INERTIAL INSTRUMENT SYSTEM FOR AERIAL SURVEYING

U.S. GEOLOGICAL SURVEY PROFESSIONAL PAPER 1390

Inertial Instrument System for Aerial Surveying

By RUSSELL H. BROWN, *with contributions by* WILLIAM H. CHAPMAN,

WILLIAM F. HANNA, CHARLES E. MONGAN, U.S. Geological Survey, *and*

JOHN W. HURSH, The Charles Stark Draper Laboratory, Inc.

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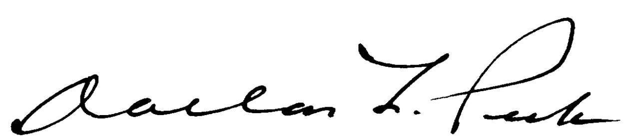
FOREWORD

In the late 1960's and early 1970's, the much-heralded and precise performance of inertial guidance systems in space exploration challenged earth scientists to seek ways of turning those systems to their advantage in the near-earth environment. Russell Brown and Charles Mongan sensed and responded to that challenge. Over a 2-year period (1972–74), they shaped a workable research task and secured the endorsement of the top U.S. Geological Survey managers. As a result, the Geological Survey was committed to the development of a pioneering inertial instrument system for aerial surveying, through contract arrangements with The Charles Stark Draper Laboratory.

The formal contract work began in mid-1974, at a time when a few military and civilian agencies were successfully operating less-precise generations of inertial guidance systems in motor vehicles and helicopters to establish control lines for topographic surveys. The Geological Survey's research commitment was to move rapidly beyond those techniques and to field a new and more precise inertial guidance system capable of obtaining similar or better control lines from a fixed-wing aircraft. The potential savings in dollars, manpower, and time in using the new system for field studies in the Geological Survey Operating Divisions more than justified this research commitment.

After nearly a decade of work, a prototype model of this new instrument system has been successfully flight tested. Its demonstrated performance has exceeded in every respect the stringent design specifications. Although the full range in performance capabilities remains to be explored, the system will provide highly efficient, cost-effective support for scientific programs of the Geological Survey, including production of topographic maps, investigations of water resources, and assessment of mineral resources and geologic hazards. The system also has immediate potential to support the research and operational missions of other Federal, State, local, and private organizations.

The Geological Survey is pleased to have played a major developmental role in this important scientific achievement.



Dallas L. Peck
Director

PREFACE

The purpose of this report is to describe, for the nonspecialist earth scientist or engineer, an inertial guidance or navigation system that will enable use of relatively light aircraft for efficient nationwide data-gathering in geology, hydrology, terrain mapping, and gravity-field mapping. Development of this airborne instrument system is a logical consequence of the phenomenal post-World-War-II growth in the science and technology of inertial guidance systems. That growth, although obviously predestined by the genius of men like Dr. Charles Stark Draper, was accelerated by the onset of the international space race in the 1960's.

By the early 1970's, new generations of gyroscopes and accelerometers—key components in inertial systems—had been designed, built, laboratory proven, and field tested sufficiently to allow application of inertial systems to certain high-precision field measurement processes in the earth sciences. Prime examples of successful applications in that period include the military development of a position and azimuth determining system, to run survey control lines, and the development for the National Aeronautics and Space Adminstration of the inertial navigators that guided the round-trip Apollo flights to the Moon. In these applications, however, the precision levels were lower than what the U.S. Geological Survey would require for its use.

The first inertially guided transcontinental flight, from Boston to Los Angeles, occurred in February 1953 with an Air Force B–29 carrying an inertial navigator that weighed more than a ton (over 900 kilograms). Dr. Draper (system designer), John Hursh (system operator), Charles Collins (pilot), and Irving Levin (flight engineer) were among The Charles Stark Draper Laboratory contingent that comprised that historic crew, and all have been involved actively in the current Geological Survey project. The present-generation inertial navigators in regular airline use individually weigh less than 100 pounds (45 kilograms) and can be accommodated in a space roughly equivalent to a desk file drawer.

In a timespan of about two decades, this kind of evolution in inertial system design, with equally graphic improvements in precision, has made the current instrument-development project technically feasible. Not only did feasibility crystallize, but the bright prospects that relate to the continuing evolutionary trend in inertial navigator design virtually guarantee that an instrument system built to satisfy the present Geological Survey specifications for precision can be upgraded periodically to meet even more stringent specifications whenever cost-effectiveness criteria dictate.

The instrument system described in this volume capitalizes not only on virtual state-of-the-art inertial guidance technology but also on similarly advanced technology for measuring distance with electromagnetic radiating devices. The distance measurement can be made with a transceiver beamed at either a cooperative target, with a specially designed reflecting surface, or a noncooperative target, such as the Earth's surface. The instrument system described herein features components that use both techniques. Thus, a laser tracker device, which updates the inertial guidance unit or navigator in flight, makes distance measurements to a retroreflector target mounted at a ground-control point; a laser profiler device, beamed vertically downward, makes distance measurements to the Earth's surface along a path that roughly mirrors the aircraft flight path. In both technological domains, inertial guidance and electromagnetic distance measuring, the equipment components selected for use in this new instrument system were advanced well beyond levels that were then commercially available.

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ILLUSTRATIONS

TABLES

Commonly used Inch-Pound terms and their preferred SI equivalents

Multiply inch-pound units	By	To obtain SI equivalent
Length		
inch (in.)	2.540×10*[a]	millimeter (mm)
foot (ft)	3.048×10^{-1}	meter (m)**
yard (yd)	9.144×10^{-1}	meter (m)
mile (mi)	1.609	kilometer (km)
nautical mile (nmi)	1.852*	kilometer (km)
Area		
square foot (ft^2)	9.290×10^{-2}	square meter (m^2)
Volume		
cubic foot (ft^3)	2.832×10^{-2}	cubic meter (m^3)
Mass		
ounce, avoirdupois (oz)	2.835×10	gram (g)
pound, avoirdupois (lb)	4.536×10^{-1}	kilogram (kg)**
Force		
ounce-force (ozf)	2.780×10^{-1}	newton (N)
pound force (lbf)	4.448	newton (N)
Pressure		
pound per square inch (lb/in.^{2})	6.895×10^3	pascal (Pa)
pound per square foot (lb/ft^2)	4.788×10	pascal (Pa)
bar	1×10^5*	pascal (Pa)
millimeter of mercury (0°C)	1.333×10^2	pascal (Pa)
inch of mercury (0°C)	3.386×10^3	pascal (Pa)
Torque		
ounce-inch (oz·in.)	7.062×10^{-3}	newton meter (N·m)
pound-foot (lb·ft)	1.356	newton meter (N·m)
dyne centimeter (dyn·cm)	1×10^{-7}*	newton meter (N·m)
Temperature		
degree Fahrenheit (°F)	Temp °C = (Temp °F − 32)(5/9)	degree Celsius (°C)
Illumination		
foot-candle (ft-c)	1.076×10	lux (lx)

*Exact conversion factor.
**Basic SI unit.
[a]Exact conversion, except for geodetic surveying in the United States for which the conversion factor is 2.540005.

