Factors Controlling the Size and Shape of Stream Channels in Coarse Noncohesive Sands

By M. GORDON WOLMAN *and* LUCIEN M. BRUSH, Jr.

PHYSIOGRAPHIC AND HYDRAULIC STUDIES OF RIVERS

GEOLOGICAL SURVEY PROFESSIONAL PAPER 282-G

A laboratory study of the development of equilibrium channels

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1961

UNITED STATES DEPARTMENT OF THE INTERIOR

STEWART L. UDALL, *Secretary*

GEOLOGICAL SURVEY

Thomas B. Nolan, *Director*

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington 25, D.C.

CONTENTS

ILLUSTRATIONS

TABLES

SYMBOLS

a coefficient representing width at unit discharge
A cross sectional area
b an exponent in the equation relating width to discharge
d mean depth, defined as ratio of cross-sectional area to width
d_L centerline depth
f an exponent in the equation relating mean depth and discharge
$\boldsymbol{f}$ Darcy-Weisbach resistance factor
$\boldsymbol{F}$ Froude number
k a coefficient
K ratio of tractive force on channel bank and bed
m an exponent in the equation relating mean velocity and discharge
n computed roughness
q water discharge per unit width
q_s discharge of sediment per unit width
Q water discharge, in cubic feet per second
R hydraulic radius, in feet
s slope of water surface
t time, in hours and minutes
v velocity
w width of top of channel
$\overline{w}$ mean width of top of channel
γ specific weight of water
τ total shear or tractive force
τ_c critical shear or tractive force
λ wavelength of pseudomeander

FACTORS CONTROLLING THE SIZE AND SHAPE OF STREAM CHANNELS IN COARSE NONCOHESIVE SANDS

By M. Gordon Wolman and Lucien M. Brush, Jr.

ABSTRACT

The size and shape of equilibrium channels in uniform, noncohesive sands, 0.67 mm and 2.0 mm in diameter, were studied experimentally in a laboratory flume 52 feet long in which discharge, slope, sediment load, and bed and bank material could be varied independently. For each run a straight trapezoidal channel was molded in the sand and the flume set at a predetermined slope. Introduction of the discharge was accompanied by widening and aggradation until a stable channel was established. By definition a stable equilibrium existed when channel width, water surface slope, and rate of transport became constant. The duration of individual runs ranged from 2 to 52 hours depending upon the time required for establishing equilibrium.

Stability of the banks determined channel shape. In the 2.0 mm sand at a given slope and discharge, only one depth was stable. At this depth the flow was just competent to move particles along the bed of the channel. An increase in discharge produced a wider channel of the same depth and thus transport per unit width remained at a minimum. Channels in the 0.67 mm sand were somewhat more stable and permitted a 1.5 fold increase in depth above that required to start movement of the bed material. An increased transport was associated with the increase in depth. The rate of transport is adequately described in terms of the total shear or in terms of the difference between the total shear and the critical shear required to begin movement.

In these experiments the finer, or 0.67 mm, sand, began to move along the bed of the channel at a constant shear stress. Incipient movement of the coarser, or 2.0 mm, sand, varied with the shear stress as well as the mean velocity. At the initiation of movement a lower shear was associated with a higher velocity and vice versa.

Anabranches of braided rivers and some natural river channels formed in relatively noncohesive materials resemble the essential characteristics of the flume channels. For a given slope and size of bed material the discharge per unit width in the laboratory channels was similar to that computed for anabranches and river channels measured in the field. Unlike most natural channels, despite impressive bank erosion, the channels in the laboratory only meandered at supercritical flows associated with very steep slopes. These conditions involving shallow depths, high velocity, and steep slopes are uncommon in most natural rivers.

INTRODUCTION

The description of natural river channels based upon measurements of selected hydraulic and channel parameters has led to the formulation of a number of empirical relations describing the average behavior of a wide variety of rivers. Field studies suggest that in many rivers the downstream increase in width, depth, and velocity may be generally described by simple power functions of the discharge at a given frequency of flow (Leopold and Maddock, 1953; Wolman, 1955; Leopold and Miller, 1956). In rivers and in self-formed equilibrium canals in erodible material it has been observed (Blench, 1957) that the rate of change of width with increasing discharge is relatively conservative. Thus, the exponent b in the equation

$$w-aQ^{b} \tag{1}$$

generally has a value of approximately 0.5.

The close association of width and discharge was noted by Leopold and Wolman (1957); but because of the larger number or uncontrollable variables involved, it was not possible to establish on the basis of field data alone the basic or underlying factors controlling river width. The present experiments were designed to study the size and shape of small self-formed channels under ideal laboratory conditions in which discharge, slope, and both bed and bank material could be controlled. To make the analysis as simple as possible, the bed and bank were made of uniform sand. A series of experimental runs was made in each of which an initial artificial channel molded in the sand was permitted to develop an equilibrium cross section appropriate to the discharge, slope, and particle size fixed by the experimenters. By varying each factor independently, a simple relation could be established between channel form and the independent variables. Because of the simplicity of the experimental conditions, it was felt that the results might provide a useful basis for analyzing the similar but far more complex conditions found in natural river channels. To test this supposition, following the presentation of the laboratory procedures and results, the channels produced in the flume are compared with similar channels observed in large braided rivers.

The study was made in the General Hydrology Branch, under the general supervision of C. C. McDonald, chief. The experimental runs were made in a structural steel flume built by the Geological Survey in the hydraulic laboratory at the University of Maryland, College Park, Md. We are indebted to S. S. Steinberg, former dean of engineering at the university, for making the construction of the flume possible. The successful pursuit of the work was made possible through the willing active cooperation of A. A. Plusch, C. L. Jones, and C. E. McCalester, in charge of the various laboratory shop facilities at the university, and through the help of R. D. Gaynor, in charge of the laboratory maintained by the Sand and Gravel Institute at the university.

The authors also thank their colleagues in the Geological Survey, Thomas Maddock, Jr., L. B. Leopold, S. A. Schumm, R. F. Hadley, N. J. King, R. W. Carter, W. S. Eisenlohr, Jr., and R. A. Bagnold of Great Britain, for their critical review of the manuscript. Many of the suggestions made by the reviewers have been incorporated in the text, but as always, the authors alone retain title to errors both of commission and omission.

EQUIPMENT AND EXPERIMENTAL PROCEDURE

THE FLUME

The flume used for the study was 52 feet long. It had a maximum width of 4 feet and a maximum depth of 8 inches. Pivoted at the upper end and supported by two chain hoists at the lower end, it could be readily raised or lowered to change the slope.

Water was provided from a constant-head tank and the discharge measured at pipe elbows which were tapped and connected to a manometer containing carbon tetrachloride colored with iodine. Each elbow used was rated in the laboratory. Water entered the flume from a stilling basin and was conducted through an approach channel 3 feet long. The width of the transition could be adjusted to provide a smooth contraction to the molded sand channel. Below the exit from the flume the flow passed into a stilling tank and thence to a large sump from which the water was recirculated to the constant-head tank.

Sediment could be fed into the approach channel at desired rates by means of a vibrating feeder. The outflowing sediment was weighed continuously as it accumulated in a tank in which the volume of water was kept constant through continuous overflow. The tank was suspended from a lever arm which was pivoted at one end and supported at an intermediate point by a vertical support resting on a scale. The position of the vertical support along the lever arm was designed to magnify the sediment weight by a factor of 1.61 thus converting from weight under water to dry weight. This magnification was useful in readily indicating the weight of sediment to be fed at the upper end, in facilitating future computations of bed-load movement, and particularly in making it possible to measure the small quantities of sediment transported.

All slope and depth determinations were made with a point gage mounted on a moving carriage suspended from overhead rails. The datum of the gage was horizontal and independent of the flume. Velocity distributions in two runs were measured with a bubbler gage and transducer.

RUN PROCEDURE

The flume was filled with sand to a depth of 4 or 5 inches and before each run an initial channel and "flood plain" were molded in the sand by means of a wooden templet mounted on a rolling carriage that travelled the length of the flume (see fig. 111). The slope of the channel was then set by adjusting the chain hoists at the lower end of the flume. A longitudinal profile of the bed was measured with the point gage following which the flow was introduced into the channel. After determining a discharge at which the bed material just began to move, the discharge was increased rapidly to the desired value for the run (see table 7) and kept constant thereafter.

During each run, water-surface slope, top width of the channel, and sediment load were measured periodically. The top width and water-surface elevation were measured at 4-foot intervals along the length of the flume. Mean top width, $\overline{w}$, was computed, and an average water-surface profile was plotted from the data.

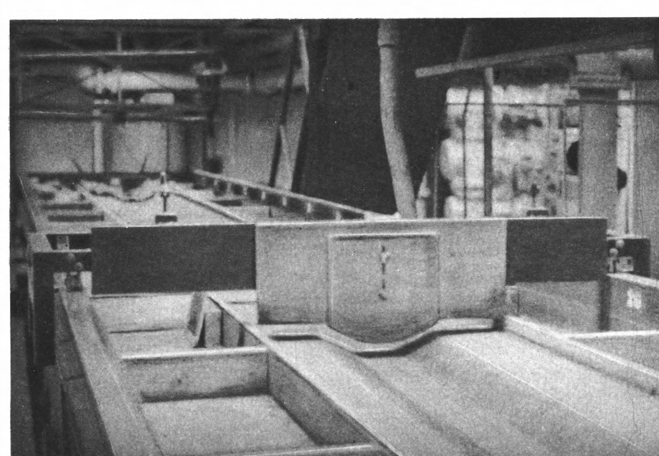

FIGURE 111.—Templet mounted on rolling carriage used to mold initial channel and flood plain in sand.

The inflow of sediment was adjusted to equal as nearly as possible the outflow rate as indicated by the weight of the accumulating sediment.

The length of run depended in part upon the rapidity with which equilibrium conditions were attained and ranged from about 8 to 54 hours. Figure 112 shows

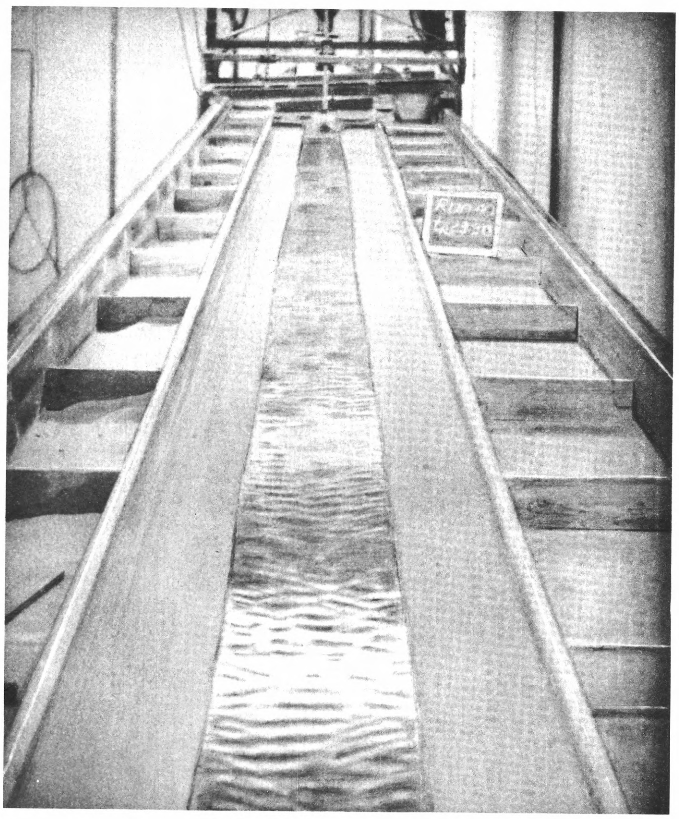

FIGURE 112.—Typical stable equilibrium channel. Run 40, in 0.67 mm sand after 23 hours 30 minutes.

the final condition of the channel after 23½ hours in run 40. Equilibrium was considered to exist when the following conditions had been established:

1. Rate of bed-load movement became constant.
2. Changes in channel shape approached zero.
3. The longitudinal profile of the water surface was regular and nearly constant.

The templet channel was always made somewhat smaller than the expected size of the equilibrium channel. Thus the templets varied from about 5 to 10 inches in width. In the early stages of a run, changes in width and the movement of bed load were usually extremely rapid. The rate of change of width and the rate of movement declined with time as equilibrium was approached. Widening in almost every run was accompanied by aggradation on the bed. Sediment from the banks moved toward the center of the channel where the rate of transport was insufficient to carry it downstream. Computations for several runs show that more than 75 percent of the material removed from the banks was deposited on the bed. The remainder collected in the weighing tank, a large part of it in the early stages of the run. This invariant process by which the narrow deep channel became wider and shallower is depicted diagrammatically in figure 113.

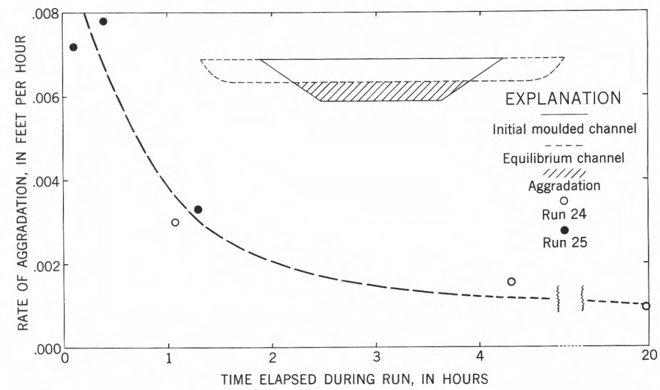

FIGURE 113.—Graph showing decreasing rate of aggradation with time during a run. Inset shows typical mode of channel widening with concomitant aggradation of channel bed.

The rate of aggradation, then, as well as the rate of widening and change in rate of transport, is indicative of the gradual approach to equilibrium. In runs 24 and 25 (fig. 113) successive longitudinal profiles of the bed indicate a progressive decline in the rate of aggradation from an initial high of about 0.01 foot per hour to about 0.002 after 4 hours of running time. Thereafter the rate of aggradation became exceedingly slow. The last point on figure 113 actually represents the end of an interval of time greater than 15 hours.

Figure 114 shows how plots of width and rate of bed-load movement against time were used to determine the condition of equilibrium. In run 40 it may be seen that after 6 hours, there was, in effect, no additional change in width. Similarly, the plot of load against time (fig. 114) shows that after 2 hours, the rate of change of load with time becomes constant. During this run the water-surface slope was in effect constant. Therefore in run 40 after 6 hours the channel was considered to have reached a state of equilibrium.

