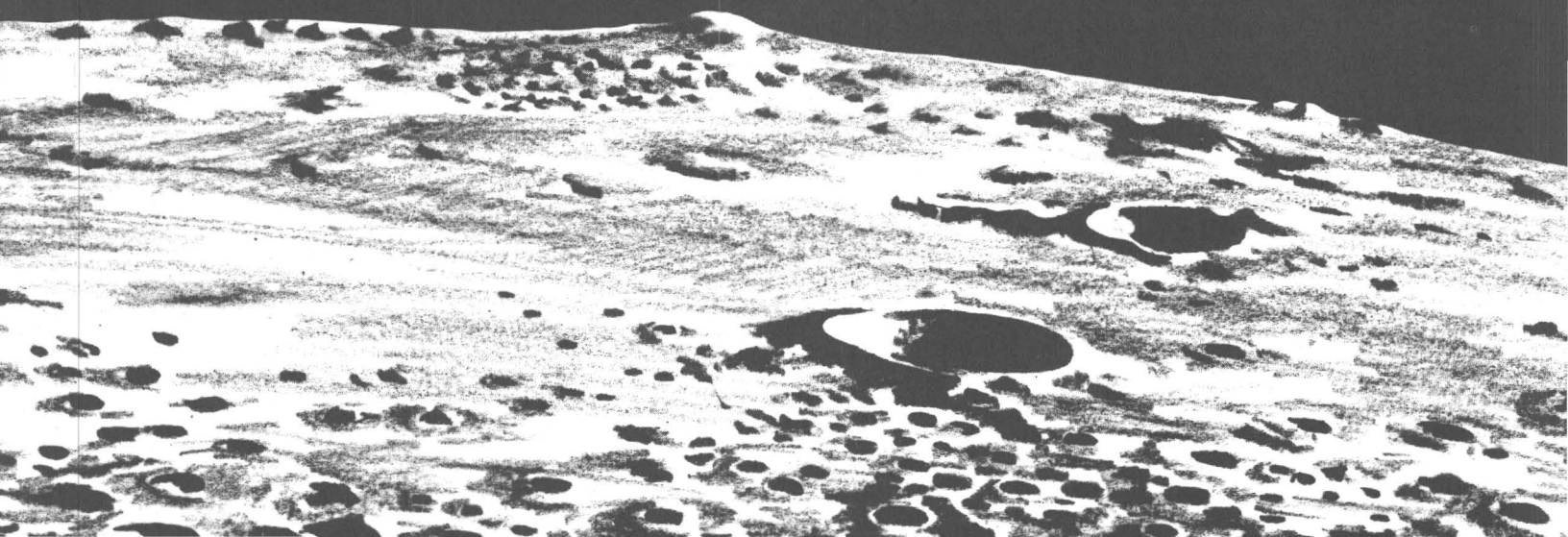


# DETECTION OF SH-TYPE SEISMIC SHEAR WAVES BY MEANS OF ANGULAR ACCELEROMETERS

CONTRIBUTIONS  
TO  
ASTROGEOLOGY



GEOLOGICAL SURVEY PROFESSIONAL PAPER 599-D





# Detection of $SH$ -Type Seismic Shear Waves by Means of Angular Accelerometers

By JAMES H. WHITCOMB

CONTRIBUTIONS TO ASTROGEOLOGY

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GEOLOGICAL SURVEY PROFESSIONAL PAPER 599-D

*A method for detecting and recording seismic  
shear waves by means of angular accelerometers*



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## CONTENTS

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	Page
Abstract .....	D1
Introduction .....	1
Theory .....	1
Field trials .....	2
Conclusions .....	5
References .....	6

## ILLUSTRATIONS

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	Page
FIGURE 1. Response curve of Statham angular accelerometers to a constant sinusoidal acceleration .....	D3
2. Recordings of the angular accelerometer AA 17 on the Kaibab Limestone test site .....	3
3. Time-distance plot of <i>P</i> - and <i>SH</i> -wave arrivals corresponding to data of figure 2 .....	3
4. Recordings of the angular accelerometer AA 14 on the Kana-a lava flow test site .....	4
5. Time-distance plot of <i>P</i> - and <i>SH</i> -wave arrivals corresponding to data of figure 4 .....	4
6. Recordings from linear array of six AA 20 and eight vertical sensors at Amboy flow test site .....	5
7. Time-distance plot of <i>P</i> - and <i>SH</i> -wave arrivals corresponding to data of figure 6 .....	5
8. Three-component geophone recordings at the Amboy flow test site .....	6



