Apparent Resistivity of a Single Uniform Overburden

GEOLOGICAL SURVEY PROFESSIONAL PAPER 365

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By IRWIN ROMAN

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An analysis of the two-boundary resistivity problem with tables for numerical applications

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APPARENT RESISTIVITY OF A SINGLE UNIFORM OVERBURDEN

By Irwin Roman

ABSTRACT

The interpretation of resistivity observations can be facilitated by the development of theoretical formulas and corresponding curves. A development for a single overburden of uniform thickness has been made and formulas derived that involve infinite series. The series have been evaluated for the geophysical case in which the measuring configuration is located on the surface of the earth. A set of curves that can be superimposed on the field observations has been prepared to permit direct determination of the resistivity and thickness of the overburden, and the resistivity of the underlying medium. Auxiliary tables are given to simplify the numerical evaluations for configurations not covered by the prepared curves.

INTRODUCTION

Geophysical methods of exploration in the United States have developed steadily since the importation of German instruments about 1923. Many methods have been tested and equipment has been developed to a high degree of proficiency. Theoretical development has paralleled field techniques of measuring and each has indicated improvements to be made in the other. When theoretical conclusions were available, interpretation of field data and techniques of observation have been guided by them. When theoretical conclusions were lacking, the interpreter has used empirical methods or intuitive and qualitative conclusions.

Specifically, in the use of electrical methods, the work of Fox, prior to 1830, and of Barus, in 1882, did not develop exploratory methods. In 1915 Wenner furnished the key to the exploratory difficulties and in 1925 Rooney and Gish demonstrated the possibilities of the method of resistivity exploration. The theoretical analyses gave little promise until the work of Hummel (1929 a, b) which was followed by other theoretical papers, and the use of field measurements was accelerated thereafter. Although results have never been spectacular, successful resistivity measurements have been made by investigators in many parts of the world (Heiland, 1940, p. 28 and p. 707–763; Jakosky, 1950, p. 465–579; Roman, 1951).

In 1931, Roman published a mathematical analysis and a set of tables for use in the interpretation of resistivity measurements made at the surface of the earth, which was assumed to consist of a uniform overburden of constant thickness overlying a uniform bed of infinite thickness. The present report is the result of a study intended to improve the analysis and the tables of the earlier paper. Terminology has been clarified by a systematic description or definition of terms used. The upper limit of the argument in the tables has been increased in extent from 5 to 30, corresponding to lateral separations of current poles from potential electrodes of 10 and 60 times, respectively, the thickness of the overburden. Local expansions for small arguments have been augmented by asymptotic expansions for large arguments. The algebraic determination of the image series has been replaced by a geometric method, without sacrificing adherence to fundamental relations. The modified potential has been tabulated to eight decimals instead of five.

The increase in the range of the argument was the result of inadequacy of the earlier tables in investigating new methods of exploration. For the Wenner and the Lee partitioning methods, they usually were adequate. Over the years, problems arose in which greater ranges were needed. Among these were the single-probe method and multiple layering. In the former, the potential-reference electrode or that electrode and one current stake are remotely located. Although the contributions of these points to the measured potential differences are small, and sensibly constant over the area of survey, their being considered as negligible is not justified for all purposes, especially if the measured potential difference is small. In the latter, a distance that is a few times the depth of the lowest contact can be many times the thickness of the top layer.

The need for more significant figures also arose in the investigation of new methods of exploration, especially when the potential differences are much smaller than the separate potentials involved. This condition arises when the potential difference is measured between two points that are closely spaced with respect to their distances from the current poles, as can happen in the single-probe method or the potential-gradient method.

The increase in range of argument led to investigations resulting in greater accuracy of the calculated values. New formulas and methods of calculation were developed and these made the inclusion of additional significant digits relatively simple and rapid.

In numbering equations, decimal grouping is used. For example, the term "equations 2.4" refers to both of the equations 2.41 and 2.42. Likewise "equation 1" refers to the entire set from 1.1 to 1.552, whereas "equations 1.5" refers to the set from 1.51 to 1.552.

As bibliographies are available in the references cited, references are limited to those specifically needed.

The author thanks his colleagues who assisted in the execution of the project; in particular, Mrs. Mary E. Knettles and Mr. W. W. Schwendinger, each of whom checked some of the algebraic and numerical details and assisted in testing alternative methods from which the present methods were selected. Mr. Vincent Latorre wired the control panels of the punched card computing machines used in differencing tables calculated near the end of the project. Mrs. V. N. Rice cooperated enthusiastically in the application of punched-card equipment as an aid in the numerical calculations. Punched-card equipment was not available until the project was almost completed.

PHYSICAL PROBLEM

HOMOGENEOUS SPACE

If electric current is introduced into a medium, an electric field is created. In this field, a difference of electrical potential exists between two points. The character of this field depends on the properties of the space and on the current introduced.

POTENTIAL FUNCTION

In a homogeneous isotropic space, the potential due to an isolated current source of strength I has the value

$$U=\frac{kI}{r}+B, \qquad (1.1)$$

where r is the distance from the current source to the field point at which the potential is evaluated, and where k and B are constants for the field. As r increases indefinitely, U approaches the value B, which is usually considered as zero. If ρ is the resistivity of the medium, the constant k is $\rho/(4\pi)$ so that the potential function may be written

$$U=\frac{\rho I}{4\pi r}+B. \qquad (1.2)$$

As the potential can not be determined at an infinite distance, actual observation usually involves the measurement of the excess of the potential at the test point, H, over the potential at a reference point, L. If the reference point is at the distance s from the current source, equation 1.1 shows that the potential at the test point is

$$U_s=\frac{kI}{s}+B, \qquad (1.3)$$

where k and B have the same values as in equation 1.1. Accordingly, the excess of the potential at H over that at L is

$$V=U-U_s=kI(1/r-1/s), \qquad (1.4)$$

independently of B.

FOUR-ELECTRODE CONFIGURATION

In actual measurements, in addition to two potential points, test-point and reference point, there must be at least one current source, at which the current enters the medium, and at least one sink, at which the current leaves the medium. In the simplest form (see fig. 1)

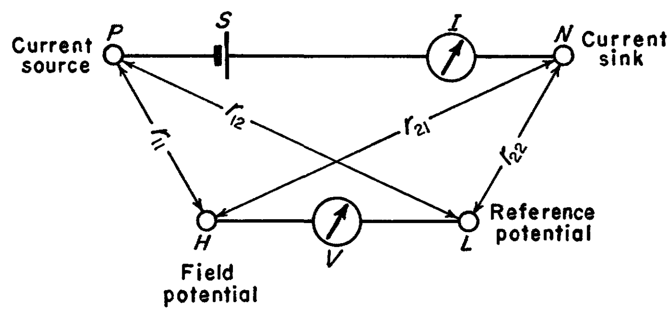

FIGURE 1.—Four-electrode configuration. A current of I amperes enters the medium at P and leaves it at N under the influence of a source S. A drop in potential of V volts from H to L results. The four points P, N, H, and L may have arbitrary locations and need not be coplanar.

there is one current source, P, one current sink, N, and two potential points, H and L; this arrangement is usually designated as a four-electrode configuration. The four points may be located arbitrarily and need not be in the same plane.

MODIFIED SUM

It is convenient to use the concept of a modified sum in discussing potential problems. Let x be an arbitrary value associated with a current pole and a potential point, jointly. The modified sum is denoted by $S\{x\}$, which represents the sum of all possible values of x, each with its own sign if a current source is paired with a potential testpoint or a current sink is paired with the potential reference point, and with its sign reversed in the opposite pairings. This notation is convenient because a current source corresponds to a positive pole or a positive current strength, a current sink corresponds to a negative pole or a negative current strength, and the total potential at the potential reference point is subtracted from the total potential at the testpoint to obtain a measured value.

GENERALIZED CONFIGURATIONS

Either the source or the sink or both may consist of multiple contacts. Thus the current may be fed into the opposite corners of a quadrilateral and drained from the remaining corners, or contacts may be made at several points along a line. The only restriction is that no two points may coincide. Each observation must be of a single potential difference, so that the potential points cannot be multiple.

FORM FACTOR

The potential difference, V, for a selected configuration has the value $S\{U\}$ where U is given by equation 1.1, so that it may be written as

$$V=S\{U\}=S\left\{\frac{\rho I}{4\pi r}+B\right\}. \quad (1.51)$$

As B occurs an equal number of times with each sign, it disappears in the sum. As ρ and 4π are common to all pairs, equation 1.51 may be written

$$V=\frac{\rho}{4\pi}S\left\{\frac{I}{r}\right\}. \quad (1.52)$$

For a four-electrode configuration, there are two current poles of the same strength so that I is common to all pairs and

$$V=\frac{\rho I}{4\pi}S\left\{\frac{1}{r}\right\}. \quad (1.53)$$

If a "form factor", F, is defined by

$$\frac{1}{F}=S\left\{\frac{1}{r}\right\}, \quad (1.54)$$

equation 1.53 may be written as

$$V=\frac{\rho I}{4\pi F}, \quad (1.551)$$

or

$$\rho=4\pi FV/I. \quad (1.552)$$

UNIFORM GEOPHYSICAL SPACE

DEFINITIONS

For purposes of the present paper, a "geophysical space" is defined as a space in which a medium of infinite resistivity is separated by a plane from a medium of finite resistivity, which may be constant or may vary in an arbitrary manner; it may even include one or more sections of infinite resistivity, if these sections are not in contact with the insulating medium other than at isolated points or along isolated lines. For convenience the medium of infinite resistivity may be called the "upper" medium, the second the "lower" medium.

In a "uniform geophysical space" the lower medium is uniform where the word "uniform" implies homogeneity and isotropy. A homogeneous medium has the same resistivity in a selected direction at every point. An isotropic medium has the same resistivity at a selected point in every direction. Accordingly, in a uniform medium, the resistivity is the same for all points and all directions. Physical properties other than resistivity do not enter the present discussion.

The ideally uniform medium has the same resistivity for every volume regardless of size. In practice, this limiting condition is not necessary. A body may be considered as uniform if variations from ideal uniformity are not detectable by measurements. Thus, a mixture of clay, sand, and gravel is not ideally uniform, but the distribution may be such that the medium may be considered as uniform for a selected accuracy of surveying and a selected spacing of electrodes.

CONFIGURATIONS

The configurations and concepts of homogeneous space apply to a uniform geophysical space, with all current poles and potential points in the lower medium. The form factor is calculated by the same equation 1.54, but the potential function of equation 1.2 must be determined for each specific arrangement of current poles. If the configuration has all of its points on the separating surface, the value of k in equations 1 becomes $\rho/(2\pi)$ instead of $\rho/(4\pi)$, which is equivalent to replacing the current I by $2I$. Thus, for a surface configuration the potential drop is

$$V=\frac{\rho}{2\pi}S\left\{\frac{I}{r}\right\}. \quad (2.1)$$

For a four-electrode surface configuration, the potential drop is

$$V=\frac{\rho I}{2\pi}S\left\{\frac{1}{r}\right\}=\frac{\rho I}{2\pi F} \quad (2.21)$$

and the resistivity is

$$\rho=2\pi FV/I \quad (2.22)$$

If the four-electrode surface configuration has the source at P (fig. 1), the sink at N, the potential reference point at L, and the potential testpoint at H, the form factor is F, where

$$\frac{1}{F}=S\left\{\frac{1}{r}\right\}=\frac{1}{r_{11}}-\frac{1}{r_{12}}-\frac{1}{r_{21}}+\frac{1}{r_{22}}. \quad (2.3)$$

SINGLE-PROBE CONFIGURATION

If the point L is remote from the other three, the distances r_{12} and r_{22} are large and the form factor, F, is determined by

$$\frac{1}{F}=\frac{1}{r_{11}}-\frac{1}{r_{21}}+A, \quad (2.41)$$

where

$$A=\frac{1}{r_{22}}-\frac{1}{r_{12}} \quad (2.42)$$

is small and may be assumed to be zero as a first approximation. This configuration has only one moving point and the potential of H is such that the potential vanishes at infinity.

If the points L, N, and P are kept fixed the value of A is constant, but not necessarily negligible. There is only one moving point and the potential at infinity is $\rho IA/(2\pi)$.

COLLINEAR CONFIGURATION

If N has the coordinates $(-l, 0, 0)$ P the coordinates $(l, 0, 0)$, H the coordinates $(x, 0, 0)$, and L the coordinates $(\lambda, 0, 0)$, the system is collinear as well as surficial. (fig. 2).

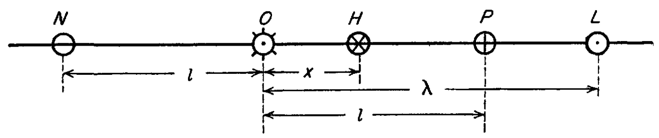

FIGURE 2. Collinear configuration. $O=(0, 0, 0)$=origin of abscissae; $N=(-l, 0, 0)$ =negative current pole; $P=(l, 0, 0)$= positive current pole; $H=(x, 0, 0)$=field potential test-point; $L=(\lambda, 0, 0)$=potential reference point.

For this configuration, the form factor has the value F, where

$$\frac{1}{F}=\frac{2x}{l^2-x^2}-\frac{2l}{\lambda^2-l^2} \tag{2.5}$$

WENNER CONFIGURATION

If the linear surface configuration has the points in the order $NLHP$ with common separation l, it is the usual Wenner configuration of figure 3, for which the

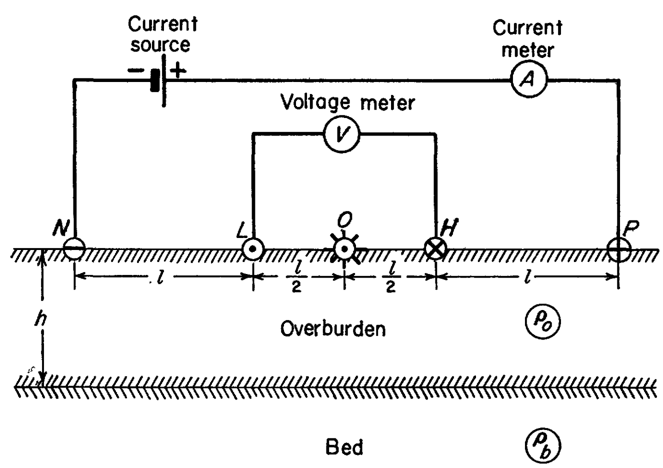

FIGURE 3. Wenner and Lee partitioning configurations. Notation as in figures 1 and 2, with $\lambda=-\frac{l}{2}$ and $x=\frac{l}{2}$; h, thickness of overburden; ρ_0, resistivity of overburden; ρ_b, resistivity of bed.

form factor is

$$F=l. \tag{2.61}$$

uniform geophysical space, the resistivity is

$$\rho=2\pi lV/I. \tag{2.62}$$

LEE PARTITIONING CONFIGURATION

If a third potential point O is located at the center of the Wenner configuration and the potential drop is measured from O to L for the "left" resistivity and from H to O for the "right" resistivity, the form factor for each half has the value

$$F=2l \tag{2.71}$$

and the resistivity for a uniform geophysical space is

$$\rho=4\pi lV/I. \tag{2.72}$$

BASIC CONFIGURATIONS

For a selected set of points in a four-electrode system, there are only three distinct groups. Interchanging the two current stakes or the two potential electrodes leaves the magnitude of the potential drop unchanged, but changes the sign. Interchanging the pair of current stakes with the pair of potential electrodes leaves the magnitude of the potential drop unchanged in uniform geophysical space. Ignoring the sign of the potential drop, for four selected locations considered in a specific sequence, there are the three groups:

1. *PNHL, NPHL, PNLH, NPLH, HLPN, HLNP, LHPN, LHNP.*
2. *PHNL, NHPL, PLNH, NLPH, HPLN, HNLP, LPHN, LNHP.*
3. *PHLN, PLHN, NHLP, NLHP, HPNL, HNPL, LPNH, LNPH.*

In the first group, the two current poles are consecutive and the potential electrodes are consecutive. In the second group, the current and potential poles alternate. In the third group, one type of electrode is interior, the other exterior. Experimentally, the order of PN determines the order of HL and when P is interchanged with N, H is automatically interchanged with L, if the sign of the measured potential drop is to remain unchanged.

NONUNIFORM GEOPHYSICAL SPACE

INTERPRETATION

If the medium of finite resistivity in a geophysical space is not uniform, the simple formulas of the uniform case do not apply. However, the departure of the measured resistivity from that calculated for the uniform case furnishes a basis of exploration. If the measured resistivity is the same for every station point and every direction, the medium of finite resistivity may be considered as uniform.

NORMAL RESISTIVITY

If the formulas for uniform geophysical space are applied to a potential drop as measured in a nonuniform medium, the resistivity will depend on the type and size of the measuring configuration as well as on its location and direction. If a selected type of configuration is made smaller, the limiting value of the resistivity so determined, if independent of direction, is a value that may be called the "normal" resistivity at that point. The normal resistivity of a uniform medium is independent of the location, direction, or dimensions of the measuring configuration.

RIGID CONFIGURATIONS

If a selected configuration of current and potential points is considered as a unit that may be translated or rotated in space without altering the respective distances between its member points, it may be considered as a "rigid configuration". The "station" may be a point of the configuration determined by a selected method applied to the elements of the configuration and the "direction" of the configuration may be a line and sense determined by a second method. For example, the station may be the midpoint of the current poles, or the midpoint of the potential points, or a point determined by a more complicated method, such as the point the sum of whose distances from all points of the configuration is a minimum. Likewise, the direction of the configuration may be the direction from the current sink to the current source, the reverse direction, the direction from the current source to the reference potential point, the direction from the station to the current source, or some other direction, associated with the configuration, but changing as the rigid configuration is translated or rotated or both.

In the Wenner configuration, the station usually is selected as the midpoint between the two current poles and the direction as that from the current sink to the current source. In the Lee partitioning configuration, the station usually is taken as that of the corresponding Wenner configuration, and each direction as that from the station to the second potential point of that potential drop measurement. These choices are not critical, but some specific choices must be made for clarity and definiteness in referring the configuration to the earth, in field surveys.

The "size" of a configuration is the distance between two selected points of it. In a Wenner system, the size may be taken as the "separation", which is the distance between adjacent elements, or as the "spread", which is the distance between the outer points and is three times the separation. In the Lee partitioning system, the size may be selected as that of the corresponding Wenner configuration, disregarding the central electrode.

For the single-probe arrangement, the station may be taken as the local current pole, the direction, that from the station to the second current pole or to the potential reference point and the size as the distance between current poles. As this system is not necessarily collinear, a fourth quantity is involved; this quantity may be taken as the angle at the local current pole from the direction of the second current pole to the direction of the potential test point. Unless the potential reference point is very remote, the angle at the local pole from the direction of the second current pole to the direction of the potential reference point is a fifth constant.

Other configurations are possible. In each, there are constants of the configuration and parameters which determine the position and direction of the specific configuration. In a selected rigid configuration the size is fixed.

APPARENT RESISTIVITY

If the potential drop for a selected current in a selected configuration is substituted in the formula for a uniform geophysical space, the computed resistivity is an "apparent resistivity" for that configuration. By equation 2.22, if F is the form factor for the configuration used, V is the measured potential drop, and I is the current strength, the apparent resistivity has the value

$$\rho_a = 2\pi F V/I. \tag{3.11}$$

If ρ_0 is the normal resistivity of the medium near the station, a uniform medium having the resistivity ρ_0 would cause a "normal" potential V_n where

$$\rho_0 = 2\pi F V_n/I. \tag{3.12}$$

Any variation of ρ_a from ρ_0 indicates a nonuniformity of the medium.

DISTURBING FACTOR

Because of the diagnostic value of the apparent resistivity for a selected configuration, it is convenient to define a disturbing factor as

$$M = \frac{\rho_a}{\rho_0} = \frac{V}{V_n}. \tag{3.2}$$

Thus M is the ratio of the apparent resistivity to the normal resistivity of the lower medium near the station. It is also the ratio of the measured potential drop to the potential drop which would have been measured if the entire lower medium had been uniform, with its resistivity that of the overburden.

If the disturbing factor is unity for all configurations a uniform lower medium is indicated. If the disturbing factor exceeds unity for a selected configuration, there must be a region of resistivity higher than that near the station somewhere in the lower medium but close enough to the station to influence the value of the potential drop. This indicates the presence of a relative insulator

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within the lower medium. Similarly, a disturbing factor below unity indicates the presence of a relative conductor within the lower medium.

RESISTIVITY SURVEYING METHODS

By varying the parameters of a system, the size of the system, or by using various systems, the resistivity properties of the earth below, but near its surface, often can be detected and the variations in the apparent resistivity can be used for purposes of interpreting geological variations.

In well-logging, a set of electrodes with a selected configuration is lowered into a drill hole. In some applications, part of a configuration is located underground and part at the surface. The theoretical solutions of such problems can be obtained from the formulas of this paper, but in this paper numerical methods are applied only to surface configurations in a geophysical space, and the expression "surveying" is restricted to such applications.

HORIZONTAL SURVEYING

In horizontal surveying, the direction and size of a system are kept fixed and the station position is changed. A variation in the apparent resistivity indicates the presence of a discontinuity or change whose position horizontally can be determined approximately with respect to the station positions. Such a change may be a finite body, a change in thickness of the overburden, a horizontal change in material of the earth, or a change in water content. However, a horizontal surveying cannot detect changes associated only with varying depth.

The simplest method of horizontal surveying is horizontal profiling, in which the configuration of electrodes is collinear and all stations are located on the common line of the electrodes. The configurations most frequently used for horizontal profiling are the Wenner and the Lee partitioning, with fixed separations.

VERTICAL SURVEYING

In vertical surveying, the station of a system is kept fixed and the size or the size and direction are varied. As the size is increased, the depth of penetration of the current lines usually is increased so that vertical surveying usually can detect changes in the earth as the depth changes. Vertical surveying is affected by horizontal variations in the earth, so that depth effects can be masked by lateral effects.

The Wenner and Lee partitioning systems are the systems most commonly used for vertical surveying, and usually the direction of the systems is kept fixed. In vertical surveying, the direction of the system is that of the line along which the stations are placed and a set of apparent resistivities is determined for each station. However, these resistivities can be rearranged later to furnish a set of horizontal profiles for as many separations as desired. Horizontal profiling is not used ordinarily to furnish vertical profiles, as it involves too much moving of equipment.

AZIMUTHAL SURVEYING

In azimuthal surveying, the direction of a system is varied, for a selected station. The size may be kept constant or a set of sizes may be used for each azimuth. Although usually slower in field operation than vertical or horizontal surveying, azimuthal surveying is helpful in some types of exploration, such as locating or identifying faults, inclined beds, or relatively small bodies.

SINGLE-OVERBURDEN EARTH

NOTATION

In the single-overburden earth, the separating plane of a geophysical space coincides with the surface of the earth, the upper medium is air of resistivity $\rho=\infty$, and a plane at depth h (fig. 4) separates a uniform medium

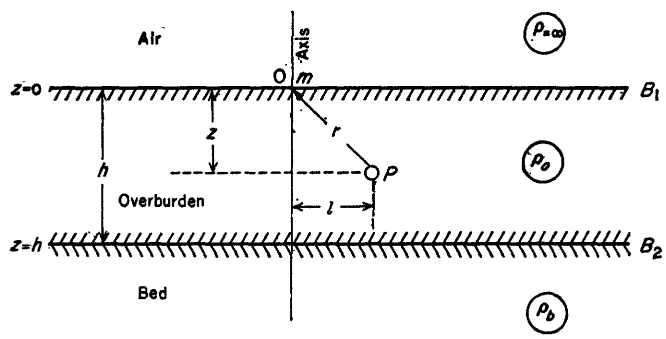

FIGURE 4.—Single-overburden earth. O, origin of coordinates; m, current pole; P, test point; B_1, upper boundary, between air and overburden; B_2, lower boundary between overburden and bed; h, thickness of overburden; z, depth of P below O; l, axial distance of P from O (horizontally); r, distance of P from O; $\rho=\infty$, resistivity of air; ρ_0, resistivity of overburden; and ρ_b, resistivity of bed.

of resistivity ρ_0 from a second uniform medium of resistivity $\rho_b \neq \rho_0$. A current pole of strength m is located at the surface of the earth. The origin is taken at the pole, the z-axis is directed downward, and the x-axis and y-axis are taken as mutually perpendicular in the surface of the earth. Except as bounded by the planes $z=0$ and $z=h$ each medium extends indefinitely in every direction.

NORMAL POTENTIAL

The normal potential at the testpoint P corresponding to a current source I at the origin is

$$U_n=\frac{\rho_0 I}{2\pi r}, \tag{4.11}$$

where r is the distance of the testpoint from the pole. For multiple poles, the normal potential drop between two points is

$$V_n=\frac{\rho_0}{2\pi} S\left\{\frac{I}{r}\right\}. \tag{4.12}$$

For the four-electrode system

$$V_n=\frac{\rho_0 I}{2\pi}S\left\{\frac{1}{r}\right\}. \qquad (4.13)$$

MODIFIED POTENTIAL

For the single overburden earth, the potential drop for a surface configuration is shown in the development of the mathematical problem to have the value:

$$V=V_n+\frac{\rho_0}{2\pi h}S\{IW(Q,a)\}, \qquad (4.21)$$

where Q is the reflection factor, whose value is

$$Q=\frac{\rho_b-\rho_0}{\rho_b+\rho_0}, \qquad (4.22)$$

$$a=\frac{l}{2h}, \qquad (4.23)$$

and $W(Q, a)$ is a "modified potential" which can be calculated, and values of which are shown in table 38. For the four-electrode surface configuration, the potential drop is

$$V=\frac{\rho_0 I}{2\pi}\left[S\left\{\frac{1}{r}\right\}+\frac{1}{h}S\{W\}\right]. \qquad (4.24)$$

TABLES OF MODIFIED POTENTIAL

Table 38, page 59, contains numerical values of the modified potential for values of the reflection factor from -1 to $+1$ by steps of 0.1, omitting the trivial case $Q=0$, and for values of the ratio of the distance between current pole and potential point to twice the thickness of the overburden by various steps from 0 to 30. The tabulated values may be rounded before calculation if full accuracy is not needed. In applications, the forcing errors may result in an error of several units in the last retained decimal place, so that one extra decimal should be used for evaluations and rounded out in final results.

INTERPOLATION

For the reflection factor of each, table 38 shows selected values of the argument, the corresponding values of the modified potential, and three differences which are convenient for interpolating within the table. For interpolation within each table, the spacings have been selected to permit interpolation to orders not exceeding the fourth. Interpolation between tables for intermediate values of the reflection factor follows the usual rules, but the differences must be calculated as needed and there is no assurance that the spacings between tables will justify interpolation of the fourth order or lower.

The formula for interpolation to the fourth order may be accepted as adequate. However, it frequently saves time in preparing calculations, to estimate the order needed for interpolation before starting the calculation. For convenience, use may be made of the working rule:

if $|\Delta^4_{-2}|>43$, use quartic interpolation;

if $|\Delta^4_{-2}|\leqq 43$ and $|\Delta^3_{-1}|>16$, use cubic interpolation;

if $|\Delta^4_{-2}|\leqq 43$, $|\Delta^3_{-1}|\leqq 16$, and $|\Delta^2_{-1}|>8$, use quadratic interpolation;

if $|\Delta^4_{-2}|\leqq 43$, $|\Delta^3_{-1}|\leqq 16$, and $|\Delta^2_{-1}|\leqq 8$, use linear interpolation; where the table has been rounded to the desired extent before the criteria are applied. For derivation of this rule see the discussion of Lagrangean interpolation (page 41).

In interpolation, the argument lies between two tabulated values. Let the desired argument be a and let a lie between a_0 and a_0+h; where a_0 is the largest tabulated argument below a and h is the tabular spacing from a_0 to the next larger tabulated argument. Then $x=(a-a_0)/h$ is the "phase" of the interpolated argument and lies between zero and one.

For linear interpolation, the interpolant is

$$f_1=f(x)=f(0)+x[f(1)-f(0)], \qquad (4.311)$$

which may be written

$$f_1=f(x)=xf(1)+(1-x)f(0). \qquad (4.312)$$

As the expressions 4.31 are so simple, there is no need to tabulate the first difference, which is $f(1)-f(0)$. If linear interpolation is adequate, the difference $f(1)-f(0)$ is small. For use with a computing machine, equation 4.312 is direct and does not use the first difference explicitly.

For quadratic interpolation, the linear interpolant is increased by $F_2\Delta^2_{-1}$ where $F_2=-\frac{1}{2}x(1-x)$ is tabulated in table 37 (p. 59) for two decimal places in x. As x lies between 0 and 1, the quadratic contribution has a sign opposite that of the second difference.

For cubic interpolation, the quadratic interpolant is increased by $F_3\Delta^3_{-1}$ where $F_3=-\frac{1}{6}(1+x)x(1-x)$ is tabulated in table 37. The cubic contribution has a sign opposite that of the third difference.

For quartic interpolation, the cubic interpolant is increased by $F_4\Delta^4_{-2}$ where $F_4=\frac{1}{24}(1+x)x(1-x)(2-x)$ is tabulated in table 37. The quartic contribution has the same sign as the fourth difference.

For direct calculation, quartic interpolation leads to

$$f_4=f(x)=xf(1)+(1-x)f(0)+F_2\Delta^2_{-1}+F_3\Delta^3_{-1}+F_4\Delta^4_{-2}. \qquad (4.32)$$

If the value of F_k is needed between those tabulated in table 37 (p. 59), it may be calculated directly or obtained by interpolation. Linear interpolation is usually adequate. The values of F_k may also be found in more extensive published values of Lagrangean interpolation coefficients (see Mathematical Tables Project, 1944).

ASYMPTOTIC EXPANSIONS

The arguments in table 38, page 59, extend to 30, corresponding to distances between the current pole and potential point of not exceeding 60 times the thickness of the overburden. For larger values of the argument, asymptotic expansions are available.

If the bed is a perfect conductor, the reflection factor is $Q=-1$ and for $a \geqq 30$, the modified potential can be shown to have the value, to at least eight decimals,

$$W(-1,a)=\log_e 2-\frac{1}{2a}, \tag{4.411}$$

where

$$\log_e 2=0.693\ 147\ 181. \tag{4.412}$$

If the bed is a perfect insulator, $Q=+1$ and for $a \geqq 30$, to at least eight decimals, the modified potential can be shown to have the value

$$W(+1,a)=K-\log_e(2a)-\frac{1}{2a}, \tag{4.421}$$

where

$$K=0.809\ 078\ 696. \tag{4.422}$$

If the bed is neither a perfect conductor nor a perfect insulator, the reflection factor has a numerical value less than unity. For these cases $Q^2<1$, and the modified potential for large values of a can be shown to have the value

$$W=\log_e(1-Q)-\sum_{n=1}^{\infty} B_n a^{1-n}, \tag{4.431}$$

where the values of B_n for the first five values of n are shown in table 1. In this expansion, B_n eventually increases as the value of n becomes larger so that for a selected value of a, the terms eventually increase in numerical value. Such a series, in which the terms initially decrease but pass through a minimum value and eventually increase, is an asymptotically divergent series, which can be of practical utility if the minimum term has a negligible value. In such series, the error usually does not exceed twice the first neglected term, so that the expansion should stop after the term immediately preceding the term having the minimum absolute value. For $Q \leqq 0.6$, asymptotic expansions check the tabulated values at $a=30$, and hence may be used for all larger values of a to the eighth decimal place, except for forcing errors. For $Q=+0.7$ the agreement is to six decimals, for $Q=+0.8$ to four decimals, and for $Q=+0.9$, to only two decimals. For values of a much larger than 30, the agreement is better, but for values slightly larger than 30 caution is advisable.

For two points close to each other, it may be more accurate to calculate the difference directly, in the form

$$W(a_1)-W(a_2)=\sum_{n=1}^{\infty} B_n(a_2^{1-n}-a_1^{1-n}) \tag{4.432}$$

as the minimum term in this difference may be negligible. In this form, the logarithmic term disappears.

TABLE 1.—*Coefficients in asymptotic expansions*

Q	B_1	B_2	B_3	B_4	B_5
−0.9	+0.473 684 210	−0.006 560 723	−0.009 800 194	−0.034 513 792	−0.218 709 985
−.8	+.444 444 444	−.013 717 421	−.020 195 092	−.069 727 872	−.430 814 191
−.7	+.411 764 706	−.021 371 871	−.030 560 296	−.101 350 283	−.594 086 090
−.6	+.375 000 000	−.029 296 875	−.039 825 439	−.122 916 698	−.653 030 165
−.5	+.333 333 333	−.037 037 037	−.046 296 296	−.126 028 807	−.553 876 600
−.4	+.285 714 286	−.043 731 778	−.047 524 841	−.102 465 050	−.283 957 451
−.3	+.230 769 231	−.047 792 444	−.040 510 459	−.050 577 938	+.060 482 081
−.2	+.166 666 667	−.046 296 296	−.023 148 148	+.011 252 572	+.233 178 298
−.1	+.090 909 091	−.033 809 166	+.000 209 561	+.031 692 485	+.059 492 146
+.1	−.111 111 111	+.075 445 816	−.140 413 047	+.655 170 329	−6.059 494 754
+.2	−.250 000 000	+.234 375 000	−.834 960 938	+8.042 907 715	−152.164 363 9
+.3	−.428 571 429	+.568 513 120	−3.559 008 151	+61.356 233 83	−2 074.095 551
+.4	−.666 666 667	+1.296 296 296	−13.935 186 19	+414.904 835 4	−24 214.693 53
+.5	−1.000 000 000	+3.000 000 000	−56.250 000 00	+2 926.875 000	−298 503.515 6
+.6	−1.500 000 000	+7.500 000 000	−258.750 000 0	+24 789.843 75	−4 655 051.953
+.7	−2.333 333 333	+22.037 037 04	−1 559.120 370	+306 389.802 0	−118 011 731.7
+.8	−4.000 000 000	+90.000 000 00	−16 267.500 00	+8 167 556.250	−8 037 473 112.
+.9	−9.000 000 000	+855.000 000 0	−693 191.250 0	+1 561 123 659.	$-6.890\ 926 \times 10^{12}$

APPARENT RESISTIVITY

For a single overburden earth, the apparent resistivity of a surface configuration has the value

$$\rho_a=\left[1+\frac{S\{IW\}}{hS\left\{\frac{I}{r}\right\}}\right]\rho_0. \tag{4.5}$$

DISTURBING FACTOR

For a single overburden earth, the disturbing factor of a surface configuration has the value

$$M=1+\frac{S\{IW\}}{hS\left\{\frac{I}{r}\right\}}, \tag{4.61}$$

which reduces for a four-electrode surface configuration to

$$M=1+\frac{S\{W\}}{hS\left\{\frac{I}{r}\right\}}. \tag{4.62}$$

WENNER OR LEE CONFIGURATION

For the Wenner Configuration of separation l, the distances are $r_{11}=r_{22}=l$ and $r_{12}=r_{21}=2l$. By equation 4.23, the corresponding arguments of the modified potential are $a_{11}=a_{22}=l/(2h)$ and $a_{12}=a_{21}=l/h$. Accordingly, the disturbing factor for a Wenner configuration has the value

$$M=1+\frac{2l}{h}\left\{W\left[Q,\frac{l}{2h}\right]-W\left[Q,\frac{l}{h}\right]\right\}. \tag{4.63}$$

For each half of the Lee partitioning configuration, the disturbing factor is given by equation (4.63). Although the applicable distances and arguments of W are not those of the Wenner configuration, the final form of the disturbing factor is the same.

For large values of l, the asymptotic expansions of W lead to approximations for these two configurations:

For $$Q=-1,\ M=0. \tag{4.641}$$

For $$Q=+1,\ M=\frac{l}{h}\log_e 4. \tag{4.642}$$

For $$Q^2<1,\ M=\frac{1+Q}{1-Q}-2\sum_{n=1}^{\infty}(2^{2n+1}-1)B_{n+1}\left(\frac{h}{l}\right)^{2n}. \tag{4.643}$$

The errors in these expressions must be determined by comparison with the calculated values if l is not large with respect to h. Thus, for $Q=-1$, $M<0.01$, by direct calculation, if $l>4.4h$. If the asymptotic expansions are sufficiently close to the direct values for a tested value of l/h, they will also be sufficiently close for every larger value of l/h.

SUPERPOSITION

If the apparent resistivities as measured match the values predicted by theory, the characteristics of the theoretical medium may be assumed as approximating those of the earth. If the observations are made by the Wenner or partitioning configurations, as in vertical surveying, the modified potential makes it possible to determine the disturbing factor, M, as a function of l/h and the observations determine the apparent resistivity, ρ_a, as a function of l.

As the disturbing factor is the ratio of the apparent resistivity to the resistivity of the overburden,

$$\log M=\log\rho_a-\log\rho_0. \tag{4.711}$$

Directly,

$$\log(l/h)=\log l-\log h. \tag{4.712}$$

Accordingly, if $\log M$ is plotted against $\log(l/h)$ on a reference chart, and $\log\rho_a$ is plotted against $\log l$ on an observation chart, either may be interpreted as a map of the other, if the proper value of Q has been chosen for the theoretical curve and if the earth approximates the assumption of a single overburden. The notation $u=\log(l/h)$, $v=\log M$, $x=\log l$, $y=\log\rho_a$ leads to the curves $v=f(u)$ and $y=g(x)$ and to the mapping functions $u=x-\log h$, $v=y-\log\rho_0$ where $\log h$ and $\log\rho_0$ are constants. Accordingly, the uv-plane and the xy-plane are identical except for translation of the uv origin to the point $x=\log h$, $y=\log\rho_0$. Corresponding curves on the two planes are superposed exactly after the translation.

On the reference sheet, cross-sectional lines should not appear, except for the resistivity index, for which $M=1$ or $\log M=0$ and the depth index, for which $l=h$ or $\log(l/h)=0$. The index point, which is the intersection of the two index lines, should be designated distinctly. If the reference curves are prepared on transparent sheeting, a small hole may be cut at the index point to permit marking through the sheet directly on the observation graph sheet. On the observation sheets, the cross-sectional lines should be retained, and the observations should be shown isolated without attempting to pass a curve through them.

In interpretation, the superposition and the observation curves are shifted, maintaining the disturbing factor axis parallel to the apparent (measured) resistivity axis. If a satisfactory agreement can be found between the observed curve and one of the reference curves, the earth may be considered as having a single overburden. The reflection factor, Q, is determined by the curve of fit. The resistivity of the overburden is determined as the observed resistivity corresponding to the resistivity index, and the depth is determined as the observed depth corresponding to the depth index.

If the superposition sheet is laid over the observation sheet, the intersection of the two index lines can be shown directly on the observation sheet, or read and recorded. Finally the resistivity of the bed is determined by

$$\rho_b=\left(\frac{1+Q}{1-Q}\right)\rho_0. \tag{4.72}$$

Values to be used in preparing a superposition sheet are shown as the disturbing factors in tables 2 and 3 with the arguments Q and l/h. The Wenner or partitioning configuration is assumed. Superposition curves, plotted from tables 2 and 3, are shown in figures 5 and 6, in which each curve is labeled with both the reflection factor and the ratio of bed resistivity to overburden resistivity for which it is plotted.

TABLE 2.—*Disturbing factor, Wenner or Lee configuration, buried conductor*

[Items marked with asterisk (*) not needed for figure 5]

l/h	Disturbing factor for indicated values of Q									
	−0.1	−0.2	−0.3	−0.4	−0.5	−0.6	−0.7	−0.8	−0.9	−1.0
0.2	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	0.9948
0.4	(*)	(*)	(*)	(*)	0.9804	(*)	(*)	(*)	(*)	.9629
0.6	(*)	(*)	0.9654	(*)	.9439	(*)	(*)	0.9134	(*)	.8942
0.8	0.9768	0.9543	.9326	0.9114	.8909	0.8710	0.8516	.8326	0.8141	.7960
1.0	.9633	.9279	.8938	.8609	.8290	.7981	.7682	.7391	.7108	.6833
1.2	.9490	.9002	.8534	.8084	.7650	.7233	.6829	.6439	.6061	.5696
1.4	.9350	.8732	.8142	.7579	.7039	.6522	.6024	.5546	.5085	.4640
1.6	.9220	.8482	.7783	.7118	.6485	.5881	.5304	.4751	.4222	.3714
1.8	.9101	.8257	.7462	.6711	.5999	.5324	.4682	.4071	.3488	.2932
2.0	.8996	.8059	.7182	.6358	.5582	.4850	.4157	.3502	.2880	.2289
2.2	.8904	.7887	.6940	.6056	.5229	.4452	.3723	.3036	.2387	.1773
2.4	.8823	.7738	.6734	.5801	.4934	.4124	.3367	.2657	.1991	.1364
2.6	.8753	.7610	.6558	.5587	.4689	.3854	.3078	.2354	.1677	.1044
3.0	.8639	.7406	.6283	.5256	.4316	.3452	.2655	.1918	.1236	.0604
3.2	.8593	.7324	.6175	.5130	.4176	.3304	.2503	.1766	.1085	.0457
3.6	.8519	.7195	.6005	.4934	.3965	.3083	.2282	.1548	.0876	.0259
4.0	.8461	.7097	.5882	.4794	.3817	.2935	.2136	.1412	.0751	.0146
4.4	.8416	.7022	.5790	.4693	.3712	.2833	.2041	.1323	.0674	.0082
5	.8366	.6941	.5691	.4587	.3607	.2733	.1950	.1245	.0610	.0034
6	.8311	.6856	.5590	.4483	.3508	.2644	.1874	.1186	.0567	.0008
7	.8278	.6804	.5532	.4425	.3455	.2598	.1838	.1159	.0550	.0002
8	.8255	.6771	.5496	.4390	.3423	.2572	.1818	.1146	.0543	.0000
9	.8240	.6749	.5471	.4366	.3402	.2555	.1805	.1137	.0539	.0000
10	.8229	.6733	.5454	.4350	.3388	.2544	.1797	.1132	.0536	.0000
12	.8215	.6712	.5432	.4330	.3371	.2530	.1787	.1125	.0533	.0000
14	.8206	.6700	.5419	.4318	.3361	.2522	.1781	.1121	.0531	.0000
16	.8200	.6692	.5411	.4310	.3354	.2516	.1777	.1119	.0530	.0000
18	.8196	.6687	.5406	.4305	.3350	.2513	.1774	.1117	.0529	.0000
20	.8194	.6683	.5402	.4301	.3346	.2510	.1772	.1116	.0529	.0000
22	.8192	.6680	.5399	.4298	.3344	.2509	.1771	.1115	.0528	.0000
24	.8190	.6678	.5396	.4296	.3342	.2507	.1770	.1114	.0528	.0000
26	.8189	.6676	.5395	.4295	.3341	.2506	.1769	.1114	.0528	.0000
30	.8187	.6674	.5392	.4293	.3339	.2505	.1768	.1113	.0527	.0000
50	.8184	.6669	.5387	.4288	.3335	.2502	.1766	.1112	.0527	.0000
100	.8182	.6667	.5385	.4286	.3334	.2500	.1765	.1111	.0526	.0000
∞	.8182	.6667	.5385	.4286	.3333	.2500	.1765	.1111	.0526	.0000

TABLE 3.—*Disturbing factor, Wenner or Lee configuration, buried insulator*

[Items marked with an asterisk (*) not needed for figure 6]

l	Disturbing factor for indicated values of Q									
	+0.1	+0.2	+0.3	+0.4	+0.5	+0.6	+0.7	+0.8	+0.9	+1.0
0.2	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	1. 0070
0.4	(*)	(*)	(*)	(*)	1. 0226	(*)	(*)	(*)	(*)	1. 0511
0.6	(*)	(*)	1. 0380	(*)	1. 0658	(*)	(*)	1. 1131	(*)	1. 1512
0.8	1. 0241	1. 0490	1. 0750	1. 1022	1. 1307	1. 1607	1. 1926	1. 2268	1. 2640	1. 3062
1.0	1. 0383	1. 0782	1. 1200	1. 1639	1. 2104	1. 2596	1. 3123	1. 3694	1. 4322	1. 5044
1.2	1. 0534	1. 1095	1. 1686	1. 2312	1. 2977	1. 3689	1. 4458	1. 5298	1. 6235	1. 7329
1.4	1. 0685	1. 1408	1. 2176	1. 2995	1. 3872	1. 4819	1. 5851	1. 6991	1. 8278	1. 9810
1.6	1. 0828	1. 1708	1. 2649	1. 3660	1. 4753	1. 5942	1. 7251	1. 8713	2. 0385	2. 2410
1.8	1. 0959	1. 1987	1. 3094	1. 4292	1. 5597	1. 7031	1. 8624	2. 0424	2. 2510	2. 5083
2.0	1. 1078	1. 2243	1. 3505	1. 4883	1. 6395	1. 8072	1. 9954	2. 2103	2. 4627	2. 7799
2.2	1. 1186	1. 2475	1. 3882	1. 5429	1. 7143	1. 9059	2. 1230	2. 3737	2. 6721	3. 0540
2.4	1. 1281	1. 2683	1. 4225	1. 5932	1. 7838	1. 9989	2. 2450	2. 5321	2. 8782	3. 3294
2.6	1. 1367	1. 2871	1. 4536	1. 6395	1. 8486	2. 0865	2. 3613	2. 6852	3. 0808	3. 6056
3.0	1. 1508	1. 3190	1. 5077	1. 7210	1. 9646	2. 2462	2. 5772	2. 9757	3. 4743	4. 1593
3.2	1. 1567	1. 3325	1. 5309	1. 7567	2. 0164	2. 3189	2. 6776	3. 1133	3. 6653	4. 4364
3.6	1. 1668	1. 3558	1. 5715	1. 8200	2. 1096	2. 4518	2. 8640	3. 3746	4. 0360	4. 9907
4.0	1. 1748	1. 3747	1. 6053	1. 8737	2. 1905	2. 5698	3. 0338	3. 6184	4. 3922	5. 5452
4.4	1. 1813	1. 3903	1. 6336	1. 9199	2. 2613	2. 6751	3. 1886	3. 8464	4. 7348	6. 0997
5	1. 1888	1. 4091	1. 6683	1. 9775	2. 3517	2. 8130	3. 3965	4. 1614	5. 2250	6. 9315
6	1. 1976	1. 4313	1. 7110	2. 0506	2. 4702	3. 0003	3. 6898	4. 6252	5. 9839	8. 3177
7	1. 2034	1. 4465	1. 7411	2. 1039	2. 5599	3. 1475	3. 9306	5. 0251	6. 6787	9. 7041
8	1. 2074	1. 4573	1. 7630	2. 1439	2. 6292	3. 2653	4. 1308	5. 3728	7. 3171	11. 090
9	1. 2103	1. 4652	1. 7794	2. 1745	2. 6838	3. 3609	4. 2990	5. 6771	7. 9055	12. 477
10	1. 2124	1. 4711	1. 7919	2. 1985	2. 7275	3. 4396	4. 4418	5. 9452	8. 4495	13. 863
12	1. 2153	1. 4792	1. 8095	2. 2329	2. 7922	3. 5599	4. 6691	6. 3941	9. 4421	16. 636
14	1. 2170	1. 4844	1. 8210	2. 2559	2. 8369	3. 6463	4. 8402	6. 7527	10. 265	19. 408
16	1. 2182	1. 4879	1. 8288	2. 2720	2. 8688	3. 7101	4. 9720	7. 0439	11. 002	22. 181
18	1. 2190	1. 4903	1. 8343	2. 2836	2. 8924	3. 7586	5. 0754	7. 2834	11. 650	24. 953
20	1. 2196	1. 4921	1. 8384	2. 2923	2. 9103	3. 7961	5. 1580	7. 4827	12. 223	27. 726
22	1. 2201	1. 4934	1. 8415	2. 2989	2. 9242	3. 8257	5. 2250	7. 6502	12. 732	30. 498
24	1. 2204	1. 4944	1. 8439	2. 3040	2. 9351	3. 8495	5. 2800	7. 7923	13. 187	33. 271
26	1. 2207	1. 4952	1. 8458	2. 3081	2. 9439	3. 8688	5. 3254	7. 9138	13. 596	36. 044
30	1. 2211	1. 4964	1. 8486	2. 3141	2. 9569	3. 8980	5. 3959	8. 1091	14. 296	41. 589
50	1. 2218	1. 4987	1. 8540	2. 3262	2. 9837	3. 9602	5. 5554	8. 5961	16. 387	69. 315
100	1. 2221	1. 4997	1. 8563	2. 3315	2. 9958	3. 9897	5. 6367	8. 8825	18. 060	138. 63
∞	1. 2222	1. 5000	1. 8571	2. 3333	3. 0000	4. 0000	5. 6667	9. 0000	19. 000	∞

If the logarithm of the disturbing factor is plotted against the natural value of the reflection factor for a selected set of values of the relative separation, l/h, a set of smooth curves results and graphical interpolation permits the plotting of superposition curves for reflection factors intermediate between those shown in figures 5 and 6. The sets of such curves shown in figures 7 and 8 may be used for this purpose, or new curves may be drawn on cross-section paper to permit direct reading with better accuracy.

ILLUSTRATIVE EXAMPLES

SUPERPOSITION

To illustrate the method of superposition, consider the values shown in figure 9, in which the observations are indicated by circles and the curve for $Q=+1.0$

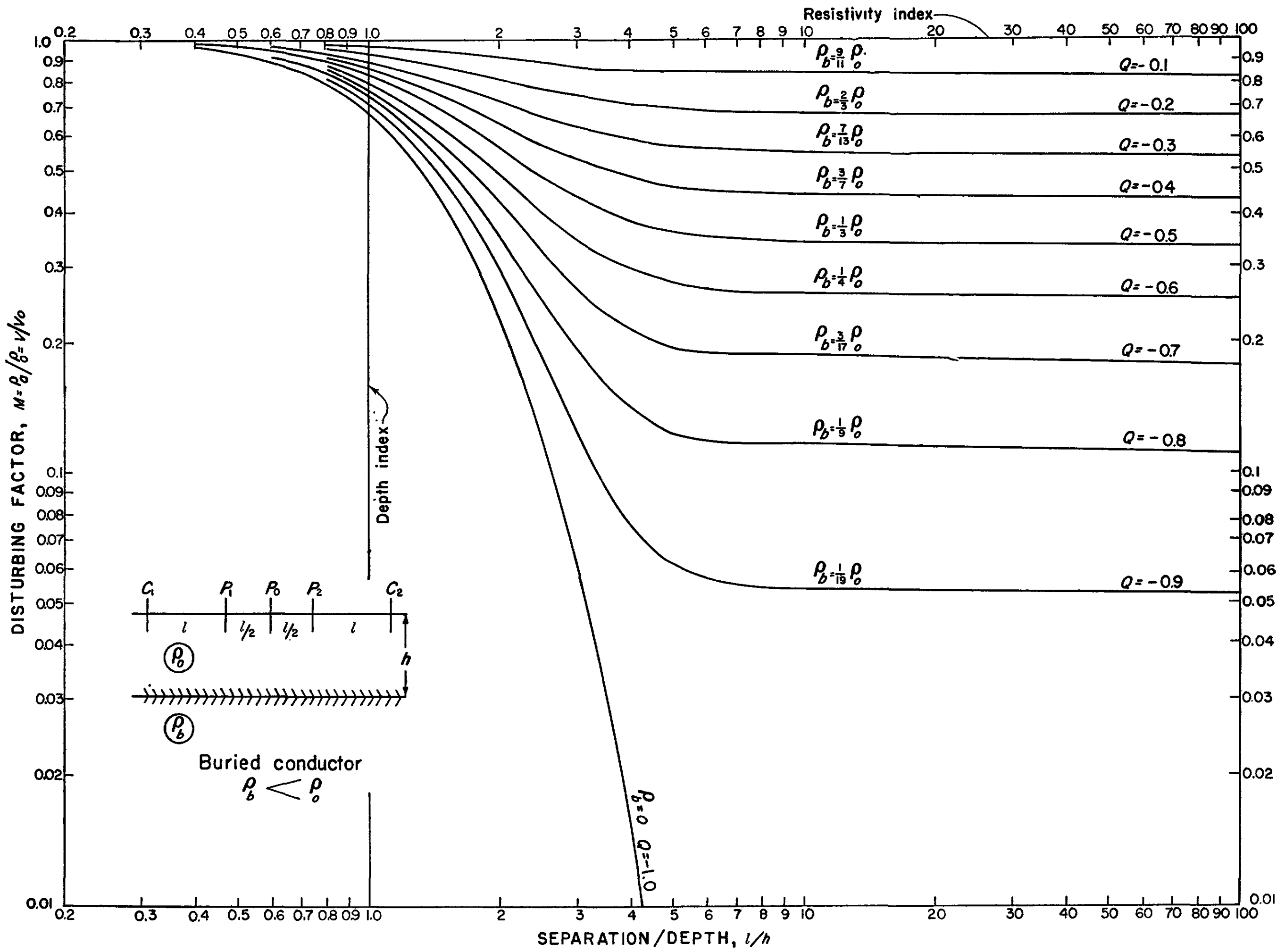

FIGURE 5.—Superposition curves, buried conductor, Wenner or Lee configuration.

has been copied from a family of superposition curves. The circles show the values of ρ_a plotted against l on double logarithmic paper. As the apparent resistivity increases with increasing spread, a buried insulator is indicated. The family of curves for a buried insulator (fig. 6) was laid over the graph and translated without rotation to obtain the curve of best fit. In this example, the curve $Q=+1.0$ fits best. The index was indicated through the reference sheet, showing a depth of 6.0 feet and an overburden resistivity of 2,440 ohm-cm. As $Q=+1$, the bed resistivity is infinite.

FOUR-ELECTRODE SURFACE CONFIGURATION

As an example of the four-electrode surface configuration, let the current source lie at the origin, the current sink at (2000,0,0), the potential reference point at (1500, 500, 0), and the potential test point at (400, 300, 0). Then $l_{11}=PH=500$, $l_{12}=PL=500\sqrt{10}$, $l_{21}=NH=100\sqrt{265}$, and $l_{22}=NL=500\sqrt{2}$. The normal potential for a pole of strength 1000 units is 2.167 462 913. For the approximation in which N and L are assumed to lie at infinite distances, the potential is 2 units so that the approximation error amounts to about 8.4 percent.

If a perfect conductor is taken at a depth of 100, $Q=-1$ and the calculations are as follows:

$a_{11}=2.5,$		$W_{11}=0.4934\ 8923$
$a_{12}=2.5\sqrt{10}$	$=7.905\ 694\ 150,$	$W_{12}=0.6299\ 0162$
$a_{21}=(1/2)\sqrt{265}$	$=8.139\ 410\ 300,$	$W_{21}=0.6317\ 1767$
$a_{22}=2.5\sqrt{2}$	$=3.535\ 533\ 905,$	$W_{22}=0.5517\ 3698.$

Thus $S\{W\}=-0.2163\ 9308$ and $V=+0.003\ 5321$. Assuming, N and L at infinity, $l_{12}=l_{21}=l_{22}=\infty$ and by equations (4.41), $W_{12}=W_{21}=W_{22}=\log_e 2=0.6931\ 4718$ so that $V=0.003\ 4205$, the difference being about 3.2 percent. It may be noted that the use of eight significant figures in the values of the modified potential leads to only five significant figures in the calculated value of the potential.

If the bed has a resistivity 19 times that of the overburden, $Q=+0.9$, the calculated potential is

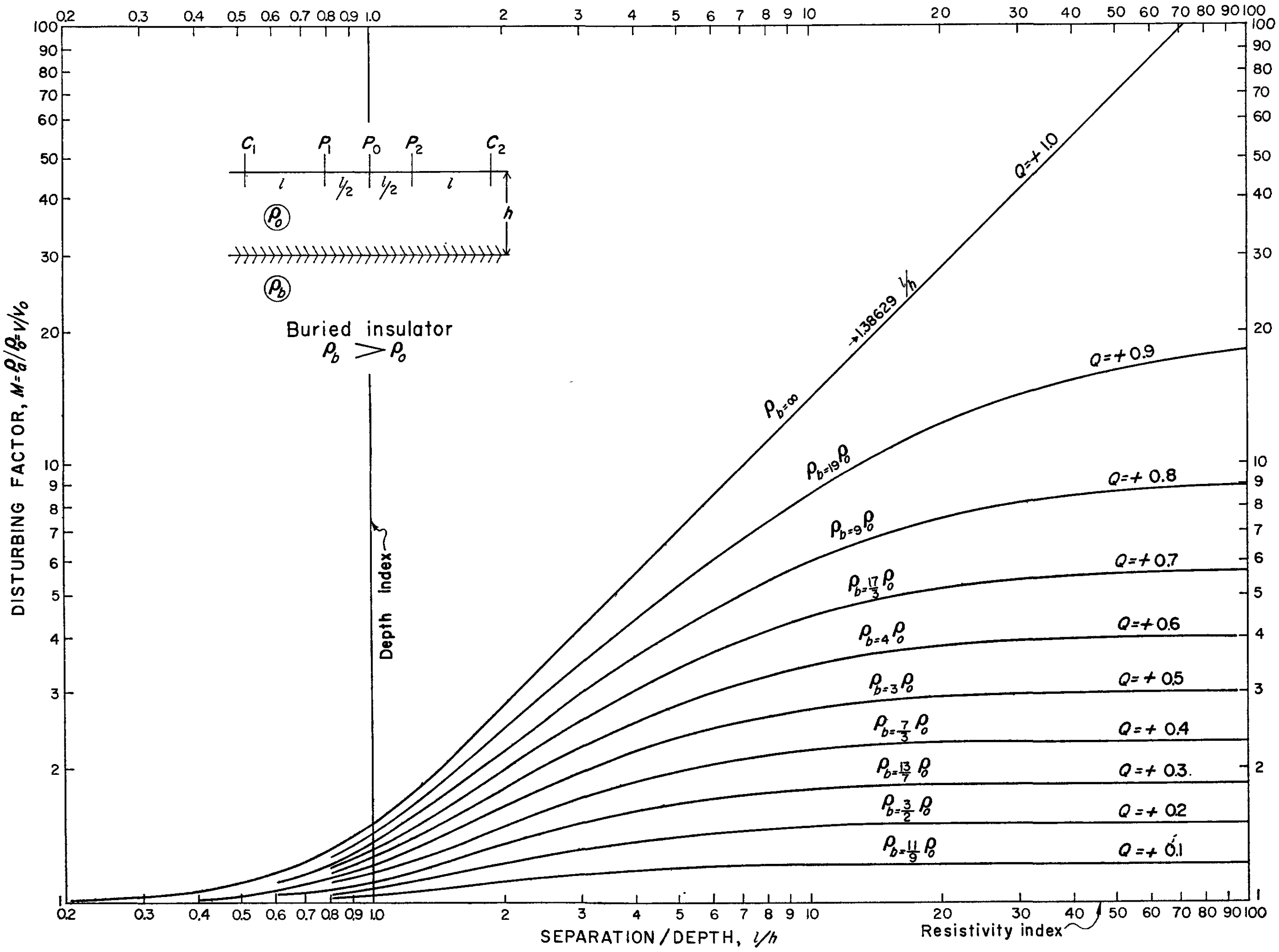

FIGURE 6.—Superposition curves, buried insulator, Wenner or Lee configuration.

13.736 310 and the limiting value as N and L recede to infinity is 16.893 981. Accordingly, neglecting of the contributions of N and L introduces an error of about 25 percent.

INTERPOLATED VALUES OF THE MODIFIED POTENTIAL

The use of interpolation in table 38 (p. 59) may be illustrated by specific examples. Let $Q=-0.8$ and let the argument lie between 13.5 and 14.0, which appear in the table. Then $a_0=13.5$, $a_1=14.0$ and the tabular spacing is $h=0.5$ so that the phase is $x=(a-13.5)/0.5 =2(a-13.5)$.

The table shows that $W(a_0)=0.5548\ 7048$ and that $W(a_1)=0.5560\ 4567$. The second, third, and fourth order differences, with the unit position in the eighth decimal place of W, are $\Delta^2_{-1}=-9035$, $\Delta^3_{-1}=+934$, $\Delta^4_{-2}=-149$. Let L be the result of interpolating linearly between the tabulated values. Then

$$L=xW(a_1)+(1-x)W(a_0)=0.5560\ 4567x+0.5548\ 7048(1-x)$$

The correction to L is, in the proper decimal position,

$$C=F_2\Delta^2_{-1}+F_3\Delta^3_{-1}+F_4\Delta^4_{-2},$$

or a reduced form.

Example 1

Let $a=13.71$ and W be to 4 decimals. As the values of the differences to 4 decimals are each less than 1, linear interpolation is adequate. The phase is $0.21/0.5=0.42$ so that

$$W(13.71)=0.42(0.5560)+0.58(0.5549)=0.5554.$$

Example 2

Let $a=13.71$ and W be to 6 decimals. Here the differences are listed as

$$\Delta^2_{-1}=-90,\ \Delta^3_{-1}=+9 \text{ and } \Delta^4_{-2}=-1,$$

and the phase is $x=0.42$. As the fourth difference is less than 43, the third less than 16 and the second

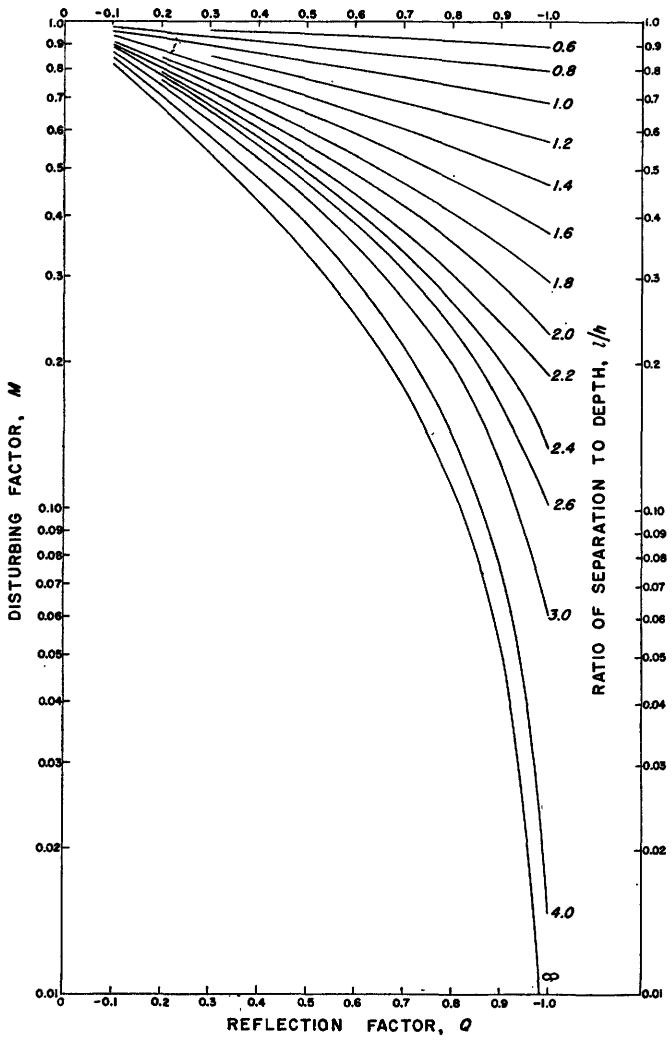

FIGURE 7.—Interpolation chart, buried conductor.

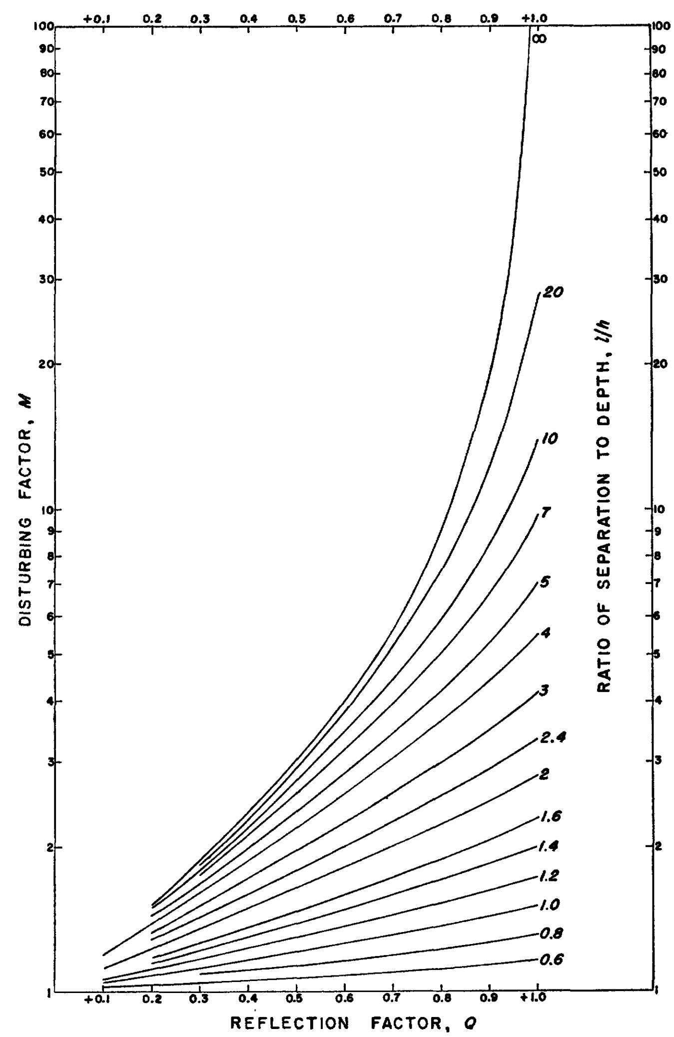

FIGURE 8.—Interpolation chart, buried insulator.

greater than 8, quadratic interpolation should be used so that $C=F_2\Delta^2_{-1}$. By table 37, page 59, the value of $F_2=-0.12180$. The linear interpolant is $L=0.555\ 364$, and the quadratic correction is $+11$ so that $W(13.71)=0.555\ 375$.

Example 3

Let $a=13.71$, and W be to 8 decimals. As $\Delta^4_{-2}>43$ quartic interpolation is required. The phase is $x=0.42$. Table 37, page 59, shows that $F_2=-0.12180$, $F_3=-0.05765$, and $F_4=+0.0228$. Accordingly,

$$L=(0.42)(0.5560\ 4567)+(0.58)(0.5548\ 7048)=0.5553\ 6406$$

and

$$C=(-0.12180)(-9035)+(-0.05765)(+934)+(0.0228)(-149)=+1043$$

in the eighth decimal place, so that

$$W(13.71)=0.5553\ 7449$$

with the last digit subject to forcing errors of rounding.

Example 4

Let $a=13.713\ 962$ and W be to 8 decimals.
The phase is $x=0.427\ 924$.
By direct calculation

$$F_2=-(1/2)x(1-x)=-0.1224\ 0253$$
$$F_3=+(1/3)(1+x)F_2=-0.0582\ 6050$$
$$F_4=-(1/4)(2-x)F_3=+0.0228\ 9748$$
$$L=0.5553\ 7337$$
$$C=+1048$$
$$W=0.5553\ 8385$$

As the differences are to 4, 3, and 3 significant digits, the tabulated values of F_k may be interpolated linearly

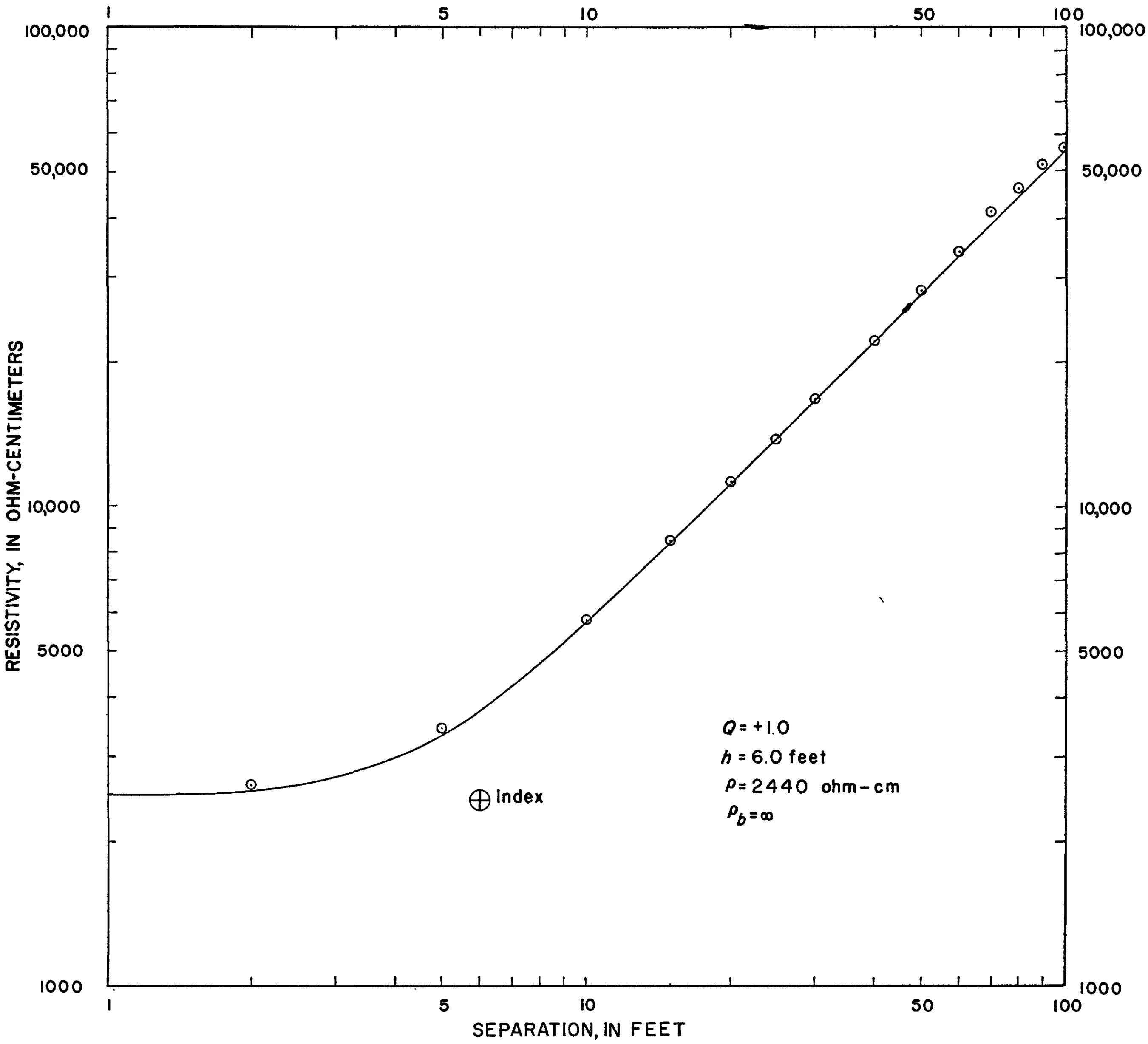

FIGURE 9.—An illustrative example of superposition.

to the same accuracy, using the phase $x=0.7924$ and leading to

$F_2=-0.1224$, $F_3=-0.0583$, $F_4=+0.0229$, $C=+1048$ and $W=0.5553\ 8385$. The difference is within the forcing error of rounding. If the values for F_k are taken directly from the table for $a=0.43$, they are

$F_2=-0.1226$, $F_3=-0.0584$, and $F_4=+0.0229$ so that $C=1050$ and $W=0.5553\ 8387$, also within the forcing error of the refined value.