Upon the completion of each run a longitudinal profile of the bed was plotted and a cross section measured in a reach considered typical for the run. The measurements of this cross section were used in the subsequent computations. Only one representative section was measured after measurements of a number of cross sections in several runs indicated that the variation in dimensions of the cross sections in a given stable equilibrium channel was small. The measurements of the representative cross sections in each run and the hydraulic computations based upon them are given in table 7, along with the mean of 11 measurements of

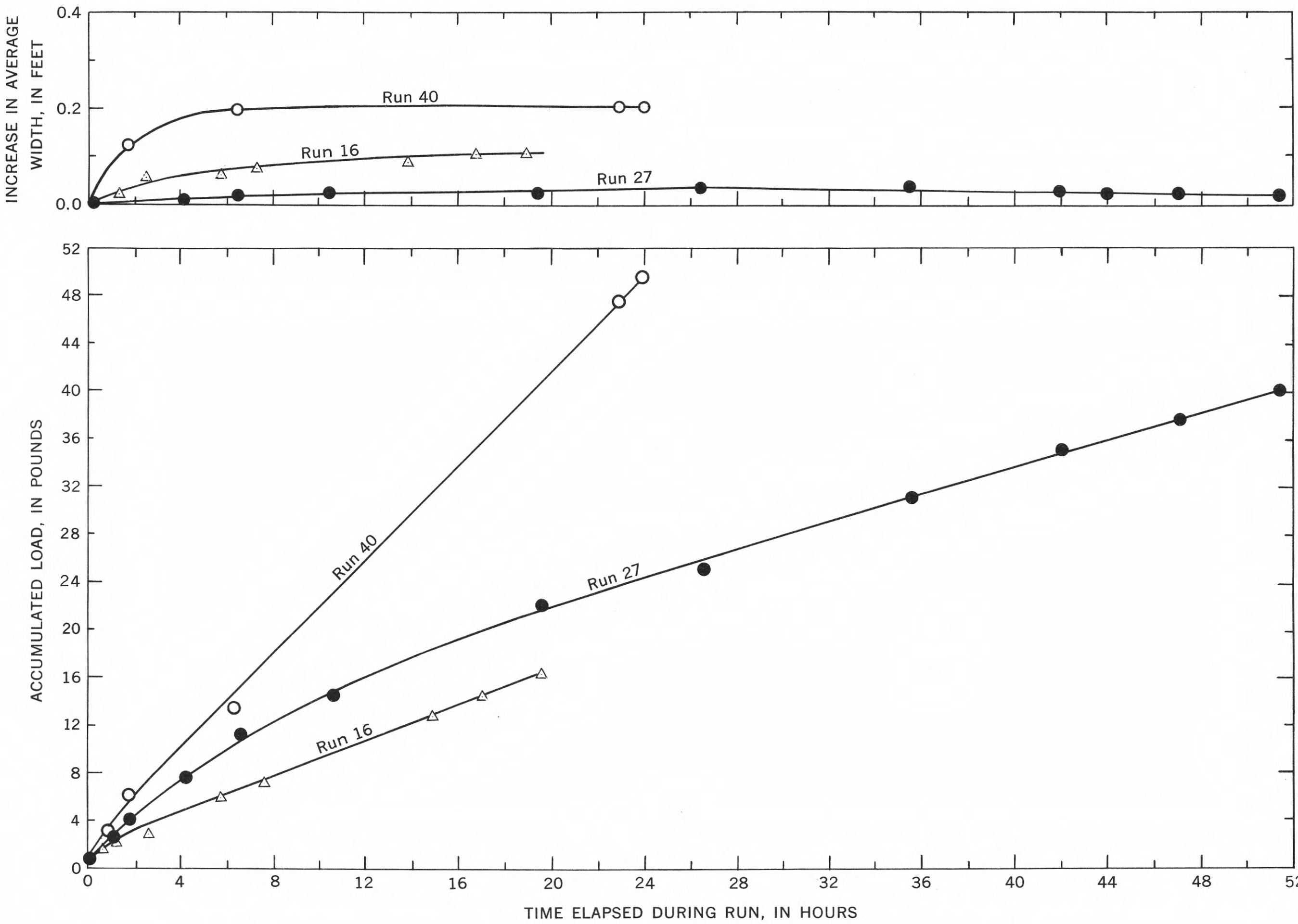

FIGURE 114.—Progressive establishment of equilibrium channel width and steady rate of transport in several channels.

the top width, $\overline{w}$, made at 4-foot intervals along the final stable channel. For most of the runs the total variation in width was on the order of 5 percent or less Four feet at the entrance and four at the exit were not included in the computation of the average width.

SIZE OF BED AND BANK MATERIAL

Two sands having median diameters of 0.67 mm and 2.0 mm were used in the experiments. Their size characteristics are summarized in table 1, and figure 115. The sands were predominantly quartz although the very coarse 2.0 mm sand contained some feldspar and other rock fragments. For the purposes of computation a specific gravity of 2.65 was assumed. Neither sphericity nor roundness was measured, but megascopically the material may be described as subequant and subrounded. No sorting by the flow took place, as demonstrated by samples collected from the bed of the channel and from the settling tank following runs 25 and 27.

TABLE 1.—*Size characteristics of bed material*

	Coarse sand, 0.67 mm	Very coarse sand, 2.0 mm
Phi median $Md_\phi=\phi_{50}$	0. 58	−1. 0
Phi median $M_\phi=\frac{1}{2}(\phi_{16}+\phi_{84})$	. 56	−1. 0
Phi deviation $\sigma_\phi=\frac{1}{2}(\phi_{84}-\phi_{16})$	. 26	. 50
Phi skewness $\gamma_\phi=\frac{M_\phi-Md_\phi}{\sigma_\phi}$	−. 08	0
2d phi skewness $\gamma_{2\phi}=\frac{\frac{1}{2}(\phi_5+\phi_9{}^5)-Md_\phi}{\sigma_\phi}$	−. 15	−. 12
Phi kurtosis $\beta_\phi=\frac{\frac{1}{2}(\phi 95-\phi_5)-\sigma_\phi}{\sigma_\phi}$	. 54	. 64

$\phi=-\log_2$ (mm)

RESULTS OF EXPERIMENTS

CONDITIONS OF INCIPIENT MOVEMENT OF BED MATERIAL

For all runs in the coarser (2.0 mm) sand and several in the finer (0.67 mm) sand, the discharge at the start was too low to cause particles on the bed of the channel

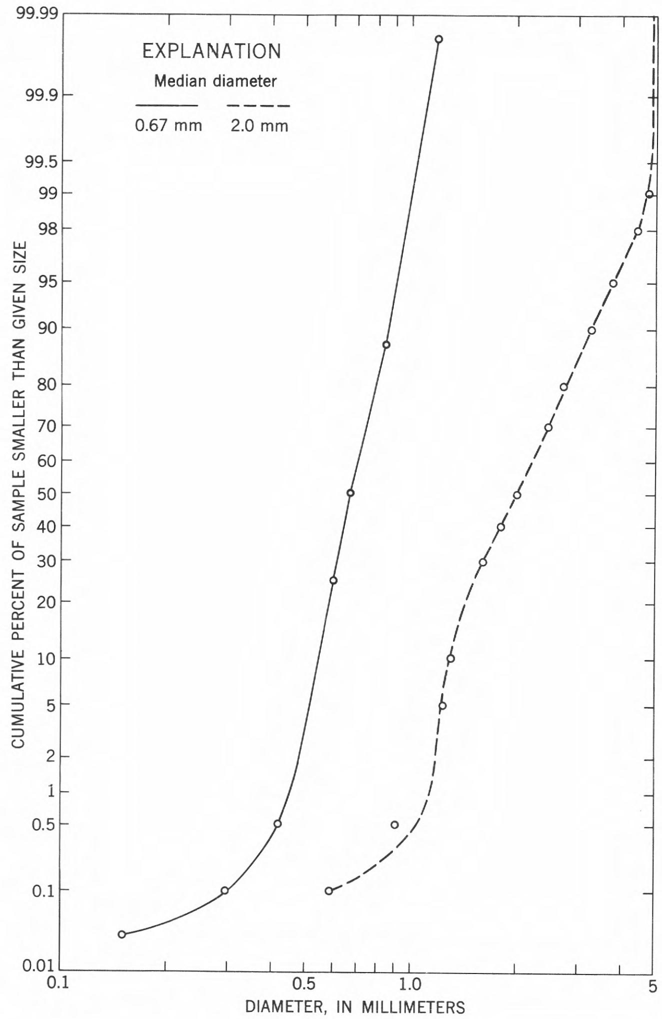

FIGURE 115.—Size distributions of the two sands used in laboratory studies.

to begin to move on the slope provided. The discharge was gradually increased until movement began. A measured rate of movement of 3 to 10 particles per minute from the bed along a given reach 1 foot long was defined as the critical point or condition of incipient motion. Discharge, area of cross section, water-surface slope, and rate of particle movement were measured, and then the discharge was increased rapidly to the quantity desired for the run. The conditions of incipient motion for runs 22, 26, 28, and 30 in the 0.67 mm sand and for runs 48, 50, 21, 53, and 54 in the coarser material are given in Table 7. Because the duration of the run is of no significance in the determination of the condition of incipient motion, this information is not given for all those runs in Table 7.

The critical tractive force at which movement began was computed from the equation

$$\tau_c = \gamma R s \qquad (2)$$

where γ is the specific weight of water, 62.4 pounds per cubic foot, R is the hydraulic radius in feet, and s the slope. It can be seen from the data that the critical shear required to start movement of the 0.67 mm sand ranged from 0.0047 to 0.0074 pound per square foot; the mean value was 0.006. For the coarser 2.0 mm sand, the critical tractive force ranged from a low of 0.024 to a high of 0.039 pound per square foot; the mean was 0.032. Although in both sands the median values for the critical shear are close to those given by Shields (see Brown, 1950, p. 790), in neither sand was the tractive force a constant at the beginning of movement and in both a progressive decrease in critical tractive force was accompanied by a progressive in crease in velocity, and, vice versa; the change in velocity being very much greater in the coarser sand.

This relation between velocity and critical tractive force was pointed out by Rubey (1938) who showed, using the Gilbert (1914) data for 6 uniform sands, that for particles 5 mm and larger, velocity rather than the depth-slope product became the primary factor controlling the beginning of sediment movement. For particles less than about 1.7 mm Rubey (1938) found that the influence of velocity was reduced and the beginning of movement was determined primarily by the depth-slope product alone. Figure 116 shows the relation between the "critical" depth-slope product and velocity. Results of this investigation are also shown on the graph, as are data from Bogardi and Yen (p. 49),[1] which extend the data to very large particles.

Although the finer sand moved at a somewhat lower tractive force, or depth-slope product, the results for these two sands are seen to agree closely with those obtained by Rubey (1938). The relation is best shown by the position and slope of the line for the 2.0 mm sand inasmuch as the beginning of movement for Gilbert's two smallest sizes is not well defined. The 2.0 mm line falls between the 1.71 mm and 3.17 mm size of Gilbert and has an intermediate slope as it should. The initial movement of Bogardi and Yen's 15.5 mm particles appears to be wholly related to velocity and not to tractive force.

It should be pointed out that although the tractive force required to begin movement did vary, the mean values of it for both the 2.0 mm and the 0.67 mm sands are close to those predicted by Shields (Brown, 1950, p. 790). Although the variation of critical tractive force for the coarser sands appears to be considerably greater than for the finer sands, when the variation is expressed as a percent of the mean value, the variation is found to be roughly the same in the two sizes.

[1] Bogardi, I., and Yen, C. H., 1938, Traction of pebbles by flowing water: Thesis for Dept. Mechanics and Hydraulics, State Univ. Iowa.

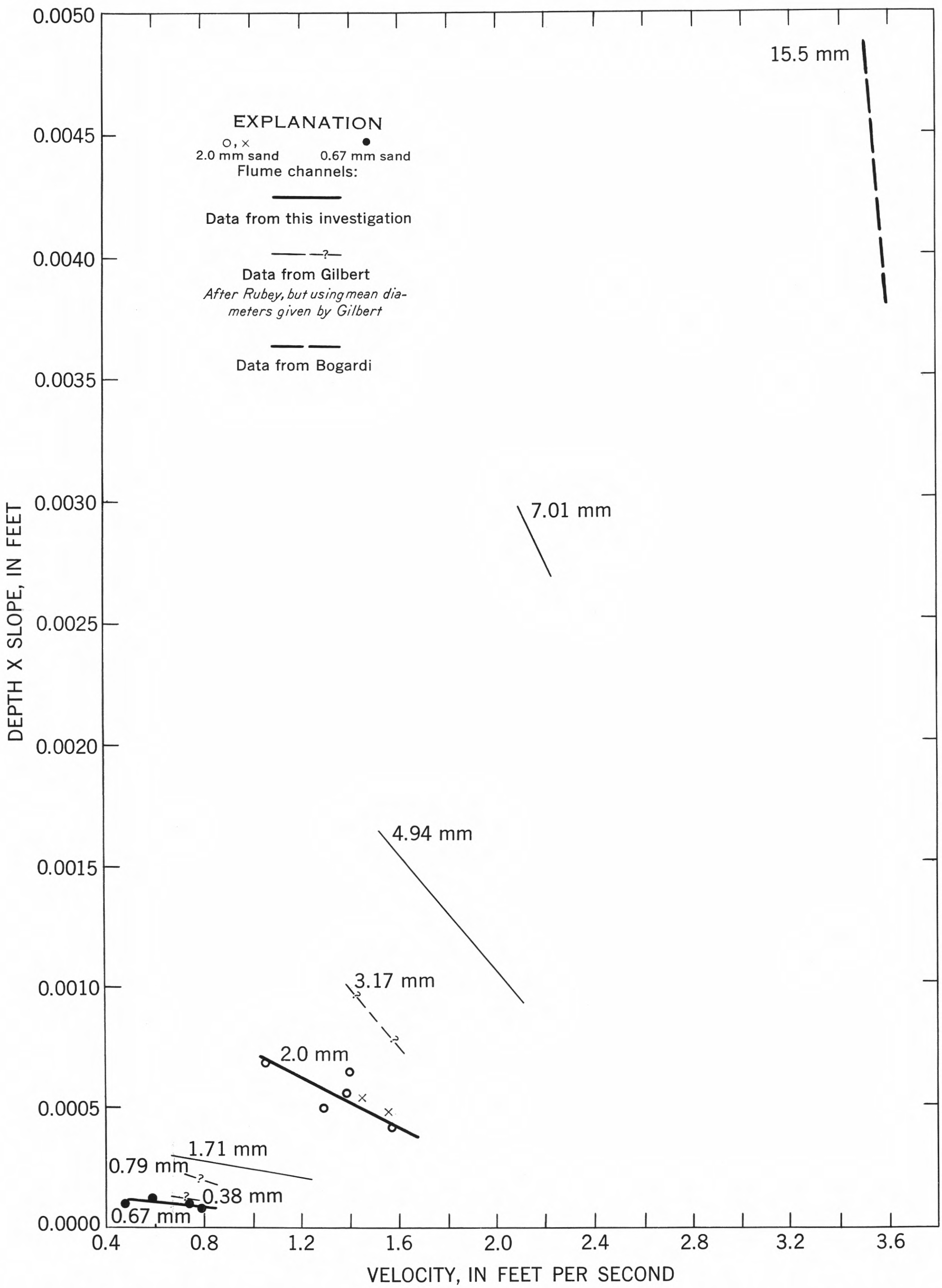

FIGURE 116.—Conditions of incipient movement of particles of different sizes determined by depth-slope product and mean velocity. Influence of velocity increases with increase in particle size. Original from Rubey, (1938, fig. 20, p. 128) based on Gilbert (1914) data. For 2.0 mm sand, points designated by x's indicate conditions at end of runs when bedload movement had been reduced to a minimum.

Despite these two qualifications however, within the conditions of the experiments (Ralph A. Bagnold, written communication, has called attention to the high Froude numbers), these data do appear to support Rubey's (1938, p. 129) conclusions that the force required to start particle movement depends on both the velocity of a stream and on the depth-slope product, and that the movement of the large particles is more sensitive to changes of velocity and movement of the small particles to the depth-slope product, or tractive force.

It was noted that in the coarser (2.0 mm) sand at the beginning of particle movement the product of slope and centerline depth remained constant. To test the significance of this relation an additional run was made in a channel having a movable bed composed of the same 2.0 mm sand but having fixed walls of smooth galvanized metal and a width of 0.83 foot. The slope and discharge in this run (run 54 in table 7) were made the same as in run 53a, and it was found that bed movement began at the same values of tractive force, γRs (0.024 pound per square foot) and mean velocity (1.56 fps). In contrast, however, because of the smooth walls, the centerline depth and, hence, the centerline shear ($\gamma d_L s$), was lower than in the previous critical runs for the 2.0 mm sand. This observation indicates that the combination of velocity and shear, computed from the product of hydraulic radius and slope, rather than the shear for any particular depth, best describes the hydraulic conditions in these experiments at the beginning of movement of the bed material.

Assuming again that channel shape might have a pronounced effect on the conditions of incipient movement of the bed material (Keulegan, 1938), we reanalyzed the Gilbert data for sand G (4.94 mm), using the hydraulic radius of the bed instead of the mean depth used by Rubey. Use of the hydraulic radius of the bed produced no change in the results. It was also possible to compare conditions at the beginning of movement in run 51 with those at the end of run 53a when bedload movement had been reduced to a minimum. The depth-slope product is the same in both channels although the width-to-depth ratio is twice as large in run 53a. A comparison of runs 48a and 53a indicates that when transport had been reduced to near the threshold value, a lower shear in run 53a was associated with a higher mean velocity, as in the runs used to determine the beginning of movement, despite the fact that the width-to-depth ratio in both runs exceeded 12.

In the Gilbert experiments, as in the sand channels reported here, the increasing velocity and decreasing tractive force for a constant discharge were associated with a decrease in the width-to-depth ratio. If we assume that the total shear must actually be a function of the velocity distribution, it follows that the effect of the channel shape is to modify the distribution of shear, a narrow channel producing a low apparent shear and a high mean velocity. Thus, although the true shear required to move a given particle might have remained constant, in terms of the depth, slope, and velocity usually measured it is necessary here to include the velocity as well as the depth and slope in describing the conditions prevailing at the inception of movement of the particles on the bed.