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### DETECTION OF *SH*-TYPE SEISMIC SHEAR WAVES BY MEANS OF ANGULAR ACCELEROMETERS

By JAMES H. WHITCOMB

#### ABSTRACT

An unsuccessful attempt to record *SH*-type seismic shear waves by means of triaxial geophones led to the development of a method for recording *SH* waves with angular accelerometers. Maximum angular acceleration in a homogenous medium is

$$\ddot{\phi}_{\max} = \frac{A \omega^3}{c}$$

where  $\ddot{\phi}$  is the maximum angular acceleration due to sinusoidal shear waves,  $A$  is the maximum displacement amplitude,  $\omega$  is the frequency (radians per second), and  $c$  is the shear-wave velocity. Tests of three types of Statham angular accelerometers are reported. The accelerometers are coupled to the ground at two points in line with the direction of wave propagation.

In areas where *SH*-waves are difficult to record, such as in areas of lava flows, clastic rocks, or volcanic ash, angular accelerometers are more effective than triaxial geophones because they discriminate against energy of *P*, *SV*, and Rayleigh waves. The angular accelerometer technique could be used on the moon for determining the *SH*-wave velocity of lunar surface materials. Before the method is applied to lunar studies, however, the relation of the *SH*-wave velocity to the physical parameters of the transmitting material needs further study.

#### INTRODUCTION

Shear-wave studies of near-surface in situ terrestrial materials may provide elastic parameters that would aid in the description and identification of both terrestrial and lunar near-surface geologic materials. Shear-wave velocity, compressional-wave velocity, and density uniquely determine the theoretical elastic moduli (Poisson's ratio, shear modulus, Young's modulus, bulk modulus, and the Lamé constants), and these moduli, if they were cataloged for terrestrial materials that are possible lunar analogs, could be compared with lunar elastic moduli as an aid in identifying lithology and structure of lunar rocks. However, one of the parameters, shear-wave velocity, is difficult to measure in certain types of geologic materials.

White, Heaps, and Lawrence (1956), Cook (1965), and Nicholls and Hooker (1962) successfully recorded *SH* (horizontal-plane polarized shear) waves by using triaxial detectors; they generated *SH* energy using,

respectively, a swinging weight, a plate charge against a trench wall, and buried explosives. The author used these and other methods in an attempt to record *SH*-waves with triaxial geophones on limestone outcrops, volcanic ash, aa lava flows, and blocky lava flows. The recordings were made with spreads less than 80 meters long; this distance corresponds to a maximum penetration depth of about 15 meters. The *SH* wave received by the transverse geophone was immersed in a large-amplitude *P*-wave signal which prevented a consistent pick of the *SH*-wave arrival time. Similar effects were seen by Markus Båth (oral commun., December 1966) using triaxial geophones to record a mine explosion near Kiruna, Sweden, in 1949. Båth found that, even at distances of 9 kilometers, the amplitude of the *P* wave recorded by the transverse geophone was nearly equal to the vertical and in-line amplitudes.

Thus, near-surface refraction studies required a technique that was insensitive to *P* waves so that *SH* waves could be detected. The development of such a technique is described in this paper.

#### THEORY

Shear-waves can be divided into two groups according to their orientations: *SH* (horizontal-plane polarized) and *SV* (vertical-plane polarized). Other orientations perpendicular to the propagation direction are possible, but they can be completely described by the combination of *SH* and *SV*.

*SH* waves were chosen for study because of the complications that *SV* waves present if velocity layering is present. For example, *SV* waves are readily converted into *P* waves, and vice versa, when they impinge on horizontal velocity boundaries. As a result, several combinations of converted waves may arrive at nearly the same time, making detection of *SV* waves difficult. Another problem is that the geophone orientation needed to measure *SV* is also favorable for recording Rayleigh or "ground roll" surface waves, whose velocity is close to the *SV*-wave velocity. Although *SV*-wave velocity can readily be found from Rayleigh- and *P*-

wave velocities in the case of a homogeneous and isotropic half space (Knopoff, 1952), simple half-space structures are uncommon for the terrestrial surface because of weathering and sedimentary layering. The layered velocity structures introduce dispersion into the Rayleigh wave and make inversion from Rayleigh-wave dispersion curves back to *SV* velocity too time consuming in regular fieldwork.

