Trend-Surface Analysis of the Basin and Range Province, and Some Geomorphic Implications

GEOLOGICAL SURVEY PROFESSIONAL PAPER 500-D

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By LAWRENCE K. LUSTIG

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A quantitative study of Basin and Range topography which provides a basis for considerations of the nature of a physiographic province, and of the relief age of ranges, the origin of pediments, and the development of mountain drainage systems in the Western United States

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By Lawrence K. Lustig

ABSTRACT

The literature on the Basin and Range province provides many qualitative assessments of the size, shape, and arrangement of mountain ranges in the region, and it is well known that the topography is nonuniform within the province. This report represents an attempt to examine quantitatively the existing regional topographic variations. Eleven topographic parameters are used for this purpose, namely (1) area of ranges to total area, (2) range length, (3) range width, (4) range height, (5) range relief, (6) range volume, (7) cumulative length of trends, (8) cumulative deviation of trends, (9) range width to length, (10) range width to height, and (11) range length to height. Measurements of these parameters were obtained from the 46 (mapped at 1:250,000 scale) topographic quadrangles that cover the Basin and Range province.

Four analytical methods of data treatment are described—manual contouring, relative-entropy function, Fourier analysis, and trend-surface analysis. The last method is used to analyze the topographic data, and trend-surface maps of first, second, and third degree are presented for each of the 11 parameters. The results show that each topographic parameter is not uniformly distributed throughout the region but varies widely with respect to range of value on a given surface of best fit. Average values of parameters on the surfaces of best fit range from 100 percent to 1,400 percent between the southern and the northern parts of the Basin and Range region.

The analytical results indicate that three areas in the Basin and Range province exhibit sufficient topographic distinctiveness to warrant delineation. These are southwestern Arizona and southeastern California, northwestern Nevada and adjacent parts of eastern California, and the eastern part of the entire region, which consists of southeastern Arizona, southwestern New Mexico, northeastern Nevada, and northwestern Utah.

This topographic distinction is the first of four geomorphic implications that are discussed. The writer concludes that the question of whether these areas should be designated as sections within the Basin and Range province or as separate provinces in their own right involves two additional questions. The first of these is the nature of hierarchical classification systems in general and the magnitude of internal variance associated with increasing rank in the system. The second is a more specific question; namely, the magnitudes of within-province variance and between-province variance that can be considered to be average values applicable to any physiographic province. Until these values and their respective ranges are assessed by quantitative means, it cannot be determined whether the within-province variance shown to exist in the Basin and Range province is normal or excessive for any province. Further work is indicated, but the results of this report suggest that any physiographic province, or finite area of the earth's surface, should be amenable to quantitative analysis by methods similar to those presented herein.

Whether the three areas within the Basin and Range region are actually sections or separate provinces, however, the magnitudes of variance of the topographic parameters are clearly sufficient to suggest three additional geomorphic implications of a regional nature.

The first of these, or second geomorphic implication, concerns the relief ages of ranges in the Basin and Range province. The variance of the regional topographic data that is indicated by the trend-surface maps implies that the topography reflects different erosional histories of ranges and, hence, different relief ages of ranges. Relief ages are not identical with radiometric dates because radiometric dates do not generally coincide precisely with dates of orogenic activity throughout the Basin and Range region. The available geological data suggest that substantial topographic differences probably existed by the end of the Tertiary and that the nonuniform distribution of Quaternary block faulting in the region served to emphasize many of the preexisting topographic distinctions. The present regional topographic variations coincide well with the known distribution of historic earthquake activity, with the Quaternary block faulting, and with the regional distribution of Precambrian and lower Paleozoic outcrops. The areas characterized by lower values of the area of ranges to total area ratio, range width, length, height, relief, and volume, and by higher values of cumulative deviation of range trend from north, are areas which by implication include ranges that have the greatest relief ages and longest erosional histories.

The known distributions of fans and pediments in the Western United States also accord well with the topographic distinctions that are made. In general, fans are more abundant in those areas characterized by large average values of range width, length, height, relief, volume, and area of ranges to total area, whereas pediments predominate in those areas where these values diminish. It is argued that the various hypotheses of pediment formation dependent upon processes that are operative on existing pediments are not relevant because they focus upon the wrong landform. Pediments are a natural and inevitable consequence of mountain mass reduction through time, and the only real "pediment problem" is the question of how this mass reduction is accomplished. The hypothesis offered in this report is that weathering predominates on the steep mountain fronts in interfluvial areas, which are probable loci of inselberg formation, whereas fluvial processes predominate in the drainage basins which are the loci of maximum mountain-mass reduction. According to this view, pediments are simply

a product of drainage-basin evolution in the mountains through time, as controlled by local base-level constraints.

Finally, it is suggested that regional drainage distinctions that are compatible with the topographic, relief age, and fan-pediment distinctions must also exist because each is a function of time. It is argued that the numbers of drainage systems and their order numbers are a function of the size, or mass, of a given range, in the absence of constraints of shape and lithology. As the range is reduced in mass through time, the average values of these variables should diminish. Drainage systems should accord with the steady state that is predicted by stochastic models and by hydraulic principles in the older and smaller ranges of greater tectonic stability, but may not yet have achieved this state within the younger and larger ranges characterized by more recent tectonic activity. The trend-surface maps of this report provide a basis for the future examination of such drainage distinctions within the Basin and Range province.

INTRODUCTION

The area discussed in this report is commonly called the Basin and Range province of the Western United States. It extends from the Sierra Nevada on the west to the Wasatch Range on the east, and from the Mexican border on the south to approximately lat. 42° N., which is the northern boundary of California, Nevada, and Utah. The boundary cannot be defined precisely, but that shown in figure 1 approximately delineates the Basin and Range province.

The term "Basin and Range province" has appeared in a legion of geologic, geographic, hydrologic, and other reports, and the general intent of any user is widely understood. Nevertheless, some contradictions exist between the standard definition of the Basin and Range province and the facts that have been set forth or surmised by many who have worked in the region. The "Glossary of Geology and Related Sciences" (American Geological Institute, 1957, p. 26) defines "Basin and range landscape" as "Landscape consisting of fault-block mountains and intervening basins." The supplement to this volume (American Geological Institute, 1960, p. 50) expands upon this by defining any physiographic province as a "Region of similar structure and climate that has had a unified geomorphic history." By combination of definitions, then, the Basin and Range province is a region of fault-block mountains and intervening basins within which the geologic structure, climate, and geomorphic history are nearly uniform.

The fact that Fenneman (1931) suggested nearly 40 years ago that the Basin and Range province be divided into five sections clearly indicates that the nonuniform characteristics of the region have long been known. In discussing various distinctions, for example, Fenneman (1931, p. 328) stated:

> The Great Basin lies north of latitude 35°30′. In this the space taken by the mountains is about half the total * * *. South of this, in California and southwestern Arizona * * * ranges are smaller and perhaps older occupying perhaps one-fifth of the space.

In a similar vein, Lobeck stated in his geomorphology text (1939, p. 557):

> In the southern part of the Basin and Range province, notably in southern California, southern Arizona, and New Mexico, the basin ranges have been almost annihilated by erosion. The presence of faults can only be inferred. The ranges are more or less symmetrical in shape and reveal none of the diagnostic features cited as topographic evidence of block faulting.

Finally, Eardley's work on the structural geology of North America includes the following summary remarks on the southern Arizona Rockies (1962 p. 425):

> A glance at the Geologic Map of the United States will show that the ranges of southern California and Arizona and southwestern New Mexico are smaller, more irregular in shape, less linear and parallel, and separated by relatively wider basins than those of western Utah and Nevada. Hence, the inclusion of the Sonoran Desert of Arizona in the Basin and Range province from a structural point of view must be made with reservations. The crisp boundaries imparted to ranges by block faulting are generally absent, and if the region is one of extensive block faulting, then the faults are older than those in Utah and Nevada, and erosion has beaten the fault scarps back considerable distances to form broad flanking pediments.

The essential burden of the foregoing statements is that substantial differences in topography and geologic structure are thought to exist between the southern and northern portions of the Basin and Range province and, by inference, that the erosional history is, likewise, nonuniform. In addition, it has been stated by many authors that although the rocks of the region range in age from Precambrian to Quaternary, Precambrian outcrops are far more abundant in the south than elsewhere. The facts, therefore, seem to be at variance with the definition of a province, and it might be asked whether the broad similarities within the Basin and Range region outweigh the differences that exist.

One purpose of this report is to examine the topographic differences that exist within the Basin and Range province in greater detail than previously attempted. It is noteworthy that the statements of the authors quoted above each contain some generalization or qualifying remarks that were necessary because of a lack of quantitative data at the time. Fenneman (1931) stated that the southern ranges occupy perhaps one-fifth of the total area; Lobeck (1939) stated that the southern ranges are more or less symmetrical in shape; and Eardley (1962) remarked that the size and shape and the degree of parallelism of these same ranges is also based on qualitative assessment to some extent. Each of these observations, however, is amenable to quantitative examination in terms of such questions as: How much smaller are the southern ranges? Are the

FIGURE 1.—Index map of the Western United States, showing the approximate boundary of the Basin and Range province and the distribution of Precambrian and lower Paleozoic (Cambrian and Ordovician) outcrops. The province area shown in central New Mexico and the westernmost part of Texas was omitted from this study. Note the correspondence of the outcrop areas to areas of greatest relief age discussed in text and to those exhibiting distinctive values of topographic parameters on the trend-surface maps (figs. 9–41).

lengths, widths, and heights of these ranges uniformly proportional? What is the distribution of ranges, in areal percentage? Do they occupy 20 percent of the southern part of the region and 50 percent of the northern part, as surmised by Fenneman? Do the trends of ranges resemble the often-cited "army of caterpillars" crawling along a north-south meridian? The intent of this report is to answer these and other questions in quantitative terms. By such means, the regional generalizations that have been made can be more soundly based.

An additional purpose of this report is to demonstrate that surficial features can be quantified on a provincewide, or regional basis. For this reason, possible applications of analytical techniques other than trend-surface analysis are also discussed, and methods of map measurement and sources of error are described in some detail.

Further, significant topographic differences are shown to exist within the Basin and Range province and to be of a magnitude sufficient to suggest that perhaps three provinces exist, rather than one. This implication, and certain aspects of the distribution of fans and pediments, the age of ranges, and the drainage systems in these ranges are discussed in the concluding section of this report.

ACKNOWLEDGMENTS

In 1964 it occurred to the writer that a quantitative analysis of Basin and Range topography might profitably be undertaken. It was hypothesized that the results of such an analysis would be applicable to the various questions stated in the section of this report entitled "Geomorphic Implications." In addition, no previous attempt had been made to treat an entire physiographic province by an analytical method such as trend-surface analysis. At that time the writer assumed responsibility for the teaching of a course designated as "Quantitative Geomorphology" at the University of Arizona. This course, among other things, was intended to familiarize the students with the methods of analysis of topographic-map data.

Of the several ways of accomplishing this end, one of the most common is to assign a drainage basin for study by means of a standard Horton-type analysis. Other common alternatives include the quantitative description of drumlins, alluvial fans, beaches, or some other landform which has been previously studied and reported on in the literature. All such problems are useful as instructive devices, but the principal common failing is that the student virtually "knows" the answer in advance. Although there can be no substitute for actual data gathering and analysis in order to gain familiarity with the associated problems and procedures, such problems and procedures are clearly minimized if anything as well known as a Horton-type drainage analysis is used. From a purely pedagogic viewpoint, it would seem far better to set a problem that has not been undertaken previously and to which the best approach, as well as the answer, is unknown.

For this reason the writer's students were set the task of gathering the raw data on topographic parameters in the Basin and Range region. Mutual benefit was derived from the ensuing discussions of what parameters to measure and how these might best be defined. The students' many false starts proved far more instructive in the long run than any standard exercise possibly could have been, and, hopefully, these students also gained some useful insight to the joys of research by attempting the unknown.

Hence, the first acknowledgment here is to my students for their many man-hours of labor at the outset of this project and, in several instances, for their stimulating discussions of subtle points and problems related to the data gathering that could not have been foreseen in advance. Their assistance is gratefully and collectively acknowledged. Specific mention is made of the contributions by Neil Jones, Frank Anderson, and George Richardson.

The writer checked all results, compiled basic data on nine topographic parameters from several sheets that had not been assigned, computed mean values, and prepared these data for computer analysis. Because the computer system then available accepted only Fortran II programs, the standard trend-surface program available in Fortran IV language required conversion. Technical problems too numerous to cite here occurred, partly as a result of this conversion, and the assistance of Bart Cross was of inestimable value in obtaining satisfactory results. Computer time for the trend-surface analysis was generously provided by the University of Arizona and its Numerical Analysis Laboratory.

Several previously published illustrations germane to this report have been reproduced, with some slight modifications. Permission to do so was kindly granted by the several publishers involved. Acknowledgment is extended here to the Massachusetts Institute of Technology Press for the use of figures 3 and 4 in this report, to the Columbia University Press for figure 6, to the Kansas Geological Survey for figure 7, to the Geological Society of America for figures 43 and 44, and to the Princeton University Press for figure 45.

Because of the somewhat varied nature of the topics treated in this report, the advice and opinions of several reviewers were sought. John Harbaugh, Thomas Maddock, Jr., Mark Melton, Richard Chorley, Robert Sharp, Stanley Schumm, James Gilluly, and William Bull each contributed helpful comments and suggestions on the

manuscript. Additionally, Paul Damon provided an enlightening discussion of the chronology of events in the Basin and Range province, and, as previously stated, Bart Cross kindly resolved several computer-programing problems. The writer is extremely grateful to each of these individuals for the aid rendered. As often happens when opinions are sought from several quarters, however, some conflicting views were obtained. For this reason the writer necessarily accepts sole responsibility for the views set forth in this paper.

STUDY AREA

As previously stated, the Basin and Range province extends from the Sierra Nevada on the west to the Wasatch Range on the east and from the Mexican border on the south to the northern boundaries of California, Nevada, and Utah, approximately, as shown in figure 1. The locations of the topographic quadrangles of the study area are shown in figure 2. The total number of maps represented is 46, but because the boundaries of the Basin and Range region do not, of course, coincide

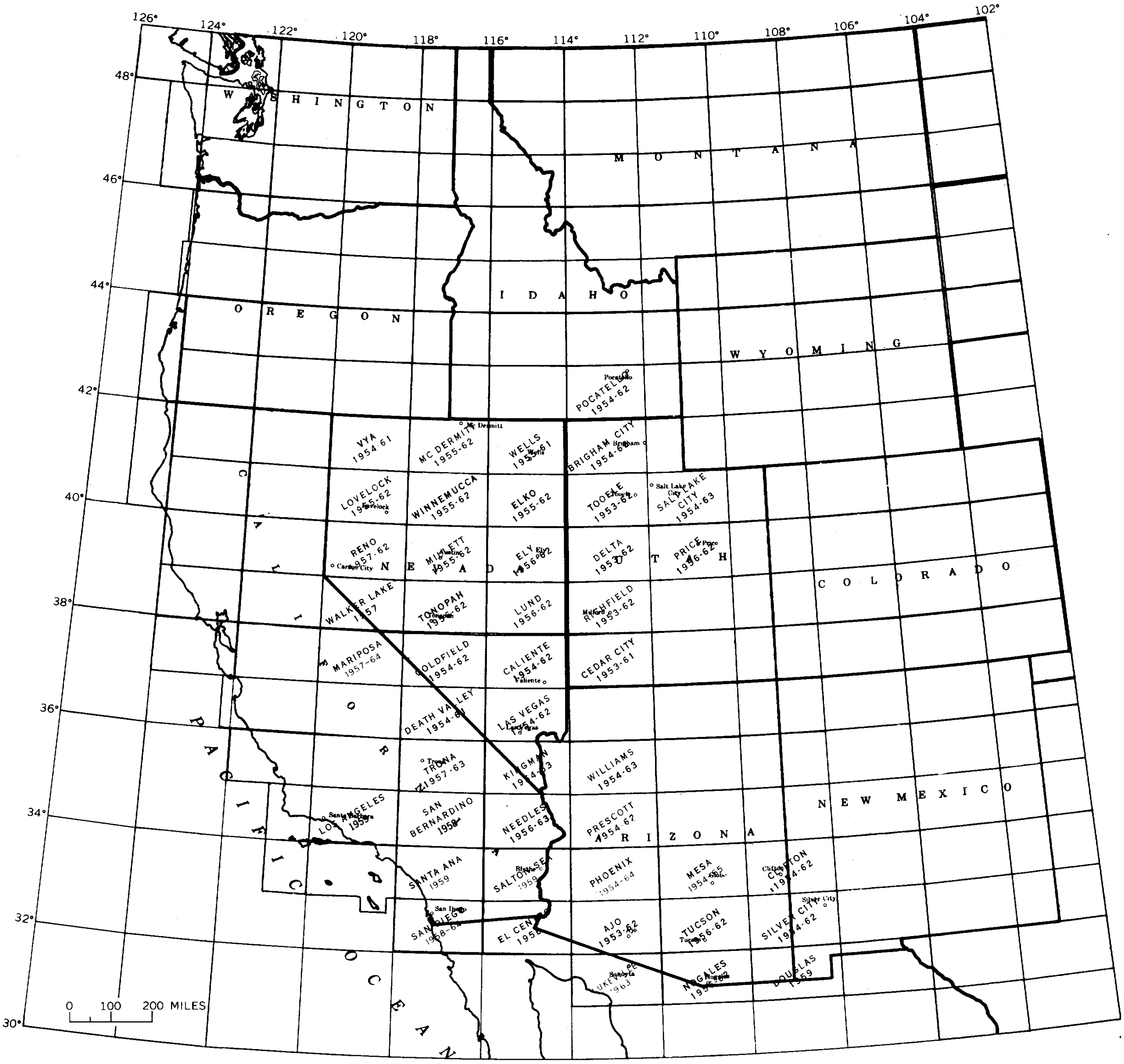

Figure 2.—Index map of the Western United States, showing the locations of the topographic quadrangles, the maps of which were used for this study. Mean values of all topographic parameters were plotted at the centers of the indicated quadrangle maps. These served as an orthogonal data grid for the trend-surface maps (figs. 9–41).

precisely with the quadrangle boundaries, certain variables were analyzed using only 43 or 44 of the maps. Only one of the topographic quadrangle maps, namely that of Pocatello, Idaho (fig. 2) represents an area north of lat. 42° N.

QUANTITATIVE METHODS

The quantitative methods used in this study were of two types: those uesd in the actual assemblage of raw data from the topographic maps, and those used in the treatment of these data by analytical mapping procedures. Because the methods of study and the degree of reproducibility of results are obviously central to this report, these twin concerns are discussed in the sections entitled "Measurements," and "Analytical Techniques," respectively.

MEASUREMENTS

The first problem that arises in quantitative map study is what to measure. The parameters or variables selected must not only be measurable on maps of the scale selected but must be equally significant to the study under consideration. In this study the essential problem was to determine whether significant topographic differences exist in the Basin and Range region. Hence, the following parameters were selected for investigation: (1) Area of ranges to total area, (2) range length, (3) range width, (4) range height, (5) range relief, (6) range volume, (7) range trend, and three derived ratios, namely (8) range width to length, (9) range width to height, and (10) range length to height. Each parameter reflects a topographic characteristic and is therefore relevant to the study. Subsequently, it became desirable to apply two parameters for the description of range trend, and a total of 11 variables were ultimately examined.

The second problem encountered in any such study is how to measure the selected parameters. This is far less straight forward than it might appear to the uninitiated. Each variable must be defined, and criteria must be established for its measurement. Regardless of the effort expended on these initial considerations, one often finds that seemingly satisfactory criteria cannot be applied with reproducible results once the work has begun. For this reason, the measurement procedures are partly trial and error by nature. This will be made clear in the following discussion of the criteria and definitions used in this study.

CRITERIA AND DEFINITIONS

Area of ranges to total area

Three criteria were required to define the first topographic parameter, namely range area, in order to measure ratios of range area to total area. The ranges are, of course, represented by topographic-contour closures on the maps, and their locations can be seen at a glance. A slope criterion must arbitrarily be chosen at the outset, however, to provide a range boundary and to thus permit measurement of a given range area. Regardless of the slope value chosen, one is still confronted with the risk of including too great or too small a proportion of actual range in the nonrange area, and with the decision as to which is preferable. For example, if the topographic quadrangle maps of the southern part of the Basin and Range region are examined, one would be inclined to choose a slope value of about 200 feet per mile to define the range boundaries. However, the slopes of some alluvial fans in the northern and western parts of the Basin and Range region are as much as 800 feet per mile, and in such areas the application of a 200 feet per mile criterion would cause much basin area to be incorporated in the measured-range areas. Conversely, the application of 800 feet per mile in the southern areas would entirely eliminate some known bedrock outcrops of much gentler overall slope and thus yield too small a value for total range area. A compromise between these extremes was necessary, for the use of variable slope values would involve much subjectivity; no criteria could be devised that would permit reproducibility. Thus, for purposes of this study, a value of 500 feet per mile was chosen. A given range is therefore defined as an area including slopes that are equal to or greater than 500 feet per mile. It is not argued here that this slope criterion is everywhere the most satisfactory, and it should be noted that the use of another value would alter the range percentages given in this report.

That this single criterion for the measurement of range area was necessary but not sufficient soon became apparent. In the southern part of the study area, for example, many of the ranges are divided into segments that are separated by passes or topographic lows. The application of the definition of a range given above to these areas would require that each such segment be considered a separate range. Because mean range area was of some concern, and because the establishment of range area affects the determined values of range width, length, and other variables to be difined later, a second criterion was necessary to avoid representing a single range of A square miles by three ranges with an average area of approximately $A/3$ square miles, for example. For these reasons, the definition of a given range was expanded by application of a second criterion, namely that a range include all apparent outcrops or topographic highs with slopes that are equal to or greater than 500 feet per mile and that are separated by no more than 2 miles. Note again that the value chosen for the separation distance is arbitrary, and the choice of another value would alter the magnitudes of the reported values of range area and other variables in certain places.

Finally, it was necessary to apply a third criterion because of the occurrence of isolated knobs and hills of small size. Many volcanic plugs, for example, are represented by topographic-contour closures around areas of perhaps 1–2 square miles. Application of the two criteria for the definition of a range discussed above would require that each of these small features be measured and computed as a separate range. Again, because mean values of several variables were of concern, the inclusion of these small features would clearly produce erroneous results. The final criterion adopted to prevent this was that any range be greater than 4 square miles in area.

These three criteria were applied to define and measure range areas on each of 44 topographic quadrangle maps. The total area of each map was then determined, and the area of ranges to total area, or percentage of range area, was calculated. All measurements of area were made with a compensating polar planimeter.

Range length

The length of a given range was initially defined as the distance between the points of intersection of the range area boundary and the longest contained straight line that could be constructed. This definition had the virtue of reproducibility, and, in addition, the trends of ranges could have been determined objectively by simply measuring the azimuth of a given length line. Unfortunately, the population of ranges that occurs in the Basin and Range region will not accommodate such a definition. Certain ranges mapped in the northern part of the region are amoebalike in configuration and extend across parts of several 1:250,000 scale topographic guadrangle maps. Even in the southern part of the region, horn-shaped ranges are not too uncommon, and their occurrence precludes the possibility of representing range length meaningfully by the use of straight lines. The definition used in this study, therefore, was that the length of a given range is equal to the length of a line constructed along the midpoints of a range, as determined by the range area boundary.

This definition does not permit precise reproducibility unless the number of midpoints and their spacing is specified. The degree of error thus introduced is negligible, however, compared with other factors to be later discussed in "Sources of Error."

Range width

In this report, the width of a given range is defined as the area of that range divided by its length, as the area and length are defined above. This definition, like all, is admittedly arbitrary and can only be considered to be an approximation of true range width. If one considers a range that is circular in plan view, the maximum error involved in so measuring range width can readily be calculated. The area of a circle A is equal to πr^2, where r is radius, or $\pi D^2/4$, where D is diameter. According to the definition adopted here, however, range length L would equal D; therefore $A/L=\pi D^2/4L=\pi L/4$, which would be the computed value of range width W. For a circle, however, width should equal length and it is obvious that $W\neq\pi L/4$; the magnitude of the error is equal to $1-\pi/4$, or about 21 percent, for maximum width. If average width of the circle is considered, then no error exists. Most ranges in the Basin and Range region are elongate rather than circular, and because many of them taper toward their ends and exhibit other irregularities, any alternate definition of range width will present some difficulties of application and will be equally arbitrary. The definition adopted here is simple to apply and yields results that are both reproducible and fairly consistent throughout the study area.

Range height

The height of a given range, as defined in this report, is the difference in elevation between the base of a range and its crest area. The use of this definition requires, in turn, the establishment of two criteria to permit measurement of base- and crest-area elevations. The base of a range will, in most places, occur at different elevations, and an average of these must be obtained. The mean base elevation was arbitrarily defined as the means of the elevations of (1) the two points of intersection of the range length line and the range area boundary, and (2) the eight points of intersection of four lines that are perpendicular to the range length line and that divide it into equal segments with the range area boundary. Alternatively, the mean elevation of the base, or the range area boundary, can be determined by averaging the elevations of a large number of points, separated by fixed intervals, along its perimeter. But this would have been more time consuming, and in several tests the means of the 10 base-elevation points defined above were found to closely approximate the alternative values.

The elevation of the crest area of a given range was determined from the means of (1) the elevation of the highest peak, and (2) the elevations of points spaced 1 mile apart along the divide, on either side of the highest peak. In practice, considerable difficulty was met in attempting to strictly follow this, or any alternate, definition. Specification of elevation determinations at fixed intervals along the divides of ranges causes mean values for small ranges to be based on fewer points than is true of large ranges. Moreover, in many ranges the highest peak occurs near one end of the range rather than in the middle, and this prevents determination of values on both sides of the peak. Range height is, in fact, rather difficult to define satisfactorily in any manner. It seemed clear to the writer that any such definition should include the elevation difference in the zone occupied by the highest peak. Because this was accomplished, the range height comparisons to be later presented are deemed meaningful, but as with all the parameters discussed thus far, the use of another definition would undoubtedly alter the absolute value of the height of any given range.

Range relief

Range heights, in addition to being difficult to define adequately, do not correspond very well to range relief because the base elevations of ranges are quite disparate from average basin elevations. Hence, a separate parameter was defined to express range relief. It was defined as the difference in elevation between the crest height of a given range and the mean value of the lowest elevations in the two adjacent basins that can be obtained along two lines that intersect, and are perpendicular to, the range length line. This definition serves to restrict relief values to the immediate vicinity of the range, but these values are not identical with relief as it is commonly defined. The parameter is easily measurable, however, and provides a meaningful basis for comparison within the Basin and Range region.

Range volume

Range volume is a characteristic of considerable significance in light of one of the theses of this report, namely that the ranges in the study area have been subjected to periods of erosion of different durations. The volume of a mountain block will, of course, bear some relationship to its areal extent, but the relationship need not be everywhere the same. For this reason, an independent measure of range volume was sought.

Volume is an exceedingly difficult topographic parameter to measure, and the approach used for purposes of this report is based upon a geometric simplification. If one considers a mountain block to roughly approximate a triangle in cross section, the area of a given cross section is equal to twice that of a right triangle, or hb, where h is height and b is the base of the right triangle. The area of such cross a section is therefore approximated by the product of range height and one-half the range width, as these measures previously were discussed. To obtain range volume, such cross sections must be cumulated along the length of a given range, and for this reason the approximation used for range volume in this report is ½ (range width × range height × range length). Although this would seem a fairly reasonable procedure, even granting that many ranges may deviate from the ideal, regular, triangular-solid model described here, one significant fact should be noted. Because range width was defined as range area divided by range length, the width-height-length product is actually equal to ½ (range area × range height). Thus, the range volume parameter is not truly independent of range area, and it involves range height which, as previously indicated, presented considerable measurement problems. The volume data given in this report are therefore considered to be valid only as first approximations.

Range trend

Range trend, as used in this report, refers to straight-line escarpments that are not less than 1 inch (equalling approximately 4 miles) on the 1:250,000 scale topographic quadrangle maps used. Aside from this minimum length, the only additional criterion imposed was that the deviation of a given escarpment from the superimposed straight line not exceed one-tenth of an inch on the map. The bearings of each such escarpment trend were measured and weighted in accord with their length; a 2-inch trend length is weighted twice as heavily as a 1-inch trend length, for example.

The results were cumulated according to total lengths of trends and total deviations of trends from north for each of the 46 1:250,000 topographic quadrangle maps. Although the trends are measured from maps and the term "escarpment" is used above, it seems probable to the writer that these measures provide an adequate representation of large-scale structural trends throughout the Basin and Range province.

Derived ratios

Three derived ratios, involving the lengths, heights, and widths of ranges were computed for comparative purposes in this report. They are range width to length, range width to height, and range length to height. Each ratio contains, of course, the uncertainties and approximations that are involved in measurement of its respective components. The principal merit of the derived ratios is that they provide information on the manner in which the respective components vary together throughout the region.

SOURCES OF ERROR

Four principal sources of error are involved in the assemblage of raw data. The first source is inherent in the measurement of the topographic parameters chosen. As indicated previously certain of the variables used are difficult to measure with consistency, are only crude approximations of the topographic property sought, or involve arbitrary criteria which directly affect the absolute magnitudes of the data obtained.

The second general source of error is operator error, which undoubtedly comprises some component of the total variance. Although this is true of nearly all such studies, in this study the magnitude of the operator-error factor may be greater because several individuals were involved in the initial compilation of map data. No attempt was made to determine the effects of this source of error.

The third source of error arises from the fact that a few of the 1:250,000 topographic quadrangle maps used are part of an older map series on which the contour interval is as much as 500 feet. This tended to produce inaccuracies both in range delineation and in all those parameters which are dependent upon range boundaries for their measurement. Fortunately, these maps were few in number and of somewhat scattered locations. For the latter reason, it was possible to correct several of the boundaries of ranges on maps which, in part, extended onto newer maps with a smaller contour interval. Nevertheless, some subjectivity was involved and should be noted as a possible source of error. The locations of the older maps generally coincide with the southernmost part of the study area.

The fourth and last source of error arises from the areal-plotting procedures that were used—this is, the geographic locations at which the data on each variable were plotted prior to areal mapping. Ideally, each attribute of a given range would have been plotted at the centroid of that range in order to map its areal distribution throughout the region. This could not be accomplished, however, because suitable base maps are lacking; the various small-scale special-purpose maps of the Western States do not adequately reflect the topography that is apparent on the 1:250,000 topographic quadrangle maps used in this study. The best alternative, of course, would have been to plot the data at the centroids of ranges on the 46 1:250,000 topographic quadrangle maps and then have them all reduced to a single scale-stable base map of suitable dimensions. Time and cost considerations mitigated against this choice, and the data were therefore plotted as mean values for each map on an orthogonal grid that corresponds to the coordinates of the centers of the maps. This simplified plotting procedure led to nonuniform results for the 11 variables considered.

No particular problem arises with respect to the first parameter, namely area of ranges to total area. One would need to choose some unit area to compute this ratio in any case, and the procedure used simply means that the area of a 1:250,000 topographic quadrangle map is that unit area. On each map the ranges present were outlined by application of the range-boundary criteria previously discussed, the areas of these ranges were measured, and the total range area present was divided by the total area of that map. The latter values vary slightly with latitude, and the total area of each map was computed separately. The area of ranges to total area parameter cannot, therefore, even be considered as resulting from an averaging procedure on each map; the unit area is the map area, and the value of the parameter could not be plotted in the range locations.

The two variables used to reflect range trend—namely cumulative length and cumulative deviation from north—could have been plotted at the centroids of the ranges measured, however. For these variables, plotting the mean values at the center of each map does represent an averaging procedure over the areas involved.

The chief sources of error with respect to plotting, however, arise from the application of this procedure to values of the other parameters, namely range width, length, and height, the derived ratios of these variables, and range volume and relief. Each of these is a property associated with a given range and because several of the ranges extend across map boundaries, a problem of considerable magnitude occurs. If one is concerned with the area of ranges to total area on a given map, and a given range extends onto another map, then one simply measures the range area present on each map and assigns the proper values to the respective maps. If one is concerned with a parameter such as range length, however, then it is obviously not valid to assign portions of the total length of a given range to the maps on which those parts occur. This would result in an overall reduction of range length wherever this situation occurred. The most reasonable solution to this problem, with respect to range length and the other individual range attributes cited, was to assign the value of a given range variable to that map on which the largest part of the range was shown. Then, values were averaged for each map, and the mean value for each variable was plotted at the central position of each map as before.

The procedure cited obviously leads to some errors, but these are not as great as one might suspect at first. Most of the very large ranges which do extend over several maps are located in the northern part of the Basin and Range region. Commonly, in this part of the study area a range may trend roughly north-south across three 1:250,000 topographic quadrangle maps, and the ends of it curve somewhat and thus occupy parts of two other maps. By use of the procedure cited above, the value of range length, width, or other attribute would be assigned to the central map of the three maps showing the north-south extent of that range (if the largest part of the range area were shown on that map) and the value would be incorporated into the mean value plotted for the map. Two facts should be noted as to the possible errors involved. First, if the centroids of ranges had been used for plotting locations, then almost every centroid location would occur somewhere within the boundaries of the map employed for plotting according to the procedure used in this report. In general, the maximum error in location is the distance from the center of a given map to one of its corners. And because these location errors are not all in the same direction across the region, they are partly compensatory; the average location error is thought to be much less than this distance. The second fact is that more than one of these very large ranges cannot, in general, fall upon or occupy the same map by reason of space limitations. For this reason also, cumulation of the location errors does not occur. For example, if the attributes of a very large range are assigned to a single map, the values of length, width, height, volume, relief, and the derived ratios will, in general, all be high for that map. But this results because the magnitudes of these values are great and not because of the plotting procedures. The main point is that the locations of the various regional highs and lows discussed and later shown in this report do not coincide precisely with geographic locations. They may be displaced by some fraction of one-half the diagonal distance of a 1:250,000 topographic quadrangle map.

Finally, some scale distortion is involved in the various trend maps presented later. The orthogonal grid used to plot mean values for the centers of the topographic maps consisted of points equally spaced in both coordinate directions. This procedure necessarily results in map elongation in a north-south direction because the topographic maps are essentially rectangles, rather than squares, and their long dimensions are oriented east-west. The geographic locations of selected topographic maps are shown on the various trend maps to avoid the conceptual difficulty that this scale-distortion problem may present to the reader.

ANALYTICAL TECHNIQUES

Once the raw data have been obtained for a regional study by measurements, such as those previously described, some analytical technique must be selected to depict their areal distribution. The choice of technique is to some extent arbitrary and depends upon the assumptions made by the investigator, for a given set of data often can be represented by more than one, unique mathematical function. As an example, one might consider a set of data which represents the relation between stream discharge and channel slope in a downstream direction. In the absence of previous knowledge of the nature of this relationship, a given investigator might attempt to plot these two variables on arithmetic, semilogarithmic, and double-logarithmic graph paper, successively. He would then choose, in all probability, that graph which provided the best fit, calculate the regression expression for this graph, and present the relation between stream discharge and channel slope in terms of a linear equation, an exponential equation, or a power function, as might be appropriate. In reality, however, it is entirely possible that some other mathematical function more closely reflects the true relationship involved. The equation of the cycloid, for example, has never been used to represent the longitudinal profiles of river channels, despite the fact that it is the solution to the Bernoulli problem in the calculus of variations of the least travel-time between two points. Hence, a profile that corresponds to a cycloid path or some portion thereof might reflect the least-work principle (Leapold and Langbein, 1962) that is operative in river systems under certain conditions. A given investigator therefore assumes the nature of the relationship between two variables by the very act of choosing the type of graph paper on which the data are plotted. In effect, he ultimately determines whether the relationship can be approximated by the mathematical function involved but not necessarily whether his choice of that function is the best among several alternative possibilities.

The same reasoning that applied to two-dimensional graphic representation of data also applies to regional mapping, which is a three-dimensional problem. For this reason, four possible analytical techniques, each of which involves mapping, are described and discussed below according to their applicability to the topographic data on the Basin and Range region. These techniques are (1) simple manual contouring, (2) the relative-entropy function, (3) Fourier analysis, and (4) trend-surface analysis.

MANUAL CONTOURING

Manual contouring is a time-honored method of depicting the areal distribution of a set of variables and is the simplest of the four methods discussed here. Because mean values of each of the 11 variables used in this study were plotted at the centers of the appropriate 1:250,000 topographic quadrangle maps of the region (fig. 2), an orthogonal array of data values was available for each of these variables. Manual contouring of any such set of derived data values requires only the choice of an appropriate contour interval and the construction of contour lines of equal value that are plotted by interpolation between adjacent points in the data array. Every reader of this report is familiar with this method but the subjectivity involved is not, perhaps, often specifically cited.

The simplest presentation would consist of an orthogonal array of data values in which each particular horizontal or vertical line of points had the same value and was adjacent to another line of points of different value, the difference in value being equal to the selected contour interval. No interpolation between adjacent data points would be required, and the resulting contour map would consist solely of a series of equally spaced horizontal or vertical isopleths. The important point is that no subjectivity would be involved in the construction of the map; thus, maps produced by any number of investigators would necessarily be identical.