RELATION OF DISCHARGE, AREA OF CROSS SECTION, AND ROUGHNESS

The channel dimensions and flow characteristics for equilibrium runs in which the bed and bank material was 0.67 mm in diameter are shown in Table 7. Discharge in these runs was varied from 0.011 cfs to 0.069 cfs, and slope from 0.001 to 0.0071 foot per foot. In the coarser 2.0 mm sand, discharge ranged from 0.032 cfs to 0.280 cfs, and slope from 0.0024 to 0.010 foot per foot. The lower limit of the slope was determined by the flow conditions required for transport of the sediment and the upper limit was determined by the stability of the erodible banks of noncohesive sand. Because of the erodibility of the banks, the depth and, hence, the transport per unit width could not be increased simply by increasing the discharge at a given slope as is done in flumes with fixed walls.

Figure 117 shows the relation between the discharge, an independent parameter, and cross-sectional area, a dependent one, for runs in both sands. As one would expect, the size of the channel is closely related to the quantity of flow. Although occupying different regions of the graph, the runs in 0.67 mm sand and those in the coarser sand are both roughly described by the equation

$$A = kQ^{0.87} \tag{3}$$

where A is the cross-sectional area in square feet, Q the discharge in cubic feet per second, and k, a coefficient. With velocity added, the graph simply shows the relation $Q = Av$. The points for the coarser sand lie to the right, at a somewhat larger k value, indicating the higher velocities and discharges in runs with the coarser sand. The higher discharges and the absence of runs on the flatter slopes reflect the greater depths needed to produce movement of the larger particles. The limited range of cross-sectional area for a given grain size at a constant discharge indicates the small variation in mean velocity associated with several combinations of slope and depth.

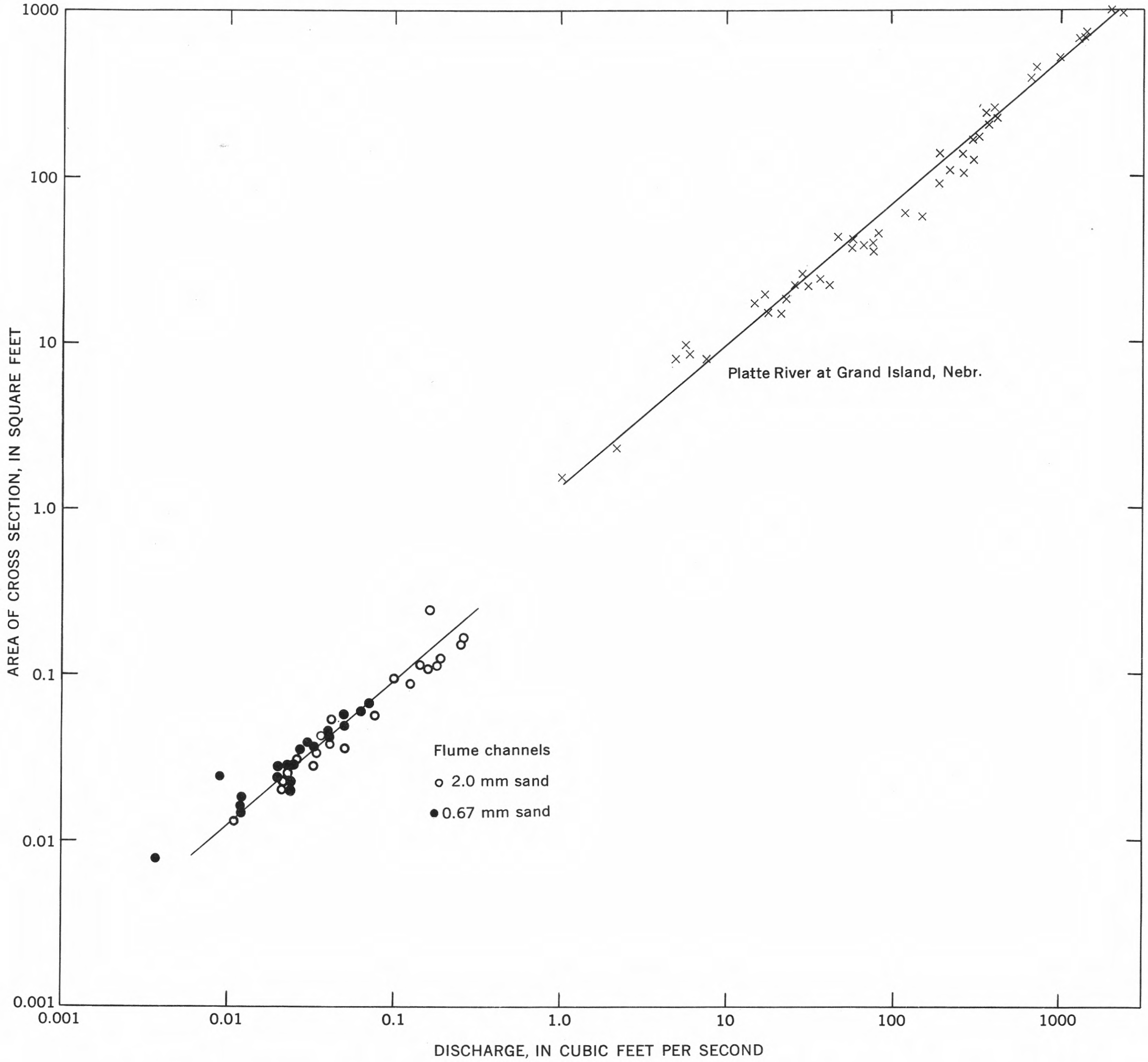

FIGURE 117—Relation of discharge to cross-sectional area in experimental channels and in anabranches of braided section of Platte River at Grand Island, Nebr.

The effect on rugosity of differences in the sand size is brought out in a comparison of the roughness or n values computed from the Manning equation, in which

$$n=\frac{1.49}{v}s^{1/2}R^{2/3} \tag{4}$$

where n is the computed roughness,
v the mean velocity, in feet per second,
s the slope, and
R the hydraulic radius in feet.

For the runs in which the median sand size was 0.67 mm, n varied from 0.008 to 0.016, and had a mean of 0.012. Fifty percent of the n values fell between 0.010 and 0.012. In contrast, half of the n values for the very coarse sand, 2.0 mm, were between 0.013 and 0.024 with a mean of 0.015. The maximum of 0.024 here, as well as the maximum in the finer sand, was observed in runs in which the discharge was small, the flow in the original molded channel having been increased only to the point of incipient motion of the bed material. The mean n values vary with grain size to about the

⅙ power which is close to the ⅙ power relation described by Strickler (1923) despite the wide range of values in each size presumably caused by variations in channel shape. A similar comparison of the values of the dimensionless Darcy-Weisbach resistance factor (table 7)

$$f=\frac{8gRs}{v^2} \tag{5}$$

for runs in the two different sands shows 60 percent of the runs in the 0.67 mm sand have values from 0.03 to 0.049, whereas 50 percent of the values for runs in the coarser material lie between 0.04 and 0.069.

On the average the roughness of the coarser sand exceeds that of the finer sand by 20 percent. With respect to the shape of "self-formed" channels this difference in roughness is important in channels where the flow is inadequate to move the bed and bank material on the slope provided. In this case, the "shape of the channel" is simply the shape of the prism of flowing water. For a given discharge, slope, and width, the rougher channel will have a lower velocity and hence a greater depth. On the other hand, as discussed below, if the flow is capable of moving the sediments in the bed and banks, the critical conditions of movement rather than the roughness (although the two may be related) determine the channel shape.

RELATION OF SLOPE, DISCHARGE, AND WIDTH; COARSER (2 MM) SAND

The results of all the runs in the coarser of the two sands are readily shown by the single curve in figure 118 in which discharge per unit width is plotted against slope. From a low discharge per unit width at the point of incipient motion, the discharge per unit width was raised to a maximum at the start of each run. As shown by the arrows tracing the progress of run 48a (fig. 118), from the maximum ($t=0$), by progressive erosion of the banks, coupled with aggradation as described earlier, the channel widened and shallowed its bed at a rate dependent upon the discharge and slope, until the depth at the end of the run ($t=20{:}05$) was reduced to near the critical required for sediment movement. Along with the increase in width went a progressive decrease in the rate of sediment transport. The slopes of the lines in figure 119 show this decline in transport with time for several runs. It can be seen that in runs 48a and 49, during which no sediment was fed, transport declined rapidly to the point where it was almost unmeasurable, whereas for runs 50a, 51a, and 53a, in which sediment was fed, the final rate of movement was somewhat greater. In each of these runs a few particles could be seen in motion along the bed at the end of the run although the quantity was too small to be measured in the weighing tank at the outlet.

All of the final equilibrium channels had similar shapes (fig. 120). The angle of the sloping banks increased in steepness from a value of about 20° at the bed to 35° at the water surface. The average angle of the side slope ranged from 18° to 30° and the mean of 25° was somewhat less than the angle of repose, which for this sand in still water is 30°. The latter value agrees in general with those for uniform sand under water given by Van Burkalow (1945) and Lane (1955). These authors also show, however, that the angle of repose of uniform sand is affected by the shape of the particles and that angularity alone may cause variations of 15° for a given grain size (Lane, 1955).

For the coarser sand the sloping banks were determined by the angle of repose of the sand and by the component of gravity acting down the side slope and in the direction of flow (see below). Similarly, because the banks are readily erodible, a given slope and given discharge will, when channel equilibrium has been established, produce an unvariable width and channel form. At a given slope but with increasing discharge, width will also increase. In both cases just one depth develops—a depth that remains constant. As the sediment being transported was the same size as that comprising the bed and banks, the low threshold of erosion of the noncohesive banks resulted in the formation of a shallow channel capable only of transporting the bed material at the minimum rate; that is, at a rate equivalent to that at the critical or incipient stage of movement.

At slopes greater than 0.011, depths of flow became exceedingly small, and at the transition from tranquil to rapid or shooting flow, straight channels became sinuous. These sinuous channels were not considered in equilibrium in these experiments and are described in a separate section below.

RELATION OF SLOPE, DISCHARGE, AND WIDTH; FINER (0.67 MM) SAND

The conditions of equilibrium for the 0.67 mm experimental sand are fundamentally like those already described for the coarser sand. However, instead of a single stable channel having a single discharge per unit width, *in the finer sand, channels at different depths and unit discharges were observed to be in equilibrium on a given slope.*

The range of stable equilibrium channels is shown by the lower graph in figure 118. At a slope of 0.0025, for example, the channel associated with the beginning of movement of the bed material had a discharge per unit width of 0.025 (run 26). On the same slope the

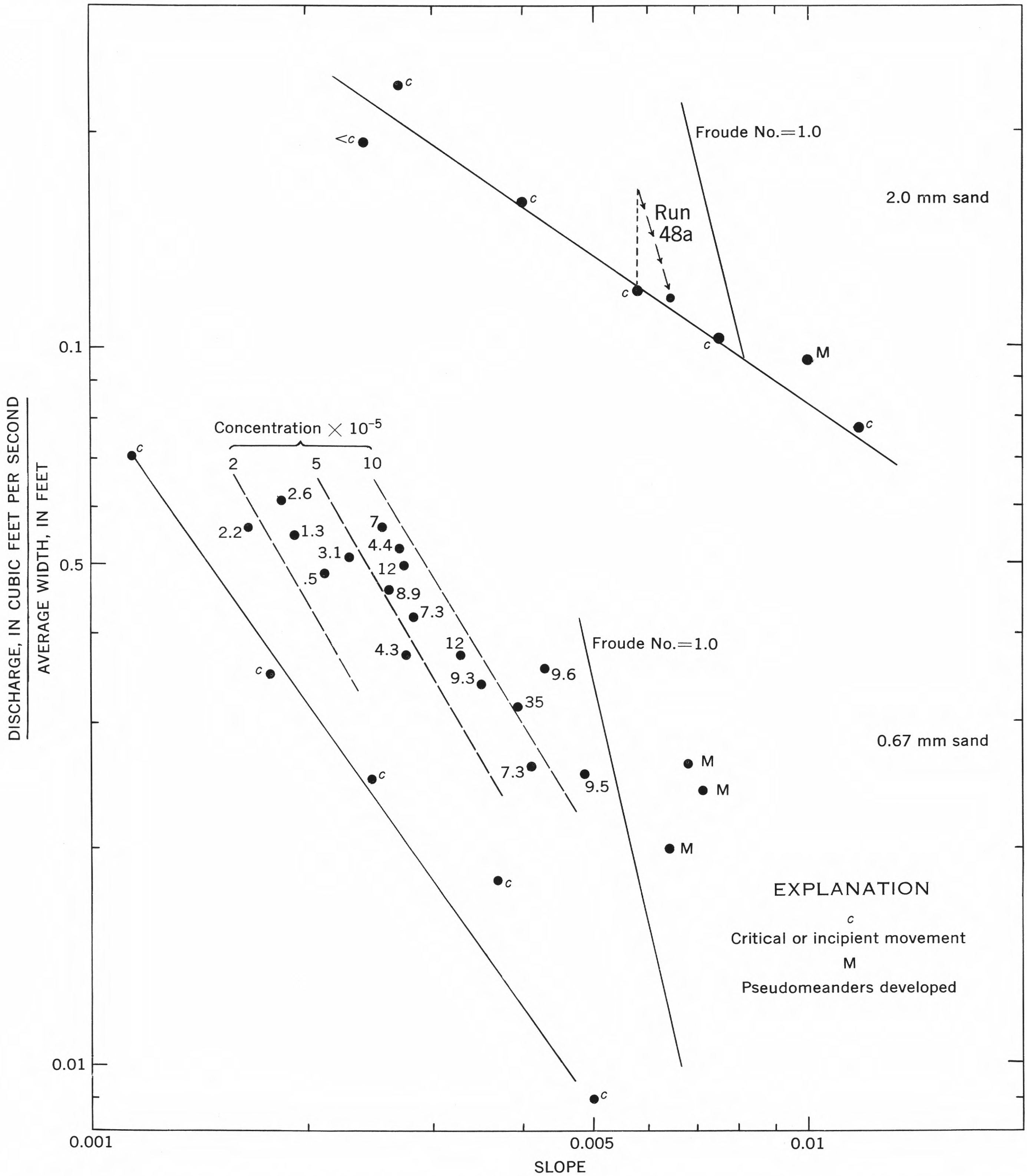

FIGURE 118.—Relation of discharge per unit width to slope for flume channels in 0.67 and 2.0 mm sand. Dashed lines are isograms of constant concentration of bed load, numbers give actual concentrations. Arrows show progress of run 48a from determination of critical shear (run 48) through beginning of high discharge ($t=0$) to end of run ($t=20{:}05$).

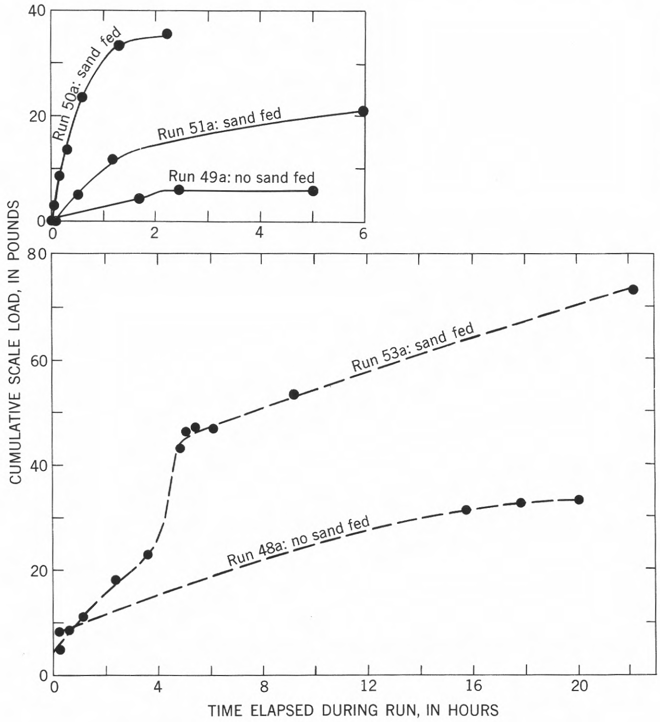

FIGURE 119.—Decreasing rate of transport with time during several runs in 2.0 mm sand.

equilibrium channel in run 17 became stable at a discharge per unit width of 0.051. The difference represented an increase of about 1.5 times in both the depth and velocity. The increase in depth and velocity was associated with an increase in the rate of sediment transport.

The conditions of incipient motion for the 0.67 mm sand are given for runs 22, 26, 28, and 30 in table 7. As explained above, velocity has less influence on the beginning of motion in the finer sizes, and thus the slope of the "critical" curve for the 0.67 mm sand differs from that for the 2.0 mm sand on the graph relating discharge per unit width and slope (fig. 118).

