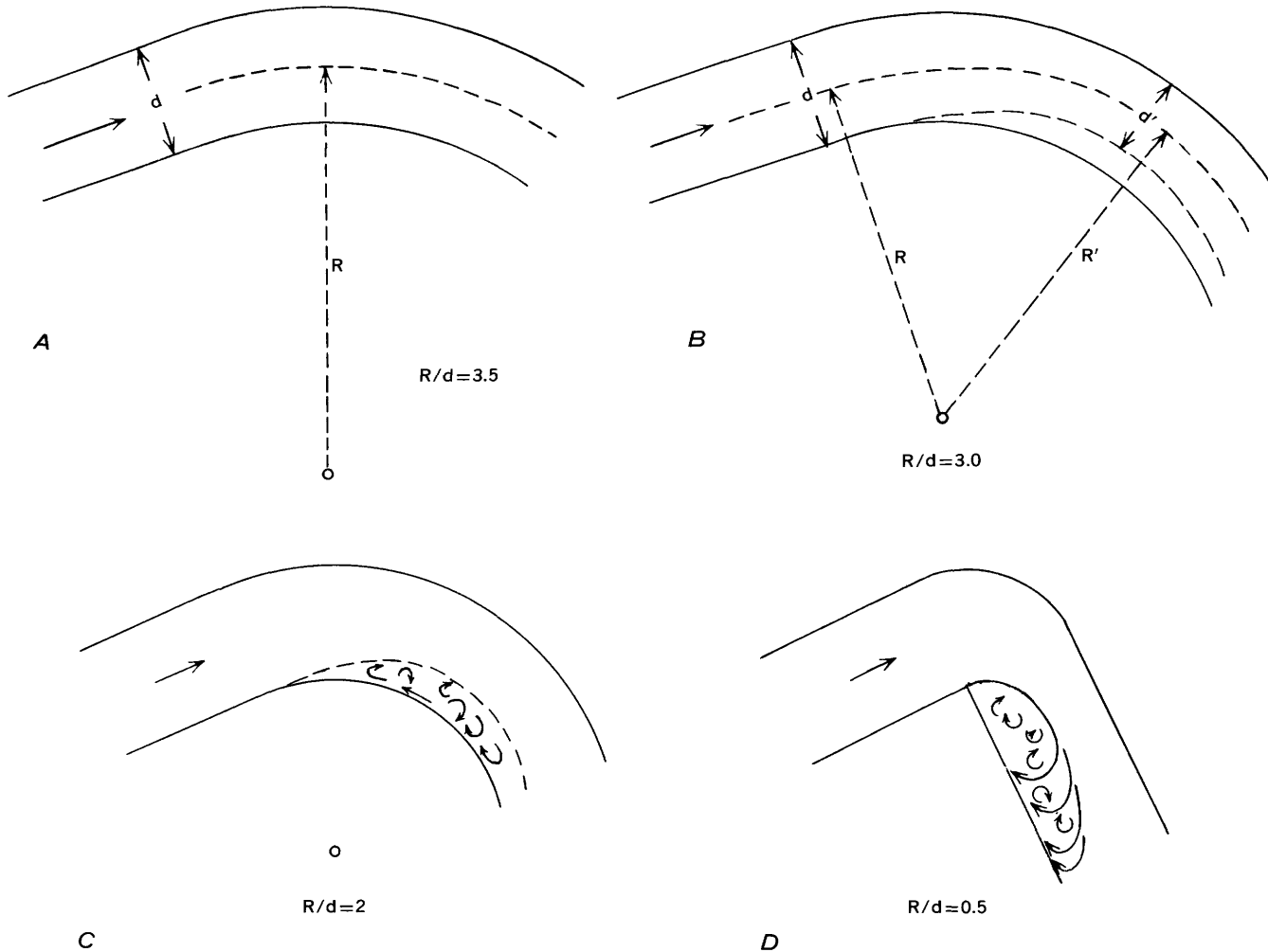


FIGURE 84.—Diagrammatic relation between local radius of curvature of water filaments and radius of curvature of pipe bend.