List Of Abbreviations

Abbreviation	Explanation
APT(S)	aerial profiling of terrain (system)
DC	direct current
DMA	direct memory access
GaAs	gallium arsenide
gyro	gyroscope
Hz	hertz
IMU	inertial measurement unit
I/O	input/output
kHz	kilohertz
MHz	megahertz
ms	millisecond
µs	microsecond
ns	nanosecond
NAD	North American Datum
NGVD	National Geodetic Vertical Datum
rpm	revolutions per minute
TV	television
V	volt
W	watt

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ABSTRACT

An inertial guidance system for aerial surveying has been developed by The Charles Stark Draper Laboratory, Inc., under contract to the U.S. Geological Survey. This prototype system, known as the aerial profiling of terrain system, is designed to measure continuously points along an aircraft flight path or a terrain profile to a positional accuracy of ±0.5 foot (15 centimeters) vertically and ±2 feet (61 centimeters) horizontally at the 90-percent reliability level. Thus, the system's objective is to accomplish rapidly and accurately from a fixed- or rotary-wing aircraft what traditionally would be accomplished from ground-based topographic surveys combined with aerial photography and photogrammetry.

The strategy for measuring the terrain profile directly beneath the aircraft flight path entails (1) continuous determination of the three-coordinate position of the aircraft, once it departs from its initial calibration and alinement point, and (2) continuous measurement of the vertical distance between the aircraft in flight and the land surface so that the desired terrain profile can be computed directly.

The first, and by far the most complex, part of the strategy is achieved by use of an inertial navigator that contains an inertial measurement unit, in combination with an eye-safe pulsed laser tracker, both mounted on a common base. The inertial measurement unit features a gimbaled three-axis inertial platform, stabilized in the desired orientation by three high-precision gyroscopes, on which is mounted an orthogonal array of three high-precision accelerometers. The gyroscopes provide a three-coordinate scheme of reference for measuring the rotational angles through which the aircraft moves; the accelerometers provide a continuous measurement of specific force acting on the aircraft in three dimensions, from which a continuous record of aircraft position can be computed if the Earth's gravity field along the flight path is known or modeled. Largely because of error buildup in the determination of the vertical coordinate of position, the inertial measurements must be supplemented with independent positional or velocity information at approximately 200-second intervals. This requirement is met by a gimbaled two-axis laser tracker, which measures distances and directions to a select number of ground-based retroreflectors which have been surveyed previously.

The second part of the strategy is achieved using an eye-safe pulsed laser profiler, which is, in effect, a high-precision altimeter. Boresighted with this is a color television camera which continuously records an image of the flight-path trace over the terrain. An onboard computer accepts data from the sensors in the aerial profiling of terrain system and performs the computations necessary for calibration and alinement of the inertial measurement unit, navigation of the aircraft, and position and velocity updating of the inertial navigator. The computer logs its results in an onboard magnetic tape recorder; selected results are deliverable as well to the sytem operator's keyboard and display unit. Postflight computer processing of the recorded data involves such functional steps as editing, renavigating, compressing, introducing a refined gravity model, state-space Kalman filtering, back filtering, optimal smoothing, and recombining. The results are used to generate a new magnetic tape of terrain-profile data in coordinates of latitude, longitude, and elevation, which are time-correlated with a magnetic tape containing the color video imagery.

A typical mission is accomplished with the aerial profiling of terrain system installed in a DeHavilland Twin Otter aircraft, flown at a speed of 115 miles per hour (105 knots) and at a height above the terrain of 2,000 feet (610 meters) over, for example, a 20- by 20-mile (32- × 32-kilometer) land area of gently rolling topography. An advance field-survey party sets out and surveys a minimum of three retroreflectors in a triangular array; but the number set out may go as high as 15, spaced in a way that will allow retroreflector-tracking gaps of no more than 300 seconds throughout the intended flight.

Upon completion of a mission and the subsequent postflight data processing, the terrain-profile data and video imagery are viewed simultaneously on the screen of a video receiver to permit identification and deletion of any unwanted anomalies which might have been caused by dense vegetation or manmade structures. Special flight missions, restricted to using the aerial profiling of terrain system as a means to establish horizontal and vertical control at sites of unsurveyed retroreflectors or as a means to control or monitor the position and orientation of other onboard remote sensing instruments, do not require operation of the laser profiler.

Performance-evaluation tests of the aerial profiling of terrain system indicate that the ±0.5-foot (15-centimeter) vertical and ±2-foot (61-centimeter) horizontal accuracy specifications have been achieved. The principal source of error is attributable to uncertainties of gravity anomalies. If future refinements are made for controlling the thermal environment of the aerial profiling of terrain system, with special attention to the several electronics packages, then gravity-disturbance data may be routinely gleaned from the flight-mission data. This, in turn, would permit improvements in the currently achieved accuracies. Also, the fully implemented Global Positioning System of satellites may offer potential benefits to the aerial profiling of terrain system, specifically with regard to providing an additional independent measurement of velocity for the inertial navigator.

Use of brand, firm, and trade names in this report is for identification purposes only and does not constitute endorsement by the U.S. Geological Survey.

INTRODUCTION

Developmental Incentives

The U.S. Geological Survey incentives for the instrument-development work described in this paper were first (mid-1960's) identified with the hydrologist's and cartographer's search for a more efficient, quick, and economical way to collect precise terrain-profile data at innumerable (tens of thousands) field sites. The hydrologist's need related to the ongoing work of defining more precisely the geometry of stream valleys nationwide; the cartographer's need keyed to the perennial quest for improved mapmaking tools and techniques.

With the National Flood Insurance Act of 1968, as amended by the Flood Disaster Protection Act of 1973, the U.S. Congress created the National Flood Insurance Program, which was aimed at improving the management of property development in the flood plains of streams. The Act required the Secretary of Housing and Urban Development to provide each of the more than 20,000 flood-prone communities with a detailed flood-insurance study by August 1, 1983. Each such study was to include maps showing the aerial extent and elevation of the 100-year (1-percent chance of occurrence in any given year) flood. The potential workload impact of this Congressional mandate upon hydrologist and cartographer alike was formidable.