Because the noncohesive banks in the finer sand were capable of supporting more than one depth and velocity, some range in the rate of sediment transport could also be observed. Thus lines of equal concentration have been drawn through the points for the 0.67 mm sand in figure 118. The flume channels transporting sediment of the same size as that composing the noncohesive bed and banks were capable of transporting at best only a very low concentration of material. Thus the rate of sediment transport throughout the entire range of equilibrium channels is low, the highest concentrations (by volume) ranging from 1 x 10^{-5} to 35 x 10^{-5}.

The measured rates of sediment transport can be described by equations in which q_s, the rate of movement per unit width, is a function of the total shear, τ (Einstein, 1942, Bagnold, 1956), or by equations of the form $q_s \propto \tau - \tau_c$ (Brown, 1950), where τ_c is the tractive force required to begin movement. For the limited data available, it appears that the several points for the 2.0 mm sand fit the more abundant data for the finer sand if an equation of the latter type is used where τ_c is based upon results of these experiments given in figure 116. The relation is shown in figure 121. Because transport of the coarser sand was negligible, however, no great weight is attached to this mode of expression, and the observed transport in the 0.67 mm sand is equally well described by an equation in which q_s is related to total shear. In these experiments up to the point at which the equilibrium channels become unstable, the rate of sediment transport is in a sense independent of the factors controlling the channel form. The upper limit of transport, however, in both sands is fixed by the upper limit of stability of the noncohesive bank.

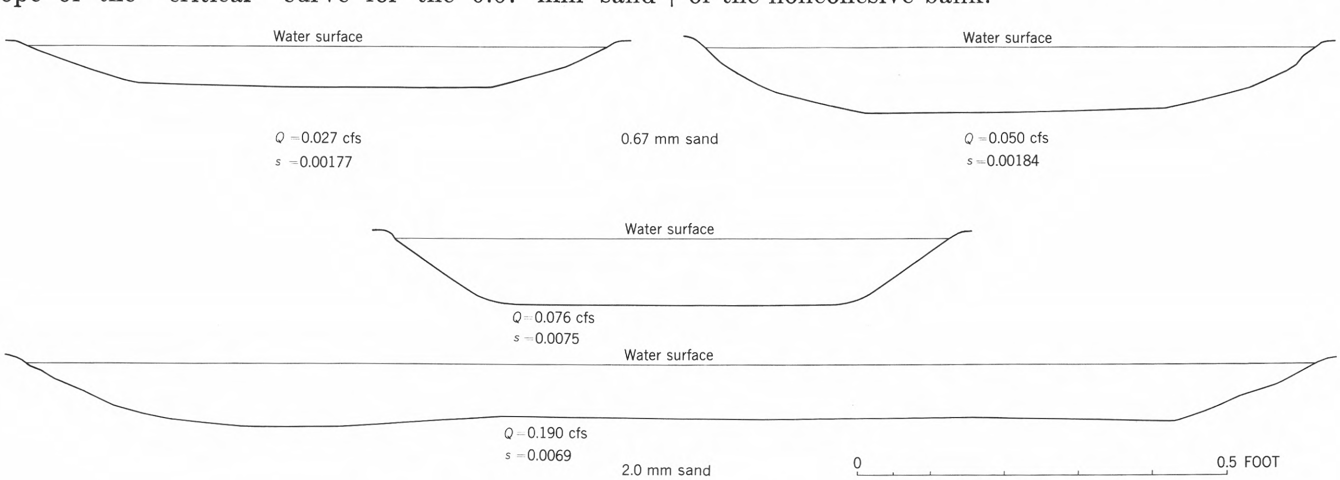

FIGURE 120.—Cross sections in 0.67 mm sand and in 2.0 mm sand. Sections show similarity in shape of channels.

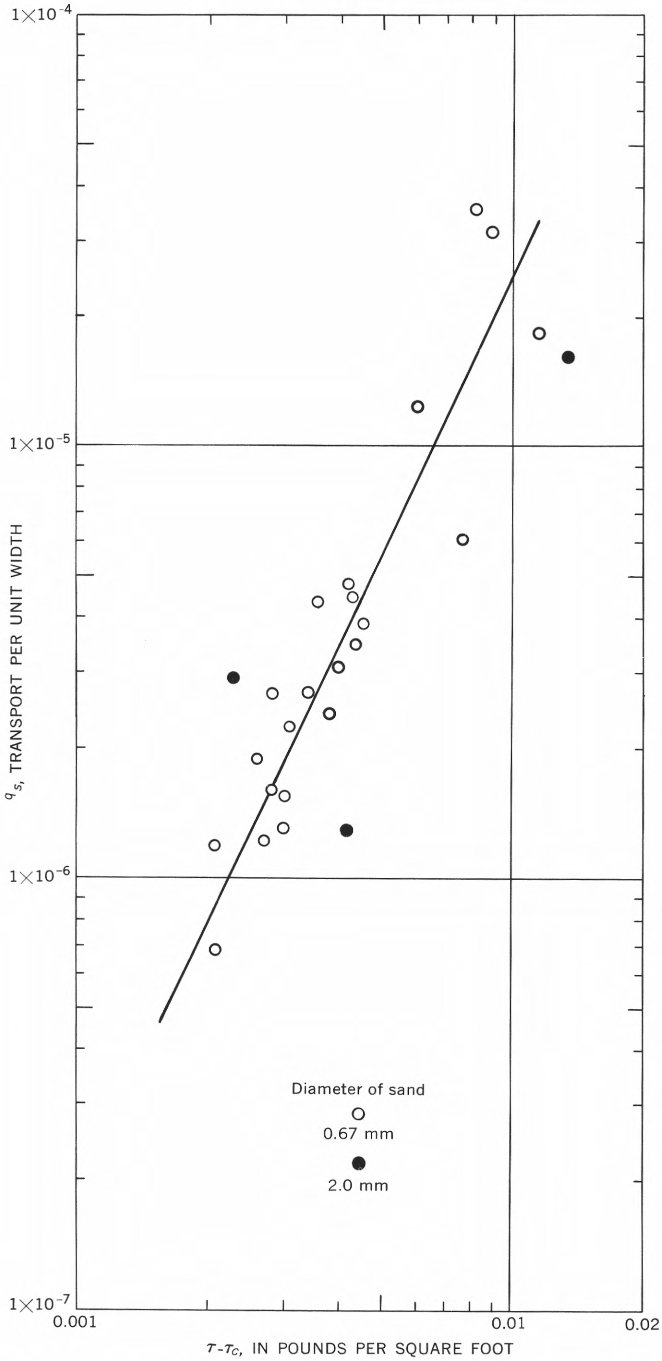

FIGURE 121.—Rate of bed load transport in equilibrium channels as function of total shear and critical tractive force.

On the flatter slopes from 0.001 to 0.002 in the 0.67 mm sand there is actually little or nor range in sediment concentration in the equilibrium channels (figs. 118 and 126). This is attributable to the fact that in order to start sediment movement relatively large depths were required at the outset. A high discharge per unit width necessitated in turn an initial channel virtually triangular in shape. As the steep banks were readily eroded by the flow, the sand from the banks collected on the bed in the apex of the triangle, rapidly reducing the depth. Thus, the maximum and sole rate of transport occurred only during the stage of incipient motion. At higher slopes with increasing velocity a greater discharge per unit width was obtainable without causing erosion of the banks.

The angle of repose of the finer 0.67 mm sand was the same as in the coarser 2.0 mm sand. The channel banks were also similarly concave, with decreasing slope at the base. The greater stability of the 0.67 mm sand which permitted it to maintain a 50 percent increase in depth (fig. 120) is perhaps attributable to several factors: (1) The greater permeability of the coarser sand may have reduced its stability by permitting greater seepage into and out of the bank. (2) The increase in velocity associated with an increase in depth may not have influenced the stability of the finer bank because of the fact that velocity had little effect on the movement of finer material. (3) The apparent cohesiveness due to surface tension or capillary effects may have been greater in the finer (0.67 mm) sand.

Because of the small size of the experimental channels and the instability of the noncohesive sand, it was exceedingly difficult to measure the velocity distributions adequately in them, particularly at meaningful points adjacent to the boundaries. An attempt was made, however, to compare values of the tractive force computed from velocity distributions (Leighly, 1932) in stable and eroding channels in the 2.0 mm sand. These values are shown in figure 122 along with the critical tractive force at each of the selected points on the boundary. The critical values were determined from figure 116 by using the observed mean velocity at each vertical.

In the stable channel the computed tractive force along the boundary nowhere exceeded the critical tractive force required to start movement (fig. 122), whereas in the channel still in the process of widening, the critical value is exceeded at several points. These results indicate that for a given sand the shape of the channel is related to the shear or tractive force at the boundaries as determined by the velocity distribution. Because of the uncertainties in these measurements and the significance of small differences in velocity near the bed, evidence from measurements of the overall geometry of laboratory channels is perhaps more significant in determining the factors responsible for the size and shape of these channels. As discussed below, the shape and size of the channels is in general accord with theoretical analysis, laboratory experimentation in larger channels, and with field observations of comparable natural channels.

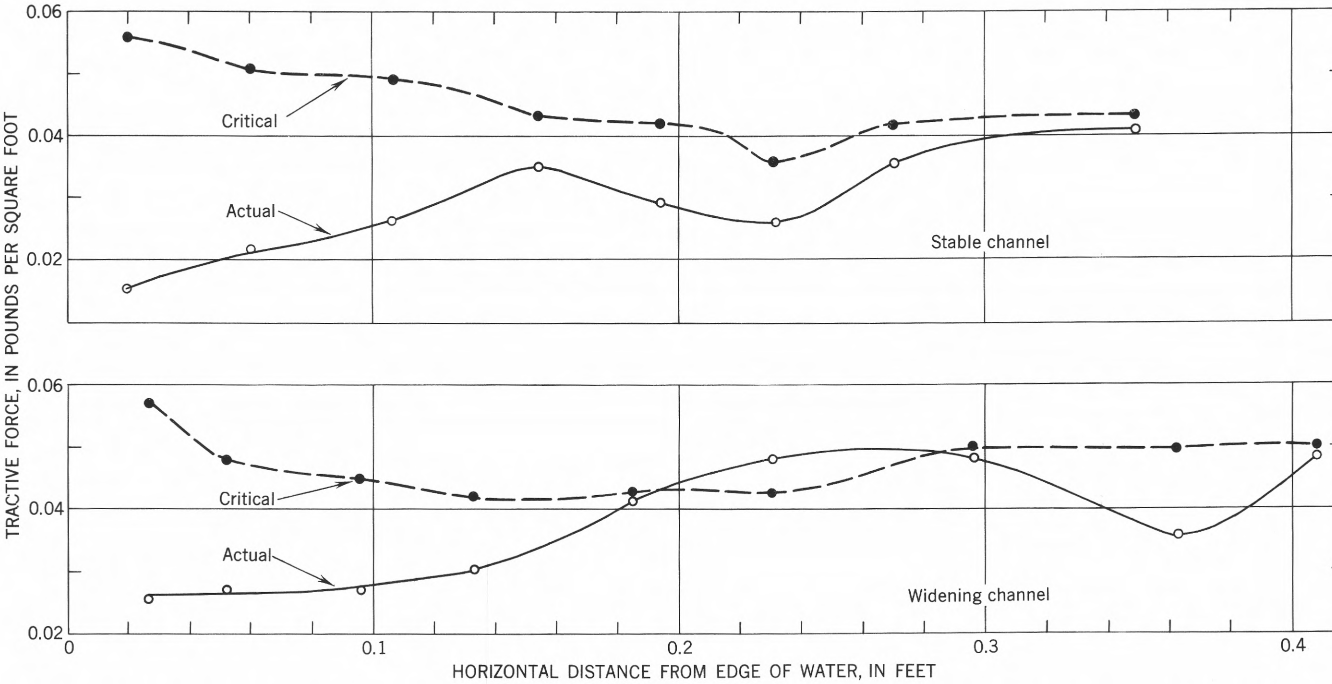

FIGURE 122.—Tractive force at successive points along channel boundary. Computed from velocity distribution in stable channel and in channel in which widening is taking place. Critical tractive force required to start movement at each position was determined from figure 116 on the basis of known mean velocity. Right margin of graph is approximate centerline of channel.

In summary, it is apparent that in the finer sand as well as in the coarser, channel shape is controlled by the characteristics of the bank material. These characteristics determine the upper limit of force which the bank can withstand. Thus control is exercised fundamentally by the depth limitation imposed upon the channel by the erodibility of the noncohesive bank. To convey a given discharge on a given slope requires a specified cross-sectional area for a particular roughness. The latter is determined here by the size of the granular material. If the available discharge in a channel of a given conveyance requires a depth and velocity which exceed the threshold of erosion of the bank, then the width of the resultant channel will, in effect, be determined by the depth and velocity which the banks are capable of withstanding. The overall width in turn is a function of the total discharge and the erodibility of the banks. The sloping angle of the banks, primarily a detail of the shape when the width is large, is determined by the angle of repose of the bank material and by the downstream component of the gravitational force acting along the side slope.

These results for the straight equilibrium channels are in general agreement with the analysis made by Koechlin (1924) and with the experimental results of Lane (1955). Koechlin (p. 99), basing his analysis on the angle of repose of the material and on the forces tending to move a particle from the bank, predicted that the side slopes of a channel in noncohesive material approximate a parabola. Analysis of the data from several runs in the 2.0 mm sand indicates that the nonrectangular or side areas of the cross section considered together have a parabolic shape and that this combined area differs from that of a parabola by less than 5 percent. Similarly, in several runs for which the comparison was made, the ratio of the depth of flow at successive points in the cross section to the value predicted by Koechlin is a constant.

The equation is not applicable at shallow depths because the angle of the side slope near the water surface in the experimental channels exceeded the angle of repose. It is possible that this steep angle was maintained because of apparent cohesion in the moist sand near the surface produced by surface tension of thin films of water (Van Burkalow, 1945, p. 692). Koechlin's analysis provides a channel in which motion of the grains begins at the same time at every point on the perimeter. In this case, on a given slope, only one side slope and depth are possible for a given cohesionless material and thus the equation would not apply as well to the results in the finer (0.67 mm) sand where there was a small range in depth. In general, the experimental stable equilibrium channels appear to be formed by erosion which proceeds until the channel shape is such that resistance to movement of the material at each point on the perimeter just balances the applied tractive force at that point.

EVIDENCE OF SECONDARY CIRCULATION

In several runs in the 0.67 mm sand, longitudinal sand ridges formed on the bed of the straight channels. With crests only 0.005 to 0.010 foot above the bed, these parallel ridges on occasion extended unbroken for distances as great as 16 feet (fig. 123). They

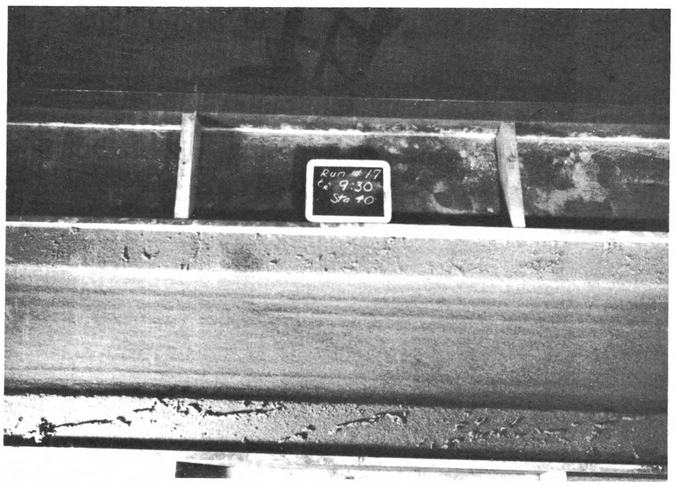

FIGURE 123.—Photograph showing parallel sand ridges along channel bed. Flow is from left to right.

were observed to form in channels where the rate of transport was low, the ridges appearing any time from 2 to 20 hours after the beginning of the run. In most runs they did not persist throughout the run. Although both the distance between crests and the depth of flow varied considerably, table 2 shows that the ratio of the two was relatively constant, varying from 1.74 to 2.36 and having a mean of about 2.1. The parallel sand ridges appear to result from pairs of vortices rotating in the plane perpendicular to the flow, adjacent vortices having opposite directions of rotation. The downward component of each vortex occurs midway between adjacent crests.

