Scour and Fill in Sand-Bed Streams

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By BRUCE R. COLBY

SEDIMENT TRANSPORT IN ALLUVIAL CHANNELS

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DEFINITIONS

Antidunes.—Roughly symmetrical sand waves that are about in phase with similar water waves. Definition includes as antidunes roughly symmetrical sand waves that may exist below quiet and symmetrical standing waves on water surface.

Bars.—Extensive raised areas or ridges, larger than dunes in same flow, on bed of stream whose bottom shifts readily, at least at high flow.

Bed elevation.—Vertical position of surface of streambed above arbitrary datum. For flow over sand bed, streambed is lowest position at which sand grains are generally moving at a particular instant. Perhaps sand grains shift somewhat, but without moving far, below streambed.

Bed material.—Sediment of particle sizes found in appreciable quantity in streambed. Term is based on particle size rather than on mode of transportation or location in cross section.

Dunes.—Somewhat irregularly spaced mounds of loose sand whose upstream faces are usually gently inclined as compared with their much steeper downstream faces.

Fill.—Net deposition of sediment on stream boundary. Fill may be measured in volume of sediment added to a channel reach, in average depth of sediment accumulation over an area, in average change of depth at a cross section, or in change of depth at a point.

Fine sediment.—Term used in this paper for sediment finer than 0.062 mm.

Flat bed.—Term applied in this paper to general configuration of beds of assumed channels. Does not imply absence of dunes, antidunes, or bars.

Measured concentration of sediment or of sands.—Concentration based on depth-integrated sediment samples which generally are obtained by sampling flow from water surface to within 0.3 or 0.4 ft of streambed.

Measured discharge of sediment or discharge of sands.—Discharge that is usually determined directly from streamflow and depth-integrated sediment samples. Depth integrated samples obtained by sampling flow from water surface to within 0.3 to 0.4 ft of streambed.

Plane bed.—Bed that is free of dunes and antidunes and, consequently, has low resistance to flow. Usually is smoother and firmer than bed of sand-bed streams of other configurations.

Pool-and-riffle stream.—Stream characterized by alternate pools and riffles.

Sand.—Sediment particles that have diameters between 0.062 and 2.0 mm.

Sand-bed stream.—Reach of stream whose bed consists almost entirely of cohesionless shifting sand and whose channel is not confined by walls of rough rock or other material that would cause unusually high turbulence throughout much of flow.

Scour.—Net removal of sediment from stream boundary by action of fluid flow. Scour may be measured in volume of sediment removed from a channel reach, in average depth of sediment removal from an area, in average change of depth at a cross section, or in change of depth at a point.

Unmeasured discharge of sands.—Difference between total discharge of sands and measured discharge of sands at a cross section.

SCOUR AND FILL IN SAND-BED STREAMS

By Bruce R. Colby

ABSTRACT

The surface of a sand bed of a stream rises when more sand is deposited on it than is removed from it and falls when more sand is removed from it than is deposited on it. The average elevation of the surface is constant whenever the volume of sand being deposited equals the volume of sand being removed. Hence, two basic principles generally are required to explain or to understand scour and fill in sand-bed channels. One principle is continuity of sediment discharge along a stream reach—that is, for a sand-bed stream, a balance between the volume of sand-size particles deposited on and removed from a sand bed. The other principle is that a relation exists between discharge of sands and characteristics of flow and available sands. If no such relation were to exist, the difference between the discharges of sands into and out of a channel reach would be wholly indeterminate except by measurement.

Although the basic principles of scour and fill are not new, they are frequently disregarded or misunderstood, partly because many observations of scour and fill in streams have been misinterpreted. These misinterpretations are due largely to the common assumptions that scour or fill at a point is representative, both in quantity and in timing, of average scour or fill at a cross section and that average scour or fill at a cross section is representative of scour or fill in a long reach of channel.

The principles of scour and fill when applied to assumed channels define general patterns of streambed changes for certain types of channel. These principles indicate fairly definite limits on the rate of net scour or net fill over long reaches of channel.

Observed changes in bed elevation, defined by streamflow measurements on the Elkhorn River at Waterloo, Nebr., the Missouri River near Yankton, S. Dak., and the Colorado River at Lees Ferry and near Grand Canyon, Ariz., show, respectively, the different general patterns of scour and fill in a stream that has a generally stable sand bed, in a stream at a laterally constricted cross section, and in a pool-and-riffle stream. These patterns are explainable in terms of the basic principles of scour and fill, although scour and fill in a pool-and-riffle stream may be difficult to compute theoretically.

Conclusions indicated by the study, but not universally accepted, include the following:

1. At any instant, the quantity of moving sand, even in deep and swift flows, is usually equivalent to an average depth of deposit of only a few hundredths of a foot on the streambed.
2. In a straight sand-bed channel that has uniform cross section, slope, roughness, and bed material, the streambed does not either scour or fill appreciably during the passage of a flood.
3. In a straight channel having uniform cross section but varying slope or roughness along the channel, fill during a rising stage is usually followed by fill on the falling stage, and scour during a rising stage is usually followed by scour on the falling stage.
4. If a change in roughness or in streamflow requires a change in the depth of flow, the depth generally adjusts through a change in elevation of the water surface and not through a change in elevation of the streambed.

INTRODUCTION

Sand on the bed of a stream usually shifts readily, and unless the velocity of flow near the bed is very low, some sand grains are moving at every instant. If the velocity near a sand bed is high, many grains move from a particular area during each short period of time and are replaced by many other grains. Because of the continual shifting of individual grains, the average elevation of the sand bed of a stream frequently is assumed to be very unstable.

The assumption of instability seems to be supported by observations that are commonly made by stream gagers, among others, who, when wading on a sand bed where velocities are high, feel the sand wash out around and under their feet. Also, cross-sectional areas of a series of streamflow measurements that are made while a flood wave was passing a measuring section generally show scour during high and rising stages and fill during falling and low stages. When the series of measurements is plotted against gage height, the slope of the average relation may indicate that the flow increased more rapidly with stage than it would have for a channel having rigid boundaries. Thus, on the basis of general experience, many stream gagers and hydraulic engineers assume that the average elevation of a streambed fluctuates widely from time to time, especially during periods of high and rapidly changing flow. The limitations imposed on changes of the average elevation of the bed by continuity of quantity of the bed material along the stream are frequently overlooked.

The purpose of this paper is to show in reasonably simple and practical terms the general principles that govern scour and fill in sand-bed streams at cross sections that are free from obstructions such as piers or pilings. These principles are not new, but they are sometimes disregarded or misapplied. In particular, the application of the principle of continuity is stressed.

Determinations of scour and fill and possible misinterpretations of them are discussed first; then the principles of scour and fill are explained; and, finally, the principles are applied to some assumed channel reaches and are illustrated by the behavior of a few types of natural streams.

Some unpublished information on concentrations and particle sizes of sediment and on elevations of the streambed of the Colorado River at Lees Ferry and near Grand Canyon, Ariz., was furnished by J. M. Stow, district chemist, Albuquerque, N. Mex.

DETERMINATIONS OF SCOUR AND FILL

Several direct or indirect methods for determining scour and fill are sometimes used in flumes or in natural streams. A few of these methods are mentioned briefly, and some possibly misleading interpretations of results of the methods are pointed out.

PHOTOGRAPHY

The general shape and movements of sand-bed features that are formed by flows in a flume can often be best shown by photographs. For example, the topography of a sand bed can be shown by a photograph taken after the water has been drained from the flume, profiles of dunes or antidunes can be photographed through observation ports of flumes during runs, and the changing features of a sand bed can often be well shown by moving pictures that are taken while water is flowing in a flume. Ordinarily, however, photographic observations cannot be made satisfactorily of the sand beds in field streams.

CROSS-SECTION SOUNDINGS

Soundings by either mechanical or sonic methods at several or many verticals of a cross section provide a direct measure of the average position of the streambed at a particular time. Thus, cross-section soundings made periodically furnish reasonably good determinations, by difference, of the average scour or fill at a cross section. However, even in a fairly uniform channel, the changes in average bed elevation may differ from one cross section to another. Much present information on scour and fill in stream channels is based on cross-section soundings at bridges or cableways where streamflow measurements are made, but, unfortunately, streamflow measurements are usually made at narrower-than-average channel sections of sand-bed streams. As will be shown later, in the sections entitled "Constriction in a straight, generally uniform channel" and "Narrow section of a natural sand-bed stream," the changes in average bed elevation at narrow cross sections are likely to be much greater than those at nearby cross sections of the same stream. Hence, although scour and fill can generally be determined with reasonable accuracy at streamflow-measuring sections, the interpretation of the determined scour and fill may be, and often is, misleading.

Even though streamflow measurements are made occasionally at wide cross sections, scour and fill information based on soundings for these measurements can seldom be interpreted correctly unless amounts of scour and fill are determined for the whole width between banks rather than for only the width of the water surface. Water-surface width may vary greatly with time and, for some wide sections of sand-bed streams, may even be less at high flows than at some much lower flows. Therefore, scour and fill generally should be determined for shifting cross sections by defining periodically the position of the surfaces of temporarily unsubmerged parts of the cross section at the same times that the position of the submerged streambed is defined. Of course, determinations for the whole width between banks are made regularly at few, if any, streamflow-measuring sections of wide sand-bed streams.

STAGE-DISCHARGE RELATIONS

Major changes in the bed configuration of sand-bed streams may cause large changes in resistance to flow (Vanoni and Brooks, 1957; Brooks, 1958; Liu and Hwang, 1961; Simons, Richardson, and Haushild, 1962). If a sand bed is in the form of dunes at low flow and in the form of a plane or antidune bed at high flow, the stage-discharge relation may show a large increase in flow for a small increase in stage, according to Einstein and Chien (in Brooks, 1958, p. 556–557, figs. 12, 14), a two-part stage-discharge relationship (Dawdy, 1961), or even a discontinuity (Colby, 1960). This large increase in flow for a small increase in stage has sometimes erroneously been assumed to be an indirect measure of scour of the streambed rather than a measure of decrease in resistance to flow.

SONIC SOUNDINGS

Sonic sounders, although they have not yet been used extensively by the U.S. Geological Survey, would provide much more dependable information on scour and fill at a point in many streams than is commonly obtained by using most other methods for determining stream-bed position. These instruments can be used to record not only the time and the elevation of the bed when it is at its lowest position at a point but also to define a graph of bed position at a point against time. Sonic sounders also can be used to establish profiles or cross sections of the streambed. Measurements made using the sounders should prove helpful in interpreting correctly observations of scour and fill made using other less informative procedures.

PROBING

The depth of maximum scour during a stream rise sometimes can be determined at points in a cross section or a stream reach by probing with steel rods or soil tubes after the flow has receded. Unless the properties of the recently deposited fill differ appreciably from those of the material that was not moved during the rise, the maximum depth of scour may not be discernible at some points. Even though the maximum depth of scour at each of several points in the cross section can be determined satisfactorily, it rarely occurs simultaneously at all the points. Hence, the interpretation of the maximum depth of scour as determined by probing and of the observed fill above the maximum depth of scour may be questionable, as is the interpretation of other determinations of maximum scour at a point at some unknown time during a flood.

PLASTIC RIBBONS

Maximum scour at points in an ephemeral stream can be determined after a flood by observation of plastic ribbons that had been inserted vertically into the bed of a stream before the flood (Essex, Terry, and Culbertson, written communication, 1954). If enough ribbons at a cross section can be dug out after the flood, the points at which the ribbons bend from vertical to horizontal will define the maximum depth of scour across the section. However, the ribbons may be difficult to recover after a flood, and the time of maximum scour at each point is unknown.

LOG CHAINS

Light log chains have been used in some northern Mississippi streams by the Agricultural Research Service, U.S. Department of Agriculture, to show maximum depths of scour. Each chain was attached to a ring that was slipped over the top of a steel angle that had been driven into the streambed. A pin kept the ring from coming off the top of the angle, but the ring could slide down the angle easily if scour removed the sand below the ring and the chain. The chain was initially laid at or just below the surface of the bed and was extended horizontally downstream from the angle. The position of the chain usually could be determined easily at low flow or no flow whenever knowledge of the depth of maximum scour prior to a selected time was needed. Probably the observed positions of a chain indicated the maximum depth of scour fairly accurately, although the steel angle and the chain may have affected the scour a little and the chain may have tended to settle slightly into a loose sand bed during periods of flow. Of course, the exact time of the maximum scour was not known, and the maximum scour at all points along a chain may not have occurred at the same time. Also, the maximum scour at two or more chains at the same cross section may have occurred at different times.

INTERPRETING OBSERVATIONS OF SCOUR AND FILL

Three general problems are involved in interpreting some observations of scour and fill. The first problem is whether or not an average position of the streambed at a cross section can satisfactorily be based on a few observations at the section. The second problem is whether or not an average bed position at a cross section can be determined from observations of maximum instantaneous depths of scour. The third problem is whether or not average or typical changes in elevation of the sand bed in a stream reach can be determined from observed changes in average position of the streambed at one or a few cross sections. These problems prevent the behavior of shifting streambeds from being easily determined and understood from the usual observations available for natural streams.

A few observations are inadequate to determine the average position of a shifting streambed at cross sections where the bed position fluctuates widely. Such sections include those on the San Juan River near Bluff, Utah, on the Colorado River at Grand Canyon, Ariz., on the Rio Grande near Bernalillo, N. Mex. (Leopold and Maddock, 1953, figs. 22, 24, 26), and on the Elkhorn River near Waterloo, Nebr. (Beckman and Furness, 1962, figs. 8, 9). For beds as unstable at individual points as those at the above sections, many measurements obviously are required to define accurately the bed position or changes in the position even for a single cross section. Stream gagers usually make soundings at 20 or more points in most cross sections to be reasonably sure of defining satisfactorily the cross section and the velocity distribution. Changes in bed elevation probably should be based also on measurements at about 20 or more points for most cross sections. If only a few soundings at a cross section are available for a given time, the average bed position based on these soundings should be considered to be inexact unless a study has been made to find if fewer than 20 points are likely to define accurately a bed position for the section.

The second problem, that of determining an average bed position at a cross section from observations of maximum instantaneous depths of scour, is usually difficult and frequently cannot be solved satisfactorily. Suppose that the minimum elevations at 20 points in a cross section of a straight, uniform reach were determined after a flood from the positions of plastic ribbons or log chains that had been installed before the flood. At comparatively low flows at the beginning of the flood, the assumed bed was covered with sand dunes; at high flows the bed was a combination of plane bed near the banks and antidune bed along the middle of the channel. A schematic diagram (fig. 1) for 6 of the 20 points gives a general idea of the difference between the scour that is indicated by the ribbons or chains and the actual lowest average position of the bed at the cross section. As will be shown later, the average position of the streambed in a straight, uniform channel that has uniform bed material usually remains nearly constant during the passage of a flood. This stability of the average bed position was assumed at each point when the schematic diagram was prepared.

Even though the average position of the streambed remains almost constant during the passage of a flood, the maximum scour at each point is roughly 1 foot in figure 1. Thus, an entirely fictitious average scour at sometime during the passage of the flood and an equally fictitious average fill during the recession of the flood (a fairly uniform and even bed with ripples usually is left when the flow ceases or becomes very low in a straight, uniform channel) seem to be indicated by the observed minimum elevations of the streambed. Also, if the average position of the streambed is virtually constant at a cross section, the stability of the average

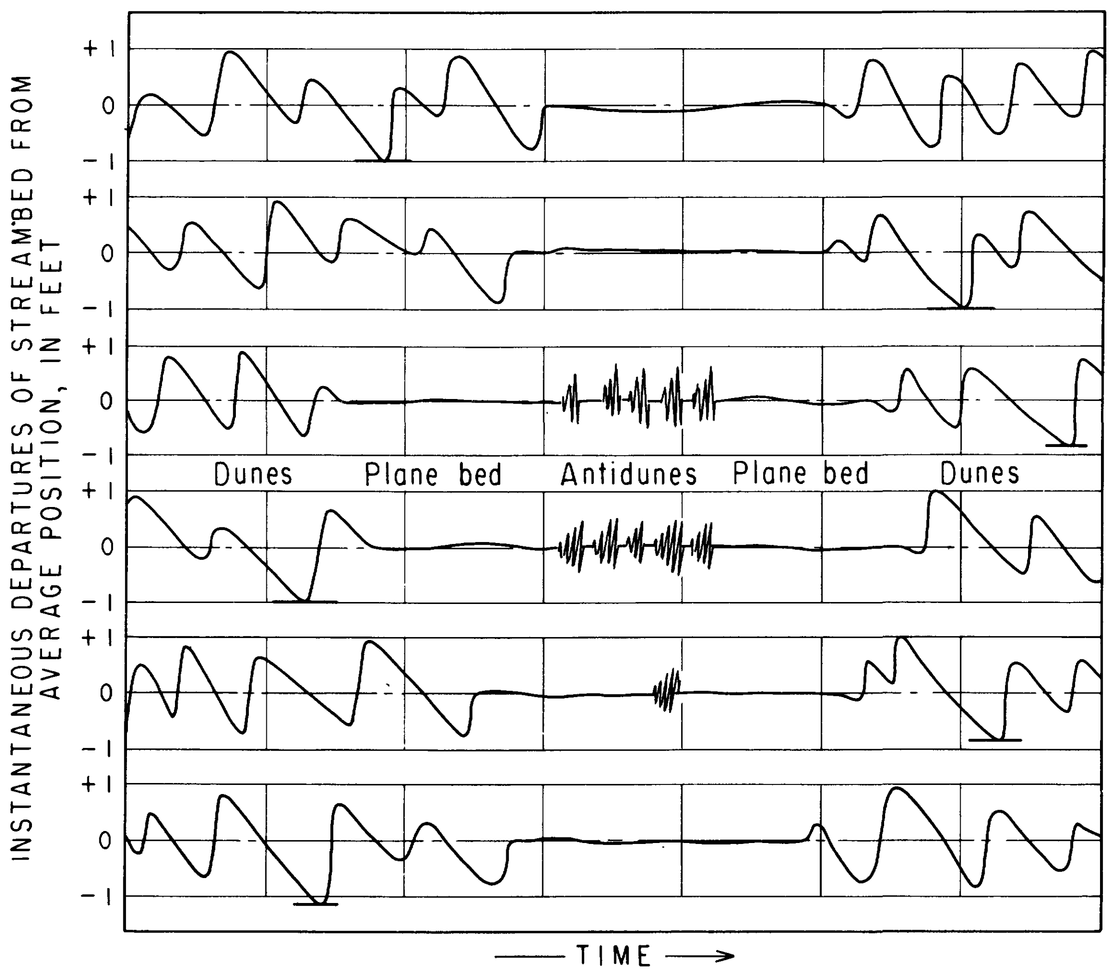


FIGURE 1.—Schematic diagram of instantaneous bed positions at six points in a stream cross section. Time of changes not to scale.

position might not be shown when the bed was rough unless observations were made at many points in the cross section (Beckman and Furness, 1962, p. 12 and figs. 8, 9). For the assumed fairly stable time-averaged elevation at each point, the maximum point scour does not occur at the highest flow. In fact, the least amount of point scour would be indicated at the peak flow at points where the highest flow was over a plane bed. According to the schematic diagram (fig. 1), the maximum point scour does not occur at the peak flow even at a point where the highest flow is over antidunes; it occurs when the bed is the roughest, which is usually when the bed has the largest dunes. In general, the largest dunes are formed at about the deepest flows at which dunes exist. Dunes may exist at the peak flow if the velocity is slow enough in relation to the depth, bed material, water temperature, and concentration of fine sediment; but for many sand-bed streams, at least part of the cross section will have a plane or an antidune bed at the highest flows.

