Bed Topography Inferred from Airborne Radio-Echo Sounding of Columbia Glacier, Alaska
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>Speed of light in air</td>
<td>(300 \text{ m/\mu s})</td>
</tr>
<tr>
<td>H</td>
<td>Airplane altitude above glacier surface</td>
<td>m</td>
</tr>
<tr>
<td>km</td>
<td>Kilometer.</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>Meter.</td>
<td></td>
</tr>
<tr>
<td>mm</td>
<td>Millimeter.</td>
<td></td>
</tr>
<tr>
<td>MHz</td>
<td>Megahertz.</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>Index of refraction of ice</td>
<td>dimensionless</td>
</tr>
<tr>
<td>q</td>
<td>Length of ice-segment of ray</td>
<td>m</td>
</tr>
<tr>
<td>R</td>
<td>Radius of curvature of reflection locus</td>
<td>m</td>
</tr>
<tr>
<td>r</td>
<td>Length of air-segment of ray</td>
<td>m</td>
</tr>
<tr>
<td>s</td>
<td>Second.</td>
<td></td>
</tr>
<tr>
<td>(\mu s)</td>
<td>Microsecond.</td>
<td></td>
</tr>
<tr>
<td>(t)</td>
<td>Time</td>
<td>(\mu s)</td>
</tr>
</tbody>
</table>

| \(t_h\) | Arrival time for echo from glacier bed | \(\mu s\) |
| \(t_s\) | Arrival time for echo from glacier surface | \(\mu s\) |
| u      | Horizontal distance from airplane position to refraction point | m |
| x      | Horizontal coordinate, positive to east | m |
| y      | Horizontal coordinate, positive to north | m |
| z      | Vertical coordinate, positive upward | m |
| \(\alpha\) | Coefficients of plane locally approximating glacier surface | dimensionless |
| \(\beta\) | Inclination of ice-ray from the vertical | degrees |
| \(\phi\) | Inclination of air-ray from the vertical | degrees |
ABSTRACT

The first airborne radio-echo sounding of a temperate glacier was performed in 1978 at Columbia Glacier, a large (1,100-square-kilometer), grounded, iceberg-calving glacier 38 kilometers west of Valdez, Alaska. The sounding system used a low frequency (about 1.5 megahertz) to overcome scatterings from water-filled voids in the ice, a short pulse, and an untuned receiver. Transverse and longitudinal profiles were flown over the lower 7 kilometers of the glacier. The received signal, the horizontal position of the airplane, and its altitude above the glacier surface were recorded by an FM tape recorder. For the data analysis, pictures of received energy from each flight profile were reconstructed from the taped data using an oscilloscope. Use of intersecting profiles allowed an internal consistency check to determine whether the correct bed reflection had been chosen. A three-dimensional geometric method of determining the envelope of the reflection lobes was developed for interpreting the data, instead of the differential method used by previous investigators. This analysis provided bedrock altitude determinations at every node of a 200-meter square grid. The probable error in the inferred bed altitudes was estimated to be 30 meters; the greatest depth was 370 meters below sea level.

INTRODUCTION

Columbia Glacier (fig. 1) is a large, grounded, calving glacier near Valdez, Alaska, the terminus of the trans-Alaska pipeline. In 1978 it was 66.6 km long and 1,100 km² in area. Many large tributaries coalesce to form the lower trunk of the glacier, which fills a deep fiord and ends in a grounded, calving terminus that stands up to 90 m above sea level. From the time of the first recorded observation in 1794 (Vancouver, 1798) until 1978, the terminus had been relatively stable, ending partly on Heather Island (fig. 2) and partly on a submerged moraine shoal in water less than 100 m deep. Upglacier from the terminus, however, the bed is nearly 400 m below sea level.

Nearly all grounded, iceberg-calving glaciers in Alaska, such as Columbia Glacier, have experienced large-scale, asynchronous advances and retreats; Columbia Glacier, however, was the only glacier that extended in 1978 to its neoglacial moraine. The large-scale advances and retreats of calving glaciers are due not solely to climatic variations; water depth at the terminus is a critical factor. When a glacier retreats into a deep basin or fiord, instability results and the calving flux increases (Post, 1975; W.O. Field, oral commun., 1980; Brown and others, 1982).

In 1974, Columbia Glacier appeared to be entering a period of retreat (Post, 1975; Sikonia and Post, 1980). Since 1977, the U.S. Geological Survey has documented an accelerated rate of thinning in the terminus region and a trend of increasingly larger embayments forming, as an even greater portion of the terminus retreats from the crest of the shoal each summer and fall. Once the glacier has backed off the moraine shoal into the deep water upglacier, rapid and irreversible retreat is expected to begin. In October of 1984, all of the terminus was 1–2 km behind the shoal in water averaging 120 m deep; all evidence indicates that the large-scale recession has begun.

A direct consequence of rapid retreat is an increasing flux of icebergs. Even prior to 1974, small icebergs from the glacier drifted toward and occasionally into the shipping lanes in Valdez Arm (Kollmeyer and others, 1977); these shipping lanes are used by tankers transporting oil from the Alaska pipeline terminal at Valdez.