TABLE 2—*Spacing of crests of longitudinal sand ridges in relation to depth of the flow*

Run No.	Local depth (feet)	Distance between ridge crests (feet)	Ratio of distance between crests of sand ridges to depth of flow
5	0. 041	0. 08	1. 95
14	. 045	. 08	1. 78
15	. 055	. 13	2. 36
16	. 065	. 12	1. 75
17	. 069	[1] . 12	1. 74
19	. 051	. 12	2. 35
30	. 027	. 06	2. 21

[1] Approximate.

Similar ridges attributed to secondary currents have been observed in several laboratory experiments (see Leliavsky, 1955, p. 185 for a summary of these observations). Comparison at roughly similar discharges and slopes of runs with and without ridges (runs 15 and 4, table 7) indicates that these secondary circulations had no observable effect on the size or shape of the equilibrium channel. Similarly, the consistent relation between the width and shape of channels in which sand ridges formed and the width and shape of all the other straight channels also suggests that the secondary currents did not affect the form of the channel.

PSEUDOMEANDERS

As the slope of the flume was increased, the channels in both sands became progressively wider and shallower, with a resultant increase in the ratio of velocity to depth. At a sufficiently high slope the Froude number, as defined by the equation:

$$F=\frac{v}{\sqrt{gR}} \tag{6}$$

where v is the mean velocity,
R the hydraulic radius, and
g the acceleration of gravity,

approached or exceeded a value of 1.0. At this value the transition from tranquil to shooting flow takes place and standing waves formed on the surface of the water. Trains of surface waves parallel to the channel banks formed, with the fronts of the waves perpendicular to the channel walls. These initially straight wave trains gradually became sinuous, and on the moving bed of sand could be observed alternating diagonal shoals in phase with the sinuous trains of surface waves. Eventually meanders developed which, because of their mode of formation and association with supercritical flows, we have called pseudomeanders. Most meandering streams in nature have Froude numbers of perhaps 0.2 to 0.5, and there is little evidence to indicate that the meanders owe their origin to the prior existence of supercritical conditions. The Mississippi River, for example, has velocities perhaps 4 to 5 times those in the experimental channels, but depths 200 to 400 times as great.

The successive stages of development of the pseudomeanders, beginning with the formation of standing waves, are sketched in figure 124. As the curvature of the flow increased, the banks of the channel at first remained straight, while the width of the channel and the spacing between the shoals increased and while the shoals moved progressively downstream. The fronts of the advancing diagonal shoals at their upstream ends became parallel to the channel banks at a distance from

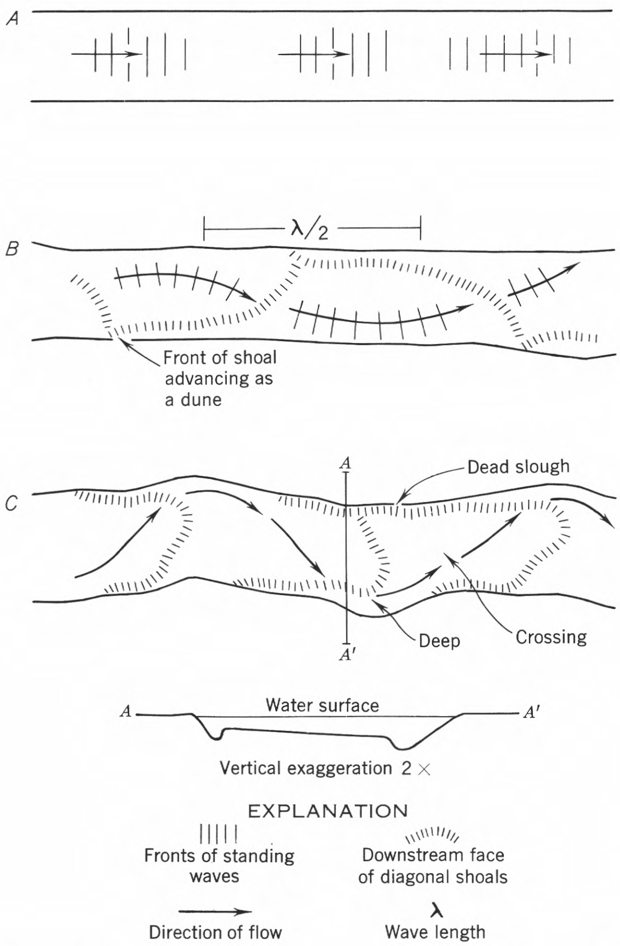

FIGURE 124.—Diagrammatic sketch of successive stages in formation of pseudo-meanders.

the banks of roughly 0.2 foot. The narrow defile between the bar and bank was a region of nondeposition, in contrast to the center of the channel where aggradation was associated with active transport and with progressive movement of the diagonal shoals.

As successive bars moved downstream, a dune front on the channel margin also moved downstream partly or wholly filling the narrow corridor (fig. 124*B*). Several rates of movement of this front are given in table 3. The direction of flow and the direction of movement of the bed material became more sinuous until the flow eventually was directed against the channel banks. The banks were gradually eroded (fig. 124*C*) and the channel developed the asymetric cross section similar to the bend section in true meanders (fig. 124 cross section *A*—*A*). Once erosion of the bank had begun, particles of sediment were no longer directed downstream but followed diagonal paths toward the center of the channel. This reduced the sediment supply to the narrow marginal corridors which were abandoned by the flow. In the final meandering pattern these dead sloughs can be seen shoreward of the point bars in successive bends (figs. 124 and 125). These are fundamentally areas of nondeposition and not erosional or scour features and hence are not to be compared to the chutes or sloughs often formed as parts of point bars and cutoffs in similar positions in large river meanders.

Assuming mean values of the Manning n for each sand and solving for values of s, it was possible to compute lines of Froude number equal to 1.0 (figs. 118 and 126) to show the region in which pseudomeanders formed. Runs in both sand sizes (fig. 118) verified the fact that the line for Froude number equal to 1.0 delineates the transition from straight to pseudomeandering channels.

Because the diagonal shoals and dead sloughs in the pseudomeanders (fig. 115) resembled features of some of the meanders produced in the laboratory studies described by Friedkin (1945, pl. 22), an attempt was made to compute Froude numbers for some of those experiments. As in the present experiments, velocities were not measured, and the Froude number was computed on the basis of the mean velocity. The computed values in several instances were about 0.7, similar to the average values reported here for runs 7 and 11 (table 3). The data indicate that because of

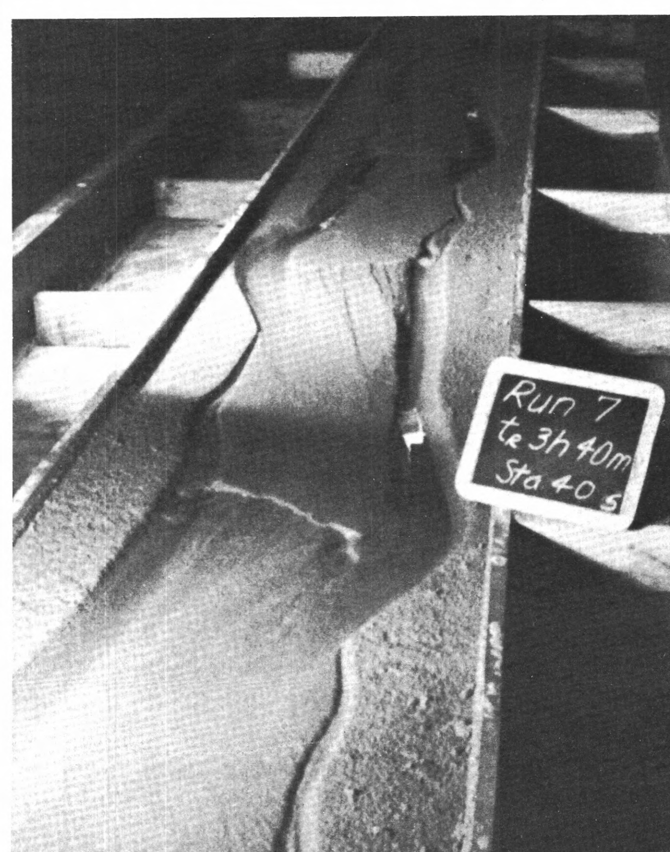

FIGURE 125.—Pseudomeander. Shows dead sloughs near channel banks, steep front of diagonal shoal, and fanlike layered bars in line of sight similar to those shown in Friedkin (1945).

TABLE 3.—*Selected data for runs with diagonal shoals and pseudomeanders*

Run No.	Discharge (cfs)	Station [1]	Froude number $\frac{v}{\sqrt{gR}}$	Wave-length of shoals or pseudo-meanders (feet)	Width of channel (feet)	Ratio of wavelength to channel width	Time elapsed during run (hours)	Remarks
3	0.020	44	0.90	8.0	0.64	12.5	8.8	Faint diagonal shoals; also sand streaks parallel to flow 0.07 ft apart.
4	.038	44	.84	6.7	.96	7.0	3.5	Diagonal shoals. Downstream movement of lateral margin of shoal, 0.09 ft per min.
5c	.039	66	-----	7.5	1.1	6.8	8.2	$s=0.0042$.
6	.011	-----	-----	4.6	.45	10.2	.8	$s=0.0069$; diagonal shoals.
	-----	-----	-----	4.8	.56	8.6	1.3	Diagonal shoals and bank cutting, downstream movement of lateral margin of shoal, 0.08 ft per min and 0.10 ft per min.
	-----	24	1.15	6.0	.70	8.6	3.1	Pseudomeanders.[2]
7	.019	24	.71	5.0	.84	6.0	1.6	Diagonal shoals; downstream movement of lateral margin of shoal, 0.23 ft per min.
	-----	-----	-----	7.3	1.30	5.6	3.6	Pseudomeanders.
8	.041	-----	-----	5.1	1.0	5.1	.7	Diagonal shoals; rate of movement, 0.12 ft per min.
	-----	-----	-----	6.7	1.1	6.1	1.2	Do.
	-----	-----	-----	7.8	1.4	6.1	1.9	Pseudomeanders.
8a	.040	-----	-----	5.2	1.01	5.2	.6	Diagonal shoals; rate of movement, 0.12 ft per min.
	-----	12	1.09	9.2	1.68	5.5	1.6	Pseudomeanders.
9	.030	-----	-----	4.6	.99	4.7	.7	Diagonal shoals, $s=0.0072$.
	-----	-----	-----	6.9	1.06	6.5	1.6	Do.
	-----	14	.94–2.1	7.8	1.43	5.5	2.5	Pseudomeanders, $s=0.0072$, $A=0.033$ sq ft, $w=1.0$ ft.
11	.025	28	.72	3.0	.68	4.5	.4	Diagonal shoals, $s=0.0063$, $A=0.039$ sq ft, $w=1.41$–1.46.
	-----	-----	-----	4.0	.75	5.3	.8	Do.
	-----	-----	-----	5.4	.81	6.7	1.3	Pseudomeanders.
	-----	-----	[3] .7–1.24	7.4	.93	8.0	2.2	Do.
12	.019	31.5	-----	5.8	.69	-----	1.1	$s=0.0067$, diagonal shoals.
	-----	-----	[4] 2.36	7.2	.97	-----	3.1	Velocity by dye over shoal at depth 0.011 ft, 1.4 fps, pseudomeanders.
	-----	-----	[5] 1.54	7.2	.97	-----	3.1	Cross section in bend. $A=0.022$ sq ft, $w=1.27$ ft.
	-----	-----	[5] 1.35	-----	-----	-----	3.1	$w=0.97$ ft.
13	.021	8	1.13	1.4	.48	2.9	0.01	Diagonal shoals, $s=0.0062$.
	-----	-----	-----	2.2	.57	3.9	0.25	Do.
	-----	-----	-----	3.2	.61	5.2	0.78	Do.
	-----	26	1.37	2.9	.64	4.5	1.2	Do. Slight waviness in banks, rate of movement of lateral margin of shoal. 0.31 ft per min.
	-----	-----	-----	6.3	.72	8.7	2.3	Pseudomeanders begin.
	-----	14–48	1.13	6.8	.89	7.6	4.1	Pseudomeanders.
	-----	-----	1.18	-----	.74	-----	4.1	Do. $R=0.0273$ ft, $v=1.11$ fps.
21	.065	-----	-----	6.0	1.43	4.2	19.5	Diagonal shoals, $s=0.00271$, concentration$=0.000095$.
32	.024	44	.96	7.6	.75	10.1	3.7	Diagonal shoals, $s=0.0051$.
50a [6]	.163	41	-----	6.0	1.4	4.3	.4	Diagonal shoals, $s=0.010$.
	-----	10	1.05	9.0	1.32	6.8	3.4	Do.

[1] Where station is not given a mean of several cross sections was used.
[2] Pseudomeander: one in which overall channel pattern is meandering.
[3] Represents approximately difference between shallow sections and deep sections.
[4] Maximum.
[5] Mean.
[6] Median grain size 2.0 mm; all other runs are with median grain size 0.67 mm.

the asymmetry of the cross sections, however, a Froude number based on the average depth and velocity may be somewhat misleading (table 2) because locally, and often over a large part of the section (see runs 11 and 12, table 3), depths were exceedingly low and velocities correspondingly high. This is by no means true of all experiments, for many of the Friedkin (1945) experiments were run at moderate Froude numbers. In simple trough experiments, however, there is sometimes a tendency to use steep slopes and small quantities of flow in producing what appear to be bona fide meanders. There is some suggestion here that meanders so formed may not be characteristic of natural streams.

In this study the pseudomeandering channels cannot be compared with the equilibrium straight channels. Because of the channel width and because the amplitude of the pseudomeanders was too large, no pseudomeanders were permitted to develop fully, but in all cases the channel shapes and widths were not in keeping with the values obtained for the straight channels. The data from run 21 suggest that a different mechanism

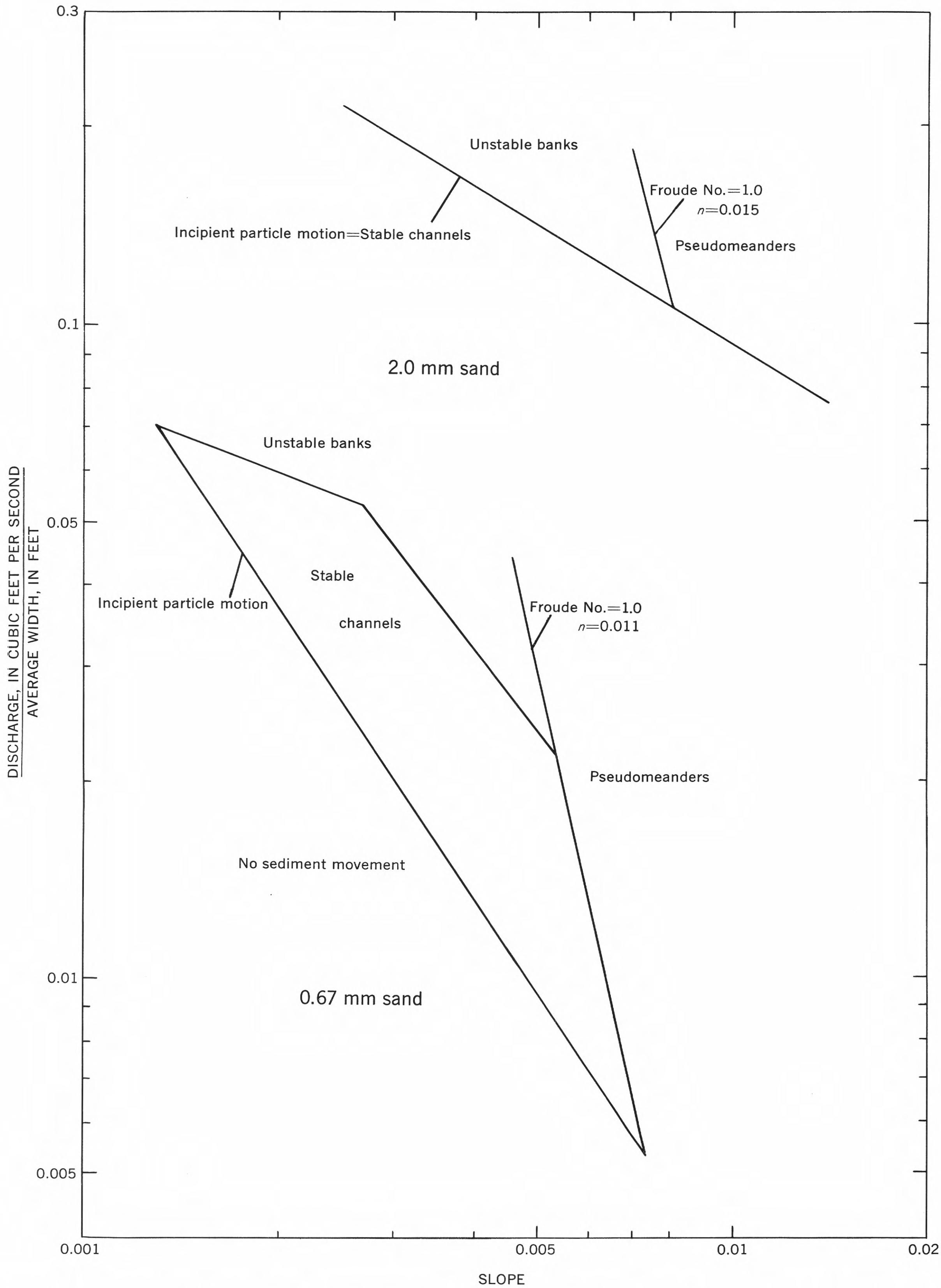

FIGURE 126.—Diagrammatic relation between discharge per unit width and slope for experimental channels showing region of stable equilibrium for coarse and very coarse sand. Region of stability bounded by no transport, eroding banks, and formation of pseudomeanders.