If the assumed cross section had been at a narrow segment of a straight channel, the average bed position at the cross section generally would have been lower at high flows than at low flows. Hence, the observed maximum scour at individual points might have occurred when the bed was plane, if a plane bed existed at the highest flows at the point, or when antidunes were forming and breaking, if antidunes existed at the highest flows. However, the problem is further complicated because the average maximum scour for a narrow section usually occurs after the peak flow.

Sometimes an adjustment might be applied to approximate the average position of the streambed from observations of instantaneous maximum scour at a point. If the streambed at a point was known to be at its lowest position when the bed was covered with dunes, a very rough adjustment to the observed maximum scour might be made by adding an approximation of half of the probable dune height to the observed lowest position at the point. If the bed at a point was known to be plane throughout the passage of the flood (an unusual condition), the observed maximum instantaneous scour may require no adjustment. If the bed position at the point was known to be lowest when the flow was over antidunes, the observed position at maximum scour might be increased by approximately half of the probable maximum amplitude of the antidunes on the bed. Obviously, adjustments such as these, if they can be made at all, are usually inaccurate; but they may at least indicate the probable inaccuracies in the maximum observed point scour during a flood.

The third problem, that of determining average or typical changes in bed elevation for reaches of a stream from changes at a cross section, requires a consideration of the general principles of scour and fill and the application of these principles to different types of channel. This problem will be considered at length in the following sections. In general, it will be shown that alternate scour and fill at a cross section usually indicate that the observed scour and fill are not representative of a long reach of channel even though based on accurate soundings at many points in the cross section. Therefore, alternate scour and fill at a cross section may lead to an incorrect assumption of changes in average bed position in a reach of channel in much the same way that alternate scour and fill at a point may incorrectly be interpreted to mean that the average position of the bed changed at the cross section.

Also, some observed changes in the average bed elevation at a cross section may not be representative or significant even at the cross section. A log or a pile of driftwood at a section or immediately upstream from if may affect the cross-sectional area at least occasionally. A field stream may shift its alinement somewhat from time to time, and a change in alinement may be associated with changes in the shape and size of the cross section at a given flow. Sometimes a narrow and deep channel may form, at least temporarily, in a sand-bed stream (Lane and Borland, 1954, p. 1088–1089). Sandbars may shift on the streambed and cause somewhat unrepresentative short-time changes in cross-sectional area. Therefore, some observed variations in average bed position at a cross section may be due to chance rather than be directly related to changes in flow or in the available sediment.

GENERAL PRINCIPLES OF SCOUR AND FILL

Two simple principles can be used to explain scour and fill in sand-bed streams. One principle is that a relation exists between the discharge of bed material, which is sediment of sizes found in appreciable quantity in the streambed, and the characteristics of the flow and available sediment. Without such a relation, the discharge of sands into or out of a reach of sand-bed channel would be wholly indeterminate except by measurement. The other principle is continuity of the sediment discharge along a stream reach. Each principle is obviously applicable even though some details of its use may be obscure or questionable.

Simple logic indicates that the discharge of sediment of particle sizes that are readily available in the streambed must, for a steady state, be limited by the characteristics of the flow and sediment. If the sediment discharge were not so limited, the available sediment would not remain in the bed. Also, the reproducibility of sediment discharges in flume investigations at steady flow suggests a relation of sediment discharge to the characteristics of the flow and sediment.

Although steady flow is rare in natural streams, an approximate relation of discharge of sands in sand-bed streams to mean velocity, width, depth, median particle size of the bed sands, and viscosity or apparent viscosity exists in natural streams (Colby, 1964). If the effects of all the characteristics of the flow and sediment could be accurately evaluated and if all experimental error could be removed, a good relationship could certainly be defined between the discharge of sands and these characteristics for naturally changing rates of flow as well as for constant flows.

Continuity as applied to scour or fill of a channel reach requires that the difference between the discharge of sediment into and out of the reach be balanced by a gain or loss of accumulated sediment within the reach. If fine sediment accumulates within a reach during a period of time, the inflow of fine sediment must have exceeded the outflow of fine sediment during the time. However, in general, the silt and clay that enter a sand-bed reach also leave the reach; so the principle of continuity need be applied to the sands only. Hence, the concentrations of fine sediment in sand-bed streams are usually not directly related to scour and fill. Of course, inflow or outflow of sands or of sediment of all sizes may occur either at the ends of the reach or through tributaries or diversions along the reach. Also, a change in the quantity of sediment in a stream reach may result in a change either in the average elevation of the streambed or in the quantity of sediment that is moving above the streambed. (The weight per unit volume of the sediment in the streambed is assumed to remain unchanged.)

The principles of scour and fill are more easily stated and applied if the quantity of sand being transported can be shown to be negligible. Persons aware of the low concentrations of sands in natural streams may already realize that the amount of sands being transported can usually be disregarded in the application of the principle of continuity. Others may want some rough approximation of the amount of sands being moved at different combinations of depth and velocity.

QUANTITY OF SANDS BEING TRANSPORTED

The depth of all sands in transit, if instantaneously deposited, could be computed from the product of a concentration by weight; the depth of flow; and the ratio, by weight, of a cubic foot of water-sediment mixture to a cubic foot of deposited sands. Unfortunately, the type of concentration c_1 that is needed for such a computation is seldom available, because it is the ratio, by weight, of the sediment to the water-sediment mixture above a particular area of the bed. In contrast, the concentration c_2 that is usually determined from sediment samples is a weight ratio of the discharged sediment to the discharged water-sediment mixture. In terms of sampling at a vertical, the first concentration (c_1) would be obtained if an equal volume of water-sediment mixture could be collected from each small equal increment of depth, including the bed layer, and if all the equal volumes could be mixed and sampled. The second concentration (c_2) would be obtained if the volume from each small equal increment of depth could be collected in proportion to the flow velocity through the increment and if all the volumes could be mixed and sampled. Because the concentration of sands is greatest where the velocity is slowest in the vertical, the first concentration is larger than the second.

For sediment as coarse as sands, the difference between the two types of concentration is large and variable (table 1). Six computations of the ratio of $c_1 : c_2$ ranged from 1.2 to 2.6 and averaged 1.7. Of course, the computations on which information in table 1 is based require extensions, usually inexact, of velocity and concentration of sands from points of observation in the flow to the streambed. However, the inaccuracy is not objectionable for estimating the depth of all sands in transit, if instantaneously deposited from a flow, because the maximum indicated depth of deposit for the flows and concentrations in table 1 is less than 0.01 foot.

TABLE 1.—*Differences between two types of concentration of transported sands at a vertical*

Selected vertical	Date	Median size of bed material (mm)	Flow		Concentration of sands (ppm)		Ratio $c_1:c_2$
			Depth (ft)	Velocity (fps)	c_1	c_2	
Assumed flow		0. 25	5. 00	3. 5	1, 330	980	1. 4
Barton and Lin (1955), run 22		. 18	. 76	3. 2	2, 870	1, 090	2. 6
Mississippi River at St. Louis, Mo	June 7, 1954	. 3	33. 5	5. 8	292	242	1. 2
Do	April 24, 1956	. 3	27. 0	4. 0	344	185	1. 9
Do	May 5, 1956	. 3	32. 8	4. 1	161	97	1. 7
Do	May 13, 1959	. 45	36. 8	5. 0	159	100	1. 6

Approximate depths of average deposits (fig. 2) that might result from the instantaneous settling of moving sands in a reach can be based on a graph by Colby (1964, fig. 21). The graph shows approximate c_2 concentrations of sands for a wide range of mean velocities and depths, a median grain-size diameter of 0.30 millimeter for the bed sands, and a water temperature of 60°F. These c_2 concentrations were determined from an anal-

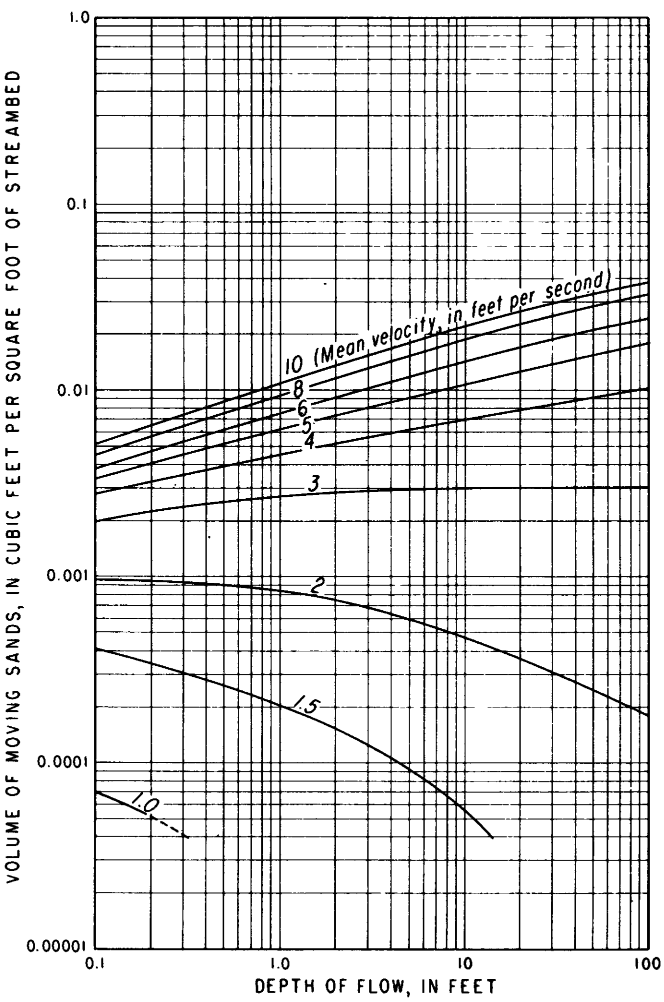


FIGURE 2.—Approximate volume of moving sands per unit area of streambed for 0.30-millimeter median diameter of bed material and 60° F water temperature. Assumed weight per unit volume of sands is 85 lb per cu ft.

ysis of many available records of the discharge of sands in flumes and in field streams. They should be increased somewhat (Colby, 1964, fig. 24) for high concentrations of fine sediment in the flow. The c_2 concentrations from the graph were doubled (table 1) to obtain approximate c_1 concentrations. A weight of 85 lb per cu ft (pounds per cubic foot) was assumed for the deposited sands; and the approximate volume of moving sand per unit area of the streambed was computed from the product of the c_1 concentration, the depth of flow, and the ratio of 62.4 lb per cu ft to 85 lb per cu ft. The indicated depths of average deposits (fig. 2) are only a few hundredths of a foot, even for deep and fast flows; for many flows the indicated depth is less than 0.01 foot. These amounts of moving sand are so small that they can usually be disregarded in studies of scour and fill.

SOME PREVIOUS STATEMENTS OR USES OF THE PRINCIPLES

Straub (1934) computed scour for constant flow at a laterally contracted cross section from his form of the Du Boys equation and from the principle of continuity of the bedload discharge. He concluded that observed amounts of scour at narrower-than-average cross sections of a stream probably were not representative of long reaches of a river.

In 1948, the scour and fill in the Missouri River at bridge cross sections where streamflow measurements had been made were reported to be greater than the scour and fill away from the bridges (B. R. Colby and R. E. Oltman, written communication). The conclusion was based on continuity of the sediment discharge and on relatively smaller fluctuations in the stage-discharge relation than in the average elevation of the streambed at the bridge sections.

In 1952, Billy Mitchell and A. J. Harrison, in separately discussing a paper by Leopold and Maddock (1952, p. 172, 173, 176), noted the significance of continuity of the sediment sizes in the streambed. Unfortunately, Mr. Harrison's brief and excellent statement of the principles of scour and fill apparently received little attention, perhaps because it differed from the ideas presented by the authors of the paper.

E. M. Laursen (1952) stated well the fundamental principles of scour and fill. He applied the principles more fully to scour by jets and to scour around bridge piers and abutments than to general scour or fill in a channel reach.

Lane and Borland (1954) studied scour and fill, particularly in the middle Rio Grande, and concluded that the streambed did not generally degrade appreciably during floods although scour was pronounced at some narrow sections. In spite of this excellent paper by Lane and Borland and the statements of principles by Straub, Mitchell, and Harrison, the inherent stability of the average elevation of sand beds, especially in reasonably uniform channels, is still largely unrecognized.

In 1959, John Fisher Kennedy (oral communication) pointed out the stringent limitations imposed on scour and fill in a uniform channel by the principle of continuity and the principle of a relation between the discharge of bed material and the characteristics of the flow and available sediment. His reasoning was almost exactly the same as that previously used by the present writer in unpublished reports.

In 1960, the stability of a sand bed during rises was emphasized in a study of the stage-discharge relationship for Pigeon Roost Creek (Colby, 1960, p. 13–15). The reasons given for the stability of the average elevation of the sand bed had been arrived at independently, but they closely paralleled those given years earlier by Mitchell and Harrison (in Leopold and Maddock, 1952, p. 176).

The concepts and principles applied by the preceding workers were generally consistent, but they differ from the idea (Leopold and Maddock, 1952, 1953; Brooks, 1958, p. 553–555) that an imposed sediment load affects the resistance to flow and hence the depth of flow for a given rate of flow. According to this idea, scour or fill results from an attempt of the stream to adjust its depth of flow to a changing sediment discharge. Information on large amounts of scour and fill at the measuring section on the Colorado River near Grand Canyon, Ariz., was given in support of this idea. On the other hand, Einstein and Chien (in Brooks, 1958, p. 558) stated that the cited information did not support the idea that changes in the elevation of the streambed at the measuring section were due to changing resistance to flow. The scour and fill and the effect of changing roughness on depth at this measuring section will be examined in more detail under the heading "A pool-and-riffle stream."

DISCHARGE OF SANDS AS A FUNCTION OF FLOW CHARACTERISTICS

The discharge of sands can be roughly computed for different flows and bed-material sizes according to the procedure suggested by Einstein (1950) or according to any other of several equations or procedures that are available for determining the discharge of sands in a sand-bed stream. In this paper, approximate discharges of sands in sand-bed streams are based on relations that were obtained from an analysis of the total discharge of sands in field and laboratory streams (Colby, 1964). The relations do not apply unless the streambed consists almost entirely of nearly cohesionless sand and unless the banks are not excessively rough as rock walls might be. The relation for low concentrations of fine sediment, a water temperature of 60° F, and a median particle size of bed sands of 0.30 millimeter is given in figure 3. For the stated conditions, the discharge of sands per foot of width is determined according to figure 3 by mean velocity and depth of flow. (Relations for other median particle sizes and approximate adjustments for changes in water temperature and for the effects of high concentrations of fine sediment are also given in Colby, 1964.)

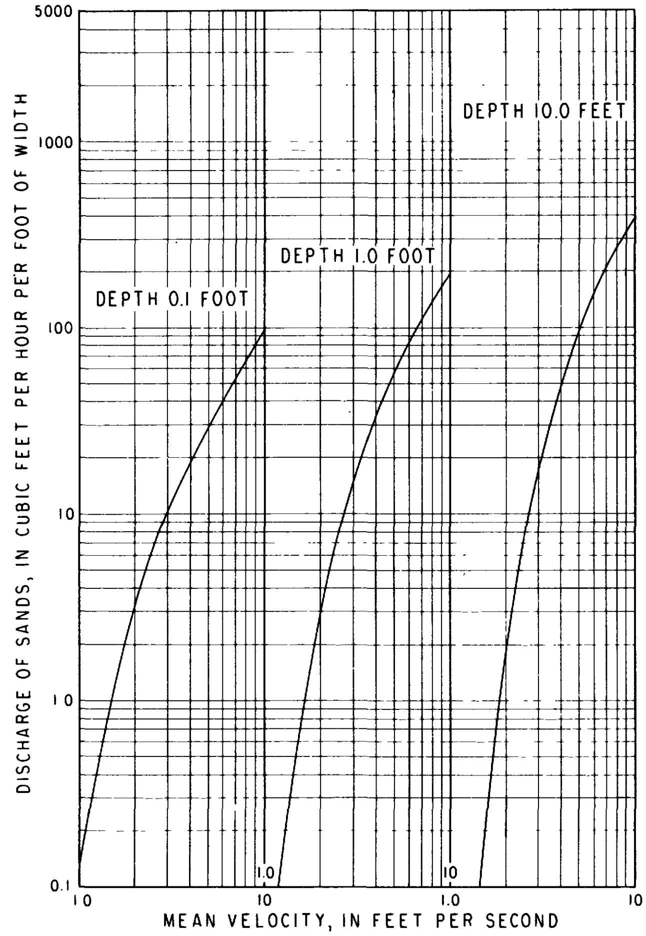


Figure 3.—Approximate discharge of sands for 0.30-millimeter median diameter of bed material and 60° F water temperature. Assumed weight per unit volume of sands is 85 lb per cu ft.

THE SIMPLE CONTINUITY EQUATION

The continuity equation for a reach of channel below the highest water-surface position that is to be considered can be written

$$Q_s' = \Delta S + Q_s''$$

or

$$\Delta S = Q_s' - Q_s'' \quad (1)$$

in which

Q_s' is the quantity of sediment that enters the stream reach during a period,

Q_s'' is the quantity of sediment that leaves the reach during the period, and

ΔS is the quantity of sediment that accumulates (or erodes if ΔS is negative) during the period within the reach and below the highest position of the water surface.

