Sediment Transport in Alluvial Channels 1963-65

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STEWART L. UDALL, *Secretary*

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[The letters designate separately published chapters]

Discharge of Sands and Mean-Velocity Relationships in Sand-Bed Streams

By BRUCE R. COLBY

SEDIMENT TRANSPORT IN ALLUVIAL CHANNELS

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Thomas B. Nolan, ***Director***

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ILLUSTRATIONS

DEFINITIONS

Antidunes are roughly symmetrical sand waves that are about in phase with similar water waves. This definition includes as antidunes the roughly symmetrical sand waves that may exist below relatively quiet and symmetrical standing waves on the water surface.

Bars are extensive raised areas or ridges, larger than dunes in the same flow, on the bed of a stream whose bottom shifts readily, at least at high flow.

Bed layer is the layer through which the bedload moves. It is usually considered to be only a few grain diameters thick.

Bedload is the sediment that moves by sliding, rolling, or skipping on or very near the streambed and is supported mainly by the bed rather than by the turbulence of the flow.

Bed material is the sediment of those particle sizes that are found in appreciable quantity in the streambed. It is classified according to particle size rather than mode of transportation or location in the cross section.

Bed sediment is the sediment on the streambed.

Bed sand is bed sediment within the size range of sand.

Concentration is a ratio of the weight of sediment to the weight of the water-sediment mixture. In this paper it is the weight ratio of the sediment that is transported through a stream cross section or part of a cross section to the water-sediment mixture that moves through the same area during the same time interval.

Dunes are somewhat irregularly spaced mounds of loose sand. Their upstream faces are usually gently inclined as compared with their much steeper downstream faces.

Fall velocity of a sediment particle is the rate at which the suspended particle settles with respect to the surrounding fluid.

Fine sediment as the term is used in this paper is the sediment that is finer than 0.062 mm.

Froude number is the ratio of the velocity of flow to the square root of the product of the gravity constant and the depth of flow.

Grain roughness is the boundary roughness that results solely from the sizes of the stationary particles without any effect from the configuration of the bed or banks.

Mean velocity is the velocity that is obtained by dividing the flow by the perpendicular area of section through which the flow passes. Usually, the mean velocity is computed for the whole cross section of a stream, but sometimes a mean velocity is computed for part of a flow.

Measured discharge of sediment or of sands is the discharge of sediment or sands that is usually determined directly from streamflow and depth-integrated sediment samples, which generally are obtained by sampling the flow from the water surface to within 0.3 or 0.4 foot of the streambed.

Median particle size is the size for which 50 percent of the sediment by weight is finer.

Plane bed is a relatively smooth and firm sand bed that is free of ripples, dunes, and antidunes and, consequently, has low resistance to flow.

Resistance to flow is an inclusive term for the total resistance to flow in a channel.

Roughness of bed or banks is a boundary irregularity and is usually expressed as a linear measure of the effective size of the irregularities of bed or banks.

Sand is sediment particles that have diameters between 0.062 and 2.0 mm.

Sand-bed stream is a reach of stream whose bed is composed almost entirely of cohesionless, shifting sand and whose channel is not confined by walls of rough rock or other material that would cause unusually high turbulence throughout much of the flow.

Sediment discharge is the rate at which weight of sediment moves through a cross section perpendicular to the direction of flow.

Shear per unit area of the streambed is the product of the specific weight of water-sediment mixture, the depth of flow, and the energy gradient.

Shear velocity is the square root of the product of the gravity constant, the depth of flow, and the energy gradient.

Shear velocity with respect to the grains or particles is a concept used by Einstein (1950) and is the shear velocity that can be computed from the energy gradient and a particular hydraulic radius R'.

The square root of the ratio of R' to the hydraulic radius R equals the ratio of the actual mean velocity to the mean velocity that would exist if all resistance to flow were due to the stationary particles on the bed.

Shear velocity computed from mean velocity is the shear velocity that can be computed from a known mean velocity with the use of an equation such as the one Einstein (1950) derived from work by Keulegan (1938).

Size class or **size range** is a definitely limited range of sediment particles that are grouped together because their behavior can be reasonably well represented by some one particle size such as the geometric mean size, which is the square root of the product of the upper and lower limiting sizes.

Suspended sediment (suspended load) or **suspended sand** is sediment or sand that is supported by the upward components of turbulent currents or by colloidal suspension if the sediment particles are very small.

Standard fall velocity is the average rate at which a particle would eventually settle if falling alone in quiescent distilled water of infinite extent and at a temperature of 24° C.

Unmeasured sediment discharge is the difference between total sediment discharge and measured sediment discharge.

SYMBOLS

A and B	Dimensionless constants used as 0.047 and 0.25, respectively, by Meyer-Peter and Müller (1948).
A_1	Dimensionless constant used by Bagnold (1956). It may be about 9.
B_1	Dimensionless constant used by Bagnold. It varies slowly with particle size.
a	Distance above the streambed.
C	Chezy C, which is equal to $\bar{u}/\sqrt{RS}$.
c	Concentration of sediment. Concentration at a is denoted by c_a and at y by c_y.
D	Particle size expressed as a diameter or the geometric mean diameter of a size range.
D_m	Effective diameter of the bed-material mixture (the particle size of uniform sediment) and equal to $\Sigma D i_b$ for bed sediment of a range of sizes.
D_{35}	Particle size for which 35 percent of the bed material by weight is finer. Subscripts 50, 65, and 90 have comparable meanings. Hence, D_{50} is the median particle size.
d	Depth of flow.
g	The gravity constant.
i_b	Fraction by weight of bed sediment in a size range.
K_r	A measure of resistance to flow and equal to the mean velocity $\bar{u}$ divided by the quantity $(R^{2/3}S^{1/2})$.
K_r'	A comparable measure of the flow resistance due only to the resistance of the stationary grains, and for metric units it can be computed from $26/(D_{90})^{1/6}$ in which D_{90} is the particle size, in meters, for which 90 percent of the bed sediment by weight is finer.
K_1, K_2, K_3	Constants.
k	The turbulence constant, which is about 0.40 for clear-water flow and certain types of boundary roughness but may vary widely for sediment-laden flow and dune roughness.
k_s	A linear measure of roughness that may be assumed to equal D_{65} if the bed roughness is due wholly to the roughness of the stationary grains.
k_1	The form of k that is computed from the vertical distribution of velocity over a dune bed or other rough bed.
L	Length of channel reach.
l_e	The mixing length.
q_B	Bedload discharge per foot of width in terms of weight of sediment per unit time.
q_B'	Bedload discharge per unit width in terms of underwater weight of sediment per unit time.
R	The hydraulic radius, which equals average depth of flow when the bank resistance is negligible.
R'	The hydraulic radius with respect to the grains or particles. It is the hydraulic radius that could be used to compute velocity if all the resistance to flow were due to the stationary sediment particles.

S The energy gradient.

u Time-averaged velocity at a point. A subscript may be used to indicate distance of the point above the streambed.

$\bar{u}$ Mean velocity, which is computed by dividing the streamflow by the area of the cross section.

u_m Shear velocity computed from mean velocity and equal to $0.40\bar{u}/[2.30 \log (12.27 xd/D_{65})]$.

u_* Shear velocity and equal to $\sqrt{gRS}$.

u_*' Shear velocity with respect to the sediment particles and equal to $\sqrt{gR'S}$.

v Average vertical velocity of the upward- or downward-moving water in turbulent flow.

W Channel width.

w Fall velocity of a sediment particle or particles.

x A dimensionless parameter to cover the transition from hydraulically smooth to hydraulically rough boundaries.

y Distance above the streambed.

z Theoretical exponent for vertical distribution of sediment of one fall velocity or a narrow range of fall velocity and equal to $w/(0.40u_*)$ for uniform flow over a plane bed.

z_m The form of z that can be computed from $w/(0.40u_m)$.

z_o The form of z for flow over dunes or other rough beds and equal to $w/(k_1u_*)$.

z_1 Exponential measure of the actual distribution of sediment with depth as determined from a logarithmic plot of concentration for a size range against $(d-y)/y$.

log Logarithm to the base 10.

$\log_e$ Logarithm to the base e.

β Declination of the bed surface below the horizontal; $\cos \beta$ generally can be considered to equal unity and be disregarded.

γ Specific weight of water-sediment mixture.

γ_f Specific weight of the fluid.

γ_s Specific weight of the sediment particles.

θ_F A dimensionless measure of the apparent tangential stress due to action of external forces on the fluid and equal to $[\rho_f/(\rho_s-\rho_f)](RS/D)$.

θ_t That value of θ_F (determined by extrapolation) at which sediment particles begin to move on a rippled sand bed.

ν Kinematic viscosity.

ρ_f Density of the fluid.

ρ_s Density of the sediment particles.

Φ Intensity of bedload transportation for uniform bed sediment.

Φ_* Intensity of bedload transportation for a size range of nonuniform bed sediment.

Ψ Function for correlating effect of flow with intensity of sediment transportation.

Ψ_* Function Ψ as adjusted by Einstein (1950, p. 37) for an individual size range.

DISCHARGE OF SANDS AND MEAN-VELOCITY RELATIONSHIPS IN SAND-BED STREAMS

By Bruce R. Colby

ABSTRACT

Theories of sediment movement and general observation provide the basis for a practical concept of sediment transportation in sand-bed streams. This concept covers bedload discharge, the concentrations of sediment of different size ranges immediately above the bed layer, and the vertical distribution of suspended sediment of different size ranges within the flow. According to the concept, the bedload discharge per unit width depends mostly on the velocity near the streambed, the fall velocities of the bed sediment, and the depth of flow. Immediately above the bed layer, the concentrations of sands in the different size ranges are determined mainly by the bedload discharge of the different size ranges, the depth of the bed layer, the velocity of flow through the bed layer, and the probable continuity of the concentrations from the bed layer to the suspended sediment immediately above the bed layer. The concentrations of suspended sands within the different size ranges at any level in the flow depend on the concentrations immediately above the bed layer and the rate at which the concentrations decrease with distance above the bed layer. The rate of decrease of sediment concentration in each size range can be computed, but only roughly, from the ratio of the fall velocity to the shear velocity.

An analysis of the relationships of discharge of sands per foot of width to the controlling factors that were indicated or suggested by the general concept of sediment transportation showed approximate major effects of several variables as follows:

1. Mean velocity or, as an alternative factor, shear velocity computed from mean velocity has a dominant effect on the discharge of sands in most sand-bed streams. At a cross section of a sand-bed stream a reasonably close relationship usually exists between discharge of sands and mean velocity unless water temperature changes greatly or unless the concentration of fine sediment is high and variable. The curve of relationship for one sand-bed stream may approximately apply for some other sand-bed streams.
2. Shear and stream power each has a dominant effect, somewhat similar to that of mean velocity, on the discharge of sands except for the uncertainty of the relationship when the resistance to flow is variable.
3. For moderate to high velocities and shallow flows, discharge of sands per foot of width varies somewhat but not greatly for changes in median diameter of the bed material from 0.30 to 0.80 mm. However, the effect of particle size on the discharge of sands may be large at low velocities in any depth of flow or at high velocities in deep flows.
4. Depth sometimes has a large effect on the relationship between discharge of sands per foot of width and any one of the four measures of the force or the energy that causes sediment movement, and the effect varies complexly with changes in velocity and particle size.
5. If other factors remain constant, an increase in viscosity as a result of a decrease in water temperature causes an increase in discharge of sands because the change in viscosity causes a decrease in the fall velocity of the sand grains. In much the same way, a high concentration of fine sediment may increase the apparent viscosity of the water and thus decrease the fall velocities of the sand grains.

Graphs based on the empirical relationships of discharge of sands to major variables were prepared for a wide range of velocity, depth, water temperature, and concentration of fine sediment; they provide a method for quickly approximating the discharge of sands in sand-bed streams. In spite of many inaccuracies in the available data and uncertainties in the graphs, about 75 percent of the sand discharges that were used to define the relationships were less than twice or more than half of the discharges that were computed from the graphs of average relationship.

The agreement of computed and observed discharges of sands for sediment stations whose records were not used to define the graphs seemed to be about as good as that for stations whose records were used.

INTRODUCTION

Two practical objectives of sedimentation research are to understand the effects of major factors on sediment discharge in streams and to develop methods for computing the sediment discharge. An understanding of the effects of the major factors is essential to evaluate correctly the somewhat incomplete and inexact information available on sediment discharge and on the behavior of sediment-laden streams. Methods for computing sediment discharge are needed in studies and computations relating to channel design and maintenance, sediment yields from drainage basins, rates of scour or fill in natural or artificial channels, and depletion of reservoir storage by sediment accretion.

Because sediment transportation varies with complex characteristics of sediment and flow, an

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accurate and always applicable method for determining sediment discharge from characteristics of drainage basins, transported sediment, and streamflow may never be developed. If such a method is ever devised for all types of sediment, drainage areas, channels, and flows, it is likely to be unmanageably complex and generally impracticable because of the large amount of field information and the complicated office procedures that would be necessary. However, the relating of sediment discharge to characteristics of streamflow is much simplified (although it still remains difficult) if the sediment discharge is computed for only those particle sizes generally present in the streambed in appreciable quantities and if only streams with beds of sand are considered.

The main purpose of this paper is to indicate in as simple, accurate, and practical a form as possible the major relationships between the discharge of sands in sand-bed streams and the characteristics of flow and sediment. Secondary purposes are to provide a rapid method for computing the discharge of sands in sand-bed streams and to show deficiencies in the information that is necessary to define more accurately the relationships between discharge of sands and characteristics of flow and sediment.

Because the relationships are complex and are inexactly defined, they have been expressed in graphs rather than in equations. The graphs are based on laboratory and field information and on a general concept of the theories and equations of sediment transportation. They express the discharge of sands for flows as shallow as 0.1 foot and as deep as 100 feet, although for the deepest flows the graphs are poorly defined extrapolations. Bed-material sizes on the graphs are shown only as median diameters and range from 0.1 to 0.8 mm. The relationships for the largest and the smallest sizes are based to a considerable degree on extrapolation.

First, a concept of sediment transportation is given on the basis of recognized theories and personal observations of water and sediment discharge. It provides a background for understanding the major factors that can be expected to affect the discharge of sands in sand-bed streams. Then, the quantitative effects of the major factors as empirically defined by available information are individually shown and discussed. Finally, graphs are given for computing the approximate discharge of sands in sand-bed streams; and the probable accuracy, limitations, and possible improvements of the graphs and relationships are suggested.

Studies on which this paper is based were made over a period of several years; some were made while the writer was working for the Agricultural Research Service, U.S. Department of Agriculture.

Unpublished data for Pigeon Roost Creek in northern Mississippi and its tributaries were used in some of the studies and were from the files of the Agricultural Research Service, University, Miss. Much unpublished information collected by the Geological Survey was made available by: D. M. Culbertson, district engineer, for streams in Nebraska and for the Mississippi River at St. Louis, Mo.; J. M. Stow, district chemist, for streams in New Mexico and for the Colorado River; D. B. Simons, project chief, for flume investigations at Fort Collins, Colo.; and T. F. Hanly, district engineer, for streams in the Bighorn River basin in Wyoming.

CONCEPT OF SEDIMENT TRANSPORTATION

An understanding of the causes of sediment movement by streamflow and the general pattern of the movement is necessary for developing sound relationships for computing sediment discharge. Such a concept cannot be precise, however, partly because our knowledge is not complete and partly because our minds cannot wholly grasp the complexities of all the complete and detailed relationships involved in sediment transportation. Fortunately, a concept of sediment movement in flowing water may be adequate for many studies if it includes approximately correct interrelationships of major factors and characteristics of the flow, the sediment particles, and the sediment discharge. An attempt will be made to state a useful concept in terms of theories, equations, and observations of streamflow and sediment movement.

The statement of this concept is divided into several parts. The complexity of sediment transportation is explained briefly; then, the assumptions made to simplify and to limit the scope of the studies and the relationships for this paper are stated. Next, each of the two general modes of sediment discharge—bedload discharge and suspended-sediment discharge—are discussed separately; and finally, the total discharge of sediment at a cross section is considered.

COMPLEXITY OF SEDIMENT AND STREAMFLOW RELATIONSHIPS

The relationships of sediment discharge to characteristics of sediment, drainage basin, and streamflow are complex because of the large number of variables involved, the problems of expressing some variables simply, and the complicated relationships among the variables. At a cross section of a stream, the sediment discharge may be considered to depend: on depth, width, velocity, energy gradient, temperature, and turbulence of the flowing water; on size, density, shape, and cohesiveness of particles in the banks and bed at the cross section and in upstream channels; and on the geology, meteorology, topography, soils, subsoils, and vegetal cover of the drainage area. Obviously, simple and satisfactory mathematical expressions for such factors as turbulence, size and shape of the sediment particles in the streambed, topography of the drainage basin, and rate, amount, and distribution of precipitation are very difficult, if not impossible, to obtain.

The concentration of the moving sediment is generally nonuniform both laterally and vertically in the cross section, and each sediment size (more precisely, sediment of each fall velocity) will have a vertical and lateral distribution that differs from the distribution of sediment of any other size. Similarly, the depth, velocity, turbulence, shear, and other characteristics of flow vary, sometimes widely, across a stream. Thus, a single value such as an average depth, a mean velocity, or a median particle size may be a somewhat unsatisfactory measure of depth, velocity, or particle size for a cross section. Furthermore, the lateral and vertical distributions of sediment and flow depend partly on nearby cross sections and on channel alinement. The flow may be over a relatively firm and smooth bed or over bars, dunes, or antidunes; and the bed configuration may vary laterally, longitudinally, and with time. In addition, sediment particles interfere with each other at high concentrations, and the chemical composition of the water may affect the degree of flocculation of fine sediment. Because the flow pulsates and the coarse sediment tends to discharge erratically both in time and in lateral position in the flow, observations of velocity, discharge of coarse sediment, and slopes of the water surface must be made carefully over a considerable period of time to obtain satisfactory averages. An adequate knowledge of these difficulties indicates that (a) sediment discharge cannot, in general, be precisely computed from characteristics of the flow, the sediment, and the drainage basin and (b) some assumptions are necessary to reduce the problem of sediment transportation to manageable proportions.

SIMPLIFYING ASSUMPTIONS

The assumptions made for this study should be well understood because they limit significantly the scope and applicability of the discussions and results. The derived relationships should only be applied to compute discharge of sands for streams and cross sections that have characteristics reasonably consistent with the assumptions.

When one cross section is compared with others, sediment discharge is assumed to be proportional to width if other major variables are constant from section to section. Thus, sediment discharge is computed and discussed in terms of discharge per unit width. The assumption does not imply a uniform distribution of sediment discharge across a section. It implies only that the sediment discharge through two sections should be about proportional to width if the sections have the same mean velocity, average depth, bed sediment, water temperature, and concentration of fine sediment.

Mean velocity, average depth, and average shear at a cross section are assumed to be acceptable measures of the actual nonuniform velocities, depths, and shears across the section. Obviously, this assumption will not always be satisfactory. Some irregular cross sections should be subdivided into parts, and separate computations of sediment discharge should be made for each part. However, the error in using a mean velocity for an entire nonuniform cross section may be partly compensated, or even overcompensated, for by the assumption that sediment discharge is proportional to stream width.

For example, if 30 feet of the width of a 100-foot section is shallow and has a very low velocity, almost the entire discharge of sands may be within the other 70 feet of width. The mean velocity may be only a few percent lower for the whole section than for the 70-foot width (mean velocity is weighted with width and depth, not with width alone). Hence, even though the discharge of sands may vary as the third or fourth power of the mean velocity, the computed decrease in discharge of sands per foot of width for the 100-foot width as compared with the 70-foot width will be partly compensated, or even overcompensated, for when multiplication is by 100 feet rather than by 70 feet.

Cross sections for determining sediment discharges are assumed to be in reasonably straight reaches of channel.

The bed sediment is considered to be adequately represented by the median particle size, although for certain somewhat extreme conditions of flow, such as the beginning of sediment movement, low discharges of sands, or deep flows, the assumption is not generally satisfactory.

The discharge of fine sediment is assumed to be more closely related to supply of fine sediment in the drainage basin than to conditions of flow at a cross section, and hence no attempt is made to compute the discharge of fine sediment from characteristics of the flow at a cross section. (However, some attention is given to the effect of high concentrations of clay on the discharge of sands.) This assumption is in line with findings of Einstein and Chien (1953) that the discharge of fine sediment theoretically follows the same laws as the discharge of sands but that limitations on sampling the fine bed sediment and on applying the laws make the computation of the discharge of fine sediment impracticable and unreliable. Theoretically, the dividing size between sediment too fine to have its discharge related satisfactorily to characteristics of flow at a cross section and sediment whose discharge can be so related should vary with mean velocity and other factors. For this paper the dividing size, which is arbitrary because no sharp division exists, has been selected as 0.062 mm., the lower limit of sand sizes.

Only stream reaches whose beds are composed almost entirely of nearly cohesionless sand and whose channels are not confined between rough rock walls are considered. Such walls may cause unnaturally high turbulence for a given mean velocity and depth.

BEDLOAD DISCHARGE

When water flows very slowly over a bed of sand, none of the sand grains may move. If velocity near the bed is slowly increased, a critical velocity will be reached at which some sand grains occasionally move along the bed for short distances and then stop. Obviously, this critical velocity is somewhat indefinite because initial movement of the grains depends on the arrangement of the grains and on local variations of velocity. Hence, Einstein (1950) related the picking up of the grains from the bed and the beginning of movement to the probability of individual grains to move. This reasonable concept of Einstein indicates that any critical velocity can only be an inexact and partly arbitrary measure of the beginning of significant bedload movement.

If the velocity near the bed is greater than critical, sand grains move intermittently by rolling, sliding, or skipping along the bed. The movement is within a very thin layer, called the bed layer, a few grain diameters thick. (The bed-layer concept is, of course, inexact, especially for a dune bed and a wide range of grain sizes in the bed sediment.) The grains which thus move in the bed layer and which are supported mainly by contact with the streambed compose the bedload. In general, the rate of travel of these grains while in motion and the frequency with which the grains begin intermittent movement depend on the velocity of flow near the bed. Obviously, the particle sizes of bed sediment and the difference in density between the sediment and the water are also significant factors that affect bedload discharge. The viscosity of the fluid probably also has an appreciable effect. There are three generally known procedures by different investigators for computing bedload discharge.

MEYER-PETER AND MÜLLER EQUATION

A reasonably simple and evidently fairly accurate formula for bedload discharge was developed by Meyer-Peter and Müller (1948). For channels having negligible bank friction, the formula may be written

$$\gamma_f(K_r/K_r')^{3/2}RS=A(\gamma_s-\gamma_f)D_m+B(\gamma_f/g)^{1/3}(q_B')^{2/3}, \quad (1)$$

in which

- γ_f and γ_s are the specific weights, respectively, of the fluid and the sediment particles;
- K_r is a measure of resistance to flow and equals the mean velocity $\bar{u}$ divided by $(R^{2/3}S^{1/2})$;
- K_r' is a comparable measure of the flow resistance due only to the resistance of the stationary grains and for metric units can be computed from $26/(D_{90})^{1/6}$. D_{90} is the particle size, in meters, for which 90 percent of the bed sediment by weight is finer;
- R is the hydraulic radius, which equals average depth of flow when the bank resistance is negligible;
- S is the energy gradient;
- A and B are dimensionless constants that may be used as 0.047 and 0.25, respectively;
- D_m is the particle size of uniform sediment and equals $\Sigma D i_b$ for bed sediment of a range of sizes; i_b is the fraction by weight of sediment of a size D in the streambed;
- g is the gravity constant; and
- q_B' is the bedload discharge per unit width by weight under water.

Equation 1 for bedload discharge, in foot-pound-second units, when γ_s is equal to 165 pounds per cubic foot, can be written as

$$(K_r/K_r')^{3/2}RS=0.077D_m+0.0050(q_B')^{2/3}.$$

For convenience, the bedload discharge per foot of width can be expressed in terms of the dry weight per foot of width (q_B) through the relationship

$$(q_B')^{2/3}=[(165-62.4)/165]^{2/3}(q_B)^{2/3}$$

and equation 1 becomes

$$(K_r/K_r')^{3/2}RS=0.077D_m+0.0036q_B^{2/3}. \quad (2)$$

According to equation 2, no bedload moves until the left-hand side exceeds $0.077D_m$. For high velocities and small particle sizes, $0.077D_m$ becomes relatively small. If it is disregarded and (K_r/K_r') is constant, the computed bedload discharge varies as $(RS)^{3/2}$ or as the third power of the mean velocity if the Chezy C is constant.

The ratio K_r/K_r' in equation 2 equals the ratio of the actual mean velocity to the mean velocity if all resistance to flow were due to the bed roughness that the grains would cause if stationary on a plane streambed. (The equation is based on assumed negligible bank friction.) A convenient way to evaluate K_r/K_r' in foot-pound-second units is to compute the ratio of the actual velocity to the velocity that can be determined from an equation of the type given by Keulegan (1938) and from a roughness equal to the D_{65} size of the bed material as suggested by Einstein (1950). In whatever way the ratio is computed, K_r/K_r' is an adjustment that makes the left side of equation 2 vary much more closely with mean velocity than with RS or with shear with respect to the particles rather than with γRS.

The close relationship between bedload discharges computed from the Meyer-Peter and Müller equation and mean velocity can readily be shown by an example for a cross section of Pigeon Roost Creek near Byhalia, Miss. Within a narrow range of depths and shears, two considerably different flows may occur at a particular depth at this cross section because of differences in configuration of the streambed. The bed sediment at the cross section is cohesionless sand that has a median diameter of 0.40 mm. D_m in equation 2 is 0.45 mm, and D_{90} is 0.66 mm. Bedload discharges were computed from equation 2 for five velocities and depths of flow; the three lowest velocities were over dune beds, and the two highest were over plane beds or a combination of plane bed and antidune bed. The computed bedload discharges increase rapidly and consistently as mean velocity increases, but a break exists in their relationship to RS. (See fig. 1.) Evidently, RS may be a poor measure of the bedload discharge unless RS is adjusted by some factor, such as K_r/K_r', that corrects for the effect of changes in resistance to flow that are due to changes in bed roughness.