The hypothetical ideal cited above seldom, if ever, actually occurs, however. The topographic data from the study reported here reflect considerable variation of each parameter across the entire study region. The construction of isopleths from such data cannot be objective because the map produced by a given investigator is not precisely reproducible. In geophysical or other studies in which the location of specific high or low values is of importance, the tendency for an investigator to produce as many contour closures as possible is inherent. If, on the other hand, the basic goal of the investigation is to determine regional trends or gradients, the opposite tendency will exist, namely to produce the minimum number of closures from the same set of data. Subconsciously or otherwise, two investigators can easily produce startlingly different maps from the same set of data by manual-contouring methods.

The use of moving averages does not really eliminate this difficulty because it essentially represents an attempt to reduce one of the components of the total variance, namely local variance, and thus emphasizes regional trends or gradients. This is particularly true if successive moving averages are taken from the original data values. Regardless of the averaging scheme employed, the isopleth maps produced will always reflect some additional degree of subjectivity that is a function of map construction. For this reason, such maps were not constructed and employed for this study, despite the fact that discrimination of regional trends was an essential goal.

RELATIVE-ENTROPY FUNCTION

The mapping of variables by the manual-contouring method can be described as a univariate method because only the areal distribution of single variables, or perhaps some simple ratios, can be reflected by isopleth maps. In the study reported here, for example each of the 11 variables can be investigated on separate distribution maps of a given type; these would be classified as univariate maps. If, however, one wishes to treat the variables as parts of a multicomponent system, use of the relative-entropy function can, in certain problems, provide a quantitative and analytical technique.

In thermodynamics, the entropy of a system is a measure of the free-energy variation, and this concept has been applied in a general way to open and closed geomorphic systems by Chorley (1962). An alternative consideration of entropy in terms of the summation of the logarithms of probabilities of the states of a system has been used by Leopold and Langbein (1962). This usage, dervied from statistical mechanics, is very close to the significance of entropy in the relative-entropy function, but a better understanding can be derived from information theory.

As shown by Raisbeck (1963), if there is some information source, such as a transmitter or an experiment, which can yield n equally likely outcomes or messages, then the information associated with any given message, n, is log n. The reason for this can be seen if one considers a simple model (fig. 3) in which an information source, A, consists of two independent sources, B and C. If the possible outcomes or messages from B and C are n_B and n_C, and if these are equally likely, then the total number of possible outcomes, n_A, is equal to the product of n_B and n_C. The information transmitted by source A, however, must equal that transmitted by the sum of sources B and C. Therefore,

$$f(n_A)=f(n_B)+f(n_C), \text{ and} \quad (1)$$
$$n_A=n_Bn_C. \quad (2)$$

One solution, then, is

$$f(n_A)=d\ log\ n_A, \quad (3)$$

where d is some constant.

With this fact in mind, if one next considers a model in which the outcomes are not equally likely, the relationship of entropy to information content becomes clear. If an information source (fig. 4) generates n equally likely outcomes which can be segregated into two unequal groups, n_A *and* n_B, then if we ask whether a given output or message will be in group n_A or group

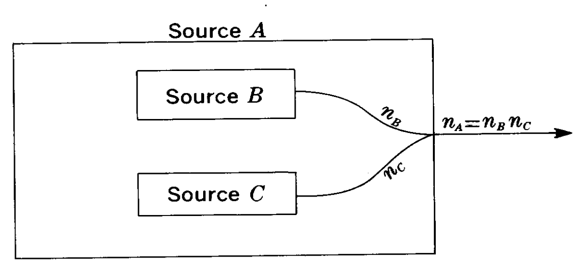

FIGURE 3.—Diagrammatic model of information source A that is composed of sources B and C, which yield equally likely outcomes n_B and n_C, respectively. (After Raisbeck, 1963, fig. 1.3.)

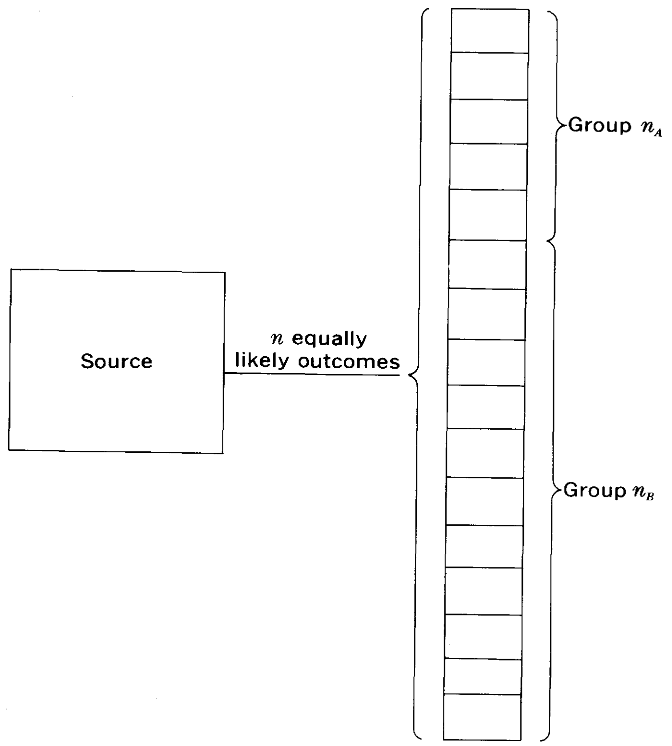

FIGURE 4.—Diagrammatic model of an information source that generates two groups of outcomes, n_A and n_B, which are not equally likely. (After Raisbeck, 1963, fig. 1.4.)

n_B, the probabilities of unequal outcomes must be investigated. The probability that a given message is within group n_A *is*

$$p_A = \frac{n_A}{n_A + n_B}, \quad (4)$$

and the probability that it is within group n_B is

$$p_B = \frac{n_B}{n_A + n_B}. \quad (5)$$

These expressions are simply statements of the fundamental definition of probability—that the probability of the occurrence of any given event is equal to the number of outcomes that will produce that event divided by the total number of possible outcomes. The information associated with any of the equally likely messages within group n_A is log n_A, and that associated with the messages within group n_B is log n_B. This information, however, can only be transmitted during a part of the total time n, namely during time n_A/n for a message in group n_A, and time n_B/n for a message in group n_B. Therefore, if information is defined as H, then

$$H = \log n - \frac{n_A}{n} \log n_A - \frac{n_B}{n} \log n_B. \quad (6)$$

Because n can be taken as unity, and because n_A/n and n_B/n are equal to the probabilities p_A and p_B, respectively,

$$H = -p_A \log p_A - p_B \log p_B, \quad (7)$$

or, in the most general form,

$$H = \sum_{i=1}^{N} -p_i \log p_i. \quad (8)$$

This expression, relating the information content of a source or system to the sum of the products of the probabilities and the logarithms of the probabilities of the components of that system, is essentially identical to the statistical definition of entropy and to the relative entropy function. The latter, as defined by Pelto (1954), is

$$100H_r = -\frac{100\sum p_i \log p_i}{H_m}, \quad (9)$$

where H_r is relative entropy, H_m is maximum entropy, and the constant 100 is simply inserted to obtain percentage results directly. For a system that consists of any finite number of components, one need only convert the values of each of these components to percentages and substitute these percentages for probabilities to obtain the $p_i \log p_i$ products in the numerator of the relative-entropy function. Maximum entropy, H_m, is a function of the number of components in the system and occurs when the probabilities $p_1, p_2, p_3, \ldots, p_n$ are equal. For a three-component system, for example,

$$H_m = -3(0.33 \log 0.33) = 1.0986. \quad (10)$$

The applicability of the relative-entropy function to three-component facies mapping has been illustrated by Forgotson (1960) and a more general discussion has also been provided by Miller and Kahn (1962). The latter applied the relative-entropy function to mapping of an eight-component foraminiferal fauna and successfully revealed certain aspects of the areal distribution of facies that were not apparent from more conventional maps. Additional illustrations of the uses of entropy-function mapping are not abundant in the literature, and to the writer's knowledge no attempt has been made to apply the technique to geomorphic problems. Nevertheless, it represents a potentially useful mapping technique and was considered as such during the conduct of the study reported here.

If one wished to consider such topographic characteristics as the lengths, widths, and heights of ranges as components within systems of ranges, it would be entirely possible to convert the absolute values of these variables to percentages, compute relative-entropy values for individual ranges or topographic maps, and produce a relative-entropy map. It is noteworthy, however, that in the context of information theory, entropy refers to the degree of intermixing of the components of a given system. That is, maximum entropy is equivalent to minimum information, and this occurs either when the degree of intermixing of end members is greatest or when the probabilities associated with each of the components are equal. Conversely, maximum information is gained when entropy is a minimum, which occurs when all of the components or end members save one are equal to zero or are absent. Therefore, the relative-entropy function is a most efficient quantitative mapping tool when treating a set of components or variables, each of which ranges from 0 to 100 percent at some point within the system, or within the region studied. The components need not, of course, reflect the entire range from 0 to 100 percent, but a fairly wide range of values must be encompassed. Figure 5 shows the entropy distribution for a three-component system. High entropy values dominate the central part of this triangular representation of the relative-entropy distribution. For maximum information and low entropy values, two of the three components must tend toward zero; otherwise a corner of the total field will not be approached.

The topographic variables that are discussed in this report do not lend themselves to the most efficient usage because any combination of components tends to occupy high or moderate values of the relative-entropy distribution field. Like isopleth mapping, relative-entropy mapping was also considered and discarded in search of a means to extract the maximum information from the data by quantitative analytical techniques.

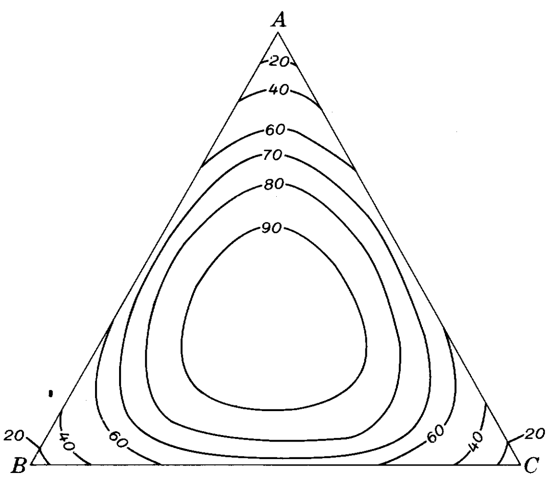

FIGURE 5.—Distribution of relative-entropy values for a three-component system. Minimum information is associated with maximum entropy values, at center of the field where the three components are equally abundant. Maximum information is associated with minimum entropy values at each of the three vertices.

FOURIER ANALYSIS

Fourier analysis is another potentially useful quantitative mapping technique which is most applicable to regional data containing oscillatory characteristics. This is most easily understood from consideration of the two-dimentional graph—that is, the graph of any function in an x, y coordinate system.

First, it should be recalled that specific functions can be approximated by expansion of terms in a Taylor or Maclaurin series. For example,

$$\frac{1}{1+x}=1-x+x^2-x^3 \ldots \text{, or} \qquad (11)$$

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots . \qquad (12)$$

In general, if $y_1=a_0+a_1x$, $y_2=a_2x^2$, $y_3=a_3x^3$, and $y_n=a_nx^n$, then the Taylor expansion of $y(x)$ is

$$y(x)=y_1+y_2+y_3+\ldots+y_n. \qquad (13)$$

This means that the graph of a function $y(x)$ can be approximated by the graphs of y_1, which is a straight line, y_2, which is a quadratic parabola, y_3, which is a cubic parabola, and other functions of successively higher order. The greater the number of terms that are considered the better the approximation to the original function, $y(x)$.

If the function is periodic, however, then it may best be approximated by a series of terms, each of which is also periodic. This was first shown by Fourier in 1807, who assumed that a function, $y=f(x)$, could be approximated by an expansion into sine and cosine terms, namely

$$f(x)=\frac{1}{2}a_0+a_1\cos x+a_2\cos 2x+a_3\cos 3x\ldots +b_1\sin x+b_2\sin 2x+b_3\sin 3x+\ldots, \qquad (14)$$

or that,

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx. \qquad (15)$$

The terms in this expansion of $f(x)$ therefore make n complete oscillations in the interval $-\pi$ to $+\pi$ and the problem involved is finding the values of a_0 and the coefficients $a_1, a_2, a_3, \ldots, a_n$, and $b_1, b_2, b_3, \ldots, b_n$. The problem is simplified by certain properties of the Fourier series which are termed "orthogonality conditions." These are—

$$\int_{-\pi}^{\pi}\cos mx\cos nx\,dx=\int_{-\pi}^{\pi}\sin mx\sin nx\,dx, \text{ which}$$
$$=0 \text{ (when } m\neq n\text{)},$$
$$=\pi \text{ (when } m=n\text{), and} \qquad (16)$$

$$\int_{-\pi}^{\pi} \cos mx \sin nx \, dx=0, \ (m, n \text{ are integers}). \quad (17)$$

The proofs of these orthogonality conditions are very simple and are based upon trigonometric identities. For example:

when $(m \neq n)$,

$$\cos mx \cos nx=\tfrac{1}{2} \cos(m+n)x+\tfrac{1}{2} \cos(m-n)x; \quad (18)$$

therefore,

$$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx=\frac{1}{2}\int_{-\pi}^{\pi} \cos (m+n)x \, dx + \frac{1}{2}\int_{-\pi}^{\pi} \cos (m-n)x \, dx \quad (19)$$

$$=\frac{1}{2(m+n)} \sin (m+n)x\Big]_{-\pi}^{\pi}+\frac{1}{2(m-n)} \sin (m-n)x\Big]_{-\pi}^{\pi} \quad (20)$$

$$=\frac{1}{2(m+n)}(0)+\frac{1}{2(m-n)}(0)=0. \quad (21)$$

When $(m=n)$,

$$\cos mx \cos nx=\cos^2 mx=\tfrac{1}{2}(1+\cos 2mx); \quad (22)$$

therefore,

$$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx=\frac{1}{2}\int_{-\pi}^{\pi}(1+\cos 2mx)dx, \text{ which} \quad (23)$$

$$=\frac{1}{2}\int_{-\pi}^{\pi} dx+\frac{1}{4m}\int_{-\pi}^{\pi} \cos 2mx \, d(2mx), \quad (24)$$

$$=\frac{1}{2} x\Big]_{-\pi}^{\pi}+\frac{1}{4m} \sin 2mx\Big]_{-\pi}^{\pi}, \text{ and} \quad (25)$$

$$=\frac{1}{2}(\pi+\pi)+\frac{1}{4m}(0)=\pi. \quad (26)$$

The third orthogonality condition (eq 17) can be proven by similar methods, and the use of these relations permits determination of a_0, the remaining coefficients, and the general simplification of the Fourier series. To determine a_0, the original expansion of $f(x)$ is multiplied by cos $0x$, which = 1, namely

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty} a_n \cos nx+\sum_{n=1}^{\infty} b_n \sin nx \quad (15)$$

$$\int_{-\pi}^{\pi} f(x)dx=\frac{1}{2}a_0\int_{-\pi}^{\pi} \cos 0x \, dx+\sum_{n=1}^{\infty}a_n\int_{-\pi}^{\pi} \cos 0x \cos nx \, dx + \sum_{n=1}^{\infty} b_n\int_{-\pi}^{\pi} \cos 0x \sin nx \, dx. \quad (27)$$

Because $(m \neq n)$, $\int_{-\pi}^{\pi} \cos 0x \cos nx \, dx=0$, and because of condition (17), $\int_{-\pi}^{\pi} \cos 0x \sin nx \, dx=0$, therefore,

$$\int_{-\pi}^{\pi} f(x)dx=\frac{1}{2}a_0\int_{-\pi}^{\pi} \cos 0x \, dx=\frac{1}{2}a_0\int_{-\pi}^{\pi} dx, \text{ and} \quad (28)$$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx. \quad (29)$$

By similar means it can be shown that

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \, dx, \text{ and} \quad (30)$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx, \quad (31)$$

and that for even functions of $f(x)$

$$b_n=0, \text{ and} \quad (32)$$

$$a_n=\frac{2}{\pi}\int_{0}^{\pi} f(x) \cos nx \, dx, \quad (33)$$

whereas for odd functions of $f(x)$

$$a_n=0, \text{ and} \quad (34)$$

$$b_n=\frac{2}{\pi}\int_{0}^{\pi} f(x) \sin nx \, dx. \quad (35)$$

The foregoing background material is sufficient to permit an illustration of the application of Fourier analysis to the two-dimensional graph. If one chooses to approximate one of the least likely mathematical functions by a summation of periodic terms, namely $y=x$, which is the graph of a straight line, the procedure is as follows:

First, because $y=x$ is an odd function, relations 34 and 35 hold, namely

$$a_n=0, \text{ and} \quad (34)$$

$$b_n=\frac{2}{\pi}\int_{0}^{\pi} f(x) \sin nx \, dx. \quad (35)$$

Substituting for $f(x)$,

$$b_n=\frac{2}{\pi}\int_{0}^{\pi} x \sin nx \, dx, \text{ and} \quad (36)$$

$$=\frac{2}{\pi n}\int_{0}^{\pi} x(-d \cos nx), \quad (37)$$

the solution of which is

$$-\frac{2}{n}\cos n\pi, \quad (38)$$

and because $\cos n\pi=+1$ when n is even and $=-1$ when n is odd,

$$b_n=(-1)^{n-1}\frac{2}{n}. \tag{39}$$

Therefore, for $(-\pi<x<\pi)$,

$$f(x)=x=2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sin nx}{n}, \tag{40}$$

and the Fourier expansion of $y=x$ for $n=1$ through $n=4$ is, respectively,

$$f(x)=2\sin x, \tag{41}$$

$$f(x)=2\left(\sin x-\frac{\sin 2x}{2}\right), \tag{42}$$

$$f(x)=2\left(\sin x-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}\right), \text{ and} \tag{43}$$

$$f(x)=2\left(\sin x-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}-\frac{\sin 4x}{4}\right). \tag{44}$$

The graphic approximation to $y=x$ that is achieved by this Fourier expansion is shown in figure 6. It is obvious that the approximation improves as successively higher order terms are considered and, also, that if the function treated was by nature periodic, instead of the straight line $y=x$, that a good fit certainly could be obtained. In

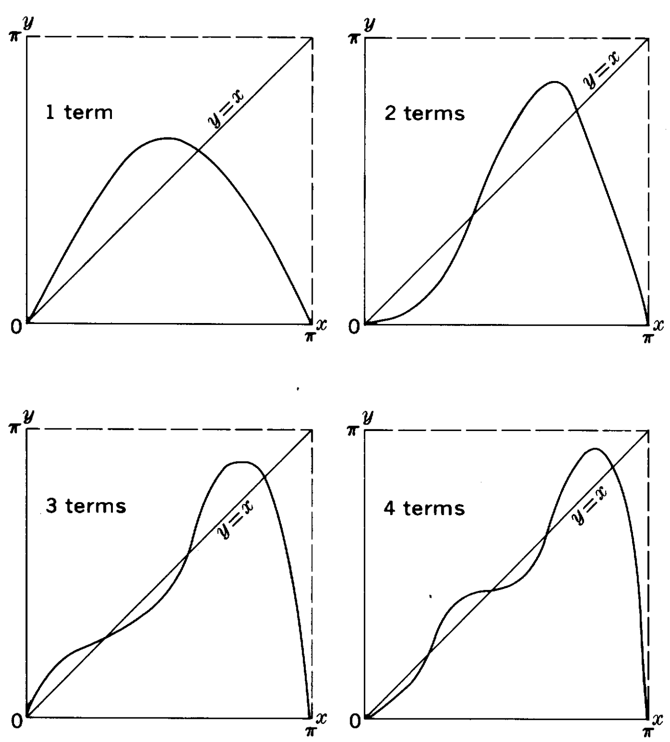

FIGURE 6.—Fourier expansion of the function $y=x$. Note the improvement of approximation to this straight-line function by the successive addition of terms. (After Salvadori and Miller, 1953, fig. 6.2.)

many practical applications of Fourier analysis there are two notable departures from the ideal case treated above. First, the functional interval involved is not the Fourier interval of $+\pi$ to $-\pi$, or the half interval 0 to π. This is readily overcome by modification of the basic relationships to

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{N}a_n\cos\frac{n\pi}{L}x+\sum_{n=1}^{N}b_n\sin\frac{n\pi}{L}x, \tag{45}$$

$$a_n=\frac{1}{L}\int_0^{2L}f(x)\cos\frac{n\pi}{L}x\,dx, \text{ and} \tag{46}$$

$$b_n=\frac{1}{L}\int_0^{2L}f(x)\sin\frac{n\pi}{L}x\,dx, \tag{47}$$

where L simply represents the fact that some finite interval ($+L$ to $-L$) is involved and that the function has a period equal to $2L$. The second difficulty is that the form of the function $f(x)$ may not be known thus precluding the possibility of analytical solution of the required integrals in order to obtain the coefficients. A large number of numerical integration schemes are available, however. Salvadori and Miller (1953) described the Runge and selected-ordinate method and Harbaugh and Preston (1965) discussed the problem and cite several appropriate papers and texts that present solutions, including the use of optical and mechanical devises. Basically, what is involved in any of the numerical integration schemes is the use of data points from the graph of a function in the absence of knowledge of the function proper. That is, for each point used,

$$y_i=\frac{1}{2}a_0+\sum_{n=1}^{N}a_n\cos\frac{n\pi}{L}x_i+\sum_{n=1}^{N}b_n\sin\frac{n\pi}{L}x_i, \tag{48}$$

where x_i, y_i represent the coordinates of any point on the curve.

With respect to quantitative mapping by Fourier analysis, one simply extends the two-dimensional treatment above to the third dimension. That is, instead of treating the oscillatory character of a plotted curve in an x,y coordinate system, the problem involved is fitting an oscillatory surface to data on some parameter, z, which is a function of two variables x and y. In the Basin and Range region under consideration, for example, any of the 11 parameters previously discussed represent a series of z values in the third dimension which are functions of geographic location or an x,y coordinate system on the ground. If the z values of these parameters oscillate with some fundamental periodicity in mutually perpendicular directions, such as north-south and east-west, a double Fourier analysis will serve to reveal the harmonics involved, and a map of the surface of best fit can be obtained. Assuming that the fundamental periods involved are $2L$ in the x-coordinate direction and $2H$ in the y-coordinate direction, the double Fourier series to be used to approximate the surface of best fit to a given set of z values is

$$z \cong f(x, y) = \sum_{m=0}^{m=M} \sum_{n=0}^{n=N} \lambda m, n \Big[a_{m,n} \cos \frac{\pi m}{L} x \cos \frac{\pi n}{H} y + b_{m,n} \sin \frac{\pi m}{L} x \cos \frac{\pi n}{H} y + c_{m,n} \cos \frac{\pi m}{L} x \sin \frac{\pi n}{H} y + d_{m,n} \sin \frac{\pi m}{L} x \sin \frac{\pi n}{H} y \Big], \quad (49)$$

where

$$\left. \begin{array}{l} \lambda m,n = \tfrac{1}{4}, \text{ for } m=n=0, \\ \lambda m,n = \tfrac{1}{2}, \text{ for } m=0,\ n>0; \text{ or } m>0,\ n=0, \text{ and} \\ \lambda m,n = 1, \text{ for } m>0,\ n>0. \end{array} \right\} \quad (50)$$

And the coefficients of the sine and cosine terms in the double Fourier expansion can be determined from the following double integrals:

$$a_{m,n} = \frac{1}{LH} \int_{-H}^{H} \int_{-L}^{L} f(x,y) \cos \frac{m\pi}{L} x \cos \frac{n\pi}{H} y \, dxdy, \quad (51)$$

$$b_{m,n} = \frac{1}{LH} \int_{-H}^{H} \int_{-L}^{L} f(x,y) \sin \frac{m\pi}{L} x \cos \frac{n\pi}{H} y \, dxdy, \quad (52)$$

$$c_{m,n} = \frac{1}{LH} \int_{-H}^{H} \int_{-L}^{L} f(x,y) \cos \frac{m\pi}{L} x \sin \frac{n\pi}{H} y \, dxdy, \text{ and} \quad (53)$$

$$d_{m,n} = \frac{1}{LH} \int_{-H}^{H} \int_{-L}^{L} f(x,y) \sin \frac{m\pi}{L} x \sin \frac{n\pi}{H} y \, dxdy. \quad (54)$$

The analogy of the double Fourier expansion series and the expressions that define the coefficients, with the two-dimensional or single series and its coefficient equations, is readily apparent. As before, the integral equations that define the coefficients cannot be determined analytically unless the function $f(x,y)$ is known. In a practical problem, such as this Basin and Range study, one virtually has at hand sets of z values at data points in an x,y coordinate system, and the form of the surface, or the $f(x,y)$ function, is not known. For this reason one must again resort to numerical integration or to other available schemes. Harbaugh and Preston (1965) and Preston and Harbaugh (1965) presented numerical summation equations that can be used for the requisite coefficient determination, for example. They also presented some examples of the application of double Fourier analysis.

The applications involve the areal variation of calcium carbonate in a magnesite deposit and an analysis of geological structure in Kansas. The paper by Harbaugh and Preston (1965), however, also contains a discussion of other possible applications, among which was the analysis of harmonics of topographic surfaces. To the writer's knowledge, the sole application of Fourier analysis to landform topography was that presented in a military-contract report on microrelief in Thailand. Harbaugh (written commun., 1967) cited a second example which consists of the application of single Fourier series, radiating from a common point, to surface topography. The suggestion that Fourier analysis is a potentially useful quantitative mapping technique in geomorphic investigations is therefore worthy of reemphasis here because it must be classified as seldom used for this purpose

Figure 7, a reproduction of an illustration that appears in both papers by Harbaugh and Preston cited

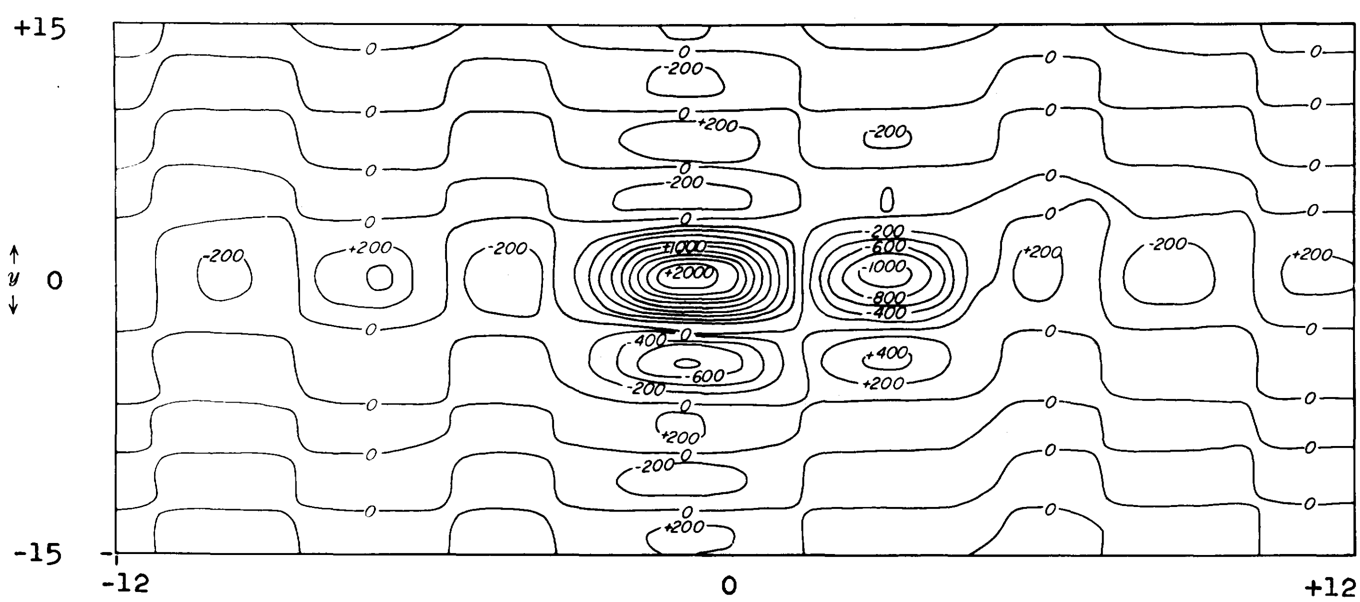

FIGURE 7.—Synthetic double Fourier series representing a wave form containing four harmonics in both coordinate directions. (After Preston and Harbaugh, 1965, fig. 2.)

above, is a map resulting from the synthetic generation of a double Fourier series containing terms as high as order $m=n=4$. In other words, the map represents a wave form containing four harmonics in both x- and y-coordinate directions. This can be considered as an ideal representation, and the symmetrical character of the doubly oscillating surface is apparent. The distortion present is the result of unequal grid spacing in both directions. The trend-surface maps of Basin and Range topography that are presented later in this report (figs. 9–41) clearly show that regional high and low values of the 11 chosen parameters exist within the area and that any surface of best fit to the z values probably has some underlying oscillatory or harmonic components in both the north-south and the east-west directions. A double Fourier analysis of these topographic variations would by no means yield a map as simple as that shown in figure 7, but the writer firmly believes that the region would be amenable to analysis by this method.

Two basic problems prevented a double Fourier analysis of the data on the Basin and Range region. First, there is no standard method for orienting the x, y coordinate system with respect to regional topography. Although it was stated above that generally regional highs and lows of topographic parameters are present in both the north-south and the east-west directions, the first-degree trend-surface maps to be later discussed clearly show that the regional "grain" is oriented northwestward or northeastward, rather than due north in most places, and that similar departures from due east or west exist. This suggests that in any future studies of regional topography by double Fourier analysis it might be most profitable to obtain first-degree trend-surface maps at the outset to orient a coordinate system for Fourier analysis perpendicular and parallel to the regional gradients. Such orientation would at least prevent the arbitrary selection of possibly unfortunate coordinate directions.

The second problem cannot be overcome so readily, given the data contained in this report, however. Because the Basin and Range region is irregular in plan (fig. 1), there is a corresponding irregularity in the numbers of topographic maps of the region, the irregularity varying according to the direction of traverse. As shown in figure 2, for example, at the narrow "waist" of the region only two maps were used for this study, namely Death Valley and Las Vegas, whereas in a north-south direction a maximum of 11 maps exist along the Pocatello-Sonoyta meridian. Because each map represents a single data point, as previously discussed, the irregularity of this available grid would be too great to yield valid regional results, particularly in an east-west direction.

A double Fourier analysis is merely a combination of single Fourier analyses in two mutually perpendicular directions. This is shown by equation 49, for example. Although there are eight times the number of sine and cosine terms in a double Fourier series, the results of a double analysis are nevertheless dependent upon the validity of the two components. This does not necessarily mean that one must deal with a square data array, but if less than some minimum number of data points are available in one of the coordinate directions, the results may well be meaningless. Considering again the fact that a single Fourier series expansion is used to approximate some function $y=f(x)$ in two dimensions, it is obvious that some oscillatory characteristics can always emerge if the basis for the graph of the function is only two or three points. Some number of harmonics may well underlie the regional variation of topographic parameters in the Basin and Range area, but valid appraisal of this possibility requires the availability of a greater number of data points in an east-west direction than is afforded by use of the 1:250,000 topographic quadrangle maps. Quantitative mapping of Basin and Range topography by means of double Fourier analysis is therefore strongly recommended as a future project, with the provision that the data analyzed be plotted at the locations of range centroids. The use of maps of larger scale than 1:250,000 would not eliminate the problem raised here concerning numbers of data points because the plotting of mean z values at the centers of larger scale maps would increase location errors intolerably. If range centroids are used for z-value locations, the resulting irregular data array can be analyzed by means of the Fortran IV computer program for double Fourier analysis recently provided by James (1966).

TREND-SURFACE ANALYSIS

Trend-surface analysis can be considered to embrace many forms of least-squares analysis and, in this sense, the fitting of double Fourier series to a set of data is actually one form of trend-surface analysis. In common usage, however, trend-surface analysis implies least-squares fitting of polynomial surfaces, and Fourier analysis and trend-surface analysis are thus distinguished in this report.

Like the application of single and double Fourier analysis previously described, trend-surface analysis is a potentially useful quantitative analytical technique that may be most readily understood by consideration of a two-dimensional analogy. If one is confronted with a set of data that approximates a linear distribution on arithmetic-graph paper, three choices of presentation are open. The investigator can (1) simply plot the data as a scatter diagram, (2) construct a straight line through the data points by visual means, or (3) construct the line of best fit through the data points by the statistical procedure of the least-squares method.

Choices (2) and (3) above both yield similar results; a linear equation of the form

$$y=mx+b \tag{55}$$

is obtained, where m is the slope of the fitted line, and b is the intercept on the y axis. There is, however, an extremely important difference between construction of a fitted straight line by visual means and that by the least-squares method. The least-squares method, as is well known, permits the construction of a unique straight line, about which the sum of the squares of the departures of the x_i,y_i observed data values and the corresponding computed values for that line is a minimum. Because this method can yield only one line of best fit for a given set of data, it is objective. Construction of such a line by visual means must, in contrast, be subjective because different investigators may well produce lines of different slopes and intercept values.

The analogy to the three-dimensional case, where one attempts to map the distribution of z values which are a function of x,y coordinates, should be obvious. One is attempting, in fact, to map a surface of best fit to the z values in three-dimensional space. This can be accomplished by simple manual contouring, but this, in turn, is analogous to the visual contruction of straight lines in the two-dimensional case. As noted above, and as previously mentioned with respect to manual contouring in general (p. D10), this method is subjective and is not necessarily reproducible. Alternatively, one can produce a three-dimensional surface of best fit to a given set of z values by mathematical techniques which will yield a unique surface that is therefore both reproducible and objective. One such technique is the method of trend-surface analysis, by means of which the Basin and Range data treated in this report were mapped.

If we return to two-dimensional examples for a moment, a straight line represents an equation of the first degree which, for reasons of consistency that will become apparent, can be written in the form

$$Y=A+Bx. \tag{56}$$

In this equation, the coefficient A is the zeroth-degree term, and Bx is the first-degree term. An equation of the second degree in an x,y coordinate system is a parabola whose equation is of the form

$$Y=A+Bx+Cx^2. \tag{57}$$

It can be noted that equation 57 differs from an equation of the first degree (equation 56) by reason of the addition of a quadratic term, namely Cx^2. Similarly, an equation of the third degree in an x,y coorinate system is a sinusoidal curve whose equation is of the form

$$Y=A+Bx+Cx^2+Dx^3. \tag{58}$$

This equation differs from that of a quadratic parabola (eq 57) by addition of the cubic term Dx^3. Equations of any given higher degree can similarly be written, requiring only the addition of an appropriate number of terms of successively increasing order. Because this report makes use of trend surfaces that are not of a degree higher than the third, however, these two-dimensional or two-variable expressions (eq 56–58) will suffice for illustrative purposes.

To understand trend surfaces one must consider the three-dimensional surfaces that correspond to first-, second-, and third-degree equations in an x,y,z coordinate system. A first-degree surface is a plane whose equation is linear and is of the form

$$Z=A+Bx+Cy. \tag{59}$$

A second-degree surface is a paraboloid, and (like the two-dimensional equivalent (eq 57)) its equation contains the zeroth- and first-degree terms of a first-degree or linear surface (eq 59) and the addition of appropriate quadratic terms, namely—

$$Z=A+Bx+Cy+Dx^2+Exy+Fy^2. \tag{60}$$

A third-degree surface is oscillatory or doubly sinusoidal in space, and its equation consists of the linear and quadratic terms contained above (eq 60) and four additional cubic terms, namely—

$$Z=A+Bx+Cy+Dx^2+Exy+Fy^2 + Gx^3+Hx^2y+Ixy^2+Jy^3. \tag{61}$$

As in the two-variable case, one can write an equation representing a three-dimensional surface of any given degree by merely adding appropriate terms of higher degree. The geometric relations of first-, second-, and third-degree equations for two and three variables are shown in figure 8.