The equation is obviously true for any time period, whether the sediment is transported in the main channel, in tributaries, or in diversions or whether the sediment enters or leaves through the action of wind or human agencies.

The three quantities of equation 1 usually can be used more conveniently for computations of scour and fill if they are expressed in terms of equivalent volumes of deposit rather than in terms of weight. The equivalent volume of deposit is the weight of sediment divided by a known or assumed weight per unit volume of deposited sediment. The weight per unit volume can generally be approximated accurately enough for sands, but it is questionable for silt and clay or for mixtures of clay, silt, and sand.

Assume that the average elevation of the streambed would remain constant if no net weight of sediment were added nor taken away during the period to which equation 1 is applied and that the amount of sands being transported at any instant is so small that changes in the amount during a computation period can be disregarded. Equation 1 for computing the depth D, in feet, of average fill on the sand bed of a stream reach during a period then becomes

$$D=\frac{\Delta S}{A}$$

$$=\frac{Q_s'-Q_s''}{A}, \qquad (2)$$

in which A is the area, in square feet, of the bed of the channel reach if the sediment quantities are in cubic feet for the period. The quantity of silt and clay being transported has not been shown to be negligible, and the weight per unit volume of fine sediment may be questionable; therefore, equation 2 should be limited to flows over sand beds. However, Q_s' and Q_s'' can generally, but not always, be volumes either of sediment of all sizes or of only sands, because the difference will usually be almost entirely a difference in volume of sands.

APPLICATIONS OF THE PRINCIPLES TO ASSUMED CHANNEL REACHES

Although the general principles are broadly applicable, discussions of scour and fill in assumed channel reaches will, for convenience, be limited to reaches without diversions or tributary inflows of water or sediment. Also, only flows within the channel banks will be considered. Because changes in the amount of sands being transported must be small (fig. 2), they usually will be disregarded.

Two extremes must be avoided if the principles and applications of scour and fill in the assumed channels are to be reasonably well understood. One extreme is a tendency to become enmeshed in questionable details and minor effects so that general and significant relations are obscured; the other is a tendency to oversimplify and hence to eliminate some significant effects.

The following assumptions, unless otherwise noted, will apply for all the computations in the assumed channels:

1. The bed material is cohesionless sand whose median diameter is 0.30 millimeter, and the sand is uniform along the reach.
2. The water temperature is 60° F.
3. The combined concentrations of silt and clay do not exceed 10,000 ppm (parts per million).
4. The standard cross section has a flat bottom 84 feet wide and 45° bank slopes (fig. 4). (The term "flat" applies to general configuration and, as used in this report, does not imply an absence of dunes and antidunes.)
5. The stage-discharge relation (stage is measured above the flat bed) for a straight reach is assumed to be that shown in figure 4.
6. For this stage-discharge relation, the velocity-discharge relation must also be that shown in figure 4 for the standard cross section.
7. The discharges of sands for different combinations of depth and mean velocity in a straight or nearly straight channel are assumed to be as defined by figure 3.

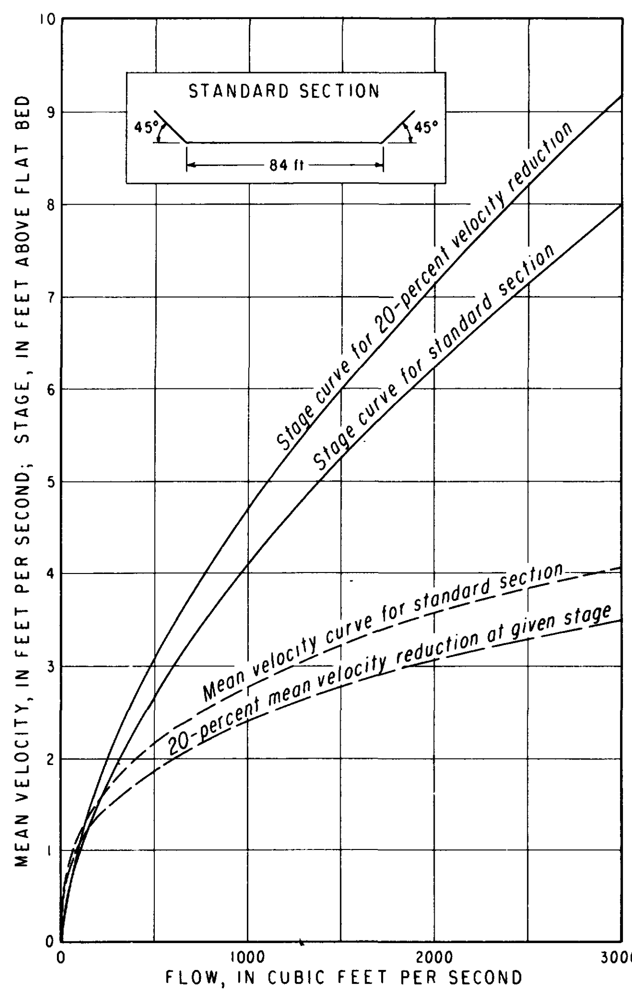


FIGURE 4.—Properties of assumed cross section.

CHANNEL DOWNSTREAM FROM A RESERVOIR

If all the sediment is removed from a flow in a reservoir, equation 1 for a reach of channel immediately downstream from the reservoir reduces to

$$\Delta S = -Q_s'', \qquad (3)$$

and the quantity of scour in the reach is known if the sediment outflow Q_s'' is known or can be computed for the downstream end of the reach. The conversion of weight to volume might be based on the weight per unit volume of deposited sands (a weight that can be reasonably accurately estimated) if the discharged sediment is entirely sands. If the discharged sediment contains appreciable quantities of silt and clay and the size distribution is approximately that of the material in place before it was scoured out, the weight per unit volume of the material before it was scoured away can be used to convert from weight to volume. Even though the quantity of sediment discharged from the reach has not been measured, the discharge of sands from the reach can be computed from such a relationship as that in figure 3. However, figure 3 is based on an assumed weight of 85 lb per cu ft for sands and on the assumption that the flow has had time to pick up its equilibrium load of sands. If a cubic foot of material before it was removed by scour contained, for example, only 55 pounds of sand and no great amount of selective sorting of particle sizes took place within the reach, the volumes from figure 3 should be increased in the ratio of 85:55 for computing quantities of scour.

Sometimes the amount of sediment released from a reservoir is known, and then this amount of sediment inflow to the reach can be subtracted from a measured or computed outflow of sediment from the reach to compute scour.

For a computation of the volume of scour, assume that the standard cross section exists in a fairly straight and uniform reach of channel at a point 1,000 feet downstream from a reservoir where all the sand is removed from the streamflow. If this distance is sufficiently long to permit the streamflow to pick up its equilibrium load of sands, the discharges of sands at the downstream end of the reach should, according to figure 3, be those given in table 2 for different constant rates of flow. If all the material scoured from the channel was sand and the weight per unit volume of sand in place was 85 lb per cu ft, the volumes in column 6 of table 2 would indicate the volume of scour directly. Perhaps only 60 percent of the eroded material was sands, and the weight per unit volume of the material in place before it was removed by the flow was 80 lb per cu ft. Then if the amount of selective sorting within the reach was not appreciable, the volumes of scour for this material could be computed from the volumes in column 6 of table 2 by multiplying by 85/48 or 1.77.

TABLE 2.—*Volumes of sand removed from a 1,000-foot reach of channel downstream from a dam at different constant rates of streamflow through the standard cross section*

Streamflow (cfs)	Depth of flow above the flat bed (ft)	Mean velocity (fps)	Width at water surface (ft)	Discharge of sands (cu ft per hr): Per foot of width (assumed 85 lb per cu ft)	Discharge of sands: For the total width, Assumed 85 lb per cu ft	Discharge of sands: For the total width, Assumed 48 lb per cu ft
(1)	(2)	(3)	(4)	(5)	(6)	(7)
100	0.97	1.21	86.0	0.11	9.5	17
500	2.68	2.15	89.4	3.7	330	580
1,000	4.10	2.76	92.2	11.5	1,060	1,880
2,000	6.20	3.58	96.4	31.5	3,030	5,360
3,000	8.00	4.08	100.0	52	5,200	9,200

The computed amounts of scour are for a 1,000-foot reach, but they would be very unequally distributed along the reach. Most of the scour probably would take place within the first 100 or 200 feet below the dam.

If the hydrograph of figure 5 represents flow through the standard cross section 1,000 feet downstream from the dam, the velocities and depths can be determined from the curves of figure 4 for many times during the period of flow. These velocities and depths and figure 3 can then be used to compute the discharges of sands at frequent enough times to determine a graph (fig. 5) of discharge of sands during the period of flow. A cumulative curve of the volume and average depth of sands removed from the reach (fig. 5) can then be drawn from this graph. Of course, the volumes indicated by the curve are for 85 pounds of sands per cubic foot and should be increased if less than this weight of sands is removed for each cubic foot of sediment that is scoured from the reach. The average depth of sands removed is based on the 1,000-foot length of reach and on the maximum water-surface width of 100 feet.

STRAIGHT, UNIFORM CHANNEL

The reason for constancy of the average elevation of the sand bed of a uniform channel should be clear from the principles of scour and fill. (The channel is assumed to be uniform in slope, bed material, and resistance to flow at a given depth and velocity as well as uniform in cross section.) For such a channel, the relation between a particular flow and the depth and mean velocity of that flow is the same at each end of a reach. If the reach is not excessively long nor the volume of flow very small in comparison to the size of the channel, the hydrograph of flow at the downstream end of the reach will lag somewhat but will otherwise differ only a little from the hydrograph of flow at the upper end of the reach. Hence, the graph of discharge of sands from the reach will also lag a little as compared with

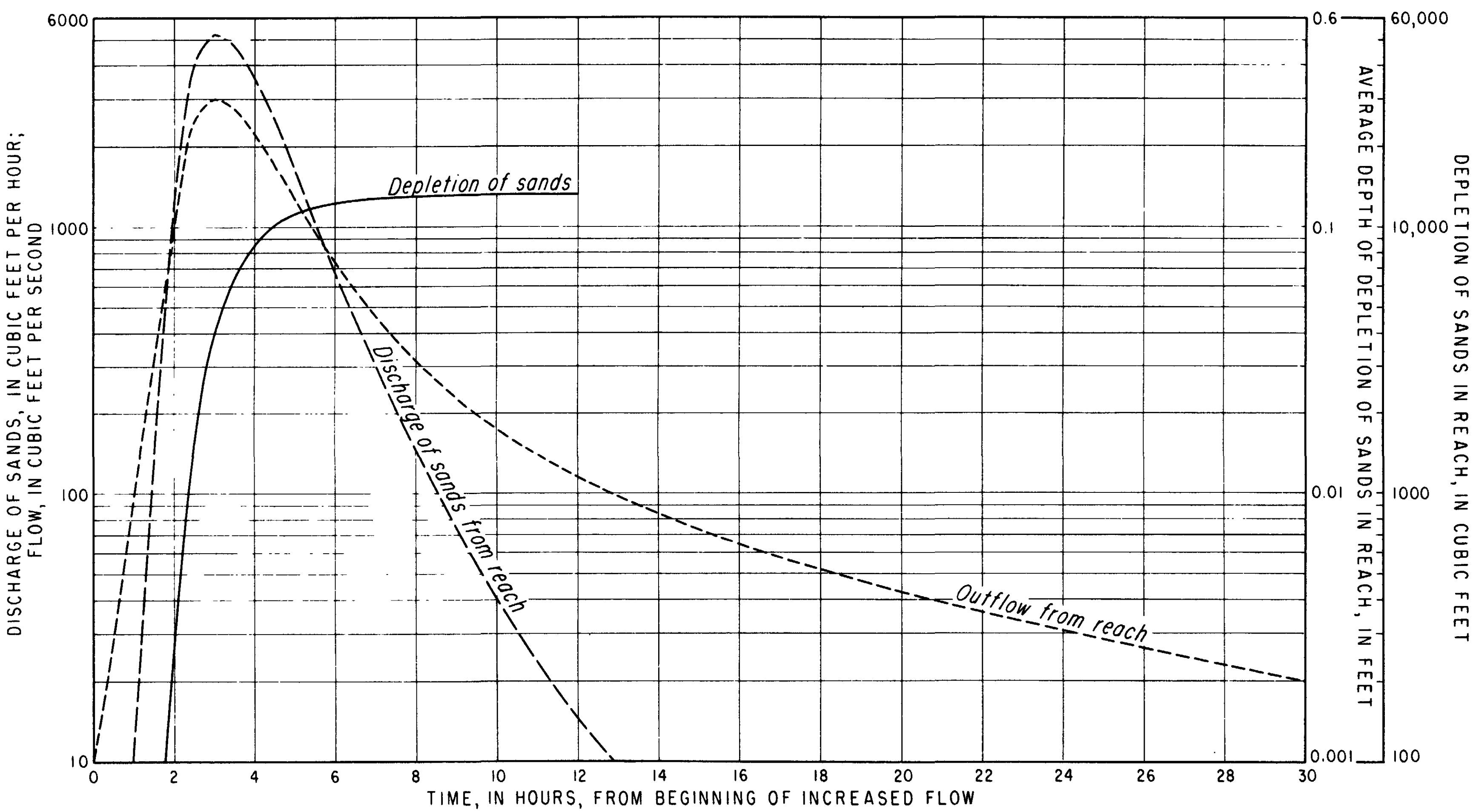


FIGURE 5.—Streamflow and discharge of sands at the downstream end of an assumed 1,000-foot channel reach immediately downstream from a dam that traps all sands in the flow.

the graph of discharge of sands into the reach, but these graphs will be otherwise about the same. Therefore, the discharges of sands into and out of the reach must be about equal during a runoff event, and the average elevation of the sand bed in the uniform channel should be about the same after a flood as before the flood. If an allowance for the lag is made in the hydrographs of flow and in the graphs of discharge of sands, the inflows and outflows of sands must be about equal during intervals of time during a flood and the average elevation of the sand bed must remain about constant within the reach throughout the flood.

Perhaps an even simpler concept of the reason for constancy of the average elevation of a sand bed in a uniform channel can be based on consideration of volumes of water within short segments of length as these segments of flow pass along the channel. If the mean velocity and average depth of each segment of flow do not change as the flow moves through a reach of channel, the sand-transporting ability of each segment does not change either, and the segments of flow neither pick up nor drop appreciable net amounts of sands. Hence, the average elevation of the sand bed does not change as the flow passes over the bed.

UNIFORM SLOPE AND ROUGHNESS

For illustrative computations, assume a straight reach of uniform channel that has at all sections the stage-discharge relation (fig. 4) for the standard cross section and the relation of streamflow to discharge of sands that is shown in figure 3. Constant flows of 100; 500; 1,000; 2,000; and 3,000 cfs (cubic feet per second) would be accompanied by discharges of sands of about 9.5; 330; 1,060; 3,030; and 5,200 cubic feet of sands per hour (table 2, column 6) at each end of the reach. Because the discharges of sands into and from the reach are equal for any constant rate of flow, the sand bed on the average neither scours nor fills within the reach.

If the hydrograph of streamflow and the graph of discharge of sands (fig. 5) are assumed to apply at the upstream end of a 5,000-foot reach of the same assumed straight, uniform channel, the hydrograph of outflow at the downstream end of the reach would be about as shown in figure 6. The relation between flow and discharge of sands (fig. 5) can then be used with the hydrograph of outflow to compute a graph of discharge of sands from the reach. Differences between the discharges of sands into and out of the reach were accumulated by short intervals of time to define the accumulation-of-sands curve (fig. 6). According to equation 1, this curve represents the accumulation of sands within the 5,000-foot reach from the beginning of the increased flow to any time covered by the curve. This accumulation may be either within the flow or on the streambed. If the accumulation of sands within the reach were distributed evenly over the area that is submerged at peak flow, 1,000 cubic feet of sands would be equivalent to an average depth of accumulation, including the accumulation in the flow, of 0.002 foot (fig. 6). If the accumulation of sands within the reach were distributed evenly over the flat part of the streambed, an area 84 feet by 5,000, the computed average depths of cumulative deposit would be increased in the ratio of 100 : 84. Even though the 84-foot width is used, the maximum accumulation of sands represents an average depth of less than 0.01 foot during the total time shown in the assumed hydrograph of flow. The accumulation of sands is greatest when the outflow of sands becomes equal to the inflow of sands.

CHANGING SLOPE OR ROUGHNESS

For the next computations, assume that the reach is the same but that the slope of the channel gradually decreases through the downstream part of the reach. The assumed change in slope begins far enough downstream so that the stage-discharge relation is not affected at the upstream end of the 5,000-foot reach, but the mean velocity at a given depth is exactly 20 percent less at the downstream end of the reach than at the upstream cross section. This assumed velocity reduction is significantly less than a 20-percent velocity reduction at a particular flow. (See fig. 4.)