As this example illustrates, the values of table 37, page 58, may be used without interpolation for most purposes and with linear interpolation for better accuracy. Only in rare calculations is it necessary to calculate the coefficients by formula. The magnitudes of the differences and their rates of change determine the validity of the simpler approximations.

MATHEMATICAL PROBLEM

NOTATION

Mathematically, the present problem is that of finding a function that determines the value of the difference in electrical potential for each pair of field-points when the locations of the current sources and sinks are known. As the potential function for a selected field is the algebraic sum of the potential functions due to the separate current poles, the fundamental problem is that of determining the potential function corresponding to a single current pole. The extension to more complicated fields is direct.

As actual measurements of electrical potential always involve the measurement of the difference in potential between two points, the potential function lacks uniqueness to the extent of an additive constant, which may be selected as convenient for each measurement of potential difference and may be altered between pairs of points when such an alteration is convenient. As a constant multiplier does not affect the fundamental properties of a potential function, the solution may be made for a unit current pole and the function may be multiplied by a factor appropriate to a specific current strength.

The basic problem is that of an infinite space separated into three sections by two parallel planes. Although rotation of the entire space may be needed in a geological application, such as exploration over a sloping plane, the separating planes may be considered as horizontal. The section between the two planes may be considered as "the overburden", one external section as "the air", and the second external section as "the bed". These terms have no mathematical significance but are convenient in discussing the regions, and the terminology transfers conveniently into geological significance. The air, in this use, may have an arbitrary value of resistivity.

The Z-axis may be taken perpendicularly to the boundary planes with its positive sense from the air toward the bed. The x- and y-axes may be taken arbitrarily, but mutually perpendicularly, in the plane separating the air from the overburden. For the fundamental problem, there is a single current source so that the field has cylindrical symmetry about the Z-axis. Let the air, the overburden, and the bed be designated as G_1, G_2, and G_3, respectively (fig. 10), and let the

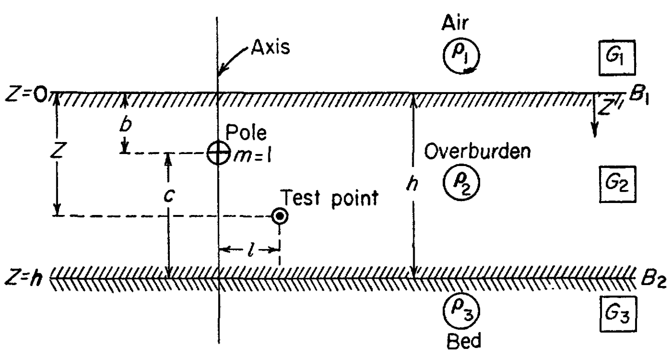

FIGURE 10.—Two-boundary space, internal pole. G_1 is the air; G_2, is the overburden; G_3, is the bed; m is the current pole of unit strength; ρ_k, is the resistivity of G_k; B_1, is the upper boundary; B_2, is the lower boundary; b, is the distance of the pole below B_1; c, is the distance of the pole above B_2; $h=b+c$, is the thickness of the overburden; z, is the depth of the test-point below B_1; l, is the axial distance of the test point (horizontally from the axis) ....

corresponding resistivities be ρ_1, ρ_2, and ρ_3, respectively. Let the plane B_1 separate the air from the overburden and the plane B_2 separate the overburden from the bed. In the geophysical case, the resistivity of the air is infinite, but this restriction may be removed for the mathematical problem. If the distance between the separating planes is h, the plane B_1 is specified by $z=0$ and the plane B_2 by $z=h$. There are two distinct cases, according to the location of the current source. For an "internal pole", the current source lies between the two planes; for an "external pole", the source lies in one of the external regions. Let the source lie at $P\equiv(0,0,b)$ and have a strength $m=1$. Let $c=h-b$. Let the test point lie at $H\equiv(x,y,z)$. Because of the cylindrical symmetry, the values of x and y enter only through $l=\sqrt{x^2+y^2}$, the distance of H from the "axis" of the space.

As the term "electric" or "electrical" is implied throughout this discussion, it will be omitted except where need for clarity or emphasis.

Unless otherwise specified, each counter will be assumed to have the range of positive integers, excluding zero and including infinity. A summation sign without limits will mean that each counter covers the range of positive integers. For example, $\sum a_j$ means $\sum_{j=1}^{\infty} a_j$, $b_k=\sum a_{jk}$ means $b_k=\sum_{j=1}^{\infty} a_{jk}$, and $b_k=\sum a_{ijk}$ means $b_k=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ijk}$. Specifically $\sum a_{j-1}$ is equivalent to $\sum_{j=0}^{\infty} a_j$.

POTENTIAL FUNCTION

The potential function due to a single pole has several properties, not mutually exclusive, each generally accepted:

1. The potential function is continuous at every point of space, except at the pole.
2. At the pole, the potential function becomes infinite in value in the order of $1/r$, where r is the distance from the pole to the field point at which the potential is measured.
3. At points infinitely remote from the pole, the value of the potential approaches a fixed value U_∞, independently of the path of recession, in such a manner that the difference $U-U_\infty$ approaches zero in the order of $1/r$, where U is the potential value at the test point.
4. At a boundary separating two media, the product of the conductivity by the derivative, in the direction perpendicular to the boundary, of the potential function is continuous. At a pole, located on the boundary, this condition is meaningless but is not needed, as it can be treated as the limiting condition as the pole approaches the boundary.

5. The derivative, in a direction parallel to a boundary, of the potential function is continuous.
6. Except at the pole and at a boundary, the derivative, in an arbitrary direction, of the potential is continuous.
7. Except at the pole, the potential is harmonic, so that the potential function satisfies Laplace's equation

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}=0. \tag{5}$$

8. A special potential function is $1/r$. Multiplication by a constant and addition of a constant are permissible.

For multiple sources, the potential function is the sum of the functions that represent the potentials of the separate sources, each multiplied by an arbitrary constant. If the number of sources is infinite, the sum becomes an infinite series, which satisfies the conditions if it converges or if it consists of two parts, of which one converges and the other diverges in a manner independent of the distance from each pole.

UNIT POLE

In order to use the unit pole for problems involving electric current, a relationship must be established between a pole of strength m and a current of strength I. If the current I enters a medium of uniform resistivity ρ, the differential form of Ohm's law is

$$\frac{\partial U}{\partial r}=-\rho I_r \tag{6.1}$$

where U is the potential, r is the distance from the source to the test point and I_r is the current density, at the testpoint, measured perpendicularly to the equipotential surface. As the total current I crosses each closed surface enclosing the source and as the equipotential surface for a single source in a uniform medium is a sphere with center at the source, the current density has the value

$$I_r=\frac{I}{4\pi r^2} \tag{6.2}$$

and equation 6.1 becomes

$$\frac{\partial U}{\partial r}=-\frac{I\rho}{4\pi r^2}, \tag{6.3}$$

so that the potential is

$$U=-\int\frac{I\rho}{4\pi r^2}\,dr=-\frac{I\rho}{4\pi}\int\frac{dr}{r^2}, \tag{6.4}$$

where the integration is taken along the path of the testpoint. The drop in potential from a point at the distance r_1 from the source to a point at the distance r_2 is independent of the path followed by the testpoint and has the value

$$V=U_1-U_2=-\frac{I\rho}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2}=\frac{I\rho}{4\pi}\left(\frac{1}{r_1}-\frac{1}{r_2}\right). \tag{6.51}$$

The drop in potential from the distance r_1 to the distance r_2 due to a pole of strength m is

$$V=m\left(\frac{1}{r_1}-\frac{1}{r_2}\right), \tag{6.52}$$

so that

$$m=\frac{I\rho}{4\pi}, \tag{6.61}$$

and a unit pole corresponds to a current of strength

$$I=\frac{4\pi}{\rho}. \tag{6.62}$$

Accordingly, the solution for a unit pole must be multiplied by $I\rho/(4\pi)$ to obtain the solution for a current of strength I.

BOUNDARY CONDITIONS

At the boundary B_1, the conditions on the potential function are:

$$z=0 \tag{7.11}$$

$$U_1-U_2=0 \tag{7.12}$$

$$\frac{1}{\rho_1}\frac{\partial U_1}{\partial z}-\frac{1}{\rho_2}\frac{\partial U_2}{\partial z}=0, \tag{7.13}$$

and at the boundary B_2, they are:

$$z=h \tag{7.21}$$

$$U_2-U_3=0 \tag{7.22}$$

$$\frac{1}{\rho_2}\frac{\partial U_2}{\partial z}-\frac{1}{\rho_3}\frac{\partial U_3}{\partial z}=0. \tag{7.23}$$

As usual, in evaluating a function for specified conditions, the conditions are imposed after all indicated operations have been performed. Specifically, in equations 7.13 and 7.23, the differentiation is performed before the values of z are substituted.

SIMPLE-IMAGE METHOD

The general aspects of the Kelvin image method have been known for about a century (Thomson, 1884, 1897, especially article 109, p. 174–176), and the method has been applied to the solution of a variety of problems. (See, for example: Jeans, 1925, p. 185–299, 341–363; Maxwell 1892, especially article 317, p. 443). The method is based on the principle that if a surface separates two media, the potential at every point of space is the same as if the medium of one side occupied all space and the surface were replaced by properly chosen fictitious poles, or images, determined by the original pole and the surface.

Applied to a space consisting of two media separated by a single plane, the images are determinable without difficulty. This fundamental case may be used to determine the image sets in spaces involving plane boundaries, provided no contradictions of the fundamental conditions are involved (see Keller, 1953). The method has been applied to nonplanar boundaries, but such boundaries are not considered in the present study.

In figure 11, let the boundary plane between the media G_1 and G_2 be defined by $z=0$, the resistivities of

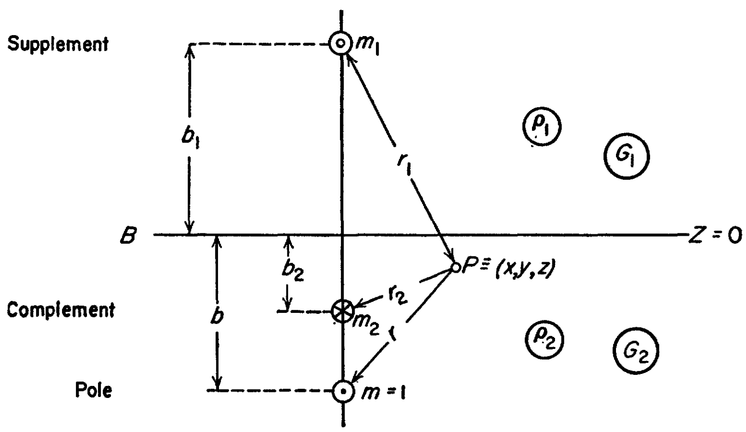

FIGURE 11.—Simple images. B is the boundary plane between media G_1 and G_2. P, is the test point; ρ_k, is the resistivity of G_k; m, is the unit pole; $b>0$, is the distance of m below B; $b_2>0$, is the distance of complement below B; m_1, is the supplementary image strength; m_2, is the complementary image strength; $b_1>0$, is the distance of supplement above B; r, is the distance from pole to test point, P; r_k, is the distance from m_k to test point, P.

G_1 and G_2 be ρ_1 and ρ_2, respectively, and a unit pole be located at $(0, 0,b)$ in G_2 where $b\geqq 0$.

The method of images replaces the medium G_1 by one of resistivity ρ_2 and two images: a supplementary image in the space occupied by G_1 and a complement in the original medium G_2. Although the potential is mathematically continuous and a single function, the analytical expression for the potential function takes a different form in the spaces occupied by G_1 and by G_2. For the space originally occupied by G_1, the actual pole, located at $(0,0,b)$ is replaced by a complementary pole of strength m_2 located in the region of G_2 at (x_2, y_2, b_2) where $b_2\geqq 0$; for the space originally occupied by G_2, the actual pole is augmented by a supplementary pole of strength m_1 located in the region of G_1 at $(x_1, y_1, -b_1)$ where $b_1\geqq 0$. The distances of the test point P from the pole, supplement, and complement are r, r_1, and r_2, respectively, where

$$r^2=x^2+y^2+(z-b)^2 \tag{8.11}$$

$$r_1^2=(x-x_1)^2+(y-y_1)^2+(z+b_1)^2 \tag{8.12}$$

$$r_2^2=(x-x_2)^2+(y-y_2)^2+(z-b_2)^2. \tag{8.13}$$

The potential in the space occupied by G_1 has the value

$$U_1=\frac{m_2}{r_2} \tag{8.21}$$

and that in the space originally occupied by G_2 has the value

$$U_2=\frac{1}{r}+\frac{m_1}{r_1}. \tag{8.22}$$

But

$$\frac{\partial r}{\partial z}=\frac{z-b}{r} \tag{8.311}$$

$$\frac{\partial r_1}{\partial z}=\frac{z+b_1}{r_1} \tag{8.312}$$

$$\frac{\partial r_2}{\partial z}=\frac{z-b_2}{r_2} \tag{8.313}$$

so that

$$\frac{\partial U_1}{\partial z}=\frac{m_2\,(b_2-z)}{r_2^3} \tag{8.321}$$

$$\frac{\partial U_2}{\partial z}=\frac{b-z}{r^3}-\frac{m_1\,(b_1+z)}{r_1^3}. \tag{8.322}$$

At the boundary, the testpoint has $z=0$, so that

$$U_1=\frac{m_2}{s_2} \tag{8.411}$$

$$U_2=\frac{1}{s}+\frac{m_1}{s_1} \tag{8.412}$$

$$\frac{\partial U_1}{\partial z}=\frac{m_2 b_2}{s_2^3} \tag{8.421}$$

$$\frac{\partial U_2}{\partial z}=\frac{b}{s^3}-\frac{m_1 b_1}{s_1^3} \tag{8.422}$$

where

$$s^2=x^2+y^2+b^2 \tag{8.431}$$

$$s_1^2=(x-x_1)^2+(y-y_1)^2+b_1^2 \tag{8.432}$$

$$s_2^2=(x-x_2)^2+(y-y_2)^2+b_2^2. \tag{8.433}$$

By equations 8.4, the boundary conditions of equations 7.1 become

$$\frac{m_2}{s_2}-\frac{1}{s}-\frac{m_1}{s_1}=0 \tag{8.511}$$

$$\frac{m_2 b_2}{\rho_1 s_2^3}-\frac{b}{\rho_2 s^3}+\frac{m_1 b_1}{\rho_2 s_1^3}=0. \tag{8.512}$$

Solution of equations (8.51) for m_1 and m_2 leads to

$$m_1=\frac{b\rho_1 s_1^3 s_2^2-b_2\rho_2 s_1^3 s^2}{\delta} \tag{8.521}$$

$$m_2=\frac{b\rho_1 s_1^2 s_2^3+b_1\rho_1 s_2^3 s^2}{\delta} \tag{8.522}$$

where

$$\delta=b_1\rho_1 s_2^2 s^3+b_2\rho_2 s_1^2 s^3. \tag{8.523}$$

To have significance, the image strengths and positions must be independent of the position of the test point. Accordingly, m_1 must be independent of x and y where the test point is $P=(x,y,0)$. Specifically,

$$\delta^2 \frac{\partial m_1}{\partial x}=0=\{b_1\rho_1 s_2{}^2 s^3+b_2\rho_2 s_1{}^2 s^3\}\begin{Bmatrix} b\rho_1[3s_1s_2{}^2(x-x_1)+2s_1{}^3(x-x_2)] \\ -b_2\rho_2[3s_1s^2(x-x_1)+2s_1{}^3x \end{Bmatrix}$$

$$-\{b\rho_1 s_1{}^3 s_2{}^2-b_2\rho_2 s_1{}^3 s^2\}\begin{Bmatrix} b_1\rho_1[2s^3(x-x_2)+3s_2{}^2sx] \\ +b_2\rho_2[2s^3(x-x_1)+3s_1{}^2sx] \end{Bmatrix}. \quad (8.531)$$

As the right member of equation 8.531 is a polynomial in ρ_1 and ρ_2, the coefficients must vanish. The vanishing of the coefficient of $\rho_1{}^2$ reduces to

$$0=3b_1bs_1s_2{}^4s[(s^2-s_1{}^2)x-s^2x_1]. \quad (8.532)$$

Hence, if $xyb\neq 0$, either $b_1=0$ or $x_1=0$ and $s_1=s$. The coefficient of $\rho_2{}^2$ reduces to

$$0=b_2{}^2s_1{}^3s^3[(s_1{}^2-s^2)x+s^2x_1]. \quad (8.533)$$

Hence, if $xyb\neq 0$, either $b_2=0$ or $x_1=0$ and $s_1=s$. If $s_1\neq s$, $b_1=b_2=0$, which is impossible for $b\neq 0$ by equation 8.512. Accordingly $x_1=0$ and $s_1=s$. Similarly, $y_1=0$, so that $b_1=b$.

Setting $x_1=y_1=0$, and $b_1=b$, $s_1=s$, equation 8.531 reduces to

$$0=\{b\rho_1 s_2^2+b_2\rho_2 s^2\begin{Bmatrix} b\rho_1[3s_2^2sx+2s^3x-2s^3x_2] \\ -b_2\rho_2[5s^3x] \end{Bmatrix}$$

$$-\{b\rho_1 s_2^2-b_2\rho_2 s^2\}\begin{Bmatrix} b\rho_1[2s^3x-2s^3x_2+3s_2^2sx] \\ +b_2\rho_2[5s^3x] \end{Bmatrix}, \quad (8.534)$$

and the coefficient of $\rho_1\rho_2$ reduces to:

$$0=2bb_2s^3[2(s^2-s_2^2)x-2s^2x_2]. \quad (8.535)$$

Hence $b_2=0$ or $x_2=0$ and $s_2=s$, with $y_2=0$ similarly.

If $b_2=0$, equation 8.512 shows that $m_1=\frac{bs_1^3}{b_1s_3}=1$, so that by equation 8.511, $m_2=\frac{2s_2}{s}$. Hence $m_2^2s^2=4s_2^2$ which leads to $m_2=\pm 2$, $x_2=y_2=0$ and $b_2=b$. Accordingly, in either choice, $b_2=b$ and $s_2=s$.

If $x_1=x_2=y_1=y_2=0$ and $b_1=b_2=b$, it follows from equations 8.43 that $s=s_1=s_2$, so that, by equations 8.52,

$$m_2-m_1=1 \quad (8.5411)$$

$$\frac{m_2}{\rho_1}+\frac{m_1}{\rho_2}=\frac{1}{\rho_2}. \quad (8.5412)$$

Equations 8.541 have the solutions:

$$m_1=\frac{\rho_1-\rho_2}{\rho_1+\rho_2} \quad (8.5421)$$

$$m_2=\frac{2\rho_1}{\rho_1+\rho_2}. \quad (8.5422)$$

The notation

$$Q=\frac{\rho_1-\rho_2}{\rho_1+\rho_2} \quad (8.551)$$

$$T=1+Q=\frac{2\rho_1}{\rho_1+\rho_2} \quad (8.552)$$

reduces the solutions of equations 8.542 to

$$m_1=Q \quad (8.561)$$

$$m_2=T. \quad (8.562)$$

The quantity Q may be called the "reflection factor" or "supplement factor," in B, of the medium G_2, and the quantity T may be called the "reflection complement," or "complement factor," in B, of medium G_2.

Substitution of equations 8.56 in equations 8.2 furnishes the two potential expressions:

$U_1=T/r$ over the space occupied by G_1 (8.61)

$U_2=1/r+Q/r_1$ over the space occupied by G_2 (8.62)

where r and r_1 are defined by equations 8.1.

Stated in words, the image method applied to a single horizontal boundary plane with a pole located in the lower medium leads to the conclusion that the upper medium may be considered as a continuation of the lower medium by the introduction of two additional poles obtained by "reflection" in the boundary. Below the boundary, the potential is that due to the original pole added to that due to the supplementary image located symmetrically above the boundary. The strength of the supplementary image is the product of the strength of the original pole and the reflection factor, in the boundary, of the lower medium. Above the boundary, the potential is that due to the complementary image located at the position of the original pole. The strength of the complementary image is the product of the strength of the original pole and the reflection complement, in the boundary, of the lower medium. As this method of analysis does not change the value of the potential at any point of space, the conclusions apply to the original space without change. It must be remembered that the solution implies the extension of medium G_2 to all space so that, by equation 6.61, a current of strength I corresponds to a pole of strength $m=\frac{I\rho_2}{4\pi}$.

As a matter of notation, if a medium G_j is reflected in the boundary that separates it from a medium G_k, the reflection factor has the value

$$Q_{jk}=\frac{\rho_k-\rho_j}{\rho_k+\rho_j} \quad (8.71)$$

and the reflection complement has the value

$$T_{jk}=1+Q_{jk}=\frac{2\rho_k}{\rho_k+\rho_j}. \qquad (8.72)$$

The images contributing to the potential over a selected region may be said to be "in" that potential, but each such image is external to the region over which the function applies. Specifically, an image contributing to the potential U_k is " in U_k" but lies externally to the region G_k over which U_k expresses the potential. In the limiting case for which the pole is on the boundary plane, both images coincide with the pole in position. However, this causes no difficulty if considered as a problem in limits.

POLE INTERNAL TO A PARALLEL PLATE

The two-boundary problem with plane parallel boundaries includes two distinct cases, according as the current source lies between the two boundaries or external to the medium between them. An internal pole lies in G_2 (fig. 10, p. 16), and external pole lies in G_1 or G_3. The problem with the pole in G_3 is omitted from this discussion as it is obtainable by a simple change in notation from the problem with the pole in G_1.

For the problem of an internal pole, figure 10, the original pole must be reflected in the boundary B_1 and in the boundary B_2. Each of these reflections introduces into the external potential a complementary image located at the position of the original pole, and also introduces into the internal potential a supplementary image located in an external region. The new space has the uniform resistivity ρ_2. At each boundary, the original pole and the two images satisfy the potential conditions at that boundary. The complementary image belongs to an external potential and its region does not touch the other external region. However, the supplementary image belongs to the internal potential, whose region does touch the other external region. Accordingly, this new image must be reflected in the boundary which did not introduce it, and the solution of the problem involves two sequences of images, each involving reflections alternately in the two boundary planes.

If a pole or image of strength m is located at the depth ζ, reflection in a plane $z=u$ leads to two images. The complement has the strength $T\ m$ and is located on the axis through the pole at the depth ζ. The supplement has the strength $Q\ m$ and is located on the same axis at the depth $(2u-\zeta)$. For the present problem, reflection is either upward in B_1 or downward in B_2. All images are located externally to the regions to whose potential function they contribute. For some plane boundaries, the application of the image theory leads to contradictions and can not be used unless modified (Keller, 1953).

For the present problem, reflection is from G_2 into G_1 or G_3. The two reflection factors are

$$Q_{21}=\frac{\rho_1-\rho_2}{\rho_1+\rho_2} \qquad (9.111)$$

$$Q_{23}=\frac{\rho_3-\rho_2}{\rho_3+\rho_2} \qquad (9.112)$$

and the two complements are

$$T_{21}=\frac{2\rho_1}{\rho_1+\rho_2} \qquad (9.121)$$

$$T_{23}=\frac{2\rho_3}{\rho_3+\rho_2}. \qquad (9.122)$$

It is convenient to write

$$Q=Q_{21}Q_{23}. \qquad (9.13)$$

The sequences of images are shown in table 4.

Inspection of table 4 shows that the images fall into four groups whose depths are

$$z_{k-1,1}=-[2(k-1)h+b] \qquad (9.21)$$

$$z_{k-1,2}=2(k-1)h+b \qquad (9.22)$$

$$z_{k-1,3}=2(k-1)h-b \qquad (9.23)$$

$$z_{k-1,4}=-[2(k-1)h-b]. \qquad (9.24)$$

The images of the three potentials are m_{jku} where j indicates the region or potential expression and k,u indicate the indices of the image. With this notation, the image strengths are

$$m_{1,k-1,2}=T_{21}Q^{k-1} \text{ at depth } z_{k-1,2} \qquad (9.311)$$

$$m_{1k3}=T_{21}Q_{23}Q^{k-1} \text{ at depth } z_{k3} \qquad (9.312)$$

$$m_{2,k-1,1}=Q_{21}Q^{k-1} \text{ at depth } z_{k-1,1} \qquad (9.321)$$

$$m_{2k2}=Q^{k} \text{ at depth } z_{k2} \qquad (9.322)$$

$$m_{2k3}=Q_{23}Q^{k-1} \text{ at depth } z_{k3} \qquad (9.323)$$

$$m_{2k4}=Q^{k} \text{ at depth } z_{k4} \qquad (9.324)$$

$$m_{3,k-1,1}=Q_{21}T_{23}Q^{k-1} \text{ at depth } z_{k-1,1} \qquad (9.331)$$

$$m_{3,k-1,4}=T_{23}Q^{k-1} \text{ at depth } z_{k-1,4}. \qquad (9.332)$$

In addition,

$$m_{1,k-1,1}=m_{1,k-1,4}=m_{3,k-1,2}=m_{3,k-1,3}=0 \qquad (9.341)$$

$$m_{103}=m_{203}=m_{204}=0 \qquad (9.342)$$

$$m_{202}=1. \qquad (9.343)$$

The notation

$$r_{k-1,u}=\sqrt{l^2+z_{k-1,u}^2} \qquad (9.41)$$

TABLE 4.—*Two-boundary image sequences for an internal pole*

Original pole is in U_2, has unit strength, and has depth b.
$Q = Q_{21}\,Q_{23}$.
Each line represents a single reflection from the preceding line.

Sequence with first reflection upward

Reflection		introduces				
from preceding—	in—	supplement in U_2		complement		
		strength	depth	in—	strength	depth
Pole	B_1	Q_{21}	$-b$	U_1	T_{21}	b
Supplement	B_2	Q	$2h+b$	U_3	$T_{23}\,Q_{21}$	$-b$
Supplement	B_1	$Q_{21}\,Q$	$-(2h+b)$	U_1	$T_{21}\,Q$	$2h+b$
Supplement	B_2	Q^2	$4h+b$	U_3	$Q_{21}\,T_{23}\,Q$	$-(2h+b)$
Supplement	B_1	$Q_{21}\,Q^2$	$-(4h+b)$	U_1	$T_{21}\,Q^2$	$4h+b$
Supplement	B_2	Q^3	$6h+b$	U_3	$Q_{21}\,T_{23}\,Q^2$	$-(4h+b)$
And so on, with the general terms:						
Supplement	B_1	$Q_{21}\,Q^{k-1}$	$-[2(k-1)h+b]$	U_1	$T_{21}\,Q^{k-1}$	$2(k-1)h+b$
Supplement	B_2	Q^k	$2kh+b$	U_3	$Q_{21}\,T_{23}\,Q^{k-1}$	$-[2(k-1)h+b]$

Sequence with first reflection downward

Reflection		introduces				
from preceding—	in—	supplement in U_2		complement		
		strength	depth	in—	strength	depth
Pole	B_2	Q_{23}	$2h-b$	U_3	T_{23}	b
Supplement	B_1	Q	$-(2h-b)$	U_1	$T_{21}\,Q_{23}$	$2h-b$
Supplement	B_2	$Q_{23}\,Q$	$4h-b$	U_3	$T_{23}\,Q$	$-(2h-b)$
Supplement	B_1	Q^2	$-(4h-b)$	U_1	$T_{21}\,Q_{23}\,Q$	$4h-b$
Supplement	B_2	$Q_{23}\,Q^2$	$6h-b$	U_3	$T_{23}\,Q^2$	$-(4h-b)$
Supplement	B_1	Q^3	$-(6h-b)$	U_1	$T_{21}\,Q_{23}\,Q^2$	$6h-b$
And so on, with the general terms:						
Supplement	B_1	Q^k	$-(2kh-b)$	U_1	$T_{21}\,Q_{23}\,Q^{k-1}$	$2kh-b$
Supplement	B_2	$Q_{23}\,Q^{k-1}$	$2kh-b$	U_3	$T_{23}\,Q^{k-1}$	$-2[(k-1)h-b]$

leads to the potential functions:

$$U_1 = T_{21}\sum\frac{Q^{k-1}}{r_{k-1,2}} + T_{21}\,Q_{23}\sum\frac{Q^{k-1}}{r_{k3}} \tag{9.421}$$

$$U_2 = Q_{21}\sum\frac{Q^{k-1}}{r_{k-1,1}} + \sum\frac{Q^{k-1}}{r_{k-1,2}} + Q_{23}\sum\frac{Q^{k-1}}{r_{k3}} + \sum\frac{Q^k}{r_{k4}} \tag{9.422}$$

$$U_3 = Q_{21}\,T_{23}\sum\frac{Q^{k-1}}{r_{k-1,1}} + T_{23}\sum\frac{Q^{k-1}}{r_{k-1,4}}, \tag{9.423}$$

where $\sum$ represents the sum in k over all positive integers, excluding zero.

For the limiting case of $b = 0$, in which the pole is located on the upper boundary, corresponding to the surface of the earth, the depths of equations 9.2 reduce to

$$z_{k-1,1} = -2(k-1)h \tag{9.511}$$

$$z_{k-1,2} = 2(k-1)h \tag{9.512}$$

$$z_{k3} = 2kh \tag{9.513}$$

$$z_{k-1,4} = -2(k-1)h \tag{9.514}$$

so that

$$z_{01} = z_{02} = z_{04} = 0 \tag{9.521}$$

$$z_{k2} = z_{k3} = 2kh \tag{9.522}$$

$$z_{k1} = z_{k4} = -2kh \tag{9.523}$$

and

$$r_{01} = r_{02} = r_{04} = \sqrt{l^2 + z^2} \tag{9.531}$$

$$r_{k2} = r_{k3} = \sqrt{l^2 + (z-2kh)^2} \tag{9.532}$$

$$r_{k1} = r_{k4} = \sqrt{l^2 + (z+2kh)^2}. \tag{9.533}$$

For this limiting case, the potential may be written

$$U_1 = \frac{T_{21}}{r_{01}} + Q_{23}T_{21}^2\sum\frac{Q^{k-1}}{r_{k2}} \tag{9.61}$$

$$U_2 = T_{21}\sum\frac{Q^{k-1}}{r_{k-1,1}} + Q_{23}T_{21}\sum\frac{Q^{k-1}}{r_{k2}} \tag{9.62}$$

$$U_3 = T_{21}T_{23}\sum\frac{Q^{k-1}}{r_{k-1,1}}. \tag{9.63}$$

If the pole lies on the lower boundary, a corresponding change of notation reduces it to the present problem.

POLE EXTERNAL TO PARALLEL PLATE

In the problem of a pole external to a parallel plate, the pole may be taken at $z=-b$, using the notation of figure 10. The first reflection is downward in B_1, leading to the supplement Q_{12} in U_1 at depth b and to the complement T_{12} in U_2 at depth $(-b)$. The supplement needs no further reflection, but the complement is in U_2 and must be reflected in B_2. This introduces a complement in U_3 and a supplement in U_2. Accordingly, the U_2 images are reflected alternately in the two planes to form a single sequence of images, each with its associated image in an external potential. The new space has a uniform resistivity ρ_1. The details are indicated in table 5, with the same notation as for an internal pole, except that the reflection factors are

$$Q_{12}=\frac{\rho_2-\rho_1}{\rho_2+\rho_1} \tag{10.111}$$

$$Q_{21}=\frac{\rho_1-\rho_2}{\rho_1+\rho_2} \tag{10.112}$$

$$Q_{23}=\frac{\rho_3-\rho_2}{\rho_3+\rho_2} \tag{10.113}$$

and the reflection complements are

$$T_{12}=\frac{2\rho_2}{\rho_1+\rho_2} \tag{10.121}$$

$$T_{21}=\frac{2\rho_1}{\rho_1+\rho_2} \tag{10.122}$$

$$T_{23}=\frac{2\rho_3}{\rho_2+\rho_3}. \tag{10.123}$$

Inspection of table 5 shows that the images fall into two groups, for which the depths are

$$z_{k-1,1}=-[2(k-1)h+b] \tag{10.21}$$

$$z_{k-1,2}=2(k-1)h+b \tag{10.22}$$

and the corresponding image strengths are

$$m_{101}=1 \text{ at depth } z_{01} \tag{10.311}$$

$$m_{102}=Q_{12} \text{ at depth } z_{02} \tag{10.312}$$

$$m_{1k2}=Q_{23}T_{12}T_{21}Q^{k-1} \text{ at depth } z_{k2} \tag{10.313}$$

$$m_{2,k-1,1}=T_{12}Q^{k-1} \text{ at depth } z_{k-1,1} \tag{10.321}$$

$$m_{2k2}=T_{12}Q_{23}Q^{k-1} \text{ at depth } z_{k2} \tag{10.322}$$

$$m_{3k1}=T_{12}T_{23}Q^{k-1} \text{ at depth } z_{k-1,1}. \tag{10.33}$$

Accordingly, the potentials for the external pole case are

$$U_1=\frac{1}{r_{01}}+\frac{Q_{12}}{r_{02}}+Q_{23}T_{12}T_{21}\sum\frac{Q^{k-1}}{r_{k2}} \tag{10.41}$$

$$U_2=T_{12}\sum\frac{Q^{k-1}}{r_{k-1,1}}+Q_{23}T_{12}\sum\frac{Q^{k-1}}{r_{k2}} \tag{10.42}$$

$$U_3=T_{12}T_{23}\sum\frac{Q^{k-1}}{r_{k-1,1}} \tag{10.43}$$

where $r_{k-1,u}$ is defined by equation 9.41.

For the limiting case $b=0$, in which the original unit pole lies on the upper boundary, the depths of equations 10.2 reduce to those of equations 9.5, so that the potentials are

$$U_1=\frac{T_{12}}{r_{01}}+Q_{23}T_{12}T_{21}\sum\frac{Q^{k-1}}{r_{k2}} \tag{10.51}$$

$$U_2=T_{12}\sum\frac{Q^{k-1}}{r_{k-1,1}}+Q_{23}T_{12}\sum\frac{Q^{k-1}}{r_{k2}} \tag{10.52}$$

$$U_3=T_{12}T_{23}\sum\frac{Q^{k-1}}{r_{k-1,1}}. \tag{10.53}$$

TABLE 5.—*Two-boundary image sequences for an external pole*

Original pole is in U_1, has unit strength, and has depth $(-b)$. $Q=Q_{21}Q_{23}$.

Each line represents a single reflection of an image from the preceding line.

Reflection from preceding—	Reflection in—	Introduces supplement in—	supplement strength	supplement depth	complement in—	complement strength	complement depth
Pole	B_1	U_1	Q_{12}	b	U_2	T_{12}	$-b$
Complement	B_2	U_2	$Q_{23}T_{12}$	$2h+b$	U_3	$T_{12}T_{23}$	$-b$
Supplement	B_1	U_2	QT_{12}	$-(2h+b)$	U_1	$Q_{23}T_{12}T_{21}$	$2h+b$
Supplement	B_2	U_2	$Q_{23}T_{12}Q$	$4h+b$	U_3	$QT_{12}T_{23}$	$-(2h+b)$
Supplement	B_1	U_2	$T_{12}Q^2$	$-(4h+b)$	U_1	$Q_{23}T_{12}T_{21}Q$	$4h+b$
Supplement	B_2	U_2	$T_{12}Q_{23}Q^2$	$6h+b$	U_3	$T_{12}T_{23}Q^2$	$-(4h+b)$
Supplement	B_1	U_2	$T_{12}Q^3$	$-(6h+b)$	U_1	$Q_{23}T_{12}T_{21}Q^2$	$6h+b$
And so on, with the general terms:							
Supplement	B_2	U_2	$Q_{23}T_{12}Q^{k-1}$	$2kh+b$	U_3	$T_{12}T_{23}Q^{k-1}$	$-[2(k-1)h+b]$
Supplement	B_1	U_2	$T_{12}Q^k$	$-(2kh+b)$	U_1	$Q_{23}T_{12}T_{21}Q^{k-1}$	$2kh+b$

If equations 9.6 are compared with equations 10.5 an apparent inconsistency is detected. However, the unit pole corresponding to the current I is $I\rho_2/(4\pi)$ for equations (9.6) and $I\rho_1/(4\pi)$ for equations 10.5. Furthermore,

$$\rho_2 T_{21}=\rho_1 T_{12}=\frac{2\rho_1\rho_2}{\rho_1+\rho_2}, \qquad (10.6)$$

so that the potentials of both limiting cases reduce to

$$U_1=\frac{I\rho_1\rho_2}{2\pi(\rho_1+\rho_2)}\left[\frac{1}{r_{02}}+Q_{23}T_{21}\sum\frac{Q^{k-1}}{r_{k2}}\right] \qquad (10.71)$$

$$U_2=\frac{I\rho_1\rho_2}{2\pi(\rho_1+\rho_2)}\left[\sum\frac{Q^{k-1}}{r_{k-1,1}}+Q_{23}\sum\frac{Q^{k-1}}{r_{k2}}\right] \qquad (10.72)$$

$$U_3=\frac{I\rho_1\rho_2 T_{23}}{2\pi(\rho_1+\rho_2)}\sum\frac{Q^{k-1}}{r_{k-1,1}}. \qquad (10.73)$$

SOME SPECIAL CASES

By choices of special values of the parameters involved, the general problem reduces to forms of interest for special purposes. The special case of $b=0$ has been considered. Except for the geophysical case, for which $z=b=0$ and $\rho_1=\infty$, special cases will be indicated without detailed consideration.

IMBEDDED PLATE

If the parallel plate G_2 is imbedded in a uniform medium of resistivity ρ_1, $\rho_3=\rho_1$ so that

$$Q_{21}=Q_{23}=\frac{\rho_1-\rho_2}{\rho_1+\rho_2} \qquad (11.11)$$

$$T_{21}=T_{23}=\frac{2\rho_1}{\rho_1+\rho_2} \qquad (11.12)$$

$$Q=Q_{21}^2 \qquad (11.13)$$

$$Q_{12}=-Q_{21} \qquad (11.14)$$

$$T_{12}=1-Q_{21}. \qquad (11.15)$$

INSULATED PLATE

If the imbedding medium is a perfect insulator, $\rho_1=\rho_3=\infty$, so that

$$Q_{21}=Q_{23}=1 \qquad (11.21)$$

$$T_{21}=T_{23}=2. \qquad (11.22)$$

The pole must be internal.

COVER PLATE

If the upper medium is a perfect insulator, the problem reduces to a conducting cover plate in contact with a second medium. For this case, $\rho_1=\infty$, so that

$$Q_{21}=1 \qquad (11.31)$$

$$T_{21}=2 \qquad (11.32)$$

$$Q=Q_{23}=\frac{\rho_3-\rho_2}{\rho_3+\rho_2} \qquad (11.33)$$

$$T_{23}=1+Q_{23} \qquad (11.34)$$

$$Q_{32}=-Q_{23} \qquad (11.35)$$

$$T_{32}=1-Q_{23} \qquad (11.36)$$

$$Q_{12}=-1 \qquad (11.37)$$

$$T_{12}=0. \qquad (11.38)$$

GEOPHYSICAL CASE

In the geophysical case, the upper medium is air and both current pole and test point lie on the upper boundary, B_1. Accordingly, for this case, $\rho_1=\infty$ and $b=z=0$, so that

$$Q_{21}=1 \qquad (12.11)$$

$$T_{21}=2 \qquad (12.12)$$

$$Q=Q_{23}=\frac{\rho_3-\rho_2}{\rho_3+\rho_2} \qquad (12.13)$$

$$T_{23}=1+Q \qquad (12.14)$$

$$r_{02}=l \qquad (12.15)$$

$$r_{k1}=r_{k2}=\sqrt{l^2+(2kh)^2}. \qquad (12.16)$$

By equations 10.7, the potential at the surface of the earth has the value

$$U=\frac{I\rho_2}{2\pi}\left[\frac{1}{l}+2\sum\frac{Q^k}{\sqrt{l^2+(2kh)^2}}\right]. \qquad (12.21)$$

The introduction of the "relative displacement"

$$a=\frac{l}{2h} \qquad (12.22)$$

changes equation 12.21 to

$$U=\frac{I\rho_2}{2\pi}\left[\frac{1}{l}+\frac{1}{h}\sum\frac{Q^k}{\sqrt{k^2+a^2}}\right]. \qquad (12.23)$$

For $Q=1$, as k increases, the summand approaches $1/k$ so that U becomes asymptotic to a harmonic series, which diverges. Accordingly, for every value of the reflection factor, Q, it is convenient to introduce the auxiliary variable

$$\psi_k(a)=\frac{1}{k}-\frac{1}{\sqrt{k^2+a^2}} \qquad (12.31)$$

so that the potential function of equation 12.23 becomes

$$U=\frac{I\rho_2}{2\pi}\left[\frac{1}{l}+\frac{1}{h}\sum Q^k\left(\frac{1}{k}-\psi_k\right)\right]. \qquad (12.32)$$

As the potential function lacks uniqueness to the extent of an additive constant and as $\Sigma Q^k/k$ is independent of a and hence of l, the potential function may be written as

$$U=\frac{I\rho_2}{2\pi}\left[\frac{1}{l}-\frac{1}{h}\sum Q^k\psi_k\right]. \qquad (12.33)$$

If $h=1/2$, then $a=l$ and $-z_{k1}=z_{k2}=k$, so that $\psi_k(a)=\frac{1}{z_k}-\frac{1}{\sqrt{z_k^2+a^2}}$ represents the excess of the reciprocal of the depth of an image over the reciprocal of the distance from the testpoint to the image. Accordingly, it may be called the "reduced reciprocal distance". It depends only on the horizontal and vertical distances between the testpoint and an image. Accordingly, the summand is the product of Q^k, which depends only on the ratio of resistivities ρ_2 and ρ_3, and on $\psi_k(a)$, which depends only on the distances involved. Each term of the summation is the product of a "resistivity factor" and a "distance factor." For many purposes, it is convenient to take the unit of length as twice the thickness of the parallel plate.

If $\rho_3=\rho_2$, the earth is homogeneous and $Q=0$ so that the potential reduces to the normal potential

$$U_n=\frac{I\rho_2}{2\pi l}. \tag{12.41}$$

Accordingly, the surface potential for a two-layer earth is

$$U=U_n-\frac{I\rho_2}{2\pi h}\sum Q^k\psi_k. \tag{12.42}$$

The medium, G_2 has been called the overburden and the medium G_3 the bed (see p. 16). Then B_1 will be the surface of the earth, assumed plane, and B_2 will be the contact of the overburden and the bed, also assumed plane. In applications, it is convenient to replace ρ_2 by ρ_0 and ρ_3 by ρ_b, as a matter of notation. Then

$$Q=\frac{\rho_b-\rho_0}{\rho_b+\rho_0}. \tag{12.5}$$

Interchanging ρ_0 and ρ_b reverses the sign of Q (see equation 12.5) but leaves its numerical value unchanged. This suggests the separation of even and odd terms in the potential, permitting simultaneous analysis of the buried conductor and buried insulator, having the two materials interchanged in the overburden and the bed. Such an interchange corresponds to an interchange of resistivities.

If the bed is more conducting than the overburden, $\rho_b<\rho_0$ and $Q<0$. For a homogeneous earth, $Q=0$.

For purposes of analysis, it is convenient to write

$$q=|Q| \tag{12.61}$$

$$W_1=\Sigma q^{2k-1}\psi_{2k-1} \tag{12.621}$$

$$W_2=\Sigma q^{2k}\psi_{2k}. \tag{12.622}$$

As ψ_k and q are both positive, W_1 and W_2 will each be positive except for the trivial case in which $Q=q=0$.

If the bed is relatively an insulator,

$$Q=q \tag{12.71}$$

$$\Sigma Q^{2k-1}\psi_{2k-1}=W_1 \tag{12.721}$$

$$\Sigma Q^{2k}\psi_{2k}=W_2 \tag{12.722}$$

$$\Sigma Q^k\psi_k=W_1+W_2 \tag{12.73}$$

$$U=U_n+\frac{I\rho_0}{2\pi h}W_i, \tag{12.741}$$

where

$$W_i=-(W_1+W_2). \tag{12.742}$$

If the bed is relatively a conductor,

$$Q=-q \tag{12.81}$$

$$\Sigma Q^{2k-1}\psi_{2k-1}=-W_1 \tag{12.821}$$

$$\Sigma Q^{2k}\psi_{2k}=W_2 \tag{12.822}$$

$$\Sigma Q^k\psi_k=W_2-W_1 \tag{12.83}$$

$$U=U_n+\frac{I\rho_0}{2\pi h}W_c, \tag{12.841}$$

where

$$W_c=W_1-W_2. \tag{12.842}$$

Inspection of equations 12.7 and 12.8 indicates that W_i is negative and W_c is positive, except for the trivial case $Q=0$. Accordingly, for a buried insulator, the actual potential is less than for a uniform earth having the resistivity of the overburden; whereas for a buried conductor, the actual potential exceeds that for a uniform earth. This paradox of a buried insulator corresponding to a decrease of potential arises from the fact that the introduction of the reduced reciprocal distance reverses the sign of the sum in the potential as given by equation 12.23 when expressed by equation 12.33. In the original notation, the potential becomes infinite for $l=0$ and approaches the value zero for $l=\infty$. In the modified notation, the potential vanishes for $l=0$ and approaches the value $-\frac{I\rho_0}{2\pi h}\sum\frac{Q^k}{k}$ for $l=\infty$. The discrepancy disappears in the difference in potential between two points of observation. Specifically, consider two points at distances l_1 and l_2 from the surface pole, so that $a_1=\frac{l_1}{2h}$ and $a_2=\frac{l_2}{2h}$. In the original equation 12.23,

$$U'=\frac{I\rho_0}{2\pi}\left[\frac{1}{l_1}+\frac{1}{h}\sum\frac{Qk}{\sqrt{a_1^2+k^2}}\right] \tag{12.911}$$

$$U''=\frac{I\rho_0}{2\pi}\left[\frac{1}{l_2}+\frac{1}{h}\sum\frac{Qk}{\sqrt{a_2^2+k^2}}\right] \tag{12.912}$$

so that

$$U=U''-U'$$

$$=\frac{I\rho_0}{2\pi}\left[\left(\frac{1}{l_2}-\frac{1}{l_1}\right)+\frac{1}{h}\sum Q^k\left(\frac{1}{\sqrt{a_2^2+k^2}}-\frac{1}{\sqrt{a_1^2+k^2}}\right)\right]. \tag{12.913}$$

In the modifiied notation of equation 12.32,

$$U'=\frac{I\rho_0}{2\pi}\left[\frac{1}{l_1}-\frac{1}{h}\sum Q^k\left(\frac{1}{k}-\frac{1}{\sqrt{k^2+a_1^2}}\right)\right] \quad (12.921)$$

$$U''=\frac{I\rho_0}{2\pi}\left[\frac{1}{l_2}-\frac{1}{h}\sum Q^k\left(\frac{1}{k}-\frac{1}{\sqrt{k^2+a_2^2}}\right)\right] \quad (12.922)$$

$$U=U''-U'=\frac{I\rho_0}{2\pi}\left[\left(\frac{1}{l_2}-\frac{1}{l_1}\right)+\frac{1}{h}\sum Q^k\left(\frac{1}{\sqrt{k^2+a_2^2}}-\frac{1}{\sqrt{k^2+a_1^2}}\right)\right]. \quad (12.923)$$

Inspection shows that equations 12.913 and 12.923 are identical.

As W_1 and W_2 depend only on q and a, tables may be prepared using one table for each value of Q and the value of a as the argument in each table.

For some purposes, it is convenient to write

$$U=U_n+\frac{I\rho_0}{2\pi h}W \quad (12.931)$$

without reference to whether the bed is a better or a poorer conductor than the overburden. For this use

$$W=-\sum Q^k\psi_k. \quad (12.932)$$

For a buried insulator $W=W_t$ and for a buried conductor $W=W_c$. For this reason it is convenient to call W the modified potential function.

FORMULAS FOR THE EVALUATION OF THE MODIFIED POTENTIAL

CALCULATION OF THE REDUCED RECIPROCAL DISTANCE

DIRECT CALCULATION

The basic calculation in evaluating the modified potential is that of evaluating the reduced reciprocal distance

$$\psi_k=\psi_k(a)=\frac{1}{k}-\frac{1}{\sqrt{k^2+a^2}}. \quad (12.31)$$

If

$$k>\frac{1}{3}\sqrt{3}a,\ \psi_k<\frac{1}{\sqrt{k^2+a^2}}$$

and the terms of

$$W=-\Sigma Q^k\psi_k \quad (12.932)$$

are respectively smaller than those of $\sum\frac{Q^k}{\sqrt{k^2+a^2}}$ that occur in equation 12.23. Furthermore ψ_k decreases more rapidly than $(a^2+k^2)^{-1/2}$ as k increases.

If an adequate table of square roots is available, the calculation is simplified. If not, the method of iteration may be used for finding the square root. If D is an approximate value of $\sqrt{N}$, better approximations are

$$\sqrt{N}=(1/2)\ (D+N/D) \quad (13.11)$$

$$\sqrt{N}=D+(N^2-D^2)/(2D). \quad (13.12)$$

If a calculating machine with adequate capacity is available, the first form is simplest. However, the second expression leads to a somewhat more accurate value, as it determines a correction to the approximation, to the same degree of accuracy as the first form determines the entire value.

After $\sqrt{k^2+a^2}$ has been determined, the value of ψ_k is calculated by some convenient routine, such as

$$\psi_k=\frac{1}{k}-\frac{1}{\sqrt{k^2+a^2}} \quad (12.31)$$

$$\psi_k=\frac{\sqrt{k^2+a^2}-k}{k\sqrt{k^2+a^2}} \quad (13.21)$$

$$\psi_k=\frac{1}{k}\left(1-\frac{k}{\sqrt{k^2+a^2}}\right) \quad (13.22)$$

$$\psi_k=\frac{1}{\sqrt{k^2+a^2}}\left(\frac{\sqrt{k^2+a^2}}{k}-1\right) \quad (13.23)$$

$$\psi_k=\frac{1}{k}\left[\frac{1}{\sqrt{k^2+a^2}}\left(\sqrt{k^2+a^2}-k\right)\right] \quad (13.24)$$

depending on the available computing machine and the preferences of the operator. Of this group equation 12.31 has the advantage that the values of $1/k$ may be preprinted on the calculation form. However, it is less occurate than the others because of forcing errors of rounding. The values should be calculated by two independent methods, as a check.

RESCALING

As ψ_k is homogeneous of order minus one in k and a jointly, it follows that $\psi_{nk}(na)=\frac{\psi_k(a)}{n}$ so that if $\psi_k(a)$ is known $\psi_{nk}(na)$ is available by division. In preparing tables of the reduced reciprocal distance, this device of "re-scaling" avoids the need for calculating a considerable number of values of ψ_k. As an example, direct calculation shows that

$$\psi_3(2)=0.055\ 983\ 235,$$

so that

$$\psi_6(4)=0.027\ 991\ 618$$

$$\psi_9(6)=0.018\ 661\ 078,$$

and so forth.

CONTROL VALUES

The modified potential is given by equation 12.932, in which the terms decrease as the counter of the term increases. As each term has a resistivity factor which is a power of q, the largest value of q needed in the calculations will determine the largest value of k needed to attain a selected accuracy. For $q=1$, the series converges very slowly. For the range $q=[0(0.1)0.9]$, the value $[q=0.9]$ determines the control on the value of k needed.

By the definition in equation 12.31, the value of the reduced reciprocal distance, ψ_k, decreases as k increases. Hence

$$\psi_{2k+2u-1}<\psi_{2k-1} \qquad (13.31)$$

$$\psi_{2k+2u}<\psi_{2k}. \qquad (13.32)$$

Equation 12.62 may be written

$$W_1=\sum_{k=1}^{u} q^{2k-1}\psi_{2k-1}+\sum_{k=1}^{\infty} q^{2u+2k-1}\psi_{2u+2k-1} \qquad (13.411)$$

$$W_2=\sum_{k=1}^{u} q^{2k}\psi_{2k}+\sum_{k=1}^{\infty} q^{2u+2k}\psi_{2u+2k}, \qquad (13.412)$$

so that

$$0<W_1-\sum_{k=1}^{u} q^{2k-1}\psi_{2k-1}<q^{2u-1}\psi_{2u-1}\sum_{k=1}^{\infty} q^{2k} \qquad (13.421)$$

$$0<W_2-\sum_{k=1}^{u} q^{2k}\psi_{2k}<q^{2u}\psi_{2u}\sum_{k=1}^{\infty} q^{2k}. \qquad (13.422)$$

But

$$\sum_{k=1}^{\infty} q^{2k}=\frac{q^2}{1-q_2}, \qquad (13.43)$$

so that

$$W_1<\sum_{k=1}^{u} q^{2k-1}\psi_{2k-1}+\left(\frac{q^2}{1-q^2}\right)(q^{2u-1}\psi_{2u-1}) \qquad (13.451)$$

$$W_2<\sum_{k=1}^{u} q^{2k}\psi_{2k}+\left(\frac{q^2}{1-q^2}\right)(q^{2u}\psi_{2u}). \qquad (13.452)$$

If W_1 or W_2 is calculated term by term for the first u terms, the last term is $q^{2u-1}\psi_{2u-1}$ or $q^{2u}\psi_{2u}$, respectively, so that the remainder, in either evaluation, can not exceed the product of $q^2/(1-q^2)$ by the last retained term. The remainder is always positive, so that if the remainder is taken as the product of the last retained term by $q^2/[2(1-q^2)]$, the error can not exceed this remainder, except for the forcing errors of rounding. This method fails for $q=1$, but is not needed, as convergence is too slow for direct calculation.

The values of the error coefficient $q^2/[2\ (1-q^2)]$ are shown in table 6. For $q=0.9$, the error coefficient is 81/38 or about 2.13. Accordingly, for $q=0.9$, the error in W_1 or W_2 can not exceed 81/38 times the last retained term. For an error not in excess of one in the ninth decimal place, the last term must not exceed the product of 1×10^{-9} by 38/81 or about 5×10^{-10}. This furnishes a control, Ψ_k, such that the last term needed in the direct calculation is the last one for which $\psi_k\geqq\Psi_k$. As $\psi_{k+1}<\psi_k, \psi_{k+1}$ need not be calculated if $\psi_k<\Psi_{k+1}$. Accordingly, direct calculation may be stopped as soon as $\psi_k<\Psi_{k+1}$. The values of the controls are shown in table 7, for nine decimal calculations, with the unit in the ninth decimal place. For $q<0.9$, the series converges more rapidly than for $q=0.9$ and the ψ_k usually vanish to the ninth decimal place before the control value is reached. If the ψ_k do not become negligible, the same control values may be used as for $q=0.9$.

TABLE 6.—*Error factors in direct evaluation of* W *for varying factors of* q

Error factors for indicated values of Q										
0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	$\frac{1}{198}$	$\frac{1}{48}$	$\frac{9}{182}$	$\frac{2}{21}$	$\frac{1}{6}$	$\frac{9}{32}$	$\frac{49}{102}$	$\frac{8}{9}$	$\frac{81}{38}$	∞

TABLE 7.—*Control values for* q=*0.9*

[Unit: 10^{-9}]

k	Ψ_k	k	Ψ_k	k	Ψ_k
21	5	66	524	111	59 987
22	6	67	582	112	66 652
23	6	68	649	113	74 057
24	7	69	719	114	82 286
25	7	70	798	115	91 429
26	8	71	887	116	101 588
27	9	72	986	117	112 875
28	10	73	1 095	118	125 417
29	11	74	1 217	119	139 352
30	12	75	1 352	120	154 835
31	14	76	1 502	121	172 039
32	15	77	1 669	122	191 155
33	17	78	1 854	123	212 394
34	18	79	2 060	124	235 993
35	20	80	2 289	125	262 215
36	23	81	2 543	126	291 350
37	25	82	2 826	127	323 721
38	28	83	3 140	128	359 691
39	31	84	3 489	129	399 656
40	34	85	3 876	130	444 063
41	38	86	4 307	131	493 403
42	42	87	4 785	132	548 225
43	47	88	5 317	133	609 139
44	52	89	5 908	134	676 821
45	58	90	6 564	135	752 024
46	64	91	7 293	136	835 582
47	71	92	8 104	137	928 424
48	79	93	9 004	138	1 031 582
49	88	94	10 005	139	1 146 203
50	98	95	11 116	140	1 273 558
51	108	96	12 351	141	1 415 065
52	120	97	13 723	142	1 572 294
53	134	98	15 248	143	1 746 993
54	148	99	16 942	144	1 941 104
55	165	100	18 825	145	2 156 782
56	183	101	20 916	146	2 396 424
57	203	102	23 240	147	2 662 694
58	226	103	25 823	148	2 958 548
59	251	104	28 692	149	3 287 276
60	279	105	31 880	150	3 652 529
61	310	106	35 422	151	4 058 365
62	344	107	39 357	152	4 509 295
63	382	108	43 730	153	5 010 327
64	425	109	48 589	154	5 567 030
65	472	110	53 988	155	6 185 589

As $\psi_k<1/k$ for every value of a, no value of k in excess of 155 will be needed for nine decimal calculations, because $(0.9)^{156}/156<5\times10^{-10}$. As ψ_k is large for small values of k, controls for $k<21$ are seldom needed and do not appear in table 7.

ASYMPTOTIC LIMITS OF W

As the value of a increases indefinitely, the value of W defined by equations 12.932 and 12.31 approaches the limit

$$W_\infty = -\sum \frac{Q^k}{k}. \tag{14.1}$$

From the customary series expansions of the logarithm,

$$\log_e (1+x) = \sum (-1)^{k-1}\left(\frac{x^k}{k}\right) = \sum \frac{x^{2k-1}}{2k-1} - \sum \frac{x^{2k}}{2k} \tag{14.211}$$

$$\log_e (1-x) = -\sum \frac{x^k}{k} = -\sum \frac{x^{2k-1}}{2k-1} - \sum \frac{x^{2k}}{2k}, \tag{14.212}$$

it follows that

$$\sum \frac{x^{2k}}{2k} = -\frac{1}{2}\log_e (1-x^2) \tag{14.221}$$

$$\sum \frac{x^{2k-1}}{2k-1} = \frac{1}{2}\log_e \left(\frac{1+x}{1-x}\right). \tag{14.222}$$

Accordingly,

$$W_\infty = \log_e (1-Q). \tag{14.31}$$

Also, by equations 12.62, as a becomes infinite W_1 and W_2 approach, respectively, the limits

$$W_{1\infty} = \frac{1}{2}\log_e \left(\frac{1+q}{1-q}\right) \tag{14.321}$$

$$W_{2\infty} = -\frac{1}{2}\log_e (1-q^2). \tag{14.322}$$

As q lies between zero and unity, $W_{1\infty}$ and $W_{2\infty}$ are positive, as are W_1 and W_2, for all positive values of a.

For $q=1$, equations 14.3 hold in a limiting sense but are of no use in computation. The numerical values of the limits are shown in table 8.

TABLE 8.—*Asymptotic limits of* W

q	$W_{1\infty}$	$W_{2\infty}$	$-W_{i\infty}$	$+W_{e\infty}$
0.1	0.100 335 348	0.005 025 168	0.105 360 516	0.095 310 180
0.2	.205 732 554	.020 410 997	.223 143 551	.182 321 557
0.3	.309 519 604	.047 155 340	.356 674 944	.262 364 264
0.4	.423 648 930	.087 176 694	.510 825 624	.336 472 237
0.5	.549 306 144	.143 841 036	.693 147 181	.405 465 108
0.6	.693 147 181	.223 143 551	.916 290 732	.470 003 629
0.7	.867 300 528	.336 672 277	1.203 972 804	.530 628 251
0.8	1.098 612 289	.510 825 624	1.609 437 912	.587 786 665
0.9	1.472 219 490	.830 365 603	2.302 585 093	.641 853 886
1.0	∞	∞	∞	.693 147 181

EULER-MACLAURIN SUMMATION FORMULAS

For reflection factors numerically less than unity, the series W_1 and W_2 converge with sufficient rapidity that direct evaluation is at least feasible. It is true that as many as 78 terms may be needed in each of these series to assure an accuracy of a few units in the ninth decimal place, but the process is definite and the error can be limited arbitrarily. However, if the numerical value of the reflection factor is unity, the terms of each series approach those of a harmonic series, which diverges, and direct evaluation is not practicable.

For this case, a group of formulas developed independently by Euler and by Maclaurin almost simultaneously (Charlier, 1907, p. 1) are applicable and permit the evaluation of the remainder after a number of initial terms have been calculated. For many series, the Euler-Maclaurin formulas permit the transformation of a slowly convergent series into an equivalent series that converges more rapidly. For this reason, one of the Euler-Maclaurin formulas is derived in a form convenient for the present problem (Roman, 1931, 1936).

In the series

$$S = \sum_{k=a}^{b} u_k, \tag{15.1}$$

let the limit a be finite and the limit b be finite or infinite. If a function $f(x)$ can be found, such that for every positive integer k from a to b,

$$f(k) = u_k, \tag{15.2}$$

the series S may be replaced by an integral, as a first approximation. All needed derivatives of $f(x)$ must exist over the range (a,b) and for values somewhat outside this range. Usually, the function $f(x)$ can be selected directly from the general term of the series.

The translation

$$x = n+h \tag{15.31}$$

$$f(x) = \phi(h) \tag{15.32}$$

permits writing Taylor's theorem in the form

$$f(x) = \phi(h) = f(n) + \sum_{k=1}^{\infty} \frac{f^{(k)}(n)}{k!} h^k. \tag{15.41}$$

Integration of $f(x)$ from $(n-h)$ to $(n+h)$ in x is equivalent to integration of $\phi(h)$ from $(-h)$ to $(+h)$ in h, so that term by term integration of equation 15.41 leads to

$$\int_{n-h}^{n+h} f(x)\,dx = \int_{-h}^{h} \phi(h)\,dh = \left[hf(n) + \sum_{k=1}^{\infty} \frac{f^{(k)}(n)}{(k+1)!} h^{k+1}\right]_{-h}^{h} \tag{15.42}$$

For odd values of k, the corresponding members of the summation have the same value at each limit so that

$$\int_{n-h}^{n+h} f(x)\,dx = 2hf(n) + 2\sum_{k=1}^{\infty} \frac{f^{(2k)}(n)}{(2k+1)!} h^{2k+1}, \tag{15.431}$$

which may be written

$$f(n) = \frac{1}{2h}\int_{n-h}^{n+h} f(x)\,dx - \sum_{k=1}^{\infty} \frac{f^{(2k)}(n)}{(2k+1)!} h^{2k}. \tag{15.432}$$

Application of this equation to $f^{(2k)}(n)$ leads to

$$f^{(2k)}(n)=\frac{1}{2h}\Big[f^{(2k-1)}(x)\Big]_{n-h}^{n+h}-\sum_{j=1}^{\infty}\frac{f^{(2j+2k)}(n)}{(2j+1)!}h^{2j}. \qquad (15.433)$$

The notation

$$a_{2k}=-\frac{1}{(2k+1)!} \qquad (15.51)$$

$$\theta_0=\frac{1}{2h}\int_{n-h}^{n+h}f(x)\,dx \qquad (15.521)$$

$$\theta_{2k}=\frac{1}{2h}\Big[\int f^{(2k-1)}(x)\Big]_{n-h}^{n+h} \qquad (15.522)$$

converts equations 15.43 to

$$f(n)=\theta_0+\sum_{k=1}^{\infty}a_{2k}f^{(2k)}(n)\,h^{2k} \qquad (15.61)$$

$$f^{(2k)}(n)=\theta_{2k}+\sum_{j=1}^{\infty}a_{2j}f^{(2j+2k)}(n)\,h^{2j}. \qquad (15.62)$$

Successive applications of equation 15.62 to 15.61 lead to

$$f(n)=\theta_0+\sum_{k=1}^{\infty}a_{2k}\theta_{2k}h^{2k}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}a_{2j}a_{2k}\theta_{2j+2k}h^{2j+2k}+\ldots. \qquad (15.71)$$

If b_{2u} represents the sum of all possible products of the a_{2k} for which the sum of the indices is $2u$, with each term having a coefficient that is the number of possible distinct permutations of the indices, equation 15.71 may be written:

$$f(n)=\theta_0+\sum_{u=1}^{\infty}b_{2u}\theta_{2u}h^{2u}. \qquad (15.72)$$

Inspection of equations 15.7 shows the structure of b_{2u}. For $u=1$, the only term of equation 15.71 which can contribute to the term involving h^2 in equation 15.72 is the one for which k is 1 in the second term. The corresponding term is

$$a_2\theta_2h^2=-\theta_2h^2/3!, \qquad (15.811)$$

so that

$$b_2=-1/6. \qquad (15.812)$$

The term involving h^4 in equation 15.71 can come from the second term for $k=2$ and from the third term for $j=k=1$. Accordingly,

$$b_4=a_4+a_2^2=-\frac{1}{5!}+\left(-\frac{1}{3!}\right)^2=\frac{7}{360}. \qquad (15.82)$$

In a similar manner, other values of b_{2u} are available. For example,

$$b_6=a_6+2a_2a_4+a_2^3 \qquad (15.83)$$

$$b_8=a_8+2a_2a_6+a_4^2+3a_2^2a_4+a_2^4. \qquad (15.84)$$

Arbitrary selection of

$$b_0=1 \qquad (15.91)$$

leads to the relation

$$b_{2u}=\sum_{v=1}^{u}a_{2v}b_{2u-2v}=-\sum_{v=1}^{u}\frac{b_{2u-2v}}{(2v+1)!}. \qquad (15.92)$$

The common fractional values of b_{2u} for values of u from zero to eight and the decimal values of b_{2u} to 18 decimals for values of u from 0 to 18 are shown in table 9. It can be shown that the ratio of b_{2u+2} to b_{2u} approaches $-1/\pi^2$ as u increases indefinitely.