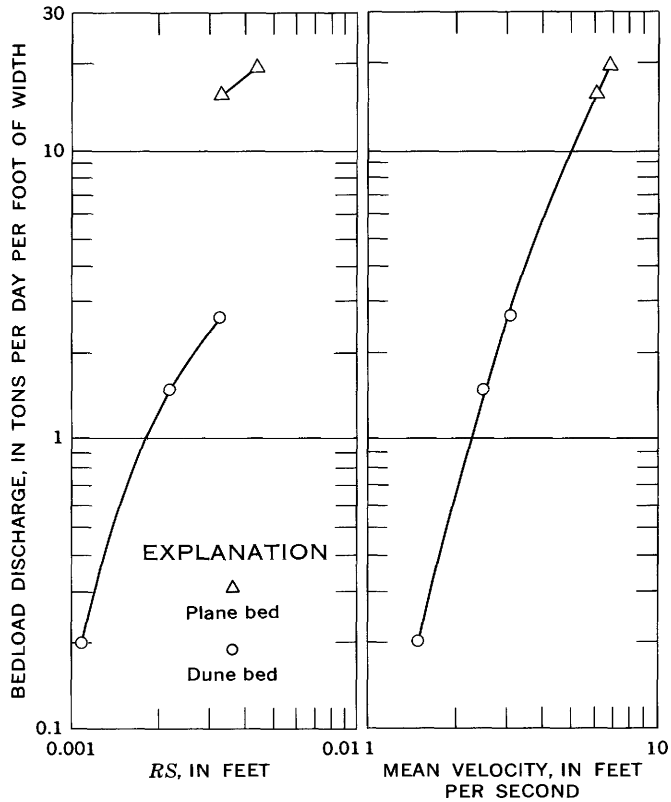


FIGURE 1.—Bedload discharges, computed with the Meyer-Peter and Müller equation, as functions of RS and mean velocity.

EINSTEIN EQUATIONS

Einstein (1950) suggested a comprehensive procedure for the computation of bed-material discharge from the characteristics of flow and bed sediment. In this procedure, the part of the bed-material discharge that is bedload was computed on the basis of the probability of movement of particles at the bed surface as related to intensity of flow. Einstein's bedload equations for uniform sediment can be written

$$\Phi=\frac{q_B}{\rho_s g}\left(\frac{\rho_f}{\rho_s-\rho_f}\frac{1}{gD^3}\right)^{1/2},$$

in which ρ_f and ρ_s are densities of the fluid and the sediment particles, respectively. Also,

$$\Psi=\frac{\rho_s-\rho_f}{\rho_f}\frac{D}{R'S}$$

and

$$\Phi=f(\Psi).$$

For constant densities and for one consistent system of units of measurement these three equations of Einstein's become

$$q_B=K_1D^{3/2}\Phi$$
$$=K_1D^{3/2}f\left(\frac{K_2D}{R'S}\right), \qquad (3)$$

and K_1 and K_2 are constants. R' is the hydraulic radius that could be used to compute velocity if all the resistance to flow were due to the roughness that might be caused by the sediment particles if stationary on the bed.

Einstein used R' in equations based on work by Keulegan (1938), but R' can be used equally well in a Manning equation; that is, R' can be stated in terms of R, the actual hydraulic radius (or average depth of flow for negligible bank friction), as

$$\bar{u}=K_r R^{2/3} S^{1/2}$$
$$=K_r'(R')^{2/3} S^{1/2};$$

hence

$$R'=R(K_r/K_r')^{3/2}.$$

Einstein's equation for q_B then may be written, in which K_3 is a constant, as

$$q_B=K_3 D^{3/2} f\left[\frac{D}{(K_r/K_r')^{3/2} RS}\right].$$

Thus, Einstein's parameter for the effect of flow on bedload discharge of uniform particle size is like that used in the Meyer-Peter and Müller equation (see eq. 2). Einstein's graph of the function (fig. 2) shows, of course, that Φ_* and, hence, q_B increase as $D/[(K_r/K_r')^{3/2} RS]$ decreases.

Einstein stressed the need for the adjustment from R to R', an adjustment that ties the bedload discharge more closely to mean velocity than to RS. However, he computed the relationship between R and R' through the use of a curve (Einstein, 1950, fig. 5) that may be a good average but sometimes does not apply well (Brooks, 1958, p. 588–589).

If the bed sediment is not uniform in size, the Einstein procedure requires considerable adjustment of Φ and Ψ. The adjusted quantities for individual size classes or size fractions of bed sediment are Φ_* and Ψ_*. Einstein's graph of Φ_* as a function of Ψ_* (fig. 2) shows a rapid decrease of bedload discharge when the velocity, as measured by $R'S$, becomes low in relation to the particle size; that is, when Ψ_* becomes large. The bedload discharge of a size fraction is directly proportional to the percentage by weight of that size fraction in the bed sediment. In other words, if the percentage of bed sediment in a size range is doubled but Φ_* remains constant, the bedload discharge of that size range is also doubled. The statement of proportionality does not mean that the bedload discharges for different size fractions are in direct proportion to the relative percentages of the size fraction in the streambed.

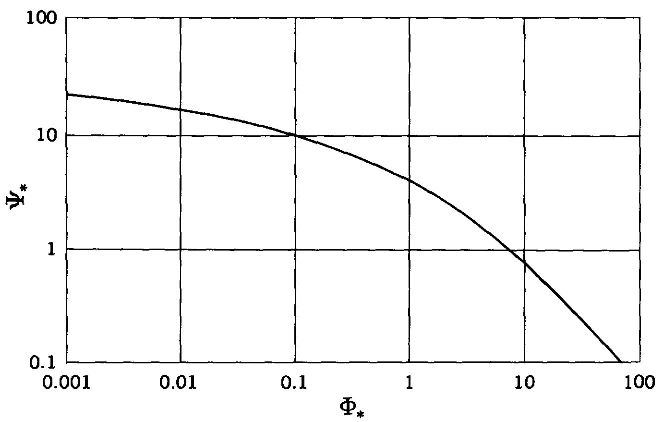


FIGURE 2.—Relationship of Φ_* to Ψ_* for individual size ranges. From Einstein (1950, fig. 11).

BAGNOLD EQUATION

Using a different approach from that of Einstein, Bagnold (1956) derived a formula for computing bedload discharge similar to Einstein's. For bed sediment of uniform size, his equation (Bagnold, 1956, eq 42c) is

$$q_B=A_1 B_1 K_1 D^{3/2}\sqrt{\cos\beta}\,(\theta_F-\theta_t)\theta_F^{1/2}, \tag{4}$$

in which

- A_1 is a dimensionless constant that may be about 9,
- B_1 is a dimensionless constant that varies slowly with particle size,
- K_1 is the same constant that was used in equation 3,
- θ_F is a dimensionless measure of the apparent tangential stress due to action of external forces on the fluid and equals $[\rho_f/(\rho_s-\rho_f)](RS/D)$,
- θ_t is that value of θ_F (determined by extrapolation) at which sediment particles begin to move on a rippled sand bed, and
- β is declination of the bed surface below the horizontal, and $\cos\beta$ generally can be considered to equal unity and be disregarded.

Except for the use of R instead of R', Bagnold's θ_F is the reciprocal of Einstein's Ψ. Of course, the bedload discharge increases as θ_F increases or as Ψ decreases. Also, for a particular grain size, for constant densities, and for moderately high velocities—for which θ_t becomes relatively insignificant—the bedload discharge according to equation 4 is about proportional to $(RS)^{3/2}$. For these stated conditions, the bedload discharge is also about proportional to the third power of the mean velocity if the Chezy C is constant.

A major difference between the Bagnold equation for computing bedload discharges and the equations of Einstein or Meyer-Peter and Müller is that Bagnold made no direct provision for the effect of changing resistance caused by changing bed configuration (if θ_t is intended to be such an adjustment, it is inadequate for most natural streams); that is, Bagnold used neither an R' instead of R nor a K_r/K_r' ratio. If RS is as inadequate a measure of bedload discharge as figure 1 indicates that it may sometimes be, Bagnold's equation may generally not be as applicable as the equations of Einstein and Meyer-Peter and Müller.

MAJOR VARIABLES

The general concept of bedload discharge and certain mathematical statements of the concept indicate that no sediment may move as bedload at very low velocities. At relatively high velocities, the bedload discharge may vary roughly with the third power of the mean velocity. Hence, mean velocity—or more exactly, velocity near the streambed—must be a highly significant variable in the computation of bedload discharge. Shear (or RS) is an alternate major variable and might be used rather than velocity, but an adjustment of the shear from total shear to shear with respect to the particles evidently is required for changing bed roughness. If mean velocity is used as a parameter of bedload discharge, depth can be expected to affect bedload discharge because for a given velocity near the streambed the mean velocity usually increases as the depth of flow increases.

The emphasis here placed on mean velocity as a measure of bedload discharge is wholly consistent with the usual concept that shear with respect to the particles is a major parameter of bedload discharge. Unfortunately, shear with respect to the particles can at present be obtained only indirectly and often inexactly except for flow over plane beds. Usually it can be computed as accurately from mean velocity and depth as in any other way. Thus, mean velocity is a practical and convenient measure of bedload discharge to substitute for shear with respect to the particles.

Another significant variable is the bed sediment. The size of the bed sediment, as measured by the median diameter and the size distribution, has three noteworthy effects on bedload discharge. One effect is that particle size largely determines the velocity that is necessary near the bed to start appreciable sediment movement. Particle size also partly determines resistance to flow because it directly determines grain roughness and affects bed configuration. A third effect is that particles of different sizes discharge at different rates even though the velocity near the bed is constant.

The properties of the fluid may also affect the discharge of sands as bedload. The density of the fluid is included in each of the three bedload equations that were discussed, but the practical effect of changes in the different densities of fluid and sediment is usually small because these densities are generally about constant in sand-bed streams. Changes in viscosity of the fluid or of the fluid-sediment mixture seem to have an effect on the relationship of shear to mean velocity because these changes may modify the bed configuration.

Thus for sand-bed streams, the discharge per foot of width of sands as bedload can be expected to vary mainly with mean velocity, depth of flow, median particle size or median fall velocity of bed sediment, and size distribution of the bed sediment. If the velocity near the bed could be satisfactorily determined, perhaps the only other major parameter that would usually have to be considered in determining bedload discharge in sand-bed streams would be the sizes of the bed sediment.

SUSPENDED-SEDIMENT DISCHARGE

Although in sand-bed streams much more sand is generally discharged as suspended sediment than as bedload, bedload discharge was discussed first because it forms a base to which the suspended sediment adjusts itself. The most direct relationship of suspended sediment to bedload results from continuity of the concentrations of suspended sediment of each size range at the upper surface of the bed layer; that is, the concentrations of suspended sediment immediately above the bed layer depend on the concentrations within the bed layer. Suspended-sediment concentrations at higher levels in the flow depend, in turn, on the concentrations immediately above the bed layer and on the vertical attenuation of the concentrations of sediment of the different sizes from the bed layer upward.

CONCENTRATION IMMEDIATELY ABOVE THE BED LAYER

If the velocity near the streambed is great enough, some sand grains, perhaps usually the lightest or the fastest moving ones, escape from the bed layer by hydraulic lift, aerodynamic lift (R. E. Glover, written communication, 1960), or other mechanical or hydraulic action. Grains that rise above the bed layer may be supported by the turbulence of the flow and thus become suspended sediment rather than bedload. Of course, if the flow contains suspended sediment, some particles of the suspended sediment are continually settling into the bed layer. The ready interchange of particles from bedload to suspended load or of suspended load to bedload means that the suspended-sediment concentration just above the bed layer adjusts itself to changes in discharge of bedload.

Einstein (1950) used the assumption that the concentration of each particle size of suspended sediment at the upper surface of its own 2-grain-diameter bed layer was equal to the concentration of that particle size in the bed layer. This assumption may be only approximate, and the idea of separate thicknesses of bed layer for each particle size may be somewhat unrealistic. However, changes in the concentration of suspended sediment of each size class at the upper surface of the bed layer should be roughly proportional to changes in concentration of particles of that size class within the bed layer. If the thickness of the bed layer and the average velocity in the bed layer remain constant, changes in the concentration of sediment of a given size class in the bed layer will be proportional to changes in bedload discharge of that size class of sediment. Hence, the same variables that determine the bedload discharge will mainly determine the suspended-sediment concentrations at the upper surface of the bed layer and immediately above it. Thus, the major variables that affect concentration of suspended sediment immediately above the bed layer are mean velocity, depth of flow, and particle sizes or fall velocities of the bed material.

VERTICAL DISTRIBUTION OF SEDIMENT CONCENTRATION

A detailed discussion of the concepts that relate to the vertical distribution of suspended sediment seems to be necessary for two reasons. The first reason is that much more sand is transported in most streams as suspended sediment than as bedload. The second reason is that the factors affecting the vertical distribution of sediment are incompletely defined and their relationships to the discharge of sands are complex. Available experimental data are generally inadequate to determine the factors and their relationships except, perhaps, for narrow ranges of the factors. Therefore, empirical analysis of the data should be based on as adequate and practical an understanding of the basic concepts as can be obtained.

By definition, suspended sediment is supported by the vertical currents in turbulent flow. These currents exist because of vertical eddies or random vertical movements of some parts of the flow relative to other parts of the flow. The amount of the vertical movement in either direction is largely a function of the time-averaged horizontal velocity, although roughness of the channel boundary and other factors have considerable effect. Hence, the mean velocity of a stream is a rough measure of the turbulent currents that suspend sediment. If the water surface and the streambed are plane and parallel, any upward movment of part of the flow must be balanced by a downward movement of another part of the flow. If the sediment particles were uniformly distributed in the turbulent flow and did not settle with respect to it, the rate of upward sediment movement, like the rate of upward water movement, would on the average equal the rate of downward sediment movement through any area of plane parallel to the water surface.

Because sediment particles settle with respect to the surrounding water, vertical equilibrium requires that the number of sediment particles raised by the upward components of flow must exceed the number lowered by the downward components of flow. Thus, equilibrium may be reached if the upward movement of water starts at a lower altitude where the concentration is comparatively high and if the downward movement of water starts from a higher altitude where the concentration is comparatively low. The sediment in both upward and downward components of the flow is assumed to move an appreciable distance before mixing with other water. If this vertical distance of movement before mixing is the same for all particle sizes, the ratio of the number of particles that starts moving upward to the number that starts moving downward is directly related to the fall velocity; that is, in a particular flow the fall velocity is a measure of the rate of change of concentration with distance above the streambed. For a constant fall velocity, an increase in turbulence means an increase in the vertical movements of the flow and results in a lower rate of change of concentration with distance above the streambed. Thus, the major parameters that determine vertical variation of concentration of sediment in a flow are fall velocity and turbulence. Mean velocity is probably the best available measure of turbulence, although relative roughness of the channel boundary is likely to affect the relationship between mean velocity and turbulence.

THE CLASSICAL EQUATION FOR SEDIMENT OF ONE FALL VELOCITY

The theoretical equation for the vertical distribution of concentration of suspended sediment of a particular fall velocity was given by Rouse (1937) and was restated by Einstein (1950, p. 17) in the familiar form

$$\frac{c_y}{c_a}=\left(\frac{d-y}{y}\,\frac{a}{d-a}\right)^z \tag{5}$$

and

$$z=\frac{w}{ku_*}, \tag{6}$$

in which

- c_y is the concentration at a distance y above the streambed,
- c_a is the concentration at a distance a above the streambed,

d is the depth of flow,
w is the fall velocity of the particles,
k is the turbulence constant, and
u_* is the shear velocity and is equal to $\sqrt{gRS}$

The exponent z is the theoretical slope of a log-log graph of concentration of sediment of one fall velocity against $(d-y)/y$ and increases as the rate of change of the concentration with depth increases. When z is zero, the vertical distribution of concentration is uniform.

Equation 5, in which z is defined as in equation 6, is the classical equation for the vertical distribution of suspended sediment. Theoretically, it can be used to compute the vertical distribution of concentration of suspended sediment of any particular fall velocity in turbulent flow over a plane bed that is parallel to the water surface if the suspended sediment concentration is known at any one distance above the streambed.

THEORETICAL AND OBSERVED DISTRIBUTIONS

Observed vertical distributions of sediment concentration in a natural stream (Anderson, 1942) and in a laboratory flume (Vanoni, 1946) checked equation 5 to the extent that the log-log graphs of concentration against $(d-y)/y$ defined approximately straight lines. However, the slope z_1 determined from a line was usually different from the theoretical slope z computed from equation 6. Vanoni (1946) and Ismail (1952) considered that the coefficient of sediment transfer is not the same as the coefficient of momentum transfer. Einstein and Chien (1954) suggested other changes in the relatively simple assumptions on which equations 5 and 6 are based in order to make z and z_1 agree better. However, they apparently restricted their explanations to flows over plane beds, whereas equation 6 is likely to be less accurate for flows over dune beds than for flows over plane beds. Their suggestions merit careful study but are generally too complicated and uncertain for use at the present time.

Because the vertical distribution of sediment concentration is an important phase of sediment transportation particularly in deep flows, equation 6 is here examined to show some possible uncertainties and inaccuracies that are involved in its use. A few of the uncertainties and inaccuracies result directly from the assumptions on which the equation is based. Each of the variables w, k, and u_* is considered individually.

The variation of z_1 with fall velocity can be defined experimentally from the concentrations and size analyses of sediment samples that are collected at several points on a vertical during a short period of time when the turbulent flow over a sand bed is reasonably constant. A value of z_1 for each size range at a vertical can readily be determined by plotting. Because ku_* is the same for all size ranges, the values of z_1 for the different size ranges should vary in proportion to the fall velocities of representative particles from each size range, if the numerator of the right side of equation 6 is correct. However, the values of z_1 thus determined for sands in the Enoree River (Anderson, 1942) did not increase as rapidly as the values of z and were generally smaller for the coarser particles than the corresponding values of z computed from equation 6. Similarly, Colby and Hembree (1955) found that the values of z_1 for different ranges of sand sizes in several natural streams varied roughly with about the 0.7 power of the fall velocity as computed from equations given by Rubey (1933). The relationship of z_1 to fall velocity was based on the slopes of lines that might be drawn through points on figure 3 for a given time and cross section. The first power of the fall velocity from the Rubey equations evidently is an inadequate measure, at least for sands, of the change in z_1 with change in particle size within a given flow. The vertical distributions of concentration of the coarse sizes of sediment are more uniform than equations 5 and 6 indicate that they should be.

The first power of the mean velocity may be an inadequate measure of the change of z_1 with particle size for any one of several reasons or for a combination of reasons. These reasons include the possible effects of using an average vertical velocity rather than a range of vertical velocities and of disregarding higher order terms for the concentration gradients of coarse sediments (Einstein and Chien, 1954). Other possible reasons are mass movement of water and sediment from the bed upward, inapplicability of a laboratory determination of fall velocity of a representative particle of a size range as a measure of the effective fall velocity in the turbulent flow of a natural stream, and inadequacy of the assumptions regarding the mixing length.

On figure 3 the wide vertical scatter, which is relatively consistent from one size range to another, indicates that $0.40u_m$ does not satisfactorily show the effect that the characteristics of the flow have on z_1, at least for these shallow flows that were mostly over dune beds. Colby and Hembree (1955, p. 75) suggested that large variations in k might explain much of this vertical scatter. Hence, the significance and variability of k, especially for flow over dune beds, may help to explain differences between z and z_1.

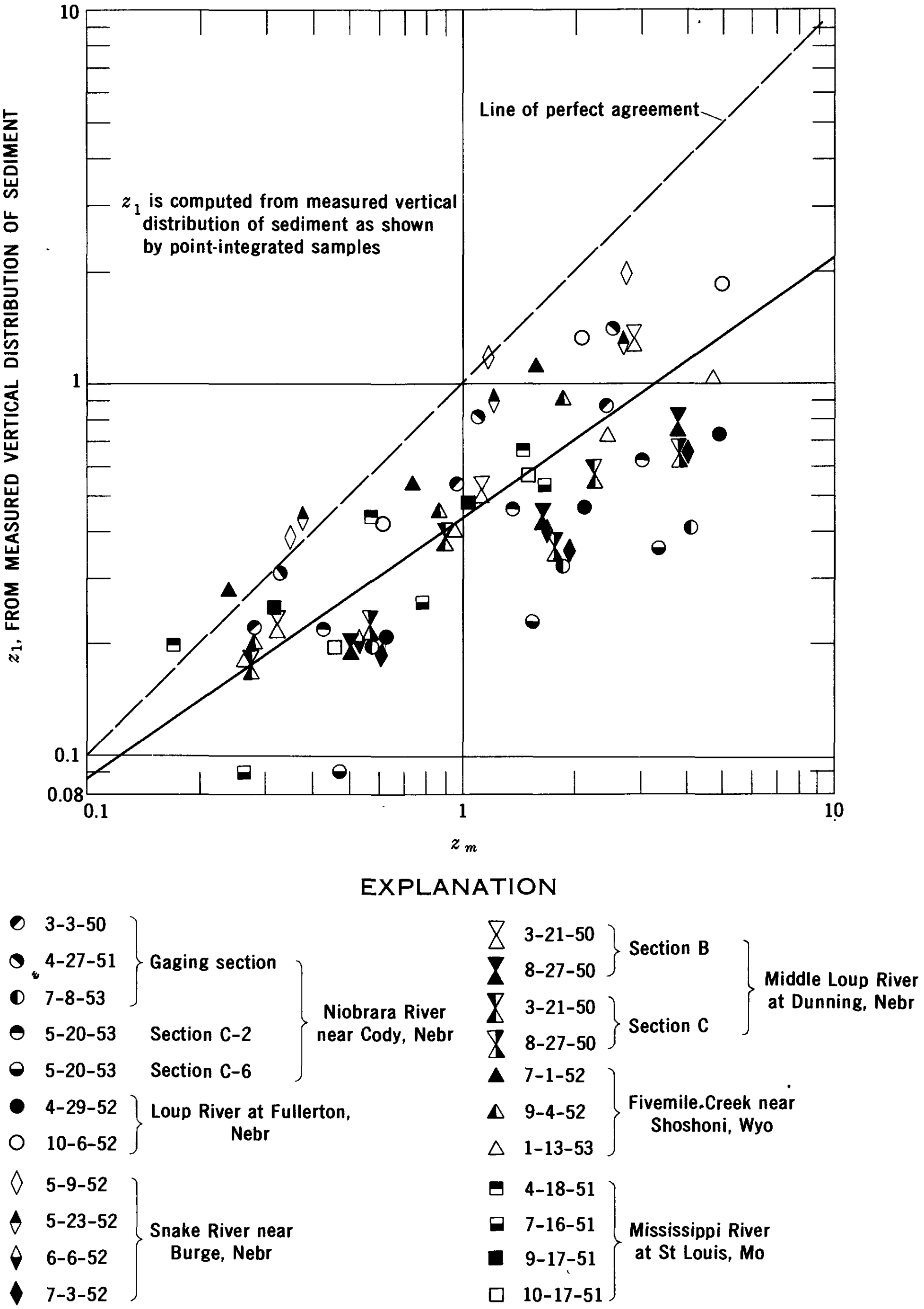


FIGURE 3.—z_1 plotted against z_m. $z_m = w/(0.40u_m)$ and $u_m = 0.40\bar{u}/[2.30 \log (12.27\, xd/D_{65})]$. From Colby and Hembree (1955, fig. 40).

EQUATION FOR FLOW OVER DUNE BEDS

The equations of the type given by Keulegan (1938) for mean velocity $\bar{u}$ and for time-averaged velocity at a point u_y may be written, respectively,

$$\bar{u}/2.30=\frac{u_*}{k}[1.09+\log (xd/k_s)] \qquad (7)$$

$$u_y/2.30=\frac{u_*}{k}[1.48+\log (xy/k_s)], \qquad (8)$$

in which

- x is a dimensionless parameter to cover the transition from hydraulically smooth to hydraulically rough boundaries, and
- k_s is a linear measure of roughness and may be taken equal to D_{65} (the size for which 65 percent of the bed material by weight is finer) if the roughness is due wholly to grain roughness.

Usually, k_s is large and is not known for flow over dune beds, and therefore these equations cannot then be used to compute velocities directly. Equation 8, however, shows that the difference in point velocity at distances $10y$ and y above the streambed is $2.30u_*/k$, which is the same as the difference for flow over a plane bed provided that equations 7 and 8 apply; that is, the vertical distribution of velocity depends on the total shear over a dune bed the same as it does over a plane bed, if equation 8 is applicable.

Three types of information indicate that equations (7) and (8) do not necessarily apply to shallow flows over dune beds. The first type shows that the vertical-velocity distribution changes (and hence k changes) as the measuring vertical is shifted with respect to a dune. For example, in the Middle Loup River at Dunning, Nebr., two values of k computed from vertical distributions of velocity at the crest of a dune were 0.32 and 0.62 as compared to k values of 0.11 and 0.19 for verticals in a trough downstream from the crest. A second type of information shows that some k's as computed from equation (8) and from observed vertical distributions of velocity are much higher than 0.40 for shallow flows over dune beds (Hubbell and Matejka, 1959, p. 71), whereas, in general, k values for flows that transport bed material in appreciable quantity probably should be somewhat lower than 0.40 (Einstein and Chien, 1954). The third type of information, obtained by Sayre and Albertson (1959) for clear-water flow, rigid boundaries, and fixed baffles, shows that the k computed from an equation similar to equation (7) differs from the k computed from an equation similar to equation (8) for the same flow if the baffles are spaced too widely apart. In other words, the vertical distribution of velocity is determined partly by position of the vertical with respect to a baffle if the spacing of baffles is wide. The variability of vertical velocity distribution and k from place to place in the clear-water flow is consistent to that for the Middle Loup River at Dunning. On the basis of these three types of information, the k that is computed from the vertical distribution of velocity in flow over a dune bed or other rough bed is here called "k_1" because it may not be a true turbulence constant.

Although k_1 may not be a true turbulence constant, it seems to have a relationship—as it logically should—to the vertical distribution of sediment concentration as was shown by Hubbell and Matejka (1959, p. 71 and fig. 36). They used values of k_1 (k in their terminology) that were computed by averaging the values of k_1 for individual verticals in the cross section and found that z_1 for shallow flows in some sand-bed streams in Nebraska varied inversely with about the 0.6 power of k_1. For this paper, the k_1 values for the cross sections were recomputed from a vertical-velocity difference and a shear velocity that were averages of figures for the individual verticals. The recomputed k_1 values give less weight to a high k_1 at an individual vertical than did the k values that were used for a cross section by Hubbell and Matejka. In general, the z_1 values vary inversely with about the 0.8 power of the recomputed k_1 values. (See fig. 4.) The quantity z_1u_* also varies inversely with about the 0.8 power of k_1 (fig. 5), whereas equation (6) indicates that it should vary inversely with the 1.0 power of k_1. The difference may result from the fact that an abnormally high k_1 is usually higher than the k_1 for the surrounding flows; that is, the vertical distribution of sediment is probably partly determined by the k_1 at nearby verticals; hence, the average or effective k_1 may often be lower than an abnormally high observed k_1 even though the observed k_1 is itself an average for perhaps three to five verticals.