In the early 1970's, hydrologists and geomorphologists were pressing the search for better quantitative expressions to relate hillslope profiles and stream gradients to watershed runoff characteristics. Corollary interests concerned attempts to expedite geologic mapping in hilly forested terrain, where important geologic features were often difficult to spot and trace. The foregoing combination of work and interest, whose progress often was thwarted by a paucity of data on longitudinal and transverse stream-valley profiles, prompted the Director in December 1971 to solicit from the several Operating Divisions of the Survey, thoughts on the technical feasibility of developing airborne instrumentation to measure terrain profiles. The director followed this invitation with the establishment in March 1972 of an ad hoc interdivisional committee to explore technical feasibility, and that committee's report in May 1972 became the stepping stone to the research effort described in this report.

Developmental Strategy

Early in 1972, when Brown and Mongan began to analyze the potential for developing an airborne instrument system capable of measuring terrain profiles, some quantitative and substantial qualitative results from a variety of earlier flight experiments were already available. Most flights had featured large multiengine aircraft that included an Air Force B–17, a civilian Douglas DC-3, and a Navy Lockheed Constellation. In all flights, the laser distance-ranging equipment that was carried was bulky and heavy and required a substantial amount of electrical power to operate. The limited quantitative results clearly showed that, unless a way were found to define very precisely the aircraft path of motion, the overall accuracy in vertical measurements would only approach ±1 ft (0.3 m) and, in horizontal position, might range from tens to hundreds of feet (3–30 m).

Very early in the development analysis, therefore, thought processes were conditioned by the following rather obvious facts:

1. Aircraft size should be held within the relatively light category [less than 12,500 lb (5,670 kg)] to minimize the logistics for long-term continuous field use in the Survey.
2. Instrument system should be compact and lightweight and should require a minimum of electrical power commensurate with the constraints imposed by the choice of aircraft.
3. Instrument system should be capable of defining continuously and precisely the aircraft three-dimensional path of motion.
4. Instrument system should be self-contained and should not require any ground-based electromagnetic radiation (radio or microwave) for its successful operation.

It was fact number 4 that almost immediately steered the development analysis into the available instrumentation literature on state-of-the-art developments for stable platforms deployed in satellites and inertial guidance systems. The explorations through the literature paid off in April 1973 when the first schematic was drafted to show what should constitute the ideal airborne terrain-profiling system. That schematic featured an inertial navigator to measure continuously the three-dimensional motion of the aircraft, a laser distance-measuring device to collect terrain-profile data, and an onboard digital computer to process and combine all measured data into the desired formats.

The literature explorations repeatedly associated the names of Dr. Charles Stark Draper and the Instrumentation Laboratory of the Massachusetts Institute of Technology, which Dr. Draper headed, with some of the most significant advances in the technology of inertial navigation and guidance and the manned space flights to the Moon. A logical and final step, therefore, in the developmental strategy for the work described herein was to seek Dr. Draper's endorsement of the proposed new airborne instrument system. Endorsement in principle was obtained in June 1973 in a meeting with Dr. Draper and one of his principal staff advisors and colleagues, John W. Hursh. Over the next year, the foundation was laid for his laboratory, which in the interim had become the independent Charles Stark Draper Laboratory, Inc., to undertake the necessary work through contract with the Survey.

DEVELOPMENTAL HISTORY

In the early 1970's, as national concerns grew over property developments in the flood plains of streams, the resolve to improve flood-plain management grew concurrently. This triggered in the minds of a few Survey hydrologists a desire to formalize a modest beginning research on improved techniques for flood-plain delineation. Thus in August 1971, Rolland W. Carter (ret.), then Chief, Surface Water Branch, and the late Roy E. Oltman, then Assistant Hydrologist for Research and Technical Coordination, arranged for the assignment of George W. Edelen, Jr., to the research problem of investigating and comparing various alternative procedures for flood-plain mapping. This investigation was to feature cost comparisons and was to include examination of the prospects for electromagnetic measurements from aircraft.

Edelen's early work coincided with the Director's expressed interest in examining the technical feasibility of developing an airborne instrument system for surveying. Thus, Edelen served on the ad hoc interdivisional committee appointed to explore the feasibility issue. In the wake of that committee's report, Edelen arranged with the U.S. Navy Oceanographic Office to test fly laser distance-measuring equipment in a Lockheed Constellation aircraft. The intent was to profile the terrain along U.S. Highway 60 west of Richmond, Va., and to monitor vertical motion of the aircraft with an accelerometer; however, severe air turbulence vitiated most results. This served mainly to underscore the merit in proceeding with an analytical appraisal of the principal features needed in an airborne instrument system geared to the Survey's unique field surveying and operating requirements. Preliminary work on such an appraisal had been started already, but, at this point, the effort was intensified with appropriate administrative sanction.

Several early milestones in this developmental history are in the two preceding introductory sections and are not repeated here. It is pertinent to note, however, that Dr. Draper's June 1973 endorsement in principle of the proposed new airborne instrument system, coupled with an opportunity to view state-of-the-art inertial guidance hardware already tested and proven by Draper Laboratory, effectively spurred the earnest work to shape the initial research tasks. Thus, at widely spaced intervals in the period from June 1973 to June 1974, several all-day technical discussion sessions were held at Draper Laboratory. Participants included a number of their professionals as well as Survey professionals from the several Operating Divisions. The result was a "Request for Proposal" (RFP No. 5507) which the Survey delivered to Draper Laboratory on April 8, 1974. The RFP described a proposed 6-month engineering analysis, or technical feasibility study, and, in essence, circumscribed the research task with the following specifications:

1. Define the prospects for designing and building an airborne instrument system capable of—
   a. Determining continuously in real time (virtually instantaneously) the three-coordinate position of the aircraft.
   b. Continuously profiling terrain along the vertical trace of flight path.
   c. Achieving absolute (referenced to local control) position and profile accuracies of ±0.5 ft (0.15 m) vertically and ±2 ft (0.61 m) horizontally.
   d. Deployment in relatively light fixed- or rotary-wing aircraft.
   e. Operating at relatively low flight altitudes, arbitrarily defined as below 3,000 ft (914 m) above the terrain.
   f. Providing vertical pointing stabilization for a small camera.
   g. Being updated and adjusted in flight with respect to its precise orientation and position.
   h. Tying aircraft position data to ground-control points in local survey area.
2. Define flight and operating techniques for the subject instrument system.