TABLE 9—*Euler-Maclaurin coefficients*

[Values of b_{2u}]

$2u$	sign	Decimal	Common fraction	
			Numerator	Demoninator
0	+	1.000 000 000 000 000 000	1	1
2	−	.166 666 666 666 666 667	1	6
4	+	.019 444 444 444 444 444	7	360
6	−	.002 050 264 550 264 550	31	15 120
8	+	.000 209 986 772 486 772	127	604 800
10	−	.000 021 336 045 641 601	73	3 421 440
12	+	.000 002 163 347 442 779	1 414 477	653 837 184 000
14	−	.000 000 219 232 713 446	8 191	37 362 124 800
16	+	.000 000 022 213 930 854	16 931 177	762 187 345 920 000
18	−	.000 000 002 250 767 479		
20	+	.000 000 000 228 021 077		
22	−	.000 000 000 023 101 422		
24	+	.000 000 000 002 340 586		
26	−	.000 000 000 000 237 149		
28	+	.000 000 000 000 024 028		
30	−	.000 000 000 000 002 435		
32	+	.000 000 000 000 000 248		
34	−	.000 000 000 000 000 025		
36	+	.000 000 000 000 000 002		

By means of the definitions in equations 15.5 equation 15.72 becomes

$$f(n)=\frac{1}{2h}\int_{n-h}^{n+h}f(x)\,dx+\frac{1}{2h}\sum_{u=1}^{\infty}b_{2u}\Big[f^{(2u-1)}(x)\Big]_{n-h}^{n+h}h^{2u}. \qquad (16.11)$$

Introduction of the notation

$$Z(x)=\frac{1}{2h}\Big[\int f(x)\,dx+\sum_{u=1}^{\infty}b_{2u}h^{2u}f^{(2u-1)}(x)\Big] \qquad (16.12)$$

transforms equation 16.11 into

$$f(n)=Z(n+h)-Z(n-h). \qquad (16.2)$$

Successive applications of equation 16.2 results in the relations

$$f[k+2sh]=Z[k+(2s+1)h]-Z[k+(2s-1)h] \qquad (16.311)$$

$$f[k+2(s+1)h]=Z[k+(2s+3)h]-Z[k+(2s+1)h] \qquad (16.312)$$

and so on to

$$f[k+2(t-1)h]=Z[k+(2t-1)h]-Z[k+(2t-3)h] \qquad (16.321)$$

$$f[k+2th]=Z[k+(2t+1)h]-Z[k+(2t-1)h]. \qquad (16.322)$$

Addition of equations 16.3 and cancellation of balancing terms leads to

$$\sum_{i=s}^{t} f(k+2hi)=Z[k+(2t+1)h]-Z[k+(2s-1)h]. \quad (16.33)$$

Several special cases of equation 16.33 are important. The basic equations may be rearranged to furnish additional results (Roman, 1936).

For $h=1$, equation 16.33 becomes

$$\sum_{i=s}^{t} f(k+2i)=Z[k+(2t+1)]-Z[k+(2s-1)] \quad (16.411)$$

where

$$Z(x)=\frac{1}{2}\left[\int f(x)\,dx+\sum_{u=1}^{\infty} b_{2u} f^{(2u-1)}(x)\right]. \quad (16.412)$$

For $k=0$, equation 16.411 takes the form

$$\sum_{i=s}^{t} f(2i)=Z(2t+1)-Z(2s-1) \quad (16.421)$$

and for $k=1$, it takes the form

$$\sum_{i=s}^{t} f(2i+1)=Z(2t+2)-Z(2s). \quad (16.422)$$

The utility of the Euler-Maclaurin summation formulas is determined by the function $Z(x)$. To be useful, the integral and odd order derivatives of $f(x)$ must exist over the interval $(s-h)$ to $(t+h)$ and be obtainable by direct calculation or by available tables. The derivatives must decrease as the order increases or at least increase so slowly that $b_{2u}f^{(2u-1)}(x)$ converges with convenient rapidity as u increases.

The error in applying the Euler-Maclaurin formula in the original form usually does not exceed twice the first neglected term (Charlier, 1907, p. 14; Scarborough, 1950, p. 184). Accordingly, if the terms begin to increase as the counter increases, the calculations should stop after the term preceding the minimum. Without proof, the present form of the Euler-Maclaurin formula may be assumed to have the same error criterion. The Euler-Maclaurin formula permits the evaluation of a divergent series if a term sufficiently small can be reached. For convergent series, the use of the formulas often increases the speed of convergence, thus reducing the amount of numerical calculations needed.

In the present study, the Euler-Maclaurin formula has proved convenient for $q=1$, but for $q<1$, it has not proved practicable because the integral has not been obtainable in closed form or in available tables. Fortunately, it is not needed.

As the Euler-Maclaurin formula uses only the difference between two values of the auxiliary function, a constant term may be dropped from that function. The formula is especially convenient if $Z(t)$ approaches a determinable constant as t increases indefinitely.

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LOCAL EXPANSIONS

LOCAL EXPANSIONS FOR $Q^2=1$

If the first n terms of the modified potentials are calculated directly, equations 12.62 may be written

$$W_1=\sum_{k=1}^{n} q^{2k-1}\psi_{2k-1}+R_{n1} \quad (17.11)$$

$$W_2=\sum_{k=1}^{n} q^{2k}\psi_{2k}+R_{n2}, \quad (17.12)$$

where

$$R_{n1}=\sum_{k=n+1}^{\infty} q^{2k-1}\psi_{2k-1} \quad (17.21)$$

$$R_{n2}=\sum_{k=n+1}^{\infty} q^{2k}\psi_{2k}. \quad (17.22)$$

The quantity R_{nk} is the remainder, or residue, of order n, for the series W_k.

For $Q^2=1$, $q=1$ and each of the series W_1, W_2 converges very slowly. The Euler-Maclaurin summation formula expedites the numerical evaluation unless the argument a is so small and the degree of accuracy needed is so low that direct calculation is adequate. By equation 12.31 the simplest choice of equivalent function is

$$f(x)=\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}. \quad (18.1)$$

Direct integration transforms equations 18.1 to

$$\int f(x)\,dx=-\log\left[1+\sqrt{1+\alpha}\right] \quad (18.2)$$

where

$$\alpha=\left(\frac{a}{x}\right)^2. \quad (18.3)$$

Differentiation shows that

$$f^{(k-1)}(x)=(-1)^{k-1}(k-1)!(1-D_kE_k)x^{-k} \quad (18.41)$$

where

$$D_k=(1+\alpha)^{\frac{1}{2}-k} \quad (18.42)$$

and

$$E_2=1 \quad (18.51)$$

$$E_4=1-\frac{3}{2}\alpha \quad (18.52)$$

$$E_6=1-5\alpha+\frac{15}{8}\alpha^2 \quad (18.53)$$

$$E_8=1-\frac{21}{2}\alpha+\frac{105}{8}\alpha^2-\frac{35}{16}\alpha^3 \quad (18.54)$$

$$E_{10}=1-18\alpha+\frac{189}{4}\alpha^2-\frac{105}{4}\alpha^3+\frac{315}{128}\alpha^4 \quad (18.55)$$

$$E_{12}=1-\frac{55}{2}\alpha+\frac{495}{4}\alpha^2-\frac{1,155}{8}\alpha^3+\frac{5,775}{128}\alpha^4-\frac{693}{256}\alpha^5 \quad (18.56)$$

$$E_{14}=1-39\,\alpha+\frac{2,145}{8}\,\alpha^2-\frac{2,145}{4}\,\alpha^3+\frac{45,045}{128}\,\alpha^4-\frac{9,009}{128}\,\alpha^5+\frac{3,003}{1,024}\,\alpha^6 \quad (18.57)$$

$$E_{16}=1-\frac{105}{2}\,\alpha+\frac{4,095}{8}\,\alpha^2-\frac{25,025}{16}\,\alpha^2+\frac{225,225}{128}\,\alpha^4-\frac{189,189}{256}\,\alpha^5+\frac{105,105}{1024}\,\alpha^6-\frac{6,435}{2,048}\,\alpha^7 \quad (18.58)$$

$$E_{18}=1-68\,\alpha+\frac{1,785}{2}\,\alpha^2-\frac{7,735}{2}\,\alpha^2+\frac{425,425}{64}\,\alpha^4-\frac{153,153}{32}\,\alpha^5+\frac{357,357}{256}\,\alpha^6-\frac{36,465}{256}\,\alpha^7+\frac{109,395}{32,768}\,\alpha^6. \quad (18.59)$$

It can be shown by mathematical induction that

$$E_u=\sum_{k=0}^{A}(-1)^k E_{uk}\left(\frac{\alpha}{4}\right)^k \quad (18.61)$$

where

$$E_{uk}=\frac{(u-1)!}{k!k!(u-2k-1)!} \quad (18.62)$$

and A is the largest integer not exceeding $(1/2)(u-1)$. In particular, $A=0$ for $u=2$ and odd values of u are not needed in the present study.

With the notation

$$g_{2k}=(2k-1)!b_{2k}x^{-2k} \quad (19.11)$$

$$D_0=\frac{1}{2}\left[1+\frac{1}{\sqrt{1+\alpha}}\right], \quad (19.12)$$

the Euler-Maclaurin function of equation 16.42 becomes, for $q=1$,

$$Z(x)=-\frac{1}{2}\left[\log_e D_0+\sum g_{2k}(1-D_{2k}E_{2k})\right]. \quad (19.2)$$

By equations 15.8 and 19.11, the early values of g_{2k} are

$$g_2=-\frac{1}{6x^2} \quad (19.31)$$

$$g_4=\frac{7}{60x^4} \quad (19.32)$$

$$g_6=-\frac{31}{126x^6} \quad (19.33)$$

$$g_8=\frac{127}{120x^8} \quad (19.34)$$

$$g_{10}=-\frac{511}{66x^{10}} \quad (19.35)$$

$$g_{12}=\frac{1\,414\,477}{16\,380x^{12}} \quad (19.36)$$

$$g_{14}=-\frac{8191}{6x^{14}} \quad (19.37)$$

$$g_{16}=\frac{118\,518\,239}{4\,080x^{16}} \quad (19.38)$$

For a selected value of a, as k increases indefinitely, the values of α and g_{2k} approach zero and the values of D_0, D_{2k}, and E_{2k} approach unity. Accordingly, equations 17.2 and 19 show that, for $q=1$,

$$R_{n1}=-Z(2n) \quad (19.41)$$

$$R_{n2}=-Z(2n+1) \quad (19.42)$$

In each remainder, the argument of the Euler-Maclaurin function $Z(x)$ is one unit greater than the counter of the last term of the direct calculation.

In an isolated application, the value of $Z(x)$ must be computed term by term until a term of negligible value is reached. In group calculations, it usually is preferable to predetermine the number of terms needed. Direct analysis leads to the values of $(1-D_{2k}E_{2k})$ shown in table 10. On each interval between the values shown for α, the expression is monotonic. Numerical values are shown for $\alpha=[1(1)25(5)50]$ and $k\leqq 5$, in table 11.

TABLE 10.—*Variations of* $(1-D_{2k}E_{2k})$

[Monotonic between values shown]

$2k$	α	$1-D_{2k}E_{2k}$
2	0	0
	∞	1
4	0	0
	2/3	1
	4/3	1. 051 532 505
	∞	1
6	0	0
	0. 217 786 631	1
	0. 450 806 662	1. 112 759 589
	2. 448 880 035	1
	3. 549 193 338	0. 998 344 737
	∞	1
8	0	0
	0. 110 117 150	1
	0. 235 232 105	1. 158 338 770
	0. 818 616 636	1
	1. 180 638 620	0. 990 473 786
	5. 071 266 214	1
	6. 584 129 275	1. 000 031 129
	∞	1
10	0	0
	0. 066 855 287	1
	0. 146 317 486	1. 192 181 694
	0. 430 721 573	1
	0. 624 921 564	0. 978 416 470
	1. 657 987 349	1
	2. 129 978 868	1. 000 509 415
	8. 511 102 456	1
	10. 432 115 413	0. 999 999 619
	∞	1

In calculating the residue, the number of terms needed depends in a complicated manner on the value of the argument and the number of terms initially calculated. A reduction in the number of initial terms usually requires an increase in the number of terms needed in the residue to obtain a selected accuracy. No

TABLE 11.—*Values of* $(1-D_{2k}E_{2k})$

α	$1-D_2E_2$	$1-D_4E_4$	$1-D_6E_6$	$1-D_8E_8$	$1-D_{10}E_{10}$
1	0.646 446 610	1.044 194 174	1.046 956 310	0.992 058 859	0.991 077 006
2	.807 549 911	1.042 766 687	1.003 563 891	.996 040 122	1.000 487 652
3	.875 000 000	1.027 343 750	.998 596 192	.999 128 342	1.000 261 620
4	.910 557 281	1.017 888 544	.998 425 808	.999 833 994	1.000 083 575
5	.931 958 618	1.012 285 249	.998 799 038	.999 995 351	1.000 026 369
6	.946 005 075	1.008 815 498	.999 134 192	1.000 028 454	1.000 008 304
7	.955 805 827	1.006 560 072	.999 375 552	1.000 030 293	1.000 002 383
8	.962 962 963	1.005 029 921	.999 542 753	1.000 025 298	1.000 000 412
9	.968 377 223	1.003 952 847	.999 658 869	1.000 019 766	.999 999 787
10	.972 589 878	1.003 171 419	.999 740 708	1.000 015 147	.999 999 629
11	.975 943 739	1.002 589 389	.999 799 444	1.000 011 585	.999 999 630
12	.978 665 377	1.002 146 086	.999 842 386	1.000 008 906	.999 999 679
13	.980 909 911	1.001 801 871	.999 874 339	1.000 006 905	.999 999 737
14	.982 786 741	1.001 530 067	.999 898 506	1.000 005 404	.999 999 788
15	.984 375 000	1.001 312 256	.999 917 060	1.000 004 271	.999 999 831
16	.985 733 198	1.001 135 420	.999 931 502	1.000 003 409	.999 999 866
17	.986 905 430	1.000 990 176	.999 942 885	1.000 002 746	.999 999 893
18	.987 925 488	1.000 869 632	.999 951 960	1.000 002 231	.999 999 915
19	.988 819 660	1.000 768 648	.999 959 270	1.000 001 828	.999 999 931
20	.989 608 672	1.000 683 330	.999 965 216	1.000 001 510	.999 999 944
21	.990 309 058	1.000 610 689	.999 970 095	1.000 001 256	.999 999 955
22	.990 934 156	1.000 548 406	.999 974 131	1.000 001 052	.999 999 963
23	.991 494 827	1.000 494 658	.999 977 495	1.000 000 886	.999 999 970
24	.992 000 000	1.000 448 000	.999 980 319	1.000 000 751	.999 999 975
25	.992 457 072	1.000 407 273	.999 982 704	1.000 000 641	.999 999 979
30	.994 206 281	1.000 265 269	.999 990 349	1.000 000 311	.999 999 991
35	.995 370 370	1.000 183 971	.999 994 149	1.000 000 166	.999 999 996
40	.996 190 884	1.000 133 693	.999 996 224	1.000 000 096	.999 999 998
45	.996 794 740	1.000 100 732	.999 997 442	1.000 000 059	.999 999 999
50	.997 254 353	1.000 078 115	.999 998 199	1.000 000 038	.999 999 999

simple working rule has been found. Usually the terms of the residue are more difficult to compute than are those of the original series.

If the value of α is small, it usually is simpler to calculate log D_0 by series than to interpolate in a table of natural logarithms. Specifically, by equation 19.12 and the usual series expansions,

$$D_0=1+\frac{1}{4}\alpha-\frac{1}{16}\alpha^2+\frac{1}{32}\alpha^3-\frac{3}{256}\alpha^4+\frac{7}{512}\alpha^5-\frac{21}{2048}\alpha^6+\frac{33}{4096}\alpha^7-\ldots \quad (20.11)$$

so that by equation 14.211

$$\log_e D_0=\frac{1}{4}\alpha-\frac{3}{32}\alpha^2+\frac{5}{96}\alpha^3-\frac{35}{1024}\alpha^4+\frac{63}{2560}\alpha^5-\frac{77}{4096}\alpha^6+\frac{429}{28\ 672}\alpha^7-\frac{6\ 435}{524\ 288}\alpha^8+\ldots \quad (20.12)$$

which may be written

$$\log_e D_0=\sum u_{k-1} \quad (20.21)$$

where

$$u_0=\frac{\alpha}{4} \quad (20.221)$$

$$\frac{u_1}{u_0}=-\frac{3\alpha}{8} \quad (20.222)$$

$$\frac{u_2}{u_1}=-\frac{5\alpha}{9} \quad (20.223)$$

$$\frac{u_3}{u_2}=-\frac{21\alpha}{32} \quad (20.224)$$

$$\frac{u_4}{u_3}=-\frac{18\alpha}{25} \quad (20.225)$$

$$\frac{u_5}{u_4}=-\frac{55\alpha}{72} \quad (20.226)$$

$$\frac{u_6}{u_5}=-\frac{39\alpha}{49} \quad (20.227)$$

$$\frac{u_7}{u_6}=-\frac{105\alpha}{128}. \quad (20.228)$$

Except for rounding errors, the errors do not exceed five units in the 10th decimal place for the approximations

$$\log_e D_0=\frac{1}{4}\alpha \qquad \text{for } \alpha<0.000\ 073\ 030 \quad (20.31)$$

$$\log_e D_0=\frac{1}{4}\alpha-\frac{3}{32}\alpha^2 \qquad \text{for } \alpha<0.002\ 125\ 317 \quad (20.32)$$

$$\log_e D_0=\frac{1}{4}\alpha-\frac{3}{32}\alpha^2+\frac{5}{96}\alpha^3 \text{ for } \alpha<0.010\ 997\ 665. \quad (20.33)$$

More generally,

$$\log_e D_0=\sum(-1)^{u-1}\frac{(2u-1)!}{2^{2u}(u!)^2}\alpha^u. \quad (20.4)$$

For $k>0$, equations 18.42 and 18.61 may be written

$$D_k=1+\sum_{u=1}^{\infty}(-1)^u\frac{(2k+2u-3)!(k-1)!}{2^{2u-1}u!(2k-2)!(k+u-2)!}\alpha^u \quad (20.51)$$

$$E_k=1+\sum_{u=1}^{A}(-1)^u\frac{(k-1)!}{2^{2u}u!u!(k-2u-1)!}\alpha^u, \quad (20.52)$$

where A is the largest integer not exceeding $(k-1)/2$. If equation 20.51 is written

$$D_k=1+\sum(-1)^u D_{ku}\alpha^u, \quad (20.53)$$

the values of D_{ku} may be calculated and tabulated as in table 12, which shows the values of D_{ku} for $u \leqq 8$ and even values of k not exceeding 10.

TABLE 12.—*Values of* D_{ku}

k	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$	$u=6$	$u=7$	$u=8$
2	$\frac{3}{2}$	$\frac{15}{8}$	$\frac{35}{16}$	$\frac{315}{128}$	$\frac{693}{256}$	$\frac{3\ 003}{1\ 024}$	$\frac{6\ 435}{2\ 048}$	$\frac{109\ 395}{32\ 768}$
4	$\frac{7}{2}$	$\frac{63}{8}$	$\frac{231}{16}$	$\frac{3\ 003}{128}$	$\frac{9\ 009}{256}$	$\frac{51\ 051}{1\ 024}$	$\frac{138\ 567}{2\ 048}$	$\frac{2\ 909\ 907}{32\ 768}$
6	$\frac{11}{2}$	$\frac{143}{8}$	$\frac{715}{16}$	$\frac{12\ 155}{128}$	$\frac{46\ 189}{256}$	$\frac{323\ 323}{1\ 024}$	$\frac{1\ 062\ 347}{2\ 048}$	$\frac{26\ 558\ 675}{32\ 768}$
8	$\frac{15}{2}$	$\frac{255}{8}$	$\frac{1615}{16}$	$\frac{33\ 915}{128}$	$\frac{156\ 009}{256}$	$\frac{1\ 300\ 075}{1\ 024}$	$\frac{5\ 014\ 575}{2\ 048}$	$\frac{145\ 422\ 675}{32\ 768}$
10	$\frac{19}{2}$	$\frac{399}{8}$	$\frac{3059}{16}$	$\frac{76\ 475}{128}$	$\frac{412\ 965}{256}$	$\frac{3\ 991\ 995}{1\ 024}$	$\frac{17\ 678\ 835}{2\ 048}$	$\frac{583\ 401\ 555}{32\ 768}$

The Euler-Maclaurin function may be written

$$Z(x)=-\sum A_k\alpha^k, \quad (20.61)$$

where

$$A_k=\sum_{u=0}^{\infty}\frac{A_{k,2u}}{x^{2u}}. \quad (20.62)$$

Values of $A_{k,2u}$ are shown in table 13 for $k \leqq 8$ and values of $2u$ not exceeding 10.

TABLE 13.—*Values of* $A_{k,2u}$

k	$2u$ equals—					
	0	2	4	6	8	10
1	$+\frac{1}{8}$	$-\frac{1}{8}$	$+\frac{7}{24}$	$-\frac{31}{24}$	$+\frac{381}{40}$	$-\frac{2555}{24}$
2	$-\frac{3}{64}$	$+\frac{5}{32}$	$-\frac{49}{64}$	$+\frac{93}{16}$	$-\frac{4191}{64}$	$+\frac{33\ 215}{32}$
3	$+\frac{5}{192}$	$-\frac{35}{192}$	$+\frac{49}{32}$	$-\frac{1705}{96}$	$+\frac{18\ 161}{64}$	$-\frac{1\ 162\ 525}{192}$
4	$-\frac{35}{2048}$	$+\frac{105}{512}$	$-\frac{2695}{1024}$	$+\frac{22\ 165}{512}$	$-\frac{1\ 906\ 905}{2\ 048}$	$+\frac{19\ 762\ 925}{768}$
5	$+\frac{63}{5\ 120}$	$-\frac{231}{1\ 024}$	$+\frac{21\ 021}{5\ 120}$	$-\frac{93\ 093}{1\ 024}$	$+\frac{6\ 483\ 477}{2\ 560}$	$-\frac{45\ 059\ 469}{512}$
6	$-\frac{77}{8\ 192}$	$+\frac{1001}{4\ 096}$	$-\frac{49\ 049}{8\ 192}$	$+\frac{527\ 527}{3\ 072}$	$-\frac{123\ 186\ 063}{20\ 480}$	$+\frac{525\ 693\ 805}{2\ 048}$
7	$+\frac{429}{57\ 344}$	$-\frac{2145}{8192}$	$+\frac{17\ 017}{2\ 048}$	$-\frac{4\ 295\ 577}{14\ 336}$	$+\frac{52\ 794\ 027}{4\ 096}$	$-\frac{2\ 714\ 296\ 585}{4\ 096}$
8	$-\frac{6\ 435}{1\ 048\ 576}$	$+\frac{36\ 465}{131\ 072}$	$-\frac{2\ 909\ 907}{262\ 144}$	$+\frac{64\ 433\ 655}{131\ 072}$	$-\frac{13\ 356\ 888\ 831}{524\ 288}$	$+\frac{203\ 572\ 243\ 875}{131\ 072}$

Noting that α_{ku} is not to be confused with α having no indices, the notation

$$\alpha_{ku}=-\frac{A_{k,2u}}{A_{2k,u-2}} \qquad (20.71)$$

$$u_{k0}=A_{k0} \qquad (20.721)$$

$$u_{k,2j}=-\frac{\alpha_{kj}u_{k,2j-2}}{x^2}, \qquad (20.722)$$

leads to the relation

$$A_k=\sum_{j=0}^{\infty} u_{k,2j}. \qquad (20.73)$$

Early values of $\alpha_{k,2j}$ are shown in table 14 with $\alpha_{k0}=A_{k0}$.

If a good table of natural logarithms is available, it may be preferable to write

$$-Z(x)=\frac{1}{2}\log_e D_0+\sum \bar{A}_k\alpha^k, \qquad (20.81)$$

where

$$\bar{A}_k=\sum_{j=1}^{\infty}\frac{A_{k,2j}}{x^{2j}}, \qquad (20.82)$$

so that, by equations 20.62 and 20.82,

$$A_k=A_{k0}+\bar{A}_k. \qquad (20.83)$$

LOCAL EXPANSIONS FOR $Q^2<1$

For points near the current pole, the axial displacement, l, of the test point, is small and the relative displacement, $a=l/(2h)$, likewise is small. For such points, the reduced reciprocal distance of equation 12.31 may be written

$$\psi_k=\sum(-1)^{u-1}H_u a^{2u}k^{-(2u+1)}, \qquad (21.11)$$

where

$$H_u=\frac{(2u)!}{2^{2u}(u!)^2} \qquad (21.121)$$

and the definition of equation 21.121 is extended to include

$$H_0=1. \qquad (21.122)$$

The value of H_u depends only on the value of u, and is shown as a common fraction for $u\leqq 24$ and as a decimal fraction for $u\leqq 32$, in table 15.

If the first $(n-1)$ terms of the modified potential are calculated directly and if w is an arbitrary constant, the notation

$$\beta=\left(\frac{a}{w}\right)^2 \qquad (21.21)$$

transforms equation 12.932 into

$$W=-\sum_{k=1}^{n-1}Q^k\psi_k+\sum_{k=n}^{\infty}Q^k\sum_{u=1}^{\infty}\frac{H_u(-1)^u w^{2u}\beta^u}{k^{2u+1}}, \qquad (21.221)$$

which is reducible to

$$W=-\sum_{k=1}^{n-1}Q^k\psi_k+\sum_{u=1}^{\infty}(-1)^u H_u w^{2u}\beta^u\sum_{j=0}^{\infty}\frac{Q^{n+j}}{(n+j)^{2u+1}}. \qquad (21.2221)$$

TABLE 14.—*Values of* $\alpha_{k,2j}$

k	$2j$ equals— 0	2	4	6	8	10
1	$+\frac{1}{8}$ +0.125	1 1	$\frac{7}{3}$ 2.333 333 333	$\frac{31}{7}$ 4.428 571 428	$\frac{1143}{155}$ 7.374 193 548	$\frac{12\,775}{1143}$ 11.176 727 91
2	$-\frac{3}{64}$ −0.046 875	$\frac{10}{3}$ 3.333 333 333	$\frac{49}{10}$ 4.9	$\frac{372}{49}$ 7.591 836 735	$\frac{1397}{124}$ 11.266 129 03	$\frac{66\,430}{4191}$ 15.850 632 30
3	$+\frac{5}{192}$ +0.026 041 667	7 7	$\frac{42}{5}$ 8.4	$\frac{1705}{147}$ 11.598 639 456	$\frac{4953}{310}$ 15.977 419 355	$\frac{89\,425}{4191}$ 21.337 389 64
4	$-\frac{35}{2048}$ −0.017 089 844	12 12	$\frac{77}{6}$ 12.833 333 333	$\frac{806}{49}$ 16.448 979 59	$\frac{2667}{124}$ 21.508 064 52	$\frac{347\,480}{12\,573}$ 27.636 999 92
5	$+\frac{63}{5120}$ +0.012 304 688	$\frac{55}{3}$ 18.333 333 333	$\frac{91}{5}$ 18.2	$\frac{155}{7}$ 22.142 857 14	$\frac{4318}{155}$ 27.858 064 42	$\frac{48\,545}{1397}$ 34.749 463 14
6	$-\frac{77}{8192}$ −0.009 399 414	26 26	$\frac{49}{2}$ 24.5	$\frac{4216}{147}$ 28.680 272 11	$\frac{21\,717}{620}$ 35.027 419 35	$\frac{178\,850}{4191}$ 42.674 779 29
7	$+\frac{429}{57\,344}$ +0.007 481 166	35 35	$\frac{476}{15}$ 31.733 333 333	$\frac{1767}{49}$ 36.061 224 49	$\frac{2667}{62}$ 43.016 129 03	$\frac{58\,765}{1143}$ 51.412 948 38
8	$-\frac{6435}{1\,048\,576}$ −0.006 136 894	$\frac{136}{3}$ 45.333 333 333	$\frac{399}{10}$ 39.9	$\frac{310}{7}$ 44.285 714 29	$\frac{32\,131}{620}$ 51.824 193 55	$\frac{255\,500}{4191}$ 60.963 970 41

TABLE 15.—*Values of* H_u

u	Decimal	Numerator	Denominator
1	0.500 000 0000	1	2
2	.375 000 0000	3	8
3	.312 500 0000	5	16
4	.273 437 5000	35	128
5	.246 093 7500	63	256
6	.225 585 9375	231	1 024
7	.209 472 6562	429	2 048
8	.196 380 6152	6 435	32 768
9	.185 470 5811	12 155	65 536
10	.176 197 0520	46 189	262 144
11	.168 188 0951	88 179	524 288
12	.161 180 2578	676 039	4 194 304
13	.154 981 0171	1 300 075	8 388 608
14	.149 445 9808	5 014 575	33 554 432
15	.144 464 4481	9 694 845	67 108 864
16	.139 949 9341	300 540 195	2 147 483 648
17	.135 833 7596	583 401 555	4 294 967 296
18	.132 060 5996	2 268 783 825	17 179 869 184
19	.128 585 3206	4 418 157 975	34 359 738 368
20	.125 370 6876	34 461 632 205	274 877 906 944
21	.122 385 6712	67 282 234 305	549 755 813 888
22	.119 604 1787	263 012 370 465	2 199 023 255 552
23	.117 004 0879	514 589 420 475	4 398 046 511 104
24	.114 566 5027	8 061 900 920 775	70 368 744 177 664
25	.112 275 1727	----	----
26	.110 116 0347	----	----
27	.108 076 8489	----	----
28	.106 146 9052	----	----
29	.104 316 7861	----	----
30	.102 578 1730	----	----
31	.100 923 6863	----	----
32	.099 346 7537	----	----

Upon isolating the term $j=0$, equation 21.2221 becomes

$$W=-\sum_{k=1}^{n-1} Q^k\psi_k+\sum_{u=1}^{\infty}(-1)^u H_u w^{2u}\beta^u\left[\frac{Q^n}{n^{2u+1}}+\sum_{j=1}^{\infty}\frac{Q^{n+j}}{(n+j)^{2u+1}}\right], \tag{21.2222}$$

which may be written

$$W=-\sum_{k=1}^{n-1} Q^k\psi_k+\sum_{u=1}^{\infty}(-1)^u\frac{H_u w^{2u}\beta^u Q^n}{n^{2u+1}}\left[1+n^{2u+1}\sum_{j=1}^{\infty}\frac{Q^j}{(n+j)^{2u+1}}\right]. \tag{21.2223}$$

As u increases indefinitely, the quantity in the square brackets approaches unity so that the summand in the second sum approaches

$$(-1)^u\left(\frac{H_u}{n}\right)\left(\frac{w}{n}\right)^{2u} Q^n\beta^u,$$

which suggests that w should be selected as having the value n, for which choice

$$W=-\sum_{k=1}^{n-1} Q^k\psi_k+\sum_{u=1}^{\infty}(-1)^u\left(\frac{H_u}{n}\right)Q^n\beta^u\left[1+n^{2u+1}\sum_{j=1}^{\infty}\frac{Q^j}{(n+j)^{2u+1}}\right] \tag{21.2224}$$

and equation 21.21 becomes

$$\beta=\left(\frac{a}{n}\right)^2. \quad (21.23)$$

For systematic calculation, it is convenient to write

$$Q=\epsilon q \quad (21.31)$$

where $\epsilon=1$ for a buried insulator and $\epsilon=-1$ for a buried conductor. The notation

$$S_m^\alpha=\sum_{j=0}^{\infty}\frac{q^{m+2j}}{(m+2j)^\alpha} \quad (21.321)$$

leads to

$$\sum_{j=0}^{\infty}\frac{Q^{n+j}}{(n+j)^{2n+1}}=\epsilon^n S_n^{2u+1}+\epsilon^{u+1}S_{n+1}^{2u+1}, \quad (21.322)$$

so that equation 21.2221 becomes, for $w=n$,

$$W=-\sum_{k=1}^{n-1}Q^k\psi_k+\sum_{u=1}^{\infty}(-1)^u H_u n^{2u}\beta^u\epsilon^n\left(S_n^{2u+1}+\epsilon S_{n+1}^{2u+1}\right). \quad (21.331)$$

With the notation

$$C_{nu}=(-1)^{u-1}H_u n^{2u}\epsilon^n(S_n^{2u+1}+\epsilon S_{n+1}^{2u+1}) \quad (21.332)$$

equation 21.331 may be written

$$-W=\sum_{k=1}^{n-1}Q^k\psi_k+\sum_{u=1}^{\infty}C_{nu}\beta^u. \quad (21.333)$$

For selected values of q and n, the values of C_{nu} may be tabulated for values of u. If powers of β are available, equation 21.333 is convenient for small values of β, as the remainder series converges rapidly, especially if n is selected as greater than a.

If powers of β are not available, a ratio form is more convenient. The notation

$$\lambda_{nu}=-\frac{C_{n,u+1}}{C_{nu}} \quad (21.411)$$

leads to

$$C_{n,u+1}\beta^{u+1}=-\lambda_{nu}(C_{nu}\beta^u)\beta, \quad (21.412)$$

which starts the recursion sequence with

$$C_{n1}=\frac{1}{2}n^2\epsilon^n(S_n^3+\epsilon S_{n+1}^3) \quad (21.413)$$

Because of equations 21.12 and 21.332, the ratio of equation 21.411 may be written

$$\lambda_{nu}=n^2\left(\frac{2u+1}{2u+2}\right)\left(\frac{S_n^{2u+3}+\epsilon S_{n+1}^{2u+3}}{S_n^{2u+1}+\epsilon S_{n+1}^{2u+1}}\right). \quad (21.42)$$

As the value of u increases indefinitely the value of C_{nu} approaches the limit

$$C_n=(-1)^{u-1}\frac{H_uQ^n}{n} \quad (21.511)$$

and the ratio λ_{nu} approaches the limit

$$\lambda_n=\frac{2u+1}{2u+2}. \quad (21.512)$$

The series for S_m^α usually converges rapidly so that it frequently is simpler to calculate

$$\mu_{nn}=n^{2u+1}\sum_{j=1}^{\infty}\frac{Q^j}{(n+j)^{2u+1}}, \quad (21.521)$$

in terms of which

$$C_{nu}=(-1)^{u-1}H_uQ^n\left(\frac{1+\mu_{nu}}{n}\right) \quad (21.522)$$

and

$$\lambda_{nu}=\left(\frac{2u+1}{2u+2}\right)\left(\frac{1+\mu_{n,u+1}}{1+\mu_{nu}}\right). \quad (21.523)$$

As u increases indefinitely, μ_{nu} approaches zero, leading again to the limits of equations 21.51.

Omitting the constant subscript n, for a selected value of v and large value of u, λ_{nu} approaches the value

$$\lambda_u=\frac{2u+1}{2u+2}=1-\phi_{u+1-v}, \quad (21.611)$$

where

$$\phi_k=\frac{1}{2(v+k)}. \quad (21.612)$$

Dropping the subscript n, equation 21.411 may be written

$$C_{u+1}=-\lambda_uC_u. \quad (21.613)$$

In particular,

$$C_{v+1}=-\lambda_vC_v=-(1-\phi_1)C_v \quad (21.621)$$

$$C_{v+2}=-\lambda_{v+1}C_{v+1}=(1-\phi_2)(1-\phi_1)C_v \quad (21.622)$$

$$C_{v+3}=-(1-\phi_3)(1-\phi_2)(1-\phi_1)C_v \quad (21.623)$$

$$C_{v+i}=(-1)^i(1-\phi_i)(1-\phi_{i-1})(1-\phi_{i-2})\ldots(1-\phi_1)C_v \quad (21.624)$$

If the first $(n-1)$ terms of the modified potential are calculated directly, the remainder has the value, by equation 21.333, upon dropping the subscript n,

$$R_v=-\sum_{u=1}^{v}C_u\beta^u-\sum_{u=v+1}^{\infty}C_u\beta^u, \quad (21.631)$$

which may be written

$$R_v=-\sum_{u=1}^{v}C_u\beta^u+C_v\beta^vS, \quad (21.632)$$

where

$$S=\sum_{i=1}^{\infty}(-1)^i(1-\phi_i)(1-\phi_{i-1})\ldots(1-\phi_2)(1-\phi_1)\beta^i. \quad (21.633)$$

The value of S is expressible as a power series in β, and the various ϕ_k. Specifically, the term free of ϕ_k has the value

$$S_o=\sum_{i=1}^{\infty}(-1)^{i-1}\beta^i. \tag{21.641}$$

For $Q^2<1$, and $n>a$, $\beta<1$, so that

$$S_0=\frac{\beta}{1+\beta}. \tag{21.642}$$

The terms of S which are linear in ϕ_k have the sum

$$S_1=\sum_{i=1}^{\infty}(-1)^i\beta^i\sum_{j=1}^{i}\phi_j. \tag{21.651}$$

Reversal of the order of summation transforms this relation to

$$S_1=\sum_{j=1}^{\infty}\phi_j\sum_{i=j}^{\infty}(-1)^i\beta^i. \tag{21.652}$$

Summation in i reduces S_1 to

$$S_1=\sum_{j=1}^{\infty}\frac{(-1)^j\phi_j\beta^j}{1+\beta}, \tag{21.653}$$

which may be written, by equation 21.612, in the form

$$S_1=\frac{1}{2(1+\beta)}\sum_{j=1}^{\infty}\frac{(-1)^j\beta^j}{v+j}. \tag{21.654}$$

Separating the even and odd terms, this equation may be written

$$S_1=-\frac{1}{2(1+\beta)}\sum\left[1-\beta+\frac{\beta}{v+2j}\right]\frac{\beta^{2j-1}}{v+2j-1}, \tag{21.655}$$

in which all of the summands are positive, and converge conveniently to zero as j increases, for small values of β.

For values of β near unity, another form is preferable. Upon retarding the counter by v, equation 21.654 becomes

$$S_1=\frac{(-1)^v}{2(1+\beta)}\sum_{j=v+1}^{\infty}\frac{(-1)^j\beta^{j-v}}{j}, \tag{21.656}$$

which may be written

$$S_1=\frac{(-1)^v}{2(1+\beta)\beta^v}\left\{\sum_{j=1}^{\infty}\frac{(-1)^j\beta^j}{j}-\sum_{j=1}^{v}\frac{(-1)^j\beta^j}{j}\right\}. \tag{21.6562}$$

But

$$\sum_{j=1}^{\infty}\frac{(-1)^{j-1}\beta^j}{j}=\log_e(1+\beta), \tag{21.6563}$$

so that

$$S_1=\frac{(-1)^{v-1}}{2(1+\beta)\beta^v}\left\{\log_e(1+\beta)-\sum_{j=1}^{v}\frac{(-1)^{j-1}\beta^j}{j}\right\}. \tag{21.6564}$$

The notation

$$\theta_v=\log_e(1+\beta)-\sum_{j=1}^{v}\frac{(-1)^{j-1}\beta^j}{j} \tag{21.6565}$$

reduces equation 21.6564 to

$$S_1=\frac{(-1)^{v-1}\theta_v}{2(1+\beta)\beta^v}. \tag{21.6566}$$

The terms of S which are quadratic in ϕ_k have the sum

$$S_2=\sum_{i=2}^{\infty}(-1)^{i-1}\beta^i\sum_{j=1}^{i-1}\phi_j\sum_{k=j+1}^{i}\phi_k. \tag{21.661}$$

Interchanging the j and k summations, this becomes

$$S_2=\sum_{i=2}^{\infty}\sum_{k=2}^{\infty}\sum_{j=1}^{k-1}(-1)^{i-1}\phi_j\phi_k\beta^i. \tag{21.662}$$

Interchanging the i and k summations, this becomes

$$S_2=\sum_{k=2}^{\infty}\sum_{i=k}^{\infty}\sum_{j=1}^{k-1}(-1)^{i-1}\phi_j\phi_k\beta^i. \tag{21.663}$$

Since the limits of i and j are mutually independent, this may be written

$$S_2=\sum_{k=2}^{\infty}\sum_{j=1}^{k-1}\sum_{i=k}^{\infty}(-1)^{i-1}\phi_j\phi_k\beta^i, \tag{21.664}$$

and the summation in i may be performed, leading to

$$S_2=\frac{1}{1+\beta}\sum_{k=2}^{\infty}\sum_{j=1}^{k-1}(-1)^{k-1}\beta^k\phi_j\phi_k. \tag{21.665}$$

By equation 21.612 this is equivalent to

$$S_2=\frac{1}{4(1+\beta)}\sum_{k=2}^{\infty}\sum_{j=1}^{k-1}\frac{(-1)^{k-1}\beta^k}{(v+j)(v+k)}. \tag{21.666}$$

For small values of β, equation 21.666 is convenient. For values of β near unity, a procedure similar to that used for S_1, is convenient. Interchanging the j and k summations, equation 21.666 becomes

$$S_2=\frac{1}{4(1+\beta)}\sum_{j=1}^{\infty}\sum_{k=j+1}^{\infty}\frac{(-1)^{k-1}\beta^k}{(v+j)(v+k)}. \tag{21.6671}$$

If the k counter is retarded by v, this becomes

$$S_2=\frac{1}{4(1+\beta)}\sum_{j=1}^{\infty}\sum_{k=v+j+1}^{\infty}\frac{(-1)^{k-v-1}\beta^{k-v}}{(v+j)k}, \tag{21.6672}$$

which may be written

$$S_2=\frac{(-1)^v}{4(1+\beta)\beta^v}\sum_{j=1}^{\infty}\frac{1}{v+j}\left\{\sum_{k=1}^{\infty}\frac{(-1)^{k-1}\beta^k}{k}-\sum_{k=1}^{v+j}\frac{(-1)^{k-1}\beta^k}{k}\right\}. \tag{21.6673}$$

By equations 21.6563 and 21.6565 this becomes

$$S_2=\frac{(-1)^v}{4(1+\beta)\beta^v}\sum_{j=1}^{\infty}\frac{\theta_{v+j}}{v+j}. \tag{21.6674}$$

Retarding the counter by v, this becomes

$$S_2=\frac{(-1)^v}{4(1+\beta)\beta^v}\sum_{j=v+1}^{\infty}\frac{\theta_j}{j}, \tag{21.6675}$$

which may be written

$$S_2=\frac{(-1)^v}{4(1+\beta)\beta^v}\left\{\sum_{j=1}^{\infty}\frac{\theta_j}{j}-\sum_{j=1}^{v}\frac{\theta_j}{j}\right\}. \tag{21.6676}$$

Summarizing, the first $(n-1)$ terms of W are calculated by the products $-Q^k\psi_k$, the next v terms are calculated by the product $-C_{nu}\beta^u$, and the residue is calculated as $-C_{nv}\beta^v(S_0+S_1+S_2)$. Additional values of S_k are not needed as the method is used only when the term $C_{nv}\beta^v$ is numerically small. The values of S_k need not be calculated with great accuracy. The value of S_0 may be calculated directly. The residue factors are independent of Q and are needed only for values of β near unity. However, Q enters the residue through the coefficient $C_{nv}\beta^v$. Table 16 shows the values of S_0, S_1, S_2, and S to 6 decimals, for a/n on the range [0.80(0.01) 1.00] for $v=30$. If intermediate values are needed, S_0 should be calculated directly, S_1 may be obtained by interpolation in table 16, and S_2 may be taken directly from the table. The value of S is the sum of S_0, S_1, and S_2.

TABLE 16.—Residue factors for $v=30$

[Last figure in columns $-S_1$ and $-S_2$ are in sixth decimal place]

a/n	$+S_0$	$-S_1$	$-S_2$	Factor
0.80	0.390 244	3886	24	0.386 334
.81	.396 172	3907	25	.392 240
.82	.402 057	3928	25	.398 104
.83	.407 899	3947	26	.403 926
.84	.413 696	3964	26	.409 706
.85	.419 448	3980	27	.415 441
.86	.425 155	3996	27	.421 132
.87	.430 816	4010	28	.426 778
.88	.436 429	4023	28	.432 378
.89	.441 995	4034	29	.437 932
.90	.447 514	4045	29	.443 440
.91	.452 984	4055	29	.448 900
.92	.458 406	4064	30	.454 312
.93	.463 778	4071	30	.459 677
.94	.469 102	4078	31	.464 993
.95	.474 376	4083	31	.470 262
.96	.479 600	4088	31	.475 481
.97	.484 775	4091	32	.480 652
.98	.489 900	4094	32	.485 774
.99	.494 975	4096	33	.490 846
1.00	.500 000	4097	33	.495 870

In preparing table 16, the values of S_0 were computed for each entry. The values of S_1 and S_2 were calculated on the range $a/n=[0.80(0.05)1.00]$. The intermediate values were interpolated from a fourth degree polynomial passing through the five points, with adjustments when needed.

ASYMPTOTIC EXPANSIONS

ASYMPTOTIC EXPANSIONS FOR $Q^2<1$

The modified potential function has been defined as

$$W(Q,a)=-\sum Q^k\psi_k(a), \tag{12.932}$$

where

$$Q=\frac{\rho_b-\rho_0}{\rho_b+\rho_0} \tag{12.5}$$

$$\psi_k(a)=\frac{1}{k}-\frac{1}{\sqrt{k^2+a^2}}. \tag{12.31}$$

$$a=\frac{l}{2h} \tag{12.22}$$

The limiting value of the modified potential as l increases indefinitely is

$$W_\infty=-\sum\frac{Q^k}{k}, \tag{14.1}$$

so that

$$W=W_\infty+\sum\frac{Q^k}{\sqrt{k^2+a^2}}. \tag{22.1}$$

For $a>k$,

$$\frac{1}{\sqrt{k^2+a^2}}=\frac{1}{a}\left[1-\frac{1}{2}\left(\frac{k}{a}\right)^2+\frac{3}{8}\left(\frac{k}{a}\right)^4-\frac{5}{16}\left(\frac{k}{a}\right)^6+\frac{35}{128}\left(\frac{k}{a}\right)^8+\ldots\right], \tag{22.2}$$

so that

$$W_\infty-W=\sum B_n a^{1-2n}, \tag{22.31}$$

where

$$B_1=-\sum Q^k \tag{22.321}$$

$$B_2=\frac{1}{2}\sum k^2Q^k \tag{22.322}$$

$$B_3=-\frac{3}{8}\sum k^4Q^k \tag{22.323}$$

$$B_4=\frac{5}{16}\sum k^6Q^k \tag{22.324}$$

$$B_5=-\frac{35}{128}\sum k^8Q^k, \tag{22.325}$$

with similar expressions for higher order terms.

Each of the coefficients B_n is of the form $\Sigma k^{2n-2}Q^k$ with a constant factor. The ratio of the $(k+1)$-th term to the k-th term in B_n is $Q[(k+1)/k]^{2n-2}$. Although the coefficient of Q in this ratio exceeds unity, it approaches unity as k increases indefinitely so that for $Q^2<1$, B_n converges. However, only the early values of n lead to values of B_n that are convenient for calculation.

Excluding the cases $Q^2=1$, the values of B_n may be evaluated directly. Specifically, B_1 is a geometric series of ratio Q and first term $-Q$. Accordingly

$$B_1=-\frac{Q}{1-Q}. \tag{22.41}$$

As B_2 involves the factor k^2, whose third difference vanishes, it is convenient to isolate the initial terms and write

$$2B_2=Q+4Q^2+9Q^3+\sum(k+3)^2Q^{k+3} \tag{22.511}$$

$$2B_2=Q+4Q^2+\sum(k+2)^2Q^{k+2} \tag{22.512}$$

$$2B_2=Q+\sum(k+1)^2Q^{k+1} \tag{22.513}$$

$$2B_2=\sum k^2Q^k. \tag{22.514}$$

The third difference of k^2 is $(k+3)^2-3(k+2)^2+3(k+1)^2-k^2$, and the exponents of the summations in equations 22.5 may be made $k+3$ by multiplying by 1, Q, Q^2, and Q^3, respectively, leading to

$$2B_2=Q+4Q^2+9Q^3+\sum(k+3)^2Q^{k+3} \tag{22.521}$$

$$-3Q(2B_2)=-3Q^2-12Q^3-\sum 3(k+2)^2Q^{k+3} \tag{22.522}$$

$$3Q^2(2B_2)=3Q^3+\sum 3(k+1)^2Q^{k+3} \tag{22.523}$$

$$-Q^3(2B_2)=-\sum k^2Q^{k+3} \tag{22.524}$$

Addition of equations 22.52 leads to

$$(1-Q)^3(2B_2)=Q+Q^2, \tag{22.531}$$

so that

$$B_2=\frac{Q(1+Q)}{2(1-Q)^3}. \tag{22.532}$$

A procedure similar to the determination of B_2 leads to

$$B_3=-\frac{3Q(1+Q)(1+10Q+Q^2)}{8(1-Q)^5} \tag{22.541}$$

$$B_4=\frac{5Q(1+Q)(1+56Q+246Q^2+56Q^3+Q^4)}{16(1-Q)^7} \tag{22.542}$$

$$B_5=-\frac{\begin{bmatrix}35Q(1+Q)(1+246Q+4{,}047Q^2\\+11{,}572Q^3+4{,}047Q^4+246Q^5+Q^6\end{bmatrix}}{128(1-Q)^9}. \tag{22.543}$$

The notation

$$v=2\left(\frac{1}{Q}-1\right) \tag{22.551}$$

$$u=Q+\frac{1}{Q} \tag{22.552}$$

transforms equations 22.53 and 22.54 to

$$B_1=-\frac{2}{v} \tag{22.561}$$

$$B_2=\frac{4(1+Q)}{Q^2v^3} \tag{22.562}$$

$$B_3=-\frac{12(1+Q)(u+10)}{Q^3v^5} \tag{22.563}$$

$$B_4=\frac{40(1+Q)(u^2+56u+244)}{Q^4v^7} \tag{22.564}$$

$$B_5=-\frac{140(1+Q)(u^3+246u^2+4{,}044u+11{,}080)}{Q^5v^9} \tag{22.565}$$

Each quantity B_n depends only on Q so that numerical values may be calculated. For $Q^2<1$, the first five values of B_n are shown in table 1, as noted on page 8.

As Q approaches unity, v approaches zero and u approaches 2, so that each value of B_n becomes infinite and the usefulness of these asymptotic expansions disappears. As Q approaches minus one, v approaches -4 and u approaches -2 so that B_1 approaches one-half and all other B_n vanish. This property is convenient for large displacements but its validity may need verification for small displacements.

The use of asymptotic expansions may lead to divergent series as the value of n increases. Usually an asymptotically divergent series has a remainder less than twice the first neglected term, so that the evaluation should stop after the term ahead of the smallest has been calculated. If the first neglected term is sufficiently small, the series may be used for calculation.

ASYMPTOTIC EXPANSIONS FOR $Q=+1$

If the underlying bed is a perfect insulator, $\rho_b=\infty$ and $Q=1$, so that the modified potential function of equation 12.932 may be written by equation 12.31,

$$W=-\sum\left(\frac{1}{k}-\frac{1}{\sqrt{k^2+a^2}}\right). \tag{23.1}$$

In calculating this function, the Euler-Maclaurin summation formula uses the integral and odd order derivatives of the function

$$f(x)=\frac{1}{\sqrt{x^2+a^2}}. \tag{23.21}$$

Direct differentiation shows that

$$f'(x)=-x(x^2+a^2)^{-3/2} \tag{23.22}$$

$$f''(x)=(2x^2-a^2)(x^2+a^2)^{-5/2} \tag{23.23}$$

$$f'''(x)=-(6x^3-9a^2x)(x^2+a^2)^{-7/2}. \tag{23.24}$$

These derivatives suggest the relation

$$f^{(k)}(x)=(-1)^k(x^2+a^2)^{-k-1/2}\sum_{u=0}^{k}A_{ku}a^ux^{k-u}, \tag{23.31}$$

where A_{ku} must be determined for $k\geqq 1$ and $0\leqq u\leqq k$. By differentiation, equation 23.31 leads to

$$f^{(k+1)}(x)=(-1)^{k+1}(x^2+a^2)^{-k-3/2}\left\{\begin{array}{l}\sum_{u=0}^{k}(2k+1)A_{ku}a^ux^{k+1-u}\\ -\sum_{u=0}^{k-1}(k-u)A_{ku}a^ux^{k+1-u}\\ -\sum_{u=0}^{k-1}(k-u)A_{ku}a^{u+2}x^{k-1-u}\end{array}\right\}. \quad (23.321)$$

Replacing k by $(k+1)$, equation 23.31 becomes

$$f^{(k+1)}(x)=(-1)^{k+1}(x^2+a^2)^{-k-3/2}\sum_{u=0}^{k+1}A_{k+1,u}a^ux^{k+1-u}. \quad (23.322)$$

Equating the right members of equations 23.32 and dividing by the common factors leads to

$$\sum_{u=0}^{k+1}A_{k+1,u}a^ux^{k+1-u}=\sum_{u=0}^{k}(2k+1)A_{ku}a^ux^{k+1-u}-\sum_{u=0}^{k-1}(k-u)A_{ku}a^ux^{k+1-u}-\sum_{u=0}^{k-1}(k-u)A_{ku}a^{u+2}x^{k+1-u}. \quad (23.331)$$

Upon retarding the counter of the final summation by 2, this becomes

$$\sum_{u=0}^{k+1}A_{k+1,u}a^ux^{k+1-u}=\sum_{u=0}^{k}(2k+1)A_{ku}a^ux^{k+1-u}-\sum_{u=0}^{k-1}(k-u)A_{ku}a^ux^{k+1-u}-\sum_{u=2}^{k+1}(k+2-u)A_{k,u-2}a^ux^{k+1-u}. \quad (23.332)$$

Upon equating coefficients of corresponding terms, this equation determines the conditions:

$$A_{k+1,0}=(k+1)A_{k0} \quad (23.341)$$

$$A_{k+1,1}=(k+2)A_{k1} \quad (23.342)$$

$$A_{k+1,u}=(k+1+u)A_{ku}-(k+2-u)A_{k,u-2} \quad (2\leqq u\leqq k) \quad (23.343)$$

$$A_{k+1,k+1}=-A_{k,k-1}. \quad (23.344)$$

For positive integral values of k, equations 23.34 are satisfied by the values

$$A_{k,2u-1}=0 \quad \text{where } 1\leqq 2u-1\leqq k \quad (23.351)$$

$$A_{k,2u}=(-1)^u\frac{(k!)^2}{(u!)^2 2^{2u}(k-2u)!} \quad \text{where } 0\leqq 2u\leqq k. \quad (23.352)$$

Specifically, equations 23.35 take the special forms

$$A_{k0}=k! \quad (23.361)$$

$$A_{2k-1,2k-1}=0 \quad (23.362)$$

$$A_{2k,2k}=(-1)^k\left[\frac{(2k)!}{2^kk!}\right]^2. \quad (23.363)$$

By equations 23.35 and 23.36 the derivative of equation 23.31 may be written

$$f^{(k)}(x)=(-1)^k(x^2+a^2)^{-k-\frac{1}{2}}\sum_{u=0}^{\frac{1}{2}k}A_{k,2u}a^{2u}x^{k-2u}. \quad (23.37)$$

The Eulerian constant is defined as

$$\Phi=\lim_{t\to\infty}\Phi_t, \quad (23.411)$$

where

$$\Phi_t=\sum_{k=1}^{t}\frac{1}{k}-\log_e t. \quad (23.412)$$

To 16 decimals (Jolley, 1925, p. 19), the value of the Eulerian constant is

$$\Phi=0.5772\,1566\,4901\,5325. \quad (23.413)$$

Equation 23.412 may be written

$$\sum_{k=1}^{t}\frac{1}{k}=\Phi_t+\log_e t, \quad (23.421)$$

so that

$$\sum_{k=s}^{t}\frac{1}{k}=\Phi_t+\log_e t-\sum_{k=1}^{s-1}\frac{1}{k}. \quad (23.422)$$

For $h=\frac{1}{2}$, $k=0$, the Euler-Maclaurin summation formula of equation 16.33 reduces to

$$\sum_{i=s}^{t}f(i)=Z\left(t+\frac{1}{2}\right)-Z\left(s-\frac{1}{2}\right), \quad (23.431)$$

where equation 16.12 reduces to

$$Z(x)=\int f(x)\,dx+\sum_{u=1}^{\infty}\frac{b_{2u}}{2^{2u}}f^{2u-1}(x). \quad (23.432)$$

Accordingly, by equations 23.21, 23.37, and 23.43,

$$\sum_{k=s}^{t}\frac{1}{\sqrt{k^2+a^2}}=Z\left(t+\frac{1}{2}\right)-Z\left(s-\frac{1}{2}\right), \quad (23.441)$$

where

$$Z(x)=\log_e(x+\sqrt{x^2+a^2})+\sum_{u=1}^{\infty}\frac{b_{2u}}{2^{2u}}(x^2+a^2)^{\frac{1}{2}-2u}\sum_{v=0}^{u=1}A_{2u-1,2v}a^{2v}x^{2u-2v-1}. \quad (23.442)$$

Combination of equations 23.42 and 23.44 leads to

$$\sum_{k=1}^{t}\psi_k=\Phi_t+\log_e t-Z\left(t+\frac{1}{2}\right)+Z\left(\frac{1}{2}\right) \quad (23.451)$$

$$\sum_{k=s}^{t}\psi_k=\Phi_t+\log_e t-\sum_{k=1}^{s-1}\frac{1}{k}-Z\left(t+\frac{1}{2}\right)+Z\left(s-\frac{1}{2}\right). \quad (23.452)$$

For large values of t, Φ_t approaches Φ and $\log_e(x+\sqrt{x^2+a^2})$ approaches $\log_e(2x)$. For specific values of a, u, and v, the summand in equation 23.442 approaches a finite constant multiplied by x^{-2u-2v} and accordingly approaches zero as x becomes infinite. Hence $Z(x)$ approaches $\log_e(2x)$, and equations 23.1 and 23.45 lead, for the limiting values, to

$$-W=\sum_{k=1}^{\infty}\psi_k=\Phi-\log_e 2+Z\left(\frac{1}{2}\right) \qquad (23.461)$$

$$\sum_{k=s}^{\infty}\psi_k=\Phi-\log_e 2-\sum_{k=1}^{s-1}\frac{1}{k}+Z\left(s-\frac{1}{2}\right). \qquad (23.462)$$

For small values of x, corresponding to large values of a, Maclaurin's expansion may be used for $Z(x)$. As

$$\frac{d}{dx}\log_e(x+\sqrt{x^2+a^2})=\frac{1}{\sqrt{x^2+a^2}}, \qquad (23.51)$$

the notation

$$\phi(x)=\log_e(x+\sqrt{x^2+a^2}) \qquad (23.521)$$

leads to the relations

$$\phi'(x)=f(x) \qquad (23.5221)$$

$$\phi^{(k+1)}(x)=f^{(k)}(x), \qquad (23.5222)$$

where $f(x)$ is the function defined by equation 23.21. Accordingly, by equations 23.35, 23.36, 23.37, 23.521 and 23.522,

$$\phi(0)=\log_e a \qquad (23.531)$$

$$\phi'(0)=1/a \qquad (23.532)$$

$$\phi^{(2u)}(0)=0 \qquad (23.533)$$

$$\phi^{(2u+1)}(0)=A_{2u,\,2u}\,a^{-2u-1}. \qquad (23.534)$$

If the definition of A_{jk} is extended to include

$$A_{00}=1, \qquad (23.541)$$

equations 23.532 and 23.534 may be combined to

$$\phi^{(2u-1)}=A_{2u-2,2u-2}\,a^{-2u+1}. \qquad (23.542)$$

Accordingly, Maclaurin's series leads to

$$\log_e(x+\sqrt{x^2+a^2})=\log_e a+\sum_{u=1}^{\infty}\frac{A_{2u-2,2u-2}}{(2u-1)!}\left(\frac{x}{a}\right)^{2u-1}. \qquad (23.55)$$

Combining of equations 23.442 and 23.55 results in the relation

$$Z(x)=\log_e a+\sum_{u=1}^{\infty}\frac{A_{2u-2,2u-2}}{(2u-1)!}\left(\frac{x}{a}\right)^{2u-1}$$
$$-\sum_{u=1}^{\infty}\frac{b_{2u}}{2^{2u}(x^2+a^2)^{2u-\frac{1}{2}}}\sum_{v=0}^{u-1}A_{2u-1,2v}a^{2v}x^{2u-2v-1}. \qquad (23.56)$$

By equations 18.42 and 20.51,

$$(1+\alpha)^{\frac{1}{2}-k}=\sum_{v=0}^{\infty}(-1)^v\frac{(2k+2v-3)!(k-1)!}{2^{2v-1}v!(2k-2)!(k+v-2)!}\alpha^v, \qquad (23.571)$$

so that

$$(x^2+a^2)^{\frac{1}{2}-2u}=a^{1-4u}\sum_{v=0}^{\infty}(-1)^v\frac{(4u+2v-3)!(2u-1)!}{2^{2v-1}v!(4u-2)!(2u+v-2)!}\left(\frac{x}{a}\right)^{2v}. \qquad (23.572)$$

The notation

$$G_{u,v-1}=(-1)^{v-1}\frac{(4u+2v-5)!(2u-1)!}{2^{2v-3}(v-1)!(4u-2)!(2u+v-3)!} \qquad (23.581)$$

reduces equation 23.572 to

$$(x^2+a^2)^{\frac{1}{2}-2u}=a^{1-4u}\sum_{v=0}^{\infty}G_{uv}\left(\frac{x}{a}\right)^{2v}, \qquad (23.582)$$

so that

$$Z(x)=\log_e a+\sum_{u=1}^{\infty}\frac{A_{2u-2,2u-2}}{(2u-1)!}\left(\frac{x}{a}\right)^{2u-1}$$
$$-\sum_{u=1}^{\infty}\sum_{w=0}^{\infty}\sum_{v=0}^{u-1}\frac{b_{2u}}{2^{2u}x^{2u}}A_{2u-1,2v}G_{uw}\left(\frac{x}{a}\right)^{4u+2w-2v-1}. \qquad (23.583)$$

If the term $u=1$ is isolated and the counter advanced by unity in the first summation, equation 23.583 becomes

$$Z(x)=\log_e a+\frac{x}{a}+\sum_{u=1}^{\infty}\frac{A_{2u,2u}}{(2u+1)!}\left(\frac{x}{a}\right)^{2u+1}$$
$$-\sum_{u=1}^{\infty}\sum_{w=0}^{\infty}\sum_{v=0}^{u-1}\frac{b_{2u}}{(2x)^{2u}}A_{2u-1,2v}G_{uw}\left(\frac{x}{a}\right)^{4u+2w-2v-1}. \qquad (23.584)$$

For $x=1/2$, this determines

$$Z\left(\frac{1}{2}\right)=\log_e a+\frac{1}{2a}+\sum_{u=1}^{\infty}\frac{A_{2u,2u}}{(2u+1)!}\left(\frac{1}{2a}\right)^{2u+1}$$
$$-\sum_{u=1}^{\infty}\sum_{w=0}^{\infty}\sum_{v=1}^{u-1}b_{2u}A_{2u-1,2v}G_{uw}\left(\frac{1}{2a}\right)^{4u+2w-2v-1}. \qquad (23.591)$$

If the counters v and w in the triple summation are retarded by unity, this equation becomes

$$Z\left(\frac{1}{2}\right)=\log_e a+\frac{1}{2a}+\sum_{u=1}^{\infty}\frac{A_{2u,2u}}{(2u+1)!}\left(\frac{1}{2a}\right)^{2u+1}$$
$$-\sum_{u=1}^{\infty}\sum_{u=1}^{u}\sum_{w=1}^{\infty}b_{2u}A_{2u-1,2v-2}G_{u,w-1}\left(\frac{1}{2a}\right)^{4u+2w-2v-1}. \qquad (23.592)$$

If the order of summation in v is reversed, this equation becomes

$$Z\left(\frac{1}{2}\right)=\log_e a+\frac{1}{2a}+\sum_{u=1}^{\infty}\frac{A_{2u,2u}}{(2u+1)!}\left(\frac{1}{2a}\right)^{2u+1}$$
$$-\sum_{u=1}^{\infty}\sum_{v=1}^{u}\sum_{w=1}^{\infty}b_{2u}A_{2u-1,2u-2v}G_{u,w-1}\left(\frac{1}{2a}\right)^{2u+2v+2w-3}. \qquad (23.593)$$

In equation 23.593, the coefficient of $1/(2a)^{2k+1}$ is

$$\frac{A_{2k,2k}}{(2k+1)!}-\sum_{u=1}^{\infty}\sum_{v=1}^{u}b_{2u}A_{2u-1,2u-2v}G_{u,k-u-v+1}.$$

The value selected for k restricts $u+v$ to the range $2\leqq u+v\leqq k+1$, so that u is restricted to the range $1\leqq u\leqq k$ and v can not exceed either u or $(k-u+1)$. Two cases exist, according as k is even or odd. If k is even, it may be written as $k=2j$. For $u\leqq j$, $k+1-$

$u \geqq j+1$, so that $u<k+1-u$ and $1 \leqq v \leqq u$. For $u \geqq j+1$, $k+1-u \leqq j$, so that $k+1-u \leqq u$ and $1 \leqq v \leqq k+1-u$. Accordingly the coefficient of $1/(2a)^{4j+1}$ in equation 23.593 is

$$\frac{A_{4j,4j}}{(4j+1)!}-\sum_{u=1}^{j}\sum_{v=1}^{u} b_{2u}A_{2u-1,2u-2v}G_{u,2j-u-v+1} - \sum_{u=j+1}^{2j}\sum_{v=1}^{2j+1-u} b_{2u}A_{2u-1,2u-2v}G_{u,2j-u-v+1}.$$

If k is odd, and greater than 3, it may be written as $k=2j+1$, where $j>1$. For $u \leqq j+1$, $k+1-u \geqq j+1$, so that $u \leqq k+1-u$ and $1 \leqq v \leqq u$. For $u \geqq j+2$, $k+1-u \leqq j$, so that $k+1-u \leqq u$ and $1 \leqq v \leqq 2j+2-u$. Accordingly, the coefficient of $1/(2a)^{4j+3}$ in equation 23.593 is

$$\frac{A_{4j+2,4j+2}}{(4j+3)!}-\sum_{u=1}^{j+1}\sum_{v=1}^{u} b_{2u}A_{2u-1,2u-2v}G_{u,2j-u-v+2} - \sum_{u=j+2}^{2j+1}\sum_{v=1}^{2j+2-u} b_{2u}A_{2u-1,2u-2v}G_{u,2j-u-v+2}.$$

For the coefficient of $1/(2a)^3$ in equation 23.593, all counters must be taken as unity so that the coefficient of $1/(2a)^3$ is $\frac{A_{22}}{3!}-b_2A_{10}G_{10}$. But by equation 23.363, $A_{22}=-1$; by equation 15.812, $b_2=-1/6$; by equation 23.361, $A_{10}=1$; and by equation 23.581, $G_{10}=1$; hence the coefficient reduces to zero. Direct calculation for $k \leqq 6$ has shown that the coefficient of $1/(2a)^{2k+1}$ vanishes, so that the modified potential of equation 23.461 reduces to

$$W=\log_e 4-\Phi-\log_e (2a)-\frac{1}{2a}, \qquad (23.61)$$

at least for powers not above the 14th, in $1/(2a)$, and except for errors in the application of the Euler-Maclaurin summation formula. To 16 decimals, $\log_e 4=$ 1.3862 9436 1119 8906 so that for large values of a, the modified potential is given approximately by the asymptotic expansion

$$W=K-\log_e(2a)-1/(2a), \qquad (23.621)$$

where

$$K=0.8090\ 7869\ 6218\ 358. \qquad (23.622)$$

Comparison with the direct series calculations shows that the asympotic expansion is valid to eight decimals if a exceeds about three.

REDUCTION FORMULA FROM POSITIVE TO NEGATIVE REFLECTION FACTOR

Displaying both arguments, the modified potential is given by

$$W(Q,a)=-\sum_{k=1}^{\infty} Q^k\psi_k(a) \qquad (24.11)$$

where

$$\psi_k(a)=\frac{1}{k}-\frac{1}{\sqrt{k^2+a^2}}. \qquad (24.12)$$

For a positive reflection factor,

$$W(q,a)=-\sum q^k\psi_k(a) \qquad (24.21)$$

and for a negative reflection factor,

$$W(-q,a)=-\sum (-1)^k q^k\psi_k(a) \qquad (24.22)$$

where

$$q=|Q|. \qquad (24.23)$$

If the terms for odd and even values of k are separated, equations 24.2 become

$$W(q,a)=-\sum q^{2k-1}\psi_{2k-1}(a)-\sum q^{2k}\psi_{2k}(a) \qquad (24.31)$$

$$W(-q,a)=\sum q^{2k-1}\psi_{2k-1}(a)-\sum q^{2k}\psi_{2k}(a). \qquad (24.32)$$

But, by equation 24.12

$$\psi_{2k}(a)=\frac{1}{2}\left[\frac{1}{\frac{1}{k}-\sqrt{k^2+\left(\frac{a}{2}\right)^2}}\right], \qquad (24.41)$$

so that

$$\psi_{2k}(a)=\frac{1}{2}\psi_k\left(\frac{a}{2}\right). \qquad (24.42)$$

Accordingly, by equations 24.3 and 24.42,

$$W(q,a)=-\sum q^{2k-1}\psi_{2k-1}(a)-\frac{1}{2}\sum q^{2k}\psi_k\left(\frac{a}{2}\right) \qquad (24.51)$$

$$W(-q,a)=\sum q^{2k-1}\psi_{2k-1}(a)-\frac{1}{2}\sum q^{2k}\psi_k\left(\frac{a}{2}\right). \qquad (24.52)$$

But

$$\sum q^{2k}\psi_k\left(\frac{a}{2}\right)=-W\left(q^2,\frac{a}{2}\right), \qquad (24.6)$$

so that

$$W(-q,a)+W(q,a)=W\left(q^2,\frac{a}{2}\right), \qquad (24.71)$$

which leads to the reduction formula

$$W(-q,a)=W\left(q^2,\frac{a}{2}\right)-W(q,a). \qquad (24.72)$$

Hence if W is available for $Q=q$, and for $Q=q^2$, a simple subtraction furnishes W for $Q=-q$. In the present study, the spacings in q are by one-tenth of a unit, so that the only useful case is that for $q=1$, for which

$$W(-1,a)=W\left(1,\frac{a}{2}\right)-W(1,a). \qquad (24.81)$$

However if tables of W are available for reflection factors on the range $Q=[-1(0.1)1]$, equation 24.71 furnishes $W(q^2,a/2)$.