In contrast, *SH* waves are not converted at horizontal boundaries, and detectors oriented to measure *SH* will be insensitive to ideal *P*-, *SV*-, and Rayleigh-wave motion. Vertical or steeply sloping boundaries can cause conversion of *SH* to *P* waves, but they are much less common than horizontal boundaries near the surface. Love-type surface waves can be generated in layered materials, and their motion is similar to that of *SH* waves. Thus, Love waves are readily detected by *SH*-oriented detectors. However, before the direct shear wave reaches the critical refraction distance of typical single-layered structures, direct *SH* arrivals predominate and Love waves have not yet formed (Knopoff, 1958). Beyond the critical refraction distance, the refracted *SH* wave arrives earlier than the first Love arrivals.

If the mathematical operation "curl" is applied to the body-wave equation of elastic motion (Bullen, 1963, p. 73), the terms associated with the dilatational or *P*-wave motion drop out leaving only the equivoluminal or shear-wave terms. Consider a *SH* wave being transmitted through a three-dimensional medium in the *x*-*y* plane. The displacement ( $\underline{\psi}$ ) of the *SH* wave is represented by the vector

$$\underline{\psi} = u\underline{i} + v\underline{j} \quad (1)$$

where *u* and *v* are displacements in the *x* and *y* directions, respectively, and  $\underline{i}$  and  $\underline{j}$  are unit vectors in the *x* and *y* directions, respectively. The three-dimensional curl of  $\underline{\psi}$  gives

$$\nabla \times \underline{\psi} = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \underline{k} \quad (2)$$

where  $\nabla$  is the del operator and  $\underline{k}$  is the unit vector in the *z* direction.

If the direction of the *SH*-wave arrival is known, as it is with artificial sources, equation 2 can be simplified. Suppose that the *SH*-wave travels only along the *y* axis. Then, *v*, and thus  $\partial v / \partial x$ , would be zero in equation 2, and one need measure only  $\partial u / \partial y$  to get the curl. Thus, if a sensor is coupled along the *y* axis at two closely spaced points compared with the widely spaced wavelength, and the displacements, *u*, are small, then  $\partial u / \partial y$  can be considered a measure (in radians) of the rotation of the vector connecting the two coupling points. Consider a sinusoidal *SH* wave traveling along

the *y* axis. The displacement is

$$u = A \sin \left( \frac{\omega y}{c} - \omega t \right) \quad (3)$$

where *A* is the maximum displacement amplitude,  $\omega$  is the frequency, *c* is *SH*-wave velocity, and *t* is time. The rotation due to the sinusoidal *SH* wave will be

$$\phi = \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left[ A \sin \left( \frac{\omega y}{c} - \omega t \right) \right] = \frac{A\omega}{c} \cos \left( \frac{\omega y}{c} - \omega t \right). \quad (4)$$

Rotational velocity will be

$$\dot{\phi} = \frac{A\omega^2}{c} \sin \left( \frac{\omega y}{c} - \omega t \right), \quad (5)$$

and rotational acceleration

$$\ddot{\phi} = -\frac{A\omega^3}{c} \cos \left( \frac{\omega y}{c} - \omega t \right). \quad (6)$$

Maximum acceleration will therefore be

$$\ddot{\phi}_{\max} = \frac{A\omega^3}{c} = \frac{8A\pi^3\nu^3}{c}, \quad (7)$$

where  $\nu$  is frequency in cycles per second.

Values of rotational acceleration to be expected in near-surface seismic refraction work with short spreads have been tabulated in a preliminary report on the angular accelerometer method (Whitcomb, 1966). These values, on the order of 1 rad per sec<sup>2</sup> (radian per second squared), were calculated using equation 7 and three-component geophone data for amplitude, frequency, and velocity; the data represent *SH* waves which have traveled through volcanic rocks and limestone.

#### FIELD TRIALS

A family of angular accelerometers is available (Statham Instruments, Los Angeles, Calif.) that measures rotation around a single axis. The basic design of the accelerometer is a strain-gauge-supported paddle immersed in a fluid (oil) which is the inertial element. These accelerometers have a low response to linear acceleration in all directions and to angular acceleration in any direction other than the sensitive axis. Their response curves are similar to the one shown in figure 1; they differ only in their natural frequency,  $\nu_n$ , and sensitivity. Damping of the accelerometers is near 0.7 of critical.

A single angular accelerometer, the AA 17, was obtained for preliminary field trials. The AA 17 has a natural frequency of approximately 7 cps (cycles per second) and a response of 11.1 mv per rad per sec<sup>2</sup> (millivolts per radian per second squared) with less than 0.05 mv per *g* (*g*=980 cm per sec<sup>2</sup>) response to linear acceleration.

