

Statistical Properties of Dune Profiles

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Statistical Properties of Dune Profiles

By CARL F. NORDIN, JR.

SEDIMENT TRANSPORT IN ALLUVIAL CHANNELS

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SYMBOLS

Symbol	Definition	Units	Symbol	Definition	Units
$A(\omega)$	Gain function.....	0	g	Acceleration due to gravity.....	Ft per sec ²
a	Dune amplitude, the maximum ordinate of y between zero crossings.....	0	h	A fixed level of the bed, measured in units of standard deviation of y from the mean bed level.....	0
c	Wave celerity.....	Fpm	H	Dune or wave height, from crest to trough.....	Ft
$c(\omega)$	Cospectrum.....	0	$H(\omega)$	Frequency response function.....	0
D	Mean flow depth.....	Ft	l_h^+	Duration or length of the upward excursion of the process y above the fixed level h	or ft
E	Expected value.....	0	L	Dune length, from trough to trough.....	Ft
F	Froude number.....	0	m_n	The n^{th} moment of the spectrum.....	0
f	Frequency number.....	Cycles per unit time	N_0	Number of zero crossings.....	0
f'	Dimensionless frequency.....	0	N_h	Number of h -level crossings.....	0
$G_{yy}(\omega)$	Spectral density function..... (Cycles per unit time) ⁻¹		P	Probability distribution function.....	0
$G_{yz}(\omega)$	Cross spectrum..... (Cycles per unit time) ⁻¹		p	Probability density function.....	0
$G(x)$	Spectral density function for the process $y=y(x)$ (Cycles per unit distance) ⁻¹		q	Water discharge per foot of width.....	Cfs
$G(t)$	Spectral density function for the process $y=y(t)$ (Cycles per unit time) ⁻¹		$q(\omega)$	Quadrature spectrum.....	0
G'	Dimensionless spectra.....	0	s	A time or distance lag.....	Min or ft
			S	Water-surface slope.....	0

CONTENTS

<i>Symbol</i>	<i>Definition</i>	<i>Units</i>	<i>Symbol</i>	<i>Definition</i>	<i>Units</i>
t	Time.....	Min	η	A dimensionless wave height.....	0
T	Wave period.....	Min	$\theta(\omega)$	Phase angle.....	0
V	Mean flow velocity.....	Fps	μ	Mean number of upcrossings per unit time.....	(Min) ⁻¹
x	Distance along channel in flow direction.....	Ft	$\rho_{yy}(s)$	Autocorrelation function.....	0
y	Bed elevation, measured from $y=0$	Ft	$\rho_{yz}(s)$	Cross-correlation function.....	0
α	Height of local maximum of $y(t)$	Ft	σ_y^2	Variance of the process y	Sq ft
$\gamma_{yz}^2(\omega)$	Coherence function.....	0	$\phi_{yy}(s)$	Covariance function.....	0
δ^2	A parameter.....	0	$\phi_{yz}(s)$	Cross-covariance function.....	0
ϵ	Wave number.....	Cycles per ft	ω	Angular frequency, $\omega = 2\pi f$	Radians per unit time
ϵ'	Dimensionless wave number.....	0			

STATISTICAL PROPERTIES OF DUNE PROFILES

BY CARL F. NORDIN, JR.

ABSTRACT

Properties of sand waves formed by subcritical unidirectional water currents are investigated by statistical analyses of records of streambed profiles. Records of bed elevation y as a function of distance x along the channel, $y=y(x)$, and time records at a fixed point of the channel, $y=y(t)$, were collected in three laboratory flumes that were 8 inches, 2 feet, and 8 feet wide and in a straight alluvial channel that was 55 feet wide. All bed material was fine sand. The continuous analog records were converted to discrete data points and were analyzed by digital computer.

The analyses show that both types of records, $y(x)$ and $y(t)$, can be approximately represented as stationary Gaussian processes. When the data are standardized and the lengths or distances are expressed as ratios of the mean duration between zero crossings of y , the statistical properties of all the flume data are similar, with no distinguishing characteristics that can be attributed to size of flume or to whether the bed forms were ripples or dunes. The field data, however, reflect the influence of large alternate bars that were not present in the flumes.

The Gaussian assumption, together with the spectral properties of the records as expressed by a dimensionless parameter, δ , permits predicting the distributions of maximum and minimum values of y between successive zeros of y . These distributions represent the probability distributions of the depth of local scour and fill due to the formation and migration of sand waves, and the parameters that specify the distributions relate approximately to flow velocity and depth.

Observed values of the number of zero and h -level crossings, the mean duration between zero crossings, and the mean duration of upward excursions of the process $y(t)$ above the fixed level h compared reasonably well with theoretical values for the Gaussian model. The distribution of the duration of upward excursions is the conditional probability distribution of the rest period of a particle, given that it is deposited on the downstream face of a ripple or dune at the level h . Observed distributions of these durations can be approximated by a gamma distribution with parameters that relate to h , where h is measured in units of standard deviation from the mean bed level. These distributions and other probability distributions that enter into stochastic models of sediment transport can be determined either from the theoretical model or empirically from the observed data. The results of the study show that, even though the bed elevation deviates somewhat from the postulated normal distribution, reasonable estimates of many properties of the bed profiles can be derived from fairly simple statistical models.

INTRODUCTION

BACKGROUND

A distinguishing characteristic of sand waves formed by unidirectional subcritical water currents is their

tendency to form "en echelon" with gently sloping upstream faces and more steeply sloping downstream faces that meet the horizontal at approximately the natural repose angle of the sand. These features migrate slowly in the mean flow direction as material is eroded from their upstream faces and deposited on their downstream faces.

Generally, these features are described as simple triangular forms, in profile, somewhat as sketched in figure 1, with a mean length from crest to crest or trough to

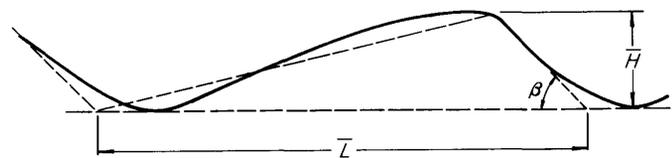


FIGURE 1.—Idealized dune shape.

trough, \bar{L} , a mean height from crest to trough, \bar{H} , and a constant angle of downstream face, β . If the waves are long-crested, or two-dimensional, their geometric properties then are considered completely specified by \bar{L} , \bar{H} , β . The ratio of mean length to mean height, \bar{L}/\bar{H} , called the ripple index, is a measure of the wave steepness.

In reality, the simple waveforms of figure 1 rarely exist. Long-crested sand waves occur apparently only under rather restricted flow conditions which will not be considered here. In the general case of flow in a wide channel with a bed of fine sand, the ripples and dunes that form are three-dimensional and highly irregular in size, shape, and spacing.

The three-dimensional properties of these features are completely described by a contour map of the bed, and with modern sounding, navigation, and computing equipment, large areas of a streambed can be mapped with ease and dispatch. However, the expense involved in obtaining detailed contour maps is generally prohibitive, and more generally, one has available only profiles of the streambed, obtained either by sounding along the channel from a boat or by sounding at some fixed point in the flow and recording changes in the bed elevation as the dunes and other bed features migrate past the sounding point.

Although the profiles give only a two-dimensional picture of the streambed, they still provide a great amount of useful information. From longitudinal records, one can determine directly the distributions of lengths and heights associated with a particular ensemble of waveforms. From time records, the average wave period is easily found, which, together with mean wave height, provides a very good estimate of the bed load transport (Simons and others, 1965). The distribution of troughs and crests indicates the amount of local scour and fill associated with the migrating sand waves; this information may be important in such practical problems as designing and maintaining navigation channels or estimating the depth to which a structure such as a pipeline or siphon should be buried beneath the mean bed level to minimize the probability of local scour exposing the structure to the current.

From a comparison of the properties of different streambed profiles, it may be possible to establish whether or not there are any essential differences, other than scale, between ripples and dunes, and whether or not there are statistical properties of the dune profiles other than scale that can be attributed to the size of the channel. Both questions have important implications in modeling alluvial channel processes.

Perhaps the potentially most useful information to be derived from streambed profiles is information that relates to stochastic models of sediment transport. For example, in their two-dimensional stochastic model for the transport and dispersion of bed-material sediment particles, Sayre and Conover (1967) require the probability that a sediment particle will be deposited at a given level in the bed and the conditional probability for the length of time a particle will remain buried in the bed (that is, that it will experience a rest period of a certain duration), given that it is deposited at a particular level. These probabilities, together with some other distributions of interest, are easily found from the bed profiles.

To be more specific, consider the short segment of profile sketched in figure 2. Assume a straight uniform channel with equilibrium flow conditions, as defined by Simons and Richardson (1966, p. J3). If y is the bed elevation,

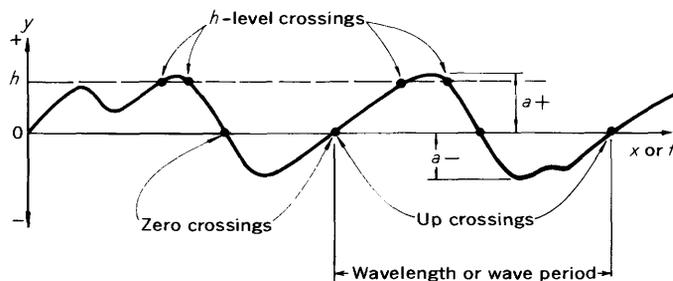


FIGURE 2.—Definition sketch of bed profile.

tion, measured from the mean bed level so that $\bar{y}=0$, and x is distance along the channel in the direction of flow, the bed profile then can be represented in the form $y=y(x, t)$, $x \in X$, $t \in T$. At any given position $x=x_0$ along the channel, one may record the change with time of bed elevation y to produce the record, $y=y(t)$. Similarly, at any given instant of time, $t=t_0$, one can sound along the channel to obtain a record of the bed profile, $y=y(x)$. In reality, of course, it is impossible to obtain instantaneously a longitudinal profile, but practically the time required to obtain a profile is small compared with the time required for a dune to shift appreciably downstream, so that the assumption $y=y(x)$ is quite reasonable.

For either $y=y(x)$ or $y=y(t)$, the bed elevation y , measured about the mean bed level, is a random variable that depends on a parameter (t or x) defined on an arbitrary parameter set (T or X , respectively). By definition, then $y=y(x, t)$ is a stochastic process (Cramer, 1964, p. 137).

In all cases, y is a continuous function; obviously, there can be no discontinuities in the sand bed of a stream. Intuitively, one also would expect that, if the mean properties of the flow, of the sediment, and of the sediment transport do not change with time or with distance along the channel, then y will represent a stochastic process which meets the requirements both of stationarity and of ergodicity.

Consider next, in figure 2, some simple definitions that will be used later. The points where the processes $y(x)$ or $y(t)$ cross the zero axis are zero crossings, and the average distance between successive upcrossings (values of y going from negative to positive) is an average wavelength or wave period, somewhat analogous to \bar{L} shown in figure 1. The maximum ordinate between zero crossings, in absolute values, is defined as the amplitude, a , and is roughly comparable to one-half of the wave height of figure 1.

Crossings of the level h are defined in a similar manner. Note that the average duration of the upward excursion of the process $y(t)$ above the fixed level h is the average rest period experienced by a particle after it is deposited on the downstream faces of the sand waves at the level h .

The probability that a particle will be deposited at the level h also can be determined from the bed profiles for at least some simple postulated depositional patterns. If a particle is equally likely to be deposited at any place on the bed, the distribution is simply the frequency distribution of the y values. If deposition occurs only on the downstream faces of the ripples or dunes, the distribution can be determined from the distribution of y values where the process $y(x)$ has a negative slope or $y(t)$ has a positive slope. If nothing is known of the previous history of a particle, that is, if it is equally likely to be

found any place in the bed above the lowest point of particle motion, then the probability that it will be found at the level h can be determined approximately as the ratio of the area bounded by the fixed level h and the upward excursions of the process $y(x)$ above h to the total area of the bed profile above the minimum y value.

Many properties of the processes $y(x)$ and $y(t)$ that are of interest can be determined empirically if suitable records of streambed profiles are available. Usually, though, it is difficult to obtain both types of records. In the laboratory, where flows can be controlled, it is possible to obtain long records of $y(t)$, but longitudinal profiles of the process $y(x)$ are limited by the effective length of the flume. On the other hand, in field studies, it may be possible to obtain suitable longitudinal profiles, but the sand waves generally move so slowly that satisfactory samples of the process $y(t)$ cannot be obtained under constant flow conditions. It has not been established that the statistical properties of the processes $y(x)$ and $y(t)$ are comparable, although some similarities have been noted (Nordin and Algert, 1966). Thus, it is extremely important to determine in what respects the two types of records are similar and to develop methods of correlating the properties of the two types of records.

Ultimately, of course, one will wish to predict something about the streambed profiles, given only information on the characteristics of the flow and the bed sediment. In order to do this, it is necessary first to determine if there are any consistent or recognizable patterns in the properties of interest, and then to attempt to relate these properties to flow and sediment parameters.

There is little theoretical or empirical basis upon which to postulate the statistical properties of $y(x)$ or $y(t)$. However, during the course of preliminary studies of the bed profiles, several facts emerged that led to the approach adopted for this investigation. First, it was noted that the y values were distributed about their mean values approximately as normal distributions, which was to be expected as most natural processes that develop under the influence of many random factors exhibit approximate Gaussian distributions. Second, many of the properties of a Gaussian process with known covariance functions, particularly the mean values of duration between zero and h -level crossings and the maximum between zero crossings, are well established from previous work on the statistical properties of random noise (Rice, 1954) and on ocean waves (Longuet-Higgins, 1958, 1962, 1963). Finally, it has been shown that properties of the covariance function near the origin relate to a simple flow parameter, at least for a limited range of flow conditions (Nordin and Algert, 1966), so that in considering the prediction problem, the assumption of a known covariance function may be rather simple to satisfy.

Therefore, the approach adopted for this study was to compare the observed properties of the bed profiles with the theoretical properties of a Gaussian process of known covariance function. In the following section, the scope and specific objectives of the study are given in more detail.

PURPOSE AND SCOPE

In broad terms, this study was designed to investigate the statistical properties of streambed profiles. Data were collected in both laboratory and field investigations, and attention was always restricted to equilibrium flow over a bed of fine sand in a straight uniform channel with either ripple or dune bed configuration. The classification of bed configurations as either ripples or dunes is according to Simons and Richardson (1966, p. J5-J7). Details of the hydraulic and sediment data are given in a later section.

Specifically, we are interested in the mean values and the distributions of the durations between zero and h -level crossings, of the durations of upward excursions of the process $y(t)$ above the fixed level h , and of the positive and negative maximums of $y(x)$ or $y(t)$ between zero crossings. In addition, it is of interest to consider whether or not the statistical properties of $y(x)$ and $y(t)$ are similar, whether or not there are any significant differences other than scale in the statistical properties of ripple and dune profiles, and whether or not or to what extent the statistical properties of dune profiles depend on the scale of the flow system.

Particular attention is paid to the spectral representations of the process y because the distributions of the amplitudes a for a Gaussian process depend to a large extent on the properties of the spectra (Cartwright and Longuet-Higgins, 1956). Some applications of cross correlation and of cross-spectral analysis are examined briefly.

As indicated above, the approach used in this study is to compare the observed properties of the bed profiles to the theoretical properties of a Gaussian process of known covariance function. Similarities and differences between the observed and the theoretical processes are noted, and some of the statistical parameters that describe the observed processes are related empirically to properties of the flow.

In the following section, a review of the properties of a Gaussian process is given, and some of the mathematical relations for spectral analysis are listed. Then the data is described and the results of the analysis are presented. Finally, the paper discusses the implication of the results, summarizes the conclusions drawn, and lists some recommendations for future research along these same lines.

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SOME PROPERTIES OF A GAUSSIAN PROCESS

ZERO AND h -LEVEL CROSSINGS

Suppose that $y(t)$ is a real stationary Gaussian random function of the continuous parameter t , $0 < t < \infty$, with zero mean and covariance function $\phi_{yy}(s)$ and possessing a spectral density function $G_{yy}(\omega)$. The covariance function is the expected value of the lagged product of $y(t)$ and $y(t+s)$, or

$$\phi_{yy}(s) = E\{y(t)y(t+s)\}. \quad (1)$$

The process $y(t)$ is assumed to be ergodic, so that the covariance function is given by

$$\phi_{yy}(s) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T y(t)y(t+s) dt \quad (2)$$

which by stationarity is a function only of s . The covariance function is related to the spectral density function by the equation

$$\phi_{yy}(s) = \int_0^\infty G_{yy}(\omega) \cos \omega s d\omega \quad (3)$$

where ω is the angular frequency.

The spectral density and covariance are Fourier transform pairs:

$$G_{yy}(\omega) = \frac{2}{\pi} \int_0^\infty \phi_{yy}(s) \cos \omega s ds. \quad (4)$$

The variance of $y(t)$ is

$$\sigma_y^2 = \text{var } y(t) = \phi_{yy}(0) = \int_0^\infty G_{yy}(\omega) d\omega. \quad (5)$$

The first moment of the spectrum about the origin is given by

$$m_1 = \int_0^\infty \omega G_{yy}(\omega) d\omega \quad (6)$$

and, in general, the r th moment, $r=0, 1, 2, \dots$, is defined as

$$m_r = \int_0^\infty \omega^r G_{yy}(\omega) d\omega. \quad (7)$$

The mean frequency of the spectrum is then

$$\bar{\omega} = m_1/m_0, \quad (8)$$

and the derivatives of the covariance function at the origin, if they exist, are

$$\phi^r(0) = (-1)^{1/2r} m_r, \quad (9)$$

where r is even, and

$$\phi^r(0) = 0,$$

where r is odd.

Apparently, there is no general solution for determining the probability distribution of the interval between zero crossings, $p\{l_0\}$, but its mean value is the reciprocal of the average number of zero crossings per unit time, $E\{N_0\}$, which was given by Rice (1954), as

$$E\{N_0\} = \frac{1}{\pi} \left[\frac{-\phi''(0)}{\phi(0)} \right]^{1/2}. \quad (10)$$

The expected number of h -level crossings is

$$E\{N_h\} = \frac{1}{\pi} e^{-h^2/2} \left[\frac{-\phi''(0)}{\phi(0)} \right]^{1/2}. \quad (11)$$

The ratio of the average number of h -level crossings to the average number of zero crossings from equations 10 and 11 is

$$E\{N_h\}/E\{N_0\} = e^{-h^2/2}. \quad (12)$$

The expected duration of an upward excursion above the level h is given by Cramer and Leadbetter (1967):

$$E\{l_h^+\} = \mu^{-1} \text{pr}\{y(0) > h\} \quad (13)$$

where μ is the mean number of upcrossings per unit time and pr denotes probability. The mean number of upcrossings is one-half of the mean number of crossings per unit time, so that, combining equations 10 and 13, the ratio of expected duration of upward excursions above the level h to expected duration of upward excursions above the zero level is

$$E\{l_h^+\}/E\{l_0^+\} = 2 \text{pr}\{y(0) > h\} e^{h^2/2}. \quad (14)$$

An interesting alternative to Rice's approach is given by Tick and Shaman (1966) for a straight line interpolation of an underlying continuous Gaussian process determined by sampling the underlying process at equispaced intervals. Again, it is assumed that $y(t)$ is a stationary Gaussian process with continuous parameter t , $-\infty < t < \infty$, and with zero mean and covariance function $\phi_{yy}(s)$, and possessing a spectral density function $G_{yy}(\omega)$ defined by equation 3. Assume that $\phi(0) = 1$ and that $y(t)$ is sampled discretely at the time points $\dots -2\Delta t, -\Delta t, 0, \Delta t, 2\Delta t, \dots$. The sampled process also is Gaussian with covariance sequence $\phi(n\Delta t)$, $n = 0, \pm 1, \pm 2, \dots$. Connecting successively the observed ordinates of the sampled sequence with straight line segments yields the interpolation process mentioned above.

The expected number of zero crossings in a record of length $k\Delta t$ is found to be

$$E\{N_0\} = k \left[\frac{1}{2} - \frac{1}{\pi} \arcsin \phi(\Delta t) \right]. \quad (15)$$

To determine the expected number of h -level crossings, choose two adjacent values, $y(n\Delta t)$ and $y(\{n+1\}\Delta t)$, and denote them by Y_1 and Y_2 . The joint distribution of Y_1 and Y_2 is bivariate normal with correlation $\phi(\Delta t)$. Then, the expected number of h -level crossing is

$$\begin{aligned} E\{N_h\} &= k [\text{pr}(Y_1 > h, Y_2 < h) + \text{pr}(Y_1 < h, Y_2 > h)] \\ &= 2k [\text{pr}(Y_1 > h, Y_2 > -\infty) - \text{pr}(Y_1 > h, Y_2 > h)]. \end{aligned} \quad (16)$$

From equations 15 and 16, the value of the ratio of the expected number of h -level to the expected number of zero crossings is seen to depend on the covariance function at one lag, $\phi(\Delta t)$. Values were computed for $\phi(\Delta t) = 0.7$ and 0.9 , and the curve for $\phi(\Delta t) = 0.7$ is plotted in figure 3 along with equation 12 for the continuous process. There is so little difference in the two curves for values of $\phi(\Delta t)$ greater than 0.7 that the simpler expression of equation 12 is to be preferred.

Although the probability distribution of the l values cannot be precisely determined in the general case,

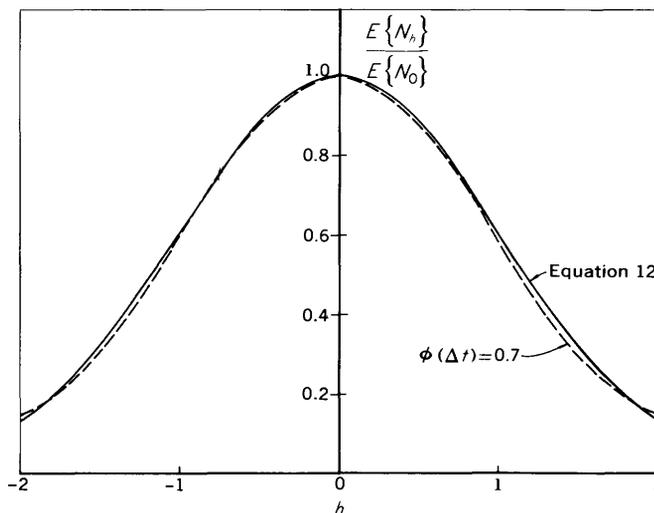


FIGURE 3.—Ratio of the expected number of h -level crossings to the expected number of zero crossings, $E\{N_h\}/E\{N_0\}$, as a function of h .

Longuet-Higgins (1962, 1963) gives some approximations for upper and lower bounds of $p(l_0)$, with particular attention to certain ideal forms of the spectra of $y(t)$, and Cramer and Leadbetter (1967) give equations for the moments of the distribution functions of the duration of upward excursions above the level h . From a practical point of view, the mean values of l_h^+ are of most interest, particularly the mean in an interval $h_2 - h_1$, which is the mean rest period of a particle deposited on the downstream face of a sand wave between the elevations h_1 and h_2 , and which can be determined easily by integrating equation 13 between appropriate limits.

WAVE HEIGHTS AND AMPLITUDES

The wave height H was defined as the difference in elevation between a crest (maximum) and the following trough (minimum) in figure 1. The statistical distribution of H generally is not known, but where $y(t)$ has a narrow spectrum, it has been established that $H/2$ is distributed according to a Rayleigh distribution (see Cartwright and Longuet-Higgins, 1956):

$$p(H/2) = \frac{H}{(m_0)^{1/2}} e^{-\left(\frac{H}{2}\right)^2 / (m_0)^{1/2}} \quad (17)$$

where m_0 , the variance of $y(t)$, is determined by equation 5 or equation 7. If the process y has unit variance, the equation simplifies to

$$p(H/2) = H e^{-(H/2)^2} \quad (18)$$

and the cumulative probability distribution of $H/2$ is given by

$$p(H/2 \leq \xi) = 1 - e^{-\xi^2}. \quad (19)$$

Although the probability distribution of H is not established, the statistical distribution of the local maxima of $y(t)$ is known. If $y(t)$ is a strictly stationary process possessing a continuous sample derivative $y'(t)$, a local maximum or crest is said to occur at $t = t_0$ if $y'(t)$ has a downcrossing of zero at t (Cramer and Leadbetter, 1967, p. 242). Define α as the difference in height between the crest and the mean level of $y(t)$. Then, for the Gaussian process considered here, the probability distribution of α depends only on $(m_0)^{1/2}$ and on a parameter δ that represents the relative width of the frequency spectrum,

$$\delta^2 = \frac{m_0 m_4 - m_2^2}{m_0 m_4}, \quad (20)$$

where $0 < \delta < 1$.

For $\delta \rightarrow 0$, the spectrum becomes infinitely narrow and the dimensionless maxima, η , tend to a Rayleigh distribution:

$$p(\eta) = \eta e^{-\frac{\eta^2}{2}} \quad (21)$$

where $\eta > 0$, and

$$p(\eta) = 0,$$

where $\eta < 0$.

where η is defined by the relation

$$\eta = \alpha / (m_0)^{1/2}. \quad (22)$$

When δ approaches its maxima value of 1, the distribution of η is Gaussian,

$$p(\eta) = \frac{1}{(2\pi)^{1/2}} e^{-\eta^2/2}, \quad -\infty < \eta < \infty. \quad (23)$$

Derivations of the above distributions are given by Rice (1954) and are discussed in detail by Cartwright and Longuet-Higgins (1956). The cumulative probability distributions of η for various values of δ are shown in figure 4.