Combination of equations 23.621 and 24.81 furnishes the result, for large values of a,

$$W(-1,a)=\log_e 2-\frac{1}{2a}, \qquad (24.91)$$

where

$$\log_e 2 = 0.6931\ 4718\ 0560. \qquad (24.92)$$

As for the case $Q=+1$, equation 24.91 must be considered as an approximation which is valid only for values of the relative displacements that are sufficiently large. To eight decimal places, the approximation of equation 23.621 is valid for values of a above 3, so that equation 24.91 is valid for values of a above 6. For fewer decimal places, the lower limit of a may be decreased.

TABLE SMOOTHING AND ERROR DETECTION

Two important uses for differences in tables are detection of gross errors of calculation and smoothing for small errors due to forcing errors of rounding. (Scarborough, 1950, p. 57; Steffensen, 1927, p. 46; Willers, 1948, p. 101). Specifically, the descending differences of $f(x)$ at $x=a$ are defined as

$$\Delta_a = f(a+h) - f(a) \qquad (25.11)$$

$$\Delta_a^n = \Delta_{a+1}^{n-1} - \Delta_a^{n-1} \text{ for } n \geqq 2, \qquad (25.12)$$

where h is the constant spacing in the argument x. If a unit error occurs at some position in a table of exact values, the successive differences have the values shown in table 17, where correct values have been subtracted from the functional values y, leaving only the error residuals, and the error shown is assumed to be the only error in this portion of the table. This example illustrates the general rule that the errors in the n-th descending differences begin on the ascending diagonal through the error in the function, end on the line of the error, and are the value of the error multiplied by the coefficients of x in the expansion of $(1-x)^n$. The errors on the ascending diagonal through the error are the same as the error in the function, and the errors on the line of the error are the error with alternating signs, beginning with the error itself. If the function is a polynomial of degree k in the variable x, the k-th difference is constant and the $(k+1)$-th difference vanishes. Hence, the presence of the error pattern in the $(k+1)$-th difference detects the error, locates it, and determines its value.

TABLE 17.—*Error patterns*

y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
0	0	0	0	0	0	0
0	0	0	0	0	0	+1
0	0	0	0	0	+1	−6
0	0	0	0	+1	−5	+15
0	0	0	+1	−4	+10	−20
0	0	+1	−3	+6	−10	+15
0	+1	−2	+3	−4	+5	−6
1	−1	+1	−1	+1	−1	+1
0	0	0	0	0	0	0

If the functional values are approximations, they are subject to forcing errors of half a unit in the last digit, if the table contains no gross errors. Hence the various differences will not approach zero, as for a polynomial, but will eventually alternate and the sum of $(k+2)$ successive values of the $(k+1)$-th difference will approximate zero. To be of much value for interpolating, the successive order of differences must decrease in numerical value except where they change signs for two successive lines. In the early order of differences, an error may not be recognizable; for large order of differences, the forcing error causes a pattern of large numerical value. For some intermediate order of differences, the error pattern may be recognizable, and the error recognized and determined. The term in error can then be verified and corrected.

If the tabulated function is subject to normal forcing errors of rounding, but contains no gross errors, the table may be "smoothed" by making arbitrary adjustments by small amounts to improve the error pattern. Although such smoothing does not necessarily improve the accuracy of the tables, it does tend to do so by producing a table that is more regular in behavior. To insure greater accuracy, the only safe method is to retain a sufficient number of decimal places in each calculation to prevent an accumulation of forcing errors from causing a tabular error in excess of half a unit in the last tabulated digit. For data obtained from observations, no such improvement is possible.

Although the forcing errors due to rounding increase the numerical errors of successive differences, the coefficients used in interpolation usually decrease with the order so that the net effects of such increased differences usually tend to decrease, and smoothing a table usually does not lead to errors which exceed the permissible tolerance associated with the table.

LAGRANGEAN INTERPOLATION

Numerous formulas are available in the literature (for example, Scarborough, 1950, chap. 3; Steffensen, 1927; Willers, 1948, chap. 2) for interpolation but the simplicity and directness of the Lagrangean formulas favor their use except for specialized problems.

The Lagrangean interpolation formulas are based on the assumption that a polynomial of a selected degree, not inconveniently high, can be used with sufficient accuracy to determine the value of a function for arguments between those in a table of that function, or only slightly outside the range of the table. As a polynomial of degree n in a single variable has $(n+1)$ terms, its determination requires $(n+1)$ tabulated values of the function. Accordingly, the degree of the polynomial must be less than the available number of functional values. Usually, a polynomial of small degree is sufficient. The arguments need not be equally spaced, but the formulas and their use are simplified if they are. Interpolation by a polynomial of degree $(n-1)$ is called "interpolation of order $(n-1)$" or "n-point interpolation".

To illustrate the method, consider a table of two entries. For such a table, the polynomial must be linear and the usual formulas of interpolation result. Let the two arguments be $x_0=a$ and $x_1=b$, and let the corresponding functional values be $y_0=f(a)$ and $y_1=f(b)$. The desired function is $y=f(x)$. Then

$$y=\alpha y_0+\beta y_1, \tag{26.11}$$

where α and β are functions of x to be determined. For $x=a$, $y=y_0$ so that $\beta=0$ for $x=a$ and β may be taken as the product of $(x-a)$ and a constant γ. For $x=b$, $y=y_1$, so that $\alpha=0$ for $x=b$ and α may be taken as the product of $(x-b)$ and a constant δ. Hence,

$$y=\delta(x-b)y_0+\gamma(x-a)y_1. \tag{26.12}$$

But, for $x=a$, $y_0=\delta(a-b)y_0$, so that

$$\delta=\frac{1}{a-b}, \tag{26.131}$$

and for $x=b$, $y_1=\gamma(b-a)y_1$, so that

$$\gamma=\frac{1}{b-a}. \tag{26.132}$$

Accordingly,

$$y=\frac{x-b}{a-b}y_0+\frac{x-a}{b-a}y_1. \tag{26.14}$$

Introducing the phase of x on the interval (a,b) as

$$\phi=\frac{x-a}{b-a}, \tag{26.151}$$

equation 26.14 may be written

$$y=(1-\phi)y_0+\phi(y_1) \tag{26.152}$$

or

$$y=y_0+\phi(y_1-y_0). \tag{26.153}$$

Equation 26.153 is the customary formula for linear interpolation without a computing machine or where the first difference $\Delta=y_1-y_0$ is tabulated or small enough to be calculated quickly. Equation 26.152 is preferable if a computing machine is available, as differencing is not needed.

For three tabulated entries, linear interpolation may be used between the first two and between the second two, if the results are sufficiently accurate. Usually quadratic interpolation on the whole interval is more reliable. Let the arguments be $x_0=a$, $x_1=b$, and $x_2=c$, and let the corresponding functional values be $y_0=f(a)$, $y_1=f(b)$, and $y_2=f(c)$, respectively. Let the function be $y=f(x)$. As $f(a)=y_0$, the coefficients of y_1 and y_2 must vanish for $x=a$ and hence, each must have the factor $(x-a)$. Similarly the coefficients of y_0 and y_2 must have the factor $(x-b)$ and the coefficients of y_0 and y_1 must have the factor $(x-c)$. Hence, the function may be written $y=\alpha(x-b)(x-c)y_0+\beta(x-a)(x-c)y_1+\gamma(x-a)(x-b)y_2$. For $x=a$, $y=y_0$ so that $(a-b)(a-c)\alpha=1$, with similar relations for β and γ. Accordingly,

$$y=\frac{(x-b)(x-c)}{(a-b)(a-c)}y_0+\frac{(x-a)(x-c)}{(b-a)(b-c)}y_1+\frac{(x-a)(x-b)}{(c-a)(c-b)}y_2. \tag{26.22}$$

Similar formulas may be derived for higher order of interpolation.

The formulas derived by the method just discussed are general and take simpler forms if the arguments are equally spaced. For most functions, the results are most accurate when the interpolating argument is as near the center of the range as possible. As a matter of notation, it is convenient to select the arguments so that the range of x is (0, 1), whence $0<x<1$, and so that where possible there are the same number of arguments below x as above for odd order interpolation and the same number of positive as of negative for even order interpolation. If the available functional values do not permit this choice, the restriction $0<x<1$ may be removed or the notation may be changed. The results may be somewhat less accurate for such conditions, as near the beginning or end of a table.

For the choices of the preceding paragraph, linear interpolation has $x_0=0$, $x_1=1, \phi=x$, so that equations 26.152 and 26.153 may be written

$$y'=(1-x)y_0+xy_1 \tag{26.311}$$

$$y'=y_0+x(y_1-y_0). \tag{26.312}$$

For quadratic interpolation, $x_0=-1$, $x_1=0$, $x_2=1$ so that equation 26.22 may be written

$$y''=(1/2)(1-x)xy_{-1}+(1-x)(1+x)y_0+(1/2)x(1+x)y_1 \tag{26.32}$$

For cubic interpolation, similar analyses show that

$$y'''=-(1/6)(2-x)(1-x)xy_{-1}+(1/2)(2-x)(1-x)(1+x)y_0 + (1/2)(2-x)x(1+x)y_1-(1/6)(1-x)x(1+x)y_2. \tag{26.33}$$

The corresponding quartic interpolation formula is:

$$y^{iv}=(1/24)(1+x)x(1-x)(2-x)y_{-2}-(1/6)(2+x)x(1-x)(2-x)y_{-1} + (1/4)(2+x)(1+x)(1-x)(2-x)y_0+(1/6)(2+x)(1+x)x (2-x)y_1-(1/24)(2+x)(1+x)x(1-x)y_2. \tag{26.34}$$

The corresponding quintic interpolation formula is

$$\begin{aligned} y^{5} = &(1/120)(1+x)x(1-x)(2-x)(3-x)y_{-2} \\ &-(1/24)(2+x)x(1-x)(2-x)(3-x)y_{-1} \\ &+(1/12)(2+x)(1+x)(1-x)(2-x)(3-x)y_0 \\ &+(1/12)(2+x)(1+x)x(2-x)(3-x)y_1 \\ &-(1/24)(2+x)(1+x)x(1-x)(3-x)y_2 \\ &+(1/120)(2+x)(1+x)x(1-x)(2-x)y_3. \end{aligned} \quad (26.35)$$

The value of the function as determined by a selected order of interpolation may be considered as the value as determined by interpolation of order one lower plus the "contribution" of the selected order. Thus, the contribution of order n is

$$L_n = y^{(n)} - y^{(n-1)}. \quad (26.41)$$

Direct calculation shows that the quadratic contribution is

$$L_2 = -(1/2)x(1-x)\Delta^2_{-1} \quad (26.421)$$

where

$$\Delta^2_{-1} = y_1 - 2y_0 + y_{-1}. \quad (26.422)$$

The cubic contribution is

$$L_3 = -(1/6)(1-x)x(1+x)\Delta^3_{-1} \quad (26.431)$$

where

$$\Delta^3_{-1} = y_2 - 3y_1 + 3y_0 - y_{-1}. \quad (26.432)$$

The quartic contribution is

$$L_4 = (1/24)(1+x)x(1-x)(2-x)\Delta^4_{-2} \quad (26.441)$$

where

$$\Delta^4_{-2} = y_2 - 4y_1 + 6y_0 - 4y_{-1} + y_{-2}. \quad (26.442)$$

The quintic contribution is

$$L_5 = (1/120)(2+x)(1+x)x(1-x)(2-x)\Delta^5_{-2} \quad (26.451)$$

where

$$\Delta^5_{-2} = y_3 - 5y_2 + 10y_1 - 10y_0 + 5y_{-1} - y_{-2}. \quad (26.452)$$

The notation

$$F_2 = -(1/2)(1-x)x \quad (26.51)$$

$$F_3 = -(1/6)(1-x)x(1+x) \quad (26.52)$$

$$F_4 = (1/24)(1+x)x(1-x)(2-x) \quad (26.53)$$

$$F_5 = (1/120)(2+x)(1+x)x(1-x)(2-x) \quad (26.54)$$

permits equation 26.35 to be written as the quintic interpolation formula

$$y = xy_1 + (1-x)y_0 + F_2\Delta^2_{-1} + F_3\Delta^3_{-1} + F_4\Delta^4_{-2} + F_5\Delta^5_{-2}. \quad (26.55)$$

The coefficients of the differences in equation 26.55 may be written

$$F_3 = (1/3)(1+x)F_2 \quad (26.61)$$

$$F_4 = -(1/4)(2-x)F_3 \quad (26.62)$$

$$F_5 = (1/5)(2+x)F_4 \quad (26.63)$$

with F_2 given by equation 26.51.

Table 37 (p. 59) shows the numerical values of F_2, F_3, and F_4 for the range $x=[0(0.01)0.99]$. The values of F_2 are exact, but the values of F_3 are rounded to five decimal places and the values of F_4 are rounded to four decimal places. If values not in the table are needed, they may be calculated directly by equations 26.5 and 26.6 or obtained by interpolation in the table. Values of F_5 are not needed in the use of table 38 (p. 59) but have been used in the selection of arguments.

Excellent tables are available for interpolating without differences (Mathematical Tables Project, 1944), but the present method usually is preferable if the necessary differences are tabulated.

Analysis of equations 26.5 is of assistance in selecting the order of interpolation to be used. The coefficient F_2 has a minimum value of $-1/8$ for $x=1/2$ so that its absolute value on the interval (0,1) cannot exceed $1/8$. The coefficient F_3 has a minimum of $-\sqrt{3}/27$ for $x=\sqrt{3}/3$ and a maximum value of $\sqrt{3}/27$ for $x=-\sqrt{3}/3$, so that its absolute value on the interval (0,1) cannot exceed $\sqrt{3}/27$. The coefficient F_4 has a minimum value of $-1/24$ at $x=1/2\ (1-\sqrt{5})$, a maximum value of 3/128 at $x=1/2$, and a minimum value of $-1/124$ at $x=1/2\ (1+\sqrt{5})$, so that its absolute value cannot exceed 3/128 on the interval (0,1). The coefficient F_5 has extrema at $x=\pm 10\sqrt{150\pm 10\sqrt{145}}$ where the signs are mutually independent. These four extrema are located at approximately $x=\pm 0.55$ and approximately $x=\pm 1.64$. On the interval (0,1), F_5 has a maximum value of $(1/6000)\sqrt{150-10\sqrt{145}}\sqrt{1+\sqrt{145}}$ at $x=(1/10)\sqrt{150-10\sqrt{145}}$. Approximately, the maximum is 0.011 822 at $x=0.543\ 912$.

The assumption that the successive differences decrease monotonically, or at least do not increase too rapidly, as the order of the differences increases, leads to the working rule:

If $|\Delta^4_{-2}|>43$, use quartic interpolation.

If $|\Delta^4_{-2}|\leqq 43$ and $|\Delta^3_{-1}|>16$, use cubic interpolation.

If $|\Delta^4_{-2}|\leqq 43$, $|\Delta^3_{-1}|\leqq 16$ and $|\Delta^2_{-1}|>8$, use quadratic interpolation.

If $|\Delta^4_{-2}|\leqq 43$, $|\Delta^3_{-1}|\leqq 16$ and $|\Delta^2_{-1}|\leqq 8$, use linear interpolation. Table 38 (p. 59) has been arranged so that quartic interpolation is always sufficient.

NUMERICAL EVALUATION OF THE MODIFIED POTENTIAL

PLAN OF CALCULATION

In the numerical evaluation of the modified potential for selected values of the reflection factor Q and the relative displacement a, various methods were tested; some of them proved to be relatively inferior to others previously developed and some proved to be superior but the ranges of superiority were so short that they were discarded. In the present discussion only those methods are considered that could be recommended for recalculation or for extensions of the present tables. The accepted values of the modified potential, $W(Q,a)$ are shown in table 38 (p. 59).

As a framework for determining the accuracy of approximation formulas, the modified potential was calculated directly for each reflection factor on the range $Q=[-1(0.1)1]$, omitting the trivial case $Q=0$, and for each relative displacement on the basic range $a=[0(0.1)$ $10(0.5)$ $15(1)$ $30]$. In addition, the values were calculated for $a=50$, $a=100$, and $a=\infty$, but these values were used only for checking and are omitted from the tables.

In making the direct calculations, the control values Ψ_k of table 7 (p. 26) were preprinted on a standardized calculation form and the values of $\psi_k(a)$ for each value of a were calculated for each value of k for which $\psi_k<\Psi_{k+1}$. The values of ψ_k were entered in a double entry table, for the selected value of a, with the arguments k and q, under the heading $q=1$. The values of $q^k\psi_k$were calculated for each pair of values (k,q). If the product did not vanish before the control value of k was reached, the remainder was calculated by the error coefficients of table 6 (p. 26) for $q<1$ and by the Euler-Maclaurin formula for $q=1$. In these calculations, the odd and even values of k were considered separately, so that there were two sets of calculations for each value of a, one for odd values of k, the other for even values. Finally, the modified potentials were calculated by algebraic addition of the contributions for the odd and for the even values of k.

In most of the computations, 10-bank desk calculators were used. The computations usually were made to nine decimal places or to ten significant figures. For purposes of checking, a few computations were made to ten decimals and occasionally more significant figures were retained, by double precision methods, to test the effect of accumulated forcing errors. For some of the auxiliary functions, as many as thirty significant figures were used.

For small values of the relative displacement, the modified potentials were calculated by the local expansion formulas; for large values, by the asymptotic formulas.

For both local and asymptotic expansions, the limits of validity were determined by comparison with the direct evaluations for relative displacements on the basic range. Between the limits of validity of the local and the asymptotic expansions, no satisfactory approximations were developed so that interpolation or special devices were used.

For each value of Q, and for each spacing in the value of a, the calculated values of W were differenced to detect gross errors and to determine the spacing of the argument needed to make fourth order interpolation valid, if that spacing did not exceed the spacing on the basic range. The criterion used was that the fifth difference Δ^5_{-2} must not exceed 425 in the ninth decimal for any line of the table. This value of Δ^5_{-2} is not included in the table, as it is not needed for the use of the table.

If the fifth difference exceeded 425, intermediate values were selected for the relative displacement and the corresponding values of the modified potential were determined by one of the approximate formulas, or by means of interpolation of orders as high as the ninth. As a guide for selecting intermediate values of the argument, a simple approximation was used: if the equal intervals of a selected table are divided into n equal parts, the fifth difference is divided by n^5, neglecting higher order differences. As the new tables were differenced, the resulting fifth differences were calculated to check the choice of n, which was usually 2 or 5. If the fifth difference exceeded 425, a larger value of n was selected.

As a guide to consistency, the tabular spacing was selected as 0.02, 0.05, 0.1, 0.2, or 0.5. The break-points are subject to personal choices and in the tables for a few values of the reflection factor, some of these spacings are omitted. The choices of breakpoints were affected by the ease of calculation, by the number of lines in the resulting tables, and by calculations previously made.

In the tables of the modified potential, the functional values and the differences are listed to eight decimals. In most of the entries, the functional values are correct to within half a unit in the eighth decimal place, as they have been rounded from values calculated to at least nine decimals. In a few cases, forcing errors may cause the values of the function to be in error by more than half a unit in the eighth decimal place, but no functional value should be in error by more than a few units. The differences may have larger errors so that interpolated values of the modified potential may have errors of several units in the eighth decimal place. The results of calculations with these tables should be reduced to fewer significant figures, depending on the use made of the table, but no general rule can be given beyond that of observing the customary precautions.

Because of forcing errors, the differences as rounded may not agree exactly with the corresponding differences calculated from the rounded values of the function. This discrepancy does not decrease the reliability of the interpolated value.

In some intervals of the tables, empirical or semi-empirical methods were used. One of these deserves specific mention. If the calculated values of the function have differences that are too large for interpolation, the remainders after subtracting the first few terms of the series used in calculating the functional values may have differences that are small enough to permit interpolation. If this condition exists, the remainders are calculated for the intermediate arguments by interpolation and added to the sums of the initial terms that are calculated directly. This device is useful because the terms of the various series used decrease rapidly at first and more slowly after the first few terms. In some cases, a single isolated term is sufficient to justify interpolation in the remainder, in other cases two isolated terms are sufficient. Rarely is there a gain in isolating more than two terms. This method applies to direct series, to local expansions, and to asymptotic expansions. The restriction to fourth order interpolation does not apply. In some parts of the tables, interpolations of orders as high as the ninth were used for remainders. Judgment as to relative difficulties and accuracy of possible methods is the only guide for details in selecting procedures.

FORMULAS FOR CALCULATIONS

In an earlier section, various results were derived for use in calculating the value of the modified potential function for selected values of the reflection factor, Q, and of the relative displacement, a. To avoid the necessity of searching for these results, they are summarized here.

The calculations fall into three groups: direct series, local expansions, and asymptotic expansions, with auxiliary devices applicable to one or more of these groups.

For the direct series form the modified potential is, by equation 12.932:

$$W=-\sum_{k=1}^{n} Q^k\psi_k+R_n \qquad (27.11)$$

where

$$\psi_k=\frac{1}{k}-\frac{1}{\sqrt{k^2+a^2}} \qquad (12.31)$$

and

$$R_n=-\sum_{k=n+1}^{\infty} Q^k\psi_k. \qquad (27.12)$$

The remainder, R_n, after isolating the initial n terms, is available by the Euler-Maclaurin formula for $q=1$ and by comparison with a geometric series for $q<1$. For purposes of calculation, it is convenient to separate the terms into two groups, one for odd, the other for even, values of the counter k. For this purpose, the modified potential is given, because of equation 12.932, by

$$-W=\epsilon W_1+W_2 \qquad (27.13)$$

where

$$W_1=\sum_{k=1}^{\infty} q^{2k-1}\psi_{2k-1} \qquad (12.621)$$

$$W_2=\sum_{k=1}^{\infty} q^{2k}\psi_{2k}. \qquad (12.622)$$

By equation 12.61,

$$\epsilon=Q/q. \qquad (27.14)$$

For $q=1$, equations 12.62 may be written

$$W_1=\sum_{k=1}^{n}\psi_{2k-1}+R_{n1} \qquad (27.211)$$

$$W_2=\sum_{k=1}^{n}\psi_{2k}+R_{n2} \qquad (27.212)$$

where

$$R_{n1}=-Z(2n) \qquad (19.41)$$

$$R_{n2}=-Z(2n+1) \qquad (19.42)$$

$$Z(x)=-\frac{1}{2}[\log_e D_0+\sum g_{2k}(1-D_{2k}E_{2k})]. \qquad (19.2)$$

The values of g_{2k} are obtainable from equations 19.1 or 19.3 The value of D_0 is given by equation 19.12. The values of D_{2k} are given by equation 18.42 and the values of E_{2k} are given by equations 18.5 or 18.6. A few values of $(1-D_{2k}E_{2k})$ are shown in tables 10 and 11, pages 30 and 31.

For $q<1$, equations 12.62 may be written

$$W_1=\sum_{k=1}^{n} q^{2k-1}\psi_{2k-1}+R_{n1} \qquad (27.311)$$

$$W_2=\sum_{k=1}^{n} q^{2k}\psi_{2k}+R_{n2}. \qquad (27.312)$$

By equations 13.45, the maximum absolute values of the remainder are

$$\left(\frac{q^2}{1-q^2}\right) q^{2n-1}\psi_{2n-1}$$

and

$$\left(\frac{q^2}{1-q^2}\right) q^{2n}\psi_{2n},$$

respectively.
If the remainders are written

$$R_{n1}=\left[\frac{q^2}{2(1-q^2)}\right] q^{2n-1}\psi_{2n-1} \qquad (27.321)$$

$$R_{n2}=\left[\frac{q^2}{2(1-q^2)}\right] q^{2n}\psi_{2n}, \qquad (27.322)$$

the error in equations 27.31 cannot exceed the applicable remainder. The second factor in each of the equations 27.32 is the last retained term of W_u, where $u=1$ or 2. The values of the first factor are shown in table 6, page 26.

For small relative displacements, local expansions are available. For $q=1$, after isolating n terms, the remainders are given by equations 19.4 where

$$Z(x)=-\sum A_k\alpha^k \qquad (20.61)$$

$$A_k=\sum_{u=0}^{\infty} A_{k,2u}x^{-2u} \qquad (20.62)$$

$$\alpha=\frac{a^2}{x^2} \qquad (18.3)$$

The values of $A_{k,2u}$ are shown in table 13, page 32.

For $q<1$, after isolating $(n-1)$ terms, the modified potential is

$$W=-\sum_{k=1}^{n-1} Q^k\psi_k+\sum_{u=1}^{\infty} C_{nu}\beta^u \qquad (21.333)$$

where

$$C_{nu}=(-1)^{u-1}H_u n^{2u}\epsilon^n(S_n^{2n+1}+\epsilon S_{n+1}^{2u+1}) \qquad (21.332)$$

$$C_{nu}=(-1)^{u-1}H_u Q^n\left(\frac{1+\mu_{nu}}{n}\right) \qquad (21.522)$$

$$C_{n,u+1}=-\lambda_{nu}C_{nu} \qquad (21.411)$$

$$\beta=\frac{a^2}{n^2} \qquad (21.23)$$

$$H_u=\frac{(2u)!}{2^{2u}(u!)^2} \qquad (21.121)$$

$$S_m^\alpha=\sum_{j=0}^{\infty}\frac{q^{m+2j}}{(m+2j)^\alpha} \qquad (21.321)$$

$$\mu_{nu}=n^{2u+1}\sum\frac{Q^j}{(n+j)^{2u+1}} \qquad (21.521)$$

$$\lambda_{nu}=n^2\left(\frac{2u+1}{2u+2}\right)\left(\frac{S_n^{2u+3}+\epsilon S_{n+1}^{2u+3}}{S_n^{2u+1}+\epsilon S_{n+1}^{2u+1}}\right) \qquad (21.42)$$

$$\lambda_{nu}=\left(\frac{2u+1}{2u+2}\right)\left(\frac{1+\mu_{n,u+1}}{1+\mu_{nu}}\right) \qquad (21.53)$$

Values of H_u are shown in table, 15 page 33. After isolating $(n-1)$ terms, the remainder of equation 21.333 has the value, by equations 21.632, 21.633,

$$R_n=\sum_{u=1}^{v} C_{nu}\beta^u-(C_{nv}\beta^v)(S_0+S_1+S_2); \qquad (27.4)$$

where the values of S_0, S_1, S_2, and the factor $(S_0+S_1+S_2)$ are shown in table 16, page 36, for $v=30$, as discussed in the summary following equations 21.667.

The local expansion for $q<1$ is also valid as a limiting case for $q=1$.

The asymptotic expansions are available for large values of the relative displacement. For $Q=+1$, the modified potential for large displacements is given by

$$-W(1,a)=\log_e(2a)+1/(2a)-K, \qquad (23.621)$$

wnere $K=0.8090\ 7869\ 6218\ 358.$ (23.622)

For $Q=-1$, the modified potential for large relative displacements is given by

$$W(-1,a)=\log_e 2-1/(2a), \qquad (24.91)$$

where

$$\log_e 2=0.6931\ 4718\ 0560. \qquad (24.92)$$

For $q<1$, and large relative displacements, the modified potential has the value

$$W=W_\infty-\Sigma B_n a^{1-2n}, \qquad (22.31)$$

where

$$W_\infty=\log_e(1-Q). \qquad (14.31)$$

The values of W_∞ are shown in table 8, page 27, and the values of B_n are given by equations 22.32, and 22.5 and are shown in table 1, page 28.

CALCULATIONS FOR SEPARATE VALUES OF Q

CALCULATIONS FOR $Q=+1$

For $Q=+1$, the local expansion may use the remainder calculated by the Euler-Maclaurin formula 19.4 and 19.2 or by one of the forms used in evaluating the remainder of equation 21.333, setting $\epsilon=1$. For $n=1$, 2, 3, and 4, the early values of S_m^α, given by equation 21.321, are shown in table 18 and the early values of C_{nu}, given by equations 21.332 and 21.522, are shown in table 19. The values of λ_{nu}, given in equations 21.42 and 21.523, are also shown in table 19.

For isolated calculations, n may be taken as 4. Increasing the value of n reduces the value of β for a selected value of a and causes the remainder series to converge more rapidly. In the calculations for the present study n was selected as the smallest integer not less than the value of a. Local expansions were used to evaluate the modified potential for relative displacements from 0 to 3.2, with various spacings.

For relative displacements of three or greater the asymptotic expansion of equation 23.621 is sufficiently accurate for eight decimal values. There is no gap between the regions of applicability of local and asymptotic expansions.

TABLE 18.—*Values of* S_m^α *for* q=+1

[D, The number in the preceding column is to be divided by this power of 10]

α	m=1	m=2	D	m=3	D	m=4	D	m=5	D
3	1. 051 799 790	1. 502 571 129	1	5. 179 979 026	2	2. 525 711 289	2	1. 476 275 323	2
5	1. 004 523 763	3. 240 399 235	2	4. 523 762 795	3	1. 153 922 348	3	4. 085 364 577	4
7	1. 000 471 549	7. 877 728 730	3	4. 715 486 524	4	6. 522 872 955	5	1. 430 128 155	5
9	1. 000 051 345	1. 957 047 642	3	5. 134 518 384	5	3. 922 642 238	6	5. 399 204 185	7
11	1. 000 005 666	4. 885 225 530	4	5. 666 051 090	6	2. 413 030 294	7	2. 102 182 063	8
13	1. 000 000 628	1. 220 852 922	4	6. 280 554 218	7	1. 497 965 669	8	8. 299 474 160	10
15	1. 000 000 070	3. 051 851 160	5	6. 972 470 313	8	9. 334 788 912	10	3. 298 375 303	11
17	1. 000 000 008	7. 629 452 798	6	7. 744 839 456	9	5. 826 719 389	11	1. 315 080 730	12
19	1. 000 000 001	1. 907 352 272	6	8. 604 441 145	10	3. 639 626 916	12	5. 251 728 516	14
21	1. 000 000 000	4. 768 373 856	7	9. 560 116 531	11	2. 274 193 701	13	2. 098 951 642	15
23	1. 000 000 000	1. 192 093 038	7	1. 062 220 241	11	1. 421 212 267	14	8. 392 273 200	17
25	1. 000. 000. 000	2. 980 232 328	8	1. 180 238 743	12	8. 882 136 200	16	3. 356 190 276	18
27	1. 000 000 000	7. 450 580 652	9	1. 311 373 995	13	5. 551 212 869	17	1. 342 329 631	19
29	1. 000 000 000	1. 862 645 153	9	1. 457 081 262	14	3. 469 474 099	18	5. 369 019 901	21
31	1. 000 000 000	4. 656 612 875	10	1. 618 978 798	15	2. 168 411 885	19	2. 147 547 056	22
33	1. 000 000 000	1. 164 153 218	10	1. 798 865 178	16	1. 355 254 810	20	8. 590 063 973	24
35	1. 000 000 000	2. 910 383 046	11	1. 998 739 026	17	8. 470 335 290	22	3. 436 000 239	25

TABLE 19.—*Local coefficients* C_{nu} *and ratios* λ_{nu} *for* Q=+1

u	Coefficients $(-1)^{u-1}C_{nu}$ where—				Ratios λ_{nu} where—			
	n=1	n=2	n=3	n=4	n=1	n=2	n=3	n=4
1	0. 601 028 452	0. 404 113 806	0. 346 756 064	0. 320 158 929	0. 646 970 883	0. 548 277 559	0. 497 357 740	0. 468 525 923
2	. 388 847 908	. 221 566 531	. 172 461 812	. 150 002 765	. 810 366 065	. 753 658 719	. 709 053 184	. 678 643 583
3	. 315 109 149	. 166 985 548	. 122 284 597	. 101 798 414	. 869 497 666	. 841 914 164	. 810 827 999	. 785 563 542
4	. 273 986 670	. 140 587 498	. 099 151 775	. 079 969 123	. 898 639 948	. 885 822 209	. 865 776 198	. 846 481 748
5	. 246 215 367	. 124 535 528	. 085 843 247	. 067 692 403	. 916 326 315	. 910 480 205	. 898 039 849	. 883 920 046
6	. 225 613 620	. 113 387 133	. 077 090 657	. 059 834 672	. 928 485 894	. 925 844 266	. 918 302 191	. 908 237 390
7	. 209 479 064	. 104 978 827	. 070 792 519	. 054 344 086	. 937 478 485	. 936 291 026	. 931 791 768	. 924 747 712
8	. 196 382 115	. 098 290 734	. 065 963 886	. 050 254 569	. 944 439 034	. 943 906 906	. 941 254 827	. 936 392 380
9	. 185 470 935	. 092 777 302	. 062 088 826	. 047 058 007	. 949 998 641	. 949 760 649	. 948 211 799	. 944 892 488
10	. 176 197 136	. 088 116 231	. 058 873 358	. 044 464 747	. 954 545 112	. 954 438 813	. 953 540 852	. 951 294 873
11	. 168 188 115	. 084 101 551	. 056 138 152	. 042 299 086	. 958 333 248	. 958 285 810	. 957 768 238	. 956 259 248
12	. 161 180 263	. 080 593 323	. 053 767 339	. 040 448 926	. 961 538 440	. 961 517 286	. 961 220 356	. 960 214 870
13	. 154 981 018	. 077 491 873	. 051 682 260	. 038 839 627	. 964 285 708	. 964 276 280	. 964 106 571	. 963 438 824
14	. 149 445 981	. 074 723 575	. 049 827 207	. 037 419 605	. 966 666 666	. 966 662 464	. 966 565 761	. 966 124 740
15	. 144 464 448	. 072 232 475	. 048 161 272	. 036 152 006	. 968 750 000	. 968 748 128	. 968 693 159	. 968 403 081
16	. 139 949 934	. 069 975 075	. 046 653 495	. 035 009 714	. 970 588 235	. 970 587 401	. 970 556 216	. 970 366 082
17	. 135 833 760	. 067 916 926	. 045 279 839	. 033 972 239	. 972 222 222	. 972 221 852	. 972 204 188	. 972 079 909
18	. 132 960 600	. 066 030 320	. 044 021 249	. 033 023 731	. 973 684 210	. 973 684 046	. 973 674 052	. 973 593 081
19	. 128 585 321	. 064 292 669	. 042 862 348	. 032 151 676	. 975 000 000	. 974 999 926	. 974 994 279	. 974 941 617
20	. 125 370 688	. 062 685 348	. 041 790 544	. 031 346 007	. 976 190 471	. 976 190 444	. 976 187 255	. 976 153 071
21	. 122 385 671	. 061 192 837	. 040 795 397	. 030 598 501	. 977 272 731	. 977 272 712	. 977 270 913	. 977 248 755
22	. 119 604 179	. 059 802 090	. 039 868 155	. 029 902 347	. 978 260 868	. 978 260 863	. 978 259 849	. 978 245 554
23	. 117 004 088	. 058 502 044	. 039 001 415	. 029 251 838	. 979 166 668	. 979 166 664	. 979 166 091	. 979 156 865
24	. 114 566 503	. 057 283 252	. 038 188 863	. 028 642 138	. 980 000 000	. 979 999 999	. 979 999 674	. 979 993 672
25	. 112 275 173	. 056 137 586	. 037 425 074	. 028 069 114	. 980 769 230	. 980 769 230	. 980 769 049	. 980 765 193
26	. 110 116 035	. 055 058 017	. 036 705 354	. 027 529 210	. 981 481 480	. 981 481 481	. 981 481 379	. 981 478 909
27	. 108 076 849	. 054 038 424	. 036 025 621	. 027 019 339	. 982 142 855	. 982 142 857	. 982 142 799	. 982 141 199
28	. 106 146 905	. 053 073 453	. 035 382 299	. 026 536 806	. 982 758 621	. 982 758 621	. 982 758 589	. 982 757 562
29	. 104 316 786	. 052 158 393	. 034 772 260	. 026 079 146	. 983 333 334	. 983 333 333	. 983 333 314	. 983 332 655
30	. 102 578 173	. 051 289 086	. 034 192 724	. 025 644 512	. 983 870 968	. 983 870 968	. 983 870 958	. 983 870 534

CALCULATIONS FOR $Q=-1$

In preparing the tables for $Q=-1$, shown in table 38, page 59, methods similar to those used for $Q=+1$ were used. In fact, the calculations for the direct series are the same for $Q=q$ and for $Q=-q$, except that the sum of the odd terms is subtracted from the sum of the even terms instead of being added as for $Q=+1$. Likewise, the values of S_m^α, C_{nu}, and λ_{nu} are obtained from the same basic data in the two cases.

By equation 24.81, the modified potential for $Q=-1$, and the argument a, is the excess of the modified potential for $a/2$ over that for a, each calculated for $Q=1$. Accordingly the values of S_m^α, C_{nu}, and λ_{nu} are omitted from the tables.

CALCULATIONS FOR $Q=+0.1$

For $q=+0.1$, the early values of S_m^α are shown in table 20, the early values of C_{nu} in table 21, and the early values of λ_{nu} in table 22.

TABLE 20.—*Values of* S_n^α *for* q=*0.1*

[*D*, The value in the column preceding *D* is to be divided by this power of 10]

α	n=1	D	n=2	D	n=3	D	n=4	D	n=5	D
3	1. 000 371 173	1	1. 251 567 149	3	3. 711 732 996	5	1. 567 149 261	6	8. 029 292 449	8
5	1. 000 041 184	1	3. 125 977 852	4	4. 118 432 304	6	9. 778 515 700	8	3. 205 966 899	9
7	1. 000 004 574	1	7. 813 110 709	5	4. 573 754 925	7	6. 107 092 648	9	1. 281 216 362	10
9	1. 000 000 508	1	1. 953 163 157	5	5. 081 038 591	8	3. 815 690 302	10	5. 122 480 679	12
11	1. 000 000 056	1	4. 882 836 345	6	5. 645 234 120	9	2. 384 461 544	11	2. 048 506 052	13
13	1. 000 000 006	1	1. 220 704 615	6	6. 272 336 674	10	1. 490 192 703	12	8. 193 032 502	15
15	1. 000 000 001	1	3. 051 758 744	7	6. 969 204 708	11	9. 313 438 457	14	3. 277 010 683	16
17	(1)		7. 629 395 113	8	7. 743 537 483	12	5. 820 825 174	15	1. 310 762 993	17
19			1. 907 348 669	8	8. 603 921 215	13	3. 637 995 218	16	5. 242 967 735	19
21			4. 768 371 605	9	9. 559 908 733	14	2. 273 741 313	17	2. 097 169 905	20
23			1. 192 092 897	9	1. 062 211 932	14	1. 421 086 738	18	8. 388 644 539	22
25			2. 980 232 240	10	1. 180 235 421	15	8. 881 787 714	20	3. 355 450 657	23
27			7. 450 580 597	11	1. 311 372 666	16	5. 551 116 100	21	1. 342 178 802	24
29			1. 862 645 149	11	1. 457 080 730	17	3. 469 447 223	22	5. 368 712 226	26
31			4. 656 612 873	12	1. 618 978 585	18	2. 168 404 420	23	2. 147 484 282	27
33			1. 164 153 218	12	1. 798 865 093	19	1. 355 252 737	24	8. 589 935 885	29
35			2. 910 383 046	13	1. 998 738 992	20	8. 470 329 531	26	3. 435 974 101	30
37			7. 275 957 614	14	2. 220 821 102	21	5. 293 955 936	27	1. 374 389 589	31
39			1. 818 989 404	14	2. 467 579 002	22	3. 308 722 455	28	5. 497 558 249	33
41			4. 547 473 509	15	2. 741 754 447	23	2. 067 951 533	29	2. 199 023 278	34
43			1. 136 868 377	15	3. 046 393 830	24	1. 292 469 707	30	8. 796 093 068	36
45			2. 842 170 943	16	3. 384 882 033	25	8. 077 935 670	32	3. 518 437 218	37
47			7. 105 427 358	17	3. 760 980 037	26	5. 048 709 794	33	1. 407 374 885	38
49			1. 776 356 839	17	4. 178 866 707	27	3. 155 443 621	34	5. 629 499 538	40
51			4. 440 892 099	18	4. 643 185 230	28	1. 972 152 263	35	2. 251 799 814	41
53			1. 100 223 025	18	5. 159 094 700	29	1. 232 595 164	36	9. 007 199 256	43
55			2. 775 557 562	19	5. 732 327 445	30	7. 703 719 778	38	3. 602 879 702	44
57			6. 938 893 904	20	6. 369 252 716	31	4. 814 824 861	39	1. 441 151 881	45
59			1. 734 723 476	20	7. 076 947 463	32	3. 009 265 538	40	5. 764 607 523	47

[1] For $\alpha \geqq 17$, $S_1^\alpha = 0.1$ to 9 decimal places.

For $n=7$, the value of C_{nu} is

$$C_{7u}=(-1)^{u-1}\left(\frac{H_u}{7}\right)\times 10^{-7}. \tag{28.11}$$

For $n \geqq 8$, $|C_{nu}| \leqq 7\times 10^{-10}$,

so that

$$W=-\sum_{k=1}^{7} Q^k \psi_k, \tag{28.12}$$

to nine decimal places.

As the number of isolated terms increases, the terms of the remainder converge more rapidly, so that less accuracy is needed in C_{nu} and especially in λ_{nu}.

TABLE 21.—*Local coefficients* C_{nu} *for* $Q=+0.1$

Terminal digit of C_{nu} is in ninth decimal place.
Sign is that of $(-1)^{u-1}$.
For $u>15$, $C_{nu}=(-1)^{u-1} H_u Q^n/n$

u	n=1	n=2	n=3	n=4	n=5	n=6
1	5064 4342	257 7369	17 4080	13180	1062	89
2	3761 8769	190 0297	12 8068	9695	782	65
3	3127 4559	157 1770	10 5587	7981	643	54
4	2734 9105	137 0771	9 1840	6930	558	47
5	2461 0578	123 1897	8 2381	6206	499	42
6	2255 8869	112 8511	7 5375	5671	455	39
7	2094 7330	104 7603	6 9918	5256	422	35
8	1963 8077	98 2003	6 5510	4921	395	33
9	1854 7062	92 7395	6 1850	4644	372	31
10	1761 9706	88 1003	5 8746	4409	353	29
11	1681 8810	84 0948	5 6070	4207	337	28
12	1611 8026	80 5904	5 5633	4032	322	27
13	1549 8102	77 4906	5 1663	3876	310	26
14	1494 4598	74 7230	4 9817	3737	299	25
15	1444 6445	72 2322	4 8155	3612	289	24

TABLE 22.—*Local ratios* λ_{nu} *for* Q=+0.1

u	Values of λ_{nu} where—						
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
1	0. 742 802 985	0. 737 301 229	0. 735 681 818	0. 747 685	0. 73613	0. 7369	0. 737
2	. 831 355 200	. 827 117 762	. 824 463 968	. 832 043	. 82273	. 8226	. 823
3	. 874 484 103	. 872 119 485	. 869 606 840	. 874 153	. 86734	. 8669	. 867
4	. 899 867 780	. 898 687 776	. 897 205 037	. 895 577	. 89456	. 8966	. 893
5	. 916 633 054	. 916 076 902	. 914 958 513	. 913 789	. 91284	. 9120	. 910
6	. 928 562 923	. 928 305 637	. 927 600 761	. 926 725	. 92588	. 9253	. 925
7	. 937 502 147	. 937 380 872	. 936 949 785	. 936 305	. 93562	. 9350	. 935
8	. 944 443 904	. 944 391 144	. 944 133 053	. 943 675	. 94314	. 9425	. 943
9	. 949 999 864	. 949 976 177	. 949 823 964	. 949 505	. 94908	. 9487	. 948
10	. 954 545 421	. 954 534 818	. 954 446 008	. 954 227	. 95391	. 9535	. 954
11	. 958 333 324	. 958 328 589	. 958 277 210	. 958 129	. 95789	. 9577	. 957
12	. 961 538 460	. 961 536 348	. 961 506 786	. 961 407	. 96123	. 9610	. 961
13	. 964 285 713	. 964 284 773	. 964 267 854	. 964 202	. 96406	. 9638	. 963
14	. 966 666 667	. 966 666 246	. 966 661 004	. 966 613	. 96652	. 9664	. 967
15	. 968 750 000	. 968 749 813	. 968 744 325	. 968 715	. 96864	. 9686	. 969
16	. 970 588 235	. 970 588 152	. 970 585 037	. 970 567	. 97052	. 9704	. 971
17	. 972 222 222	. 972 222 184	. 972 220 418	. 972 207	. 97217	. 9721	. 972
18	. 973 684 211	. 973 684 192	. 973 683 197	. 973 675	. 97365	. 9736	. 974
19	. 975 000 000	. 974 999 995	. 974 999 429	. 974 994	. 97498	. 9750	. 975
20	. 976 190 476	. 976 190 470	. 976 190 152	. 976 186	. 97617	. 9762	. 976
21	. 977 272 727	. 977 272 727	. 977 272 547	. 977 270	. 97726	. 9773	. 977
22	. 978 260 870	. 978 260 870	. 978 260 786	. 978 260	. 97825	. 9783	. 978
23	. 979 166 667	. 979 166 667	. 979 166 612	. 979 166	. 97916	. 9792	. 979
24	. 980 000 000	. 980 000 000	. 979 999 967	. 979 999	. 98000	. 9800	. 980
25	. 980. 769 231	. 980 769 231	. 980 769 211	. 980 769	. 98077	. 9808	. 981
26	. 981 481 481	. 981 481 481	. 981 481 472	. 981 480	. 98148	. 9815	. 981
27	. 982 142 857	. 982 142 857	. 982 142 848	. 982 143	. 98214	. 9821	. 982
28	. 982 758 621	. 982 758 621	. 982 758 619	. 982 759	. 98276	. 9828	. 983
29	. 983 333 333	. 983 333 333	. 983 333 332	. 983 333	. 98333	. 9833	. 983
30	. 983 870 968	. 983 870 968	. 983 870 968	. 983 871	. 98387	. 9839	. 984

As seven initial terms are always sufficient and as the basic range furnishes values of the modified potential for which fourth order interpolation is adequate, local expansions were used for $a<9$, and asymptotic expansions were used for $a\geqq 9$ on the basic range to check the direct calculations. For immediate values above $a=10$, interpolation was used.

CALCULATIONS FOR $Q=-0.1$

For $Q=-0.1$, the values of S_m^α are the same as for $Q=+0.1$ and are shown in table 20. However, the values of C_{nu} and of λ_{nu} are different for the two cases for small values of u. The values of C_{nu} are shown in table 23 and of λ_{nu} in table 24.

TABLE 23.—*Local coefficients* C_{nu} *for* Q=−0.1

[Terminal digit in ninth decimal place. Sign is that of $(-1)^{n+u-1}$]

u	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
1	4939 2775	242 8900	15 9976	1 1895	946	78
2	3738 4320	185 0876	12 2127	9080	721	59
3	3122 5727	155 3475	10 2805	7653	608	50
4	2733 8423	136 3658	9 0470	6746	537	45
5	2460 8175	122 9052	8 1688	6100	486	40
6	2255 8319	112 7352	7 5018	5609	447	37
7	2094 7202	104 7124	6 9731	5219	416	35
8	1963 8047	98 1803	6 5411	4899	391	33
9	1854 7054	92 7311	6 1797	4630	370	31
10	1761 9704	88 0967	5 8718	4401	351	29
11	1681 8810	84 0932	5 6055	4202	336	28
12	1611 8026	80 5898	5 3723	4028	322	27
13	1549 8102	77 4904	5 1658	3874	310	26
14	1494 4598	74 7229	4 9814	3736	299	25
15	1444 6445	72 2322	4 8154	3611	289	24

TABLE 24.—*Local ratios* λ_{nu} *for* Q=−0.1

u	Values of λ_{nu} where—						
	n=1	n=2	n=3	n=4	n=5	n=6	n=7
1	0.756 878 312	0.762 022 477	0.763 410 135	0.763 322	0.762 71	0.7619	0.760
2	.835 262 665	.839 318 535	.841 783 145	.842 800	.843 23	.8431	.842
3	.875 509 583	.877 811 233	.880 023 802	.881 464	.882 27	.8827	.883
4	.900 131 459	.901 290 959	.902 921 342	.904 280	.905 23	.9058	.906
5	.916 700 192	.917 252 772	.918 346 551	.919 475	.920 37	.9211	.921
6	.928 579 925	.928 835 786	.929 531 158	.930 385	.931 19	.9318	.932
7	.937 502 145	.937 618 754	.938 046 038	.938 679	.939 34	.9351	.940
8	.944 444 985	.944 497 650	.944 754 255	.945 206	.945 74	.9426	.946
9	.950 000 136	.950 023 795	.950 175 443	.950 492	.950 90	.9487	.952
10	.954 545 489	.954 556 083	.954 644 668	.954 861	.955 17	.9535	.956
11	.958 333 342	.958 338 076	.958 389 385	.958 536	.958 76	.9575	.959
12	.961 538 463	.961 540 577	.961 570 104	.961 668	.961 85	.9610	.963
13	.964 285 714	.964 286 657	.964 303 567	.964 370	.964 51	.9638	.965
14	.966 666 667	.966 667 087	.966 676 734	.966 721	.966 81	.9664	.967
15	.968 750 000	.968 750 187	.968 755 676	.968 785	.968 86	.9686	.969
16	.970 588 235	.970 588 319	.970 591 434	.970 609	.970 66	.9704	.971
17	.972 222 222	.972 222 259	.972 224 025	.972 237	.972 27	.9721	.972
18	.973 684 211	.973 684 227	.973 685 226	.973 693	.973 72	.9749	.974
19	.975 000 000	.975 000 007	.975 000 572	.975 006	.975 02	.9750	.975
20	.976 190 476	.976 190 479	.976 190 798	.976 194	.976 21	.9762	.976
21	.977 272 727	.977 272 727	.977 272 909	.977 276	.977 28	.9773	.977
22	.978 260 870	.978 260 870	.978 260 972	.978 262	.978 27	.9783	.978
23	.979 166 667	.979 166 667	.979 166 724	.979 168	.979 18	.9792	.979
24	.980 000 000	.980 000 000	.980 000 033	.980 001	.980 00	.9800	.980
25	.980 769 231	.980 769 231	.980 769 251	.980 769	.980 77	.9808	.981
26	.981 481 481	.981 481 481	.981 481 490	.981 482	.981 48	.9815	.981
27	.982 142 857	.982 142 857	.982 142 866	.982 143	.982 14	.9821	.982
28	.982 758 621	.982 758 621	.982 758 623	.982 759	.982 76	.9828	.983
29	.983 333 333	.983 333 333	.983 333 334	.983 333	.983 33	.9833	.983
30	.983 870 968	.983 870 968	.983 870 968	.983 871	.983 87	.9839	.984

The values of the modified potential for $Q=-0.1$ were calculated by local expansions for values of the relative displacement from 0 to 6 and by asymptotic expansions from 5 to 30; the overlapping values were used for adjustments.

CALCULATIONS FOR $Q=+0.2$

For $Q=+0.2$, the use of μ_{nu} given by equation 21.521 is preferable to that of S_m^α as μ_{nu} converges to zero rapidly. The early values of μ_{nu} are shown in table 25, the values of the local coefficients in table 26, and the values of the local ratios in table 27. In each of these tables, the number of retained decimal places is reduced as the value of n increases.

TABLE 25.—*Values of* μ_{nu} *for* Q=+0.2

u	Values of μ_{nu} where—							
	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8
1	0. 026 620 979	0. 064 839 148	0. 094 160 60	0. 115 977 48	0. 132 5926	0. 145 600	0. 15607	0. 1646
2	. 006 422 979	. 027 676 633	. 050 847 14	. 071 347 20	. 088 6719	. 103 221	. 11550	. 1260
3	. 001 581 301	. 012 031 926	. 027 883 64	. 044 457 02	. 059 9378	. 073 841	. 08616	. 0971
4	. 000 392 689	. 005 282 767	. 015 436 50	. 027 939 37	. 040 8225	. 053 176	. 06465	. 0751
5	. 000 097 828	. 002 332 080	. 008 596 20	. 017 660 10	. 027 9534	. 038 484	. 04873	. 0585
6	. 000 024 439	. 001 032 584	. 004 804 70	. 011 206 39	. 019 2153	. 027 954	. 03685	. 0456
7	. 000 006 106	. 000 457 961	. 002 691 75	. 007 130 08	. 013 2454	. 020 361	. 02794	. 0357
8	. 000 001 526	. 000 203 298	. 001 510 22	. 004 544 80	. 009 1486	. 014 861	. 02124	. 0279
9	. 000 000 381	. 000 090 295	. 000 848 11	. 002 900 54	. 006 3282	. 010 865	. 01617	. 0219
10	. 000 000 095	. 000 040 116	. 000 476 56	. 001 852 76	. 004 3819	. 007 952	. 01232	. 0172
11	. 000 000 024	. 000 017 826	. 000 267 89	. 001 184 17	. 003 0366	. 005 826	. 00938	. 0136
12	. 000 000 006	. 000 007 922	. 000 150 62	. 000 757 17	. 002 1055	. 004 270	. 00718	. 0107
13	. 000 000 001	. 000 003 520	. 000 084 70	. 000 484 28	. 001 4605	. 003 132	. 00548	. 0084
14	. 000 000 000	. 000 001 565	. 000 047 64	. 000 309 80	. 001 0134	. 002 298	. 00419	. 0066
15		. 000 000 695	. 000 026 79	. 000 198 21	. 000 7033	. 001 687	. 00320	. 0051
16		. 000 000 309	. 000 015 07	. 000 126 83	. 000 4893	. 001 239	. 00245	. 0041
17		. 000 000 137	. 000 008 48	. 000 081 16	. 000 3389	. 000 909	. 00188	. 0032
18		. 000 000 061	. 000 004 77	. 000 051 93	. 000 2353	. 000 668	. 00143	. 0026
19		. 000 000 027	. 000 002 68	. 000 033 24	. 000 1634	. 000 490	. 00109	. 0020
20		. 000 000 012	. 000 001 51	. 000 021 27	. 000 1135	. 000 360	. 00084	. 0016
21		. 000 000 005	. 000 000 85	. 000 013 61	. 000 0788	. 000 264	. 00064	. 0013
22		. 000 000 002	. 000 000 48	. 000 008 71	. 000 0547	. 000 194	. 00049	. 0010
23		. 000 000 001	. 000 000 27	. 000 005 58	. 000 0379	. 000 143	. 00038	. 0008
24		. 000 000 000	. 000 000 15	. 000 003 57	. 000 0264	. 000 105	. 00029	. 0006
25			. 000 000 08	. 000 002 28	. 000 0183	. 000 077	. 00022	. 0005
26			. 000 000 05	. 000 001 46	. 000 0104	. 000 057	. 00017	. 0004
27			. 000 000 03	. 000 000 94	. 000 0088	. 000 042	. 00013	. 0003
28			. 000 000 02	. 000 000 60	. 000 0061	. 000 031	. 00010	. 0002
29			. 000. 000 01	. 000 000 38	. 000 0043	. 000 022	. 00007	. 0002
30			. 000 000 01	. 000 000 24	. 000 0030	. 000 016	. 00006	. 0002

TABLE 26.—*Local coefficients* C_{nu} *for* Q=+0.2

Terminal digit of the coefficient is in ninth decimal place.
Sign is that of $(-1)^{u-1}$

u	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8
1	10266 2098	1064 8391	145 8880	22 3195	36243	6109	1057	186
2	7548 1723	770 7575	105 0847	16 0702	26128	4413	765	135
3	6259 8831	632 5200	85 6569	13 0557	21199	3579	620	110
4	5470 8975	549 7640	74 0423	11 2431	18214	3072	532	95
5	4922 3565	493 3353	66 1891	10 0176	16190	2726	472	84
6	4511 8291	451 6378	60 4453	9 1245	14715	2473	428	75
7	4189 4787	419 1372	56 0098	8 4386	13584	2279	394	69
8	3927 6183	392 8410	52 4473	7 8909	12683	2126	367	65
9	3709 4130	370 9747	49 5007	7 4403	11945	1999	344	60
10	3523 9413	352 4082	47 0083	7 0610	11326	1894	326	57
11	3363 7620	336 3822	44 8622	6 7355	10797	1804	311	55
12	3223 6052	322 3631	42 9879	6 4521	10338	1726	297	53
13	3099 6203	309 9631	41 3318	6 2022	9933	1658	285	50
14	2988 9196	298 8925	39 8542	5 9797	9575	1598	274	48
15	2889 2890	288 9291	38 5249	5 7797	9253	1544	265	46
16	2798 9987	279 9000	37 3206	5 5987	8961	1495	257	45
17	2716 6752	271 6675	36 2226	5 4340	8696	1450	248	43
18	2641 2120	264 1212	35 2162	5 2827	8454	1410	241	42
19	2571 7064	257 1706	34 2895	5 1436	8230	1373	235	41
20	2507 4138	250 7414	33 4323	5 0149	8025	1337	229	40
21	2447 7134	244 7713	32 6362	4 8955	7834	1305	224	39
22	2392 0836	239 2084	31 8944	4 7842	7655	1276	219	38
23	2340 0818	234 0082	31 2011	4 6802	7488	1248	214	37
24	2291 3301	229 1330	30 5511	4 5827	7332	1222	209	37
25	2245 5035	224 5503	29 9400	4 4910	7186	1198	205	36
26	2202 3207	220 2321	29 3643	4 4046	7047	1175	201	35
27	2161 5370	216 1537	28 8205	4 3231	6917	1153	198	35
28	2122 9381	212 2938	28 3058	4 2459	6793	1132	194	34
29	2086 3357	208 6336	27 8178	4 1727	6676	1113	191	33
30	2051 5635	205 1563	27 3542	4 1031	6565	1094	188	33

In calculating the values of the modified potential, for $Q=+0.2$, local expansions were used for relative displacements from 0 to 8 and asymptotic expansions were used for relative displacements of 14 or more. For the gap between 8 and 14, with some overlapping, the asymptotic expansion of $(-W)$ through B_5/a^9 was augmented by the quantity $355.7/a^{10}$, determined empirically by comparison with the direct series values on the applicable intervals of the basic range.

CALCULATIONS FOR $Q=-0.2$

For the reflection factor $Q=-0.2$, the early values of μ_{nu}, C_{nu}, and λ_{nu} are shown in tables 28, 29, and 30, respectively. In calculating values of the modified potential, local expansions were used for $a \leqq 6$ and asymptotic expansions for $a \geqq 7.3$. On the range $4.8 \leqq a \leqq 7.2$, the asymptotic expansion values of W through $B_5 a^9$ were decreased by $0.208/a^{10}$, determined empirically. Overlapping of ranges served as a check on the calculations.

TABLE 27.—*Local ratios for* Q=+0.2

u	Values of λ_{nu} where—							
	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8
1	0.735 244 311	0.723 825 262	0.7203 1048	0.7200 059	0.7209 158	0.722 255	0.72368	0.7252
2	.829 324 351	.820 647 187	.8151 2303	.8124 249	.8113 386	.811 141	.81141	.8119
3	.873 961 607	.869 164 696	.8644 0420	.8611 622	.8592 199	.858 161	.85768	.8574
4	.899 734 730	.879 358 337	.8939 3732	.8910 001	.8888 721	.887 445	.88654	.8861
5	.916 599 400	.915 478 233	.9132 2075	.9108 534	.9088 746	.907 372	.90628	.9055
6	.928 554 405	.928 038 401	.9266 1878	.9248 282	.9231 324	.921 712	.92059	.9198
7	.937 495 706	.937 261 363	.9363 9529	.9350 935	.9337 095	.932 447	.93139	.9305
8	.944 443 363	.944 337 740	.9438 2006	.9428 986	9418 048	.940 725	.93976	.9389
9	.949 999 728	.949 952 335	.9496 4732	.9490 075	.9481 626	.947 262	.94640	.9456
10	.954 545 387	.954 524 179	.9543 4636	.9539 084	.9532 670	.952 532	.95178	.9512
11	.958 333 316	.958 323 742	.9582 2098	.9579 246	.9574 437	.956 850	.95622	.9555
12	.961 538 457	.961 534 229	.9614 7509	.9612 763	.9609 196	.960 449	.95991	.9593
13	.964 285 713	.964 283 829	.9642 4997	.9641 175	.9638 552	.963 484	.96306	.9626
14	.966 666 667	.966 665 826	.9666 4652	.9665 588	.9663 672	.966 077	.96571	.9652
15	.968 750 000	.968 749 626	.9687 0945	.9687 431	.9685 428	.968 317	.96802	.9678
16	.970 588 235	.970 588 069	.9705 8184	.9705 439	.9704 422	.970 268	.97004	.9697
17	.972 222 222	.972 222 148	.9722 1861	.9721 938	.9721 215	.971 988	.97178	.9716
18	.973 684 211	.973 684 178	.9736 8218	.9736 660	.9736 142	.973 511	.97335	.9731
19	.975 000 000	.974 999.985	.9749 9886	.9749 883	.9749 513	.974 873	.97476	.9746
20	.976 190 476	.976 190 469	.9761 8984	.9761 830	.9758 518	.976 096	.97599	.9759
21	.977 272 727	.977 272 724	.9772 7237	.9772 679	.9772 491	.977 205	.97712	.9770
22	.978 260 870	.978 260 869	.9782 6066	.9782 578	.9782 445	.978 211	.97815	.9781
23	.979 166 667	.979 166 666	.9791 6655	.9791 647	.9790 541	.979 130	.97908	.9790
24	.980 000 000	.980 000 000	.9799 9993	.9799 987	.9799 921	.979 973	.97993	.9799
25	.980 769 231	.980 769 231	.9807 6920	.9807 684	.9807 615	.980 749	.98072	.9807
26	.981 481 481	.981 481 481	.9814 8146	.9814 810	.9814 658	.981 466	.98144	.9814
27	.982 142 857	.982 142 857	.9821 4284	.9821 425	.9821 402	.982 132	.98211	.9820
28	.982 758 621	.982 758 621	.9827 5861	.9827 584	.9827 568	.982 750	.98273	.9826
29	.983 333 333	.983 333 333	.9833 3333	.9833 332	.9833 320	.983 327	.98332	.9831
30	.983 870 968	.983 870 968	.9838 7097	.9838 702	.9838 701	.983 870	.98387	.9839

TABLE 28.—*Values of* $(-\mu_{nu})$ *for* Q=−0.2

u	Values of $(-\mu_{nu})$ where—					
	n=1	n=2	n=3	n=4	n=5	n=6
1	0.023 632 035	0.054 718 586	0.076 623 86	0.091 865 39	0.102 8770	0.111 142
2	.006 092 729	.025 163 338	.044 579 54	.060 710 89	.073 6253	.083 983
3	.001 544 678	.011 405 454	.025 635 59	.039 749 59	.052 2958	.063 071
4	.000 388 623	.005 126 353	.014 628 72	.025 852 10	.036 9343	.047 137
5	.000 097 489	.002 293 000	.008 305 68	.016 733 66	.025 9729	.035 093
6	.000 024 389	.001 022 817	.004 700 16	.010 794 94	.018 2058	.026 049
7	.000 006 101	.000 455 519	.002 654 12	.006 947 29	.012 7307	.019 290
8	.000 001 526	.000 202 688	.001 496 68	.004 463 58	.008 8860	.014 260
9	.000 000 381	.000 090 142	.000 843 23	.002 864 45	.006 1943	.010 526
10	.000 000 095	.000 040 078	.000 474 81	.001 836 71	.004 3136	.007 761
11	.000 000 024	.000 017 816	.000 267 26	.001 177 05	.003 0017	.005 718
12	.000 000 006	.000 007 919	.000 150 40	.000 754 00	.002 0877	.004 210
13	.000 000 001	.000 003 520	.000 084 62	.000 482 87	.001 4514	.003 098
14	.000 000 000	.000 001 565	.000 047 61	.000 309 17	.001 0088	.002 279
15		.000 000 695	.000 026 78	.000 197 93	.000 7010	.001 676
16		.000 000 309	.000 015 07	.000 126 71	.000 4859	.001 232
17		.000 000 137	.000 008 48	.000 081 10	.000 3383	.000 906
18		.000 000 061	.000 004 77	.000 051 91	.000 2350	.000 665
19		.000 000 027	.000 002 68	.000 033 22	.000 1632	.000 489
20		.000 000 012	.000 001 51	.000 021 27	.000 1134	.000 360
21		.000 000 005	.000 000 85	.000 013 61	.000 0787	.000 264
22		.000 000 002	.000 000 48	.000 008 71	.000 0547	.000 194
23		.000 000 001	.000 000 27	.000 005 58	.000 0379	.000 143
24		.000 000 000	.000 000 15	.000 003 57	.000 0264	.000 105
25			.000 000 08	.000 002 28	.000 0183	.000 077
26			.000 000 05	.000 001 46	.000 0104	.000 057
27			.000 000 03	.000 000 94	.000 0088	.000 042
28			.000 000 02	.000 000 60	.000 0061	.000 031
29			.000 000 01	.000 000 38	.000 0043	.000 022
30			.000 000 01	.000 000 24	.000 0030	.000 016

TABLE 29.—*Local coefficients* C_{nu} *for* $Q=-0.2$

Terminal digit of the coefficient is in ninth decimal place.
Sign is that of $(-1)^{n+u-1}$.
For $u>22$, the value of C_{nu} is the same as that shown in table 24.

u	Values of C_{nu} where— $n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
1	9763 6797	9452 814	1231 168	181 627	28 708	4740
2	7454 3045	7311 275	955 420	140 893	22 233	3664
3	6240 3458	6178 716	811 970	120 031	18 954	3123
4	5466 6247	5440 715	718 500	106 547	16 854	2780
5	4921 3952	4910 589	650 799	96 791	15 341	2533
6	4511 6088	4507 104	598 736	89 260	14 175	2343
7	4189 4275	4187 545	557 111	83 207	13 235	2191
8	3927 6063	3926 816	522 898	78 201	12 456	2065
9	3709 4102	3709 078	494 171	73 975	11 796	1957
10	3523 9407	3523 800	469 636	70 350	11 228	1864
11	3363 7618	3363 702	448 382	67 196	10 732	1784
12	3223 6052	3223 605	429 749	64 423	10 294	1712
13	3099 6203	3099 620	413 248	61 962	9 905	1648
14	2988 9196	2988 915	398 504	59 760	9 555	1590
15	2889 2890	2889 287	385 229	57 775	9 240	1538
16	2798 9987	2798 999	373 194	55 973	8 953	1491
17	2716 6752	2716 675	362 220	54 332	8 690	1448
18	2641 2120	2641 212	352 162	52 821	8 450	1408
19	2571 7064	2571 706	342 893	51 432	8 228	1371
20	2507 4138	2507 414	334 321	50 147	8 023	1337
21	2447 7134	2447 713	326 362	48 953	7 832	1305
22	2392 0836	2392 084	318 944	47 842	7 655	1276

TABLE 30.—*Local ratios* λ_{nu} *for* $Q=-0.2$

u	Values of λ_{nu} where— $n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
1	0. 763 472 871	0. 773 449 562	0. 7760 2757	0. 7757 295	0. 7744 546	0. 772 916
2	. 837 146 609	. 845 094 179	. 8498 5655	. 8519 301	. 8525 205	. 852 357
3	. 876 013 113	. 880 557 600	. 8848 8440	. 8876 637	. 8891 831	. 889 881
4	. 900 262 122	. 902 563 158	. 9057 7522	. 9084 244	. 9102 436	. 911 376
5	. 916 733 682	. 917 833 677	. 9199 9941	. 9222 031	. 9239 764	. 925 259
6	. 928 588 411	. 929 098 745	. 9304 8029	. 9321 832	. 9337 500	. 935 015
7	. 937 504 289	. 937 737 137	. 9385 8799	. 9398 448	. 9411 509	. 942 308
8	. 944 445 525	. 944 550 759	. 9450 6251	. 9459 615	. 9470 093	. 948 022
9	. 950 000 272	. 950 047 566	. 9503 5029	. 9509 792	. 9517 978	. 952 654
10	. 954 545 523	. 954 566 705	. 9547 4366	. 9551 763	. 9558 032	. 956 510
11	. 958 333 350	. 958 342 818	. 9584 4535	. 9587 392	. 9592 119	. 959 787
12	. 961 538 467	. 961 542 692	. 9616 0172	. 9617 994	. 9621 516	. 962 612
13	. 964 285 715	. 964 287 599	. 9643 2140	. 9644 533	. 9647 131	. 965 079
14	. 966 666 667	. 966 667 508	. 9666 8681	. 9667 742	. 9669 645	. 967 251
15	. 968 750 000	. 968 750 374	. 9687 6134	. 9688 190	. 9689 624	. 969 181
16	. 970 588 235	. 970 588 402	. 9705 9464	. 9706 325	. 9707 316	. 970 904
17	. 972 222 222	. 972 222 296	. 9722 2583	. 9722 506	. 9723 226	. 971 988
18	. 973 684 211	. 973 684 244	. 9736 8624	. 9737 024	. 9737 541	. 973 513
19	. 975 000 000	. 975 000 015	. 9750 0114	. 9750 117	. 9750 486	. 974 874
20	. 976 190 476	. 976 190 483	. 9761 9112	. 9761 980	. 9762 244	. 976 284
21	. 977 272 727	. 977 272 730	. 9772 7309	. 9772 775	. 9772 962	. 977 341
22	. 978 260 870	. 978 260 871	. 9782 6108	. 9782 639	. 9784 252	. 978 311
23	. 979 166 667	. 979 166 668	. 9791 6679	. 9791 686	. 9791 780	. 979 204
24	. 980 000 000	. 980 000 000	. 9800 0007	. 9800 013	. 9800 794	. 980 027
25	. 980 769 231	. 980 769 231	. 9807 6926	. 9807 700	. 9807 769	. 980 789
26	. 981 481 481	. 981 481 481	. 9814 8150	. 9814 820	. 9814 831	. 981 496
27	. 982 142 857	. 982 142 857	. 9821 4287	. 9821 432	. 9821 456	. 982 154
28	. 982 758 621	. 982 758 621	. 9827 5863	. 9827 588	. 9827 604	. 982 768
29	. 983 333 333	. 983 333 333	. 9833 3334	. 9833 335	. 9833 346	. 983 339
30	. 983 870 968	. 983 870 968	. 9838 7097	. 9838 702	. 9338 701	. 983 870

CALCULATIONS FOR $Q=+0.3$

For the reflection factor $Q=+0.3$, early values of μ_{nu}, C_{nu}, and λ_{nu} are shown in tables 31, 32, and 33, respectively. In calculating values of the modified potential on the basic range, the direct series values were checked by local expansions for $a \leqq 8$, by asymptotic expansions from $a=16$ to $a=30$, and by asymptotic expansions of $(-W)$ through B_5/a^9 augmented by $6650/a^{10}$ from $a=10$ to $a=16$. The direct value for $a=9$ was checked only approximately.