An added stipulation, implicit in the foregoing specifications, limits field use of the proposed airborne instrument system to nonmountainous terrain. Furthermore, the specifications were drawn primarily from requirements of the flood-plain delineation problem, which entails fitting floods of given magnitudes, such as the 100-year flood or the 50-year flood, into the local stream-valley geometry. The fitting exercise uses precisely measured valley cross sections at a sequence of selected sites and processes the relevant data through what a hydrologist terms a "step-backwater computational procedure." In the field-investigation programs of the Survey alone, about 10,000 stream-valley areas require such flood-plain definition—a singular need that drives the stakes quite high for success in developing airborne instrumentation that can offer substantial work savings in manpower, time, and cost. Reflections of this particular Survey need are evident in each of several other Federal agency programs.

With the research task defined, the heart of the proposed instrument system was seen to be the inertial navigator which establishes the three-coordinate reference-carrying capability. This suggested a versatility in using the system that would transcend the rather limited variety of field problems involving terrain profiling. Some expression of the range of field uses, identified as time progressed, is given in the section titled "Uses for the Instrument System" and is seen to embrace the study programs of all Field-Operating Divisions in the Survey. Furthermore, the time and costs projected by Draper Laboratory for the necessary contract work clustered principally around the inertial navigator. The laser profiler, therefore, was recognized, even at this early juncture, as a low-cost and relatively simple adjunct to the heart of the instrument system that would enable early flight testing on important problems that involved terrain profiling. Thus, when the need for a system acronym arose, project personnel coined "APT" to signify Aerial Profiling of Terrain.

Effective June 24, 1974, the Survey contracted with Draper Laboratory (contract no. 14–08–0001–14548) for the referenced 6-month engineering analysis, and the author was designated Technical Officer for this contract. His counterpart at Draper Laboratory, John W. Hursh, was designated Principal Investigator. The Survey monitored the advancing work regularly and closely and stayed involved through technical discussions and field data inputs as questions arose. The Draper Laboratory study culminated in a 225-page open-file report (Desai and others, 1975).

The significant findings in the report are summarized as follows:

1. An airborne instrument system can be built that will perform to the stipulated precision.
2. The airborne system will require update (position and velocity vector) information at 3-minute intervals during the flight mission.
3. The update information required by the system can be supplied by an onboard tracker component.
4. The inertial and tracker components should be built to function as one integral unit.
5. Inflight calibration of the system will require an inexpensive retroreflector mounted on each of three presurveyed ground-control points. These will be tracked, one at a time, by the tracker.
6. The gyroscopes and accelerometers in the system should be state-of-the-art quality (Draper Laboratory supplied) if the Survey's high-precision specifications are held; they can be available commercial quality if lower precision specifications can be tolerated.

The Survey arranged for outside technical reviews by selected scientists in the Harry Diamond Laboratories, part of the Army Materiel Command, in Washington, D.C. Through those Laboratories arrangements also were made for reviews by scientists at the Army Missile Laboratory, Huntsville, Ala. The reviews focused on two fundamental areas: The mathematical developments that undergird the concept of a tracker device for updating the inertial navigator in flight, and the performance capabilities of the Draper Laboratory gyroscopes (gyros) and accelerometers in terms of meeting the overall precision specified for the complete instrument system. The reviews completely supported the Draper Laboratory findings.

To anticipate the ultimate need for a decision and commitment on when and how to continue contract work at Draper Laboratory, a briefing was arranged for the Survey Executive Committee, which comprised the Director, principal administrators in the Director's office, and the Division Chiefs. The briefing was given on December 18, 1974, and highlighted work progress and outlook. After a followup briefing on May 7, 1975, the committee recommended that the contract effort at Draper Laboratory be extended through completion of the general design for the full instrument system. That work phase, estimated to require 18 months, officially began June 28, 1975, with approval of the contract papers and allocation of an initial sum of money. In the hiatus that otherwise would have occurred between completion of the engineering analysis and start of the general design for the overall instrument system, the Survey contracted with Draper Laboratory for a series of laboratory experiments with laser devices. Primary purpose of the experiments was to accumulate pertinent operational data that would lead to the best design for the laser profiler. In this work, Draper Laboratory was aided by the cooperative consulting efforts of personnel in the Electronics Branch of Harry Diamond Laboratories. Results of this 8-month work phase are documented in the report by Mamon and others (1976).

Concurrent with their recommendation for continuing the Draper Laboratory contract effort, the Executive Committee requested that cost-effectiveness data be assembled, based on the potential Survey uses of the proposed airborne instrument system. During summer 1975, those data were assembled and analyzed. Justification was shown for completing the design and venturing into the costly fabrication phase of the instrument-system development as budgetary constraints would allow. Highlights of the cost-effectiveness analysis were given to the Executive Committee in a briefing on November 12, 1975; the Survey-envisioned uses for the instrument system came from that analysis.

The contract effort progressed through a series of distinct work phases, beginning with the described engineering analysis, as shown in figure 1. The figure illustrates the sequence and time frames within which the phases were accomplished and also shows costs of the contract work by phase and fiscal year. When completed, each phase was documented in an open-file manuscript report which is included in the list of references.

To stay abreast of the evolving contract work and to recognize the diversity in potential field applications for the proposed airborne instrument system, the Survey chose to monitor progress directly through monthly or bimonthly review sessions at Draper Laboratory. These sessions commonly ran 1 or 2 days and involved the Survey Contract Technical Officer and at least one colleague each from the Geologic, National Mapping, and Water Resources Divisions, as well as a colleague from what was then the Survey's Office of Land Information Analysis. The individuals who have comprised that review team throughout most, if not all, of the contract work phases shown in figure 1 are Russell H. Brown, William H. Chapman, John M. De Noyer, William F. Hanna, and Charles E. Mongan. The indicated mix of Survey representation, commonly combined with one or two representatives, most consistently Zoltan G. Sztankay, from the Electronics Branch of Harry Diamond Laboratories, brought top-level review expertise to bear in the very relevant scientific domains of cartography, geodesy, geology, geophysics, hydrology, mathematics, physics, seismology, and sophisticated instrumentation design and development, the last involving especially the fields of acoustics, electromagnetics, electronics, and optics. Each review session featured an informal, but detailed, exchange of ideas, identification of problem areas, and discussion of the most promising avenues for resolution. On-the-spot decisions usually were made, thereby forestalling delays that inevitably would have accompanied attempts at resolution by correspondence or telephone.

At about the two-thirds point in the 18-month design phase of the contract work, Draper Laboratory held a design-review session. On June 22 and 23, 1976, Draper Laboratory project personnel described their progress on the primary facets of the design effort. Handout paperbound text and illustration materials totaled more than 400 sheets.

In addition to Survey attendance at the review session, representatives from six other Federal agencies and from the Geodetic Survey of Canada attended. The discussions surrounding the design review reemphasized and strengthened the favorable determinations of feasibility that had been developed earlier.