In figure 2, a was defined as the maximum y value between zero crossings, and it can assume either positive or negative values. From a practical point of view, it may be advantageous to consider separately the distributions of positive $a+$ and negative $a-$ values, in the event that $y(t)$ is not symmetric about its mean level. The theoretical Gaussian model is assumed symmetric, so that in this case it is permissible to consider only the the positive a values, keeping in mind the symmetry of

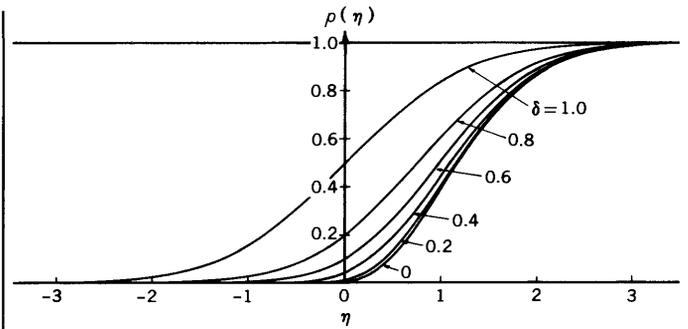


FIGURE 4.—Cumulative probability distribution functions for dimensionless maxima, η , as a function of δ .

the distribution about zero when negative values are of concern.

Clearly, for narrow-band random noise, the distribution of $a+$ would approach the distribution of local maxima, the Rayleigh distribution. (See, for example, Bendat and Piersol, 1966, p. 17.) Generally, however, there apparently is no theoretical derivation for the probability distribution of the a values. For purposes of this study, therefore, the observed distributions of a simply will be compared with the theoretical distributions of local maxima shown in figure 4. At least intuitively, one would expect the distribution of a values for a process with a broad spectrum to approach a form of the normal distribution, but there is no reason to expect the distribution to depend only on δ and m .

THEORY OF TIME SERIES ANALYSIS

In the preceding section, the covariance function, $\phi(s)$, or alternatively, the spectral density function, $G(\omega)$, was assumed known. Values of these functions are required to predict mean durations between zero and h -level crossings and to estimate probability distributions of maxima for the Gaussian model. For the actual streambed profiles analyzed in this study, values of $\phi(s)$ and $G(\omega)$ were computed and the observed properties of the profiles were then compared with properties of a Gaussian process having the same covariance and spectral density functions.

Apart from the problem of predicting the zero and h -level crossings, the covariance functions and spectra together with the probability distributions of the y values provide a great deal of information about the process $y(t)$, and these functions will be found particularly useful in later sections when we consider the similarities and the differences of the streambed profiles that can be attributed to scale of the flow system or to channel size. Therefore, it is appropriate at this point to review briefly the theory of time series analysis and to mention some of the properties of the covariance functions and spectral density functions that were found useful in analyzing the dune profiles.

For this purpose, consider $y(t)$ to be a real continuous random function of time t , $-\infty < t < \infty$. The process $y(t)$ is assumed to be stationary and ergodic. Stationarity implies time invariance of the general statistical properties of the process $y(t)$, and ergodicity insures that averages across an ensemble are equivalent to averages over time along a single sample function or realization of infinite extent. (For a more detailed discussion of these concepts, see Yaglom, 1962 or Cr amer and Leadbetter, 1967.)

By stationary, both the mean value, \bar{y} , and the variance, σ_y^2 , are constants, so that it is always possible to form a new time series, $[y(t) - \bar{y}]/\sigma_y$, that has zero mean and unit variance. Unless otherwise specified, it will be assumed in the following discussion that this transformation has been made. Then the correlation function, defined as

$$\rho_{yy}(s) = \phi_{yy}(s)/\phi_{yy}(0), \quad (24)$$

is identical to the covariance function.

Both $\phi(s)$ and $G(\omega)$ are even functions of their respective arguments, and the usual equations for these functions are

$$G_{yy}(\omega) = \frac{1}{\pi} \int_0^{\infty} \phi_{yy}(s) e^{-i\omega s} ds \quad (25)$$

$$\phi_{yy}(s) = \int_0^{\infty} G_{yy}(\omega) e^{i\omega s} d\omega \quad (26)$$

where negative values of both time and frequency are considered. Because $y(t)$ is a real function, the correlation and spectral density functions often are given as a cosine transform pair, equations 3 and 4, where the G and ϕ values are modified by the necessary constants to apply only to positive t and ω values. Bendat and Piersol (1966, p. 77-84) give a complete discussion of the "one-sided" correlation and spectral density functions that generally are used in practical computations.

The autocovariance function measures the degree of dependence between the observed quantity y at one time and at another time s units later. It is especially useful in defining periodicities in the process $y(t)$.

The power spectrum or spectral density function describes the general frequency composition of the data in terms of the density of its mean square value, that is, $G_{yy}(\omega)d\omega$ represents the contribution to the variance of the process from the frequencies between ω and $(\omega + d\omega)$. Thus, the spectra show directly similarities and differences in the streambed profiles and provide a quantitative basis for comparing the ripple bed forms with the

dune forms or for evaluating the effect of scale on the statistical properties of the streambed profiles.

The treatment is extended to two stochastic processes as follows. The cross correlation between two series $y(t)$ and $z(t)$ is given by

$$\rho_{yz}(s) = \frac{\phi_{yz}(s)}{\sqrt{\phi_{yy}(0)\phi_{zz}(0)}} \quad (27)$$

where $\phi_{yz}(s)$ is the cross covariance between the series and is defined as the expected value of the product $y(t)z(t+s)$. Both $y(t)$ and $z(t)$ are ergodic, so that the cross covariance is

$$\begin{aligned} \phi_{yz}(s) &= E\{y(t)z(t+s)\} \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} y(t)z(t+s) dt. \end{aligned} \quad (28)$$

The cross spectrum is defined as the Fourier transform of the cross-covariance function

$$G_{yz}(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \phi_{yz}(s) e^{-i\omega s} ds = c(\omega) + iq(\omega) \quad (29)$$

where ω represents the angular frequency, $c(\omega)$ is the cospectrum, a measure of the in-phase covariance, and $q(\omega)$ is the quadrature spectrum, a measure of the out-of-phase covariance. The cospectrum measures the contributions of oscillations at the lag zero between two time series. The quadrature spectrum measures the contribution of the different harmonics to the total cross covariance between the series when all the harmonics of the series $y(t)$ are delayed by a quarter period but the series $z(t)$ remains unchanged.

The real quantity defined as coherence, $\gamma_{yz}^2(\omega)$, is a direct measure of the square of the correlation of the amplitudes of frequency ω of the processes $y(t)$ and $z(t)$, or

$$\gamma_{yz}^2(\omega) = \frac{c_{yz}^2(\omega) + q_{yz}^2(\omega)}{G_y(\omega)G_z(\omega)}, \quad 0 \leq \gamma(\omega) \leq 1, \quad (30)$$

where $G_y(\omega)$ and $G_z(\omega)$ represent the spectra of $y(t)$ and $z(t)$, respectively.

Even if the amplitudes are fully correlated, it is possible that the corresponding frequency components will have different phases. The phase lag at each frequency is given by

$$\theta(\omega) = \arctan \frac{q(\omega)}{c(\omega)} \quad (31)$$

where $\theta(\omega)$ is called the phase function.

Another quantity sometimes useful in cross-spectral analysis is the frequency-response function, $H(\omega)$, calculated from the relation

$$H(\omega) = \frac{G_{zy}(\omega)}{G_z(\omega)} = A(\omega)e^{i\theta(\omega)}. \quad (32)$$

The quantity $A(\omega)$ is the gain function or amplitude gain of the system and measures the ratio of the amplitude of the frequency components of the series $y(t)$ and $z(t)$ at each frequency ω .

The cross covariance measures the dependence between two time series at the given lag, s , and for this study, it was applied to investigate the three-dimensional properties of the sand waves, their mean rate of shifting, and the time or distance required for the waves effectively to lose their identity. The cross-spectral functions, equations 29–32, were used in conjunction with the cross covariance.

The above theory is presented for a continuous process with the parameter t representing time. The frequency $f = \omega/2\pi$ is then given in cycles per unit time. The parameter t is simply a member of an arbitrarily specified parameter set, and it can be any quantity that permits the set of y or z values to be ordered linearly. If time t is replaced by distance x , the frequency f is replaced by wave number ϵ and the wave period $T = 1/f$ is replaced by wavelength $L = 1/\epsilon$.

DATA AND ANALYSIS

BASIC DATA

Profiles from four different channels were selected for these analyses. Table 1 gives a summary of the records. Three of the channels were recirculating laboratory flumes located at the Research Center Hydraulics Laboratory, Colorado State University, Fort Collins, Colo. The other channel was Atrisco Lateral near Bernalillo, N. Mex., a conveyance channel with a sand bed and with banks stabilized by clay and vegetation. The dimensions of the flumes were: 0.67 foot wide by 30 feet long, 2 feet wide by 60 feet long, and 8 feet wide by 200 feet long. Atrisco Lateral was approximately 55 feet wide, and the profiles were obtained about midway in a straight reach 12,000 feet long. The median diameters of bed material and flow parameters are shown in table 1.

Fifty-four records representing six different flow conditions were selected for analysis. Runs 1, 2, and 3 were data collected from Atrisco Lateral on three different days, but with similar flows. Records 4 through 19 and 20 through 39 are from the 8-foot flume for two different

flow conditions, and like the field data for the first three runs, the bed configuration was dunes. The records for runs 40–43 are for a ripple bed in the 2-foot flume, and runs 44–47 are for identical flow conditions. Runs 48–51 are for a dune-bed flow in the 2-foot flume, and runs 52–54 are for a ripple bed in the 0.67-foot flume, corresponding to the experiments reported by Rathbun and Guy (1967). Run 54 subsequently was discarded because of suspected errors in the basic data, so analysis eventually was carried out on 53 profiles from the six different flow conditions.

Profiles of the bed elevations were obtained with the sonic depth sounder described by Karaki, Gray, and Collins (1961) except for the smallest flume where the profiles were traced on a stripchart from the plastic side-walls of the flume. All data were digitized with an analog-to-digital converter at the intervals shown in table 1. The flume data, runs 4 through 54, were standardized with zero mean and unit variance after removing a straight line trend to account for the possibility that the sand bed in the flume was not parallel to the instrument carriage rails supporting the sonic sounder.

At first, the trend was not removed from the Atrisco Lateral data because the sounder was mounted on a boat at a constant depth below the water surface. However, initial analyses showed some long-term trends in the data, so that parts of records from run 2 were selected for trend removal. These shorter records from Atrisco Lateral are shown as runs 55 through 57 in table 1 and are discussed in detail in a later section.

Only longitudinal profiles were available for Atrisco Lateral and the smallest flume; for the 2- and 8-foot-wide flumes, time records $y = y(t)$ were available along with the longitudinal profiles. All the computations described in this and subsequent sections were accomplished on the CDC 6400 computer at Colorado State University.

For the discrete data used in this study, the values in equations 25 through 32 were approximated by the estimates presented by Granger and Hatanaka (1964), using the Blackman and Tukey spectral estimates with a Hanning window (Blackman and Tukey, 1958, p. 34). Formulas for the digital calculations of the covariance functions and spectra and some guidelines for estimating the length of record required for the various calculations are given in the appendix. A detailed discussion of the calculations and an excellent review of spectral theory are given in the above references and in a recent book by Bendat and Piersol (1966). Rodriguez-Iturbe (1967) investigated the application of cross-spectral analysis to hydrologic data and gave a thorough discussion of the computational procedures for discrete data. The procedures used in this study are identical to those listed by him (Rodriguez-Iturbe, 1967, p. 5–7).

TABLE 1.—Summary of basic data

[Transverse station: CL, centerline; L and R, left and right third points of the channel; 2, 4, and 6, stations from the left wall of the 8-ft flume. Lag interval: given in minutes for runs 4, 19, 33, 40, 44, and 48]

Channel and run	Date	Time	Time or station	Transverse station	No. of data points	Lag interval (ft or min)	Mean depth (ft)	Mean velocity (fps)	Slope	Water temperature (° C)	Median diam of bed material (mm)	Bed configuration						
Atrisco Lateral:																		
1966																		
Run 1.....	June 23	0-4000	CL	12,924	0.333	2.20	2.16	0.00055	19	0.23	Dunes.						
2.....	June 22	0-4000	CL	13,116	.333	2.30	2.11	.00055	20	.23	Do.						
3.....	June 21	1000-4000	CL	9,864	.333	2.29	2.08	.00058	20	.23	Do.						
8-ft flume:																		
Run 4.....	Mar. 15	936 min	CL	936	1.00	2.80	2.09	.00063	20	.24	Do.						
5.....	Mar. 17	1115	60-180	2	924	.130												
6.....	do.....	1115	180-60	4	924	.130												
7.....	do.....	1115	60-180	6	912	.132												
8.....	do.....	1255	60-120	4	432	.139												
9.....	do.....	1300	60-120	4	444	.135												
10.....	do.....	1305	60-120	4	432	.139												
11.....	do.....	1310	60-120	4	432	.139												
12.....	do.....	1315	60-120	4	444	.135												
13.....	do.....	1320	60-120	4	444	.135												
14.....	do.....	1325	60-120	4	456	.131												
15.....	do.....	1330	60-120	4	444	.135												
16.....	do.....	1530	60-180	2	876	.137												
17.....	do.....	1536	60-180	4	900	.133												
18.....	do.....	1540	60-180	6	900	.137												
19.....	do.....	888 min	CL	888	1.0												
20.....	July 12	1732	60-180	4	468	.256							2.36	2.01	.00056	24	.24	Do.
21.....	do.....	2114	60-180	4	736	.163												
22.....	do.....	0930	60-180	4	984	.122												
23.....	do.....	0924	60-180	4	720	.167												
24.....	do.....	0935	60-180	4	876	.137												
25.....	do.....	0946	60-180	4	612	.196												
26.....	do.....	0950	60-180	4	480	.250												
27.....	do.....	0953	60-180	4	528	.228												
28.....	do.....	1113	60-180	4	480	.250												
29.....	do.....	1308	60-180	4	468	.256												
30.....	do.....	1328	60-170	4	792	.152												
31.....	do.....	1329	170-60	4	969	.158												
32.....	do.....	1727	60-180	4	852	.141												
33.....	do.....	1735	505 min	4	672	.750												
34.....	July 13	0907	60-180	4	864	.139	2.36	2.01	.00053	25	.24	Do.						
35.....	do.....	0912	60-180	4	480	.250												
36.....	do.....	1012	60-180	4	864	.134												
37.....	do.....	1016	60-180	4	480	.250												
38.....	July 14	0810	60-180	4	1,188	.101												
39.....	do.....	1435	60-180	4	1,176	.102												
40.....	do.....	4												
2-ft flume:																		
1967																		
Run 40.....	Feb. 23	395 min	CL	395	1.0	.518	1.10	.00088	20	.35	Ripples.						
41.....	Feb. 21	1110	5-55	CL	668	.075												
42.....	Feb. 22	2350	5-55	L	667	.075												
43.....	Feb. 23	1412	5-55	R	766	.070	.522	1.07	.00088	20	.35	Do.						
44.....	Mar. 23	532 min	CL	532	1.0												
45.....	Mar. 21	1720	5-55	CL	640	.078												
46.....	Mar. 22	0908	5-55	CL	696	.072	.521	1.62	.00212	20	.35	Dunes.						
47.....	do.....	2156	5-55	L	640	.078												
48.....	Apr. 12	1640	1160 min	CL	776	1.50												
49.....	Apr. 11	1400	5-55	CL	416	.120	.521	1.62	.00212	20	.35	Dunes.						
50.....	Apr. 12	1403	5-55	L	407	.123												
51.....	do.....	1536	5-55	L	393	.127												
0.67-ft flume:																		
1965																		
Run 52.....	Nov. 30	1000	0-7.2	L	306	.0208	.174	.508	.00148	20	.30	Ripples.						
53.....	do.....	1000	9-16	L	348	.0208	.174	.508	.00148	20	.30	Do.						
54.....	Dec. 1	1000	0-7.2	L	318	.0208	.174	.508	.00148	20	.30	Do.						
Atrisco Lateral:																		
1966																		
Run 55.....	June 22	0-2000	CL	6,000	.333	2.30	2.11	.00055	20	.23	Dunes.						
56.....	do.....	0-200	CL	600	.333	2.30	2.11	.00055	20	.23	Do.						
57.....	do.....	200-400	CL	600	.333	2.30	2.11	.00055	20	.23	Do.						

ZERO AND *h*-LEVEL CROSSING ANALYSES

In this section, mean values of the durations between zero and *h*-level crossings, the mean durations of upward excursions above the level *h*, and some of the probability distributions of interest in the two-dimensional stochastic model of particle movement (Sayre and Conover, 1967) will be considered.

First, it is noted that the bed elevation *y*, measured from the mean bed level follows approximately a Gaussian distribution (fig. 5). Intuitively, an approximate normal distribution is expected, because physical phenomena governed by the complex interaction of many factors often exhibit such a distribution. Logically, the distribution can be only approximate; in a finite flow

depth, the variation of the bed elevation about the mean can never assume infinite values. In addition, the preferred orientation of the dune forms, with the characteristic steeply sloping downstream faces, suggests a pattern more regular than normal distribution. Figure 5 shows examples of the distributions from each of the four channels, and although the sets of values plot around the straight line of a normal distribution, each shows some departures from normality.

The cumulative distribution curves of figure 5 tend to smooth out irregularities of the data and are not really a good indication of the normality or lack of normality of the data. A better criterion, perhaps, is to compare the skewness of the data with the skewness of a normal dis-

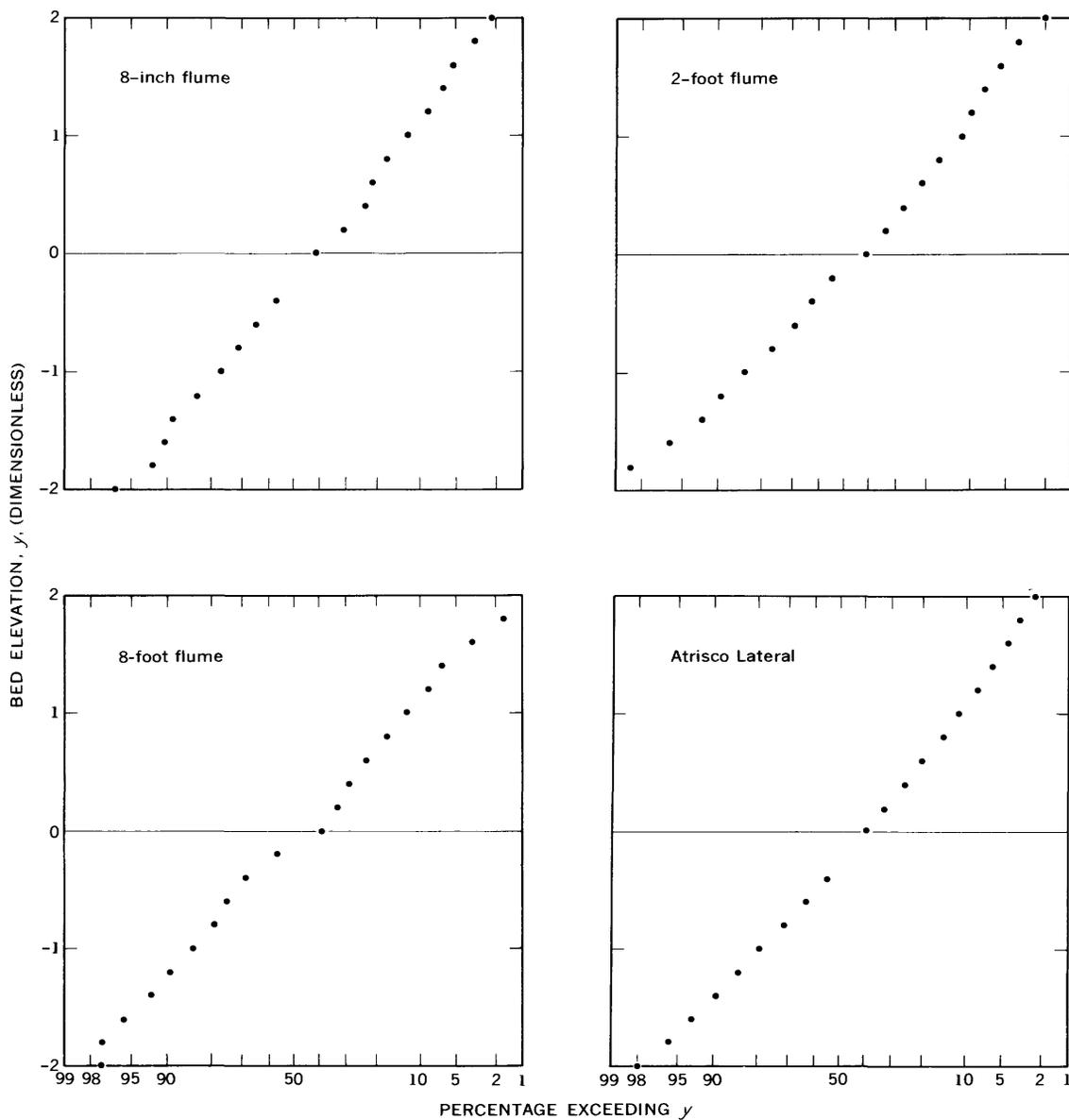


FIGURE 5.—Distributions of bed elevations.

tribution, which is zero. Table 2 lists the properties of the raw data and shows that the skewness varied from -0.3 to about 1.7, with a preponderance of values on the positive side. Both the skewness and kurtosis show a considerable range values with no recognizable pattern.

TABLE 2.—Statistical properties of raw data

Run	Variance	Standard deviation	Skewness	Excess of kurtosis
1.....	6.70×10^{-2}	2.59×10^{-1}	6.38×10^{-1}	1.07×10^0
2.....	7.63×10^{-2}	2.76×10^{-1}	5.09×10^{-1}	9.47×10^{-1}
3.....	7.28×10^{-2}	2.70×10^{-1}	2.77×10^{-1}	3.35×10^{-1}
4.....	4.00×10^{-2}	2.00×10^{-1}	1.71×10^0	9.91×10^0
5.....	2.09×10^{-2}	1.44×10^{-1}	4.04×10^{-1}	-8.12×10^{-3}
6.....	1.58×10^{-2}	1.26×10^{-1}	4.92×10^{-1}	6.45×10^{-2}
7.....	1.77×10^{-2}	1.33×10^{-1}	1.48×10^{-1}	-2.87×10^{-1}
8.....	2.62×10^{-2}	1.62×10^{-1}	6.06×10^{-2}	-2.03×10^{-1}
9.....	1.90×10^{-2}	1.38×10^{-1}	2.59×10^{-1}	-5.62×10^{-1}
10.....	3.12×10^{-2}	1.77×10^{-1}	5.70×10^{-1}	3.35×10^{-1}
11.....	3.59×10^{-2}	1.89×10^{-1}	4.12×10^{-1}	-3.15×10^{-1}
12.....	3.38×10^{-2}	1.84×10^{-1}	3.38×10^{-1}	-2.58×10^{-1}
13.....	2.96×10^{-2}	1.72×10^{-1}	5.20×10^{-1}	-1.49×10^{-1}
14.....	3.80×10^{-2}	1.95×10^{-1}	6.07×10^{-1}	-3.23×10^{-1}
15.....	3.83×10^{-2}	1.96×10^{-1}	5.57×10^{-1}	-4.36×10^{-1}
16.....	3.22×10^{-2}	1.79×10^{-1}	8.66×10^{-1}	2.32×10^0
17.....	4.68×10^{-2}	2.16×10^{-1}	1.65×10^{-1}	-5.84×10^{-1}
18.....	3.37×10^{-2}	1.84×10^{-1}	2.06×10^{-1}	-1.53×10^{-1}
19.....	1.14×10^{-2}	1.07×10^{-1}	1.29×10^{-1}	-1.67×10^{-1}
20.....	2.16×10^{-2}	1.47×10^{-1}	4.50×10^{-1}	5.03×10^{-1}
21.....	1.23×10^{-2}	1.11×10^{-1}	2.28×10^{-1}	4.46×10^{-1}
22.....	1.94×10^{-2}	1.39×10^{-1}	3.17×10^{-1}	-2.21×10^{-1}
23.....	1.88×10^{-2}	1.37×10^{-1}	1.47×10^{-1}	-2.20×10^{-1}
24.....	1.96×10^{-2}	1.40×10^{-1}	5.09×10^{-1}	-8.21×10^{-2}
25.....	2.61×10^{-2}	1.62×10^{-1}	8.33×10^{-1}	8.11×10^{-1}
26.....	2.77×10^{-2}	1.67×10^{-1}	6.97×10^{-1}	4.20×10^{-1}
27.....	2.98×10^{-2}	1.73×10^{-1}	6.91×10^{-1}	4.41×10^{-1}
28.....	1.76×10^{-2}	1.33×10^{-1}	4.06×10^{-1}	7.06×10^{-1}
29.....	1.44×10^{-2}	1.20×10^{-1}	5.37×10^{-1}	1.66×10^0
30.....	1.07×10^{-2}	1.03×10^{-1}	9.84×10^{-2}	8.41×10^{-3}
31.....	1.17×10^{-2}	1.08×10^{-1}	-9.77×10^{-2}	5.49×10^{-1}
32.....	2.11×10^{-2}	1.45×10^{-1}	8.07×10^{-1}	1.55×10^0
33.....	1.45×10^{-2}	1.20×10^{-1}	1.75×10^{-1}	2.57×10^{-1}
34.....	1.22×10^{-2}	1.10×10^{-1}	2.04×10^{-1}	-1.79×10^{-1}
35.....	1.23×10^{-2}	1.11×10^{-1}	1.16×10^0	9.00×10^0
36.....	1.03×10^{-2}	1.01×10^{-1}	1.60×10^{-1}	2.91×10^{-1}
37.....	1.04×10^{-2}	1.02×10^{-1}	-2.71×10^{-2}	-4.17×10^{-2}
38.....	1.25×10^{-2}	1.12×10^{-1}	7.66×10^{-1}	1.12×10^0
39.....	8.52×10^{-3}	9.23×10^{-2}	1.03×10^0	1.71×10^0
40.....	1.29×10^{-3}	3.60×10^{-2}	6.78×10^{-1}	4.76×10^{-1}
41.....	1.91×10^{-3}	4.37×10^{-2}	2.81×10^{-1}	1.98×10^{-1}
42.....	2.25×10^{-3}	4.74×10^{-2}	-1.85×10^{-1}	-7.54×10^{-3}
43.....	2.09×10^{-3}	4.57×10^{-2}	3.33×10^{-1}	-4.25×10^{-1}
44.....	1.20×10^{-3}	3.46×10^{-2}	-2.29×10^{-1}	-4.87×10^{-1}
45.....	1.64×10^{-3}	4.04×10^{-2}	7.92×10^{-2}	-5.28×10^{-1}
46.....	1.50×10^{-3}	3.87×10^{-2}	2.15×10^{-1}	-4.16×10^{-1}
47.....	1.79×10^{-3}	4.23×10^{-2}	2.40×10^{-1}	-1.78×10^{-1}
48.....	5.62×10^{-3}	7.50×10^{-2}	9.27×10^{-1}	6.70×10^{-1}
49.....	5.48×10^{-3}	7.39×10^{-2}	1.03×10^0	1.64×10^0
50.....	5.54×10^{-3}	7.44×10^{-2}	3.60×10^{-1}	2.68×10^0
51.....	3.23×10^{-3}	5.68×10^{-2}	1.46×10^{-1}	-8.08×10^{-1}
52.....	2.16×10^{-3}	4.64×10^{-2}	4.44×10^{-1}	-4.99×10^{-1}
53.....	1.19×10^{-3}	3.44×10^{-2}	2.84×10^{-1}	-4.33×10^{-1}
55.....	8.09×10^{-2}	2.84×10^{-1}	5.41×10^{-1}	3.83×10^0
56.....	8.72×10^{-2}	2.96×10^{-1}	1.16×10^0	2.49×10^0
57.....	3.95×10^{-2}	1.99×10^{-1}	4.81×10^{-2}	-5.73×10^{-2}

NOTE.—Run 54 not used.