The use of u_* in equation 6 was not accepted by Einstein (1950), although he obtained u_* in his derivation. He computed z from the equation

$$z=w/(0.40u_*'). \qquad (9)$$

In this equation, u_*' or the shear velocity with respect to the particles is the shear velocity that can be computed from the energy gradient and the part of the hydraulic radius that relates to the movement of sediment particles on the bed; that is, Einstein used the mean velocity equation

$$\bar{u}=\frac{2.30u_*'}{0.40}\log (12.27xR'/D_{65}). \qquad (10)$$

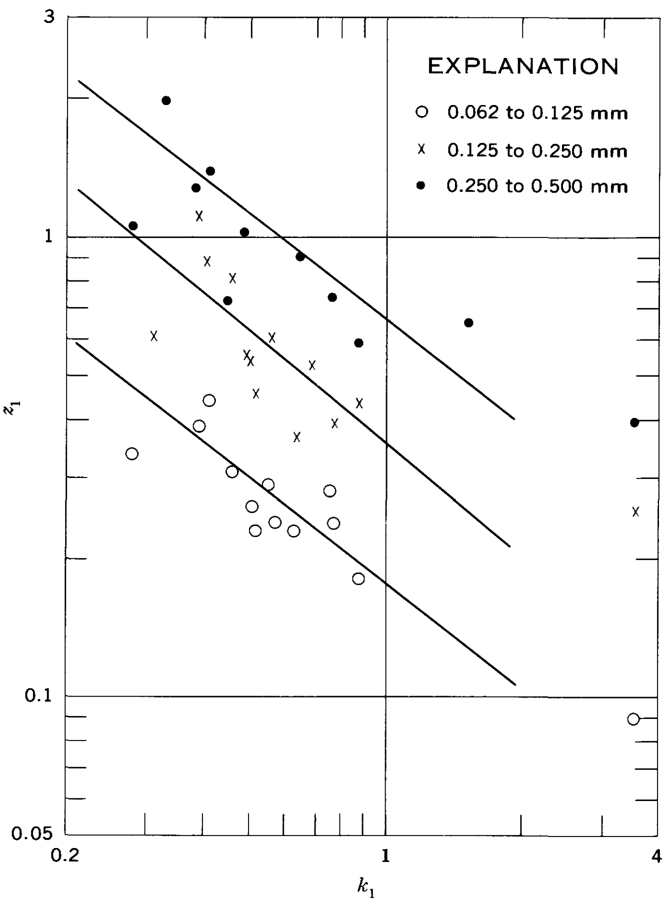


FIGURE 4.—z_l plotted against k_l for shallow streams in Nebraska.

As D_{65} is only a small fraction of the total roughness of a dune bed, a comparison of equations 7 and 10 shows that $u_*'/0.40$ must be much smaller for flow over dune beds than u_*/k_1 (k_1 rather than k because the constant may not be a true turbulence constant). Whether k_1 is about equal to 0.40 or is larger than 0.40, u_* must be considerably larger than u_*' for flow over dune beds, and for such flow the two kinds of shear velocity are by no means equivalent.

A reexamination of Einstein's development (1950, p. 14–17) of equation 6 does not seem to indicate that u_*' should be substituted for u_* in this equation. Einstein used the equation for the vertical distribution of suspended sediment at equilibrium in the form

$$\frac{dc_y}{d_y}=\frac{-c_y w}{vl_e/2}.$$

He then evaluated the product of the average vertical water movement v and the mixing length l_e from the equation for momentum exchange to obtain

$$\frac{dc_y}{c_y}=\frac{-wddy}{u_*^2(d-y)}\frac{du_y}{dy}.$$

By differentiation of equation 8 with k_1 rather than k

$$du_y/dy=u_*/(k_1y).$$

Hence,

$$\frac{dc_y}{c_y}=\frac{-w}{k_1u_*}\frac{ddy}{y(d-y)}. \qquad (11)$$

For convenience define z_o by the equation

$$z_o=w/(k_1u_*). \qquad (12)$$

Equation 11 can now be integrated from a distance a to a distance y above the streambed to obtain

$$\frac{c_y}{c_a}=\left(\frac{d-y}{y}\frac{a}{d-a}\right)^{z_o},$$

which is the same as the classical equation 5 except for the change from k to k_1 in the exponent. If equation 8 applies, k_1 equals k, and the classical equation should apply to flow over a dune bed provided that the equation for momentum exchange also applies to flow over a dune bed. Of course, the equation for momentum exchange is somewhat questionable for flow over a dune bed, especially if the dune height is relatively large in comparison to the depth of flow.

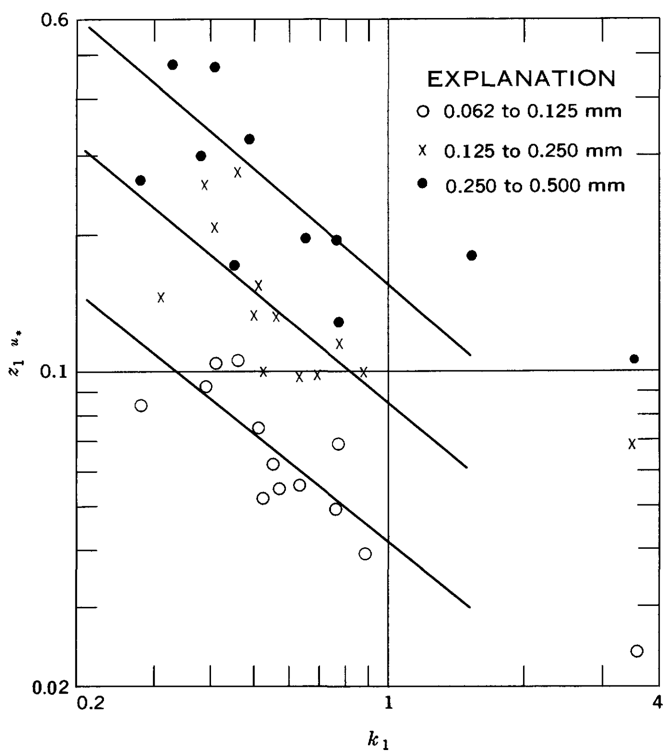


FIGURE 5.—z_lu_* plotted against k_l for shallow streams in Nebraska.

The difference between z_o from equation 12 and z from equation 9 is usually large for flow over dune beds. If k_1 differs appreciably from k, it is usually larger than 0.40. Also, u_* for flow over dune beds is usually much larger than u_*'. Hence, the product k_1u_* is usually much larger than 0.40 u_*', and the computed vertical distributions of sediment concentration that are based on k_1u_* are much more uniform than those that are based (as Einstein suggested) on $0.40u_*'$.

In summary, the vertical distribution of sediment concentration of a particular size range becomes more uniform as the shear velocity and k or k_1 increase and as the fall velocity decreases. However, the classical equation for the vertical distribution holds only inexactly and is especially questionable for flow over dune beds. Hence, a procedure like Einstein's (1950) for computing the discharge of bed material, although based on generally sound concepts, is unlikely to give consistently accurate computations of suspended-sediment discharge particularly for flow over dune beds.

TOTAL SEDIMENT DISCHARGE

Some general relationships involved in the computation of the total sediment discharge at a cross section can be visualized from figure 6, which is consistent with the preceding explanations of a concept of sediment transportation. The figure was prepared for flow at a depth of 5.0 feet and at a mean velocity of 3.5 feet per second over a dune bed and for a bed material in which D_{35} and D_{65} are 0.0007 and 0.001 foot, respectively. (The same depth and energy gradient might be associated with a plane bed, but the velocity would then be much greater and the vertical distributions of concentration would be different.) The vertical distribution of sediment concentration varies widely from one particle size to another.

The complicated interrelationships indicated by the general concept of sediment transportation and partly illustrated by figure 6 preclude any simple modeling in a flume of the sediment transportation in a river. Although many phases of sediment transportation can well be studied in the laboratory, flume investigations, in the absence of satisfactory model laws, are basically studies of sediment transportation in small channels rather than model studies of the sediment transportation in natural streams.

The complexity of sediment and flow relationships and the difficulties of modeling sediment transportation suggest extreme caution in the use of compound parameters, whether dimensionless or not, in studies of sediment transportation. Even the simple ratio d to D can be highly misleading if used as a parameter, because the effect of doubling the depth d is often much different from the effect of dividing the particle size D by 2.

GENERAL METHODS OF COMPUTATION

The total sediment discharge at a cross section of a stream is the sum of the bedload discharges and the suspended-sediment discharges of all particle sizes. The bedload discharge either can be computed directly as a whole or can be obtained by adding the computed bedload discharges for the different size ranges. It must be computed by individual size ranges if the concentrations of suspended sediment by size ranges are to be based on the bedload discharges. The computation of suspended-sediment discharge can be made from defined distributions, such as those shown on figure 6, by multiplying average velocity and average concentration within each of several layers of flow, adding the products for all the layers of flow, and converting the sum of the products to a convenient rate, such as tons per day. Thin layers of flow should be selected at depths where the concentration and velocity change rapidly as the distance above the bed changes.

FINE SEDIMENT

The discharge of the fine sediment can seldom be computed from the characteristics of the bed sediment and of the flow at a cross section by the general procedure that can be used, though inexactly, for coarser sediment such as sand. As Einstein and Chien (1953) have pointed out, the two main reasons preventing this type of computation for fine sediments are that (a) a representative sample of the fine sediment in the upper layer of the streambed can seldom be obtained especially under high flows and (b) the relative amount of the fine sediment in this upper layer changes rapidly. (Silt and clay together are roughly equivalent to "wash load" as the term was used by Einstein and Chien.) Thus, the relative amount of fine sediment at the surface of the streambed is seldom known; hence, the bedload discharge of the fine sediment usually cannot be computed with any reasonable accuracy. The bedload discharge of the fine sediment is so small as compared to the discharge of suspended fine sediment that it could be disregarded, except that the only theoretical way of computing the concentration and discharge of the suspended fine sediment is based directly on the bedload discharge. As figure 6 indicates, a small percentage of fine sediment at the streambed may provide a base for much suspended fine sediment.

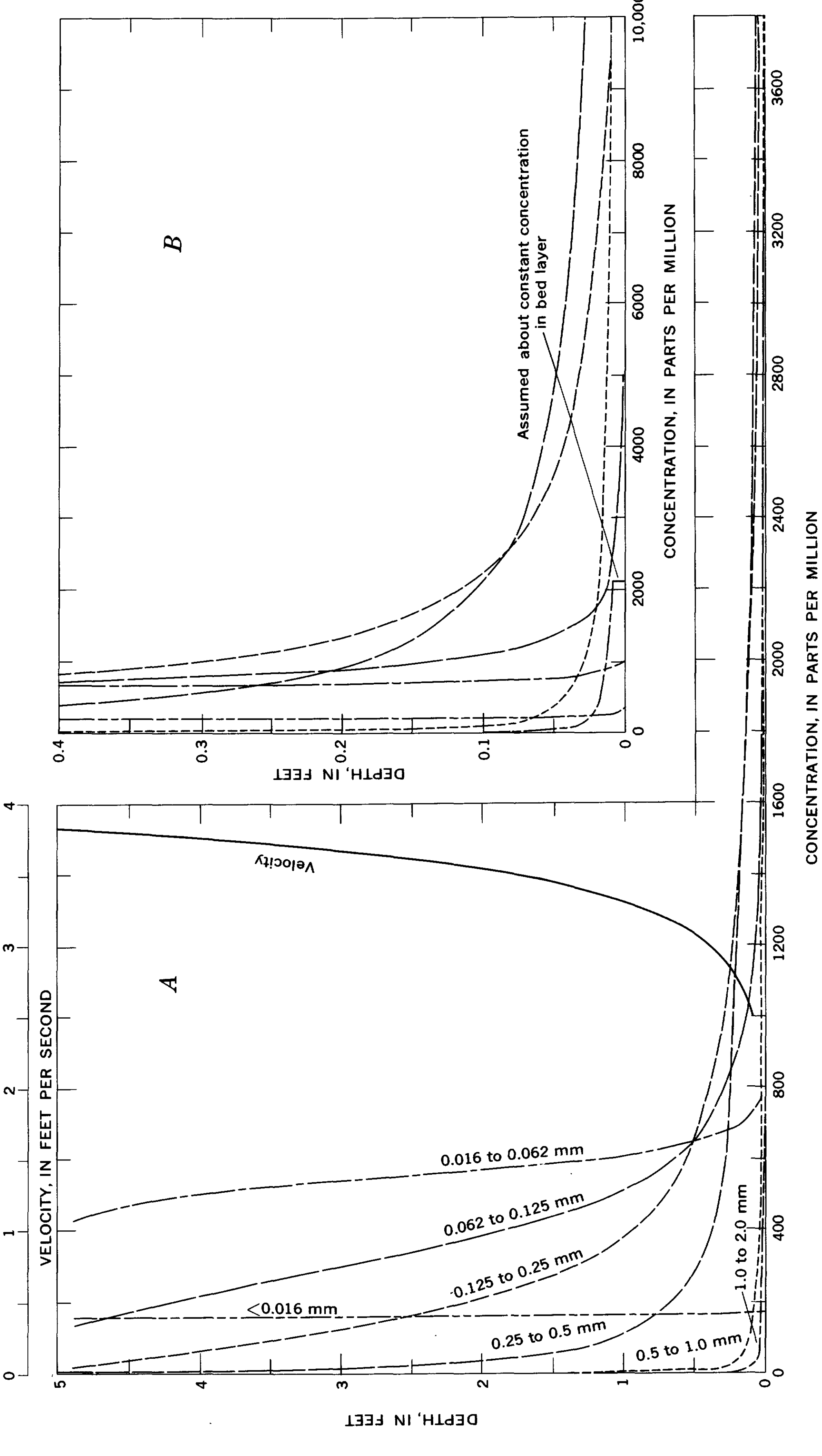


FIGURE 6.—Distribution of concentrations of different sediment sizes at a stream vertical.

High concentrations of fine sediment in a flow increase the discharge of the coarser sediment because they decrease the fall velocity of the suspended sediment and may modify the bed configuration (D. B. Simons, E. V. Richardsons, and W. L. Haushild, written comm.). The fine sediment seems to affect sediment transportation by changing the apparent viscosity of the mixture of water and sediment. The effect of the fine sediment should be comparable to the effect of a decrease in water temperature, but the effect of high concentrations of fine sediment may be much greater than the effect of usual changes of the temperature of natural streams.

EFFECTS OF MAJOR FACTORS

The preceding discussion of the concept of sediment transportation indicates the major variables that probably control the discharge of sands in sandbed streams through the effect of these variables on bedload discharges, suspended-sediment concentrations immediately above the bed layer, and vertical distributions of suspended-sediment concentrations. The quantitative effect of individual variables can be approximately defined, at least for some flows and streams cross sections, by a type of graphical analysis. An important part of the analysis is to determine which of four possibly interchangeable variables—namely, mean velocity, shear, shear velocity computed from mean velocity, or stream power—shows the closest and most useful relationship to the discharge of sands. Of course, shear velocity computed from mean velocity is the type of shear velocity with respect to the particles that is computed from a known mean velocity with the use of an equation such as equation 10.

Three general assumptions or limitations should be kept in mind in the following discussions. One is that the assumed bed material of the streams is relatively cohesionless sands. Another is that discharge of sands in these streams is equivalent to the sum of the bedload discharge and the suspended discharge of bed material. The third is that discharge of sands is proportional to the width of stream; this assumption, in effect, excludes some highly irregular cross sections from the studies but does permit the use of discharge of sands per foot of width as the usual unit for expressing the rate of transportation of sands through a cross section.

MEAN VELOCITY

Probably the simplest measure of the force that moves sediment in a stream is the mean velocity. Studies by Gilbert (1914), Colby (1957), and Rottner (1959) have shown an approximate relationship between the discharge of bed material in shallow flows or near the streambed and roughly the third power of the mean velocity, except for low velocities. Hence, the relationship of discharge of sands per unit width of stream to mean velocity was investigated in detail for natural streams and less extensively for flumes.

NATURAL STREAMS

Total discharges of sands for Niobrara River near Cody, Nebr., show a close relationship to mean velocity. (See fig. 7.) (For this and other curves that involve a relationship of discharge of sediment to mean velocity or to other variables, the sediment discharge is considered to be the statistically dependent variable; that is, each curve is drawn to show, as well as possible, the average sediment discharge for each of several narrow ranges of the independent variable.) These total discharges of sands were measured at a contracted section 1,900 feet downstream from the gaging-station section where velocities, depths, and widths of flow and particle sizes of the bed sediment were determined (Colby and Hembree, 1955). The median size of the bed sediment was about 0.28 mm. The total discharges of sands have the usual inaccuracies inherent in sampling suspended sands at a somewhat unsatisfactory cross section. Also, at times some appreciable gain or loss of sands must have occurred between the two sections. No adjustment was applied for change in water temperature. In spite of the inaccuracies, the scatter of points from the average curves of figure 7 is not excessive.

An average curve of discharge of sands per foot of width against mean velocity was also defined (fig. 8) from information given by Hubbell and Matejka (1959) for Middle Loup River at Dunning, Nebr. Sections *A*, *B*, *C*, and *E*, at which velocities, depths, and widths of flow and particle sizes of the bed sediment were determined, have average widths of roughly 280, 70, 80, and 150 feet, respectively. Section A is about 1.4 miles upstream from the turbulence flume where total sediment discharges were measured (Benedict, Albertson, and Matejka, 1955), and section *E* is about 700 feet downstream from the flume. Section *B* is about 1,200 feet and section *C* is less than 100 feet upstream from the flume. Median particle size of the bed material at the four sections is about 0.32 mm. Rough adjustments for changes in water temperature were applied before the total discharges of sands were plotted on figure 8. A single curve represents the average relationship fairly well for all four sections.

Total discharges of sands (data from the files of the Agricultural Research Service, U.S. Dept. of

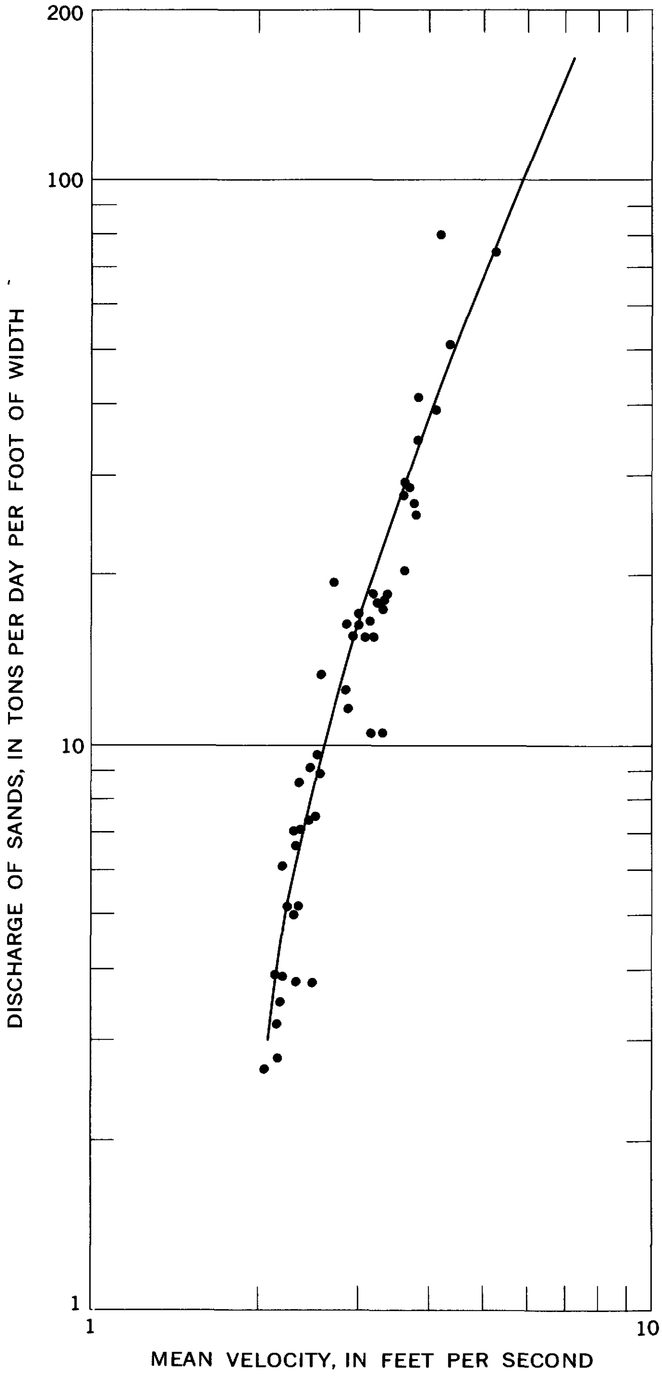


FIGURE 7.—Discharge of sands plotted against mean velocity for Niobrara River near Cody, Nebr. (Data unadjusted for water temperature.)

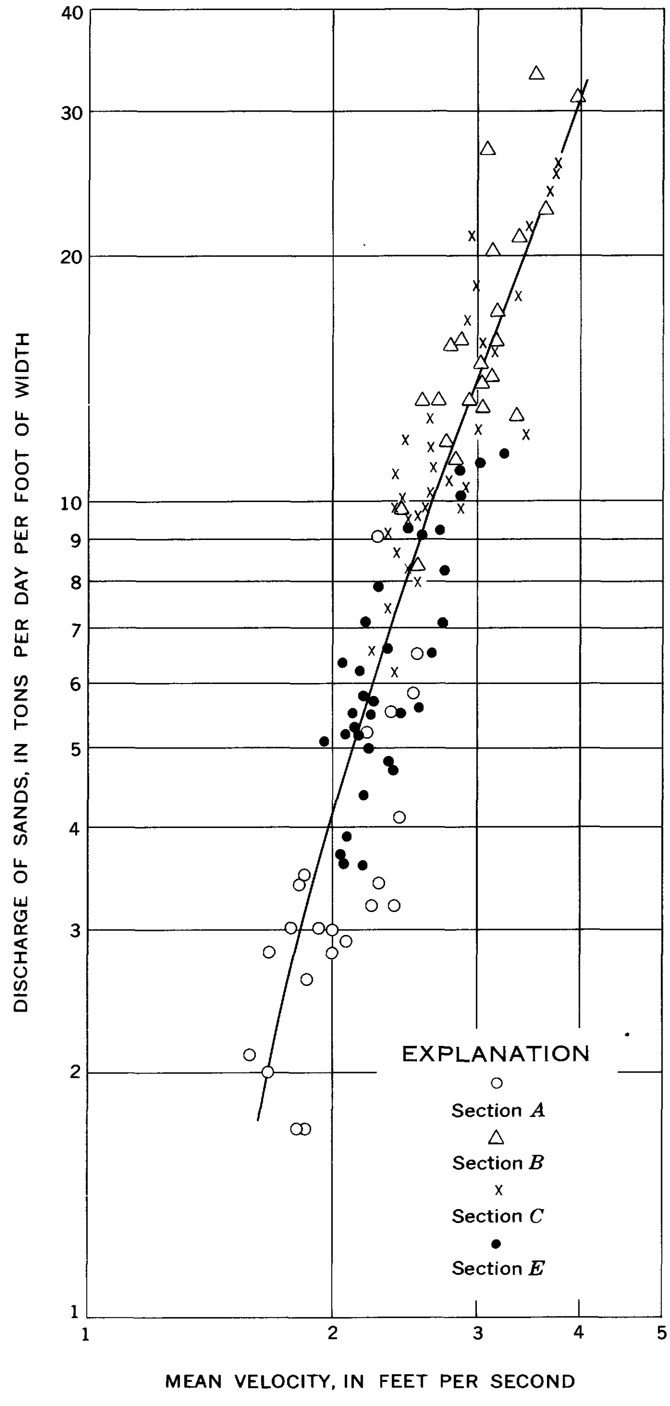


FIGURE 8.—Discharge of sands plotted against mean velocity for Middle Loup River at Dunning, Nebr. (Data roughly adjusted for water temperature.)

Agriculture, University, Miss.) for several stations on Pigeon Roost Creek or its tributaries in Mississippi were approximately determined by adding estimates of unmeasured sediment discharges to measured discharges of sands. These total discharges were plotted against mean velocity to define the six average curves that are indicated by broken lines on figure 9. Each of five of the broken curves is for an individual sediment station, and the sixth is a composite curve that is based on only a few determinations for each of several sediment stations. Median diameter of the bed sands is about 0.40 mm. at all stations. The unmeasured sediment discharges, mostly of sands, were estimated from a graphical

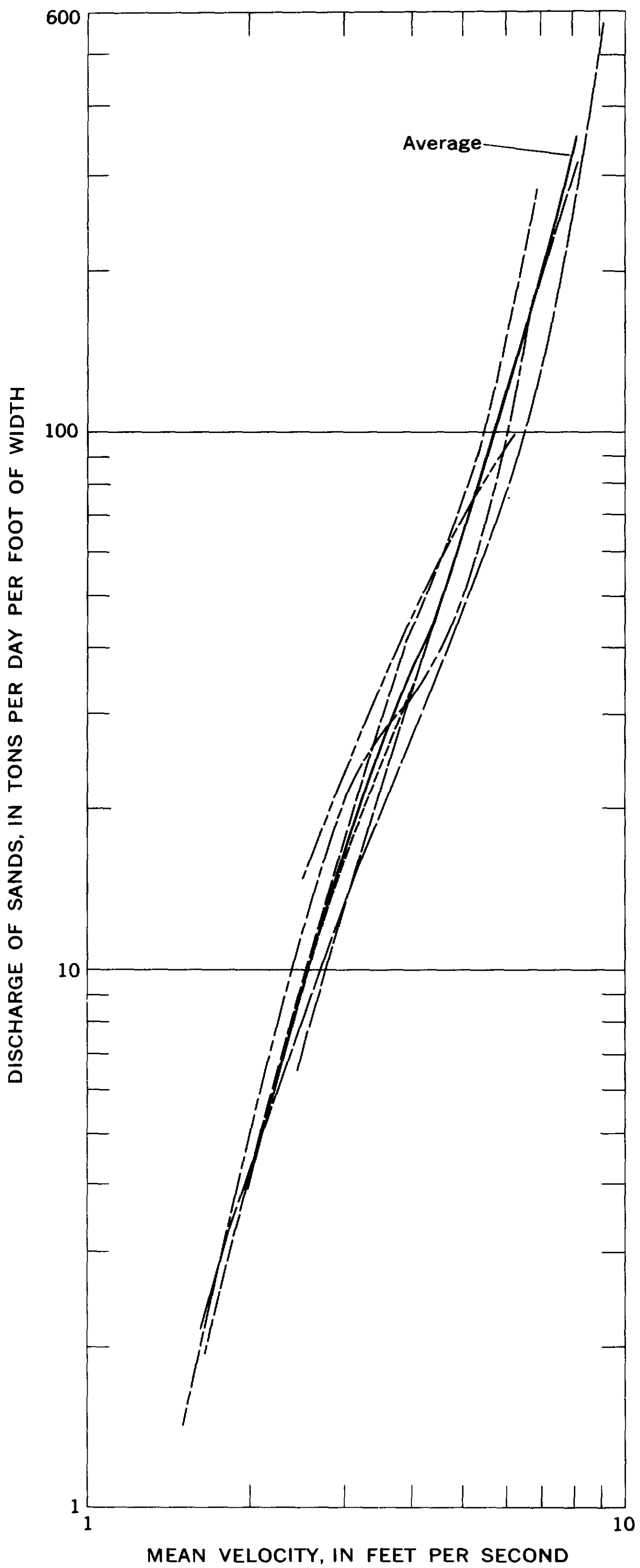


FIGURE 9.—Curves of relationship between discharge of sands and mean velocity for stations on Pigeon Roost Creek in Mississippi. (Data unadjusted for water temperature.)

relationship to mean velocity (Colby, 1957). Hence, the unmeasured portion of the total discharge of sands was based partly on mean velocity. Also, the mean velocities for the high flows at several of the stations were determined by measuring from bridges where the streambed scoured locally. Most measurements of high flows were made on falling stages when the mean velocity under a bridge may have been appreciably lower than the mean velocity away from the bridge. In general, the six curves are reasonably consistent and may be roughly expressed by the single continuous curve of figure 9.