Volumes and depths of fill in the assumed reach can be computed readily. Discharges of sands into the reach at constant rates of flow of 100; 500; 1,000; 2,000; and 3,000 cfs can be taken from column 6 of table 2. The discharges of sands from the reach can be computed from the properties of the flow at the downstream end of the reach. A curve of relation between flow and depth at the downstream end of the reach was drawn in figure 4 by reducing the flows for the upstream end by 20 percent for each of enough stages to define the stage-discharge curve for the downstream end of the reach. The stages directly determine the cross-sectional areas from the standard cross section (fig. 4) and hence the mean velocities for different flows at the downstream end of the reach. The mean velocities and the depths of flow can then be used to determine the discharges of sands per foot of width from figure 3. These discharges of sands multiplied by the water-surface widths are the total discharges of sands from the reach. At each of the five constant rates of flow, the computed differences between discharges of sands into and out of the reach gives, according to equation 1, the rate of accumulation of sands (expressed in terms of volume) within the 5,000-foot reach. (See table 3.) The volume of sand accumulation can also be expressed as an average depth of fill on the total submerged area, which is

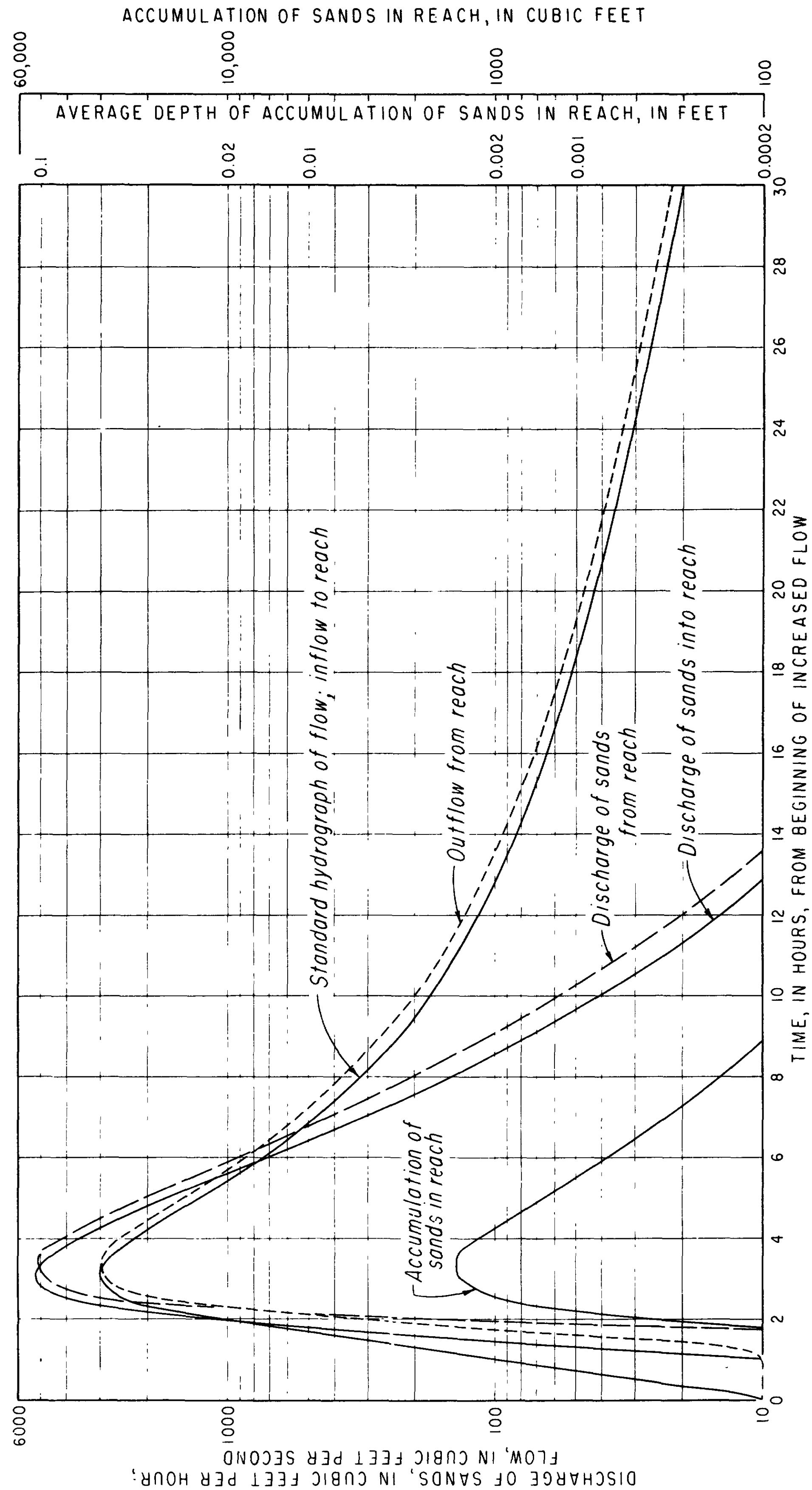


FIGURE 6.—Streamflow, discharge of sands, and accumulated sands in an assumed 5,000-foot reach of straight, uniform channel. The channel is uniform in bed material (median diameter 0.30 mm), slope, and cross section.

TABLE 3.—*Fill in an assumed 5,000-foot reach of straight channel of the standard cross section if velocities at a given depth are 20 percent lower at the downstream end of the reach than at the upstream end*

Streamflow (cfs)	Discharge of sands (cu ft per hr)			Average depth of fill (ft per hr)	
	Upstream end (table 2)	Downstream end	Difference	On horizontal submerged area	On flat-bed area (420,000 sq ft)
100	9.5	1.5	8.0	0.00002	0.00002
500	330	150	180	.00040	.00043
1,000	1,060	600	460	.00099	.0011
2,000	3,030	1,830	1,200	.0025	.0029
3,000	5,200	3,350	1,850	.0037	.0044

the product of the length of the reach and the average of the surface widths at the ends of the reach, or on the area of the flat bed of the reach, which is 5,000 feet by 84 feet. Of course, the volume of fill is greater for the larger constant flows, but a smaller percentage of the incoming sands remains in the reach at the larger flows.

The fill in the same assumed channel reach of 5,000 feet was computed for the inflow that is represented by the hydrograph of figure 5. The computed approximate hydrograph of flow at the downstream end of the reach, where the slope is flatter, is shown in figure 7. The depths and velocities at many times during the flow were computed for the downstream cross section, and these depths and velocities were then used with figure 3 to compute a graph of discharge of sands at the downstream end of the reach through the period of flow. The curve of volume of sand accumulation (fig. 7) within the reach was determined from the differences between the inflow and the outflow of sands during short increments of time. This cumulative curve is also shown as depth of average deposit on figure 7. Similar sediment accumulation would be caused if the velocity at the downstream end of the reach was reduced 20 percent for a given depth of flow by increased resistance such as might result from growth of small willows in the channel. In a straight, uniform channel, a change of slope or of roughness that causes fill while the flow is decreasing can also be expected to cause fill while the flow is increasing.

Of course, large amounts of fill over a long period of time are inconsistent with the maintenance of a uniform channel. A uniform channel was assumed to simplify the study of the principles of scour and fill as much as possible.

Obviously, an assumed increase in slope or decrease of roughness toward the downstream end of a 5,000-foot reach of uniform channel of the standard cross section would result in computed scour during periods of both increasing and decreasing flow. Also, the amount of the scour in the reach would, except for the usually small effect of the change in the hydrograph of flow as the water passes through the reach, be about the same as the amount of fill shown in figure 7, if the hydrographs of flow and the graphs of discharge of sands were assumed to apply at the opposite ends of the reach—that is, if the 20-percent lower velocities for a given depth existed at the upstream end of the reach rather than at the downstream end.

CONSTRICTION IN A STRAIGHT, GENERALLY UNIFORM CHANNEL

Consider, next, the scour and fill at a narrower-than-average cross section in a long straight reach that has the standard cross section (fig. 4) except at the contraction. At a cross section A in the uniform channel upstream from the narrow section, the relations of depth and mean velocity to flow are assumed to be those shown in figure 4, and the discharges of sands are assumed to be given by the curves in figure 3. Also, these relations and figure 3 are assumed to apply at a cross section C that is downstream from the narrow section and 1,000 feet downstream from section A. A suitable distance downstream from section A, the width of the channel slowly and smoothly decreases to an assumed bottom width of either 75 or 66 feet (9 or 18 ft less than the bottom width at section A). The bottom widths of 75 or 66 feet are assumed to be constant for a short distance before the channel gradually widens to the standard cross section and bottom width of 84 feet. The channel banks are assumed to have a 45° slope throughout the reach. Some computations of scour and fill will be based on a cross section B that is 200 feet downstream from section A and in the narrow section. Because the transitions are slow and smooth and because section B is in a short reach of straight channel, the relationship of mean velocity to turbulence at section B will be assumed to be normal for a sand-bed reach at the computed depths and velocities. Hence, figure 3 will be assumed to apply at section B as well as at sections A and C.

If sections A and C are far enough upstream and downstream from the narrow reach, the net scour and fill between these sections would be the same as for a straight, uniform 1,000-foot reach. At constant flow, neither net scour nor net fill would occur in the reach. If the flow indicated by the hydrograph of figure 5 passes through the reach, the net accumulation of sands (expressed in terms of volume) within the reach would be about one-fifth as much as that for the 5,000-foot reach (fig. 6), but the average net depth of accumulation of sands in the reach would be about the same as that for the 5,000-foot reach. The average elevation of the streambed in the 1,000-foot reach would be about the same after the flow has passed through the reach as before the flow arrived.

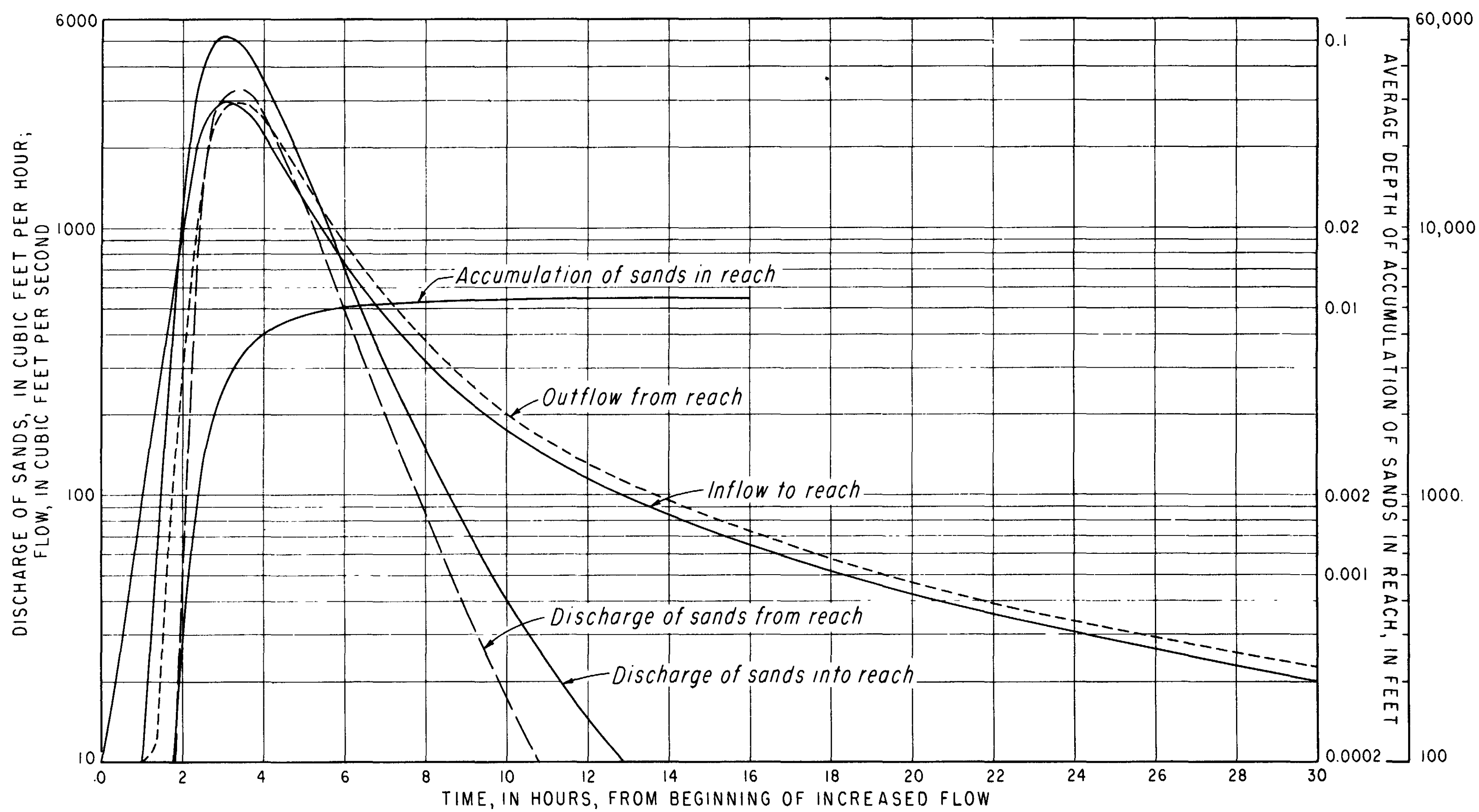


FIGURE 7.—Streamflow, discharge of sands, and accumulated sands in an assumed 5,000-foot reach of straight channel that is uniform in bed material (median diameter 0.30 mm) and cross section but has a gradually decreasing slope toward the downstream end. Mean velocities at a given depth of flow are 20 percent lower at the downstream end of the reach than at the upstream end.

Two types of information were computed for five different rates of constant flow. One type is the initial rate of volume of net scour in the reach between sections A and B. This initial rate of net scour is numerically equal to the initial volume rate of net fill in the reach between sections B and C. The initial rate of scour or of fill, respectively, means the difference between the discharges of sands from and into or into and from the reach, provided that the discharges of sands are based on positions of the streambed before these positions are changed by the flow. Thus, if a streambed is generally flat (although ripples, dunes, bars, or antidunes may cause local irregularities) before flow begins, the initial rate of scour in a reach between a wide and a narrow cross section is the difference between the discharges of sands at the narrow section and at the wide section before the streambed scoured noticeably at the narrow section. The significance of the initial rate of scour or fill is that it represents a maximum rate of removal or accumulation of sands in the reach for a particular rate of streamflow. The second type of information is the approximate depth of average scour to be expected at section B after flow has continued at the constant rates until an equilibrium cross section or bed position has been reached at section B.

The computed initial rates of scour in the 200-foot reach between section A and section B (table 4) are based on the assumptions that the sand that is removed from the channel bed weighs 85 lb per cu ft and that the depths of flow at sections A and B are equal at the beginning of each flow at a constant rate. The steps in the computation of the initial scour are shown in table 4 and are rather obvious. The discharges of sands (table 2, column 6) at the constant rates of flow had already been computed for sections such as A and C. The depths of flow (table 2, column 2) and the geometry of section B determine the surface widths and the cross-sectional areas of section B. The streamflow divided by the area gives the mean velocity, and the mean velocity and the depth define the discharges of sands per foot of width in figure 3. The rate of initial scour in the reach between section A and B is the difference in discharges of sands at the ends of the reach. The net average depth of scour per hour is obtained by dividing the volume of scour by the product of the length of the reach and an effective bottom width of the reach. The effective bottom widths were assumed to be 78 and 72 feet for the two different reductions in bottom width. Of course, the scour at section B probably would be at least twice the net average depth of scour and may well be somewhat more than twice the average.

A precise effective width of channel subject to scour, fortunately, is not necessary for computing approximate average depths of scour in the reach for different flows and reductions in channel width. In the computations for table 4, only the flat bed of the channel was assumed to be subject to scour. For a rough computation, both the depth of scour and the reduction in channel width can be assumed to vary directly with distance from section A. Under these assumptions, the effective width of flat bed subject to scour in the reach would be $W_1-2(W_1-W_2)/3$ or 78 and 72 feet, respectively, for the reductions of 9 and 18 feet in channel width. (W_1 and W_2 are the bottom widths at sections A and B.) The

TABLE 4.—*Computed initial rates of scour between cross sections A and B at different flows for which the depths of flow above the flat beds are assumed to be equal at the two sections*

[Net average depth of scour between sections is based on product of length of reach (200 ft) and effective bottom widths of 78 ft for 9-ft contraction and 72 ft for 18-ft contraction]

Streamflow (cfs)	Section A	Section B, bed not scoured by flow					Scour between sections or difference in discharges of sands (rounded)	
	Discharge of sands (cu ft per hr)	Depth of flow above the flat bed (ft)	Area (sq ft)	Mean velocity (fps)	Surface width (ft)	Discharge of sands (cu ft per hr)	Cu ft per hr	Ft of net average scour per hr
Bottom width 9 ft less at section B than at section A								
100	9.5	0.97	73.7	1.36	77.0	20	10	0.0006
500	330	2.68	208	2.40	80.4	506	180	.012
1,000	1,060	4.10	325	3.08	83.2	1,500	440	.028
2,000	3,030	6.20	504	3.97	87.4	3,760	730	.047
3,000	5,200	8.00	664	4.52	91.0	6,280	1,080	.069
Bottom width 18 ft less at section B than at section A								
100	9.5	0.97	64.9	1.54	68.0	41	31	0.002
500	330	2.68	184	2.72	71.4	770	440	.031
1,000	1,060	4.10	288	3.47	74.2	2,150	1,090	.076
2,000	3,030	6.20	448	4.46	78.4	4,940	1,910	.133
3,000	5,200	8.00	592	5.07	82.0	7,870	2,670	.185

effective width can be multiplied by the length of the reach and by the average depth of scour to obtain the volume of scour under the stated assumptions. This effective width is less than the average bottom width because the depth of scour increases as the width decreases. Of course, the net average depth of scour would be a little less if the width of water surface had been used rather than the width of the flat bed.

The 18-foot decrease of bottom width, which represents roughly a 20-percent decrease in area, produced initial rates of scour between sections A and B that were about 2.5–3.0 times the rates of scour that were caused by the 9-foot decrease in bottom width, which represents roughly a 10-percent decrease in area. (See table 4.)

Of course, the initial rates of scour would begin to decrease immediately because the scour would reduce the mean velocity at section B and hence decrease the difference in discharges of sands between sections B and A. The equilibrium bed positions at section B, the positions for which the discharges of sands at sections A and B are equal, were computed for the same decreases in bottom width and the same five constant flows that were used in the computation (table 4) of initial rates of scour.

The computation of average depth of scour at equilibrium bed position at section B is indicated in table 5 and begins with the assumption that the discharges of sands are to be the same at sections A and B. The discharge of sands at each constant rate of flow is taken from column 6 of table 2 and is divided by the surface width at section B (the water-surface slope is assumed to be the same as though the width were not contracted) to obtain the equilibrium discharges of sands per foot of width at section B. The difference in discharges of sands per foot of width at sections B and A requires an increase in mean velocity from section A to section B, an increase that can be approximated from figure 3, and the mean velocity at section B is computed from that at section A (table 2, column 3) by adding this increase. The flow is divided by the mean velocity to obtain the cross-sectional area necessary to discharge the flow. If the banks are considered to remain stable at section B and all scour is assumed to occur at the flat bed, the average depth of flow above the flat bed can be computed from the geometry of section B. The difference between this depth of flow and the depth of flow above the flat bed at section A (table 2, column 4) is the indicated average depth of scour of the flat bed at section B.

The computations of average depth of scour at equilibrium indicate at least three significant points. One is that at the narrow section the depths of scour, like the initial rates of scour between sections, increase fairly rapidly as the flow increases. A second point is that at a given flow the depth of scour is about directly proportional to the reduction in width. A third point is that the length of the reach does not influence the computations, and hence the equilibrium depth of average scour at the narrow section is independent of the length of reach between the sections unless the reach is so short and the change in cross section and streamlines so abrupt that the depths and mean velocities cannot be satisfactorily determined or that the relations of figure 3 do not apply.