To find the equation of the surface of best fit of any given degree, the coefficients of the appropriate polynomial expression must be determined and, like the two variable case, the departures of the observed z values from the corresponding computed z values on the fitted surface must be minimized by a least-squares procedure. One can obtain the required coefficients by the manual solution of simple matrices, as described by Krumbein and Graybill (1965), or by relaxation or other successive-approximation procedures. In fact, an actual trend-

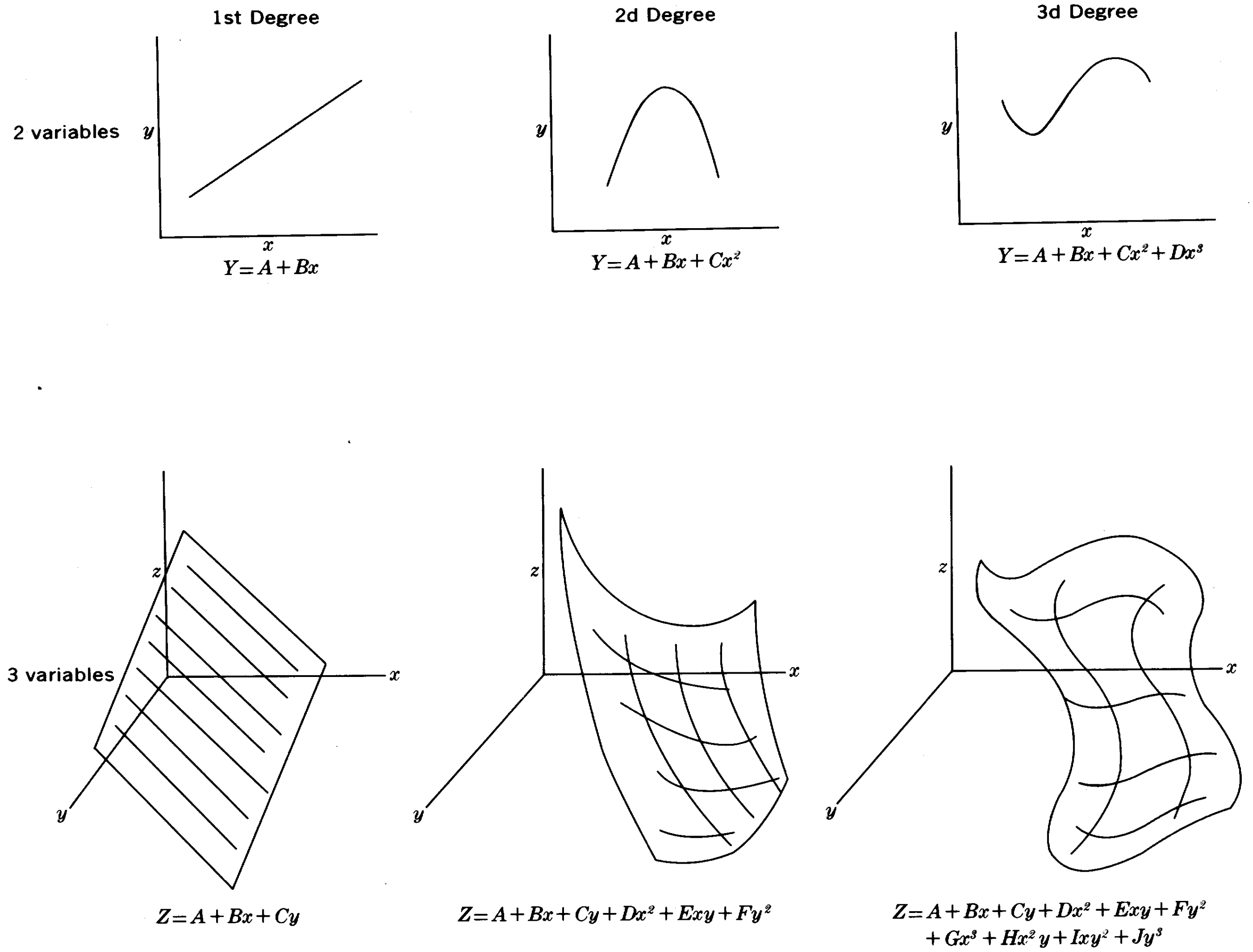

FIGURE 8.—Geometric relations of first-, second-, and third-degree equations for two and three variables.

surface map can be produced by taking successive moving averages of sets of mutually perpendicular profiles across the data array (Krumbein, 1956). This method is simple, but it involves some subjectivity in the smoothing of the profiles and does not provide the equation of a given surface. Moreover, it is time consuming.

The most rapid method of generating the surfaces of best fit and their corresponding equations is to use computers. O'Leary, Lippert, and Spitz (1966) recently provided a program in Fortran IV which will compute and plot trend surfaces of first through sixth degree. In addition to the contoured trend surfaces, the output includes the equations of each surface of best fit, a tabular summary of z values for each surface of a given degree, and the corresponding departures, or residuals, from the z observations, the coefficient matrix, plots of original data, plots of residuals for each surface of a given degree, and several useful statistical-error measures.

The latter include the total variance V, which is the sum of the squares of the departures of the z values, z_i, from the mean observed z value, namely

$$V=\sum_{i=1}^{N}(z_i-\overline{z})^2. \tag{62}$$

The z_i represents original data, and this statistic is identical with that involved in calculating the deviation of x values about their mean, $\overline{x}$, in a two-variable data system. The total variance V is therefore a constant value for a set of data and is not related to the degree of a given surface that may be fitted to the data.

The variance that is not explained by the surface is, therefore, defined as

$$U=\sum_{i=1}^{N}(z_{i(o)}-z_{i(c)})^2, \tag{63}$$

where $z_{i(o)}$ refers to an observed z value, or the original data, and $z_{i(c)}$ refers to a corresponding computed z

value on a surface of best fit of a given degree. In a two-variable system the analogy to this statistic is the deviation or scatter of data values about the least-squares regression line of best fit. Some points will occur above the line, and some below, and because the arithmetic sum of these departures will equal zero, the departures must be squared. The reasoning is precisely the same with respect to z values. Some will occur above the surface of best fit, and some will occur below. Squaring the departures and taking their sum provides a measure of the scatter about a surface of best fit of a given degree and, therefore, provides information on the unexplained variance.

The variance that is explained by a given surface, E, is simply the difference between the total variance and the unexplained variance. Therefore,

$$E=V-U. \tag{64}$$

If the variance that is explained by a given surface is divided by the total variance, it provides the variance that is explained as a percentage. This is defined as the coefficient of determination, namely,

$$D=\frac{E}{V}. \tag{65}$$

The coefficient of correlation is the square root of the coefficient of determination, or

$$R=D^{1/2}. \tag{66}$$

Finally, a measure of the standard deviation of the data for each surface of a given degree is also generated by this computer program. The statistic, or error measure, is again analogous to the corresponding statistic for a set of data in a two-variable system. That is,

$$\sigma=\left(\frac{U}{N}\right)^{1/2}. \tag{67}$$

The program discussed above (O'Leary and others, 1966) was modified and rewritten in Fortran II because Fortran II machine language was required by the computer system available at the time the Basin and Range study was made. The computer output corresponds exactly to that described above, except that trend surfaces and associated data of degrees higher than third were not generated. In a subsequent section of this report, the trend-surface maps and certain error measures for first-, second-, and third-degree surfaces of best fit are presented for each of the 11 variables used in this study. Before considering these results, however, a few additional remarks should be made about trend-surface mapping and the previous work done in this field.

With respect to previous work, in recent years Merriam and Lippert (1964, 1966), Merriam and Harbaugh (1964), and Merriam and Sneath (1966) have applied trend-surface analysis to geological structure problems in Kansas. Harbaugh (1964a, b) has treated oil-field and facies data in both three- and four-component systems; computer programs other than that discussed above have been written by Harbaugh (1963) and by Sampson and Davis (1966a, b); and Spitz (1966), among others, has treated the problem of generating orthogonal polynomials for an irregularly spaced array of original data points. Read and Merriam (1966) investigated the thickness of stratigraphic units in Scotland by trend-surface analysis, and a classic petrological and mineralogical study of the Donegal granite by trend-surface methods was reported by Whitten in 1959. Since that time, Whitten (1961, 1962) has obtained similar results which delineated previously obscure regional trends in other European granite bodies. The general application of trend-surface analysis to sedimentary environment studies was discussed by Miller in 1956. All the reports cited here contain additional references to previous studies that involved trend-surface analysis, and many pertinent papers are also cited in the texts by Miller and Kahn (1962) and Krumbein and Graybill (1965), each of which provide a good general discussion of the method. The total body of literature on the subject suggests that applications have been directed primarily toward problems of stratigraphy and sedimentation, structure and petrology, and geochemistry and ore deposits. Surficial deposits, such as soils and sands, have been treated (Chorley, 1964), but regional topographic variations and general applications to goemorphology have not been studied previously by trend-surface analysis.

Certain of the work on surficial sands and soils well illustrates a fundamental point concerning trend analyses in general, however, and this should be emphasized here. Chorley, Stoddart, Haggett, and Slaymaker (1966) wished to investigate the origins of surficial sands in an area of 1,000 square kilometers in eastern England. Accordingly, they undertook a trend-surface analysis of sand size, where z values were sand size, in mm, and x,y coordinates represented the geographic locations of sampling stations. This analysis revealed a regional trend, consisting of highs to the northeast and a dominant low to the southwest, but the third-degree trend surface of best fit explained only 21.5 percent of the total variance. The authors therefore designed a hierarchical, or nested, sampling technique on six levels which utilized the knowledge of regional trends, and by an analysis of variance they examined the parts of the unexplained total variance that could be attributed to local variance, corresponding to each of these six levels of sampling. This technique enabled them to distinguish several possible environmental factors that, in combination, could account for the overall variability of sand size.

The fundamental point is that in any study of natural phenomena which are not constant in a given region, the total variance is commonly a function of variability at several levels. Trend-surface analysis is a means of distinguishing regional variation, or the part of the total variance that is attributable to the highest level of variability. In a study of range heights in the Basin and Range region, for example, one would not expect to find that all ranges at a given latitude are of precisely the same height nor that a uniform decrease in height occurs with latitudinal variation. If this were true, every z value for range height would fall precisely on a plane of best fit, or first-degree surface, which had a uniform dip. As previously stated, one seldom, if ever, obtains a perfect fit to either least-squares regression lines or regional trend surfaces. The departures of the observed values from the computed values on the line or surface represent the unexplained variance U (63). In trend surfaces, this unexplained variance is attributable to more localized causes of variability. If one is interested primarily in the question of whether regional variation exists and is persistent, then trend-surface mapping is a quantitative analytical technique that can provide the answer. If one wishes to examine all possible causes of the total variance, then one must engage in a secondary analysis of the unexplained variance revealed by regional trend analysis. In the study reported here, the primary goal was to determine whether the magnitudes of regional variation were compatible with the hypothesis that significant topographic differences exist within the Basin and Range region. Trend-surface analysis showed that this was true, and there was no reason to examine the various sublevels which would account for the residual unexplained variance for each topographic parameter. Returning to range heights, for example, within any given subregion that exhibits predominantly high or low values of range height, some ranges will obviously depart from these general subregional values. This constitutes a part of the total variance of range heights that may be ascribed to a lower level of variability, namely the individual range.

Troeh (1965) has argued that the complexities of a series of hills and valleys are so great that it is impossible to describe them by a three-dimensional equation. This should not be construed as a denigration of trend surface techniques, however, because again the ultimate goals must be kept in mind. If the goal is to describe some specific landform, such as the alluvial fans described by Troeh (1965), the drumlins by Chorley (1959), or the headland-bay beaches by Yasso (1965), then of course one attempts to fit some specific descriptive function to the landform in question. If, however, one is concerned with regional variation among groups of landforms, one of the most appropriate means of separating the relatively large scale systematic variations from more localized and smaller scale variations is indeed the method of trend-surface analysis.

With this background, the results of the trend-surface analysis of each of the 11 topographic parameters within the Basin and Range region can now be considered.

TREND-SURFACE RESULTS

The trend-surface results presented here consist of contour maps of the first-, second-, and third-degree surfaces of best fit for each of the 11 topographic parameters used in this study. The statistics associated with these results are presented in the section entitled "Summary of Trend-Surface Results" (p. D51); however, a few general points previously covered are restated here for convenience.

First, the criteria employed for measurement of any given variable are arbitrary and a choice of different criteria would result in different absolute z-values than those determined for this report. Second, of the four sources of error mentioned previously (p. D8), the most important is that which results from plotting procedures. Because mean z-values of a given parameter were plotted at the centers of each of the 1:250,000 topographic quadrangle maps used (fig. 2), instead of plotting the population of z-values at the centroids of each of the ranges that make up this population, some dislocation of high and low regional values occurs. The dislocation has no effect on the area of ranges to total area ratio and the range trend parameters; nonetheless, the average plotting error is thought to be less than one-half of the diagonal distance on any given 1:250,000 topographic quadrangle map. Third, the previous discussion of trend-surface analysis (p. D17) indicates that the maps of the first-, second-, and third-degree surface of best fit for each of the topographic parameters represent planes, paraboloids, and oscillatory surfaces of best fit, respectively, to the z-values that were obtained. (See eq 59–61; fig. 8.) Finally, this previous discussion emphasized that the trend-surface results generally reflect the part of the total variance that is attributable to regional causes. For each of the topographic parameters discussed here, there is some fraction of the total variance that is attributable to more localized causes, and no effort has been made to determine the latter. Regional variation, alone, is of concern here.

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AREA OF RANGES TO TOTAL AREA

Area of ranges to total area is, perhaps, one of the most significant of the topographic parameters used. Fenneman, Lobeck, and Eardley, each of whom is quoted on page D2 of this report, clearly invoked this parameter in discussing regional differences within the Basin and Range area (fig. 1). Moreover, Fenneman's statement includes the estimate that the ratio of area of ranges to total area is 50 percent in the northern part of the region and 20 percent in the southern part of the region. Although the absolute values of any parameter will vary with the measurement criteria used, it is noteworthy that values of 50 percent or more were obtained for only two topographic maps, namely Mariposa and Death Valley in the north and that values of this parameter in the southern part of the region are less than 10 percent in the area covered by the Santa Ana–Salton Sea–Ajo–Sonoyta maps (fig. 2).

The trend-surface maps reflect, of course, the absolute values used in their preparation. Figure 9 is the first-degree trend-surface map of the area of ranges to total area. The values of this parameter on the plane of best fit increase to the northwest. The regional values range from 3 percent in the south to 39 percent in the extreme northwest corner of the map; thus, the regional variation is 13-fold for this parameter.

The second-degree trend-surface map (fig. 10) shows that a paraboloid of best fit to the regional data has a similar northwest-southeast orientation. This second-degree surface may be likened to the part of a tablespoon that is concave upward, with its bowl in the northwest and tip in the southeast. The margins of the paraboloid (or the rim of the hypothetical tablespoon) extend through the Mohave Desert area to the southwest and the Salt Lake desert area to the northeast, where comparable map values occur. In other words, the northeastern part of the Basin and Range region exhibits greater similarity to the southern part of this region than to the northern part, on the basis of the variation of this topographic parameter.

The third-degree trend-surface map of the area of ranges to total area ratio (fig. 11) reinforces this suggestion to some extent. Edge effects are troublesome along the east margins of all the maps presented because of a lack of data points (fig. 2); nevertheless, a large region in the northeast can be seen to exhibit values comparable to those in the south, and a regional high occurs in the northwest. Trend-surface maps of several parameters confirm these regional distinctions. The regional high shown in figure 11 delineates an area of central and western Nevada and eastern California, within the Basin and Range province, which is distinctly different from the rest of the region. The precise location of the contour closure is not always the same on the maps of the several variables presented but it is invariably in the northwestern part of the region; the total migration distance of the closure on the several maps is not great.

RANGE LENGTH

The first-degree trend-surface map of range length is shown in figure 12. The orientation of the plane of best fit is seen to be more nearly north-south than was true for the area of ranges to total area parameter, and the average lengths of ranges vary from 8 miles in the south to more than 30 miles in the north. Figure 13 shows the second-degree surface or paraboloid of best fit. The axis of this surface is oriented northwest-southeast to some extent. As before (fig. 10), comparable values of this topographic parameter occur in the northeastern and southern parts of the region; the dominant high is to the northwest. The third-degree trend-surface map of range length (fig. 14) clearly shows the location of this regional high, the innermost contour closure of which is north of that shown in figure 11 for the area of ranges to total area parameter, but which reflects the same regional distinction. The northern and southern parts of the region differ markedly with respect to range length, and, if edge effects are disregarded, the northeastern part of the region more closely resembles the southern than the northwestern part of the region.

RANGE WIDTH

The regional characteristics of range width appear to be similar on the basis of the first-degree trend-surface map (fig. 15). Again, a plane of best fit trends slightly northwest, and average values increase in this direction; in this example the increase is from 2.2 miles in the southeast to 5.2 miles in the northwest, or more than 100 percent. The second-degree (fig. 16) and third-degree (fig. 17) trend-surface maps of range width, however, reveal that this parameter increases in value both to the northeast and to the northwest. Of these two trends, the northeastward increase in range width is less real than it appears because there is far more variability in the data from this area. That is, the majority of the ranges to the northwest exhibit large values of range width, whereas a lesser percentage of exceptionally wide ranges in the northeast have affected the mean values in a similar manner because, in general, the ranges are less abundant in that area. It can again be said, however, that the northern and southern parts of the Basin and Range region are disparate, with respect to this topographic parameter.

RANGE HEIGHT

As indicated in the discussion of the criteria used for the measurement of range height (p. D7), the range

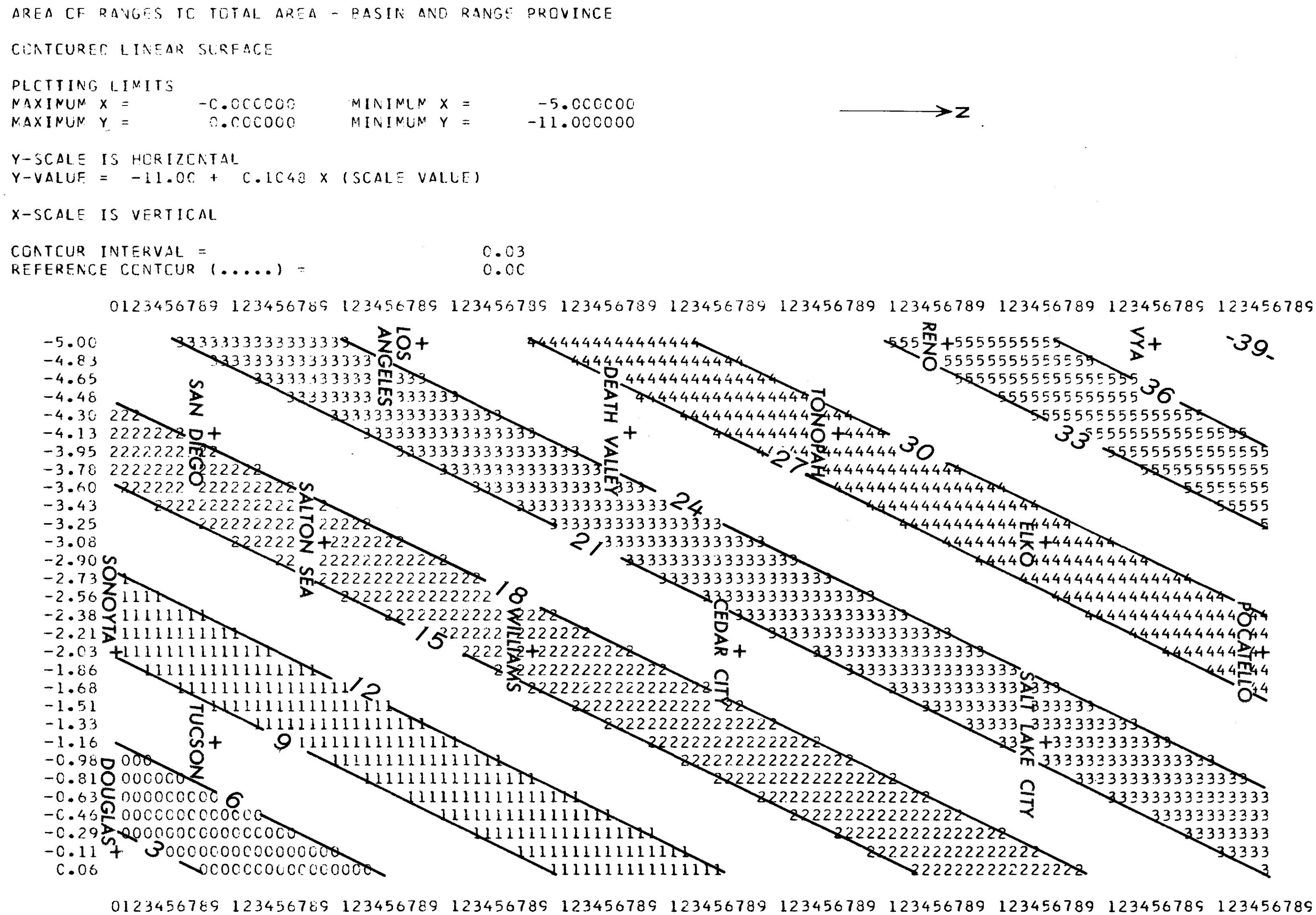

FIGURE 9.—First-degree trend-surface map of area of ranges to total area. Contour interval is 3 percent. The geographic names shown correspond to the locations of the 1:250,000 topographic maps.

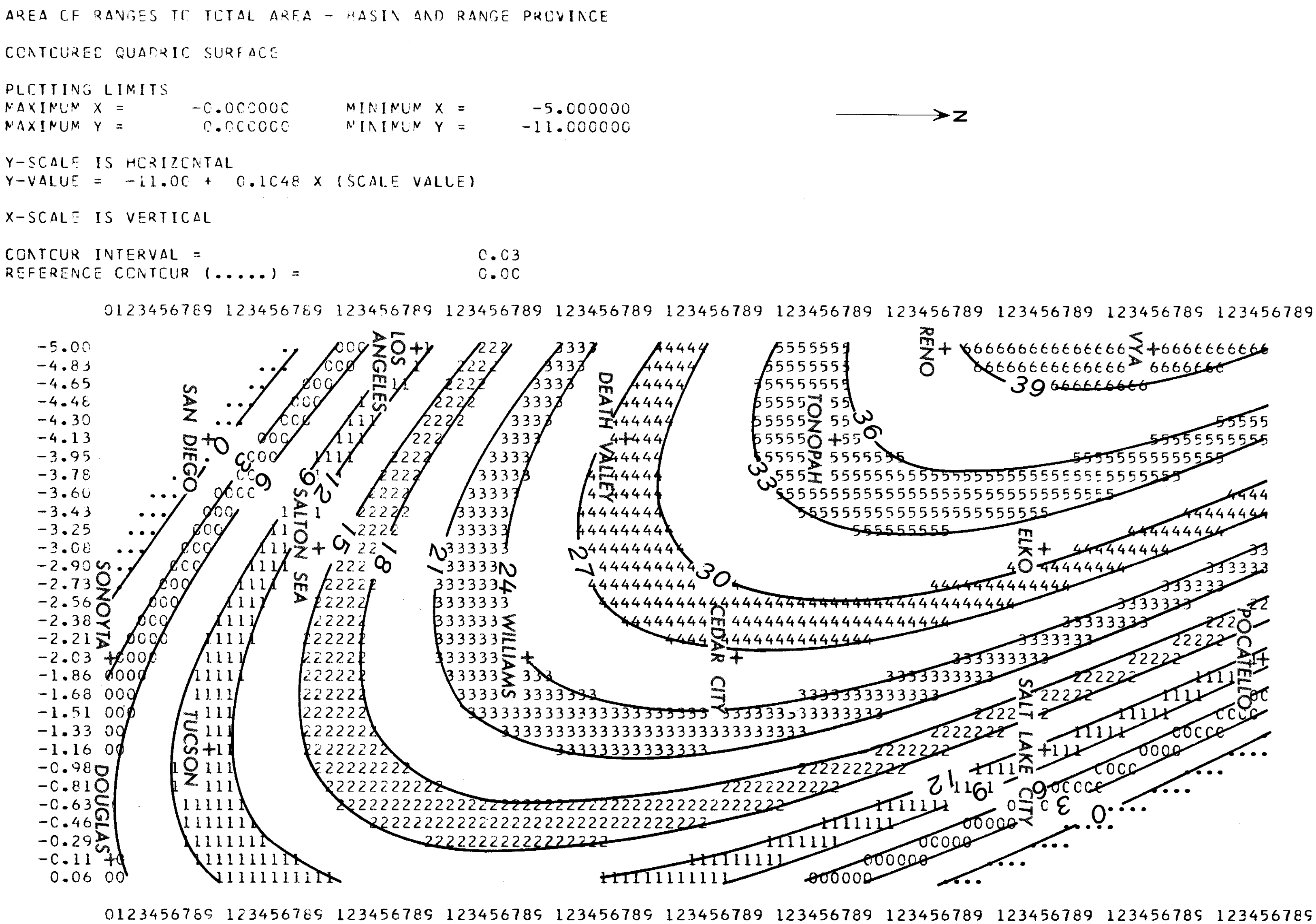

FIGURE 10.—Second-degree trend-surface map of area of ranges to total area. Contour interval is 3 percent. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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AREA OF RANGES TC TOTAL AREA - BASIN AND RANGE PROVINCE

CONTCUREC CUBIC SURFACE

PLCTTING LIMITS
MAXIMUM X =        -0.000000      MINIMUM X =       -5.000000
MAXIMUM Y =         0.000000      MINIMUM Y =      -11.000000

Y-SCALE IS HORIZCNTAL
Y-VALUE =  -11.0C +  0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTCUR INTERVAL =                        0.03
REFERENCE CCNTOUR (.....) =               0.00
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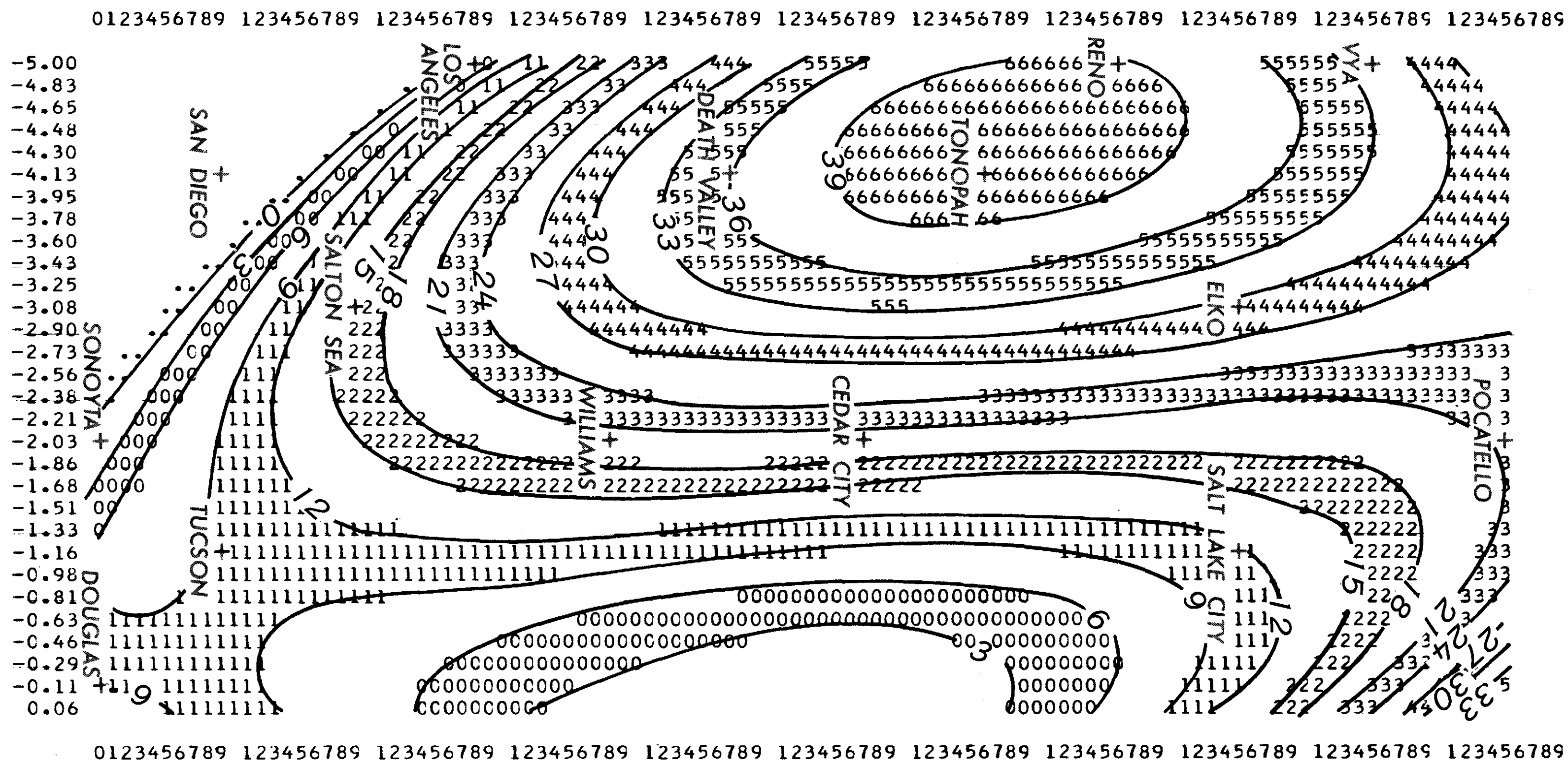

Figure 11.—Third-degree trend-surface map of area of ranges to total area. Contour interval is 3 percent. The geographic names shown correspond to the locations of the 1 : 250,000 topographic quadrangle maps.

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MEAN RANGE LENGTH (MILES) - BASIN AND RANGE PROVENCE

CONTOURED LINEAR SURFACE

PLOTTING LIMITS
MAXIMUM X =        -0.000000      MINIMUM X =       -5.000000
MAXIMUM Y =         0.000000      MINIMUM Y =      -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =   -11.00 +   0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                          2.00
REFERENCE CONTOUR (.....). =                0.00
```

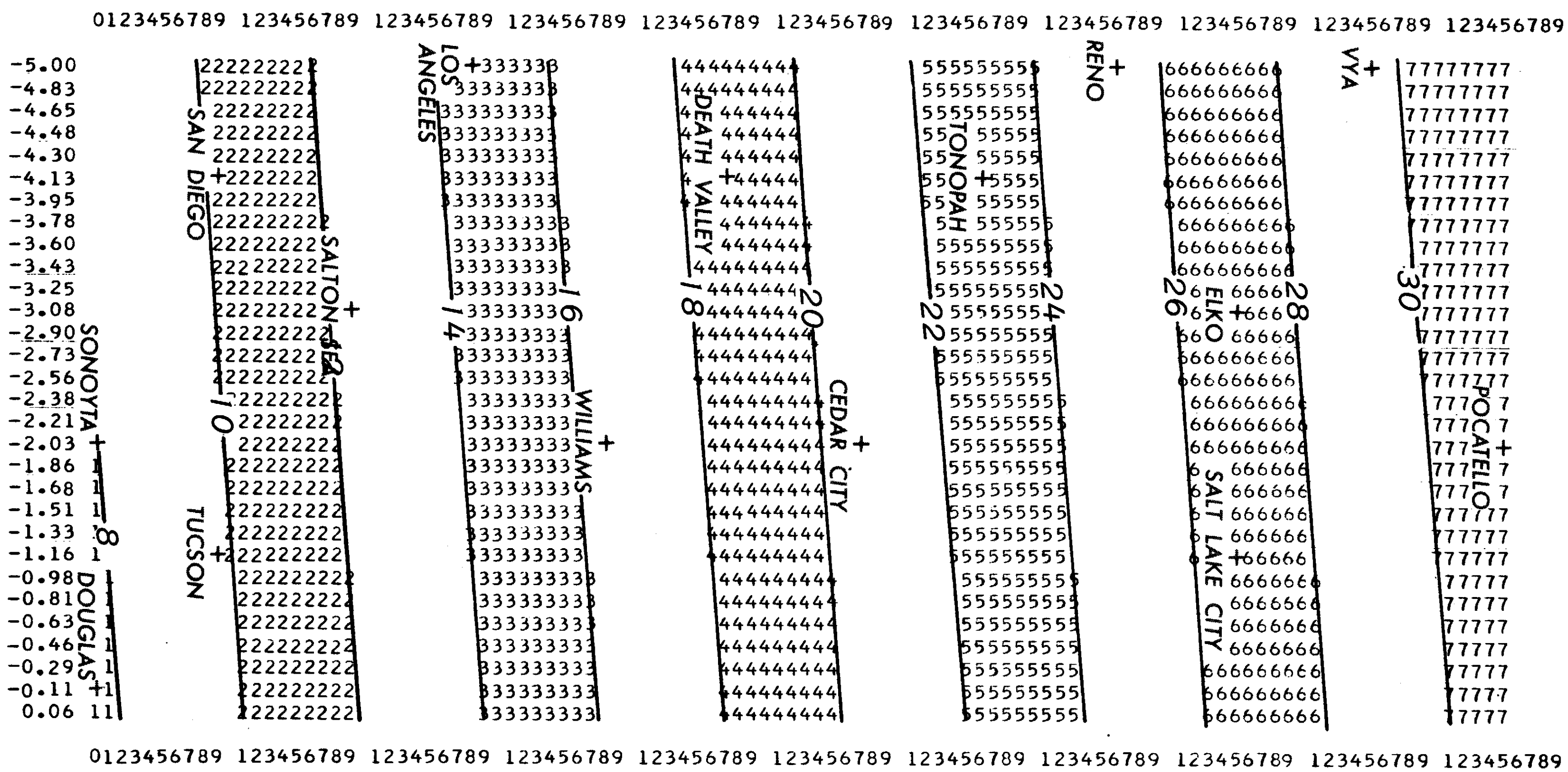

FIGURE 12.—First-degree trend-surface map of mean range length. Contour interval is 2 miles. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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MEAN RANGE LENGTH (MILES) - BASIN AND RANGE PROVENCE

CONTOURED QUADRIC SURFACE

PLOTTING LIMITS
MAXIMUM X =     -0.000000     MINIMUM X =      -5.000000
MAXIMUM Y =      0.000000     MINIMUM Y =     -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =  -11.00 +  0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                          2.00
REFERENCE CONTOUR (.....) =                 0.00
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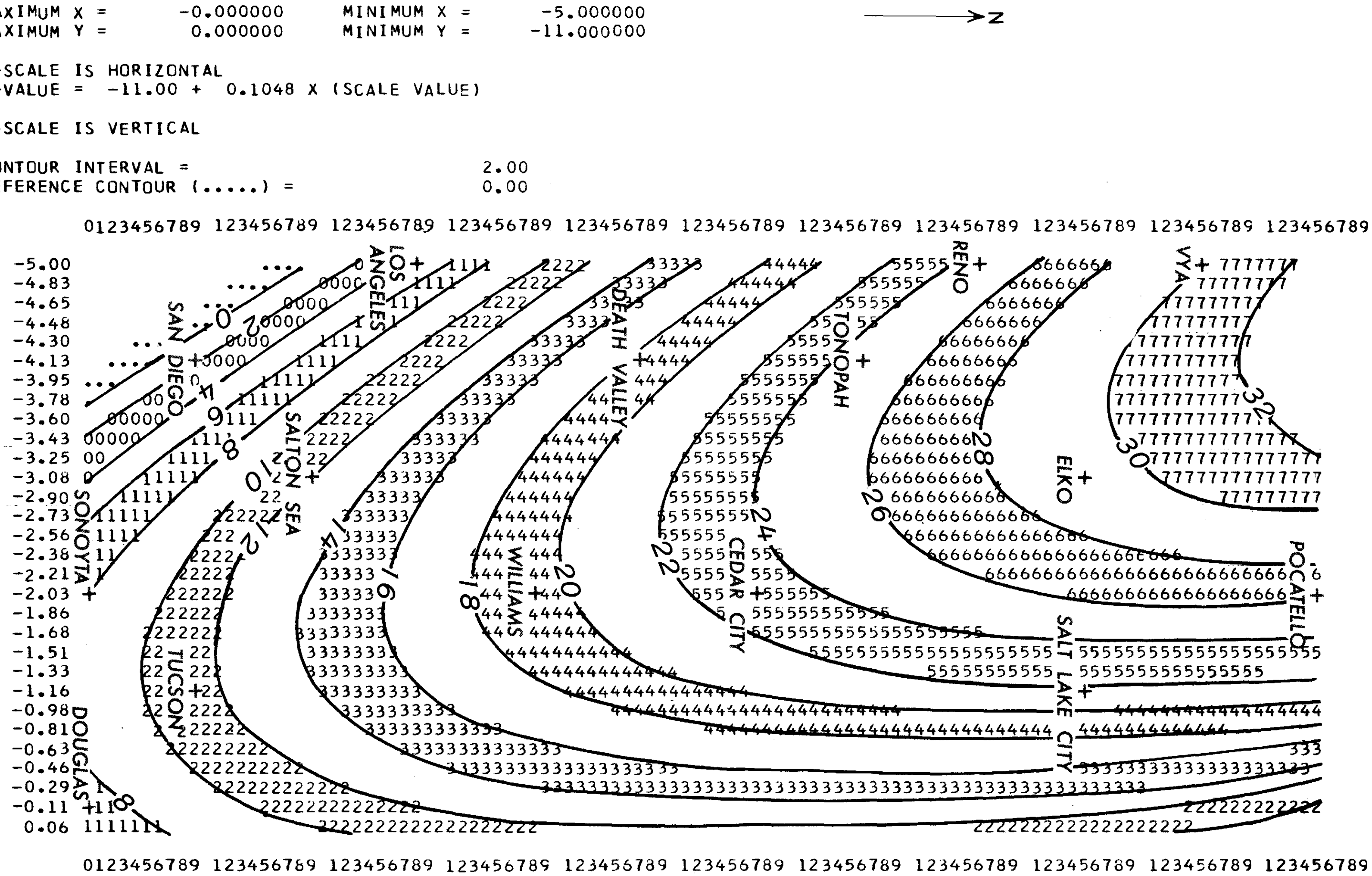

FIGURE 13.—Second-degree trend-surface map of mean range length. Contour interval is 2 miles. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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MEAN RANGE LENGTH (MILES) - BASIN AND RANGE PROVENCE

CONTOURED CUBIC SURFACE

PLOTTING LIMITS
MAXIMUM X =        -0.000000      MINIMUM X =       -5.000000
MAXIMUM Y =         0.000000      MINIMUM Y =      -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =  -11.00 +  0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                        2.00
REFERENCE CONTOUR (.....) =               0.00
```