Even though the distributions of the data depart from normality, the relation of equation 10 provides a good estimate of the mean duration or the mean distance between zero crossings, l_0 , as shown in figure 6. Although

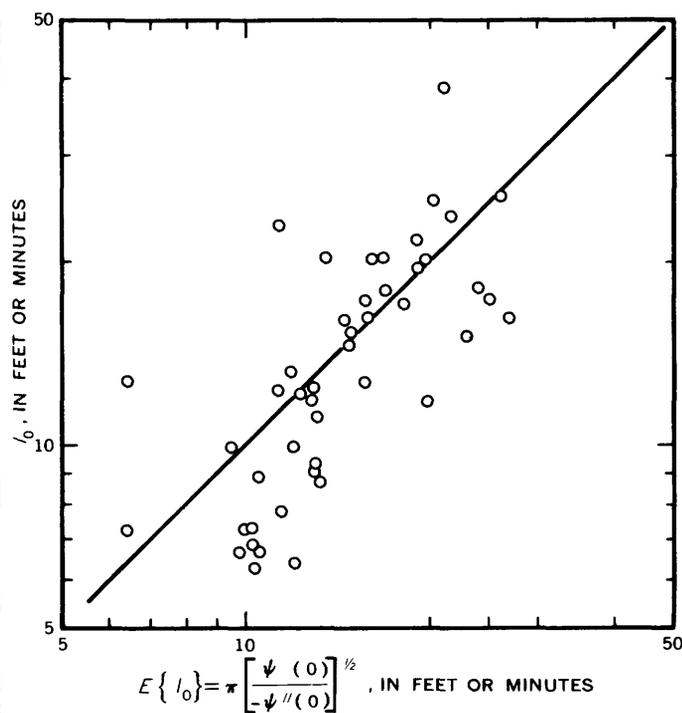


FIGURE 6.—Comparison of the expected length or duration between zero crossings for a Gaussian process, $E\{l_0\}$, with the observed values, l_0 .

there is considerable scatter, the data group around the line of perfect agreement. No consistent trends in the scatter could be attributed to flume size or type of bed form, so that most of the scatter is assumed to be due to the shortness of the records of the flume data.

The ratio of the expected number of h -level crossings to the expected number of zero crossings was given for both the continuous process and the discrete approximation to the continuous process in figure 3. Because the two curves are so similar, the simpler expression of equation 12 will be used. Observed values of the ratio N_h/N_0 are plotted in figure 7, along with the curve representing equation 12. Values for 13 profiles were plotted. Six of the profiles, runs 4, 19, 33, 40, 44 and 48, are of the process $y=y(t)$, and seven of the profiles, runs 16, 32, 41, 45, 53 and 56, are of the process $y=y(x)$. For positive values of h , the points scatter symmetrically about the curve of equation 12, but for negative values of h , most of the points fall above the curve for values of h from 0 to -1 and below the curve for values of h from -1 to -2.

Figure 7 shows clearly that there are more crossings below the mean bed elevation than above, which would be expected from consideration of flow conditions over a dune. Below the mean bed elevation, the reverse flow in the trough and the flow impinging on the back of the dune result in lower velocities which promote the growth of small-scale features. Above the mean bed elevation, the converging flow up the back of the dune results in a

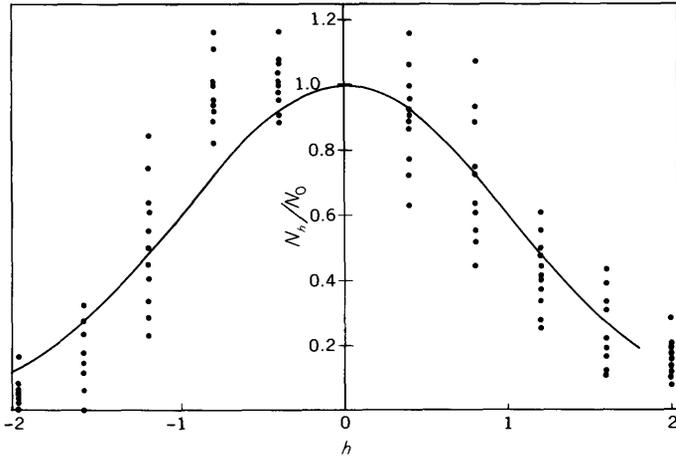


FIGURE 7.—Observed values of N_h/N_0 as a function of h .

higher-than-average velocity and shear stress near the bed, and small-scale features cannot form.

Figure 8 shows the average of all observations, with the points connected by dashed lines to give an indication of the shape of the distribution of N_h/N_0 values and with averages of $y(t)$ and $y(x)$ differentiated. The figure indicates that records of both $y(t)$ and $y(x)$ yield values of N_h/N_0 that agree very well with the theoretical curve for $h > 0$, but that deviate appreciably from the curve of equation 12 for $h \leq 0$, with the values from the process $y=y(t)$ showing the greatest deviation. Figure 8 suggests that there may be some slight differences in the properties of $y(x)$ and $y(t)$. However, distributions of the raw data and the spectral analyses of the processes, discussed in a later section, indicate that there are no appreciable

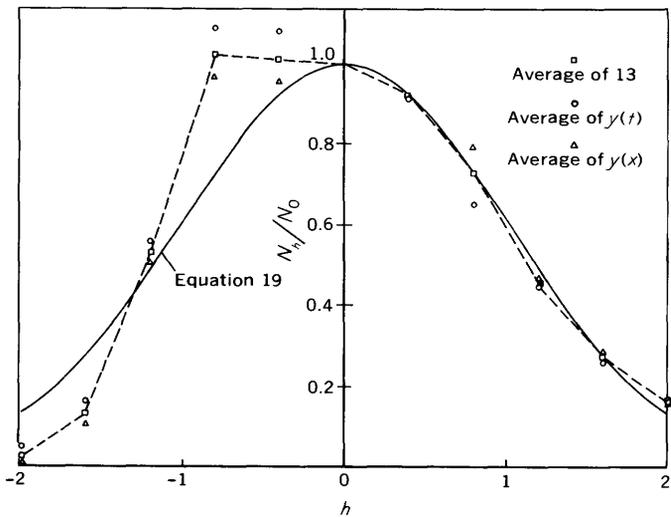


FIGURE 8.—Average values of N_h/N_0 as a function of h .

differences, so that the greater deviation of the values for the process $y=y(t)$ probably is fortuitous.

No consistent differences in the deviations of the values from the curve in figure 7 could be attributed either to flume size or to bed forms, where both ripples and dunes are represented.

The mean duration of an upward excursion of the process $y(t)$ above the level h , $E\{l_h^+\}$, was given by equation 13, and equation 14 is the ratio of mean duration of upward excursions at the level h to mean duration of upward excursions at the zero level. Equation 14 is plotted in figure 9 along with observed values of the ratio l_h^+/l_0^+ . Again, as in figure 7, there is systematic deviation from the curve for values of h from 0 to -1 , and there is considerable scatter at the higher values of h . However, this is not too disturbing because the number of upward excursions that are observed in the relatively short flume records probably is too small to get reliable average values of l_h^+ .

The average rest period of a particle deposited between the levels h_1 and h_2 can be computed from equation 14 or estimated graphically from figure 9. Theoretically, there are no limitations to equation 14; it is applicable between any two levels of the bed, $-\infty < h < +\infty$. The ratio l_h^+/l_0^+ approaches zero as h assumes large positive values and approaches an infinite value as h assumes large negative values. From a practical point of view, these extremes

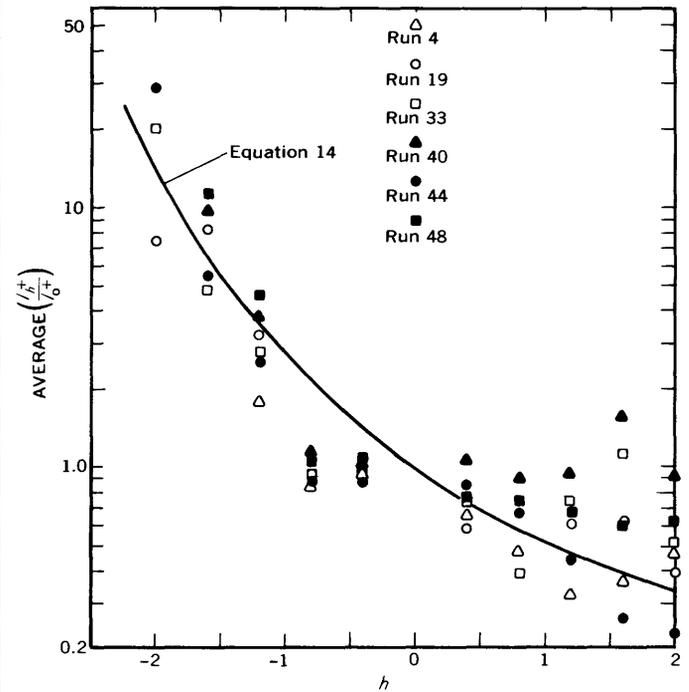


FIGURE 9.—Observed values of the ratio l_h^+/l_0^+ plotted against h .

rarely would be of interest. The major transport of bed material occurs in the activity shifting part of the bed, and it is unlikely that one would need to consider anything beyond two standard deviations of the mean bed level to account for the bulk of the sediment movement. Within these limits, a reasonable estimate of an average rest period can be determined from figure 9 or equation 14.

Equation 13 gives the mean rest period, l_h^+ , of a particle deposited on the downstream face of a dune at the level h . We are interested in not only the mean value but also the distribution of l_h^+ , the conditional probability distribution of rest periods, given that a particle is deposited at the level h . As indicated previously, there is no theoretical basis for predicting a distribution of the l_h^+ values, and, unfortunately, the records of $y(t)$ were not long enough to establish the rest period distributions.

However, it may be useful to establish some of the properties of the distributions for l_h^+ values of the process $y(x)$, even though these values do not represent rest periods, because if the statistical properties of $y(x)$ and $y(t)$ are similar, the distribution of crossings and other features of interest should be the same. (For Gaussian processes with zero means, similar covariance functions insure similarity of all other properties.) For this purpose, the Atrisco Lateral records were selected because they were the longest available.

Figure 10, a bar graph of l_h^+ distributions for run 1, shows that shapes of the distributions vary with h and suggests that at the level $h=0$ the lengths of upward excursions follow an exponential distribution. Figure 11 shows as solid lines exponential distributions with the same means as observed mean values of l_0^+ for runs 1, 2 and 3. The plotted points represent the observed values. Obviously, the exponential distribution is only a rough approximation.

W. W. Sayre (oral commun., 1967) suggested that perhaps a gamma distribution would serve as a model for the distribution of l_h^+ values over any practical range of bed elevations that are of interest. To investigate this possibility, consider the gamma distribution with parameters $b > 0$, $\lambda > 0$,

$$p(l) = \frac{\lambda}{\Gamma(b)} (\lambda x)^{b-1} e^{-\lambda x}, \quad (33)$$

where $x > 0$, and

$$p(l) = 0,$$

where $x < 0$.

When $b=1$, this is the negative exponential distribution shown in figure 11, with $\lambda=1/l_0$. The variance of the

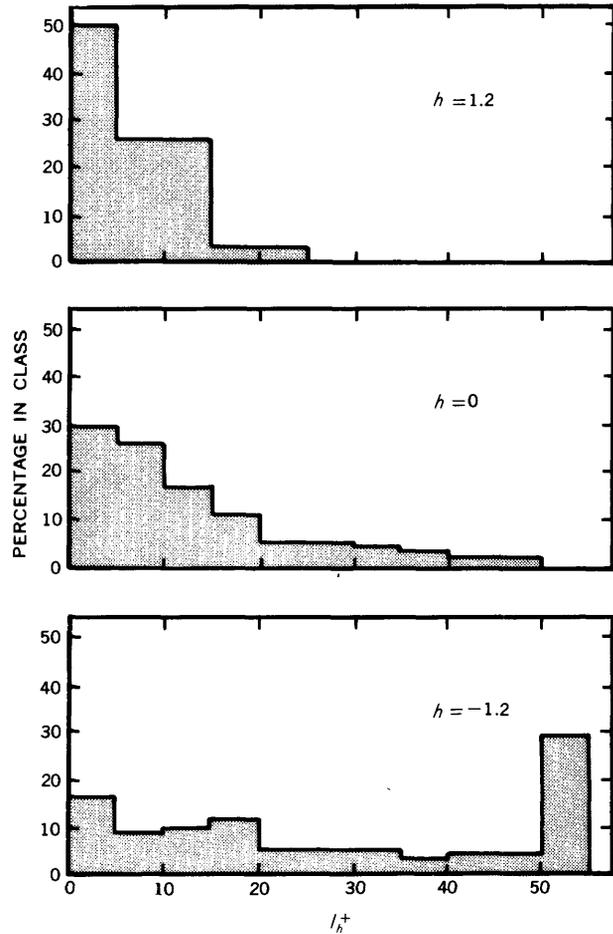


FIGURE 10.—Observed distributions of l_h^+ for run 1.

gamma distribution is b/λ^2 , and its coefficient of variation C_v , the standard deviation divided by the mean, is $1/\sqrt{b}$.

Figure 12 shows observed values of C_v for the distributions of l_h^+ values plotted as functions of h . The trend line sketched through the plotted points is positioned with $C_v=1$ at $h=0$, corresponding to the exponential distribution of figure 11. Thus, the postulated gamma distributions of the lengths or durations of positive excursions above the level h can be determined directly from figures 12 and 9, with λ and b computed from the following equations:

$$b = 1/C_v^2 \quad (34)$$

$$\lambda = (1/C_v^2) \langle l_h^+ \rangle_{\text{avg.}} \quad (35)$$

Figure 13 shows bar graphs of the observed l_h^+ distributions for run 1, with points plotted at the midpoint of each class interval representing the frequency for that class from a gamma distribution with parameters given by equations 34 and 35. The observed mean value of l_0^+

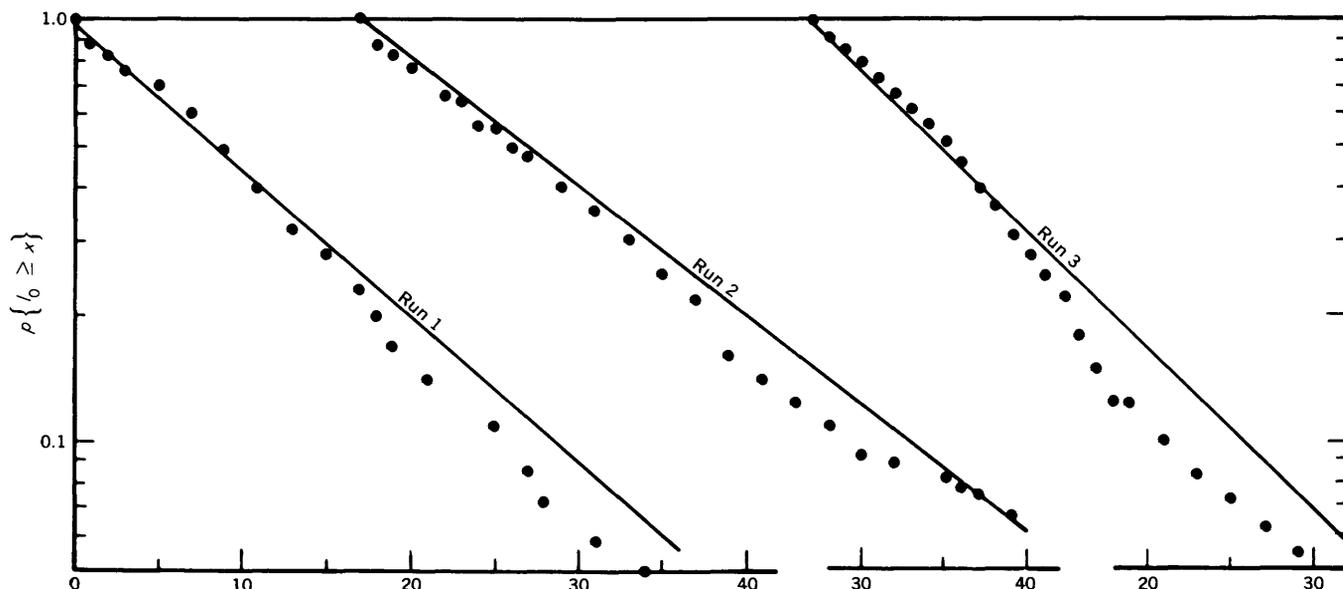


FIGURE 11.—Observed distributions of l_0^+ for runs 1, 2, and 3. Solid lines are exponential distributions with the same mean values as the observed means. The lengths are in multiples of the lag interval, 0.333 feet.

was used to compute λ for $h = 0$, and all other parameters were determined from figures 9 and 12. The results of this example certainly are encouraging but not conclusive.

It should be noted that the data from one time record, run 19, are plotted in figure 12. These data deviate more from the trend line of the C_v values than do the data from the longitudinal profiles, but the number of crossings observed in the time records was too small to provide reliable statistical estimates of the distribution of the l_h^+ values. Thus, the approach outlined above appears to be equally applicable to records of $y(x)$ or $y(t)$, but confi-

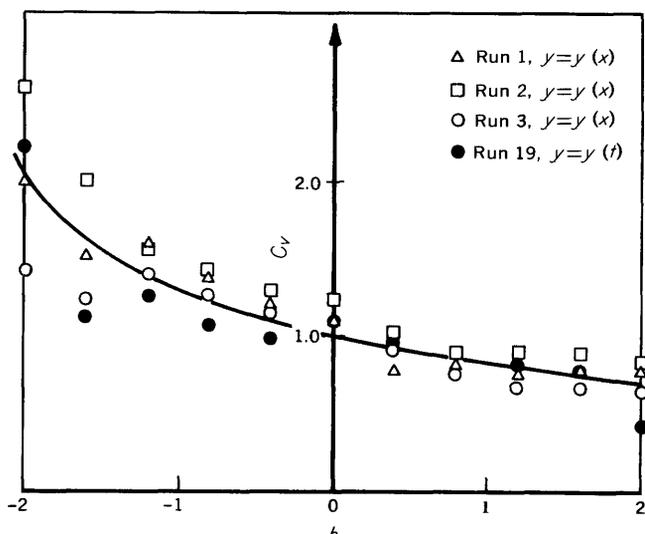


FIGURE 12.—Observed values of the coefficient of variation, C_v , for the distributions of l_h^+ plotted as functions of h .

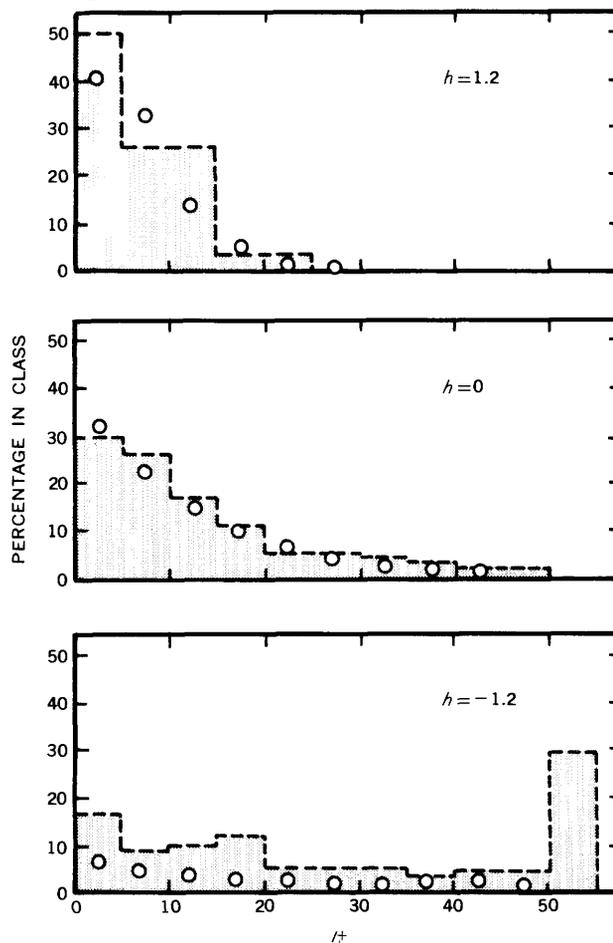


FIGURE 13.—Comparison of observed l_h^+ distributions with the theoretical class frequencies from a gamma distribution. The plotted points are from the theoretical distribution.

dence levels for estimating the distributions of rest periods cannot be established until longer records of the process $y=y(t)$ become available.

One other distribution of interest that will be considered in this section is the probability that a particle is residing in the bed at the level h . This is not the same as the probability that a particle is deposited at the level h , for deposition occurs on the downstream faces and in the troughs of the sand waves; this latter probability no doubt depends on flow conditions. The distributions derived empirically by investigating the area bounded by the curve $y=y(t)$ or $y=y(x)$ above a level h give the percent of particles found in the bed above the level h and represent the probability of finding a particle at the level h if nothing is known of its previous history. Figure 14 shows the computed values for $y(x)$ and $y(t)$ records from the 2-ft flume, with a smooth curve drawn by eye to indicate the trend of the distributions. An approximate equation for $P(h)$, the probability that a particle is residing in the bed above the level h , is given by

$$P(h) = 1 - e^{-0.157(h+1.75)} \tag{36}$$

Equation 36 is a good approximation in the range $0.1 < P < 0.9$, but it is not applicable for extreme values of h .

DISTRIBUTION OF WAVE HEIGHTS AND AMPLITUDES

The absolute maximum value of y between zero crossings was defined in figure 2 as the wave amplitude, a , and it was noted that the distribution of positive amplitudes, $a+$, represented the probability distribution of local deposition and that the distribution of negative values, $a-$, represented the probability distribution of local depth of scour associated with the formation and migration of sand waves. If the processes $y(x)$ and $y(t)$ were symmetric about their mean values, the distribution of positive and negative a values would be identical, and for a Gaussian process, it was postulated that the distributions of a values would approximate the distribution of crest heights, η , shown in figure 4.

All of the records analyzed exhibited relatively broad spectra, with values of δ^2 from equation 20 varying from 0.83 to 0.99 (see table 3) and with most of the values greater than 0.9. Figures 15 through 17 show distributions of positive and negative a values for runs 1 through 3. The solid curves on the figures represent the Gaussian distribution from figure 4 of crest heights for a process with a broad-band spectrum ($\delta^2=1.0$). Two points of interest should be noted: (1) the distributions are not symmetric, and (2) the negative values, $a-$, follow more closely the normal distribution than do the positive values.

The positive amplitudes, $a+$, followed approximately an exponential distribution, as shown in figure 18. In this figure, $P(a)$ is the cumulative probability distribution of a , and for this form of plotting, a Rayleigh distribution (eq 21) would have a slope of two and an exponential distribution, a slope of one. Clearly, except for very small values of a , the slope is one, indicating an approximate exponential distribution.

As discussed previously, there is no basis for estimating the probability distribution of wave heights, H , the trough-to-crest height, and there is no reason to expect similarities in the distribution of H and a values. Figure 19 shows the distribution of wave heights to approach the Rayleigh distribution, rather than the exponential distribution. The same data were used to prepare figures 18 and 19, but in figure 19 the raw data were not standardized and the wave heights are given in feet rather than in units of standard deviations.