Total discharges of sands for stations on the Rio Grande or diversions from it in New Mexico were obtained from unpublished computations that were made by J. K. Culbertson and D. Q. Matejka according to the modified Einstein procedure (Colby and Hembree, 1955) and were used to define a curve of relationship to mean velocity. (See fig. 10.) Two total discharges of sands in the Rio Grande near Socorro, N. Mex., and one near San Antonio, N. Mex., were made when most of the flow originated in the drainage basin of the Rio Puerco and contained very high concentrations of clay and silt. The plotted points for the three determinations when the concentration of fine sediment was very high are circled on figure 10. The circled points indicate discharges of sands that are roughly three to eight times greater than the discharges of sands from the average curve for times when the concentrations of fine sediment were low.

Total discharges of sands for Rio Puerco near Bernardo, N. Mex., where concentrations of silt plus clay are on the order of 100,000 ppm (parts per million) or more on many rises, have only a rough relationship to mean velocity (fig. 11). They are generally very high as compared with total discharges indicated by an estimated curve for which factors such as velocities, depths, and bed-materials sizes are similar but the concentration of fine sediment is low. The effect of the high concentrations of fine sediment apparently is very large. It is reasonably consistent with the laboratory findings of D. B. Simons, E. V. Richardson, and W. L. Haushild (written communication).

For a large stream, Mississippi River at St. Louis, Mo., the total discharges of sands plot fairly close to an average curve of relationship to mean velocity. (See fig. 12.) They were based on measured discharges of sands plus unmeasured sediment discharges that were estimated from the same graphs used for the unmeasured sediment discharges of Pigeon Roost

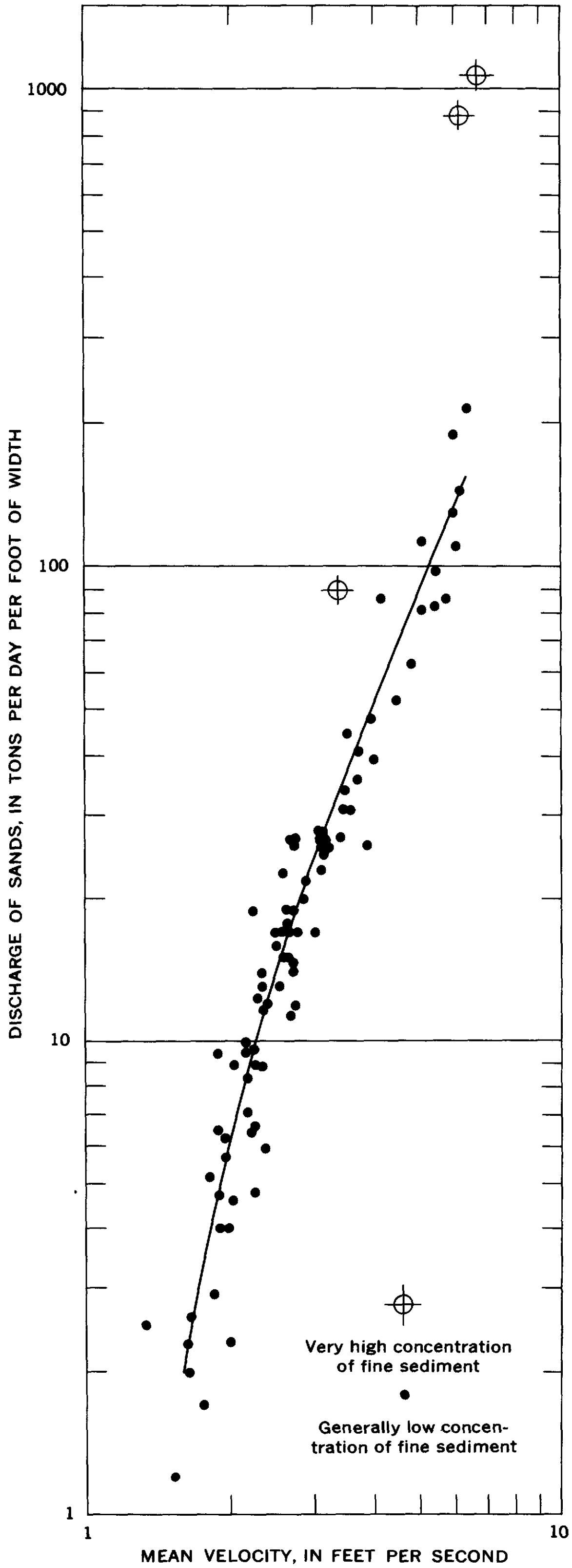


FIGURE 10.—Discharge of sands plotted against mean velocity for the Rio Grande in New Mexico and for some diversions from it. (Data unadjusted for water temperature.)

Creek. Hence, these unmeasured sediment discharges are related to mean velocity. However, the relationship of measured discharge of sands to mean velocity (P. R. Jordan, written communication) seems to be equally as consistent as the relationship of figure 12.

Scatter of total discharges of sands from an average relationship such as that of figures 7, 8, 10, or 12 is generally less for a single and reasonably uniform cross section — for example, the cross section of Niobrara River near Cody, Nebr. — than for a group of cross sections or for an irregular cross section.

Curves showing relationship between discharge of sands per foot of stream width and mean velocity for the Niobrara River, the Middle Loup River, Pigeon Roost Creek, the Rio Grande, and the Mississippi River are rather surprisingly consistent (fig. 13) and can be used to estimate quickly and roughly the discharge of sands in many sand-bed streams. The average water temperature is about 60° F. for each curve. Of course, the somewhat steeper slope of the curve for the Mississippi River is due to the fact that any particular mean velocity occurs at greater depths in the Mississippi River than in the shallower streams. However, in general, a sand-bed stream seems to have not only a reasonable close relationship between discharge of sands per foot of width and mean velocity at a cross section, but the relationship defined for one cross section may apply approximately for other sand-bed streams if bed-material sizes and depths are not greatly different and if the concentrations of silt plus clay are not unusually high.

FLUMES

The discharge of sands per foot of width for a series of flume experiments with the same bed sediment and constant water temperature usually shows a fairly close relationship to mean velocity. (See fig. 14). However, in a flume whose slope can be adjusted, a particular mean velocity may occur over a wide percentage range of depths. Thus, some scattering of points on figure 14 is caused by changes in depth, but some is caused by changes in water temperature, by unidentified factors, or by experimental error. Except for the effect of depth, the scatter of the observed discharges of sands about an average curve would probably be considerably less for most flume experiments in which a constant bed material is used that it would be for similar mean velocities at a cross section of a natural stream where the determinations are likely to be less accurate and where the lateral distributions of depth, velocity, and sediment discharge generally are less uniform. If the mean velocities in a flume are much lower than in a natural stream, they may be in

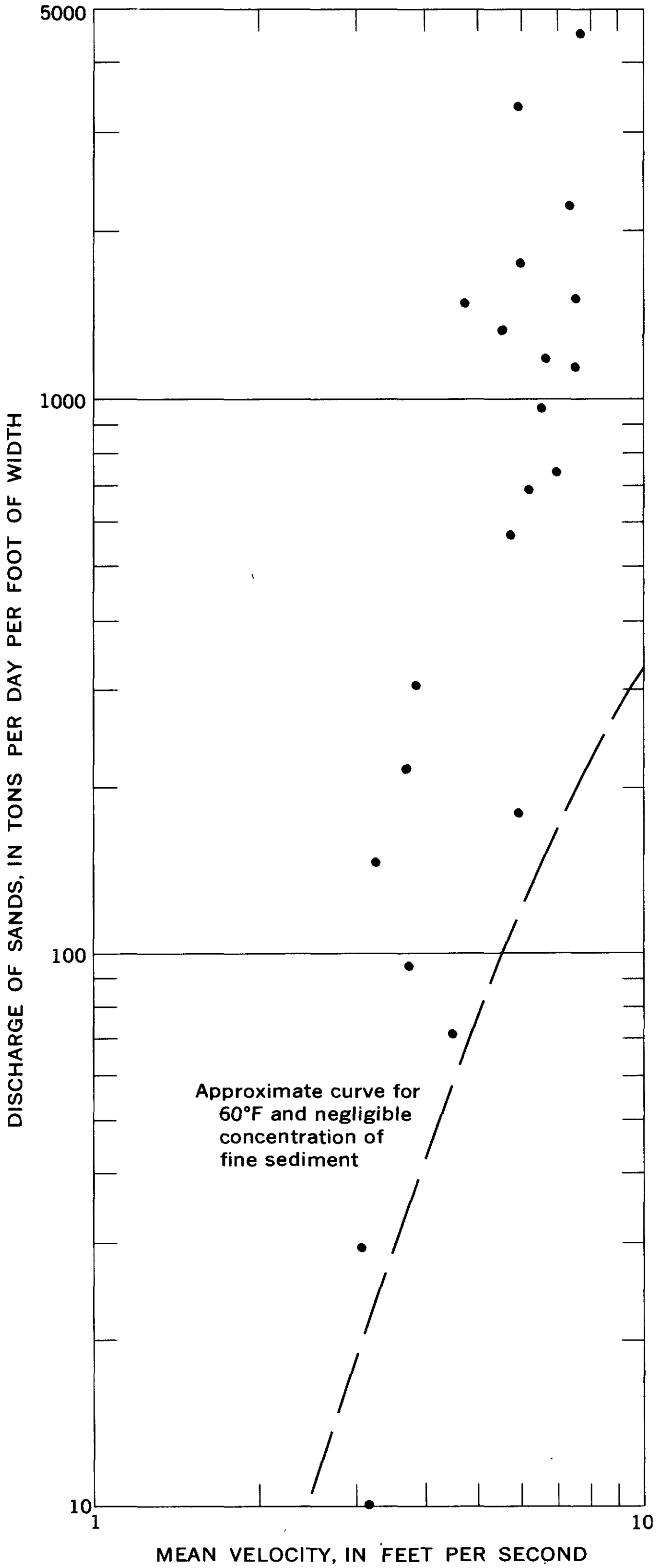


FIGURE 11.—Discharge of sands plotted against mean velocity for Rio Puerco near Bernardo, N. Mex. (Data unadjusted for water temperature.)

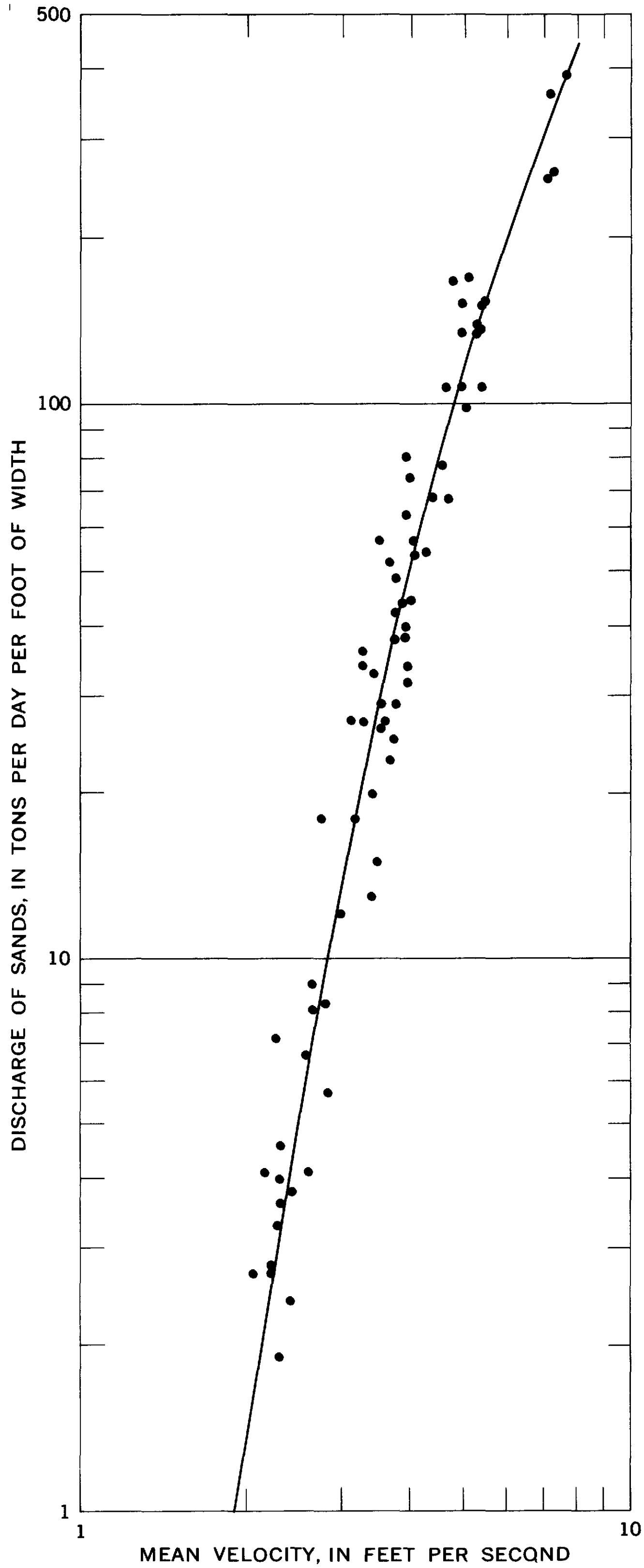


FIGURE 12.—Discharge of sands plotted against mean velocity for Mississippi River at St. Louis, Mo. (Data unadjusted for water temperature.)

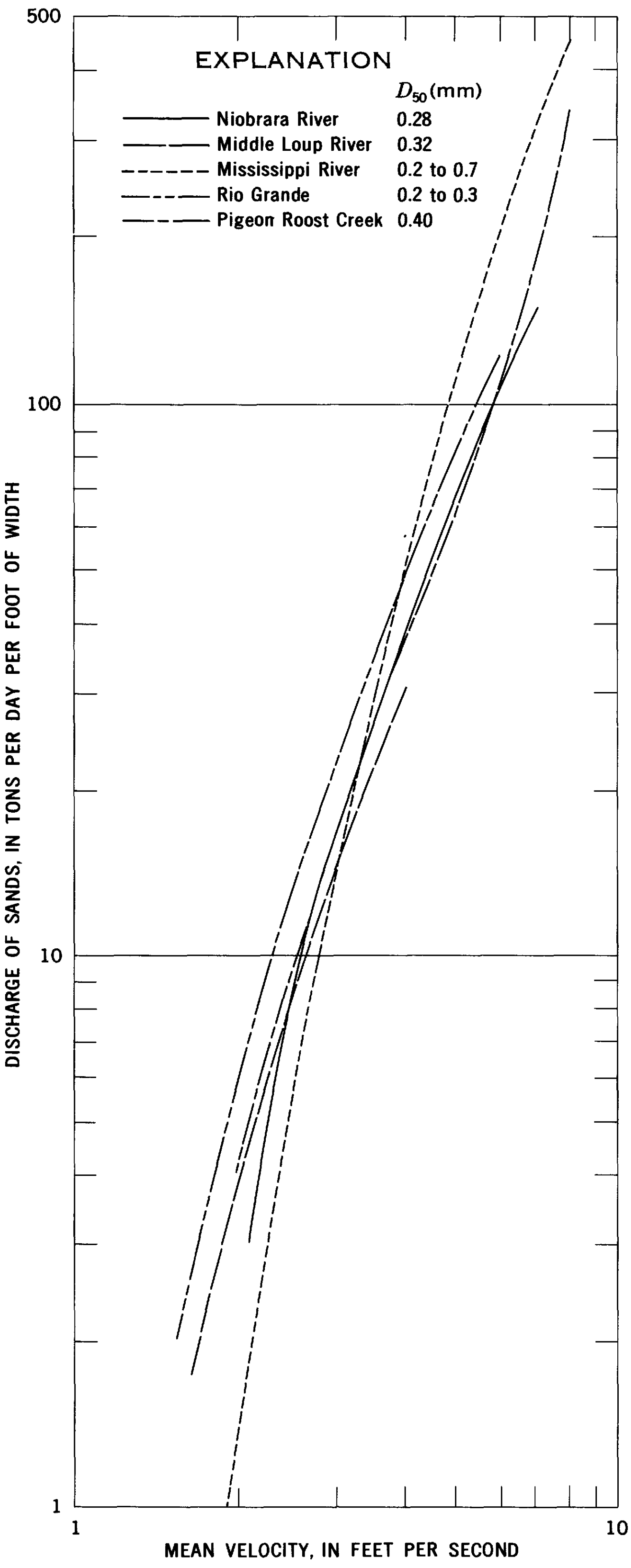


FIGURE 13.—Curves showing relationship between discharge of sands and mean velocity for five sand-bed streams at average temperatures of about 60° F.

the range for which the discharge of sands is very sensitive to small changes in velocity, depth, and size distribution of the bed sediment.

The main reason for the curvature of the lines showing average relationship on figure 14 is that the discharges of sands are plotted against mean velocity rather than against mean velocity minus the critical velocity at which movement of significant amounts of sand begins. If 0.8 and 0.7 foot per second are subtracted from the mean velocities for the left and right curves of figure 14, respectively, straight lines can be substituted for the curves.

The average relationships of discharges of sands to mean velocity for flume experiments differ considerably among themselves and especially for shallow flows are rather inconsistent with the relationship for Niobrara River near Cody, Nebr. (See fig. 15.) Much of the disagreement is due to differences in sizes of bed sediment or in depth. Because of the divergence among the curves for flumes, the comparative consistency of the curves for natural streams (fig. 13) probably would not have been predicted from flume experiments.

SHEAR

Shear or tractive force is an obvious measure of the force that may cause sediment to move and has been widely used as a parameter of sediment discharge. Shear in the form of a shear velocity is directly proportional to mean velocity if the Chezy C is constant. Thus, if resistance to flow as expressed by C were always constant, mean velocity and shear velocity could be used interchangeably as parameters of discharge of sands in sand-bed streams. The choice between those two parameters should be made, therefore, mainly on the basis of which one has the more consistent relationship to the discharge of sands when resistance to flow is variable.

Other factors should also be considered in making a choice between mean velocity and shear as a measure of the discharge of sands. Shear can be used directly in the design of a channel, whereas mean velocity for channel design normally must be computed from the shear. However, if the discharge of sands is more closely related to mean velocity than to shear, the problem of the relationship of mean velocity to shear cannot be avoided. Also, canals usually are designed for velocities that are slow enough to be in the range of flow over dune beds, and in this range the major change of roughness at the transition from dunes to a smoother bed does not enter the computations. In a natural stream

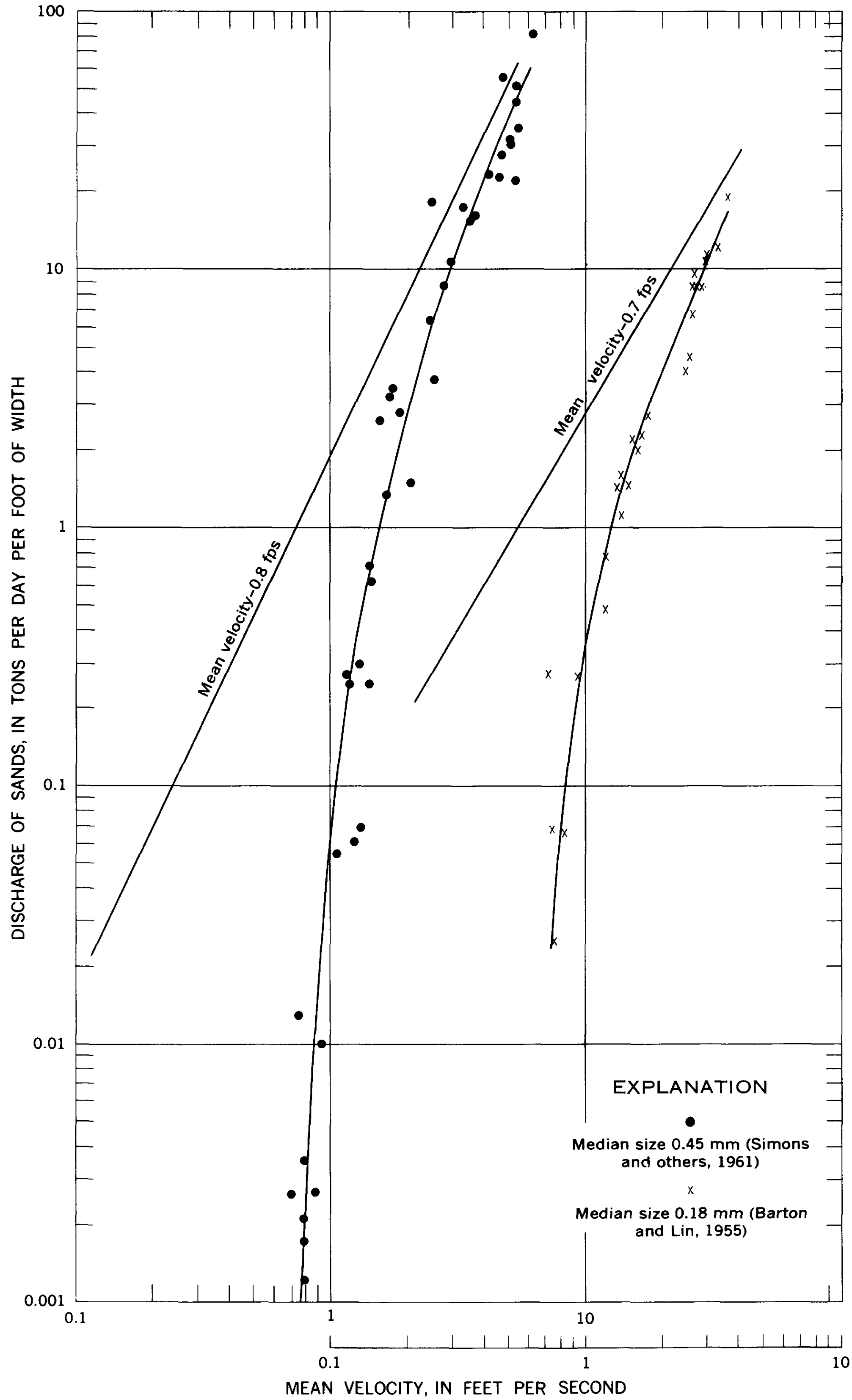
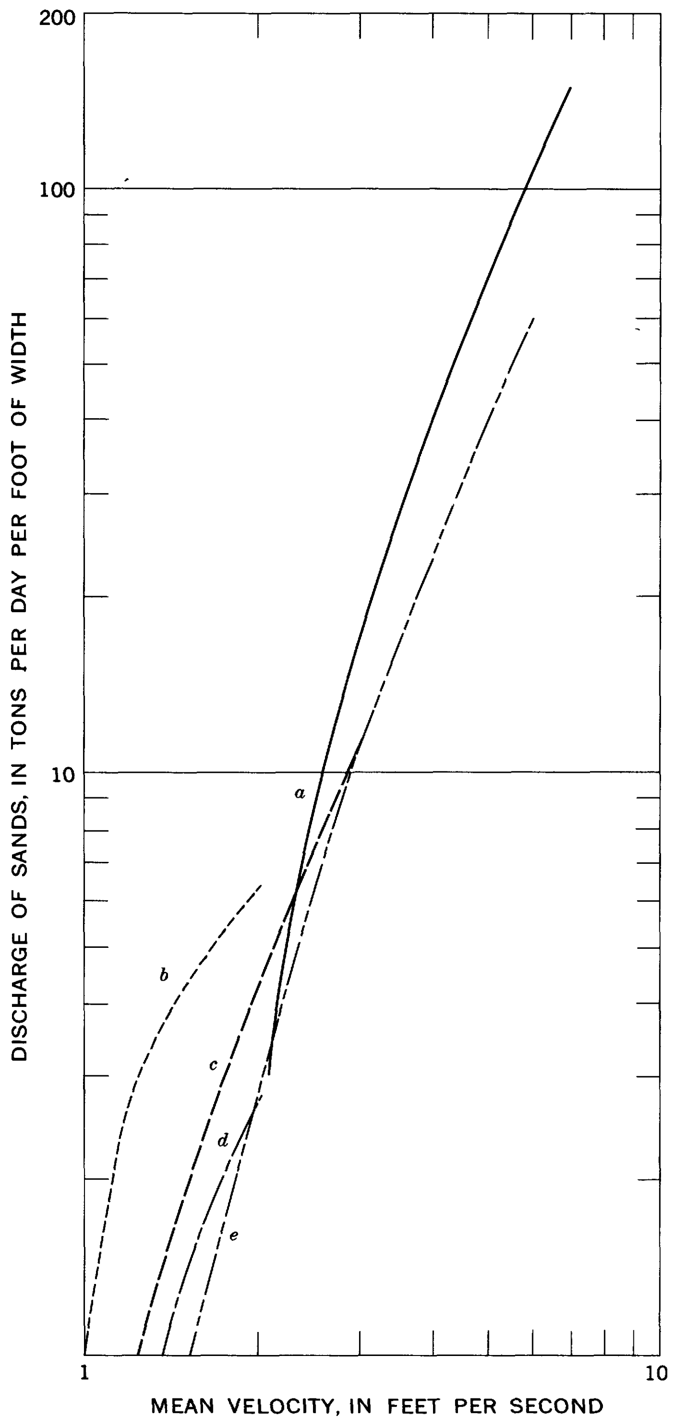


Figure 14.—Curves showing relationship between discharge of sands and mean velocity for some flume experiments with two sands. (Data unadjusted for water temperature and depth.)