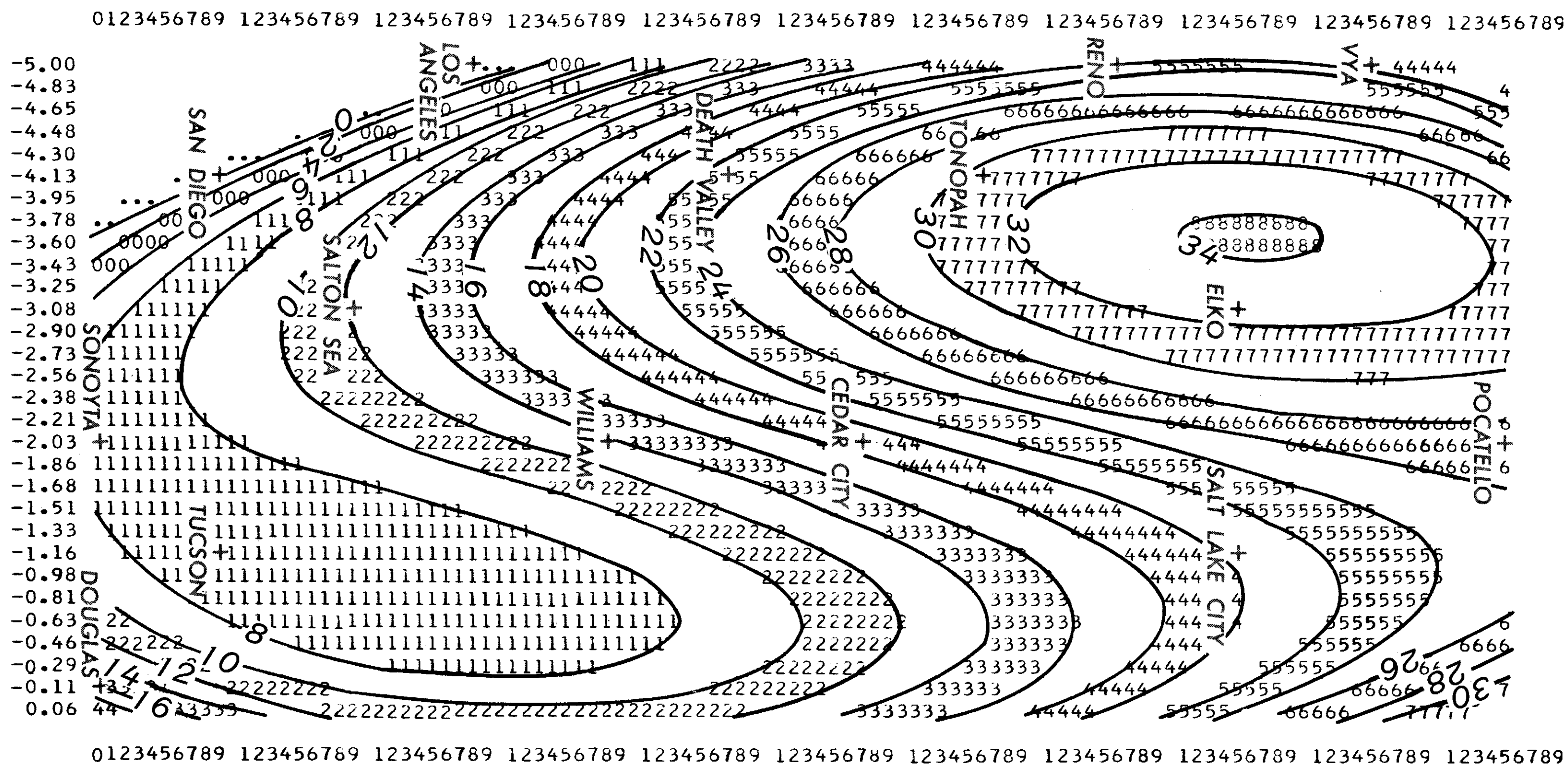

FIGURE 14.—Third-degree trend-surface map of mean range length. Contour interval is 2 miles. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

MEAN RANGE WIDTH (MILES) - BASIN AND RANGE PROVINCE

CONTOURED LINEAR SURFACE

PLOTTING LIMITS
MAXIMUM X = -0.000000 MINIMUM X = -5.000000
MAXIMUM Y = 0.000000 MINIMUM Y = -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE = -11.00 + 0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL = 0.20
REFERENCE CONTOUR (.....) = 0.00

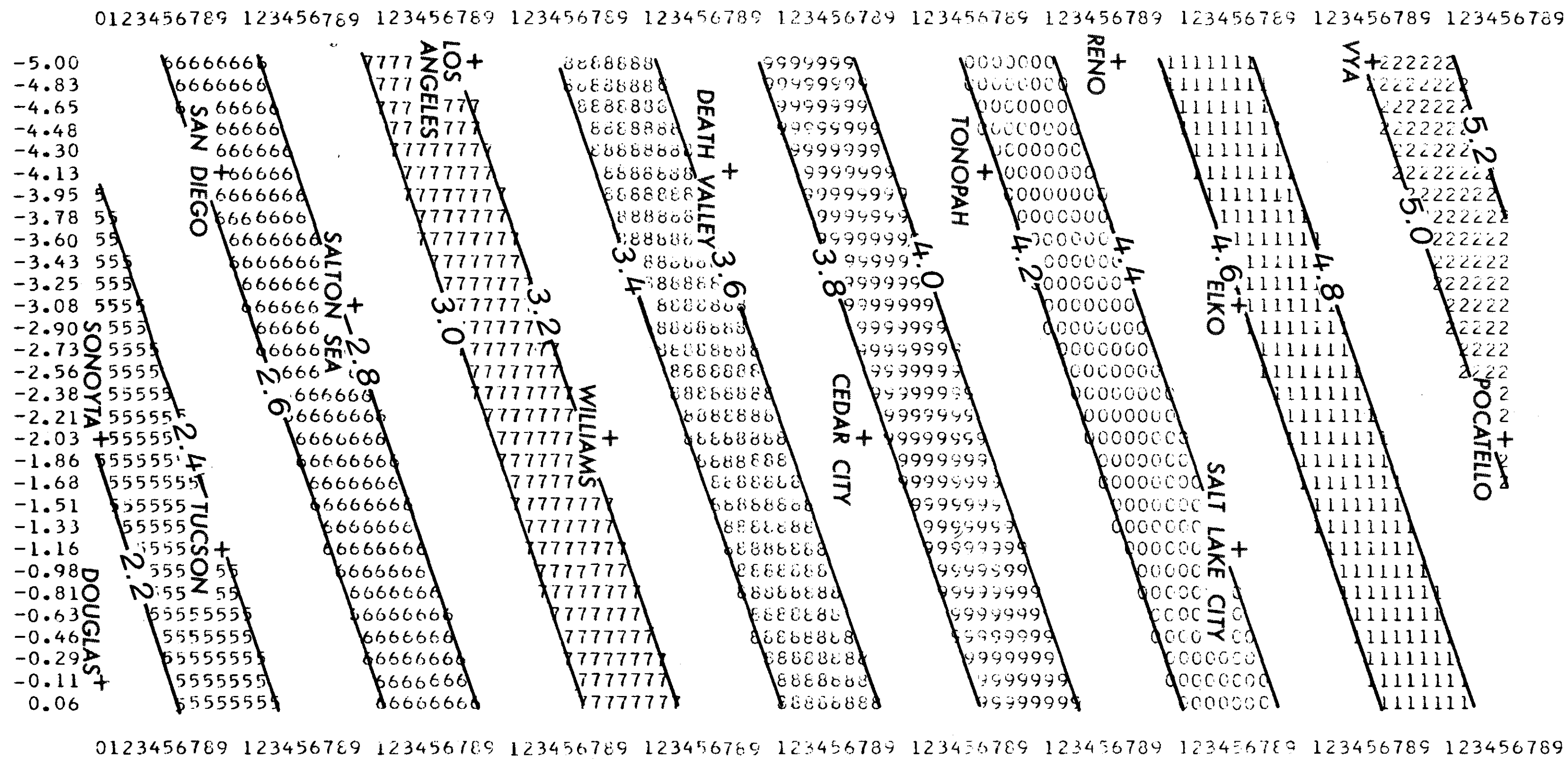

FIGURE 15.—First-degree trend-surface map of mean range width. Contour interval is 0.2 mile. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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MEAN RANGE WIDTH (MILES) - BASIN AND RANGE PROVENCE

CONTOURED QUADRIC SURFACE

PLOTTING LIMITS
MAXIMUM X = -0.000000 MINIMUM X = -5.000000
MAXIMUM Y = 0.000000 MINIMUM Y = -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE = -11.00 + 0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL = 0.20
REFERENCE CONTOUR (.....) = 0.00

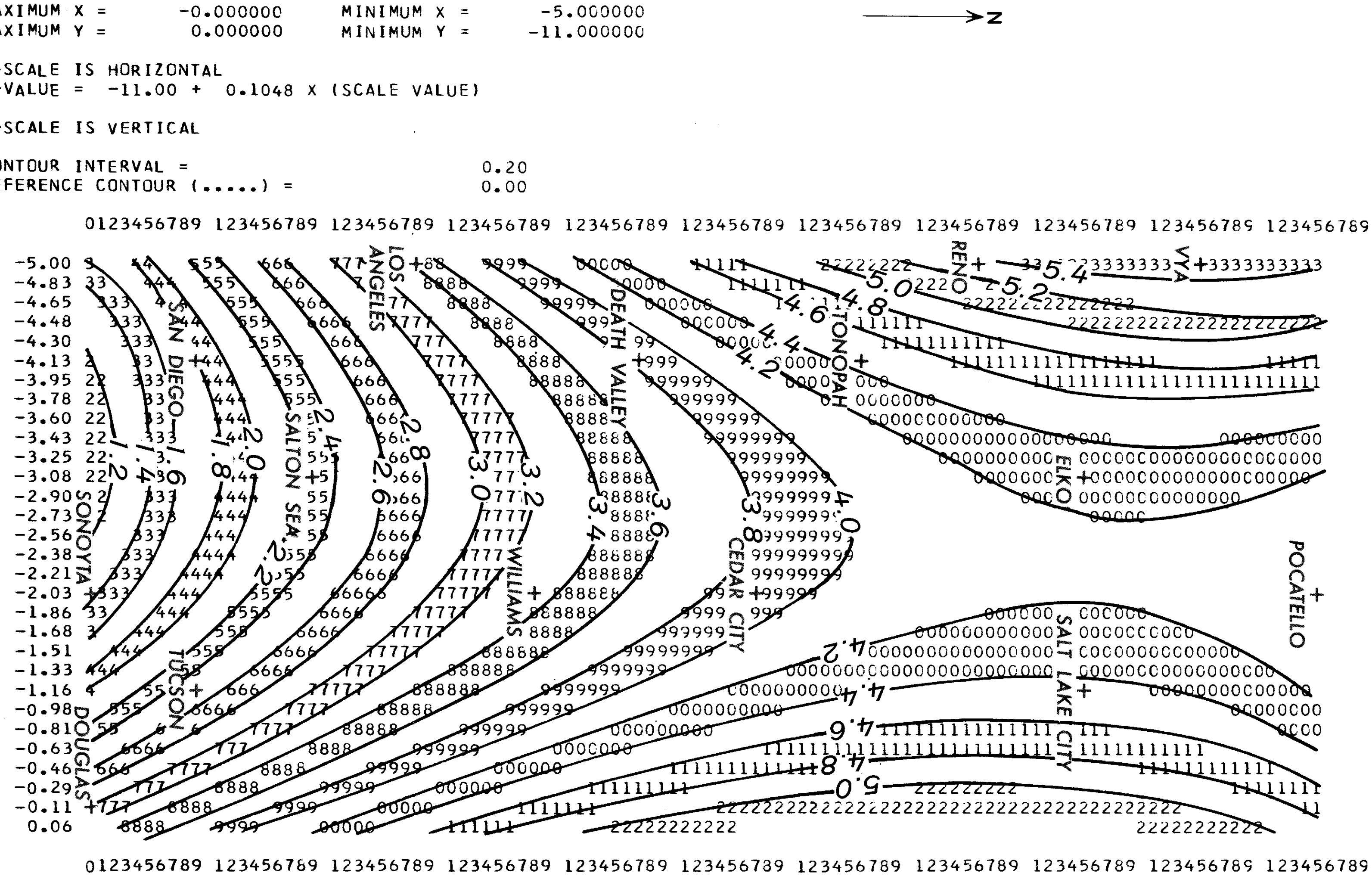

FIGURE 16.—Second-degree trend-surface map of mean range width. Contour interval is 0.2 mile. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

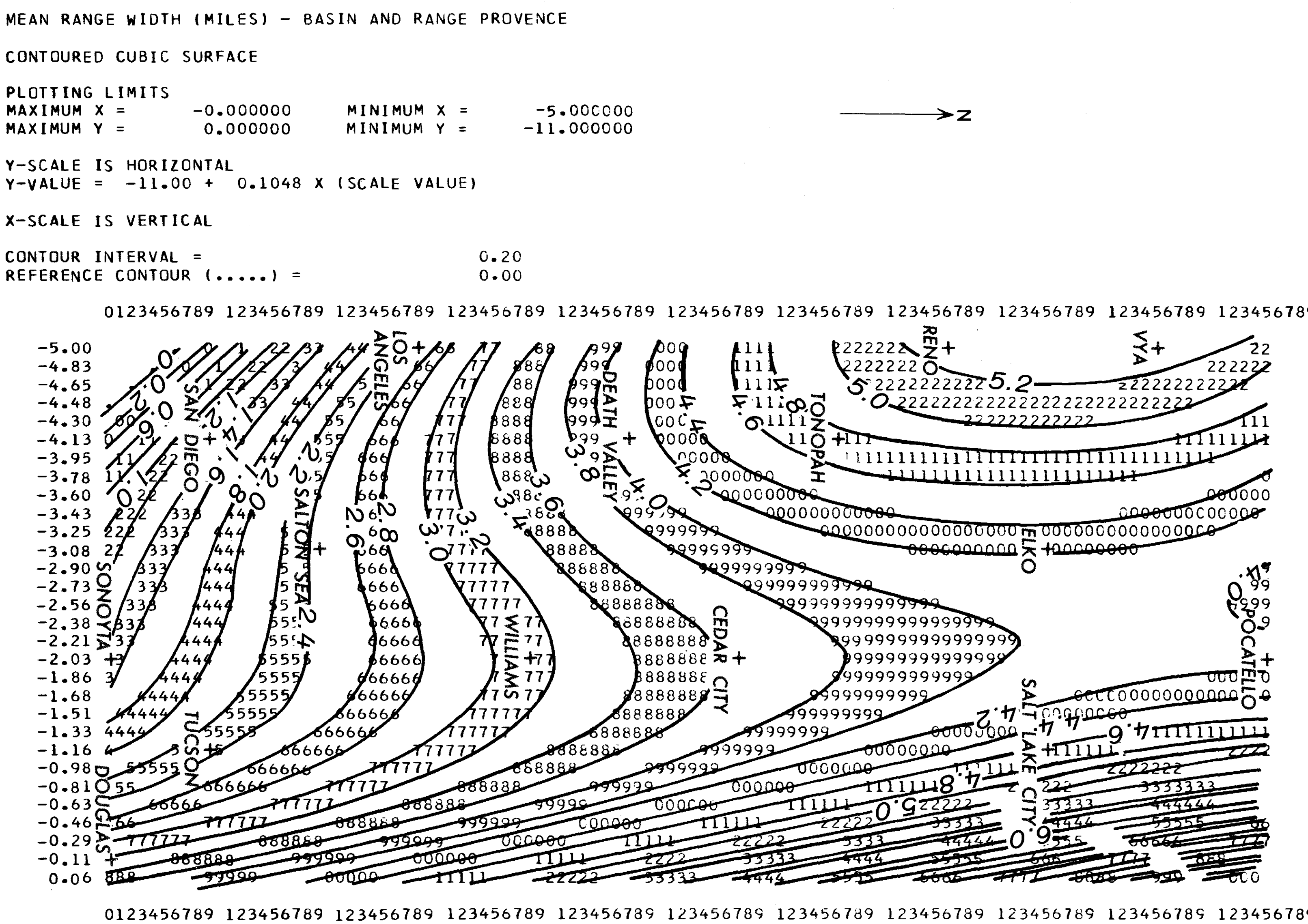

FIGURE 17.—Third-degree trend-surface map of mean range width. Contour interval is 0.2 mile. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

height parameter is difficult to measure in an entirely satisfactory manner. The trend-surface results for range height are therefore subject to some question, but the maps are reasonably consistent with those of range width (figs. 15–17). Figure 18 shows the first-degree trend-surface map of range height, and the plane of best fit is oriented to the northeast; values on this surface range from 1,200 feet in the southwest to more than 2,100 feet in the northeast. The second-degree trend-surface map (fig. 19) shows a paraboloid of best fit whose axis is oriented approximately north-south, and the third-degree trend-surface map (fig. 20) deviates only slightly from this general configuration. Range height, like range width (fig. 17), increases both to the northeast and to the northwest. As stated in connection with range width, however, greater uniformity, or persistence of values, prevails in the northwest, and the large area occupied by the 1,900-foot and 2,000-foot contours attests to this fact. The variation of range height on this third-degree surface of best fit (fig. 20) is approximately 100 percent between the northern and southern parts of the Basin and Range region.

RANGE RELIEF

In measuring range relief an essential distinction is that whereas range height involves the difference in elevation between the crest and base areas of a given range, the range relief is measured from the lowest points in the basins adjacent to a given range (p. D8). For this reason, the absolute values obtained for range relief in this study, and the average values on the trend surfaces of best fit to the regional data, better accord with most workers' concepts of the "size" of the ranges than do the previously discussed values of range height. The first-degree trend-surface map of range relief (fig. 21) shows that, like range height, this parameter increases from southwest to northeast and that on the plane of best fit the magnitude of this increase is approximately 100 percent. The second-degree trend-surface map of range relief (fig. 22) reveals a regional high to the north and to the west and reveals far a more rapid decrease of values to the southwest than to the southeast. The third-degree trend-surface map (fig. 23) depicts a complex hyperbolic surface of best fit. In the southern part of the region, a prominent low is centered about Ajo-Sonoyta (fig. 2), and values increase eastward into New Mexico, westward to the Mohave Desert, and continuously northward toward the dominant high with a contour-closure value of 4,000 feet in Neveda. This third-degree surface clearly shows a marked difference in range relief in the northern and southern parts of the Basin and Range region. Moreover, the decrease to the northeast, although partly induced by edge effects, tends to support the area of ranges to total area ratio and the range length parameter because the northeastern values of range relief are more closely comparable to values in the south than in the north.

RANGE VOLUME

Range volume was chosen as a descriptive topographic parameter for several reasons, but it is obvious that the regional variation in range volume should be roughly compatible with the regional variation in the area of ranges to total area ratio. This expectation is sustained by the trend-surface maps of range volume. The first-degree surface of best fit is shown in figure 24. The plane exhibits a regional trend to the northwest, and values on the surface range from less than 2.5 cubic miles in the southeast to more than 32.5 cubic miles in the northwest. This range of values, namely 13–14 times the minimum volumes obtained is closely comparable to the 13-fold increase in the area of ranges to total area ratio (fig. 9). The second-degree trend-surface map (fig. 25) shows that the values on this surface of best fit to the range volume data follow an arcuate path from the high in the northwest, through the central part of the region, to the southwest, where they decrease to less than 2.5 cubic miles in southern California and southwestern Arizona. The third-degree trend-surface map (fig. 26) reveals the persistent high in the northwest that is accentuated by the 35-cubic-mile contour closure and by the marked trough to the south. This map is very similar to the third-degree trend-surface map of area of ranges to total area (fig. 11) in this regard. The chief difference occurs in the northeast; range volume attains maximum values in the northeast corner of the area shown in figure 26. This is partly due to edge effects, but it also reflects the fact that high width (fig. 17) and height (fig. 20) values exist in this area; both parameters are involved in the computation of range volume.

The third-degree trend-surface map of range volume (fig. 26) shows that regional differences clearly exist within the Basin and Range area, with respect to this parameter, and that, in addition to a lesser mean range area in the south, there is a lesser mean range volume. The implications of this fact will be discussed later in this report.

CUMULATIVE LENGTH OF TRENDS

Cumulative length is the first of two parameters that were devised to represent escarpment trends in the Basin and Range region. Although somewhat variable because of the measurement criteria used (p. D8), maximum values of comulative length of trends will tend to occur in areas where topographic escarpments are longest, whereas minimum values will tend to be associated with areas containing the shortest escarpments. Accordingly,

```
MEAN RANGE HEIGHT (FEET) - BASIN AND RANGE PROVENCE

CONTCUREC LINEAR SURFACE

PLCTTING LIMITS
MAXIMUM X =       -0.000000      MINIMUM X =       -5.000000
MAXIMUM Y =        0.000000      MINIMUM Y =      -11.000000

Y-SCALE IS HORIZCNTAL
Y-VALUE =   -11.00 +   0.1C48 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTCUR INTERVAL =                          100.0C
REFERENCE CCNTCUR (.....) =                   0.00
```

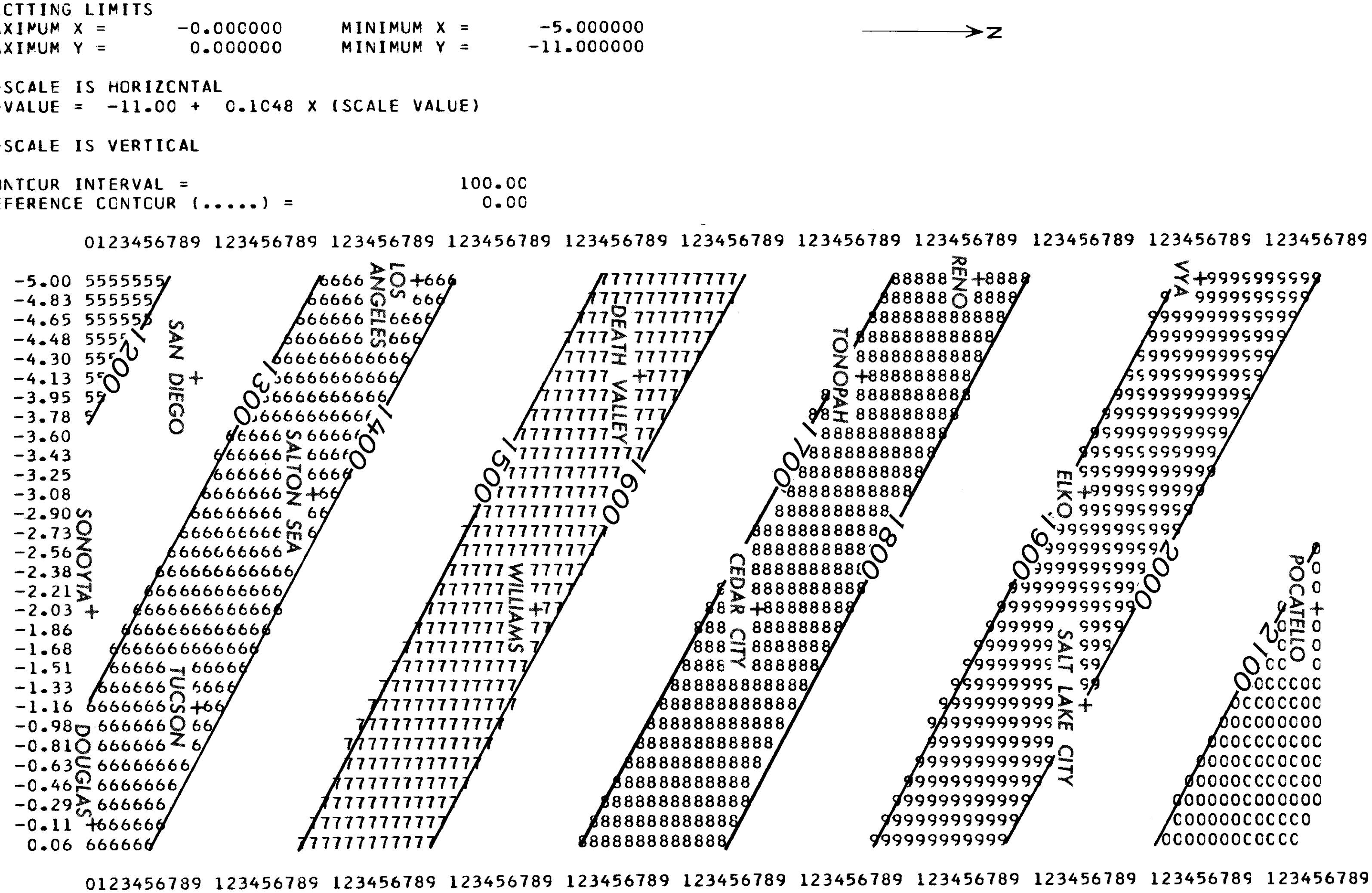

FIGURE 18.—First-degree trend-surface map of mean range height. Contour interval is 100 feet. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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MEAN RANGE HEIGHT (FEET) - BASIN AND RANGE PROVENCE

CONTOURED QUADRIC SURFACE

PLOTTING LIMITS
MAXIMUM X =       -0.000000      MINIMUM X =      -5.000000
MAXIMUM Y =        0.000000      MINIMUM Y =     -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =  -11.00 +  0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                        100.00
REFERENCE CONTOUR (.....) =                 0.00
```

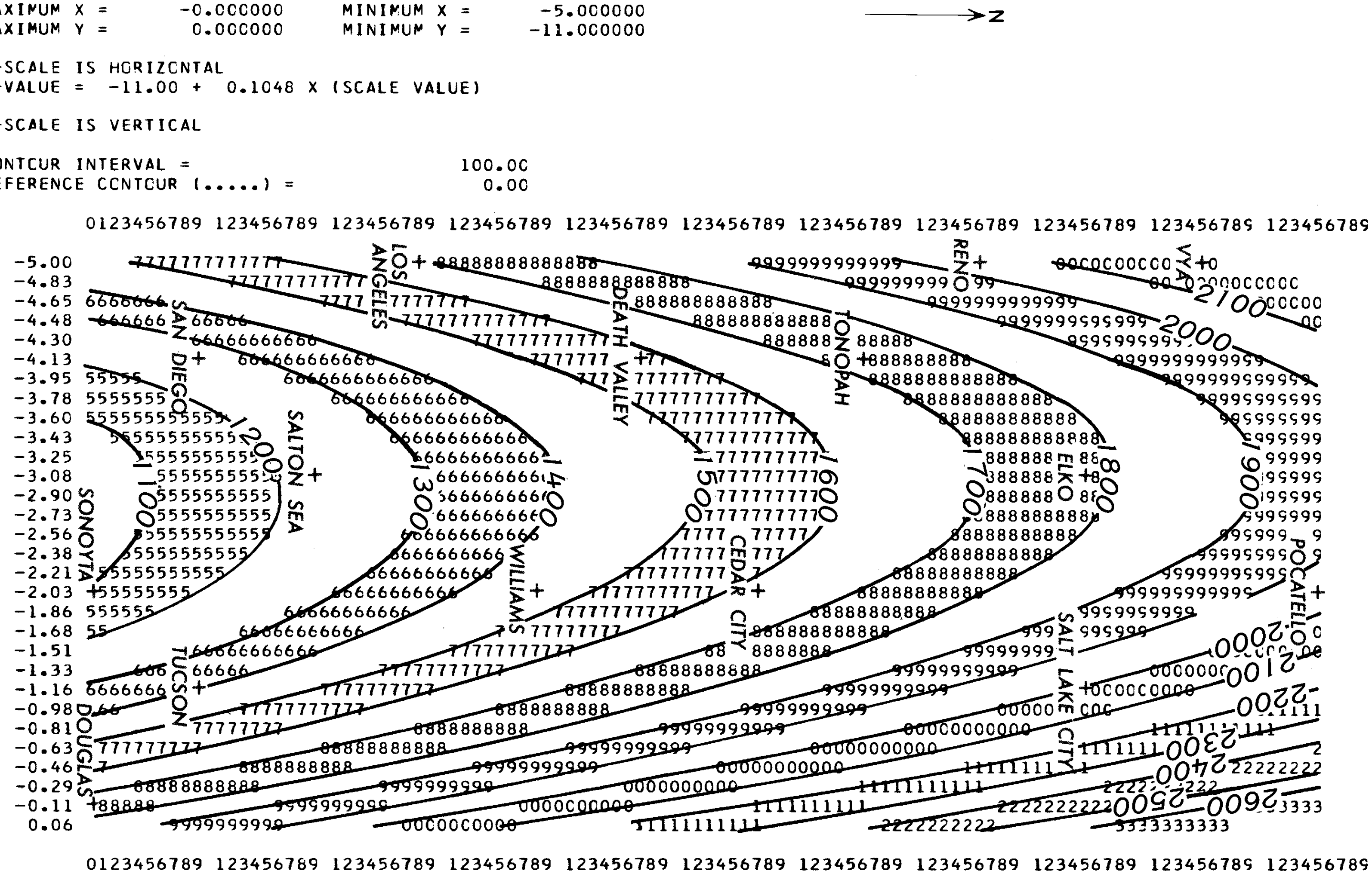

FIGURE 19.—Second-degree trend-surface map of mean range height. Contour interval is 100 feet. The geographic names shown correspond to the locations of the 1:250,000 topographic maps.

```
MEAN RANGE HEIGHT (FEET) - BASIN AND RANGE PROVENCE

CONTCURED CUBIC SURFACE

PLCTTING LIMITS
MAXIMUM X =      -0.000000     MINIMUM X =      -5.000000
MAXIMUM Y =       0.000000     MINIMUM Y =     -11.000000

Y-SCALE IS HORIZCNTAL
Y-VALUE =  -11.0C +  0.1C48 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTCUR INTERVAL =                  100.0C
REFERENCE CONTCUR (.....) =           0.0C
```

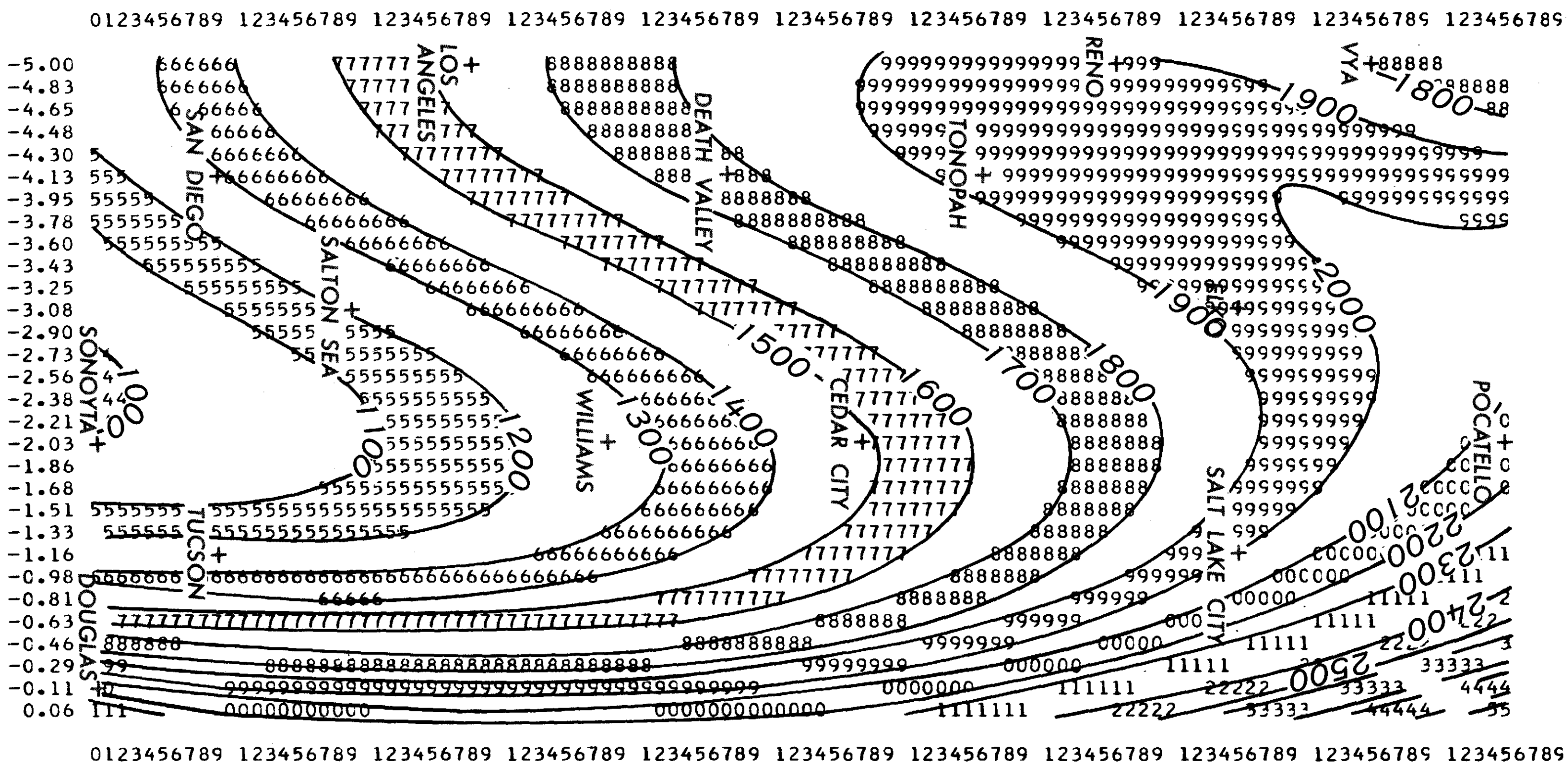

FIGURE 20.—Third-degree trend-surface map of mean range height. Contour interval is 100 feet. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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MEAN RANGE RELIEF (FEET) - BASIN AND RANGE PROVENCE

CONTOURED LINEAR SURFACE

PLOTTING LIMITS
MAXIMUM X =      -0.000000     MINIMUM X =      -5.000000
MAXIMUM Y =       0.000000     MINIMUM Y =     -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =  -11.00 +  0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                 200.00
REFERENCE CONTOUR (.....) =          0.00
```

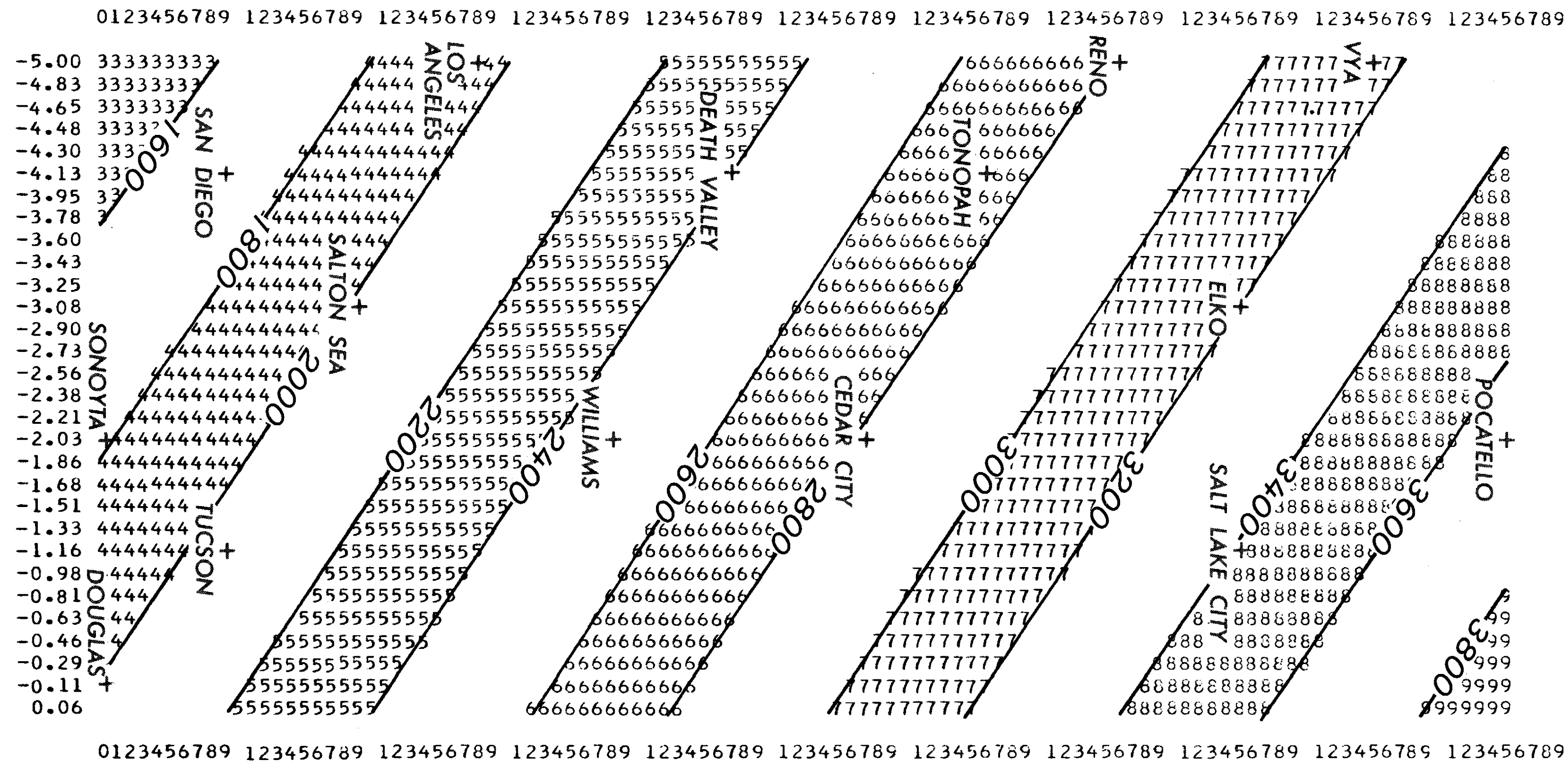

FIGURE 21.—First-degree trend-surface map of mean range relief. Contour interval is 200 feet. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