Even though the distributions are different, the mean values of $a+$ and H relate reasonably well (fig. 20) as do the mean values of distances between successive up-crossings of y and mean dune lengths, L , (trough-to-trough distance) (fig. 21). Note, however, that values of $2a$ and of the distance between successive up-crossings are not strictly comparable to H and L because entire waveforms occur above or below the mean bed level, and their lengths and heights are not reflected in the average

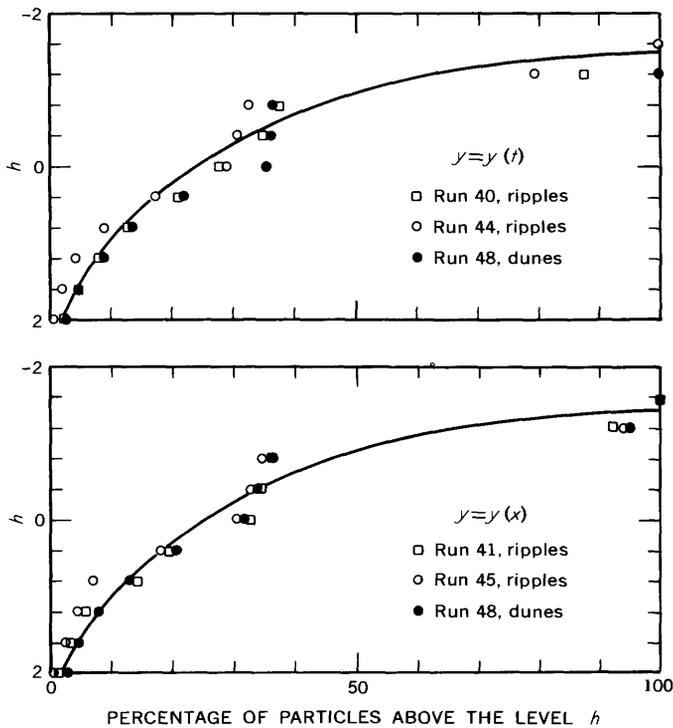


FIGURE 14.—Percentage of particles in the bed above the level h .

TABLE 3.—Miscellaneous properties from the zero-crossing and spectral analysis

[Type of profile is $y(x)$ except type for runs 4, 19, 33, 40, 44, and 48, which is $y(t)$]

Length: L_0 is the average distance in feet, or time in minutes for runs 4, 19, 33, 40, 44, and 48, between successive upcrossings of $y=0$.

Amplitude: $\bar{a}+$ is the average of maximum y values between successive zeros.

σ_y : From eq 5.

δ^2 : From eq 20.

$1/m_1$: Mean wavelength in feet, or wave period in minutes for runs 4, 19, 33, 40, 44, and 48, from spectra, eq 8.

Run	Length		Amplitude		σ_y (Ft)	δ^2	$1/m_1$
	L_0	C_r	$\bar{a}+$ (Ft)	C_r			
1.....	8.96	0.577	0.277	0.850	0.259
2.....	9.22	.589	.287	.913	.276
3.....	7.79	.560	.302	.782	.270
4.....	57.3	.504	.386	1.46	.200	0.933	26.0
5.....	4.97	.599	.164	.868	.144	.978	6.38
6.....	4.94	.681	.158	.886	.126	.968	4.91
7.....	5.37	.618	.156	.673	.133	.973	5.75
8.....	5.24	.631	.180	.712	.162	.964	4.46
9.....	6.13	.379	.225	.573	.138	.952	3.85
10.....	5.78	.511	.221	.687	.177	.966	5.11
11.....	6.59	.596	.223	.684	.189	.972	5.56
12.....	3.39	.877	.119	1.21	.184	.975	5.31
13.....	6.39	.481	.243	.690	.172	.970	4.91
14.....	4.74	.789	.192	1.02	.195	.982	6.37
15.....	7.38	.186	.306	.467	.196	.977	6.24
16.....	6.21	.512	.256	1.09	.176	.960	4.52
17.....	8.45	.384	.274	.747	.216	.983	7.59
18.....	6.96	.358	.226	.555	.184	.979	6.48
19.....	29.3	.591	.099	.885	.107	.974	42.5
20.....	4.68	.448	.169	.800	.147	.931	5.35
21.....	4.34	.648	.138	.834	.111	.946	4.01
22.....	4.21	.575	.189	.648	.139	.974	4.47
23.....	4.19	.623	.167	.708	.137	.946	4.07
24.....	5.04	.416	.190	.705	.140	.966	4.43
25.....	5.61	.412	.201	.843	.162	.952	5.15
26.....	5.62	.598	.212	.836	.167	.938	5.85
27.....	5.79	.530	.227	.702	.173	.944	5.68
28.....	3.70	.635	.137	.928	.133	.909	4.72
29.....	4.04	.706	.144	1.00	.120	.876	3.48
30.....	3.98	.620	.091	.899	.103	.968	5.37
31.....	5.69	.576	.129	.716	.108	.966	5.86
32.....	4.78	.644	.157	.853	.145	.970	5.44
33.....	16.6	.817	.103	.815	.120	.961	26.8
34.....	4.67	.662	.132	.732	.110	.965	4.95
35.....	5.78	.453	.179	1.06	.111	.830	3.02
36.....	3.20	.709	.093	.836	.101	.961	4.31
37.....	4.12	.756	.106	.755	.102	.923	5.25
38.....	4.20	.552	.072	1.01	.112	.983	6.21
39.....	2.79	.757	.088	1.23	.0923	.962	3.32
40.....	14.3	.989	.0237	1.16	.0360	.958	29.7
41.....	.968	.635	.0432	.857	.0437	.886	1.36
42.....	1.16	.666	.0483	.666	.0474	.895	1.61
43.....	1.36	.542	.0512	.680	.0457	.921	1.75
44.....	27.4	.630	.0279	.963	.0346	.970	38.8
45.....	1.14	.546	.0493	.619	.0404	.874	1.33
46.....	1.01	.515	.0430	.652	.0387	.881	1.24
47.....	1.13	.679	.0431	.820	.0423	.904	1.58
48.....	25.5	.562	.0885	.817	.0750	.927	32.4
49.....	2.56	.472	.0952	.901	.0739	.948	3.00
50.....	2.77	.610	.0734	1.03	.0744	.927	2.37
51.....	3.16	.665	.0653	.687	.0568	.956	3.56
52.....	.636	.534	.0384	.920	.0464	.980	1.08
53.....	.782	.460	.0416	.449	.0344	.964	.81

distance or average of maximum ordinates between zero crossings.

An interesting correlation was found to exist between the average of maximum ordinates between zero crossings and the standard deviation of the bed elevation

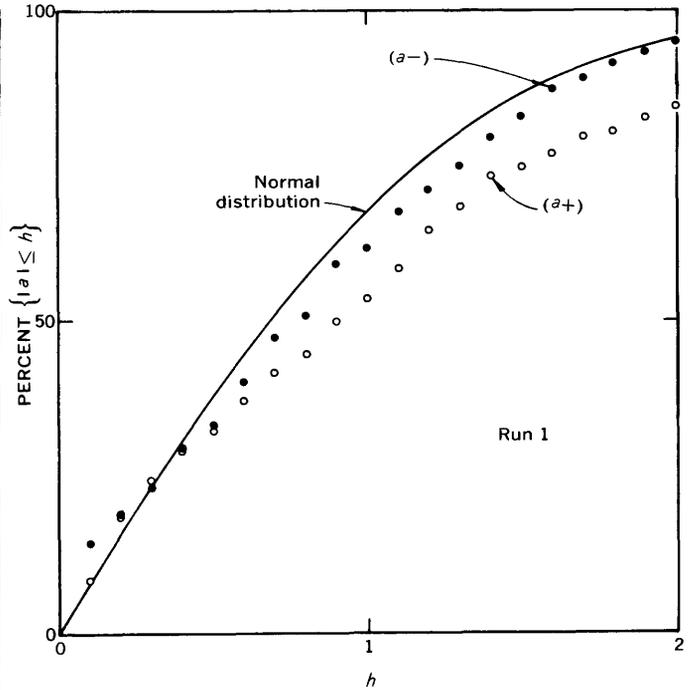


FIGURE 15.—Distribution of positive and negative a values for run 1.

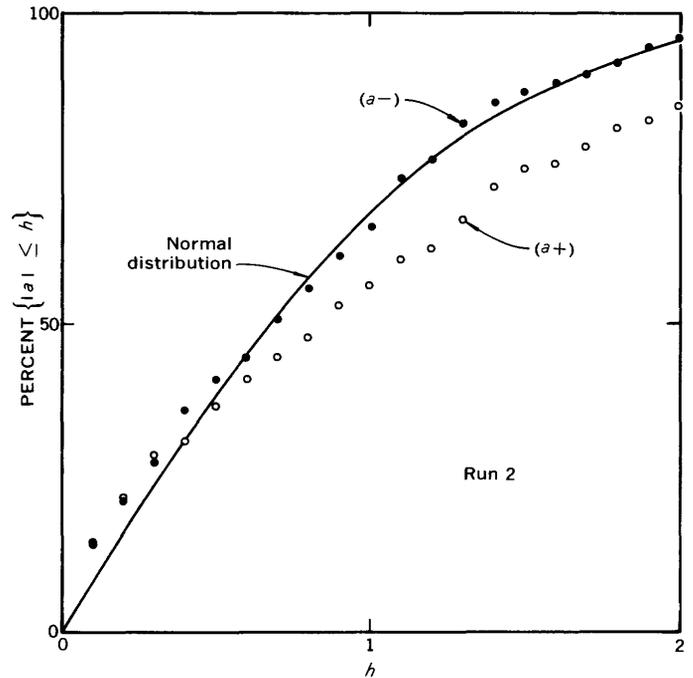


FIGURE 16.—Distribution of positive and negative a values for run 2.

(fig. 22). For the smaller features, a direct linear relation applies, with the standard deviation of the bed elevation approximately equal to the average amplitude of the sand waves. This is precisely the relation predicted for the mean value of local maxima given by equation 23 and

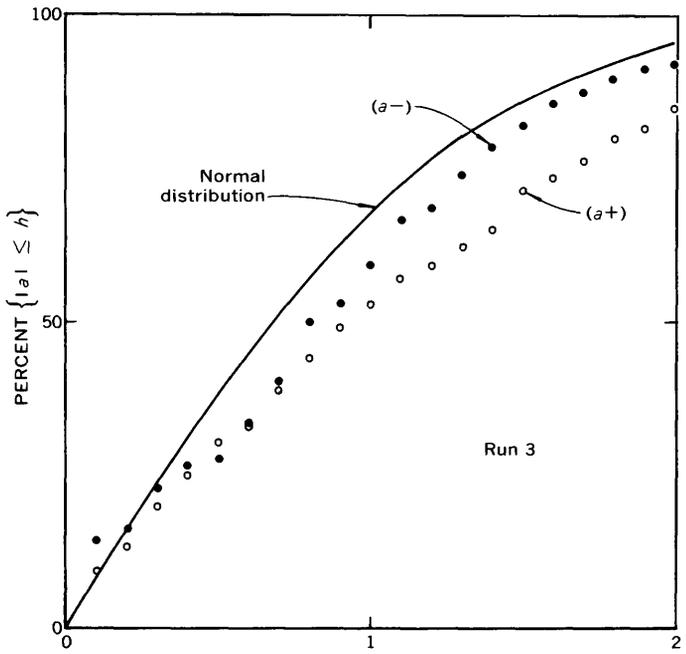


FIGURE 17.—Distribution of positive and negative a values for run 3.

shown in figure 4 as the curve corresponding to $\delta^2 = 1.0$. The relation of figure 22, then, supports the assumption that the distribution of a values should be similar to the distribution of the local maxima. The deviation from the line of agreement at the larger a values might be attributed to the fact that some of the details of the record

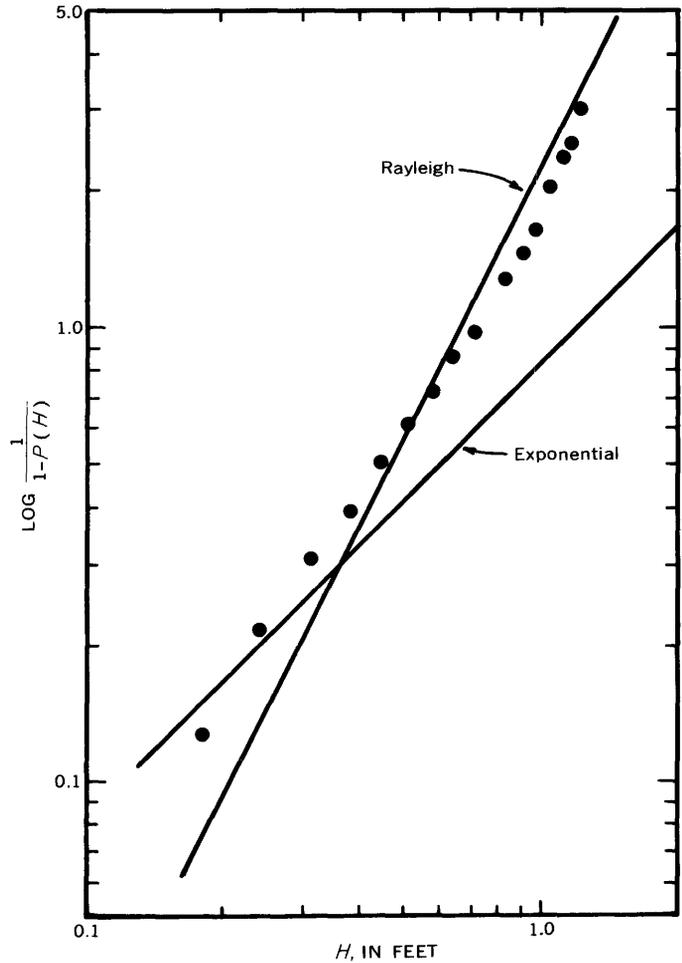


FIGURE 19.—Distribution of wave heights, H , for run 55.

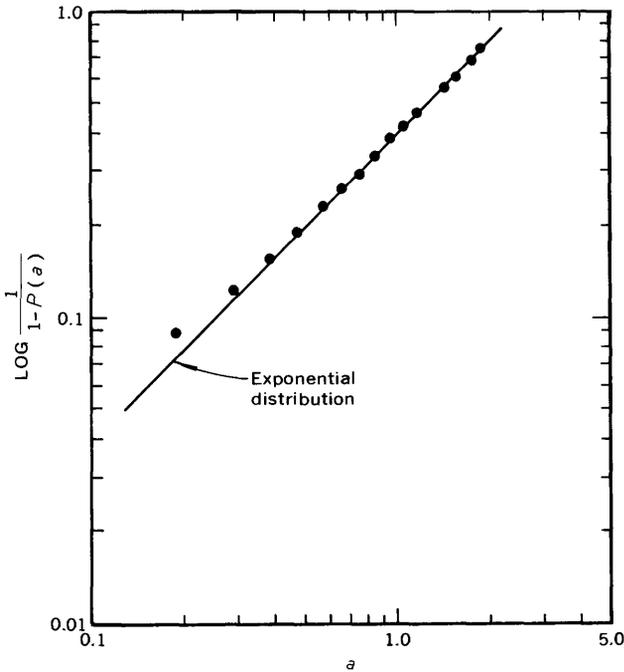


FIGURE 18.—Approximate exponential distribution of positive a values for run 55.

are lost in digitizing the continuous trace of bed elevation. The larger dunes are not more regular than the smaller features; in fact, the reverse is true. If the bed profile is represented as a succession of identical triangles, similar to the idealized dune form of figure 1, with ripple index, \bar{L}/\bar{H} , of 15 and $\tan \beta = 0.6$, the relation of mean amplitude to standard deviation of y is given by $a = 0.64 \sigma_y$. If this condition were to hold for the larger dunes represented in figure 22, the plotted points would fall to the left of the line of agreement.

Summarizing, the average amplitude a is approximately equal to the standard deviation of the bed elevation, and the distributions of a values predicted by the relations in figure 4 appear to give reasonable agreement with the observed distributions. However, the bed profile is not symmetric about its mean level, and the distribution of negative values $a-$ agrees more closely with the predicted normal distribution than do the positive values $a+$. For the positive values, an exponential distribution can be shown to apply (fig. 18).

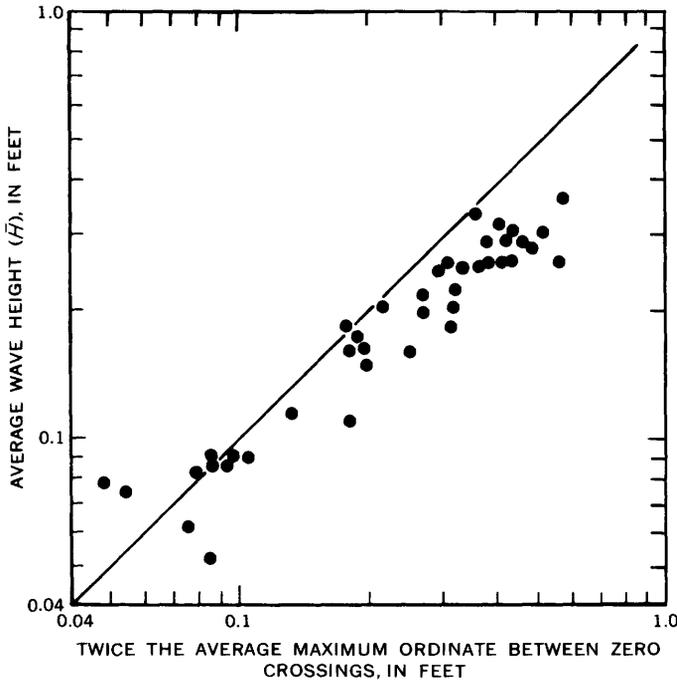


FIGURE 20.—Relation between twice the average maximum ordinate between zero crossings, $2a+$, and \bar{H} .

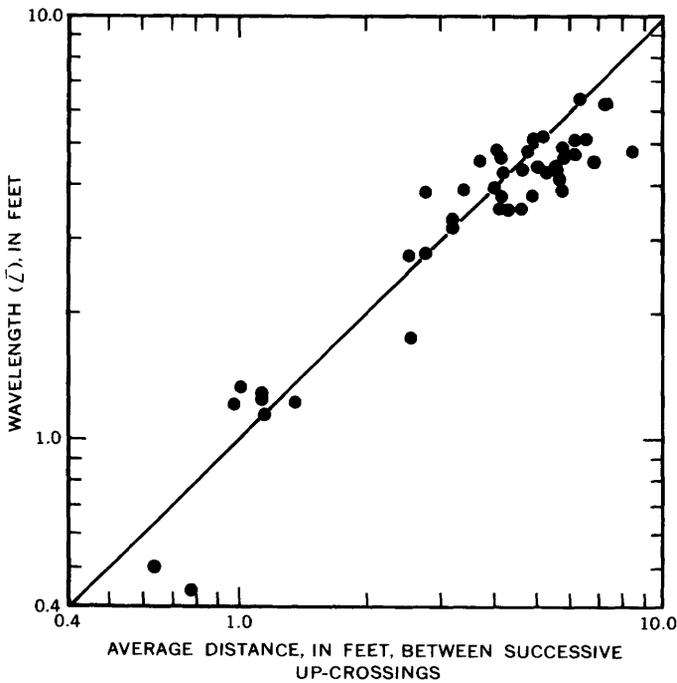


FIGURE 21.—Relation between mean values of the distances between successive zero upcrossings of y and the mean dune length, \bar{L} .

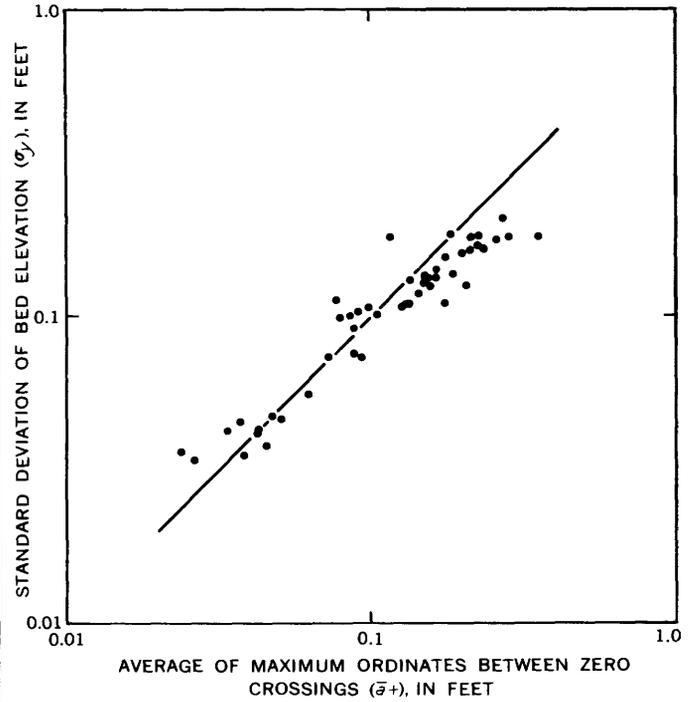


FIGURE 22.—Relation between average of maximum ordinates between zero crossings, $\bar{a}+$, and standard deviation of bed elevation, σ_y .

SPECTRAL ANALYSIS

In the previous discussion, the autocovariance functions or the spectral density functions for the processes $y(x)$ and $y(t)$ were assumed known. These functions were required to predict the number of zero and h -level crossings for the model Gaussian process, and for the experimental data the functions were known because they were computed. In the following sections, the properties of the covariance functions and spectra will be considered in somewhat more detail.

STATIONARITY AND EQUILIBRIUM FLOW

A critical assumption in spectral analysis is that the processes under consideration are stationary, at least to the second order. At the initiation of this study, it seemed intuitively obvious that stationarity of the processes $y(t)$ and $y(x)$ would be a direct consequence of equilibrium flow conditions. If the mean characteristics of the flow, the sediment, and the transport do not change with time or with distance along the channel, then surely the statistical properties of records of the bed profile should be invariant with respect to shifts in the origin of the records.

For the longer field records, it can be demonstrated that the assumption of weak stationarity is justified. Figure 234 shows mean values and standard deviations for eight short segments from the record of run 3 plotted with their respective 90 percent significance levels. The standard

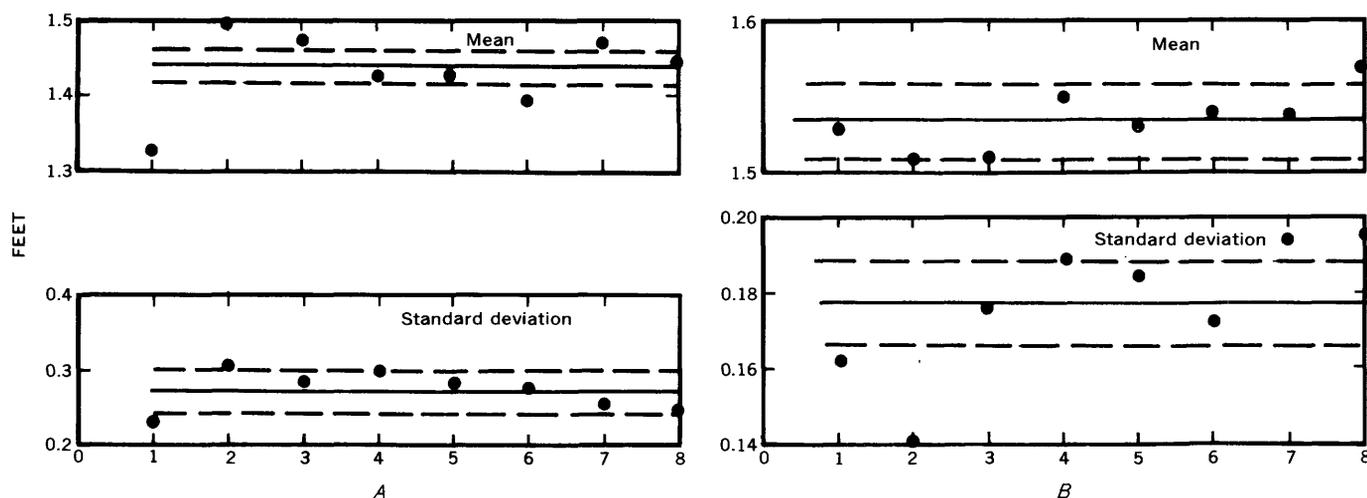


FIGURE 23.—Comparisons of means and standard deviations, in feet: *A*, for eight short segments of the record of run 3, Atrisco Lateral; *B*, for eight longitudinal profiles during equilibrium flow in the 8-foot flume, runs 8 through 15. The solid line represents the average value and the dashed lines indicate a 90-percent significance level.

deviations do not vary significantly. The rather large variations in the mean values resulted from large alternate bars and a meandering thalweg that existed in the channel and that introduced apparent trends in the short segments of the record. The bed profiles were obtained by sounding from a boat with the water surface as a datum, and the depth of flow varied somewhat systematically along the channels. However, when the bed elevations were measured from the mean bed level established by a linear trend line through short segments of the reach, the assumption of second-order stationarity for these short segments of record was satisfactory. For longer segments of the record, on the order of 40 to 50 times the mean channel width, no significant differences were noted in the means and variances.

The flume records presented a somewhat different problem. It was assumed that, for equilibrium flow conditions, any two records of the bed profile would show approximately the same statistical properties. However, this was not found to be true. Figure 23*B* shows that the standard deviations for runs 8 through 15, all of which were taken during apparent equilibrium flow conditions over an identical reach of the flume (see table 1), vary significantly.

Two factors appear responsible for the large variations in the flume records. First, the flume records for $y(x)$ are short relative to the number of dunes that are observed, and the short records introduce inherently large variations in the statistics from one observation to the next. Second, the concept of equilibrium flow implies a time-averaged stability that may not necessarily apply to any single observation. Simons and Richardson (1966, p. J3) indicate that equilibrium flow obtains when the time-averaged

water-surface slope and bed slope are parallel and constant and the time-averaged sediment discharge is constant. Rathbun and Guy (1967, p. 111) have shown that extremely large variations in sediment transport rates are to be expected in recirculating flumes, and probably these variations are reflected in changes in the properties of the bed profiles from one observation to the next.