The standard hydrograph of flow and the graph of discharge of sands (fig. 5) were assumed at section A and also at section C except for a few minutes lag

TABLE 5.—*Average scour at section B after five constant flows have established equilibrium bed positions*

Streamflow (cfs)	Section A	Section B		Difference between sections A and B		Section B			
	Discharge of sands (cu ft per hr)	Surface width (ft)	Discharge of sands (cu ft per hr per ft of width)	Discharge of sands (cu ft per hr per ft of width)	Mean velocity (fps)	Mean velocity (fps)	Cross sectional area (sq ft)	Depth of flow above the flat bed (ft)	Average scour of flat bed (ft)
Bottom width 9 ft less at section A than at section B									
100	9. 5	77. 0	0. 123	0. 013	0. 01	1. 22	82	1. 08	0. 11
500	330	80. 4	4. 10	. 40	. 05	2. 20	227	2. 93	. 25
1, 000	1, 060	83. 2	12. 7	1. 2	. 06	2. 82	355	4. 50	. 40
2, 000	3, 030	87. 4	34. 6	3. 1	. 09	3. 67	545	6. 75	. 55
3, 000	5, 200	91. 0	57. 2	5. 2	. 11	4. 19	717	8. 70	. 70
Bottom width 18 ft less at section A than at section B									
100	9. 5	68. 0	0. 14	0. 03	0. 03	1. 24	81	1. 20	0. 23
500	330	71. 4	4. 60	. 9	. 10	2. 25	222	3. 25	. 57
1, 000	1, 060	74. 2	14. 3	2. 8	. 14	2. 90	345	4. 99	. 89
2, 000	3, 030	78. 4	38. 6	7. 1	. 19	3. 77	530	7. 46	1. 26
3, 000	5, 200	82. 0	63. 4	11. 4	. 25	4. 33	693	9. 56	1. 56

for the passage of the flow through the 1,000-foot reach; scour was computed at section B for reductions in bottom width of 9 and 18 feet. At the beginning of the standard hydrograph, the depth of the low flow was assumed to be the same at all three cross sections. Computations were made for 20-minute intervals before the peak discharge at section A and for 30-minute or longer intervals after the peak. No time lag was applied for the 200-foot reach between sections A and B, and scour was assumed to be limited to the flat bed. The average depth of scour in the reach was based on effective bottom widths of 72 and 78 feet for the two different reductions in bottom width, and the depth of scour at section B was assumed to be double the average depth of scour for the reach.

The same type of computation was made for each successive time interval. The depths and streamflows at section B at the beginning and at the end of the first time interval and the channel geometry were used to compute the cross-sectional areas and the velocities at section B at the beginning and end of the interval. From the depths, velocities, and widths at section B, the discharges of sands at the beginning and end of the time interval were computed from figure 3. The difference between the discharges of sands from and into the reach during the interval was used to compute the volume of scour and the average depth of scour in the 200-foot reach. This average depth was then doubled to obtain an assumed, but perhaps somewhat too small, depth of scour at section B. Next, a similar computation was made for the second time interval except that the discharge of sands at section B at the beginning of this time interval did not have to be computed unless the computed scour at the end of the first interval was large enough to require a recomputation of the discharge of sands. The same procedure was followed for each successive interval except that as the scour became appreciable, the amount of scour at the end of the interval at section B was estimated in advance. A recomputation was necessary only if the computed scour differed significantly from the estimated scour. The details of the computations for the reduction of 18 feet in bottom width are shown in table 6.

The cumulative depths of computed scour at section B (fig. 8), although inexact, indicate qualitatively the general behavior of scour and fill at a contracted section during the passage of an assumed variable flow. The average elevation of the bed at the narrow section is lowest an appreciable time after the peak of the flow when the discharge of sands at the narrow section has decreased until it exactly equals the discharge of sands upstream from the narrow section. The depths of scour are roughly in proportion to the percentage reductions of bottom width. Also, much of the scour is not replaced by fill during the recession of this particular hydrograph of flow. In general, the recession side of a hydrograph of flow is long; and in many sand-bed streams, even low flows transport appreciable quantities of sands; hence, soon after a flood has passed, the streambed at a narrow section may have filled as much as it scoured during the earlier part of the flood. Narrow cross sections of a sand-bed stream are likely to be characterized by alternate scour and fill with the passage of each flood, but the actual amounts of scour and fill depend partly on the form and size of the hydrograph of flow and partly on the initial position of the streambed.

The maximum computed depth of scour at the narrow section is roughly 40 percent of the equilibrium depth of scour for the peak flow of 3,000 cfs (table 5). The figure of 40 percent, as well as the amount of fill after the flow has passed and the time between the peak of the flow and the maximum depth of scour, is largely a function of the shape, size, and duration of the hydrograph of flow and of the graph of sands.

Although little scour or fill occurs at representative cross sections such as A and C of the assumed uniform channel, a pattern of alternate scour and fill is characteristic at a narrow section. (Of course, the scour and fill that are discussed here are based on an average bed elevation across a channel; such very local irregularities as sand dunes are assumed to be smoothed out in the computation of the average bed elevation at a cross section.) Because most information on scour or fill has been obtained at streamflow measuring sections, which are usually at narrower-than-average parts of the channel or at structures that constrict the flow laterally, the untypical alternation of scour and fill is often thought to be representative for sand-bed streams (Lane and Borland, 1954). On the other hand, a sand bed whose average elevation is stable during periods when the flow changes greatly indicates uniformity of width of flow and of available bed sand with distance along the channel.

The principles of scour and fill apply, of course, to the reach between sections B and C. If this 800-foot reach is long enough for normal flow and discharge of sands to become reestablished in the straight, uniform channel, the discharge of sands at section C differs from that at section A only by a few minutes lag. Slight adjustment can be made for this lag, or it can be disregarded. In simple terms and with reasonable accuracy, any net volumes of sands that were removed from the reach between sections A and B can be considered to be balanced by equal volumes of net deposit in the reach between sections B and C. This broad relation exists in spite of the facts that the bed at section

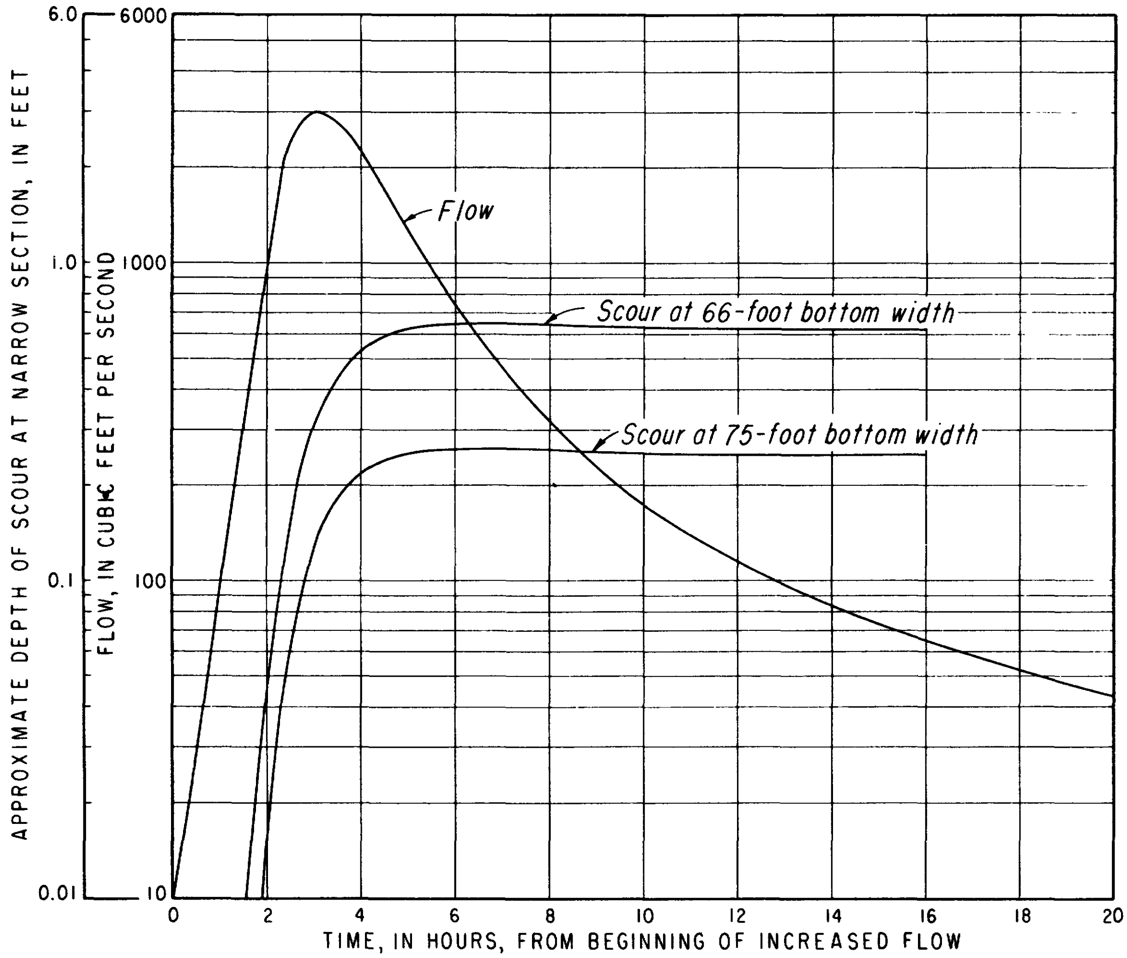


FIGURE 8.—Depths of computed scour at narrow section of an assumed 1,000-foot reach of straight channel.

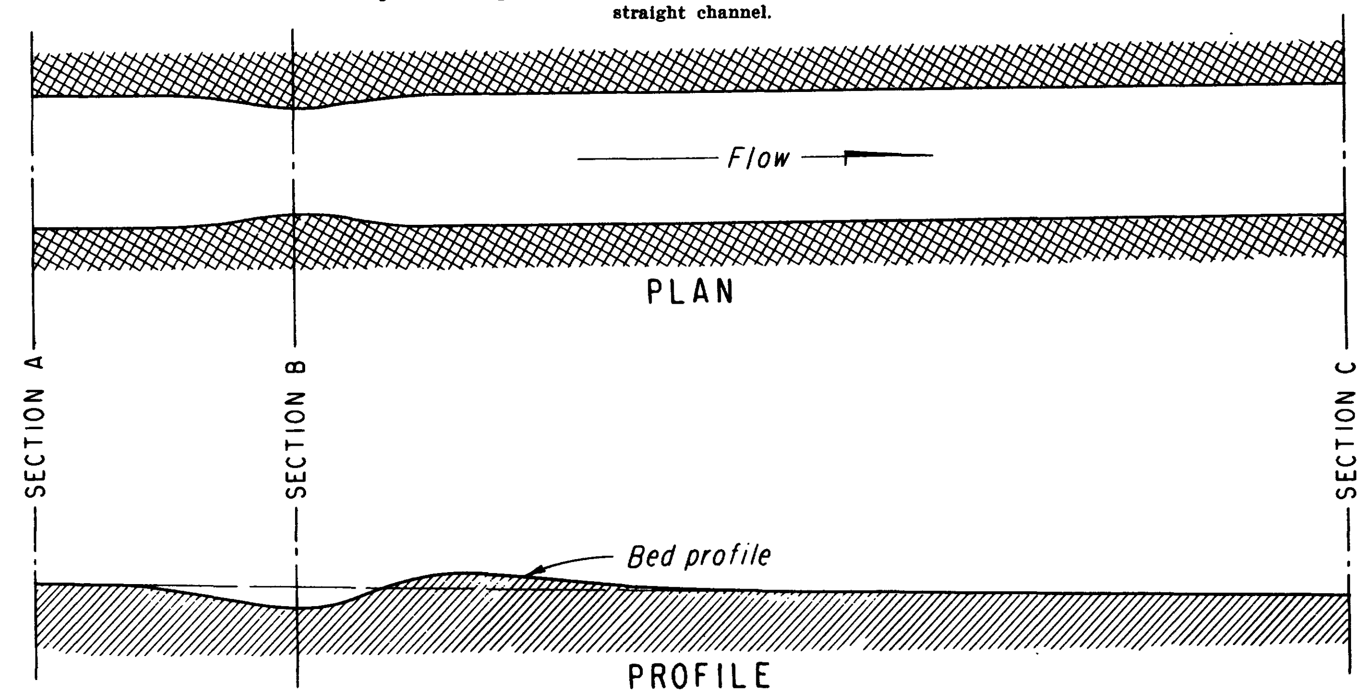


FIGURE 9.—Schematic profile of bed position at and near a narrow section of an assumed straight sand-bed channel after a short period of constant flow.

TABLE 6.—*Computed scour at cross section where bottom width is 66 feet in channel whose uncontracted bottom width is 84 feet*

Column 1: Time from beginning of increased flow.
Column 2: Flow from hydrograph on figure 5.
Column 3: Depth, or stage, above flat bed from figure 4.
Column 4: Mean velocity from figure 4.
Column 5: Discharge of sands per foot of width from figure 3.
Column 6: Surface width of wide section is bottom width of 84 feet plus twice the depth.
Column 7: Discharge of sands per foot of width times the surface width.
Column 8: Surface width of narrow section is bottom width of 66 feet plus twice the depth at the wide section or the surface width at the wide section minus 18 feet.
Column 9: Bottom width of 66 feet plus the depth at the wide section.
Column 10: The average width from column 9 minus the computed scour from column 22. For a first trial, the computed scour is estimated.
Column 11: The same depth as given in column 3 for the wide section.
Column 12: The depth from column 11 plus the computed scour from column 22.
Column 13: The product of the average width from column 10 and the depth from column 12.
Column 14: The velocity is obtained by dividing the flow from column 2 by the area from column 13.
Column 15: The discharge of sands per foot of width from figure 3 for the depth from column 12 and the mean velocity from column 14.
Column 16: The discharge of sands from column 15 multiplied by the surface width from column 8.
Column 17: The difference in discharge of sands from column 16 and from column 7.
Column 18: Graphically estimated difference in discharge at a time halfway between the times given in column 1. (Figures used only for rapidly increasing or high flows.)
Column 19: Volume removed from reach during period that ended at the time given in column 1. Volume is equal to the product of the length of period in hours and the average difference in discharge of sands in cubic feet per hour, based on figures in columns 17 and 18.
Column 20: Cumulative volume of sands removed from reach.
Column 21: Average depth of scour in reach is computed from the cumulative volume from column 20 divided by the product of the 200-foot length of the reach and an estimated average bottom width of 72 feet.
Column 22: The average depth of scour in the reach, from column 21, multiplied by 2. This assumes a straight-line variation of the scour from zero at the upper and wide end of the reach to a maximum at the narrow section.

Time (hr)	Upstream section, 84-foot bottom width						Downstream section, 66-foot bottom width									Difference in discharges of sands between sections				Depth of scour	
	Flow (cfs)	Depth above flat bed (ft)	Mean velocity (fps)	Discharge of sands per foot of width (cu ft per hr)	Surface width (ft)	Discharge of sands through section (cu ft per hr)	Surface width (ft)	Average width: No scour (ft)	Average width: Scoured (ft)	Depth above flat bed: No scour (ft)	Depth above flat bed: Scoured (ft)	Area of scoured section (sq ft)	Mean velocity (fps)	Discharge of sands per foot of width (cu ft per hr)	Discharge of sands through section (cu ft per hr)	At stated time (cu ft per hr)	Estimate between times (cu ft per hr)	During period (cu ft)	Cumulative (cu ft)	Average in 200-ft reach (ft)	Estimated at narrow section (ft)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
0:00	10	0.22	0.54	--------	84.4	--------	66.4	66.2	------	0.22	------	14.6	0.68	--------	--------	--------		--------	--------	--------	--------
0:20	20	.35	.68	--------	84.7	--------	66.7	66.4	66.4	.35	0.35	23.2	.86	--------	--------	--------	--------	--------	--------	--------	--------
0:40	44	.58	.90	--------	85.2	--------	67.2	66.6	66.6	.58	.58	38.6	1.14	0.07	5	5	--------	0.4	0.4	0.00003	0.00006
1:00	100	.97	1.21	0.11	85.9	9.5	67.9	67.0	67.0	.97	.97	65.0	1.54	.60	41	31	12	5	5	.00035	.0007
1:20	210	1.56	1.59	.72	87.1	63	69.1	67.6	67.6	1.56	1.56	105	2.00	2.55	176	113	70	24	29	.002	.004
1:40	460	2.53	2,10	3.3	89.1	294	71.1	68.5	68.5	2.53	2.54	174	2.64	9.5	675	381	200	74	103	.007	.014
2:00	1,000	4.10	2.76	11.5	92.2	1,060	74.2	70.1	70.1	4.10	4.14	290	3.45	28	2,080	1,020	610	218	321	.022	.044
2:20	2,000	6.20	3.58	31.5	96.4	3,030	78.4	72.2	72.1	6.20	6.31	456	4.38	65	5,100	2,070	1,500	507	828	.057	.11
2:40	2,700	7.50	3.93	47	99.0	4,630	81.0	73.5	73.3	7.50	7.71	565	4.78	83	6,720	2,090	2,080	693	1,521	.106	.21
3:00	3,000	8.00	4.08	52	100.0	5,200	82.0	74.0	73.7	8.00	8.31	612	4.90	88	7,210	2,010	2,050	683	2,204	.153	.31
3:30	2,800	7.70	4.00	48	99.4	4,770	81.4	73.7	73.3	7.70	8.13	596	4.70	79	6,430	1,660	1,860	924	3,128	.217	.43
4:00	2,230	6.70	3.70	37	97.4	3,600	79.4	72.7	72.2	6.70	7.23	521	4.28	60	4,760	1,160	1,410	705	3,833	.265	.53
4:30	1,700	5.70	3.40	27	95.4	2,570	77.4	71.7	71.1	5.70	6.29	448	3.80	41	3,170	600	840	430	4,263	.297	.59
5:00	1,280	4.80	3.00	16.5	93.6	1,550	75.6	70.8	70.2	4.80	5.43	381	3.36	25	1,890	340	--------	235	4.498	.312	.62
5:30	960	4.00	2.75	11.5	92.0	1,060	74.0	70.0	69.4	4.00	4.64	322	2.98	15.7	1,160	100	--------	110	4,608	.320	.64
6:00	740	3.40	2.50	7.6	90.8	690	72.8	69.4	68.8	3.40	4.04	278	2.66	9.7	706	16	--------	29	4,637	.322	.64
7:00	470	2.60	2.12	3.4	89.2	304	71.2	68.6	68.0	2.60	3.24	220	2.14	3.7	264	−40	--------	−12	4,625	.321	.64
8:00	320	2.00	1.85	1.7	88.0	150	70.0	68.0	67.4	2.00	2.64	178	1.80	1.45	102	−48	--------	−44	4,581	.318	.64
9:00	230	1.63	1.63	.86	87.3	75	69.3	67.6	67.0	1.63	2.26	151	1.52	.57	40	−35	--------	−42	4,539	.315	.63
10:00	175	1.36	1.50	.50	86.7	43+	68.7	67.4	66.8	1.36	1.99	134	1.31	.21	14	−29	--------	−32	4,507	.313	.63
11:00	140	1.20	1.37	.27	86.4	23	68.4	67.2	66.6	1.20	1.82	121	1.16	.08	5	−18	--------	−24	4,483	.311	.62
12:00	115	1.06	1.28	.17	86.1	15−	68.1	67.1	66.5	1.06	1.68	112	1.03	.022	2	−13	--------	−16	4,467	.310	.62
14:00	84	.86	1.14	.07	85.7	6.0	67.7	66.9	66.3	.86	1.48	98	.86	--------	--------	−6	--------	−19	4,448	.308	.62
16:00	65	.74	1.03	.02	85.5	1.7	67.5	66.7	66.1	.74	1.36	90	.72	--------	--------	−1.7	--------	−8	4,440	.308	.62
18:00	52	.64	.96	--------	85.3	--------	67.3	66.6	66.0	.64	1.26	83	.63	--------	--------	--------	--------	--------	4,440	.308	.62
20:00	43	.56	.90	--------	85.1	--------	67.1	66.6	66.0	.56	1.18	77.8	.55	--------	--------	--------	--------	--------	4,440	.308	.62
24:00	31	.46	.80	--------	84.9	--------	66.9	66.5	65.9	.46	1.08	71.2	.44	--------	--------	--------	--------	--------	4,440	.308	.62
30:00	20	.35	.68	--------	84.7	--------	66.7	66.4	65.8	.35	.97	63.8	.31	--------	--------	--------	--------	--------	4,440	.308	.62
36:00	13	.26	.58	--------	84.5	--------	66.5	66.3	65.7	.26	.88	57.8	.22	--------	--------	--------	--------	--------	4,440	.308	.62
42:00	10	.22	.54	--------	84.4	--------	66.4	66.2	65.6	.22	.84	55.1	.18	--------	--------	--------	--------	--------	4,440	.308	.62

Note.—The computed scour from column 22 was first needed in the computations for columns 10 and 12. At that point in the computations, the scour is estimated and the computations of the computed scour are completed. If the estimated scour and the computed scour do not agree very closely, the computations from column 10 to column 22 are repeated with either the first computed scour or an estimate that is based on the first computed scour.