For values of the relative displacement not on the basic range, values were obtained by local expansions for $a<2.2$, by asymptotic values for $a>16$, by augmented asymptotic expansion for $13<a<16$, and by interpolation in the first order direct remainder for $10<a<13$.

TABLE 31.—*Values of* μ_{nu} *for* $Q=+0.3$

u	Values for μ_{nu} where							
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$
1	0. 041 333 926	0. 1022 3805	0. 1501 779	0. 1865 849	0. 214 783	0. 23716	0. 25530	0. 2703
2	. 009 774 692	. 0426 3397	. 0791 724	. 1121 053	. 140 394	. 16449	. 18508	. 2027
3	. 002 386 664	. 0183 0979	. 0428 000	. 0687 922	. 093 423	. 11583	. 13593	. 1539
4	. 000 590 618	. 0079 8699	. 0234 895	. 0427 995	. 062 937	. 08247	. 10082	. 1178
5	. 000 146 999	. 0035 1343	. 0130 110	. 0268 719	. 042 766	. 05919	. 07532	. 0907
6	. 000 036 678	. 0015 5265	. 0072 482	. 0169 749	. 029 237	. 04273	. 05658	. 0703
7	. 000 009 161	. 0006 8787	. 0040 522	. 0107 672	. 020 075	. 03098	. 04270	. 0547
8	. 000 002 290	. 0003 0518	. 0022 705	. 0068 488	. 013 827	. 02253	. 03232	. 0427
9	. 000 000 572	. 0001 3550	. 0012 740	. 0043 647	. 009 545	. 01643	. 02452	. 0334
10	. 000 000 143	. 0000 6019	. 0007 155	. 0027 852	. 006 599	. 01200	. 01864	. 0261
11	. 000 000 036	. 0000 2674	. 0004 020	. 0017 790	. 004 568	. 00877	. 01420	. 0205
12	. 000 000 009	. 0000 1188	. 0002 261	. 0011 370	. 003 165	. 00643	. 01082	. 0161
13	. 000 000 002	. 0000 0528	. 0001 270	. 0007 270	. 002 202	. 00471	. 00825	. 0127
14	. 000 000 001	. 0000 0235	. 0000 715	. 0004 649	. 001 522	. 00345	. 00630	. 0100
15	. 000 000 000	. 0000 0104	. 0000 401	. 0002 974	. 001 056	. 00253	. 00482	. 0079
16	----------	. 0000 0046	. 0000 226	. 0001 903	. 000 733	. 00186	. 00368	. 0062
17	----------	. 0000 0021	. 0000 127	. 0001 219	. 000 509	. 00136	. 00282	. 0049
18	----------	. 0000 0009	. 0000 071	. 0000 779	. 000 354	. 00100	. 00216	. 0039
19	----------	. 0000 0004	. 0000 040	. 0000 498	. 000 245	. 00074	. 00166	. 0031
20	----------	. 0000 0002	. 0000 023	. 0000 319	. 000 170	. 00054	. 00126	. 0024
21	----------	. 0000 0001	. 0000 013	. 0000 204	. 000 118	. 00040	. 00096	. 0019
22	----------	. 0000 0000	. 0000 007	. 0000 131	. 000 082	. 00029	. 00074	. 0015
23	----------	----------	. 0000 004	. 0000 084	. 000 057	. 00021	. 00056	. 0012
24	----------	----------	. 0000 002	. 0000 054	. 000 040	. 00016	. 00043	. 0009
25	----------	----------	. 0000 001	. 0000 034	. 000 027	. 00011	. 00033	. 0007
26	----------	----------	. 0000 001	. 0000 022	. 000 019	. 00008	. 00025	. 0006
27	----------	----------	. 0000 000	. 0000 014	. 000 013	. 00006	. 00019	. 0005
28	----------	----------	----------	. 0000 009	. 000 009	. 00005	. 00015	. 0004
29	----------	----------	----------	. 0000 006	. 000 006	. 00003	. 00011	. 0003
30	----------	----------	----------	. 0000 004	. 000 004	. 00002	. 00009	. 0002

TABLE 32.—*Local coefficients* C_{nu} *for* Q=+0.3

Terminal digit of the coefficient is in ninth decimal place. Sign is that of $(-1)^{u-1}$

u	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8
1	15620 0089	2480 0356	5175 801	1201 417	295 192	75159	19609	5208
2	11359 9653	1749 4448	3642 207	844 505	207 837	53056	13884	3698
3	9397 3750	1431 9981	2932 875	676 345	166 064	42367	11090	2956
4	8207 9699	1240 2965	2518 744	577 410	141 255	35963	9404	2506
5	7383 8978	1111 3127	2243 661	511 731	124 717	31670	8268	2201
6	6767 8263	1016 7129	2044 989	464 566	112 840	28580	7447	1980
7	6284 2373	943 2754	1892 893	428 749	103 848	26239	6824	1812
8	5891 4320	883 9825	1771 439	400 395	96 761	24398	6333	1679
9	5564 1206	834 7307	1671 362	377 217	90 999	22905	5937	1572
10	5285 9124	792 9344	1586 908	357 793	86 197	21665	5608	1483
11	5045 6431	756 8666	1514 302	341 187	82 112	20614	5330	1407
12	4835 4078	725 3198	1450 950	326 761	78 582	19709	5090	1343
13	4649 4305	697 4183	1395 006	314 065	75 487	18919	4882	1287
14	4483 3794	672 5085	1345 110	302 769	72 742	18221	4698	1237
15	4333 9334	650 0907	1300 232	292 628	70 284	17596	4535	1194
16	4198 4980	629 7750	1259 577	283 453	68 066	17036	4388	1155
17	4075 0128	611 2520	1222 520	275 097	66 049	16526	4256	1119
18	3961 8180	594 2728	1188 553	267 444	64 204	16061	4135	1087
19	3857 5596	578 6339	1157 273	260 398	62 507	15635	4024	1057
20	3761 1206	564 1681	1128 339	253 884	60 940	15241	3922	1030
21	3671 5701	550 7355	1101 472	247 836	59 486	14876	3828	1006
22	3588 1254	538 2188	1076 439	242 201	58 133	14536	3740	982
23	3510 1226	526 5184	1053 037	236 935	56 867	14219	3658	960
24	3436 9951	515 5493	1031 099	231 998	55 681	13922	3581	940
25	3368 2552	505 2383	1010 477	227 358	54 567	13643	3509	922
26	3303 4810	495 5222	991 044	222 985	53 517	13380	3441	904
27	3242 3055	486 3458	972 692	218 856	52 526	13132	3378	886
28	3184 4072	477 6611	955 322	214 947	51 587	12898	3316	870
29	3129 5036	469 4255	938 851	211 241	50 698	12674	3259	855
30	3077 3452	461 6018	923 204	207 721	49 853	12463	3205	841

CALCULATIONS FOR $Q=-0.3$

For the reflection factor $Q=-0.3$, early values of μ_{nu}, C_{nu}, and λ_{nu} are shown in tables 34, 35, and 36, respectively. On the basic range, direct values of the modified potential were checked by local expansions for $a \leqq 6.9$ and by asymptotic expansions for $a \geqq 6.4$. For intermediate values, local expansions were used for $a<2.2$ and asymptotic expansions were used for $a>10$.

CALCULATIONS FOR $0.4 \leqq Q \leqq 0.9$

For reflection factors Q, where $-0.9 \leqq Q \leqq -0.4$ or $0.4 \leqq Q \leqq 0.9$, local expansion involve so many terms that other methods were developed, and special devices were used. For the sake of conciseness, these 12 cases are discussed in a condensed notation. For each case, the values of W on the basic range were calculated by direct series, and the summary omits such values.

TABLE 33.—*Local ratios* λ_{nu} *for* Q=+0.3

u	Values for λ_{nu} where—							
	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8
1	0. 727 270 091	0. 7094 4337	0. 7036 992	0. 7029 240	0. 704 073	0. 70595	0. 70805	0. 7101
2	. 827 236 241	. 8138 9204	. 8052 466	. 8008 775	. 799 010	. 79851	. 79877	. 7995
3	. 873 432 201	. 8661 2996	. 8587 968	. 8537 203	. 850 604	. 84884	. 84795	. 8476
4	. 899 600 979	. 8960 0570	. 8907 858	. 8862 535	. 882 921	. 88064	. 87916	. 8782
5	. 916 565 554	. 9148 7558	. 9114 520	. 9078 318	. 904 773	. 90242	. 90069	. 8996
6	. 928 545 878	. 9277 6966	. 9256 250	. 9229 034	. 920 305	. 91811	. 91637	. 9150
7	. 937 493 558	. 9371 4147	. 9358 364	. 9338 656	. 931 758	. 92981	. 92817	. 9268
8	. 944 442 821	. 9442 8423	. 9435 054	. 9421 143	. 940 455	. 93880	. 93730	. 9360
9	. 949 999 592	. 9499 2846	. 9494 701	. 9485 060	. 947 228	. 94586	. 94455	. 9433
10	. 954 545 353	. 9545 1352	. 9542 464	. 9535 877	. 952 619	. 95150	. 95039	. 9493
11	. 958 333 307	. 9583 1909	. 9581 648	. 9577 192	. 956 994	. 95611	. 95514	. 9542
12	. 961 538 455	. 9615 3211	. 9614 432	. 9611 447	. 960 615	. 95990	. 95910	. 9583
13	. 964 285 713	. 9642 8288	. 9642 322	. 9640 332	. 963 631	. 96308	. 96243	. 9617
14	. 966 666 666	. 9666 6540	. 9666 363	. 9665 048	. 966 217	. 96578	. 96525	. 9647
15	. 968 750 000	. 9687 4944	. 9687 330	. 9686 463	. 968 437	. 96810	. 96766	. 9672
16	. 970 588 235	. 9705 8800	. 9705 786	. 9705 219	. 970 371	. 97010	. 96976	. 9693
17	. 972 222 222	. 9722 2210	. 9722 168	. 9721 794	. 972 071	. 97187	. 97158	. 9712
18	. 973 684 211	. 9736 8416	. 9736 812	. 9736 569	. 973 578	. 97343	. 97319	. 9729
19	. 975 000 000	. 9749 9998	. 9749 983	. 9749 825	. 974 927	. 97480	. 97460	. 9743
20	. 976 190 476	. 9761 9047	. 9761 895	. 9761 792	. 976 139	. 97605	. 97591	. 9757
21	. 977 272 727	. 9772 7272	. 9772 721	. 9772 656	. 977 238	. 97716	. 97706	. 9769
22	. 978 260 870	. 9782 6087	. 9782 606	. 9782 563	. 978 237	. 97818	. 97808	. 9780
23	. 979 166 667	. 9791 6667	. 9791 665	. 9791 637	. 979 150	. 97912	. 97904	. 9789
24	. 980 000 000	. 9800 0000	. 9799 999	. 9799 980	. 979 987	. 97995	. 97990	. 9798
25	. 980 769 231	. 9807 6923	. 9807 692	. 9807 681	. 980 761	. 98074	. 98069	. 9807
26	. 981 481 481	. 9814 8148	. 9814 814	. 9814 807	. 981 475	. 98146	. 98142	. 9814
27	. 982 142 857	. 9821 4286	. 9821 429	. 9821 424	. 982 139	. 98213	. 98210	. 9820
28	. 982 758 621	. 9827 5862	. 9827 586	. 9827 583	. 982 756	. 98274	. 98272	. 9827
29	. 983 333 333	. 9833 3333	. 9833 333	. 9833 331	. 983 331	. 98332	. 98331	. 9832
30	. 983 870 968	. 9838 7097	. 9838 710	. 9838 712	. 983 872	. 98389	. 98389	. 9839

TABLE 34.—*Values of* $(-\mu_{nu})$ *for* Q=−0.3

u	Values for $(-\mu_{nu})$ where—						
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
1	0.034 533 220	0.0791 1414	0.1099 658	0.1311 404	0.146 263	0.15752	0.16620
2	.009 028 680	.0369 4084	.0649 351	.0878 799	.106 039	.12047	.13208
3	.002 304 150	.0168 9604	.0377 174	.0581 282	.076 077	.09135	.10424
4	.000 581 464	.0076 3460	.0216 675	.0380 853	.054 142	.06879	.08179
5	.000 145 981	.0034 2545	.0123 565	.0247 828	.038 296	.05153	.06387
6	.000 036 564	.0015 3066	.0070 128	.0160 480	.026 961	.03843	.04968
7	.000 009 149	.0006 8238	.0039 675	.0103 557	.018 916	.02856	.03853
8	.000 002 288	.0003 0381	.0022 400	.0066 660	.013 235	.02118	.02980
9	.000 000 572	.0001 3516	.0012 630	.0042 835	.009 243	.01567	.02300
10	.000 000 143	.0000 6010	.0007 116	.0027 492	.006 445	.01157	.01773
11	.000 000 036	.0000 2673	.0004 007	.0017 629	.004 490	.00853	.01364
12	.000 000 009	.0000 1188	.0002 256	.0011 298	.003 125	.00629	.01049
13	.000 000 002	.0000 0528	.0001 269	.0007 238	.002 173	.00464	.00806
14	.000 000 001	.0000 0235	.0000 714	.0004 635	.001 511	.00342	.00618
15	.000 000 000	.0000 0104	.0000 401	.0002 968	.001 051	.00251	.00474
16	.000 000 000	.0000 0046	.0000 226	.0001 900	.000 730	.00184	.00363
17	.000 000 000	.0000 0021	.0000 127	.0001 216	.000 507	.00136	.00279
18	.000 000 000	.0000 0009	.0000 071	.0000 778	.000 352	.00100	.00214
19	.000 000 000	.0000 0004	.0000 040	.0000 498	.000 245	.00073	.00164

For $u>19$, the absolute value of μ_{nu} is the same as that shown in table 31.

TABLE 35.—*Local coefficients* C_{nu} *for* Q=−0.3

Terminal digit of the coefficient is in ninth decimal place. Sign is that of $(-1)^{n+u-1}$

u	Values of C_{nu} where—						
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
1	144 820 017	2071 9932	4005 154	879 720	207 458	51181	13025
2	111 484 274	1625 1623	3155 844	692 641	162 924	40073	10169
3	93 533 986	1382 4899	2706 420	596 028	140 321	34501	8745
4	81 983 552	1221 0747	2407 616	532 623	125 696	30938	7844
5	73 817 347	1103 6285	2187 476	485 990	115 022	28359	7198
6	67 673 307	1013 5829	2016 035	449 481	106 679	26356	6981
7	62 841 222	941 9838	1877 774	419 789	99 878	24724	6293
8	58 914 050	883 4443	1763 467	395 020	94 178	23355	5952
9	55 641 142	834 5048	1667 127	373 969	89 306	22182	5662
10	52 859 108	792 8390	1584 645	355 818	85 080	21160	5407
11	50 456 427	756 8262	1513 086	339 981	81 372	20261	5183
12	48 354 077	725 3026	1450 295	326 021	78 089	19460	4983
13	46 494 305	697 4109	1394 652	313 610	75 157	18743	4803
14	44 833 794	672 5053	1344 918	302 488	72 521	18096	4640
15	43 339 334	650 0893	1300 128	292 454	70 136	17508	4492
16	41 984 980	629 7744	1259 521	283 345	67 966	16973	4356
17	40 750 128	611 2518	1222 488	275 030	65 982	16482	4232
18	39 618 180	594 2726	1188 537	267 402	64 158	16029	4117
19	38 575 596	578 6339	1157 263	260 372	62 477	15612	4010
20	37 611 206	564 1681	1128 333	253 868	60 920	15225	3912
21	36 715 701	550 7355	1101 470	247 826	59 472	14864	3820
22	35 881 254	538 2188	1076 437	242 195	58 123	14528	3734
23	35 101 226	526 5184	1053 037	236 931	56 861	14213	3654
24	34 369 951	515 5493	1031 099	231 996	55 677	13918	3577
25	33 682 552	505 2383	1010 477	227 356	54 565	13639	3507
26	33 034 810	495 5222	991 044	222 985	53 515	13378	3439
27	32 423 055	486 3458	972 692	218 856	52 524	13130	3376
28	31 844 072	477 6611	955 322	214 947	51 587	12896	3316
29	31 295 036	469 4255	938 851	211 241	50 698	12674	3259
30	30 773 452	461 6018	923 204	207 721	49 853	12463	3205

As a matter of convenience, certain expressions are adopted:

$W[\alpha(h)\beta]$ means that the function $W(Q,a)$ for the specified value of Q was calculated and tabulated by steps of h from $a=\alpha$ to $a=\beta$.

$W[\gamma(k)\delta]$ were interpolated in $W[\alpha(h)\beta]$ means that the values of W on the range $\gamma \leqq a \leqq \delta$ were calculated for steps of k by interpolation in the values on the basic range, in which the step is h.

$W[\gamma(k)\delta]$ were interpolated in $R_n[\alpha(h)\beta]$ means that:

1. The values of the remainder of order n, (retaining the first n terms of the series) are calculated, tabulated, and differenced on the range (α, β) as far as needed to justify the interpolation.
2. The remainder table is smoothed by correction of errors and adjustment.
3. The range (α, β) is extended at each end so that the differences will be complete over the range (α, β).
4. The value of R_n is calculated by interpolation for each value on the range $a=[\gamma(k)\delta]$ from the smoothed table for the range $a=[\alpha(h)\beta]$.

TABLE 36.—*Local ratios* λ_{nu} *for* Q = −0.3

u	Values for λ_{nu} where—						
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
1	0. 769 812 598	0. 7843 4734	0. 7879 457	0. 7873 425	0. 785 336	0. 78298	0. 78069
2	. 838 988 164	. 8506 7806	. 8575 898	. 8605 152	. 861 263	. 86092	. 86006
3	. 876 510 744	. 8832 4303	. 8895 941	. 8936 199	. 895 773	. 89673	. 89693
4	. 900 392 252	. 9038 1738	. 9085 655	. 9124 463	. 915 078	. 91669	. 91757
5	. 916 766 981	. 9184 0953	. 9216 264	. 9248 770	. 927 471	. 92933	. 93057
6	. 928 596 887	. 9293 6033	. 9314 191	. 9339 433	. 936 248	. 93810	. 93946
7	. 937 506 432	. 9378 5515	. 9391 260	. 9409 953	. 942 929	. 94462	. 94601
8	. 944 446 065	. 9446 0377	. 9453 692	. 9467 097	. 948 265	. 94976	. 95106
9	. 950 000 408	. 9500 7132	. 9505 245	. 9514 638	. 952 683	. 95396	. 95512
10	. 954 545 865	. 9545 7731	. 9548 425	. 9554 895	. 956 424	. 95749	. 95852
11	. 958 333 359	. 9583 4756	. 9585 012	. 9589 411	. 959 647	. 96050	. 96139
12	. 961 538 469	. 9615 4481	. 9616 334	. 9619 293	. 962 456	. 96314	. 96391
13	. 964 285 715	. 9642 8854	. 9643 392	. 9645 369	. 964 925	. 96548	. 96612
14	. 966 666 668	. 9666 6793	. 9666 970	. 9668 279	. 967 113	. 96755	. 96807
15	. 968 750 000	. 9687 5056	. 9687 670	. 9688 535	. 969 061	. 96940	. 96984
16	. 970 588 235	. 9705 8848	. 9705 978	. 9706 546	. 970 804	. 97106	. 97141
17	. 972 222 222	. 9722 2234	. 9722 276	. 9722 648	. 972 373	. 97257	. 97285
18	. 973 684 211	. 9736 8426	. 9736 872	. 9737 115	. 973 788	. 97394	. 97417
19	. 975 000 000	. 9750 0002	. 9750 017	. 9750 175	. 975 073	. 97519	. 97538
20	. 976 190 476	. 9761 9049	. 9761 915	. 9762 017	. 976 241	. 97633	. 97647
21	. 977 272 727	. 9772 7274	. 9772 733	. 9772 799	. 977 308	. 97738	. 97748
22	. 978 260 870	. 9782 6087	. 9782 612	. 9782 655	. 978 285	. 97834	. 97844
23	. 979 166 667	. 9791 6667	. 9791 669	. 9791 696	. 979 184	. 97922	. 97930
24	. 980 000 000	. 9800 0000	. 9800 001	. 9800 020	. 980 013	. 98005	. 98010
25	. 980 769 231	. 9807 6923	. 9807 692	. 9807 704	. 980 777	. 98080	. 98085
26	. 981 481 481	. 9814 8148	. 9814 815	. 9814 823	. 981 487	. 98150	. 98154
27	. 982 142 857	. 9821 4286	. 9821 429	. 9821 433	. 982 147	. 98215	. 98218
28	. 982 758 621	. 9827 5862	. 9827 586	. 9827 589	. 982 762	. 98278	. 98280
29	. 983 333 333	. 9833 3333	. 9833 333	. 9833 335	. 983 335	. 98334	. 98335
30	. 983 870 968	. 9838 7097	. 9838 710	. 9838 708	. 983 870	. 98387	. 98385

5. The resulting values of R_n are tabulated, differenced, and smoothed, with necessary corrections and adjustments.
6. The value of W is calculated by evaluating the first n terms directly and adding the corresponding value of R_n.

The nth direct residue means the residue or remainder of the direct series after isolation of the first n terms.

The nth asymptotic residue means the residue of the asymptotic expansion, after isolation of the terms through B_n/a^{2n-1}.

Interpolation in the nth specified residue means that the specified residue is evaluated over the indicated range, that this residue is interpolated to complete the new range, and that the values of the isolated initial terms are added to the residues to obtain the values of the function over the new range. The expanded table is smoothed and corrected by differences.

With the foregoing condensations, the calculations of $W(Q, a)$ may be summarized for each case.

$Q=+0.4$:

1. The values of $-W[0.00(0.05)2.50]$ were interpolated in the first direct residue $R_1[0.0(0.1)\text{-}2.5]$.
2. The values of $-W[0.00(0.02)1.40]$ were interpolated in $-W[0.00(0.05)2.50]$.
3. The values of $-W[10.0(0.2)14.0]$ were interpolated in $-W[10.0(0.5)14.0)]$.
4. The values of $W[15.0(0.5)20]$ were interpolated in the third asymptotic residue $R_3[15(1)20]$.
5. For $a>20$, asymptotic expansions are adequate.

$Q=-0.4$:

1. The values of $W[0.00(0.02)1.30]$ were interpolated in the first local residue, $R_1[0.0(0.1)1.3]$.
2. The values of $W[1.30(0.05)2.20]$ were interpolated in $W[1.3(0.1)2.2]$.
3. For $a \geqq 5.8$, fifth order asymptotic expansions are adequate.

$Q=+0.5$:

1. Values of $-W[0.00(0.02)1.60]$ were interpolated in the first direct residue $R_1[0.0(0.1)1.6]$.
2. Values of $-W[1.60(0.05)3.00]$ were interpolated in the first direct residue, $R_1[1.6(0.1)3.0]$.
3. Values of $-W[15.5(1.0)29.5]$ were interpolated in the fourth asymptotic residue $R_4[15(1)30]$.
4. Values of $-W[10.1(0.1)10.4(0.2)15.8]$ were interpolated in $-W[10.0(0.5)16.0]$.

$Q=-0.5$:

1. Values of $W[0.00(0.02)1.40(0.05)2.20]$ were interpolated in the first direct residue, $R_1[0.0(0.1)2.2]$.
2. For a >6.4 asymptotic expansions are adequate.

$Q=+0.6$:

1. Values of $-W[0.00(0.02)1.60(0.05)3.00]$ were interpolated in the first direct residue, $R_1[0.0(0.1)3.4]$.
2. The value of $-W(15.5)$ was inserted between $-W[10.0(0.5)15]$ and $-W[16.0(0.5)30.0]$ by smoothing.
3. Values of $-W[10(0.2)16.0]$ were interpolated in $-W[10.0(0.5)16.0]$.
4. Values of $-W[16.0(0.5)30.0]$ were interpolated in the fourth asymptotic residue $R_4[16(1)30]$.

$Q=-0.6$:

1. Values of $W[0.00(0.02)1.40(0.05)3.20]$ were interpolated in the first direct residue, $R_1[0.0(0.1)3.2]$.
2. For $a \geqq 6.7$, asymptotic expansions are adequate.

$Q=+0.7$:

1. Values of $-W[0.0(0.02)1.60(0.05)2.40]$ were interpolated in the first direct residue, $R_1[0.0(0.1)2.4]$.
2. Values of $-W[15.0(0.5)30.0]$ were interpolated in $-W[15(1)30]$.
3. Values of $-W[10.0(0.2)17.0]$ were interpolated in $-W[10.0(0.5)17.0]$.

$Q=-0.7$:

1. Values of $W[0.00(0.02)1.40(0.05)3.20]$ were interpolated in the first direct residue, $R_1[0.0(0.1)3.2]$.
2. For $a \geqq 6.5$, asymptotic expansions are adequate.

$Q=+0.8$:

1. Values of $-W[0.00(0.02)1.60(0.05)2.50]$ were interpolated in the first direct residue, $R_1[0.0(0.1)2.5]$.
2. Values of $-W[15.0(0.5)30.0]$ were interpolated in the second direct residue, $R_2[15(1)30]$.
3. Values of $-W[10.0(0.02)18.0]$ were interpolated in $-W[10.0(0.5)18.0]$.

$Q=-0.8$:

1. Values of $W[0.00(0.02)1.50]$ were interpolated in the first direct residue, $R_1[0.0(0.1)1.5]$.
2. Values of $W[1.50(0.05)3.40]$ were interpolated in the second direct residue, $R_2[1.5(0.1)3.4]$.
3. For $a \geqq 6.6$ asymptotic expansions are adequate.

$Q=+0.9$:

1. Values of $-W[0.00(0.02)1.60(0.05)2.4]$ were interpolated in the first direct residue, $R_1[0.0(0.1)2.4]$.
2. Values of $-W[15.0(0.5)33.0]$ were interpolated in the second direct residue, $R_2[12(1)33]$.
3. Values of $-W[10.0(0.2)19.0]$ were interpolated in $-W[10.0(0.5)19.0]$.

$Q=-0.9$:

1. Values of $W[0.00(0.02)1.70(0.05)3.60]$ were interpolated in the first direct residue, $R_1[0.0(0.1)4.0]$.
2. For $a \geqq 6.7$, asymptotic expansions are adequate.

NUMERICAL VALUES OF THE MODIFIED POTENTIAL

The results of the analysis described in this report are summarized in tables 37 and 38. Table 37 (p. 59) shows the coefficients to be used for interpolating between arguments shown in table 38 (p. 59). Table 38 shows values of the modified potential for reflection factors from -1 to $+1$ by steps of one-tenth (except for the trivial case of zero) and for relative displacements not exceeding 30, spaced so that fourth order interpolation is always adequate.

In the use of table 38, the following observations should be noted:

Values of $\pm W$ are complete, except for forcing errors of calculation. Differences are expressed in units of one in the eighth decimal place. Values have been rounded so that apparent inconsistencies occur, but the values of $\pm W$ are usually within half a unit in the terminal digit and within one unit except in rare cases. The 2nd and 3rd differences are usually within 1 unit and the 4th difference within 2 units.

TABLE 37.—*Difference coefficients for interpolation*

$y = xy_1 + (1-x)\, y_0 + F_2\Delta^2_{-1} + F_3\Delta^3_{-1} + F_4\Delta^4_{-2} + \ldots$

$F_2 = -x\,(1-x)/2$ $\qquad$ $\Delta^2_{-1} = y_1 - 2y_0 + y_{-1}$

$F_3 = -(1+x)\,x\,(1-x)/6$ $\qquad$ $\Delta^3_{-1} = y_2 - 3y_1 + 3y_0 - y_{-1}$

$F_4 = (1+x)\,x\,(1-x)\,(2-x)/24$ $\qquad$ $\Delta^4_{-2} = y_2 - 4y_1 + 6y_0 - 4y_{-1} + y_{-2}$

x	$-F_2$	$-F_3$	$+F_4$	x	$-F_2$	$-F_3$	$+F_4$
0.00	0.00000	0.00000	0.0000	0.50	0.12500	0.06250	0.0234
.01	.00495	.00167	.0008	.51	.12495	.06289	.0234
.02	.00980	.00333	.0016	.52	.12480	.06323	.0234
.03	.01455	.00500	.0025	.53	.12455	.06352	.0233
.04	.01920	.00666	.0033	.54	.12420	.06376	.0233
.05	.02375	.00831	.0041	.55	.12375	.06394	.0232
.06	.02820	.00996	.0048	.56	.12320	.06406	.0231
.07	.03255	.01161	.0056	.57	.12255	.06413	.0229
.08	.03680	.01325	.0064	.58	.12180	.06415	.0228
.09	.04095	.01488	.0071	.59	.12095	.06410	.0226
.10	.04500	.01650	.0078	.60	.12000	.06400	.0224
.11	.04895	.01811	.0086	.61	.11895	.06384	.0222
.12	.05280	.01971	.0093	.62	.11780	.06361	.0219
.13	.05655	.02130	.0100	.63	.11655	.06333	.0217
.14	.06020	.02288	.0106	.64	.11520	.06298	.0214
.15	.06375	.02444	.0113	.65	.11375	.06256	.0211
.16	.06720	.02598	.0120	.66	.11220	.06208	.0208
.17	.07055	.02751	.0126	.67	.11055	.06154	.0205
.18	.07380	.02903	.0132	.68	.10880	.06093	.0201
.19	.07695	.03052	.0138	.69	.10695	.06025	.0197
.20	.08000	.03200	.0144	.70	.10500	.05950	.0193
.21	.08295	.03346	.0150	.71	.10295	.05868	.0189
.22	.08580	.03489	.0155	.72	.10080	.05779	.0185
.23	.08855	.03631	.0161	.73	.09855	.05683	.0180
.24	.09120	.03770	.0166	.74	.09620	.05580	.0176
.25	.09375	.03906	.0171	.75	.09375	.05469	.0171
.26	.09620	.04040	.0176	.76	.09120	.05350	.0166
.27	.09855	.04172	.0180	.77	.08855	.05224	.0161
.28	.10080	.04301	.0185	.78	.08580	.05091	.0155
.29	.10295	.04427	.0189	.79	.08295	.04949	.0150
.30	.10500	.04550	.0193	.80	.08000	.04800	.0144
.31	.10695	.04670	.0197	.81	.07695	.04643	.0138
.32	.10880	.04787	.0201	.82	.07380	.04477	.0132
.33	.11055	.04901	.0205	.83	.07055	.04304	.0126
.34	.11220	.05012	.0208	.84	.06720	.04122	.0120
.35	.11375	.05119	.0211	.85	.06375	.03931	.0113
.36	.11520	.05222	.0214	.86	.06020	.03732	.0106
.37	.11655	.05322	.0217	.87	.05655	.03525	.0100
.38	.11780	.05419	.0219	.88	.05280	.03309	.0093
.39	.11895	.05511	.0222	.89	.04895	.03084	.0086
.40	.12000	.05600	.0224	.90	.04500	.02850	.0078
.41	.12095	.05685	.0226	.91	.04095	.02607	.0071
.42	.12180	.05765	.0228	.92	.03680	.02355	.0064
.43	.12255	.05842	.0229	.93	.03255	.02094	.0056
.44	.12320	.05914	.0231	.94	.02820	.01824	.0048
.45	.12375	.05981	.0232	.95	.02375	.01544	.0041
.46	.12420	.06044	.0233	.96	.01920	.01254	.0033
.47	.12455	.06103	.0233	.97	.01455	.00955	.0025
.48	.12480	.06157	.0234	.98	.00980	.00647	.0016
.49	.12495	.06206	.0234	.99	.00495	.00328	.0008

For interpolation, use

$$f(x) = xf(1) + (1-x)f(0) + F_2\Delta^2_{-1} + F_3\Delta^3_{-1} + F_4\Delta^4_{-2}$$

with the line selected so that $0 < x < 1$. Except for forcing errors, sufficient accuracy is obtainable by:

linear interpolation $\quad$ if $|\Delta^4_{-2}| \leqq 43, |\Delta^3_{-1}| \leqq 16$, and $|\Delta^2_{-1}| \leqq 8$.
quadratic interpolation if $|\Delta^4_{-2}| \leqq 43, |\Delta^3_{-1}| \leqq 16$, but $|\Delta^2_{-1}| > 8$.
cubic interpolation $\quad$ if $|\Delta^4_{-2}| \leqq 43$ but $|\Delta^3_{-1}| > 16$.
quartic interpolation $\quad$ if $|\Delta^4_{-2}| > 43$.

For two additional digits in the argument, beyond those tabulated, the values of F_k are shown in table 37. For more than two additional digits, F_k may be calculated from the formulas shown in table 37, or taken from more extensive tables of Lagrangean interpolants. For some purposes linear interpolation in table 37 is adequate.

TABLE 38.—*Modified potential*

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q = −1.0; tabular spacing = 0.02				
0.00	0.0000 0000	+3 6050	−70	−140
.02	.0001 8025	3 5980	−209	−139
.04	.0007 2030	3 5771	−346	−137
.06	.0016 1807	3 5425	−479	−133
.08	.0028 7008	3 4946	−608	−129
.10	.0044 7157	3 4338	−730	−122
.12	.0064 1643	3 3608	−846	−116
.14	.0086 9737	3 2762	−954	−108
.16	.0113 0593	3 1808	−1052	−98
.18	.0142 3257	3 0756	−1141	−89
.20	.0174 6676	2 9614	−1220	−79
.22	.0209 9710	2 8394	−1289	−69
.24	.0248 1138	2 7105	−1347	−58
.26	.0288 9671	2 5758	−1395	−48
.28	.0332 3962	2 4364	−1432	−37
.30	.0378 2617	2 2932	−1458	−26
.32	.0426 4204	2 1474	−1475	−17
.34	.0476 7265	2 0000	−1482	−7
.36	.0529 0326	1 8518	−1480	+2
.38	.0583 1905	1 7038	−1470	10
.40	.0639 0522	1 5568	−1452	18
.42	.0696 4707	1 4115	−1428	25
.44	.0755 3007	1 2688	−1397	31
.46	.0815 3995	1 1291	−1360	36
.48	.0876 6275	9931	−1319	41
.50	.0938 8485	8612	−1274	45
.52	.1001 9307	7338	−1225	49
.54	.1065 7467	6113	−1174	52
.56	.1130 1740	4939	−1120	53
.58	.1195 0953	3819	−1066	54
.60	.1260 3984	2754	−1010	56
.62	.1325 9769	1744	−953	57
.64	.1391 7298	+791	−896	57
.66	.1457 5618	−105	−840	56
.68	.1523 3834	−945	−785	55
.70	.1589 1103	−1730	−730	55
.72	.1654 6643	−2460	−677	53
.74	.1719 9722	−3138	−625	52
.76	.1784 9664	−3763	−575	50
.78	.1849 5843	−4338	−527	48
.80	.1913 7684	−4865	−480	46
.82	.1977 4660	−5345	−436	45
.84	.2040 6291	−5781	−394	42
.86	.2103 2141	−6175	−354	40
.88	.2165 1816	−6528	−315	39
.90	.2226 4963	−6843	−279	36
.92	.2287 1266	−7122	−245	34
.94	.2347 0448	−7367	−213	32
.96	.2406 2263	−7580	−183	30
.98	.2464 6498	−7763	−155	28

TABLE 38.—*Modified potential*—Continued

Q = −1.0; tabular spacing = 0.02—Continued

a	*W*	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
1.00	0.2522 2971	−7918	−128	27
1.02	.2579 1526	−8046	−104	24
1.04	.2635 2035	−8150	−82	22
1.06	.2690 4394	−8232	−60	21
1.08	.2744 8522	−8292	−41	19
1.10	.2798 4357	−8333	−23	18
1.12	.2851 1859	−8357	−7	16
1.14	.2903 1005	−8364	+8	15
1.16	.2954 1787	−8356	22	14
1.18	.3004 4212	−8334	34	13
1.20	.3053 8304	−8300	46	11
1.22	.3102 4096	−8254	56	10
1.24	.3150 1633	−8198	65	9
1.26	.3197 0972	−8133	73	8
1.28	.3243 2178	−8060	81	8
1.30	.3288 5323	−7979	87	7
1.32	.3333 0489	−7892	93	6
1.34	.3376 7763	−7799	98	5
1.36	.3419 7238	−7701	103	5
1.38	.3461 9013	−7598	107	4
1.40	.3503 3190	−7491	110	3
1.42	.3543 9876	−7381	113	3
1.44	.3583 9180	−7268	115	2
1.46	.3623 1216	−7153	116	1
1.48	.3661 6099	−7037	118	2
1.50	.3699 3945	−6918	119	1
1.52	.3736 4873	−6799	120	0
1.54	.3772 9001	−6679	121	1
1.56	.3808 6450	−6558	120	0
1.58	.3843 7341	−6438	121	0
1.60	.3878 1794	−6317	120	0
1.62	.3911 9930	−6197	120	−1
1.64	.3945 1869	−6077	119	−1
1.66	.3977 7730	−5958	118	−1
1.68	.4009 7633	−5840	117	−1
1.70	.4041 1696	−5723	117	−1
1.72	.4072 0035	−5607	114	−2
1.74	.4102 2767	−5493	113	−1
1.76	.4132 0006	−5380	112	−1
1.78	.4161 1866	−5268	110	−2
1.80	.4189 8457	−5158	108	−2
1.82	.4217 9892	−5049	107	−2
1.84	.4245 6277	−4942	105	−2
1.86	.4272 7719	−4837	103	−2
1.88	.4299 4325	−4734	102	−2
1.90	.4325 6196	−4632	100	−2
1.92	.4351 3436	−4533	98	−2
1.94	.4376 6142	−4435	96	−2
1 96	.4401 4414	−4339	94	−2
1.98	.4425 8347	−4245	92	−2
2.00	.4449 8035	−4152	90	−2
2.02	.4473 3571	−4062	88	−2
2.04	.4496 5045	−3974	87	−2
2.06	.4519 2546	−3887	85	−2
2.08	.4541 6159	−3802	83	−2
2.10	.4563 5971	−3719	81	−2
2.12	.4585 2063	−3638	79	−2
2.14	.4606 4518	−3558	78	−2
2.16	.4627 3414	−3481	76	−1
2.18	.4647 8829	−3405	74	−2
2.20	0.4668 0839	−3331	72	−2
2.22	.4687 9519	−3259	71	−1
2.24	.4707 4939	−3188	69	−2
2.26	.4726 7172	−3119	68	−2
2.28	.4745 6287	−3051	66	−2
2.30	.4764 2350	−2985	64	−2
2.32	.4782 5428	−2921	63	−2
2.34	.4800 5586	−2858	62	−1
2.36	.4818 2884	−2797	60	−2
2.38	.4835 7387	−2737	58	−2
2.40	.4852 9152	−2678	57	−2
2.42	.4869 8239	−2621	56	−1
2.44	.4886 4705	−2566	54	−1
2.46	.4902 8605	−2512	53	−1
2.48	.4918 9993	−2459	52	−1
2.50	.4934 8923	−2407	51	−1
2.52	.4950 5446	−2356	49	−2
2.54	.4965 9612	−2307	48	−1
2.56	.4981 1472	−2259	47	−1
2.58	.4996 1072	−2212	46	−1
2.60	.5010 8460	−2167	45	−1
2.62	.5025 3681	−2122	43	−1
2.64	.5039 6781	−2079	43	−1
2.66	.5053 7802	−2036	42	−1
2.68	.5067 6786	−1994	40	−2
2.70	.5081 3777	−1954	39	−1
2.72	.5094 8813	−1915	39	−1
2.74	.5108 1934	−1876	38	−1
2.76	.5121 3178	−1839	37	−1
2.78	.5134 2584	−1802	36	−1
2.80	.5147 0188	−1767	35	0
2.82	.5159 6026	−1732	34	−1
2.84	.5172 0132	−1698	33	−1
2.86	.5184 2540	−1664	32	−1
2.88	.5196 3284	−1632	32	0
2.90	.5208 2395	−1600	31	−1
2.92	.5219 9907	−1569	30	−1
2.94	.5231 5849	−1539	30	0
2.96	.5243 0252	−1510	29	−1
2.98	.5254 3146	−1481	28	−1
3.00	.5265 4558	−1453	27	0
3.02	.5276 4517	−1426	27	0
3.04	.5287 3051	−1399	26	−1
3.06	.5298 0186	−1373	26	0
3.08	.5308 5948	−1347	25	−1
3.10	.5319 0363	−1322	24	−1
3.12	.5329 3456	−1298	24	0
3.14	.5339 5250	−1274	23	−1
3.16	.5349 5771	−1251	23	−1
3.18	.5359 5040	−1228	22	0
3.20	.5369 3082	−1206	22	−1
3.22	.5378 9917	−1184	21	0
3.24	.5388 5568	−1163	21	0
3.26	.5398 0055	−1142	20	−1
3.28	.5407 3400	−1122	19	−1
3.30	.5416 5623			

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-1.0$; tabular spacing=0.1				
3.3	0.5416 5623	−2 7587	2313	−276
3.4	.5461 0578	−2 5274	2068	−245
3.5	.5503 0258	−2 3206	1854	−214
3.6	.5542 6733	−2 1352	1666	−188
3.7	.5580 1856	−1 9686	1500	−166
3.8	.5615 7293	−1 8186	1353	−147
3.9	.5649 4544	−1 6833	1224	−129
4.0	.5681 4962	−1 5609	1110	−114
4.1	.5711 9772	−1 4499	1008	−102
4.2	.5741 0083	−1 3491	917	−91
4.3	.5768 6903	−1 2574	836	−81
4.4	.5795 1148	−1 1738	764	−72
4.5	.5820 3655	−1 0974	700	−64
4.6	.5844 5187	−1 0275	641	−58
4.7	.5867 6445	−9633	589	−52
4.8	.5889 8069	−9044	542	−47
4.9	.5911 0649	−8502	500	−42
5.0	.5931 4728	−8002	461	−38
5.1	.5951 0804	−7541	427	−34
5.2	.5969 9339	−7114	395	−32
5.3	.5988 0760	−6719	366	−29
5.4	.6005 5462	−6353	340	−26
5.5	.6022 3811	−6012	316	−24
5.6	.6038 6148	−5696	295	−22
5.7	.6054 2789	−5401	274	−20
5.8	.6069 4029	−5127	256	−18
5.9	.6084 0142	−4870	240	−17
6.0	.6098 1385	−4631	224	−16
6.1	.6111 7997	−4407	210	−14
6.2	.6125 0202	−4197	197	−13
6.3	.6137 8210	−4000	185	−12
6.4	.6150 2218	−3816	173	−11
6.5	.6162 2410	−3642	163	−10
6.6	.6173 8960	−3479	154	−10
6.7	.6185 2032	−3326	145	−9
6.8	.6196 1777	−3181	136	−8
6.9	.6206 8341	−3045	129	−8
7.0	.6217 1861	−2916	122	−7
7.1	.6227 2465	−2795	115	−6
7.2	.6237 0274	−2680	109	−6
7.3	.6246 5403	−2571	103	−6
7.4	.6255 7961	−2468	98	−5
7.5	.6264 8052	−2371	92	−5
7.6	.6273 5771	−2278	88	−5
7.7	.6282 1212	−2191	83	−4
7.8	.6290 4462	−2108	79	−4
7.9	.6298 5604	−2029	75	−4
8.0	.6306 4718	−1953	71	−4
8.1	.6314 1878	−1882	68	−3
8.2	.6321 7157	−1814	65	−3
8.3	.6329 0622	−1749	62	−3
8.4	.6336 2337	−1687	59	−3
8.5	.6343 2365	−1629	56	−3
8.6	.6350 0765	−1572	54	−2
8.7	.6356 7592	−1519	51	−2
8.8	.6363 2900	−1468	49	−2
8.9	.6369 6740	−1419	47	−2
9.0	.6375 9162	−1372	45	−2
9.1	.6382 0213	−1327	43	−2
9.2	.6387 9936	−1284	41	−2
9.3	.6393 8374	−1243	39	−2
9.4	.6399 5569	−1204	38	−2

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-1.0$; tabular spacing=0.1—Continued				
9.5	0.6405 1560	−1166	36	−2
9.6	.6410 6385	−1130	35	−1
9.7	.6416 0079	−1096	33	−1
9.8	.6421 2677	−1063	32	−1
9.9	.6426 4213	−1031	31	−1
10.0	.6431 4718			
$Q=-1.0$; tabular spacing=0.2				
10.0	0.6431 4718	−4002	231	−19
10.2	.6441 2757	−3771	214	−17
10.4	.6450 7026	−3557	198	−16
10.6	.6459 7737	−3360	183	−14
10.8	.6468 5088	−3176	170	−13
11.0	.6476 9264	−3006	158	−12
11.2	.6485 0432	−2848	147	−11
11.4	.6492 8753	−2701	137	−10
11.6	.6500 4373	−2563	128	−9
11.8	.6507 7430	−2435	120	−8
12.0	.6514 8051	−2316	112	−8
12.2	.6521 6357	−2203	105	−7
12.4	.6528 2460	−2098	98	−6
12.6	.6534 6464	−2000	92	−6
12.8	.6540 8468	−1908	87	−5
13.0	.6546 8564	−1821	81	−5
13.2	.6552 6839	−1740	77	−5
13.4	.6558 3375	−1663	72	−4
13.6	.6563 8248	−1590	68	−4
13.8	.6569 1530	−1522	64	−4
14.0	.6574 3290	−1458	61	−4
14.2	.6579 3591	−1397	58	−3
14.4	.6584 2496	−1340	54	−3
14.6	.6589 0060	−1286	51	−3
14.8	.6593 6340	−1234	49	−2
15.0	.6598 1385	−1185	46	−3
15.2	.6602 5244	−1139	44	−2
15.4	.6606 7965	−1095	41	−3
15.6	.6610 9590	−1054	40	−2
15.8	.6615 0161	−1014	37	−2
16.0	.6618 9718	−977	36	−2
16.2	.6622 8298	−941	34	−2
16.4	.6626 5938	−907	32	−2
16.6	.6630 2670	−875	31	−2
16.8	.6633 8528	−844	29	−2
17.0	.6637 3542			
$Q=-1.0$; tabular spacing=0.5				
17.0	0.6637 3542	−5093	424	−53
17.5	.6645 7575	−4668	378	−46
18.0	.6653 6940	−4290	339	−40
18.5	.6661 2015	−3951	304	−34
19.0	.6668 3139	−3647	274	−31
19.5	.6675 0616	−3374	247	−27
20.0	.6681 4718	−3127	223	−23
20.5	.6687 5694	−2904	203	−21
21.0	.6693 3766	−2701	184	−18
21.5	.6698 9137	−2517	168	−16
22.0	.6704 1991	−2349	153	−14
22.5	.6709 2496	−2196	140	−13
23.0	.6714 0805	−2056	128	−12

TABLE 38.—*Modified potential*—Continued

a	*W*	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=−1.0; tabular spacing=0.5—Continued				
23. 5	0. 6718 7058	−1927	118	−10
24. 0	. 6723 1385	−1809	109	−10
24. 5	. 6727 3902	−1701	100	−9
25. 0	. 6731 4718	−1601	92	−8
25. 5	. 6735 3934	−1508	85	−7
26. 0	. 6739 1641	−1423	79	−6
26. 5	. 6742 7926	−1344	74	−5
27. 0	. 6746 2866	−1270	68	−5
27. 5	. 6749 6536	−1202	63	−5
28. 0	. 6752 9004	−1139	59	−4
28. 5	. 6756 0332	−1080	55	−4
29. 0	. 6759 0580	−1025	51	−4
29. 5	. 6761 9803	−974	48	−3
30. 0	. 6764 8051			
Q=−0.9; tabular spacing=0.02				
0. 00	0. 0000 0000	3 2735	−63	−126
. 02	. 0001 6368	3 2672	−189	−126
. 04	. 0006 5407	3 2483	−312	−123
. 06	. 0014 6930	3 2171	−432	−120
. 08	. 0026 0624	3 1739	−549	−116
. 10	. 0040 6058	3 1190	−659	−111
. 12	. 0058 2681	3 0531	−764	−104
. 14	. 0078 9836	2 9768	−861	−97
. 16	. 0102 6758	2 8907	−950	−89
. 18	. 0129 2587	2 7957	−1030	−80
. 20	. 0158 6373	2 6927	−1102	−72
. 22	. 0190 7086	2 5825	−1163	−62
. 24	. 0225 3624	2 4662	−1216	−53
. 26	. 0262 4824	2 3446	−1259	−43
. 28	. 0301 9469	2 2186	−1293	−34
. 30	. 0343 6301	2 0894	−1317	−24
. 32	. 0387 4027	1 9577	−1332	−15
. 34	. 0433 1330	1 8245	−1338	−6
. 36	. 0480 6879	1 6907	−1337	+1
. 38	. 0529 9334	1 5570	−1328	9
. 40	. 0580 7359	1 4241	−1313	16
. 42	. 0632 9625	1 2928	−1291	22
. 44	. 0686 4819	1 1638	−1263	28
. 46	. 0741 1651	1 0375	−1230	32
. 48	. 0796 8859	9145	−1193	37
. 50	. 0853 5211	7952	−1153	40
. 52	. 0910 9515	6799	−1109	44
. 54	. 0969 0617	5690	−1062	46
. 56	. 1027 7410	4627	−1015	48
. 58	. 1086 8830	3613	−966	49
. 60	. 1146 3862	2647	−915	50
. 62	. 1206 1542	1732	−864	51
. 64	. 1266 0954	868	−813	51
. 66	. 1326 1234	+55	−763	50
. 68	. 1386 1568	−708	−713	50
. 70	. 1446 1195	−1421	−664	49
. 72	. 1505 9400	−2085	−616	48
. 74	. 1565 5521	−2701	−569	47
. 76	. 1624 8941	−3270	−524	45
. 78	. 1683 9090	−3794	−481	43
. 80	. 1742 5446	−4275	−438	42
. 82	. 1800 7526	−4714	−399	40
. 84	. 1858 4894	−5112	−361	38
. 86	. 1915 7148	−5473	−324	37
. 88	. 1972 3931	−5797	−290	34

TABLE 38.—*Modified potential*—Continued

a	*W*	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=−0.9; tabular spacing=0.02—Continued				
0. 90	0. 2028 4916	−6086	−257	33
. 92	. 2083 9815	−6343	−226	31
. 94	. 2138 8372	−6569	−197	29
. 96	. 2193 0358	−6767	−170	27
. 98	. 2246 5579	−6937	−145	25
1. 00	. 2299 3862	−7082	−121	24
1. 02	. 2351 5064	−7203	−99	22
1. 04	. 2402 9063	−7302	−78	20
1. 06	. 2453 5761	−7380	−60	19
1. 08	. 2503 5078	−7440	−42	18
1. 10	. 2552 6957	−7482	−26	16
1. 12	. 2601 1353	−7507	−11	15
1. 14	. 2648 8243	−7518	+3	14
1. 16	. 2695 7614	−7515	15	13
1. 18	. 2741 9471	−7500	27	11
1. 20	. 2787 3827	−7473	37	10
1. 22	. 2832 0710	−7436	46	9
1. 24	. 2876 0157	−7390	55	9
1. 26	. 2919 2214	−7335	63	8
1. 28	. 2961 6937	−7272	69	7
1. 30	. 3003 4386	−7203	75	6
1. 32	. 3044 4633	−7128	81	6
1. 34	. 3084 7751	−7047	85	5
1. 36	. 3124 3823	−6962	90	4
1. 38	. 3163 2933	−6872	93	4
1. 40	. 3201 5170	−6779	96	3
1. 42	. 3239 0630	−6682	99	3
1. 44	. 3275 9406	−6584	101	2
1. 46	. 3312 1599	−6482	103	2
1. 48	. 3347 7310	−6380	104	1
1. 50	. 3382 6641	−6275	106	1
1. 52	. 3416 9696	−6170	106	1
1. 54	. 3450 6582	−6064	107	+1
1. 56	. 3483 7404	−5957	107	0
1. 58	. 3516 2269	−5850	107	0
1. 60	. 3548 1284	−5743	107	0
1. 62	. 3579 4556	−5636	106	0
1. 64	. 3610 2191	−5530	106	0
1. 66	. 3640 4297	−5424	105	−1
1. 68	. 3670 0978	−5320	105	0
1. 70	. 3699 2340			
Q=−0.9; tabular spacing=0.05				
1. 70	0. 3699 2340	−3 2596	1600	−37
1. 75	. 3769 8155	−3 0995	1553	−47
1. 80	. 3837 2976	−2 9442	1499	−54
1. 85	. 3901 8354	−2 7942	1440	−59
1. 90	. 3963 5791	−2 6502	1379	−62
1. 95	. 4022 6725	−2 5123	1315	−64
2. 00	. 4079 2535	−2 3808	1252	−64
2. 05	. 4133 4538	−2 2557	1188	−63
2. 10	. 4185 3983	−2 1369	1126	−62
2. 15	. 4235 2060	−2 0243	1065	−61
2. 20	. 4282 9894	−1 9178	1007	−59
2. 25	. 4328 8550	−1 8171	950	−56
2. 30	. 4372 9035	−1 7221	896	−54
2. 35	. 4415 2299	−1 6325	844	−52
2. 40	. 4455 9238	−1 5480	796	−49
2. 45	. 4495 0698	−1 4684	750	−46

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.9$; tabular spacing=0.05—Continued				
2.50	0.4532 7472	−1 3935	706	−44
2.55	.4569 0312	−1 3229	664	−42
2.60	.4603 9923	−1 2565	625	−39
2.65	.4637 6968	−1 1940	589	−37
2.70	.4670 2073	−1 1351	554	−35
2.75	.4701 5828	−1 0797	522	−32
2.80	.4731 8785	−1 0276	491	−30
2.85	.4761 1466	−9784	463	−28
2.90	.4789 4364	−9321	436	−27
2.95	.4816 7940	−8885	411	−25
3.00	.4843 2630	−8474	388	−23
3.05	.4868 8847	−8086	366	−22
3.10	.4893 6977	−7721	345	−21
3.15	.4917 7386	−7376	326	−19
3.20	.4941 0420	−7050	308	−18
3.25	.4963 6404	−6742	291	−17
3.30	.4985 5646	−6451	275	−16
3.35	.5006 8437	−6176	260	−14
3.40	.5027 5053	−5916	246	−14
3.45	.5047 5752	−5670	233	−13
3.50	.5067 0782	−5436	221	−12
3.55	.5086 0376	−5215	210	−12
3.60	.5104 4754			
$Q=-0.9$; tabular spacing=0.1				
3.6	0.5104 4754	−2 0033	1551	−173
3.7	.5139 8693	−1 8483	1397	−154
3.8	.5173 4149	−1 7086	1262	−134
3.9	.5205 2519	−1 5823	1144	−119
4.0	.5235 5066	−1 4680	1038	−106
4.1	.5264 2933	−1 3642	942	−95
4.2	.5291 7158	−1 2700	858	−84
4.3	.5317 8683	−1 1842	783	−75
4.4	.5342 8366	−1 1058	717	−67
4.5	.5366 6991	−1 0342	656	−60
4.6	.5389 5275	−9685	602	−54
4.7	.5411 3873	−9083	554	−49
4.8	.5432 3388	−8530	510	−44
4.9	.5452 4373	−8020	470	−40
5.0	.5471 7338	−7550	434	−36
5.1	.5490 2754	−7117	401	−32
5.2	.5508 1052	−6715	372	−29
5.3	.5525 2636	−6343	345	−27
5.4	.5541 7876	−5998	321	−24
5.5	.5557 7118	−5677	298	−23
5.6	.5573 0683	−5379	277	−21
5.7	.5587 8868	−5102	259	−18
5.8	.5602 1952	−4843	241	−17
5.9	.5616 0193	−4602	226	−15
6.0	.5629 3831	−4376	212	−14
6.1	.5642 3094	−4164	198	−13
6.2	.5654 8192	−3966	186	−13
6.3	.5666 9325	−3780	174	−12
6.4	.5678 6676	−3607	163	−10
6.5	.5690 0421	−3443	154	−9
6.6	.5701 0723	−3289	145	−9
6.7	.5711 7736	−3144	137	−8
6.8	.5722 1605	−3008	129	−8
6.9	.5732 2465	−2879	121	−7

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.9$; tabular spacing=0.1—Continued				
7.0	0.5742 0447	−2758	115	−7
7.1	.5751 5672	−2643	108	−6
7.2	.5760 8253	−2534	103	−6
7.3	.5769 8301	−2432	97	−6
7.4	.5778 5917	−2334	92	−5
7.5	.5787 1198	−2242	87	−5
7.6	.5795 4238	−2155	83	−4
7.7	.5803 5121	−2072	79	−4
7.8	.5811 3933	−1994	74	−4
7.9	.5819 0750	−1919	71	−3
8.0	.5826 5649	−1848	68	−3
8.1	.5833 8700	−1780	64	−4
8.2	.5840 9970	−1716	61	−3
8.3	.5847 9524	−1655	58	−3
8.4	.5854 7422	−1597	56	−3
8.5	.5861 3725	−1541	53	−2
8.6	.5867 8486	−1488	51	−2
8.7	.5874 1760	−1437	48	−2
8.8	.5880 3596	−1389	46	−2
8.9	.5886 4043	−1343	44	−2
9.0	.5892 3148	−1298	42	−2
9.1	.5898 0955	−1256	41	−2
9.2	.5903 7505	−1215	39	−2
9.3	.5909 2840	−1177	37	−2
9.4	.5914 6999	−1140	35	−2
9.5	.5920 0018	−1104	34	−1
9.6	.5925 1932	−1070	33	−2
9.7	.5930 2777	−1037	32	−1
9.8	.5935 2584	−1006	30	−2
9.9	.5940 1386	−976	29	−1
10.0	.5944 9213			
$Q=-0.9$; tabular spacing=0.2				
10.0	0.5944 9213	−3788	218	−18
10.2	.5954 2053	−3569	202	−16
10.4	.5963 1324	−3367	187	−15
10.6	.5971 7228	−3180	173	−14
10.8	.5979 9951	−3007	161	−12
11.0	.5987 9668	−2846	150	−11
11.2	.5995 6538	−2696	139	−10
11.4	.6003 0712	−2557	130	−10
11.6	.6010 2329	−2427	121	−8
11.8	.6017 1519	−2306	113	−8
12.0	.6023 8404	−2192	106	−7
12.2	.6030 3096	−2086	99	−7
12.4	.6036 5702	−1987	93	−7
12.6	.6042 6321	−1894	88	−5
12.8	.6048 5046	−1806	82	−5
13.0	.6054 1965			
$Q=-0.9$; tabular spacing=0.5				
13.0	0.6054 1965	−1 0791	1156	−193
13.5	.6067 6886	−9635	996	−159
14.0	.6080 2171	−8639	864	−133
14.5	.6091 8818	−7775	752	−112
15.0	.6102 7690	−7023	658	−94
15.5	.6112 9539	−6364	578	−80
16.0	.6122 5023	−5786	511	−68
16.5	.6131 4722	−5276	452	−59
17.0	.6139 9145	−4824	402	−50

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.9$; tabular spacing=0.5—Continued				
17.5	0.6147 8745	−4422	358	−44
18.0	.6155 3923	−4063	321	−38
18.5	.6162 5038	−3742	288	−33
19.0	.6169 2410	−3455	259	−28
19.5	.6175 6328	−3196	234	−25
20.0	.6181 7050	−2962	211	−22
20.5	.6187 4810	−2750	192	−19
21.0	.6192 9820	−2558	174	−18
21.5	.6198 2272	−2384	159	−15
22.0	.6203 2340	−2225	145	−14
22.5	.6208 0183	−2080	133	−12
23.0	.6212 5946	−1947	122	−11
23.5	.6216 9762	−1826	112	−10
24.0	.6221 1752	−1714	103	−9
24.5	.6225 2028	−1611	95	−8
25.0	.6229 0694	−1516	87	−7
25.5	.6232 7843	−1429	81	−6
26.0	.6236 3564	−1348	75	−6
26.5	.6239 7936	−1273	69	−6
27.0	.6243 1036	−1204	64	−5
27.5	.6246 2932	−1139	60	−4
28.0	.6249 3689	−1079	56	−4
28.5	.6252 3367	−1023	52	−4
29.0	.6255 2022	−971	48	−4
29.5	.6257 9705	−923	46	−3
30.0	.6260 6466			
$Q=-0.8$; tabular spacing=0.02				
0.00	0.0000 0000	2 9365	−56	−112
.02	.0001 4683	2 9309	−168	−112
.04	.0005 8675	2 9141	−278	−110
.06	.0013 1808	2 8863	−386	−108
.08	.0023 3805	2 8478	−489	−103
.10	.0036 4279	2 7989	−587	−98
.12	.0052 2742	2 7401	−681	−93
.14	.0070 8605	2 6720	−767	−86
.16	.0092 1190	2 5953	−847	−80
.18	.0115 9728	2 5107	−918	−72
.20	.0142 3372	2 4188	−982	−64
.22	.0171 1204	2 3206	−1038	−56
.24	.0202 2243	2 2168	−1084	−47
.26	.0235 5450	2 1084	−1123	−38
.28	.0270 9741	1 9961	−1153	−30
.30	.0308 3993	1 8808	−1175	−22
.32	.0347 7054	1 7634	−1188	−14
.34	.0388 7748	1 6446	−1194	−6
.36	.0431 4888	1 5251	−1194	+1
.38	.0475 7279	1 4058	−1186	8
.40	.0521 3728	1 2872	−1172	14
.42	.0568 3049	1 1700	−1152	19
.44	.0616 4070	1 0548	−1128	25
.46	.0665 5639	9420	−1099	29
.48	.0715 6628	8321	−1066	33
.50	.0766 5939	7255	−1030	36
.52	.0818 2504	6225	−992	39
.54	.0870 5294	5233	−950	41
.56	.0923 3318	4283	−908	43
.58	.0976 5624	3375	−864	44

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.8$; tabular spacing=0.02—Continued				
0.60	0.1030 1304	2511	−819	45
.62	.1083 9496	1692	−774	45
.64	.1137 9380	918	−729	45
.66	.1192 0181	+189	−684	45
.68	.1246 1171	−496	−640	44
.70	.1300 1665	−1135	−596	44
.72	.1354 1024	−1732	−553	43
.74	.1407 8651	−2285	−512	41
.76	.1461 3994	−2797	−472	40
.78	.1514 6540	−3269	−433	39
.80	.1567 5817	−3702	−396	37
.82	.1620 1393	−4097	−360	36
.84	.1672 2871	−4458	−326	34
.86	.1723 9892	−4784	−294	32
.88	.1775 2129	−5077	−263	31
.90	.1825 9289	−5340	−234	29
.92	.1876 1108	−5574	−207	27
.94	.1925 7354	−5780	−180	26
.96	.1974 7819	−5961	−156	24
.98	.2023 2324	−6117	−134	22
1.00	.2071 0711	−6251	−113	21
1.02	.2118 2847	−6364	−93	20
1.04	.2164 8619	−6456	−74	18
1.06	.2210 7936	−6531	−58	17
1.08	.2256 0721	−6588	−42	16
1.10	.2300 6919	−6630	−27	14
1.12	.2344 6486	−6657	−14	14
1.14	.2387 9397	−6671	−1	12
1.16	.2430 5637	−6672	+10	11
1.18	.2472 5204	−6663	20	10
1.20	.2513 8109	−6643	29	10
1.22	.2554 4370	−6614	38	9
1.24	.2594 4018	−6576	45	7
1.26	.2633 7091	−6531	52	7
1.28	.2672 3632	−6478	58	6
1.30	.2710 3695	−6420	64	6
1.32	.2747 7338	−6356	69	5
1.34	.2784 4625	−6287	73	4
1.36	.2820 5625	−6214	77	4
1.38	.2856 0410	−6137	80	3
1.40	.2890 9059	−6057	83	3
1.42	.2925 1651	−5974	86	3
1.44	.2958 8269	−5888	88	2
1.46	.2991 8998	−5800	89	1
1.48	.3024 3928	−5711	91	2
1.50	.3056 3146			
$Q=-0.8$; tabular spacing=0.05				
1.50	0.3056 3146	−3 5125	1441	43
1.55	.3133 6804	−3 3683	1459	+17
1.60	.3207 6778	−3 2225	1456	−2
1.65	.3278 4528	−3 0768	1438	−18
1.70	.3346 1510	−2 9330	1408	−30
1.75	.3410 9161	−2 7922	1369	−39
1.80	.3472 8890	−2 6553	1324	−46
1.85	.3532 2065	−2 5230	1274	−50
1.90	.3589 0012	−2 3956	1221	−53
1.95	.3643 4002	−2 2735	1167	−54

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.8$; tabular spacing=0.05—Continued				
2.00	0.3695 5258	−2 1568	1112	−55
2.05	.3745 4945	−2 0457	1057	−55
2.10	.3793 4176	−1 9400	1003	−54
2.15	.3839 4006	−1 8397	950	−53
2.20	.3883 5439	−1 7447	899	−51
2.25	.3925 9425	−1 6548	850	−49
2.30	.3966 6863	−1 5699	802	−47
2.35	.4005 8601	−1 4896	757	−45
2.40	.4043 5444	−1 4139	714	−43
2.45	.4079 8147	−1 3425	674	−41
2.50	.4114 7426	−1 2751	635	−39
2.55	.4148 3954	−1 2116	598	−36
2.60	.4180 8366	−1 1517	564	−35
2.65	.4212 1260	−1 0954	532	−32
2.70	.4242 3201	−1 0422	501	−30
2.75	.4271 4720	−9921	472	−29
2.80	.4299 6319	−9449	445	−27
2.85	.4326 8468	−9004	420	−25
2.90	.4353 1614	−8584	396	−24
2.95	.4378 6176	−8188	374	−22
3.00	.4403 2550	−7814	353	−21
3.05	.4427 1111	−7461	333	−19
3.10	.4450 2210	−7128	314	−19
3.15	.4472 6182	−6813	297	−17
3.20	.4494 3340	−6516	281	−16
3.25	.4515 3982	−6235	266	−16
3.30	.4535 8390	−5969	252	−14
3.35	.4555 6829	−5717	238	−13
3.40	.4574 9550			
$Q=-0.8$; tabular spacing=0.1				
3.4	0.4574 9550	−2 1928	1759	−203
3.5	.4611 8783	−2 0169	1583	−176
3.6	.4646 7846	−1 8586	1426	−157
3.7	.4679 8324	−1 7160	1287	−139
3.8	.4711 1642	−1 5874	1164	−123
3.9	.4740 9086	−1 4710	1055	−109
4.0	.4769 1820	−1 3655	958	−96
4.1	.4796 0900	−1 2697	872	−87
4.2	.4821 7282	−1 1825	795	−77
4.3	.4846 1839	−1 1031	725	−69
4.4	.4869 5365	−1 0305	664	−61
4.5	.4891 8586	−9641	609	−55
4.6	.4913 2166	−9032	558	−50
4.7	.4933 6714	−8474	514	−45
4.8	.4953 2788	−7960	474	−40
4.9	.4972 0901	−7486	437	−37
5.0	.4990 1529	−7049	404	−33
5.1	.5007 5108	−6645	373	−31
5.2	.5024 2041	−6272	346	−27
5.3	.5040 2702	−5927	322	−24
5.4	.5055 7436	−5605	299	−23
5.5	.5070 6565	−5306	277	−22
5.6	.5085 0389	−5029	258	−18
5.7	.5098 9185	−4770	242	−17
5.8	.5112 3210	−4528	226	−16
5.9	.5125 2707	−4303	210	−15