To determine when this rapid retreat might begin, and by how much the iceberg discharge might be increased, an intensive study was begun by the Geological Survey in 1977 (Meier and others, 1978). A prediction was made (Meier and others, 1980) that the glacier would begin an irreversible retreat within a few years. Details of each phase of this study are being published as a series of scientific papers, of which this is one. The other papers discuss the relationship of calving speed to water depth and other variables (Brown and others, 1982); surface topography (Rasmussen and Meier, 1982); and bedrock topography (Brown and others, 1982).
Figure 1.—Index map of Columbia Glacier, Alaska. Arrows indicate direction of ice flow. Main ice stream is indicated by longer arrows and dots at 2-kilometer intervals along the longitudinal coordinate system. Boxed area is region with inferred bed topography from the airborne radio-echo sounding.
Glacier thickness is a principal variable in the equation of continuity, in the flow law, and in calving relationships; thus, it is crucial in any dynamic glacier model, such as those developed for predicting the behavior of Columbia Glacier. Thickness cannot easily be measured directly; drilling in the lower reach of Columbia Glacier is expensive, relatively slow, and prohibitively dangerous (fig. 3). Thickness, however, can be determined indirectly from the rather easily measured surface topography and an estimated bed topography. Currently, polar ice bodies are sounded by radio-echo methods. When it became necessary to know the thickness of Columbia Glacier, efforts to sound this
large, temperate glacier were begun. This was a formidable task, as radio-echo sounding techniques of temperate (nonpolar) ice bodies are relatively new and are still very much in the developmental stage. This paper outlines the development of radio-echo sounding methods for temperate glaciers, describes the collection and analyses of data in this first airborne radio-echo sounding of a temperate glacier, and explains how the bed topography, and thus the glacier thickness, were inferred from these data.

**RADIO-ECHO SOUNDING DEVELOPMENT**

Measurements of glacier thickness by radio methods were first attempted with some success in 1927 and 1928 (Evans, 1963c). Since the first reported radio sounding of polar ice in Antarctica in 1957 (Evans, 1963a), sounding of polar ice bodies by radio-echo methods has continued in Greenland and Antarctica (see, for instance, Evans, 1963b, 1967; Swithinbank, 1968; Robin and others, 1970; Morgan and Budd, 1975) and elsewhere. Recently, much emphasis has been placed on the interpretation of these returns (see, for instance, Harrison, 1970; Oswald, 1975; and Robin, 1975).

Research in applying radio-echo sounding methods to temperate glaciers first had to address the problems of sounding ice at the melting temperature, ice that is less homogeneous than polar ice. Smith and Evans (1972) determined that the difficulty of sounding temperate glaciers was due not to the high temperature, impurity content, or ice fabric but to the masking of the

---

**FIGURE 3.** Oblique aerial photograph showing the roughness of the Columbia Glacier surface. View is to the west, about 2 kilometers above the terminus. Average relief from summits to valleys is about 25 meters; local relief may range up to 40–50 meters. USGS photograph by M.F. Meier, August 14, 1984.
true bottom return by the diffuse return from the englacial scatterers, or inhomogeneities. Shortly thereafter, Watts and England (1976) correctly attributed these inhomogeneities to water-filled voids in the ice, determined that frequencies less than 10 MHz were needed to overcome the scatterings, and suggested that a short-pulse radar with an untuned receiver could be used at these low frequencies. This system differs from radars typically used for polar ice, which employ a modulated carrier signal and a tuned radio receiver. Vickers and Bollen (1974) introduced the technology of resistively loaded antennas, from which there is practically no reflection from the ends of the arms, to the radio-echo sounding of temperate glaciers. With this system, they successfully sounded depths of more than 200 m on South Cascade Glacier, a temperate glacier in the North Cascade Mountains of Washington, using 5 MHz frequency and then applied the system to Columbia Glacier (Vickers and Bollen, 1974). Since then, scientists at the University of Iceland developed a profiling system based on these principles, employing frequencies of 2 to 10 MHz, and used it successfully on the Vatnajökull and Myrdalsjökull icecaps (Ferrari and others, 1976; Björnsson and others, 1977; Sverrison and others, 1978; Bishop and others, 1979; Björnsson, 1982). The system was towed behind a motorized sled and was the first successful attempt at continuous profiling on the surface of a temperate glacier.

**THE AIRBORNE RADIO-ECHO SOUNDING SYSTEM**

**DEVELOPMENT OF THE SYSTEM**

Sounding work was begun on Columbia Glacier during the summer of 1974 (Vickers and Bollen, 1974), when approximately 70 surface radar and gravimetric spot measurements were made, all in the lower 30 km of the main trunk glacier. For the purposes of numerical modeling, however, data from a much denser network of points were necessary. A continuous profiling system could not be operated on the glacier surface because of the many locations rendered inaccessible by the highly crevassed surface (fig. 3).

On the basis of successful sounding of South Cascade and Columbia Glaciers with a surface system, Watts and Wright (1981) modified their surface monopulse generator for the airborne work; they were able to use transmitting and receiving antennas similar in principle to those used in the surface system but modified because they had to be fed in the center, yet could be attached to the airplane only at the ends. Because of these modifications and the proximity of the metal airplane, the emitted wave train contained about six cycles instead of the more desirable single sinusoid (Churchill and Wright, 1978).

An oscilloscope on board the airplane sampled the received signal and transformed it to an audio frequency that was recorded on a seven-track FM tape recorder. The oscilloscope also provided an output, recorded on another channel, that gave an absolute time correlation between echoes and their delay from the initial transmitted pulse. Photographs (fig. 5) were taken of an oscilloscope composite reconstruction made by playing back the magnetic tapes. Time delay from the original transmitted pulse is plotted vertically, and the horizontal position represents movement above the surface; brightness was modulated in proportion to received (echo) voltage.

**AIRPLANE POSITIONING**

The horizontal position of the airplane was recorded in the local \(x, y\) coordinate system devised for Columbia Glacier which is related to the geodetic UTM (Universal Transverse Mercator) system according to

\[
\begin{align*}
(x) &= \frac{1}{0.9996} \left( \text{UTM Easting—490 km} \right) \\
y &= \text{UTM Northing—6,750 km}
\end{align*}
\]  

(1)

The \(x,y\) position was determined by an automated navigation system using two microwave transponders located at known points off the edge of the glacier. It measured the times of the responses from the transponders, and a small computer on the airplane then calculated the position. The distances to the transponders, and thus the \(x,y\) position, were recorded every 0.5 s.

