River Channel Bars and Dunes—Theory of Kinematic Waves

GEOLOGICAL SURVEY PROFESSIONAL PAPER 422-L

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By WALTER B. LANGBEIN *and* LUNA B. LEOPOLD

PHYSIOGRAPHIC AND HYDRAULIC STUDIES OF RIVERS

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CONTENTS

	Page
Abstract	L1
Introduction	1
Flux-concentration relations	2
Relation of particle speed to spacing—a flume experiment	4
Transport of sand in pipes and flumes	5
Flux-concentration curve for pipes	6
Flume transport of sand	7
Effect of rock spacing on rock movement in an ephemeral stream	L9
Waves in bed form	12
General features	12
Kinematic properties	15
Gravel bars	17
Summary	19
References	19

ILLUSTRATIONS

		Page
FIGURE	1. Flux-concentration curve for traffic	L3
	2. Sketch of flume	4
	3. Graph showing relation between speed of beads and linear concentration	5
	4, 5. Flux-concentration curves for—	
	4. Beads	5
	5. Transport of sand in a 1-inch pipe	6
	6. Graph showing relation between mean water velocity and mean particle speed	8
	7. Flux-concentration curve for transport of sand	9
	8. Location map showing Arroyo de los Frijoles near Santa Fe, N. Mex	10
	9, 10. Graphs showing—	
	9. Data on rock movement and percentage of rocks moved during two individual flows	11
	10. Representative plots of the effect of spacing on the percentage of rocks moved by different discharges	13
	11. Generalized diagram showing discharge required to move all rocks of given size as a function of spacing	14
	12. Graphs showing field data for rock movement in Arroyo de los Frijoles and data for bead experiments in a flume	15
	13. Sketch of tilting flume used to observe movement of beads at different spacings	16
	14–18. Graphs showing—	
	14. Variation in car density along a highway	16
	15. Generation of waves in a random model	17
	16. Length of sand waves in a flume in relation to mean water velocity, mean wave amplitude, and depth of water	18
	17. Suggested transport relations in tranquil flow	18
	18. Suggested transport relations in rapid flow	18
	19. Suggested flux-concentration curves for river gravel	19

TABLES

		Page
TABLE	1. Observations on 0.185-inch beads	L5
	2. Data from Blatch's experiments on transport of 0.6-millimeter sand in a 1-inch pipe	6
	3. Data on flume transport of 0.19-millimeter sand	8
	4. Observations in miniature flume of slopes at which 25 percent of beads moved	14

RIVER CHANNEL BARS AND DUNES—THEORY OF KINEMATIC WAVES

By WALTER B. LANGBEIN and LUNA B. LEOPOLD

ABSTRACT

A kinematic wave is a grouping cf moving objects in zones along a flow path and through which the objects pass. These concentrations may be characterized by a simple relation between the speed of the moving objects and their spacing as a result of interaction between them.

Vehicular traffic has long been known to have such properties. Data are introduced to show that beads carried by flowing water in a narrow flume behave in an analogous way. The flux or transport of objects in a single lane of traffic is greatest when the objects are spaced about two diameters apart; beads in a single-lane flume as well as highway traffic conform to this property.

By considering the sand in a pipe or flume to a depth affected by dune movement, it is shown that flux-concentration curves similar to the previously known cases can be constructed from experimental data. From the kinematic point of view, concentration of particles in dunes and other wave bed forms results when particles in transport become more numerous or closely spaced and interact to reduce the effectiveness of the ambient water to move them.

Field observations over a 5-year period are reported in which individual rocks were painted for identification and placed at various spacings on the bed of ephemeral stream in New Mexico, to study the effect of storm flows on rock movement. The data on about 14,000 rocks so observed show the effect of variable spacing which is quantitatively as well as qualitatively comparable to the spacing effect on small glass beads in a flume.

Dunes and gravel bars may be considered kinematic waves caused by particle interaction, and certain of their properties can be related to the characteristics of the flux-concentration curve.

INTRODUCTION

Natural river channels are neither smooth nor regular in form. As seen on a map or from an airplane, their sinuous curves convey only one aspect of their changing form. The streambed has a patterned or textured irregularity which is composed not only of the grains or cobbles making up the bed surface but also of wavelike undulations of larger magnitude. Ripples and dunes are characteristic of sand beds except for high flow. As for the majority of rivers which have beds composed of gravel of heterogeneous size, the alternation of deeps and shallows—what we have called pools and riffles—is ubiquitous. These forms result from the accumulation of gravel in bars spaced along the stream at distances equal to five to seven channel widths.

Each of these bed forms—ripples, dunes, and gravel bars—is composed of groups of noncoherent particles piled up in some characteristic manner.

As compared with the surface texture presented by the bed grains themselves, these bed forms provide a larger scale bed rugosity and account for an important part of the total roughness. The character of the bed and the overall pattern of sinuosity of the channel as well as the hydraulics of the system are influenced by the undulatory forms assumed by bed particles.

These accumulations of grains, however, are by no means composed always of the same particles, for at those stages of flow when bed grains are in motion, there is a continual trading of particles as some are swept away only to be replaced by others. The form of ripple, dune, or bar exists independently of the grains which compose them and may move upstream, downstream, or remain stationary in the channel though the particles move into and out of it in their more rapid downstream progress. In a typical pool and riffle sequence of a gravel bed stream in eastern United States we painted individual cobbles on a riffle or bar at low flow. After a high discharge the painted rocks were gone, but the form and the position of the bar were unchanged. In flows of bankfull stage or within the banks, the trading process apparently involves only those particles lying at or very near the bar surface. In those rivers which we have studied in detail, the surface layer of gravel on a riffle bar will be in motion and participate in the trading process when the flow reaches about three-quarters bankfull stage.

In a fundamental paper, Lighthill and Whitham (1955a) direct attention to a class of wave motions which they distinguish as being different from the classical wave motions found in dynamic systems. These waves they call kinematic because their major properties may be described by the equation of continuity and a velocity-relation. These define the association between the flux or transport of the objects or particles (quantity per unit of time) and the concentration (quantity per unit of space), which association will be called the flux-concentration relation. Although Lighthill and Whitham imply a contrast between these waves and dynamic waves, it is well to recognize, as they do, that dynamic considerations are involved in the flux-concentration relation, but that the latter relation may be defined experimentally without reference to the dynamics.

Automobile traffic is one of the most easily visualized examples of kinematic waves cited by Lighthill and Whitham (1955b), because it is experienced in everyday living. How exasperatingly often we encounter a slowly moving line of cars. Even on the open highway concentrations of cars occur, for it is a usual driving experience to overtake and finally progress through or pass a considerable number of cars that are close to one another, even after one has driven several miles without passing any. Such concentrations of cars are a form of kinematic waves, for through them move the objects (automobiles in this example) composing them.

These concentrations of automobiles are related to a well-known driving axiom that at high speeds a driver stays farther behind the car in front than he does when traffic is moving slowly. The spacing of cars, in other words, is dependent in part on the speed at which they are moving. It is an interaction between cars governed by the drivers' judgments of braking distance as well as other conditions along the road, such as likelihood of stopping.

The queue of persons waiting at a bus stop is also analogous. The queue may remain at the boarding point even though some passengers get on the bus, for other prospective passengers take a place at the end of the waiting line.

A flood wave in a river is a hydrologic example of kinematic waves. The concentration of water particles is a function of water depth, for the latter is proportional to the volume of water per unit area of channel. The flux-concentration curve for riverflow is the familiar discharge rating, as defined from current-meter measurements.

In addition to flood waves, other interesting wave phenomena in hydrology have been explained by the properties of a flux-concentration curve. Waves in glacial ice (Nye, 1958) are an example.

In the preceding two examples—flood waves and waves in glacial ice—flux or discharge increases with increase in concentration or depth. In these examples, as the depth increases, the velocity also increases. Their flux-concentration curves are monotonically increasing and do not show the bell shape that is characteristic of the flow of discrete noncoherent particles. In the flow of discrete particles in a given environment, as their number per unit of space increases, their mutual interference increases and their velocity decreases.

This paper, therefore, treats of the flow of noncoherent discrete particles. Several examples will be presented. The first is a simple experiment of the single-file movement of beads in a small flume. Others will deal with the transport of sand and gravel in pipes, flumes, and rivers. It is out thesis in the present paper that ripples, dunes, and bars on the bed of a river are kinematic waves composed of bed particles that concentrate temporarily in the bed form as in a queue and are later carried downstream.

FLUX-CONCENTRATION RELATIONS

Some properties of kinematic waves are readily noted by inspection of a graph in which flux is plotted in relation to linear concentration. These terms find easiest introduction in the single-lane traffic case. Flux refers to the number of cars passing a particular point on the route per unit of time. Linear concentration or linear density in a given zone is the average number of cars per unit distance of traffic lane. The reciprocal of linear density is the traffic spacing or average distance between cars. Flux is plotted in relation to linear concentration in figure 1. The attributes of the relation shown in figure 1 have been presented by Lighthill and Whitham (1955b).

If the cars on a highway are very far apart—that is, when linear concentration approaches zero—it seems clear that the flux approaches zero, and so the curve goes through the origin of the graph. Experience indicates further that, as traffic density has increased to the bumper-to-bumper condition, flux again becomes zero. Thus the curve relating flux to density is of the shape shown in figure 1.

To illustrate more specifically, consider the movement of cars on the single-lane highway. Many data show that the mean speed, v, and mean linear concentration, k, follow a linear relationship (Highway Research Board, 1950) that has been expressed as follows:

$$v=v_0[1-(k/k')], \qquad (1)$$

where v_0 is the maximum car speed for the given highway and k' is the maximum linear density which equals the reciprocal of car length. This equation states that mean speed decreases linearly from v_0 when a car is alone on the highway to zero when cars are bumper-to-bumper, which exists when $k/k'=1.0$.

Since the product of mean speed by mean linear

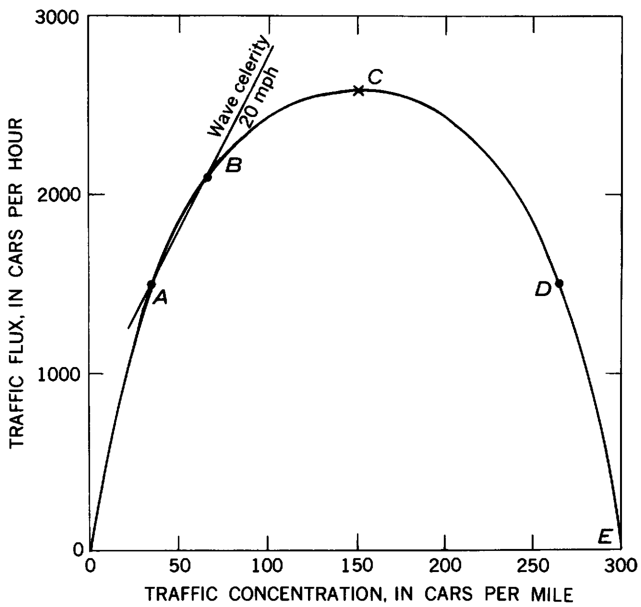

FIGURE 1.—Flux-concentration curve for traffic.

concentration equals mean rate of transport, T, or cars per unit of time,

$$T = vk = v_0 k[1-(k/k')]. \quad (2)$$

This equation, which is called a flux-concentration relationship, is graphed on figure 1. The equation defines a maximum transport rate, which occurs when the spatial concentration is half that when traffic is bumper-to-bumper, or in other words, when the cars are spaced two car lengths apart.