- A. Curvature without appreciable asymmetry of the flow.
 B. Flow asymmetry creates a relatively stagnant but still stable zone, and a decrease in the effective value of R/d .
 C. Breakaway occurs, at which the stagnant zone becomes unstable and dissipates energy in eddying.
 D. When the bend ultimately forms a right angle, the unstable zone is large and very much pronounced.

boundary must cease to be true at some critical value of R/d considerably greater than 0.5. For a stage must be reached at which the flow along the inner boundary becomes unstable and breaks away from the boundary, leaving an intervening space occupied by a zone of unstable and confused fluid motion, figures 84C and D.

That such a stage must be reached is a matter of common experience. A real fluid will not flow smoothly round a sharp projecting curve. But it is not possible to predict quantitatively just what that stage will be in the case of a real frictional fluid. Nor is it possible to predict what effect the change in the flow condition will have on the overall resistance to flow.

The factors most likely to determine the breakaway stage are the ratio R/d , the degree of turbulence in the entry flow, and the velocity distribution over the entry cross section. Under comparable conditions as regards

the two last factors, the breakaway stage is likely to be determined only by the ratio R/d .

But there are no grounds for assuming that the critical value of R/d at which breakaway occurs will coincide with the critical value at which the resistance becomes a minimum by the considerations leading to equation (2). So, it is to be expected that breakaway may occur at a larger or a smaller value of R/d .

The study of the effects of breakaway has been confined largely to conditions of diverging flow. As the divergence is increased, the flow is found to become suddenly unstable in the neighborhood of the diverging boundaries, with a consequent sudden increase in the rate of energy dissipation.

But in the case of a bend in a channel of uniform width there is no initial divergence anywhere. As the radius of bend curvature is reduced, the velocity distri-

bution becomes progressively more asymmetrical.¹ The flow tends to recede away from the inside region of greatest curvature and towards the outer boundary. Consequently, in the region near the inner boundary the shear rate is reduced, and with it both the shear stress in the fluid and the pressure gradient in the direction of flow.

A final abrupt change occurs when the pressure gradient along the inner boundary is so far reduced as to allow a backward leakage to take place. Flow in the inside region now becomes unstable. Local eddying results, accompanied by increased energy dissipation and increased flow resistance associated with breakaway.

But if we confine attention to conditions that exist just prior to this breakdown of the inside flow into large eddies, we have: (a) A reduction in boundary resistance at the inner side. This is, however, probably compensated by a corresponding increase at the outer side where the velocity gradient is increased. (b) A restriction of the flow proper to the outer side of the channel because of inertia effects, while as yet the inside flow is still stable and no dissipatory eddying has set in.

As the critical value of the ratio R/d of channel boundary curvature is closely approached, and the asymmetry of the velocity distribution increases, the still stable and nondissipatory body of relatively un-sheared fluid on the inner side may be expected to increase in width, thereby narrowing the effective width of the flow proper from d to d' , and increasing² its effective radius from R to R' , as sketched in figure 84B.

Consistent with this notion we have therefore immediately prior to the breakdown of the inside flow a stage in which frictional dissipation has not appreciably increased, but in which the flow proper has an effective curvature ratio R'/d' increasingly larger than the channel value R/d .

As a result, we should expect the value of the first term of equation (2), which represents the inertia re-

¹ The experimental evidence is somewhat confused as regards the changes in the velocity distribution which takes place within a pipe bend. There is general agreement that the distribution within the bend is principally asymmetrical, the effective flow being confined to a proportion only of the channel cross section. But whether the zone of high velocity is found to hug the outer or the inner wall of the bend at any particular cross section of it appears to depend on a combination of factors, such as velocity distribution at entry, angular distance from entry, and the curvature ratio R/d .

The reasoning which follows is, however, very largely independent of whether the effective flow hugs the outer or the inner boundary, being based only on the generally accepted fact that the effective flow is constricted to occupy a less width than that of the channel. Hence the words "outer" and "inner" may be interchanged without affecting the general sense.

² Interchanging if necessary the words "inner" and "outer," the effective radius R' is decreased. But the change in radius is small compared to that in width. Hence the effect of a decrease in width, from either the inner or the outer boundary, from d to d' is always an increase in the effective curvature ratio from R/d to R'/d' . And the changes are approximately proportional, the error becoming more appreciable as R/d approaches its limit at 0.5.