Despite the rather large variations in the flume records due to the fact that equilibrium flow conditions do not prevail over short time intervals, the spectral properties of the individual records are remarkably similar for a given flow condition, provided the data are standardized to zero mean and unit variance. Figure 24 shows the spectral ordinates for runs 8, 10, 12 and 14 as points plotted against a dimensionless wave number, ϵ/ϵ_{\max} , where ϵ_{\max} is the maximum wave number for which the computations were carried out; the solid line represents an average curve for the spectra and the dashed lines represent a 90-percent confidence band based on the procedure given by Blackman and Tukey (1958, p. 21-23). Only two of the 80 points fall outside the confidence band; the general shapes of the spectra are therefore quite reproducible from one observation to the next. The approximation used in this study, therefore, was to treat each individual record as if it were weakly stationary. The moments of the spectra for the individual records could then be used to estimate the number of zero crossings by equation 10 to compare with observed values. However, in attempting to relate the statistical properties, especially the variance, of the bed profiles to flow parameters, it will be found convenient to work with average values over a consistent set of flow conditions.

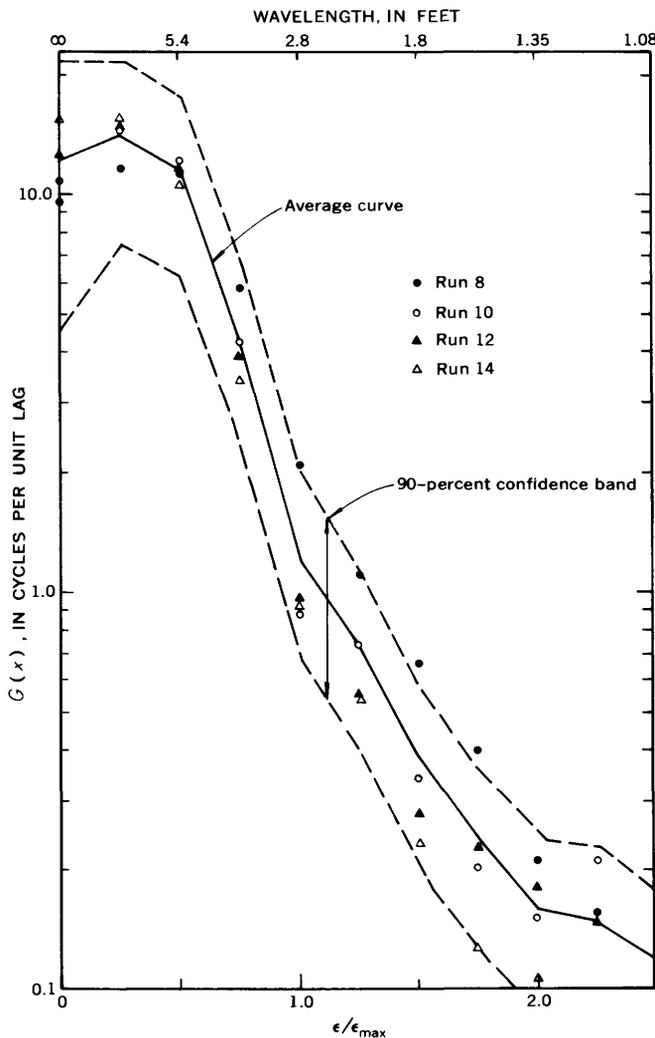


FIGURE 24.—Spectra for indicated flume records.

THE MARKOV MODEL FOR DUNE PROFILES

A major question with regard to spectral analysis is whether or not the properties of the covariance functions or the spectra can be predicted from only the characteristics of the flow and the sediment. Some work already has been accomplished along these lines. Algert (1965) observed that the general shapes of the autocorrelation functions and of the spectra for the process $y = y(x)$ were similar to that of a second-order Markov process, and he developed a model for the spectra based on previous work of Siddiqui (1962). Nordin and Algert (1966) presented virtually the same development, based on Algert's 1965 study, but added some observations on the process $y = y(t)$. Ashida and Tanaka (1967) noted that a second-order Markov process fits the observed spectra only for dunes and postulated a higher-order linear regressive scheme for other bed forms but did not develop a model or apply the higher order scheme to any of their data.

In the studies by Algert (1965) and Nordin and Algert (1966), the parameters upon which the model of the spectra is based are the values of the covariance function at zero, one, and two lag intervals, and these values were shown to relate roughly to the flow parameter, unit discharge (fig. 25). We can use figure 25 and equation 15 to predict the average length between zero crossings of the process $y(x)$ by noting that the spacing of the discrete data points used by Nordin and Algert (1966) in figure 25 was approximately given by

$$\Delta x \approx 4\sigma_y = 4C_0^{1/2}. \tag{37}$$

Here Δx is equivalent to Δt in equation 15, C_1/C_0 is $\phi(\Delta t)$, and C_i is $\sigma_y \phi(i\Delta t)$. Substituting these relations in equation 15 and reading values of C_0 and C_1 from figure 25 for values of unit discharge given in table 1, the average distances between zero crossings were computed and are compared with observed values in figure 26. For the dune bed configuration, the observed values of l_0 fall between plus and minus 40 percent of the expected value, $E\{l_0\}$. For the ripple bed configuration, the variations are considerably greater, probably because the size of ripples is more dependent on grain size than flow conditions, so that figure 25 perhaps is not applicable. In view of the

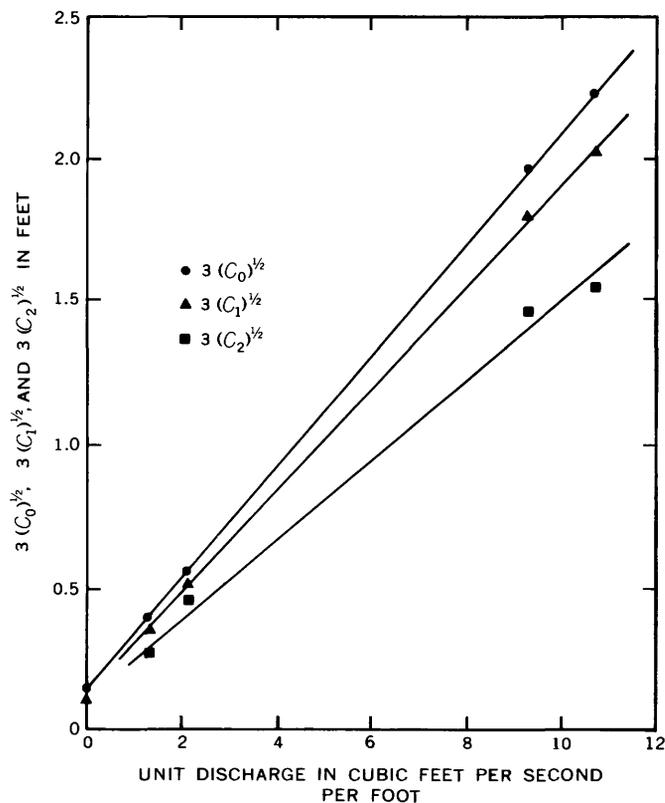


FIGURE 25.—Relation of C_0 , C_1 , and C_2 to unit water discharge (from Nordin and Algert, 1966).

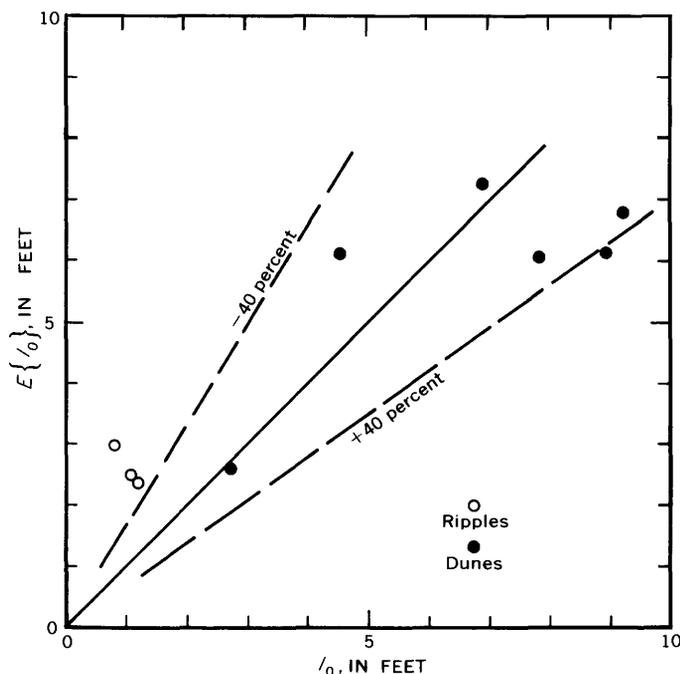


FIGURE 26.—Comparison of estimated and observed average distance in feet between zero crossings.

gross assumptions that went into estimating $E\{l_0\}$, the results of figure 26 are considered quite good.

There are a number of problems involved in using the second-order Markov model and the relations of figure 25 for predictive purposes. Both the model and the relations of figure 25 are strongly dependent on the lag interval at which the continuous records are digitized. In addition, spectral analyses by the writer and by D. R. Dawdy and N. C. Matalas (personal commun., 1966) have shown that the Markov model does not apply for many of the dune profiles examined. Finally, Plate (1967) has raised some serious questions with regard to the lack of physical basis of this model. It is desirable, therefore, to consider an alternative approach to describe the properties of the spectra and to relate their properties to characteristics of the flow.

DIMENSIONLESS SPECTRA

Autocovariance functions and spectra were computed for all the data listed in table 1. Figure 27 shows spectra for the process $y=y(x)$ for runs 17, 32, 43, 46 and 49. Spectra of the process $y=y(t)$ for identical flow conditions, runs 19, 33, 40, 44 and 48, are shown in figure 28. The data include dune flows in the 8-ft flume and both ripples and dunes in the 2-ft flume. The abscissa in both figures is dimensionless, with each value of frequency or wave number divided by the maximum value. All the spectra are remarkably similar in general shape, and there is little in the two figures to differentiate the processes

$y(t)$ and $y(x)$; this fact suggests that the statistical properties of the two types of records are similar.

Figure 29 shows a comparison of spectra for ripples and dunes for the 2-ft flume, with a 90-percent confidence band estimated according to the method given by Blackman and Tukey (1958). Here, as in figures 27 and 28, the similarity in the spectra is apparent. For both ripples and dunes, the major part of the variance is contributed by the longer wavelength components, greater than 2 feet. Both spectra show peaks at about 0.3 cycle per foot and again at about 0.8 cycle per foot. Both show additional peaks in the range from 2 to 4 cycles per foot, but within the confidence limits the peaks on the dune spectra in this range probably are not significant, whereas those for the ripples are. The rather wide band of the confidence limits leaves open to question the significance of most of the peaks at the lower wave numbers, but there is no question as to the general shape of the spectra.

The similarity of shape of the spectra in figures 27 and 28 suggests that a model incorporating flow parameters might be developed from dimensional considerations. For a first approximation, consider the major part of the spectra, that part which contributes all but a small percent of the variance, to be a function (ψ) of only four variables:

$$G(t) = \psi(f, g, D, V) \tag{38}$$

$$G(x) = \psi(\epsilon, g, D, V) \tag{39}$$

where D is mean flow depth, V is mean flow velocity, g is the acceleration due to gravity, and other symbols are as defined previously. Dimensionless spectra G' are then given in terms of a dimensionless frequency f' or wave number ϵ' and the square of the Froude number, F , as follows:

$$G'(t) = \psi'(f', F^2) \tag{40}$$

$$G'(x) = \psi'(\epsilon', F^2) \tag{41}$$

where

$$G'(t) = Gg/V, \tag{42}$$

$$f' = fV/g, \tag{43}$$

$$G'(x) = Gg/V^2, \tag{44}$$

$$\epsilon' = \epsilon V^2/g, \text{ and} \tag{45}$$

$$F^2 = V^2/gD. \tag{46}$$

Neither fluid nor sediment properties enter into equations 38 and 39 on the assumption that water temperature and particle size for the data analyzed were approximately

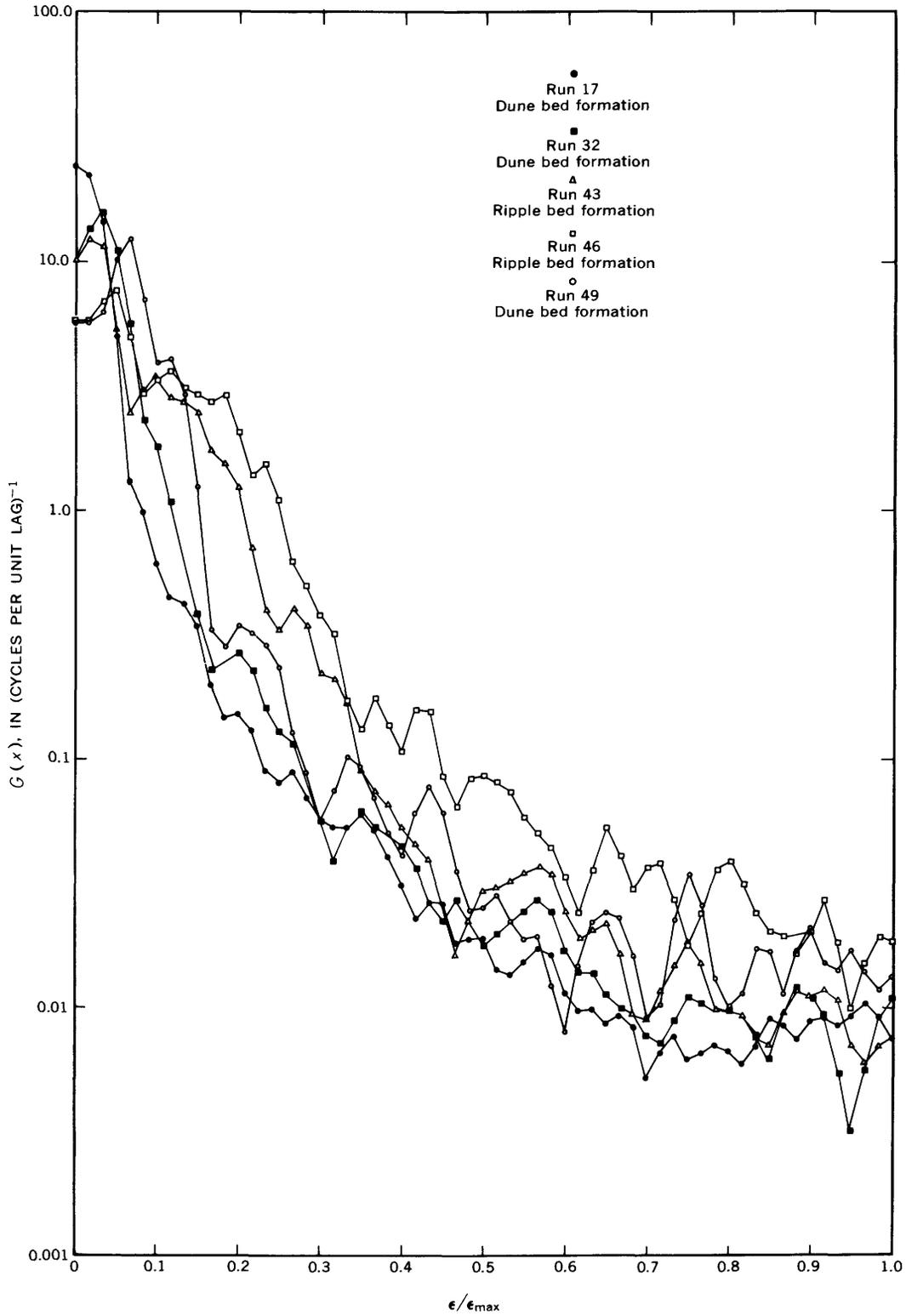


FIGURE 27.—Spectra of the process $y=y(x)$.

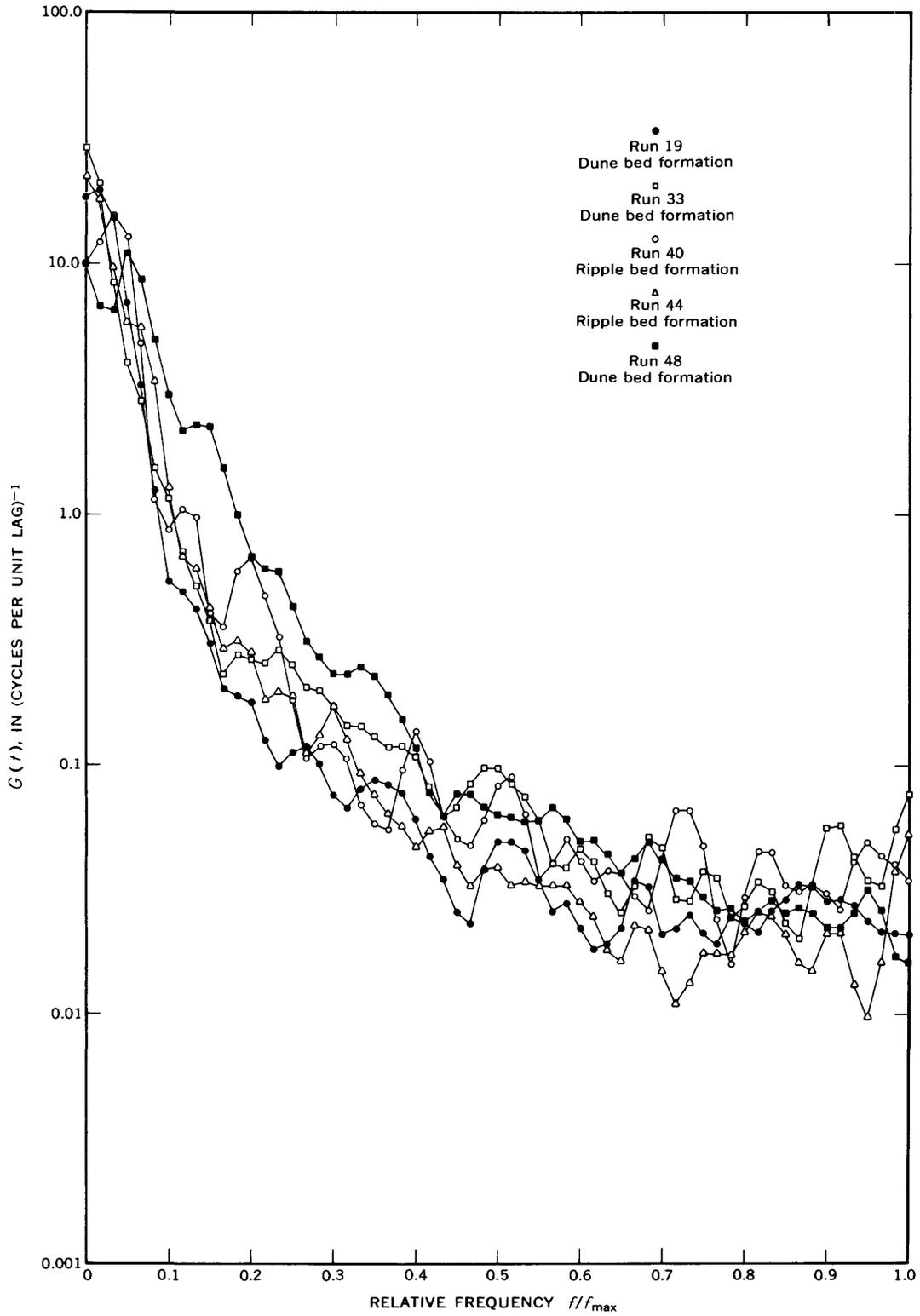


FIGURE 28.—Spectra of the process $y=y(t)$.

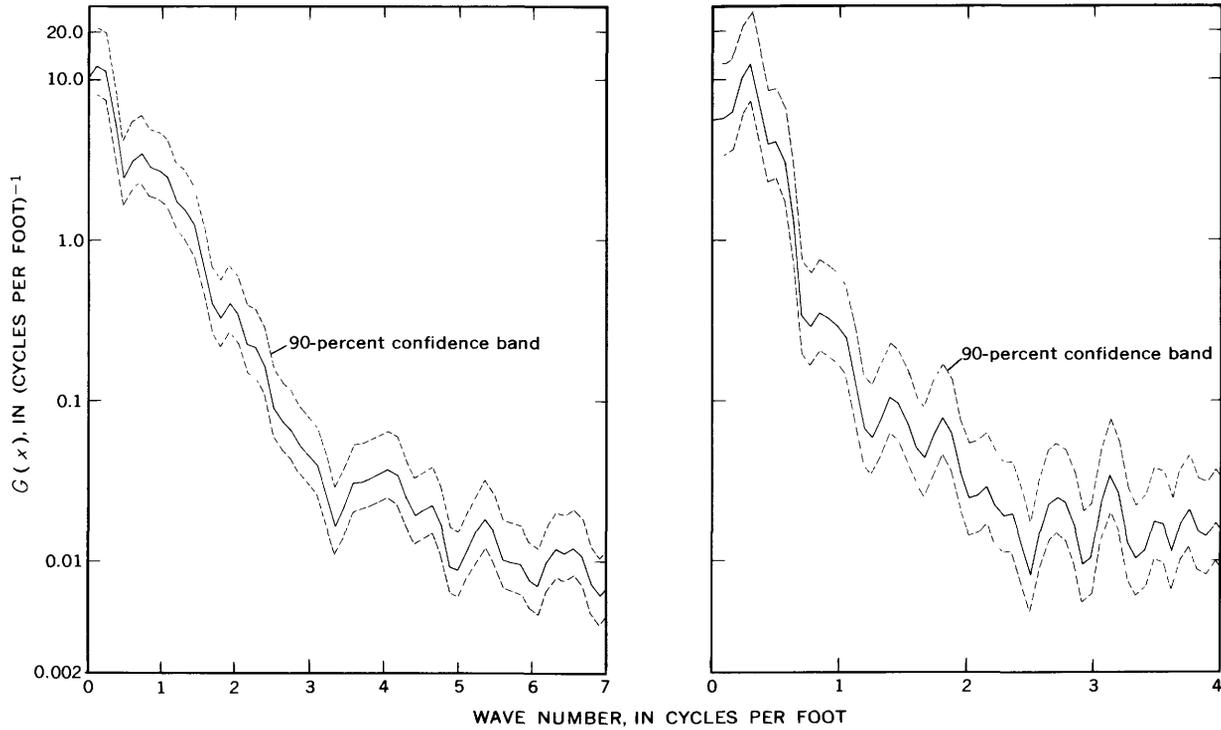


FIGURE 29.—Comparison of ripple (left) and dune (right) spectra, 2-foot flume.

constant. Then, the sediment transport rate and the bed forms are primarily functions of velocity and depth (Colby, 1964, Nordin and others, 1965). The mean flow depth enters equations 40 and 41 only through the Froude number. It is assumed that the equilibrium height of the bed configurations depends on flow depth, and it was shown in the previous section that the mean height of the bed irregularities and the variance of the process $y(x)$ are closely related. Thus, because all the records were previously standardized in terms of the variance, the Froude number should be a parameter of only minor importance in the spectra, and for the first approximation, G' should be a function only of the dimensionless frequency or wave number.

Figure 30 shows the relation between G' and ϵ' for the same data given in figure 27. For values of dimensionless wave numbers greater than 0.03, all data follow the relation

$$G' = 0.0001(\epsilon')^{-3.2}. \tag{47}$$

For lower wave numbers, each set of data follows a separate curve without a consistent pattern. It does not appear possible to collapse the family of curves into a single curve on the basis of Froude number. The dashed curve on the figure is intended only to show the general trend of the data, which seems always to show a maximum for values of ϵ' less than 0.025.

The dimensionless spectra for the process $y=y(t)$ are shown in figure 31, where the general relation for dimensionless frequencies less than 0.001 is given by

$$G' = 0.000015(f')^{-2.1}. \tag{48}$$

In figure 31 as in figure 30, there is considerable scatter; each set of data defines a general trend which tends to parallel equation 48. Again, it was not possible to sort the curves in terms of the third parameter, Froude number, which fact supports the assumption that depth should be a variable of minor importance in this analysis.

Equation 47 by no means characterizes the entire spectrum, but if the maximum values of G' and the dimensionless wave numbers at which they occur were either constant or functions of the flow and sediment, the spectra would be reasonably well described. Table 4 summarizes pertinent data for such an analysis for the process $y=y(x)$. The maximum spectral ordinate relates roughly to flow velocity, as shown in figure 32 (upper). The same weak relation appears to hold between the maximum $G'(x)$ and the dimensionless wave number, ϵ' , at which the maximum occurs, as shown in figure 32 (lower). No apparent relation was found between values of ϵ' corresponding to maximum values of $G'(x)$ and any of the flow or sediment characteristics, but it simply

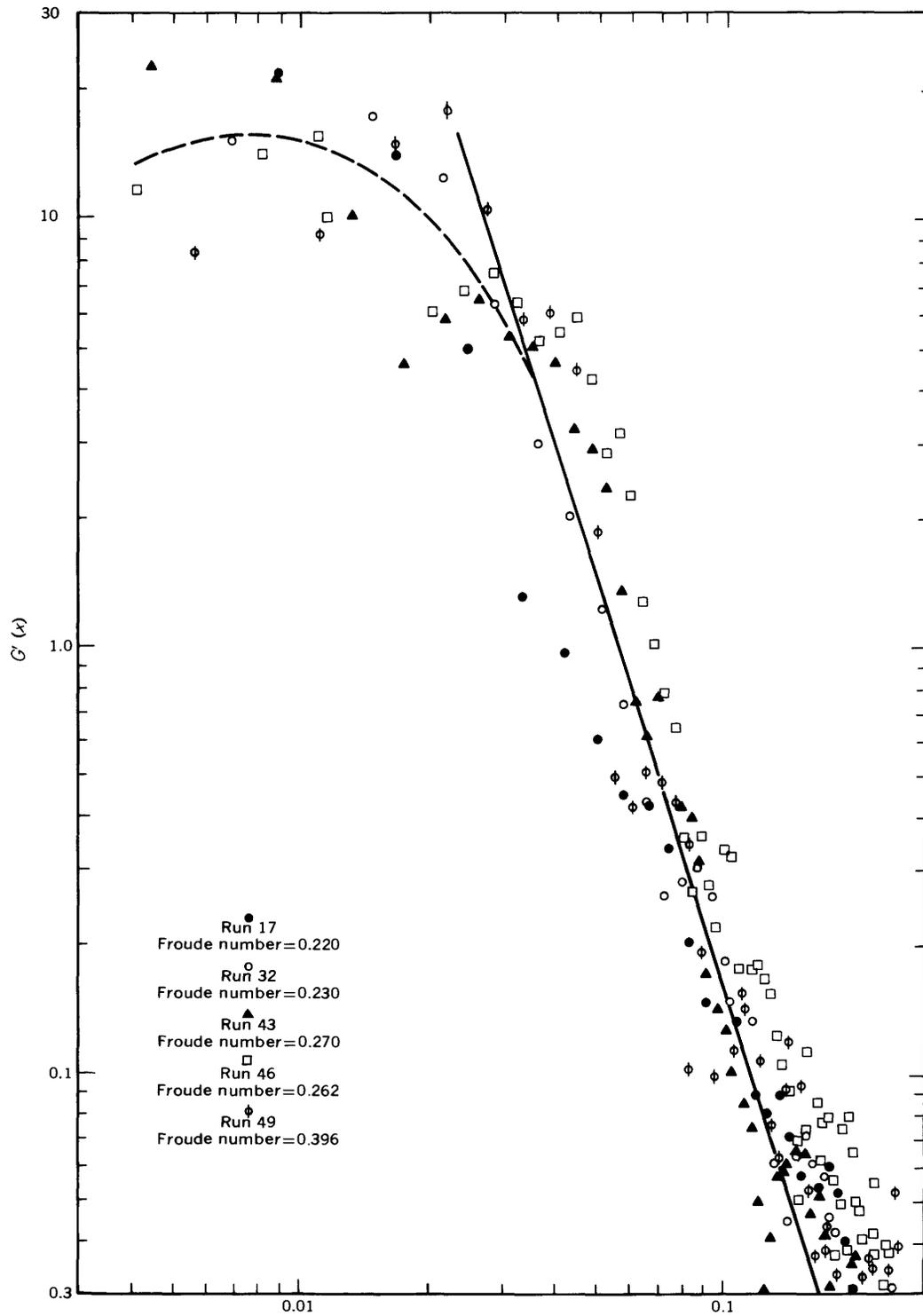


FIGURE 30.—Dimensionless spectra for the process $y=y(x)$.

may be that there was an insufficient number of observations to establish any trends.