The design phase terminated virtually on schedule with submittal of a 401-page open-file report (Draper Laboratory, 1977). The report gives the principal general design features for the overall proposed airborne instrument system, with significant backup detail to support important design choices. Part of this detail for the laser profiler evolved from two sets of limited flight tests jointly organized by the Survey and Draper Laboratory in September 1976 and April 1977, respectively. The flight-test results are in a key report by Youmans (1977).

On March 31, 1977, the Survey contracted with Draper Laboratory for the design-verification phase of the contract effort. The phase was programmed originally for a 10-month period, with the Survey entitled to fund the work incrementally. Due to unavoidable budget delays, this phase was not completed until Septem-

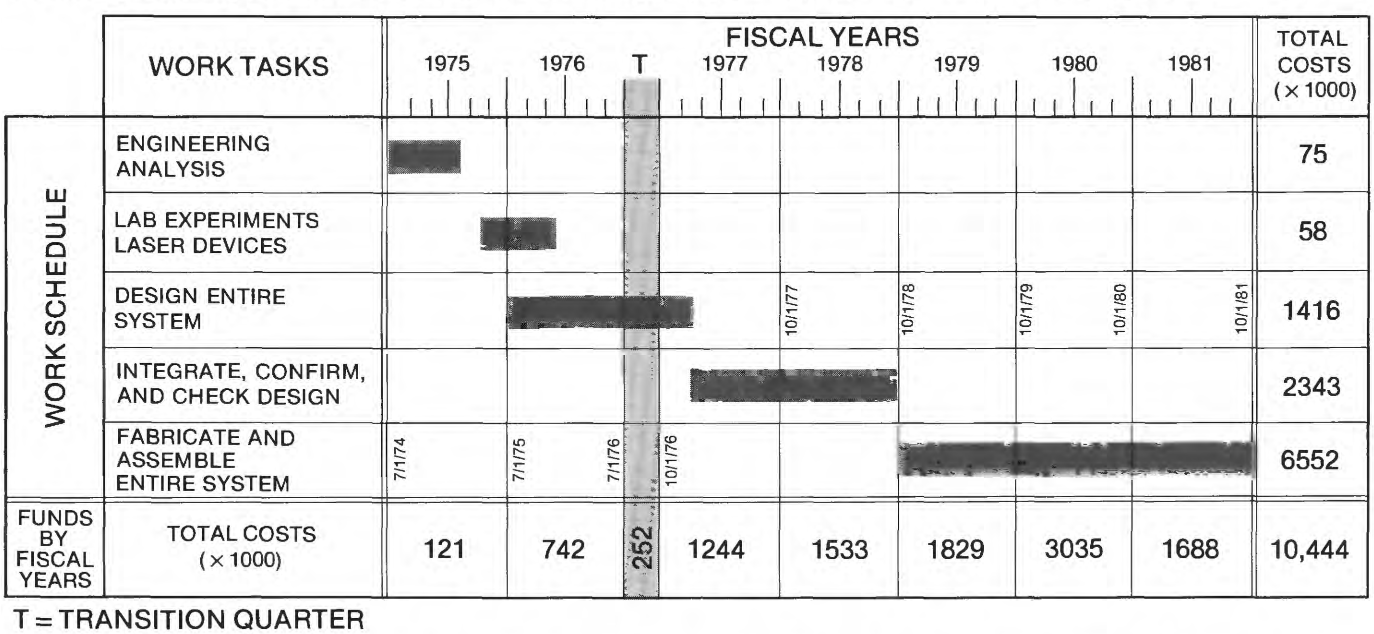

FIGURE 1.—Fiscal year history of work on U.S. Geological Survey contract by Draper Laboratory, Cambridge, Mass., 1975–81.

ber 30, 1978. Effective July 13, 1978, Lowell E. Starr was named the Survey Contracting Officer's Technical Representative, and Russell H. Brown and William H. Chapman were named his Technical Designees.

The design-verification phase represented the last necessary step before undertaking the fabrication phase. Design verification embraced complete integration and checking of all general design features, as well as detailed laboratory testing of key components, circuits, and interface electronics. The end products were the drawings, assembly and test procedures, and process specifications ready for submittal to the fabrication facilities.

The design-verification phase is documented in a narrative report published by Draper Laboratory (1978). Companion to the narrative report is a "Technical Data Package" that contains all approved drawings, source control drawings, process specifications, and software documentation. The package is part of the official body of open-file information maintained at Draper Laboratory, Cambridge, Mass., and at the Survey National Center, Reston, Va.

With no break in time, the fabrication phase began and spanned the 3 fiscal years from October 1, 1978, to September 30, 1981 (see fig. 1). In the negotiations for this phase, it became evident very quickly that if Draper Laboratory fabricated, in the absence of any commercial suppliers, the principal inertial components (three gyros and three accelerometers), the contract costs quickly would overrun the Survey's limited budget resources. Conferences between the Survey and Draper Laboratory elicited an alternate and less costly fabrication plan for the prototype APT system, which was based on using gyros and accelerometers that were emerging as "surplus" to an Air Force contract. In adopting this plan, however, some instrument design constraints were relaxed, with no relaxation in overall precision, to accommodate the existing gyro and accelerometer hardware. The principal relaxed constraint concerned the operating temperature environment for the gyros and accelerometers, and this, thereby, invoked a need to build cooling coils into the housing of the inertial navigator unit. This, in turn, mandated a decision to air condition the aircraft cabin, with the end result that the complete prototype APT system would need to be fielded in an aircraft larger than that envisioned in the Draper Laboratory design report. Thus, this first system was designed to be flown, through its test, demonstration, and early operational phases, in a DeHavilland Twin Otter aircraft rather than the earlier envisioned Rockwell Commander twin-engine aircraft. The fabrication phase terminated on schedule and is documented in a five-volume report (Draper Laboratory, 1982).

ACKNOWLEDGMENTS

The authors acknowledge the insights and effective support of top managers and scientists in the Survey, Harry Diamond Laboratories of the U.S. Army Materiel Command, and Draper Laboratory.