MEAN RANGE RELIEF (FEET) - BASIN AND RANGE PROVENCE

CONTOURED QUADRIC SURFACE

PLOTTING LIMITS
MAXIMUM X = -0.000000 MINIMUM X = -5.000000
MAXIMUM Y = 0.000000 MINIMUM Y = -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE = -11.00 + 0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL = 200.00
REFERENCE CONTOUR (.....) = 0.00

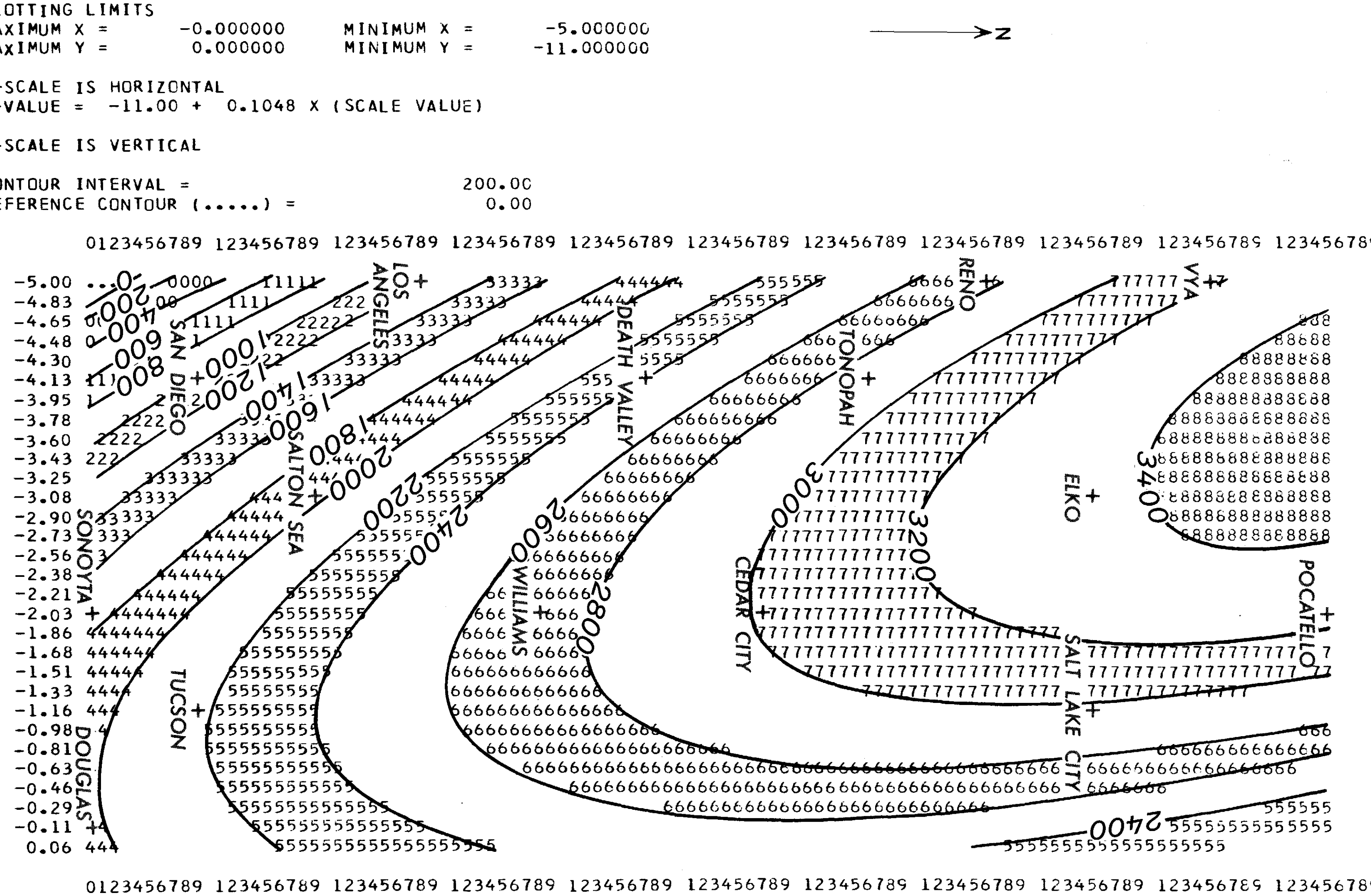

FIGURE 22.—Second-degree trend-surface map of mean range relief. Contour interval is 200 feet. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

MEAN RANGE RELIEF (FEET) - BASIN AND RANGE PROVENCE

CONTOURED CUBIC SURFACE

PLOTTING LIMITS
MAXIMUM X = -0.000000 MINIMUM X = -5.000000
MAXIMUM Y = 0.000000 MINIMUM Y = -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE = -11.00 + 0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL = 200.00
REFERENCE CONTOUR (.....) = 0.00

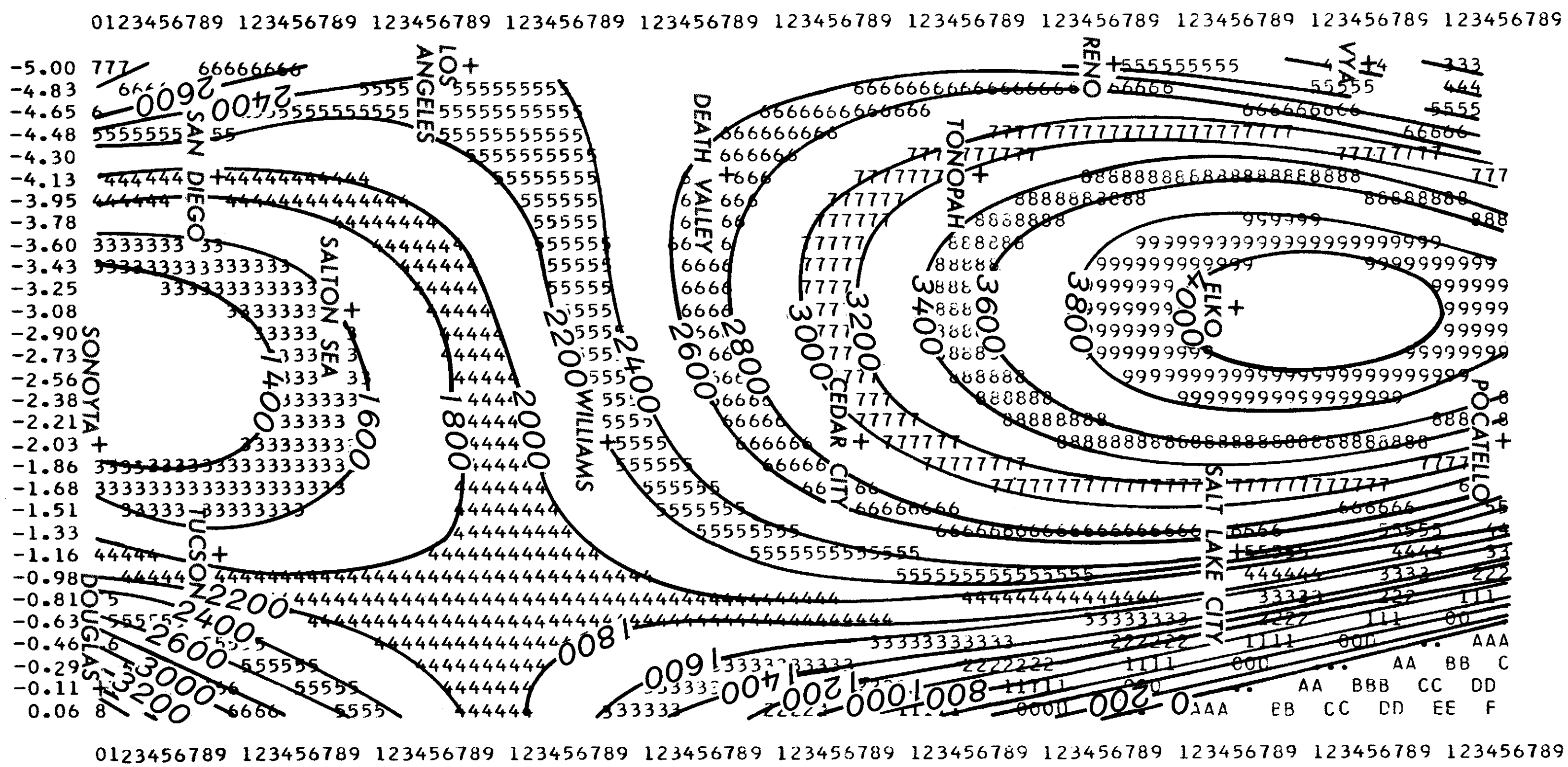

FIGURE 23.—Third-degree trend-surface map of mean range relief. Contour interval is 200 feet. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

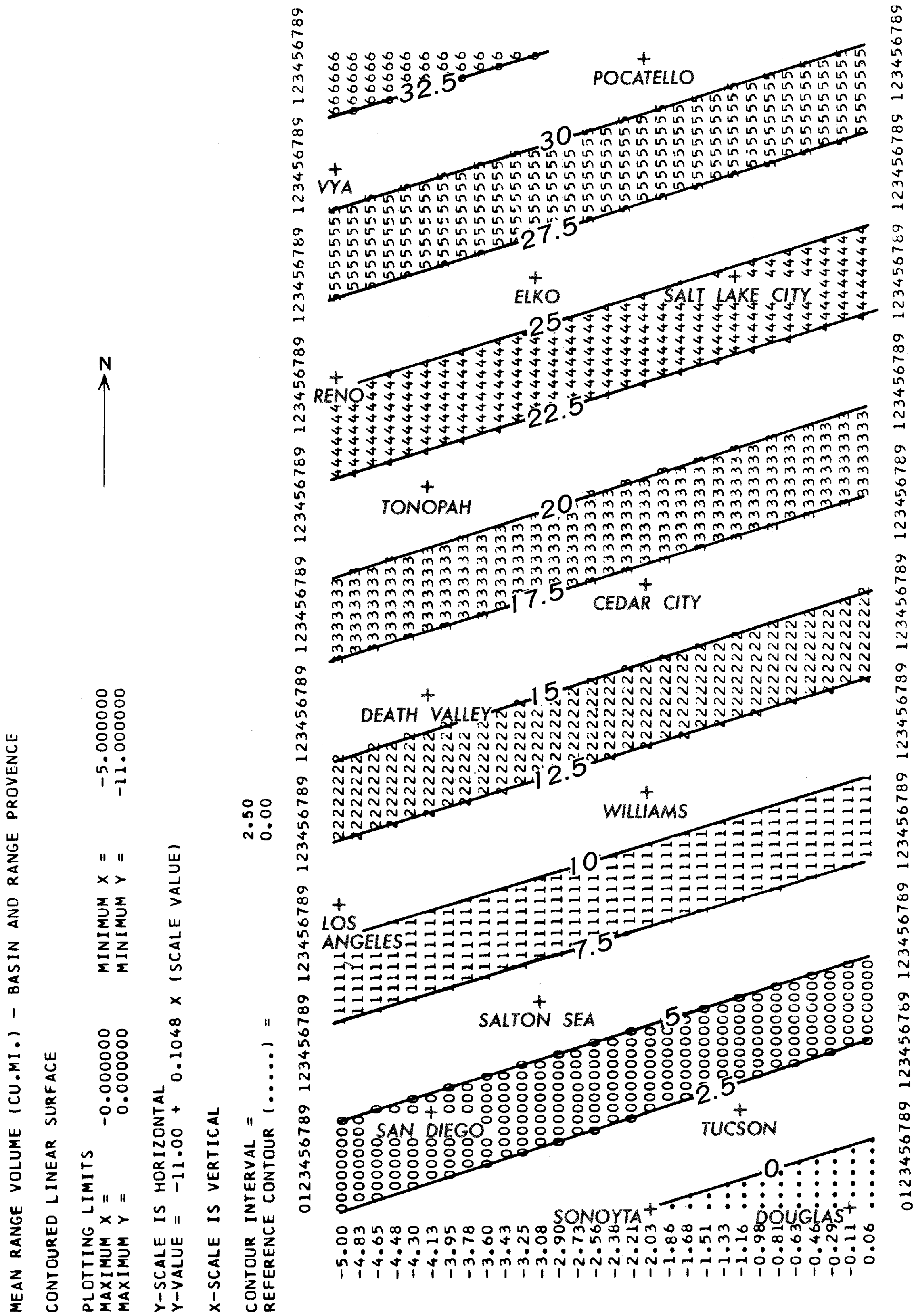

FIGURE 24.—First-degree trend-surface map of mean range volume. Contour interval is 2.5 cubic miles. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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MEAN RANGE VOLUME (CU.MI.) - BASIN AND RANGE PROVENCE

CONTOURED QUADRIC SURFACE

PLOTTING LIMITS
MAXIMUM X =     -0.000000      MINIMUM X =      -5.000000
MAXIMUM Y =      0.000000      MINIMUM Y =     -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =  -11.00 +  0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                        2.50
REFERENCE CONTOUR (.....) =               0.00
```

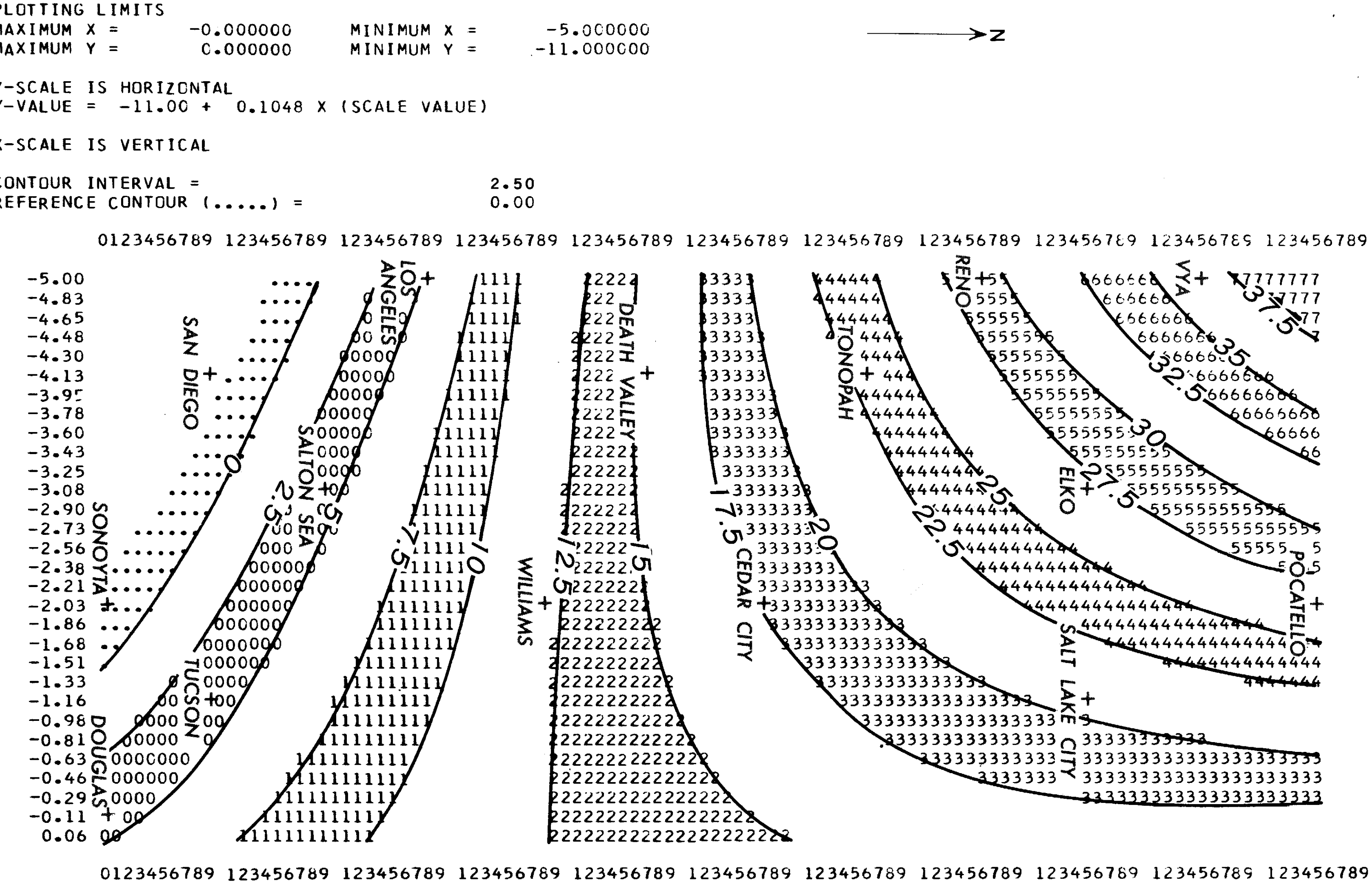

FIGURE 25.—Second-degree trend-surface map of mean range volume. Contour interval is 2.5 cubic miles. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

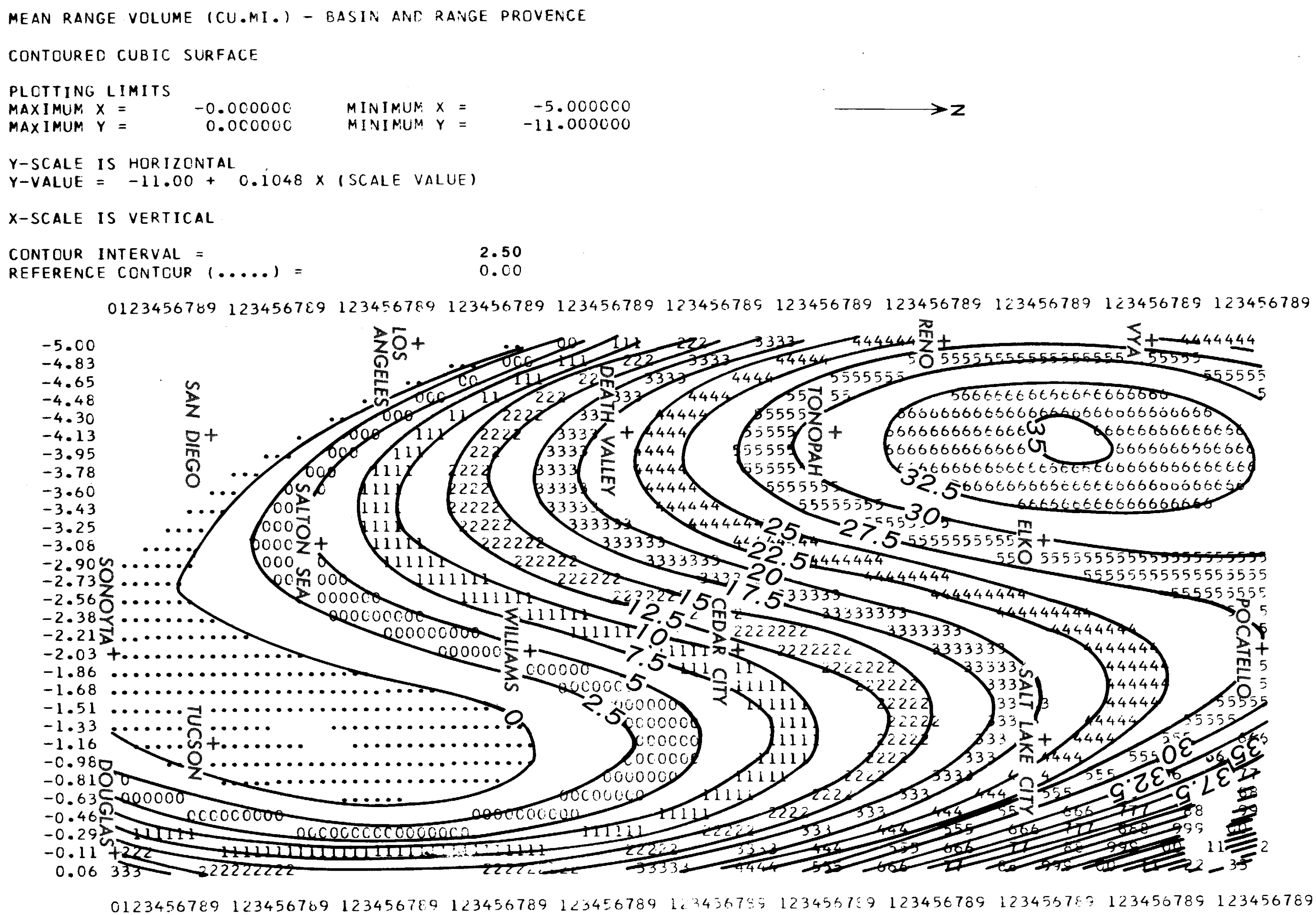

FIGURE 26.—Third-degree trend-surface map of mean range volume. Contour interval is 2.5 cubic miles. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

the first-degree trend-surface map of this parameter (fig. 27) shows that longer escarpments and greater cumulative lengths occur in the western and northwestern parts of the region. The range of values on this plane of best fit, from less than 14 miles in the southeast to more than 77 miles in the northwest, or approximately six times the minimum value, is greater than the range of values associated with range width, height, and relief, and less than that associated with range area and volume.

The second-degree trend-surface map of cumulative length of trends (fig. 28) provides better definition of the regional variation of this parameter. The paraboloid of best fit can be seen to trend southeast-northwest, and minimum values are attained to the northeast, southeast, and southwest. The third-degree trend-surface map (fig. 29) reinforces this impression and provides further definition of regional variation. The pattern is similar to that obtained for the area ratio (fig. 11), range length (fig. 14), range relief (fig. 23) and range volume (fig. 26) parameters. The pattern tends to support the idea that the area occupied by the 91-mile cumulative-length high in the northwest (fig. 29) is one containing longer, larger, and more massive ranges than generally occur elsewhere in the region. Thus, the trend-surface maps of cumulative length of trends also suggest that marked topographic differences exist within the Basin and Range region.

CUMULATIVE DEVIATION OF TRENDS

In addition to the lengths of trends of escarpments and the regional variation of cumulative length, it is obviously useful to specify the orientation of such trends. The parameter expressing the cumulative deviation of trends from due north serves to accomplish this purpose. Maximum values of this parameter in a given area reflect maximum deviation from the north-south direction within that area. Figure 30 shows the first-degree trend-surface map of cumulative deviation of trends. The values on the plane of best fit increase from less than 12° in the northeast to 42° in the northwest. Because the qualitative assessment of range trends in terms of "An army of caterpillars marching to Mexico * * * would necessarily coincide with a cumulative deviation of 0° these quantitative results obviously render specious such a description.

The second-degree trend-surface map of cumulative deviation of trends (fig. 31) provides further definition of the regional variations. Minimum deviation occurs in the north-central part of the region, and the values on the paraboloid of best fit increase to the south, southeast, and southwest. Several additional refinements appear on the third-degree trend-surface map for this parameter (fig. 32). The high values to the southwest are misleading because they are produced, in part, by the inclusion in the average of one or two topographic maps in the southwest which were dominated by a single range trending nearly east-west. Similarly, the decrease in cumulative deviation to zero in the northeast is more apparent than real. But the 15° contour closure in the northwest is quite real, and the coincidence of location of this regional low and the several regional high values for parameters previously discussed will be apparent to the reader. Also, the general range of values from about 30° to 40° across the southern part of the Basin and Range region reflects the topographic conditions rather faithfully. This third-degree map indicates that ranges in the northern part of the Basin and Range region deviate from due north much less than do the ranges in the southern part. In other words, consideration of the cumulative deviation of trends, like the previous parameters discussed, suggests that very substantial quantitative differences in topography exist in the Basin and Range region.

RANGE WIDTH TO LENGTH

Range width to length is the first of three derived ratios that were used to further characterize regional topography in the Basin and Range region. Any such ratio obviously serves to compound the sins of which its component variables may be guilty. That is, if either width or length values are suspect for some reason in any given local area, a ratio of these two variables will not obviate such difficulty. The width to length ratio, like the other two derived ratios yet to be discussed, should merely be considered as an alternative means of representing regional topographic variations. They are not independent of their individual components, but these ratios do reveal the manner in which a given variable changes in relation to a second variable throughout the region.

Figure 33 shows the first-degree trend-surface map of range width to length. From previous knowledge of the general distribution of range width (fig. 15) and length (fig. 12) values, one would anticipate this outcome. In general, ranges tend to increase both in length and in width from south to north, but as shown by the plane of best fit to the data, the magnitude of the increase of range length is greater. For this reason range width to length ratios decrease northward. The range of values on this first-order surface of best fit (fig. 33) is from 0.12 to 0.24, or 100 percent. This range of values is similar to the minimum range for most of the parameters considered thus far.

The second-degree trend-surface map of range width to length (fig. 34) shows that the paraboloid of best

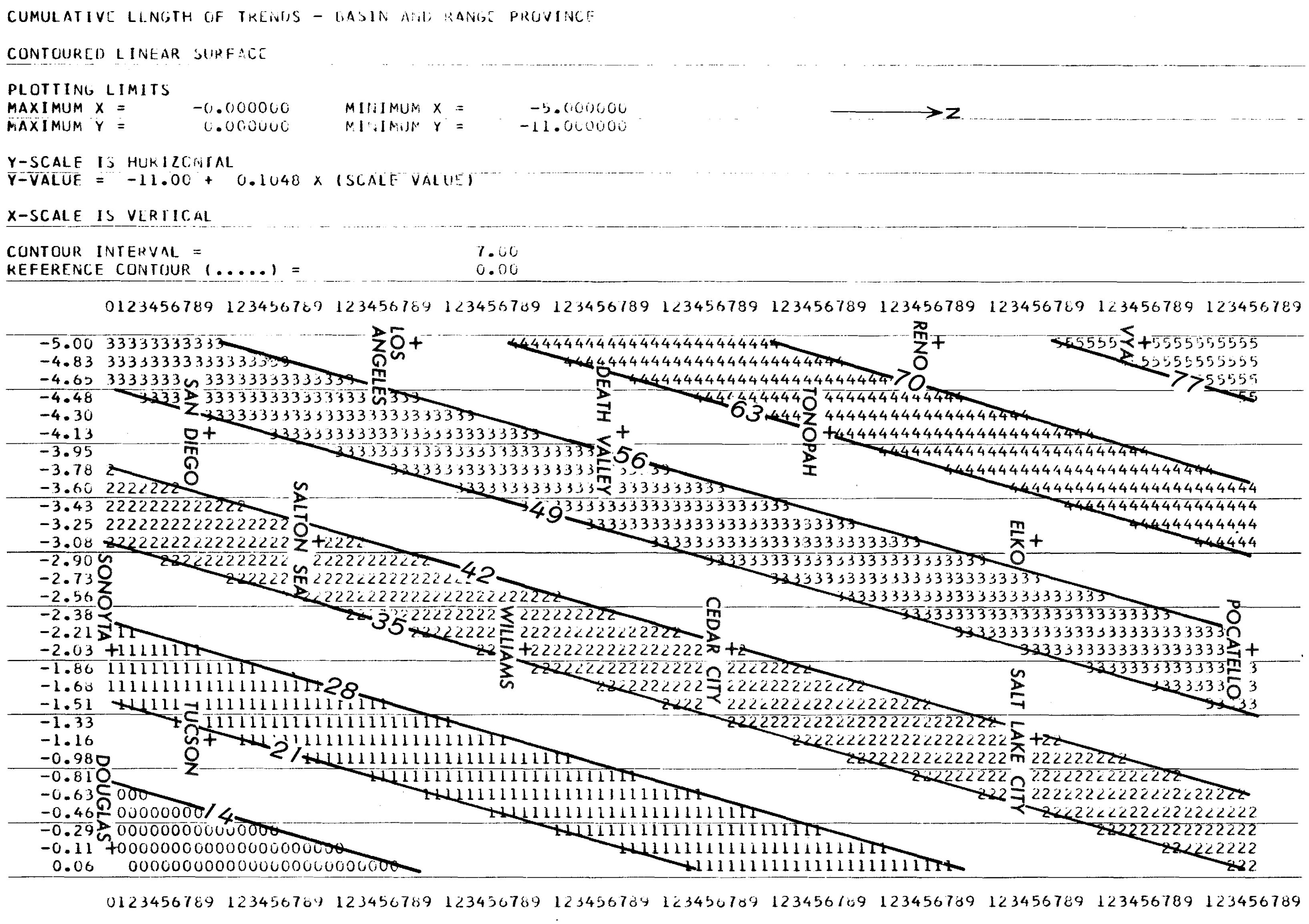

FIGURE 27.—First-degree trend-surface map of cumulative length of trends. Contour interval is 7 miles. The geographic names shown correspond to the locations of the 1 : 250,000 topographic quadrangle maps.

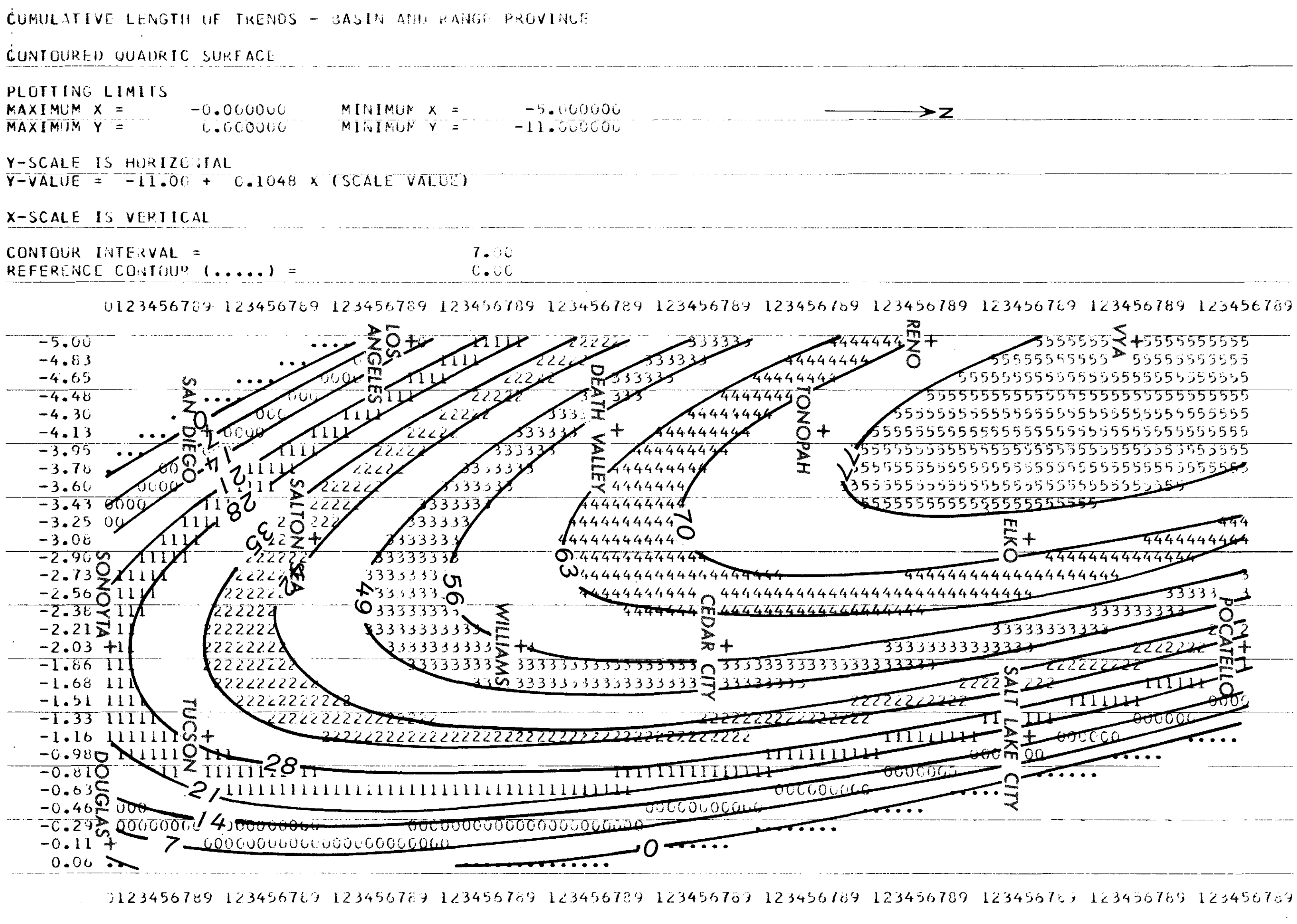

FIGURE 28.—Second-degree trend-surface map of cumulative length of trends. Contour interval is 7 miles. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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CUMULATIVE LENGTH OF TRENDS - BASIN AND RANGE PROVINCE

CONTOURED CUBIC SURFACE

PLOTTING LIMITS
MAXIMUM X =        -0.000000       MINIMUM X =        -5.000000
MAXIMUM Y =         0.000000       MINIMUM Y =       -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =  -11.00 +  0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                   7.00
REFERENCE CONTOUR (.....) =          0.00
```

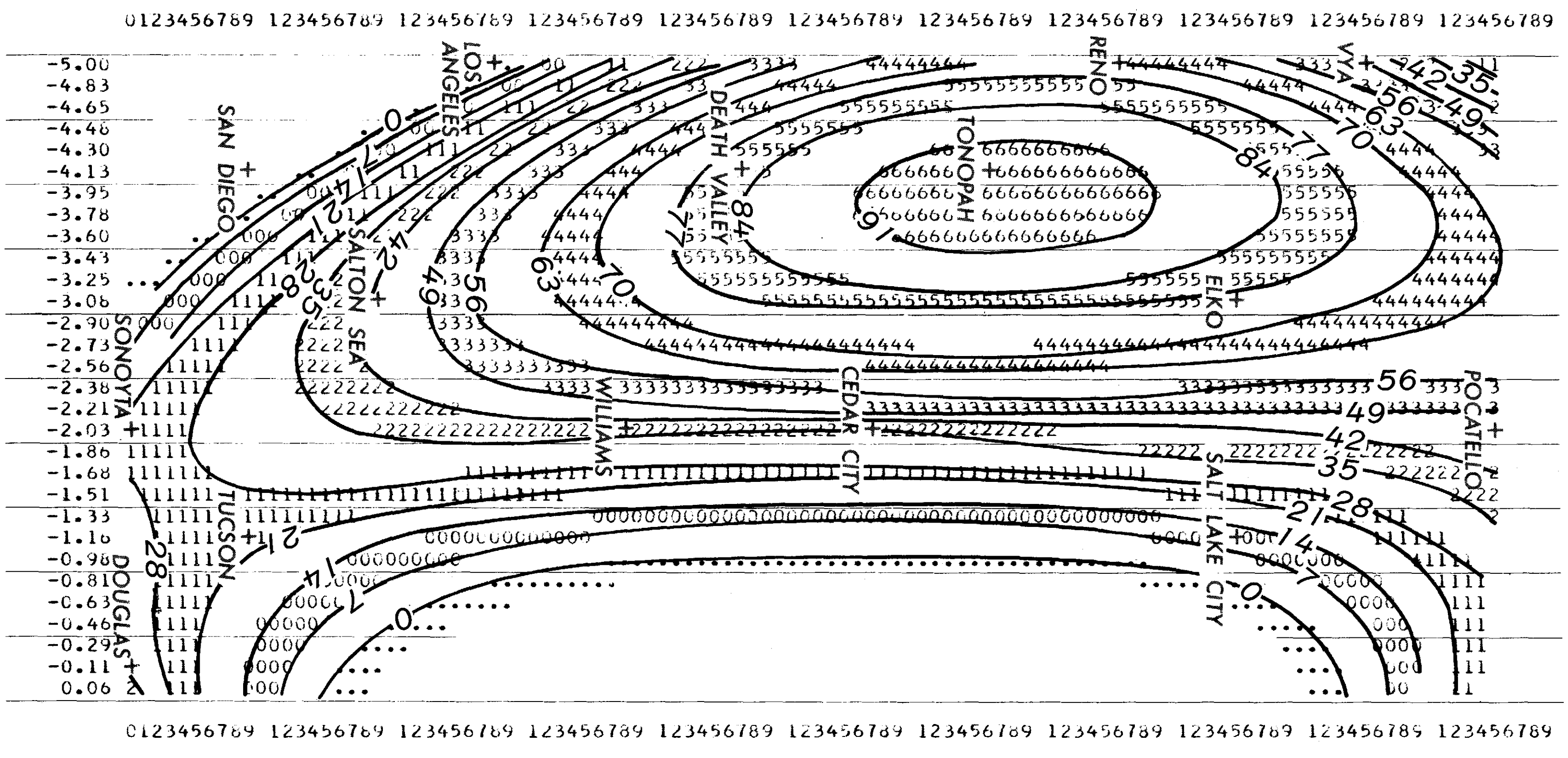

FIGURE 29.—Third-degree trend-surface map of cumulative length of trends. Contour interval is 7 miles. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