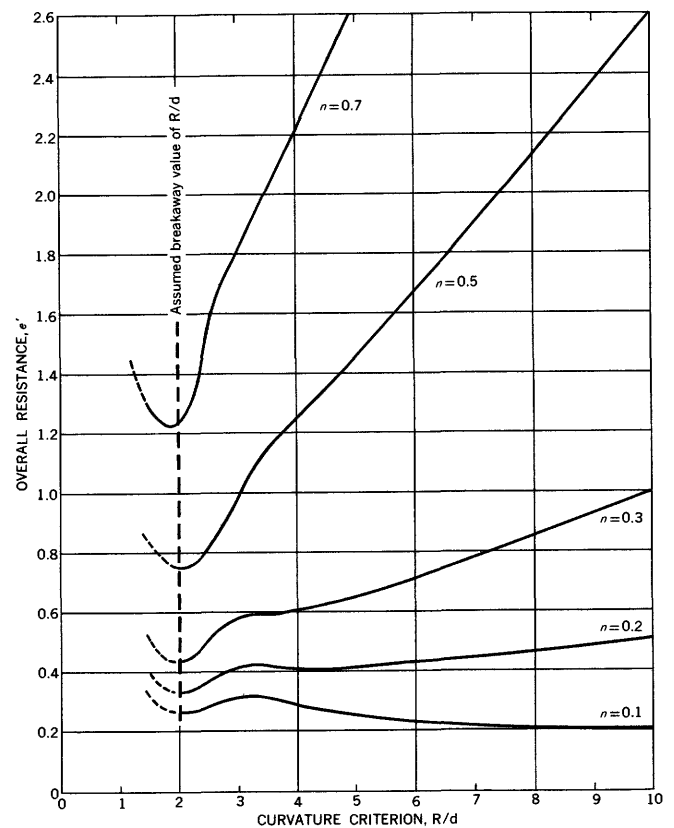


FIGURE 85.—Theoretical relation between total pipe resistance and curvature criteria (equations 2 and 3) modified on the assumption that the effective values d'/R' of the ratio d/R becomes reduced to half the channel value d/R just before the breakaway in stability occurs.

sistance, to decrease from d/R to d'/R' , while the second term representing the boundary friction, to remain more or less unchanged at R/d . On this view the overall resistance represented by σ' in equation (2) should begin to depart from the values indicated by figure 83 at some value of R/d not far exceeding the critical value at which the breakdown of the inner fluid zone occurs. And the overall resistance should reach a local minimum value at the point of breakdown, followed when the breakdown has actually occurred by a rapid and progressive rise due to the well-known throttling effect of an unstable flow zone.

In figure 85 the curves of equation (2) as shown in figure 83 have been modified on the assumption that as the abscissa value, channel ratio, R/d is reduced, (a) the inner zone becomes unstable and breaks down at $R/d = 2$; (b) as the abscissa ratio is reduced towards 2 the effective flow ratio R'/d' in the first term of equation (2) increases to a maximum value of 4 attained at the point of breakdown, while R'/d' in the second term remains unchanged at R/d ; (c) for still smaller abscissa ratios (actual channel dimensions) the overall resistance increases rapidly.

All the curves show a minimum resistance at the assumed breakdown ratio $R/d=2$. But those for small values of n (smooth channels) also retain the original minimum at the larger value of R/d .

Now compare figure 85 with figure 86, which reproduces the summary diagram given by Hofmann (1929, p. 45) of the experimental data on the resistance of 90° pipe bends of varying roughness.

The experimental curves appear inconsistent among themselves, presumably because of differences in the experimental conditions, most probably in the conditions of entry flow.³

In spite of the apparent mutual inconsistencies disclosed in figure 86, the correspondence in general pattern between figure 86 and figure 85 is remarkable in view of the gross simplifying assumptions introduced into the reasoning on which figure 85 is based.