An area north of Flagstaff, Ariz., was chosen for testing the AA 17. The Kaibab Limestone of Permian age is exposed at the test site, and the formation there



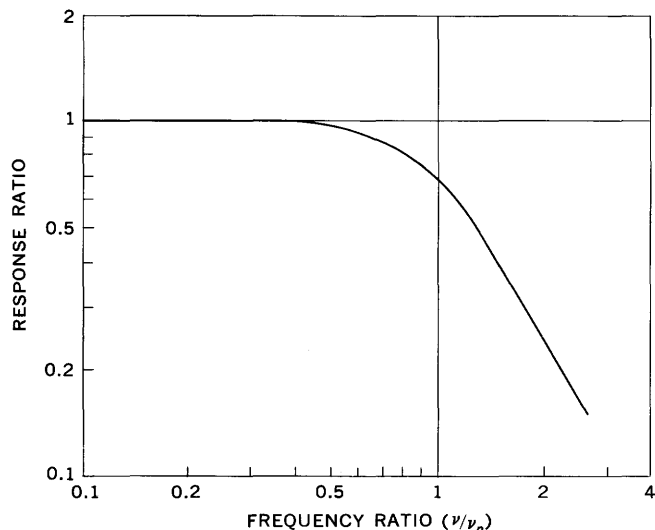


FIGURE 1.—Response curve of Statham angular accelerometers to a constant sinusoidal acceleration. The natural frequency is  $\nu_n$ ; the damping ratio is about 0.7 of critical. (From Statham Instruments, Inc.)

consists of horizontal beds of limestone, sandstone, and dolomite totaling approximately 24 meters in thickness. The AA 17 was placed at successive 10-meter intervals for a series of shots made at a common point, and a composite record section was prepared from these recordings (fig. 2). A small buried explosive generated adequate *SH*-wave energy, as was found by Nicholls and Hooker (1962). Ticks (fig. 2) indicate arrival times of the *P* and *SH* waves. Conventional vertical geophones recorded arrival times of the *P* wave. Past the fourth trace, the beginning of the *SH* wave was difficult to identify. This difficulty may have been caused by the sources or the transmission paths which varied the character of each trace. A time-distance plot of *P*-

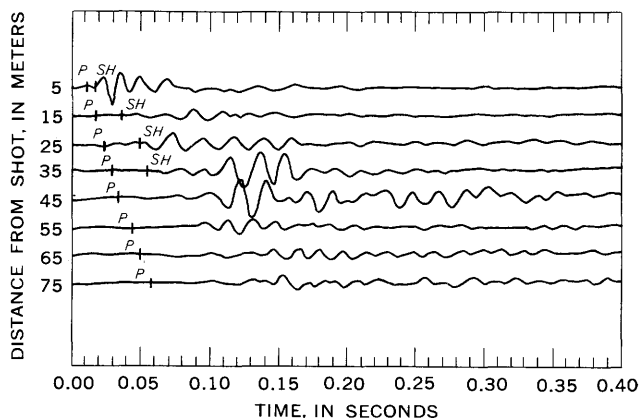


FIGURE 2.—Recordings of the angular accelerometer AA 17 on the Kaibab Limestone test site. The AA 17 was placed at successive 10-meter intervals for a series of shots made at a common point. The *P* arrivals included for reference were recorded by conventional geophones.

and *SH*-wave arrival times is shown in figure 3, in which near-surface and refracted velocities are shown corresponding to lines connecting the data points. These lines are dashed because the irregularity of the arrivals did not conform to a linear plot predicted by a simple layer over a half-space structure. Depth calculations from the *SH*-wave refraction indicate a high-velocity boundary at 11 meters, and *P*-wave refraction data indicate a high-velocity boundary at 13 meters. Cores from a drill hole on this spread have been analyzed (Watkins, 1967, p. 267) to give the following velocity data:

Depth (meters)	$V_p$ (meters per sec)	$V_s$ (meters per sec)
2.5.....	5,800	3,400
5.0.....	4,800	2,900
7.2.....	2,700	1,700
8.3.....	4,500	2,800
11.2.....	5,600	3,300
15.2.....	3,700	2,400
17.9.....	3,900	2,300
18.7.....	2,600	1,800
22.3.....	3,900	2,300