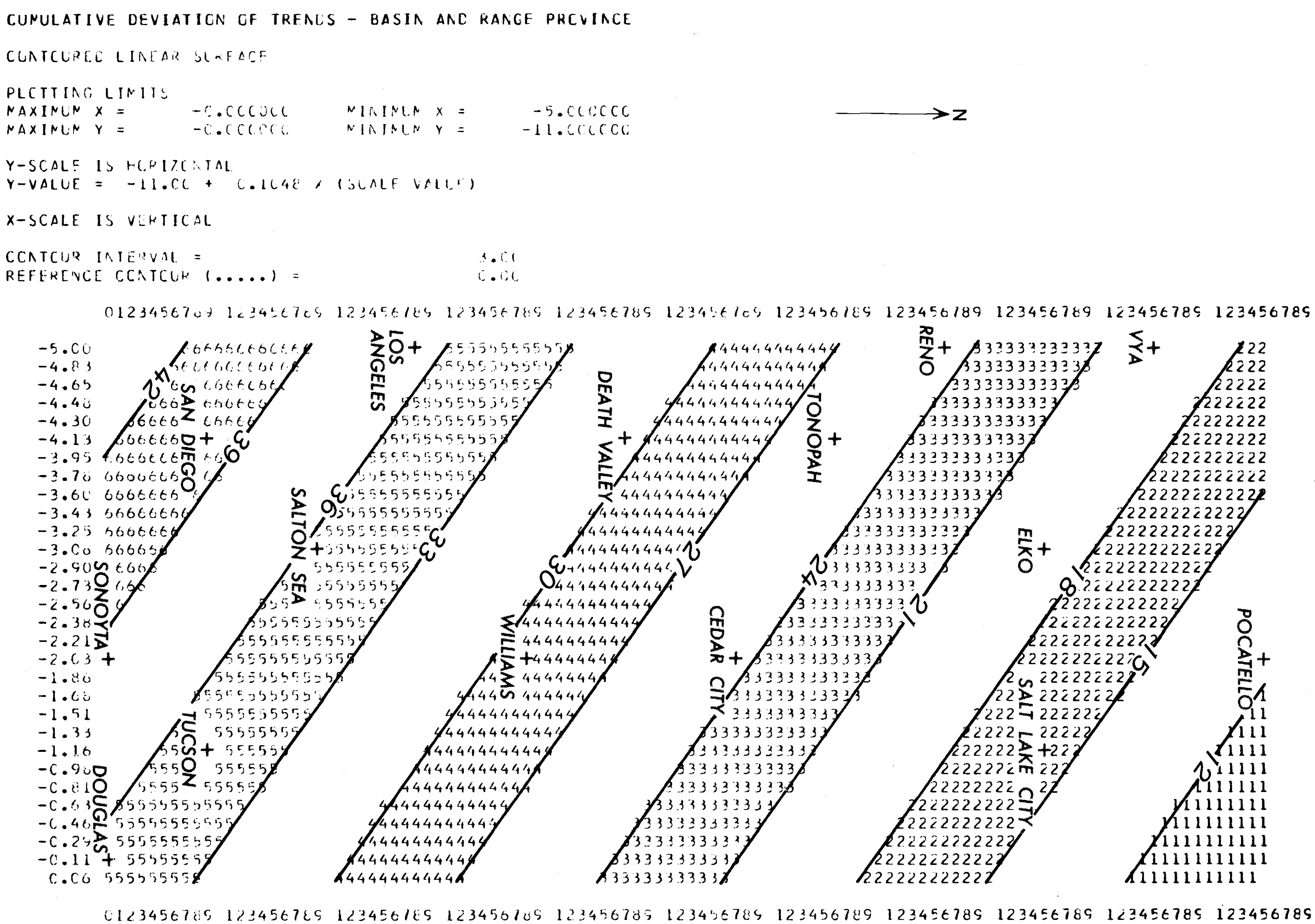

FIGURE 30.—First-degree trend-surface map of cumulative deviation of trends. Contour interval is 3°. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

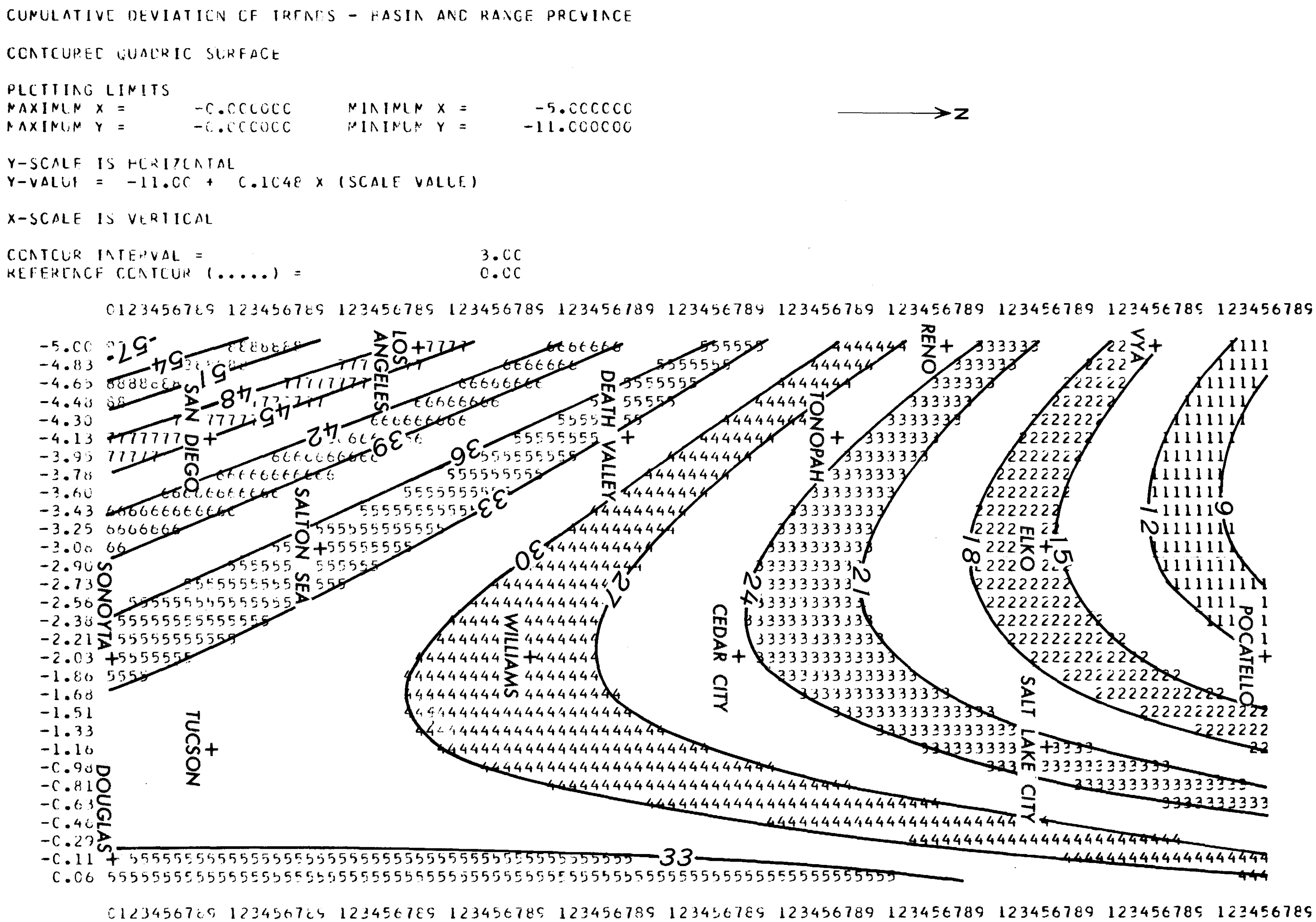

FIGURE 31.—Second-degree trend-surface map of cumulative deviation of trends. Contour interval is 3°. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

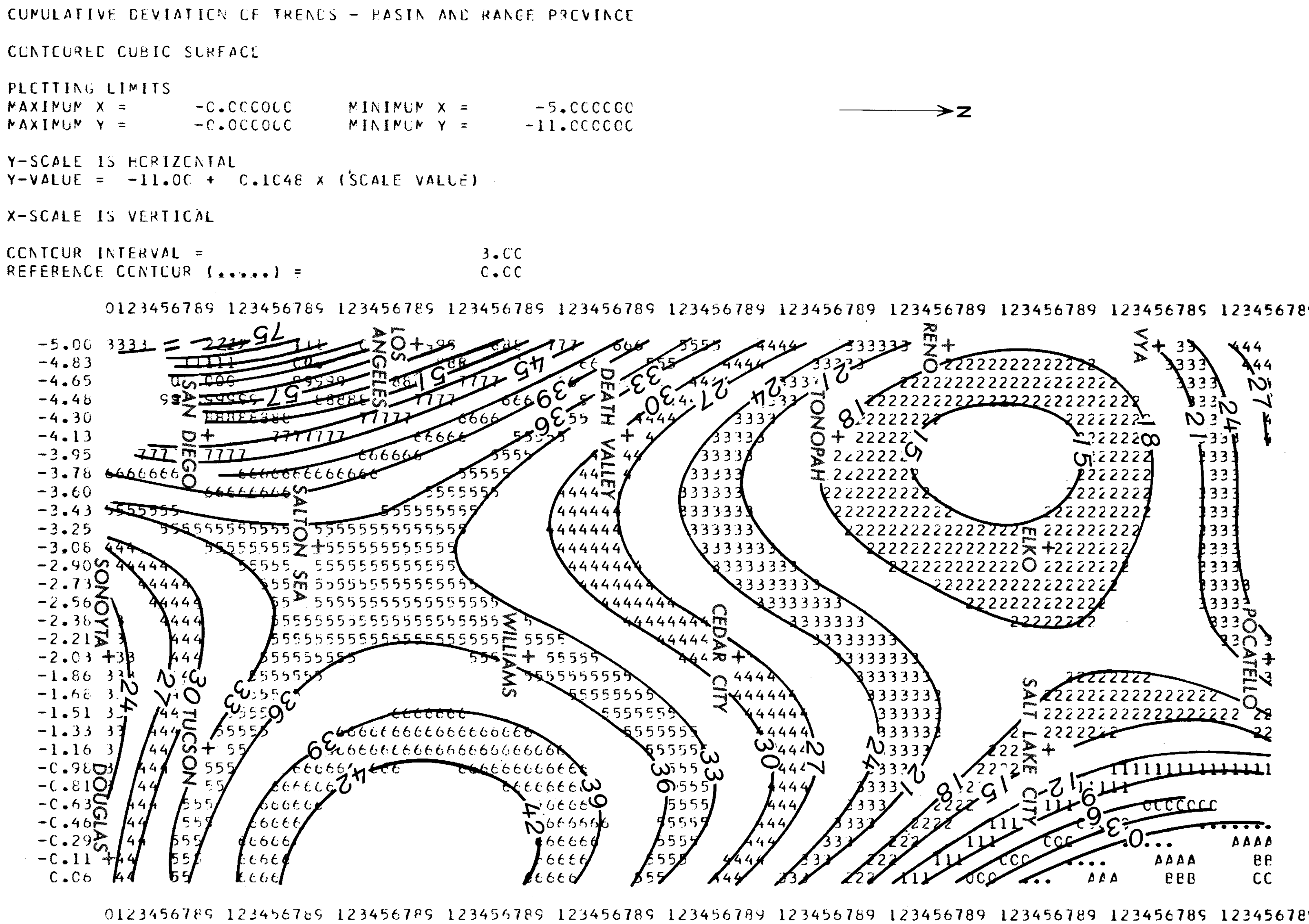

FIGURE 32.—*Third-degree trend-surface map of cumulative deviation of trends. Contour interval is 3°. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.*

```
RANGE WIDTH TO LENGTH - BASIN AND RANGE PROVENCE

CONTOURED LINEAR SURFACE

PLOTTING LIMITS
MAXIMUM X =        -0.000000     MINIMUM X =      -5.000000
MAXIMUM Y =         0.000000     MINIMUM Y =     -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =  -11.00 +  0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                          0.02
REFERENCE CONTOUR (.....) =                 0.00
```

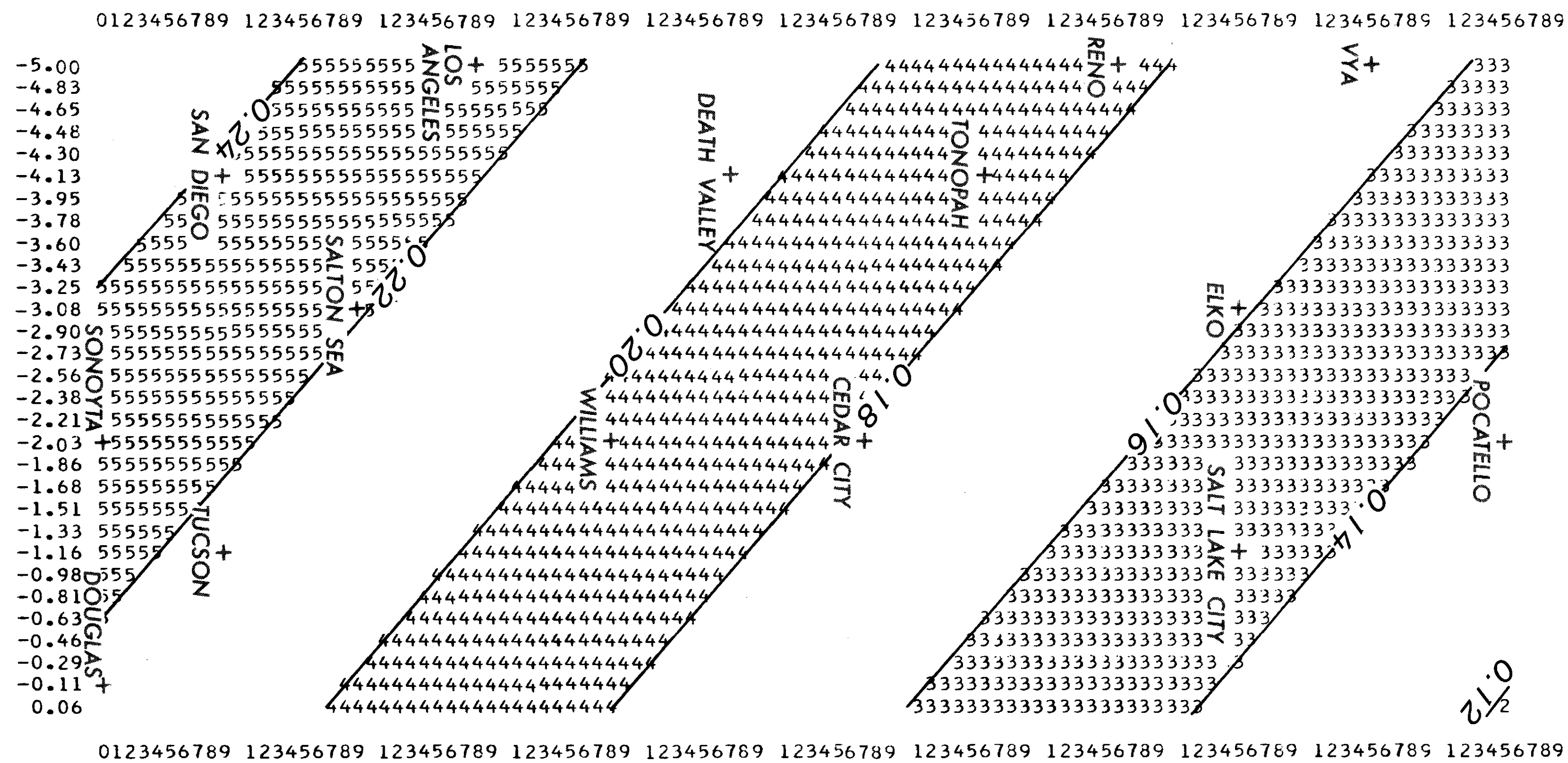

FIGURE 33.—First-degree trend-surface map of range width to length. Contour interval is 0.02. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

RANGE WIDTH TO LENGTH - BASIN AND RANGE PROVENCE

CONTOURED QUADRIC SURFACE

PLOTTING LIMITS
MAXIMUM X = -0.000000 MINIMUM X = -5.000000
MAXIMUM Y = 0.000000 MINIMUM Y = -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE = -11.00 + 0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL = 0.02
REFERENCE CONTOUR (.....) = 0.00

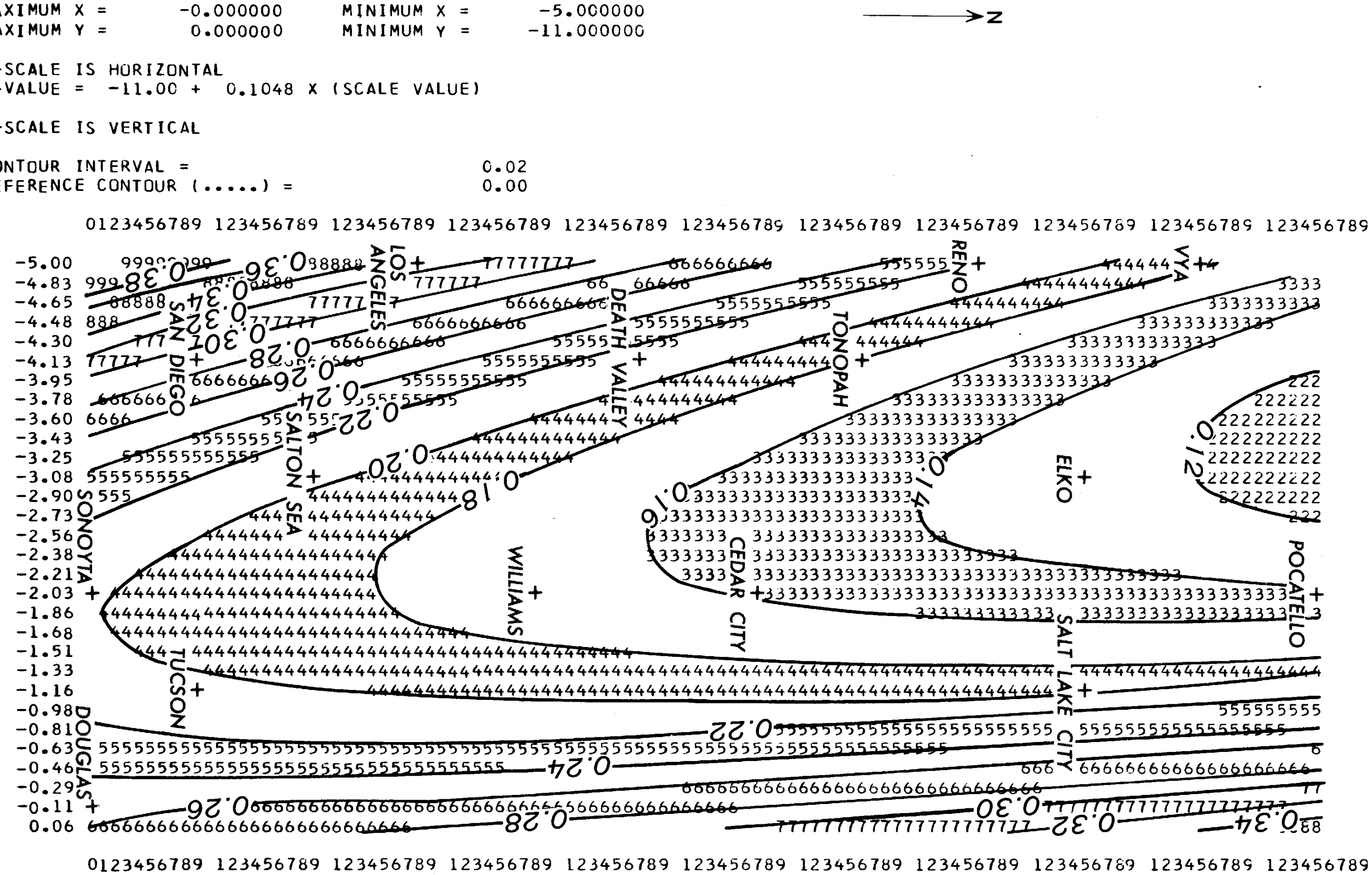

FIGURE 34.—Second-degree trend-surface map of range width to length. Contour interval is 0.02. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

fit has a slight northwest-southeast orientation. A regional low exists in the central and northwestern parts of the Basin and Range region, where the range lengths are great, and the high values of the width to length parameter occur elsewhere. This is particularly true in the Mohave Desert to the southwest and along the eastern margin of the region, where values as large as 0.30 or more occur. The third-degree trend-surface map of this topographic parameter (fig. 35) shows an accentuation of these same regional trends. The north-central and northwestern parts of the Basin and Range region clearly differ from the rest of the area on the basis of width to length ratios.

RANGE WIDTH TO HEIGHT

Range width to height is the second of the three derived topographic parameters considered here. An increase in this ratio can be caused in several ways—namely by an increase in width, by a decrease in height, or by some combination of effects, such as an increase in both variables whereby width increases at a greater rate than height. The general distribution of range width (fig. 15) and range height (fig. 18) values on surfaces of best fit in the Basin and Range region indicates that both the widths and heights of ranges tend to increase from south to north. The first-degree trend-surface map of range width to height (fig. 36) shows, however, that this ratio increases from 8.8 in the southeast to more than 13.2 in the northwest, on the plane of best fit. The second-degree trend-surface map (fig. 37) of this parameter more clearly delineates the regional high to the northwest and the widespread low in the southern part of the region, which decreases to a minimum value in the southwest. The range in values on the paraboloid of best fit is greater than 100 percent of the minimum, and the results clearly demonstrate that the ranges in the northwest are distinctive because the range widths increase more rapidly with range heights than elsewhere in the region.

The third-degree trend-surface map of range width to height ratios (fig. 38) is more complex. The trough of regional low values in the northeast on the second-degree map (fig. 37) has migrated to the north-central region, but the northwest regional high is still in evidence. South of the central field, which is characterized by range width to height values of about 12:1, a general decrease in this ratio is apparent. The extreme low to the southwest, like the high to the northeast, is partly induced by edge effects and by peculiarities of one or two sheets which gave somewhat anomalous z-values. The general regional trends are very real, however, and this map of the third-degree surface of best fit (fig. 38) indicates that substantial regional differences exist in the width to height parameter.

RANGE LENGTH TO HEIGHT

The last of the three derived topographic ratios, and the last of the 11 parameters considered is range length to height. As was true of range width to length and range width to height ratios, the regional variation in range length to height values is caused by the fact that, although both component variables increase from south to north (figs. 12, 18), they do not increase uniformly. The magnitude of the increase in average range length in a northward direction is much greater than the increase in height, and maximum length to height ratios therefore occur in the northern part of the region.

The northward increase in average range length is shown by the first-degree trend-surface map of range length to height (fig. 39); values on the plane of best fit range from less than 40 in the southeast to more than 75 in the northwest. Figure 40 shows the second-degree paraboloid of best fit which accentuates a large regional high in the northwestern part of the area. This high is further emphasized on the third-degree trend-surface map of range length to height (fig. 41); this part differs markedly from the rest of the Basin and Range region on the basis of length to height ratios. Some rather long ranges exist in eastern Arizona and western New Mexico, and elsewhere along the eastern margin of the region, and the crest of values from 30 to 50 on the third-degree map reflect this fact. The southern and particularly the southwestern part of the area, however, is generally represented by values of this topographic parameter that are one-half the values in the northern part.

SUMMARY OF TREND-SURFACE RESULTS

The statistical measures that are associated with each of the trend-surface maps discussed above include: the total variance, V; the variance not explained by the surface, U; the variance explained by the surface, E; the coefficient of determination, D; the coefficient of correlation, R; and the standard deviation, σ. These statistics are defined by equations 62 through 67, respectively, but for purposes of discussion here only the total variance, V, the explained variance E, and the correlation coefficient R, need be considered. These measures are incorporated in table 1 for the first-, second-, and third-degree trend surfaces of each of the 11 topographic parameters that were treated.

The E/V ratios are equivalent to the percentage of total variance that is explained by a surface of given degree for each listed parameter. These percentages and the associated correlation coefficients permit some summary generalizations to be made. In terms of either E/V ratios or the values of the correlation coefficient associated with the third-degree trend surface (table 1),

```
RANGE WIDTH TO LENGTH - BASIN AND RANGE PROVENCE

CONTOURED CUBIC SURFACE

PLOTTING LIMITS
MAXIMUM X =        -0.000000      MINIMUM X =        -5.000000
MAXIMUM Y =         0.000000      MINIMUM Y =       -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =  -11.00 +   0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                      0.02
REFERENCE CONTOUR (.....) =             0.00
```

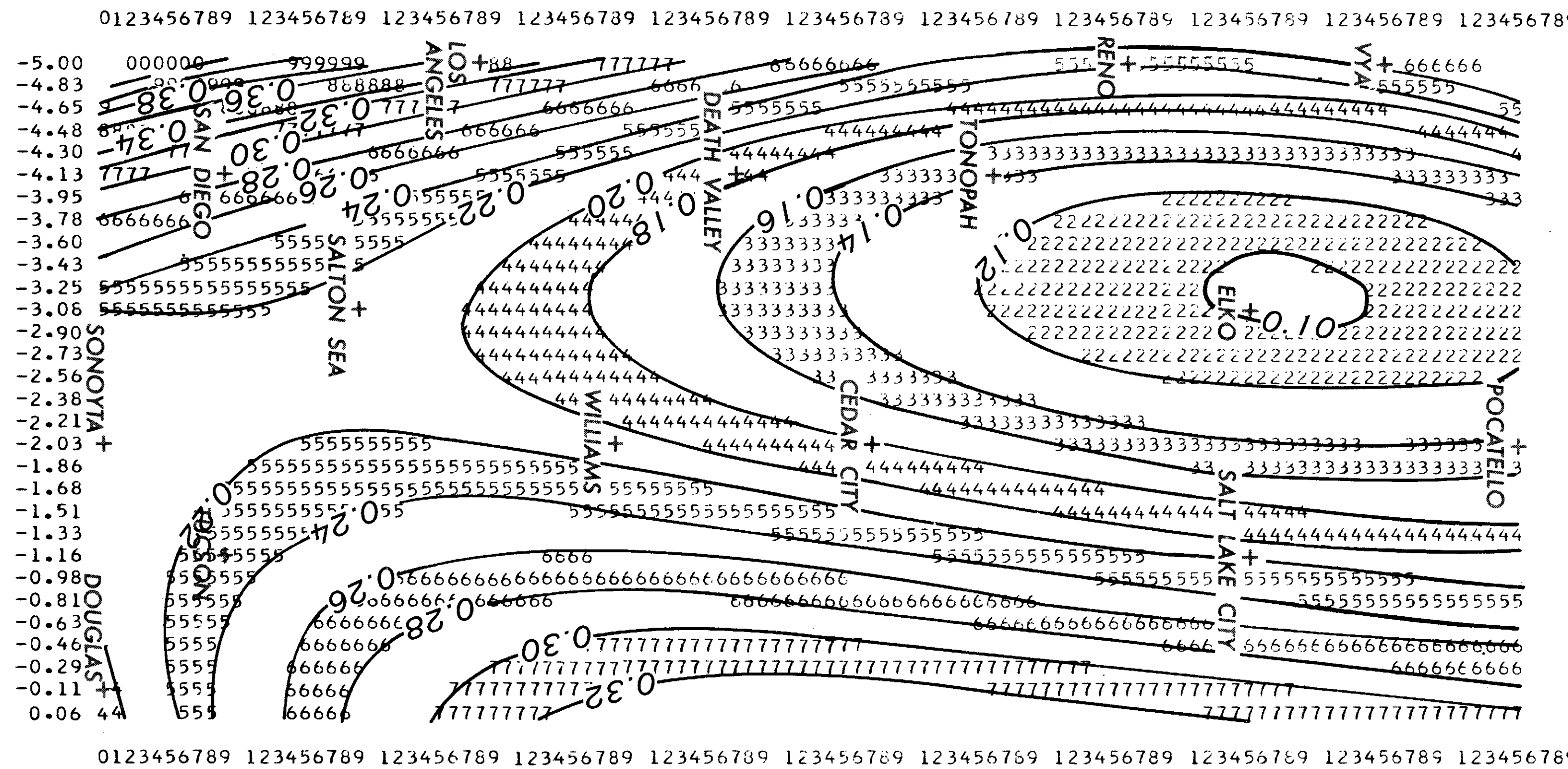

FIGURE 35.—Third-degree trend-surface map of range width to length. Contour interval is 0.02. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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RANGE WIDTH TO HEIGHT - BASIN AND RANGE PROVENCE

CONTCUREC LINEAR SURFACE

PLCTTING LIMITS
MAXIMUM X =      -0.0CC000      MINIMUM X =      -5.000000
MAXIMUM Y =       C.0CC000      MINIMUM Y =     -11.0C0000

Y-SCALE IS HORIZCNTAL
Y-VALUE =  -11.0C +  C.1C48 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTCUR INTERVAL =                         0.40
REFERENCE CCNTCUR (.....) =                0.00
```

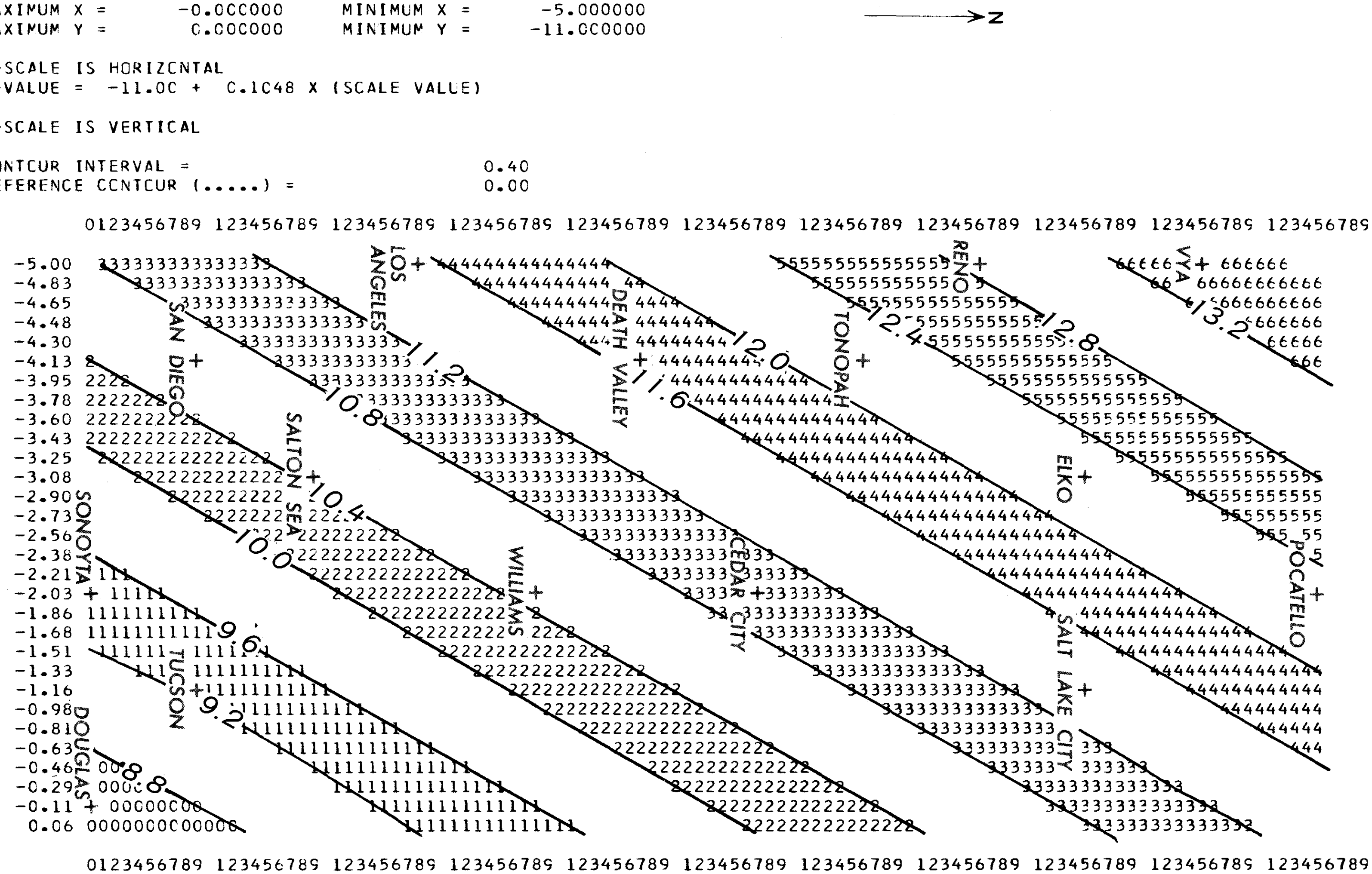

FIGURE 36.—First-degree trend-surface map of range width to height. Contour interval is 0.40. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

RANGE WICTH TO HEIGHT - BASIN AND RANGE PROVENCE

CONTCUREC QUACRIC SURFACE

PLCTTING LIMITS
MAXIMUM X = -C.OCCOCO MINIMUM X = -5.0C0000
MAXIMUM Y = C.OCCOCO MINIMUM Y = -11.000000

Y-SCALE IS HCRIZCNTAL
Y-VALUE = -11.0C + 0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTCUR INTERVAL = 0.4C
REFERENCE CCNTCUR (.....) = 0.0C

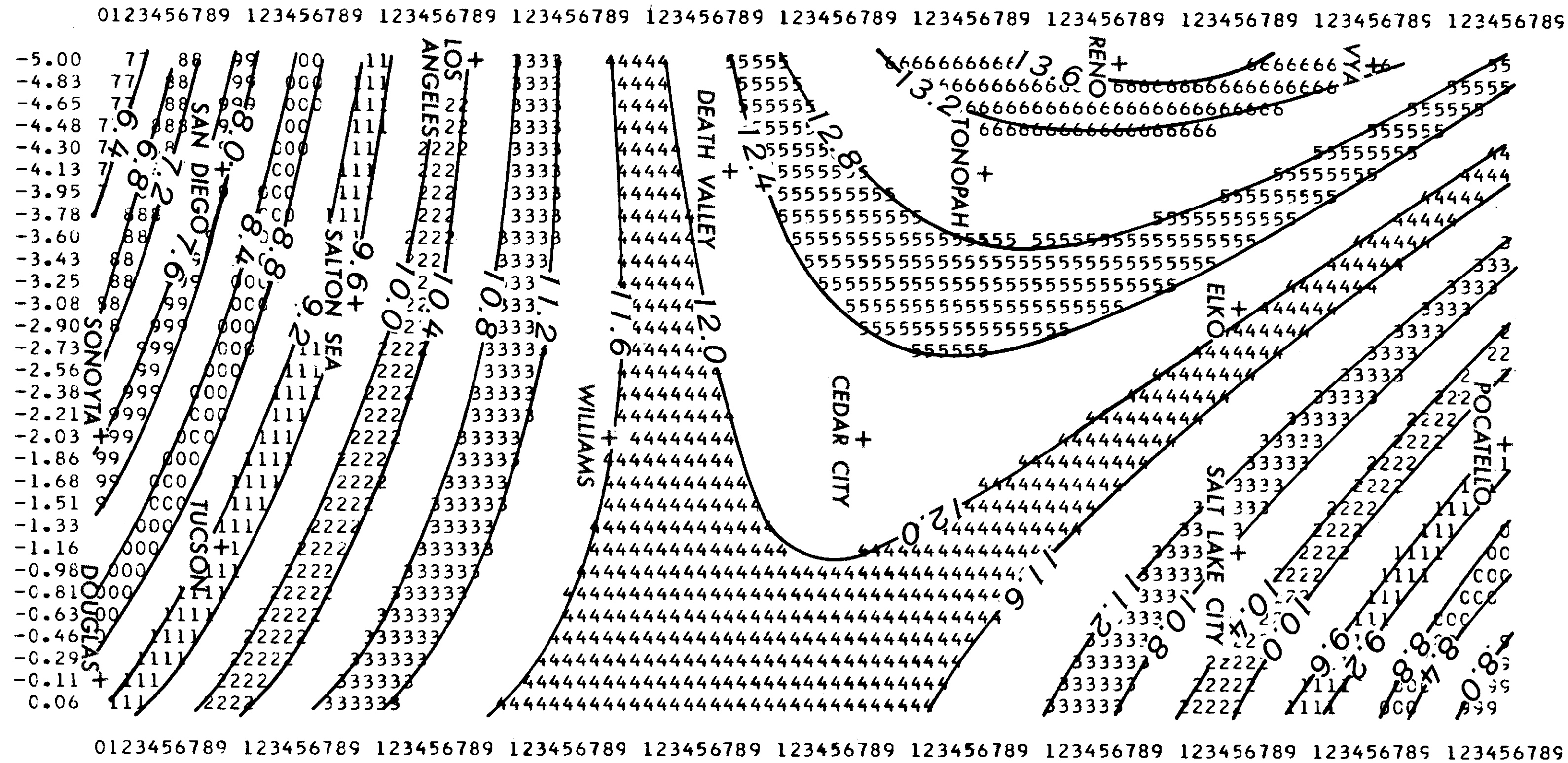

FIGURE 37.—Second-degree trend-surface map of range width to height. Contour interval is 0.40. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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RANGE WICTH TC HEIGHT - BASIN AND RANGE PRCVENCE

CONTCUREC CUBIC SURFACE

PLCTTING LIMITS
MAXIMUM X =       -0.0CC000      MINIMUM X =      -5.000000
MAXIMUM Y =        0.CCC0C0      MINIMUM Y =     -11.0C0C00

Y-SCALE IS HCRIZCNTAL
Y-VALUE =  -11.0C +  C.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTCUR INTERVAL =                          0.4C
REFERENCE CCNTCUR (.....) =                 0.0C
```