In particular, all the experimental curves show either a sharp minimum or else an abrupt upturn in the value of the resistance coefficient to occur within the same narrow range from 2 to 3 in the ratio R/d , irrespective of large variations in the boundary roughness and irrespective of the variations in other experimental conditions which are evident from the disparity in the shapes of the different curves.

It would seem difficult to account for this nearly constant critical value of R/d and for the pronounced changes in the resistance value associated with it by any other explanation than that of the phenomenon of breakaway.

Whatever the explanation offered, the experimental evidence on the resistance of closed 90° pipe bends indicates that with the possible exception of very smooth bends, the resistance undergoes a decrease to a pronounced minimum value when R/d is between 2 and 3.

If the foregoing explanation is correct, this marked decrease in resistance should occur within the same range of R/d whatever the subtended angle, provided it exceeds some minimum angle, and whatever the boundary roughness, provided the roughness of the channel upstream of the bend is of the same order.

CONDITIONS IN A CURVED OPEN CHANNEL

The conditions of flow round a bend in an open channel can, as a first approximation, be regarded as those of the flow in the lower half of a closed pipe bend. The single transverse flow can be regarded as that of one only of the two equal and opposite circulations in the case of a closed pipe.

Differences exist, however, which may or may not have an influence on the occurrence and effects of breakaway. The pattern of the internal turbulence cannot be the same because the free surface of the open channel flow prevents any internal fluid motion taking place through it in the vertical direction. Again, in any natural open channel the cross section is far from semicircular, being greatly flattened. And, whereas, experiment shows that the transverse peripheral flow extends over the whole of each semiperiphery from the outside to the inside of a circular pipe, in the same type of experiment on flow in a shallow open channel (Leopold and others, 1960) the transverse peripheral flow inwards did not extend much beyond the centerline. Towards the inner bank the transverse motion appears to become confused by merging in a random way with the main flow. This suggests that the degree of turbulence increases towards the inner bank in a way it does not do towards the inner wall of a circular pipe bend.

Now experiments on the conditions under which the flow breaks away from the boundary, in the case of rectilinear diverging flow, show that the occurrence of breakaway can be appreciably delayed by increasing the degree of turbulence near the boundary. This has the effect, by increasing the shear stress in the fluid in the downstream direction, of tearing away any incipient eddy as it begins to form. This follows from the fact that the greater the downstream shear, the greater the stability of the flow against the effects of a local upstream pressure gradient, or a local reversed gravity slope in the open channel case. That is, in terms of the inverse ratio b/r_m used in a previous paper (Leopold and others, 1960) on the results of relevant open channel experiments, the critical value of b/r_m might be nearer 0.5 than 0.33, where b is the channel width and r_m the mean radius of the channel centerline.

On this view it would not be unexpected that the breakaway stage should occur in a shallow open channel at a rather smaller value of R/d than in a circular pipe. The critical value might well be nearer 2 than 3.

Apart from this possible change, no clear reason is apparent for any radical difference in the shape of the curves of resistance coefficient against R/d when the closed pipe bends are exchanged for open channel bends which subtend the same central angle. But the experimental curves are likely to have a like diversity among themselves, as is apparent in Hofmann's curves for the closed bends, on account of differences in experimental entry conditions. In particular, some special peculiarities are to be expected in the case of a succession of alternately reversed bends, because the

³ For the results of subsequent work, none of which discloses any radical disagreement with figure 86, see Yarnell and Woodward, 1936; Robertson, 1944; Mockmore, 1944.