Haight (1963, p. 82) lists several theoretical and empirical relations, including the empirical equation 2, which is his Case VI. The following theoretical equation, also included in Haight's list as Case III, is based on a car-following theory:

$$T = v_0 k \ ln \left(\frac{k'}{k}\right).$$

Although this relation does not satisfy the condition that $v = v_0$ when k is zero, the function indicates that transport is a maximum when linear concentration, k/k', is 0.37 or when average car spacing equals 2.7 car lengths.

In either model, transport rates less than the maximum value can, theoretically, be carried at one of two traffic densities. Only one of these rates, however, is stable. Consider the effect of a perturbation at a point A on the rising limb of the curve (fig. 1). If the concentration or density should increase locally, the transport capacity would exceed the actual average transport rate, and the concentration would tend to return toward the average transport rate. Similarly, if the concentration should decrease locally, the capacity would be less than the average transport rate, and concentration would tend to increase toward the average. Thus, the rising limb of the flux-concentration curve is stable. The highway is said to be uncrowded.

A similar analysis on the descending curve, say point D, will show that a perturbation, which would locally increase the concentration, would decrease the flux and so tend to ultimately increase the concentration to a bumper-to-bumper condition. Similarly, a perturbation, which would locally decrease concentration, would tend toward further decrease in concentration, and the concentration would decrease toward the rising limb. Thus, the descending limb is unstable. The highway is said to be crowded.

The instability characteristic of the descending limb of a flux-concentration curve is piquantly illustrated by the patronage of a honky-tonk bar. People who seek amusement of the sort offered are hardly attracted to the cold emptiness of Monday mid-morning. Rather, they want company. So, if an inspection from the door discloses a lively crowd, they go in and try to find a table; but if they find it virtually empty, they turn away and seek elsewhere. The more people a bar has gathered, the more inviting it appears to prospective customers, and thus it tends either to be crowded or empty. The proprietor, well aware of the effect if not the mathematics of the unstable queue, tries to dampen the oscillation by such devices as "ladies tables," "cocktail hour specials," or myriad mirrors to make the place look more crowded, or at least inviting, even when sparsely patronized.

Note that it is the interaction among individual units that causes changes in concentration and thus gives these concentrations the properties of kinematic waves. In the traffic case interaction between automobiles follows from driver reaction to the longer braking and maneuvering distance required at high speeds. In the example of patronage at a bar, it is the interaction of present and potential patrons that gives the relation its wave properties. The discussions of waves in the transport of grains by fluids will indicate how interaction among grains leads to analogous wave properties.

In figure 1, a line drawn from the origin to any point on the curve has a slope expressed by:

$$\frac{\text{Number of cars}}{\text{Unit time}} \div \frac{\text{Number of cars}}{\text{Unit distance}} = \frac{\text{Distance}}{\text{Time}} = \text{Speed},$$

or the slope is equal to the average speed of cars at that place represented by the point on the curve.

In traffic flow there are, of course, local concentrations of vehicles. Behind and in front of such a concentration is a zone of lower density, and the alternation along the path may be considered to be a wave, in this case a kinematic wave. The zones of low density are troughs on either side of a wave peak. Since the spatial concentration in the trough is less than at the wave peak, transport or flux is also less than at the peak. Thus, there is a loss in volume between trough and peak, and the "upstream" part of the wave therefore decreases in volume and in height because of the net transport of cars away from the region. The opposite is true of the "downstream" part, which increases in volume by the exact amount lost by the "upstream" part. Thus, the wave form progresses at the rate $\Delta T/\Delta k$.

The slope of a chord connecting two points on the flux-concentration curve equals the celerity (that is, speed) of a kinematic wave whose peak and trough are represented by the two points, as for example, points B and A on figure 1. The derivation of this statement follows from the equation of continuity exactly parallels the derivation of the celerity of a flood wave as described by Seddon (1900).

A wave whose peak and trough correspond to B and A would move more slowly than the cars within the wave but in the same direction. In the example shown the wave celerity is 20 miles per hour. Car speed in the troughs (point A) is 44 miles per hour and at the peak (point B), 32 miles per hour.

A tangent at point D has a negative slope, and so the wave moves "upstream" as a shock. A tangent at point C is horizontal, and so the wave is stationary relative to the roadway.

RELATION OF PARTICLE SPEED TO SPACING—A FLUME EXPERIMENT

The flume diagrammed on figure 2 was built only for determining whether the speed of discrete particles moved by water can be related to the distance between them and whether a flux-concentration curve can be developed for this kind of transport. Observations were made of spherical glass beads in a single-lane trough.

The flume was 2 feet long, 2 inches high, and 0.27 inch wide and had one side of clear plastic. Glass spheres 0.185 inch in diameter were transported by the water flow on a smooth, painted bed. Thus, the channel was wide enough to permit easy passage of the glass spheres but not wide enough for the spheres to pass each other.

Water was fed into the upper end through a wire-screen baffle; the flow and tailgate positions were adjusted to obtain a uniform depth of about seven-eighths inch. When so adjusted, the speed of a single bead on the bed was about 0.25 foot per second. Speeds were determined by timing over a 1-foot reach centrally located. The experiment was not designed to measure the relationship between water depth, velocity, or slope; so, no observations were made of these quantities. Water velocity, depth, and rate of flow remained constant throughout, as shown in headnote of table 1.

A number of beads were dropped in at the upper end of the flume at approximately equalintervals. A sufficient number of beads was used in a run to cover at least an 8-inch reach at the indicated spacing. A bead selected at random was timed over a 1-foot reach. Different beads moved with different speeds depending on the size of their group or as they left one group and moved ahead to overtake another. To obtain an average speed for a given average spacing each "run" was repeated several times. Table 1 lists the data obtained for beads of 0.185-inch diameter.

Figure 3 shows a plot of linear concentration, k/k', in relation to observed mean speed of beads. The points establish a well-defined line until the linear concentration reached about 0.5 bead per diameter. As will be explained, no observations could be obtained for closer spacings. The line is extended to zero speed observed when the beads would be in contact with each other. The data define the line $v=v_0\,[1-(k/k')]$, which corresponds to the linear equation for highway traffic given previously.

The value of v_0, the speed of a single particle on the bed of a stream of flowing water, in relation to the hydraulic properties of the stream has been reported by Krumbein (1942) and Ippen and Verma (1953); so, no study was made of this relation.

The retarding action of close spacing may be viewed as being due to the shielding action of one particle

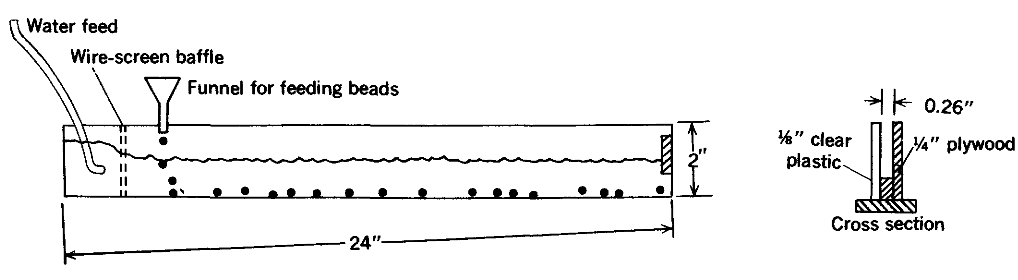

FIGURE 2.—Sketch of flume.

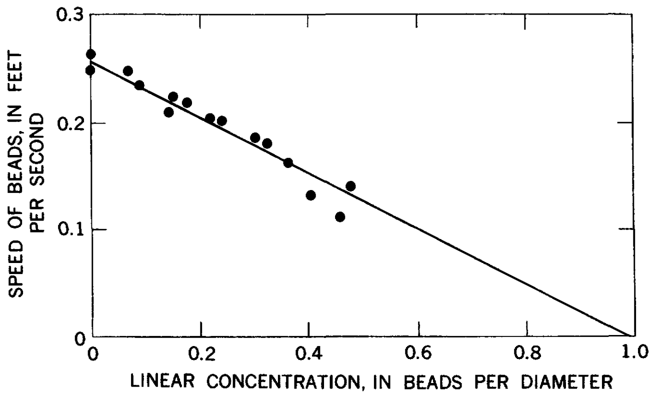

FIGURE 3.—Relation between speed of beads and linear concentration.

TABLE 1.—*Summary of observations with 0.186-inch beads*

[Hydraulic conditions: Discharge=0.0057 cfs; width=0.022 ft; depth=0.073 ft; velocity =0.35 ft per sec]

Mean linear concentration, k (beads per inch) (1)	Spacing (inches) (2)	Speed of beads (ft per sec) (3)	Transport rate or flux (beads per sec) (4)	Linear concentration, k/k' (beads per diam) (5)
2.6	0.39	0.14	4.3	0.48
2.5	.4	.11	3.3	.46
2.2	.46	.13	3.4	.41
2.0	.50	.167	4.0	.37
1.67	.6	.18	3.6	.31
1.67	.60	.185	3.7	.31
1.25	.80	.20	3.0	.23
1.25	.80	.20	3.0	.23
.90	1.1	.215	2.24	.17
.83	1.2	.22	2.2	.15
.74	1.35	.21	1.87	.14
.39	2.6	.25	1.15	.07
.50	2.0	.235	1.41	.09
0	∞	.26	0	0
0	∞	.25	0	0

upon its neighbors downstream; that is, a particle tends to restrict the linkage of its neighbor with the water in which the particles are entrained. This shielding action was noted when two beads were placed in the flume about an inch apart. The upstream bead would move faster and overtake the downstream bead, and then both would move more slowly as a group.

The speed of a bead selected at random at the beginning of the 1-foot reach depended on whether the bead became a part of a group or whether it remained separate over the 1-foot measured course. The beads did not move uniformly but formed in groups rather quickly. Single beads sometimes left the front of one group and overtook the group ahead. As the rate of feed was increased, the size of the groups increased; and the average speed decreased so that ultimately a point was reached where a group became so large that a jam formed and all motion halted. The analogy to the traffic case is evident.

Figure 4 shows the results in the form of a flux-concentration graph. This graph shows transport rate

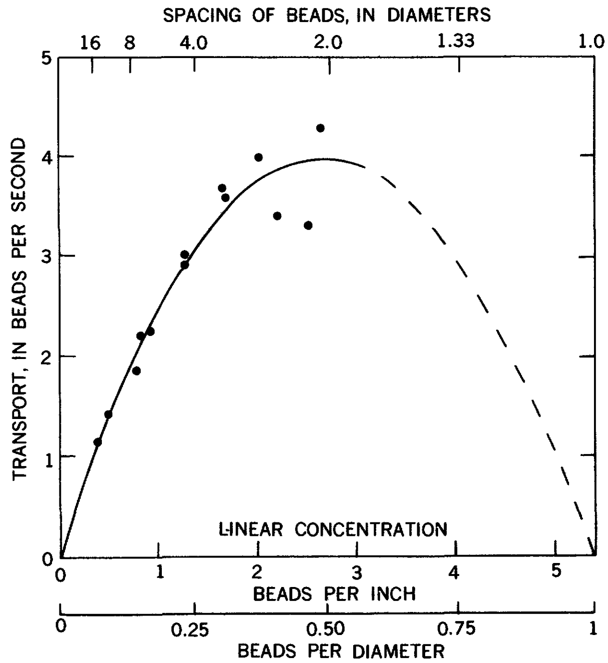

FIGURE 4.—Flux-concentration curve for beads.

in beads per second in relation to the average linear concentration, in beads per inch, as calculated from the curve on figure 3. The transport rate (col. 4, table 1) equals the product of the linear concentration, k, and the speed (col. 3) converted to inches per second.