		D_{50} (*mm*)	*Approx depth* (*ft*)
a	Niobrara River	0.28	1.0 -6.0
b	Brooks (1958)	.088	0.15-0.3
c	Barton and Lin (1955)	.18	0.3 -1.4
d	Brooks (1958)	.145	0.15-0.3
e	Simons and others (1961)	.45	0.25-1.0

FIGURE 15.—Curves showing relationship between discharges of sands and mean velocity for some flume experiments. (Data unadjusted for water temperature and depth.)

at a particular time, the mean velocity usually can be determined experimentally more easily than the shear. For channels that are not uniform laterally, the average depth of flow ordinarily used to compute shear is weighted with lateral distance in contrast to mean velocity, which is weighted with the product of lateral distance and depth; that is, average depth is equal to the area divided by the width, but mean velocity is equal to the streamflow divided by the area, which is the product of the average depth and the width. Thus, the areas of highest velocity in a cross section usually are given more weight in computing the mean velocity than in computing the average depth because the velocities generally are highest where the flow is deepest.

The discharges of sands reported by Simons, Richardson, and Albertson (1961) and plotted against mean velocity on figure 14 indicate an approximate relationship (fig. 16) to shear. For easy comparison with stream power, the discharges of sands are replotted against the 3/2 power of the shear and the cube of the mean velocity. At a shear of about 0.07 to 0.10 pound per square foot (a shear to the 3/2 power of about 0.018 to 0.030), the range in discharge of sands at a particular shear is large. From independent experiments that were made mostly within a similar range of uncertain relationship, Brooks (1958) concluded that neither the velocity nor the sediment discharge is a single-valued function of the shear. Although the conclusion applies to only a narrow range of shears in most natural streams if the bed material, water temperature, and energy gradient are constant, within this range the shear is not an acceptable measure of the mean velocity or of the discharge of sands. The range of uncertain relationship between discharge of sands and shear differs considerably from one stream to another and cannot be acccurately predicted at the present time from characteristics of the channel. Hence, the shear is at times an undependable measure of the discharge of sands.

A conclusion by Barton and Lin (1955, p. 21, 37) that the bed-material concentration may be a function only of resistance to flow is a further indication of the relatively poor relationship of discharge of sands to shear. Of course, the reason for some relationship between bed-material discharge and resistance to flow is that mean velocity depends both on resistance to flow and on shear, and the relationship either to resistance to flow or to shear is an incomplete part of the more basic relationship to mean velocity. In other words, shear at constant resistance to flow is a measure

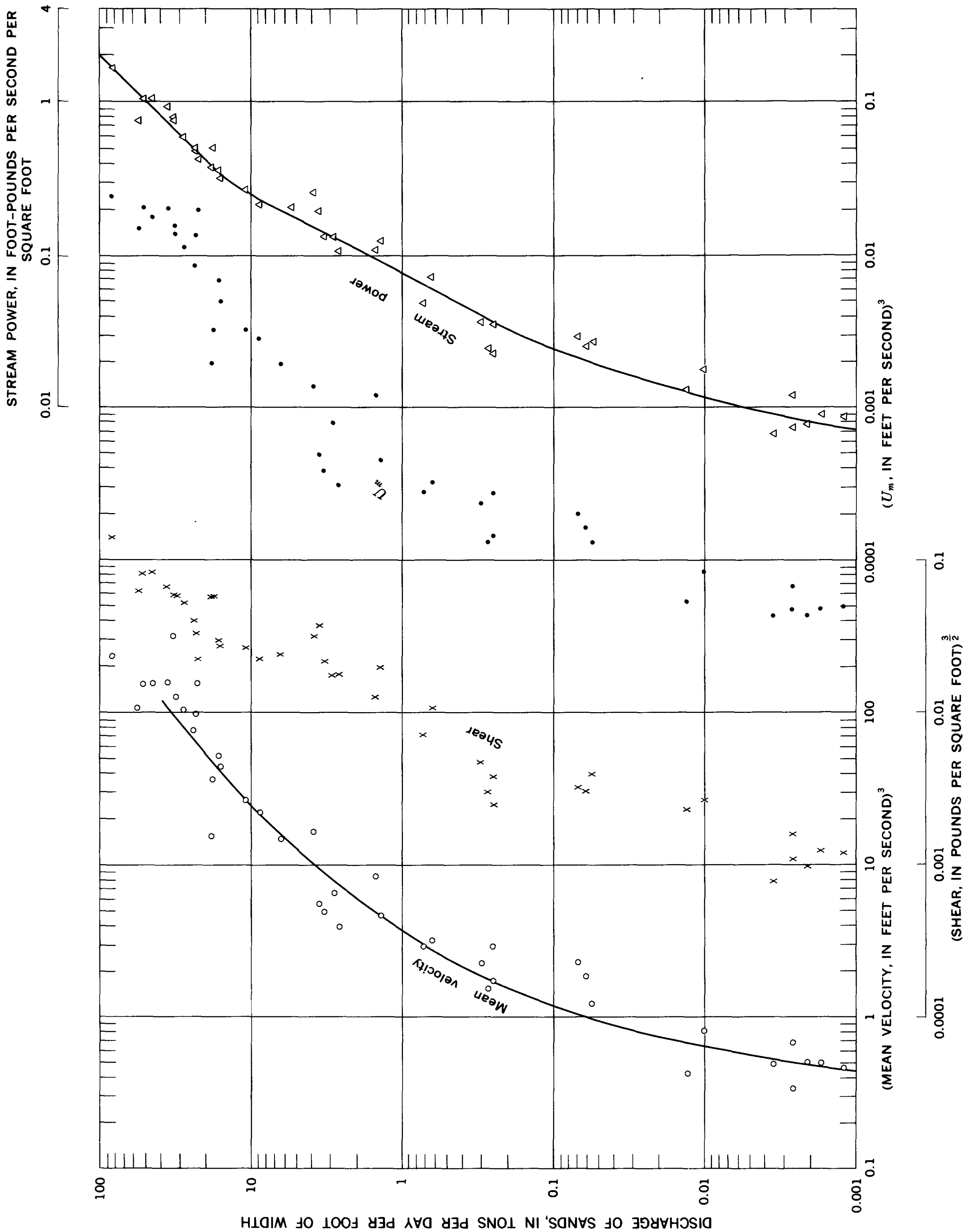


FIGURE 16.—Discharges of sands plotted against parameters that are related to the force which moves sediment particles. (Data unadjusted for water temperature and depth.) u_m is shear velocity computed from mean velocity.

of mean velocity and discharge of bed material, and resistance to flow at constant shear is also a measure of mean velocity and discharge of bed material.

SHEAR VELOCITY COMPUTED FROM MEAN VELOCITY

Einstein (1950, p. 9–10) used a shear velocity with respect to the grains, which is the shear velocity that can be computed from the hydraulic radius with respect to the grains. This hydraulic radius with respect to the grains or particles is the hydraulic radius that can be used with the energy gradient to compute mean velocity from equation 10. However, the use of this hydraulic radius, R', under the logarithm sign in the equation is questionable. The shear velocities used in this paper were computed from known mean velocities and from equation 10 except that d or R was substituted for R' under the logarithm sign.

The same discharges of sands that were plotted against the mean velocity cubed and against the 3/2 power of the shear show no discontinuity in their relationship to the shear velocities computed from mean velocity. Hence, this type of shear velocity has a major advantage over shear as a measure of the discharge of bed material. The scatter about an average curve is, of course, approximately the same whether mean velocity or shear velocity computed from mean velocity is used as the independent variable. However, as will be shown later, the effect of depth on the relationship between discharge of sands and shear velocity computed from mean velocity is somewhat different from that on the relationship between discharge of sands and mean velocity. Except for the difference in effect of depth, the choice between mean velocity and shear velocity computed from mean velocity is based mostly on convenience and simplicity of use.

STREAM POWER

"Stream power" is a term for the rate at which a stream loses energy. It has been discussed as a measure of the discharge of sediment by Cook (1935) and Bagnold (1960), but its relationship to sediment discharge is not simple, as Cook indicated. Specifically, the stream power of a volume of water in a reach of river L units long and W units wide is equal to the product of the weight of the water in the reach and the loss of energy head per unit time. Hence, the stream power for low concentrations of sediment and of dissolved solids and in pound-foot-second units is 62.4 $LWd\bar{u}S$. For unit width and length of channel the stream power is 62.4 $dS\bar{u}$ or the product of shear and mean velocity.

If the Chezy C were constant, the stream power would be directly proportional to the third power of the mean velocity and would be neither a better nor a worse parameter than mean velocity. However, when C varies, the shear is a somewhat inadequate measure of the discharge of sands. Hence, the stream power also is a poor measure of the discharge of sands when C varies widely, as it does within the range of shear at which a dune bed changes to a plane or antidune bed. In other words, the discontinuity in the relationship between discharge of sands and shear at about a shear of 0.07 to 0.10 pound per square foot also exists in the relationship between discharge of sands and stream power but is partly obscured by the inclusion in stream power of mean velocity.

Any continuous curve that might be drawn to represent the experimental relationship between discharge of sands and stream power on figure 16 obviously cannot be as smooth and consistent as the curve for the relationship to the cube of the mean velocity. A comparison of the two curves of figure 16 suggests some conclusions with respect to stream power.

At velocities high enough to cause plane or antidune beds, the resistance to flow should be reasonably constant; hence, all curves showing relationship between discharge of sands and one of the independent variables of figure 16 should have about the same slope. Curves showing relationship of discharge of sands to mean velocity or to shear velocity computed from mean velocity are smooth and continuous and merge gradually into the slope for high velocity. In fact, as was shown (fig. 14) for two sands, the relationship between discharge of sands per foot of width and mean velocity minus the velocity at which sediment begins to move significantly is about a straight-line relationship. No constant quantity can be subtracted from the cube of the mean velocity to obtain a straight-line relationship to the discharge of sands. Similarly, no constant stream power can be subtracted from the stream power on figure 16 to define a straight-line relationship between discharge of sands and stream power. This fact and objections based on theoretical physics make very questionable the usefulness of Bagnold's suggestion (1960, p. 4) that a power threshold should be used for the beginning of sediment movement.

The relationship between discharge of sands and stream power is poor within the range of stream power from about 0.10 to 0.30 foot-pound per second per square foot, because within this range the shear has little relationship either to mean velocity or to discharge of sands. Above and below this range, the curve showing the relationship of discharge of sands to stream power is much like that to the cube of the mean velocity. The break in the relationship occurs at the transition from dunes below this range to a plane bed or antidunes above this range and is caused by the unsatisfactory independent variable. It can be eliminated by using mean velocity or shear velocity computed from mean velocity as the independent variable.

At high discharges of sands, the scatter of points about a curve showing average relationship is less from stream power than for any of the other three independent variables that were used on figure 16. This relatively good agreement for one set of experiments at high discharges of sands may or may not be characteristic.

A reasonably constant concentration of sands is indicated by the approximately 45° slope of the curve showing the relationship between the discharge of sands and stream power at high flows and at high discharges of sands. At a particular reach of a sand-bed stream, the slope of the energy gradient usually varies comparatively little with changes in depth. Hence, if the depth is quadrupled, the mean velocity should double provided that the Chezy C remains constant, and the flow per foot of width should be eight times greater for the deeper flow. For the concentration of sands to remain constant, the discharge of sands should increase eightfold or in proportion to the cube of the mean velocity.

DEPTH

The effect of depth on the relationship of discharge of sands to any one of the four independent variables —namely, mean velocity, shear, shear velocity computed from mean velocity, and stream power—is different from the effect of depth on the relationship of discharge to any of the other three variables. It can be defined more accurately for the relationships of discharge of sands to mean velocity or to shear velocity computed from mean velocity than for the relationships to shear or to stream power.

The complicated effect of depth of flow on the relationship of discharge of sands to mean velocity at different water temperatures can be shown qualitatively by the results of many computations of discharge of sands. The computations were made according to a procedure that was based mostly on the work of Einstein (1950) and for a natural bed of unconsolidated sand whose D_{35}, D_{50}, and D_{65} sizes were about 0.35, 0.40, and 0.45 mm, respectively. They were made for many different combinations of depth and mean velocity and usually for water temperatures of 40°, 60°, and 80°F. The main departures from the Einstein procedure were the use of shear velocities computed from the mean velocities and of some assumptions with regard to the vertical distribution of sediment concentrations. These assumptions were probably fairly accurate for flow over dune beds but resulted in too great discharges of sands at high velocities. A qualitative graph based on these computations (fig. 17) indicates the dominant general relationship between discharge of sands and mean velocity. It also indicates a complex effect of depth on this dominant relationship. At constant low velocities, an increase in depth is accompanied by a decrease in discharge of sands; but at constant high velocities, an increase in depth is accompanied by an increase in discharge of sands. At some intermediate constant velocities, depth has little effect on the relationship between discharge of sands and mean velocity. At constant mean velocity, the effect of depth varies somewhat, but not greatly, as the water temperature changes.

The general reason for the variable effect is easily understood in broad outline, although the actual computation of bed-material discharges for different depths is inexact and complicated. If mean velocity is low—perhaps 2 feet per second—and if water flows in a shallow stream over a sand bed whose median diameter is 0.3 or 0.4 mm, a certain amount of bed material (as generally indicated by the curves of velocity and concentration on figure 18) will be discharged, mostly near the streambed. If the mean velocity is constant at 2 feet per second and if the flow is deep, the velocity near the streambed will be lower and the amount of bed material discharged will be less, both at the bed and throughout the original shallow depth of flow, than they were in the shallow stream. Only a little bed material will be discharged in suspension in the additional depth of flow. Hence, the total bed-material discharge is likely to be less for a given low mean velocity when the flow is deep than when it is shallow. On the other hand, if the mean velocity is high—perhaps 6 feet per second—the bed-material discharge is greater in a shallow flow than in a layer of equal depth at the bottom of a deep flow because of the difference in velocity near the streambed. However, the difference in the bed-material discharge in this layer near the bed is more than balanced by the discharge of bed material in the superposed layer of the deeper flow. (See fig. 18.)

If enough information is available for one bed material and one water temperature and for a wide range of depths and mean velocities, the effects of mean velocity and depth on the discharge of bed material can be determined by the empirical definition of a graph similar to figure 17. Even information for different water temperatures and different bed materials can be analyzed by trial-and-error procedures to

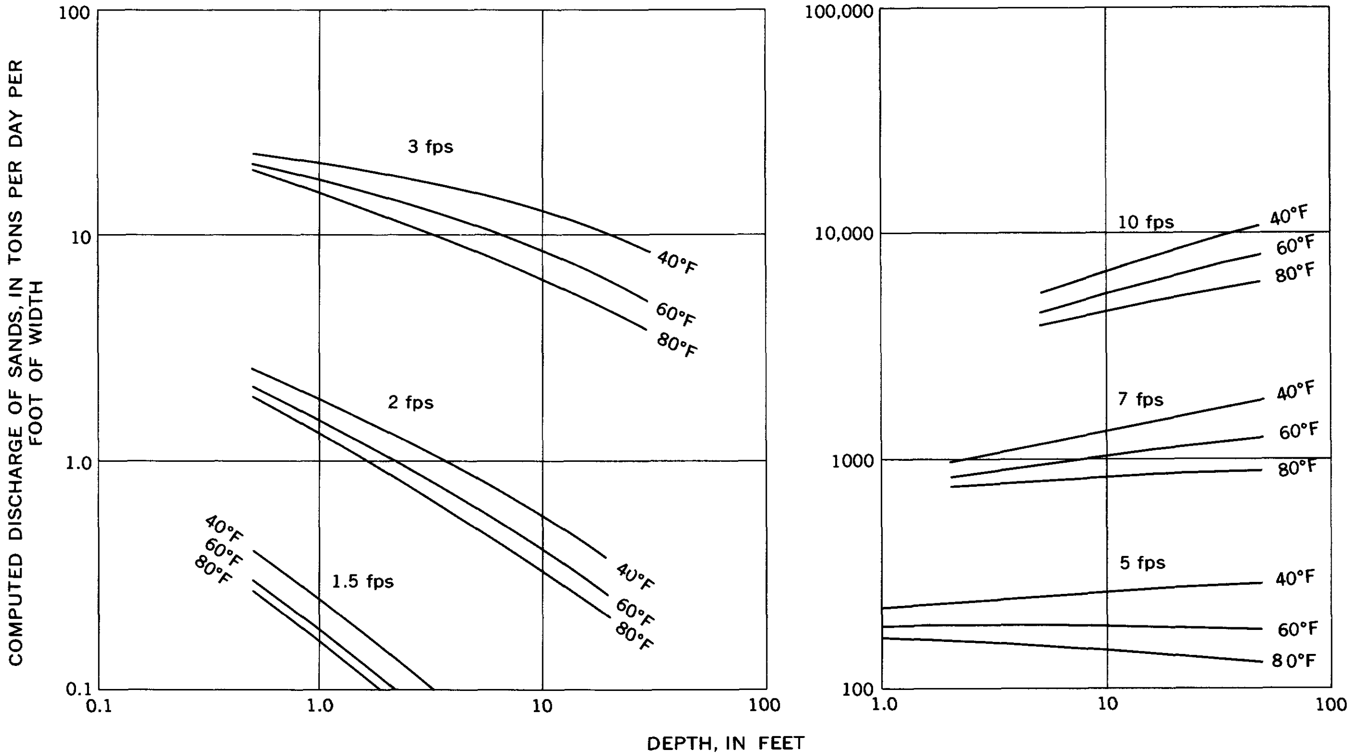


FIGURE 17.—Effect of depth and water temperature on the relationship between discharge of sands (0.40 mm median diameter of the bed sands) and mean velocity, computed from a somewhat revised Einstein procedure. From Colby (1961, fig. 1).

define a graph of empirical relationship of discharge of sands to mean velocity and to depth. (See p. 34–35.) Such a relationship for a water temperature at 60° F and bed sediment having a 0.30-mm median diameter is shown on figure 19. Qualitatively, the relationships of figures 17 and 19 are similar in spite of the difference in particle size and in method of determination; but quantitatively, figure 19 is much more accurate than figure 17. The fundamental soundness of the Einstein procedure is well shown, at least for the assumed bed sand, by the general qualitative agreement of the computed effects with the empirically defined effects of mean velocity, depth, and water temperature.

For a median particle size of 0.30 mm and a water temperature of 60°F, about the same discharge of sands is indicated for a mean velocity of 3.0 feet per second whether the depth is 1.0 foot or 100 feet. Of course, if the line on figure 19 for a mean velocity of 3.0 feet per second were defined more accurately, it might have a somewhat different shape and position.

The effect of depth varies somewhat with changes in particle size. The mean velocities at which depth has little apparent effect on the relationship between discharge of sands and mean velocity are about 1.6, 2.2, and 3.5 feet per second for bed sediments of 0.10–, 0.20–, and 0.40–mm median diameters, respectively, according to empirically defined graphs similar to figure 19. To some extent an increase in mean velocity at constant particle size has about the same effect as a decrease of particle size at constant mean velocity.

Only a small apparent effect of depth on discharge of sands in natural streams was indicated by the relative consistency of the relationship between discharge of sands and mean velocity for different streams. (See fig. 13.) These streams generally have mean velocities, depths, and sizes of bed sediments for which changes in depth should have comparatively little effect, according to figure 19. However, compared with shallower streams, the deeper flows of the Mississippi River at St. Louis result in lower discharges of sands at low velocities and higher discharges of sands at high velocities.

Some inconsistencies of the relationships between discharge of sands in flumes and mean velocity (fig. 15) are explainable on the basis of the effect of depth and particle size. Not only are the depths in flume experiments usually shallow, but they may decrease as the velocities increase. At a high velocity the discharge of sands, especially fine sands, is likely to be far lower in a shallow flow in a flume than at the same mean velocity in a deep river. In contrast, at a low velocity the discharge of sands is likely to be higher in a shallow flow in a flume than at the

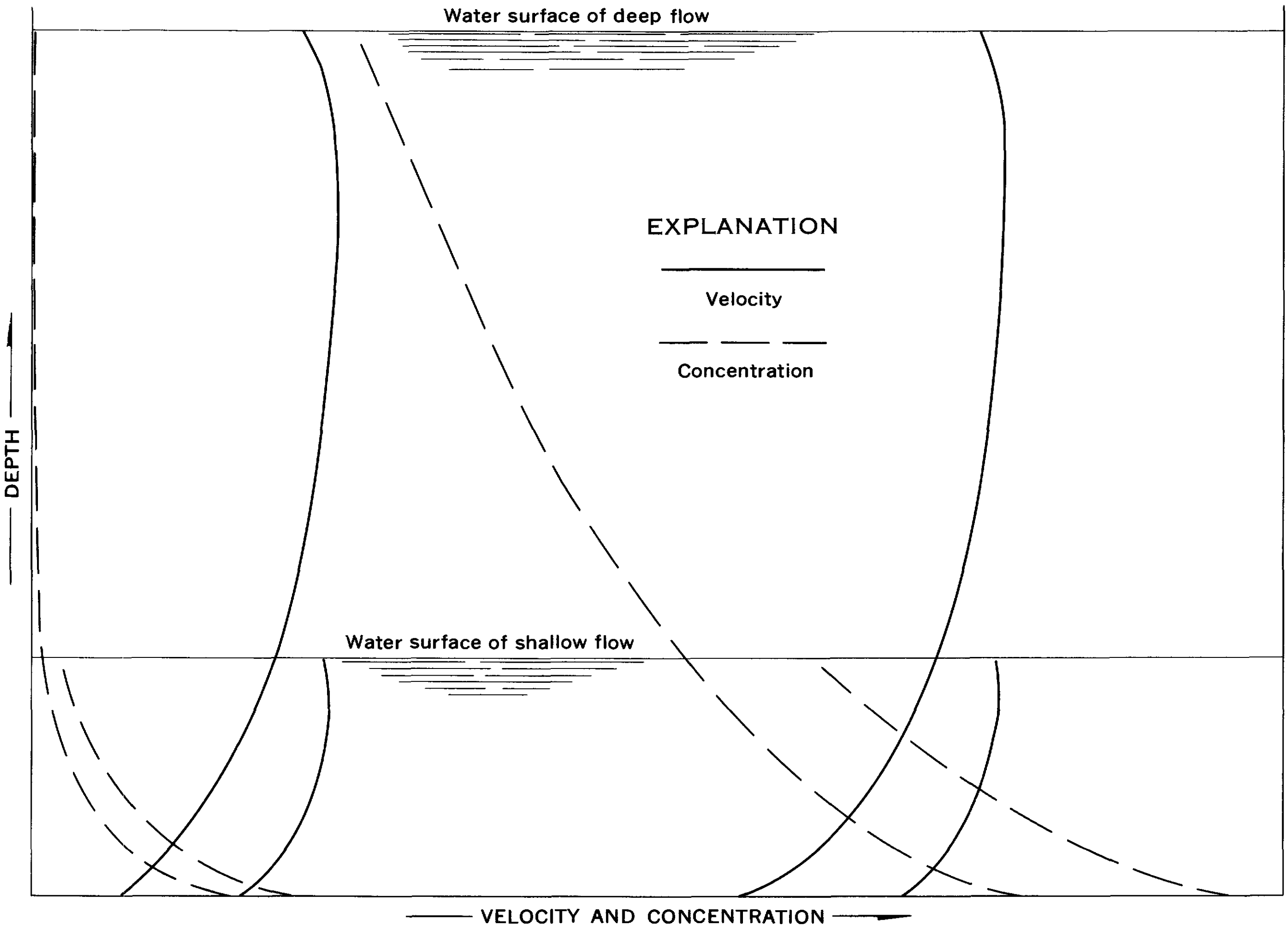


FIGURE 18.—Schematic diagram showing the general reasons for variability of effect of depth on the relationship of bed-material discharge to velocity. From Colby (1961, fig. 3).

same mean velocity in a deep river. Thus, the rate at which the discharge of sands increases with respect to the mean velocity is usually greater in a natural stream than in a flume and in a deep river than in a shallow stream. Also, the depth effect becomes increasingly large at low velocities, and the velocities in flumes—particularly for a given type of bed configuration—are generally low as compared with those in rivers.

The effect of depth on the relationship between discharge of sands and shear velocity computed from mean velocity differs somewhat from the effect on the relationship between discharge of sands and mean velocity but can be computed readily from figure 19 for a median diameter of 0.30 mm and a water temperature of 60°F. The computations require the solving of equation 10 by graphic or algebraic trial and error to obtain values of the shear velocity computed from the mean velocity for many different combinations of depth and velocity. The discharges of sands corresponding to these combinations can then be plotted against depth and shear velocity computed from mean velocity to obtain a reasonably simple pattern of depth effect. (See fig. 20.) One advantage of this depth effect is that it is always in the same direction; that is, at constant shear velocity computed from mean velocity, the discharge of sands increases as the depth increases if there is any appreciable discharge of sands. However, the rate of increase in the discharge of sands is slow at low shear velocities computed from mean velocities, but the rate becomes progressively greater as the shear velocity computed from the mean velocity increases. In many flume experiments, the effect of depth on the relationship between discharge of sands and shear velocities computed from mean velocities would be simpler to use than the effect of depth on the relationship between discharge of sands and mean velocities. In computations for natural streams, the latter effect of depth is smaller and, hence, is generally easier to apply with reasonable accuracy.

The effect of depth on the relationship between discharge of sands and shear is somewhat indefinite except when shear is about proportional to mean velocity or to shear velocity computed from mean velocity. When shear has this relationship, the effect of depth on the relationship of discharge of sands to mean velocity or to shear velocity computed from mean velocity can be applied.