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The use of brand names in this report is for identification purposes only and does not imply endorsement by the U.S. Geological Survey.
Figure 4.—Flight paths and sounding positions from which the Columbia Glacier bed topography was inferred. Dots represent the sounding positions; their location corresponds to the point where the echo arrival times $t_b$ were read off the reconstructed profiles at approximately 5-millimeter intervals. If the maximum value of $t_b$ did not occur at one of these roughly equally spaced positions, its position was also included. Note the close proximity of the Boom Boom, Barefoot, and Easy ridges; contours are given in meters above sea level. The location of the region depicted is indicated by the box in figure 1.
FIGURE 5.—Reconstructed airborne radio-echo profiles across the lower part of Columbia Glacier: (A) profile at $y = 14.4$ kilometers, illustrating a reasonably good return; (B) profile at $y = 12.9$ kilometers, illustrating a poor return and the difficulties in determining $t_b$. Each profile is shown twice, once with and once without the surface return and the selected bed return identified.
Twelve east-west profiles and seven north-south profiles were flown (fig. 4). The east-west profiles are termed the “North family,” owing to their northward progression at 500-m increments from their baseline, and the north-south profiles are similarly termed the “West family.”

The airplane’s radar altimeter reading was also recorded every 0.5 s, giving \( H \), the altitude in meters above the glacier surface. This was the only altitude recorded on the tape. The altitude at which the airplane was to fly each profile was predetermined and logged prior to flight; this was the barometric altitude to the nearest 100 ft (30 m). The horizontal distance along the flight path from the beginning of each profile was also recorded for use in locating the photographed arrival times in real space.

**DATA ANALYSIS**

**SIGNAL EXTRACTION**

The value of \( t_b (x, y, z) \), the round-trip signal time to the bed from a particular airplane position, was measured manually from a photograph of an oscilloscope reconstruction of the profile. Figure 5 shows the reconstruction of both a good profile and a poor one; most were somewhere in between in quality. Obviously there were several difficulties to overcome before \( t_b \) could be measured. First was the problem of correlating the several bands (cycles) of the reflection with those of the transmitted pulse. Not only were there multiple bands, but each usually was not continuous across the profile. Sophisticated off-line signal processing could be used to compress these multiple bands, making it easier to distinguish the bottom echo. There also was the problem of the valley wall echoes masking returns from the glacier bed, particularly on the west edge of the glacier against the Boom Boom and Barefoot ridges (fig. 4). These valley wall returns likewise could have been removed by signal processing or eliminated at the outset by flying much closer to the glacier surface.

Owing to the electronic saturation of the airborne receiving system by the transmitted pulse, no echoes could be detected during the first several microseconds. Thus, variations in \( t_b \) were recorded but the absolute arrival time \( t_b \) of the echo from the surface was not recorded and the \( t_b \)-datum line, commonly referred to as the flightline, does not appear on the reconstructed profile pictures (the lines at the very top of the pictures are the tail end of the recovery from the saturation of the electronics). This flightline is necessary to measuring \( t_b \), and reconstructing it was difficult. To approximate its position on the picture, the surface reflection corresponding to the selected bed return first had to be determined. In many cases there was not a one-to-one correspondence between the several surface bands and the several bands from the bed return. Usually the brightest and widest band was chosen; from the internal consistency check (below) it could be determined whether the corresponding bed reflection had been chosen correctly. Two points then were plotted, one at each end of the profile above the selected surface band at a distance corresponding to the average altitude of the airplane above the surface. This distance was determined by knowing the number of microseconds per division on the oscilloscope and by knowing the signal speed in air. A line drawn between these points was used as the \( t_b \)-datum line. The error in using this \( t_b \)-datum line was determined to be well within the 30-m error in airplane altitude \( H \).

**INTERNAL CONSISTENCY**

The final phase of the data reduction was checking for internal consistency by comparing the two \( t_b \) values at each intersection of two profiles. Each \( t_b \) value was adjusted to account for possible differences in altitude at which the two profiles were flown. This was done by reducing each \( t_b \) by \( t_H \), the amount of time for the airwave to travel the distance \( H \):

\[
t' = t_b - t_H = t_b - 2H/c ,
\]

in which \( c = 300 \text{ m/\mu s} \) is the signal speed in air. The difference \( |t'_{N} - t'_{W}| \) between the reduced \( t_b \) values for the east-west profile and the north-south profile was calculated for each of the 80 profile intersections. Although this formulation embodies the strict assumption that the echo comes from the nadir, the difference \( |t'_{N} - t'_{W}| \) is little affected if the echo comes from elsewhere, so long as the echoes sensed on the two profiles come from approximately the same location. That is, the difference between the two nadir distances to the glacier surface is used to approximate the difference between the two slant distances actually traversed by the signal. Because the difference in \( H \) was generally small compared with \( H \) (fig. 6), this weaker assumption seems reasonable. As the error in measuring \( t_b \) on the photograph was a maximum of 0.45 \( \mu \text{s} \), this value was used as the maximum allowable difference value of \( |t'_{N} - t'_{W}| \). Sixty-three percent of the differences were actually less than 0.20 \( \mu \text{s} \). One difference was 0.46 \( \mu \text{s} \), for which there was no obvious explanation.