In situ and core velocities do not match; however, the core data are useful inasmuch as they indicate velocity layering. The core data confirm the *SH*-wave calculations by showing a high-velocity layer at a depth of about 11 meters, but the velocity layering is complicated by high values at the surface and lower velocities

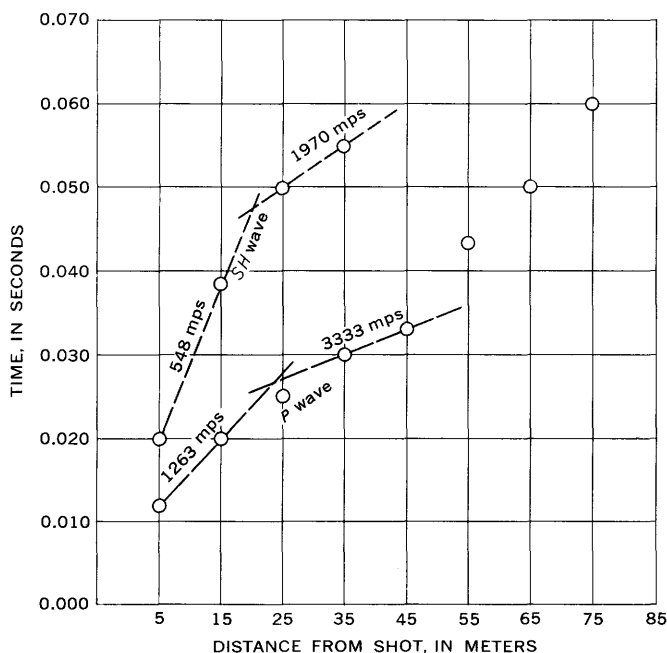


FIGURE 3.—Time-distance plot of *P*- and *SH*-wave arrivals corresponding to data of figure 2. Near-surface and refracted velocities are shown as dashed lines connecting the data points. Velocity is given in meters per second.

at 15 meters and below, a reversal of the normal layer over a half space refraction structure. This reversed structure is a possible explanation for the irregular *P*-wave arrivals shown in figure 3.

At the next field trial, a different angular accelerometer, the AA 14, was tested. The AA 14 has a natural frequency of approximately 15 cps and a response of 1.4 mv per rad per sec<sup>2</sup> with less than 0.025 mv per g response to linear acceleration. The test was conducted on the Kana-a lava flow, near Sunset Crater, about 15 miles north of Flagstaff, Ariz. The flow is a recent basaltic aa lava that is covered by loose volcanic ash and cinders. The test site was chosen to approximate a single layer (volcanic ash) over a half space (volcanic flow) structure. Eight separate shots and recordings were made at the same shot point, and after each shot the accelerometer was moved 5 meters farther from the shot point. Figure 4 shows a composite of the eight recordings. Air-wave velocity above the upper ash and cinder layer was slightly greater than *P*-wave velocity in the layer, and as a result, a higher amplitude air blast masked the *P*-wave arrivals. Air-blast and *SH*-wave arrivals are indicated in figure 4. Also recorded was a slower arrival that was probably the develop-

ment of a trapped wave. Figure 5 shows a time-distance plot of the data in figure 4. The velocity lines are drawn

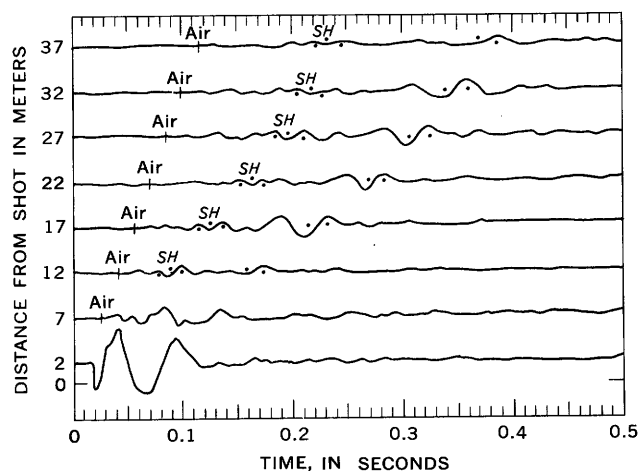


FIGURE 4.—Recordings of the angular accelerometer AA 14 on the Kana-a lava flow test site with the AA 14 placed at successive 5-meter intervals. Waves were generated by the explosion of one buried blasting cap. High amplitude air blasts masked the *P*-waves. Slower wave arrivals (indicated by two dots) are probably a trapped wave.