Many individuals and organizations contributed to the success of this instrument development project. Some are identified at pertinent places in this paper. However, in recognition of especially supportive and significant contributions, the following individuals, and their organizational affiliations, are gratefully identified:

U.S. GEOLOGICAL SURVEY

- James R. Balsley, former Assistant Director, Land Resources (ret.)[1]
- Rolland W. Carter, former Regional Hydrologist, Southeastern Region (ret.)
- Philip Cohen, Chief Hydrologist
- Joseph S. Cragwall, Jr., former Chief Hydrologist (ret.)
- Edward J. Cyran, Cartographer, National Mapping Division
- John M. De Noyer, former Chief, Earth Resources Observation System Program; currently Geologic Division
- O. Milton Hackett, former Associate Chief Hydrologist (ret.)
- Warren W. Hastings, former Assistant Chief Hydrologist, Research and Technical Coordination (ret.)
- Joseph T. Long, former Chief, Branch of Field Surveys, National Mapping Division (ret.)
- Hugh B. Loving, former Deputy Assistant Chief, Research and Technical Standards, National Mapping Division (dec.)
- Robert H. Lyddan, former Chief Topographer (ret.)
- Edward A. Moulder, former Assistant Chief Hydrologist, Research and Technical Coordination (dec.)
- Roy E. Oltman, former Assistant Chief Hydrologist, Research and Technical Coordination (dec.)
- William A. Radlinski, former Associate Director (ret.)
- Richard P. Sheldon, former Chief Geologist (ret.)
- Rupert B. Southard, former Chief, National Mapping Division (ret.)
- Lowell E. Starr, Assistant Chief for Research, National Mapping Division

HARRY DIAMOND LABORATORIES

- Lyndon S. Cox, Plans and Operations Office
- Robert G. Humphrey, Physicist, Near-Millimeter Wave Branch (ret.)

[1] ret.—retired
dec.—deceased
res.—resigned

Joseph M. Kirshner, former Chief, Fluid Systems Branch (ret.)
Clarence F. Ravilious, former Mathematician, Fluid Systems Branch (ret.)
Helmut Sommer, former Director, Research and Technology Division (ret.)
James E. Spates, former Chief, Plans and Operations Office
Zoltan G. Sztankay, Chief, Near-Millimeter Wave Branch

U.S. ARMY MISSILE LABORATORY

Charles A. Halijak, Professor, University of Alabama
Joseph S. Hunter, Aerospace Engineer, Inertial Systems Development
J. V. Johnston, Research Aerospace Engineer, Inertial Systems Development (ret.)
William W. Stripling, Supervising Aerospace Engineer, Inertial Systems Development (ret.)
H. V. White, Aerospace Engineer, Inertial Systems Development

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John H. Barker, Project Program Manager
George T. Bernard, Technical Illustrator
William G. Denhard, former Department Head (ret.)
Charles Stark Draper, Senior Scientist
William A. Drohan, former Associate Division Leader (res.)
Leo F. Hughes, former Section Chief (ret.)
Irving M. Levin, former Staff Member (ret.)
Glenn Mamon, Project Technical Director
Roger E. Medeiros, Staff Member
Benjamin S. Smith, former Section Chief (ret.)
J. Arnold Soltz, Section Chief
Douglas S. Youmans, former Staff Member (res.)

INSTRUMENT SYSTEM

Overall System Concept

Some important clues have already been given regarding the nature of the overall airborne instrument system. From these, the realization should be emerging that, if the Survey earth science measurement criteria are to be met, the heart of the instrument system, the inertial navigator, must be able to determine instantaneously, very precisely, and continuously its position and its orientation. The former implies a capability for navigation, and the latter, for pointing.

The uniqueness of an inertial navigator is that it combines both the navigation and the pointing capabilities in a computer and a mechanism known as an inertial measurement unit (IMU)[2]. The IMU is an assemblage of gyros and accelerometers. For it to function as it should, it must be able to measure or detect such physical vector quantities as gravity (the local vertical) and the Earth's rotation. The directions of these vectors are "remembered" by the gyros, which establish the needed references for pointing or orienting the whole instrument. The manner in which the references are created may be likened to an orthogonal set of x, y, z coordinate axes fixed on an inner or stable platform that actually supports three orthogonally mounted gyros. The stable platform is isolated from the disturbing influence of aircraft rotational motion (pitch, roll, and yaw) about the three axes by means of a gimbal-support structure (see section titled "Gimbal System"). The rotational movement of individual gimbals can be offset or compensated for through the actions of their individual torque motors, which, in turn, are controlled by signals from the gyros as relayed through the appropriate servoamplifiers.

An inertial navigator cannot operate perfectly, owing to inescapable built-in mechanical imperfections and imprecise knowledge of variations in the Earth's gravitational field along its path of motion (the flight path). Thus, the position and orientation data yielded by the instrument drift slowly away from the truth (the correct values). This can be overcome by providing high-quality position-data samples periodically from an independent source, and the frequency with which this is done is related to the desired overall measurement precision. To satisfy the Survey precision criteria, the requirement is to update the inertial navigator with position data at 3-minute intervals; the independent source for obtaining those data is a laser-ranging device called a tracker, mounted in the aircraft in the same structural housing as the IMU. During flight, the laser tracker locates and locks onto a retroreflector target mounted over a known ground-control point. While the target is in view (about 30 seconds), a stream of range and pointing-angle measurements is collected.

The way in which the inertial navigator and tracker complement each other and combine to make an outstandingly high-quality instrument system can best be illustrated in the following manner. First, the inertial navigator is a device that can very easily recognize and measure high-frequency-type accelerations and angular rotations—in other words, the normal motions—of the carrying vehicle (the aircraft). Its performance is corrupted, however, by very low frequency angular rotations caused by very slow and random gyro drift and other small systematic errors. Second, the tracker is an

[2]The term "inertial measurement unit" tends to mislead, in the sense that inertia cannot be measured directly. The unit simply features mechanical components (gyros and accelerometers) which, in responding to the phenomenon of inertia, deliver the measurements needed to define a "specific force" vector (containing gravitational and nongravitational accelerations) acting upon the carrying vehicle.

instrument that measures best in a way that allows good definition of the low-frequency types of rotations and disturbances. The tracker has virtually no capability for measuring or defining the high-frequency-type disturbances. Finally, for the best performance characteristics to be realized, the inertial navigator and the tracker must be optimally combined to obtain a near-ideal (minimal total error) measurement capability over the full frequency range for the expected accelerations, angular rotations, and disturbances. These several points are diagrammed schematically in figure 2.

An important attribute of the inertial navigator is that, in fulfilling its navigation capability, its IMU component directly senses or measures accelerations of the carrying vehicle, and these are then mathematically integrated once to yield velocities and twice to yield travel distances. Because integration is a smoothing process, any inherent measurement errors are greatly reduced; thus, the inertial navigator is especially good at precisely resolving high-frequency types of motion.

Implicit in the foregoing discussion is a capability for processing the sensed or measured data and performing computations; thus, an airborne digital computer is indeed a central component in the complete instrument system. Although hardware and related software details are described in the section titled "Computer," a few advance observations are pertinent here.