Matching \( t'_{N} \) and \( t'_{W} \) at each intersection provided a check for consistency between profiles. To check for consistency within a single profile, the band chosen as representing the bed reflection on the photograph from which one \( t_b \) was obtained had to be the same band used to measure the \( t_b \) values of the neighboring intersection points on the profile. At 5 (out of 138) profile intersec-
perhaps another 10 m. Therefore the aggregate error in the effect due to atmospheric pressure fluctuations, was referred to as the "average profile altitude," meaning flight is of the order of 10 m, to which must be added instrumental and reading error in altitude values during which it was predetermined the airplane should fly. It metric altitude rounded to the nearest 100 ft (30 m) at the reconstructed profiles. This altitude was the barometer used to position the two major sources of error. The first was in the altitude data acquisition and the data analysis, there were only from the surface makes this unlikely.

The good agreement between the directly measured bathymetry and the radar soundings consistent results by virtue of putting almost all the energy into a single band.

Apparent internal consistency would result if every \( t_b \) in every profile were misinterpreted in the same way. The good agreement between the \( t_b \) values and both the directly measured bathymetry and the radar soundings from the surface makes this unlikely.

ERROR IN ECHO ARRIVAL TIMES

Although there were difficulties involving both the data acquisition and the data analysis, there were only two major sources of error. The first was in the altitude used to position the \( t \)-datum line on the photographs of the reconstructed profiles. This altitude was the barometric altitude rounded to the nearest 100 ft (30 m) at which it was predetermined the airplane should fly. It was referred to as the "average profile altitude," measured above mean sea level. It is assumed that the instrumental and reading error in altitude values during flight is of the order of 10 m, to which must be added the effect due to atmospheric pressure fluctuations, perhaps another 10 m. Therefore the aggregate error in flying the predetermined altitude is assumed to be about 30 m. Thus, at any given point on a profile and, therefore, in the \( t \)-datum line, there was an error of \( \pm 30 \text{ m} \) in the altitude value used. This corresponds to \( \pm 0.20 \mu \text{s} \) on the photograph.

The second major source of error was in measuring \( t_b \) on the photographs. The bands chosen as representing the bed reflections on the profiles averaged 1 mm in width. Thus, this was the average amount by which the distance from the \( t \)-datum line to the chosen reflection could vary. By using the predetermined number of microseconds per division on the oscilloscope and the reduction factor between the oscilloscope screen and the photograph, the conversion factor on the photograph was calculated to be 0.30 \( \mu \text{s/mm} \) (microseconds per millimeter). Thus the 1 mm variation in measuring \( t_b \) corresponds to 0.30 \( \mu \text{s} \). The possible inaccuracy in measuring \( t_b \) on the photograph was less than 0.3 mm (0.09 \( \mu \text{s} \)) and is considered to be negligible in relation to the other, much larger errors.

Instrumental errors, including those in the radio-echo sounder, oscilloscope, and the transponders used to determine the \( x, y \) position of the airplane, were deemed to be insignificant compared with the other errors. Thus, if the two sources of error are considered to be independent, the error in \( t_b \) is \[ \sigma^2 = (0.20 \mu \text{s})^2 + (0.30 \mu \text{s})^2 = 0.36 \mu \text{s} \]. This error results directly in error in glacier thickness; however, as will be shown, the two are not strictly proportional. If the echo is assumed to come from the nadir, then 0.36 \( \mu \text{s} \) corresponds to 30 m.

INTERPRETATION OF AIRBORNE RADIO-ECHO SOUNDING DATA

The interpretation of the radio-echo sounding data is based on simple, known geometric principles, which are briefly reviewed here. Although they are ultimately applied three-dimensionally, they are initially described in two dimensions for ease of presentation. The simplicity of the mathematical model follows from several assumptions concerning the physics of the signal propagation and of the composition of the glacier, as well as recognition of the low quality of the original data.

Signal strength is assumed to be unreliably known, so that echo arrival time is the only variable used in inferring the distance traveled by the signal. The signal is assumed to be omnidirectional, so that no direction is ruled out as a candidate from which an echo may be received. Because of the uncertainty with which arrival times \( t_b \) were read from the reconstructed profiles, a simple geometric method of determining the envelope of the reflection lobes was selected for interpreting the
data instead of the differential method used by Har­rison (1970), which is based on the rate of change of the signal arrival time as the airplane moves along its flight path. The signal is assumed to travel in air at the rate of \( c = 300 \text{ m/\mu s} \) (meters per microsecond); within the glacier it is \( \frac{1}{n} \) times this rate, where \( n = 1.78 \) is taken to be the index of refraction of ice.

Although the glacier surface is highly irregular and fractured (fig. 3), the mathematical model assumes a much smoother, average surface at which refraction follows Snell’s law (fig. 7):

\[
\sin \phi = \frac{1}{n} \sin \theta .
\]

In fact, it assumes a piecewise planar surface between given altitudes on the nodes of a 200-m square grid. These were obtained by applying the method of optimum interpolation to photogrammetrically determined altitudes of irregularly positioned points appearing on a sequence of vertical aerial photographs taken on August 26, 1978, from about 6,800 m above the glacier surface (Rasmussen and Meier, 1985). This surface topography is represented in figure 8 by a manual contouring of the altitudes on the 200-m square grid. Because the soundings were taken in late August over the part of Columbia Glacier having surface altitudes less than 300 m above sea level, no correction—such as that used by Harrison (1970)—was applied to the passage of the signal through a firn layer, which would have an \( n \)-value smaller than that for ice. Another assumption that is made is that the arrival times that are used are from echoes occurring only at the glacier bed, rather than from englacial bodies of liquid phase water.

Change of airplane position during the period between signal transmission and reception is neglected. The 100-kn airplane speed (Watts and Wright, 1981) produces only 1 mm of displacement in 20 \( \mu \text{s} \), a time that exceeds all signal arrival times obtained over Columbia Glacier. The 1 s consumed by the sampling oscilloscope in constructing a complete sweep (R.D. Watts, written commun., 1985) corresponds to a displacement of about 50 m.