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.8$; tabular spacing=0.1—Continued				
6.0	0.5137 7901	−4092	197	−14
6.1	.5149 9002	−3896	185	−12
6.2	.5161 6208	−3710	173	−12
6.3	.5172 9704	−3537	162	−11
6.4	.5183 9663	−3375	153	−9
6.5	.5194 6246	−3222	144	−10
6.6	.5204 9608	−3078	135	−8
6.7	.5214 9892	−2943	128	−7
6.8	.5224 7232	−2815	120	−8
6.9	.5234 1758	−2695	113	−7
7.0	.5243 3589	−2582	107	−6
7.1	.5252 2839	−2474	102	−6
7.2	.5260 9614	−2373	96	−6
7.3	.5269 4017	−2277	91	−5
7.4	.5277 6143	−2186	86	−5
7.5	.5285 6083	−2100	82	−4
7.6	.5293 3923	−2018	77	−4
7.7	.5300 9745	−1941	74	−4
7.8	.5308 3626	−1867	70	−4
7.9	.5315 5639	−1797	66	−3
8.0	.5322 5855	−1731	63	−3
8.1	.5329 4340	−1668	60	−3
8.2	.5336 1158	−1608	57	−3
8.3	.5342 6368	−1550	55	−3
8.4	.5349 0027	−1496	52	−2
8.5	.5355 2190	−1444	50	−3
8.6	.5361 2910	−1394	47	−2
8.7	.5367 2236	−1347	46	−2
8.8	.5373 0214	−1301	43	−2
8.9	.5378 6892	−1258	41	−2
9.0	.5384 2312	−1217	40	−2
9.1	.5389 6516	−1177	38	−2
9.2	.5394 9542	−1139	36	−2
9.3	.5400 1430	−1103	35	−1
9.4	.5405 2214	−1068	33	−2
9.5	.5410 1931	−1035	32	−1
9.6	.5415 0613	−1003	31	−2
9.7	.5419 8293	−972	30	−1
9.8	.5424 5000	−943	28	−1
9.9	.5429 0764	−914	27	−1
10.0	.5433 5615			
$Q=-0.8$; tabular spacing=0.2				
10.0	0.5433 5615	−3550	205	−17
10.2	.5442 2680	−3346	189	−16
10.4	.5450 6399	−3156	175	−14
10.6	.5458 6962	−2981	163	−12
10.8	.5466 4543	−2819	151	−12
11.0	.5473 9306	−2668	140	−11
11.2	.5481 1401	−2528	131	−10
11.4	.5488 0968	−2397	122	−9
11.6	.5494 8138	−2275	114	−8
11.8	.5501 3032	−2162	106	−8
12.0	.5507 5765	−2055	99	−7
12.2	.5513 6442	−1956	93	−6
12.4	.5519 5164	−1863	87	−6
12.6	.5525 2022	−1776	82	−6
12.8	.5530 7104	−1694	77	−5
13.0	.5536 0493			

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=−0.8; tabular spacing=0.5				
13.0	0.5536 0493	−1 0118	1083	−181
13.5	.5548 7048	−9035	934	−149
14.0	.5560 4567	−8101	810	−124
14.5	.5571 3986	−7292	705	−104
15.0	.5581 6113	−6586	617	−88
15.5	.5591 1654	−5969	542	−75
16.0	.5600 1226	−5427	479	−64
16.5	.5608 5371	−4948	424	−55
17.0	.5616 4568	−4524	377	−47
17.5	.5623 9241	−4147	336	−40
18.0	.5630 9767	−3811	301	−35
18.5	.5637 6482	−3510	270	−31
19.0	.5643 9686	−3240	243	−27
19.5	.5649 9650	−2998	219	−24
20.0	.5655 6616	−2778	198	−21
20.5	.5661 0805	−2580	180	−18
21.0	.5666 2413	−2400	164	−16
21.5	.5671 1621	−2236	149	−15
22.0	.5675 8594	−2087	136	−13
22.5	.5680 3479	−1951	125	−11
23.0	.5684 6412	−1827	114	−11
23.5	.5688 7520	−1712	105	−9
24.0	.5692 6914	−1608	96	−8
24.0	.5696 4701	−1511	89	−8
25.0	.5700 0977	−1422	82	−7
25.5	.5703 5830	−1340	76	−6
26.0	.5706 9343	−1264	70	−6
26.5	.5710 1592	−1194	65	−5
27.0	.5713 2646	−1129	61	−4
27.5	.5716 2571	−1069	56	−4
28.0	.5719 1428	−1012	52	−4
28.5	.5721 9272	−960	49	−3
29.0	.5724 6156	−911	46	−3
29.5	.5727 2129	−866	43	−3
30.0	.5729 7236			
Q=−0.7; tabular spacing=0.02				
0.00	0.0000 0000	2 5938	−49	−98
.02	.0001 2969	2 5889	−148	−98
.04	.0005 1828	2 5742	−244	−97
.06	.0011 6428	2 5498	−338	−94
.08	.0020 6525	2 5160	−429	−91
.10	.0032 1783	2 4730	−516	−86
.12	.0046 1770	2 4215	−597	−82
.14	.0062 5973	2 3618	−673	−76
.16	.0081 3793	2 2944	−743	−69
.18	.0102 4558	2 2202	−806	−64
.20	.0125 7524	2 1395	−862	−56
.22	.0151 1885	2 0533	−911	−49
.24	.0178 6779	1 9623	−952	−41
.26	.0208 1296	1 8671	−986	−34
.28	.0239 4484	1 7685	−1012	−26
.30	.0272 5357	1 6672	−1031	−19
.32	.0307 2902	1 5641	−1044	−12
.34	.0343 6088	1 4598	−1049	−6
.36	.0381 3872	1 3548	−1048	+1
.38	.0420 5204	1 2500	−1042	7
.40	.0460 9036	1 1458	−1030	12
.42	.0502 4327	1 0428	−1013	17

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=−0.7; tabular spacing=0.02—Continued				
0.44	0.0545 0045	9415	−992	21
.46	.0588 5180	8424	−966	25
.48	.0632 8737	7457	−938	28
.50	.0677 9753	6519	−906	32
.52	.0723 7287	5613	−873	34
.54	.0770 0435	4740	−837	36
.56	.0816 8323	3903	−800	37
.58	.0864 0114	3104	−761	38
.60	.0911 5010	2342	−722	39
.62	.0959 2248	1620	−683	40
.64	.1007 1106	938	−643	39
.66	.1055 0901	+294	−604	39
.68	.1103 0991	−310	−565	39
.70	.1151 0772	−875	−527	38
.72	.1198 9677	−1402	−490	37
.74	.1246 7180	−1891	−454	36
.76	.1294 2793	−2345	−418	36
.78	.1341 6060	−2763	−384	34
.80	.1388 6564	−3147	−352	33
.82	.1435 3921	−3499	−320	31
.84	.1481 7780	−3819	−290	30
.86	.1527 7819	−4109	−262	29
.88	.1573 3750	−4371	−235	27
.90	.1618 5309	−4606	−210	25
.92	.1663 2261	−4816	−185	24
.94	.1707 4398	−5001	−163	23
.96	.1751 1533	−5164	−142	21
.98	.1794 3505	−5305	−122	20
1.00	.1837 0171	−5427	−103	18
1.02	.1879 1410	−5530	−85	18
1.04	.1920 7119	−5615	−69	16
1.06	.1961 7213	−5685	−54	15
1.08	.2002 1622	−5739	−41	14
1.10	.2042 0292	−5780	−28	13
1.12	.2081 3183	−5807	−16	12
1.14	.2120 0266	−5823	−5	11
1.16	.2158 1525	−5828	+5	10
1.18	.2195 6957	−5824	14	9
1.20	.2232 6564	−5810	22	8
1.22	.2269 0363	−5787	30	7
1.24	.2304 8374	−5758	36	7
1.26	.2340 0627	−5721	43	6
1.28	.2374 7159	−5678	48	6
1.30	.2408 8013	−5630	53	5
1.32	.2442 3237	−5577	57	4
1.34	.2475 2883	−5520	62	4
1.36	.2507 7010	−5458	65	3
1.38	.2539 5678	−5393	68	3
1.40	.2570 8954			
Q=−0.7; tabular spacing=0.05				
1.40	0.2570 8954	−3 3277	1124	103
1.45	.2646 8998	−3 2152	1193	69
1.50	.2719 6890	−3 0959	1234	41
1.55	.2789 3823	−2 9725	1253	19
1.60	.2856 1031	−2 8472	1254	+1
1.65	.2919 9768	−2 7217	1241	−13
1.70	.2981 1287	−2 5976	1218	−23

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.7$; tabular spacing=0.05—Continued				
1. 75	0. 3039 6830	−2 4758	1186	−31
1. 80	. 3095 7614	−2 3572	1149	−38
1. 85	. 3149 4827	−2 2423	1108	−41
1. 90	. 3200 9616	−2 1315	1063	−44
1. 95	. 3250 3091	−2 0252	1017	−46
2. 00	. 3297 6314	−1 9234	971	−46
2. 05	. 3343 0302	−1 8264	924	−46
2. 10	. 3386 6026	−1 7339	878	−46
2. 15	. 3428 4412	−1 6461	833	−45
2. 20	. 3468 6336	−1 5627	790	−44
2. 25	. 3507 2634	−1 4838	747	−42
2. 30	. 3544 4093	−1 4090	707	−40
2. 35	. 3580 1462	−1 3384	668	−39
2. 40	. 3614 5448	−1 2716	631	−37
2. 45	. 3647 6717	−1 2085	596	−35
2. 50	. 3679 5902	−1 1489	562	−33
2. 55	. 3710 3598	−1 0927	530	−32
2. 60	. 3740 0367	−1 0397	500	−30
2. 65	. 3768 6739	−9896	472	−28
2. 70	. 3796 3215	−9424	446	−27
2. 75	. 3823 0268	−8978	421	−25
2. 80	. 3848 8342	−8558	397	−24
2. 85	. 3873 7858	−8160	375	−22
2. 90	. 3897 9215	−7786	354	−21
2. 95	. 3921 2785	−7432	334	−20
3. 00	. 3943 8924	−7097	316	−19
3. 05	. 3965 7966	−6782	299	−17
3. 10	. 3987 0225	−6483	282	−16
3. 15	. 4007 6002	−6201	267	−15
3. 20	. 4027 5578			
$Q=-0.7$; tabular spacing=0.1				
3. 2	0. 4027 5578	−2 3749	1970	−229
3. 3	. 4065 7182	−2 1779	1767	−203
3. 4	. 4101 7007	−2 0011	1588	−179
3. 5	. 4135 6821	−1 8423	1430	−158
3. 6	. 4167 8211	−1 6993	1290	−140
3. 7	. 4198 2608	−1 5703	1166	−124
3. 8	. 4227 1303	−1 4537	1056	−110
3. 9	. 4254 5461	−1 3480	959	−97
4. 0	. 4280 6138	−1 2521	871	−88
4. 1	. 4305 4295	−1 1650	793	−78
4. 2	. 4329 0802	−1 0856	724	−69
4. 3	. 4351 6452	−1 0132	662	−62
4. 4	. 4373 1971	−9470	607	−56
4. 5	. 4393 8019	−8863	556	−50
4. 6	. 4413 5204	−8307	511	−45
4. 7	. 4432 4083	−7796	470	−41
4. 8	. 4450 5165	−7326	433	−37
4. 9	. 4467 8922	−6892	400	−33
5. 0	. 4484 5786	−6492	370	−30
5. 1	. 4500 6158	−6122	343	−27
5. 2	. 4516 0408	−5779	318	−25
5. 3	. 4530 8880	−5462	295	−23
5. 4	. 4545 1890	−5167	274	−20
5. 5	. 4558 9733	−4892	255	−19
5. 6	. 4572 2683	−4637	238	−17

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.7$; tabular spacing=0.1—Continued				
5. 7	0. 4585 0996	−4400	222	−16
5. 8	. 4597 4910	−4178	207	−15
5. 9	. 4609 4646	−3970	194	−13
6. 0	. 4621 0412	−3777	181	−13
6. 1	. 4632 2401	−3595	170	−11
6. 2	. 4643 0795	−3425	160	−11
6. 3	. 4653 5763	−3266	150	−10
6. 4	. 4663 7466	−3116	141	−9
6. 5	. 4673 6054	−2975	132	−8
6. 6	. 4683 1666	−2843	125	−8
6. 7	. 4692 4435	−2718	118	−7
6. 8	. 4701 4486	−2600	111	−7
6. 9	. 4710. 1937	−2489	105	−6
7. 0	. 4718 6899	−2385	99	−6
7. 1	. 4726 9476	−2286	93	−5
7. 2	. 4734 9766	−2193	89	−4
7. 3	. 4742 7864	−2104	84	−5
7. 4	. 4750 3858	−2020	79	−5
7. 5	. 4757 7832	−1941	75	−4
7. 6	. 4764 9865	−1866	72	−3
7. 7	. 4772 0032	−1794	68	−4
7. 8	. 4778 8406	−1726	64	−4
7. 9	. 4785 5053	−1662	61	−3
8. 0	. 4792 0039	−1600	58	−3
8. 1	. 4798 3424	−1542	56	−3
8. 2	. 4804 5268	−1486	52	−3
8. 3	. 4810 5624	−1434	51	−2
8. 4	. 4816 4547	−1383	48	−2
8. 5	. 4822 2087	−1335	46	−3
8. 6	. 4827 8292	−1289	44	−2
8. 7	. 4833 3207	−1245	42	−2
8. 8	. 4838 6878	−1203	40	−2
8. 9	. 4843 9344	−1164	38	−2
9. 0	. 4849 0648	−1125	37	−2
9. 1	. 4854 0826	−1089	35	−1
9. 2	. 4858 9915	−1053	33	−2
9. 3	. 4863 7950	−1020	32	−2
9. 4	. 4868 4966	−988	31	−1
9. 5	. 4873 0994	−957	30	−1
9. 6	. 4877 6064	−928	29	−1
9. 7	. 4882 0206	−899	27	−2
9. 8	. 4886 3450	−872	26	−1
9. 9	. 4890 5822	−846	26	0
10. 0	. 4894 7347			
$Q=-0.7$; tabular spacing=0.2				
10. 0	0. 4894 7347	−3285	190	−15
10. 2	. 4902 7959	−3096	175	−15
10. 4	. 4910 5475	−2921	162	−13
10. 6	. 4918 0070	−2759	150	−12
10. 8	. 4925 1907	−2609	140	−11
11. 0	. 4932 1135	−2469	130	−10
11. 2	. 4938 7894	−2339	121	−9
11. 4	. 4945 2313	−2219	113	−8
11. 6	. 4951 4514	−2106	105	−7
11. 8	. 4957 4608	−2001	98	−7

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=−0.7; tabular spacing=0.2—Continued				
12. 0	0. 4963 2702	−1902	92	−7
12. 2	. 4968 8893	−1811	86	−5
12. 4	. 4974 3274	−1724	81	−6
12. 6	. 4979 5929	−1644	76	−5
12. 8	. 4984 6942	−1568	71	−5
13. 0	. 4989 6386			
Q=−0.7; tabular spacing=0.5				
13. 0	0. 4989 6386	−9367	1002	−167
13. 5	. 5001 3592	−8365	864	−138
14. 0	. 5012 2433	−7500	749	−115
14. 5	. 5022 3774	−6751	653	−97
15. 0	. 5031 8364	−6098	571	−82
15. 5	. 5040 6856	−5527	502	−69
16. 0	. 5048 9820	−5025	443	−59
16. 5	. 5056 7760	−4582	392	−51
17. 0	. 5064 1117	−4190	349	−44
17. 5	. 5071 0285	−3841	311	−37
18. 0	. 5077 5612	−3530	279	−33
18. 5	. 5083 7409	−3251	250	−29
19. 0	. 5089 5955	−3001	225	−25
19. 5	. 5095 1500	−2776	203	−22
20. 0	. 5100 4270	−2573	184	−19
20. 5	. 5105 4466	−2389	167	−17
21. 0	. 5110 2272	−2223	152	−15
21. 5	. 5114 7856	−2071	138	−14
22. 0	. 5119 1369	−1933	126	−12
22. 5	. 5123 2948	−1807	115	−11
23. 0	. 5127 2720	−1692	105	−10
23. 5	. 5131 0800	−1586	97	−8
24. 0	. 5134 7294	−1489	90	−8
24. 5	. 5138 2298	−1400	82	−7
25. 0	. 5141 5903	−1318	76	−6
25. 5	. 5144 8191	−1242	70	−6
26. 0	. 5147 9236	−1171	65	−5
26. 5	. 5150 9111	−1106	60	−5
27. 0	. 5153 7879	−1046	56	−4
27. 5	. 5156 5602	−990	52	−4
28. 0	. 5159 2335	−938	49	−3
28. 5	. 5161 8129	−889	45	−3
29. 0	. 5164 3034	−844	42	−3
29. 5	. 5166 7096	−802	39	−3
30. 0	. 5169 0355			
Q=−0.6; tabular spacing=0.02				
0. 00	0. 0000 0000	2 2451	−42	−85
. 02	. 0001 1225	2 2408	−127	−84
. 04	. 0004 4859	2 2281	−210	−83
. 06	. 0010 0774	2 2072	−291	−81
. 08	. 0017 8760	2 1781	−369	−78
. 10	. 0027 8528	2 1412	−443	−74
. 12	. 0039 9708	2 0969	−513	−70
. 14	. 0054 1856	2 0456	−579	−65
. 16	. 0070 4461	1 9877	−639	−60
. 18	. 0088 6942	1 9238	−693	−54
. 20	. 0108 8661	1 8545	−741	−48
. 22	. 0130 8926	1 7804	−783	−42

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=−0.6; tabular spacing=0.02—Continued				
0. 24	0. 0154 6994	1 7021	−819	−35
. 26	. 0180 2083	1 6202	−848	−29
. 28	. 0207 3374	1 5354	−871	−23
. 30	. 0236 0020	1 4484	−887	−17
. 32	. 0266 1148	1 3596	−898	−10
. 34	. 0297 5873	1 2698	−903	−5
. 36	. 0330 3296	1 1796	−902	0
. 38	. 0364 2515	1 0893	−897	+6
. 40	. 0399 2627	9996	−887	10
. 42	. 0435 2735	9110	−872	14
. 44	. 0472 1954	8237	−854	18
. 46	. 0509 9409	7383	−833	22
. 48	. 0548 4248	6551	−808	24
. 50	. 0587 5637	5742	−782	27
. 52	. 0627 2769	4961	−752	29
. 54	. 0667 4862	4208	−722	31
. 56	. 0708 1163	3487	−690	32
. 58	. 0749 0951	2797	−657	33
. 60	. 0790 3535	2139	−624	33
. 62	. 0831 8259	1516	−590	34
. 64	. 0873 4499	926	−556	34
. 66	. 0915 1664	+370	−522	34
. 68	. 0956 9198	−153	−489	33
. 70	. 0998 6580	−642	−456	33
. 72	. 1040 3320	−1098	−424	32
. 74	. 1081 8961	−1523	−393	31
. 76	. 1123 3079	−1916	−363	30
. 78	. 1164 5281	−2279	−334	29
. 80	. 1205 5204	−2613	−306	28
. 82	. 1246 2514	−2919	−279	27
. 84	. 1286 6904	−3198	−254	26
. 86	. 1326 8096	−3452	−229	25
. 88	. 1366 5836	−3681	−206	23
. 90	. 1405 9896	−3887	−184	22
. 92	. 1445 0069	−4070	−163	21
. 94	. 1483 6172	−4234	−144	20
. 96	. 1521 8040	−4377	−125	18
. 98	. 1559 5532	−4503	−108	17
1. 00	. 1596 8521	−4611	−92	16
1. 02	. 1633 6898	−4703	−77	15
1. 04	. 1670 0573	−4780	−63	14
1. 06	. 1705 9468	−4843	−50	13
1. 08	. 1741 3520	−4894	−38	12
1. 10	. 1776 2677	−4932	−27	11
1. 12	. 1810 6903	−4959	−17	10
1. 14	. 1844 6169	−4976	−8	9
1. 16	. 1878 0459	−4984	+1	9
1. 18	. 1910 9766	−4983	9	8
1. 20	. 1943 4088	−4974	16	7
1. 22	. 1975 3437	−4958	22	6
1. 24	. 2006 7828	−4936	28	6
1. 26	. 2037 7283	−4907	34	6
1. 28	. 2068 1831	−4873	39	5
1. 30	. 2098 1505	−4835	43	4
1. 32	. 2127 6345	−4792	47	4
1. 34	. 2156 6393	−4745	50	3
1. 36	. 2185 1697	−4694	54	3
1. 38	. 2213 2306	−4641	56	2
1. 40	. 2240 8274			

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.6$; tabular spacing=0.05				
1. 40	0. 2240 8274	−2 8650	933	92
1. 45	. 2307 8262	−2 7717	995	62
1. 50	. 2372 0533	−2 6722	1033	38
1. 55	. 2433 6082	−2 5689	1053	19
1. 60	. 2492 5941	−2 4637	1056	+4
1. 65	. 2549 1164	−2 3580	1048	−8
1. 70	. 2603 2807	−2 2532	1030	−18
1. 75	. 2655 1918	−2 1502	1006	−24
1. 80	. 2704 9526	−2 0496	976	−30
1. 85	. 2752 6639	−1 9520	942	−34
1. 90	. 2798 4233	−1 8578	906	−36
1. 95	. 2842 3248	−1 7672	868	−38
2. 00	. 2884 4592	−1 6803	830	−39
2. 05	. 2924 9133	−1 5974	791	−38
2. 10	. 2963 7700	−1 5182	753	−38
2. 15	. 3001 1085	−1 4429	716	−37
2. 20	. 3037 0041	−1 3713	679	−37
2. 25	. 3071 5284	−1 3034	643	−35
2. 30	. 3104 7492	−1 2391	609	−34
2. 35	. 3136 7309	−1 1782	576	−33
2. 40	. 3167 5345	−1 1205	545	−31
2. 45	. 3197 2175	−1 0660	516	−30
2. 50	. 3225 8345	−1 0144	487	−28
2. 55	. 3253 4371	−9657	460	−27
2. 60	. 3280 0739	−9197	435	−26
2. 65	. 3305 7910	−8762	411	−24
2. 70	. 3330 6319	−8351	388	−23
2. 75	. 3354 6377	−7963	366	−22
2. 80	. 3377 8472	−7596	347	−20
2. 85	. 3400 2970	−7250	328	−19
2. 90	. 3422 0219	−6922	310	−18
2. 95	. 3443 0545	−6612	293	−17
3. 00	. 3463 4259	−6320	277	−16
3. 05	. 3483 1654	−6043	262	−14
3. 10	. 3502 3005	−5781	248	−14
3. 15	. 3520 8575	−5532	235	−14
3. 20	. 3538 8614			
$Q=-0.6$; tabular spacing=0.1				
3. 2	0. 3538 8614	−2 1203	1735	−198
3. 3	. 3573 3019	−1 9468	1560	−176
3. 4	. 3605 7956	−1 7909	1404	−156
3. 5	. 3636 4985	−1 6505	1266	−137
3. 6	. 3665 5508	−1 5238	1145	−122
3. 7	. 3693 0794	−1 4094	1036	−109
3. 8	. 3719 1986	−1 3058	940	−96
3. 9	. 3744 0119	−1 2119	854	−86
4. 0	. 3767 6134	−1 1265	777	−77
4. 1	. 3790 0885	−1 0488	708	−69
4. 2	. 3811 5147	−9780	647	−61
4. 3	. 3831 9630	−9132	593	−55
4. 4	. 3851 4980	−8540	543	−50
4. 5	. 3870 1791	−7997	498	−45
4. 6	. 3888 0605	−7499	458	−40
4. 7	. 3905 1920	−7040	422	−36
4. 8	. 3921 6195	−6618	389	−33
4. 9	. 3937 3851	−6229	360	−30
5. 0	. 3952 5279	−5869	333	−27
5. 1	. 3967 0838	−5536	308	−25

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TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.6$; tabular spacing=0.1—Continued				
5. 2	0. 3981 0860	−5228	286	−22
5. 3	. 3994 5654	−4942	266	−20
5. 4	. 4007 5506	−4676	247	−19
5. 5	. 4020 0681	−4430	230	−17
5. 6	. 4032 1427	−4200	214	−15
5. 7	. 4043 7974	−3985	200	−14
5. 8	. 4055 0535	−3785	187	−13
5. 9	. 4065 9311	−3598	175	−12
6. 0	. 4076 4490	−3423	164	−11
6. 1	. 4086 6247	−3259	154	−10
6. 2	. 4096 4744	−3105	144	−10
6. 3	. 4106 0136	−2961	135	−9
6. 4	. 4115 2567	−2826	127	−8
6. 5	. 4124 2172	−2699	120	−7
6. 6	. 4132 9079	−2579	113	−7
6. 7	. 4141 3407	−2466	106	−7
6. 8	. 4149 5269	−2359	100	−6
6. 9	. 4157 4772	−2259	95	−5
7. 0	. 4165 2015	−2165	90	−4
7. 1	. 4172 7094	−2075	85	−5
7. 2	. 4180 0098	−1990	79	−5
7. 3	. 4187 1112	−1910	76	−3
7. 4	. 4194 0216	−1834	72	−4
7. 5	. 4200 7486	−1762	68	−4
7. 6	. 4207 2993	−1694	65	−3
7. 7	. 4213 6806	−1629	62	−4
7. 8	. 4219 8990	−1567	58	−4
7. 9	. 4225 9606	−1509	55	−3
8. 0	. 4231 8713	−1454	53	−2
8. 1	. 4237 6366	−1401	51	−2
8. 2	. 4243 2618	−1350	48	−3
8. 3	. 4248 7520	−1302	45	−2
8. 4	. 4254 1120	−1257	44	−2
8. 5	. 4259 3463	−1213	42	−2
8. 6	. 4264 4592	−1171	40	−2
8. 7	. 4269 4551	−1132	38	−2
8. 8	. 4274 3377	−1094	36	−1
8. 9	. 4279 1110	−1057	35	−2
9. 0	. 4283 7785	−1022	33	−2
9. 1	. 4288 3438	−989	31	−2
9. 2	. 4292 8101	−958	31	−1
9. 3	. 4297 1807	−927	30	−1
9. 4	. 4301 4585	−898	28	−2
9. 5	. 4305 6465	−870	26	−1
9. 6	. 4309 7475	−844	26	−1
9. 7	. 4313 7642	−818	25	−1
9. 8	. 4317 6991	−793	24	−1
9. 9	. 4321 5547	−769	23	−1
10. 0	. 4325 3334			
$Q=-0.6$; tabular spacing=0.2				
10. 0	0. 4325 3334	−2987	172	−14
10. 2	. 4332 6690	−2815	159	−13
10. 4	. 4339 7232	−2656	147	−12
10. 6	. 4346 5118	−2509	136	−10
10. 8	. 4353 0494	−2372	127	−10
11. 0	. 4359 3498	−2246	118	−9
11. 2	. 4365 4257	−2128	110	−8
11. 4	. 4371 2888	−2018	102	−7
11. 6	. 4376 9501	−1916	95	−7
11. 8	. 4382 4198	−1820	90	−6

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=−0.6; tabular spacing=0.2—Continued				
12. 0	0. 4387 7075	−1731	84	−6
12. 2	. 4392 8221	−1647	78	−6
12. 4	. 4397 7720	−1569	73	−5
12. 6	. 4402 5650	−1496	69	−4
12. 8	. 4407 2084	−1427	65	−4
13. 0	. 4411 7092			
Q=−0.6; tabular spacing=0.5				
13. 0	0. 4411 7092	−8523	912	−152
13. 5	. 4422 3785	−7611	786	−126
14. 0	. 4432 2867	−6825	682	−104
14. 5	. 4441 5123	−6144	594	−88
15. 0	. 4450 1236	−5550	519	−74
15. 5	. 4458 1799	−5030	457	−62
16. 0	. 4465 7332	−4574	403	−54
16. 5	. 4472 8291	−4170	357	−46
17. 0	. 4479 5080	−3814	317	−39
17. 5	. 4485 8055	−3496	284	−34
18. 0	. 4491 7534	−3213	254	−30
18. 5	. 4497 3800	−2959	227	−26
19. 0	. 4502 7108	−2732	205	−22
19. 5	. 4507 7682	−2527	185	−20
20. 0	. 4512 5730	−2342	167	−18
20. 5	. 4517 1436	−2175	152	−16
21. 0	. 4521 4966	−2024	138	−14
21. 5	. 4525 6473	−1886	126	−12
22. 0	. 4529 6093	−1760	115	−11
22. 5	. 4533 3954	−1645	105	−10
23. 0	. 4537 0170	−1540	96	−9
23. 5	. 4540 4844	−1444	88	−8
24. 0	. 4543 8075	−1356	81	−7
24. 5	. 4546 9950	−1274	75	−6
25. 0	. 4550 0551	−1200	69	−6
25. 5	. 4552 9952	−1130	64	−5
26. 0	. 4555 8222	−1066	59	−5
26. 5	. 4558 5426	−1007	55	−4
27. 0	. 4561 1623	−952	51	−4
27. 5	. 4563 6868	−901	48	−3
28. 0	. 4566 1211	−854	44	−4
28. 5	. 4568 4700	−810	41	−3
29. 0	. 4570 7380	−769	39	−3
29. 5	. 4572 9291	−730	36	−3
30. 0	. 4575 0471			
Q=−0.5; tabular spacing=0.02				
0. 00	0. 0000 0000	1 8898	−36	−71
. 02	. 0000 9449	1 8862	−106	−70
. 04	. 0003 7760	1 8757	−175	−70
. 06	. 0008 4829	1 8582	−243	−68
. 08	. 0015 0479	1 8338	−308	−65
. 10	. 0023 4467	1 8030	−370	−62
. 12	. 0033 6486	1 7660	−429	−58
. 14	. 0045 6164	1 7231	−484	−55
. 16	. 0059 3073	1 6747	−534	−50
. 18	. 0074 6730	1 6213	−579	−46
. 20	. 0091 6599	1 5634	−620	−40
. 22	. 0110 2102	1 5014	−655	−35
. 24	. 0130 2620	1 4359	−684	−30

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=−0.5; tabular spacing=0.02—Continued				
0. 26	0. 0151 7497	1 3675	−709	−25
. 28	. 0174 6049	1 2966	−728	−19
. 30	. 0198 7566	1 2238	−742	−14
. 32	. 0224 1322	1 1496	−751	−9
. 34	. 0250 6572	1 0744	−755	−4
. 36	. 0278 2568	9989	−755	0
. 38	. 0306 8552	9234	−750	+5
. 40	. 0336 3770	8484	−742	8
. 42	. 0366 7472	7741	−730	12
. 44	. 0397 8914	7011	−715	15
. 46	. 0429 7368	6295	−697	18
. 48	. 0462 2116	5598	−677	20
. 50	. 0495 2463	4921	−655	22
. 52	. 0528 7730	4266	−631	24
. 54	. 0562 7263	3635	−605	26
. 56	. 0597 0431	3029	−579	27
. 58	. 0631 6628	2450	−552	27
. 60	. 0666 5275	1899	−524	28
. 62	. 0701 5821	1375	−496	28
. 64	. 0736 7741	879	−468	28
. 66	. 0772 0541	+412	−439	28
. 68	. 0807 3753	−28	−412	28
. 70	. 0842 6936	−440	−384	27
. 72	. 0877 9681	−824	−358	27
. 74	. 0913 1601	−1182	−332	26
. 76	. 0948 2340	−1513	−307	25
. 78	. 0983 1565	−1820	−282	24
. 80	. 1017 8970	−2102	−259	23
. 82	. 1052 4274	−2361	−236	22
. 84	. 1086 7215	−2598	−215	21
. 86	. 1120 7559	−2813	−195	20
. 88	. 1154 5091	−3008	−175	20
. 90	. 1187 9614	−3183	−157	18
. 92	. 1221 0955	−3340	−140	17
. 94	. 1253 8956	−3480	−123	17
. 96	. 1286 3477	−3603	−108	15
. 98	. 1318 4396	−3710	−94	14
1. 00	. 1350 1604	−3804	−81	13
1. 02	. 1381 5008	−3885	−67	13
1. 04	. 1412 4526	−3952	−56	11
1. 06	. 1443 0092	−4008	−45	11
1. 08	. 1473 1651	−4053	−35	10
1. 10	. 1502 9156	−4088	−26	9
1. 12	. 1532 2572	−4114	−17	9
1. 14	. 1561 1874	−4131	−9	8
1. 16	. 1589 7045	−4140	−2	7
1. 18	. 1617 8076	−4142	+5	7
1. 20	. 1645 4964	−4138	11	6
1. 22	. 1672 7714	−4127	16	6
1. 24	. 1699 6337	−4111	21	5
1. 26	. 1726 0849	−4090	26	5
1. 28	. 1752 1271	−4064	30	4
1. 30	. 1777 7629	−4034	34	4
1. 32	. 1802 9954	−4000	37	3
1. 34	. 1827 8278	−3964	40	3
1. 36	. 1852 2639	−3924	42	2
1. 38	. 1876 3076	−3881	45	2
1. 40	. 1899 9632			

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.5$; tabular spacing=0.05				
1. 40	0. 1899 9632	−2 3971	749	79
1. 45	. 1957 4343	−2 3221	804	55
1. 50	. 2012 5832	−2 2417	839	35
1. 55	. 2065 4905	−2 1578	858	18
1. 60	. 2116 2400	−2 0720	863	+6
1. 65	. 2164 9175	−1 9856	859	−4
1. 70	. 2211 6093	−1 8998	846	−13
1. 75	. 2256 4014	−1 8151	828	−18
1. 80	. 2299 3784	−1 7323	804	−23
1. 85	. 2340 6231	−1 6519	778	−26
1. 90	. 2380 2158	−1 5741	750	−29
1. 95	. 2418 2345	−1 4991	720	−30
2. 00	. 2454 7541	−1 4271	689	−31
2. 05	. 2489 8466	−1 3582	658	−31
2. 10	. 2523 5809	−1 2924	627	−31
2. 15	. 2556 0227	−1 2297	596	−30
2. 20	. 2587 2348			
$Q=-0.5$; tabular spacing=0.1				
2. 2	0. 2587 2348	−4 6834	4421	−473
2. 3	. 2646 2054	−4 2412	3978	−444
2. 4	. 2700 9347	−3 8434	3567	−411
2. 5	. 2751 8206	−3 4868	3195	−372
2. 6	. 2799 2198	−3 1673	2860	−335
2. 7	. 2843 4516	−2 8813	2558	−301
2. 8	. 2884 8021	−2 6255	2290	−268
2. 9	. 2923 5271	−2 3965	2052	−238
3. 0	. 2959 8556	−2 1913	1840	−212
3. 1	. 2993 9928	−2 0073	1652	−188
3. 2	. 3026 1228	−1 8422	1485	−166
3. 3	. 3056 4105	−1 6936	1338	−147
3. 4	. 3085 0047	−1 5598	1206	−132
3. 5	. 3112 0390	−1 4392	1090	−116
3. 6	. 3137 6341	−1 3302	987	−103
3. 7	. 3161 8990	−1 2315	895	−92
3. 8	. 3184 9325	−1 1420	812	−83
3. 9	. 3206 8239	−1 0608	739	−73
4. 0	. 3227 6544	−9869	674	−65
4. 1	. 3247 4981	−9195	615	−59
4. 2	. 3266 4223	−8580	562	−52
4. 3	. 3284 4886	−8017	516	−47
4. 4	. 3301 7530	−7502	473	−43
4. 5	. 3318 2674	−7029	434	−38
4. 6	. 3334 0788	−6594	400	−34
4. 7	. 3349 2308	−6194	369	−32
4. 8	. 3363 7634	−5826	340	−28
4. 9	. 3377 7134	−5485	315	−25
5. 0	. 3391 1149	−5170	291	−24
5. 1	. 3403 9993	−4879	270	−21
5. 2	. 3416 3958	−4610	251	−19
5. 3	. 3428 3314	−4358	233	−18
5. 4	. 3439 8311	−4126	217	−16
5. 5	. 3450 9182	−3909	202	−15
5. 6	. 3461 6145	−3707	188	−14
5. 7	. 3471 9401	−3519	176	−12
5. 8	. 3481 9138	−3343	164	−12
5. 9	. 3491 5532	−3178	154	−10

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.5$; tabular spacing=0.1—Continued				
6. 0	0. 3500 8749	−3024	144	−10
6. 1	. 3509 8941	−2880	135	−9
6. 2	. 3518 6253	−2745	127	−8
6. 3	. 3527 0820	−2618	119	−8
6. 4	. 3535 2769	−2499	112	−7
6. 5	. 3543 2220	−2387	106	−6
6. 6	. 3550 9284	−2281	99	−6
6. 7	. 3558 4067	−2182	94	−6
6. 8	. 3565 6668	−2088	88	−6
6. 9	. 3572 7182	−1999	84	−4
7. 0	. 3579 5696	−1916	79	−5
7. 1	. 3586 2295	−1837	75	−4
7. 2	. 3592 7057	−1762	71	−4
7. 3	. 3599 0057	−1691	67	−4
7. 4	. 3605 1366	−1624	64	−4
7. 5	. 3611 1051	−1560	60	−4
7. 6	. 3616 9175	−1500	57	−3
7. 7	. 3622 5799	−1443	55	−3
7. 8	. 3628 0979	−1389	52	−3
7. 9	. 3633 4771	−1337	49	−3
8. 0	. 3638 7226	−1288	47	−2
8. 1	. 3643 8392	−1241	45	−2
8. 2	. 3648 8318	−1197	43	−2
8. 3	. 3653 7046	−1154	40	−2
8. 4	. 3658 4621	−1114	38	−2
8. 5	. 3663 1082	−1075	37	−1
8. 6	. 3667 6467	−1038	35	−2
8. 7	. 3672 0814	−1003	34	−1
8. 8	. 3676 4158	−969	32	−2
8. 9	. 3680 6532	−938	31	−1
9. 0	. 3684 7969	−907	29	−1
9. 1	. 3688 8499	−878	28	−1
9. 2	. 3692 8151	−849	27	−1
9. 3	. 3696 6954	−822	26	−1
9. 4	. 3700 4936	−796	25	−1
9. 5	. 3704 2120	−772	24	−1
9. 6	. 3707 8533	−748	23	−1
9. 7	. 3711 4198	−725	22	−1
9. 8	. 3714 9138	−703	21	−1
9. 9	. 3718 3374	−682	20	−1
10. 0	. 3721 6929			
$Q=-0.5$; tabular spacing=0.2				
10. 0	0. 3721 6929	−2649	152	−13
10. 2	. 3728 2070	−2497	140	−12
10. 4	. 3734 4714	−2357	131	−10
10. 6	. 3740 5002	−2226	121	−10
19. 8	. 3746 3063	−2105	112	−9
11. 0	. 3751 9020	−1993	104	−8
11. 2	. 3757 2983	−1888	97	−7
11. 4	. 3762 5059	−1791	91	−7
11. 6	. 3767 5343	−1700	85	−6
11. 8	. 3772 3927	−1616	79	−6
12. 0	. 3777 0895	−1536	74	−5
12. 2	. 3781 6328	−1462	70	−4
12. 4	. 3786 0297	−1393	65	−4
12. 6	. 3790 2874	−1328	61	−4
12. 8	. 3794 4124	−1267	58	−3
13. 0	. 3798 4107			

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.5$; tabular spacing=0.5				
13.0	0.3798 4107	−7567	809	−134
13.5	.3807 8891	−6758	697	−111
14.0	.3816 6917	−6061	605	−93
14.5	.3824 8882	−5456	527	−78
15.0	.3832 5392	−4929	461	−66
15.5	.3839 6973	−4468	405	−56
16.0	.3846 4086	−4062	358	−48
16.5	.3852 7137	−3704	317	−41
17.0	.3858 6484	−3388	282	−35
17.5	.3864 2443	−3105	251	−31
18.0	.3869 5296	−2854	225	−26
18.5	.3874 5296	−2629	202	−23
19.0	.3879 2667	−2427	182	−20
19.5	.3883 7610	−2245	164	−18
20.0	.3888 0308	−2081	148	−16
20.5	.3892 0926	−1933	135	−14
21.0	.3895 9610	−1798	122	−13
21.5	.3899 6497	−1676	112	−11
22.0	.3903 1708	−1564	102	−10
22.5	.3906 5355	−1462	93	−9
23.0	.3909 7540	−1369	85	−8
23.5	.3912 8357	−1283	78	−7
24.0	.3915 7890	−1205	72	−6
24.5	.3918 6219	−1132	66	−6
25.0	.3921 3415	−1066	62	−5
25.5	.3923 9545	−1005	57	−5
26.0	.3926 4670	−948	52	−5
26.5	.3928 8849	−895	49	−4
27.0	.3931 2132	−846	46	−3
27.5	.3933 4568	−801	42	−3
28.0	.3935 6204	−759	39	−3
28.5	.3937 7080	−720	37	−3
29.0	.3939 7238	−683	34	−2
29.5	.3941 6712	−649	32	−2
30.0	.3943 5537			
$Q=-0.4$; tabular spacing=0.02				
0.00	0.0000 0000	1 5277	−29	−57
.02	.0000 7638	1 5248	−85	−56
.04	.0003 0525	1 5163	−140	−56
.06	.0006 8575	1 5023	−195	−54
.08	.0012 1648	1 4828	−247	−52
.10	.0018 9549	1 4580	−297	−50
.12	.0027 2030	1 4283	−344	−47
.14	.0036 8795	1 3939	−388	−44
.16	.0047 9498	1 3551	−428	−40
.18	.0060 3753	1 3122	−465	−36
.20	.0074 1130	1 2657	−497	−32
.22	.0089 1165	1 2160	−526	−29
.24	.0105 3360	1 1635	−550	−24
.26	.0122 7189	1 1085	−569	−20
.28	.0141 2103	1 0516	−585	−16
.30	.0160 7534	9931	−596	−11
.32	.0181 2895	9335	−603	−7
.34	.0202 7592	8732	−607	−3
.36	.0225 1020	8125	−607	0
.38	.0248 2574	7519	−603	+3
.40	.0272 1647	6915	−596	7
.42	.0296 7634	6319	−587	9

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.4$; tabular spacing=0.02—Continued				
0.44	0.0321 9941	5732	−575	12
.46	.0347 7979	5156	−561	14
.48	.0374 1174	4596	−545	16
.50	.0400 8964	4051	−527	18
.52	.0428 0805	3524	−508	19
.54	.0455 6170	3016	−488	21
.56	.0483 4550	2528	−466	21
.58	.0511 5458	2062	−445	22
.60	.0539 8428	1617	−422	22
.62	.0568 3015	1195	−400	23
.64	.0596 8797	795	−377	22
.66	.0625 5374	418	−355	22
.68	.0654 2368	+63	−333	22
.70	.0682 9425	−270	−311	22
.72	.0711 6212	−581	−289	22
.74	.0740 2418	−870	−269	21
.76	.0768 7753	−1139	−249	20
.78	.0797 1950	−1388	−229	20
.80	.0825 4758	−1617	−211	18
.82	.0853 5949	−1827	−192	18
.84	.0881 5313	−2020	−175	17
.86	.0909 2658	−2195	−159	16
.88	.0936 7807	−2354	−144	15
.90	.0964 0602	−2497	−129	15
.92	.0991 0900	−2626	−115	14
.94	.1017 8571	−2741	−102	13
.96	.1044 3502	−2842	−89	12
.98	.1070 5591	−2932	−78	12
1.00	.1096 4747	−3010	−67	11
1.02	.1122 0894	−3077	−57	10
1.04	.1147 3964	−3134	−47	10
1.06	.1172 3901	−3181	−39	9
1.08	.1197 0656	−3220	−30	8
1.10	.1221 4192	−3250	−23	7
1.12	.1245 4478	−3273	−16	7
1.14	.1269 1490	−3290	−10	7
1.16	.1292 5212	−3300	−4	6
1.18	.1315 5635	−3303	+1	5
1.20	.1338 2754	−3302	6	5
1.22	.1360 6572	−3296	11	4
1.24	.1382 7094	−3285	15	4
1.26	.1404 4331	−3270	19	3
1.28	.1425 8298	−3251	22	3
1.30	.1446 9014			
$Q=-0.4$; tabular spacing=0.05				
1.30	0.1446 9014	−2 0174	420	119
1.35	.1498 1733	−1 9754	510	90
1.40	.1547 4698	−1 9243	576	66
1.45	.1594 8419	−1 8667	622	46
1.50	.1640 3474	−1 8045	652	30
1.55	.1684 0484	−1 7392	670	17
1.60	.1726 0101	−1 6723	676	+6
1.65	.1766 2996	−1 6047	674	−2
1.70	.1804 9844	−1 5372	666	−8
1.75	.1842 1320	−1 4706	653	−13
1.80	.1877 8090	−1 4054	636	−17
1.85	.1912 0806	−1 3418	616	−20
1.90	.1945 0104	−1 2801	595	−22
1.95	.1976 6601	−1 2207	572	−23

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.4$; tabular spacing=0.05—Continued				
2.00	0.2007 0891	−1 1635	548	−24
2.05	.2036 3546	−1 1087	524	−24
2.10	.2064 5115	−1 0562	500	−24
2.15	.2091 6121	−1 0062	478	−22
2.20	.2117 7064			
$Q=-0.4$; tabular spacing=0.1				
2.2	0.2117 7064	−3 8363	3542	−369
2.3	.2167 0651	−3 4821	3195	−347
2.4	.2212 9416	−3 1626	2874	−321
2.5	.2255 6556	−2 8751	2581	−293
2.6	.2295 4945	−2 6170	2316	−265
2.7	.2332 7164	−2 3854	2078	−238
2.8	.2367 5529	−2 1776	1865	−213
2.9	.2400 2118	−1 9912	1674	−190
3.0	.2430 8795	−1 8238	1505	−169
3.1	.2459 7234	−1 6733	1354	−151
3.2	.2486 8941	−1 5378	1220	−134
3.3	.2512 5270	−1 4158	1101	−119
3.4	.2536 7440	−1 3057	995	−106
3.5	.2559 6553	−1 2062	901	−94
3.6	.2581 3604	−1 1162	817	−84
3.7	.2601 9493	−1 0345	742	−75
3.8	.2621 5037	−9603	675	−67
3.9	.2640 0978	−8928	615	−60
4.0	.2657 7991	−8314	561	−53
4.1	.2674 6690	−7753	513	−48
4.2	.2690 7636	−7240	470	−43
4.3	.2706 1343	−6770	431	−39
4.4	.2720 8280	−6339	396	−35
4.5	.2734 8877	−5943	364	−32
4.6	.2748 3532	−5579	335	−30
4.7	.2761 2607	−5244	309	−26
4.8	.2773 6439	−4935	286	−23
4.9	.2785 5336	−4648	265	−21
5.0	.2796 9585	−4384	245	−20
5.1	.2807 9450	−4139	228	−17
5.2	.2818 5177	−3911	211	−16
5.3	.2828 6993	−3700	196	−15
5.4	.2838 5108	−3504	183	−13
5.5	.2847 9720	−3321	171	−12
5.6	.2857 1011	−3150	159	−12
5.7	.2865 9152	−2991	148	−11
5.8	.2874 4303	−2842	139	−9
5.9	.2882 6611	−2703	130	−9
6.0	.2890 6215	−2573	122	−8
6.1	.2898 3247	−2451	114	−8
6.2	.2905 7828	−2336	108	−7
6.3	.2913 0072	−2229	101	−7
6.4	.2920 0088	−2128	95	−5
6.5	.2926 7975	−2033	90	−5
6.6	.2933 3830	−1943	84	−5
6.7	.2939 7741	−1859	79	−5
6.8	.2945 9794	−1779	75	−4
6.9	.2952 0068	−1704	71	−4
7.0	.2957 8637	−1633	67	−4
7.1	.2963 5573	−1566	63	−4
7.2	.2969 0943	−1503	60	−3
7.3	.2974 4810	−1442	57	−3
7.4	.2979 7235	−1386	54	−3

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.4$; tabular spacing=0.1—Continued				
7.5	0.2984 8274	−1331	51	−3
7.6	.2989 7982	−1280	49	−3
7.7	.2994 6410	−1232	46	−2
7.8	.2999 3606	−1186	44	−2
7.9	.3003 9617	−1141	42	−2
8.0	.3008 4486	−1099	40	−2
8.1	.3012 8256	−1060	38	−2
8.2	.3017 0967	−1022	36	−2
8.3	.3021 2656	−986	34	−2
8.4	.3025 3359	−951	33	−1
8.5	.3029 3111	−918	31	−2
8.6	.3033 1944	−887	30	−1
8.7	.3036 9891	−857	29	−1
8.8	.3040 6980	−828	27	−1
8.9	.3044 3242	−801	26	−1
9.0	.3047 8702	−775	25	−1
9.1	.3051 3388	−750	24	−1
9.2	.3054 7324	−726	23	−1
9.3	.3058 0534	−703	22	−1
9.4	.3061 3042	−681	21	−1
9.5	.3064 4868	−660	20	−1
9.6	.3067 6036	−639	20	−1
9.7	.3070 6564	−620	19	−1
9.8	.3073 6472	−601	18	−1
9.9	.3076 5779	−583	17	−1
10.0	.3079 4502			
$Q=-0.4$; tabular spacing=0.2				
10.0	0.3079 4502	−2265	130	−10
10.2	.3085 0268	−2135	120	−10
10.4	.3090 3898	−2015	111	−9
10.6	.3095 5514	−1904	103	−8
10.8	.3100 5226	−1800	96	−7
11.0	.3105 3137	−1705	89	−7
11.2	.3109 9344	−1615	83	−6
11.4	.3114 3935	−1532	78	−6
11.6	.3118 6994	−1454	72	−6
11.8	.3122 8599	−1382	68	−5
12.0	.3126 8822	−1315	64	−4
12.2	.3130 7730	−1251	59	−4
12.4	.3134 5386	−1192	55	−4
12.6	.3138 1851	−1136	52	−3
12.8	.3141 7180	−1084	49	−3
13.0	.3145 1425			
$Q=-0.4$; tabular spacing=0.5				
13.0	0.3145 1425	−6476	692	−115
13.5	.3153 2609	−5784	596	−95
14.0	.3160 8010	−5188	517	−79
14.5	.3167 8222	−4671	451	−66
15.0	.3174 3764	−4220	395	−56
15.5	.3180 5085	−3825	347	−48
16.0	.3186 2582	−3478	306	−41
16.5	.3191 6599	−3172	271	−35
17.0	.3196 7445	−2901	242	−30
17.5	.3201 5390	−2660	215	−26
18.0	.3206 0674	−2444	192	−23
18.5	.3210 3515	−2252	173	−19
19.0	.3214 4104	−2079	156	−17
19.5	.3218 2614	−1923	140	−15

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.4$; tabular spacing=0.5—Continued				
20. 0	0. 3221 9200	−1783	127	−13
20. 5	. 3225 4004	−1656	115	−12
21. 0	. 3228 7153	−1540	105	−10
21. 5	. 3231 8761	−1435	95	−10
22. 0	. 3234 8934	−1340	87	−8
22. 5	. 3237 7767	−1253	80	−7
23. 0	. 3240 5348	−1173	73	−7
23. 5	. 3243 1756	−1099	67	−6
24. 0	. 3245 7064	−1032	62	−6
24. 5	. 3248 1341	−970	57	−5
25. 0	. 3250 4647	−913	53	−4
25. 5	. 3252 7040	−861	48	−4
26. 0	. 3254 8572	−812	45	−4
26. 5	. 3256 9292	−767	42	−3
27. 0	. 3258 9245	−725	39	−3
27. 5	. 3260 8473	−686	36	−3
28. 0	. 3262 7015	−650	34	−3
28. 5	. 3264 4907	−617	31	−2
29. 0	. 3266 2182	−585	29	−2
29. 5	. 3267 8871	−556	27	−2
30. 0	. 3269 5005			
$Q=-0.3$; tabular spacing=0.02				
0. 00	0. 0000 0000	1 1582	−21	−43
. 02	. 0000 5791	1 1561	−64	−43
. 04	. 0002 3143	1 1497	−106	−42
. 06	. 0005 1991	1 1391	−146	−41
. 08	. 0009 2231	1 1244	−186	−40
. 10	. 0014 3714	1 1058	−224	−38
. 12	. 0020 6257	1 0835	−259	−35
. 14	. 0027 9634	1 0576	−292	−33
. 16	. 0036 3586	1 0284	−322	−30
. 18	. 0045 7823	9961	−350	−27
. 20	. 0056 2021	9612	−374	−24
. 22	. 0067 5830	9237	−396	−22
. 24	. 0079 8877	8842	−414	−18
. 26	. 0093 0766	8428	−428	−15
. 28	. 0107 1082	8000	−440	−12
. 30	. 0121 9399	7560	−448	−8
. 32	. 0137 5275	7111	−454	−6
. 34	. 0153 8262	6656	−457	−2
. 36	. 0170 7905	6200	−457	0
. 38	. 0188 3748	5743	−454	+2
. 40	. 0206 5334	5288	−450	5
. 42	. 0225 2208	4839	−442	7
. 44	. 0244 3921	4396	−434	9
. 46	. 0264 0030	3962	−423	11
. 48	. 0284 0102	3539	−411	12
. 50	. 0304 3713	3128	−398	13
. 52	. 0325 0453	2731	−384	14
. 54	. 0345 9923	2347	−368	16
. 56	. 0367 1741	1979	−352	16
. 58	. 0388 5539	1627	−336	16
. 60	. 0410 0963	1291	−319	17
. 62	. 0431 7678	972	−302	17
. 64	. 0453 5365	669	−286	17
. 66	. 0475 3722	384	−269	17
. 68	. 0497 2462	+115	−252	17
. 70	. 0519 1317	−137	−236	16
. 72	. 0541 0035	−373	−220	16

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.3$; tabular spacing=0.02—Continued				
0. 74	0. 0562 8380	−593	−204	16
. 76	. 0584 6132	−797	−189	15
. 78	. 0606 3088	−986	−174	15
. 80	. 0627 9058	−1160	−160	14
. 82	. 0649 3867	−1320	−147	13
. 84	. 0670 7356	−1467	−134	13
. 86	. 0691 9378	−1601	−122	12
. 88	. 0712 9798	−1723	−110	12
. 90	. 0733 8495	−1833	−99	11
. 92	. 0754 5359	−1932	−88	10
. 94	. 0775 0292	−2020	−79	10
. 96	. 0795 3204	−2099	−69	10
. 98	. 0815 4018	−2168	−60	9
1. 00	. 0835 2662	−2229	−53	8
1. 02	. 0854 9079	−2281	−45	8
1. 04	. 0874 3214	−2326	−38	7
1. 06	. 0893 5023	−2364	−31	6
1. 08	. 0912 4468	−2395	−25	6
1. 10	. 0931 1518			
$Q=-0.3$; tabular spacing=0.05				
1. 10	0. 0931 1518	−1 5109	−244	223
1. 15	. 0976 8516	−1 5353	−62	182
1. 20	. 1021 0161	−1 5415	+85	147
1. 25	. 1063 6390	−1 5330	202	117
1. 30	. 1104 7290	−1 5128	293	91
1. 35	. 1144 3061	−1 4835	362	69
1. 40	. 1182 3997	−1 4473	413	51
1. 45	. 1219 0459	−1 4060	450	37
1. 50	. 1254 2861	−1 3610	474	24
1. 55	. 1288 1652	−1 3136	489	15
1. 60	. 1320 7308	−1 2648	495	+6
1. 65	. 1352 0315	−1 2152	495	0
1. 70	. 1382 1170	−1 1657	490	−5
1. 75	. 1411 0368	−1 1167	482	−8
1. 80	. 1438 8399	−1 0685	470	−12
1. 85	. 1465 5744	−1 0215	457	−14
1. 90	. 1491 2874	−9758	442	−15
1. 95	. 1516 0246	−9317	425	−16
2. 00	. 1539 8301	−8892	408	−17
2. 05	. 1562 7465	−8484	391	−17
2. 10	. 1584 8144			
$Q=-0.3$; tabular spacing=0.1				
2. 1	0. 1584 8144	−3 2386	2925	−273
2. 2	. 1626 5601	−2 9462	2657	−268
2. 3	. 1665 3596	−2 6805	2404	−254
2. 4	. 1701 4786	−2 4401	2169	−235
2. 5	. 1735 1575	−2 2232	1953	−216
2. 6	. 1766 6132	−2 0279	1757	−196
2. 7	. 1796 0411	−1 8522	1580	−176
2. 8	. 1823 6167	−1 6942	1422	−158
2. 9	. 1849 4980	−1 5520	1280	−142
3. 0	. 1873 8274	−1 4240	1153	−126
3. 1	. 1896 7328	−1 3087	1040	−113
3. 2	. 1918 3295	−1 2047	939	−101
3. 3	. 1938 7215	−1 1108	849	−90
3. 4	. 1958 0026	−1 0259	769	−80

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=−0.3; tabular spacing=0.1—Continued				
3. 5	0. 1976 2579	−9490	698	−72
3. 6	. 1993 5643	−8792	634	−64
3. 7	. 2009 9914	−8158	576	−58
3. 8	. 2025 6027	−7582	525	−51
3. 9	. 2040 4558	−7057	480	−45
4. 0	. 2054 6031	−6577	438	−42
4. 1	. 2068 0928	−6139	401	−37
4. 2	. 2080 9685	−5738	368	−33
4. 3	. 2093 2704	−5370	338	−30
4. 4	. 2105 0352	−5032	311	−27
4. 5	. 2116 2969	−4722	286	−24
4. 6	. 2127 0863	−4436	264	−22
4. 7	. 2137 4322	−4172	244	−20
4. 8	. 2147 3610	−3928	225	−18
4. 9	. 2156 8970	−3702	209	−16
5. 0	. 2166 0627	−3494	194	−15
5. 1	. 2174 8791	−3300	180	−14
5. 2	. 2183 3654	−3120	167	−13
5. 3	. 2191 5398	−2953	155	−12
5. 4	. 2199 4189	−2798	145	−10
5. 5	. 2207 0182	−2652	135	−10
5. 6	. 2214 3523	−2517	126	−9
5. 7	. 2221 4347	−2391	118	−8
5. 8	. 2228 2780	−2273	110	−8
5. 9	. 2234 8939	−2163	104	−6
6. 0	. 2241 2936	−2059	97	−6
6. 1	. 2247 4874	−1962	91	−6
6. 2	. 2253 4851	−1871	85	−6
6. 3	. 2259 2956	−1785	80	−5
6. 4	. 2264 9277	−1705	76	−4
6. 5	. 2270 3892	−1629	72	−4
6. 6	. 2275 6878	−1557	67	−4
6. 7	. 2280 8308	−1490	63	−4
6. 8	. 2285 8247	−1427	60	−3
6. 9	. 2290 6759	−1367	57	−3
7. 0	. 2295 3904	−1310	54	−3
7. 1	. 2299 9739	−1256	50	−3
7. 2	. 2304 4317	−1206	48	−2
7. 3	. 2308 7690	−1158	46	−2
7. 4	. 2312 9904	−1112	43	−2
7. 5	. 2317 1006	−1069	41	−3
7. 6	. 2321 1040	−1028	39	−2
7. 7	. 2325 0045	−989	37	−2
7. 8	. 2328 8061	−952	35	−2
7. 9	. 2332 5125	−917	33	−2
8. 0	. 2336 1272	−884	32	−2
8. 1	. 2339 6535	−852	31	−1
8. 2	. 2343 0947	−821	29	−2
8. 3	. 2346 4537	−792	28	−2
8. 4	. 2349 7336	−765	26	−1
8. 5	. 2352 9370	−738	25	−1
8. 6	. 2356 0665	−713	24	−1
8. 7	. 2359 1247	−689	23	−1
8. 8	. 2362 1140	−666	22	−1
8. 9	. 2365 0367	−644	21	−1
9. 0	. 2367 8949	−623	20	−1
9. 1	. 2370 6909	−603	19	−1
9. 2	. 2373 4265	−584	18	−1
9. 3	. 2376 1038	−565	17	−1
9. 4	. 2378 7245	−548	17	0
9. 5	0. 2381 2904	−531	17	0
9. 6	. 2383 8032	−514	16	−1
9. 7	. 2386 2646	−499	15	−1
9. 8	. 2388 6762	−484	14	0
9. 9	. 2391 0393	−470	14	0
10. 0	. 2393 3554	−456	13	−1
10. 1	. 2395 6260	−442	12	0
10. 2	. 2397 8524	−430	12	0
10. 3	. 2400 0358	−417	12	0
10. 4	. 2402 1774	−406	11	−1
10. 5	. 2404 2785	−394	11	0
10. 6	. 2406 3402	−383	10	−1
10. 7	. 2408 3635	−373	10	0
10. 8	. 2410 3496	−362	10	0
10. 9	. 2412 2995	−353	10	0
11. 0	. 2414 2140			
Q=−0.3; tabular spacing=0.5				
11. 0	0. 2414 2140	−8596	1068	−211
11. 5	. 2423 2900	−7528	898	−170
12. 0	. 2431 6132	−6630	761	−138
12. 5	. 2439 2733	−5870	649	−112
13. 0	. 2446 3465	−5220	557	−92
13. 5	. 2452 8976	−4664	480	−76
14. 0	. 2458 9824	−4183	417	−64
14. 5	. 2464 6488	−3767	363	−54
15. 0	. 2469 9386	−3404	318	−45
15. 5	. 2474 8881	−3086	280	−38
16. 0	. 2479 5289	−2806	247	−33
16. 5	. 2483 8892	−2559	218	−29
17. 0	. 2487 9936	−2340	194	−24
17. 5	. 2491 8639	−2146	174	−21
18. 0	. 2495 5197	−1972	155	−18
18. 5	. 2498 9782	−1817	140	−16
19. 0	. 2502 2550	−1678	126	−14
19. 5	. 2505 3640	−1552	113	−12
20. 0	. 2508 3179	−1439	103	−11
20. 5	. 2511 1279	−1336	93	−10
21. 0	. 2513 8042	−1243	85	−8
21. 5	. 2516 3563	−1158	77	−8
22. 0	. 2518 7925	−1081	70	−6
22. 5	. 2521 1206	−1011	65	−6
23. 0	. 2523 3475	−946	59	−6
23. 5	. 2525 4798	−887	54	−5
24. 0	. 2527 5234	−833	50	−5
24. 5	. 2529 4837	−783	46	−4
25. 0	. 2531 3656	−738	43	−3
25. 5	. 2533 1738	−695	39	−4
26. 0	. 2534 9125	−656	37	−2
26. 5	. 2536 5856	−619	34	−3
27. 0	. 2538 1969	−585	31	−2
27. 5	. 2539 7496	−554	29	−2
28. 0	. 2541 2468	−525	27	−2
28. 5	. 2542 6916	−498	25	−2
29. 0	. 2544 0867	−473	24	−1
29. 5	. 2545 4344	−449	22	−2
30. 0	. 2546 7373			

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
		$Q=-0.2$; tabular spacing=0.02		
0.00	0.0000 0000	7809	−14	−29
.02	.0000 3904	7794	−43	−28
.04	.0001 5603	7752	−71	−28
.06	.0003 5053	7681	−98	−28
.08	.0006 2184	7583	−124	−26
.10	.0009 6898	7458	−149	−25
.12	.0013 9070	7309	−173	−24
.14	.0018 8551	7136	−195	−22
.16	.0024 5167	6940	−216	−20
.18	.0030 8724	6725	−234	−18
.20	.0037 9006	6491	−250	−16
.22	.0045 5779	6240	−264	−14
.24	.0053 8792	5976	−277	−12
.26	.0062 7780	5699	−287	−10
.28	.0072 2468	5412	−295	−8
.30	.0082 2569	5118	−300	−5
.32	.0092 7787	4818	−304	−4
.34	.0103 7822	4514	−306	−2
.36	.0115 2372	4208	−306	0
.38	.0127 1128	3901	−304	+2
.40	.0139 3787	3597	−301	3
.42	.0152 0042	3296	−296	5
.44	.0164 9593	3000	−290	6
.46	.0178 2144	2709	−284	7
.48	.0191 7404	2425	−276	8
.50	.0205 5089	2150	−267	9
.52	.0219 4924	1883	−257	10
.54	.0233 6642	1626	−247	10
.56	.0247 9985	1378	−237	11
.58	.0262 4707	1142	−226	11
.60	.0277 0571	916	−214	11
.62	.0291 7351	702	−203	11
.64	.0306 4833	498	−192	11
.66	.0321 2813	306	−181	11
.68	.0336 1099	125	−170	11
.70	.0350 9511			

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
		$Q=-0.2$; tabular spacing=0.05		
0.70	0.0350 9511	−250	−2360	425
.75	.0387 9999	−2610	−1962	399
.80	.0424 7876	−4572	−1595	366
.85	.0461 1183	−6167	−1267	329
.90	.0496 8322	−7434	−978	289
.95	.0531 8027	−8412	−727	250
1.00	.0565 9320	−9139	−513	214
1.05	.0599 1475	−9652	−333	180
1.10	.0631 3978	−9986	−184	149
1.15	.0662 6494	−1 0169	−61	122
1.20	.0692 8842	−1 0231	+38	100
1.25	.0722 0959	−1 0193	117	79
1.30	.0750 2883	−1 0075	179	62
1.35	.0777 4732	−9896	227	48
1.40	.0803 6684	−9669	262	35
1.45	.0828 8968	−9407	288	26
1.50	.0853 1844	−9120	305	18
1.55	.0876 5600	−8814	316	11
1.60	.0899 0543	−8498	321	5
1.65	.0920 6987	−8177	322	+1
1.70	.0941 5255	−7854	320	−2
1.75	0.0961 5668	−7534	315	−5
1.80	.0980 8547	−7219	308	−7
1.85	.0999 4207	−6911	300	−8
1.90	.1017 2956	−6610	291	−10
1.95	.1034 5095	−6320	280	−10
2.00	.1051 0914	−6039	270	−10
2.05	.1067 0695	−5769	259	−11
2.10	.1082 4706	−5510	248	−11
2.15	.1097 3206	−5262	237	−11
2.20	.1111 6445	−5025	226	−11
2.25	.1125 4658	−4799	216	−10
2.30	.1138 8073			

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
		$Q=-0.2$; tabular spacing=0.1		
2.3	0.1138 8073	−1 8342	1605	−164
2.4	.1164 1359	−1 6737	1452	−153
2.5	.1187 7907	−1 5285	1312	−140
2.6	.1209 9171	−1 3973	1183	−128
2.7	.1230 6462	−1 2790	1067	−116
2.8	.1250 0964	−1 1722	963	−104
2.9	.1268 3743	−1 0760	869	−94
3.0	.1285 5762	−9890	784	−85
3.1	.1301 7891	−9106	709	−75
3.2	.1317 0914	−8397	642.	−67
3.3	.1331 5540	−7755	582	−60
3.4	.1345 2411	−7173	528	−54
3.5	.1358 2109	−6644	480	−48
3.6	.1370 5163	−6165	437	−43
3.7	.1382 2052	−5728	398	−38
3.8	.1393 3214	−5330	364	−35
3.9	.1403 9045	−4966	332	−31
4.0	.1413 9910	−4634	304	−28
4.1	.1423 6141	−4330	279	−25
4.2	.1432 8042	−4051	256	−23
4.3	.1441 5892	−3795	236	−20
4.4	.1449 9948	−3559	217	−19
4.5	.1458 0444	−3342	200	−17
4.6	.1465 7598	−3142	185	−15
4.7	.1473 1610	−2957	171	−14
4.8	.1480 2664	−2786	158	−13
4.9	.1487 0932	−2628	146	−12
5.0	.1493 6572	−2482	136	−10
5.1	.1499 9729	−2346	127	−10
5.2	.1506 0541	−2219	118	−9
5.3	.1511 9134	−2101	110	−8
5.4	.1517 5626	−1992	102	−7
5.5	.1523 0126	−1890	96	−7
5.6	.1528 2736	−1794	89	−6
5.7	.1533 3553	−1705	83	−6
5.8	.1538 2665	−1621	78	−5
5.9	.1543 0155	−1543	74	−4
6.0	.1547 6102	−1470	69	−5
6.1	.1552 0580	−1401	64	−4
6.2	.1556 3657	−1336	61	−4
6.3	.1560 5397	−1276	57	−4
6.4	.1564 5861	−1218	54	−4
6.5	.1568 5107	−1165	51	−3
6.6	.1572 3188	−1114	48	−3
6.7	.1576 0155	−1066	45	−3
6.8	.1579 6056	−1021	42	−3
6.9	.1583 0935	−979	40	−2