The airborne computer and its accessories must be versatile enough to perform a variety of tasks that include accepting or logging the sensed and measured data from the IMU, tracker, and any ancillary earth science measuring equipment; processing the accepted data and performing necessary computations; storing and displaying the appropriate data; and accomplishing all of this in accordance with suitably developed instructions (software). Selection of the most appropriate computer must reflect consideration not only of the foregoing tasks but also the need for some reserve capacity in processing and computing speeds, memory (storage) size, and input and output capabilities. This represents a deliberate choice to build into the instrument system greater flexibility for planning and executing a larger variety of data-gathering flight missions.

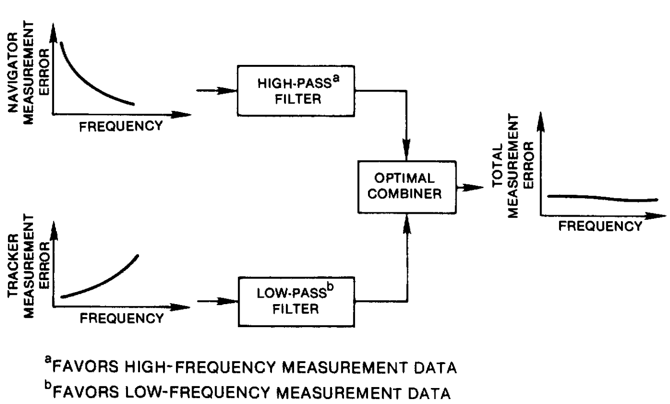


FIGURE 2.—Frequency response characteristics of inertial navigator and tracker to vehicle motion.

Any sincere attempt to detail the instrument system leads inescapably to and through descriptions of the principal components. Because their introduction to data-gathering programs in the earth sciences is relatively new, they deserve more than casual attention. In this paper, therefore, each principal component in the overall instrument system is described in detail with pertinent extracts from the supporting body of scientific principles and theory.

INERTIAL NAVIGATOR

The earth sciences community is gradually accumulating exposure to and experience with inertial-type measuring instruments with the advent of commercially available portable and land-vehicle-mounted units used in determining geographic position. The precision requirements for such units, however, have been at least an order of magnitude coarser than those specified by the Survey for the instrument system herein described. Furthermore, the Survey specifications require deployment in a light aircraft, rather than in land vehicles.

To comprehend the significance of an inertial navigator, one needs to reexamine, briefly, Newton's three basic laws of motion and the commonly used term, "inertia." These laws of motion feature three principles which have proved valid for all mechanical problems not involving speeds comparable to the speed of light and not involving atomic or subatomic particles. The first law, also know as Galileo's law of inertia, states that a particle not subjected to external forces remains at rest or moves with constant speed in a straight line; the nongravitational measurements of accelerometers contained within an inertial navigator are referenced to an inertial coordinate system in which a body moves with constant velocity as long as no force acts on it. The second law states that the acceleration of a particle is directly proportional to the resultant external force acting on the particle and is inversely proportional to the mass of the particle; a more general statement of this law is that the force equals the time rate of change of linear momentum, where momentum is the product of the mass and the velocity of the particle. The more general statement reduces to the more specific one if mass does not change with time, as is the case with an inertial navigator. The more general statement involving momentum also provides a reference for defining inertia—that property of matter which manifests itself as a resistance to any change in the momentum of a body. The third law states that, if two particles interact, the force exerted by the first particle on the second is equal in magnitude but opposite in direction to the force exerted by the second particle on the first. This law explains why the contact force exerted by the support structure on the triad of accelerometers is the inertial reaction to the vehicle acceleration and is, therefore, opposite in direction to vehicle motion.

The literature on instrumentation for inertial guidance and navigation contains many definitions of inertial navigators. Inasmuch as no single definition seems adequate for the present purpose, the following composite was collected piecemeal with interpretations from several inertial-navigation textbooks (see "Selected References"):

> "An inertial navigator is a self-contained instrument package designed to track continuously the velocity and position of the vehicle that carries it. Its proper functioning requires no external electromagnetic radiation. It consists of accelerometers that measure a combination of inertially referenced specific force (force per unit mass) and gravitational force; gyros that provide the inertially referenced three-coordinate scheme by virtue of precessing (spin axis slowly changing direction) in reaction to disturbing torques or vehicle rotations at right angles to the precessions; an assumed model of gravitational force to permit extraction of inertially referenced specific force from accelerometer measurements; a computer to integrate with respect to time the specific force data, to obtain the navigator velocity and position; an updating device to correct or compensate for errors in navigator velocity and position caused by time-dependent drifts of gyros and other instrument components; and housing materials to interconnect components and to stabilize and regulate the physical environment of the instrument package."

The specific force, equivalent to force per unit mass, may be seen to have units of acceleration by expressing Newton's second law of motion in the simple form of force (F) equals mass (M) times acceleration (a)—$F = Ma$—and by subsequently dividing both sides of the equation by the mass. As previously noted, the measured specific force is opposite in direction to vehicle motion as a consequence of Newton's third law of motion.

The variety of Survey field applications envisioned for the APT system contains the common element of field surveys of some kind in a relatively localized geographic area. This prompted selection of a geographic coordinate system of latitude and longitude, based on a reference ellipsoid and specifically upon the North American Datum of 1927 (NAD of 1927), and elevation, based on a geopotential surface termed the "geoid" and specifically on the National Geodetic Vertical Datum of 1929 (NGVD of 1929). This Earth-fixed coordinate system serves as the reference for all fully processed position data. Other coordinate systems are used, however, in implementing the inertial navigator; for example, the stable platform established by the three gyros in the APT system is mechanized in a tangent-plane orientation, convenient for inertial navigation when individual flight missions will operate over relatively small areas of the Earth—areas, for example, that have a radius of less than 200 mi (322 km). This means that the x-y plane of the platform is maintained normal (and the z axis parallel) to the direction of the actual plumb line at the point where the system is calibrated and alined before a flight mission. To maintain this initial tangent-plane orientation throughout the flight mission, the system need only compensate for Earth rotation.

The tangent-plane coordinate system was judged to be superior to a locally level coordinate system because, in the latter system, knowledge of changing torque rates required to maintain the platform in the system could not be recovered for postsurvey data processing (postprocessing). The locally level coordinate system, however, is used as a reference for the processed velocity data, having been transformed to this system from the tangent-plane system. In the locally level system, the x–y plane of the stable platform is maintained tangent to the surface of a mathematically defined reference ellipsoid, and the z axis is maintained parallel to the ellipsoid normal. Gyrocompassing permits the x-axis to be oriented along geographic north and the y-axis along geographic east. The direction of the z-axis departs from the direction of actual gravity approximately by a quantity known as the deflection of the vertical.