REFRACTION AND REFLECTION REPRESENTED IN TWO DIMENSIONS

The dependence of the ice angle \( \phi \) on the air angle \( \theta \) that arises from using \( n = 1.78 \) in equation 3 is shown in figure 9. This relationship gives a maximum value of \( \phi = 34.18^\circ \) when \( \theta = 90^\circ \), and the slope \( d\phi/d\theta \) decreases continuously from \( \frac{1}{n} \) at \( \theta = 0^\circ \), to 0 at \( \theta = 90^\circ \).

The distance equation for an echo with arrival time \( t_b \) is

\[
ct_b = 2 (r + nq) ,
\]

in which \( r \) is the length of the air leg and \( q \) is the length of the ice leg (fig. 7). It is easily shown that the ray that obeys Snell’s law is the one that minimizes the travel time between \((x,z)\) and \((0,H)\).

A parametric representation of the \( x,z \) locus is easily obtained from equations 3 and 4 in terms of \( \theta \):

\[
\begin{align*}
x &= \frac{1}{n^2} \left[ (n^2 - 1)H \frac{ct_b}{2} \sin \theta \right] \\
z &= \frac{1}{n^2} \left[ \frac{H}{\cos \theta} \frac{ct_b}{2} (n^2 - \sin^2 \theta)^{1/2} \right]
\end{align*}
\]

This formulation is valid only for an idealized glacier surface lacking curvature. Although it is not possible here to give a closed-form expression \( z = f(x) \) for the locus, equations 5 may easily be transformed so that...
Figure 8.—Surface topography of lower part of Columbia Glacier on August 26, 1978. The location of this region is indicated by the box in figure 1. The contour interval on the glacier is 10 meters, with occasional 5-meter dashed contours. Datum is sea level.
the parameter is $r$ or $u$ or the locus slope $dz/dx$. Half of each of the $x,z$ loci for three different airplane altitudes $H=0, 200$, and $800$ m is shown in figure 10; all three loci are scaled to have the same $z=-393$ m depth (corresponding to $t_b=10$ µs when $H=800$ m, which is typical of the Columbia Glacier data) at $x=0$. The $H=0$ locus is constructed on the assumption that the antennas are in dielectric contact with the ice and, therefore, that no refraction occurs.

The $x,z$ curve is strictly just a locus, in the sense that any point on it could be the source of the echo having the arrival time that determines the curve. If several loci were available from closely spaced airplane positions, then a glacier bed could be inferred from their envelope. No part of the bed can be above the envelope because, for one or more of the observation positions, an echo would have been received with an earlier arrival time than the one from which the locus was constructed. Several such loci, and a bed profile that is consistent with them but that is not uniquely deter-

mined by them, are shown in figure 11. Any other curve that was tangent to each of the several loci, and that nowhere rose above their envelope, might also be the bed profile.

Glaciological and geological knowledge must influence the bed inference, for radio-echo sounding can only impose some constraints on the bed topography, not determine it exactly. For example, the envelope itself is consistent with all the data; however, on those grounds, the locus-intersection cusps would not be interpreted as bed features. A dense array of loci, from a continuum of observation points along the flight path, would eliminate these cusps, but it would still yield only an upper-bound constraint on the actual bed topography.

The locus (eqs. 5) may be differentiated to give its slope, a convenient expression for which is

$$
\frac{dz}{dx} = \tan \phi = \sin \theta \left( n^2 \sin^2 \theta \right)^{-1/2}.
$$

The maximum value $dz/dx=0.679$ occurs at the maximum value $\phi=34.18^\circ$. A second differentiation yields
the radius of curvature, which may be written in the form

\[ R = \left[ 1 + \left( \frac{dz}{dx} \right)^2 \right] \left\{ nH \left[ 1 - (n^2 - 1) \left( \frac{dz}{dx} \right)^2 \right]^{-3/2} \right\} - z. \quad (7) \]

At \( x = 0 \), where \( \frac{dz}{dx} = 0 \), the radius is \( R = nH - z \). This is significant because the radius increases at the same rate as the depth increases, but it increases \( n \) times faster than the airplane altitude increases. For a surface sounding \((H = 0)\), the radius of curvature at \( x = 0 \) equals the depth. The ratio of these two, \( R(0, z)/R(H, z) \), as a function of the ratio of the airplane altitude to the depth, is shown in figure 12. The radius of curvature may also be considered a function of the slope at some particular depth. The ratio \( R(H, z, 0)/R(H, z, \frac{dz}{dx}) \) as a function of the slope is shown in figure 13 for selected values of the ratio of the airplane altitude to the depth. These two diagrams demonstrate that as airplane altitude increases, the radius of curvature of the locus increases very rapidly—and so does the smallest scale of bed features that can be inferred from the data.

Errors in the calculation of the bed altitude can be related to errors in the original data. It is easily seen from equations 5 that at \( x = 0 \) and error of \( \delta z = 8.43 \) m
Figure 13.—Effect of the locus slope on the radius of curvature of the reflection locus \( R \). Two reflection loci at depth \( z \) are both from airplane altitude \( H \) but from different airplane positions, so that one has locus slope \( \frac{dz}{dx} \) and the other has zero slope. The radius-of-curvature ratio is given as a function of slope for selected \( H-/|z| \) ratios, including the special \( H=0 \) case of surface soundings (eq. 7). A horizontal glacier surface is assumed.

Figure 14.—Effects of errors in echo arrival time and airplane altitude. For the three reflection loci shown in figure 10, the underestimate \( \delta z \) of the glacier thickness is shown as a function of horizontal distance, for errors in arrival time \( \delta t_b = -0.1 \) microseconds and in airplane altitude \( \delta H = +15 \) meters. Only part of the \( H=0 \) curve is shown; this error increases very rapidly, because the locus from a surface sounding becomes so steep as the locus approaches the glacier surface (fig. 10).