B generally scours when the reach from A to B scours and the bed at section C generally shows little net scour or net fill. Therefore, the volume of fill in the reach between sections B and C must roughly balance both the volume of scour in the upstream end of the reach between sections B and C and the volume of scour in the reach between sections A and B. If the reach downstream from the narrow section is longer than the reach upstream from the section, the net average depths of fill in the downstream reach must be less than the net average depths of scour in the upstream reach. The bed profile for a constant flow must have the general shape indicated in figure 9. Obviously, the reach between sections B and C must be long, or the flow and discharge of sands at its downstream end may be affected appreciably by the contraction and expansion of the flow.

WINDING STREAMS

The discharge of sands at bends of a sand-bed stream may be significantly different from that indicated by figure 3 because the turbulence for a given depth, mean velocity, water temperature, and bed material may be different from the turbulence in a straight reach of channel. Although a relation may exist between the discharge of sands at a bend and the characteristics of flow and sediment, the relation would be complex and has not yet been defined. Hence, one of the principles of scour and fill cannot now be used at a stream bend; but the principle of continuity does, of course, apply. Also, both principles can be applied at reasonably straight reaches of channel between bends provided that these reaches are long enough for the flow to entrain about its equilibrium discharge of sands. Thus, in some ways a channel reach containing a bend is like a straight channel reach that contains a narrow section.

The amount of scour and fill at a bend may be difficult to determine, but the net volume of scour or fill within a reach that includes the bend can be computed if the difference in sediment discharges at the ends of the reach can be measured or computed. A general restriction on the scour of sands from a long reach of winding channel is, of course, imposed by the total discharge of sands at any cross section. Thus, the total discharge of sands should be about 13,000 cubic feet (fig. 5) or about 130 cubic feet per foot of surface width at maximum stage for the standard cross section in a fairly straight reach of channel (0.30-mm median diameter of the bed sediment) and for the hydrograph of flow and the discharge of sands that are given in figure 5. In a reach of channel 1 mile long that has no inflow of sands, the indicated average depth of net scour during the passage of the flow would be only 0.02–0.03 foot. For five constant rates of flow, the discharges of sand per hour per foot of width of the standard cross section would be approximately the same as those given in table 2, column 5. At the highest of the constant flows, 3,000 cfs, the indicated net scour would be about 0.01 foot per hour for a reach 1 mile long having no inflow of sand. Volumes of discharge of sand, in cubic feet per hour per foot of width, are given in figure 3 for many combinations of depth and mean velocity in sand-bed channels and for water temperatures of 60° F, bed sediment of 0.30-millimeter median diameter, and low concentrations of fine sediment. These volumes of sand can be converted to net scour, in feet per hour, of the sand bed of uniform-width channels by subtracting the inflow of sands per foot of width and dividing the difference by the length of the channel reach, in feet. Of course, if a channel reach cannot be considered to have about a uniform width, the volumes of sand per foot of width per hour at different flows can be multiplied by the channel width at the downstream end of the reach, any inflow of sands per hour can be subtracted, and the difference can be divided by the area of the bed of the reach to obtain a net depth of average scour, in feet per hour. Obviously, large depths of scour or fill in a winding channel like those in a straight channel usually are not typical of long reaches of streams.

SCOUR AND FILL IN NATURAL STREAMS

Most natural streams differ considerably from the assumed channels for which computations of scour and fill have been made for this report. The comparatively few streams that have continuous sand beds usually have somewhat variable widths, uneven lateral distributions of depth and velocity, and some change in particle size with distance along the channel. Many natural streams are pool-and-riffle streams and have mean velocities, widths, depths, and particle sizes that change widely and rapidly with distance along the channel. Although the pools may have sand beds much or all of the time, the pool-and-riffle streams as a whole are not really sand-bed streams.

The differences in pattern of scour and fill will be shown for only three natural streams.

A GENERALLY STABLE SAND-BED STREAM

Changes of bed position in a generally stable sand-bed stream have been defined by many streamflow measurements or cross-section soundings under two cableways over the Elkhorn River near Waterloo, Nebr. (Beckman and Furness, 1962, table 2). The general stability of the sand bed implies few, if any, large changes in bed elevation for a stream reach of appreciable length. This stability exists in spite of frequent and rapid changes in bed elevation at individual points and even appreciable changes in average bed elevation at entire cross sections (Beckman and Furness, 1962, p. 12). The average bed positions at the two cableways, which are about 1,670 feet apart, were usually determined at about the same time. The width of the water surface at the upstream cableway varied, partly with stage and partly with time, from 224 to 283 feet over a period of 2 or 3 years. Usually the width was at least 245 feet; therefore, this width was used in computing changes in the bed elevation. Changes in bed elevation outside of this width were disregarded. The width of the water surface at the downstream cableway was usually 285–290 feet, although some widths were a little outside of this range. Because the range in width was small, the changes in bed elevation at the downstream cableway were based on the reported area and width at the time of each measurement. The width of the channel at the upper cableway was probably significantly less than the average width for the reach of channel, and the width of channel at the lower cableway was probably about average for the reach near the cableways.

The cableways are in a reach of channel whose bed is composed almost entirely of shifting sand that had a median diameter of about 0.24 millimeter in 1952 when many of the observations were made. The bed shifts readily. Driftwood occasionally lodges on or near the cableways and somewhat distorts the streambed and the pattern of the flow. Also, scour and fill at the upstream end of the reach may be affected somewhat by the shifting shape and elevation of a large sand bar a few hundred feet upstream from the upper cableway and by sometimes active erosion of the right bank about opposite the bar. Computed average changes of 0.1 or 0.2 foot in elevation of the wide and shifting streambed at either cableway section may be practically meaningless as measures of scour or fill in the reach.

As Beckman and Furness (1962, p. 12) concluded earlier, no clearcut relationship is apparent between flow or changes in flow and net scour or fill (figs. 10–13). Of course, the position of the streambed generally changes faster at high flow than at low flow, but the direction of the change is not consistent from one cableway section to the other. The change in average bed elevation at the downstream cableway was determined from consecutive cross-section soundings for 34 times when the difference in flow between the times of the soundings was 5 percent or more. Of the 11 times when the flow increased 5 percent or more, the computed bed elevation increased 3 times at the upstream cableway and 8 times at the downstream cableway. If only changes greater than 0.20 foot are considered, the computed bed elevation increased one time out of eight at the upstream cableway and five times out of five at the downstream cableway. Of the 23 times when the flow decreased 5 percent or more, the computed bed elevation increased 16 times at the upstream cableway and 9 times at the downstream cableway. If only changes greater than 0.20 foot are considered, the computed bed elevation for decreases of flow increased 12 times out of 15 at the upstream cableway and 3 times out of 7 at the downstream cableway. Thus, whether all changes of the bed elevation or only those greater than 0.20 foot are considered, the tendency is for the slightly narrower upstream cross section to scour on increasing flow and to fill on decreasing flow, as narrow sections are gen-

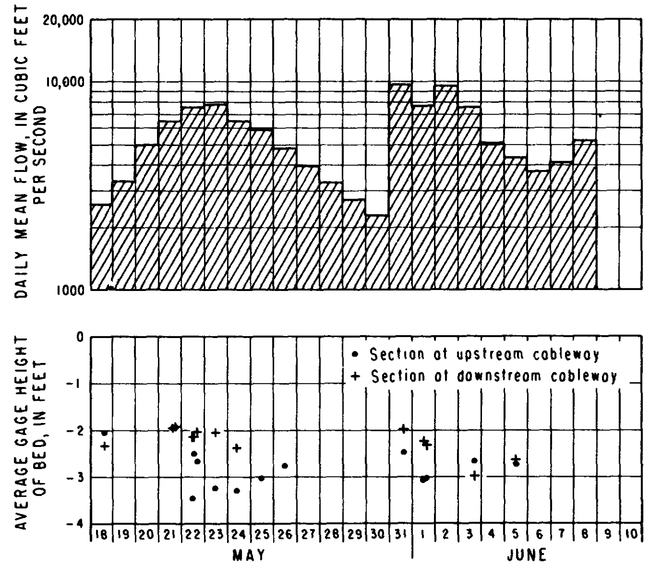


FIGURE 10.—Changes in average gage height of the streambed of the Elkhorn River near Waterloo, Nebr., during a period of high flow in 1951.

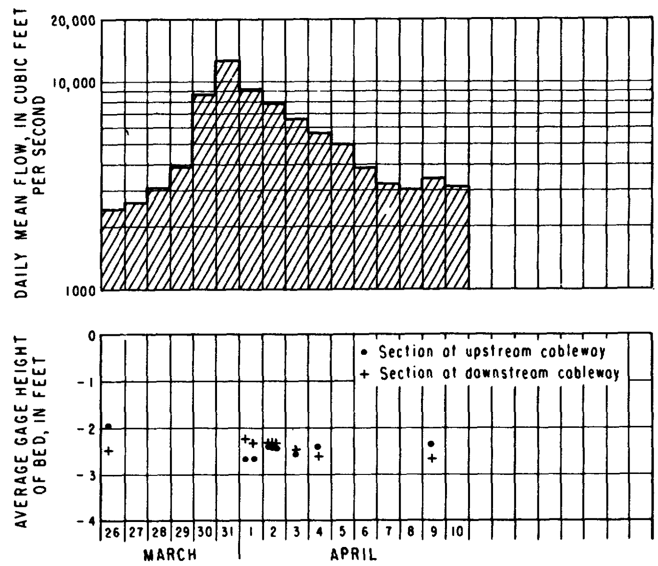


FIGURE 11.—Changes in average gage height of the streambed of the Elkhorn River near Waterloo, Nebr., during a period of high flow in 1952.

erally likely to do. The wider downstream cross section shows a slight tendency to fill on increasing flow and to scour on decreasing flow, but this tendency may not be significant.

The lack of agreement between the computed net scour and net fill at the two cableways is clear from a graph of change in bed elevation at one cableway (fig. 14) against change in bed elevation at the other. Of 42 comparisons, the computed change in average bed position at the downstream cableway was in the same direction as that at the upper cableway 22 times, and once a large fill at the upper cableway was accompanied by no change at the lower cableway. A reasonable conclusion from figure 14 seems to be that the two cross sections do not show any significant tendency to scour or to fill at the same time. Hence, measured scour or fill at either cross section is unlikely to represent average scour or fill along a channel reach near the cableways. Also, the lack of consistency indicates that net scour or fill is not governed directly by changes either in flow or in sediment concentration, because the rate of flow and the concentration of sediment are nearly the same at the two cross sections. The upstream cross section does show somewhat greater amounts of net scour and fill, as a narrower section is likely to, than the downstream section. At the downstream section, changes of the average bed position seem to be nearly random, as they should be for a uniform sand-bed

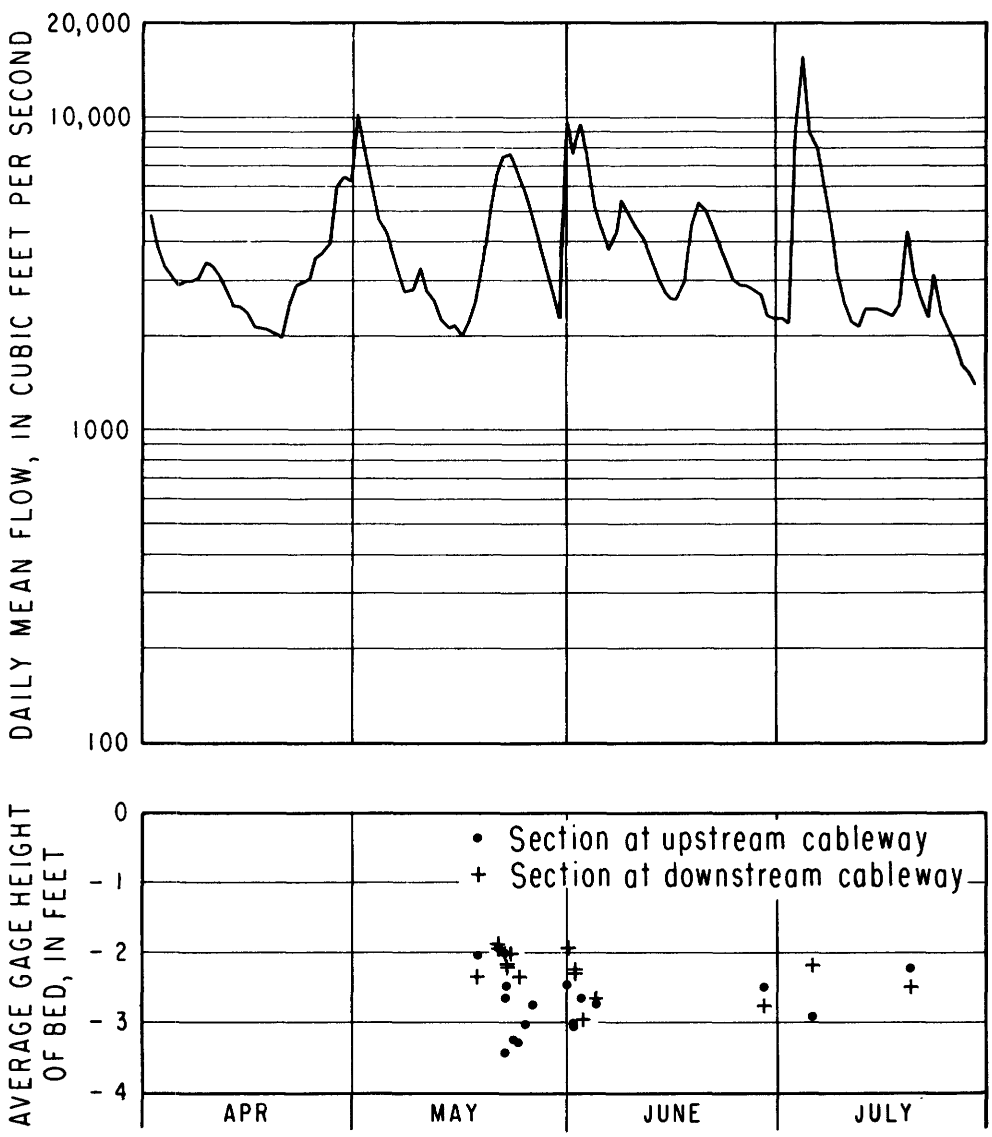


FIGURE 12.—Changes in average gage height of the streambed of the Elkhorn River near Waterloo, Nebr., May 18–July 19, 1951.