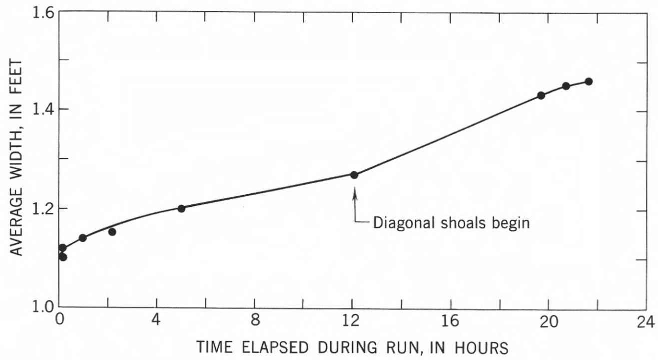

FIGURE 127.—Change of channel width with time. Shows increase in width due to formation of diagonal shoal during run 21.

may perhaps control the width and form of these or similar meandering channels. A plot of mean channel width against time for run 21 (fig. 127) shows the characteristic rapid increase in width at the beginning of the run and the subsequent leveling off with time (see fig. 114). At time 12 hours, however, the rate of change of width increases abruptly for a second time. This change in width is coincident with the appearance of pronounced surface waves and diagonal shoals. With the passage of time the width curve (fig. 127) can be seen to flatten again, gradually approaching a constant value of about 1.46 feet. Because the channel impinged on the wall of the flume the run was stopped after 21.5 hours. There is some indication that between 8 and 12 hours the width was approaching a stable value determined by the flow and bank material in accord with the equilibrium principles for straight channels described earlier. In contrast, the final width of the sinuous channel which was being approached at 21.5 hours may be determined by a different set of conditions in which the erosion of the concave bank is balanced by deposition on the opposite convex bar.

Although the final ratio of wave length to channel width in the pseudomeanders ranges from about 5 to 10, approximating the value of 7 observed by Leopold and Wolman (1957) in small rivers, the examples in figure 128 show that in the pseudomeanders the ratio is not constant but changes progressively with time. This may be due in part to the fact that width had not become truly stabilized in the meanders before the end of the run when the bands impinged upon the walls of the flume.

With the inclusion of the pseudomeanders in the experimental data, the limiting conditions for equilibrium of the straight self-formed stable channels can be shown on a diagrammatic graph of discharge per unit width against slope (fig. 126). The upper limit of stability of the equilibrium experimental channels in both sands is determined by the erodibility of the bank material. The threshold of sediment movement

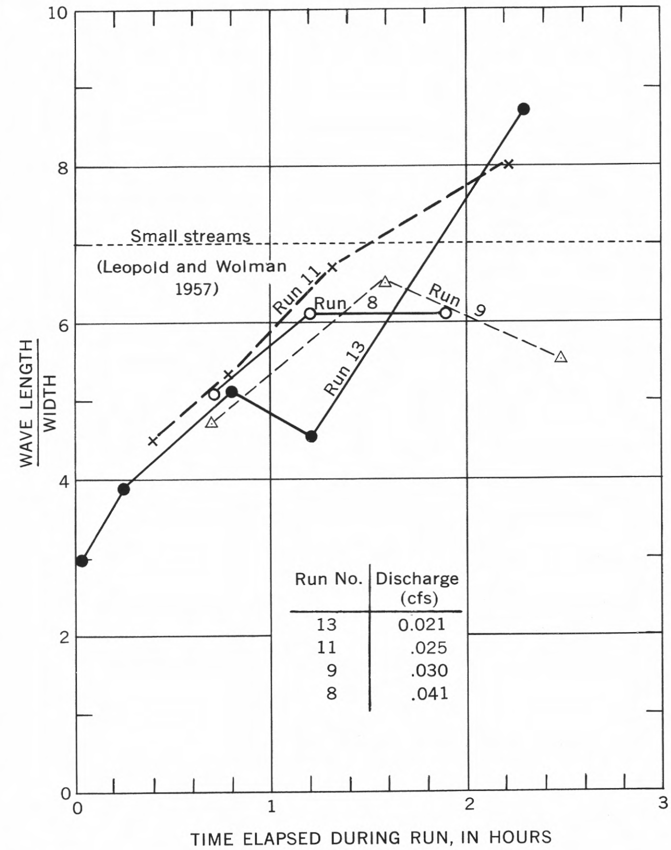

FIGURE 128.—Increase in ratio of channel width to wavelength of diagonal shoals with time during several runs.

may be thought of as the lower limit of the equilibrium channel in the sense that below this threshold, although the channel may be stable, its size and shape do not represent equilibrium conditions adjusted to the imposed discharge, load, slope, and particle size. In these experimental channels an additional factor, the development of pseudomeanders at Froude numbers greater than 1, defined the third limiting condition of the adjusted equilibrium channels.

COMPARISON WITH FIELD OBSERVATIONS OF NATURAL CHANNELS

ANABRANCHES, RIVER CHANNELS, AND FLUME CHANNELS COMPARED

The self-adjusted channels in the flume are not models of larger channels but are themselves small stream channels similar to those in nature. They represent an ideal case, however, inasmuch as there are few natural channels which are straight and in which the bed and banks are composed of material of the same size which material is nearly unigranular and noncohesive. There are fewer still in which the hydraulic roughness is solely a function of the size of the bed material and not a function of the configuration of dunes which form on the channel bed. Individual anabranches of braided channel systems appeared likely to provide conditions as close to those in the flume as it was possible to find in nature, and for this reason the authors sought to measure a number of these channels in the field. A typical anabranch is shown on the photograph in figure 129.

FIGURE 129.—Typical braided channel, Coldwater Creek near Lisco, Nebr., looking upstream (see fig. 130, table 4).

The dimensions and hydraulic characteristics of several natural channels are given in table 4. It was impossible to obtain discharge measurements at the bankfull stage, the stage which appears to correspond most closely to the conditions established in the laboratory; hence it was necessary to resort to an estimate of this velocity based on the equation (Leopold and Wolman, 1957, fig. 49, p. 65).

$$\frac{d}{D_{84}}=\left(\frac{8gds}{v^2}\right)^{0.5} \tag{7}$$

in which d=the mean depth at the bankfull stage,
s=the mean slope of the water surface at low flow,
D_{84}=that size of the bed material derived from a pebble count such that 84 percent of the material on the cumulative curve is finer than the size D_{84}, and
v=the mean velocity.

We are aware of the large error of estimate in this empirical relation but, in the absence of a more satisfactory measure of field conditions, this estimate of velocity must be considered adequate for present purposes.

Besides measurements of anabranches, measurements were available of the width, depth, and discharge of several natural channels transporting fine material within relatively erodible banks. The stage of incipient motion was determined from the known grain size and slope by using the relation between critical tractive force and grain size given by Lane (1955, p. 1253). Knowing the depth, we took the associated discharge and width from curves relating measurements of width, depth, and velocity to discharge at the river cross section (Leopold and Maddock, 1953). At shallow or moderate stages these rivers wander within self-formed low-water channels of relatively noncohesive material. At the bankfull stage and well beyond the condition of incipient motion, which characterized the bankfull stage in the flume channels, the banks contain sufficient quantities of silt and clay to make them quite cohesive. Thus, in contrast to the large anabranches, it was assumed that in these rivers, the readily adjustable low-water channels most closely approximated the flume channels.

The relation between two sets of laboratory channels and the channels in nature is given in figure 130 where

TABLE 4.—*Estimated bankfull discharge, width, and slope in selected natural channels including anabranches of braided rivers in relatively noncohesive materials* [1]

River and location	Discharge (cfs)	Width (ft)	Discharge / width	Slope	Type of channel	Remarks
Greybull River near Basin, Wyo	109	81.0	1.35	0.0070	Anabranch	D_{50}=0.16 foot.
Wind River near Dubois, Wyo	70	41.0	1.71	.0037	Channel	Between point bars Section C, near Sheridan ranger station, D_{50}=0.089 foot.
Du Noir Creek near Dubois, Wyo	538	66.0	8.15	.00098	Main channel	At bar level, D_{50}=0.033 foot.
Lava Creek near Moran, Wyo	17	20.0	.86	.0135	Anabranch	D_{50}=0.161 foot.
Pacific Creek near Moran, Wyo	26	41.5	.64	.0033	do	D_{50}=0.250 foot.
New Fork River near Pinedale, Wyo	36	20.0	1.80	.0100	do	D_{50}=0.12 foot.
Do	1,300	54.0	24.0	.0029	Undivided channel.	D_{50}=0.12 foot American Gothic reach (Leopold and Wolman, 1957, p. 77, 79, 80).
Green River at Daniel, Wyo	2,000	70.0	29.0	.0005	do	D_{50}=0.056 foot (Idem, p. 77, 79).
Coldwater Creek near Lisco, Nebr	.23	1.4	.16	.0069	Anabranch	D_{50}=0.0089 foot.
White River below Emmons Glacier, Wash.	.29	1.5	.19	.013+	do	Tiny channel, D_{50}=0.0135 foot.

[1] Data for other river sections in figure 130 given in Leopold and Wolman, 1957, app. H.

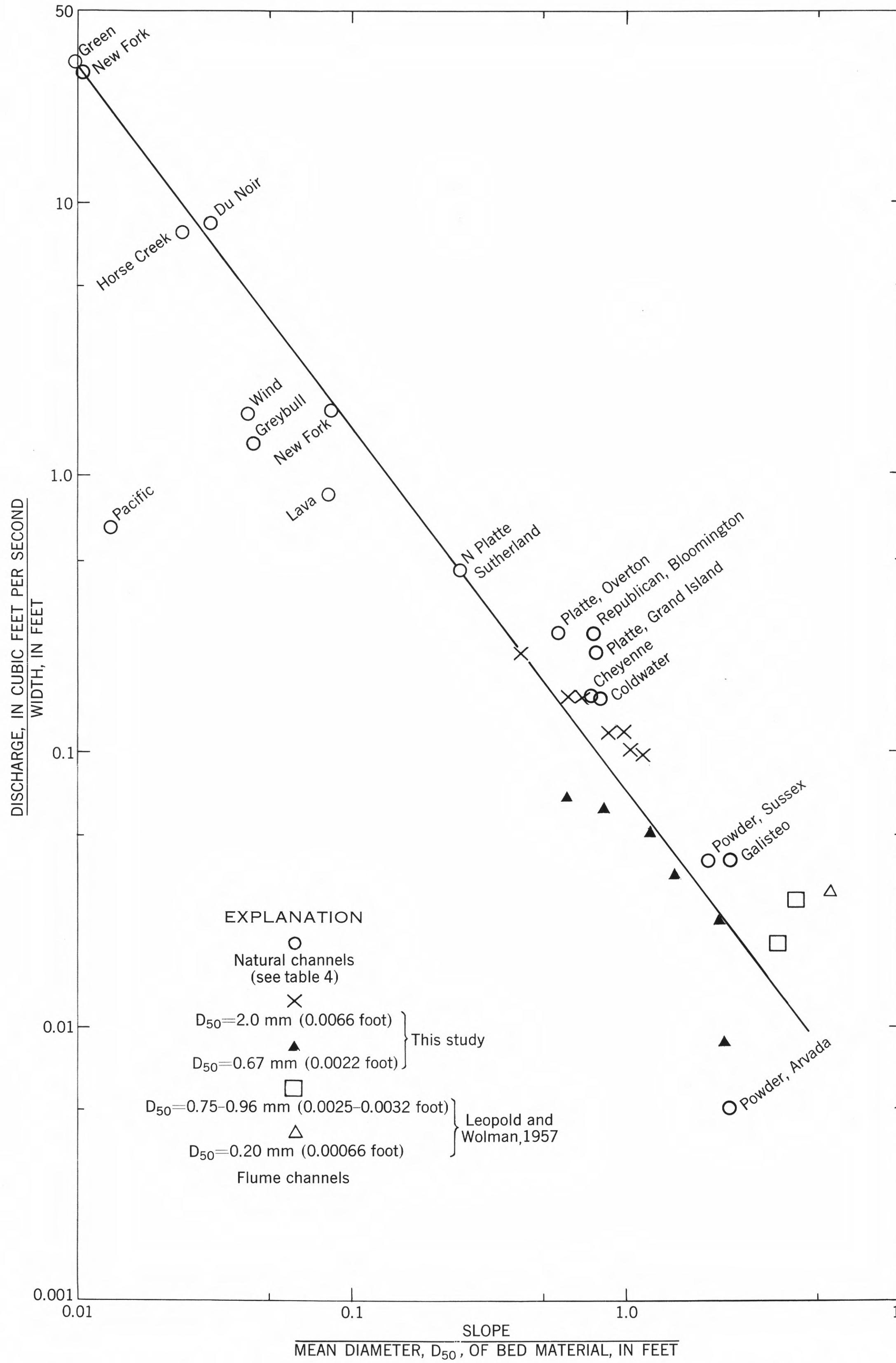

FIGURE 130.—Relation of discharge per unit width to quotient of slope and bed material size in experimental channels and natural channels and natural channels in relatively noncohesive materials. Natural channels include anabranches and undivided reaches of braided channels and low-water sand channels in large rivers.

the unit discharge is related to the quotient of bed material size and slope. This is simply a transformation of figures 118 and 126. D_{50} is the median diameter of the bed material. Sufficient laboratory data are given to define the range of stable equilibrium channels in each grain size. The greater scatter of the data for the laboratory channels in the 0.67 mm sand as contrasted with the channels in the coarser, 2 mm sand, reflects again the range in unit discharge that can be accommodated by the equilibrium channels in the finer sand. Despite the crude and limited character of the field data, it is interesting to note that for a wide range of particle size and slope, the discharge per unit width which was obtained where the banks are virtually noncohesive is quite limited. The increase from the sand in the flume to the 50 mm gravel in the bed and 25 mm gravel in the bank of Lava Creek is a 10- to 25-fold increase in particle size. The discharge per unit width, however, is 5 times as great. No explanation for the position of the point for Pacific Creek has been found although it appears that the large bed material may not actually move at the bankfull stage according to the graph in Lane (Lane, 1955, p. 1253). The behavior of these natural channels in relatively noncohesive material appears to be quite similar to the behavior of the very much smaller laboratory sand channels.

In braided-river channels in the coarse material studied, the diameter of the bank material is approximately one-half the diameter of the bed material (table 5). On Wind River, near Dubois, Wyo., section C, for example, the size, D_{75}, of the bed material is about 41 mm and the bank is 19 mm. The tractive force required to move the bank material here is about 0.38 of that required to move the bed material (Lane, 1955, p.1253). This value agrees closely with the expected value of 0.40 from Lane's (1955, fig. 3, p. 1245) curve based on the angle of repose of the bank material (31°) and the known angle of the side slope (28°) measured in the field. Predicted ratios of tractive force on the bed to tractive force on the side are based on measured angle of slope of bank (table 5, column 9), and calculated ratios are based on tractive force required to move particle size found in bank (table 5, column 6). Comparative values of the two are also given for the 7 additional sections in coarse noncohesive sediments.

The agreement in some cases is excellent, but it is apparent that in natural channels the heterogeneity of the materials, particularly the admixture of even small amounts of cohesive silts and clays, produces considerably steeper side slopes than would be expected in noncohesive materials. This is illustrated by the Wind River near Dubois, Wyo., (sec. A) where the angle of the sloping bank is 34°. Had the side slope been 30°, closer to the average value observed in the field, the estimated ratio of the tractive force on the sloping side to that on the bed would have been about 0.45, very close to the observed or calculated one.