The peak of the graph on figure 4 represents the maximum rate of transport in the given flume under the given hydraulic factors. The descending branch of the curve represents only a kind of statistical average between cases when (1) concentrations of beads break up or become more open and transport takes place at the maximum rate and (2) the beads jam and transport is zero. The greater the spatial concentration, the greater the frequency of jamming; and when linear concentration is such that the beads are in continuous contact, transport is zero.

Note that the transport rate increases in a nearly linear fashion with density or concentration until the concentration reaches 0.7 to 1.0 bead per inch, which corresponds to a bead spacing of five to seven diameters. At closer spacing, the transport rate increases at less than the linear rate. It can be said, then, that the effect of close spacing does not become very large until the particles are as close as about seven diameters apart.

TRANSPORT OF SAND IN PIPES AND FLUMES

The traffic example and bead experiments serve to illustrate the nature of the flux-concentration curve for kinematic waves composed by discrete particles. As already stated, these flux-concentration curves differ greatly from those of water waves. For example, the flux-concentration wave for water in a channel shows increasing velocity for increasing concentration (=stage or depth). The same is true of the flow of glacial ice (Nye, 1958).

In the traffic and bead examples, particles interact so as to reduce the mean speed with increasing concentration. As will be shown in this section, sand movement in pipes and flumes behaves similarly, even though as may be readily recognized, the movement of grains of sand is unlike that of cars on a single-lane road, or of beads in a single-lane flume. Sediment grains can readily pass one another; they start, stop, and exchange positions in rather complex fashion. The analogy is rather on bulk considerations. The interaction is considered not only between particular grains but between groups of grains. Grains grouped together move less readily than those widely or thinly spaced, and one type of group includes grains lying in a ripple or dune and thus temporarily covered by other grains.

The analogy suggests that similar flux-concentration curves will exist, provided that concentration is interpreted in terms of weight per unit of space, rather than particles per unit of space.

There is a further consideration, In the traffic and bead examples, environmental conditions remained unchanged; as for example, in the bead experiment, the rate of flow and depth of water were held constant as transport was varied. However, the pipe and flume experiments were conducted for a different purpose: to define transport in relation to gradients, water velocity, and other hydraulic factors. In these experiments, therefore, constant environmental conditions could only be inferred from the data, by the methods explained.

FLUX-CONCENTRATION CURVE FOR PIPES

Data on sediment transport in pipes are available from several experiments by Blatch (1906), Howard (1939), and Durand (1953). Blatch, whose experiments are among the best available and who perhaps ranks as the Gilbert among those who have studied sand movement in pipes, prepared a diagram (Blatch, 1906, p. 401) showing loss of head in a 1-inch pipe for various velocities and various percentages of sand. The "percent sand" is the volume of sand transported in ratio to the total mixture discharged. Figure 5 shows the bulk transport of sand (product of percent sand $\times$ mean velocity $\times$ cross-sectional area of pipe) in relation to concentration estimated as described in the following paragraph. Data plotted in figure 5 are listed in table 2, as read from Blatch's diagram for a constant head-loss rate of 30 feet per hundred, which is sufficiently great to insure that any deposition within the pipe was local and temporary.

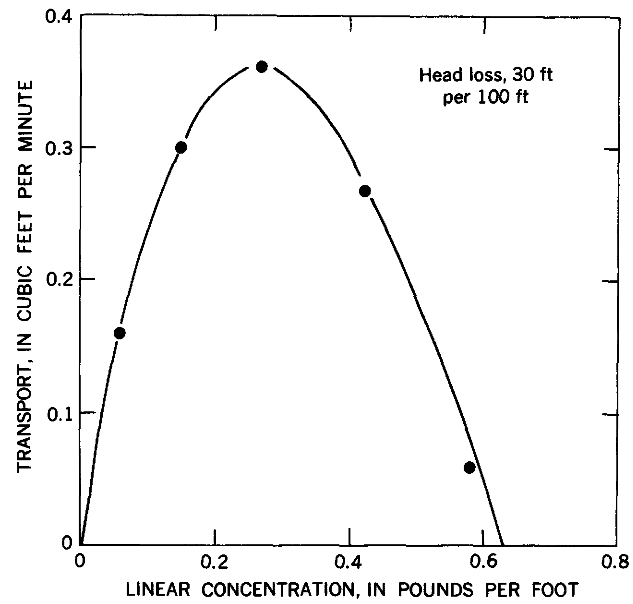

FIGURE 5.—Flux-concentration curve for transport of sand in a 1-inch pipe (data from Blatch, 1906).

In the single-lane flume, as on the highway, linear concentration meant the number of particles per unit length. In the transport of sand in pipes, one deals with bulk volumes of material, and so concentration will be defined in terms of weight of sand per unit length of pipe. As in other sediment examples discussed in the present paper, concentration is defined to include moving plus deposited sediment temporarily at rest. It is necessary to emphasize along with Guy and Simons (1964) that this definition of concentration is not the same as that usually ascribed to sediment in transport—mass of sediment per unit mass of water—sediment mixture.

TABLE 2.—*Data from Blatch's experiments on transport of 0.6-millimeter sand in a 1-inch pipe for a head loss of 30 feet per 100 feet of pipe*

[From N. S. Blatch (1906, p. 401)]

Velocity (ft per sec)	Percent of sand	Transport (cu ft per min)	Linear concentration (lb per ft)
1	16. 5	0. 06	0. 58
4	19	. 27	. 43
6. 7	15	. 36	. 27
8. 2	10	. 30	. 15
8. 95	5	. 16	. 06
9. 25	0	0	0

NOTE.—Pipe area 1.04 inch diam = 0.006 sq ft; spatial concentration

$$=\left[\frac{v}{v'}\bar{c}+0.7\left(1-\frac{v}{v'}\right)\right]0.9 \text{ lb per ft},$$

where v' = 9.25 ft per sec and $\bar{c}$ = percent of sand/100.

The concentration in the terms used in the present discussion was not measured, but it can be estimated. When all said is carried in turbulent suspension, the concentration is cAw, where c is the measured proportion of sand in the effluent per unit volume of mixture, A is the area of pipe, and w is the unit weight of the sediment material. At the other extreme, when the pipe is clogged, sediment concentration is $(1-\eta)Aw$, the factor η being the porosity, about 0.3 for sand particles. Within these limits the concentration is assumed to vary linearly with $(1-v/v')$, where v is the reported mean velocity and v' is the mean velocity for the same rate of head loss for clear water. Other variations could have been assumed, but the results are not sensitive to this factor. When sand transport is light, $v=v'$; when the pipe is clogged, $v=0$. Hence, lineal concentration in the pipe has been estimated for medium sands by the following formula:

$$k=\left[0.7\left(1-\frac{v}{v'}\right)+\bar{c}\frac{v}{v'}\right]Aw.$$

At low rates of transport, v is near v', and the values of k approach the value of $\bar{c}Aw$. However, as v decreases relative to v', the value of k tends to exceed $\bar{c}Aw$ and to approach $0.7Aw$.

A check is available from a curve showing distribution of medium sand across a 4-inch pipe (Howard, 1939, p. 1339). Integration indicates that the concentration, k, equals 0.28 Aw; value of the parameter, $\bar{c}$, percentage of sands, was given as 14.4 percent and mean velocity of flow in pipe, as 8.88 feet per second. From the head-loss curves, the value of v' is about 11 feet per second. Equation 4 gives a computed Aw value of 0.25 whereas the actual value is 0.28. Data on the cross-sectional distribution of solids compiled by Durand (1953, p. 101–102) provide a corresponding check.

Referring again to figure 5, we note that the data define a typical flux-concentration curve, although the mechanics are different from the traffic or bead flume cases. According to Blatch (1906), the data representing the points on the ascending branch of this curve correspond to complete suspension of the transported sand. The descending branch indicates that sand is also being dragged along the bottom of the pipe. A layer of sand is built up on the bottom, and the sand is transported much in the same manner as in an open flume. As load increases, there is spasmodically a blockage, and transport exists only in an average sense. Transport ceases entirely as spatial concentration approaches its maximum, 0.63 pound per foot. Significantly, however, maximum transport occurs when concentration is at about 45 percent of its maximum value. Thus, the pipe-transport example and the previous bead example—two examples of the kinematic wave theory—demonstrate that the average speed of particle movement decreases with an increase in concentration.

FLUME TRANSPORT OF SAND

Transport of noncoherent particles expressed in terms of kinetic theory, as derived from the traffic analogy and the bead experiments, is given in equation 2. In this equation v_0 is the velocity of a single particle, k is the linear concentration (particles per unit length), and k' is the reciprocal of the linear dimension of a single particle.

To apply this formula to the transport of sand by flowing water in an open flume, T is defined in pounds of sand carried per second per foot of width and k, in terms of areal concentration—pounds of sand in motion per square foot. Thus,

$$T=v_0W\left(1-\frac{W}{W'}\right), \tag{5}$$

where v_0 is the velocity of particles as concentration approaches zero. The value of v_0 is a function of the velocity of water (depth and size of particles are assumed to be constant), and W is the areal concentration—weight of sand per square foot of channel. According to the theory, the factor in parentheses of equation 5 decreases as concentration W increases, reaching zero as $W=W'$ where W' is the areal concentration when transport ceases. The value of W' conceptually corresponds to a state represented by dunes whose height is so great as to block the flow in the flume. The value of W' should therefore vary with water depth, but it is assumed to be a constant in a set of flume runs in which depths vary over relatively conservative range.

As in the pipe example, concentration is the weight of moving plus deposited sediment temporarily at rest. Concentration includes sediment moving in suspension or in bed forms. In the flume moving bed forms account for nearly all transport. An estimate of W can be made from the reported height of dunes or sand waves on the flume bed (if suspended sediment is neglected). Inasmuch as the average depth of sand in motion is about half the dune height and the weight-volume ratio is about 100 pounds per cubic foot, therefore W in pounds per square foot numerically equals 50 times dune height in feet. That this estimate is correct may be judged from the experiments with radioactive tracer sands reported by Hubbell and Sayre (1964). They show that depth of zone of particle movement as determined by sampling the bed is closely the same as that computed from the bed forms. The validity of equation 5 can be tested using the data for 0.19 tested using the data for 0.19-millimeter sand as reported by Guy, Simons, and Richardson (1966) and listed in table 3.

Transport is, in effect, the product of two factors—mean particle velocity and areal concentration, W. Mean particle velocity, as previously stated, is a function of the water velocity and, as in the previous examples, of the concentration as well. Thus, there are two independent factors—water velocity and concentration. Their effects can be separated by multiple regression analysis. The results of the regression analysis by a converging graphical process (Ezekiel and Fox, 1959) are shown in figures 6 and 7.