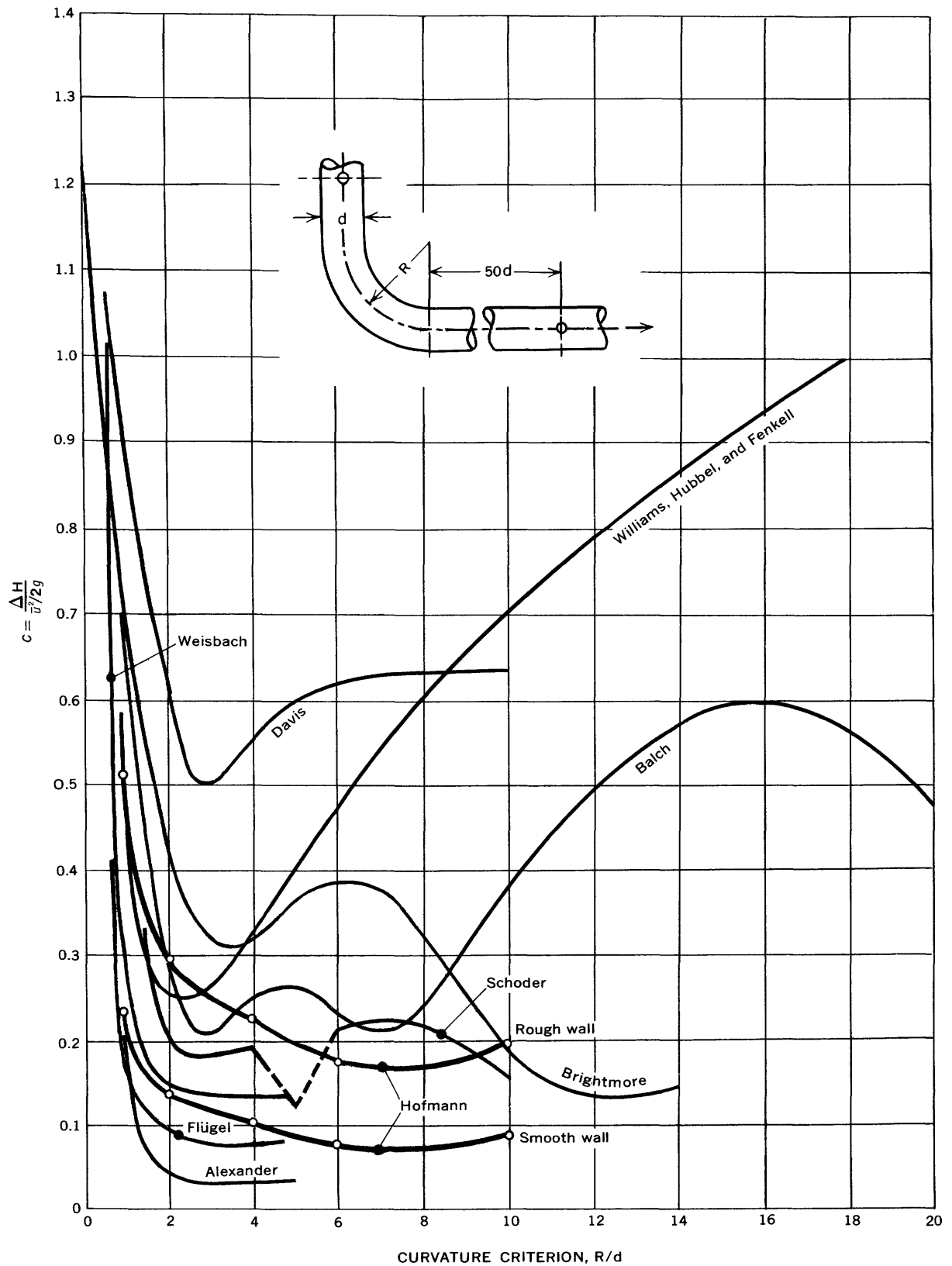


FIGURE 86.—Experimental relation of resistance as a function of curvature criterion for 90° bends in pipes (after Hofmann, 1929).

entry flow into each bend will be asymmetrical about the centerline of the channel.

APPLICATION TO SUCCESSIVE BENDS IN CONSTANT CHANNEL LENGTH

It must be remembered that the existence of the resistance minimum shown in figure 83 arises solely on account of the definition of the bend as subtending a constant central angle. The increase of resistance with increasing R/d towards the right of the figure is due to the increase in frictional resistance as the wall area increases with R .

The conditions become very different in the case of a succession of bends contained within a constant length of channel. The second term in equation (1) then becomes constant; and the equation takes the simple form

$$e = a \frac{d}{R} + e_s \quad (3)$$

where e_s is constant and is measurable by the resistance of a straight channel of identical cross section and roughness. The curve of e against R/d is now that of a rectangular hyperbola.⁴ And if the theory is sound, an experimental plot of e against R/d should conform to this curve for all values of R/d except in the immediate neighborhood of the critical value at which breakaway occurs.