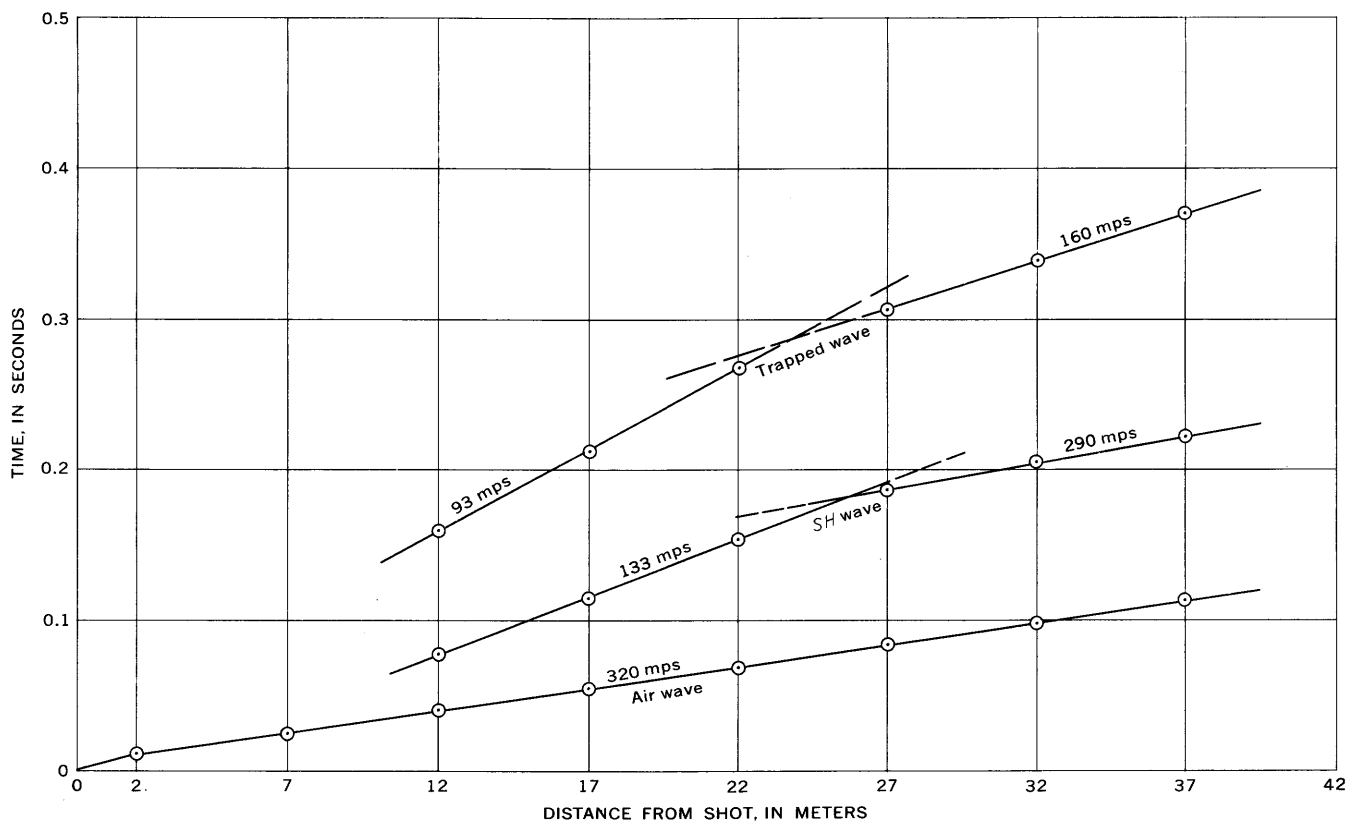


FIGURE 5.—Time-distance plot of *P*- and *SH*-wave arrivals corresponding to the data of figure 4. Trapped wave is also plotted.

through the data by eye. According to *SH*-wave calculations, the refracting layer, which is assumed to be horizontal, is about 7 meters below the surface. Subsequent drilling near the site indicated that the top of the basaltic lava is about 7.6 meters deep at the spread.