Inferring the bed topography from radio-echo sounding data is an inverse problem, many characteristics of which are readily revealed by considering the forward problem. A hypothetical bed topography, having no variation in the other horizontal direction and underlying a horizontal glacier surface, is shown in figure 15. It also shows the envelope that would be obtained from loci constructed from arrival times that would be received from that bed along flight paths at \( H=200 \) and \( 800 \) m above the glacier surface and at the surface itself. A slight idealization is invoked at each end of the bed profile (points A and M), where it is assumed that a high curvature feature exists having the capability of reflecting back along the path from which it came a ray that might have come from any direction; otherwise, an echo would not be received at sounding positions near that same end, for the ray could not be normal to the bed at its endpoints. Were this not assumed, no echoes would be received toward the ends of the sounding profiles.

As would be expected from equation 7, the greater the altitude from which the sounding is done, the less detail is obtained in the envelope and the greater the error is. In fact, the 800-m profile is controlled by just short stretches of each of the five bed prominences \( (A, C, E, G, M) \); they control only part of the 200-m profile, which also receives echoes from bed points between the five prominences, and they control even less of the profile of soundings taken from the glacier surface. The range of control of each of these five stretches is indicated by the refracted ray making up the beginning and the end of each range.

The bed profile in figure 15 was composed so as to emphasize several different effects (eq. 7) on the curva-
Figure 15.—Hypothetical bed profile and profiles inferred from echoes received at different altitudes. Over those segments where an inferred profile is not indicated, it is understood to be coincident with the bed. The ranges of control of each of the five prominences, which collectively control the entire record of soundings from 800 meters, are indicated by the refracted rays making up the beginning and end of each range. The discussion in the text is keyed to the lettered features.

ture of the reflection locus. Because of the effect of airplane altitude, the depression at B is detected in the 200-m data but not in the 800-m data. Because of the effect of depth below the surface, the depression at B is detected in the 200-m data but the one at F, which has the same curvature, is not; the same is true of the depressions at D and I with respect to the data from soundings at the surface. The depression at D—which is at the same depth as the one at B but has greater curvature—is not detected in the 200-m data, whereas the one at B is. The depression at L has the same depth and the same curvature as the one at B, but it is inclined from the horizontal; thus, the one at B is detected in the 200-m data but the one at L is not.

A separate effect occurs at H, J, and M, where the bed slope exceeds the maximum value of 0.679 that a locus from an aerial sounding can achieve. Therefore, these steep stretches could not be detected by any aerial data. The separation of the surface-sounding envelope from the bed at H and J is not caused by the steepness of the bed slope, but rather by the curvature of the depression at I.

Shown also in figure 15 is the bed profile that would be obtained from the 800-m data were the echo assumed to come always from the nadir. If the appropriate linear transformation ($ct = 800n - z$) is made to the vertical scale, then the 800-m nadir curve becomes the $t_b(x)$ curve of the original data. The curves arising from making the same assumption with the 200-m and surface data are omitted for reasons of graphical clarity, but all six different bed inferences are briefly summarized in table 1.

Persistently reflecting points on the glacier bed generate hyperbolic segments (Harrison, 1970) on the signal profile $t_b(x)$, whether soundings are taken from the glacier surface or from above it. Such a hyperbolic segment is the nadir-method curve from $x = 2,140$ to $2,900$ m of figure 15, which is the same as the $t_b(x)$ profile if the appropriate linear transformation is made to the vertical scale. Five such hyperbolas are shown in figure 16 for persistently reflecting points at depths of 200 and 500 m as detected from the glacier surface and from altitudes of 200 and 800 m above it. The slope $dt_b/dx$ is zero at $x = 0$, where the sounding is taken from directly above the reflecting point. It may be shown that the limiting slope, at $x = \infty$, is $dt_b/x = 2/c = 6.7 \mu s/\text{km}$ for surface soundings, and is $dt_b/x = 2n/c = 11.9 \mu s/\text{km}$ for soundings taken from above the surface.
TABLE 1.—Summary of errors in inferring bed profile of figure 15, using each of the two methods, on data from each of three indicated altitudes (m, meters)

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REFRACTION AND REFLECTION
REPRESENTED IN THREE DIMENSIONS

The two-dimensional representation of the refraction-reflection geometry is easily extended to three dimensions. For the case of a horizontal glacier surface, the complete locus is the body of revolution formed by rotating the $x,z$ locus (fig. 10) about the $z$-axis. If the surface is not horizontal, but is planar, the rotation is about the normal to the plane.

Snell’s law (eq. 3) is easily expressed in three dimensions. The point $(x_1,y_1,z_1)$ on the air ray and the point $(x_2,y_2,z_2)$ on the ice ray are each a unit distance from the origin, which is where the refraction occurs (fig. 17).

The plane $\alpha x + \beta y + \gamma z = 0$ has its coefficients scaled as to magnitude and sign so that, respectively, $\alpha^2 + \beta^2 + \gamma^2 = 1$ and $\alpha x_1 + \beta y_1 + \gamma z_1 = \cos \theta > 0$. Then,

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + n \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = k \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix},$$

in which

$$k = \cos \theta - (n^2 - \sin^2 \phi)^{1/2} = (1 - n^2 \sin^2 \phi)^{1/2} - n \cos \phi$$

and

$$-\cos \phi = \alpha x_2 + \beta y_2 + \gamma z_2.$$

FIGURE 16.—Arrival-time hyperbolas caused by hypothetical persistently reflecting points on the glacier bed. Each $t_i(x)$ hyperbola is identified by the depth $z$ of the point below the horizontal glacier surface and by the altitude $H$ above it from which the point is detected. The horizontal distance from the point is $x$.