Table 3.—*Observed data for flume transport, 0.19-millimeter sand*

[Source: Guy, Simons, and Richardson (1966)]

Run	Velocity (ft per sec)	Mean depth (feet)	Mean amplitude of sand waves (feet)	Mean length of sand waves (feet)	Speed of sand waves (ft per sec)	Transport T (lbs per sec per ft)
22a	0.78	0.48	0.028	0.57	----------	0.
25	.87	.93	.034	.44	----------	.000015
20	1.11	1.00	.032	.62	0.0004	.00026
1	.74	.58	.023	----------	.000014	.000026
31	1.30	1.02	.038	.58	----------	.0024
27	.93	.55	.033	.53	----------	.000125
5	1.54	1.03	.045	.92	.0013	.012
23	.85	.44	.031	.45	.000049	.000046
32	1.79	.95	.096	3.97	.004	.03
8	1.99	.93	.18	5.39	.003	.06
28	1.04	.54	.038	.58	----------	.0012
33	1.96	1.06	.65	8.0	.002	.11
29	1.13	.56	.038	.60	----------	.0024
3	1.18	.55	.035	.70	.00053	.0034
11	2.35	1.09	.39	11.6	.0035	.21
13	3.09	.89	.13	13.4	.0033	.21
14	3.22	.86	.17	18.8	----------	.26
15	3.46	.79	.04	20.5	----------	.34
34	1.68	.52	.044	1.67	.0043	.027
12	2.69	1.02	.32	17.7	.0033	.22
6	1.67	.61	.18	5.06	.0017	.055
7	1.78	.68	.31	7.4	.004	.094
10	2.89	.51	.10	24.0	.013	.23
9	2.10	.49	.20	5.19	.0025	.077
17	4.14	.67	.10	4.4	----------	.80
18	4.33	.64	.10	4.9	----------	1.6
35	1.81	.52	.13	4.5	.004	.059

Figure 6 shows the variation in v_0, the imputed velocity of particles as concentration approaches zero, with mean water velocity. Values of v_0 equal the quantity $\frac{T}{W}(1-\frac{W}{W'})$. It should be kept in mind that the curve of figure 6 expresses a dynamic relation that applies only to the particular set of experiments with unigranular sands. Note that values of v_0 are small relative to mean water velocity but somewhat greater than wave speeds listed in table 3.

Figure 6 applies where transport is in the bed forms and where suspended sediment represents saltation of grains from one position of rest to another. The data listed in table 3 include all those runs made by D. B. Simons and E. V. Richardson for which bed-form data were reported. This information was not given for runs made at steep slopes. Descriptions of these runs indicate that the bed was composed of "chutes and pools" and that sediment transport was in dispersed or suspended form. In these steep-slope runs, spatial concentration was very low, and $v_0 = v$.

Figure 7 shows the values of T/v_0 corresponding to various values of areal concentration, W. The curve corresponds to an imputed flux-concentration curve if water velocity in each run were the same (v_0=constant), but the runs differ in flux (transport) and in concentration (dune heights). Figure 7 is defined by multiple regression in that the values of v_0 are read off the graph of figure 6. Thus, points in figure 7 correspond to different runs, each with different water velocities and therefore different conditions. Specifically, ordinate values of figure 7 represent the ratio of transport as measured in a given run to the value of

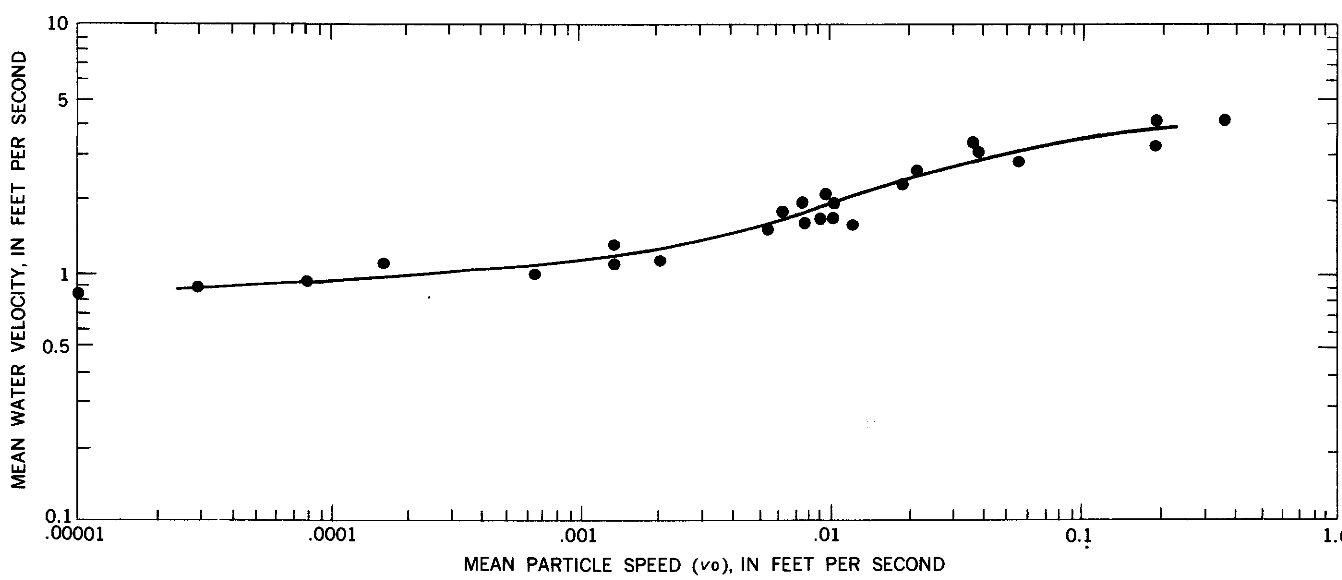

Figure 6.—Relation between mean water velocity and mean particle speed, 0.19-millimeter sand.

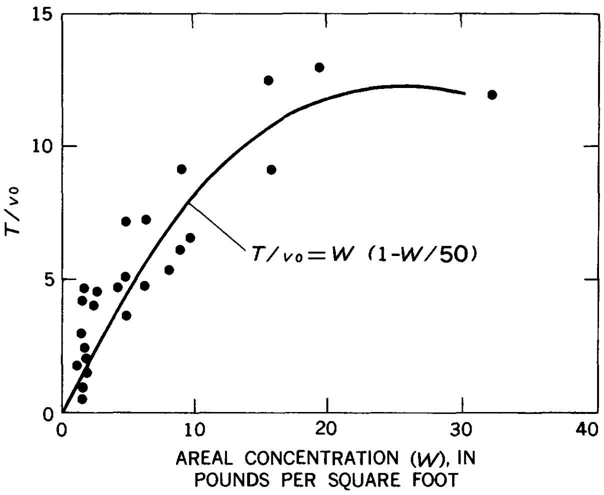

FIGURE 7.—Flux-concentration curve for transport of sand.

v_0 read from figure 6 for the mean water velocity measured in that run. In this way the observed transport data are adjusted for different water velocities as mentioned above.

The ordinate of figure 7 has the dimension of concentration as does the abscissa. The slope of the curve is 1 to 1 relative to the scales used at the origin. Thus, at the origin (concentration is low), mean particle speed equals v_0. As concentration increases, mean particle speeds decrease and thus the curve deviates from a 1 to 1 slope and flattens; in other words, the rate of increase in transport lessens as concentration increases.

There is a suggested maximum for a value of W of about 25 pounds per square foot, and W' may be estimated at 50 pounds per square foot. The data shown in figure 7 follow the curve

$$\frac{T}{v_0}=W(1-\frac{W}{50}).$$

Hence transport in this set of flume experiments may be described by

$$T=v_0\,W(1-\frac{W}{50}) \tag{6}$$

where v_0 is defined by figure 6. The inference is that transport would be zero when $W=0$ and when $W=50$. This equation indicates that transport for a given depth and size of material varies with the water velocity in accord with previous findings by Colby (1964), and with areal concentration as well. Mean particle speed, T/W, decreases linearly with W to a value of zero when $W=W'$.

However, in a sand-channel flume, unlike the bead flume, water velocities differ at troughs and crests of dunes; therefore different flux-concentration curves apply at such points. Therefore the imputed flux-concentration in figure 7 should not be used to define wave celerities. This matter is developed further in the section on "Waves in bed form."

THE EFFECT OF ROCK SPACING ON ROCK MOVEMENT IN AN EPHEMERAL STREAM [1]

Even before the experiments were set up to test the effect of spacing of beads in a small flume, we had conjectured from consideration of the highway traffic analysis that the interaction between individual rocks must be an important factor in the formation of gravel bars on a streambed. Observations of rock movement on a streambed, which were made before some of the other research discussed in this paper had been conducted, suggested the mode of investigation to be followed. The design of the field experiment perhaps, in retrospect, could have been made somewhat more efficient for present purposes. However, despite their limitations, the results are in keeping with the observations of the bead and flume experiments.

In streambeds that are covered by gravel of heterogeneous size, alternation of pools and riffles is an obvious characteristic. But the undulation of bed elevation is small or not apparent in those ephemeral channels that so commonly rise in the foothills in the semiarid countries and that either debouch in alluvial basins or reach master streams of a perennial character rising in nearby mountains. These sandy washes range in size from a few feet to several hundred feet in width and characteristically exhibit a bed profile of nearly uniform slope through reaches of moderate length. They have no deep hollows or deposited bars typical of gravelly streams of perennial flow. Walking along a typical sandy arroyo, one has the impression of walking on a nearly level surface free of bed undulations either along the length or across a main channel.

Such washes are ubiquitous in New Mexico in a broad area that is drained westward from foothills along the base of the Sangre de Cristo Range. One of these washes—the Arroyo de los Frijoles, a tributary of the Santa Fe River and thereby of the Rio Grande—was chosen for an intensive study. Its location, about 10 miles northwest of Santa Fe, N. Mex., is shown in figure 8. It is in a semarid area having about 12 inches of precipitation annually. Other investigations in the

[1] The fieldwork described in this section was initiated by L. B. Leopold with the help of the late John P. Miller and was pursued with the assistance of Robert M. Myrick, William W. Emmett, and personnel of the U.S. Geological Survey at Santa Fe, N. Mex.; all are sincerely thanked for this effort. The interest and the work of Wilbur L. Heckler, Leon A. Wiard, Leo G. Stearns, Louis J. Reiland, Kyle D. Medina, and Charlie R. Sieber are particularly acknowledged for they not only helped with the painted rock experiments but made the field surveys and office computations of the flow discharges. Other aspects of the field experiment are discussed by Leopold (Leopold and others, 1966).

same area are reported by Leopold and Miller (1956). The arroyo rises in low and rounded foothills that are underlain by Pleistocene fan gravels and the Santa Fe Formation, a poorly consolidated mixture of rounded gravel and sand. Despite the large amounts of gravel which tend to pave some of the hillslopes nearby, the ephemeral washes that drain the area are composed of sand and only minor amounts of gravel. Detailed planetable mapping shows gravel accumulations occurring in a regular pattern along the channel length at a spacing comparable to that observed in the pool and riffle sequence of perennial streams. Furthermore, remapping at yearly intervals showed the location of these gravelly accumulations remained more or less constant from year to year.

A large number of holes were dug in the streambed, both on the gravel accumulations which we will here call bars and on the intergravel sandy areas. Nearly without exception it was found that the gravel occurred only as a thin skin, 2 to 4 cobble diameters in thickness, lying on top of the sand.

Flows which occur in these washes average in number about three per year and are caused by summer thunderstorm rainfall only. Snowmelt in spring produces no surface runoff. During a flow the channel typically scours to a depth averaging at least one-quarter of the maximum depth of the flowing water.

Individual rocks were painted and placed on the streambed in groups with varied spacing between the individual cobbles. The experiment was designed to determine the effect of spacing as well as of discharge and rock size on the ability of flowing water to move the rocks downstream. The rocks selected to be painted were obtained from the streambed itself. They were individually weighed and placed in groups of 24 rocks each. On each rock was painted a number that represented its weight in grams. Because the numbers generally consisted of four digits, the number also identified each rock.