The successful completion of the preliminary tests with the AA 17 and AA 14 led to the construction of a linear array of eight AA 20 angular accelerometers for regular field use. The AA 20 has a natural frequency of

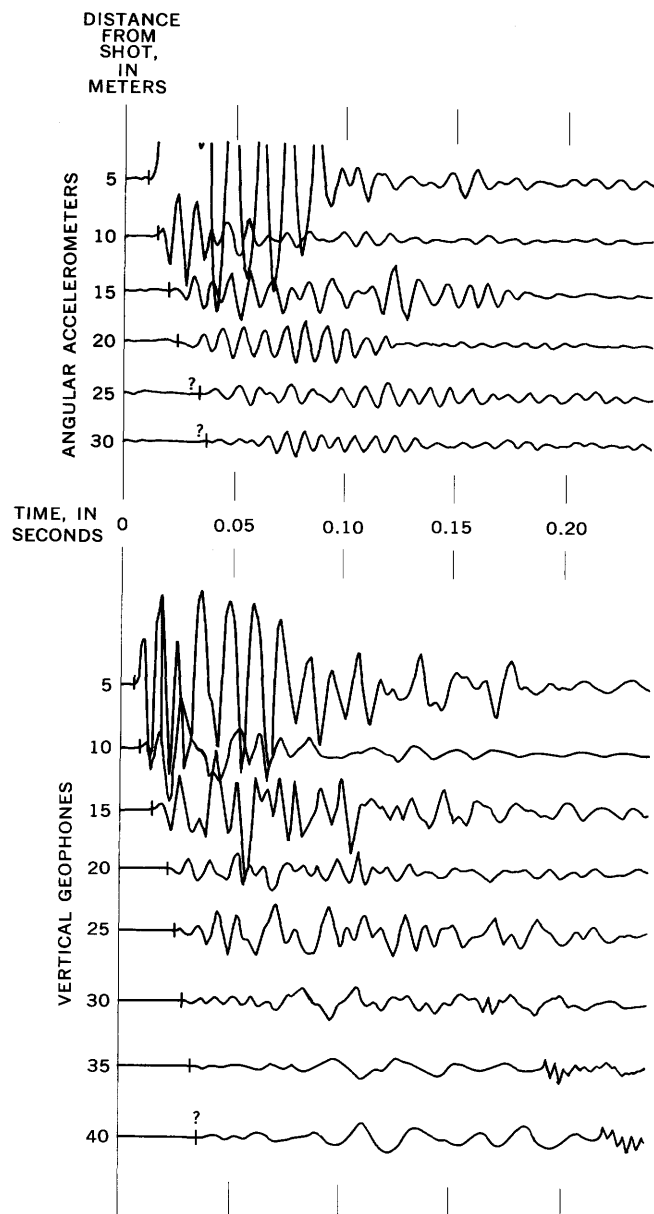


FIGURE 6.—Recordings from linear array of six AA 20 and eight vertical sensors at Amboy flow test site. Group spacing was 5 meters. Waves were generated by explosion of a buried blasting cap.

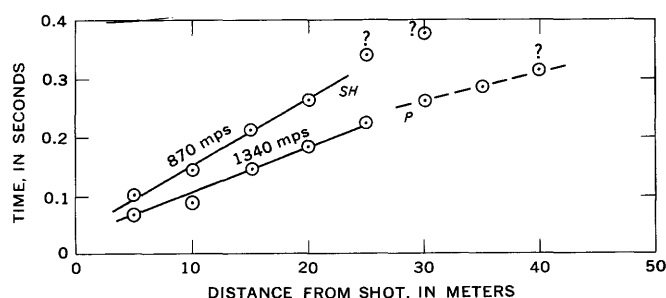


FIGURE 7.—Time-distance plot of *P*- and *SH*-wave arrivals corresponding to the data of figure 6. Questionable readings are indicated by circles with question marks. Velocity is in meters per second.

about 80 cps and a sensitivity of 0.106 mv per rad per sec<sup>2</sup> with less than 0.01 mv per g sensitivity to linear acceleration. A record from six AA 20 (the last two traces of the eight were lost in recording) and eight vertical sensors is shown in figure 6. The recording was made at the Amboy flow test site, California, on a pahoehoe lava at least 17 meters in thickness (Watkins, 1967). Figure 7 shows traveltime readings of *SH* and *P*. The direct *SH* wave was followed to only 20 meters because of rapidly diminishing amplitude and the possible emergence of a refraction arrival. The velocities from the direct arrivals, 870 mps (meters per second) for *SH* and 1,340 mps for *P*, give a Poisson's ratio of 0.14 for the surface material. For comparison, a recording from three-component geophones is shown in figure 8. This record was made at the same Amboy spread with an explosive source but with larger geophone spacing. It is evident from this recording that the transverse geophones did not separate the *SH* wave from arrivals as expected from theory.