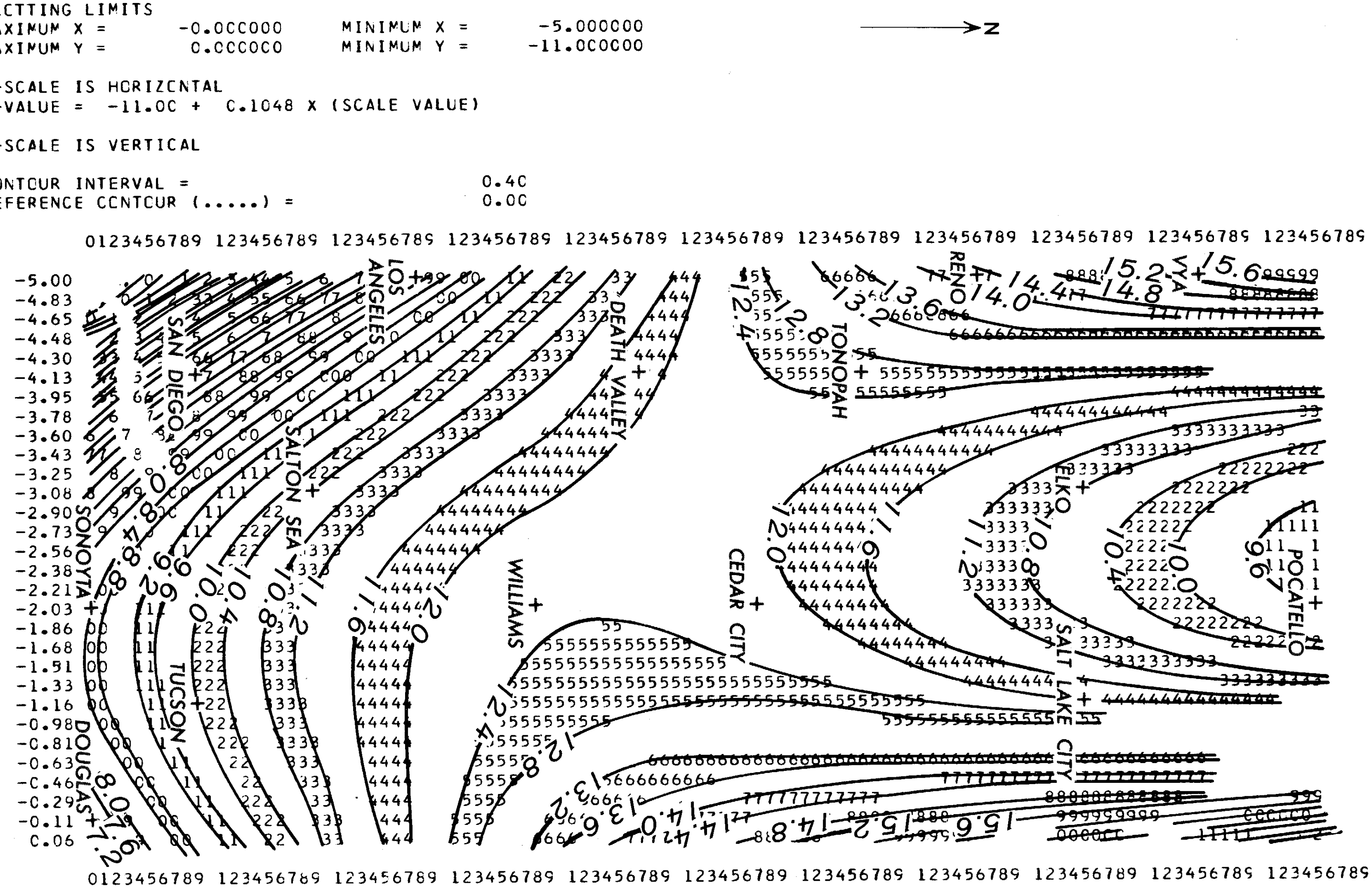

FIGURE 38.—Third-degree trend-surface map of range width to height. Contour interval is 0.40. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