The effect of depth on the relationship between discharge of sands and stream power is a combination of the effects on the relationships to mean velocity and to shear. It is, therefore, both complex and sometimes indfinite. If the Chezy C is constant and if a change in hydraulic radius is exactly compensated by an inverse change in energy gradient so that RS remains constant, the depth effect on the relationship of discharge of sands to stream power can be determined from figure 19 for a median particle size of 0.30 mm and a water temperature of 60°F. If the mean velocity is 3.0 feet per second, median diameter is 0.30 mm, and shear and water temperature are constant, the indicated discharge of sands remains nearly unchanged as the depth increases from about 1 foot to 100 feet. For a

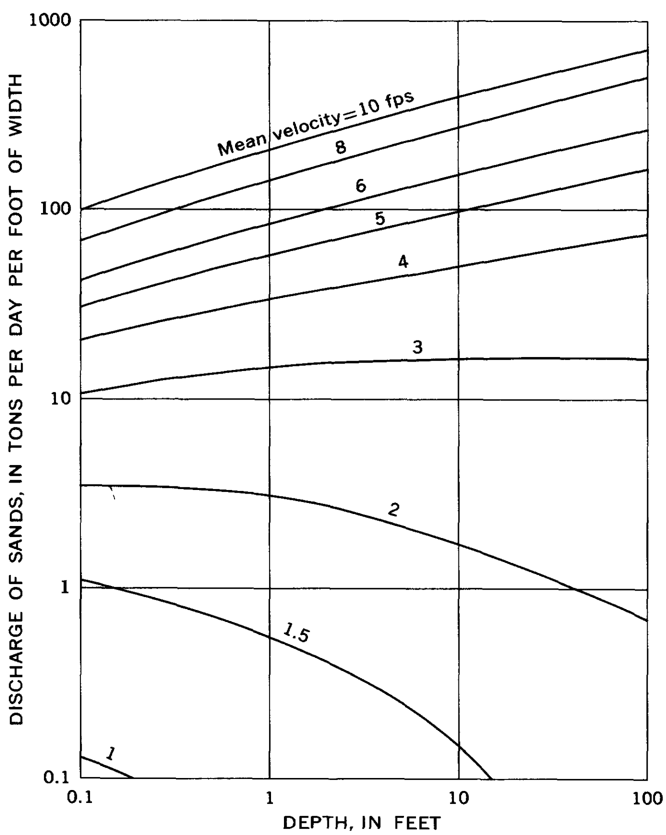


FIGURE 19.—Empirically determined graph of the effect of depth on the relationship between discharge of sands and mean velocity. (Water temperature 60 °F, and median diameter of bed sands 0.30 mm.)

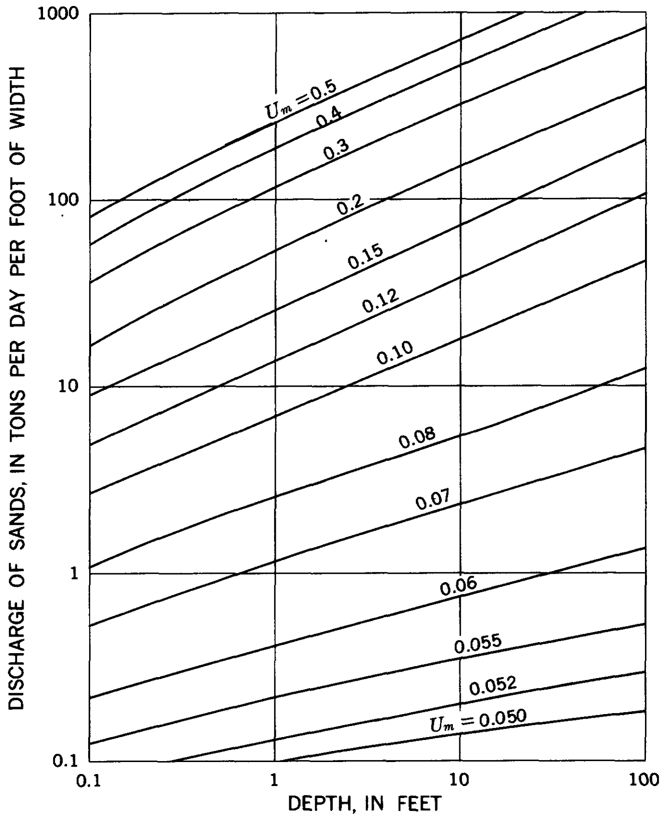


FIGURE 20.—Effect of depth on the relationship between discharge of sands and shear velocity computed from mean velocity. (Water temperature 60°F, and median diameter of bed sands 0.30 mm.)

lower mean velocity such as 1.5 feet per second, the discharge of sands decreases rapidly as the depth increases at constant shear. If the Chezy C varies, as it does when the bed configuration changes, the effect of depth on the relationship between discharge of sands and stream power becomes more complex and is somewhat indefinite. It is indefinite because the compound parameter stream power is also somewhat indefinite in the sense that doubling the shear may cause a different effect than doubling the mean velocity.

The effect of depth on relationships that involve concentrations of sands should also be understood because it seriously limits the usefulness of concentration of sands as a measure of the rate of sediment movement. The effect of depth on the relationship between concentration of sands and mean velocity for flow over a sand bed whose median particle size is 0.30 mm is large and variable. (See fig. 21, which is plotted directly from the information on fig. 19.) Also, the effect of depth on the relationship between the concentration of sands and mean velocity obviously will differ considerably from one particle size to another. The effect of depth on the relationship of concentration of sands to mean velocity is large and

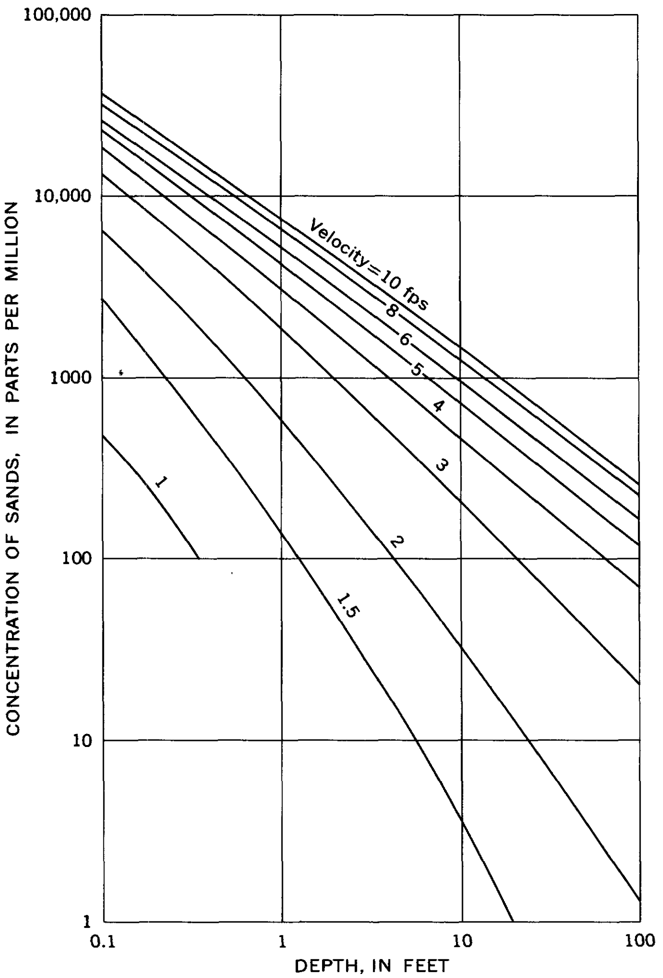


FIGURE 21.—Effect of depth on the relationship of concentration of sands to mean velocity. (Water temperature 60°F, and median diameter of bed sands 0.30 mm.)

variable; therefore, the concentration of sands is a relatively meaningless parameter of rates of sediment transportation as compared to discharge of sands per foot of width, unless both particle size and depth of flow are stated or are kept within narrow ranges. Hence, the concentration of sands may be significant in a flume study of about equal depths of flow over a particular bed of sand, but any attempts to relate concentrations of sands at a cross section in a flume to concentrations at a cross section in a natural stream are likely to be unfruitful. The effect of depth is also large and variable on the relationships of concentration of sands to shear, to shear velocity computed from mean velocity, or to stream power.

VISCOSITY

According to the concept of sediment transportation, viscosity has an effect on the thickness of the laminar sublayer, but its major effect on the relationship between discharge of sands and mean velocity is through its influence on the fall velocities of the particles and thus on the vertical distributions of sediment of different sizes. The percentage effects of changes in water temperature on the discharge of sands obviously varies with velocity, depth, and particle size. If the concentrations of the different particle sizes and the vertical distributions of the concentrations could be computed accurately, the effect of a change in viscosity could also be determined accurately, although the computations would be rather complex and tedious. Unfortunately, the vertical distributions of concentration cannot now be accurately computed, especially for flows over dune beds. Until vertical distributions of sands are more accurately predictable, somewhat oversimplified adjustments based mainly on empirical analysis of available field and flume information may be more practical measures of the effect of viscosity on the discharge of sands than are more theoretical adjustments.

The viscosity of water or the apparent viscosity of a water-sediment mixture can be changed appreciably by variations in water temperature or in concentrations of fine sediment. The relationship between the viscosity of distilled water and water temperature is accurately known. In contrast, high concentrations of fine sediment have a large and less readily defined effect on the apparent viscosity of the water-sediment mixture (D. B. Simons, E. V. Richardson, and W. L. Haushild, written commun.; J. C. Mundorff, written commun.). The effect varies not only with concentration of the fine sediment but with characteristics of the fine sediment. The effects of changes in water temperature and in concentration of fine sediment on discharge of sands are sometimes too large to be disregarded.

WATER TEMPERATURE

The effect of water temperature at a particular cross section can be approximated with trial-and-error multiple correlation. Usually, the size and the composition of the bed sediment at a particular cross section of a sand-bed stream are fairly constant with respect to time and flow, and either the relationship between depth and velocity is reasonably constant or the velocity is in the range for which depth has little effect on the relationship between discharge of sands and mean velocity. Hence, the effect of changes in depth at a particular mean velocity is likely to be small, and the discharge of sands at the cross section is controlled mainly by mean velocity and water temperature if concentrations of fine sediment are not high. An average curve drawn through the observed discharges of sands plotted against mean velocities for Niobrara

River near Cody, Nebr. (fig. 7), gives an approximation of the effect that a change in mean velocity has on the average discharge of sands. The ratio of each observed discharge of sands to the average discharge of sands from the curve of figure 7 at that mean velocity can be computed and then plotted against water temperature. (See fig. 22.) Although the points scatter widely, in general the discharge of sands decreases as the temperature increases.

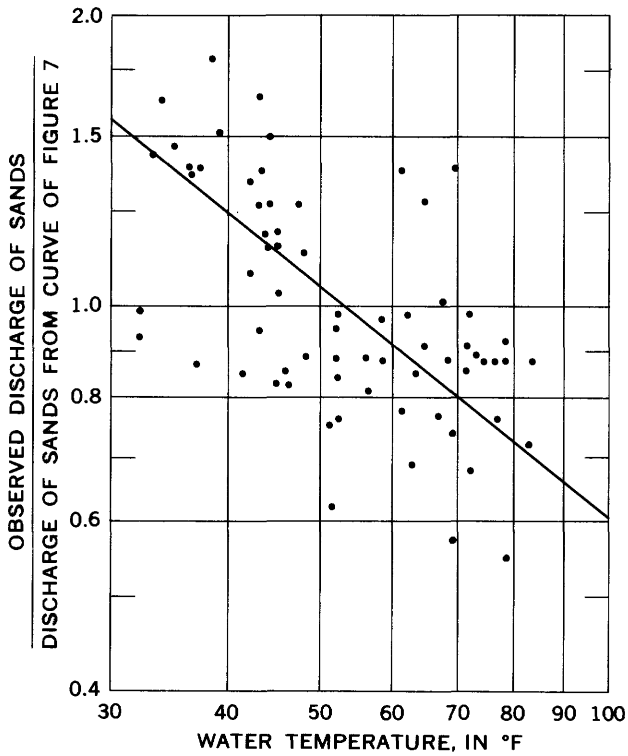


FIGURE 22.—Adjustment for effect of water temperature on the discharge of sands, Niobrara River near Cody, Nebr.

The curves of figure 7 and 22 are only first approximations of the relationships that they represent. Second approximations can be obtained by using the curve of figure 22 to adjust the observed discharges of sands to equivalent sand discharges for a constant temperature of, for example, 60°F. The temperature-adjusted discharges of sands can then be plotted to obtain a second approximation of an average curve of relationship of discharge of sands to mean velocity. This second approximation is, of course, for a temperature of 60°F. Next, the ratio of each observed discharge of sands (unadjusted) to the average discharge of sands from the curve of second approximation can be plotted against water temperature to obtain the second approximation of the effect of changes in water temperature. For the discharge of sands in Niobarara River near Cody, the second approximation for the effect of temperature changes did not seem to differ significantly in slope from the curve of figure 22.

The effect of water temperature on discharge of sands, defined by trial-and-error multiple correlation, for three sediment stations is surprisingly consistent. (See fig. 23.) It also agrees approximately with the computed effect of temperature (fig. 17) for usual combinations of depth and velocity (for example, 10 feet and 7.0 feet per second or 2.0 feet and 3.0 feet per second) in natural flows over a bed material whose median diameter is 0.40 mm. However, the percentage effect shown on figure 17 is generally a little smaller for the 0.40-mm bed sand than for the somewhat finer bed sands at the three sediment stations for which the temperature effect was empirically defined.

A complete expression of the temperature effect might be made in terms of several graphs, each for a different median size of bed material and each similar to figure 17. However, presently available information is inadequate to define such graphs accurately either empirically or theoretically.

In the absence of such detailed and accurate graphs, an oversimplified relationship indicated by the dashed curves on figure 24 is suggested as a practical measure of the effect of water temperature on the discharge of sands in sand-bed streams. It is based on figures 18 and 23 and on general considerations of the effect of viscosity on the discharge of sands. It may be satis-

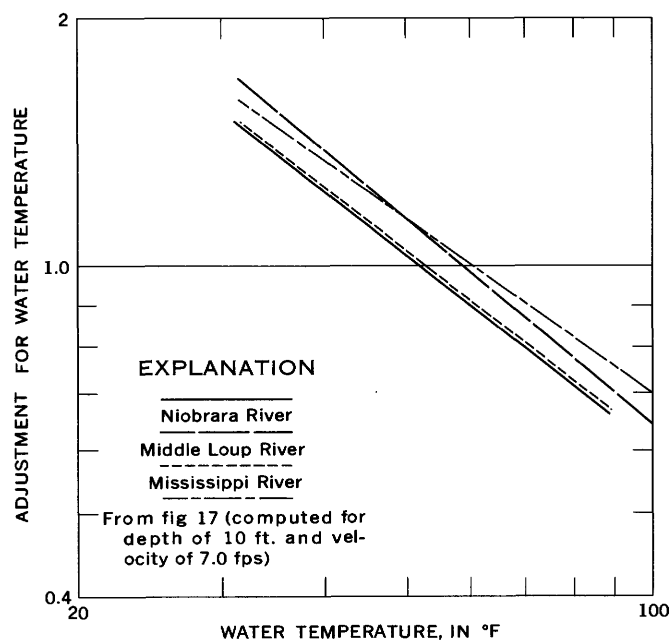


FIGURE 23.—Effect of water temperature on the discharge of sands in four natural streams.

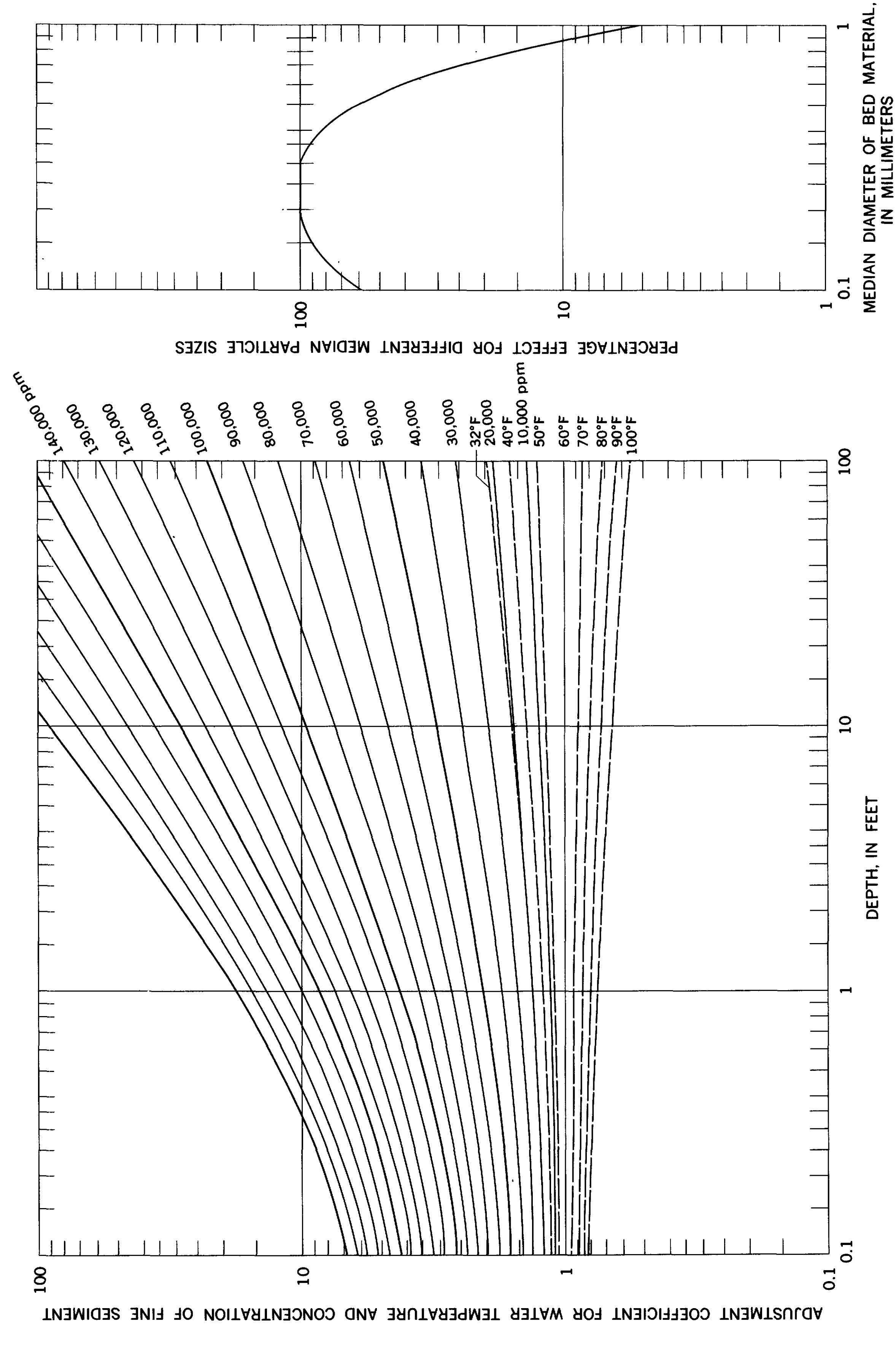


FIGURE 24.—Approximate effect of water temperature and concentration of fine sediment on the relationship of discharge of sands to mean velocity.

factorily accurate for most natural streams whose bed sands have median diameters of 0.20 to 0.40 mm but may be very inaccurate for some other particle sizes, for very low velocities, or for very deep flows. A theoretically accurate temperature correction would be different for each particle size of sand in suspension and presumably cannot be made without a more exact knowledge of the vertical distribution of sediment than is now available. Fortunately, the temperature correction generally is rather small, and a large percentage error in the correction usually causes only a moderate percentage error in discharge of sands.

A curve for making rough estimates of adjustments for different particle sizes is also given on figure 24, but it may be very inexact for some particle sizes. This curve and its usage will be discussed in more detail later.

CONCENTRATION OF FINE SEDIMENT

According to D. B. Simons, E. V. Richardson, and W. L. Haushild (written commun.), a high concentration of fine sediment has an effect on the fall velocity of sands that is explainable in terms of the change that the fine sediment causes in the apparent viscosity of the water-sediment mixture. Their laboratory investigations showed that the median fall diameter of each of three sands decreased as the concentration of suspended fine sediment increased. Roughly four times as high a concentration of kaolin as of "bentonite" was required to decrease the median fall diameter by equal amounts. (However, the fine material that Simons, Richardson, and Haushild called "bentonite" contained about 25 percent of particles coarser than 0.004 mm and about 3 percent coarser than 0.062 mm. In this paper the term "bentonite" when inclosed in quotation marks refers specifically to the fine sediment that they used.) Although these laboratory findings show the great effect that high concentrations of fine sediment have on the discharge of sands, they cannot be used directly to indicate the effect in natural streams.

To some extent the effect of an increase in concentration of fine sediment is equivalent to a decrease of water temperature. Median fall diameters of 0.19 and 0.29 mm were decreased about as much by an increase of 40,000 ppm of "bentonite" as by a change in water temperature from 100° to 32°F. The same general equivalence can be established between the change in concentration of "bentonite" that will cause a given change in apparent viscosity of the water-sediment mixture and the change in water temperature that will cause an equal change in the viscosity of distilled water.

For concentrations of "bentonite" on the order of 50,000 ppm or less, an adjustment cofficient for their effect on the discharge of sands can be estimated on the basis of the indicated equivalent effect of a particular concentration of "bentonite" and a given change in water temperature; that is, if the temperature and the concentration of fine sediment affect the relationship of discharge of sands to mean velocity only through changes in viscosity or apparent viscosity, the adjustment coefficient can be approximated on figure 24 on the basis of the curves already drawn on that figure for water temperature. However, the extension of the graph to high concentrations of "bentonite" would obviously be very questionable if based only on the relationship to water temperature.

A crude idea of the adjustment coefficients for high concentrations of fine sediment can be obtained from information for Rio Puerco near Bernardo, N. Mex., whose sediment concentrations frequently are very high. Usually, the discharge of sands at a given mean velocity, especially a high mean velocity, is far greater (fig. 11) than that for flows over a bed of roughly similar sand in the Rio Grande or other streams. However, discharges of sands may be unusually high (fig. 10) in the Rio Grande at infrequent times when the concentrations of fine sediment are high. The clay carried by the Rio Puerco and the Rio Grande is probably bentonite or similar fine material, but appreciable quantities of the fine sediment are silt. Because 25 percent of the "bentonite" used in the laboratory studies consisted of particles coarser than clay, the fine material of the Rio Puerco and the Rio Grande probably is similar to the "bentonite." The ratios of observed discharges of sands (roughly adjusted to a water temperature of 60°F) for Rio Puerco near Bernardo to discharges of sands from the estimated curve for low concentrations of fine sediment (fig. 11) were plotted on a draft of figure 24. The ratios together with extrapolations from the curves for lower concentrations of fine sediment on figure 24 were used to define the adjustment coefficients for high concentrations of "bentonite" or of the fine sediment that is transported by the Rio Puerco.

Discharges of sands computed from figure 24 average roughly the same as the observed discharges of sands for the Rio Puerco, but the observed discharges tend to be somewhat higher at high flows and lower at low flows. (See fig. 25.) Although the percentage differences between computed and observed discharges of sands are rather large, the adjustments of figure 24 can be used to remove most of the wide departures of the observed discharges of sands in the Rio Puerco from the approximate curve (fig. 11) for low concentrations of fine sediment. Of course, the adjustments of figure 24 are unlikely to apply as well to other streams as to the Rio Puerco for which they were defined.

Presumably, the adjustment coefficients for water temperature and for concentration of fine sediment can be multiplied together to obtain a single adjustment coefficient for the combined effects of water temperature and concentration of fine sediment on the relationship between discharge of sands and mean velocity.

Adjustment coefficients similar to those on figure 24 will certainly vary with changes in bed-material size, but the relative effects of the different factors that affect the coefficients have not been satisfactorily evaluated. Hence, the right-hand diagram of figure 24 is only a guess at the relative effect of concentration of fines or of water temperature for different median sizes of bed sediment. The effect is stated in percent, and the adjustments on the main diagram of figure 24 are assumed to be for a median size of bed sediment of 0.20 to 0.30 mm and to represent 100 percent. Of course, the adjustment coefficients from the main diagram of figure 24 are not to be directly multiplied by percentages from the right-hand diagram; rather, the adjustment coefficients minus 1.00 are to be multiplied by the percentages. For example, if an adjustment coefficient from the main diagram is 1.50 and the median diameter of the bed sediment is 0.50 mm, the adjustment for the effect of viscosity would be 60 percent of 0.50 or 30 percent. The final adjustment coefficient would be 1.30.

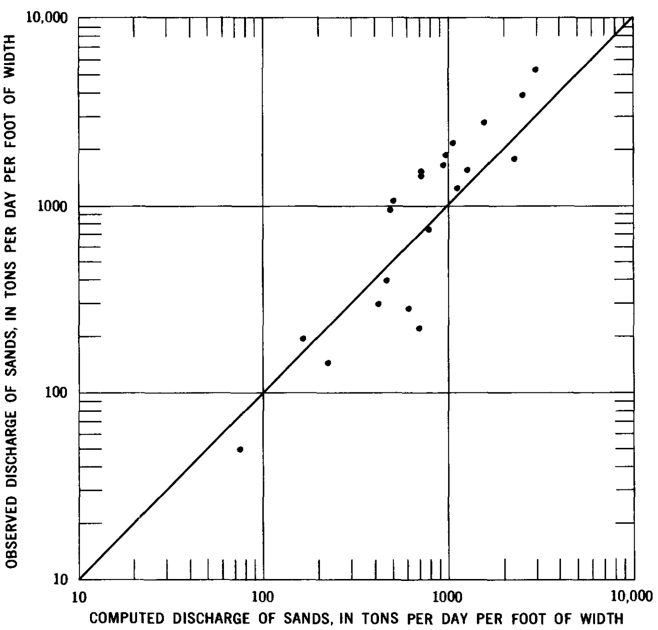


FIGURE 25.—Comparison of computed and observed discharges of sands for Rio Puerco near Bernardo, N. Mex.