Considering both the difficulties involved in determining the angle of repose and the extreme variability of natural bank materials, it is perhaps significant that as many as one-half of the sections in this limited sample show rather close agreement between Lane's (1955) predicted values and those computed from the field measurement of the size of bed and bank material and the angle of the sloping banks of the natural channels. That some bedload transport does occur in channels in the 0.67 mm sand without erosion of the banks, however, does indicate that in this narrow range the behavior of the experimental channels is not described by

TABLE 5.—*Size of bed and bank materials in river sections in relatively noncohesive material and critical tractive force required to initiate movement*

River and location	Size of bed material D_{75} (mm)	Tractive force required to move bed material [1] (lb per ft²)	Size of bank material D_{75} (mm)	Tractive force required to move bank material (lb per ft²)	Ratio of bank tractive force to bed tractive force	Angle of sloping bank (degrees)	Angle of repose of bank material (degrees)	Computed ratio of tractive forces[1] (K)	Remarks
Lava Creek near Moran, Wyo.	80	1. 3	30	0. 41	0. 32	28	34	0. 53	
New Fork near Pinedale, Wyo.	57	. 87	40	. 61	. 70	29	36	. 53	
Pacific Creek near Moran, Wyo.	105	1. 6	63	1. 0	. 63	22–30	38	. 80–. 59	
Little Snake River near Baggs, Wyo.	111	1. 7	64	1. 1	. 65	30	38	. 59	Bank material is angular, all other rounded.
Greybull River near Basin, Wyo.	--------	--------	--------	--------	--------	27	--------	--------	
Wind River Dubois, Wyo	37	. 56	34	. 47	. 84	34	35	. 21	Section A, some cohesiveness to bank material.
Do	77	1. 2	20	. 27	. 22	21	32	. 52	Section B, same.
Do	41	. 62	19	. 24	. 38	28	31	. 40	Section C, noncohesive.

[1] From Lane, 1955, p. 1253.

Lane's (1955) equation. As figure 130 shows, however, the variability in natural channels fully encompasses the range of transport without bank erosion in the 0.67 mm sand.

The channels in the flume with movable bed and banks have been described by us as being in "stable" equilibrium. Under the experimental conditions they are indeed stable and would so remain indefinitely provided the controlling conditions were unchanged. Their counterparts in nature with which they have been compared, the anabranches of braided rivers, however, are best known because of their instability. This paradox must be attributed in part to the fact that the natural channels, otherwise quite similar to the flume channels, are characterized by fluctuating discharges, whereas the discharge in any given flume channel is constant. The potential effect of a fluctuating discharge is suggested by the curve showing the progressive change in channel width with time for a run in which the discharge was halted twice (fig. 131). After each interval in which the discharge was stopped and the bank became partly dry, the resumption of flow resulted in an abrupt increase in channel width. Repeated fluctuations in discharge, coupled with overflow stages such as occur in nature, would undoubtedly produce in easily eroded noncohesive bank material the shifting, unstable channels found in nature. The form of these transient channels, however, as we have seen, is similar to the form of the stable equilibrim channels deveploed in the flume.

The low unit rate of load transport in the flume channels has been described. In this brief comparison of the flume and natural channels, it is worth noting that in many braided river valleys, as in the flume channels, the capacity for transport of sediment is likely to be quite low. Because of their great width that is due to the instability of the banks, the individual anabranches are very shallow. This instability of the banks and resultant loss in capacity for transport may lead to the aggradation sometimes associated with braiding. A surfeit of load, implying usually a large volume of large particles, is often given as the cause of such aggradation. In a sense, the true or perhaps immediate cause lies in the nature of the available sediment which produces channels with unstable banks. The latter, in turn, result in channel shapes inefficient in transporting sediment, thus providing the combination of factors responsible for the so-called overloading. On the other hand, the mere appearance of a braided river is not prima facie evidence of "overloading" (Leopold ad Wolman, 1957). Like the flume channels, the channels of the braided river may be so adjusted that they are fully capable of transporting the size of material and quantity of load provided. Because the rate of transport per unit width in such channels is low however, it is not uncommon to find reaches of meandering channels on relatively flat slopes readily transporting the total load received from broad, braided, upstream reaches on steeper slopes. Stricklin (1957) has described one such case on the Brazos River in Texas in which both the braided reach and the meandering reach are stable and not aggrading.

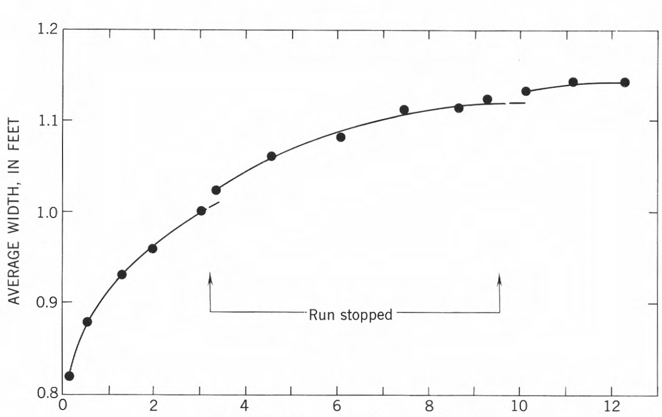

FIGURE 131.—Change of width with time showing abrupt increase in width after resumption of flow following periods in which flow was cut off.

RELATION OF BANK MATERIAL TO AT-A-STATION AND DOWNSTREAM VARIATION OF WIDTH WITH DISCHARGE

Runs at different discharges on the same slope in the flume are similar to the changes which take place at a river cross section with increasing discharge. Similarly, the experimental channels in runs with different discharges and variable slopes may represent the changes which take place with increasing discharge in the downstream direction in a drainage basin. It has been shown that in a large number of river sections the change in width with increasing discharge is small, while, on the average, the width increases in the downstream direction with increasing discharge to the 0.5 power. (Leopold and Maddock, 1953; Leopold and Miller, 1956; Wolman, 1955, Blench, 1957). It has been suggested that the variations in the exponent of discharge are related to the characteristics of the bed and bank material (Blench, 1956, p. 21). Analogy with the experimental channels in noncohesive sands described here indicates that the distinction between at-a-station and downstream relations is primarily a function of the cohesiveness and size of the bank material. At a constant slope, the width of the flume channels in the finer (0.67 mm) sand varies approximately as the 0.7 power of the discharge, reflecting the small range in depth (see runs 17, 19, 20, 21, 26, 27, and 35). In the coarser (2.0 mm) sand, as previously observed, there is almost no range in depth for a given slope, and thus there is virtually no at-a-station relation of depth to discharge. Width, like cross-sectional area (fig. 117), varies roughly as the 0.9 power of the discharge. Thus, in both sands the hydraulic geometry describing the at-a-station and downstream relations (Leopold and Maddock, 1953) consists of only one line for each of the three variables, width, depth, and velocity, owing to the lack of cohesion in the bank material.

We should expect this same continuity of at-a-station and downstream curves in natural rivers which are nearly comparable to the experimental conditions; that is, ones in which the bed and bank material can be considered noncohesive. Obviously, few rivers possess these ideal characteristics. Blue Creek, for example, a stream flowing exclusively from the Sand Hills of Nebraska has a bed of 0.4 to 0.7 mm sand almost devoid of finer sizes but a mixture of finer material in the bank with the help of vegetation, produces a cohesive bank whose particles have a median diameter of approximately 0.1 mm. If the bank material alone is considered, perhaps the closet parallel to the flume channels for which records are available in this country are several stations on the Platte River system. Figure 132 shows the rates of change of width with discharge at six stations on the North Platte River in Nebraska. The braided channel at all of these stations is made up of shifting and stable islands. At some stations the banks are covered with coarse grass and contain considerable fine material (note Lisco, fig. 132). At others, Sutherland, for example, the material is more uniform and sandy. Despite departures from the ideal or flume condition, it can be seen that at these sections the rates of change of width with discharge are comparable to the 0.5 power which is the average rate of change of width with increasing discharge in the downstream direction (Leopold and Maddock, 1953, p. 16). Coupled with the experimental data, these curves appear to support the view that the hydraulic characteristics of equilibrium or regime channels are closely associated with the characteristics of the bank material.

Aside from the fact that the hydraulic roughness of the Platte River at all but low stages is a function of the configuration of the bed and not of the absolute size of the bed material, the Platte River at Grand Island is not unlike the flume channels. Data from discharge measurements on the Platte River, often described as, "A mile wide and an inch deep," provide an additional basis for comparison. For discharges up to several thousand cubic feet per second, the cross-sectional area of individual channels in the sand increases as the 0.85 power of the discharge, a rate equivalent to that observed in the flume channels. The close agreement between the two is shown in figure 117. As in the experimental channels, because there is little

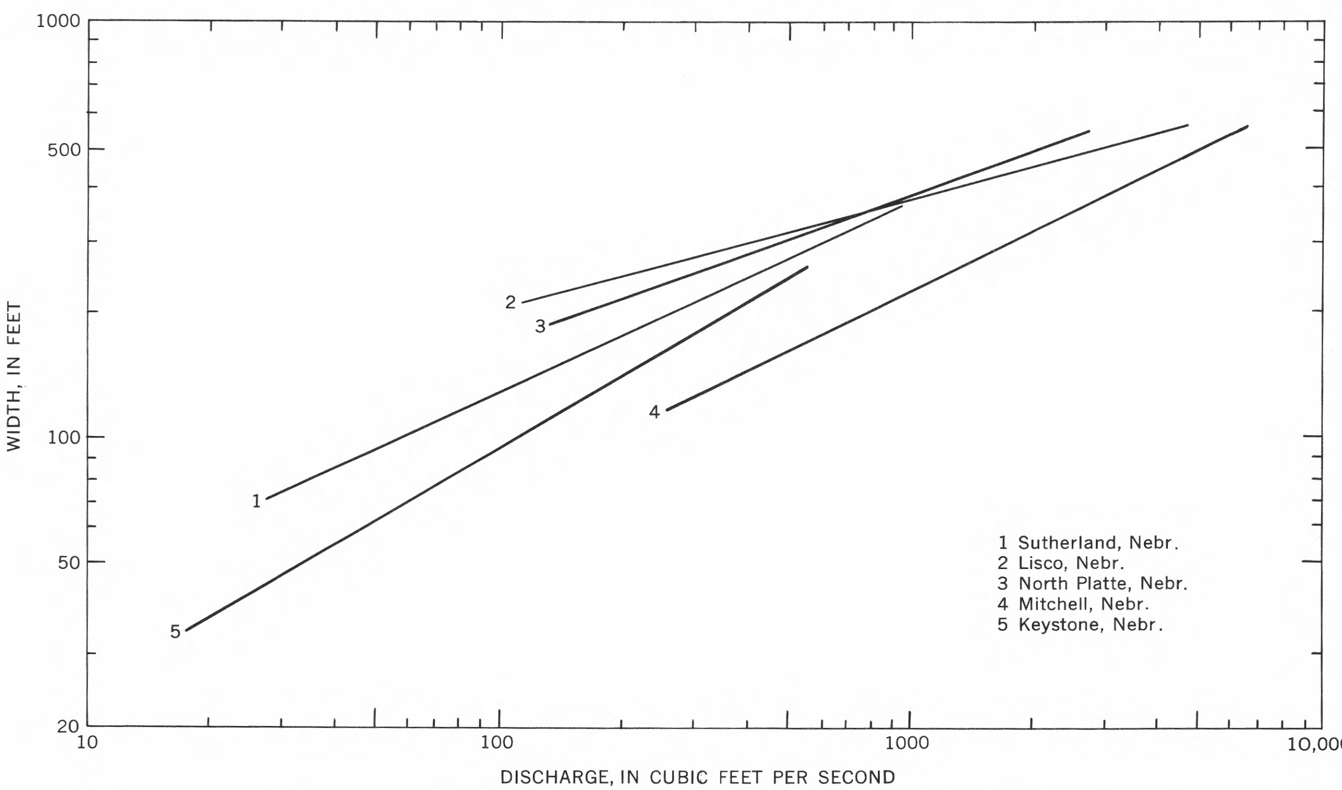

FIGURE 132.—Increase of width with discharge at several stations in the Platte River system.

or no increase in depth, most of the increase in area represents an increase in width. It is interesting to note that White (1940) has previously shown from dimensional considerations alone that the area of a cross section will vary as the 0.85 power of the discharge if the settling velocity of the bed material remains constant.

On the basis of the foregoing laboratory data and field relations, a simplified classification of downstream and at-a-station relations can be constructed (table 6). For noncohesive materials where the flow is at or above the critical required for transport the at-a-station and downstream relations will be the same (table 6). This condition is perhaps best developed in the 2.0 mm sand channels in the laboratory experiments where for a given slope the depth varies but little and the cross-sectional area is roughly proportional to discharge to the 0.9 power ($A \propto Q^{0.9}$) and width is also proportional to discharge to the 0.9 power ($w \propto Q^{0.9}$) for various slopes. The Platte River at Grand Island at moderate stages is an example of a channel in noncohesive material in which to a limited extent the shape of the lowest water channel at a given cross section follows the shape made at slightly higher discharges which were at or above the critical stage of incipient motion (table 6). Although area is proportional to discharge to the 0.9 power in this noncohesive material, the change of width with discharge and stage below the point of erosion of the bank material is a function of the angle of the sloping bank formed in the noncohesive sand. At a given section where the grain size and slope remain constant, with increasing depth, the following relations can be expected ($\propto$ is a sign of proportionality). From the Manning equation $V \propto d^{0.67}$. If $A \propto Q^{0.9}$ then $V \propto Q^{0.1}$, and $d \propto Q^{0.15}$. From the relation $wd=A$, and from the relations between d, A, and Q, it follows that $w \propto Q^{0.75}$. This is the relation between width and discharge for the anabranches of the Platte River at Grand Island, Nebr. (table 6).

In cohesive materials essentially the same principles apply with the exception that at-a-station the rate of change of width and depth cannot be predicted easily because the slope of the channel banks and the bed and bank roughness are highly variable. The downstream and at-a-station relations differ owing to the fact that the downstream curves are for flows either at or above the critical required to erode the banks or the downstream comparisons of lower flows reflect conditions primarily determined at higher stages. That is, each point on the downstream curve of width and discharge represents a width determined by some flow capable of eroding the channel banks. Thus, the rate of change of width in the downstream direction is nearly the same for various frequencies (that is, comparable stages) of flow. Because the erodibility of the bank as well as the bed is in part a function of the slope, at a given upstream section on a steep slope in cohesive material the channel width will be determined when a given quantity of water flowing at a given depth produces an erosive force just sufficient to balance the forces tending to hold the bank together.

In most river channels the slope and size of the bed material decrease rather markedly in the downstream direction, whereas changes in the character of the bank material are more limited (Wolman, 1955). To erode the same bank material at a downstream section on a flatter slope at a comparable frequency of discharge will require a greater depth than was required at the upstream section. We have already seen that if the velocity is to increase as the 0.1 power of the discharge in accord with the increase in cross-sectional area, any increase in depth must be at the expense of an increase in width. If the channel bed were composed of unerodible concrete, the increase in depth and thus the

TABLE 6.—*Generalized relations between discharge, width, depth, and area of cross section at a given cross section and in the downstream direction*

	Noncohesive bank material		Coehesive bank material	
	At-a-station	Downstream direction	At-a-station	Downstream direction
At or above critical point of incipient motion.	$A \propto Q^{0.9}$ $w \propto Q^{0.9}$ (Flume in 2.0 mm sand)		$w \propto Q^{0+}$ $d \propto Q^{f}$ $v \propto Q^{m}$ $f \doteq m$ (Brandywine Creek, Pa.)	$A \propto Q^{0.9}$ $w \propto Q^{0.75}$ or $w \propto Q^{0.5}$ (Many rivers, Leopold and Maddock, 1953)
Below critical point of incipient motion.	$A \propto Q^{0.9}$ $w \propto Q^{0.75}$ (Platte River at Grand Island, Nebr.)	Same as for cohesive material in downstream direction.		Determined by above; that is, at or above point of incipient motion.

increase in width would be directly proportional to the decrease in slope. If, as in nature, the erodibility of the bed tends to increase somewhat in the downstream direction, the rate of change of width will tend to decrease as the rate of change of depth is increased.