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.2$; tabular spacing=0.1—Continued				
7.0	0.1586 4836	−938	38	−2
7.1	.1589 7799	−900	36	−2
7.2	.1592 9862	−864	34	−2
7.3	.1596 1062	−830	33	−1
7.4	.1599 1431	−797	31	−2
7.5	.1602 1004	−766	29	−1
7.6	.1604 9810	−737	28	−2
7.7	.1607 7879	−709	26	−1
7.8	.1610 5239	−683	25	−1
7.9	.1613 1915	−658	24	−1
8.0	.1615 7934	−634	22	−2
8.1	.1618 3320	−611	22	−1
8.2	.1620 8094	−589	21	−1
8.3	.1623 2278	−569	20	−1
8.4	.1625 5895	−549	19	−1
8.5	.1627 8962	−530	18	−1
8.6	.1630 1499	−512	17	−1
8.7	.1632 3524	−495	17	−1
8.8	.1634 5053	−478	16	−1
8.9	.1636 6105	−463	15	−1
9.0	.1638 6693	−448	14	0
9.1	.1640 6834	−433	14	−1
9.2	.1642 6542	−420	13	0
9.3	.1644 5830	−406	13	0
9.4	.1646 4711	−394	12	−1
9.5	.1648 3199	−382	12	0
9.6	.1650 1306	−370	11	0
9.7	.1651 9042	−359	11	0
9.8	.1653 6420	−348	10	0
9.9	.1655 3450	−338	10	0
10.0	.1657 0142	−328	9	−1
10.1	.1658 6506	−318	9	0
10.2	.1660 2552	−309	9	0
10.3	.1661 8289	−300	9	0
10.4	.1663 3726	−292	8	0
10.5	.1664 8871	−283	8	−1
10.6	.1666 3733	−276	8	0
10.7	.1667 8319	−268	7	0
10.8	.1669 2637	−261	7	0
10.9	.1670 6694	−254	6	−1
11.0	.1672 0497			
$Q=-0.2$; tabular spacing=0.5				
11.0	0.1672 0497	−6186	767	−151
11.5	.1678 5936	−5420	645	−122
12.0	.1684 5955	−4774	547	−98
12.5	.1690 1200	−4228	467	−80
13.0	.1695 2218	−3761	400	−66
13.5	.1699 9475	−3360	346	−55
14.0	.1704 3371	−3015	300	−46
14.5	.1708 4253	−2715	262	−38
15.0	.1712 2419	−2453	229	−33
15.5	.1715 8133	−2224	201	−28
16.0	.1719 1622	−2023	178	−23
16.5	.1722 3087	−1845	158	−20
17.0	.1725 2707	−1688	140	−18
17.5	.1728 0640	−1548	125	−15
18.0	.1730 7025	−1423	112	−13
18.5	.1733 1987	−1311	101	−11

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.2$; tabular spacing=0.5—Continued				
19.0	0.1735 5639	−1210	90	−10
19.5	.1737 8080	−1120	82	−9
20.0	.1739 9402	−1038	74	−8
20.5	.1741 9686	−964	67	−7
21.0	.1743 9005	−897	61	−6
21.5	.1745 7428	−836	56	−6
22.0	.1747 5015	−780	51	−5
22.5	.1749 1822	−730	46	−4
23.0	.1750 7899	−683	43	−4
23.5	.1752 3293	−640	39	−4
24.0	.1753 8046	−601	36	−3
24.5	.1755 2199	−565	33	−3
25.0	.1756 5786	−532	31	−3
25.5	.1757 8840	−501	28	−2
26.0	.1759 1394	−473	26	−2
26.5	.1760 3474	−447	24	−2
27.0	.1761 5107	−422	23	−2
27.5	.1762 6318	−400	21	−2
28.0	.1763 7129	−379	20	−1
28.5	.1764 7561	−360	18	−1
29.0	.1765 7633	−341	17	−1
29.5	.1766 7364	−324	16	−1
30.0	.1767 6772			
$Q=-0.1$; tabular spacing=0.02				
0.00	0.0000 0000	3950	−7	−14
.02	.0000 1975	3943	−21	−14
.04	.0000 7893	3922	−35	−14
.06	.0001 7733	3886	−49	−14
.08	.0003 1459	3837	−62	−13
.10	.0004 9022	3774	−75	−12
.12	.0007 0360	3700	−87	−12
.14	.0009 5397	3613	−98	−12
.16	.0012 4047	3515	−108	−10
.18	.0015 6211	3407	−117	−9
.20	.0019 1783	3289	−126	−8
.22	.0023 0643	3164	−133	−7
.24	.0027 2667	3031	−139	−6
.26	.0031 7722	2892	−144	−5
.28	.0036 5669	2748	−148	−4
.30	.0041 6364	2600	−151	−3
.32	.0046 9659	2449	−152	−2
.34	.0052 5404	2297	−154	−1
.36	.0058 3445	2143	−154	0
.38	.0064 3630	1989	−153	+1
.40	.0070 5804	1836	−151	2
.42	.0076 9815	1685	−149	2
.44	.0083 5510	1536	−146	3
.46	.0090 2743	1390	−143	4
.48	.0097 1365	1247	−138	4
.50	.0104 1234	1109	−134	4
.52	.0111 2213	975	−130	4
.54	.0118 4166	845	−124	5
.56	.0125 6965	721	−119	5
.58	.0133 0484	602	−114	5
.60	.0140 4605	488	−108	6
.62	.0147 9214	380	−102	6
.64	.0155 4202	277	−97	6
.66	.0162 9467	180	−92	6
.68	.0170 4912	89	−86	6

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.1$; tabular spacing=0.02—Continued				
0. 70	0. 0178 0446	+3	−80	5
. 72	. 0185 5983	−78	−75	6
. 74	. 0193 1442	−152	−70	5
. 76	. 0200 6749	−222	−65	5
. 78	. 0208 1833	−287	−60	5
. 80	. 0215 6630	−347	−55	5
. 82	. 0223 1079	−402	−51	4
. 84	. 0230 5126	−453	−46	4
. 86	. 0237 8720	−500	−42	4
. 88	. 0245 1814	−542	−38	4
. 90	. 0252 4367	−580	−35	4
. 92	. 0259 6339	−615	−31	3
. 94	. 0266 7697	−646	−28	4
. 96	. 0273 8408	−674	−25	3
. 98	. 0280 8446	−698	−22	3
1. 00	. 0287 7785	−720	−19	3
1. 02	. 0294 6404	−740	−16	3
1. 04	. 0301 4283	−756	−14	2
1. 06	. 0308 1407	−770	−12	2
1. 08	. 0314 7761	−782	−10	3
1. 10	. 0321 3332	−792	−8	2
1. 12	. 0327 8112	−799	−6	2
1. 14	. 0334 2093	−805	−5	1
1. 16	. 0340 5268	−810	−3	2
1. 18	. 0346 7634	−813	−1	2
1. 20	. 0352 9186	−814	0	1
1. 22	. 0358 9924	−814	+1	1
1. 24	. 0364 9848	−814	2	1
1. 26	. 0370 8958	−812	3	1
1. 28	. 0376 7256	−808	4	0
1. 30	. 0382 4746	−805	5	1
1. 32	. 0388 1431	−800	6	1
1. 34	. 0393 7316	−795	6	0
1. 36	. 0399 2406	−789	7	1
1. 38	. 0404 6708	−782	7	1
1. 40	. 0410 0228	−775	8	0
1. 42	. 0415 2973	−767	8	0
1. 44	. 0420 4951	−759	8	0
1. 46	. 0425 6170	−750	9	0
1. 48	. 0430 6639	−742	9	0
1. 50	. 0435 6366	−733	9	0
1. 52	. 0440 5360	−724	10	0
1. 54	. 0445 3631	−714	10	0
1. 56	. 0450 1187	−704	10	0
1. 58	. 0454 8039	−694	10	0
1. 60	. 0459 4197	−685	10	0
1. 62	. 0463 9670	−675	10	0
1. 64	. 0468 4467	−665	10	0
1. 66	. 0472 8600	−655	10	0
1. 68	. 0477 2078	−644	10	0
1. 70	. 0481 4912	−635	10	0
1. 72	. 0485 7110	−625	10	0
1. 74	. 0489 8685	−614	10	0
1. 76	. 0493 9644	−605	10	0
1. 78	. 0497 9999	−595	10	0
1. 80	. 0501 9759	−585	10	0
1. 82	. 0505 8934	−575	10	0
1. 84	. 0509 7534	−566	10	0
1. 86	. 0513 5568	−556	10	0
1. 88	. 0517 3046	−546	10	0
1. 90	. 0520 9979			

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.1$; tabular spacing=0.1				
1. 9	0. 0520 9979	−1 3433	1127	−68
2. 0	. 0538 6765	−1 2306	1047	−80
2. 1	. 0555 1246	−1 1259	964	−82
2. 2	. 0570 4467	−1 0295	882	−82
2. 3	. 0584 7393	−9414	803	−79
2. 4	. 0598 0905	−8611	728	−74
2. 5	. 0610 5806	−7882	660	−69
2. 6	. 0622 2826	−7223	597	−63
2. 7	. 0633 2622	−6626	540	−57
2. 8	. 0643 5792	−6086	488	−52
2. 9	. 0653 2876	−5598	442	−46
3. 0	. 0662 4362	−5156	400	−42
3. 1	. 0671 0692	−4756	362	−38
3. 2	. 0679 2267	−4394	329	−33
3. 3	. 0686 9448	−4065	299	−30
3. 4	. 0694 2564	−3766	272	−27
3. 5	. 0701 1914	−3494	248	−24
3. 6	. 0707 7770	−3246	226	−22
3. 7	. 0714 0380	−3021	206	−20
3. 8	. 0719 9970	−2815	189	−17
3. 9	. 0725 6744	−2626	173	−16
4. 0	. 0731 0893	−2453	158	−15
4. 1	. 0736 2588	−2295	146	−13
4. 2	. 0741 1988	−2149	134	−12
4. 3	. 0745 9239	−2016	123	−11
4. 4	. 0750 4474	−1893	114	−9
4. 5	. 0754 7817	−1779	105	−9
4. 6	. 0758 9381	−1674	97	−8
4. 7	. 0762 9271	−1577	90	−7
4. 8	. 0766 7585	−1487	83	−6
4. 9	. 0770 4411	−1404	78	−6
5. 0	. 0773 9834	−1326	72	−6
5. 1	. 0777 3931	−1254	67	−5
5. 2	. 0780 6773	−1188	62	−4
5. 3	. 0783 8428	−1125	58	−4
5. 4	. 0786 8958	−1067	54	−4
5. 5	. 0789 8420	−1013	51	−4
5. 6	. 0792 6870	−962	47	−3
5. 7	. 0795 4357	−915	44	−3
5. 8	. 0798 0930	−870	42	−3
5. 9	. 0800 6631	−829	39	−2
6. 0	. 0803 1504	−790	36	−3
6. 1	. 0805 5587	−753	34	−2
6. 2	. 0807 8917	−719	32	−2
6. 3	. 0810 1527	−687	30	−2
6. 4	. 0812 3451	−656	29	−1
6. 5	. 0814 4719	−627	27	−2
6. 6	. 0816 5360	−600	25	−2
6. 7	. 0818 5400	−575	24	−1
6. 8	. 0820 4866	−551	23	−1
6. 9	. 0822 3781	−528	22	−1
7. 0	. 0824 2168	−506	20	−1
7. 1	. 0826 0050	−486	20	−1
7. 2	. 0827 7445	−466	18	−1
7. 3	. 0829 4375	−448	18	−1
7. 4	. 0831 0856	−430	17	−1
7. 5	. 0832 6907	−414	16	−1
7. 6	. 0834 2544	−398	15	−1
7. 7	. 0835 7783	−383	14	−1
7. 8	. 0837 2639	−369	13	−1
7. 9	. 0838 7125	−356	13	−1

TABLE 38.—*Modified potential*—Continued

a	W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=-0.1$; tabular spacing=0.1—Continued				
8.0	0.0840 1256	−343	12	0
8.1	.0841 5043	−330	12	−1
8.2	.0842 8500	−319	11	−1
8.3	.0844 1639	−308	11	0
8.4	.0845 4470	−297	10	−1
8.5	.0846 7003	−287	10	0
8.6	.0847 9250	−277	9	−1
8.7	.0849 1219	−268	9	0
8.8	.0850 2920	−259	8	−1
8.9	.0851 4362	−251	8	0
9.0	.0852 5554	−242	8	−1
9.1	.0853 6503	−235	7	0
9.2	.0854 7217	−228	7	0
9.3	.0855 7703	−220	7	0
9.4	.0856 7970	−213	7	0
9.5	.0857 8023	−207	6	0
9.6	.0858 7869	−201	6	0
9.7	.0859 7515	−195	6	0
9.8	.0860 6966	−189	6	0
9.9	.0861 6228	−183	5	0
10.0	.0862 5308			
$Q=-0.1$; tabular spacing=0.5				
10.0	0.0862 5308	−4455	599	−130
10.5	.0866 8137	−3856	497	−103
11.0	.0870 7112	−3359	415	−81
11.5	.0874 2727	−2944	350	−66
12.0	.0877 5398	−2594	296	−53
12.5	.0880 5476	−2298	253	−43
13.0	.0883 3256	−2044	217	−36
13.5	.0885 8991	−1827	188	−30
14.0	.0888 2900	−1640	163	−25
14.5	.0890 5168	−1477	142	−21
15.0	.0892 5959	−1335	124	−17
15.5	.0894 5416	−1210	110	−15
16.0	.0896 3662	−1101	96	−13
16.5	.0898 0806	−1004	86	−11
17.0	.0899 6947	−919	76	−9
17.5	.0901 2168	−843	68	−8
18.0	.0902 6547	−775	61	−7
18.5	.0904 0152	−714	55	−6
19.0	.0905 3042	−659	49	−6
19.5	.0906 5274	−610	44	−5
20.0	.0907 6895	−566	40	−4
20.5	.0908 7951	−525	36	−4
21.0	.0909 8483	−489	33	−3
21.5	.0910 8525	−455	30	−3
22.0	.0911 8112	−425	28	−2
22.5	.0912 7274	−398	25	−2
23.0	.0913 6039	−372	23	−2
23.5	.0914 4431	−349	21	−2
24.0	.0915 2475	−328	20	−2
24.5	.0916 0190	−308	18	−2
25.0	.0916 7598	−290	16	−2
25.5	.0917 4716	−273	15	−1
26.0	.0918 1560	−258	14	−1
26.5	.0918 8146	−244	13	−1
27.0	.0919 4489	−230	12	−1
27.5	.0920 0602	−218	12	−1
28.0	.0920 6497	−207	11	−1
28.5	.0921 2185	−196	10	−1
29.0	.0921 7677	−186	9	−1
29.5	.0922 2983	−177	9	−1
30.0	.0922 8113			

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.1$; tabular spacing=0.02				
0.00	0.0000 0000	4050	−7	−15
.02	.0000 2025	4043	−21	−14
.04	.0000 8093	4022	−36	−14
.06	.0001 8183	3986	−50	−14
.08	.0003 2259	3936	−63	−13
.10	.0005 0271	3874	−75	−13
.12	.0007 2157	3798	−87	−12
.14	.0009 7841	3711	−99	−12
.16	.0012 7236	3612	−109	−10
.18	.0016 0242	3503	−118	−9
.20	.0019 6752	3385	−126	−8
.22	.0023 6646	3258	−134	−7
.24	.0027 9799	3125	−140	−6
.26	.0032 6077	2985	−145	−5
.28	.0037 5340	2840	−149	−4
.30	.0042 7442	2691	−152	−3
.32	.0048 2235	2538	−154	−2
.34	.0053 9566	2385	−155	−1
.36	.0059 9282	2230	−156	−1
.38	.0066 1228	2074	−154	+1
.40	.0072 5248	1920	−153	1
.42	.0079 1189	1767	−151	2
.44	.0085 8896	1617	−148	3
.46	.0092 8220	1469	−144	4
.48	.0099 9013	1325	−140	4
.50	.0107 1130	1184	−136	4
.52	.0114 4432	1048	−131	5
.54	.0121 8782	917	−126	5
.56	.0129 4049	790	−121	6
.58	.0137 0107	670	−115	6
.60	.0144 6834			
$Q=+0.1$; tabular spacing=0.05				
0.60	0.0144 6834	3478	−1654	215
.65	.0164 0837	1824	−1437	217
.70	.0183 6663	+387	−1225	212
.75	.0203 2876	−838	−1025	200
.80	.0222 8250	−1863	−841	184
.85	.0242 1761	−2704	−676	165
.90	.0261 2568	−3381	−531	146
.95	.0279 9994	−3912	−404	127
1.00	.0298 3508	−4315	−295	108
1.05	.0316 2707	−4610	−204	91
1.10	.0333 7296	−4814	−127	77
1.15	.0350 7071	−4941	−63	64
1.20	.0367 1905	−5004	−12	51
1.25	.0383 1735	−5016	+30	42
1.30	.0398 6548	−4987	63	33
1.35	.0413 6375	−4924	88	26
1.40	.0428 1278	−4836	108	20
1.45	.0442 1345	−4727	123	14
1.50	.0455 6685	−4604	133	10
1.55	.0468 7421	−4471	141	8
1.60	.0481 3687	−4330	145	4
1.65	.0493 5622	−4185	147	+2
1.70	.0505 3372	−4038	148	0
1.75	.0516 7085	−3890	147	−1
1.80	.0527 6907	−3744	144	−2
1.85	.0538 2985	−3599	142	−3
1.90	.0548 5464			

TABLE 38.—*Modified potential*—Continued

$Q=+0.1$; tabular spacing=0.1

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
1.9	0.0548 5464	−1 3834	1087	−56
2.0	.0568 0188	−1 2747	1019	−68
2.1	.0586 2165	−1 1728	944	−75
2.2	.0603 2415	−1 0783	870	−74
2.3	.0619 1881	−9912	797	−73
2.4	.0634 1436	−9115	728	−69
2.5	.0648 1874	−8388	664	−64
2.6	.0661 3925	−7724	604	−60
2.7	.0673 8253	−7120	549	−55
2.8	.0685 5461	−6570	500	−50
2.9	.0696 6099	−6070	455	−45
3.0	.0707 0666	−5616	414	−41
3.1	.0716 9617	−5202	377	−37
3.2	.0726 3367	−4824	344	−33
3.3	.0735 2293	−4480	314	−30
3.4	.0743 6738	−4166	287	−27
3.5	.0751 7017	−3880	263	−24
3.6	.0759 3416	−3617	240	−22
3.7	.0766 6198	−3376	221	−20
3.8	.0773 5603	−3156	203	−18
3.9	.0780 1853	−2953	186	−17
4.0	.0786 5149	−2767	171	−15
4.1	.0792 5679	−2596	158	−13
4.2	.0798 3613	−2437	146	−12
4.3	.0803 9109	−2291	135	−11
4.4	.0809 2314	−2156	124	−10
4.5	.0814 3364	−2032	116	−9
4.6	.0819 2381	−1916	108	−8
4.7	.0823 9482	−1808	100	−8
4.8	.0828 4776	−1709	92	−7
4.9	.0832 8360	−1616	86	−6
5.0	.0837 0328	−1530	81	−6
5.1	.0841 0766	−1450	75	−6
5.2	.0844 9754	−1375	70	−5
5.3	.0848 7368	−1305	66	−4
5.4	.0852 3676	−1239	61	−5
5.5	.0855 8746	−1178	57	−4
5.6	.0859 2637	−1121	54	−3
5.7	.0862 5407	−1067	50	−4
5.8	.0865 7110	−1017	47	−3
5.9	.0868 7796	−970	45	−2
6.0	.0871 7513	−925	42	−3
6.1	.0874 6305	−883	40	−2
6.2	.0877 4213	−844	37	−3
6.3	.0880 1278	−807	35	−2
6.4	.0882 7537	−772	33	−2
6.5	.0885 3024	−739	31	−2
6.6	.0887 7772	−707	30	−2
6.7	.0890 1813	−678	28	−2
6.8	.0892 5176	−650	26	−2
6.9	.0894 7889	−624	25	−1
7.0	.0896 9978	−599	24	−1
7.1	.0899 1469	−575	22	−1
7.2	.0901 2385	−552	21	−1
7.3	.0903 2749	−531	20	−1
7.4	.0905 2582	−511	19	−1
7.5	.0907 1904	−492	18	−1
7.6	.0909 0735	−473	18	−1
7.7	.0910 9093	−456	16	−1
7.8	.0912 6995	−439	16	−1
7.9	.0914 4458	−423	15	−1

TABLE 38.—*Modified potential*—Continued

$Q=+0.1$; tabular spacing=0.1—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
8.0	0.0916 1497	−408	14	−1
8.1	.0917 8128	−394	14	−1
8.2	.0919 4366	−380	13	−1
8.3	.0921 0223	−367	13	0
8.4	.0922 5713	−355	12	−1
8.5	.0924 0849	−342	11	−1
8.6	.0925 5642	−331	11	0
8.7	.0927 0103	−320	11	0
8.8	.0928 4245	−310	10	−1
8.9	.0929 8076	−300	10	0
9.0	.0931 1608	−290	9	0
9.1	.0932 4849	−281	9	0
9.2	.0933 7808	−272	9	0
9.3	.0935 0496	−264	8	−1
9.4	.0936 2920	−256	8	0
9.5	.0937 5088	−248	8	0
9.6	.0938 7007	−241	7	0
9.7	.0939 8686	−233	7	−1
9.8	.0941 0132	−227	7	0
9.9	.0942 1352	−220	6	0
10.0	.0943 2351			

$Q=+0.1$; tabular spacing=0.5

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
10.0	0.0943 2351	−5352	713	−153
10.5	.0948 4262	−4639	592	−120
11.0	.0953 1535	−4046	496	−97
11.5	.0957 4761	−3550	418	−78
12.0	.0961 4437	−3132	356	−63
12.5	.0965 0981	−2777	304	−52
13.0	.0968 4748	−2473	261	−43
13.5	.0971 6042	−2212	226	−35
14.0	.0974 5125	−1986	196	−30
14.5	.0977 2221	−1790	171	−25
15.0	.0979 7528	−1619	150	−21
15.5	.0982 1216	−1468	132	−18
16.0	.0984 3436	−1336	117	−15
16.5	.0986 4319	−1220	104	−13
17.0	.0988 3983	−1116	92	−11
17.5	.0990 2530	−1024	82	−10
18.0	.0992 0054	−941	74	−9
18.5	.0993 6636	−868	66	−7
19.0	.0995 2351	−802	60	−6
19.5	.0996 7264	−742	54	−6
20.0	.0998 1435	−688	49	−5
20.5	.0999 4918	−639	44	−4
21.0	.1000 7762	−595	40	−4
21.5	.1002 0012	−554	37	−4
22.0	.1003 1707	−518	34	−3
22.5	.1004 2884	−484	31	−3
23.0	.1005 3578	−453	28	−3
23.5	.1006 3818	−425	26	−3
24.0	.1007 3633	−399	24	−2
24.5	.1008 3048	−375	22	−2
25.0	.1009 2089	−353	20	−2
25.5	.1010 0776	−333	19	−1
26.0	.1010 9129	−314	18	−1
26.5	.1011 7169	−297	16	−2
27.0	.1012 4911	−281	15	−1

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.1$; tabular spacing=0.5—Continued				
27. 5	0. 1013 2373	−266	14	−1
28. 0	. 1013 9569	−252	13	−1
28. 5	. 1014 6513	−239	12	−1
29. 0	. 1015 3218	−227	11	−1
29. 5	. 1015 9697	−215	11	−1
30. 0	. 1016 5960			
$Q=+0.2$; tabular spacing=0.02				
0. 00	0. 0000 0000	8210	−14	−28
. 02	. 0000 4105	8196	−43	−29
. 04	. 0001 6407	8153	−72	−28
. 06	. 0003 6861	8081	−99	−28
. 08	. 0006 5396	7982	−126	−27
. 10	. 0010 1914	7856	−152	−26
. 12	. 0014 6287	7704	−175	−24
. 14	. 0019 8364	7529	−198	−23
. 16	. 0025 7971	7331	−219	−21
. 18	. 0032 4908	7112	−237	−18
. 20	. 0039 8958	6875	−254	−17
. 22	. 0047 9884	6621	−268	−14
. 24	. 0056 7430	6353	−281	−12
. 26	. 0066 1328	6072	−291	−10
. 28	. 0076 1299	5781	−300	−8
. 30	. 0086 7050	5481	−306	−6
. 32	. 0097 8282	5176	−309	−4
. 34	. 0109 4690	4866	−312	−2
. 36	. 0121 5964	4555	−312	0
. 38	. 0134 1793	4243	−311	+1
. 40	. 0147 1866	3933	−307	3
. 42	. 0160 5871	3625	−303	4
. 44	. 0174 3501	3322	−297	6
. 46	. 0188 4454	3025	−290	7
. 48	. 0202 8432	2735	−283	8
. 50	. 0217 5144	2452	−274	9
. 52	. 0232 4310	2178	−264	10
. 54	. 0247 5653	1914	−254	10
. 56	. 0262 8911	1660	−244	10
. 58	. 0278 3828	1416	−233	11
. 60	. 0294 0161	1183	−222	11
. 62	. 0309 7677	960	−211	11
. 64	. 0325 6153	749	−199	12
. 66	. 0341 5379	550	−189	11
. 68	. 0357 5154	361	−178	11
. 70	. 0373 5289			
$Q=+0.2$; tabular spacing=0.05				
0. 70	0. 0373 5289	+1175	−2482	424
. 75	. 0413 6069	−1306	−2082	400
. 80	. 0453 5542	−3388	−1714	368
. 85	. 0493 1628	−5102	−1383	331
. 90	. 0532 2611	−6485	−1091	292
. 95	. 0570 7109	−7577	−837	254
1. 00	. 0608 4030	−8414	−618	218
1. 05	. 0645 2537	−9032	−434	184
1. 10	. 0681 2013	−9466	−280	154
1. 15	. 0716 2023	−9746	−151	129
1. 20	. 0750 2286	−9897	−47	104

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.2$; tabular spacing=0.05—Continued				
1. 25	0. 0783 2653	−9944	+38	84
1. 30	. 0815 3075	−9907	105	68
1. 35	. 0846 3591	−9802	158	53
1. 40	. 0876 4305	−9643	199	41
1. 45	. 0905 5376	−9444	230	31
1. 50	. 0933 7002	−9214	252	22
1. 55	. 0960 9414	−8963	268	16
1. 60	. 0987 2863	−8695	278	10
1. 65	. 1012 7617	−8418	283	6
1. 70	. 1037 3953	−8134	285	+1
1. 75	. 1061 2155	−7850	284	−1
1. 80	. 1084 2508	−7566	281	−3
1. 85	. 1106 5295	−7285	276	−5
1. 90	. 1128 0798	−7009	270	−6
1. 95	. 1148 9291	−6739	262	−7
2. 00	. 1169 1044	−6477	254	−8
2. 05	. 1188 6321	−6223	246	−9
2. 10	. 1207 5375	−5977	237	−8
2. 15	. 1225 8452	−5740	229	−8
2. 20	. 1243 5789			
$Q=+0.2$; tabular spacing=0.1				
2. 2	0. 1243 5789	−2 2054	1721	−142
2. 3	. 1277 4149	−2 0333	1583	−138
2. 4	. 1309 2177	−1 8750	1450	−132
2. 5	. 1339 1454	−1 7300	1326	−124
2. 6	. 1367 3432	−1 5974	1211	−115
2. 7	. 1393 9436	−1 4763	1105	−106
2. 8	. 1419 0676	−1 3658	1008	−97
2. 9	. 1442 8259	−1 2650	920	−88
3. 0	. 1465 3191	−1 1730	839	−80
3. 1	. 1486 6393	−1 0891	767	−72
3. 2	. 1506 8704	−1 0124	702	−66
3. 3	. 1526 0892	−9422	642	−60
3. 4	. 1544 3657	−8780	588	−54
3. 5	. 1561 7644	−8192	539	−49
3. 6	. 1578 3438	−7652	496	−44
3. 7	. 1594 1580	−7157	456	−40
3. 8	. 1609 2565	−6701	419	−37
3. 9	. 1623 6850	−6282	386	−33
4. 0	. 1637 4852	−5895	357	−30
4. 1	. 1650 6960	−5538	329	−27
4. 2	. 1663 3528	−5209	305	−25
4. 3	. 1675 4888	−4904	282	−22
4. 4	. 1687 1343	−4622	262	−21
4. 5	. 1698 3176	−4361	243	−19
4. 6	. 1709 0648	−4118	226	−17
4. 7	. 1719 4002	−3892	210	−16
4. 8	. 1729 3464	−3682	195	−14
4. 9	. 1738 9244	−3487	182	−13
5. 0	. 1748 1537	−3305	170	−12
5. 1	. 1757 0526	−3135	159	−11
5. 2	. 1765 6379	−2976	149	−10
5. 3	. 1773 9258	−2827	139	−10
5. 4	. 1781 9309	−2688	130	−8
5. 5	. 1789 6672	−2558	122	−8
5. 6	. 1797 1477	−2436	115	−8
5. 7	. 1804 3846	−2321	107	−7
5. 8	. 1811 3895	−2214	101	−6
5. 9	. 1818 1730	−2112	95	−6

TABLE 38.—*Modified potential*—Continued

a	−W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=+0.2; tabular spacing=0.1—Continued				
6.0	0.1824 7453	−2017	90	−6
6.1	.1831 1159	−1927	85	−5
6.2	.1837 2939	−1842	80	−5
6.3	.1843 2876	−1762	75	−5
6.4	.1849 1050	−1687	71	−4
6.5	.1854 7537	−1616	68	−3
6.6	.1860 2408	−1549	64	−4
6.7	.1865 5729	−1485	60	−4
6.8	.1870 7566	−1425	57	−4
6.9	.1875 7978	−1368	54	−2
7.0	.1880 7022	−1314	52	−3
7.1	.1885 4752	−1262	49	−3
7.2	.1890 1220	−1214	46	−3
7.3	.1894 6474	−1168	44	−2
7.4	.1899 0561	−1123	42	−3
7.5	.1903 3524	−1082	40	−2
7.6	.1907 5406	−1042	38	−1
7.7	.1911 6246	−1004	36	−2
7.8	.1915 6083	−968	34	−2
7.9	.1919 4952	−933	33	−1
8.0	.1923 2888	−901	32	−1
8.1	.1926 9923	−869	30	−2
8.2	.1930 6088	−839	28	−2
8.3	.1934 1415	−811	27	−1
8.4	.1937 5931	−783	26	−1
8.5	.1940 9664	−757	25	−1
8.6	.1944 2640	−732	24	−1
8.7	.1947 4883	−708	23	−1
8.8	.1950 6418	−686	22	−1
8.9	.1953 7267	−664	21	−1
9.0	.1956 7452	−643	20	0
9.1	.1959 6995	−622	19	−1
9.2	.1962 5916	−603	18	−1
9.3	.1965 4233	−585	18	−1
9.4	.1968 1965	−567	17	0
9.5	.1970 9130	−550	16	−1
9.6	.1973 5746	−534	16	−1
9.7	.1976 1827	−518	15	−1
9.8	.1978 7391	−503	15	0
9.9	.1981 2453	−488	14	−1
10.0	.1983 7026	−474	13	−1
10.1	.1986 1126	−461	13	0
10.2	.1988 4764	−448	13	0
10.3	.1990 7955	−435	12	−1
10.4	.1993 0711	−423	11	0
10.5	.1995 3044	−412	11	0
10.6	.1997 4966	−400	11	0
10.7	.1999 6487	−389	10	0
10.8	.2001 7618	−379	10	−1
10.9	.2003 8371	−369	10	0
11.0	.2005 8754	−360	10	0
11.1	.2007 8779	−350	9	0
11.2	.2009 8453	−341	9	0
11.3	.2011 7786	−332	8	0
11.4	.2013 6787	−324	8	0
11.5	.2015 5464			
Q=+0.2; tabular spacing=0.5				
11.5	0.2015 5464	−7904	926	−170
12.0	.2024 4270	−6978	787	−139

TABLE 38.—*Modified potential*—Continued

a	−W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=+0.2; tabular spacing=0.5—Continued				
12.5	0.2032 6097	−6191	674	−114
13.0	.2040 1733	−5517	580	−94
13.5	.2047 1852	−4937	502	−78
14.0	.2053 7034	−4436	436	−66
14.5	.2059 7780	−3999	381	−55
15.0	.2065 4527	−3618	334	−47
15.5	.2070 7656	−3284	294	−40
16.0	.2075 7500	−2990	260	−34
16.5	.2080 4355	−2729	231	−29
17.0	.2084 8480	−2498	206	−26
17.5	.2089 0107	−2292	184	−22
18.0	.2092 9442	−2109	165	−19
18.5	.2096 6668	−1944	148	−17
19.0	.2100 1950	−1796	133	−15
19.5	.2103 5436	−1663	121	−12
20.0	.2106 7259	−1542	109	−11
20.5	.2109 7541	−1433	99	−10
21.0	.2112 6390	−1334	90	−9
21.5	.2115 3905	−1243	82	−8
22.0	.2118 0177	−1161	75	−7
22.5	.2120 5287	−1086	69	−6
23.0	.2122 9312	−1017	63	−6
23.5	.2125 2320	−954	58	−5
24.0	.2127 4373	−896	54	−4
24.5	.2129 5531	−842	49	−4
25.0	.2131 5847	−793	45	−4
25.5	.2133 5369	−748	42	−3
26.0	.2135 4143	−706	39	−3
26.5	.2137 2212	−667	36	−3
27.0	.2138 9614	−630	33	−3
27.5	.2140 6386	−597	32	−2
28.0	.2142 2561	−566	29	−2
28.5	.2143 8170	−536	27	−2
29.0	.2145 3243	−509	25	−2
29.5	.2146 7807	−484	24	−2
30.0	.2148 1886			
Q=+0.3; tabular spacing=0.02				
0.00	0.0000 0000	1 2492	−22	−44
.02	.0000 6246	1 2470	−65	−43
.04	.0002 4963	1 2406	−108	−43
.06	.0005 6085	1 2298	−150	−42
.08	.0009 9505	1 2148	−190	−40
.10	.0015 5073	1 1959	−228	−39
.12	.0022 2600	1 1731	−264	−36
.14	.0030 1858	1 1466	−298	−33
.16	.0039 2582	1 1169	−329	−32
.18	.0049 4474	1 0839	−358	−28
.20	.0060 7206	1 0482	−382	−24
.22	.0073 0419	1 0100	−404	−22
.24	.0086 3733	9696	−423	−19
.26	.0100 6742	9272	−438	−15
.28	.0115 9023	8834	−451	−13
.30	.0132 0138	8383	−460	−9
.32	.0148 9635	7922	−466	−6
.34	.0166 7055	7456	−470	−3
.36	.0185 1931	6987	−470	−1
.38	.0204 3794	6517	−468	+2
.40	.0224 2174	6049	−463	4
.42	.0244 6603	5585	−457	6

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.3$; tabular spacing=0.02—Continued				
0.44	0.0265 6617	5128	−448	9
.46	.0287 1759	4680	−438	10
.48	.0309 1582	4242	−427	12
.50	.0331 5645	3815	−414	13
.52	.0354 3524	3401	−400	14
.54	.0377 4804	3002	−385	15
.56	.0400 9086	2617	−369	16
.58	.0424 5985	2248	−353	16
.60	.0448 5132	1895	−336	16
.62	.0472 6175	1559	−320	17
.64	.0496 8777	1240	−303	17
.66	.0521 2618	937	−286	16
.68	.0545 7396	650	−270	17
.70	.0570 2825	381	−253	16
.72	.0594 8634	+128	−237	16
.74	.0619 4571	−109	−222	15
.76	.0644 0399	−331	−207	15
.78	.0668 5895	−538	−192	15
.80	.0693 0854	−729	−178	14
.82	.0717 5084	−907	−164	14
.84	.0741 8407	−1071	−151	13
.86	.0766 0659	−1222	−138	13
.88	.0790 1689	−1360	−127	12
.90	.0814 1359	−1487	−116	11
.92	.0837 9542	−1602	−105	11
.94	.0861 6124	−1707	−95	10
.96	.0885 0997	−1802	−85	10
.98	.0908 4069	−1887	−76	8
1.00	.0931 5254	−1963	−68	9
1.02	.0954 4476	−2031	−60	8
1.04	.0977 1666	−2091	−53	7
1.06	.0999 6765	−2144	−46	7
1.08	.1021 9721	−2190	−39	7
1.10	.1044 0486	−2229	−33	6
1.12	.1065 9023	−2262	−28	5
1.14	.1087 5298	−2290	−23	5
1.16	.1108 9282	−2313	−18	5
1.18	.1130 0954	−2331	−13	5
1.20	.1151 0295			
$Q=+0.3$; tabular spacing=0.05				
1.20	0.1151 0295	−1 4640	−107	159
1.25	.1202 3372	−1 4748	+22	129
1.30	.1252 1702	−1 4726	125	103
1.35	.1300 5306	−1 4601	207	82
1.40	.1347 4308	−1 4394	270	63
1.45	.1392 8917	−1 4124	318	48
1.50	.1436 9402	−1 3806	353	35
1.55	.1479 6082	−1 3452	379	26
1.60	.1520 9309	−1 3073	396	16
1.65	.1560 9464	−1 2678	406	10
1.70	.1599 6940	−1 2272	410	+4
1.75	.1637 2146	−1 1861	410	0
1.80	.1673 5490	−1 1451	407	−3
1.85	.1708 7383	−1 1044	401	−6
1.90	.1742 8232	−1 0643	393	−8
1.95	.1775 8439	−1 0250	383	−9
2.00	.1807 8396	−9866	372	−11
2.05	.1838 8486	−9494	361	−12

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.3$; tabular spacing=0.05—Continued				
2.10	0.1868 9082	−9133	350	−12
2.15	.1898 0546	−8784	336	−13
2.20	.1926 3225			
$Q=+0.3$; tabular spacing=0.1				
2.2	0.1926 3225	−3 3801	2544	−199
2.3	.1980 3568	−3 1257	2348	−195
2.4	.2031 2652	−2 8909	2159	−190
2.5	.2079 2828	−2 6750	1980	−179
2.6	.2124 6254	−2 4770	1815	−165
2.7	.2167 4910	−2 2955	1661	−154
2.8	.2208 0610	−2 1294	1520	−142
2.9	.2246 5016	−1 9775	1391	−128
3.0	.2282 9648	−1 8383	1274	−117
3.1	.2317 5896	−1 7110	1167	−107
3.2	.2350 5034	−1 5942	1070	−97
3.3	.2381 8231	−1 4872	982	−88
3.4	.2411 6555	−1 3890	902	−80
3.5	.2440 0990	−1 2987	830	−73
3.6	.2467 2436	−1 2158	764	−66
3.7	.2493 1726	−1 1394	704	−60
3.8	.2517 9621	−1 0689	650	−55
3.9	.2541 6827	−1 0040	600	−49
4.0	.2564 3993	−9440	555	−45
4.1	.2586 1720	−8884	514	−41
4.2	.2607 0562	−8370	476	−38
4.3	.2627 1034	−7894	442	−34
4.4	.2646 3611	−7452	411	−31
4.5	.2664 8737	−7041	382	−29
4.6	.2682 6821	−6659	356	−27
4.7	.2699 8246	−6304	332	−24
4.8	.2716 3367	−5972	309	−23
4.9	.2732 2517	−5663	289	−20
5.0	.2747 6004	−5374	270	−19
5.1	.2762 4117	−5104	253	−18
5.2	.2776 7126	−4851	236	−16
5.3	.2790 5284	−4615	222	−14
5.4	.2803 8827	−4393	208	−13
5.5	.2816 7977	−4185	196	−13
5.6	.2829 2942	−3989	184	−12
5.7	.2841 3918	−3805	173	−11
5.8	.2853 1089	−3633	163	−10
5.9	.2864 4628	−3470	154	−9
6.0	.2875 4696	−3316	145	−9
6.1	.2886 1448	−3172	136	−8
6.2	.2896 5028	−3035	129	−7
6.3	.2906 5573	−2906	122	−7
6.4	.2916 3211	−2785	115	−6
6.5	.2925 8065	−2669	109	−6
6.6	.2935 0250	−2560	103	−6
6.7	.2943 9874	−2457	98	−5
6.8	.2952 7041	−2359	93	−5
6.9	.2961 1849	−2266	88	−5
7.0	.2969 4391	−2178	84	−5
7.1	.2977 4755	−2094	80	−4
7.2	.2985 3025	−2015	76	−4
7.3	.2992 9280	−1939	72	−4
7.4	.3000 3595	−1868	69	−3
7.5	.3007 6043	−1799	65	−4
7.6	.3014 6692	−1734	62	−3

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.3$; tabular spacing=0.1—Continued				
7.7	0.3021 5607	−1672	59	−2
7.8	.3028 2851	−1612	57	−3
7.9	.3034 8482	−1556	54	−3
8.0	.3041 2557	−1502	51	−2
8.1	.3047 5130	−1450	49	−2
8.2	.3053 6253	−1402	47	−2
8.3	.3059 5975	−1354	45	−2
8.4	.3065 4342	−1309	42	−2
8.5	.3071 1400	−1267	41	−1
8.6	.3076 7192	−1225	40	−1
8.7	.3082 1758	−1186	38	−2
8.8	.3087 5139	−1148	36	−2
8.9	.3092 7372	−1112	35	−1
9.0	.3097 8493	−1077	33	−1
9.1	.3102 8537	−1044	32	−1
9.2	.3107 7537	−1012	31	−2
9.3	.3112 5526	−981	29	−1
9.4	.3117 2533	−952	28	−1
9.5	.3121 8589	−924	28	−1
9.6	.3126 3721	−896	26	−1
9.7	.3130 7958	−870	25	−1
9.8	.3135 1324	−845	24	−1
9.9	.3139 3845	−820	23	−1
10.0	.3143 5546			
$Q=+0.3$; tabular spacing=0.2				
10.0	0.3143 5546	−3190	176	−14
10.2	.3151 6578	−3014	164	−12
10.4	.3159 4596	−2850	152	−12
10.6	.3166 9764	−2698	142	−10
10.8	.3174 2233	−2557	132	−10
11.0	.3181 2146	−2425	123	−9
11.2	.3187 9634	−2302	115	−8
11.4	.3194 4819	−2187	108	−7
11.6	.3200 7818	−2080	100	−7
11.8	.3206 8737	−1979	94	−6
12.0	.3212 7677	−1885	88	−6
12.2	.3218 4732	−1796	83	−5
12.4	.3223 9990	−1714	78	−5
12.6	.3229 3535	−1636	73	−5
12.8	.3234 5445	−1562	69	−4
13.0	.3239 5792			
$Q=+0.3$; tabular spacing=0.5				
13.0	0.3239 5792	−9343	975	−157
13.5	.3251 5268	−8368	845	−130
14.0	.3262 6377	−7523	735	−110
14.5	.3272 9962	−6788	643	−92
15.0	.3282 6759	−6145	565	−78
15.5	.3291 7412	−5580	498	−67
16.0	.3300 2484	−5083	441	−57
16.5	.3308 2473	−4642	391	−49
17.0	.3315 7820	−4251	348	−43
17.5	.3322 8917	−3902	312	−37
18.0	.3329 6111	−3591	280	−32
18.5	.3335 9714	−3311	251	−28
19.0	.3342 0006	−3060	226	−25
19.5	.3347 7238	−2834	205	−22

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.3$; tabular spacing=0.5—Continued				
20.0	0.3353 1637	−2629	186	−19
20.5	.3358 3406	−2443	168	−17
21.0	.3363 2733	−2275	154	−15
21.5	.3367 9784	−2121	140	−14
22.0	.3372 4715	−1981	128	−12
22.5	.3376 7664	−1853	117	−11
23.0	.3380 8759	−1736	108	−10
23.5	.3384 8119	−1629	99	−8
24.0	.3388 5849	−1530	91	−8
24.5	.3392 2050	−1439	84	−7
25.0	.3395 6812	−1355	77	−6
25.5	.3399 0219	−1278	72	−6
26.0	.3402 2348	−1206	67	−5
26.5	.3405 3272	−1139	62	−5
27.0	.3408 3057	−1077	57	−4
27.5	.3411 1764	−1020	53	−4
28.0	.3413 9451	−967	50	−4
28.5	.3416 6172	−917	46	−4
29.0	.3419 1976	−871	43	−3
29.5	.3421 6909	−826	40	−3
30.0	.3424 1014			
$Q=+0.4$; tabular spacing=0.02				
0.00	0.0000 0000	1 6910	−29	−58
.02	.0000 8455	1 6881	−87	−58
.04	.0003 3791	1 6794	−144	−57
.06	.0007 5922	1 6650	−200	−56
.08	.0013 4702	1 6450	−254	−54
.10	.0020 9932	1 6196	−305	−51
.12	.0030 1358	1 5891	−354	−48
.14	.0040 8674	1 5537	−399	−45
.16	.0053 1529	1 5139	−440	−42
.18	.0066 9522	1 4698	−478	−38
.20	.0082 2214	1 4220	−512	−34
.22	.0098 9125	1 3708	−541	−30
.24	.0116 9745	1 3167	−566	−25
.26	.0136 3532	1 2601	−588	−21
.28	.0156 9919	1 2013	−604	−16
.30	.0178 8320	1 1409	−616	−12
.32	.0201 8130	1 0793	−625	−8
.34	.0225 8732	1 0168	−629	−4
.36	.0250 9503	9539	−630	0
.38	.0276 9813	8909	−627	+2
.40	.0303 9033	8282	−622	6
.42	.0331 6534	7660	−613	9
.44	.0360 1696	7048	−602	11
.46	.0389 3906	6446	−588	14
.48	.0419 2562	5858	−573	16
.50	.0449 7075	5285	−555	17
.52	.0480 6873	4730	−537	19
.54	.0512 1401	4193	−517	20
.56	.0544 0122	3676	−496	21
.58	.0576 2520	3180	−475	22
.60	.0608 8097	2705	−453	22
.62	.0641 6379	2252	−430	22
.64	.0674 6914	1822	−408	22
.66	.0707 9271	1414	−386	22
.68	.0741 3041	1028	−364	22
.70	.0774 7840	664	−342	22
.72	.0808 3303	322	−321	21

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
	Q=+0.4; tabular spacing=0.02—Continued			
0. 74	0. 0841 9088	+2	−300	21
. 76	. 0875 4875	−298	−280	20
. 78	. 0909 0364	−578	−260	19
. 80	. 0942 5274	−838	−241	19
. 82	. 0975 9347	−1079	−223	18
. 84	. 1009 2340	−1302	−206	17
. 86	. 1042 4031	−1508	−189	17
. 88	. 1075 4214	−1697	−173	16
. 90	. 1108 2700	−1870	−158	15
. 92	. 1140 9316	−2028	−144	14
. 94	. 1173 3904	−2172	−130	14
. 96	. 1205 6320	−2303	−118	13
. 98	. 1237 6433	−2420	−106	12
1. 00	. 1269 4125	−2526	−94	11
1. 02	. 1300 9292	−2621	−84	11
1. 04	. 1332 1838	−2705	−74	10
1. 06	. 1363 1679	−2779	−65	9
1. 08	. 1393 8741	−2844	−56	8
1. 10	. 1424 2960	−2900	−48	8
1. 12	. 1454 4278	−2948	−41	7
1. 14	. 1484 2649	−2989	−34	7
1. 16	. 1513 8031	−3023	−27	6
1. 18	. 1543 0390	−3050	−22	6
1. 20	. 1571 9699	−3072	−16	5
1. 22	. 1600 5936	−3088	−11	5
1. 24	. 1628 9086	−3099	−6	5
1. 26	. 1656 9136	−3105	−2	4
1. 28	. 1684 6082	−3107	+2	4
1. 30	. 1711 9920	−3105	5	4
1. 32	. 1739 0653	−3100	9	3
1. 34	. 1765 8286	−3091	12	3
1. 36	. 1792 2828	−3080	14	2
1. 38	. 1818 4290	−3065	17	3
1. 40	. 1844 2687			
	Q=+0.4; tabular spacing=0.05			
1. 40	0. 1844 2687	−1 9046	319	87
1. 45	. 1907 5385	−1 8728	386	67
1. 50	. 1968 9355	−1 8342	436	50
1. 55	. 2028 4984	−1 7905	472	36
1. 60	. 2086 2707	−1 7433	498	25
1. 65	. 2142 2998	−1 6935	513	16
1. 70	. 2196 6353	−1 6422	521	8
1. 75	. 2249 3286	−1 5901	524	+2
1. 80	. 2300 4318	−1 5377	521	−2
1. 85	. 2349 9973	−1 4856	515	−6
1. 90	. 2398 0772	−1 4340	506	−9
1. 95	. 2444 7231	−1 3834	496	−11
2. 00	. 2489 9856	−1 3338	483	−13
2. 05	. 2533 9143	−1 2856	469	−14
2. 10	. 2576 5574	−1 2387	454	−14
2. 15	. 2617 9618	−1 1933	439	−15
2. 20	. 2658 1729	−1 1494	424	−16
2. 25	. 2697 2346	−1 1070	408	−16
2. 30	. 2735 1894	−1 0662	393	−15
2. 35	. 2772 0779	−1 0269	377	−16
2. 40	. 2807 9395	−9892	363	−15
2. 45	. 2842 8120	−9529	348	−14
2. 50	. 2876 7315			

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
	Q=+0.4; tabular spacing=0.1			
2. 5	0. 2876 7315	−3 6737	2619	−226
2. 6	. 2941 8498	−3 4118	2407	−212
2. 7	. 3003 5563	−3 1712	2210	−197
2. 8	. 3062 0916	−2 9502	2029	−181
2. 9	. 3117 6767	−2 7472	1864	−166
3. 0	. 3170 5146	−2 5608	1711	−152
3. 1	. 3220 7917	−2 3897	1573	−139
3. 2	. 3268 6791	−2 2324	1446	−126
3. 3	. 3314 3340	−2 0878	1331	−115
3. 4	. 3357 9011	−1 9547	1227	−104
3. 5	. 3399 5134	−1 8321	1131	−96
3. 6	. 3439 2937	−1 7190	1044	−88
3. 7	. 3477 3551	−1 6146	965	−79
3. 8	. 3513 8018	−1 5181	893	−72
3. 9	. 3548 7305	−1 4288	826	−66
4. 0	. 3582 2303	−1 3462	766	−60
4. 1	. 3614 3839	−1 2696	712	−55
4. 2	. 3645 2680	−1 1984	661	−50
4. 3	. 3674 9535	−1 1323	614	−47
4. 4	. 3703 5068	−1 0709	572	−42
4. 5	. 3730 9891	−1 0137	533	−39
4. 6	. 3757 4578	−9603	498	−36
4. 7	. 3782 9661	−9106	465	−33
4. 8	. 3807 5639	−8641	434	−30
4. 9	. 3831 2976	−8206	406	−28
5. 0	. 3854 2106	−7800	381	−26
5. 1	. 3876 3437	−7419	357	−23
5. 2	. 3897 7348	−7062	335	−22
5. 3	. 3918 4197	−6727	314	−21
5. 4	. 3938 4320	−6412	295	−19
5. 5	. 3957 8030	−6117	278	−17
5. 6	. 3976 5623	−5839	262	−16
5. 7	. 3994 7377	−5576	247	−15
5. 8	. 4012 3555	−5330	232	−15
5. 9	. 4029 4403	−5097	220	−12
6. 0	. 4046 0153	−4878	208	−12
6. 1	. 4062 1026	−4670	196	−12
6. 2	. 4077 7230	−4474	185	−11
6. 3	. 4092 8959	−4289	176	−9
6. 4	. 4107 6399	−4113	166	−10
6. 5	. 4121 9727	−3947	158	−9
6. 6	. 4135 9108	−3789	149	−8
6. 7	. 4149 4699	−3640	142	−8
6. 8	. 4162 6651	−3498	135	−7
6. 9	. 4175 5105	−3363	128	−7
7. 0	. 4188 0195	−3236	122	−6
7. 1	. 4200 2050	−3114	116	−6
7. 2	. 4212 0790	−2998	110	−6
7. 3	. 4223 6533	−2888	105	−5
7. 4	. 4234 9388	−2783	100	−5
7. 5	. 4245 9459	−2683	95	−5
7. 6	. 4256 6847	−2588	91	−5
7. 7	. 4267 1647	−2497	87	−4
7. 8	. 4277 3950	−2410	83	−4
7. 9	. 4287 3842	−2327	79	−4
8. 0	. 4297 1407	−2248	75	−4
8. 1	. 4306 6724	−2173	73	−2
8. 2	. 4315 9868	−2100	69	−3
8. 3	. 4325 0911	−2031	66	−4
8. 4	. 4333 9923	−1965	64	−2
8. 5	. 4342 6970	−1902	61	−3
8. 6	. 4351 2115	−1841	58	−3

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.4$; tabular spacing=0.1—Continued				
8. 7	0. 4359 5419	−1783	56	−2
8. 8	. 4367 6940	−1727	54	−2
8. 9	. 4375 6735	−1673	52	−2
9. 0	. 4383 4856	−1622	49	−2
9. 1	. 4391 1355	−1573	47	−2
9. 2	. 4398 6281	−1526	46	−2
9. 3	. 4405 9682	−1480	44	−2
9. 4	. 4413 1603	−1436	42	−2
9. 5	. 4420 2088	−1394	40	−2
9. 6	. 4427 1179	−1354	39	−2
9. 7	. 4433 8916	−1315	37	−2
9. 8	. 4440 5339	−1277	36	−1
9. 9	. 4447 0484	−1241	35	−1
10. 0	. 4453 4388			
$Q=+0.4$; tabular spacing=0.2				
10. 0	0. 4453 4388	−4827	263	−21
10. 2	. 4465 8610	−4565	244	−18
10. 4	. 4477 8266	−4320	227	−17
10. 6	. 4489 3603	−4093	212	−16
10. 8	. 4500 4847	−3881	198	−14
11. 0	. 4511 2209	−3684	184	−13
11. 2	. 4521 5888	−3499	172	−12
11. 4	. 4531 6068	−3327	162	−11
11. 6	. 4541 2921	−3165	151	−10
11. 8	. 4550 6609	−3014	142	−9
12. 0	. 4559 7283	−2872	133	−9
12. 2	. 4568 5084	−2739	125	−8
12. 4	. 4577 0147	−2614	118	−7
12. 6	. 4585 2596	−2496	111	−7
12. 8	. 4593 2548	−2386	104	−6
13. 0	. 4601 0115	−2281	98	−6
13. 2	. 4608 5401	−2183	93	−6
13. 4	. 4615 8504	−2090	88	−5
13. 6	. 4622 9517	−2002	83	−5
13. 8	. 4629 8528	−1920	78	−4
14. 0	. 4636 5619			
$Q=+0.4$; tabular spacing=0.5				
14. 0	0. 4636 5619	−1 1518	1116	−164
14. 5	. 4652 5455	−1 0402	977	−139
15. 0	. 4667 4889	−9424	860	−118
15. 5	. 4681 4898	−8565	758	−102
16. 0	. 4694 6343	−7807	672	−86
16. 5	. 4706 9980	−7135	598	−74
17. 0	. 4718 6483	−6538	533	−65
17. 5	. 4729 6447	−6005	476	−56
18. 0	. 4740 0407	−5529	428	−49
18. 5	. 4749 8837	−5101	385	−43
19. 0	. 4759 2167	−4716	347	−38
19. 5	. 4768 0781	−4369	314	−33
20. 0	. 4776 5026	−4055	285	−30
20. 5	. 4784 5216	−3770	259	−26
21. 0	. 4792 1636	−3511	236	−23
21. 5	. 4799 4545	−3275	215	−21
22. 0	. 4806 4178	−3060	197	−18
22. 5	. 4813 0751	−2864	180	−16
23. 0	. 4819 4460	−2683	165	−15

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.4$; tabular spacing=0.5—Continued				
23. 5	0. 4825 5487	−2518	152	−13
24. 0	. 4831 3995	−2366	140	−12
24. 5	. 4837 0138	−2225	129	−11
25. 0	. 4842 4056	−2096	120	−10
25. 5	. 4847 5877	−1976	111	−9
26. 0	. 4852 5723	−1866	102	−8
26. 5	. 4857 3702	−1763	95	−7
27. 0	. 4861 9919	−1668	88	−7
27. 5	. 4866 4468	−1579	82	−6
28. 0	. 4870 7437	−1497	77	−6
28. 5	. 4874 8910	−1420	72	−5
29. 0	. 4878 8962	−1349	67	−5
29. 5	. 4882 7665	−1282	63	−4
30. 0	. 4886 5086			
$Q=+0.5$; tabular spacing=0.02				
0. 00	0. 0000 0000	2 1482	−36	−73
. 02	. 0001 0741	2 1446	−110	−73
. 04	. 0004 2928	2 1336	−181	−71
. 06	. 0009 6452	2 1156	−251	−70
. 08	. 0017 1131	2 0905	−318	−68
. 10	. 0026 6716	2 0586	−383	−64
. 12	. 0038 2886	2 0204	−444	−61
. 14	. 0051 9261	1 9760	−500	−57
. 16	. 0067 5396	1 9260	−553	−52
. 18	. 0085 0791	1 8707	−600	−47
. 20	. 0104 4892	1 8107	−642	−42
. 22	. 0125 7102	1 7465	−680	−37
. 24	. 0148 6776	1 6785	−711	−32
. 26	. 0173 3235	1 6074	−737	−26
. 28	. 0199 5768	1 5337	−758	−21
. 30	. 0227 3638	1 4578	−774	−15
. 32	. 0256 6087	1 3805	−785	−11
. 34	. 0287 2340	1 3020	−790	−6
. 36	. 0319 1613	1 2229	−791	−1
. 38	. 0352 3115	1 1438	−788	+3
. 40	. 0386 6056	1 0650	−781	7
. 42	. 0421 9645	9868	−771	10
. 44	. 0458 3103	9097	−757	14
. 46	. 0495 5659	8340	−740	17
. 48	. 0533 6554	7600	−721	19
. 50	. 0572 5050	6880	−700	21
. 52	. 0612 0426	6180	−676	23
. 54	. 0652 1981	5504	−651	25
. 56	. 0692 9041	4852	−625	26
. 58	. 0734 0952	4227	−599	26
. 60	. 0775 7091	3628	−571	28
. 62	. 0817 6858	3057	−544	28
. 64	. 0859 9681	2513	−516	28
. 66	. 0902 5018	1997	−488	28
. 68	. 0945 2351	1509	−460	28
. 70	. 0988 1194	1049	−433	27
. 72	. 1031 1085	615	−406	27
. 74	. 1074 1591	+209	−381	26
. 76	. 1117 2306	−172	−355	26
. 78	. 1160 2849	−527	−331	24
. 80	. 1203 2865	−858	−307	24
. 82	. 1246 2023	−1165	−285	23
. 84	. 1289 0015	−1450	−263	22
. 86	. 1331 6558	−1713	−242	21
. 88	. 1374 1387	−1955	−222	20

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_1	Δ^3_1	Δ^4_2
$Q=+0.5$; tabular spacing=0.02—Continued				
0.90	0.1416 4263	−2177	−204	19
.92	.1458 4961	−2380	−185	18
.94	.1500 3280	−2566	−168	17
.96	.1541 9032	−2734	−153	16
.98	.1583 2051	−2887	−138	15
1.00	.1624 2183	−3024	−124	14
1.02	.1664 9291	−3148	−110	13
1.04	.1705 3250	−3258	−98	12
1.06	.1745 3952	−3356	−86	12
1.08	.1785 1298	−3442	−75	11
1.10	.1824 5202	−3517	−65	10
1.12	.1863 5588	−3583	−56	9
1.14	.1902 2392	−3639	−47	9
1.16	.1940 5557	−3686	−39	8
1.18	.1978 5036	−3725	−32	7
1.20	.2016 0791	−3756	−25	7
1.22	.2053 2790	−3781	−18	6
1.24	.2090 1008	−3799	−12	6
1.26	.2126 5427	−3811	−7	5
1.28	.2162 6034	−3818	−2	5
1.30	.2198 2824	−3820	+3	5
1.32	.2233 5793	−3817	7	4
1.34	.2268 4945	−3810	11	4
1.36	.2303 0287	−3800	14	4
1.38	.2337 1829	−3786	17	3
1.40	.2370 9585	−3768	20	3
1.42	.2404 3572	−3748	23	3
1.44	.2437 3812	−3725	25	2
1.46	.2470 0326	−3700	27	2
1.48	.2502 3140	−3673	29	2
1.50	.2534 2280	−3644	31	2
1.52	.2565 7777	−3613	32	2
1.54	.2596 9660	−3581	34	1
1.56	.2627 7963	−3547	34	1
1.58	.2658 2718	−3513	36	2
1.60	.2688 3960			
$Q=+0.5$; tabular spacing=0.05				
1.60	0.2688 3960	−2 1729	580	34
1.65	.2762 1934	−2 1149	603	23
1.70	.2833 8759	−2 0546	616	13
1.75	.2903 5037	−1 9930	622	+6
1.80	.2971 1386	−1 9309	621	0
1.85	.3036 8425	−1 8688	616	−5
1.90	.3100 6777	−1 8071	608	−9
1.95	.3162 7058	−1 7463	596	−12
2.00	.3222 9875	−1 6867	582	−14
2.05	.3281 5826	−1 6285	567	−15
2.10	.3338 5491	−1 5718	551	−16
2.15	.3393 9439	−1 5167	533	−17
2.20	.3447 8220	−1 4634	516	−17
2.25	.3500 2367	−1 4118	498	−18
2.30	.3551 2397	−1 3620	480	−18
2.35	.3600 8806	−1 3140	463	−18
2.40	.3649 2076	−1 2677	445	−18
2.45	.3696 2669	−1 2232	428	−17
2.50	.3742 1030	−1 1804	412	−17
2.55	.3786 7587	−1 1392	395	−16
2.60	.3830 2752	−1 0997	380	−16
2.65	.3872 6920	−1 0618	364	−15
2.70	.3914 0470	−1 0253	350	−14

TABLE 38.—*Modified potentinl*—Continued

a	$-W$	Δ^2_1	Δ^3_1	Δ^4_2
$Q=+0.5$; tabular spacing=0.05—Continued				
2.75	0.3954 3768	−9903	336	−14
2.80	.3993 7162	−9568	322	−13
2.85	.4032 0989	−9245	309	−13
2.90	.4069 5571	−8936	297	−13
2.95	.4106 1217	−8639	285	−12
3.00	.4141 8224			
$Q=+0.5$; tabular spacing=0.1				
3.0	0.4141 8224	−3 3428	2145	−184
3.1	.4210 7449	−3 1283	1977	−168
3.2	.4276 5391	−2 9306	1824	−153
3.3	.4339 4028	−2 7482	1684	−140
3.4	.4399 5184	−2 5798	1556	−128
3.5	.4457 0541	−2 4242	1439	−117
3.6	.4512 1657	−2 2803	1332	−107
3.7	.4564 9969	−2 1471	1234	−97
3.8	.4615 6810	−2 0237	1145	−89
3.9	.4664 3414	−1 9092	1063	−82
4.0	.4711 0926	−1 8029	988	−75
4.1	.4756 0408	−1 7041	920	−68
4.2	.4799 2849	−1 6121	856	−63
4.3	.4840 9169	−1 5265	798	−58
4.4	.4881 0224	−1 4466	745	−53
4.5	.4919.6813	−1 3721	697	−49
4.6	.4956 9680	−1 3025	651	−46
4.7	.4992 9523	−1 2374	609	−42
4.8	.5027 6992	−1 1764	571	−38
4.9	.5061 2697	−1 1193	536	−35
5.0	.5093 7208	−1 0658	503	−33
5.1	.5125 1062	−1 0155	472	−31
5.2	.5155 4762	−9683	444	−28
5.3	.5184 8778	−9239	418	−26
5.4	.5213 3556	−8821	394	−24
5.5	.5240 9512	−8427	371	−23
5.6	.5267 7042	−8057	350	−21
5.7	.5293 6514	−7707	330	−20
5.8	.5318 8280	−7376	312	−18
5.9	.5343 2670	−7064	295	−17
6.0	.5366 9994	−6770	279	−16
6.1	.5390 0550	−6490	265	−15
6.2	.5412 4615	−6226	250	−15
6.3	.5434 2454	−5976	237	−13
6.4	.5455 4316	−5739	226	−12
6.5	.5476 0440	−5513	214	−12
6.6	.5496 1050	−5300	202	−11
6.7	.5515 6361	−5097	193	−10
6.8	.5534 6575	−4904	184	−9
6.9	.5553 1884	−4721	175	−9
7.0	.5571 2472	−4546	166	−9
7.1	.5588 8515	−4380	158	−8
7.2	.5606 0177	−4222	151	−7
7.3	.5622 7618	−4071	144	−7
7.4	.5639 0988	−3927	137	−7
7.5	.5655 0430	−3790	131	−6
7.6	.5670 6083	−3659	125	−5
7.7	.5685 8077	−3534	120	−6
7.8	.5700 6538	−3414	114	−5
7.9	.5715 1585	−3300	109	−5
8.0	.5729 3332	−3190	105	−5
8.1	.5743 1888	−3086	100	−5

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=+0.5: tabular spacing=0.1—Continued				
8.2	0.5756 7359	−2986	96	−4
8.3	.5769 9844	−2890	92	−4
8.4	.5782 9440	−2798	88	−4
8.5	.5795 6238	−2710	84	−4
8.6	.5808 0327	−2625	81	−3
8.7	.5820 1790	−2544	78	−4
8.8	.5832 0710	−2466	75	−3
8.9	.5843 7163	−2392	72	−3
9.0	.5855 1225	−2320	69	−3
9.1	.5866 2967	−2251	67	−2
9.2	.5877 2458	−2184	64	−3
9.3	.5887 9765	−2121	61	−2
9.4	.5898 4951	−2060	59	−2
9.5	.5908 8078	−2001	57	−2
9.6	.5918 9204	−1944	55	−2
9.7	.5928 8386	−1889	53	−2
9.8	.5938 5680	−1836	51	−2
9.9	.5948 1137	−1786	49	−2
10.0	.5957 4809			
Q=+0.5; tabular spacing=0.2				
10.0	0.5957 4809	−6948	371	−28
10.2	.5975 6990	−6577	345	−25
10.4	.5993 2593	−6232	322	−24
10.6	.6010 1965	−5910	300	−22
10.8	.6026 5426	−5610	280	−20
11.0	.6042 3276	−5330	262	−18
11.2	.6057 5797	−5068	245	−17
11.4	.6072 3250	−4822	230	−15
11.6	.6086 5880	−4592	216	−14
11.8	.6100 3919	−4376	202	−13
12.0	.6113 7581	−4174	190	−12
12.2	.6126 7069	−3984	180	−11
12.4	.6139 2574	−3804	169	−10
12.6	.6151 4274	−3636	159	−10
12.8	.6163 2338	−3477	150	−9
13.0	.6174 6925	−3327	141	−8
13.2	.6185 8185	−3186	134	−8
13.4	.6196 6259	−3052	126	−7
13.6	.6207 1281	−2926	120	−6
13.8	.6217 3376	−2807	113	−6
14.0	.6227 2666	−2694	107	−6
14.2	.6236 9261	−2586	102	−6
14.4	.6246 3270	−2485	96	−5
14.6	.6255 4794	−2388	92	−5
14.8	.6264 3930	−2297	87	−4
15.0	.6273 0769	−2210	82	−4
15.2	.6281 5398	−2127	78	−4
15.4	.6289 7900	−2049	75	−4
15.6	.6297 8353	−1974	71	−4
15.8	.6305 6832	−1903	68	−4
16.0	.6313 3408			
Q=+0.5; tabular spacing=0.5				
16.0	0.6313 3408	−1 1478	977	−124
16.5	.6331 6960	−1 0500	870	−108
17.0	.6349 0011	−9630	777	−93
17.5	.6365 3433	−8854	696	−80
18.0	.6380 8000	−8157	625	−71