From equations 47 and 48, the spectral density function G is approximately proportional to the square of the

wave period, $T=1/f$, or to the third power of the wave length, $L=1/\epsilon$. The wave celerity c is defined as $L/T=f/c$, so that from the equations for the dimensionless spectra, we can conclude that wave celerity is directly proportional

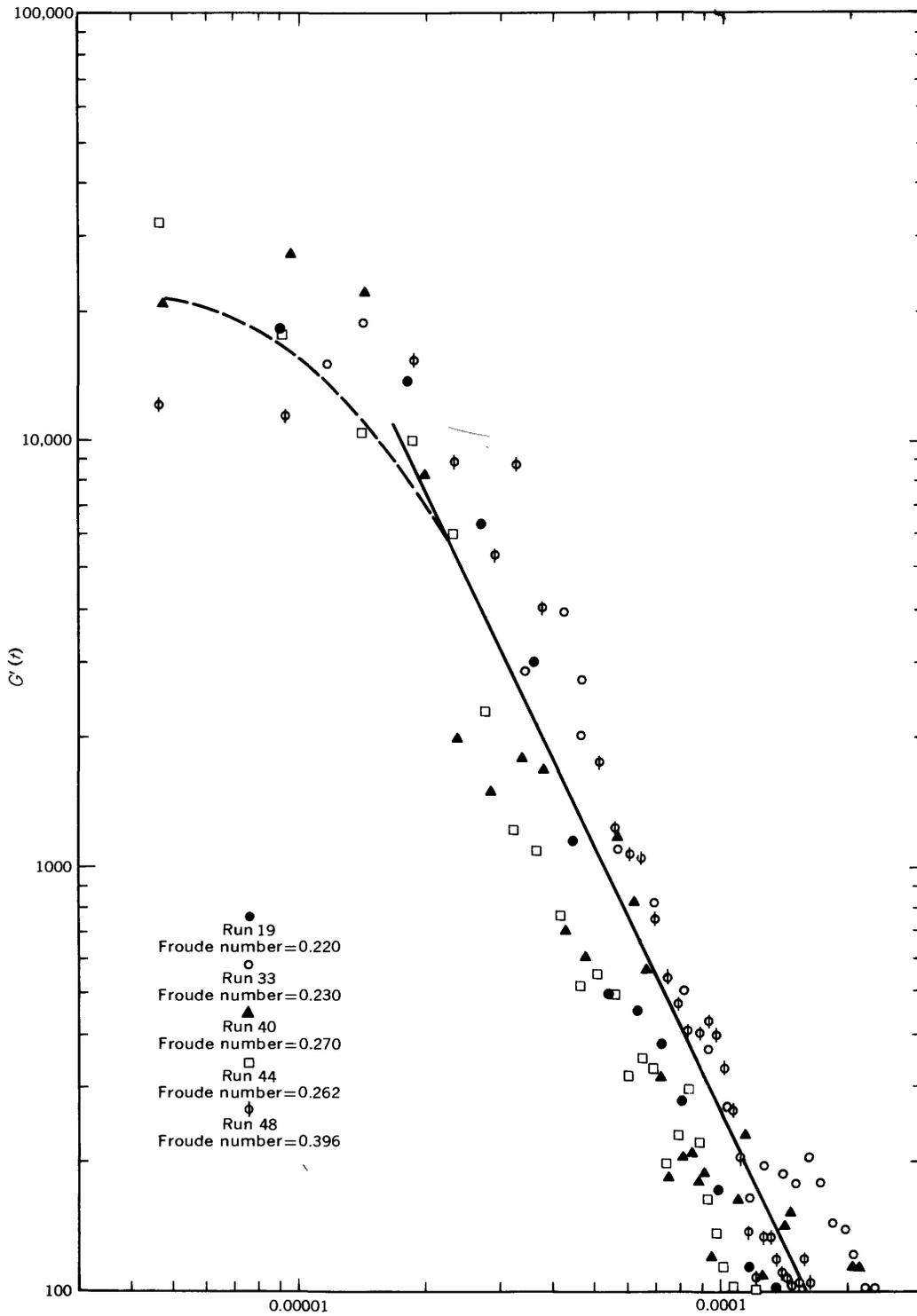


FIGURE 31. — Dimensionless spectra for the process $y = y(t)$.

to $\epsilon^{1/2}$. Thus, for constant flow conditions, the small waves move faster than the large waves. This conclusion is supported by Simons, Richardson, and Albertson (1961), who state, "Smaller dunes with their higher velocities overtake the larger dunes."

One arrives at the same conclusion by considering a simple model for sediment transport based on continuity principles, such as the one given by Simons, Richardson, and Nordin (1965), which states that, for ripples or dunes, the transport rate of bedload, q_b , is directly proportional

TABLE 4.—Summary of flow characteristics for dimensionless spectra

Run	Mean velocity (fps)	Mean depth (ft)	Median particle diameter (mm)	F	Maximum G'	ε' for maximum G'
53.....	0.593	0.174	0.30	0.246	29.0	0.0086
46.....	1.07	.522	.35	.262	15.5	.012
43.....	1.10	.518	.35	.270	22.8	.004
49.....	1.62	.521	.35	.396	17.8	.023
32.....	2.01	2.36	.24	.230	17.3	.015
17.....	2.09	2.80	.24	.220	21.8	.009

to wave height, *H*, and wave celerity, *c*. Suppose that the bed forms are dunes with ripples superposed, and that the dune movement is due entirely to the ripples overtaking the dune crest and depositing material on its downstream face. Then, continuity requires that $q_b \sim c_1 H_1 \sim c_2 H_2$, which states simply that the larger features move at a smaller velocity.

Note, however, that equations 47 and 48 say nothing about how the mean wave celerity might vary with varying flow conditions. This matter is considered in the next

section in connection with mean frequencies and wave numbers of the spectra computed from equation 8.

Equations 47 and 48 describe only the high frequency or high wave number parts of the spectra. The generality of the results are supported by Hino's (1968) study of sand wave spectra, in which he showed from dimensional considerations that the wave-number spectra at high wave numbers should follow the wave number, ϵ , to the minus third power and that the frequency spectra should vary as the minus second power of the frequency.

In considering sand wave properties in relation to channel roughness, however, it is the low frequency part of the spectrum that is important, and the dimensionless spectra of figure 30 do not reduce the low frequency part to a single relation. Engelund (1969) derived a universal dimensionless wave-number spectrum from similitude criteria that is of the form

$$\frac{2gDS}{V^2} \frac{G(x)}{\sigma} = \psi \frac{\sigma \epsilon V^2}{2gDS} \tag{49}$$

where ϵ is wave number in cycles per foot, $G(x)$ is in (cycles per foot)⁻¹, ψ is an arbitrary function, *S* is the water-surface slope, and the quantity $(2gDS)/V^2$ is the friction factor.

Engelund computed the relation of equation 49 for selected data from this report and plotted the results to define the general shape of the function as shown in figure 33. The results are encouraging, but additional data for greater flow depths probably need to be collected before it will be possible to determine whether or not the relation of figure 33 is superior to the relation of figure 30.

OTHER PROPERTIES OF THE SPECTRA

The probability distributions of the *y* values, the characteristics of the zero and *h*-level crossings, and the consistent shapes of the dimensionless spectra all point to some remarkable similarities of the bed profiles, irrespective of the size of the channel or of whether the bed forms were ripples or dunes. Indeed, the analysis to this point has been directed primarily toward exposing such similarities, and the procedures for standardizing the raw data to zero mean and unit variance and for nondimensionalizing the spectra really are nothing more than procedures for applying appropriate scale factors to the data to facilitate comparison. Quite often, however, one is more interested in the differences than in the similarities of the bed profiles, and if this is the case, it is more appropriate to compare the spectra of the records directly in terms of the actual frequency or wave number compo-

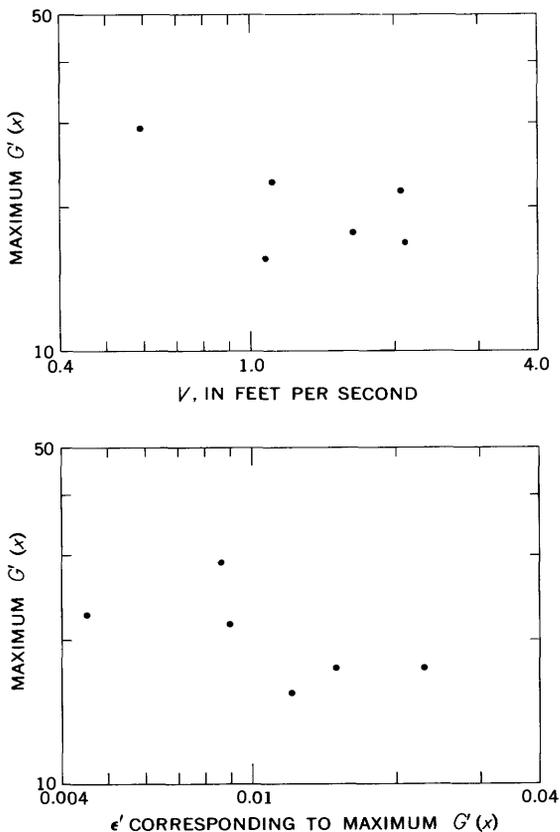


FIGURE 32.—Relation of maximum value of $G'(x)$ to mean velocity and to ϵ' .

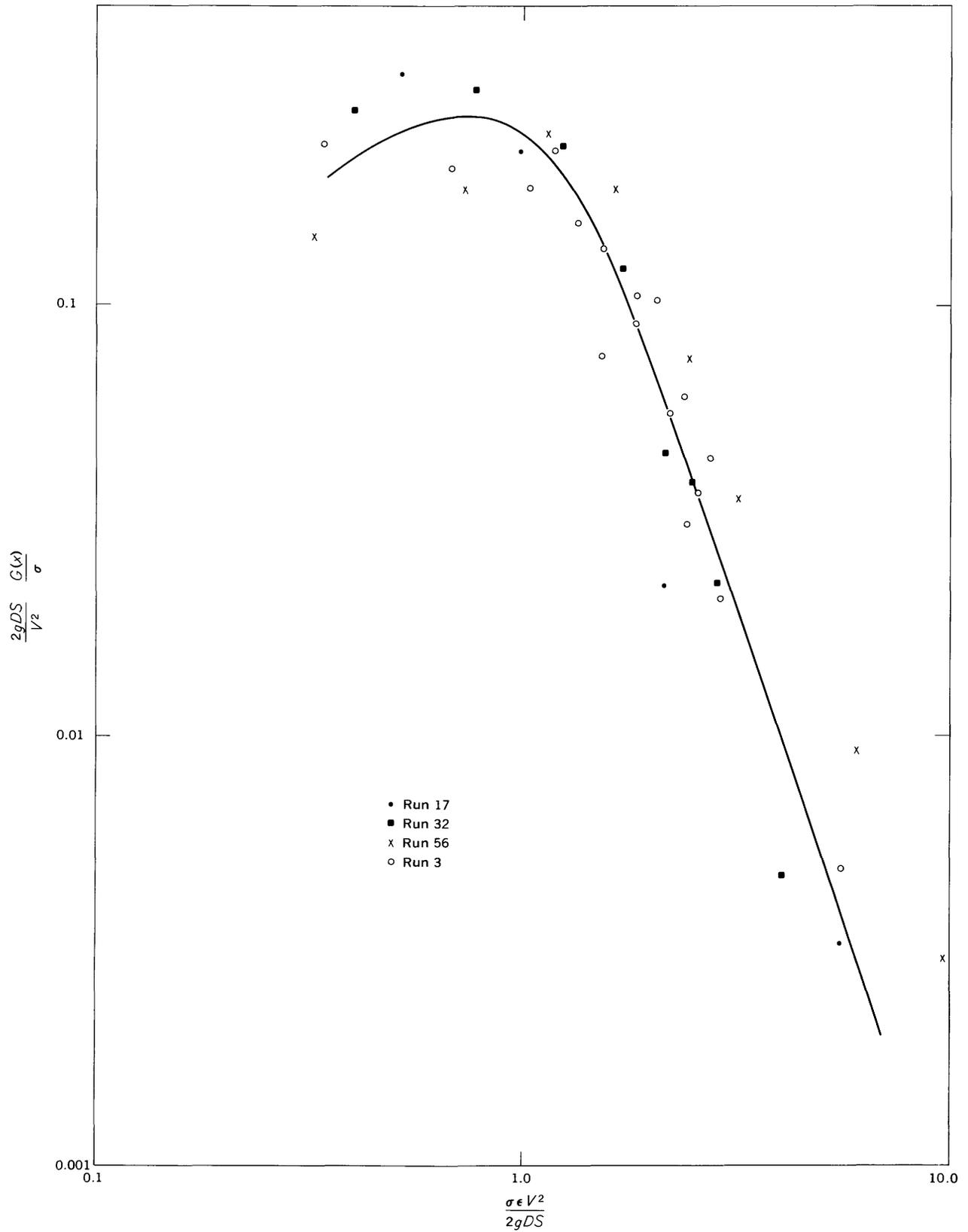


FIGURE 33.—Dimensionless spectrum based on similitude criteria (modified from Engelund, 1969).

nents. Figure 34 is an example of such a comparison. Spectra of longitudinal profiles, $y=y(x)$, from each of the four channels, are plotted against wave numbers in cycles per foot. The one-sided spectra were computed from the standardized data, so that the area under each curve is equal to 0.5. The standard deviation, σ_y , for each record is shown in the figure. The effects of channel size on the distribution of the variance over the various wave number components is obvious.

Several features of figure 34 are particularly interesting. First, note that the flow conditions for run 56 and run 24 are quite similar (table 1), yet there are appreciable differences in the spectra of the dune profiles. Run 56 is a 200-ft reach of Atrisco Lateral, and the major differences in fluid and flow characteristics between the field data and flume data are given below. In the field the velocity is about 5 percent higher and the temperature is lower by 4°C. In addition, a fine sediment load was associated with the field data that was not present in the

flume. For Atrisco Lateral, the observed sediment concentration from suspended sediment samples was 1440 mg/l (milligrams per liter) of which 220 mg/l were in the sand sizes, greater than 0.062 mm. For the flume data, the sediment concentration was 164 mg/l, all of which was sand. It is rather surprising that the dune profiles would be so markedly different with only the minor differences in mean flow properties. However, the effects of channel width, of the 4°C difference in temperature, of the 1,200 mg/l fine suspended sediment, and of the minor velocity variations are all undefined. Their combined effect on the bed forms, though, is appreciable, and comparison of the spectra indicates that, even if the mean velocity, depth, and slope are similar in the laboratory and the field, there may exist important differences in bed forms.

A second point of interest concerns the comparison of run 46 and run 49. Here, the difference in the distribution of variance is due strictly to bed configuration and not to channel size. The only difference in the flows for these two runs is the increase in velocity, slope, and sediment transport rate associated with the dune bed configuration; all other factors—depth, width, water temperature, and particle size of bed material—were constant. For the dune bed configuration, values of the standard deviation of the bed elevation, the mean wave amplitude, and the mean length between zero crossings are approximately double the values observed for the ripples. Surprisingly, the flow resistance as measured by the dimensionless Chezy coefficient is very similar for these two runs.

Finally, a third interesting feature of figure 34 is the indication that, for all the dune records, the variance is distributed generally over components of wavelength greater than 2 feet, whereas for the ripple bed configuration, the variance is distributed fairly uniformly over a greater range of wave numbers. The higher wave-number, or shorter wavelength, components contribute an appreciable part to the total variance of the profiles of the ripple bed configurations, but their contribution to the variance of the dune profiles is almost negligible. The question of whether the transition from ripples to dunes is gradual or abrupt cannot be answered from these data, but if additional experimentation were to show the transition to be abrupt, then the distribution of variance over the wave-number components would serve as a reliable criterion for distinguishing between ripples and dunes.

Spectral analyses of the longer records from Atrisco Lateral also are of particular interest because they permit better definition of the spectra at the lower wave numbers.

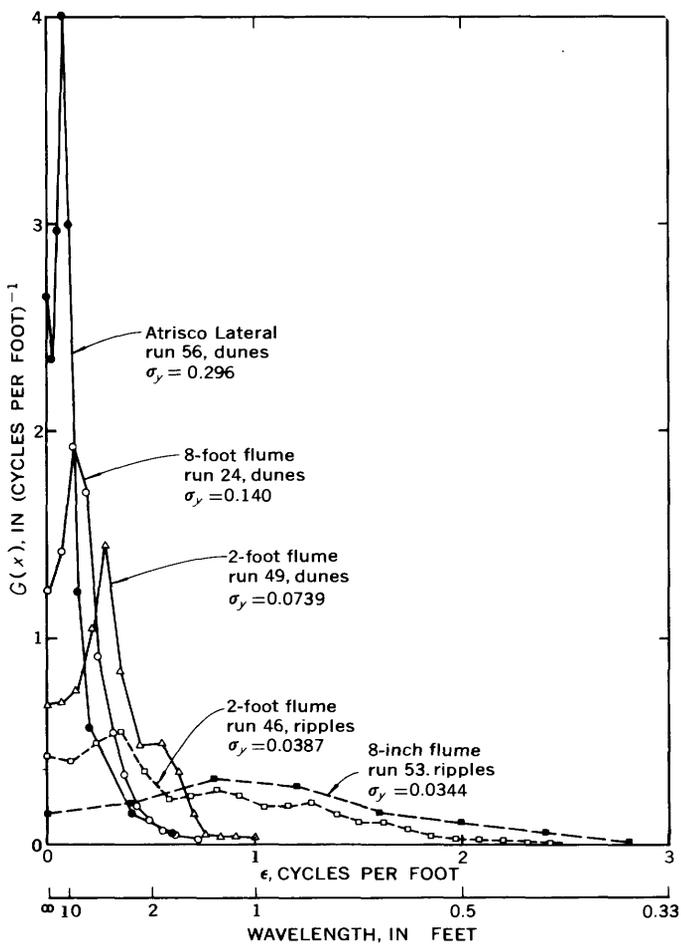


FIGURE 34.—Spectra of longitudinal profiles, $y=y(x)$, showing effect of channel size and bed configuration.

An example of these analyses is shown for run 3 in figure 35. In part *A*, the autocovariance function shows an apparent cyclic trend with a wavelength approximating the maximum lag of 600 or 200 feet. Inspection of cross sections of the channel every 50 feet along the length of the reach revealed the existence of a meandering thalweg and of alternate bars spaced approximately three to five times the channel width through the entire 4,000-ft reach. The trend in the autocovariance function (fig. 35*A*) is almost certainly due to the presence of these alternate bars. The spectra for these longer records show the generally decreasing values of $G(x)$ with increasing wave

numbers (fig. 35*B*) and are very similar in shape to the spectra shown in figures 27 and 28, but afford much greater detail at the lower wave numbers. Figure 35*C* shows the spectrum for wave numbers less than 0.1, corresponding to wavelengths of 10 feet or greater, with the wavelengths of the spectral peaks noted. The chi-square test with 20 degrees of freedom to establish a 90-percent confidence band on the spectral estimates indicates that the average ordinate should fall between $1.85 G(x)$ and $0.66 G(x)$. The peak at 200 feet may or may not be related to the meandering thalweg, but there is clearly a significant peak in the spectrum at wave num-

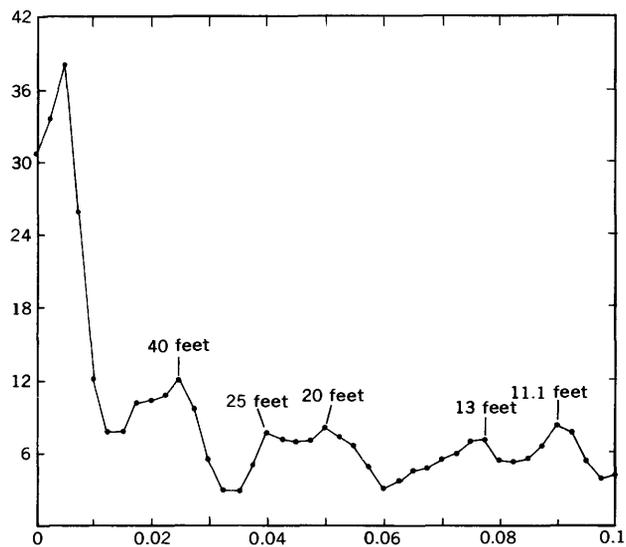
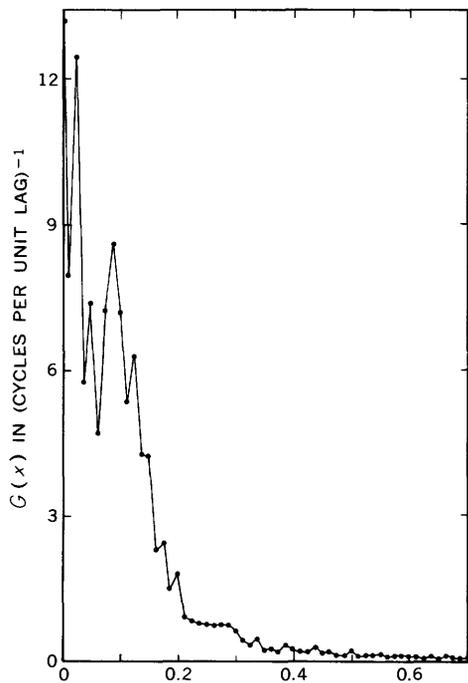
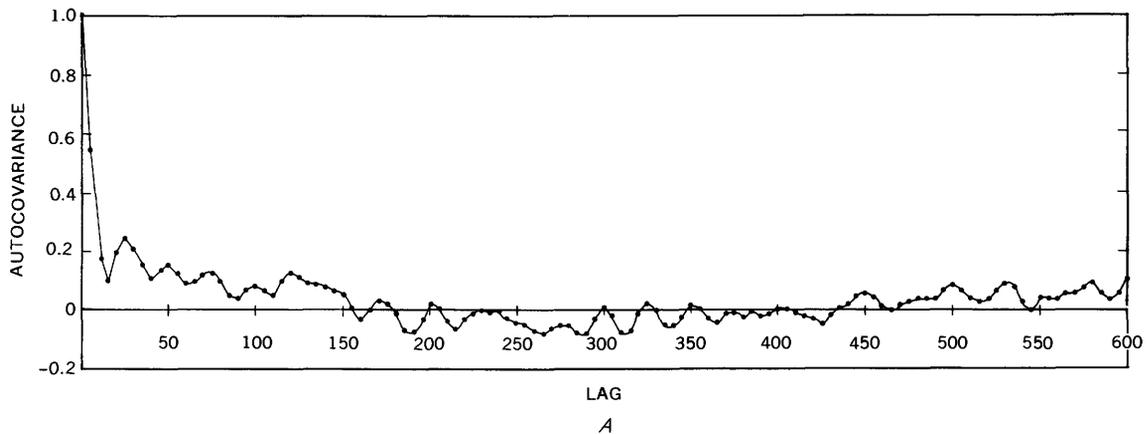


FIGURE 35.—Autocovariance function (*A*) and spectrum (*B* and *C*) of longitudinal profile, $y = y(x)$, for run 3, Atrisco Lateral. *C* is an expanded version of part of *B* to show greater detail for the low wave numbers.

bers less than 0.01. All other peaks shown on the spectrum of figure 35C are considered significant with the exception of the one at 13 feet, but the wavelengths at which these significant peaks occur were not found to correlate with any recognizable features of the bed profile. There is, of course, no reason to suspect that these peaks in the spectrum should relate to the lengths of the physical waves on which the analysis is being performed, although it has been observed that for dune records there sometimes is a correlation between the wavelength corresponding to the peak in the spectra and the average dune length (Nordin and Algert, 1966; Ashida and Tanaka, 1967).