But by inference from the results of the pipe experiments, we should expect an abrupt departure from this curve in the order of $R/d=2$ to 3, in the form of a marked fall in resistance that is due to conditions immediately preceding the occurrence of breakaway.

COMPARISON WITH EXPERIMENT IN OPEN CHANNELS

In the results of laboratory experiments on flow resistances in open sinuous channels (Leopold and others, 1960) the flow resistances derived from measurements made during three of the many runs appeared anomalous. The data respecting two of these three were omitted from the analyses in that paper on grounds of relatively inadequate coverage by individual measurements. The data were, however, published in the appendix of that paper (p. 134), and those data are included in figure 87 of the present paper. All three results previously thought to be anomalous now appear in the light of the present discussion to be wholly consistent.

⁴ Robertson (1944) having plotted in his figure 18 the measured resistance coefficients of bends in both pipes and rectangular open channels against R/d , suggested this hyperbolic relationship on empirical grounds. Unfortunately his experimental range of R/d was confined to small values. So the anomalous kink disclosed by the present figure 87 was not apparent.

Figure 87 shows all the resistance values obtained (appendix Leopold and others, 1960) without omission, plotted⁵ against the channel curvature ratio $r_m/b (= R/d)$.

The three previously anomalous resistance values are those lying together at a value of r_m/b in the order of 1.75. By inference from the pipe bend results, the low resistances here indicate the special conditions obtaining when breakaway is closely approached. The circles in figure 87 represent the results of runs at full depth, and the crosses, the results at reduced depth. The broken curve is that of the hyperbola

$$e = 0.0624d/R + 0.0026 \quad (4)$$

the numerical parameters having been chosen to give the best fit.

The only discrepancy from the theory is that in order to make the hyperbola given by equation (3) fit the plotted experimental curve, the asymptote e_s must be given the value 0.0026, whereas the values of e_s as measured for the straight channel were 0.0165 for full flow depth and 0.0160 for reduced flow depth. These measured values are in fact very close to those obtained in the experimental curved channel of least curvature (r_m/b 4.55 and 5.25), the channel curvature being then still very appreciable.

It appears, on the face of it, that channels of less curvature ($r_m/b > 5$) offer no greater resistance to flow than a straight channel. This is just possible on the assumption that no transverse flow is set up, the internal shear involved in simple tangential flow being permitted without any increase of resistance by some change towards organization in the otherwise random internal turbulent motion.⁶

It is of interest that in figure 87 the plotted points marked by a cross (reduced depth) give by themselves

⁵ In this plot the ordinate is the overall square-law coefficient $\tau \frac{1}{\rho \bar{u}^2}$ (which is equal to s/\bar{F}^2 because in the experiment the hydraulic mean depth was kept constant) given by the constant slope of the lower portion of each of the graphs, figures 70 and 71, of the above-mentioned paper. For comparison of the numerical values of the ordinates of the present figures 86 and 87, it should be noted that the functional definitions differ by a factor of 2. The coefficient e of figure 87 can be written $\frac{\tau}{\rho \bar{u}^2} = \frac{hsg}{\bar{u}^2}$, h being the hydraulic mean depth, whereas the coefficient c of Hofmann's pipe bend diagram, figure 86, is defined as $c = \frac{\Delta H}{\bar{u}^2/2g}$.

⁶ The above discrepancy appears consistent with the notion that the resistance of a straight channel may, for some purposes, be usefully regarded as comprised of two separate elements: a skin element proper due to energy dissipation arising directly from and proceeding very close to the boundary and an indirect body element proceeding internally throughout the general body of the flow. On this view the hyperbola of equation (4) shown by the broken curve of figure 87 would be expected to be asymptotic not to the whole straight channel resistance but to the lesser skin resistance whose contribution to e is only 0.0026.