Six categories of rock size were chosen, ranging from 300 to 13,000 grams, and each individual group of 24 rocks contained four of each of the six weight categories. The 24 rocks were arranged in a rectangle—six rocks in the direction of the channel and four rocks in the cross-channel direction. The rocks within each group were arranged in a latin square so that in a given row two rocks of the same size did not occur and so that is each column of six rocks all the size classes were represented. Each group was arranged with one of three spacings; that is, in a given group the rocks were placed a distance apart equal to 0.5 foot, 1 foot, or 2 feet. Typical rock groups placed on the streambed and rock movement resulting from a small flow are shown in figure 152 of Professional Paper 352–G.

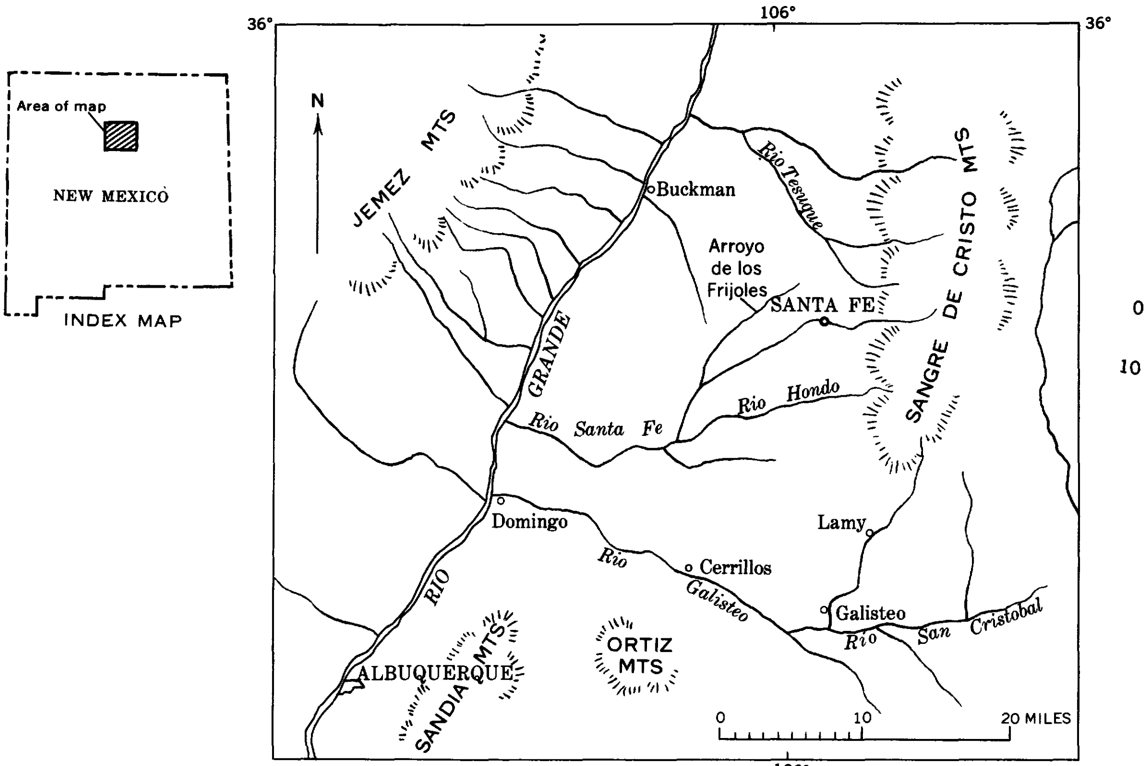

FIGURE 8.—Arroyo de los Frijoles near Santa Fe, N. Mex.

The groups were placed in three reaches of the channel—the most upstream reach was 9 feet wide, the intermediate reach was 35 feet, and the downstream reach was 100 feet. The number of rocks in place before a given flow averaged about 900 during the 5 years of experiment.

After each flow the streambed was searched the whole distance affected by that flow. The location of each rock found was recorded and related to its position before the flow occurred. After a small flow, 70 90 percent of the rocks was usually recovered; and after a large flow, 50 to 80 percent recovery was usual. In one big flood, however, 98 percent was not found and was presumably carried out of the 7-mile study reach.

In the period from 1958 to 1962, a total of 14 flows occurred; but because of the scattered nature of thunderstorm rainfall, each flow did not necessarily affect all rock groups nor each of the three principal locations where the rock groups were placed. Discharges were measured by indirect or slope-area methods, the cross-sectional area being determined by measurement of scour depth registered by buried chains. Because of the difference in channel width at the various locations, discharge was tabulated for purposes of analysis as cubic feet per second per foot of channel width.

In the experiment, therefore, there were four variables—discharge, rock size, rock spacing, and percentage of rocks moved by a given flow. Owing to the fact that even in a single cross section a given flow may not have a uniform effect on each rock group, the data are understandably scattered, and the problem in analysis was to choose a method of plotting which smoothed the scatter in some consistent and reasonable way. Smoothing of the data is justified because the total number of observations of painted rocks affected by a flow during the 5-year period was about 14,000.

The analysis began by treating each flow event separately and for a given flow plotting the percentage of rocks of a given size which moved from their original positions as a function of rock size and spacing. Two typical plots of this relationship are shown in figure 9 which shows that in a given flow a larger percentage of small rocks was moved than of large and

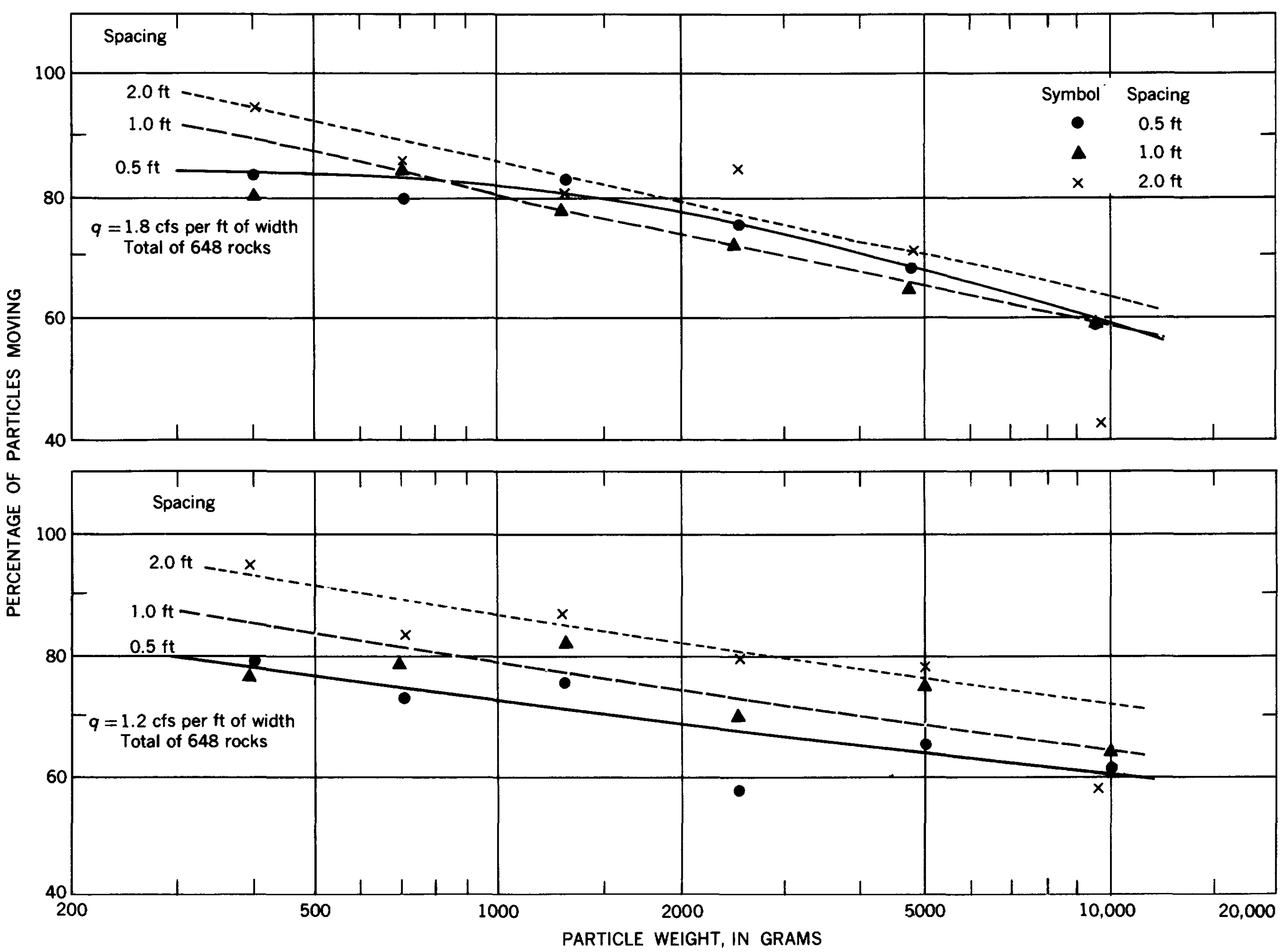

FIGURE 9.—Original data on rock movement plotted as percentage of rocks moved during two individual flows represented in the two separate graphs. Each graph represents data from 648 painted rocks affected by the flow.

that rocks in groups having wider spacing (smaller value of linear concentration) moved more readily than rocks which were close together.

By considering all flows and only those rocks of a given size, we constructed plots from values read off the smooth curves of figure 9; the percentage of rocks moved was expressed as a function of discharge with rock spacing as a third variable. Examples for two rock sizes are presented in figure 10. The lines in figure 10 were drawn as envelopes so that, other than groups where 100 percent of the rocks moved, at least 80 percent of the individual points lies above the respective line. For example, for rocks averaging 1,000 grams, a line representing a spacing of one-half foot envelops or lies below all points except two.

The abscissa value, where a curve intersects the ordinate value of 100 percent of rocks moved, is an estimate of the discharge required to move all rocks of a given weight and at a given spacing.

Though the positions of these lines must perforce be subjective, a pattern that emerged in figure 11 was obtained from the intercepts on the discharge scale for 100 percent of rocks moving.

The abscissa on the figure has two scales—the *b*-axis diameter and weight of particles. The ordinate also has two scales—a scale of discharge in cubic feet per second per foot of width required to move 100 percent of the rocks and a scale of shear corresponding to the values of discharge using field measurements of stream slope. The family of lines represents spacing converted from feet to particle diameter. The graph shows that 100 percent of the rocks of a particular size will be moved only by increasingly larger discharges as the spacing between the rocks decreases. In agreement with the data on glass beads, particle interaction is negligible for spacing greater than about eight diameters.

Another plot can be constructed from straight-line graphs exemplified by those in figure 11. For different values of particle size the discharge required for movement of 100 percent of the rocks was plotted in relation to rock concentration (rocks per diameter), as shown in figure 12*A*. If these lines are extrapolated to the point where each intersects the ordinate axis, the values then represent the theoretical discharge necessary to move all the rocks of a given size at zero concentration—that is, for a single particle. The values of these discharges for zero concentration were used as the denominator in the ratio

$$\frac{\text{Discharge for the individual event}}{\text{Discharge for zero concentration (single particle)}},$$

which will be referred to as the discharge ratio. This discharge ratio could then be plotted against spacing in diameters to result in the nondimensional diagram shown in figure 12*B*.

Comparable date were obtained in a miniature tilting flume shown on figure 13 by using glass beads of 0.12- and 0.18-inch diameter. (See table 4.) Beads were placed at various spacings, and the flume was filled with flowing water sufficient to cover them by a uniform depth of at least three diameters. With discharge constant, the flume was then slowly tilted until the beads began to move. Readings were made of the slope at which 25 percent of the beads moved at various linear concentrations for a fixed discharge. A 25-percent value was used because the single-track flume could not be tilted sufficiently to obtain 100-percent movement when beads were closely spaced.