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=+0.5; tabular spacing=0.5—Continued				
18.5	0.6395 4410	−7532	563	−62
19.0	.6409 3288	−6969	509	−54
19.5	.6422 5198	−6460	461	−48
20.0	.6435 0647	−5999	418	−43
20.5	.6447 0096	−5581	380	−38
21.0	.6458 3964	−5201	347	−34
21.5	.6469 2632	−4854	317	−30
22.0	.6479 6445	−4538	290	−27
22.5	.6489 5720	−4248	265	−25
23.0	.6499 0748	−3982	244	−21
23.5	.6508 1794	−3738	225	−19
24.0	.6516 9101	−3513	207	−18
24.5	.6525 2896	−3306	191	−16
25.0	.6533 3384	−3115	177	−14
25.5	.6541 0757	−2938	164	−13
26.0	.6548 5191	−2775	152	−12
26.5	.6555 6851	−2623	141	−11
27.0	.6562 5888	−2482	132	−9
27.5	.6569 2444	−2351	122	−10
28.0	.6575 6648	−2229	114	−8
28.5	.6581 8623	−2115	106	−7
29.0	.6587 8484	−2009	99	−7
29.5	.6593 6335	−1910	93	−6
30.0	.6599 2276			
Q=+0.6; tabular spacing=0.02				
0.00	0.0000 0000	2 6233	−44	−88
.02	.0001 3116	2 6189	−132	−88
.04	.0005 2422	2 6057	−218	−86
.06	.0011 7784	2 5839	−302	−84
.08	.0020 8985	2.5537	−384	−82
.10	.0032 5724	2.5154	−461	−77
.12	.0046 7616	2.4693	−534	−74
.14	.0063 4201	2 4158	−603	−68
.16	.0082 4945	2 3556	−666	−63
.18	.0103 9244	2 2890	−723	−57
.20	.0127 6432	2 2167	−774	−51
.22	.0153 5788	2 1393	−819	−45
.24	.0181 6536	2 0574	−857	−38
.26	.0211 7858	1 9717	−889	−32
.28	.0243 8897	1.8828	−914	−26
.30	.0277 8764	1 7914	−933	−19
.32	.0313 6544	1 6980	−946	−13
.34	.0351 1304	1 6034	−954	−7
.36	.0390 2098	1 5080	−955	−1
.38	.0430 7973	1 4126	−952	+3
.40	.0472 7973	1 3174	−943	8
.42	.0516 1147	1 2231	−931	12
.44	.0560 6552	1 1300	−914	17
.46	.0606 3256	1 0386	−894	20
.48	.0653 0346	9492	−871	23
.50	.0700 6928	8620	−846	26
.52	.0749 2130	7774	−818	28
.54	.0798 5106	6956	−788	30
.56	.0848 5039	6168	−758	31
.58	.0899 1140	5411	−725	32
.60	.0950 2651	4685	−693	33
.62	.1001 8848	3993	−660	33
.64	.1053 9037	3333	−626	34
.66	.1106 2560	2707	−593	33
.68	.1158 8789	2114	−560	33

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.6$; tabular spacing=0.02—Continued				
0. 70	0. 1211 7132	1554	−527	32
. 72	. 1264 7030	1027	−495	32
. 74	. 1317 7955	532	−464	31
. 76	. 1370 9412	+68	−434	31
. 78	. 1424 0937	−366	−404	29
. 80	. 1477 2096	−770	−376	28
. 82	. 1530 2486	−1146	−349	27
. 84	. 1583 1729	−1494	−323	26
. 86	. 1635 9478	−1817	−297	25
. 88	. 1688 5410	−2114	−274	24
. 90	. 1740 9228	−2388	−251	23
. 92	. 1793 0657	−2639	−230	22
. 94	. 1844 9447	−2869	−209	20
. 96	. 1896 5368	−3078	−190	19
. 98	. 1947 8210	−3268	−172	18
1. 00	. 1998 7784	−3440	−155	17
1. 02	. 2049 3918	−3595	−139	16
1. 04	. 2099 6457	−3734	−124	15
1. 06	. 2149 5261	−3858	−110	14
1. 08	. 2199 0207	−3968	−97	13
1. 10	. 2248 1185	−4065	−85	12
1. 12	. 2296 8098	−4150	−73	12
1. 14	. 2345 0861	−4223	−63	10
1. 16	. 2392 9401	−4286	−53	10
1. 18	. 2440 3655	−4339	−44	10
1. 20	. 2487 3571	−4382	−35	8
1. 22	. 2533 9104	−4418	−28	7
1. 24	. 2580 0220	−4445	−20	7
1. 26	. 2625 6891	−4466	−14	7
1. 28	. 2670 9096	−4479	−8	6
1. 30	. 2715 6821	−4487	−2	6
1. 32	. 2760 0060	−4489	+3	5
1. 34	. 2803 8809	−4486	8	5
1. 36	. 2847 3072	−4478	12	5
1. 38	. 2890 2857	−4466	16	4
1. 40	. 2932 8176	−4450	19	4
1. 42	. 2974 9044	−4431	23	4
1. 44	. 3016 5482	−4408	26	3
1. 46	. 3057 7510	−4383	28	3
1. 48	. 3098 5157	−4354	31	2
1. 50	. 3138 8449	−4324	33	2
1. 52	. 3178 7417	−4291	35	2
1. 54	. 3218 2094	−4256	36	2
1. 56	. 3257 2515	−4220	38	1
1. 58	. 3295 8716	−4182	39	2
1. 60	. 3334 0734			
$Q=+0.6$; tabular spacing=0.05				
1. 60	0. 3334 0734	−2 5892	640	46
1. 65	. 3427 7741	−2 5252	671	31
1. 70	. 3518 9496	−2 4582	690	20
1. 75	. 3607 6670	−2 3891	701	11
1. 80	. 3693 9951	−2 3190	704	+3
1. 85	. 3778 0043	−2 2486	701	−3
1. 90	. 3859 7648	−2 1785	694	−7
1. 95	. 3939 3468	−2 1091	683	−11
2. 00	. 4016 8196	−2 0409	669	−14
2. 05	. 4092 2516	−1 9740	653	−15
2. 10	. 4165 7096	−1 9087	636	−17
2. 15	. 4237 2589	−1 8451	617	−18
2. 20	. 4306 9631	−1 7834	598	−19

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.6$; tabular spacing=0.05—Continued				
2. 25	0. 4374 8839	−1 7235	579	−19
2. 30	. 4441 0813	−1 6656	559	−20
2. 35	. 4505 6130	−1 6097	540	−20
2. 40	. 4568 5351	−1 5557	521	−19
2. 45	. 4629 9015	−1 5036	502	−19
2. 50	. 4689 7643	−1 4534	483	−19
2. 55	. 4748 1736	−1 4052	465	−18
2. 60	. 4805 1778	−1 3586	447	−18
2. 65	. 4860 8234	−1 3139	430	−17
2. 70	. 4915 1550	−1 2709	414	−16
2. 75	. 4968 2157	−1 2295	398	−16
2. 80	. 5020 0468	−1 1898	382	−15
2. 85	. 5070 6882	−1 1515	368	−15
2. 90	. 5120 1781	−1 1148	353	−14
2. 95	. 5168 5532	−1 0794	340	−14
3. 00	. 5215 8488			
$Q=+0.6$; tabular spacirg=0.1				
3. 0	0. 5215 8488	−4 1832	2564	−210
3. 1	. 5307 3364	−3 9268	2371	−193
3. 2	. 5394 8972	−3 6897	2194	−177
3. 3	. 5478 7682	−3 4703	2032	−162
3. 4	. 5559 1690	−3 2671	1883	−149
3. 5	. 5636 3027	−3 0788	1746	−136
3. 6	. 5710 3575	−2 9042	1622	−125
3. 7	. 5781 5082	−2 7420	1507	−115
3. 8	. 5849 9168	−2 5914	1402	−105
3. 9	. 5915 7341	−2 4512	1306	−96
4. 0	. 5979 1003	−2 3206	1216	−90
4. 1	. 6040 1458	−2 1989	1135	−82
4. 2	. 6098 9925	−2 0854	1061	−74
4. 3	. 6155 7537	−1 9794	991	−70
4. 4	. 6210 5355	−1 8802	927	−64
4. 5	. 6263 4371	−1 7875	868	−59
4. 6	. 6314 5512	−1 7007	815	−53
4. 7	. 6363 9646	−1 6192	765	−50
4. 8	. 6411 7588	−1 5428	717	−47
4. 9	. 6458 0102	−1 4710	674	−43
5. 0	. 6502 7906	−1 4036	635	−39
5. 1	. 6546 1674	−1 3401	598	−37
5. 2	. 6588 2041	−1 2803	563	−35
5. 3	. 6628 9605	−1 2240	531	−32
5. 4	. 6668 4929	−1 1709	501	−30
5. 5	. 6706 8544	−1 1208	474	−28
5. 6	. 6744 0951	−1 0734	448	−26
5. 7	. 6780 2624	−1 0287	424	−24
5. 8	. 6815 4011	−9863	401	−22
5. 9	. 6849 5534	−9462	380	−21
6. 0	. 6882 7595	−9082	360	−20
6. 1	. 6915 0573	−8723	342	−18
6. 2	. 6946 4829	−8381	324	−17
6. 3	. 6977 0703	−8057	308	−16
6. 4	. 7006 8521	−7750	293	−15
6. 5	. 7035 8589	−7457	278	−14
6. 6	. 7064 1200	−7179	265	−13
6. 7	. 7091 6632	−6914	252	−13
6. 8	. 7118 5151	−6662	240	−12
6. 9	. 7144 7008	−6422	229	−11
7. 0	. 7170 2443	−6193	218	−11
7. 1	. 7195 1685	−5975	209	−9

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.6$; tabular spacing=0.1—Continued				
7.2	0.7219 4952	−5766	199	−10
7.3	.7243 2453	−5568	190	−9
7.4	.7266 4386	−5378	182	−8
7.5	.7289 0941	−5196	174	−8
7.6	.7311 2300	−5023	166	−8
7.7	.7332 8635	−4857	159	−7
7.8	.7354 0114	−4698	152	−7
7.9	.7374 6894	−4546	145	−7
8.0	.7394 9127	−4401	140	−6
8.1	.7414 6960	−4261	134	−6
8.2	.7434 0532	−4128	128	−6
8.3	.7452 9976	−3999	123	−5
8.4	.7471 5422	−3876	118	−5
8.5	.7489 6990	−3758	113	−5
8.6	.7507 4802	−3644	109	−4
8.7	.7524 8969	−3536	105	−4
8.8	.7541 9600	−3431	101	−4
8.9	.7558 6801	−3330	97	−4
9.0	.7575 0671	−3233	93	−4
9.1	.7591 1309	−3140	90	−4
9.2	.7606 8806	−3050	86	−3
9.3	.7622 3253	−2964	83	−3
9.4	.7637 4736	−2881	80	−3
9.5	.7652 3338	−2801	77	−3
9.6	.7666 9138	−2724	74	−3
9.7	.7681 2216	−2649	72	−2
9.8	.7695 2643	−2578	69	−3
9.9	.7709 0494	−2508	67	−2
10.0	.7722 5835			
$Q=+0.6$; tabular spacing=0.2				
10.0	0.7722 5835	−9768	507	−38
10.2	.7748 9259	−9262	473	−34
10.4	.7774 3420	−8789	442	−31
10.6	.7798 8792	−8348	413	−29
10.8	.7822 5817	−7935	386	−26
11.0	.7845 4907	−7549	362	−24
11.2	.7867 6448	−7187	339	−22
11.4	.7889 0802	−6848	319	−21
11.6	.7909 8308	−6529	299	−19
11.8	.7929 9286	−6230	281	−18
12.0	.7949 4034	−5948	265	−16
12.2	.7968 2833	−5684	250	−15
12.4	.7986 5948	−5434	235	−14
12.6	.8004 3630	−5199	222	−14
12.8	.8021 6113	−4977	210	−12
13.0	.8038 3619	−4767	198	−11
13.2	.8054 6358	−4569	188	−11
13.4	.8070 4527	−4382	177	−10
13.6	.8085 8315	−4204	168	−9
13.8	.8100 7899	−4036	159	−9
14.0	.8115 3447	−3877	151	−8
14.2	.8129 5118	−3726	144	−8
14.4	.8143 3064	−3582	136	−7
14.6	.8156 7427	−3446	129	−7
14.8	.8169 8345	−3317	123	−6
15.0	.8182 5945	−3194	117	−6
15.2	.8195 0352	−3076	111	−6

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.6$; tabular spacing=0.2—Continued				
15.4	0.8207 1682	−2965	106	−5
15.6	.8219 0048	−2859	101	−5
15.8	.8230 5555	−2757	97	−5
16.0	.8241 8304			
$Q=+0.6$; tabular spacing=0.5				
16.0	0.8241 8304	−1 6642	1393	−173
16.5	.8268 8737	−1 5249	1242	−151
17.0	.8294 3922	−1 4006	1112	−130
17.5	.8318 5099	−1 2894	999	−113
18.0	.8341 3383	−1 1895	899	−101
18.5	.8362 9771	−1 0996	810	−88
19.0	.8383 5164	−1 0186	734	−77
19.5	.8403 0370	−9452	665	−68
20.0	.8421 6123	−8787	605	−61
20.5	.8439 3090	−8182	551	−54
21.0	.8456 1874	−7632	503	−48
21.5	.8472 3027	−7129	460	−43
22.0	.8487 7051	−6669	421	−39
22.5	.8502 4406	−6248	386	−35
23.0	.8516 5512	−5862	356	−31
23.5	.8530 0758	−5506	328	−28
24.0	.8543 0498	−5178	302	−26
24.5	.8555 5059	−4876	279	−22
25.0	.8567 4745	−4597	259	−21
25.5	.8578 9833	−4338	239	−19
26.0	.8590 0584	−4099	222	−17
26.5	.8600 7236	−3877	206	−16
27.0	.8611 0011	−3670	192	−14
27.5	.8620 9116	−3478	179	−14
28.0	.8630 4744	−3299	167	−12
28.5	.8639 7072	−3132	156	−11
29.0	.8648 6269	−2976	146	−10
29.5	.8657 2490	−2830	137	−9
30.0	.8665 5881			
$Q=+0.7$; tabular spacing=0.02				
0.00	0.0000 0000	3 1194	−52	−103
.02	.0001 5597	3 1142	−154	−102
.04	.0006 2336	3 0988	−255	−101
.06	.0014 0064	3 0733	−354	−99
.08	.0024 8525	3 0379	−449	−95
.10	.0038 7365	2 9930	−540	−91
.12	.0055 6135	2 9390	−626	−86
.14	.0075 4296	2 8765	−706	−80
.16	.0098 1221	2 8058	−780	−74
.18	.0123 6204	2 7278	−847	−67
.20	.0151 8466	2 6431	−907	−60
.22	.0182 7159	2 5524	−960	−53
.24	.0216 1377	2 4565	−1004	−45
.26	.0252 0159	2 3560	−1042	−38
.28	.0290 2502	2 2518	−1072	−30
.30	.0330 7362	2 1446	−1094	−23
.32	.0373 3670	2 0352	−1110	−15
.34	.0418 0329	1 9242	−1118	−9
.36	.0464 6230	1 8124	−1121	−3
.38	.0513 0256	1 7003	−1116	+4

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.7$; tabular spacing=0.02—Continued				
0. 40	0. 0563 1284	1 5887	−1107	9
. 42	. 0614 8200	1 4780	−1093	14
. 44	. 0667 9895	1 3686	−1074	20
. 46	. 0722 5276	1 2613	−1050	23
. 48	. 0778 3269	1 1562	−1024	26
. 50	. 0835 2825	1 0538	−995	30
. 52	. 0893 2918	9543	−962	33
. 54	. 0952 2555	8581	−928	34
. 56	. 1012 0773	7653	−892	36
. 58	. 1072 6643	6761	−855	37
. 60	. 1133 9275	5906	−817	38
. 62	. 1195 7814	5090	−778	39
. 64	. 1258 1441	4312	−739	39
. 66	. 1320 9381	3573	−701	38
. 68	. 1384 0894	2872	−662	39
. 70	. 1447 5279	2210	−624	38
. 72	. 1511 1874	1587	−587	37
. 74	. 1575 0056	1000	−550	36
. 76	. 1638 9238	+450	−515	35
. 78	. 1702 8869	−65	−480	35
. 80	. 1766 8436	−546	−448	33
. 82	. 1830 7457	−993	−416	32
. 84	. 1894 5484	−1409	−385	31
. 86	. 1958 2103	−1794	−356	29
. 88	. 2021 6928	−2150	−328	28
. 90	. 2084 9602	−2478	−302	27
. 92	. 2147 9799	−2780	−277	25
. 94	. 2210 7215	−3056	−253	24
. 96	. 2273 1575	−3310	−230	23
. 98	. 2335 2625	−3540	−209	21
1. 00	. 2397 0136	−3749	−189	20
1. 02	. 2458 3898	−3938	−171	19
1. 04	. 2519 3722	−4109	−153	18
1. 06	. 2579 9438	−4261	−136	16
1. 08	. 2640 0892	−4398	−121	15
1. 10	. 2699 7948	−4519	−107	14
1. 12	. 2759 0486	−4626	−93	14
1. 14	. 2817 8398	−4719	−81	12
1. 16	. 2876 1591	−4800	−70	11
1. 18	. 2933 9985	−4869	−58	11
1. 20	. 2991 3509	−4927	−48	10
1. 22	. 3048 2106	−4976	−40	9
1. 24	. 3104 5728	−5015	−31	8
1. 26	. 3160 4334	−5046	−23	8
1. 28	. 3215 7893	−5069	−16	7
1. 30	. 3270 6383	−5085	−9	6
1. 32	. 3324 9788	−5094	−3	6
1. 34	. 3378 8100	−5097	+2	5
1. 36	. 3432 1313	−5095	8	6
1. 38	. 3484 9432	−5087	12	4
1. 40	. 3537 2464	−5074	16	4
1. 42	. 3589 0422	−5058	21	4
1. 44	. 3640 3322	−5037	24	3
1. 46	. 3691 1184	−5013	27	3
1. 48	. 3741 4033	−4986	30	3
1. 50	. 3791 1896	−4956	33	3
1. 52	. 3840 4803	−4923	35	2
1. 54	. 3889 2787	−4888	37	2
1. 56	. 3937 5883	−4851	39	2
1. 58	. 3985 4128	−4812	41	2
1. 60	. 4032 7562			

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.7$; tabular spacing=0.05				
1. 60	0. 4032 7562	−2 9815	673	58
1. 65	. 4149 0365	−2 9142	714	41
1. 70	. 4262 4026	−2 8428	741	27
1. 75	. 4372 9259	−2 7687	758	16
1. 80	. 4480 6804	−2 6930	765	+8
1. 85	. 4585 7420	−2 6165	766	0
1. 90	. 4688 1871	−2 5399	761	−5
1. 95	. 4788 0923	−2 4638	751	−10
2. 00	. 4885 5337	−2 3887	739	−13
2. 05	. 4980 5865	−2 3148	724	−15
2. 10	. 5073 3244	−2 2424	706	−17
2. 15	. 5163 8200	−2 1718	687	−19
2. 20	. 5252 1437	−2 1030	668	−20
2. 25	. 5338 3644	−2 0363	648	−20
2. 30	. 5422 5489	−1 9715	627	−21
2. 35	. 5504 7618	−1 9088	606	−21
2. 40	. 5585 0660			
$Q=+0.7$; tabular spacing=0.1				
2. 4	0. 5585 0660	−7 3944	4609	−327
2. 5	. 5740 1887	−6 9336	4292	−317
2. 6	. 5888 3779	−6 5044	3987	−304
2. 7	. 6030 0626	−6 1057	3700	−287
2. 8	. 6165 6417	−5 7357	3432	−268
2. 9	. 6295 4850	−5 3925	3183	−249
3. 0	. 6419 9358	−5 0742	2951	−232
3. 1	. 6539 3124	−4 7791	2739	−212
3. 2	. 6653 9098	−4 5053	2543	−196
3. 3	. 6764 0019	−4 2510	2361	−181
3. 4	. 6869 8431	−4 0149	2196	−165
3. 5	. 6971 6694	−3 7953	2043	−153
3. 6	. 7069 7004	−3 5909	1902	−141
3. 7	. 7164 1405	−3 4008	1774	−128
3. 8	. 7255 1798	−3 2234	1655	−119
3. 9	. 7342 9957	−3 0579	1545	−110
4. 0	. 7427 7537	−2 9035	1445	−100
4. 1	. 7509 6082	−2 7590	1352	−93
4. 2	. 7588 7037	−2 6238	1265	−87
4. 3	. 7665 1755	−2 4973	1186	−79
4. 4	. 7739 1500	−2 3786	1114	−73
4. 5	. 7810 7458	−2 2673	1046	−68
4. 6	. 7880 0744	−2 1627	983	−63
4. 7	. 7947 2403	−2 0644	925	−58
4. 8	. 8012 3418	−1 9719	870	−54
4. 9	. 8075 4714	−1 8849	821	−50
5. 0	. 8136 7161	−1. 8028	774	−47
5. 1	. 8196 1580	−1 7254	730	−44
5. 2	. 8253 8744	−1 6524	690	−40
5. 3	. 8309 9386	−1 5834	653	−37
5. 4	. 8364 4193	−1 5181	618	−35
5. 5	. 8417 3820	−1 4563	584	−33
5. 6	. 8468 8884	−1 3979	554	−31
5. 7	. 8518 9968	−1 3425	525	−29
5. 8	. 8567 7628	−1 2900	498	−27
5. 9	. 8615 2388	−1 2402	473	−25
6. 0	. 8661 4746	−1 1928	450	−24
6. 1	. 8706 5176	−1 1479	427	−23
5. 2	. 8750 4127	−1 1052	406	−21
6. 3	. 8793 2026	−1 0646	387	−19
6. 4	. 8834 9280	−1 0259	369	−18

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.7$; tabular spacing+0.1—Continued				
6. 5	0. 8875 6275	−9890	351	−18
6. 6	. 8915 3380	−9539	334	−16
6. 7	. 8954 0946	−9204	320	−15
6. 8	. 8991 9308	−8885	305	−15
6. 9	. 9028 8785	−8580	291	−14
7. 0	. 9064 9682	−8289	278	−12
7. 1	. 9100 2291	−8010	266	−12
7. 2	. 9134 6890	−7744	254	−12
7. 3	. 9168 3744	−7490	243	−11
7. 4	. 9201 3108	−7247	234	−10
7. 5	. 9233 5225	−7013	223	−10
7. 6	. 9265 0329	−6790	213	−10
7. 7	. 9295 8643	−6576	205	−8
7. 8	. 9326 0380	−6371	197	−8
7. 9	. 9355 5746	−6174	189	−8
8. 0	. 9384 4938	−5986	181	−8
8. 1	. 9412 8144	−5805	174	−7
8. 2	. 9440 5546	−5631	168	−7
8. 3	. 9467 7316	−5463	161	−7
8. 4	. 9494 3624	−5302	154	−7
8. 5	. 9520 4628	−5148	148	−6
8. 6	. 9546 0485	−5000	143	−5
8. 7	. 9571 1342	−4857	138	−6
8. 8	. 9595 7341	−4719	132	−6
8. 9	. 9619 8622	−4587	127	−5
9. 0	. 9643 5316	−4459	124	−4
9. 1	. 9666 7551	−4336	118	−5
9. 2	. 9689 5450	−4218	114	−4
9. 3	. 9711 9131	−4104	110	−3
9. 4	. 9733 8709	−3993	107	−4
9. 5	. 9755 4293	−3887	102	−4
9. 6	. 9776 5991	−3784	99	−4
9. 7	. 9797 3904	−3685	96	−3
9. 8	. 9817 8132	−3590	93	−3
9. 9	. 9837 8771	−3497	89	−3
10. 0	. 9857 5912			
$Q=+0.7$; tabular spacing=0.2				
10. 0	0. 9857 5912	−1 3635	679	−48
10. 2	. 9896 0057	−1 2956	635	−44
10. 4	. 9933 1246	−1 2321	595	−40
10. 6	. 9969 0115	−1 1726	557	−37
10. 8	1. 0003 7257	−1 1169	523	−34
11. 0	1. 0037 3231	−1 0646	491	−32
11. 2	1. 0069 8559	−1 0155	461	−30
11. 4	1. 0101 3731	−9694	434	−27
11. 6	1. 0131 9210	−9260	409	−26
11. 8	1. 0161 5429	−8851	385	−23
12. 0	1. 0190 2797	−8466	363	−22
12. 2	1. 0218 1700	−8102	343	−20
12. 4	1. 0245 2500	−7759	324	−19
12. 6	1. 0271 5541	−7435	306	−18
12. 8	1. 0297 1147	−7129	290	−16
13. 0	1. 0321 9625	−6838	274	−16
13. 2	1. 0346 1264	−6564	260	−14
13. 4	1. 0369 6339	−6304	247	−14
13. 6	1. 0392 5110	−6057	234	−12
13. 8	1. 0414 7825	−5823	222	−12
14. 0	1. 0436 4716	−5600	211	−11
14. 2	1. 0457 6008	−5389	201	−10
$Q=+0.7$; tabular spacing=0.2—Continued				
14. 4	1. 0478 1910	−5188	191	−10
14. 6	1. 0498 2625	−4997	182	−9
14. 8	1. 0517 8342	−4815	173	−9
15. 0	1. 0536 9244	−4642	165	−8
15. 2	1. 0555 5504	−4477	157	−8
15. 4	1. 0573 7287	−4319	150	−7
15. 6	1. 0591 4751	−4169	143	−7
15. 8	1. 0608 8045	−4026	137	−6
16. 0	1. 0625 7314	−3889	131	−6
16. 2	1. 0642 2694	−3758	125	−6
16. 4	1. 0658 4316	−3633	120	−6
16. 6	1. 0674 2305	−3513	114	−5
16. 8	1. 0689 6781	−3399	110	−5
17. 0	1. 0704 7858			
$Q=+0.7$; tabular spacing=0.5				
17. 0	1. 0704 7858	−2 0570	1591	−181
17. 5	1. 0741 1389	−1 8979	1432	−159
18. 0	1. 0775 5941	−1 7546	1293	−139
18. 5	1. 0808 2947	−1 6253	1170	−123
19. 0	1. 0839 3699	−1 5084	1061	−109
19. 5	1. 0868 9368	−1 4022	965	−96
20. 0	1. 0897 1015	−1 3058	879	−86
20. 5	1. 0923 9604	−1 2179	802	−77
21. 0	1. 0949 6013	−1 1377	734	−68
21. 5	1. 0974 1046	−1 0643	672	−61
22. 0	1. 0997 5435	−9971	617	−55
22. 5	1. 1019 9853	−9354	568	−50
23. 0	1. 1041 4918	−8786	523	−45
23. 5	1. 1062 1196	−8263	483	−40
24. 0	1. 1081 9212	−7780	446	−37
24. 5	1. 1100 9448	−7334	413	−33
25. 0	1. 1119 2350	−6922	382	−30
25. 5	1. 1136 8330	−6539	355	−28
26. 0	1. 1153 7771	−6184	330	−25
26. 5	1. 1170 1028	−5854	307	−23
27. 0	1. 1185 8430	−5547	286	−21
27. 5	1. 1201 0286	−5261	267	−20
28. 0	1. 1215 6881	−4994	249	−17
28. 5	1. 1229 8482	−4745	233	−16
29. 0	1. 1243 5337	−4512	218	−15
29. 5	1. 1256 7681	−4294	204	−14
30. 0	1. 1269 5731			
$Q=+0.8$; tabular spacing=0.02				
0. 00	0. 0000 0000	3 6414	−59	−119
. 02	. 0001 8207	3 6355	−177	−117
. 04	. 0007 2770	3 6178	−293	−116
. 06	. 0016 3510	3 5886	−406	−113
. 08	. 0029 0137	3 5480	−515	−109
. 10	. 0045 2243	3 4964	−620	−104
. 12	. 0064 9313	3 4345	−718	−99
. 14	. 0088 0728	3 3627	−810	−92
. 16	. 0114 5770	3 2816	−895	−85
. 18	. 0144 3629	3 1921	−972	−77
. 20	. 0177 3408	3 0949	−1041	−69
. 22	. 0213 4136	2 9908	−1102	−60
. 24	. 0252 4771	2 8806	−1154	−52
. 26	. 0294 4213	2 7652	−1197	−43
. 28	. 0339 1307	2 6456	−1231	−34

TABLE 38.—*Modified potential*—Continued

a	−*W*	Δ^2_{-1}	Δ^2_{-1}	Δ^4_{-2}
Q=+0.8; tabular spacing=0.02—Continued				
0. 30	0. 0386 4856	2 5224	−1257	−26
. 32	. 0436 3630	2 3967	−1275	−18
. 34	. 0488 6371	2 2692	−1285	−10
. 36	. 0543 1804	2 1407	−1288	−3
. 38	. 0599 8644	2 0118	−1284	+4
. 40	. 0658 5602	1 8834	−1274	10
. 42	. 0719 1395	1 7561	−1257	16
. 44	. 0781 4749	1 6303	−1236	21
. 46	. 0845 4406	1 5067	−1210	26
. 48	. 0910 9130	1 3858	−1179	30
. 50	. 0977 7712	1 2678	−1146	33
. 52	. 1045 8972	1 1532	−1109	37
. 54	. 1115 1764	1 0423	−1070	39
. 56	. 1185 4980	9353	−1029	41
. 58	. 1256 7547	8324	−987	42
. 60	. 1328 8439	7336	−943	44
. 62	. 1401 6666	6393	−899	44
. 64	. 1475 1287	5494	−855	44
. 66	. 1549 1401	4638	−811	44
. 68	. 1623 6154	3828	−767	44
. 70	. 1698 4734	3060	−724	44
. 72	. 1773 6375	2337	−681	42
. 74	. 1849 0352	1656	−640	41
. 76	. 1924 5986	1016	−599	40
. 78	. 2000 2635	+417	−560	39
. 80	. 2075 9701	−143	−522	38
. 82	. 2151 6624	−665	−486	36
. 84	. 2227 2881	−1151	−451	35
. 86	. 2302 7987	−1602	−418	33
. 88	. 2378 1491	−2020	−386	32
. 90	. 2453 2974	−2406	−356	30
. 92	. 2528 2052	−2761	−327	29
. 94	. 2602 8369	−3088	−299	27
. 96	. 2677 1598	−3387	−274	26
. 98	. 2751 1439	−3661	−249	24
1. 00	. 2824 7620	−3910	−226	23
1. 02	. 2897 9890	−4137	−205	22
1. 04	. 2970 8022	−4342	−185	20
1. 06	. 3043 1813	−4527	−166	19
1. 08	. 3115 1078	−4693	−148	18
1. 10	. 3186 5649	−4841	−132	17
1. 12	. 3257 5380	−4973	−116	15
1. 14	. 3328 0138	−5089	−102	14
1. 16	. 3397 9807	−5191	−88	14
1. 18	. 3467 4285	−5279	−76	12
1. 20	. 3536 3484	−5355	−65	11
1. 22	. 3604 7328	−5420	−54	11
1. 24	. 3672 5752	−5474	−44	10
1. 26	. 3739 8701	−5518	−35	9
1. 28	. 3806 6132	−5553	−27	8
1. 30	. 3872 8011	−5580	−19	8
1. 32	. 3938 4309	−5599	−12	7
1. 34	. 4003 5009	−5610	−5	7
1. 36	. 4068 0099	−5615	+1	6
1. 38	. 4131 9573	−5614	6	5
1. 40	. 4195 3433	−5608	12	5
1. 42	. 4258 1685	−5597	16	4
1. 44	. 4320 4340	−5581	20	4
1. 46	. 4382 1414	−5561	24	4
1. 48	. 4443 2927	−5537	27	3

TABLE 38.—*Modified potential*—Continued

a	−*W*	Δ^2_{-1}	Δ^2_{-1}	Δ^4_{-2}
Q=+0.8; tabular spacing=0.02—Continued				
1. 50	0. 4503 8904	−5509	30	3
1. 52	. 4563 9372	−5479	33	3
1. 54	. 4623 4361	−5445	36	3
1. 56	. 4682 3905	−5409	38	2
1. 58	. 4740 8039	−5371	40	3
1. 60	. 4798 6802			
Q=+0.8; tabular spacing=0.05				
1. 60	0. 4798 6802	−3 3314	674	72
1. 65	. 4941 0479	−3 2640	726	52
1. 70	. 5080 1516	−3 1914	762	37
1. 75	. 5216 0639	−3 1152	787	24
1. 80	. 5348 8611	−3 0365	800	13
1. 85	. 5478 6218	−2 9565	805	+5
1. 90	. 5605 4259	−2 8760	804	−1
1. 95	. 5729 3541	−2 7956	798	−6
2. 00	. 5850 4866	−2 7158	787	−11
2. 05	. 5968 9034	−2 6371	773	−14
2. 10	. 6084 6831	−2 5598	757	−16
2. 15	. 6197 9029	−2 4841	739	−18
2. 20	. 6308 6386	−2 4102	720	−19
2. 25	. 6416 9642	−2 3382	700	−20
2. 30	. 6522 9515	−2 2682	680	−20
2. 35	. 6626 6706	−2 2002	658	−21
2. 40	. 6728 1895	−2 1344	638	−21
2. 45	. 6827 5740	−2 0706	617	−21
2. 50	. 6924 8878			
Q=+0.8; tabular spacing=0.1				
2. 5	0. 6924 8878	−8 0379	4691	−327
2. 6	. 7113 5482	−7 5689	4375	−316
2. 7	. 7294 6397	−7 1314	4074	−301
2. 8	. 7468 5999	−6 7239	3792	−282
2. 9	. 7635 8361	−6 3447	3529	−264
3. 0	. 7796 7277	−5 9918	3284	−245
3. 1	. 7951 6274	−5 6635	3057	−227
3. 2	. 8100 8636	−5 3578	2847	−210
3. 3	. 8244 7420	−5 0731	2653	−194
3. 4	. 8383 5473	−4 8079	2474	−179
3. 5	. 8517 5447	−4 5605	2309	−165
3. 6	. 8646 9816	−4 3296	2156	−153
3. 7	. 8772 0890	−4 1139	2016	−140
3. 8	. 8893 0824	−3 9123	1887	−129
3. 9	. 9010 1636	−3 7236	1767	−120
4. 0	. 9123 5211	−3 5469	1657	−110
4. 1	. 9233 3318	−3 3812	1555	−102
4. 2	. 9339 7614	−3 2256	1461	−95
4. 3	. 9442 9653	−3 0796	1373	−88
4. 4	. 9543 0897	−2 9423	1292	−81
4. 5	. 9640 2718	−2 8131	1217	−75
4. 6	. 9734 6408	−2 6914	1147	−70
4. 7	. 9826 3184	−2 5767	1082	−65
4. 8	. 9915 4193	−2 4685	1022	−61
4. 9	1. 0002 0517	−2 3663	965	−56
5. 0	1. 0086 3178	−2 2698	913	−52
5. 1	1. 0168 3141	−2 1785	864	−49
5. 2	1. 0248 1320	−2 0920	819	−46
5. 3	1. 0325 8578	−2 0102	776	−43
5. 4	1. 0401 5734	−1 9326	736	−39

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+0.8$; tabular spacing=0.1—Continued				
5. 5	1. 0475 3564	−1 8590	699	−37
5. 6	1. 0547 2804	−1 7890	663	−36
5. 7	1. 0617 4154	−1 7227	630	−33
5. 8	1. 0685 8277	−1 6596	600	−30
5. 9	1. 0752 5803	−1 5996	571	−29
6. 0	1. 0817 7333	−1 5425	544	−27
6. 1	1. 0881 3438	−1 4881	519	−26
6. 2	1. 0943 4663	−1 4362	494	−24
6. 3	1. 1004 1525	−1 3868	472	−23
6. 4	1. 1063 4519	−1 3396	450	−21
6. 5	1. 1121 4117	−1 2946	430	−20
6. 6	1. 1178 0770	−1 2516	411	−19
6. 7	1. 1233 4907	−1 2104	393	−18
6. 8	1. 1287 6939	−1 1711	376	−16
6. 9	1. 1340 7261	−1 1335	360	−16
7. 0	1. 1392 6247	−1 0975	345	−16
7. 1	1. 1443 4259	−1 0630	330	−15
7. 2	1. 1493 1640	−1 0300	317	−13
7. 3	1. 1541 8722	−9983	304	−13
7. 4	1. 1589 5821	−9679	292	−12
7. 5	1. 1636 3241	−9387	280	−12
7. 6	1. 1682 1273	−9108	268	−12
7. 7	1. 1727 0198	−8839	258	−10
7. 8	1. 1771 0283	−8581	248	−10
7. 9	1. 1814 1787	−8333	239	−10
8. 0	1. 1856 4958	−8094	230	−9
8. 1	1. 1898 0036	−7864	220	−9
8. 2	1. 1938 7249	−7644	213	−8
8. 3	1. 1978 6818	−7432	205	−8
8. 4	1. 2017 8955	−7226	197	−8
8. 5	1. 2056 3866	−7029	190	−7
8. 6	1. 2094 1747	−6839	183	−7
8. 7	1. 2131 2790	−6656	176	−7
8. 8	1. 2167 7177	−6479	170	−6
8. 9	1. 2203 5085	−6309	164	−6
9. 0	1. 2238 6685	−6144	159	−5
9. 1	1. 2273 2140	−5985	154	−5
9. 2	1. 2307 1610	−5831	148	−6
9. 3	1. 2340 5249	−5684	143	−5
9. 4	1. 2373 3204	−5541	139	−4
9. 5	1. 2405 5618	−5402	134	−5
9. 6	1. 2437 2630	−5268	129	−4
9. 7	1. 2468 4374	−5139	125	−4
9. 8	1. 2499 0980	−5013	121	−4
9. 9	1. 2529 2572	−4892	117	−4
10. 0	1. 2558 9272			
$Q=+0.8$; tabular spacing=0.2				
10. 0	1. 2558 9272	−1 9104	894	−59
10. 2	1. 2616 8460	−1 8210	840	−55
10. 4	1. 2672 9438	−1 7370	788	−51
10. 6	1. 2727 3046	−1 6582	741	−47
10. 8	1. 2780 0072	−1 5841	698	−43
11. 0	1. 2831 1258	−1 5143	657	−41
11. 2	1. 2880 7300	−1 4486	620	−37
11. 4	1. 2928 8857	−1 3866	585	−35
11. 6	1. 2975 6548	−1 3281	553	−32
11. 8	1. 3021 0959	−1 2728	522	−30
$Q=+0.8$; tabular spacing=0.2—Continued				
12. 0	1. 3065 2642	−1 2206	494	−28
12. 2	1. 3108 2120	−1 1712	468	−26
12. 4	1. 3149 9885	−1 1244	443	−25
12. 6	1. 3190 6408	−1 0800	420	−23
12. 8	1. 3230 2130	−1 0380	399	−22
13. 0	1. 3268 7473	−9981	379	−20
13. 2	1. 3306 2834	−9602	360	−19
13. 4	1. 3342 8593	−9242	342	−18
13. 6	1. 3378 5110	−8900	326	−16
13. 8	1. 3413 2727	−8575	310	−16
14. 0	1. 3447 1769	−8265	295	−15
14. 2	1. 3480 2546	−7970	282	−14
14. 4	1. 3512 5354	−7688	268	−13
14. 6	1. 3544 0474	−7420	256	−12
14. 8	1. 3574 8174	−7164	244	−12
15. 0	1. 3604 8710	−6920	234	−11
15. 2	1. 3634 2326	−6686	223	−10
15. 4	1. 3662 9257	−6463	213	−10
15. 6	1. 3690 9725	−6250	204	−9
15. 8	1. 3718 3943	−6045	196	−9
16. 0	1. 3745 2116	−5850	187	−8
16. 2	1. 3771 4439	−5663	179	−8
16. 4	1. 3797 1099	−5484	172	−7
16. 6	1. 3822 2275	−5312	165	−7
16. 8	1. 3846 8140	−5147	158	−7
17. 0	1. 3870 8857	−4989	152	−6
17. 2	1. 3894 4585	−4838	146	−6
17. 4	1. 3917 5476	−4692	140	−6
17. 6	1. 3940 1674	−4552	134	−5
17. 8	1. 3962 3321	−4418	129	−5
18. 0	1. 3984 0550			
$Q=+0.8$; tabular spacing=0.5				
18. 0	1. 3984 0550	−2 6817	1887	−194
18. 5	1. 4036 5129	−2 4930	1715	−172
19. 0	1. 4086 4780	−2 3214	1563	−152
19. 5	1. 4134 1215	−2 1652	1427	−136
20. 0	1. 4179 5999	−2 0225	1304	−122
20. 5	1. 4223 0558	−1 8921	1195	−109
21. 0	1. 4264 6196	−1 7726	1098	−98
21. 5	1. 4304 4107	−1 6628	1009	−88
22. 0	1. 4342 5391	−1 5619	929	−80
22. 5	1. 4379 1055	−1 4690	858	−72
23. 0	1. 4414 2030	−1 3832	793	−65
23. 5	1. 4447 9172	−1 3039	734	−59
24. 0	1. 4480 3276	−1 2305	680	−54
24. 5	1. 4511 5075	−1 1625	631	−49
25. 0	1. 4541 5249	−1 0994	587	−45
25. 5	1. 4570 4429	−1 0407	546	−41
26. 0	1. 4598 3202	−9861	509	−37
26. 5	1. 4625 2113	−9353	475	−34
27. 0	1. 4651 1672	−8878	443	−32
27. 5	1. 4676 2352	−8435	414	−29
28. 0	1. 4700 4598	−8021	388	−26
28. 5	1. 4723 8823	−7633	363	−25
29. 0	1. 4746 5415	−7269	341	—22
29. 5	1. 4768 4738	−6928	320	—21
30. 0	1. 4789 7133			

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_1	Δ^3_1	Δ^4_2
$Q=+0.9$; tabular spacing=0.02				
0. 00	0. 0000 0000	4 1975	−67	−133
. 02	. 0002 0988	4 1909	−200	−133
. 04	. 0008 3884	4 1708	−331	−131
. 06	. 0018 8488	4 1378	−458	−128
. 08	. 0033 4471	4 0920	−582	−124
. 10	. 0052 1373	4 0337	−700	−118
. 12	. 0074 8612	3 9637	−811	−111
. 14	. 0101 5488	3 8826	−916	−104
. 16	. 0132 1190	3 7910	−1012	−96
. 18	. 0166 4803	3 6899	−1099	−87
. 20	. 0204 5314	3 5800	−1177	−78
. 22	. 0246 1625	3 4623	−1245	−68
. 24	. 0291 2559	3 3378	−1304	−59
. 26	. 0339 6870	3 2074	−1353	−49
. 28	. 0391 3255	3 0720	−1393	−40
. 30	. 0446 0360	2 9328	−1422	−29
. 32	. 0503 6793	2 7906	−1443	−21
. 34	. 0564 1131	2 6463	−1455	−12
. 36	. 0627 1933	2 5008	−1458	−4
. 38	. 0692 7743	2 3550	−1454	+4
. 40	. 0760 7104	2 2096	−1442	11
. 42	. 0830 8561	2 0654	−1425	18
. 44	. 0903 0672	1 9229	−1401	24
. 46	. 0977 2012	1 7829	−1372	29
. 48	. 1053 1181	1 6457	−1338	34
. 50	. 1130 6807	1 5120	−1300	37
. 52	. 1209 7553	1 3819	−1259	41
. 54	. 1290 2118	1 2560	−1216	44
. 56	. 1371 9243	1 1344	−1170	46
. 58	. 1454 7712	1 0175	−1122	47
. 60	. 1538 6355	9052	−1074	49
. 62	. 1623 4051	7979	−1024	50
. 64	. 1708 9725	6955	−974	50
. 66	. 1795 2354	5980	−925	50
. 68	. 1882 0963	5055	−875	50
. 70	. 1969 4627	4180	−827	48
. 72	. 2057 2471	3353	−779	48
. 74	. 2145 3667	2574	−732	47
. 76	. 2233 7438	1841	−687	46
. 78	. 2322 3049	1154	−643	44
. 80	. 2410 9815	+511	−601	42
. 82	. 2499 7092	−89	−560	41
. 84	. 2588 4280	−649	−520	39
. 86	. 2677 0818	−1169	−483	38
. 88	. 2765 6188	−1652	−447	36
. 90	. 2853 9906	−2099	−413	34
. 92	. 2942 1524	−2512	−380	33
. 94	. 3030 0632	−2892	−350	31
. 96	. 3117 6847	−3242	−321	29
. 98	. 3204 9820	−3562	−293	28
1. 00	. 3291 9232	−3855	−267	26
1. 02	. 3378 4788	−4123	−243	24
1. 04	. 3464 6221	−4366	−220	23
1. 06	. 3550 3289	−4586	−199	21
1. 08	. 3635 5771	−4785	−179	20
1. 10	. 3720 3468	−4964	−160	19
1. 12	. 3804 6202	−5124	−142	18
1. 14	. 3888 3812	−5266	−126	16
1. 16	. 3971 6157	−5392	−111	15
1. 18	. 4054 3108	−5504	−97	14
1. 20	. 4136 4556	−5600	−84	13
1. 22	. 4218 0404	−5684	−72	12

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_1	Δ^3_1	Δ^4_2
$Q=+0.9$; tabular spacing=0.02—Continued				
1. 24	0. 4299 0567	−5756	−60	11
1. 26	. 4379 4974	−5817	−50	10
1. 28	. 4459 3564	−5867	−40	10
1. 30	. 4538 6287	−5907	−32	9
1. 32	. 4617 3103	−5939	−23	8
1. 34	. 4695 3980	−5962	−16	8
1. 36	. 4772 8894	−5978	−9	7
1. 38	. 4849 7831	−5987	−2	6
1. 40	. 4926 0780	−5990	+3	6
1. 42	. 5001 7739	−5987	9	6
1. 44	. 5076 8712	−5978	14	5
1. 46	. 5151 3706	−5965	18	4
1. 48	. 5225 2736	−5947	22	4
1. 50	. 5298 5819	−5925	26	4
1. 52	. 5371 2977	−5899	29	3
1. 54	. 5443 4236	−5870	32	3
1. 56	. 5514 9625	−5838	35	3
1. 58	. 5585 9176	−5804	37	2
1. 60	. 5656 2922			
$Q=+0.9$; tabular spacing=0.05				
1. 60	0. 5656 2922	−3 6034	636	86
1. 65	. 5829 7148	−3 5398	701	65
1. 70	. 5999 5975	−3 4698	748	47
1. 75	. 6166 0105	−3 3950	780	32
1. 80	. 6329 0284	−3 3170	801	21
1. 85	. 6488 7294	−3 2368	812	11
1. 90	. 6645 1936	−3 1556	816	+4
1. 95	. 6798 5022	−3 0740	814	−2
2. 00	. 6948 7369	−2 9926	807	−7
2. 05	. 7095 9790	−2 9119	796	−11
2. 10	. 7240 3091	−2 8323	782	−14
2. 15	. 7381 8069	−2 7542	766	−16
2. 20	. 7520 5506	−2 6776	748	−18
2. 25	. 7656 6166	−2 6028	729	−19
2. 30	. 7790 0798	−2 5299	709	−20
2. 35	. 7921 0131	−2 4590	689	−20
2. 40	. 8049 4874			
$Q=+0.9$; tabular spacing=0.1				
2. 4	0. 8049 4874	−9 5623	5268	−326
2. 5	. 8299 3328	−9 0355	4943	−325
2. 6	. 8540 1426	−8 5412	4627	−316
2. 7	. 8772 4112	−8 0785	4325	−302
2. 8	. 8996 6013	−7 6460	4039	−286
2. 9	. 9213 1455	−7 2421	3770	−269
3. 0	. 9422 4476	−6 8651	3518	−251
3. 1	. 9624 8846	−6 5132	3286	−233
3. 2	. 9820 8084	−6 1847	3070	−216
3. 3	1. 0010 5475	−5 8777	2869	−201
3. 4	1. 0194 4089	−5 5908	2682	−186
3. 5	1. 0372 6795	−5 3226	2510	−172
3. 6	1. 0545 6275	−5 0716	2352	−158
3. 7	1. 0713 5039	−4 8363	2206	−147
3. 8	1. 0876 5439	−4 6158	2070	−136
3. 9	1. 1034 9682	−4 4088	1944	−126
4. 0	1. 1188 9836	−4 2145	1827	−116
4. 1	1. 1338 7846	−4 0317	1719	−108

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=+0.9; tabular spacing=0.1—Continued				
4.2	1.1484 5539	−3 8598	1619	−100
4.3	1.1626 4633	−3 6979	1527	−92
4.4	1.1764 6748	−3 5452	1441	−86
4.5	1.1899 3412	−3 4011	1360	−81
4.6	1.2030 6065	−3 2651	1285	−75
4.7	1.2158 6067	−3 1366	1216	−69
4.8	1.2283 4703	−3 0150	1152	−64
4.9	1.2405 3189	−2 8998	1091	−61
5.0	1.2524 2676	−2 7907	1034	−57
5.1	1.2640 4257	−2 6873	981	−53
5.2	1.2753 8966	−2 5892	932	−49
5.3	1.2864 7782	−2 4959	886	−46
5.4	1.2973 1640	−2 4073	843	−43
5.5	1.3079 1424	−2 3230	802	−41
5.6	1.3182 7979	−2 2428	764	−38
5.7	1.3284 2105	−2 1664	728	−36
5.8	1.3383 4568	−2 0936	694	−33
5.9	1.3480 6094	−2 0242	663	−32
6.0	1.3575 7377	−1 9580	633	−30
6.1	1.3668 9082	−1 8947	604	−28
6.2	1.3760 1839	−1 8342	578	−26
6.3	1.3849 6254	−1 7764	553	−25
6.4	1.3937 2905	−1 7212	529	−24
6.5	1.4023 2344	−1 6683	506	−22
6.6	1.4107 5100	−1 6176	486	−21
6.7	1.4190 1680	−1 5691	466	−20
6.8	1.4271 2569	−1 5225	446	−19
6.9	1.4350 8233	−1 4779	428	−18
7.0	1.4428 9118	−1 4351	411	−16
7.1	1.4505 5652	−1 3940	396	−16
7.2	1.4580 8246	−1 3544	379	−16
7.3	1.4654 7297	−1 3165	364	−15
7.4	1.4727 3182	−1 2800	351	−13
7.5	1.4798 6267	−1 2449	338	−13
7.6	1.4868 6903	−1 2111	325	−13
7.7	1.4937 5428	−1 1786	313	−12
7.8	1.5005 2166	−1 1473	302	−12
7.9	1.5071 7432	−1 1171	291	−11
8.0	1.5137 1526	−1 0881	280	−10
8.1	1.5201 4740	−1 0600	271	−10
8.2	1.5264 7354	−1 0329	261	−10
8.3	1.5326 9638	−1 0068	252	−9
8.4	1.5388 1853	−9817	243	−8
8.5	1.5448 4252	−9574	235	−8
8.6	1.5507 7077	−9338	227	−8
8.7	1.5566 0564	−9111	220	−8
8.8	1.5623 4940	−8892	212	−7
8.9	1.5680 0423	−8679	205	−7
9.0	1.5735 7227	−8474	198	−7
9.1	1.5790 5558	−8276	192	−6
9.2	1.5844 5612	−8084	187	−6
9.3	1.5897 7583	−7897	180	−6
9.4	1.5950 1657	−7717	174	−6
9.5	1.6001 8014	−7542	169	−5
9.6	1.6052 6829	−7373	164	−5
9.7	1.6102 8271	−7209	159	−5
9.8	1.6152 2504	−7050	154	−5
9.9	1.6200 9687	−6896	150	−4
10.0	1.6248 9974			

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=+0.9; tabular spacing=0.2				
10.0	1.6248 9974	−2 6988	1145	−70
10.2	1.6343 0457	−2 5843	1079	−66
10.4	1.6434 5096	−2 4764	1018	−61
10.6	1.6523 4970	−2 3746	961	−57
10.8	1.6610 1098	−2 2785	909	−52
11.0	1.6694 4441	−2 1876	860	−49
11.2	1.6776 5908	−2 1017	814	−46
11.4	1.6856 6358	−2 0203	771	−43
11.6	1.6934 6605	−1 9432	732	−39
11.8	1.7010 7420	−1 8700	694	−38
12.0	1.7084 9536	−1 8005	659	−35
12.2	1.7157 3647	−1 7346	627	−32
12.4	1.7228 0411	−1 6719	596	−31
12.6	1.7297 0457	−1 6123	568	−29
12.8	1.7364 4380	−1 5555	541	−27
13.0	1.7430 2747	−1 5014	515	−26
13.2	1.7494 6101	−1 4499	492	−24
13.4	1.7557 4956	−1 4007	469	−23
13.6	1.7618 9803	−1 3538	448	−21
13.8	1.7679 1112	−1 3090	428	−20
14.0	1.7737 9332	−1 2662	409	−19
14.2	1.7795 4889	−1 2253	392	−18
14.4	1.7851 8193	−1 1861	374	−17
14.6	1.7906 9635	−1 1487	359	−16
14.8	1.7960 9591	−1 1128	344	−15
15.0	1.8013 8419	−1 0784	330	−14
15.2	1.8065 6462	−1 0455	316	−14
15.4	1.8116 4050	−1 0139	303	−13
15.6	1.8166 1500	−9836	291	−12
15.8	1.8214 9113	−9545	279	−12
16.0	1.8262 7181	−9266	269	−11
16.2	1.8309 5984	−8997	258	−11
16.4	1.8355 5789	−8739	248	−10
16.6	1.8400 6856	−8491	239	−10
16.8	1.8444 9431	−8252	230	−9
17.0	1.8488 3754	−8023	221	−8
17.2	1.8531 0055	−7801	213	−8
17.4	1.8572 8554	−7588	205	−8
17.6	1.8613 9465	−7383	198	−8
17.8	1.8654 2992	−7185	191	−7
18.0	1.8693 9334	−6995	184	−7
18.2	1.8732 8682	−6811	178	−6
18.4	1.8771 1218	−6633	171	−6
18.6	1.8808 7121	−6462	165	−6
18.8	1.8845 6562	−6297	160	−6
19.0	1.8881 9706			
Q=+0.9; tabular spacing=0.5				
19.0	1.8881 9706	−3 8371	2352	−212
19.5	1.8970 1054	−3 6018	2163	−190
20.0	1.9054 6384	−3 3856	1992	−171
20.5	1.9135 7858	−3 1864	1837	−155
21.0	1.9213 7469	−3 0027	1698	−139
21.5	1.9288 7052	−2 8329	1572	−126
22.0	1.9360 8306	−2 6758	1457	−115
22.5	1.9430 2802	−2 5301	1352	−104
23.0	1.9497 1997	−2 3949	1257	−95
23.5	1.9561 7243	−2 2692	1170	−87

TABLE 38.—*Modified potential*—Continued

a	−W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=+0.9; tabular spacing=0.5—Continued				
24.0	1.9623 9796	−2 1521	1091	−79
24.5	1.9684 0829	−2 0430	1018	−73
25.0	1.9742 1431	−1 9412	952	−67
25.5	1.9798 2621	−1 8460	890	−62
26.0	1.9852 5351	−1 7570	834	−56
26.5	1.9905 0510	−1 6736	782	−52
27.0	1.9955 8934	−1 5954	734	−48
27.5	2.0005 1403	−1 5221	690	−44
28.0	2.0052 8651	−1 4531	649	−41
28.5	2.0099 1368	−1 3883	610	−38
29.0	2.0144 0202	−1 3272	575	−35
29.5	2.0187 5765	−1 2697	542	−33
30.0	2.0229 8630			
Q=+1.0; tabular spacing=0.02				
0.00	0.0000 0000	4 8070	−74	−149
.02	.0002 4035	4 7995	−223	−149
.04	.0009 6065	4 7772	−369	−146
.06	.0021 5868	4 7403	−512	−143
.08	.0038 3074	4 6892	−650	−138
.10	.0059 7171	4 6242	−781	−132
.12	.0085 7511	4 5461	−906	−124
.14	.0116 3311	4 4555	−1022	−116
.16	.0151 3667	4 3533	−1129	−107
.18	.0190 7555	4 2404	−1227	−97
.20	.0234 3847	4 1177	−1314	−88
.22	.0282 1316	3 9863	−1391	−77
.24	.0333 8648	3 8472	−1456	−66
.26	.0389 4453	3 7016	−1512	−55
.28	.0448 7273	3 5504	−1556	−44
.30	.0511 5598	3 3948	−1589	−33
.32	.0577 7870	3 2359	−1613	−24
.34	.0647 2502	3 0747	−1626	−14
.36	.0719 7881	2 9120	−1630	−4
.38	.0795 2380	2 7490	−1626	+4
.40	.0873 4370	2 5863	−1614	12
.42	.0954 2222	2 4249	−1594	20
.44	.1037 4324	2 2655	−1568	26
.46	.1122 9080	2 1086	−1536	32
.48	.1210 4923	1 9550	−1499	37
.50	.1300 0316	1 8051	−1458	41
.52	.1391 3760	1 6593	−1413	45
.54	.1484 3797	1 5180	−1364	49
.56	.1578 9014	1 3816	−1314	50
.58	.1674 8046	1 2502	−1261	52
.60	.1771 9582	1 1241	−1207	54
.62	.1870 2358	1 0034	−1153	54
.64	.1969 5169	8881	−1097	56
.66	.2069 6860	7784	−1042	55
.68	.2170 6336	6742	−988	54
.70	.2272 2553	5754	−934	54
.72	.2374 4524	4820	−881	52
.74	.2477 1315	3939	−829	52
.76	.2580 2045	3110	−779	50
.78	.2683 5884	2331	−730	48
.80	.2787 2054	1601	−683	48
.82	.2890 9824	918	−637	46
.84	.2994 8513	+281	−594	43
.86	.3098 7483	−313	−552	42
Q=+1.0; tabular spacing=0.02—Continued				
0.88	0.3202 6140	−865	−512	40
.90	.3306 3932	−1377	−474	38
.92	.3410 0347	−1851	−438	36
.94	.3513 4911	−2289	−404	34
.96	.3616 7186	−2693	−372	32
.98	.3719 6769	−3064	−341	31
1.00	.3822 3287	−3405	−312	29
1.02	.3924 6400	−3717	−285	27
1.04	.4026 5795	−4002	−260	25
1.06	.4128 1188	−4262	−236	24
1.08	.4229 2319	−4498	−213	22
1.10	.4329 8952	−4711	−192	21
1.12	.4430 0873	−4904	−173	20
1.14	.4529 7891	−5076	−155	18
1.16	.4628 9833	−5231	−138	17
1.18	.4727 6544	−5369	−122	16
1.20	.4825 7886	−5491	−107	15
1.22	.4923 3737	−5598	−93	14
1.24	.5020 3991	−5691	−81	12
1.26	.5116 8554	−5772	−69	12
1.28	.5212 7346	−5841	−58	11
1.30	.5308 0297	−5899	−48	10
1.32	.5402 7349	−5947	−39	9
1.34	.5496 8455	−5986	−30	9
1.36	.5590 3574	−6016	−22	8
1.38	.5683 2678	−6038	−15	7
1.40	.5775 5743	−6054	−8	7
1.42	.5867 2755	−6062	−2	6
1.44	.5958 3705	−6064	+3	5
1.46	.6048 8590	−6061	8	5
1.48	.6138 7414	−6053	13	5
1.50	.6228 0186	−6040	18	4
1.52	.6316 6917	−6022	21	4
1.54	.6404 7626	−6001	25	4
1.56	.6492 2334	−5977	28	3
1.58	.6579 1065	−5949	31	3
1.60	.6665 3848	−5918	34	3
1.62	.6751 0712	−5884	36	2
1.64	.6836 1693	−5848	38	2
1.66	.6920 6825	−5811	40	2
1.68	.7004 6146	−5771	42	2
1.70	.7087 9696	−5729	43	1
1.72	.7170 7518	−5686	44	2
1.74	.7252 9652	−5642	45	1
1.76	.7334 6146	−5597	47	2
1.78	.7415 7042	−5550	48	1
1.80	.7496 2389	−5502	48	0
1.82	.7576 2233	−5454	49	1
1.84	.7655 6623	−5406	49	0
1.86	.7734 5608	−5356	50	0
1.88	.7812 9236	−5307	50	0
1.90	.7890 7557	−5257	50	0
1.92	.7968 0622	−5206	50	0
1.94	.8044 8480	−5156	50	0
1.96	.8121 1183	−5105	51	0
1.98	.8196 8780	−5055	51	0
2.00	.8272 1322	−5004	50	0
2.02	.8346 8860	−4954	50	0
2.04	.8421 1445	−4903	50	0
2.06	.8494 9126	−4853	50	0
2.08	.8568 1954	−4803	50	0

TABLE 38.—*Modified potential*—Continued

a	−W	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
Q=+1.0; tabular spacing=0.02—Continued				
2. 10	0. 8640 9980	−4753	50	0
2. 12	. 8713 3251	−4704	49	0
2. 14	. 8785 1819	−4655	49	0
2. 16	. 8856 5733	−4606	49	0
2. 18	. 8927 5040	−4557	48	1
2. 20	. 8997 9791			
Q=+1.0; tabular spacing=0.1				
2. 2	0. 8997 9791	−11 2745	5860	−241
2. 3	. 9343 6857	−10 6885	5579	−281
2. 4	. 9678 7038	−10 1306	5277	−302
2. 5	1. 0003 5912	−9 6030	4969	−308
2. 6	1. 0318 8757	−9 1060	4666	−303
2. 7	1. 0625 0541	−8 6394	4373	−294
2. 8	1. 0922 5932	−8 2021	4095	−278
2. 9	1. 1211 9301	−7 7926	3831	−264
3. 0	1. 1493 4744	−7 4095	3584	−247
3. 1	1. 1767 6092	−7 0510	3354	−230
3. 2	1. 2034 6929	−6 7156	3139	−215
3. 3	1. 2295 0610	−6 4018	2940	−199
3. 4	1. 2549 0274	−6 1078	2755	−185
3. 5	1. 2796 8860	−5 8323	2583	−172
3. 6	1. 3038 9122	−5 5740	2424	−159
3. 7	1. 3275 3644	−5 3316	2277	−147
3. 8	1. 3506 4850	−5 1039	2140	−137
3. 9	1. 3732 5017	−4 8899	2014	−126
4. 0	1. 3953 6285	−4 6885	1897	−117
4. 1	1. 4170 0668	−4 4988	1788	−109
4. 2	1. 4382 0063	−4 3200	1687	−101
4. 3	1. 4589 6258	−4 1514	1593	−94
4. 4	1. 4793 0939	−3 9921	1505	−87
4. 5	1. 4992 5699	−3 8416	1424	−82
4. 6	1. 5188 2044	−3 6992	1348	−76
4. 7	1. 5380 1397	−3 5644	1278	−71
4. 8	1. 5568 5107	−3 4366	1211	−66
4. 9	1. 5753 4451	−3 3155	1150	−62
5. 0	1. 5935 0640	−3 2005	1092	−58
5. 1	1. 6113 4824	−3 0913	1038	−54
5. 2	1. 6288 8096	−2 9874	988	−51
5. 3	1. 6461 1493	−2 8887	940	−48
5. 4	1. 6630 6003	−2 7947	896	−44
5. 5	1. 6797 2567	−2 7051	854	−42
5. 6	1. 6961 2080	−2 6197	815	−40
5. 7	1. 7122 5396	−2 5382	778	−37
5. 8	1. 7281 3330	−2 4604	743	−35
5. 9	1. 7437 6660	−2 3861	710	−33
6. 0	1. 7591 6129	−2 3151	679	−31
6. 1	1. 7743 2447	−2 2471	650	−29
6. 2	1. 7892 6294	−2 1821	623	−28
6. 3	1. 8039 8320	−2 1198	597	−26
6. 4	1. 8184 9147	−2 0601	572	−25
6. 5	1. 8327 9374	−2 0029	549	−23
6. 6	1. 8468 9571	−1 9480	527	−22
6. 7	1. 8608 0288	−1 8954	506	−21
6. 8	1. 8745 2051	−1 8448	486	−20
6. 9	1. 8880 5366	−1 7961	467	−19
7. 0	1. 9014 0720	−1 7494	449	−18
7. 1	1. 9145 8580	−1 7045	433	−17
7. 2	1. 9275 9396	−1 6612	416	−16
7. 3	1. 9404 3598	−1 6196	401	−15
7. 4	1. 9531 1605	−1 5795	386	−15
Q=+1.0; tabular spacing=0.1—Continued				
7. 5	1. 9656 3817	−1 5409	373	−14
7. 6	1. 9780 0620	−1 5036	359	−14
7. 7	1. 9902 2388	−1 4677	347	−12
7. 8	2. 0022 9478	−1 4330	334	−12
7. 9	2. 0142 2238	−1 3996	323	−12
8. 0	2. 0260 1003	−1 3673	312	−11
8. 1	2. 0376 6094	−1 3361	301	−11
8. 2	2. 0491 7825	−1 3059	292	−10
8. 3	2. 0605. 6496	−1 2768	282	−10
8. 4	2. 0718 2400	−1 2486	273	−9
8. 5	2. 0829 5818	−1 2213	264	−9
8. 6	2. 0939 7022	−1 1949	256	−8
8. 7	2. 1048 6278	−1 1694	247	−8
8. 8	2. 1156 3839	−1 1446	240	−8
8. 9	2. 1262 9954	−1 1207	232	−8
9. 0	2. 1368 4862	−1 0974	225	−7
9. 1	2. 1472 8795	−1 0749	218	−7
9. 2	2. 1576 1980	−1 0531	212	−7
9 3	2. 1678 4633	−1 0319	206	−6
9. 4	2. 1779 6966	−1 0114	199	−6
9. 5	2. 1879 9186	−9914	194	−6
9. 6	2. 1979 1492	−9721	188	−6
9. 7	2. 2077 4076	−9533	183	−5
9. 8	2. 2174 7128	−9350	177	−5
9. 9	2. 2271 0829	−9173	172	−5
10. 0	2. 2366 5358			
Q=+1.0; tabular spacing=0.2				
10. 0	2. 2366 5358	−3 6006	1323	−77
10. 2	2. 2554 7581	−3 4683	1251	−72
10. 4	2. 2739 5121	−3 3432	1185	−66
10. 6	2. 2920 9230	−3 2246	1124	−62
10. 8	2. 3099 1092	−3 1123	1066	−58
11. 0	2. 3274 1830	−3 0057	1012	−54
11. 2	2. 3446 2512	−2 9045	962	−50
11. 4	2. 3615 4149	−2 8083	915	−47
11. 6	2. 3781 7703	−2 7168	871	−44
11. 8	2. 3945 4090	−2 6296	830	−41
12. 0	2. 4106 4180	−2 5466	792	−38
12. 2	2. 4264 8804	−2 4675	755	−36
12. 4	2. 4420 8754	−2 3920	721	−34
12. 6	2. 4574 4784	−2 3198	689	−32
12. 8	2. 4725 7616	−2 2509	659	−30
13. 0	2. 4874 7938	−2 1850	630	−29
13. 2	2. 5021 6410	−2 1220	604	−27
13. 4	2. 5166 3662	−2 0616	578	−26
13. 6	2. 5309 0298	−2 0038	554	−24
13. 8	2. 5449 6896	−1 9484	532	−22
14. 0	2. 5588 4010	−1 8952	510	−22
14. 2	2. 5725 2172	−1 8442	490	−20
14. 4	2. 5860 1891	−1 7952	471	−19
14. 6	2. 5993 3659	−1 7481	452	−18
14. 8	2. 6124 7945	−1 7029	435	−17
15. 0	2. 6254 5202	−1. 6594	419	−16
15. 2	2. 6382 5865	−1 6175	403	−16
15. 4	2. 6509 0353	−1 5772	388	−15
15. 6	2. 6633 9068	−1 5384	374	−14
15. 8	2. 6757 2399	−1 5010	360	−14
16. 0	2. 6879 0721	−1 4650	348	−12
16. 2	2. 6999 4392	−1 4302	336	−12

TABLE 38.—*Modified potential*—Continued

a	$-W$	Δ^2_{-1}	Δ^3_{-1}	Δ^4_{-2}
$Q=+1.0$; tabular spacing=0.2—Continued				
16. 4	2. 7118 3762	−1 3966	324	−12
16. 6	2. 7235 9166	−1 3642	313	−11
16. 8	2. 7352 0928	−1 3330	302	−11
17. 0	2. 7466 9359			
$Q=+1.0$; tabular spacing=0.5				
17. 0	2. 7466 9359	−8 1450	4452	−397
17. 5	2. 7748 4080	−7 6998	4097	−355
18. 0	2. 8022 1802	−7 2900	3779	−318
18. 5	2. 8288 6624	−6 9121	3493	−286
19. 0	2. 8548 2325	−6 5629	3234	−258
19. 5	2. 8801 2398	−6 2394	3002	−233
20. 0	2. 9048 0076	−5 9392	2790	−212
20. 5	2. 9288 8362	−5 6602	2598	−192
21. 0	2. 9524 0045	−5 4004	2424	−174
21. 5	2. 9753 7723	−5 1581	2264	−160
22. 0	2. 9978 3821	−4 9317	2118	−146
$Q=+1.0$; tabular spacing=0.5—Continued				
22. 5	3. 0198 0602	−4 7199	1985	−133
23. 0	3. 0413 0183	−4 5214	1862	−123
23. 5	3. 0623 4550	−4 3352	1749	−112
24. 0	3. 0829 5565	−4 1603	1646	−104
24. 5	3. 1031 4976	−3 9957	1550	−96
25. 0	3. 1229 4431	−3 8407	1461	−89
25. 5	3. 1423 5478	−3 6946	1380	−82
26. 0	3. 1613 9579	−3 5566	1304	−76
26. 5	3. 1800 8114	−3 4262	1234	−70
27. 0	3. 1984 2387	−3 3029	1168	−65
27. 5	3. 2164 3631	−3 1861	1107	−61
28. 0	3. 2341 3014	−3 0754	1050	−57
28. 5	3. 2515 1643	−2 9703	998	−53
29. 0	3. 2686 0569	−2 8706	948	−49
29. 5	3. 2854 0790	−2 7757	902	−46
30. 0	3. 3019 3253			

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