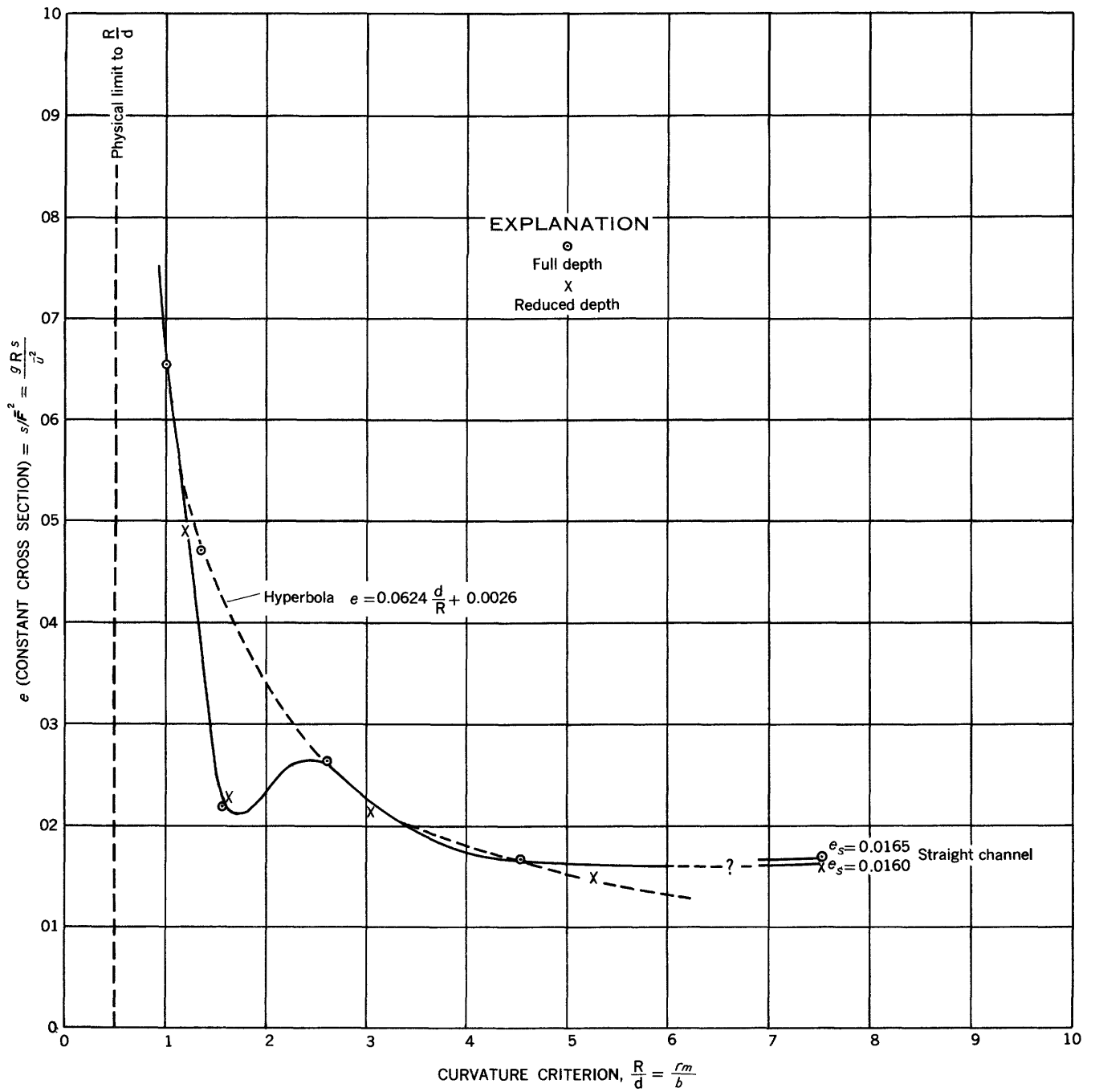


FIGURE 87.—Experimental relation between resistance and curvature criterion for sinuous open channels of constant cross section. Data from Leopold and others (1960).

a curve which is systematically displaced downwards from the curve given by the full depth plot marked by circles. And the ratio of the reduction in e is 1 to 1.03 approximately between r_m/b 3 to 5. The corresponding ratio of the two straight-channel values of e , was almost identical with this, being 1 to 1.04.