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RANGE LENGTH TO HEIGHT - BASIN AND RANGE PROVENCE

CONTOURED LINEAR SURFACE

PLOTTING LIMITS
MAXIMUM X =        -0.000000      MINIMUM X =       -5.000000
MAXIMUM Y =         0.000000      MINIMUM Y =      -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE =  -11.00 +  0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL =                    5.00
REFERENCE CONTOUR (.....) =           0.00
```

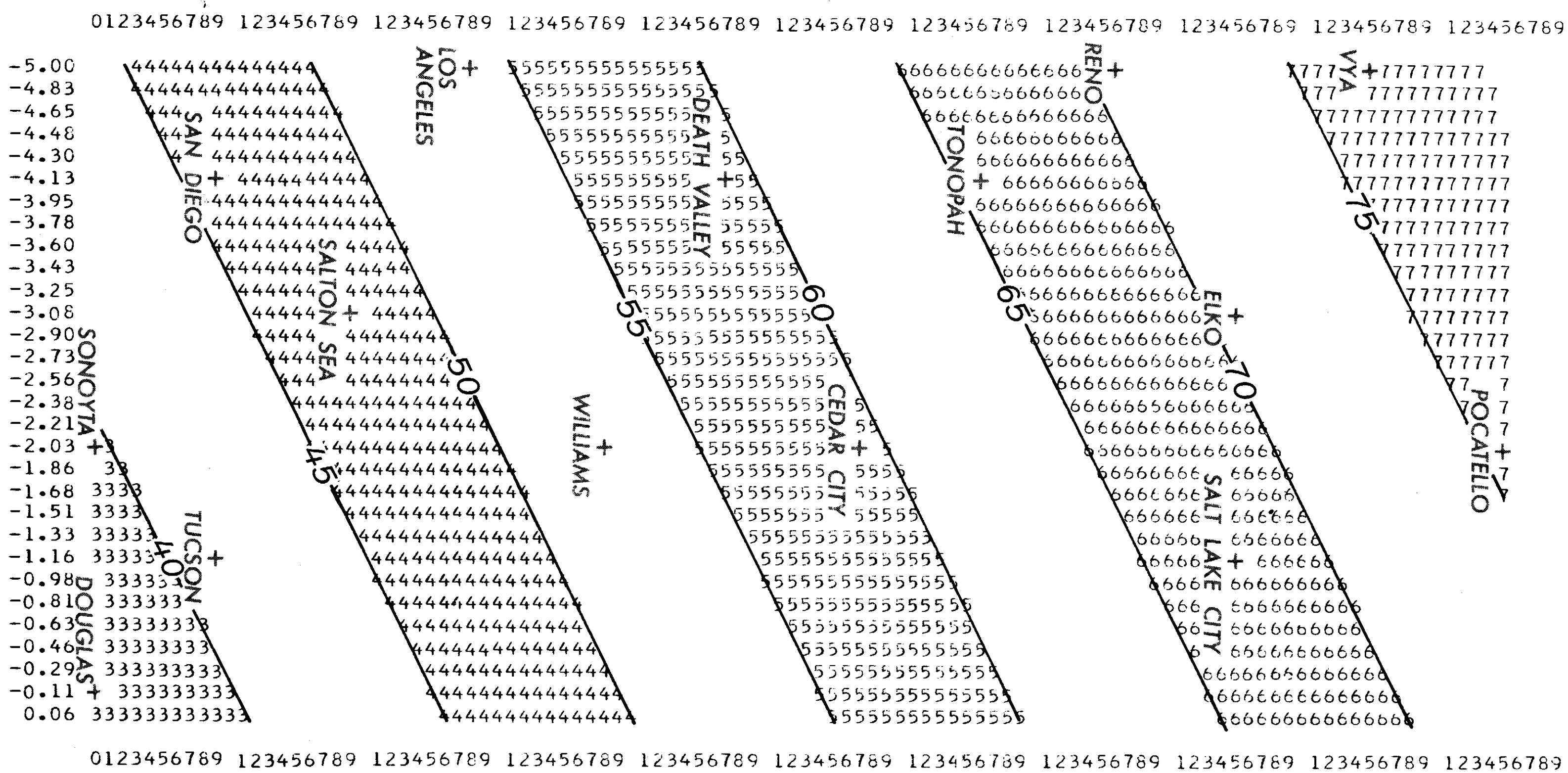

FIGURE 39.—First-degree trend-surface map of range length to height. Contour interval is 5.0. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

RANGE LENGTH TO HEIGHT - BASIN AND RANGE PROVENCE

CONTOURED QUADRIC SURFACE

PLOTTING LIMITS
MAXIMUM X = -0.000000 MINIMUM X = -5.000000
MAXIMUM Y = 0.000000 MINIMUM Y = -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE = -11.00 + 0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL = 5.00
REFERENCE CONTOUR (.....) = 0.00

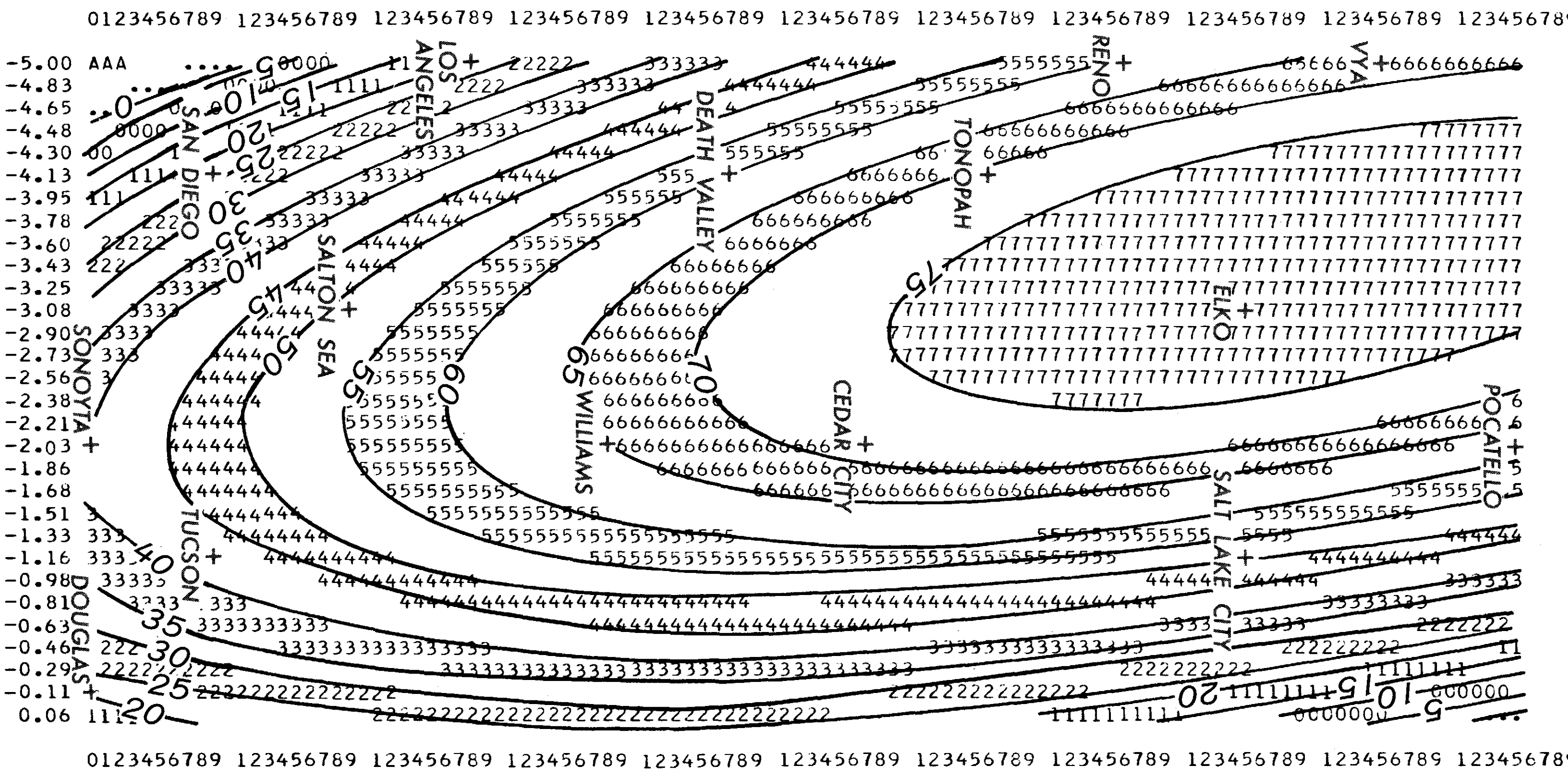

FIGURE 40.—Second-degree trend-surface map of range length to height. Contour interval is 5.0. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

RANGE LENGTH TO HEIGHT - BASIN AND RANGE PROVENCE

CONTOURED CUBIC SURFACE

PLOTTING LIMITS
MAXIMUM X = -0.000000 MINIMUM X = -5.000000
MAXIMUM Y = 0.000000 MINIMUM Y = -11.000000

Y-SCALE IS HORIZONTAL
Y-VALUE = -11.00 + 0.1048 X (SCALE VALUE)

X-SCALE IS VERTICAL

CONTOUR INTERVAL = 5.00
REFERENCE CONTOUR (.....) = 0.00

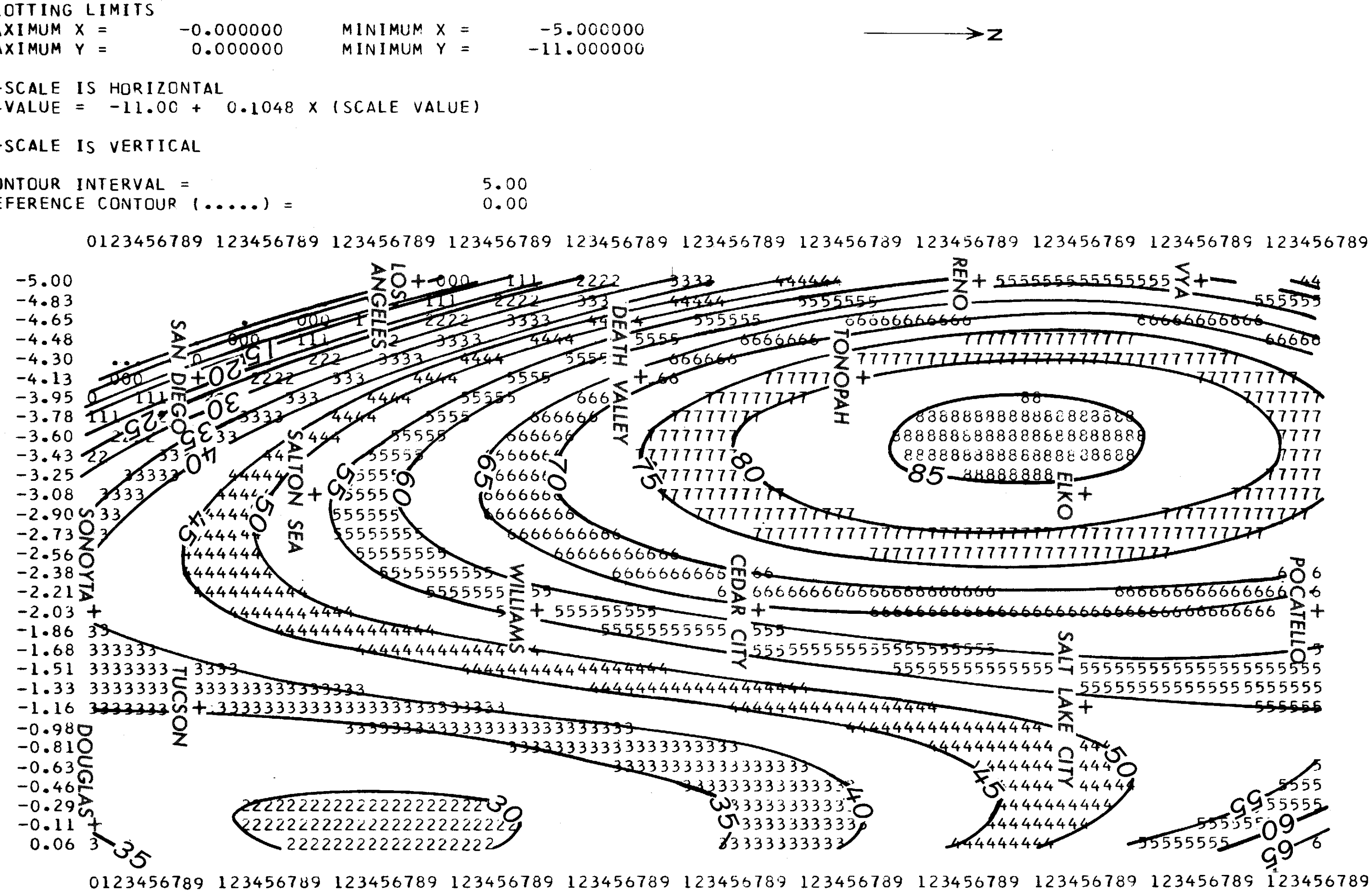

FIGURE 41.—Third-degree trend-surface map of range length to height. Contour interval is 5.0. The geographic names shown correspond to the locations of the 1:250,000 topographic quadrangle maps.

TABLE 1.—*Statistical measures associated with trend-surface results*
[*E*, explained variance; *V*, total variance; *R*, correlation coefficient]

Parameter	First degree		Second degree		Third degree	
	E/V	*R*	*E/V*	*R*	*E/V*	*R*
Area of ranges to total area	37	0.608	60	0.779	71	0.842
Range length	34	.586	39	.625	51	.708
Range width	41	.639	54	.738	56	.748
Range height	30	.545	44	.664	58	.759
Range relief	27	.516	31	.557	58	.758
Range volume	27	.518	29	.540	36	.602
Cumulative length of trends	29	.540	56	.751	74	.858
Cumulative deviation of trends	37	.609	45	.672	66	.812
Range width to length	12	.337	26	.509	37	.611
Range width to height	20	.452	34	.586	45	.674
Range length to height	19	.433	32	.563	37	.607

a relative ranking of the topographic parameters is possible.

The R values associated with the third-degree trend surfaces permit a tripartite division to be made. In the first category, the area of ranges to total area, and cumulative length and cumulative deviation of trends, each yield a correlation coefficient greater than 0.800. A second category includes range length, width, height, and relief, each of which is associated with a correlation coefficient greater than 0.700 but less than 0.800. And last, range volume and the three derived ratios, namely range width to length, width to height, and length to height, each are associated with correlation coefficients that range from 0.600 to 0.700. In terms of percentage explanation by the third-degree surfaces, the values for these three categories are approximately 66–74 percent, 50–58 percent, and 36–45 percent, respectively.

To designate these values as rankings of significance would be somewhat misleading because each of the topographic parameters does exhibit regional variation to some degree, or percentage. The results cited simply indicate that local variations are relatively greater for certain topographic parameters than for others. Howarth (1967) has shown, however, that percentage explanations of approximately 6, 12, and 16 percent can be considered as the lower limits of reliability at the 95-percent confidence level for trend surfaces of first, second, and third degree, respectively. This was determined by fitting trend surfaces to randomly generated data and analyzing the resulting sums of squares. Because all the percentages of explanation (table 1) are greater than these limiting values, the results reported here can be considered to be valid. Also, there is a definite improvement in E/V ratios and R values (table 1) for successively higher degree surfaces for each parameter. This might be expected because, as previously stated in this report, a first-degree surface, or plane of best fit, for example, will never explain a large percentage of the total variance, unless the z values for a given parameter are uniformly distributed. That is, only if ranges were of the same dimensions along a given parallel of latitude, for example, and were to exhibit a uniform decrease in these dimensions from one parallel to the next would a plane of best fit closely coincide with the distribution of z values. From this point of view, and from knowledge of the results shown in table 1, there is no question that trend surfaces of degree higher than the third would satisfactorily explain a greater percentage of the total variance as regional variance than is indicated above.

This statement is based upon the fact that the percentage variance that is explained by the third-degree surfaces represents a considerable improvement beyond the percentage variance that is explained by the second-degree surfaces for several parameters. This improvement in goodness of fit is 27 percent for range relief, 21 percent for the cumulative deviation of trends, 18 percent for the cumulative length of trends, 14 percent for range height, and 11 percent for range length, range width to length, range width to height, and area of ranges to total area. These data suggest that the percentage variance explained would increase further if trend surfaces of degree higher than the third were fitted to the z values of these parameters. This is probably not true for range width, range volume, and range length to height. The increase in percentage explanation for these parameters, from the second- to third-degree surfaces, is 7 percent or less, and the law of diminishing returns would seem to apply.

In summary, the trend-surface results indicate that each of the 11 topographic parameters exhibit marked regional variations within the Basin and Range area, and these variations have been shown quantitatively on the several trend-surface maps. A fairly large percentage of the total variance remains unexplained by the third-degree surface; this can be attributable to local variations, but the regional differences indicated are real. In the discussions of trend-surface results for each of the topographic variables, it was noted that the range of values on the surfaces of best fit was at least 100 percent for most of the parameters, and as great as 1,300–1,400 percent for range volume and the area ratio, and that this range of values reflected the general difference between the northern and southern parts of the Basin and Range area. Clearly, the magnitudes of regional differences are not minor. The range of values cited should not be construed as the range between extreme z values within the region. The extreme z values never occur on the surfaces of best fit, and the magnitude of the range of extreme values would be substantially greater than any percentage indicated.

In addition to this general distinction between the northern and southern parts of the Basin and Range region, several other observations were previously made.

The persistent contour closures or regional high values that recur to the northwest on nearly all the trend-surface maps clearly suggest that this area, which embraces central and western Nevada and parts of eastern California, is topographically different from the rest of the region. It is set apart by reason of the much greater lengths, widths, heights, areas, and volumes of the ranges than those which occur elsewhere and because of the more nearly north-south orientation of the ranges. The topographic characteristics of southern California and southwestern Arizona form the other extreme, with respect to the sizes and attitudes of ranges. In several respects the Salt Lake desert area resembles this southern region; at least it resembles this area more closely than it does the central and western Nevada area. As a summary generalization, however, the northeastern part of the Basin and Range area may be grouped with eastern Arizona and western New Mexico. Recourse to the trend-surface maps of the several topographic parameters supports this intermediate grouping and, indeed, supports this three-fold division in general.

Finally, the regional distinctions drawn from the trend-surface data are simply that, regional distinctions. It is by no means claimed that every range in the northwestern area delineated here is of great dimensions, nor that every range in the southwestern area is of small dimensions. There are indeed some ranges in any of these areas that depart from the mean size and from the other range attributes that have been defined. These exceptional ranges are one of the causes of the existence of lower level, or more local, variance components. That the distribution of differences of the average topographic attributes can be and has been demonstrated to be regional in extent is significant, and these regional differences give rise to several geomorphic implications which will be discussed below.

GEOMORPHIC IMPLICATIONS

Quantitative methods and statistical analysis of data in general basically answer the question "what?," not "why?" This is obviously true of the Basin and Range topographic data that have been presented in this report. Regional topographic distinctions have been sustained on the basis of quantitative data rather than by qualitative assessments, and the distinction of regions within the Basin and Range area has been accomplished by the objective method of trend-surface analysis. All of this, however, simply tells us what the topography of the area is like, within relatively narrow limits of error. To attempt to explain why the topographic variations occur is virtually to enter the realm of inference and deduction. Nonetheless, the writer believes that some implications do follow from the data that have been presented. The four topics most closely related to this analysis of Basin and Range topography are (1) the nature and boundaries of physiographic provinces in general, (2) the age of the ranges within the region, (3) the origin of pediments, and (4) the drainage distinctions that may exist in the ranges; they are discussed in the order given.

PHYSIOGRAPHIC PROVINCES

At the outset of this report the American Geological Institute (1957, 1960) definition of a physiographic province was given; a province is said to be a region within which the structure, climate, and geomorphic history are similar or uniform. Thornbury (1965) provided a fairly extensive introductory chapter to his book on regional geomorphology which treated the subject of physiographic provinces, their definition, and the means of recognition of "geomorphic units." Much of this material is pertinent here and is summarized briefly, but for additional details the reader should consult Thornbury (1965). In Thornbury's introductory chapter, Bowman (1911) is quoted as defining a province as "a tract in which the topographic expression is in the main uniform." Fenneman (1928) stated that "all orders of (geomorphic) divisions rest ultimately on existing differences in topography and elevation." Hinds (1952) said that geomorphic provinces were characterized, among other things, by "more or less uniform relief features or combination of features throughout its area." The definition favored by Thornbury (1965) is "A physiographic unit is an area or division of the land in which the topographic elements of altitude, relief, and type of landforms are characteristic throughout and as such is set apart or contrasted with other areas or units with different sets of characteristic topographic elements." In addition, most of the physiographic maps of individual States invoke topographic criteria in some way to justify the province or division boundaries shown.

Clearly, all classifications of physiographic provinces or geomorphic units are based primarily, if not solely, on topographic characteristics. This fact has been obscured occasionally by reference to the "unified geomorphic history" that is thought to prevail within a given province. Thornbury (1965, p. 9) stated, however, that "given a certain geologic framework, the topographic condition or expression of an area is largely determined by its geomorphic history." This means, essentially, that similarity of geomorphic history is inferred, if similarity of topography exists.

Obviously, then, physiographic provinces and their boundaries can best be determined from quantitative analysis of topographic parameters in a given region.

Such province classification or boundary delineation, by methods akin to those described in this report, or others, must remain somewhat arbitrary. That is, some arbitrary choice of value for some parameter or combination of parameters must be made to fix any required boundary. The chief virtues of such procedure, however, are objectivity, reproducibility, and the facilitation of quantitative comparison of within-province variation and between-province variation.

Only within-province variation has been treated in this report. As previously stated, this writer would tend to define three physiographic provinces within the Basin and Range region on the basis of the quantitative data obtained. Others might prefer to term these three areas "sections," as did Fenneman (1931), who described five subdivisions in the Basin and Range province. The discrepancy in numbers of subdivisions is not at issue here; rather, the choice between the use of the terms "section" and "province."

The choice is not merely a matter of semantics but one that basically involves the very nature of classifitions and of hierarchical systems in general. In any such system, the within-class variance will inevitably increase with an increase in order or rank. Therefore, the total internal variance at the highest levels, such as within the kingdom ranking of the zoologist, will be large indeed. But beyond some critical magnitude of total internal variance, the utility of a given classification may be destroyed.

To classify physiographic provinces and sections one basically needs to know the magnitudes of internal topographic variance allowable within each rank. The present writer has shown that average values of such parameters as range volume and the area of ranges to total area ratio exhibit a variation that is as much as 1,300 to 1,400 percent. The rest of the parameters exhibit a minimum variation of 100 percent. The question to be answered is whether such topographic variance is common to the province-level rank. If so, then the areas discussed in this report can be designated as "sections" in a hierarchical scheme. If this variance is excessive, however, the Basin and Range province as we know it should be subdivided into several provinces.

The answer to the question posed is beyond the scope of this report. There are no quantitative treatments of physiographic provinces that would enable one to state the average magnitude of topographic variance associated with the province rank and thereby to determine whether the known variance in the Basin and Range region is excessive. The writer recommends, therefore, that a province-by-province evaluation be undertaken to determine average values of within-province variation and between-province variation. In addition to the problem of proper subdivision of the Basin and Range region, several other questions of interest might be resolved.

To examine the degree of topographic similarity between the folded mountains of the Appalachian region, for example, and the Coast Ranges of California would be useful. Despite known differences in the geological histories of these two regions, the landform map of the National Atlas Series (U.S. Geological Survey, 1966) indicates that topographic similarity exists. Quantitative examination of these two areas might therefore lead to some assessment of the validity of the inference that similarity of topography implies similarity of geomorphic history. In another vein, the Basin and Range province of North America might be compared to its counterparts elsewhere in the world. The Dasht-e-Lut and Dasht-e-Kavir of Iran, or the basin and range regions of Afghanistan and West Pakistan, are among the more obvious choices for such an undertaking. This exercise would have more direct bearing upon the range of topographic values that exist within such areas.

In summary, further work is indicated, but the results of this report suggest that any physiographic province or finite area of the earth's surface is amenable to quantitative examination. One of the desirable goals of such efforts should be the establishment of quantitative standards of within-province and between-province variation, as well as the objective delineation of province boundaries.

AGE OF RANGES

The treatment of topographic data presented in this report supports the view of many workers that the ranges in the Basin and Range province are probably of different ages. This conclusion is indirectly implied by the topographic data, but it is supported by many other lines of evidence, some of which are discussed here. First, it should be recalled that the ranges in central and western Nevada and in part of eastern California are longer, wider, higher, and more closely spaced than are those of southwestern Arizona and southeastern California. An idealized cross section through a basin and two adjacent ranges in each of these two general categories of regions would resemble that shown in figure 42. The question is why this difference exists.

Among the possible explanations, three are most prominent. First, the difference in topographic relief and range spacing could be attributable to differences in depth of basin fill. Figure 42 shows that an increase in depth of basin fill would, for the larger ranges, produce topographic expression similar to that presently exhibited by the smaller ranges. Essentially, burial of

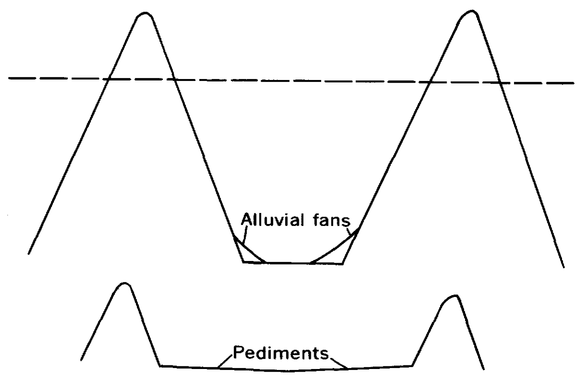

FIGURE 42.—Idealized cross section of two ranges and an intervening basin in two different parts of the Basin and Range region. Upper part of sketch shows ranges of large dimension and close spacing which occur in north-central and northwestern Nevada and eastern California, where alluvial fans predominate. Lower part of sketch shows ranges of small dimension and wide spacing which occur in southwestern Arizona and southeastern California, where pediments predominate. Dashed line in upper part of sketch illustrates how these topographic differences could be achieved by varying depths of basin fill. Area between ranges in lower part of sketch, however, is known to contain surface or near-surface pediments, and the depth-of-fill hypothesis is therefore untenable.

the larger ranges by alluvium would leave only their tops exposed, and the apparent range spacing and size would change accordingly. Aside from consideration of the range erosion necessary to accomplish this end, the basic question is whether the existing data on depths of fill in the Basin and Range area will support such an interpretation. Such data are very sparse, as indicated by the recent basement map of North America (American Association of Petroleum Geologist and United States Geological Survey, 1967), which is largely blank over the part of the Western United States considered here. The various geophysical surveys that have been made in the northern and southern parts of the Basin and Range area, however, will certainly not sustain the idea that depths of fill differ drastically. Moreover, as is well known, the southern basins have a history of through-flowing drainage, whereas the northern basins do not, and this has permitted the transportation of sediment out of the southern basins at various times. This also mitigates against the suggestion that the smaller and more widely spaced ranges are a consequence of greater depths of basin fill. The depth-of-fill explanation seems therefore to be untenable on a regional basis and may be disregarded.

A second possibility is that the differences in topographic attributes are related to basic structural differences. Specifically, the spacing of faults and the magnitudes of uplift differed in the several parts of the Basin and Range area to such an extent that the topographic differences are directly related to structural conditions. The rather voluminous geological literature on the Basin and Range area clearly suggests that this hypothesis is equally improbable. In general, faults and fault scarps are more commonly observed in the northern part of the region, and they are in zones that are reasonably close to the mountain blocks. This is not true of the southern part of the region, where broad bedrock pediments are far more common, and evidence of faulting is obscure. The argument would require the existence of fault planes between the pediments and the ranges which they abut; faults in such locations have not been observed. If this second hypothesis is disregarded, the remaining alternative is that the ages of the ranges generally differ in the Basin and Range area.

Reference to the "age" of ranges in the context of this discussion means relief age or the time span since topographic expression was achieved. Clearly, a given relief age need not be coincident with the radiometric age that may exist for the plutonic or volcanic rocks in a given range. In fact, in this context a radiometric age can only serve as a limiting value for the relief age of a range. That is, the radiometric age must be greater, for it represents either the time of crystallization of a specific part of a given magma or the time of extrusion of a given lava. Damon and Mauger (1966) and Damon (1967) have discussed the chronology of events in the Basin and Range province in such a way that the question of the relief ages of ranges is somewhat obscured. Some discussion of this point is therefore warranted.

Basically, these authors presented potassium-argon dates for volcanic and hypabyssal plutonic rocks that have a bimodal distribution in time. One peak of the distribution occurs 60–70 million years ago and this represents Late Cretaceous or Laramide time. The second peak occurs 20–30 million years ago, in Oligocene to Miocene, or late Eogene to early Neogene time. They generally concluded that these radiometric-age peaks are coincident with times of orogeny throughout the Basin and Range province and related the two time peaks to the classical pulse-of-the-earth hypothesis, which demands synchroneity of magmatism and orogeny. Whether such synchroneity exists depends upon the scale of events that is considered and upon whether the equation of magmatism with orogenic activity is actually justified.

Related to the first of these factors, the scale of events, each of the radiometric age-distribution peaks is associated with a chronological range of values of considerable magnitude. Thus, the Laramide magmatism (Damon and Mauger, 1966; Damon, 1967) occurred between 50 and 90 million years ago, and the Tertiary episode occurred between 5 and 45 million years ago. The bimodal distribution of ages is not questioned here, but the range of values about each mode—namely 40 million years—indicates why the scale of events under consideration is important. These radiometric results can be interpreted broadly as indicative of two general magmatic episodes that occurred throughout the Basin and Range province. Considered in detail, however, the rocks associated with either episode can obviously differ drastically in absolute age. The potassium-argon data for Laramide plutons between Nogales, Mexico, and a point approximately 100 miles to the north, for example, show that even in this small part of southern Arizona the absolute ages of rocks range from 59 million years to 75 million years (Damon, 1967). In general, if the potassium-argon dates are accepted as limiting dates of actual orogenic activity, the relief ages of ranges in the Basin and Range province may differ by as much as 40 million years for either of the two magmatic episodes. This span of time is certainly sufficient for substantial modification of the older ranges to have occurred by erosion.

The second factor, namely the equation of magmatism with orogenic activity, has been discussed at length by Gilluly (1965), who reviewed the available data on volcanism, plutonism, and tectonism in the Western United States. Basically, Gilluly concluded that plutonism and orogenic activity have not been synchronous throughout the region and, in support of the view expressed here, that radiometric age determinations do not necessarily date orogenies. Figures 6 and 7 from Gilluly's paper are reproduced here as figures 43 and 44, respectively. In absolute age, Eogene corresponds to a time span ranging from 30 to 70 million years ago in Gilluly's usage. Hence, figure 43 is referable to conditions in the Western United States during a substantial part of the Laramide magmatic episode (50 to 90 million years ago) in Damon's (1967) view. This depiction of Eogene conditions (fig. 43) shows that putonism and volcanism, or magmatism in general, are not everywhere coincident with orogenic activity. Evidence for tectonism over most of central and western Nevada is lacking, despite the occurrence of siliceous volcanic rocks in this area. Orogenic activity became more pronounced in Nevada during Neogene time (fig. 44), but this coincided with diminished tectonism in southern Arizona. The data on which these maps are based would appear to indicate that the synchroneity of magmatism and tectonism remains at least debatable, if not, indeed, in the category of unproven assumptions.

If relief ages of ranges in the Basin and Range province differed by as much as 40 million years because of nonuniform tectonic activity during Late Cretaceous to Tertiary time, then pre-Quaternary topography must have been nonuniformly distributed. Many authors have stated that the present relief features of the

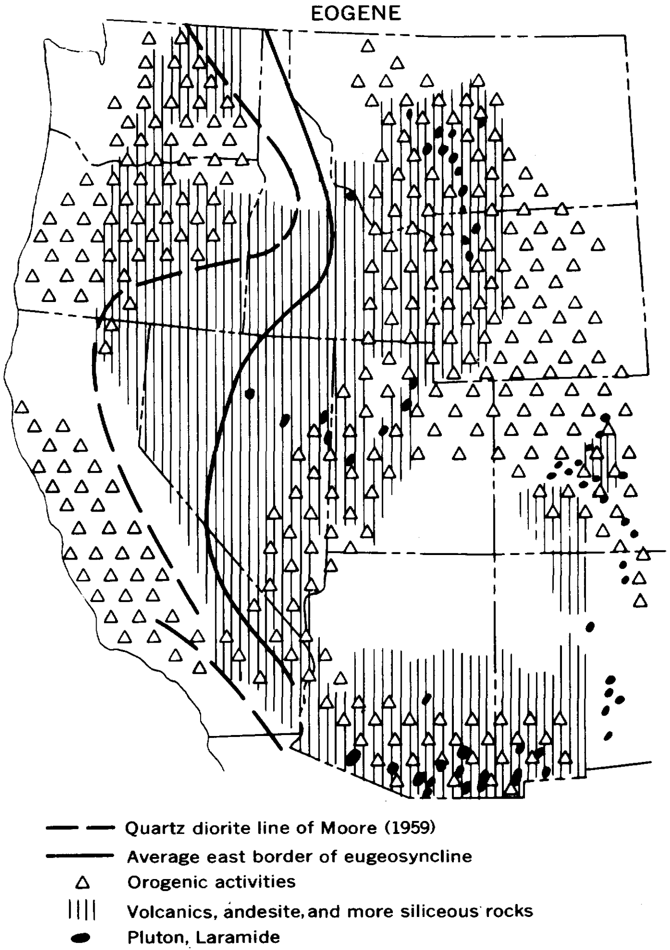

FIGURE 43.—Volcanism, tectonism, and plutonism during the Eogene in the Western United States. (After Gilluly, 1965, fig. 6.)

Basin and Range province are closely related to the block faulting of Quarternary time that was superimposed on preexisting topography. Because Quarternary tectonic activity was also distributed in a nonuniform manner, the preexisting topographic differences were accentuated in many places. King (1965) provided the most recent review article pertinent to this subject, which relies to some extent upon the distribution of recorded earthquake epicenters in the United States, as well as upon field evidence. King (1965, fig. 2) contoured the epicenter distribution; his map is shown in figure 45.

This map of epicenter distribution in the Western United States (fig. 45) clearly shows their nonuniform occurrence. The greatest concentrations occur in the northwestern part of the study area, and along the east margin of the Basin and Range area. Quiescence, or stability, is indicated for southern Arizona, eastern Nevada, and westernmost Utah. King (1965) stated that this pattern closely coincides with the known regional distribution of Quaternary block faulting, which is most extensive in western Nevada, eastern California,

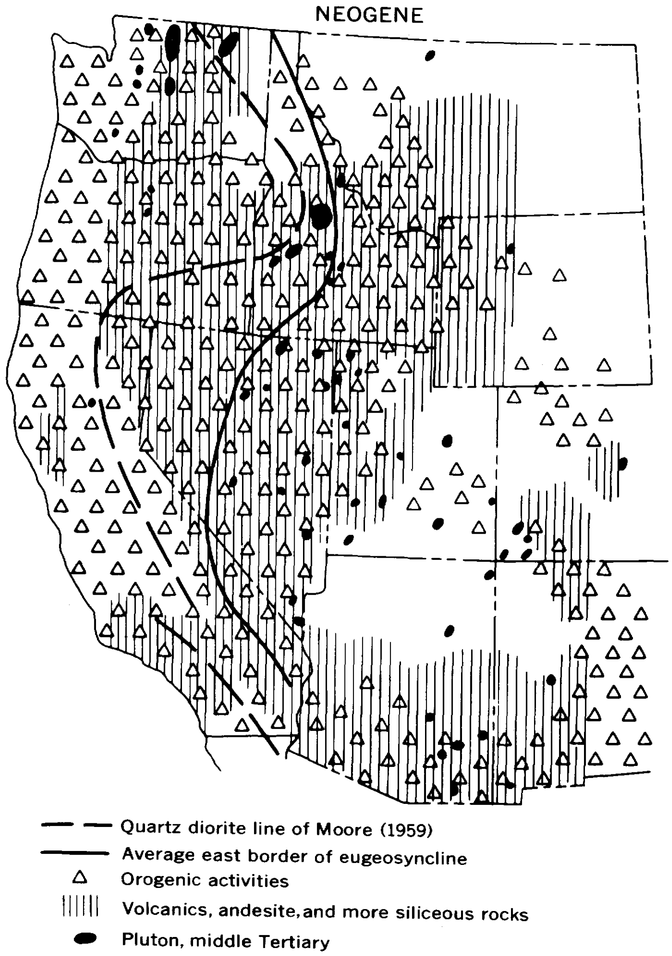

FIGURE 44.—Volcanism, tectonism, and plutonism during the Neogene in the Western United States. (After Gilluly, 1965, fig. 7.)

and in western Utah and New Mexico. The quantitative topographic data and trend-surface maps (figs. 9–41) presented in this report also coincide notably well with these regional structural patterns.

As a final point bearing upon the relief age of ranges in the Basin and Range area, figure 1 is instructive. Although highly generalized and locally inaccurate because of the small scale of the compilation (1:7,500,000), this map (fig. 1) clearly shows that the distribution of Precambrian and lower Paleozoic rocks in the Basin and Range area is nonuniform. It can be seen that outcrops of rocks of these ages occur primarily in the southern part of the region and in eastern Nevada and western Utah. These are precisely the areas which correspond to minimal recorded earthquake activity (fig. 45) and Quaternary block faulting, and the logic of Occam's razor would suggest that the relief ages of ranges in these areas are greatest. That is, these stable regions have been subjected to the longest time spans of erosion; hence, the oldest rocks are exposed within the ranges that exist.

In summary, the geomorphic implication that can most reasonably be drawn from the trend-surface maps of Basin and Range topography is that the relief ages of ranges in the region are different. Differing time spans of erosion is the simplest explanation for the variance of length, width, height, area, volume, relief, and trend of ranges that was noted previously, and this explanation accords well with the known geological history of the region. Because durations of topographic expression and of consequent erosional periods have varied within the region, a third inference can be drawn from the topographic data. This concerns the origin and distribution of pediments, which are discussed below.

ORIGIN OF PEDIMENTS

The subject of pediments and theories of their origin occupies a vast number of papers in the literature on the Basin and Range area and other parts of the world. Most of these papers have been cited elsewhere (Lustig, 1967) and will not be specifically discussed here, but a recent review of the pediment literature by Hadley (1967), which appeared after the text of this report was written, is worthy of note. Several of the better-known theories of pediment formation are probably incorrect. As an example of this generalization, consider the often-cited claims that sheetflooding is the process responsible for pediment production. Such claims are based solely on two or three reports of early vintage which described wagons overtaken by vast "walls of water" in the Western United States. The wagons in question were all traversing roads that bordered the lower reaches of a pediment surface, in low areas where the effect of channelized flows on the pediment surface combined to produce high water. This writer has never observed an unconfined "wall of water" on a gently sloping desert surface and is unacquainted with any who have observed such a phenomenon. Moreover, to attribute the origin of bedrock surfaces to sheetflows is to confuse cause and effect, even supposing that the shallower channels on these surfaces are occasionally overtopped during intense runoff events. The surfaces must necessarily predate the runoff events in question.

Lateral erosion is another frequently offered explanation. This hypothesis primarily treats the migration of channels that exist on pediment surfaces. No doubt, channels will migrate with time on these surfaces, and some erosion will therefore be accomplished, but the part of any pediment that abuts the mountain front cannot be explained in this manner. The hypothesis virtually requires that streams emerge from a given mountain range and, on occasion, turn sharply to one side or the other to "trim back" the mountain front in interfluvial areas. Such stream paths, nearly perpendic-

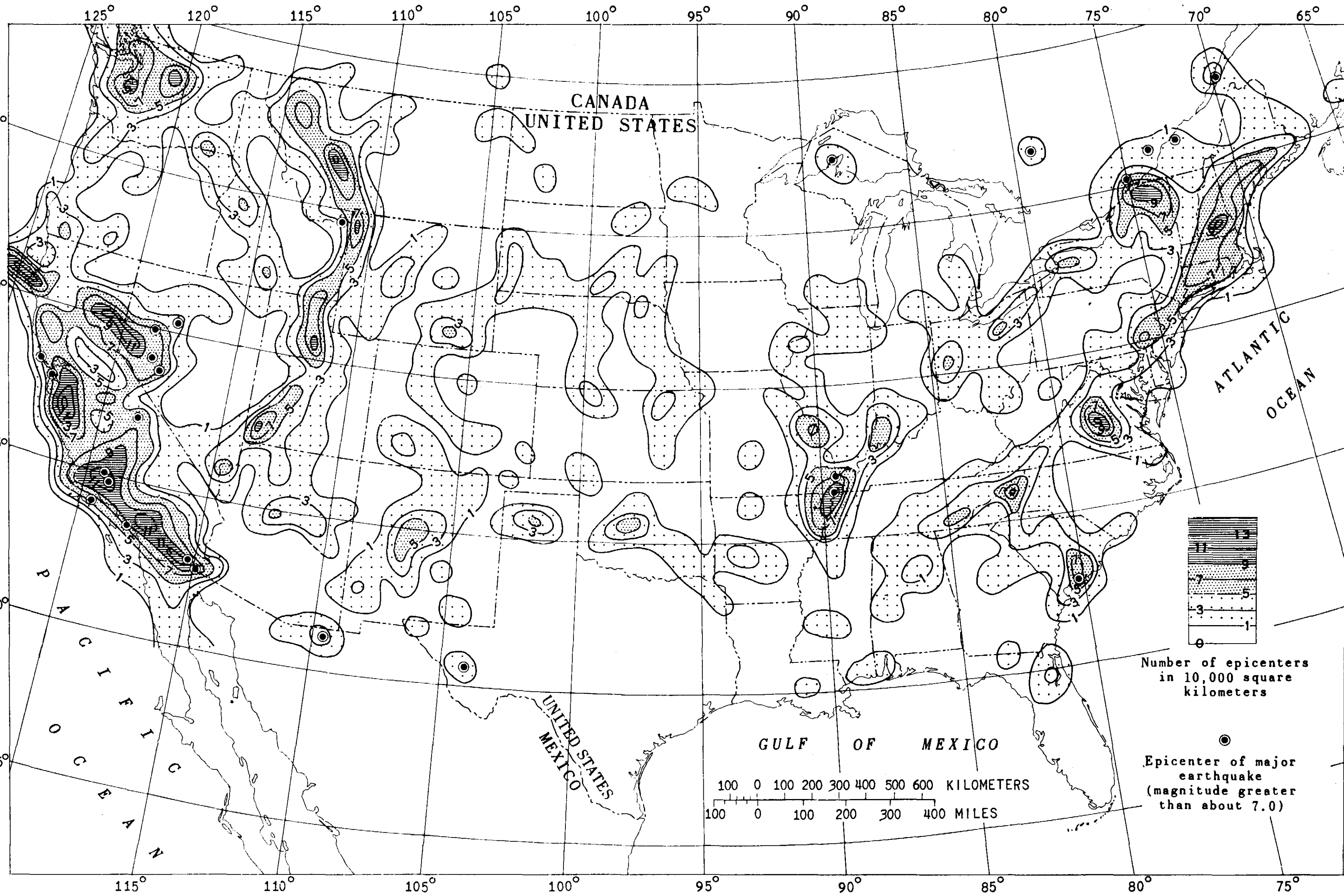

FIGURE 45.—Distribution of recorded earthquake epicenters, as of 1957, in the United States. (After King, 1965, fig. 2.)

ular to a sloping surface, would defy the laws of gravity and have not been observed except those in areas where drastic tilting has occurred. This is not surprising, because knowlege of the triangle of probabilities (fig. 46) suggests that the least probable path in a two-dimensional consideration is precisely the right-angle path required of a stream emerging from a given range. This triangle can be constructed by moving a marker to either the left or the right of any given origin in successive steps. Each given probability is equal to one-half the sum of the two numbers above it, and the probability at any point can be computed from the following general formula:

$$Z_nk=\tfrac{1}{2}\left[Z_{n-1}(k-1)+Z_{n-1}(k+1)\right], \qquad (68)$$

where Z_nk is the probability that the marker will be at point k after n trials. Thus the path of greatest probability coincides with the central portion of the triangle (fig. 46). In the absence of constraints the three-dimensional natural condition would most assuredly be best represented by a stream that emerges from the mountains and flows downslope to the basin floor, or pediment margin, with minimum deviation from a central path. Hence, lateral planation undoubtedly occurs on pediment surfaces, although quantitative data indicating the effects of the process are lacking, but it cannot account for the existence of pediments as such.

It has also been proposed that pediments result from parallel slope retreat of the mountain front. Slope retreat of this type is itself a topic that occupies much literature, and the more ardent champions of this process argue for parallel retreat of cliffs on a continental scale. The writer's observations in several deserts of the world suggest that few, if any, escarpments are not dissected by prominent drainage systems. This is true even in the driest regions, such as the Namib Desert in South West Africa or the Tuwaiq escarpment of central Arabia. The precipitation is about 2 inches per year in both areas. The existence of drainage basins in the mountain ranges is of central importance to the various pediment arguments. These basins are the loci of the most effective erosional processes that operate on mountain ranges in the Basin and Range region or elsewhere. The mountain fronts may well retain some characteristic slope angle that reflects rock strength, structure, weathering characteristics, and other variables, and they may retreat at this angle. This does not prove, however, that ranges are primarily reduced by parallel retreat of escarpments. This point will be pursued later.

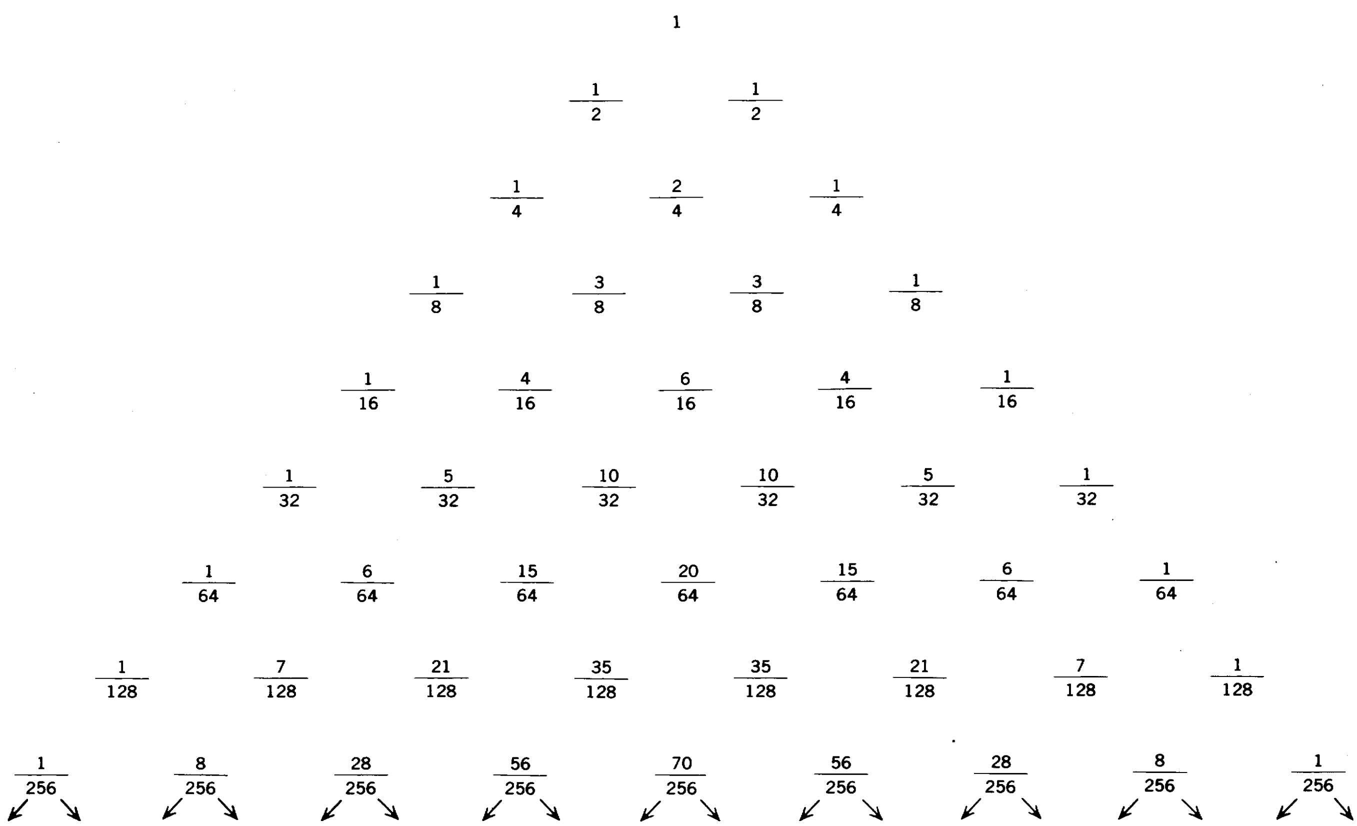

FIGURE 46.—The triangle of probabilities, showing the probability associated with any point on a two-dimensional random walk. The origin is represented by the apex of the triangle, which has a probability of 1.

Although additional arguments presented in literature concern the origin of pediments, the above examples will suffice here. The many discussions of pediment surfaces have focused upon the wrong landform in the writer's opinion. There is no question that processes of subaerial and suballuvial weathering occur on pediments today, nor that fluvial erosion also occurs. A pediment must exist prior to the onset of these processes, however, and in this sense the origin of pediments resides in the adjacent mountain mass and its reduction through time. Even if quantitative data on the rates and intensities of processes that act upon pediments were obtained, these would have little bearing upon the origin of bedrock surfaces.

To say that pediments result from the reduction of adjacent mountains through time is to give voice to a seemingly obvious and intuitive argument. But this point has often been insufficiently stressed in the past. Each range has some local base level in an adjacent basin, and it cannot be eroded to an elevation below this level. Given stability for a sufficient period of time, the consequences of mountain reduction must inevitably include the production of a pediment, whether in arid or nonarid regions. The nature of the surface produced may vary, and it may be mantled by, or free of, alluvium. However, it simply represents an area that was formerly occupied by a mountain or other bedrock topographic high. The only real "pediment problem" is how the reduction or elimination of mountain mass occurs. This question will be considered here, admittedly in a deductive manner, and the relationship of the topographic data to the pediment question will subsequently be discussed.

First, it is significant to note that the "sharp break in slope" between the pediment surface and the mountain front, which has always been emphasized in the literature, exists only in interfluvial areas. Observation shows that the course of any master stream channel from a given drainage basin in the mountains onto the pediment surface and thence to the basin floor below has no sharp break in slope. In the absence of constraints, such as recent structural disturbances, any such stream channel will exhibit a relatively smooth, concave upward, longitudinal profile that accords with the local hydraulic geometry. There are no hydraulic anomalies in nature, and none exists at the "mountain front." The interfluvial areas, however, generally do exhibit a marked change in slope, at least within a narrow zone parallel to the mountain front. The reason for the existence of such a zone is precisely that it is an interfluvial area; the dominant process that operates on the mountain front is not fluvial.

Qualitatively, it can be argued that two basic processes are operative on a given mountain mass. The steep slopes of the mountain front are interfluvial areas that are subject to weathering. Runoff on these steeply sloping surfaces is of short duration and is not concentrated. The runoff serves largely to remove the finer weathered debris that is transportable. Larger particles generally remain in place until they are reduced in size by weathering. The rates of mountain-front retreat are basically unknown, but by any reasonable assessment they are slow in relation to rates of processes that are operative in drainage basins. This is clearly true because the headwater region of any given drainage basin also consists of steep walls that are virtually indentical to those of the mountain front in interfluvial areas. In these headwater regions the same processes of weathering and of removal of debris occur. Hence, the rates of retreat of the bounding walls in the headwaters of drainage basins must be at least as great as the rate of retreat of the mountain front in the interfluvial areas. Also, however, the drainage basins represent the only parts of any mountain range that are subjected to concentration of flow and to its erosional effects, and these basins must therefore be the principal loci of mountain-mass reduction.

The entire argument may be summarized by stating that two processes are operative on mountain ranges, namely "A" and "B," and that process "B" is rapid relative to "A." If "A" operates on the mountain front, whereas both "A" and "B" operate in drainage basins then the bulk of range reduction must occur in the latter areas. Moreover, it is logical to further infer, as a corollary, that the interfluvial areas, or parts of the mountain front proper, must be "left behind" with the passage of time. These areas, which are often described as "triangular facets" in the literature, are possible loci of inselberg production. The latter landforms are mountain residuals which are left on the pediment surfaces, as these surfaces are produced by drainage-basin evolution through time.

The foregoing hypothesis appears to have the support of logic, but because quantitative data are lacking it can be considered no more than this, namely a hypothesis. Nevertheless, some tests of its merit are possible, and in this sense the topographic trend-surface data of this report are related to the pediment question.

If, as has been argued here, the origin of pediments is a direct consequence of mountain-mass reduction, the average mountain mass should be significantly smaller in certain areas that are characterized by the existence of pediments. The trend-surface results of this report have clearly indicated that regional differences do exist with respect to each of the 11 topographic parameters used. The lengths, widths, heights, relief, areas, and volumes of ranges (figs. 9–41) are, on the average, much smaller in southwestern Arizona and southeastern California than in the north-central and northwestern parts of the Basin and Range area; western New Mexico, northwestern Utah, and northeastern Nevada occupy an intermediate status with respect to most of these characteristics. As discussed in the section "Age of Ranges," the areas occupied by smaller ranges are believed to be those where the ranges are of greatest relief age and, hence, where the ranges have longer erosional histories. It is also well known that the areas occupied by significantly smaller ranges, as delineated by the trend-surface maps, are indeed those areas within the Basin and Range region which are characterized by the existence of pediments. In those areas in which the larger ranges occur, alluvial fans are far more characteristic. The generalized cross section of an ideal basin and two adjacent ranges (fig. 42) indicates why this is true. Reduction of the larger ranges, shown in the upper part of the figure, to the dimensions of the smaller ranges, shown below, must be accompanied by the production of a residual surface surrounding the ranges, the elevation and nature of which is primarily a function of local base level. This should not be construed as an argument that no pediments exist in the northwestern part of the Basin and Range region nor that no alluvial fans exist in the southwestern part of the region. As indicated for the several topographic parameters, some part of the total variance always arises from local components. But there is a significant regional variation in topography, and this coincides with the regional variation in the distribution of fans and pediments.

This coincidence does not necessarily prove that the hypothesis for pediment formation offered here is correct. It simply shows that mountain-range reduction and the occurrence of pediments go hand in hand. If pediment production is accomplished by drainage basin evolution through time, however, some regional drainage distinctions may exist within the Basin and Range province. The question of whether such distinctions exist and their nature constitutes a final geomorphic implication.

DRAINAGE DISTINCTIONS

A considerable body of literature supports the idea that drainage evolution, in the absence of constraints, accords with stochastic processes. The several random-walk models of Leopold and Langbein (1962), Schenk (1963), and Scheidegger (1967), among others, and the mathematical analysis of aspects of Horton's laws (1945) by Shreve (1966), and by Woldenberg (1966) clearly suggest that in areas of homogeneous rock type and structural stability, the drainage pattern that will develop is dendritic and, moreover, that this is the most probable pattern in nature. Combined with the far greater amount of information on the hydraulic geometry of river channels, which includes data on the profiles of drainage systems as well as their areal pattern, the status of knowledge of drainage basins and their contained drainage networks is known to be at least one order of magnitude in advance of the period when drainage was characterized as "youthful," "mature," and "old." The present writer believes, however, that so much effort has been devoted to establishing the concept that drainage systems reflect the most probable steady-state conditions that insufficient attention has been given to the study of the effects of time and whether these effects produce measurable drainage distinctions.

The drainage patterns that occur in the mountain ranges of the Basin and Range region have not been imposed by hydrologic input upon fullblown mountain blocks with planar sides, as so often depicted in older geomorphology texts and in the block diagrams of random-walk papers. Although the rates of orogeny are probably greater than the rates of denudation in the region since the Tertiary (Schumm, 1963), the existing drainage patterns must have begun to evolve at the very outset of the development of topographic relief. This means that complete equilibrium between the drainage systems and their respective host ranges may not yet be achieved in the more recently uplifted ranges, although these systems may be closer to mean steady-state conditions than might be suspected. In those ranges which have undergone long periods of stability, the drainage systems should be well adjusted and should reflect all of the attributes that are predicted by the stochastic models and by the hydraulic-geometry relations. Some measurable drainage distinctions should exist on this basis alone, however. That is, those parts of the Basin and Range region which are characterized by larger area of ranges to total area ratios, and by greater average widths, lengths, heights, volumes, and relief of ranges, might be predicted to be areas that are also characterized by some degree of departure from steady-state drainage systems.

Aside from this possibility, however, the effects of time may influence drainage systems in another way. Given a mountain block of finite size and homogeneous lithology, under specified hydrologic conditions there will be some finite number of drainage systems that can develop. Although the number of drainage systems that can coexist has never been specified, it clearly must be a function of mountain mass or the size and shape of a given range. W. B. Langbein (written commun., 1966) used random-walk models to investigate the drainage systems that will develop in square and rectangular areas in the absence of contraints. For a square area the largest stream system drained 21 percent of the total area, whereas the largest stream drained only 17 percent of the total area of a rectangle with a 2:1 side ratio. These results suggest that the problem is also amenable to testing by simple counts of the number of drainage systems in ranges of similar size, shape, lithology, and erosional history. The important question is, however, if this is true, then what effect has the passage of time on the numbers and the nature of drainage systems in a given range?

The writer has stated previously that mountain reduction is basically accomplished by the enlargement of drainage systems through time and that this results in pediment formation. But if the number of drainage systems is a function of the size of a range, this number must diminish as the mass of the range is reduced through time. Again, it might be predicted that those parts of the Basin and Range region characterized by different values of the 11 topographic parameters discussed will also be characterized by different values of mean number of drainage systems.

Finally, as the mass of a given range is reduced beyond some critical value, the nature of these drainage systems, as well as their number may be altered in some discernable manner. Clearly, the order numbers of the drainages must diminish, but, in addition, in a region characterized to some extent by orographic controls on precipitation, the hydrologic input will necessarily diminish, and weathering—rather than fluvial processes—will attain the dominant role in final mass reduction. Qualitative observation of many long-stable, residual ranges in southern Arizona and southeastern California supports this view; the lack of any prominent drainage network on the slopes of these low, narrow, and commonly linear ridges is characteristic. Thus, although this writer dislikes the use of the word "cycle," because of the many unsound or incorrect implications associated with it in geomorphology, the evolution of drainage systems in the Basin and Range region may well be termed "cyclic" in a restricted sense. Although there is undeniably an adjustment of process and form through time, predictable on the basis of the laws of probability and hydraulics, the evolution of drainage systems is intimately related to mountain mass and its ultimate reduction. The number of drainage systems and their order must diminish through time, and the relative roles of fluvial erosion and weathering must be reversed as the drainage systems diminish through time.

A search for quantitative regional drainage distinctions is therefore worthy of future efforts in the Basin and Range province. The writer suggests that these distinctions will probably coincide with the distribution of fans and pediments and with the regional topographic differences discussed in this report.

REFERENCES

American Association of Petroleum Geologists and United States Geological Survey, 1967, Basement map of North America between latitudes 24° and 60° N [Prepared by the Basement Rock Project Committee]: U.S. Geol. Survey, scale 1:500,000.

American Geological Institute, 1957, Glossary of geology and related sciences: Natl. Acad. Sci.—Natl. Research Council Pub. 501, 325 p.

——— 1960, Supplement to the glossary of geology and related sciences: Am. Geol. Inst. Rept., 72 p.

Bowman, Isaiah, 1911, Forest physiography—physiography of the United States and principles of soils in relation to forestry: New York, John Wiley & Sons, 750 p.

Chorley, R. J., 1959, The shape of drumlins: Jour. Glaciology, v. 3, no. 25, p. 339–344.

——— 1962, Geomorphology and general systems theory: U.S. Geol. Survey Prof. Paper 500–B, 10 p.

——— 1964, An analysis of the areal distribution of soil size facies on the Lower Greensand rocks of east-central England by the use of trend surface analysis: Geol. Mag., v. 101, p. 314–321.

Chorley, R. J., Stoddardt, D. R., Haggett, P., and Slaymaker, H. O., 1966, Regional and local components in the areal distribution of surface sand facies in the Breckland, eastern England: Jour. Sed. Petrology, v. 36, p. 209–220.

Damon, P. E., 1967, Correlation and chronology of ore deposits and volcanic rocks: U.S. Atomic Energy Comm. Annual Progress Rept. C00–689–76.

Damon, P. E., and Mauger, R. L., 1966, Epeirogeny-orogeny viewed from the Basin and Range province: Soc. Mining Engineers Trans., v. 235, p. 99–112.

Eardley, A. J., 1962, Structural geology of North America [2d ed.]: New York, Harper & Row, 743 p.

Fenneman, N. M., 1928, Physiographic divisions of the United States [3d ed.]: Assoc. Am. Geographers Annals, v. 18, no. 4, p. 261–353.

——— 1931, Physiography of Western United States: New York McGraw-Hill Book Co., 534 p.

Forgotson, J. M., Jr., 1960, Review and classification of quantitative mapping techniques: Am. Assoc. Petroleum Geologists Bull., v. 44, no. 1, p. 83–100.

Gilluly, James, 1965, Volcanism, tectonism, and plutonism in the Western United States: Geol. Soc. America Spec. Paper 80, 69 p.

Hadley, R.F., 1967, Pediments and pediment-forming processes: Jour. Geol. Education, v. 15, p. 83–89.

Harbaugh, J. W., 1963, Balgol program for trend-surface mapping using an IBM 7090 computer: Kansas Geol. Survey Spec. Distrib. Pub. 3, 17 p.

——— 1964a, Trend-surface mapping of hydrodynamic oil traps with the IBM 7090/94 computer: Colorado School Mines Quart., v. 59, no. 4, pt. B, p. 557–578.

——— 1964b, A computer method for four-variable trend analysis illustrated by a study of oil-gravity variations in southeastern Kansas: Kansas Geol. Survey Bull. 171, 58 p.

Harbaugh, J. W., and Preston, F. W., 1965, Fourier series analysis in geology, *in* Short course and symposium on computers and computer applications in mining and exploration: 5th Ann. Internat. Symposium, Arizona University, College of Mines, v. 1, p. R1–R46.

Hinds, N. E. A., 1952, Evolution of the California landscape: California Div. Mines and Geology Bull. 158, 240 p.

Horton, R. E., 1945, Erosional development of streams and their drainage basins hydrophysical approach to quantitative morphology: Geol. Soc. America Bull., v. 56, no. 3, p. 275–370.

Howarth, R. J., 1967, Trend-surface fitting to random data—an experimental test: Am. Jour. Sci., v. 265, no. 7, p. 619–625.

James, W. R., 1966, Fortran IV program using double Fourier series for surface fitting of irregularly spaced data: Kansas Geol. Survey Computer Contrib. 5, 19 p.

King, P. B., 1965, Quaternary tectonics in middle North America, *in* Wright, H. E., Jr., and Frey, D. G., (eds.), The Quaternary of the United States: Princeton Univ. Press, p. 831–870.

Krumbein, W. C., 1956, Regional and local components in facies maps: Am. Assoc. Petroleum Geologists Bull., v. 40, no. 9, p. 2163–2194.

Krumbein, W. C., and Graybill, F. A., 1965, An introduction to statistical models in geology: New York, McGraw-Hill Book Co., 475 p.

Leopold, L. B., and Langbein, W. B., 1962, The concept of entropy in landscape evolution: U.S. Geol. Survey Prof. Paper 500–A, 20 p.

Lustig, L. K., 1967, Inventory of research on geomorphology and surface hydrology of desert environments: Arid Lands Research Office, Arizona Univ. Press, 189 p.

Lobeck, A. K., 1939, Geomorphology—an introduction to the study of landscapes: New York, McGraw-Hill Book Co., 731 p.

Merriam, D. F., and Harbaugh, J. W., 1964, Trend-surface analysis of regional and residual components of geologic structure in Kansas: Kansas Geol. Survey Spec. Distrib. Pub. 11., 27 p.

Merriam, D. F., and Lippert, R. H., 1964, Pattern recognition studies of geologic structure using trend-surface analysis: Colorado School Mines Quart., v. 59, no. 4, pt. A, p. 237–245.

——— 1966, Geologic model studies using trend-surface analysis: Jour. Geology, v. 74, p. 344–357.

Merriam, D. F., and Sneath, P. H. A., 1966, Quantitative comparison of contour maps: Jour. Geophys. Research, v. 71, p. 1105–1115.

Miller, R. L., 1956, The relation of sediment-size parameters to current-wave systems and physiography [Calif.] pt. 1 of Trend surfaces—their application to analysis and description of environments of sedimentation: Jour. Geology, v. 64, No. 5, p. 425–446.

Miller, R. L., and Kahn, J. S., 1962, Statistical analysis in the geological sciences: New York, John Wiley & Sons, 483 p.

Moore, J. G., 1959, The quartz diorite boundary line in the United States: Jour. Geology, v. 67, p. 198–210.

O'Leary, Mont, Lippert, R. H., and Spitz, O. T., 1966, Fortran IV and map program for computation and plotting of trend surfaces for degrees 1 through 6: Kansas Geol. Survey Computer Contrib. 3.

Pelto, C. R., 1954, Mapping of multicomponent systems: Jour. Geology, v. 62, No. 5, p. 501–511.

Preston, F. W., and Harbaugh, J. W., 1965, Balgol programs and geologic application for single and double Fourier series using IBM 7090/7094 computers: Kansas Geol. Survey Spec. Distrib. Pub. 24, 72 p.

Raisbeck, Gordon, 1963, Information theory—an introduction for scientists and engineers: Cambridge, Massachusetts Inst. Technology Press, 105 p.

Read, W. A., and Merriam, D. F., 1966, Trend-surface analysis of stratigraphic thickness data from some Namurian rocks east of Stirling, Scotland: Scottish Jour. Geology, v. 2, pt. 1, p. 96–100.

Salvadori, M. G., and Miller, K. S., 1953, The mathematical solution of engineering problems: New York, Columbia Univ. Press, 245 p.

Sampson, R. J., and Davis, J. C., 1966a, Fortran II trend-surface program with unrestricted input for the IBM 1620 computer: Kansas Geol. Survey Spec. Distrib. Pub. 26, 12 p.

——— 1966b, Three-dimensional response surface program in Fortran II for the IBM 1620 computer: Kansas Geol. Survey Computer Contrib. 10, 20 p.

Scheidegger, A. E., 1967, A stochastic model for drainage patterns into an intramontane treinch: Internat. Assoc. Sci. Hydrology Bull., v. 12, No. 1, p. 15–20.

Schenk, Hilbert, Jr., 1963, Simulation of the evolution of drainage-basin networks with a digital computer: Jour. Geophys. Research, v. 68, No. 20, p. 5739–5745.

Schumm, S. A., 1963, The disparity between present rates of denudation and orogeny: U.S. Geol. Survey Prof. Paper 454–H, 13 p.

Shreve, R. L., 1966, Statistical law of stream numbers: Jour. Geology, v. 74, No. 1, p. 17–37.

Spitz, O. T., 1966, Generation of orthogonal polynomials for trend surfacing with a digital computer: Computers and Operation Research in Mineral Industries, Pennsylvania State Univ., 6th Annual Symposium.

Thornbury, W. D., 1965, Regional geomorphology of the United States: New York, John Wiley & Sons, 609 p.

Troeh, F. R., 1965, Landform equations fitted to contour maps: Am. Jour. Sci., v. 263, p. 616–627.

U.S. Geological Survey, 1966, Classes of land-surface form: National Atlas, scale 1 : 7,500,000.

Whitten, E. H. T., 1959, Composition trends in a granite; modal variation and ghost stratigraphy in part of the Donegal granite, Eire: Jour. Geophys. Research, v. 64, no. 7, p. 835–848.

——— 1961, Modal variation and the form of the Beinn an Dubhaich granite, Skye: Geol. Mag., v. 98, no. 6, p. 467–472.

——— 1962, Areal variability of alkalis in the Magsburg granite, Germany: Neues Jahrb. Mineralogie Monatsh., v. 9, p. 193–200.

Woldenberg, M. J., 1966, Horton's Laws justified in terms of allometric growth and steady-state in open systems: Geol. Soc. America Bull., v. 77, p. 431–434.

Yasso, W. E., 1965, Plan geometry of headland-bay beaches: Jour. Geology, v. 73, p. 702–714.

U. S. GOVERNMENT PRINTING OFFICE : 1969 O - 326-328