If the fine sediment transported by a stream is mostly kaolin, the adjustments of figure 24 might be used as a very rough estimate after the concentrations of fines is divided by four (D. B. Simons, E. V. Richardson, and W. L. Haushild, written commun.). The effects of high concentrations of other fine sediments or of mixtures of fine sediments are scarcely known at all. Also, the chemical character and the concentration of the dissolved solids in the flow probably control to an appreciable degree the effect of any particular high concentration of fine sediment.

At times, the fine sediment may deposit on or in the top layers of the streambed if the concentrations are very high and if the velocities near the bed are relatively low. The presence of fine sediment at the surface of the bed may reduce the discharge of sands. If the free movement of the sand particles at the surface of the bed is known to be inhibited by the fine sediment, the discharge of sands may be relatively low, and the adjustments of figure 24 should not be applied.

The adjustment for the effect of high concentrations of fine sediment on the discharge of sands could be based on the assumption that the fall velocity of a sand particle is the only measure of size that has an appreciable effect on the discharge of the sand. In other words, a sand bed whose median particle size is 0.40 mm might be assumed to have the same discharge of sands for a given condition of flow as a bed whose median particle size was 0.20 mm if the concentration of fine sediment was high enough to reduce the fall velocity of the 0.40 mm sand to that of a 0.20 mm sand. This type of adjustment was not made for two main reasons. First, in shallow flows the discharge of sands seems to increase appreciably if the concentration of fine sediment is greatly increased even for ranges of depth and velocity at which a decrease in particle size of the bed sands apparently causes little increase in the discharge of sands. Second, knowledge of the relationship of the discharge of sands to mean velocity and depth is very inexact for sands that have as low fall velocities as those of 0.25 to 0.40 mm sands when settling through water that contains very high concentrations of fine sediment.

In general, the effect of fine sediment on the relationship between discharge of sands and mean velocity is highly complex and poorly understood. The effect, although sometimes very large in natural streams, has usually been disregarded in the computation of discharges of sands or bed material. The relatively simple adjustments suggested here are intended to give only a rough idea of (a) the actual relationships and (b) the possibly large effect of concentration of fine sediment on the discharge of sands. Computed discharges of sands that are based on these adjustments for high concentrations of fine sediment are likely to be very inaccurate. However, the comparatively minor adjustments for concentrations of fine sediment lower than perhaps 50,000 ppm may usually be realistic for flows in most natural streams whose bed materials have median diameters of 0.20 to 0.40 mm.

GRAPHS FOR COMPUTING DISCHARGE OF SANDS

The complicated relationships involving major factors that control the discharge of sands in sand-bed streams can probably be better understood from graphs than from mathematical equations. Also, the discharges of sands can be computed more readily from graphs than from equations. Hence, graphs were prepared to indicate the relationships of discharge of sands to the major factors that affect the discharge of sands. The general concept of sediment transportation gives an idea of qualitative relationships to be expressed by the graphs; but until sediment transportation is better understood, the quantitative definition of the graphs must depend mainly on field and laboratory information.

BASIC INFORMATION

A graph such as the one (fig. 19) for a bed sand whose median diameter is 0.30 mm could be defined readily if enough accurate determinations of discharge of sands were available for a particular size of bed sand, for low concentrations of fine sediment, and for one water temperature. Enough determinations would require discharges of sands for a wide range of depths and velocities. In the absence of enough accurate information, a few graphs, each similar to that of figure 19 and each for a different size of bed sand, were approximated from somewhat random and sometimes questionably accurate data. None of the flume or field information was collected specifically for this analysis of the discharge of sands.

Total discharges of bed material over beds of sand were tabulated for many flume investigations, but no attempt was made to investigate all flume studies of bed-material discharge because the relationships were to be defined mainly for field streams. Particularly helpful were published data from flume experiments that were reported by Gilbert (1914), Barton and Lin (1955), Simons, Richardson, and Albertson (1961), and Brooks (1958).

Field data on the discharge of sands for several streams in different parts of the United States were tabulated and studied. Sand discharges for Middle Loup River near Dunning, Nebr. (Hubbell and Matejka, 1959), and for Niobrara River near Cody, Nebr. (Colby and Hembree, 1955), were approximately total discharges. They were based on measurements of bed-material discharge at cross sections where velocity and turbulence were high enough to suspend practically the entire sediment discharge of the stream. Widths and mean velocities were measured at nearby cross sections where the flow was over beds of sand whose median diameter was determined by sampling. For most other sand-bed streams in this study, the total discharges of sands were not so directly determined, and only the measured discharges of sands were known. For these streams, the total discharges of sands usually had been computed in local offices from the modified Einstein procedure (Colby and Hembree, 1955), but some were computed locally or by the writer from measured discharges of sands and from the relationship of unmeasured sediment discharge to mean velocity (Colby, 1957). Thus, total discharges of sands based partly on indirect computations were used for Elkhorn River near Waterloo, Nebr. (Beckman and Furness, 1962), the lower Colorado River (U.S. Bureau of Reclamation, 1958), Pigeon Roost Creek and tributaries in northern Mississippi (unpublished information, Agricultural Research Service, U.S. Dept. of Agriculture, University, Miss.), Mississippi River at St. Louis, Mo., the Cedar, Little Blue, North Loup, and South Loup Rivers in Nebraska, and the Rio Grande and the Rio Puerco in New Mexico (unpublished information, U.S. Geol. Survey, Lincoln, Nebr., or Albuquerque, N. Mex.).

GENERAL METHOD OF ANALYSIS

Graphical analysis of the information on the discharge of sands consisted in eliminating progressively the approximate effect of one variable after another. On the basis of the assumption that discharge of sands is proportional to stream width, all discharges of sands were expressed in tons per day per foot of stream width. These discharges for a single cross section or sometimes for a combination of cross sections were then usually plotted against mean velocity to define a relationship that applied specifically to each section or combination of sections. For some sections, the ratios of observed discharges of sands to the discharges from the average curve were plotted against water temperature to obtain the approximate effect of water temperature (fig. 22, for example) on the relationship between discharge of sands and mean velocity. After the effect of water temperatures had been defined for a few cross sections (fig. 23), the discharges of sands per foot of width were roughly adjusted to a water temperature of 60°F, if the water temperature were known.

The next major step in the graphical analysis was to define the separate graphs showing the effect of depth (such as fig. 19) on bed sands of different median diameters. Each temperature-adjusted discharge of sands for a median diameter of bed sands between about 0.06 and 0.90 mm was plotted against depth on the graph sheet for the selected median diameter (0.10, 0.20, 0.30, 0.40, 0.60, or 0.80 mm) that was closest to the reported median diameter. The mean velocity was noted beside each plotted point. Some discharges of sand over beds whose median diameter was, for example, about 0.25 mm were plotted on graphs for both the nearest smaller and the nearest larger median diameter. Lines for different mean velocities were drawn on each of the six graphs on the basis of the plotted points and judgment. The general form of the graph of figure 19 was used as a guide. On parts of a graph for which data were insufficient, somewhat contradictory, or entirely lacking, rough estimates were made by comparison among the six graphs or by consideration of the general concept of sediment transportation. Then the six graphs were redrawn as relationships between discharge of sands and mean velocity for each of four depths of flow; the median size of the bed sands was used as a second independent variable. The smoothing of the curves for these relationships, which were first approximations of the graphs of figure 26, removed any large inconsistencies from one median size of bed sand to another. Smoothing of the graphs was continued by replotting alternately in the form of figure 19 and in the form of figure 26 until a reasonably consistent set of curves (fig. 26) was obtained. The form of figure 26 was selected rather than that of figure 19, because the interpolation for the effect of depth can usually be made more conveniently and accurately than the interpolation for the effect of mean velocity.

The derivation of figure 24 for the effect of changing water temperature and for the effect of high concentartion of fine sediment has already been discussed.

The effect of bed configuration on the relationship between mean velocity and discharge of sands may sometimes be large and is likely to be most evident in natural streams at the cross sections that are uniform laterally. One of the dashed curves of figure 9, the curve for Pigeon Roost Creek near Byhalia, Miss., has a flatter slope at mean velocities of 3.0 to 5.0 feet per second than at either lower or higher mean velocities, and the flatter slope coincides with the transition from a dune bed to a plane bed. A distinguishing characteristic of the channel at this station is lateral uniformity, which results in a change of bed configuration at about the same time across most of the channel width. In natural streams, changes in bed configuration usually occur gradually; that is, a change usually affects only a small part or strip of the streambed at first and then spreads over more of the bed. Thus, at any one time only part of the width of the bed is likely to be plane unless the channel is unusually uniform laterally. According to D. B Simons (written commun.), flume experiments have shown considerably lower discharges of sands for a given mean velocity over a plane bed than over an antidune bed. Such a relationship might be explainable, at least in part, by the fact that the turbulence for a given mean velocity, depth, viscosity, and particle size should be less over a plane bed than over a dune or antidune bed. Although no adjustments have been applied in this paper for differences in bed configuration and none may be generally required for most natural streams, the discharge of sands should be expected to be considerably lower over a plane bed than for the same velocities and depths over irregular sand beds.

The available information was inadequate for determining the usually minor effects of such factors as the variations or scattering of particle size, velocity, or depth about the median or mean. Obviously, the variation of particle size of the bed material about the median diameter may sometimes appreciably affect the discharge of sands, perhaps especially at low flows. A nonuniform lateral distribution of velocity and depth may also have a large effect on the discharge of sands at some irregular cross sections. If an appreciable part of a cross section is shallow and has a low velocity as compared with the rest of the section, the discharge of sands may well be computed separately for each of two or more parts of the cross section.

The graphical analysis of the data, which is largely based on judgment and evaluation of complex factors, would not be exactly duplicated if the study were repeated. Obviously, the adequacy and accuracy of the analysis cannot be directly stated but can be crudely indicated by comparison of observed discharges of sands with discharges computed from the results of the graphical analysis. This comparison and an evaluation will be made as a part of the discussion of probable accuracy of sand discharges that are determined from the final graphs.

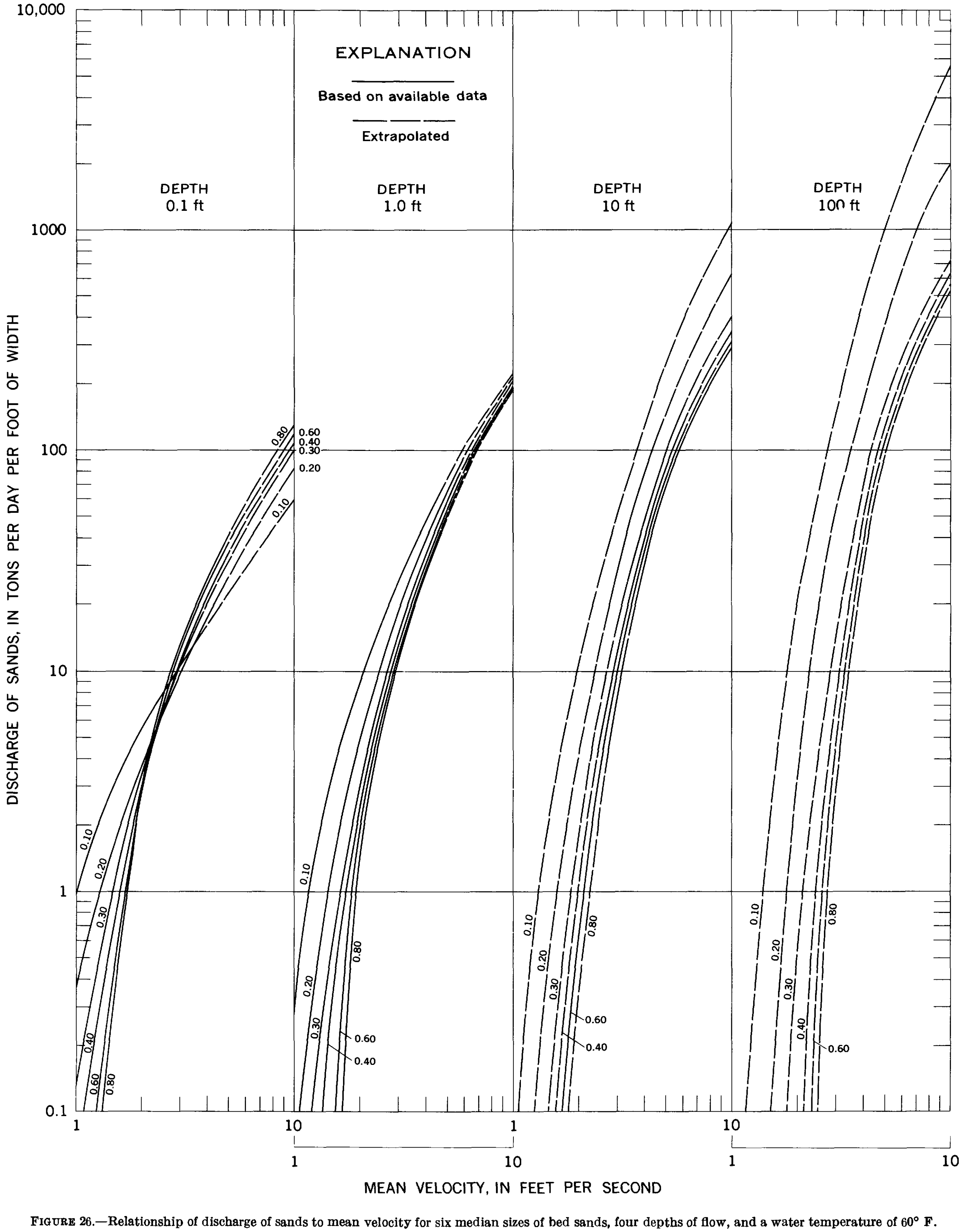


FIGURE 26.—Relationship of discharge of sands to mean velocity for six median sizes of bed sands, four depths of flow, and a water temperature of 60° F.

DETERMINING THE DISCHARGES OF SANDS FROM THE GRAPHS

The discharge of sands in a sand-bed stream can be computed from the empirical graphs according to one or more of three methods. The choice of a method depends mainly on the number of variables that need to be considered in the computation. If the velocity, depth, bed-material sizes, water temperature, and concentration of fine sediment are within the general range for which the curves of figure 13 were defined, the effect of variables other than mean velocity may perhaps be disregarded, and the discharge of sands per foot of width may be taken with reasonable accuracy from one of the curves of figure 13. However, if the depth effect is likely to be appreciable, the discharge of sands should be determined from figure 26. If concentrations of fine sediment are high and water temperatures differ considerably from 60°F, the discharge of sands from figure 26 should be adjusted according to figure 24. Examples show the computation of the discharge of sands according to each of the three methods.

First method—discharge of sands based on figure 13

Selected data for example 1:

Mean velocity ft per sec	2.70
Depth ft	3.5
Median size of bed sediment mm	0.37
Water temperature ° F	65
Concentration of fine sediment ppm	1,500

Summary of the computations: Because the size of bed material is about that for Pigeon Roost Creek, the discharge of sands per foot of stream width can be taken directly as 11 tons per day per foot of width from the curve for Pigeon Roost Creek on figure 13. (If computed by the second method, the result is about 8.5 tons per day per foot.)

Second method—discharge of sands based on figure 26

Selected data for example 2:

Mean velocity ft per sec	6.2
Depth ft	15.5
Median size of bed sediment mm	0.26
Water temperature ° F	57
Concentration of fine sediment ppm	650

Summary of the computations: From figure 26 the indicated discharges of sands are by interpolation for median size about 188 and 400 tons per day per foot for depths of 10 and 100 feet, respectively. Interpolation for the depth of 15.5 feet can be made by guess, by use of a simple diagram, or with a straightedge on a sheet of log-log paper to obtain 210 tons per day per foot. This discharge of sands can be quickly checked against a discharge of 220 tons per day per foot that might be picked directly from the curve (fig. 13) for the Mississippi River at St. Louis, Mo. Thus, the first method can usually be used as an easy check for major errors in computations by the second method.

Third method—discharge of sands based on figures 24 and 26

Selected data for example 3:

Mean velocity ft per sec	6.5
Depth ft	4.8
Median size of bed sediment mm	0.43
Water temperature ° F	75
Concentration of fine sediment, mostly bentonite ppm	33,000

Summary of the computations: From figure 26 the discharges of sands are about 92 and 150 tons per day per foot of width for depths of 1.0 and 10 feet, respectively. Hence, about 130 tons per day per foot of width can be interpolated for the depth of 4.8 feet. The adjustment coefficient from figure 24 for 75°F and a depth of 4.8 feet is 0.86 for a median diameter of 0.20 or 0.30 mm. Also, the adjustment coefficient for 33,000 ppm of fine sediment, mostly bentonite, is 1.92 for a median diameter of 0.20 to 0.30 mm, and the total adjustment coefficient is 1.92×0.86 or 1.65. According to the right-hand graph of figure 24, the effect of a change in viscosity or apparent viscosity is only 78 percent as large for the median diameter of 0.43 mm as for a median diameter of 0.20 or 0.30 mm. Therefore, 78 percent of (1.65−1.00) or 0.51 is added to 1.00 to obtain the estimated adjustment coefficient for the median diameter of 0.43 mm. The 130 tons per day per foot multiplied by 1.51 gives 196 tons per day per foot, which could well be rounded to 200 tons per day per foot because the discharge of sands ordinarily should not be determined to more than two significant figures. The discharge of sands from the curve of figure 13 for Pigeon Roost Creek is about 150 tons per day per foot.

PROBABLE ACCURACY OF THE GRAPHS

Three phases of the probable accuracy of the graphs for computing the discharge of sands need consideration. In order of their discussion, these phases are the experimental errors inherent in the basic field and laboratory information, the accuracy of the definition of the graphs for the data that were included in the study, and the applicability of the graphs to discharges of sands for streams that were not included in the study.

EXPERIMENTAL ERRORS IN BASIC INFORMATION

A major difficulty in an evaluation of the probable accuracy of the graphs is that the amount of experimental error in individual determinations of discharge of sands is seldom known satisfactorily. However, the fact of appreciable experimental error can be easily established. For example, rather wide variations of concentration of suspended sediment (mostly but not all sands) were measured at a turbulence flume in Middle Loup River at Dunning, Nebr., by Benedict, Albertson, and Matejka (1955). Also, C. F. Nordin and J. K. Culbertson (written commun.) reported two sets of suspended-sediment samples at about the same times and at nearly steady flow at each of seven sections in Rio Grande near Barnalillo, N. Mex.; these samples, although probably collected more carefully than most samples, showed considerable percentage differences in the concentrations of sands. To the nearest whole percent, the differences between the two concentrations of silt plus clay for a cross section ranged from 0 to 8 percent and averaged 3 percent for the seven cross sections. The differences between the two concentrations of suspended sands ranged from 3 to 33 percent and averaged 20 percent. Such differences as these (which probably are reasonably typical) between individual determinations of average concentration of suspended sands at a cross section indicate proportional errors in the measured discharges of the suspended sands. (Actually, these differences are, of course, measures of consistency rather than of absolute accuracy.) In addition to differences in carefully determined discharges of sands at a cross section, wide differences in measured discharges of sands may occasionally result from the collection in a suspended-sediment sample of sands from the bed layer or the streambed unless the excess sand is noted immediately and the sampling is repeated. Most experienced field men consider that sampled concentrations of suspended sands are likely to be much too large at times but are unlikely to be much too small. Usually several individual determinations of the discharge of sands may be required to establish an accurate average discharge of sands for a particular time and flow. Even in a flume experiment for which conditions of flow and sediment discharge are likely to be more constant than in field investigations, the sampling of sands may have to be continued over a period of a few hours in order to establish an accurate average discharge of sands.

Because the average discharge of sands during a period of steady flow is the discharge that should correlate with characteristics of the flow and the bed sand, departures from this average are considered to be experimental errors. Thus, the experimental error is a measure of the difference between an experimentally determined or computed discharge of sands and the average discharge of sands in the same flow at a steady state. It is not the difference between the experimentally determined or computed discharge of sands and the actual instantaneous discharge of sands at that time. Therefore, because of short-time fluctuations in the discharge of sands, the experimental error may be large even though the determination of the instantaneous discharge of sands was accurate.

At many of the high flows and sometimes at rather low flows per foot of width, the mean velocities used in this study were determined from bridges where the sand bed is affected by local scour and where piers may cause turbulence that is relatively high as compared to the mean velocity. In general, the discharges of sands per foot of width at bridge sections are likely to be larger than they would be for the same velocities in representative reaches of channel away from the bridges.

Lateral distributions of depth and velocity are sometimes so far from uniform that the reported means are not good measures of the discharge of sands. However, inaccuracies that are caused in the relationships of discharge of sands to mean velocity and to depth by poor lateral distributions are more strictly failures in analysis of the data than experimental errors.

If the slope of the water surface is used as a parameter of the discharge of sands, considerable experimental error should be expected in the determinations of slope, unless the total fall through the reach is large enough to be determined accurately and unless several observations of the slope are made and averaged.

Total discharges of sands in field streams are generally measurable only at very constricted sections or at structures that have been designed for the determination of the total sand discharges of the stream. The widths, depths, and velocities at such places are not useful measures of the discharge of sands in sand-bed streams. If useful relationships are to be obtained, the discharges of sands at these places must be correlated with widths, depths, and velocities at nearby reaches of channel where the streambed is mostly cohesionless sand. Obviously, temporary net scour or net fill and the short-time fluctuations in discharge of sands even at constant flow make uncertain any individual determinations of total discharge of sands in terms of conditions of flow at another cross section.

In most streams, only part of the discharge of sands is measured, and the rest is computed somewhat indirectly. The computation of part of the discharge of sands introduces error because the procedures for computing the unmeasured discharges of sands are by no means exact and because some of the procedures depend on the measured concentrations of sand, which are themselves inexact. The probable inaccuracy of the computation of the discharge of unmeasured sands is known only roughly.

DATA INCLUDED IN THE STUDY

The many and complicated steps involved in the graphical analysis cannot readily be reproduced or explained in detail in this paper and were partly based on judgment. Hence, the probable accuracy of the graphs on figures 24 and 26 is difficult to evaluate or to state. A practical measure of the probable accuracy for data that were included in the study can be obtained by comparing the discharges of sands computed from figures 24 and 26 with the original observed discharges of sands. (The term "observed discharges" is used for simplicity even though for many field streams the "observed discharges" of sands are partly based on indirect computations of the unmeasured discharges of sands.) Because many observations were used in the graphical analysis, an arbitrary selection was made for the test of accuracy. Within each set of field or flume data, the first, sixth, eleventh, sixteenth, etc., determinations were selected. Thus, at least one observation was included from each set of data.

Although the difference between the computed and the observed discharge of sands sometimes is large, especially for low velocities and shallow depths, the general relationship for field sand-bed streams is reasonably good. (See fig. 27.) Of about 200 indi-

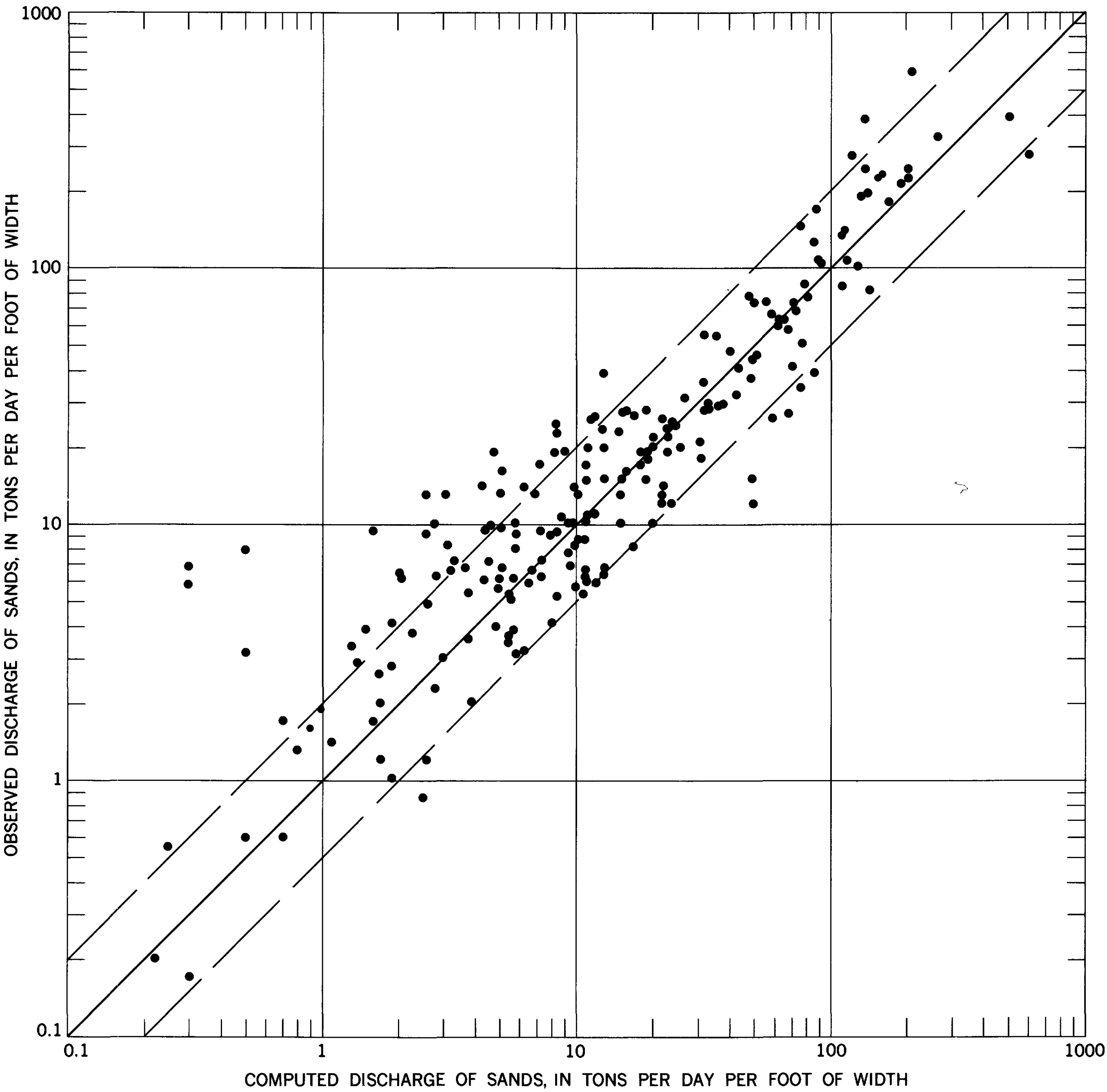


FIGURE 27.—Comparison of computed and observed discharges of sands for natural streams that were included in the study.

vidual comparisons, about three-fourths of the observed discharges of sands are within limits that are 100 percent larger and 50 percent smaller than the computed discharges of sands. The widest percentage differences and proportionately the fewest close agreements are for shallow depths and low mean velocities. Usually in field sand-bed streams the lateral distributions of depth and velocity are more uneven for shallow flows, especially perhaps in very wide channels, than for deep flows. Also, at low velocities the percentage effects of small changes in particle size or in mean velocity may be large. For example, at a depth of 1 foot and for a median particle size of 0.30 mm, a change in mean velocity from 1.50 to 1.70 feet per second slightly more than doubles the discharge of sands, according to figure 26. The differences in tonnage of the computed and the observed discharges of sands at these low velocities and shallow depths are usually insignificant in terms of the annual discharge of sands for creeks and rivers even though the percentage difference is large.