It may be expected, however, that with decreasing slope in the downstream direction where the banks are cohesive, the rate of increase of width in the downstream direction will be less than the 0.75 power of the discharge (table 6), a value applicable to the change of width with increasing discharge in noncohesive material at a constant slope. As Leopold and Maddock (1953, p. 51) point out, in many natural channels width increases as the 0.5 power of discharge, while depth increases to the 0.4 power. To the extent that this is a consistent relation, it must presumably be ascribed to the fact that *taken overall* many rivers have roughly similar bank materials and have longitudinal profiles which decrease in slope at rates which are not too dissimilar. As the data show (Leopold and Maddock, 1953, Wolman, 1955), although the average is well defined, individual rivers, and individual reaches in particular, do have varying rates of change of width and depth with increasing discharge in the downstream direction.

Inasmuch as low flows do not erode the banks of channels in cohesive materials, it follows that the rates of change of width, depth, and velocity at a given section will reflect the width and depth determined at the effective discharges. It is known (Wolman, 1958) that bank erosion occurs under a variety of discharges, but in most cohesive materials it may be expected that the bank will stand at angles greater than the angle of repose of noncohesive material (roughly 30°) up to 90° or perhaps even overhang. Thus, for a rather wide range in stage and discharge, width may remain nearly constant. Ideally, the relative change of depth and velocity, if the size of the bed material (roughness) remained constant and if no scour occurred, would be described by the relation $v \propto d^{2/3}$ from the Manning equation. Changes in width, scour, movement of bed material, vegetation, and changes in the distribution of velocity all cause departures from this ideal case. On the average in many river channel sections, velocity and depth increase with discharge to about the same power.

As the character of both bed and banks of natural river channels probably varies continuously, it is recognized that the breakdown of natural channels here into two distinct classes, cohesive and noncohesive, is quite artificial. In this analysis of an oversimplified case, however, an attempt has been made to use the ideal or extreme case represented by the flume channels in noncohesive material as an example of one end of the variety of channels observed in nature. Although it has certainly not explained this variety, it perhaps suggests the need for obtaining information on the bank materials at least comparable to that available for the longitudinal profile, bed material, and discharge.[2]

[2] While this report was in press, Prof. Paper 352–B by S. A. Schumm, "The Shape of Alluvial Channels in Relation to Sediment Type," was published. His study does begin to provide the kind of data asked for here and clearly demonstrates the importance of bank material in determining channel shape.

CONCLUSIONS

In channels composed of uniform coarse noncohesive sands, the channel size is a function of the discharge and the hydraulic roughness produced by the granular material. The shape is determined primarily by the angle of repose of the noncohesive bank material and by the force required to move the particles in the banks. The force competent to initiate movement of the particles on the bed appears to be a function of both the tractive force and velocity, the influence of the latter increasing as the size of the particles increases. Where the bank material in very coarse sand (2.0 mm) is truly noncohesive, there is for each slope one stable depth and hence one discharge per unit width (Q/w). In somewhat finer sand (0.67 mm) some cohesiveness of the bank material permitted more than one depth at a given slope. With a 4-fold increase in discharge, a depth and velocity 1.5 times greater than that required to begin movement of the bed material could be maintained within the sand (0.67 mm) banks.

Stable equilibrium flume channels determined by the stability of the noncohesive bank material possessed little capacity for transport of sediment of the same size material as that making up the bed and banks. Despite large amounts of bank erosion the adjusted channels remained straight and showed no tendency to meander unless the Froude number of the flow, defined by the mean velocity and hydraulic radius, approached a value of 1. Because the meanders developed only at supercritical velocities, they have been termed "pseudomeanders" to distinguish them from meanders in natural rivers which customarily occur at very much lower Froude numbers.

The flume conditions, including uniform noncohesive sand, similarity in bed and bank material, lack of dunes on the bed, and constant discharge, present an ideal case seldom found in nature. Braided channels in gravel and coarse sand such as often characterize glacial outwash plains provide the nearest counterparts to the experimental channels developed in the laboratory. To the normal instability of the noncohesive sands, however, must be added the unstabilizing effect of fluctuating discharges experienced in nature. Nevertheless, comparisons of several anabranch sections of braided river channels with the experimental channels indicated close agreement in

TABLE 7.—*Experimental data for stable straight channels*

Run No.	Station (distance from upstream end of flume, in ft)	Discharge (cfs) Q	Slope of water surface s	Area (sq ft) A	Width (ft) w	Wetted perimeter (ft)	Hydraulic radius (ft) R	Mean depth (ft) d	Mean velocity (fps) v	Reynolds number $\mathbf{R}$	Froude number $\mathbf{F}$	Darcy-Weisbach resistance factor f	Intensity of boundary shear (psf) τ	Water temperature (° F)	Average width along channel (ft) $\overline{w}$	Load (lb per sec)	Sediment concentration (C_s)	Duration of run hr:min	Remarks
Coarse sand: median diameter 0.67 mm																			
2	16	0.011	0.00410	0.013	0.42	0.46	0.028	0.031	0.85	8,900	0.90	0.041	0.0072	69	0.42	0.000137	0.000073	8:02	
3	44	.020	.00394	.022	.67	.69	.032	.033	.91	10,900	.90	.039	.0079	69	.64	.00046	.000141	14:12	
4	24	.038	.00387	.039	.93	.95	.041	.042	.97	14,900	.84	.043	.0099	69	1.05	.0023	.000359	6:52	
5	24	.020	.00382	.020	.58	.60	.033	.034	1.00	12,300	.97	.032	.0079	69	.60	.00049	.000147	11:10	
7	12	.019	.0064	.024	.70	.72	.033	.034	.79	9,700	.77	.087	.0132	69	.98	.00578	.00184	3:40	
8a	12	.040	.00680	.038	1.24	1.32	.029	.031	1.05	11,100	1.09	.046	.0123	67	1.58	.0077	.00123	----	
10	12	.025	.00710	.031	.81	.84	.037	.038	.81	11,100	.74	.103	.0164	68	1.07	.0033	.000790	----	
14	28	.022	.00350	.025	.67	.70	.036	.037	.88	11,100	.82	.042	.0079	65	.67	.00031	.000082	23:25	
15	28	.034	.00420	.044	.87	.90	.049	.051	.77	14,100	.61	.089	.0128	69	.97	.00097	.000175	12:30	
16	28	.041	.00229	.053	.81	.95	.056	.065	.77	15,100	.57	.056	.0080	65	.82	.00021	.000032	19:25	
17	28	.063	.00254	.059	1.13	1.17	.050	.052	1.07	19,100	.84	.029	.0079	66	1.15	.00075	.000071	13:13	
19	28	.032	.00280	.036	.72	.75	.048	.050	.89	15,800	.72	.044	.0084	68	.78	.00040	.000075	24:10	
20	----	.065	.00275	----	----	----	----	----	----	----	----	----	----	66	1.34	.00128	.000122	17:25	
21	----	.065	.00271	----	----	----	----	----	----	----	----	----	----	66	1.46	.00105	----	21:25	See figure 127.
22	32	.026	.00177	.035	.79	.81	.043	.044	.74	10,400	.63	.036	.0047	59	.78	.000001	----	2:20	Incipient movement of bed material.
22a	28.5	.049	.00184	.058	.83	.87	.067	.070	.84	18,500	.57	.045	.0077	59	.85	.00021	.000027	20:25	
22b	28.5	.039	.00211	.045	.79	.82	.055	.057	.87	15,700	.65	.039	.0072	59	.84	.000173	.000026	28:00	
23	28	.039	.00192	.044	.76	.82	.054	.058	.89	15,800	.68	.034	.0065	59	.74	.000085	.000013	23:10	
23a	----	.049	.00172	.058	.89	.92	.063	.065	.84	17,400	.59	.040	.0068	59	.90	.000178	.000022	21:55	
24	28	.049	.00175	.049	.76	.80	.061	.064	1.00	22,200	.71	.027	.0067	67	.83	.00181	.000022	5:55	
25	----	.050	.00181	----	----	----	----	----	----	----	----	----	----	67	.80	----	----	1:40	
26	36	.012	.0030	.0164	.51	.505	.032	.032	.73	8,500	.72	.046	.0060	67	----	----	----	----	Do.
26a	24	.012	.00244	.0182	----	.52	.035	----	.66	8,400	.62	.050	.0053	67	----	----	----	----	
27	24	.024	.00275	.029	.64	.67	.043	.045	.83	13,000	.71	.044	.0074	67	.66	.000176	.000045	51:40	
28	28	.009	.0038	.0153	.48	.64	.0312	.032	.59	6,700	.59	.088	.0074	67	.62	.000352	.000095	29:20	Do.
28a	16	.022	.00294	.024	.58	.60	.040	.041	.92	13,400	.81	.036	.0073	67	.59	.000352	.000095	29:20	
28a	28	.022	.00320	.024	.62	.64	.038	.039	.92	12,700	.83	.037	.0076	67	.62	.000352	.000095	29:20	
28a	40	.022	.00350	.026	.66	.67	.039	.039	.85	12,100	.76	.049	.0085	67	.66	.000352	.000095	29:20	
29	32	.022	.00375	.023	.63	.64	.036	.037	.96	12,600	.89	.038	.0084	67	.62	.000356	.000098	22:45	
30	28	.0037	.0050	.0079	.41	.42	.019	.019	.47	3,300	.60	.111	.0059	68	----	----	----	----	Do.
31	24	.012	.00485	.015	.50	.51	.029	.030	.80	8,700	.83	.057	.0088	69	.48	.000189	.000095	23:40	
32	----	.024	----	----	----	----	----	----	----	----	1.0	----	----	----	----	----	----	----	Diagonal shoals.
33	----	.07	.00	----	----	----	----	.10	----	----	----	----	----	----	----	----	----	----	Sand dunes on bed, depth is approximate.
34	40	.069	.00192	.066	1.19	1.22	.054	.055	1.05	20,600	.80	.024	.0065	67	1.20	.000436	.000039	40:00	
35	----	.049	.0026	----	----	----	----	----	----	----	----	----	----	----	1.10	.000723	.000091	----	
36	32	.049	.00372	----	1.13	----	----	----	----	----	----	----	----	----	----	----	----	8:50	Channel at exit not stable.
37	----	.049	.0033	----	----	----	----	----	----	----	----	----	----	68	1.14	----	----	5:30	Same as above.
39	----	.061	.00131	----	----	----	----	.154	----	----	----	----	----	----	----	----	----	21:00	Sand dunes on channel bed; map by Topo. Div. USGS.
40	28	.029	.00325	.039	.80	.81	.048	.049	.74	14,200	.60	.073	.0097	74	.79	.000556	.000124	24:00	

Very coarse sand: median diameter 2.0 mm

44	26	0.032	0.0137	0.028	0.66	0.69	0.041	0.042	1.14	19,100	0.99	0.111	0.0350	76	----------	----------	----------	----------	Approximate point of incipient movement.
48	28	.10	.0058	.095	.82	.88	.108	.116	1.05	47,700	.56	.146	.0390	78	0.83	----------	----------	----------	Incipient movement of bed material.
48a	16	.127	.0064	.088	1.06	1.11	.079	.083	1.44	47,900	.90	.063	.0316	78	1.07	----------	----------	20:05	
49	24	.21	.0024	.131	1.06	1.14	.115	.124	1.60	77,500	.83	.028	.0172	78	1.09	----------	----------	5:10	Very little widening.
50	26	.05	.0117	.036	.65	.69	.052	.055	1.39	30,800	1.08	.081	.0379	79	.65	----------	----------	----------	Incipient movement of bed material.
50a	10	.163	.0100	.109	1.69	1.73	.063	.064	1.50	40,200	1.05	.072	.0392	79	1.70	0.00048	0.000018	2:29	Diagonal shoals, see table 3.
51	28	.145	.0040	.113	.92	1.03	.110	.123	1.28	60,600	.68	.069	.0274	80	.91	----------	----------	----------	Incipient movement of bed material.
51a	32	.179	.0045	.114	1.15	1.21	.094	.099	1.57	63,500	.90	.044	.0263	80	1.16	.00056	.000019	6:00	
52	28	.247	.0025	.164	1.15	1.25	.131	.143	1.51	84,200	.74	.037	.0204	79	1.13	----------	----------	----------	Very little widening.
53	28	.076	.0075	.055	.75	.81	.068	.073	1.38	39,900	.93	.069	.0317	79	.75	----------	----------	----------	Incipient movement of bed material.
53a	30	.190	.0069	.123	1.78	1.82	.068	.069	1.54	44,600	1.04	.051	.0292	79	1.86	.00042	.000013	22:05	
54	32	.270	.0027	.173	1.16	1.24	.140	.149	1.56	92,900	.73	.040	.0235	79	1.17	----------	----------	0:10	Do.
55	32	.280	.0027(5)	.181	.83	1.27	.143	.218	1.55	98,300	.72	.042	.024	79	.83	----------	----------	0:30	Fixed walls of smooth galvanized sheet metal, sand bed.

essential characteristics. Comparison with other natural river sections emphasized the importance of the characteristics of the bank material in determining the shapes of natural channels. It is hoped that the analysis of the ideal case may be useful in helping to develop an understanding of the very much more complex conditions found in most natural channels.

REFERENCES CITED

Bagnold, R. A., 1956, Flow of cohesionless grains in fluids: Royal Soc. of London Phil. Trans., Series A, no. 964, v. 249, p. 235–260.

Blench, Thomas, 1956, Scale relations among sand-bed rivers including models: Am. Soc. Civil Engineers Proc. Paper 881 closure, p. 19–25.

——— 1957, Regime behaviour of canals and rivers: London, Butterworth, 138 p.

Brown, C. B., 1950, Sediment transportation, *in* Hunter Rouse, ed., Engineering hydraulics: New York, John Wiley and Sons, p. 769–843.

Einstein, H. A., 1942, Formulas for the transportation of bed load: Am. Soc. Civil Engineers Trans., v. 107, p. 561–577.

Friedkin, J. F., 1945, A laboratory study of the meandering of alluvial rivers: U.S. Waterways Expt. Sta., Vicksburg, Miss., 40 p.

Gilbert, G. K., 1914, The transportation of debris by running water: U.S. Geol. Survey Prof. Paper 86, 263 p.

Keulegan, G. H., 1938, Laws of turbulent flow in open channels: Nat. Bur. Standards Jour. Research, Research Paper 1151, v. 21, p. 707.

Koechlin, Réné, 1924, Mécanisme de l'eau: Paris, Béranger, 327 p.

Lane, E, W., 1955, Design of stable channels: Am. Soc. Civil Engineers Trans., v. 120, p. 1234–1279.

Leighly, T. B., 1932, Toward a theory of the morphologic significance of turbulence in the flow of water in streams: Univ. California Publ. in Geography, v. 6, no. 1, p. 1–22.

Leliavsky, Serge, 1955, An introduction to fluvial hydraulics: London, Constable, 257 p.

Leopold, L. B., and Maddock, Thomas, Jr., 1953, Hydraulic geometry of streams channels and some physiographic implications: U.S. Geol. Survey Prof. Paper 252, 57 p.

Leopold, L. B. and Miller, J. P., 1956, Ephemeral streams—hydraulic factors and their relation to the drainage net: U.S. Geol. Survey Prof. Paper 282–A, 45 p.

Leopold, L. B. and Wolman, M. G., 1957, River channel patterns—braided, meandering, and straight: U.S. Geol. Survey Prof. Paper 282–B, 85 p.

Rubey, W. W., 1938, The force required to move particles on a stream bed: U.S. Geol. Survey Prof. Paper 189–E, p. 121–141.

Strickler, K., 1923, Beitrage zur Frage der Geschwindigskeitsformel und der Raubigskeitzghlen fur Strom Kanale und geschlossene Leitungen: Eidgenosgische Amtes fur Wasserwirtschaft Mitt. no. 16, 77 p.

Stricklin, F. L., Jr., 1957, Geologic record of a degradational stream, Middle Brazos River, Tex. [abs.]: Geol. Soc. America Bull., v. 68, p. 1800.

Van Burkalow, Anastasia, 1945, Angle of repose and angle of friction—an experimental study: Geol. Soc. America Bull., v. 56, no. 6, p. 669–708.

White, C. M., 1940, The influence of transported solids upon rivers—British research: Comm. on Potamologie, Internat. Assoc. Hydrologie, 23 p.

Wolman, M. G., 1955, The natural channel of Brandywine Creek, Pa.: U.S. Geol. Survey Prof. Paper 271, 56 p.

——— 1959, Factors influencing erosion of a cohesive river bank: Amer. Jour. of Sci. v. 257, p. 204–216.