Values of slope representing zero concentration were estimated from the data and used to compute values of a slope ratio, defined as

$$\frac{\text{Slope of flume in individual experiment}}{\text{Slope for zero concentration (single particle)}}.$$

The data are plotted in relation to the square root of this ratio in figure 12*B*. The rationale for the square root is that discharge varies at constant depth as the square root of slope, as indicated in the Chezy equation. The ratio of the square roots of the slope is thus comparable to the discharge ratio in the arroyo. The comparison between movement of rocks in the arroyos and glass beads in the miniature flume is presented in figure 12*B*.

The rough agreement between the comparable plots for these two very different experiments may be partly a matter of coincidence. But, two types of quantitative data—rock movement in a sandy ephemeral wash and experimental findings concerning beads moving in a small flume—both show that, as the particles are spaced more closely, the slippage is greater or the linkage is weaker between the flowing water and the grains. A larger shear must be administered to move individual particles closely spaced. The relation expresses a nearly linear tendency for particles to be moved as their spacing increases.

WAVES IN BED FORM

GENERAL FEATURES

Each of the three kinds of waves—dunes, antidunes, and gravel bars—that has been observed in flumes and rivers complies with the kinematic properties of the flux-concentration curves. Dunes can result from random variations in spatial concentrations which give rise to wave forms such as traveling queues or platoons on the highway. These dunes can be explained

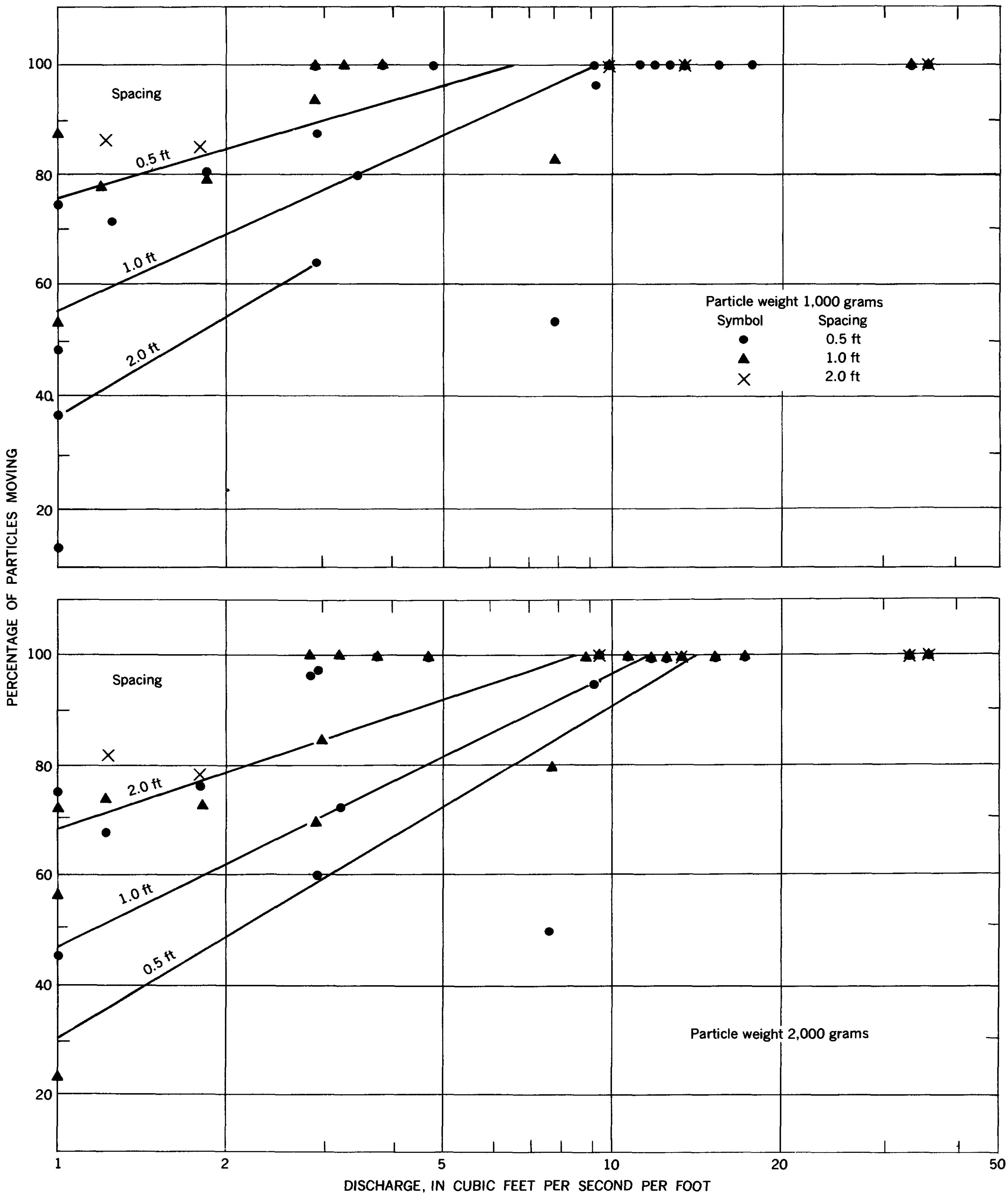

FIGURE 10.—Representative plots of the effect of spacing on the percentage of rocks moved by different discharges. The respective graphs apply to rocks of two size classes. Data on these graphs read from smooth curves of which figure 9 shows two examples.

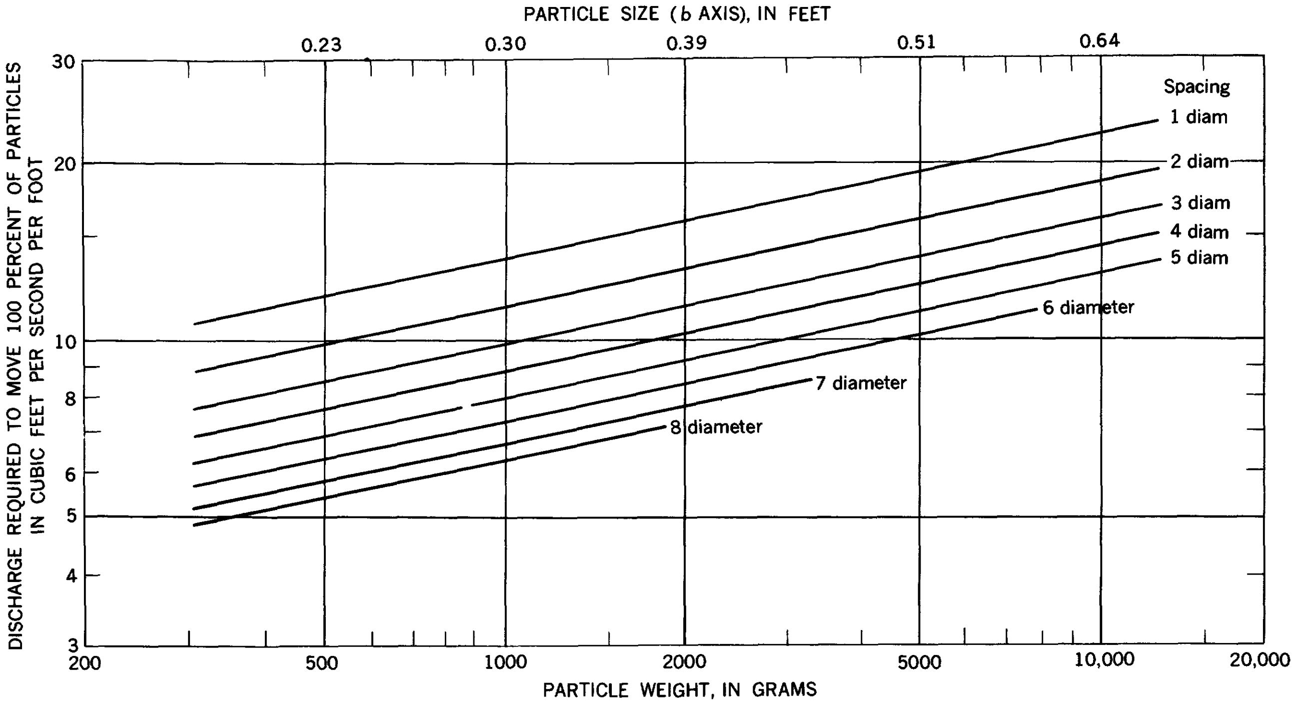

FIGURE 11.—Generalized diagram showing discharge required to move all rocks of given size as a function of spacing. Derived from values read from graphs typified by figure 10 where lines intercept ordinate value of 100 percent.

TABLE 4.—*Observations in miniature flume of slopes at which 25 percent of beads moved*

[Discharge constant for each run, but not measured]

Linear concentration (beads per diameter)	Slope	Slope ratio
Run 1 (0.18-inch beads)		
0. 50	0. 053	4. 6
. 32	. 044	3. 8
. 20	. 023	2. 0
. 13	. 02	1. 75
. 034	. 0125	1. 1
0	. 0115	1. 0
Run 2 (0.12-inch beads)		
0. 475	0. 047	4. 9
. 26	. 025	2. 6
. 053	. 011	1. 15
0	. 0096	1. 0
Run 3 (0.18-inch beads)		
0. 90	0. 10	12. 0
. 40	. 028	3. 3
. 03	. 0093	1. 1
0	. 0085	1. 0
Run 4 (0.12-inch beads)		
0. 90	0. 087	12. 0
. 53	. 043	5. 8
. 014	. 0078	1. 05
0	. 0075	1. 0

on the basis of kinematic relationships. As shown by field and laboratory evidence, the probability of a particle being moved increases as the spatial concentration decreases. Therefore, areas of low concentration (thin bed forms) tend to become thinner as areas of high concentration increase. Waves are formed that take on characteristic lengths and heights.

One of the well-known properties of highway traffic is the Poisson distribution of vehicular spacing, but even here waves—or platoons, as they are called—are formed. Figure 14, for example, shows "waves" in traffic along a 50-mile stretch of highway in Arizona. The example was obtained from a part of strip aerial photographs made along the highway from Tucson to Eloy. The vehicles measured were in a single lane in a divided four-lane highway. Such waves in highway traffic are common enough, but crossings, trucks, and towns may cause them to be very irregular.