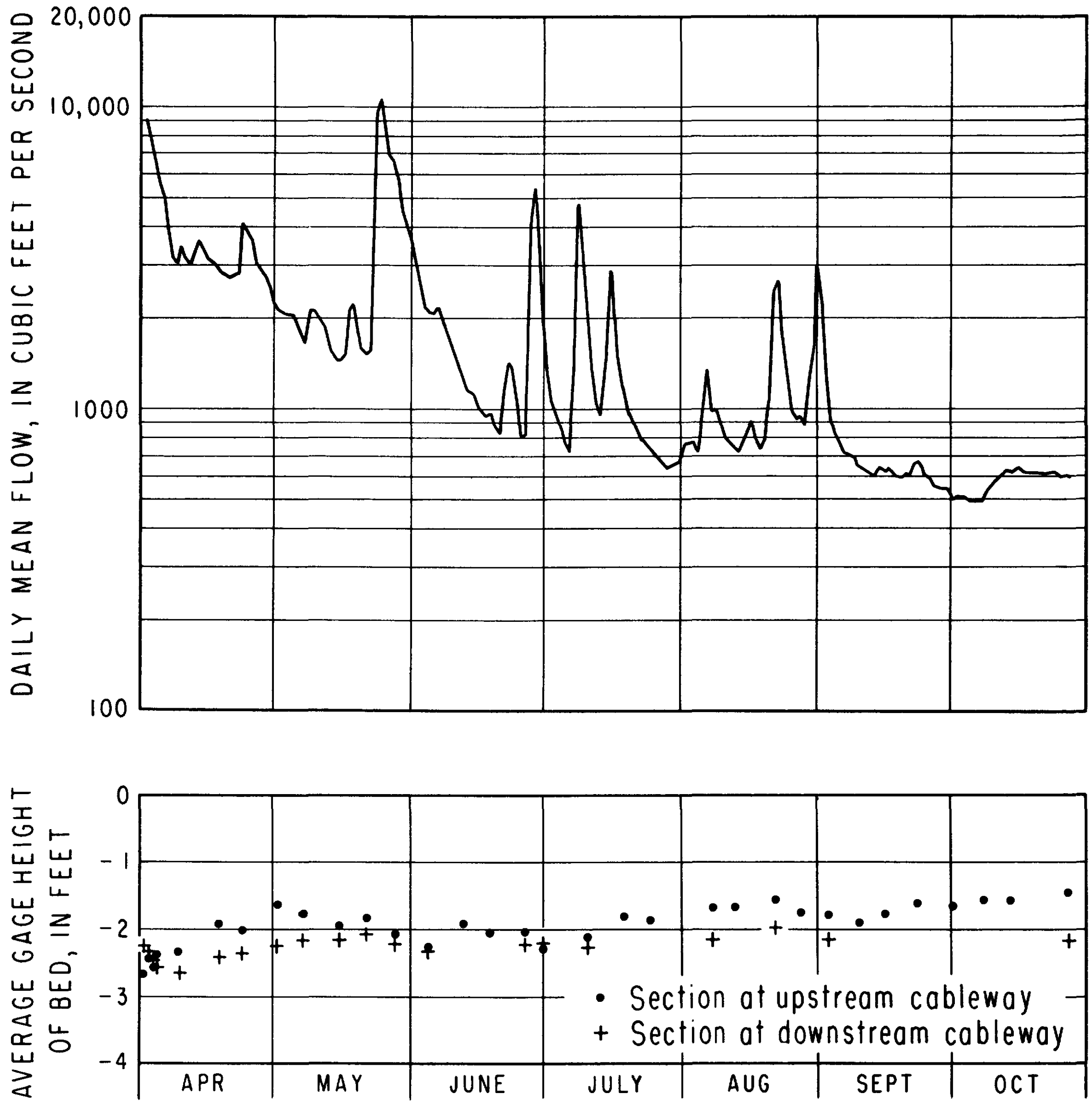


FIGURE 13.—Changes in average gage height of the streambed of the Elkhorn River near Waterloo, Nebr., April 1–October 23, 1952.

channel, except, of course, that the bed position changes faster at high flows than at low flows.

The general behavior of the streambed at the downstream cableway on the Elkhorn River near Waterloo is probably similar to that of the Colorado River at the site of Imperial Dam before the dam was constructed. At that site, too, measurements at a cableway at a laterally unconstricted section showed fluctuations of the streambed but no appreciable net scour or fill even during a high flow (Lane and Borland, 1954, p. 1075).

NARROW SECTION OF A NATURAL SAND-BED STREAM

Streamflow measurements made from a bridge over the Missouri River near Yankton, S. Dak., during 1941 show scour and fill (B. R. Colby and R. E. Oltman, written communication) that may occur at a cross section where the flow of a sand-bed stream is laterally constricted (fig. 15). Measurements were not made at or near all peak flows, but the available measurements show clearly that the streambed usually was lower than its average position whenever the flow was high. After the high flow ended, the streambed filled again to about the original position. The alternation of scour and fill is typical behavior at a narrow cross section of a sand-bed stream. A somewhat similar pattern of scour and fill was shown by the computations for the narrow section in the assumed uniform channel. The amounts of net scour or fill at this narrow cross section are not, however, representative for the reaches upstream and downstream from the measuring section.

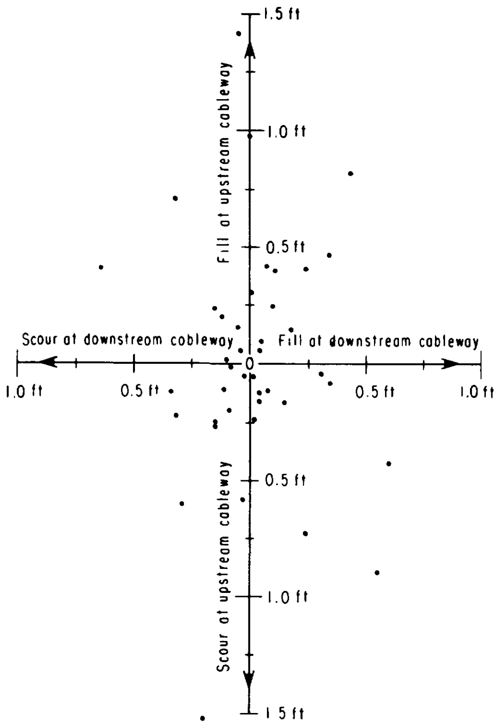


FIGURE 14.—Comparison of scour and fill computed from changes in bed position at two cableway sections on the Elkhorn River near Waterloo, Nebr.

A POOL-AND-RIFFLE STREAM

Although pool-and-riffle streams are not strictly sand-bed streams, they are discussed here because data from such streams have sometimes been used as indications of the scour and fill to be expected at sections of sand-bed streams.

A pool-and-riffle stream is one in which the water-surface slope alternates between long and comparatively level reaches (the pools) and short and comparatively steep segments (the riffles). Such a stream may have a sand bed throughout most pools, but its riffles normally have beds of material that is coarser than sand. The presence of the coarser bed material indicates that the quantity of available sand at the riffles is less than the stream can transport at the riffles. The mean velocity in a pool-and-riffle stream varies widely from one cross section to another along the channel, whereas continuity of the discharge of bed material and the abundant availability of sands prevent large changes in mean velocity from one section to another along a sand-bed stream.

STREAMFLOW, SEDIMENT CONCENTRATION, AND CHANGES IN BED ELEVATION

Leopold and Maddock (1952, 1953) and Brooks (1958) attempted to explain scour and fill at the streamflow-measuring section on the Colorado River near Grand Canyon, Ariz., on the basis of an assumed tendency for the depth of flow to adjust itself to changes

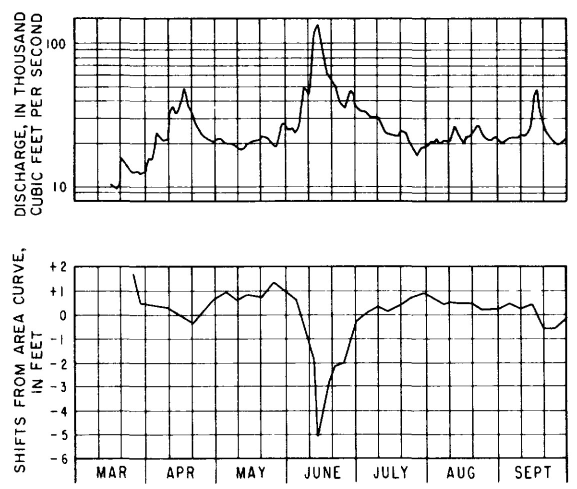
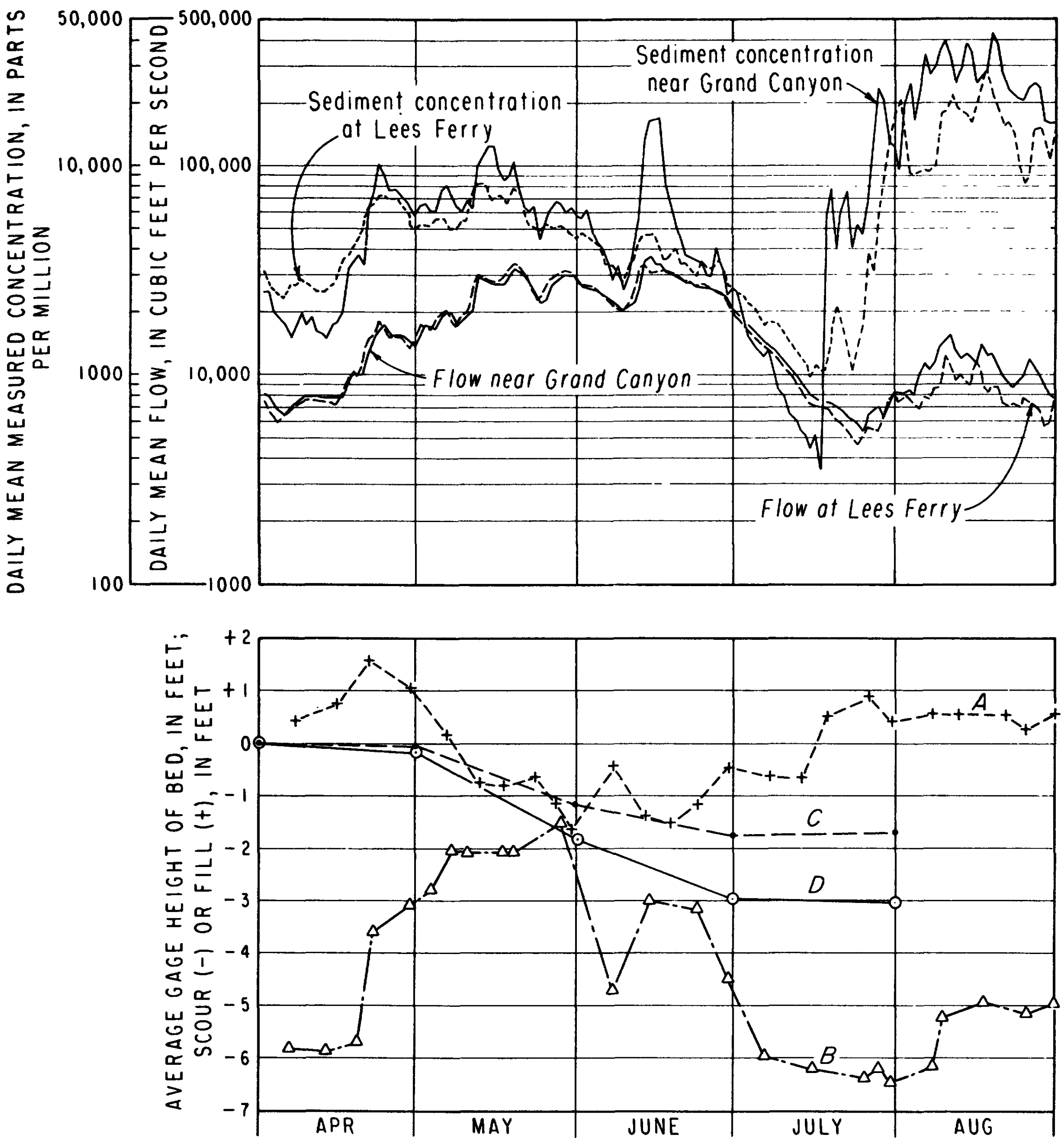


FIGURE 15.—Streamflow and changes in average bed position of the Missouri River near Yankton, S. Dak., March–September 1941. After Colby and Oltman (written communication, 1948).

in resistance to flow, changes that resulted from, or at least accompanied, changes in measured sediment concentration. These writers assumed that the pattern of scour and fill at the streamflow-measuring section of this pool-and-riffle stream was typical of a long reach of channel. Brooks' assumption (1958, p. 545) of constant water-surface slope implies a representative relation for a long reach. Actually, the slope in a pool may vary widely if the bed elevation changes appreciably. The assumptions seem to be questionable for at least four reasons, one of which is that the measuring section is at a cross section that is probably narrower than average for the pool, and hence untypically large amounts of alternate scour and fill are to be expected. Also, Einstein and Chien (in Brooks, 1958, p. 558) pointed out that according to the reasoning of Lane and Borland (1954), a change in bed elevation at this measuring section was "* * * a transient change of the local river bed during a flood." They further stated that "It should not be interpreted as an adjustment of river regimen in response to the change of independent variables imposed on the channel by the watershed."

A second reason for questioning the assumptions is that scour and fill may be inconsistent at two cross sections even though the flows and the suspended-sediment concentrations are about the same at both sections. The inconsistency of scour and fill at the two cableways on the Elkhorn River near Waterloo, Nebr., was clearly shown in figure 14. Similar inconsistency of scour and fill also exists between the measuring sections on the Colorado River at Lees Ferry and near Grand Canyon, Ariz., for periods of high flow in 1955, 1957, and 1959 (figs. 16–18). Not only are the patterns of scour and fill dissimilar from one measuring section to the other



EXPLANATION

Average gage height of bed—
A, at Lees Ferry
B, near Grand Canyon

Cumulative computed monthly change in average bed position from Grand Canyon to Lees Ferry—
C, based on difference between discharge of sands at Lees Ferry and Grand Canyon
D, based on discharge of sands at Grand Canyon (assumed no inflow of sand to reach)

Figure 16.—Streamflow, measured sediment concentrations, and changes of average bed position at Lees Ferry and near Grand Canyon, Ariz., April 1–August 31, 1955.

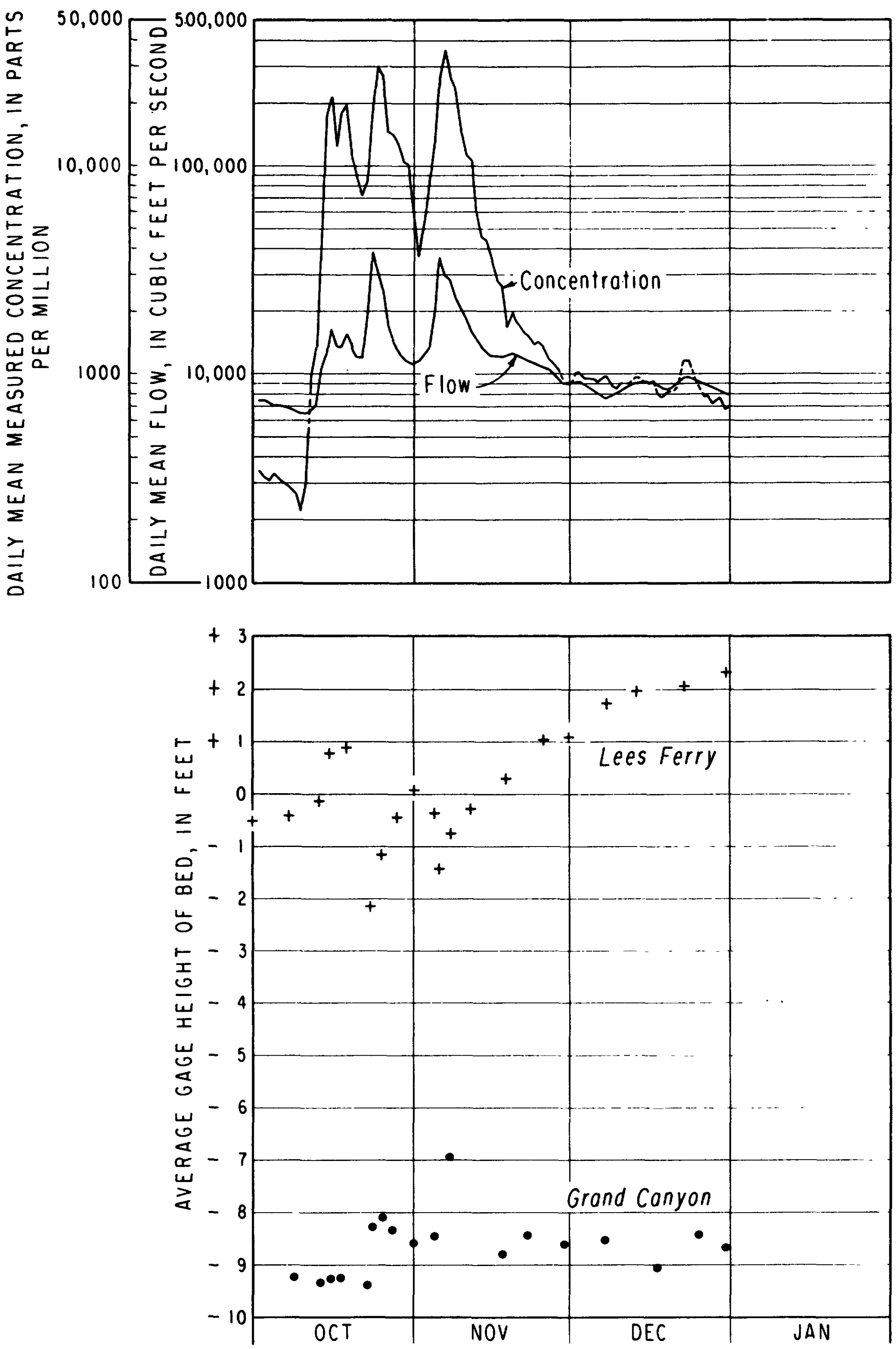


Figure 17.—Changes of average bed position at Lees Ferry and near Grand Canyon, Ariz., October–December 1957. Concentration and flow near Grand Canyon.