As a final consideration of the properties of the spectra, it might be worthwhile to explore further the relation between the characteristics of the spectra and the physical characteristics of sand waves. It was previously shown (fig. 22) that a correlation exists between the average amplitude, a , and the standard deviation of the bed elevation, σ_y , which is related to the spectrum of y through equation 5. The reciprocal of the mean frequency or wave number from equation 8 is the mean period or wavelength of the spectrum, and this value relates reasonably well to the mean period or wavelength defined from the zero-crossing analysis, as shown in figure 36. Thus, it is possible to relate empirically the spectral properties and the observed properties of the sand waves.

Ashida and Tanaka (1967) used spectral analyses for determining the propagation velocity of sand waves by plotting the wavelength of the maximum spectral ordinate

for the process $y(x)$ against the wave period for the maximum spectral ordinate of the process $y(t)$. For the records analyzed in this study, some of the corresponding records of $y(x)$ and $y(t)$ possessed maximum ordinate values of their spectra at the origin; therefore, this method could not be used. In addition, no consistent relations were found between the peaks in the spectra and the average wavelengths or periods of the ripples and dunes. Consequently, the procedure used by Ashida and Tanaka (1967) is not recommended. However, from the relation in figure 36, it should be possible to determine the mean wave celerity either from the mean length and period found in the zero-crossing analysis or from the mean wave number or frequency of the spectra by using equation 8. Table 5, which shows a comparison of mean wave celerities computed by these two methods, indicates that values of \bar{c} obtained by the two methods are comparable.

TABLE 5.—Comparison of wave celerities

Run		From zero crossing			From equation 8			Bed configuration
$y(x)$	$y(t)$	L (ft)	T (min)	$c = L/T$ (fpm)	$L = 1/\bar{f}$ (ft)	$T = 1/\bar{f}$ (min)	$c = \bar{f} \bar{\epsilon}$ (fpm)	
17.....	19...	8.45	29.3	0.288	7.62	42.5	0.179	Dunes.
32.....	33...	4.78	16.6	.288	5.45	26.8	.204	Do.
43.....	40...	1.36	14.3	.0952	1.75	29.7	.0590	Ripples.
46.....	44...	1.01	27.4	.0368	1.24	38.8	.0320	Do.
49.....	48...	2.56	25.5	.100	3.00	32.4	.0926	Dunes.

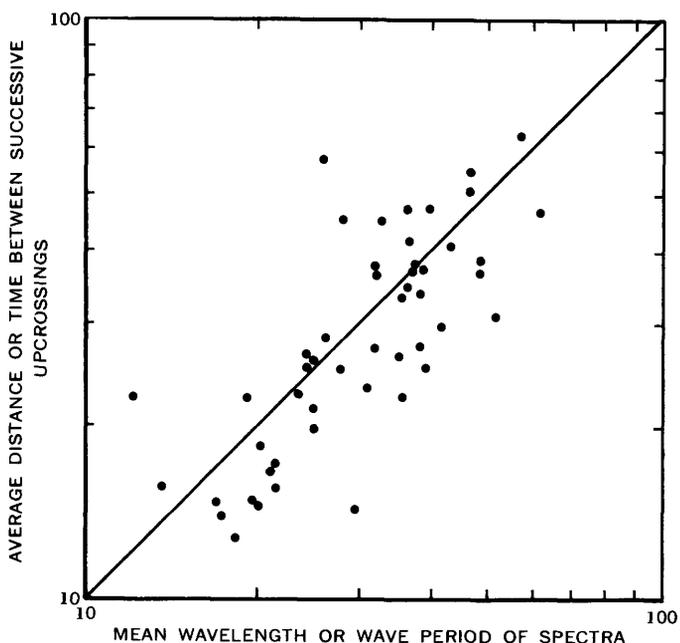


FIGURE 36.—Relation between average distance or time between successive zero up crossings and the mean wavelength or wave period of spectra from equation 8. Dimensions are in units of lag intervals.

The data in table 5 also give an indication of how the mean wave celerity varies with mean wave number for different flow conditions. In the previous section, it was established that for a constant flow condition, the wave celerity of the different wave-number components varied directly as the square root of the wave numbers. This relation is shown schematically in figure 37A. In figure 37B, the mean wave celerities computed from equation 8 are plotted against mean wave number from the spectral moments. Here we see that, with increasing flow velocity, the average wave celerity varies inversely with wave number or directly with wavelength. This implies that the mean wave celerity is independent of frequency; or on the average, for these data it took just as long for a ripple to move past a given point as it did for a dune.

The importance of the relations in figure 37 is in the fact that these relations permit determining the properties of the process, $y(t)$, from the properties of only the longitudinal profiles, $y(x)$. If the spectral properties of $y(x)$ are known, then the spectral properties of $y(t)$ can be determined from the relation in figure 37A and equations 47 and 48. If the mean wave number for the process $y(x)$ is known, then the mean frequency or wave period of $y(t)$ can be established by the relation in figure 37B,

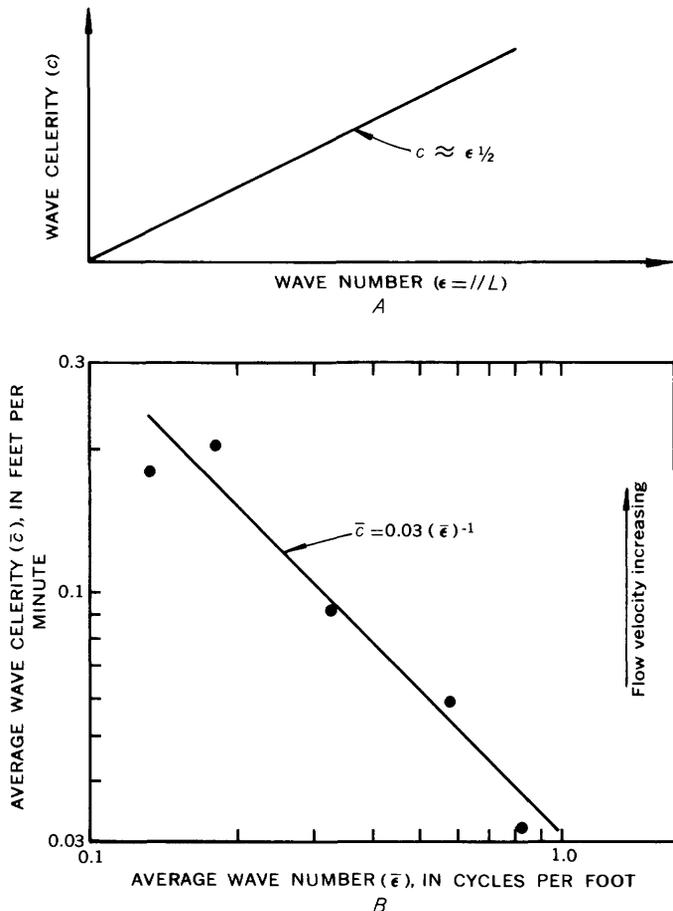


FIGURE 37.—Relation of wave celerity, \bar{c} , to wave number, ϵ . *A*, for various wave-number components of a single record, flow conditions constant. *B*, for average wave numbers from the spectral moments of records obtained under different flow conditions.

and the mean particle rest period at any level h and the conditional probability distributions for the rest periods at any level h can be determined directly from the relations in figure 12 and equations 14 and 33. Note, however, that the relation in figure 37*B* is no doubt a consequence of the sediment transport rates associated with the particular flow conditions examined here, and it is not known if the relation is generally applicable for greater flow depths.

O'Loughlin and Squarer (1967) and Squarer (1968) have urged that the standard deviation of the bed elevation and some characteristic wave length from the spectrum of the bed profiles be used to describe the geometric properties of sand waves, rather than simply the mean lengths and heights. Because no consistent terminology for describing bed configurations or methods for com-

puting average wavelengths and wave heights exists in the literature, it is very difficult to evaluate the results of different investigators. Certainly, then, it is desirable to develop some standard and rigorous method for describing the properties of the bed profiles that will permit comparison of different studies, and perhaps the standard deviation of the bed elevation and the reciprocal of the mean wave number of the spectrum from equation 8 would serve that purpose. In addition, it might be useful to specify the value of δ (eq. 20), which generally may be interpreted as a measure of the root-mean-square width of the variance spectrum (Cartwright and Longuet-Higgins, 1956, p. 216). Values of these parameters and of other measures of wavelengths and heights are given in tables 6 and 3.

CROSS CORRELATION AND CROSS-SPECTRAL ANALYSIS

Several interesting properties of the sand waves can be investigated using the techniques of cross correlation and cross-spectral analysis. Cross correlation is the simpler to use and admits to direct physical interpretations. Figure 38 shows the cross correlograms for correlations of run 8 with each of the runs 9 through 15. The profiles are all for the same reach of flume (see table 1) and were obtained 5 minutes apart. If the distance of the peak of the correlogram from the origin is plotted against time (fig. 39*A*), the slope of the trend line, 7 feet per hour, may be taken as the mean speed of movement of the dunes. An independent check on this figure is provided by the zero-crossing analysis, where the mean dune length for runs 8 through 15 is 5.7 feet and the mean period of runs 4 and 19 is 0.72 hours; these data give a dune velocity of about 8 feet per hour.

The general trend for the attenuation of the peak value of the correlation with time (fig. 39*B*) gives an indication of the rate of change in shape of the dune profiles. The dune profiles are assumed to be uncorrelated when the maximum value of the correlation function is 0.3 or less. The value of 0.3 is chosen because cross correlation of any two profiles taken at random from a group collected during approximate equilibrium flow conditions shows a maximum correlation generally less than 0.3. Then, extrapolating figure 39*B*, it is inferred that the profile virtually loses its identity in about 90 minutes, or in about the same time for the average-size dune to migrate twice its own length.

From figure 38, it is seen that the maximum value of the cross-correlation function for runs 8 through 15 is approximately the same as the maximum value for runs

TABLE 6.—*Summary of wave properties*

[Average length is trough-to-trough distance; height is difference in elevation, crest to trough]

Run	Wave characteristics			Height in feet		Ripple index	
	Average length, in feet	Average wave period, in minutes	Coefficient of variation	Average	Coefficient of variation	Average	Coefficient of variation
1.....	6.06		0.533	0.370	0.643	22.8	0.857
2.....	6.14		.535	.368	.655	22.7	.815
3.....	5.28		.535	.408	.597	17.2	.764
4.....		25.9	.763	.214	1.43	163.	.786
5.....	5.07		.658	.225	.571	25.7	.678
6.....	3.73		.700	.184	.377	21.6	.733
7.....	4.24		.604	.201	.388	22.2	.575
8.....	5.16		.408	.341	.514	22.3	.870
9.....	4.66		.553	.269	.535	21.7	.772
10.....	3.89		.742	.268	.509	17.5	1.05
11.....	5.13		.524	.305	.482	20.5	.722
12.....	3.89		.757	.251	.477	18.8	.916
13.....	6.44		.340	.384	.468	19.4	.590
14.....	4.49		.812	.264	.616	21.7	.979
15.....	6.18		.462	.364	.483	18.3	.513
16.....	5.06		.575	.315	.915	23.7	.739
17.....	4.84		.779	.265	.547	25.0	1.23
18.....	4.54		.698	.297	.483	17.8	.821
19.....		25.6	.664	.149	.229	175.	.674
20.....	4.31		.475	.261	.473	18.6	.429
21.....	3.55		.502	.221	.506	18.4	.671
22.....	3.49		.564	.265	.476	16.2	.826
23.....	3.67		.574	.257	.524	16.0	.584
24.....	4.40		.471	.296	.578	19.2	.770
25.....	4.31		.595	.316	.484	15.8	.627
26.....	4.34		.532	.300	.546	19.0	.897
27.....	4.89		.489	.367	.506	16.5	.668
28.....	4.55		.515	.276	.538	18.7	.536
29.....	4.77		.528	.253	.494	23.9	.704
30.....	3.85		.649	.165	.347	25.6	.727
31.....	4.12		.754	.161	.450	27.5	.868
32.....	4.72		.445	.260	.506	21.3	.577
33.....		16.3	.941	.133	.216	127.	.929
34.....	4.36		.521	.200	.411	27.4	.735
35.....	4.59		.562	.255	.645	25.5	.783
36.....	3.27		.538	.173	.381	21.6	.647
37.....	4.64		.428	.204	.446	26.7	.576
38.....	4.28		.510	.175	.380	26.5	.559
39.....	3.78		.545	.185	.457	21.6	.549
40.....		30.2	.426	.0783	.277	399.	.426
41.....	1.20		.484	.0922	.328	14.7	.648
42.....	1.15		.568	.0902	.288	13.4	.579
43.....	1.24		.606	.0913	.307	14.3	.679
44.....		40.4	.744	.0750	.353	577.	.802
45.....	1.24		.534	.0856	.268	14.9	.545
46.....	1.34		.542	.0827	.216	15.9	.422
47.....	1.28		.518	.0874	.318	15.7	.583
48.....		17.7	.514	.111	.491	183.	.574
49.....	2.73		.408	.167	.454	18.7	.642
50.....	2.75		.424	.149	.482	24.4	.836
51.....	3.15		.630	.114	.369	27.4	.589
52.....	.498		.353	.0609	.752	13.7	1.08
53.....	.440		.485	.0520	.752	14.3	.697

8 and 9. Even though the maximum cross correlations are about the same, it is assumed that the distributions of variance over the wave-number components should be appreciably different for the two series of runs because

the waves change shape somewhat as they shift downstream. In other words, the general shapes of the cross correlograms are not the same even though their maximums are equal. Plots of the coherence function, γ^2 ,

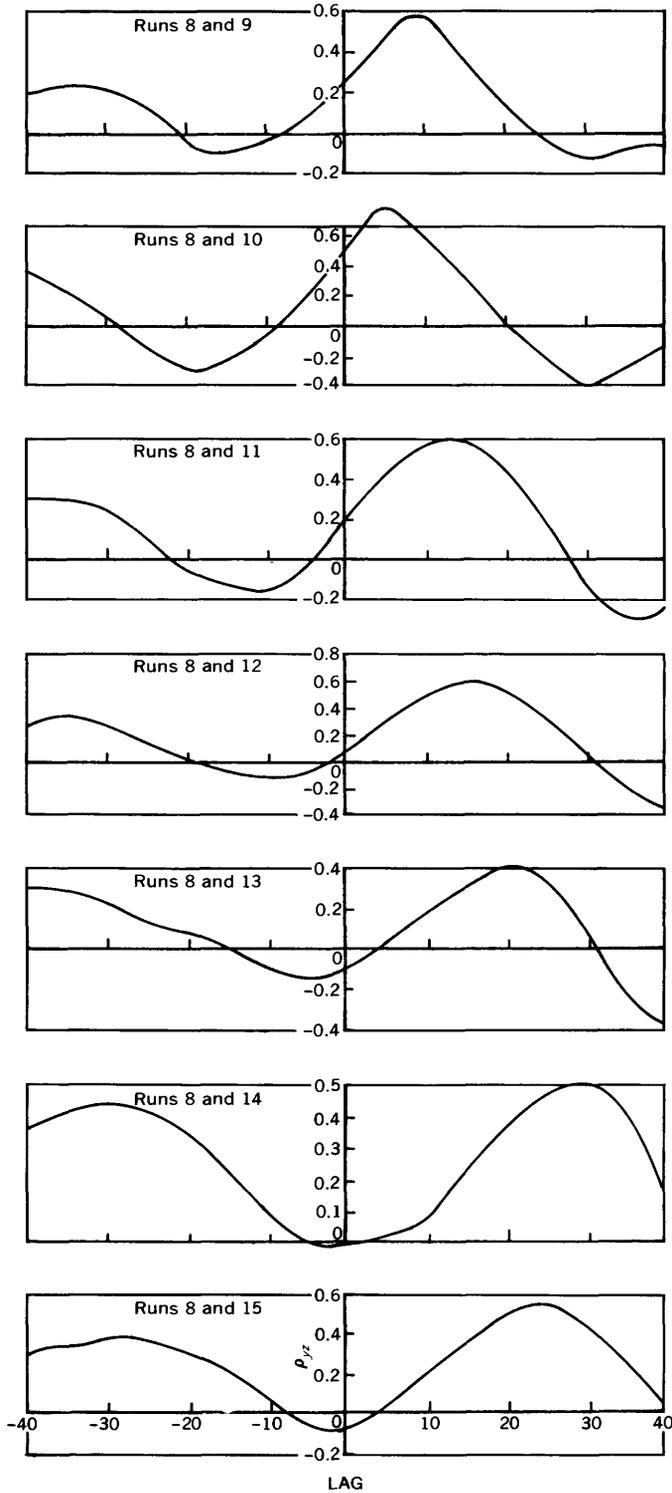


FIGURE 38.—Cross correlograms for run 8 with runs 9 through 15.

and the gain function, *A*, in figure 40 support this assumption. The general decrease in both coherence and gain from the cross-spectral analysis of runs 8 and 15, as opposed to the values for runs 8 and 9, shows that the

wave-number components of the two records 8 and 15 are virtually uncorrelated.

The cross correlation and cross-spectral analysis also proved useful in investigating the three-dimensional properties of the dunes. Run 17 was a profile $y(x)$ taken down the centerline of the 8-ft flume. Runs 16 and 18 were profiles taken 2 feet on either side of the centerline. The correlograms and the spectra for the individual profiles (fig. 41) do not suggest any appreciable differences in the profiles. A major part of the variance in all profiles is contributed by the shorter wave-number components corresponding to wavelengths generally greater than 4 feet.

The cross correlogram for runs 16 and 17 (fig. 42) shows that the two profiles, taken 2 feet apart along the same reach of flume, are practically uncorrelated, indicating that the sand waves certainly are not long crested. However, the cross correlogram for runs 17 and 18 (fig. 42)

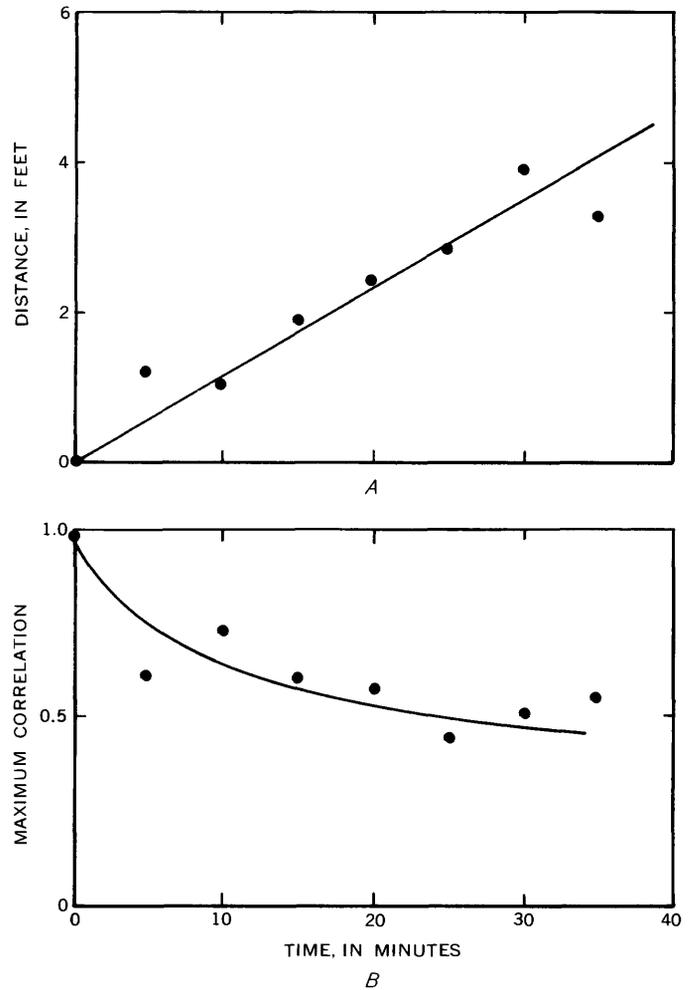


FIGURE 39.—*A*, Distance from the origin of the maximum cross correlation as a function of time. *B*, Change in maximum correlation with time.

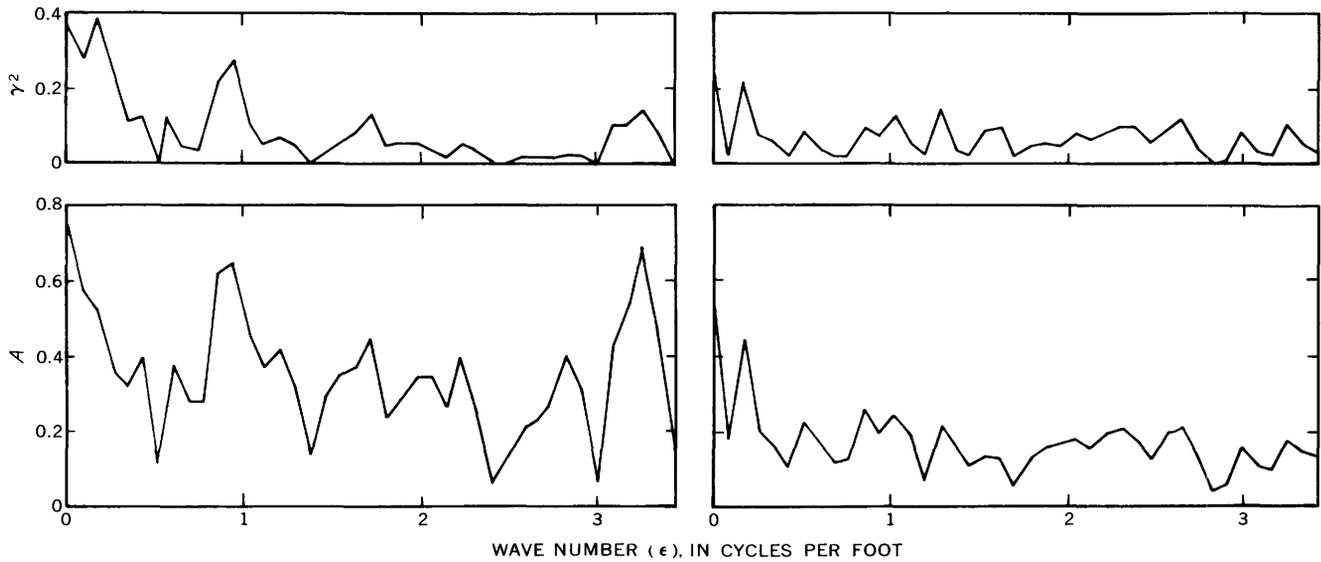


FIGURE 40.—Coherence, γ^2 , and gain functions, A , for (left) runs 8 and 9, (right) runs 9 and 15.

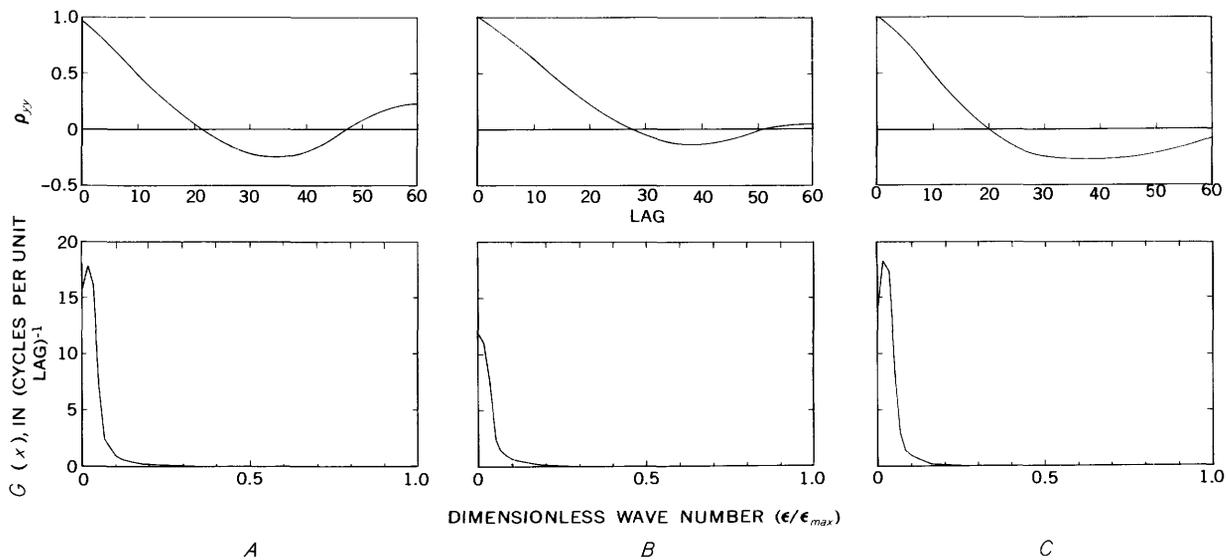


FIGURE 41.—Correlograms and spectra of $y=y(x)$. *A*, Run 16, 2 feet left of centerline. *B*, Run 17, centerline. *C*, Run 18, 2 feet right of centerline.

shows a somewhat better correlation, suggesting that the dunes along the centerline of the flume extend to or at least influence the dunes along the right side of the flume. The coherence diagrams for the spectral analyses of the two sets of data (fig. 43) show the same effect; values of the coherence function γ^2 for runs 17 and 18 are approximately twice those for runs 16 and 17.

PREDICTION

In previous sections, the problem of predicting the properties of the bed profile from only the characteristics of the flow and the sediment was considered briefly in

terms of the Markov model and the dimensionless spectra. Neither of these approaches was found to be completely satisfactory. At this point, some additional aspects of predicting the properties of the bed profiles will be considered.