#### CONCLUSIONS

Although some investigators have recorded *SH* waves with linear triaxial geophone systems, the technique fails in areas of limestone, volcanic rocks, or ash because of immersion of the *SH* wave in earlier arriving waves on the transverse geophone. The shortness of spreads (<80 m) did not permit the separation of the *P*- and *SH*-wave trains, which usually aids in the picking of *SH*. Angular transducers separate *SH* from *P* motion by measuring the curl of the ground motion.

High *SH* attenuations, measured by White, Heaps, and Lawrence (1956), make it difficult to follow *SH* waves on many records. The author was unable to measure *SH* attenuations because the *SH*-wave pattern was commonly nonuniform from trace to trace. This lack of uniformity may result from alteration of the wave shape by the accelerometer, by the wave path, or by the accelerometer coupling.

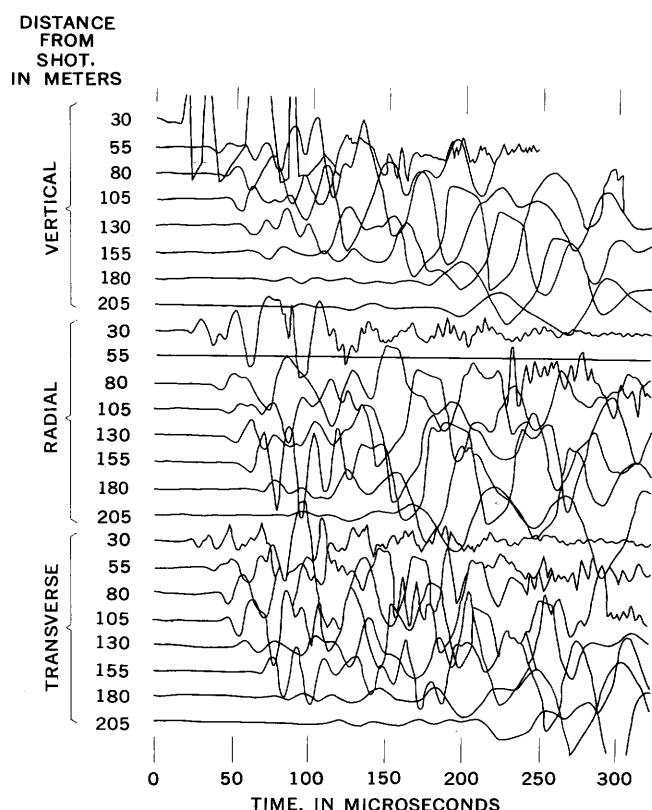


FIGURE 8.—Three-component geophone recordings at the Amboy flow test site.

Coupling is an important factor in measuring the curl, which is a space derivative of the ground motion. Ideally, the transducer should be connected to a part of the medium where the elastic properties are space continuous; in other words, the medium should be unbroken between the two coupling points. Also, in order that the shear strain may be transmitted from the propagating wave to the area where the transducer is coupled, the elastic properties of the material surrounding the transducer must be continuous. Blocky volcanic flows do not meet these requirements. However, in the lava flows on which the tests were conducted, the distance between fractures is much smaller than the wavelengths, and the fractures are assumed to be an intrinsic part of the medium's properties which affect velocity.

Calculation of Poisson's ratio for the velocities in figure 3 gives 0.38; for the velocities in figure 7, 0.14. It is logical to ask if these values are reasonable for the type of material they represent. Walsh (1965) gives at least a partial answer of what to expect; he shows

that almost any of the theoretically possible values of Poisson's ratio can be measured in a given rock; the values depend on the number of cracks in the rocks and on the confining pressure. Thus, considering the varying state of in situ materials, any value of Poisson's ratio within the theoretical limits is possible.

Because the elastic properties of a rock depend in part upon its fabric (Brace, 1965), the *SH* and *SV* velocities measured in the field may not be identical. However, because of the many different types of geologic structure and the lack of attention given the problem, little is known about the relationship between *SH* and *SV*. Therefore, if the elastic properties of a material are calculated from the *SH* wave, anisotropy must generally be assumed and the calculated properties must be considered as related only to the *SH*-wave orientation in the material.

The manner in which elastic waves propagate through shallow materials requires further study. The usual assumptions of elasticity and isotropy cannot be expected to hold for loosely aggregated and highly fractured materials in the low-pressure environment near the surface. This is not only shown by the work of Walsh (1965) and Brace (1965), but is substantiated by the common observation that in situ velocities and core velocities of the same materials are very different.

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