Fairly regular waves can be generated by a random process, as illustrated by the following model. Consider initially a distribution of particles along a line with average concentration of k particles per unit length. These particles are subject to the condition that, where particles happen to be far apart, they are more easily moved than where particles are close. Thus, the movements will collect the particles into groups having wavelike form. The results of a trial are shown in figure 15. Particles that have an average concentration k are

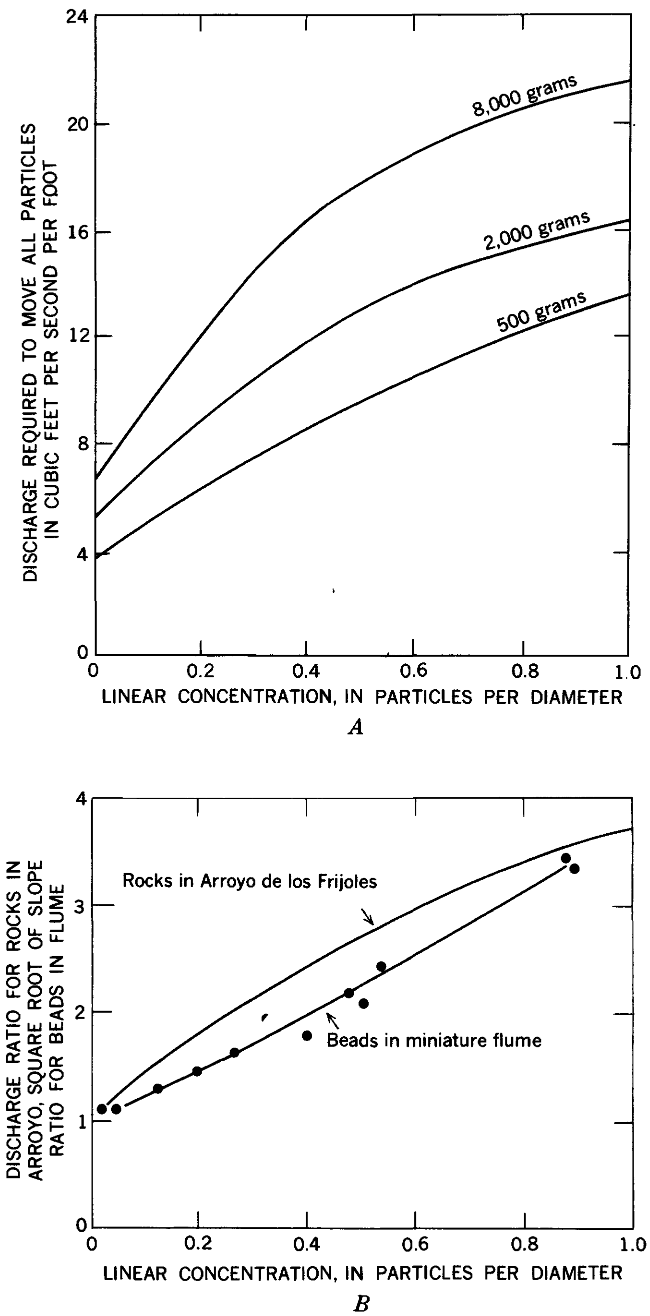

FIGURE 12.—*A*, Field data for rock movement in Arroyo de los Frijoles. *B*, Comparison of movement of rocks in arroyo with beads in flume.

arranged in random position in the first column (marked "Time interval 0"). Only one particle is permitted in a single position or box. Each column represents a successive unit of time, and the direction of motion is down the sheet. The diagram shown is merely the central part of a longer sheet on which a very long sequence of wave groups was developed. The whole suite of particles may be moving with some speed which does not concern this analysis of relative speeds and positions.

The following rule governs this relative movement. A particle cannot move if there is one in the space immediately ahead. A particle at the front of a group is free to move; and if it does so, it will move and overtake a particle in the group ahead. A particle free to move in a particluar time interval is selected to do so at random, its probability of selection being p. The process here of a particle taking off at random and overtaking the one ahead is not unlike that observed in the single-lane flume. (See p. L5.)

Figure 15 shows an experimental example where $k=0.33$ and $p=0.50$. After 10 time intervals, a well-defined pattern of groups is established owing to the interaction between particles. The lengths of such wave patterns would be proportional to the mean concentration and speed. Figure 16 shows, for example, the relation of length of sand waves to water velocity and to the ratio of amplitude of waves to depth of water over dunes as observed in the set of experiments on 0.19-millimeter sand in a flume at Fort Collins, Colo., listed in table 3 (Guy and others, 1966). Because these relations are of a random nature rather than of dynamic origin like the unique relation between velocity and length of antidunes also shown in figure 16, precise relationships cannot be found.

KINEMATIC PROPERTIES

The flux-concentration curve, shown in figure 7, for transport of sand in a flume describes the relation between transport and average concentration along the flume for different runs. Consider a single run. Transport and concentration differ along the flume.

Unfortunately, flume experiments only report averages for each run, and so the nature of the flux-concentration curves along the flume can only be inferred as on figures 17 and 18.

Flux-concentration curves, as suggested in figures 17 and 18, can be useful in explaining the development of modes of transport, dunes, plane bed, or antidunes as usually observed in flume experiments. A perturbation in the bed creates a difference in depth of sediment, in velocity, and in rate of transport at that point. When velocities are subcritical (that is, less than $\sqrt{gD}$), the velocity over the top of a protuberance in the bed is greater than in a trough. One may infer a different flux-concentration curve for the crest and for the trough. However, the actual relationship is probably a loop as shown.

The relations for dunes are indicated by points 1 and 2 in figure 17. At point 1, representing the crest, velocities are higher than at point 2, representing the trough, and transport is greater over the crest than in the trough. Thus, the action tends to decrease the

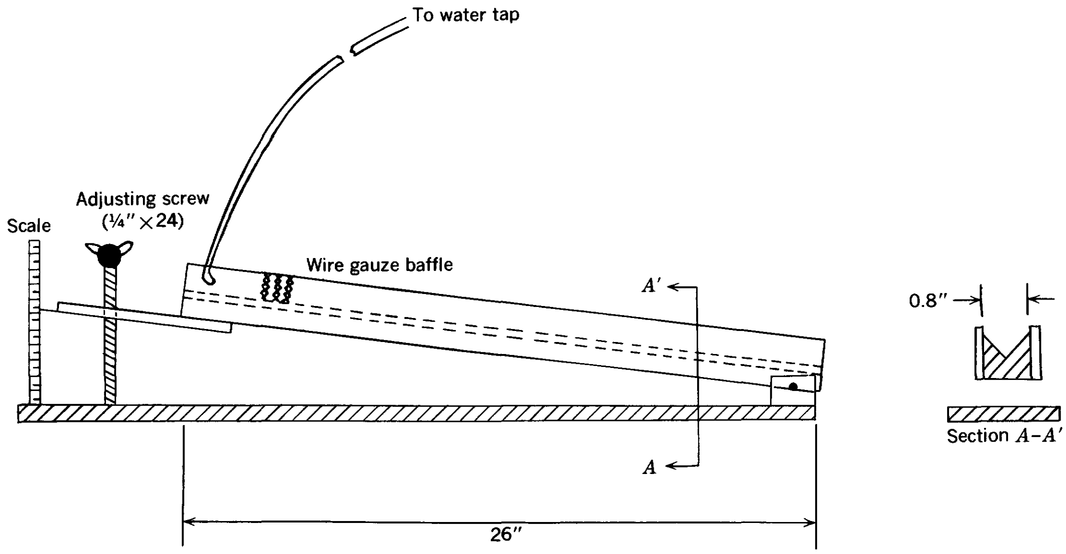

FIGURE 13.—Tilting flume used to observe movement of beads at different spacings.

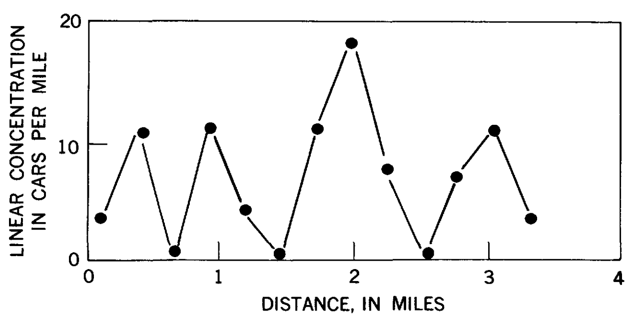

FIGURE 14.—Variation in car density along a highway.

crest of the protuberance and to deposit particles in the trough. However, because of the forward motion, the cutting and filling action is continuous and leads to the formation of dunes in forward motion as previously explained. The rate of forward motion of the dunes equals the slope $\Delta T/\Delta k$ of the chord 1–2 in figure 17, which is positive—that is, downstreamward—in this phase.

When transport is low, chord 1–2 is short; when transport increases, the chord moves up the graph and increases in steepness. Dune speed increases, but the dune height (the horizontal projection of chord 1–2) decreases. Ultimately, dune height decreases to zero, and the plane-bed phase is reached.

As velocity increases and approaches $\sqrt{gD}$, standing hydraulic waves are formed. However, it should be noted that, although the lengths of antidunes or standing waves are controlled dynamically (Kennedy, 1963), their speed can be inferred from the kinetic properties of the flux-concentration curve. These waves induce perturbations in the bed, as shown in figure 18. When velocities are supercritical (that is, greater than $\sqrt{gD}$), the depth of water over the tops of the bed forms is greater than the depth of water over the troughs; therefore, the water velocity over the tops is less than in the troughs. Hence, the slope of the chord connecting points 1 (crest) and 2 (trough) will be either flat (standing waves with zero speed) or negative (antidunes). The corresponding points are shown in figure 18. Transport is greater in the trough than at the peaks, and the sand waves grow in height, eventually become unstable and break, and throw large parts of the load into turbulent suspension. The slope of chord 1–2 is negative; hence, wave movement is upstream, and the waves are therefore called antidunes.

As explained on page L4, the rate of movement of dunes equals the slope of a chord connecting points on the flux-concentration curves for troughs and the peaks. Dune speed, c, therefore equals the ratio

$$c=\frac{T_1-T_2}{W_1-W_2},$$

where the subscripts 1 and 2 refer to transport, T, and areal concentration, W, at the crest and trough, respectively. Since $W_1-W_2=w(h_1-h_2)$ and $h_1-h_2=h$, mean amplitude of dunes, then

$$chw=T_1-T_2$$

where w is the unit weight of the sand. For those runs in which wave speed, c, and mean height of dunes were observed, the left-hand side of the equation may be calculated, but data on T_1 and T_2 are not available. It would be possible to report these quantities by observing changing transport rates as dunes move out of the flume.

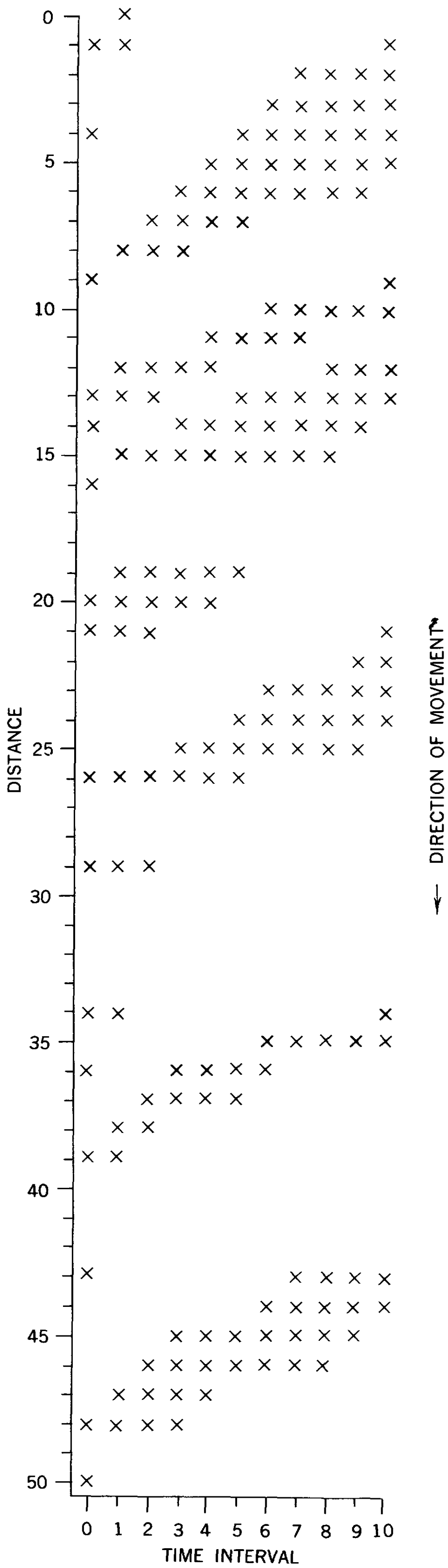

FIGURE 15.—Generation of waves in a random model.