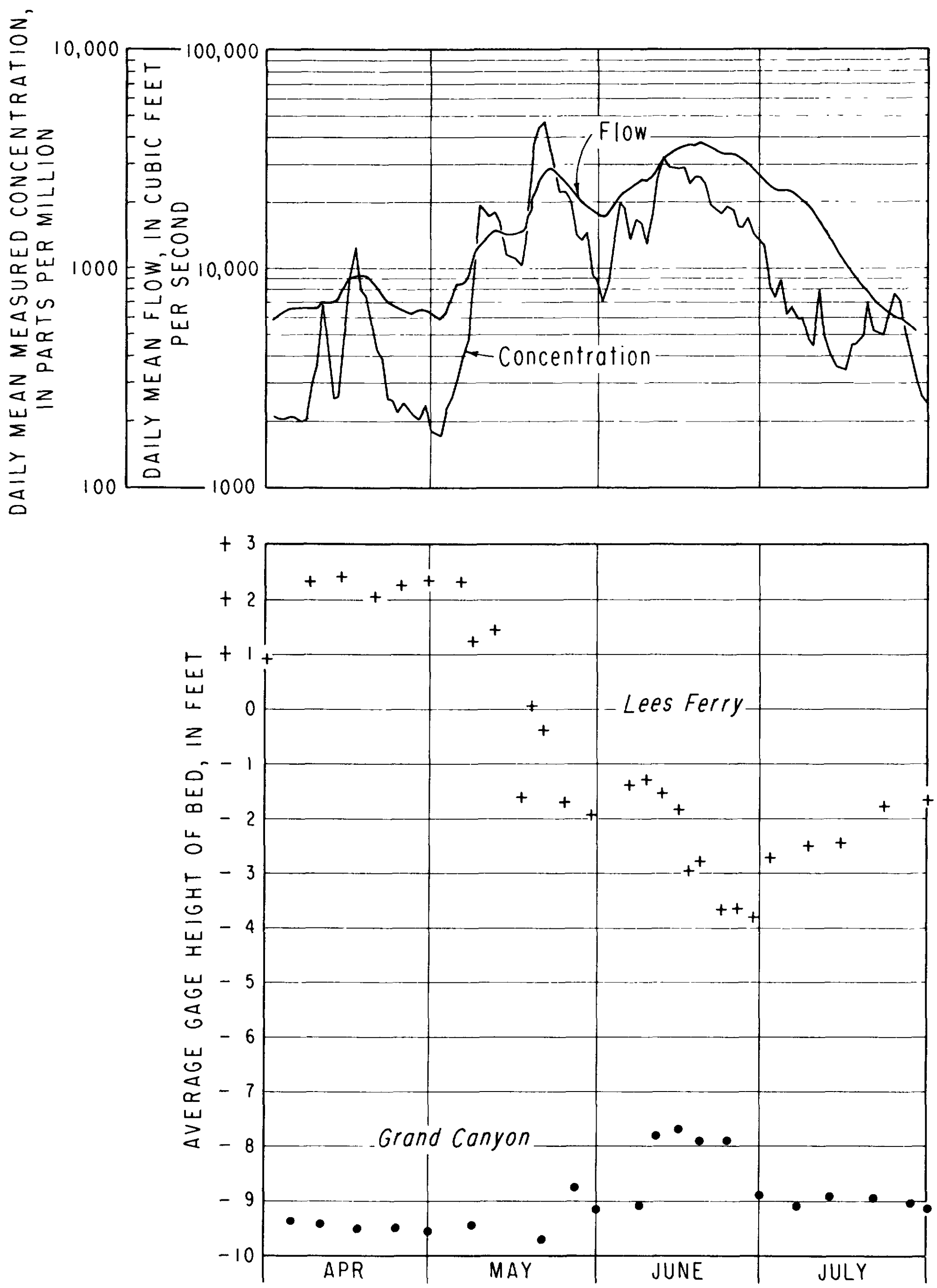


FIGURE 18.—Changes of average bed position at Lees Ferry and near Grand Canyon, Ariz., April–July 1959. Concentration and flow near Grand Canyon.

but also from one period of high flow to another at either measuring section, particularly the measuring section near Grand Canyon. The flow and the measured concentration of sediment are given (fig. 16) for both sections for the high flow in 1955, when they probably differed more than in 1957 and 1959. Obviously, neither the flow nor the measured concentration is a satisfactory guide to scour or fill at some cross sections.

A third reason for questioning the assumptions is that changes in resistance to flow in a pool usually do not cause large changes in depth of flow at a measuring section that is only a comparatively short distance upstream from a riffle. Suppose that the energy loss is 0.20 foot between a measuring section and the beginning of a fairly stable riffle downstream. This riffle is assumed to determine a stage-discharge relation that is independent of changes in roughness in the upstream pool. Now, if at a particular streamflow the Chezy C for this reach of channel between the measuring section and the beginning of the riffle was to be decreased 50 percent or to be doubled, the energy loss through the reach would be about 0.80 or 0.05 foot, respectively. The indicated change in depth at the measuring section would be about 0.60 or 0.15 foot. Even for the large assumed changes in resistance to flow through the reach, only those rather small changes in depths would be required. Experimentally, the small effect of changes in resistance to flow on depth at the Grand Canyon measuring section is shown by the agreement (Leopold and Maddock, 1953, fig. 23) between the stage-discharge relations for the same flows at different times regardless of whether the stage was rising or falling. (The gage is at the measuring section.)

The change in depth because of a given change in resistance depends on distance upstream from the riffle. If a pool extended far enough upstream along a uniform channel, the depth effect at the head of the pool would approach the depth effect in a long uniform channel. In such a channel, a change in resistance to flow along the channel results in a change in depth because the slope of the energy gradient in the uniform channel (uniform resistance is, of course, assumed along the channel) must be nearly constant at all steady flows. If the channel is wide and the width is constant as the depth changes in the channel, a decrease of 50 percent in Chezy C requires an increase in depth of about 60 percent. This adjustment of depth in a uniform sand-bed channel is the sort of effect that Brooks (1958) found in his flume investigations.

A fourth reason for questioning the assumptions, rather an obvious one even without the preceding discussion, is that any adjustment required in depth of flow by changes in roughness, flow, or energy gradient will generally appear as a difference in elevation or slope of the water surface rather than in elevation of the streambed. In other words, if the water surface is not somehow fixed in position, it can adjust readily in response to changes in flow, roughness, or energy gradient; but the streambed adjusts only as sediment accumulates or erodes. Thus, a useful general guide is that changes in bed elevation should be explained on the basis of the principles of scour and fill whereas changes in depth of flow that result mainly from changes in water-surface elevation should be explained in terms of changes in rate of flow, energy gradient, or resistance to flow.

Depth adjustments resulting from changes in water-surface elevation may be correctly called adjustments in cross section of flow but not adjustments of the stream channel. Also, in a channel of uniform roughness and cross section, the adjustment in the water surface, at least at constant flow, will usually be an adjustment in elevation rather than in water-surface slope. On the other hand, in a pool upstream from a riffle that controls the stage-discharge relation, the adjustment in the water surface will usually be mainly in slope rather than in elevation.

Briefly summarized, scour and fill at the streamflow-measuring section on the Colorado River near Grand Canyon represent local changes that are untypical of long reaches of channel; neither concentration nor flow is generally a satisfactory guide to scour and fill at the Grand Canyon or Lees Ferry sections; changes in resistance in a pool cause little depth change immediately upstream from a riffle that controls the stage-discharge relationship; and when adjustment of depth is required by change in flow, roughness, or energy gradient, the adjustment is usually in water-surface elevation rather than in bed elevation.

The variability of the relation between discharge of bed material and flow in sand-bed streams should be noted before any attempt is made to apply the principles of scour and fill to a pool-and-riffle stream such as the Colorado River between Lees Ferry and Grand Canyon. If neither the banks of a pool nor the riffle at the downstream end of the pool shifts appreciably, any major change in the average elevation of the streambed of the pool must cause an appreciable change in the mean velocity through the pool for a given rate of flow. For an abundant supply of bed material of a given particle size, even a small change in mean velocity usually causes a significant change in discharge of sands (fig. 3). Hence, the relation between discharge of bed material and flow usually changes, sometimes widely, if the mean velocity at a particular flow is changed because of scour or fill in a pool. Furthermore, the scour that might reduce the mean velocity at a particular flow is usually accompanied by an increase in particle size of the bed material in the pool and, perhaps, by a decrease in the proportion of the streambed that is covered with sand. Thus, when the elevation of the streambed is low in a pool, three factors—the velocity decrease, the particle-size increase, and the decrease of sands on the bed—tend to decrease the discharge of sands for a given flow. Of course, factors other than these also affect somewhat the relation between measured discharge of sands and flow.

The large overall effect of the difference in elevation of the streambed at the measuring section near Grand Canyon and in upstream measuring sections probably caused the large difference (figs. 16, 17) in measured discharge of sands between 1955 and the fall of 1957 for given flows (fig. 19). The long-continued high flows during the summer of 1957 scoured the bed of the pool at the measuring section near Grand Canyon and prob-

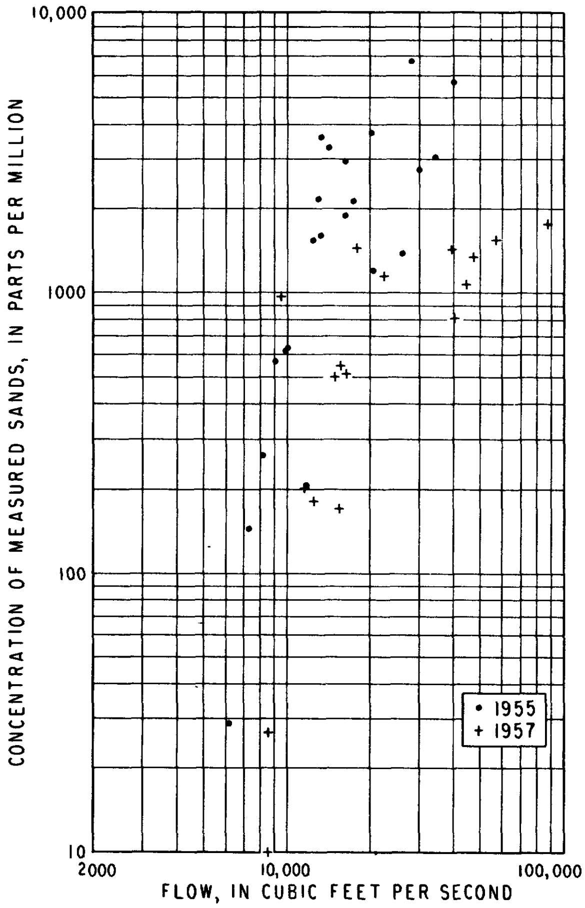


FIGURE 19.—Variable relation of concentration of measured sands to flow of the Colorado River near Grand Canyon, Ariz.

ably also scoured the beds of many pools upstream. These pools could not refill except as sediment, presumably nearly all sands, was trapped in them. Also, flows high enough to bring in appreciable quantities of sand may also be high enough to carry much of the incoming sand through the pools. Hence, during periods of high flow in the fall of 1957 and the summer of 1959, the deeply scoured pool near Grand Canyon filled very little even though the flows and the sediment concentrations were moderately high (figs. 17, 18).

APPLICATION OF PRINCIPLES OF SCOUR AND FILL

The principle of a relation between discharge of sands and characteristics of the flow and available sands applies to a pool-and-riffle stream, but the relation existing at a particular time may be impossible to determine. In such a stream, the quantity and particle size of the available sediment generally change with time, as the particle size of the bed material did in the Colorado River near Grand Canyon (U.S. Geological Survey, 1961, p. 168) in 1956, and with distance along the stream. Because of changing configuration within a short distance and perhaps because of unusually rough walls, the mean velocity is not so good a measure of discharge of sands in a pool-and-riffle stream as it is in a sand-bed stream where turbulence and mean velocity are closely related for a particular roughness and depth of flow. Hence, an equilibrium relation such as that given between discharge of sands and parameters of flow in figure 3 for sand-bed streams can seldom be established for permanent use at a section of a pool-and-riffle stream. In general, neither theoretical nor empirical relations can be used to compute the discharge of sands at many cross sections of pool-and-riffle streams; the discharge of sands at these sections often must be based at least in part on sediment sampling of the suspended sands.

In a pool-and-riffle stream the principle of continuity applies directly, the same as it does in sand-bed streams. This statement does not mean that the bed-material discharges are about the same from one cross section to

TABLE 7.—*Computed average scour or fill by months during 1955 water year in 88-mile reach from Lees Ferry to Grand Canyon, Ariz.*

[Assumed average width of reach, 400 ft; assumed weight of sands, 85 lb per cu ft]

Month	Computed total discharge of sands near Grand Canyon (1,000 tons)	Average scour in reach, based on column 2 (ft)	Computed total discharge of sands at Lees Ferry (1,000 tons)	Difference in discharge of sands from columns 4 and 2 (1,000 tons)	Average scour (−) or fill (+) in reach, based on column 5 (ft)
(1)	(2)	(3)	(4)	(5)	(6)
October	370	−0.05	700	+330	+0.04
November	22	−.003	240	+220	+.03
December	9	−.001	150	+140	+.02
January	4	−.0005	110	+110	+.01
February	6	−.0008	98	+92	+.01
March	790	−.10	1,040	+250	+.03
April	1,470	−.19	1,220	−250	−.03
May	13,500	−1.71	4,700	−8,800	−1.11
June	9,340	−1.18	4,810	−4,530	−.57
July	500	−.06	940	+440	+.06
August	660	−.08	690	+30	+.004
September	33	−.004	120	+90	+.01
Water year		−3.38			−1.50

another along the stream but only that equation 1 applies.

Continuity of bed-material discharge limits the average scour in the 88-mile reach of the Colorado River between the measuring sections at Lees Ferry and near Grand Canyon, Ariz. Daily discharges of sands were computed roughly for the 1955 water year from records of flow, measured concentration of suspended sediment, and size distributions of the measured suspended sediment for Lees Ferry and for Grand Canyon (U.S. Geological Survey, 1959, p. 135–137, 153–156). These daily discharges were added by months and were increased by 20 percent for unmeasured discharge of sands. The estimate of 20 percent was an average of seven computations of unmeasured sediment discharge for the measuring section at Grand Canyon. The seven computed unmeasured sediment discharges varied from 12 to 28 percent of the measured discharge of sands without any definite trend of the percentage with flow. If only the discharge of sands near Grand Canyon is considered—that is, no inflow of sands to the reach is assumed—the computed monthly scour (table 7, column 3) shows an average depth for the 88-mile reach of less than 0.01 foot for each of 5 months of the 1955 water year. The computed average depth of scour is 1.71 feet during May and 1.18 feet during June and totals 3.38 feet for the water year. If scour and fill in the reach are based on the difference in discharges of sands at the ends of the reach (table 7, column 6), average fill of 0.06 foot or less is shown for each of the first 6 and the last 3 months of the 1955 water year. Computed average scour totals 1.71 feet for the combined months of April, May, and June; and the net average scour for the water year is 1.50 feet. Of course, computed amounts of fill would be increased and computed amounts of scour would be decreased if the discharge of sands into the reach from the Paria and the Little Colorado Rivers and from other tributaries was known and could be included in the computations. Cumulative scour from April 1 to July 31 is shown in figure 16 and is based on the computed figures of scour from columns 3 and 6 of table 7.

The same sort of limitation that was imposed on scour and fill by continuity of bed-material discharge during the 1955 water year also applies in general. If the total discharge of sands into the 88-mile reach were known for any month or year, a limit could be established for the volume of sands that might be deposited within the reach for the month or year. If both the total discharge of sands into and out of the reach were known for any period of appreciable length, the volume of scour or fill could be reasonably closely determined for the period. Usually, such a determination would be more accurate than an estimate of volume of scour for a reach from observations of change in bed elevation at one or two cross sections along the reach.

CONCLUSIONS

Observations of scour and fill in field streams that have sand beds are frequently misinterpreted for several reasons. The position of the streambed may be defined at too few points to determine a satisfactory average elevation at a cross section or at too few cross sections to establish an accurate average bed elevation for a channel reach. Many available observations at points represent approximately the lowest positions of the bed at some unknown time during a flood. This time may have been when dunes were largest and the flow was fairly low. Also, when the bed was lowest at one point, it may have been much higher at several nearby points. Thus, many observations of scour and of subsequent fill tend to represent maximum amounts at points rather than average amounts for a cross section or an area. Also, changes in average bed elevation at narrow cross sections are almost always greater than changes at representative cross sections of channel reaches, and many streamflow-measuring sections are narrower than the average width of the nearby channel.

The sand being transported even in deep and swift flows would amount to an average depth of deposit of only a few hundredths of a foot if it could all be instantaneously deposited on the streambed.

Two principles are helpful in understanding scour and fill in sand-bed streams. One is the principle of continuity of volume of the bed material along a stream reach. This principle indicates that changes in elevation of an area of sand bed result from a difference in the rates at which sand reaches and leaves the area. The second principle is that a relation exists between the discharge of sands and the characteristics of flow and available sands.

These two principles indicate that the average bed elevation in a reach of uniform sand-bed channel is usually fairly stable during the passage of a flood because the discharge of sands into the upper end of the reach tends to be about balanced by the discharge of sands from the lower end of the reach.

A straight channel of uniform cross section may either scour or fill during rising flow because of a change in bed slope or in resistance to flow along the channel. In such a channel, scour while flow is increasing is usually followed by scour while the flow is decreasing, and fill while flow is increasing also is usually followed by fill while flow is decreasing.

Because the characteristics, especially mean velocity, of a flow are different at a narrow cross section in a sand-bed channel than at more representative cross sections, the amounts of scour and fill at a narrow cross section are unrepresentative of a channel reach. The amounts usually are larger than the average for the reach; to some extent, scour tends to accompany high or increasing flows whereas fill tends to accompany low or falling flows.

In spite of usually small and somewhat random changes in the characteristics of the streambed, the sand bed of the Elkhorn River near Waterloo, Nebr., showed general stability on the basis of many determinations of bed elevation at two cableways. At the upstream cableway, where the width was a little less than at the downstream cableway, the streambed seemed to have a slight tendency to lower during increasing flow and to rise during decreasing flow; but the streambed showed no significant tendency to change in the same direction at the downstream cableway as at the upstream cableway.

Streamflow measurements on the Missouri River near Yankton, S. Dak., showed that the streambed at the laterally constricted measuring section usually was much lower at high flow than at low flow, a pattern of bed change that characterizes many narrow cross sections in sand-bed streams.

Although the Colorado River is a pool-and-riffle stream rather than a sand-bed stream, many of its pools have sand beds. Measurements during three periods of high flow showed that the pattern of scour and fill as related to flow at Lees Ferry was different from that near Grand Canyon, Ariz., and that the pattern was inconsistent from one period to another near Grand Canyon. Changes in bed elevation at the measuring section near Grand Canyon and in upstream pools changed the relations of discharge of sands to flow and to measured sediment concentration.

Neither flow nor measured sediment concentration showed any clearcut relation to the diverse patterns of scour and fill for the Elkhorn River near Waterloo, a sand-bed stream, or for the Colorado River, a pool-and-riffle stream, at Lees Ferry or near Grand Canyon. Because bed changes at one cross section were inconsistent with those at another section on the same stream, observed scour and fill at a cross section are, at least sometimes, poor indications of scour and fill in a channel reach.

When a change in depth of flow is required because the resistance, slope, or streamflow changes, the water surface rather than the streambed adjusts to the new depth of flow. A useful general guide is that changes in bed elevation should be explained on the basis of the principles of scour and fill whereas changes in depth of flow that result mainly from changes in water-surface elevation should be explained in terms of changes in rate of flow, energy gradient, or resistance to flow.

In a pool, changes in depth of flow caused by changes in resistance to flow in the pool are usually small and decrease with nearness to a downstream riffle.

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