It is also of interest to note that the magnitude of the reduction in the resistance coefficient e at $R/d=1.75$ in figure 87 is consistent with the view that this reduction is due to the increase in the effective curvature ratio R'/d' in equation (3), for the magnitude of the reduction found experimentally appears to be predictable from a reasonable assumption regarding the relative constriction of the effective flow width.

Let us assume that the stagnant but still stable and nondissipatory zone occupies approximately the same proportion jd of the whole channel width d as when at a slightly increased curvature it breaks down into an easily observable dissipatory eddy zone. The effective flow width d' is then $d(1-j)$. And the effective flow radius R' is $R(1+\frac{1}{2}jd/R)$. Where

$$\frac{R'}{d'} = k \frac{R}{d} = \frac{R}{d} \frac{1+\frac{1}{2}jd/R}{1-j}$$

and

$$k = \frac{1+\frac{1}{2}jd/R}{1-j}$$

The first (inertia resistance) term in equations (1) and (3) will be reduced by a factor $1/k$. From observations made during the sinuous channel experiments, the effective flow width was reduced by about $0.35d$; that is $j=0.35$. Then, if breakaway occurs at $R/d=1.75$, the factor $1/k=0.59$. And the fall in resistance from what it could have been if the breakaway phenomenon had been absent will be $0.41 ad/R$. Taking a as 0.0624 from equation (4), and $d/R=0.57$, the fall in the value of the resistance coefficient e should be 0.015 approximately.

The experimental fall in e , along the ordinate $R/d=1.75$, from the hyperbola curve to the lowest plotted points is 0.016 approximately, which is in astonishingly good agreement.

It should also be noted that the relative amount of the fall in resistance, which takes place just before the final rise in the pipe results, figure 86, is of just the same order as that in the open channel results, figure 87, taking the different critical values of R/d into consideration in the above calculation.

A pronounced fall in the resistance to flow, confined within the same narrow range of boundary curvature, is thus found to occur in the same degree in both closed-pipe bends and open-channel bends. Hence, evidently it cannot be associated with the instability of the free

surface of an open channel as defined by the Froude number criterion. There can be little doubt that this fall in resistance must be associated directly with the breaking away of the flow from a boundary of the bend. If so, since breakaway always occurs at some stage as the channel curvature is increased, whatever the shape of the channel cross section and whatever the degree of turbulence, the fall in resistance appears to be a very general property of the flow of real fluids, a property in which neither Froude number nor Reynolds number is essentially involved.

The precise channel curvature, as defined by the ratio R/d or b/r_m , at which the resistance minimum occurs is likely, however, to be influenced by the flow conditions immediately upstream of the channel curvature. But provided these conditions are similar, for example, if in comparable cases the channel consists of a succession of alternately reversed bends as in a meandering river, the critical value of b/r_m is likely to remain of the same order.

Thus the critical value found in the small-scale sinuous open channel experiments seem likely to be of the same order as that to be found for the largest meandering river flowing at a low Froude number and for a small stream flowing at a very high Froude number in a meandering melt channel cut in the steep ice surface of a glacier.

RELATION TO RIVER MEANDERS

Leopold and Wolman (1960) have compiled considerable evidence that when other conditions—as yet unknown—cause a stream of any size which flows in a deformable channel to develop a meander pattern, the ratio r_m/b of the meander bends does in fact tend to a constant common value between 2 and 3. One might be tempted to conclude that a satisfactory explanation has now been found.

But such a conclusion would gloss over several important steps in the reasoning. Strong evidence has been found that the resistance to flow in a channel of uniform cross section falls to a sharply defined minimum within the narrow range of the curvature ratio r_m/b between 2 and 3 approximately. But we have yet to show that the same minimum of resistance does in fact occur in a natural meandering channel whose cross section is not uniform.

If the same resistance minimum is found to occur in a natural channel, we have still to explain why the configuration of the natural channel should tend towards that giving minimum resistance. The reason is far from being self-evident.

Any explanation which may be put forward must remain speculative until the nature of the general dynamic mechanism is understood whereby flow in a

deformable channel tends to mould its channel to a certain preferred configuration.

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