Letting $T_1 = b\ T$, where T is the mean rate of transport which is also assumed here to be the arithmetic mean of T_1 and T_2, then

$$b = 1 - (c/v_P),$$

where v_P, the mean particle speed, equals T/W. The factor, b, can be calculated in the set of experiments cited in table 3 for those runs in which dune speed, c, was observed. It ranges from a negative quantity for those runs with low velocity and increases to over 0.7 for those runs with high velocity. The significance of this range is that for low-velocity runs the interdune transport (net transport from one dune to the dune ahead) is zero or even upstream, but it increases as velocity increases. The interdune transport, for any given velocity also increases as dune height decreases. These results are in accord with the observation that as velocities increase the difference between crest and transport decreases and residence time in the dunes decreases.

GRAVEL BARS

Because of the limited supply of gravel, the spatial concentration of gravel is almost entirely in "wave" form. The "crests" of these waves are riffles or channel bars. The flux-concentration curve for the bar may be as suggested in figure 19. The flux is derived entirely by erosion of the bar deposits, where, in the kinetic terminology, spatial concentration decreases as river flow increases—points 1, 2, and 3 in figure 19. The flux-concentration curve for low riverflow, zero gravel transport, coincides with the horizontal axis. On the other hand, the flux-concentration curve for the troughs (where concentration is near zero) virtually coincides with the vertical axis.

The flux is indicated to be the same in the troughs and the bars, and so the slopes of the chords 1–1, 2–2, and 3–3 are horizontal; these slopes indicate that wave velocity is zero. That is, the bars are fixed relative to the channel.

Mean particle velocity in the troughs is high; it is low in the bar. (Again, when we speak of mean particle velocity, it is the mean speed of all particles having concentration, k, even though not all are in motion at a particular time.)

A small-flume experiment confirmed earlier observations by Gilbert (1914, p. 243). A supply of sand formed dunelike deposits separated by bare reaches approximately equal in length. In the flume, particles moved rapidly from the toe of one dune to the heel of the one

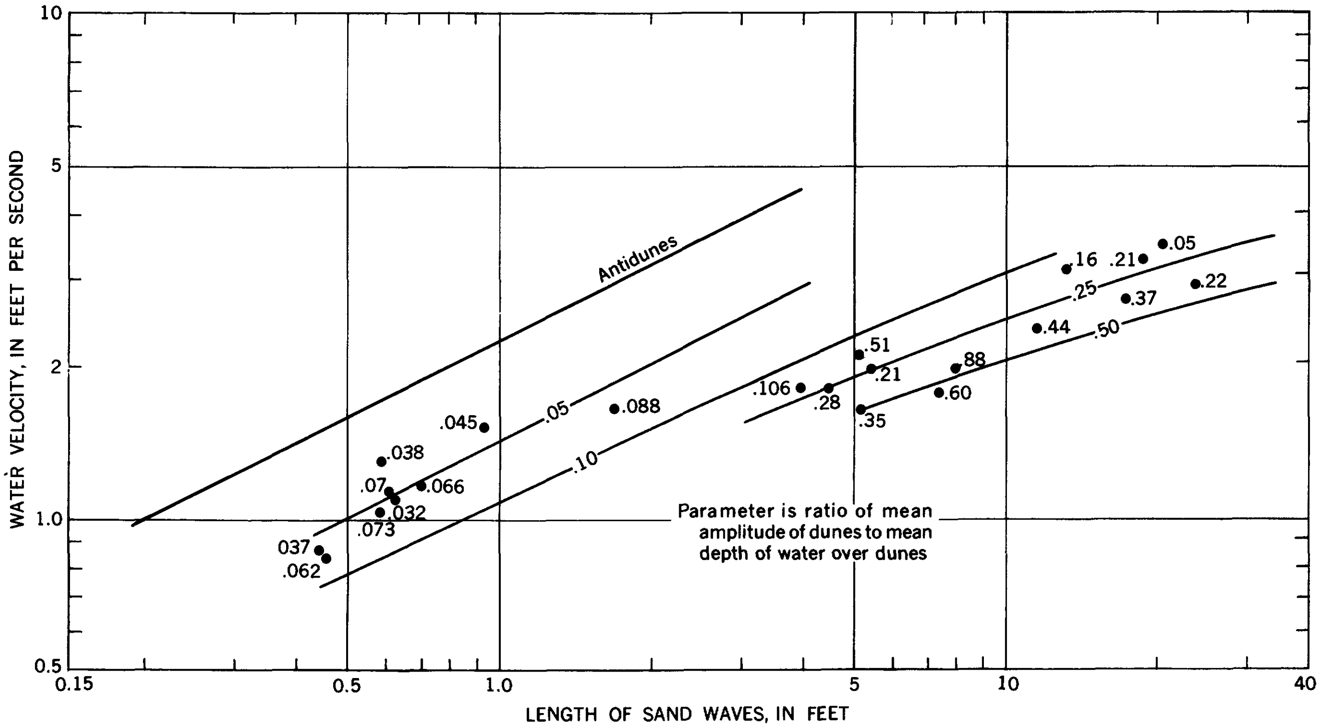

FIGURE 16.—Length of sand waves in a flume in relation to water velocity and to ratio of mean amplitude of dunes (0.19-mm sand) to mean depth of water over dunes.

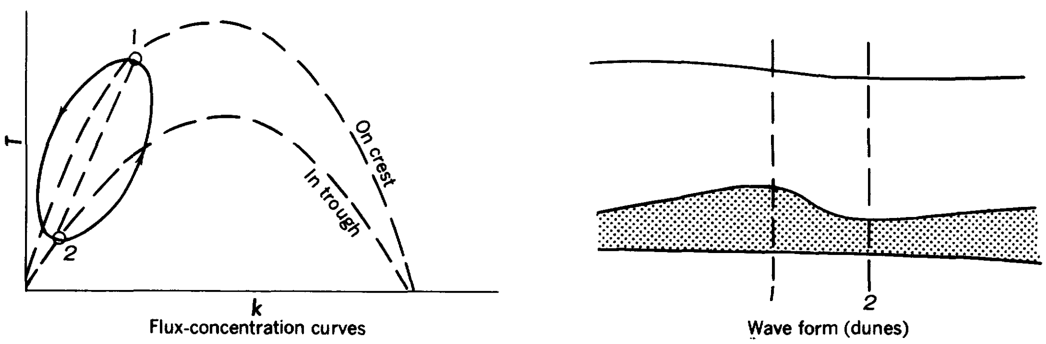

FIGURE 17.—Suggested transport relations in tranquil flow.

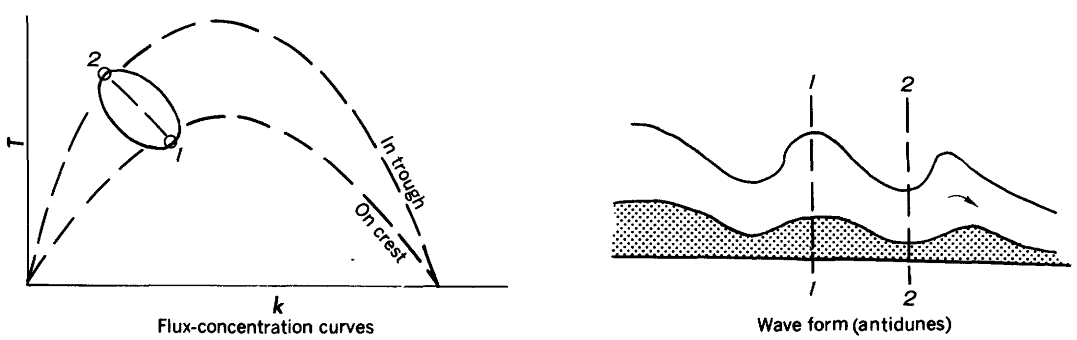

FIGURE 18.—Suggested transport relations in rapid flow.

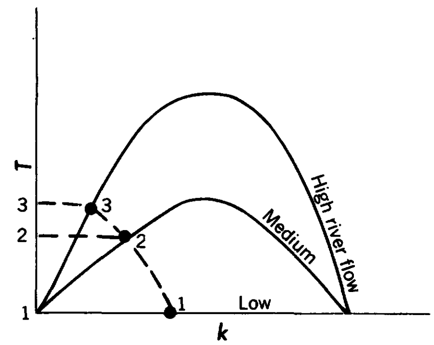

FIGURE 19.—Suggested flux-concentration curves for river gravel.

downstream. In this steady state of water and sediment input, the dunes moved slowly downstream. However, when the flow of water was increased, the dunes were slowly eroded away, but they remained fixed in position and did not then migrate down streamward.

As observed in rivers, gravel bars are spaced more or less irregularly at distances that average five to seven channel widths. The spacing of bars is seemingly controlled by the channel dimensions and remains fixed even though the velocity changes. To the extent that the theory relating to dunes applies, an increase in the velocity would tend to increase the spacing. However, the accompanying increase in depth of water and a decrease in height of the bar by scour has the opposite effect; therefore, the two conditions may combine so as to maintain bars at fixed distances apart.

SUMMARY

The theory and experimental evidence presented show that interaction among the individual units moving in a continuous flow pattern causes velocity of the units to vary with their distance apart. Concentrations of these units have certain wave properties encompassed under the term "kinematic waves." With these properties it is unlikely that such particles can remain uniformly distributed for any distance along the flow path, for random perturbations lead to the formation of groups or waves which take the form of the commonly observed dunes in sandy channels and the pool and riffle sequence in gravel river beds. The rationale may be summarized as follows.

Particles of sand or gravel which can be moved by the flowing water are seldom all in motion simultaneously unless their number is very small. There is some probability of one being set in motion instead of its neighbor. Once set in motion, the speed of a given particle is affected by its proximity to neighboring moving grains; the mean grain speed is slower when the grain density is higher. The result of this interaction is to intensify any initial random grouping whereas open spaces or places where grains are sparse tend to become even more open—have even smaller density of grains. Thus, any random influx of grains into a reach of channel will not continue to be random, for as the grains travel downstream their interaction will set up waves—that is, groups of grains separated by relatively open spaces will tend to attain a more or less regular spacing along the direction of flow.

By such a process sand collects in dunes which are small compared with channel width. The same principle will apply in a transverse section as in the longitudinal profile just described. Transverse to the current the dunes will also alternate with open spaces or troughs. Thus, there will be built the pattern of dunes so characteristic of sandy channels, in which the whole surface is covered with dunes separated by troughs; the dunes will be arranged in a rather uniform and random pattern, with some dunes arranged in echelon but others appearing to be uniformly irregular.

In the situations where the particles composing the kinematic wave are only one to several diameters thick, as in beads in a single lane flume or a veneer of cobbles on the sandy bed of an arroyo, the linear concentration is easily visualized because nearly all the particles potentially in motion are on or near the surface and visible. Concentration can then be measured by the spacing of particles on the surface. In the case of dunes and riffle bars, the concentration is not so apparent at a glance because it is measured by the weight or number of particles on a unit area to a depth of the sand or gravel above the plane representing the base of the moving dune—that is, the depth down to which particles are participating in the motion as the dune or bar is eroded or moves.

With regard to pool and riffle sequences in gravel-bed streams, the kinematics suggest that spacing of the riffles is related in part to the thin veneer of gravel set in motion by the flow. The bar represents an interaction between two opposing factors: increasing water velocity, which tends to increase wavelength, and decreasing amplitude, due to erosion, which tends to decrease wavelength. Such a balance results in riffle bars which do not appreciably move downstream.

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