FIGURE 17.—Snell’s law in three dimensions. The angles $\theta$ and $\phi$ obey equation 3. The points $(x_1,y_1,z_1)$ and $(x_2,y_2,z_2)$, each a unit distance from the refraction point, are related by equation 8.
Either the θ-form or the φ-form of \( k \) is used, according to whether \( (x_2,y_2,z_2) \) is found from \( (x_1,y_1,z_1) \) or the other way around. The plane determined by the two rays is normal to the refracting plane. The signal's travel time between \( (x_1,y_1,z_1) \) and \( (x_2,y_2,z_2) \) is less when passing through the origin than it would be were it to pass through any other point in the plane. In the analysis of the Columbia Glacier data, a plane was used to approximate the surface topography lying between an airplane position and a location where the altitude of the reflection locus is desired.

If the glacier bed is idealized as a differentiable, mathematical surface, then a normal exists at every point on the bed. A ray that strikes the bed at some point by following some path other than along the normal will be reflected, but it will not return along the same path. Because the airplane's motion is negligible compared with the speed at which the signal travels, only a ray that travels along a normal to the bed will produce an echo that reaches the airplane. Equivalently, an echo will be received only from points where the three-dimensional reflection locus is tangent to the bed. For any particular point on the bed, then, an echo can be received at only one particular airplane position at any particular altitude. Because the ray path must be normal to the bed, its direction cosines are uniquely determined by the direction cosines of the bed at that point. This unique ray path through the ice is refracted at the glacier surface, according to Snell's law, into a unique ray path through the air.

It is also assumed that the laws of geometric optics apply; that is, the wavelength of the signal is much smaller than the roughness elements on the bed. Small-scale roughness, which could not be analyzed, could cause an unknown amount of diffraction or other effects that might complicate the analysis.

COLUMBIA GLACIER DATA

The echo arrival times \( t_0 \) for the 678 airplane positions (fig. 4) are tabulated in appendix A. The arrival times for the airplane-bed-airplane round trip are in microseconds. A frequency distribution of the altitude \( H \) of the airplane above the glacier surface appears as figure 6. Arrival times were read at about every 5 mm along the reconstructed profile; if the maximum value of \( t_0 \) for the east-west flight paths did not occur at one of these equally spaced positions, an extra point at the maximum \( t_0 \) location was also included because it gives the greatest depth to the bed.

The reflection lobes implied by the set of coordinates \( (x, y, z, t_0) \) for each of the 678 airplane positions were constructed by using a plane to approximate the glacier surface locally. At each node of a 200-m square grid, the altitudes of all lobes reaching that node were compared, and the deepest one was taken to be the envelope altitude there. The bed topography was inferred by contouring the unadjusted envelope altitudes (fig. 18) except near the terminus, where directly measured bathymetry was used.

A vertical cross section (fig. 19) shows the relationship between the reflection lobes and their envelope for one flight path. The section, along \( y = 15.4 \text{ km} \), was chosen because the contours are nearly north-south in direction, so that there is little transverse bed slope. It demonstrates that the normality condition is satisfied in the \( x, z \) plane, because where the refracted rays reach the envelope they are normal to it. An indication that the normality condition is satisfied in the \( x, y \) plane is provided by figure 20, which shows at each grid node which one of the reflection lobes forms the envelope there; the straight line joining the center of the reflection lobe and the node is generally normal to the contour there. The fact that the envelope is reached by almost all of the reflection lobes is evidence of the high internal consistency of the data.

The envelope near the terminus is made up of lobes from airplane positions that are about a kilometer up-glacier (figs. 4, 20). Because the echoes having the arrival times that determine these lobes are likely from bed locations near the flight path, the envelope near the terminus is artificially shallow. This is confirmed by the direct bathymetric measurements, some of which were made since the terminus retreated behind its August 26, 1978, position (Post, 1975; Post, written commun., 1979–1984). The inferred bed topography (fig. 18) is taken to be the directly measured bathymetry—where it exists—and in a slender zone up-glacier from there it is taken to be an interpolation between it and the envelope of the reflection lobes. The adjustment made to the envelope (fig. 21) is greatest near the terminus, where the directly measured bathymetry exists and where the airborne coverage is poor, and fades to zero at the up-glacier margin of the interpolation zone.

The difference between the nadir method and the envelope method is shown in figure 22, which also shows the difference between the envelope method and the reflection lobes from some surface soundings. Where the envelope directly beneath an airplane position is horizontal, the nadir method and the envelope method give the same result; if the bed has any inclination, the nadir method gives a shallower estimate of the bed altitude. Whereas the envelope-method bed is contoured from values on the 200-m square grid, the difference between the nadir method and the envelope method exists only directly beneath an airplane position; therefore, the contouring of the difference is less...
STUDIES OF COLUMBIA GLACIER, ALASKA

10 7 8 9 10

1 1

12

x, IN KILOMETERS

y, IN KILOMETERS

x, IN KILOMETERS
Figure 19.—Vertical cross section along $y = 15.4$ kilometer showing glacier surface and reflection lobes for 31 positions along flight path N2500. For each airplane position are shown (A) the vertical cross section through its reflection lobe as well as the air rays from the intersections of the lobe and the glacier surface, and (B) the air ray refracted at the glacier surface into the ice ray that reaches envelope of lobes. In (B) are shown the envelope and those rays reaching it at integral multiples of 200 meters along the $x$-axis; also shown are some rays at intermediate points on the envelope from airplane positions for which the lobe does not form the envelope at any 200-meter multiple. For clarity, the lobes from other flight paths are not shown; the actual envelope is slightly deeper, by as much as 5 meters at $x = 5.8, 6.4,$ and 7.8 kilometers.