At low depths and mean velocities, a somewhat better average agreement between computed and observed discharges of sands in natural streams could be obtained by revising the curves of figures 26, but such a change is probably not worthwhile until the reasons for the differences at low flows are better understood. Perhaps some differences are due to nonuniformity of the cross sections and could be partly removed by subdividing the cross sections into parts and then computing the discharge of sands separately for each of the parts. Disagreement may also be caused by different bed configurations at the same mean velocity.

The agreement between the computed and the observed discharges of sands is a little better for flows over sand beds in flumes (fig. 28) than in natural streams. Any change in figure 26 that would make the computed discharges of sands agree better with the observed discharges of sands from 1 to 10 tons per day per foot of width in natural streams would make the agreement poorer for the discharges of sands in flumes. Obviously, separate graphs like those of figure 26 could be prepared for natural streams and for flumes, or an arbitrary adjustment could be applied to the computed discharges of sands for low flows in natural streams. However, such procedures may not be desirable at this time.

STREAM REACHES AND FLUME INVESTIGATIONS NOT INCLUDED IN THE STUDY

Of course, really significant tests of the accuracy of the derived graphs should be based on the total discharge of sands at stream reaches for which information was not used to define the graphs. Some unpublished information for such reaches of natural streams was available in the files of the Geological Survey at Lincoln, Nebr., and Worland, Wyo. Also, two computations of total discharges of sands at individual verticals in the Missouri River at Omaha, Nebr., were furnished by Lloyd C. Fowler, U.S. Army Corps of Engineers, Omaha, Nebr. The observed discharges of sands in natural streams were determined by the modified Einstein procedure. The flume investigations by the U.S. Waterways Experiment Station (1935) were not used to define the graphs, so about every fifth run that had an observed discharge of sands greater than 0.1 ton per day per foot of width was selected as a check on the probable accuracy of the graphs for flume flows.

The comparison between the computed discharges of sands and the observed discharges of sands for cross sections that were not used to define the graphs is generally good for the cross sections that are relatively uniform and moderately deep and that have dependable basic information. The comparison was relatively much poorer for some of the other cross sections. (See fig. 29.) The computed and observed discharges of sands agree rather closely for the individual verticals of Missouri River at Omaha and for the cross sections of Niobrara River near Valentine, Nebr., Snake River near Burge, Nebr., and Bighorn River at Thermopolis, Wyo. The Popo Agie River near Riverton, Wyo., is not a sand-bed stream, although the beds of many pools are mostly sand. Also, the samples on which the size of the bed material for the Popo Agie River is based were collected more than a year after the dates for which the discharges of sands were observed. Probably because of the incomplete sand cover, the observed discharges of sands are somewhat lower than 50 percent of the computed discharges of sands. In general, shallow cross sections, such as those of Fivemile, Muddy, and Badwater Creeks and Pavillion drain, have considerably greater observed discharges of sands than computed discharges of sands. Beaver Creek near Arapahoe, Wyo., although its flows were rather shallow, does have a close relationship between observed and computed discharges of sands. The cross sections of Badwater Creek at Bonneville, Wyo., in addition to being shallow, are nonuniform laterally at low flow and have very high concentrations of fine sediment. In general, the graphs of figures 24 and 26 seem to represent the discharges of sands as well for these stream reaches that were not included in their definition as for the stream reaches that were included.

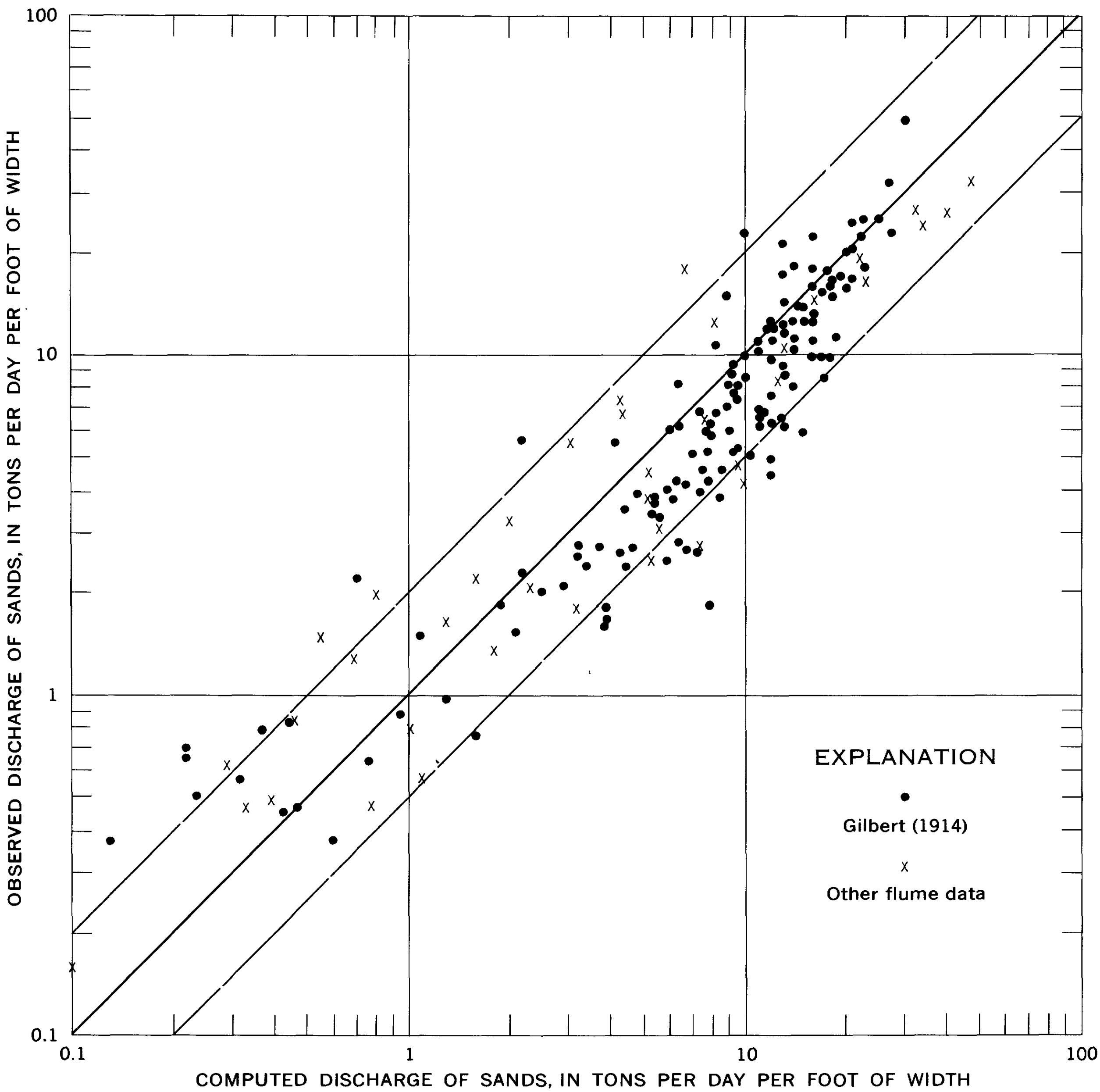


FIGURE 28.—Comparison of computed and observed discharges of sands for flume data that were included in the study.

For flume investigations, the graphs of figures 24 and 26 do not apply as well to the discharges of sands that were reported by the U.S. Waterways Experiment Station (fig. 30) as they did to the discharges of sands that were used to define the graphs. Perhaps the observed discharges of sands from the Waterways Experiment Station studies were lower than those from other flume studies because of relatively more coarse particles in several of their sands. The inconsistencies may be due, at least partly, to differences in bed configuration at about the same shallow depths and velocities. Sand 6 (U.S. Waterways Experiment Station, 1935), for example, had very little sand that was appreciably finer than the median particle size but had considerable sand that was considerably coarser than the median particle size. Observed discharges of sand 6 were very low as compared with the computed discharges of sands.

On the basis of figures 27 through 30, the graphs of figures 24 and 26 probably provide a means of computing fairly consistent discharges of sands at a particular cross section. However, especially for shallow flows and low velocities, the effects of some factors that vary from one stream or flume to another or from one sand to another are not yet satisfactorily represented by the graphs. Although the graphs

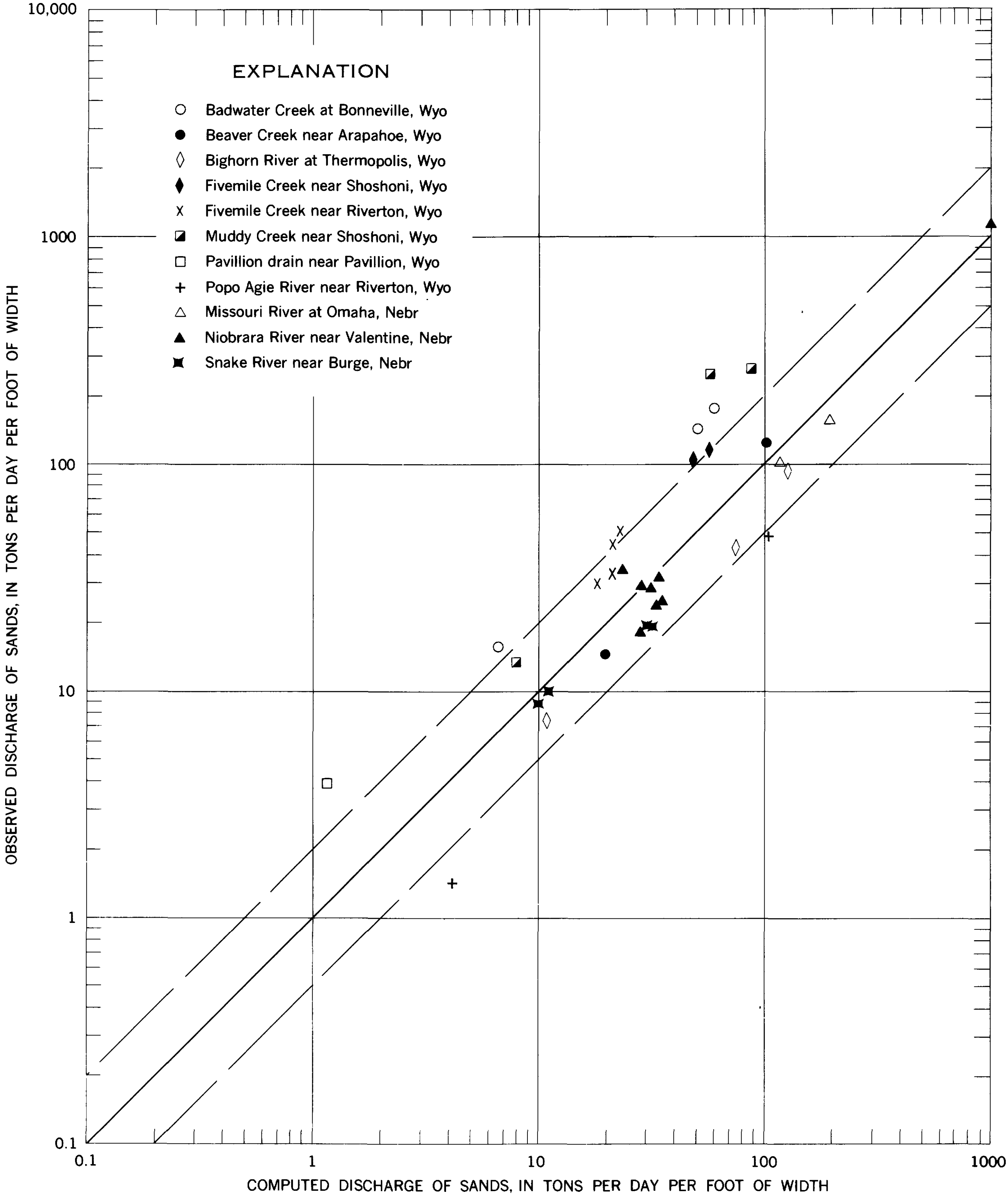


FIGURE 29.—Comparison of computed and observed discharges of sands for stream reaches that were not included in the study.

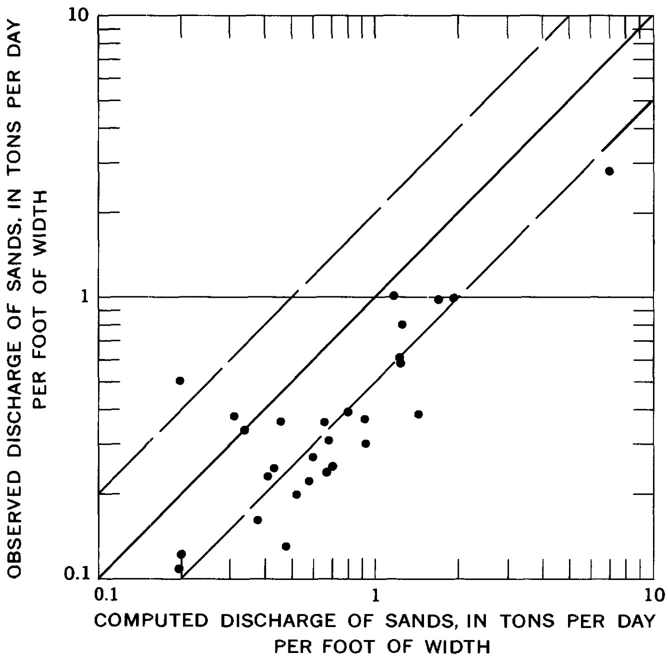


FIGURE 30.—Comparison of computed and observed discharges of sands for flume data (U.S. Waterways Experiment Station, 1935) that were not included in the study.

cannot be used to obtain highly accurate discharges of sands in sand-bed streams, they provide a quick and reasonably accurate procedure for computing the average discharge of sands for many sand-bed streams.

The graphs are applicable only to sand-bed streams that do not have unusually rough confining walls or distinct pools and riffles (except, perhaps, at very low flow). Even though the bed of a stream is composed of cohesionless sand, the graphical relationships may not apply well at narrow sections that are confined between rough rock walls. Rough rock walls, such as those at the cableway on the Colorado River near Grand Canyon, Ariz., may cause considerably more turbulence than would normally exist for a combination of velocity and depth; as was previously pointed out, the vertical distribution of the sands is determined by the turbulence, which seems to be at least partly dependent on the total shear rather than on the shear with respect to the sediment particles.

APPLICATIONS

The empirically defined graphs have two obvious general uses. One is to show the effect on the discharge of sands of a change in any major variable. The other is to compute the approximate discharge of sands in flows over sand beds.

In addition to the two general uses, more specialized applications of the graphs also can be made. For example, the average relationship at a particular cross section between discharge of sands and mean velocity for a water temperature of about 60°F can be readily approximated from the graphs of figure 26 if the median diameter of the bed sands and the widths, depths, and mean velocities are known for one or two low, medium, and high streamflows.

The change is velocity from a wide to a narrow cross section along a reach of stable channel can also be approximated on the basis of continuity of flow and of discharge of sands. Because a large change in discharge of sands usually accompanies a small change in mean velocity, the mean velocity must change comparatively little from one cross section to another along a stable reach of channel. This relative constancy of mean velocity has sometimes been noted in terms of a tendency for the area of cross section to remain about constant along a reach of sand-bed channel at steady flow. According to tht curves of figure 13, an increase of 100 percent in width might be compensated (a constant discharge of sands might be maintained from one cross section to the other) by a 25– or 30–percent decrease in mean velocity.

The large changes in discharge of sands for small changes in mean velocity help to explain channel configuration. For example, a particular flow per foot of width can occur at a wide range of depths and mean velocities; but, if that flow transports a given discharge of sands per foot of width, the velocity and depth become fixed within narrow limits for a given water temperature and for low concentrations of fine sediment. In broad terms, a particular discharge of sands per foot of width largely determines the mean velocity that must exist in a stable reach of a sand-bed stream, which has a particular size of bed sands. The mean velocity and the rate of flow per foot of width largely determine the depth of flow and hence the slope if the resistance to flow is known. (However, within certain ranges of depth, velocity, particle size, and water temperature, the resistance to flow may change widely and be somewhat indeterminate.)

In a channel reach that is not at equilibrium, the probable rate of net scour or net fill may be approximated from the graphs, provided that the flow-duration curves, the channel widths, and the velocities for given rates of streamflow can be predicted for each end of the reach.

An understanding of the effect of a few major factors on the discharge of sands may aid in understanding the effect of other factors. For example, the effect of high concentrations of fine sediment on the discharge of sands in natural streams was not generally recognized for many years because the relationship of the discharge of sands per foot of width to mean velocity had not been defined for low concentrations of fine sediment. As soon as the dominant relationship to mean velocity was known, the effect of the high concentrations of fine sediment became obvious for such a stream as the Rio Puerco.

DEFICIENCIES OF BASIC INFORMATION

Because of the number of variables, the wide ranges in value of the variables, and the complex interrelationships among the variables, the empirical determination of the major graphs in this paper should have been based on a large amount of accurate information that was collected specifically for this study. Unfortunately, the available field and flume data were all collected for purposes other than this study and are deficient in coverage of the ranges of the variables, in applicability to the specific relationships to be defined, and to some extent in accuracy. Lack of accuracy was due partly to the necessity for computing the unmeasured part of the discharge of sands for most sediment stations but probably more to differences between measured discharge of sands and the correct time average for the cross section when samples were collected. A correct time average of discharge of sands is an average over a long enough period of time to eliminate both random and short-term cyclical variations in discharge of sands.

Adequate field information for a particular median diameter of bed sand should cover at least the usual range and combinations of depths and velocities in sand-bed streams in enough detail to define a graph like that of figure 19. The information is definitely inadequate if significant parts of such a figure must be based on estimates, interpolations, or extrapolations as was necessary to some extent for all the different median sizes of bed sands. For most particle sizes, available information is inadequate or entirely lacking especially for high velocities and for deep flows. Therefore, discharges of sands for Mississippi River at St. Louis are particularly helpful because they are for deep flows and some are for moderately high velocities.

Field information on total discharges of sands is wholly inadequate to define a graph like that of figure 19 for median diameters of bed sands smaller than 0.10 mm or larger than 0.50 mm for most combinations of depth and mean velocity.

The effect of water temperature on the relationship of discharge of sands to mean velocity has been defined empirically for only a few streams and then only for the usual combinations of depths and mean velocity in those streams. The somewhat similar but sometimes much larger effect of concentration of fine sediment on the discharge of sands is based mostly on a few laboratory studies and on the discharges of sands in Rio Puerco near Bernardo. More experimental data on the effect of water temperature and of high concentrations of fine sediment in field streams are badly needed.

Information applicable and sufficiently accurate to define possible effects of such usually minor factors as the lateral variation of velocity at a given mean velocity is almost entirely lacking. Also, the effects of deviation of sizes of the bed sands from the median size have been obscured by the scarcity of precise and pertinent information. The bed configuration is seldom known from available information for field streams; hence, the effect of bed configuration on the relationship between discharge of sands and mean velocity is difficult to establish.

The empirical determination of the relationships of discharges of sands in sand-bed streams to the major factors that control such discharges could be made more exact and dependable by a planned program for obtaining the needed coverage and accuracy of field information. Such a program should include at least the following procedures:

1. Determine the probable accuracy of the basic information on the discharge of sands in natural streams either by studying the accuracy of sediment discharges in general or by repeating individual field observations, particularly of concentration of sands, often enough to obtain a practical measure of the consistency of measured discharges of sands.
2. Determine by measurement the total discharges of sands in streams other than the Niobrara and the Middle Loup Rivers; that is, obtain more determinations of the discharge of sands at cross sections where all or nearly all the sands are in suspension and can be sampled and then relate these total discharges of sands to characteristics of the flow and the bed sands at nearby cross sections where the stream has a sand bed. These other streams should preferably have other bed-material sizes and combinations of depth and mean velocity than the Niobrara and the Middle Loup Rivers.
3. Use the total discharges of sands from procedure 2 to confirm or to revise the present methods of computation so that discharges of unmeasured sands can be determined with acceptable accuracy in any sand-bed stream for which suitable information on flow and measured sediment discharge is available.

4. Use the total discharges of sands from procedure 2 as part of the basis for determining or improving individual graphs similar to the graph of figure 19 for bed sands whose median diameters are 0.10, 0.20, 0.30, 0.40, 0.60, 0.80, and 1.00 mm.
5. Complete the definition of these graphs. Obtain measured discharges of sands for particle sizes, depths, and mean velocities for which total discharges of sands could not be measured; add to the measured discharges the computed unmeasured discharges of sands; and define the graphs from the computed total discharges of sands.
6. Obtain accurate determinations of discharges of total sands (either measured or partly unmeasured) at the same cross sections and flows but at widely different water temperatures to define the effect of water temperature through the range of usual combinations of depth, velocity, and bed-material size.
7. Investigate in the laboratory the fall velocities of sand grains in water that contains high concentrations of different combinations of clays and silts of usual mineralogical compositions. Also, study the effect of changes in the dissolved solids in the flow on the effect of high concentrations of different fine sediments on the discharge of sands.
8. Check the effect of fine sediments as indicated by the laboratory studies against the apparent effect on the discharge of sands in natural streams.

After the major effects have been clearly defined, develop a program for the determination of the effects of usually minor variables on the discharge of sands. Presumably, such a program may require the careful control and evaluation of all major factors in order that relatively minor effects can be discerned.

CONCLUSIONS

The discharge of sands in a sand-bed stream depends on many variables whose effects are complicated and interrelated. Hence, no precise relationship between discharge of sands and characterics of the flow and sediment can be expected.

Flume investigations can provide much helpful information on sediment transportation; but, until scale effects are understood more completely, flume investigations of the discharge of sands are not model studies of the discharge of sands in field streams.

The use of discharge of sands per foot of width as a measure of the rate of transportation of sands is based directly on the assumption that, if other factors remain constant, the discharge of sands is in proportion to stream width. Although this assumption is not precisely correct, it seemed to afford a reasonable basis for comparison of sand discharges in streams of greatly different widths.

Concentration of sands is a poor basis for comparison of discharge of sands with the factors that control that discharge in natural streams because the concentration varies greatly with depth of flow. However, for a narrow range of depths in a set of flume experiments, concentration of sands may be a fairly satisfactory measure of transportation of sands.

For the velocities and depths at which most of the sand is transported in many natural streams, the discharge of sands per foot of width does not vary greatly with changes in the depth of flow.

Shear, which has been widely used as a measure of the discharge of sediment, is sometimes a poor measure. When the roughness of the channel changes, the discharge of the coarse sediment generally is more closely related to the mean velocity than to the shear. However, the vertical distribution of sands may vary more nearly with total shear than with shear velocity with respect to the particles.

When discharge of sands per foot of width at one cross section is plotted against mean velocity, a reasonably good relationship is usually defined for that cross section, unless the concentration of fine sediment is high and variable.

The relationships between discharge of sands per foot of width and mean velocity may be much the same for one cross section as for another, provided that differences in depth, water temperature, concentration of fine sediment, or size of bed sand are not excessively large. The agreement in relationships indicates that mean velocity is a dominant measure of the discharge of sands per foot of width in many sand-bed streams.

The relationship to mean velocity implies that mean velocity must be relatively constant from one cross section to another along a reach of sand-bed channel. If mean velocity were to change appreciably, the discharge of sands per foot of width would change much more than the mean velocity. Such a velocity change would require very large percentage differences in width. Otherwise more sands would be discharged at one section that the other, and the channel would aggrade or degrade.

Shear velocity computed from mean velocity correlates with discharge of sands about as well as mean velocity. The effect of depth on the relationship between discharge of sands and shear velocity computed from mean velocity is, however, somewhat different than on the relationship between discharge of sands and mean velocity. The effect of depth indicates that for flume investigations shear velocity computed from mean velocity may be a more convenient parameter (because of a smaller and more consistent depth effect) than mean velocity and that for field streams mean velocity is the more convenient parameter because of the smaller effect of depth for many combinations of depth and mean velocity.

Stream power, the product of shear and mean velocity, may be a good measure of discharge of sands when roughness is about constant, but it is unsatisfactory for variable roughness.

The effect of depth on the relationships between discharge of sands per foot of width and mean velocity, shear, shear velocity computed from mean velocity, or stream power is complex and variable and may be large. The effect of depth is usually easier to apply to the relationship of discharge of sands to mean velocity or to shear velocity computed from mean velocity than to the relationship of discharge of sands to shear or to stream power. The effect of depth also largely determines the use of discharge of sands per foot of width rather than concentration as a measure of the discharge of sands.

Qualitatively, the effects of mean velocity, depth, and water temperature on the discharge of sands can be computed, for some streams at least, from the Einstein procedure.

Median particle size of the bed sands has an appreciable effect on the discharge of sands especially at low velocities in shallow flows and at either low or high velocities in deep flows.

The discharge of sands increases as the water temperature decreases, but the change is usually small as compared with changes that may be caused by variations in mean velocity or in depth.

High concentrations of fine sediment, especially of bentonite, may greatly increase the discharge of sands per foot of width even though velocity, depth, and water temperature are kept constant.

The method of analysis that is suggested for determining empirically the effects of major factors on the discharge of sands is limited by the available information, but it gives relationships that should be helpful in understanding the relative effects of different major factors.

The empirically defined graphs provide a basis for rapid estimation of the discharge of sands in sand-bed streams. The accuracy of the estimated discharges should be reasonably satisfactory for many sand-bed streams. Accuracy, however, may be poor for bed sands whose median diameters are outside the range of 0.15 to 0.50 mm, for unusually low or high velocities, for deep flows, for turbulence that is unusually high in comparison to mean velocity, or for high concentrations of fine sediment.

The graphs are applicable only to streams whose beds are composed of nearly cohesionless sands and whose banks do not have the unusually high roughness that might be caused by rock walls that constrict the flow.

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