Under the assumption that the bed profiles can be approximated by a random Gaussian process, only three factors are needed to predict the mean values of durations between zero and h -level crossings and the distribution of maximum and minimum y values between zero crossings. These factors are the variance of the bed profile, σ_y^2 , the second derivative of the covariance function at the origin,

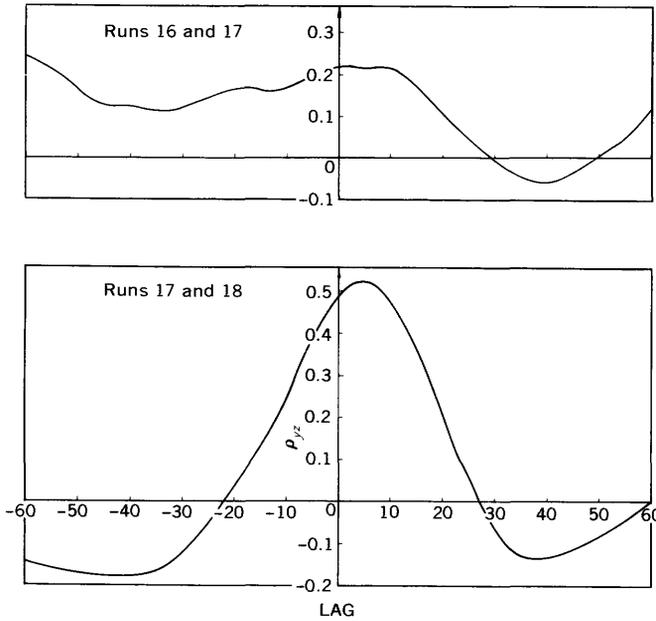


FIGURE 42.—Cross correlograms.

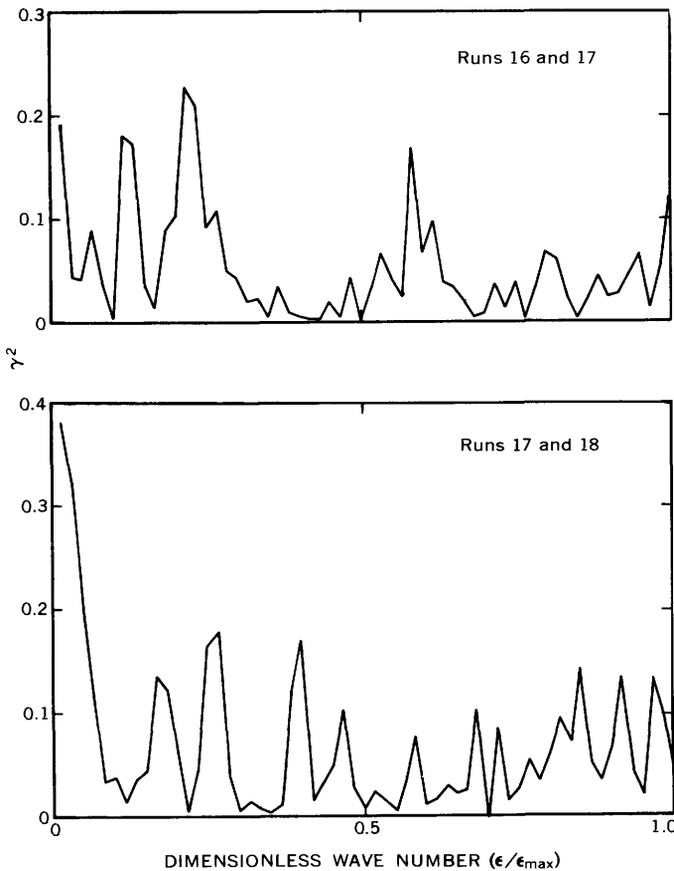


FIGURE 43.—Coherence diagrams.

$\phi''(0)$, and the parameter δ^2 . The parameter $\phi''(0)$ enters only in calculations of the mean duration between zero crossings, so that it could be replaced by $E\{l_0\}$. Values of δ^2 approached unity, and therefore as a first approximation it can be considered equal to one, and the prediction problem reduces to determining σ_y and $\phi''(0)$ or $E\{l_0\}$.

Definition of the dimensionless spectra in terms of flow parameters would permit determination of $\phi''(0)$ and $E\{l_0\}$, and this approach is favored by the writer. Some simple alternatives are to relate l_0 directly to flow parameters or to find a relation between l_0 and the standard deviation of the bed profile; such a relation would be analogous to the ripple index commonly used in geologic literature. In either event, it would still be necessary to determine σ_y , because the dimensionless spectra are based on the standardized data.

As we might expect, the standard deviation of the bed elevation, σ_y , relates roughly to flow depth (fig. 44A) or to the unit water discharge (fig. 44B). Data for these figures are summarized in table 7 and include average values from this study and values reported by Nordin and Algert (1966). It was not possible to reduce the scatter in these relations by consideration of other hydraulic or fluid variables. The point falling farthest to the left in the figures may be unrepresentative, because the bed profiles (runs 52 and 53) were collected after cessation of sediment motion and probably represent residual features from higher velocity flow. (See Rathbun and Guy, 1967, for details of this experiment.)

All things considered, the scatter of the points in figure 44 is neither unexpected nor discouraging. The data represent observations from five different flumes and two field channels. Two of the flumes used a sand feed system and three were of the recirculating type. In all the experiments except those performed by Algert (1965) in the 0.4-foot flume, information on the bed configurations was a consideration peripheral to the main objectives of the study. A number of unknown factors may contribute to the scatter in the data, such as flume entrance and exit characteristics, operating procedures in the experiment, imposed fine-sediment load (in the field case), and particle-size distribution of the bed material.

The one other property of the bed profiles that is of most interest is the probability distribution of the durations of upward excursions of the process $y(t)$ above the fixed level h , which is the conditional probability distribution of the rest period of a particle, given that it is deposited on the downstream face of a dune at the level h . The gamma distribution with parameters that relate to h , as shown in figures 12 and 13, is an attractive possibility. However, before this approach can be recommended, it is

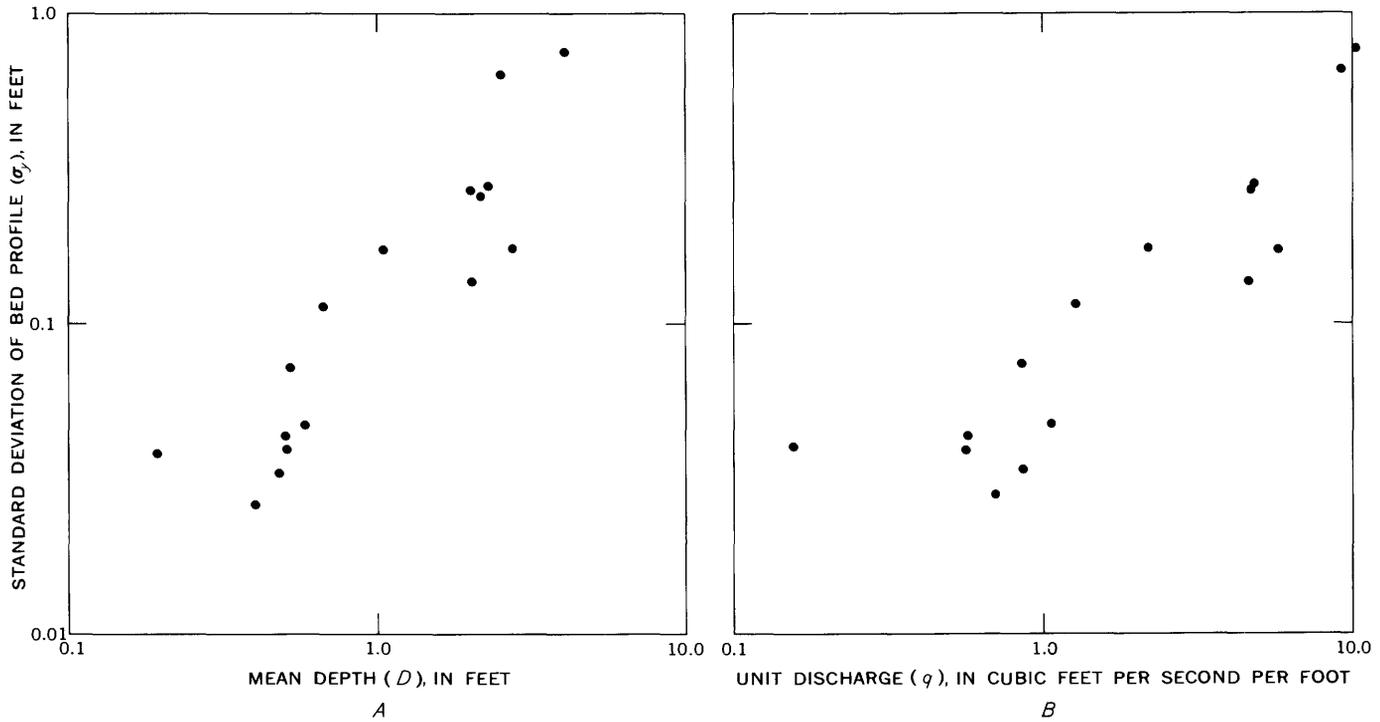


FIGURE 44.—A. Relation of standard deviation of bed profiles, σ_y , to mean flow depth, D . B. Relation of standard deviation of bed profile, σ_y , to unit water discharge, Q .

necessary to verify both the gamma distribution and the relation of figure 12 with additional observations of the process $y = y(t)$.

In summary, the problem of predicting the properties of interest of the bed profiles is still not solved, but the results of this study have suggested several promising approaches. Some crude empirical relations, such as those of figures 12, 13, 26, 37 and 44, permit approximations to be made of most of the properties of interest, but the need for both refined theoretical models and additional experimental data is obvious.

SUMMARY AND CONCLUSIONS

DISCUSSION OF RESULTS

The application of statistical techniques to analyze the properties of sand waves is a relatively recent undertaking, and therefore it is perhaps difficult to evaluate objectively some of the ramifications of this investigation. Nonetheless, three implications of the results seem to the writer to be of particular importance and to merit further discussion.

TABLE 7.—Average values of observed variables

Channel	V (fps)	D (ft)	W (ft)	S	T (°C)	d_{50} (mm)	σ_y (ft)	Avg $a +$ (ft)	l_0 (ft)	Remarks
Atrisco.....	2.16	2.20	55	0.00057	19	0.23	0.259	0.277	4.48	Dunes.
Atrisco.....	2.11	2.30	55	.00055	20	.23	.276	.287	4.61	Do.
Atrisco.....	2.08	2.29	55	.00058	20	.23	.270	.302	3.90	Do.
8-ft flume.....	2.09	2.80	8	.00063	20	.24	.169	.214	2.95	Dunes; recirculating system.
8-ft flume.....	2.01	2.36	8	.00056	24	.24	.127	.146	2.28	Do.
2-ft flume.....	1.62	.521	2	.00212	20	.25	.0700	.0806	1.42	Do.
2-ft flume.....	1.10	.518	2	.00088	20	.35	.0432	.0416	.58	Ripples; recirculating system.
2-ft flume.....	1.07	.522	2	.00088	20	.35	.0390	.0408	.54	Do.
0.67-ft flume.....	.80	.197	.67	.00148	20	.30	.0404	.0400	.36	Ripples; sand feed system.
Bernardo*	3.62	2.60	70	.00058	8.34	.23	.647	Dunes.
Bernardo*	2.48	4.15	70	.00058	8.34	.23	.745	Do.
8-ft flume*	2.11	1.05	8	.00134	16.7	.28	.175	Dunes; recirculating system.
8-ft flume*	1.91	.670	8	.00136	17.8	.28	.115	Do.
0.4-ft flume*	1.81	.580	.4	.0044	18.4	.34	.0478	Sand feed system.
0.4-ft flume*	1.78	.485	.4	.0038	15.6	.34	.0330	Do.
0.4-ft flume*	1.74	.400	.4	.0037	17.8	.34	.0270	Do.

*Data are from Nordin and Algert (1966).

The first important conclusion drawn from these investigations is that the properties of the profiles obtained in the laboratory flumes are all very similar, regardless of the size of the flume or of whether the bed configurations are ripples or dunes, provided that scale effects are properly taken into consideration. The scale effects seem to be completely accounted for by standardizing the raw data to zero mean and unit variance and by expressing the length of h -level crossings as a ratio of the mean length between zero crossings, or by forming dimensionless parameters in terms of the flow properties, as in the case of the spectra. This means that certain simple properties, such as the average values of the conditional probability-density functions of rest periods, can be modeled in a ripple bed configuration or in a very small flow system, and the results can be extrapolated to dune configurations of much larger flow systems.

Similarity of the properties of the ripple and dune profiles does not imply that there are no differences in these features but only implies that their general shape and method of movement are similar, as noted by Taylor and Brooks (1961). In fact, the comparisons of distributions of variance over the wave-number components shown in figure 34 support very strongly the notion that there are appreciable differences. The major differences between ripples and dunes are described in detail by Simons and Richardson (1966) and need not be considered here.

In addition, there were important differences between the flume and field data for dune bed configurations. The longer records from Atrisco Lateral reflected the influence of a meandering thalweg and of accompanying large alternate bars that generally were not present in the flumes. Simons and Richardson (1966) have noted that alternate bars do form in flumes, but for the flume data analyzed here, the width-to-depth ratio was not great enough to permit the bars to develop significantly.

A second important implication arises in the developments leading to figures 30 through 33 of the dimensionless spectra. These figures show that the spectral representations of the processes can be determined as unique functions of the flow, fluid, and sediment properties. It was fortunate for these studies that the water temperatures and sediment sizes varied over rather narrow ranges, because this permitted expressing the dimensionless spectra in extremely simple form. On the other hand, because of this limited range of conditions, it was not possible to define adequately the shapes of the spectra. Nevertheless, the results are encouraging, so far as they were carried, and they point the way to future experimental studies that will permit a better definition of the spectra.

However, the most important result by far is the demonstration in figures 6–9 and 15–17 that the properties of the profiles can be estimated by theoretical considerations of fairly simple models. Certainly, the assumption that the processes $y(x)$ and $y(t)$ are Gaussian is a crude approximation, but the results obtained using this model are reasonable and consistent.

In one important aspect, this study was unsuccessful. It was not possible to relate uniquely the statistical properties of the bed profiles to the characteristics of the flow. However, some simple empirical relations were considered that will permit predicting approximately many of the properties of interest. The most important relations in this regard are shown in figure 44, which gives σ_y as a function of either depth or unit water discharge, and figure 25, from Nordin and Algert (1966, p. 109), which shows values of the correlation function at zero, one, and two lag intervals as functions of unit water discharge. These values, along with the length of profile record, permit calculations of most of the factors considered in the zero and h -level crossing analysis and the approximations of l_h^*/l_0^* , as given by equations 15 and 16. However, the relations of figure 25 apply only if the lag interval is selected so that it bears a constant ratio to the standard deviation, σ_y .

CONCLUSIONS

In this study statistical properties of streambed profiles from four different size channels are compared by the techniques of time-series analyses and by a consideration of the mean values and distributions of zero and h -level crossings, the durations of upward excursions of the records $y(x)$ and $y(t)$ above a fixed level h , and the distribution of maximum y values between successive zero crossings. The principal conclusions drawn from the study are the following:

1. No appreciable differences in the statistical properties of the profiles from the flumes could be attributed to flume size or to whether the bed forms were ripples or dunes, provided that the raw data were standardized to zero mean and unit variance and that the length scales were expressed as ratios of the mean length between zero crossings. In addition, only minor differences were noted between the properties of the longitudinal profiles, $y=y(x)$, and the properties of the profiles, $y=y(t)$, obtained by sounding continuously in time at a fixed point. Longer records of both types, particularly the time records, are required to establish if the differences are real or apparent.
2. Spectra of the longer records from Atrisco Lateral appeared to reflect the influences of a meandering

thalweg and large alternate bars that were not present in the flume data.

3. By dimensional analyses, forms of dimensionless spectra were derived which describe reasonably well the observed forms in the higher frequency and wave number regions. From the equations of the dimensionless spectra, it was established that, for constant flow conditions, the celerities of the individual wavelength components vary directly as the square root of the wave numbers.
4. From the mean spectral moments, it was shown that the mean wave celerity varied inversely as the mean wave number for the different flow conditions considered. This relation, together with the relation for celerity of wave-number components under constant flow conditions, permits determining the properties of the process, $y(t)$, from the properties of the longitudinal profiles, $y(x)$.
5. The techniques of cross-spectral analysis were found useful for defining the mean rate of shifting of ripples and dunes and investigating their three-dimensional properties.
6. Values of the bed elevation follow an approximate Gaussian distribution. For a Gaussian process of known covariance function, the expected number of zero or h -level crossings, the expected length between crossings, and the mean duration of upward excursions of the process $y(t)$ above the fixed level h can be computed. The comparison of observed and computed values shows good agreement for positive values of h and indicates systematic deviations for values of h below the mean bed elevation. The mean duration of upward excursions of the process $y(t)$ is the mean rest period of a particle at the level h .
7. The distributions of the distances between zero crossings, l_0 , are approximately exponential. Values of the durations of upward excursions of y above the level h follow a gamma distribution with parameters that relate to h , as shown in figures 10 through 13.
8. The distributions of maximum values of y between zero crossings represent the distributions of scour and fill associated with the formation and migration of sand waves. The sand waves are not symmetric about the mean bed elevation. The positive maximums, $a+$, are distributed exponentially and the negative maximums, $a-$, are distributed according to a Gaussian distribution. Both distributions are functions of the standard deviation of the bed elevation, σ_y , which relates approximately to mean flow depth or unit water discharge (fig. 44).
9. Finally, the results of this study show that some of the distributions entering the two-dimensional model of sediment transport (Sayre and Conover, 1967) and

most properties of the dune profiles that are of interest can be determined from theoretical considerations of fairly simple models.

RECOMMENDATIONS FOR FUTURE STUDIES

The results of this study suggest a number of areas for future investigations. Four, in particular, appear especially promising. These are:

1. Studies should be undertaken to include fluid and sediment properties in the dimensionless spectra and to determine more accurately the dimensionless wave numbers at which the peak spectral ordinates occur.
2. An investigation should be made of the possibility of incorporating the distribution of wave height and wavelength and the relation of wave celerity to wavelength components (fig. 37) into a continuity-type bedload transport relation based on size and rate of shifting of the sand waves.
3. Experiments should be designed to obtain longer records of $y=y(t)$ so that the probability distributions of particle rest periods (the durations of upward excursions) can be adequately defined.
4. Models other than the simple Gaussian model should be investigated as possible representations of the processes $y(x)$ and $y(t)$ to see if more accurate predictions of the properties of the bed profiles can be obtained theoretically. In addition, it should be possible to investigate some models by simulation techniques. One such possibility would be a process of the form $y(t) = \rho \cos(\lambda y + \theta)$, where ρ , λ , and θ are all random variables of some specified distribution.

In addition to the above studies, it is necessary to obtain reliable field records of bed profiles to permit extending these studies to a greater range of flow conditions. Records of the process $y=y(t)$ are especially lacking, and many existing longitudinal profiles suffer from inadequate horizontal control.

As noted in the section "Data and analysis," reliable relations between the statistical properties of the profiles and the characteristics of the flow still remain to be developed. Empirical relations exist, but they are far from satisfactory for predictive purposes.

Finally, detailed descriptions of bed profiles are of value only if they lead ultimately to a better understanding of sediment transport processes. It is hoped that the results of this study and of future studies along the lines outlined above will contribute to such an understanding.

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PLANNING OF DATA REQUIREMENTS

This appendix gives some approximate guidelines for planning the length of records required for the various analyses considered in the text. The statistical basis for determining record length and the computational procedures are given by Bendat and Piersol¹ and will not be repeated here. The computer programs used in this study are available from the writer or from Mrs. Lois Niemann, Civil Engineering Department, Colorado State University.

The computational procedures used in this study can be applied to any continuous record of streambed elevation, provided that the record meets the necessary conditions for stationarity. From a practical point of view, this means that, for longitudinal profiles, the channel cross section should not vary appreciably along the channel and that, for either longitudinal or time records, the flow and sediment transport rates should be approximately constant during the period of observation.

We consider here only flow conditions which occur where well-defined sand waves are known to be present at the streambed, that is, only lower regime flow. The bed configurations for lower regime flow are ripples, dunes, bars, and combinations of these features (Simons and Richardson²). In the following discussion the computational procedures for spectral analysis of a single record first are outlined. Next are given some rough guidelines for determining the length of record and the spacing of observations for converting a continuous record to discrete data points for spectral analysis. Finally, the record length and spacing of data points for other types of analysis, such as estimating the mean lengths between zero and h -level crossings, are considered.

COMPUTATIONAL PROCEDURES

A continuous record of bed profile, $y=y(x)$, of length L_r is converted to discrete data points by sampling the continuous records at intervals $\Delta x=h$ ft, so that the sampling rate is $1/h$ samples per ft. The entire record is converted to N discrete data points, y_i , $i=1, 2, \dots, N$. We consider the entire sequence of y_i values to have zero mean and unit variance. The covariance function corresponding to equation 2 of the text is computed by the formula

$$\phi_{yy}(s) = \frac{1}{N-s} \sum_{n=1}^{N-s} y_n y_{n+s} \quad (1)$$

for $s=0, 1, \dots, m$, where m is the maximum number of lags.

¹ Bendat, J. S., and Piersol, A. G., 1966, Measurement and analysis of random data: New York, John Wiley & Sons, Inc., p. 278-320.

² Simons, D. B., and Richardson, E. V., 1966, Resistance to flow in alluvial channels: U.S. Geol. Survey Prof. Paper 422-J, 61 p.

Next, the finite cosine series transform function of the autocovariances is computed from

$$\tilde{G}(s) = \sum_{j=0}^m \hat{\phi}(j) \cos \frac{js\pi}{m} \quad (2)$$

for $s=0, 1, 2, \dots, m$. In the above,

$$\begin{aligned} \hat{\phi}(0) &= \phi_{yy}(0), \\ \hat{\phi}(i) &= 2\phi_{yy}(i), \end{aligned} \quad (3)$$

where $i=1, 2, \dots, m-1$, and

$$\hat{\phi}(m) = \phi_{yy}(m).$$

The spectrum is computed from the equations

$$\begin{aligned} G(0) &= 0.5\tilde{G}(0) + 0.5\tilde{G}(1) \\ G(i) &= 0.25\tilde{G}(i-1) + 0.5\tilde{G}(i) + 0.25\tilde{G}(i+1), \quad i=1, 2, \dots, m-1 \end{aligned} \quad (4)$$

$$G(m) = 0.5\tilde{G}(m-1) + 0.5\tilde{G}(m).$$

These computations yield $m+1$ values for the spectrum at each of the lags $s=0, 1, \dots, m$, corresponding to the wave numbers $\epsilon_s = s/2mh$ or to the wavelengths $L_2 = 2mh/s$.

LENGTH OF RECORD AND SPACING OF DATA POINTS

The computational procedures outlined above show that, in selecting a length of continuous record for spectral analysis by digital techniques, three quantities must be considered: the record length, L_r , the sampling (digitizing) rate, h , and the maximum lag, m . The smallest wavelength for which spectral estimates are computed is $2h$ and the largest wavelength for which the spectral estimates are computed is $2mh$.

As a general rule, we should select a sampling rate such that $2h \leq L_{min}$ where L_{min} is the smallest wavelength component of interest, and a maximum lag m such that $2mh \approx L_{max}$, where L_{max} is the largest wavelength component of interest. The record length, L_r , should be no less than $10mh$ if the percentage error of the estimated spectrum is to be of reasonable size (Parzen³).

For a given bed form, such as dunes, quite satisfactory results have been obtained by selecting mh approximately equal to the mean dune length, by sampling at an interval h so that the average dune length is represented by 20 to 30 data points, and by selecting a length of record 10 to 20 times the mean dune length. If ripples are superposed on the backs of dunes, a sampling interval h of about one-fourth of the mean

ripple length is recommended. Generally, the contribution to the total variance of wave-number components greater than four cycles per foot was negligible in the case of ripples. For dunes, the contribution to the variance from components greater than one cycle per foot was negligible.

Where large alternate bars with smaller bed forms superposed are known to exist in a channel, a somewhat different procedure is called for. The record should be sufficiently long to cover ten or a dozen of the bars if the properties of the bars are of interest. However, to investigate the properties of the smaller features, it is suggested that short segments of the longer record be analyzed after trend removal. Methods for removing trends are given in Bendat and Piersol⁴ and Parzen.⁵

RECORDS FOR ZERO AND *h*-LEVEL CROSSING ANALYSIS

It has been shown by Tick and Shaman⁶ that the expected number of maximums and minimums of y and the expected number of zero and h -level crossings determined from the discrete approximation to a continuous Gaussian process in a given length of record are always less than the expected number determined from the continuous record. If the sampling interval h is selected such that $4h \leq L_{min}$, the estimated number of crossings from the discrete process will be at least 90 percent of the number in the continuous process for any level h within two standard deviations of the mean. Therefore, a sampling rate at least twice that recommended for spectral analysis is required for determining the average number of zero or h -level crossings.

Much longer records are required to determine the probability distributions of maximum and minimum y values between zero crossings, of the lengths between zero crossings, and of the durations of the positive excursions of y above the level h than are required for the spectral analysis. The writer recommends at least one hundred observations of these values to obtain reasonable estimates of the distributions. As a rule of thumb, the records should be ten times as long as the records for spectral analysis, and if consideration is to be given to h -level crossings at levels beyond one and one-half times the standard deviation from the mean, for ripples or dunes, the record should be of the order of 300 times the mean ripple or dune length. Because the ripples and dunes are not symmetric about the mean bed elevation and because the y values are not strictly Gaussian in distribution, no hard-and-fast rules can be established for the record lengths to determine the distributions of the h -level crossings. The probability distributions of maximum and minimum y values between zero crossings, as shown in figures 15-17 of the text, are the logical starting place to estimate record length requirements for a given number of observations of these extreme events.

⁴Bendat, op. cit., 390 p.

⁵Parzen, loc. cit.

⁶Tick, I. J., and Shaman, Paul, 1966, Sampling rates and appearance of stationary Gaussian processes: Technometrics, v. 8, no. 1, p. 91-106.

³Parzen, Emanuel, 1967, Time series analysis papers: San Francisco, Holden-Day, 565 p.