Figure 18.—Inferred bed topography. Contour interval is 100 meters on exposed bedrock and 20 meters elsewhere. Terminus position (heavy solid curve) is taken from August 26, 1978, vertical aerial photography. Bed topography in front of terminus and in densely ruled region behind terminus is taken from direct bathymetric soundings (Austin Post, written commun., 1979–1984). Bed topography in sparsely ruled region is taken from an extrapolation of the adjustment (fig. 21) in the densely ruled region.
Figure 21.—Adjustment of envelope-method bed according to directly measured bathymetry (Austin Post, written commun., 1979–1984), a negative value indicating lowering of the inferred altitude. Contour interval is 20 meters. The adjustment in the sparsely ruled region is an extrapolation of the adjustment in the densely ruled region, where direct bathymetry was taken. The amount of the adjustment is not defined all the way down to the August 26, 1978, terminus position because the envelope method could not be applied that far away from the airplane positions (dots) from which echo arrival times are available.

Figure 20.—Sources of reflection lobes forming envelope of lobes. For each node of the 200-meter square grid is shown the airplane position (small circle) whose reflection lobe forms the envelope at that node. At some intermediate points are shown the airplane position whose lobe forms the envelope at that point but does not form the envelope at any grid node. For a few airplane positions, the echo arrival time presumably was underestimated, so that the resulting lobe was too shallow to reach the envelope. The bed contours shown are those obtained from the envelope method without modification according to the directly measured bathymetry.
accurate than the contouring of the envelope-method bed. The maximum difference between the envelope-method bed and a surface sounding, however, does not occur directly beneath the position from which the sounding was taken unless the envelope-method bed is horizontal there. The location and the amount of the algebraically maximum height of the envelope-method bed above the sphere, which is the reflection lobe of the surface sounding, are shown in figure 22.

**DISCUSSION**

If there is a strong law that says that every echo must come from one place and that the bed there must be normal to the refracted ray, then there is probably also a weak law that says that an echo comes from only one place. Were the weak law valid, then the regions where it is likely that the bed altitude is overestimated are those regions where the envelope at many grid nodes is formed by a single reflection lobe; according to this weak law, the bed coincides with the envelope at only one point within such a region and is below it elsewhere. Several candidates for such regions are revealed by the source map (fig. 20). The most conspicuous occurrence of this is the sequence of closed depressions along the bottom of the valley, where the curvature of the bed likely exceeds that of the reflection lobes. Each of the dominating lobes here is produced by the maximum $t_b$ of an east-west profile. Were the data free of error, the true bed altitude would be given at one point per reflection lobe and the altitude of the bed would be overestimated elsewhere. The resulting surface will be much smoother than the actual bed topography. When the airplane is 800 m above ice that is 500 m thick, and the bed slope does not exceed 0.2, the radius of curvature of the reflection lobe is about 2,000 m (eq. 7; figs. 12, 13).

In addition, data error must be considered. For one thing, it may be the reason some reflection lobes apparently do not reach the envelope (fig. 20). It would be possible to adjust the $t_b$ to make such a lobe reach the envelope—which is the ultimate test of internal consistency of the data—but this would not modify the envelope, which is what the bed topography is inferred from.

The effect of errors in $t_b$ or $H$ can be determined from equations 5, as illustrated in figure 14. The estimated 0.36 µs error in $t_b$, assuming the airplane altitude to be exactly 800 m, corresponds to an error of about 30 m for the point on the lobe directly beneath the airplane, and an error of about 35 m at a horizontal distance of 1 km. Conversely, if $t_b$ is correct, the estimated 30-m error in airplane altitude corresponds to an error of about 17 m directly beneath and about 12 m one kilometer away.

A biased error distribution in the inferred bed topography results. Bed prominences control the data and cause overestimation of the bed altitude elsewhere, whereas the data error may induce an error of either sign.

This project was not only the first airborne radio-echo sounding of a temperate glacier, but it also was performed under a severe time constraint. If time had allowed testing of the entire system—including the data analysis—prior to the actual sounding of Columbia Glacier, many of the problems encountered would have been recognized and solved. With this in mind, there are a few fundamental suggestions to those interested in work on airborne radio-echo sounding of temperate glaciers.

First, in order to infer a glacier bed topography from airborne radio-echo sounding, it is necessary to have the location of the initial pulse (the flightline on the reconstructed photograph), $t_b$, and knowledge of the glacier surface topography. The last can be obtained by recording $t_s$, by having a known surface topography, or by recording both the barometric and radar altitudes. Because of the possible error due to changing atmospheric conditions, the barometric altimeter needs to be carefully calibrated before and after each flight.

Second, when attempting to sound a deep, narrow, temperate, valley glacier, the gross geometry of the wave—the radius of curvature of the reflection lobe increases at the rate of 1.78 times $H$—necessitates flying as close to the glacier surface as possible. Thus, the dead time of the system (a minimum of 3.5 µs in this case) must be reduced.

Finally, careful consideration needs to be given to the data acquisition system and to the data reduction. It is necessary to think through completely what data are desired and how they will be analyzed, so it can be...
determined what variables should be measured and in what format the measurements should be recorded. The system should also allow easy recovery of the raw recorded data.

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**APPENDIX A: ECHO ARRIVAL TIMES**

The following tables give the echo arrival times at each airplane position along the 12 east-west and 7 north-south profiles. The numbers preceded by “N” or “W” indicate flight paths shown in figure 4. The arrival times (t) are the roundtrip airplane-bed-airplane signal times, in microseconds; x and y are the horizontal coordinates of the airplane in the local Columbia coordinate system (eq. 1), and z is the vertical coordinate, in meters above sea level.

### Table 1: Echo Arrival Times

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### Table 4: Echo Arrival Times

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