The requirement for a model of the gravitational field is especially important where an inertial navigator is operating in a near-Earth environment, as in an aircraft, and where precise knowledge of the elevation position coordinate is required. Both conditions apply to the present system. Much experimentation will be warranted in determining how accurate a gravity model (gravitation plus centripetal acceleration associated with Earth rotation) is required for a given spacing of ground-surveyed control points to achieve a stated precision in vertical and horizontal position coordinates.

Inertial Measurement Unit

The principal physical component of a three-coordinate inertial navigator, such as that designed for the APT system, is the IMU composed of three gyros together with appropriate gimbals (supporting frames) and servo-drive motors to establish a stable platform that will hold the specified three-coordinate scheme of reference in the correct orientation, and three accelerometers mounted on that stable platform to measure the resultant of the specific forces referenced to each coordinate axis. Beyond these major items, the IMU includes a number of subsidiary items, such as electronics packages, pickoff and readout devices to accomplish the required measurements, and all the appropriate wiring and mechanical linkages.

An IMU can be visualized as a key component in the inertial navigator, which is designed to solve problems in geometry, inasmuch as the ultimate task is to locate points along its path of motion and to reference them very precisely (in space and time) to some specified coordinate system. The challenge, then, is to design and build a collection of physical components, the heart of which is labeled an IMU, capable of simulating the specified coordinate system, measuring the resultant of all applied forces (including gravity) referenced to that coordinate system, and processing that information in real time (or after the flight mission) into the desired position and orientation or attitude information.

An important key in guaranteeing inertial navigator performance that is dependable, predictable, and at the designed level of precision is the provision of a benign thermal environment for the inertial components; that is, the gyros and accelerometers. The goal is to simulate the stable and perfectly controlled conditions of the laboratory throughout the expected vagaries of the field-operating environment. In a number of missile or astronautical applications, this has required adjunct large and heavy refrigeration units which might have compromised the Survey stipulation that the APT system be capable of deployment in a relatively light aircraft. Thus, in the original design, Draper Laboratory designers chose a high working-level temperature inside the IMU of 128°F (53°C) stabilized to within ±0.1°F. In such a design, air would be the heat transfer medium, and aircraft cabin air, the ultimate heat sink. Thus, under most expected operating conditions, heat would need to be supplied the IMU via built-in electrical heaters. Fabrication costs for this high-temperature IMU precluded its use in the prototype system, but the concept is sound and should be implemented for any future operational system. Departures from the original-design temperatures are described at the end of the section on temperature control.

Gyroscopes

Fundamental to the successful implementation of the IMU are the gyros that provide the physical means for creating a stable platform to carry the given coordinate scheme of reference in a specified orientation in three-dimensional space. Popular qualitative concepts, harking back all the way to childhood toys, convey some of the pertinent characteristics of these intriguing devices. However, a more exact and quantitative detailing of selected gyro properties is germane to the present description.

The pertinent nomenclature relating to gyros, displayed with the elemental schematic in figure 3, is important to comprehend and properly define significant aspects of their performance characteristics. The heart of a gyro is a rotor or wheel free to spin on an axis. The spin axis is held in nearly frictionless bearings retained in a gimbal which itself is held in low-friction bearings retained by the overall supporting structure or base; for example, an aircraft. Among the design goals for a spinning gyro is a high value for its angular momentum along with small physical size. A specified angular momentum, therefore, compatible with small size, is obtained by concentrating the weight at the perimeter and spinning the wheel at a high speed; for example, 24,000 revolutions per minute (rpm) is used in the APT system and is commonplace in commercially available gyros. The APT system gyros also will include a viscous damper placed on one gimbal axis. whose function will be described later in this section.

The spin axis of a gyro will maintain its initially given orientation in space. The degree of steadfastness with which the spin axis will hold its orientation is related directly to the magnitude and constancy of the angular momentum of the spinning wheel. Furthermore, if the spin axis experiences a rotational movement, causing the direction of that axis to change, it will react in a new rotational movement (directional change) orthogonal to the first. This gyro property can be illustrated by imagining that the inner gimbal supporting the wheel (fig. 3) undergoes a slight rotational movement clockwise around the middle axis. The foregoing movement can be caused by applying a parallel

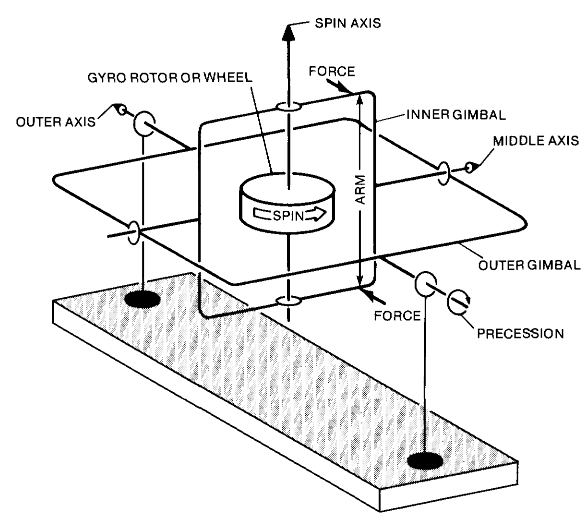


FIGURE 3.—Gyro and gimbal-support structure.

pair of equal and opposite forces to the inner gimbal as shown in figure 3. The consequent spin-axis movement can be seen to result in its changing its line of direction in the plane of the two forces and the reaction or response is a predictable and new spin-axis rotational movement, or precession, in an orthogonal plane (fig. 3). The foregoing pair of applied forces is termed a "couple," and the quantitative measure of its capability for producing rotation is termed a "torque," which is simply the product of one force times the perpendicular distance between the pair (the arm in fig. 3).

The above statement is general and not limited to the particular plane that passes through the applied forces shown in figure 3. The equation that quantifies the above relation is called the Law of Gyroscopics, which may be expressed in the convenient form,

[Torque] = [(Wheel Inertia) (Spin Velocity)] [Precession Rate][3].

(N·m) (m^2kg) (rad/s) (rad/s)

Shown parenthetically below each quantity are the SI (International System) units in which it is commonly expressed (see also list of "Symbols and Dimensions"). Because the product of wheel inertia and spin velocity is angular momentum, a simpler, more convenient form of the equation is

[Torque] = [Angular Momentum] [Precession Rate]. (1)

(m^2kg/s^2) (m^2kg/s) (rad/s)

Both of the above equations relate the indicated terms in magnitudes only.

Equation 1 is extremely useful in a two-way sense in that, for a given or applied torque (magnitude and direction), a particular rate and direction of precession is determinable, and, conversely, if a specific rate and direction of precession is observed or created, then the magnitude and direction of the particular related torque is determinable. The principle actually is used in both senses as the APT system operates.

The described performance characteristics immediately suggest the possibility for using gyros to set up the desired three-coordinate-reference axis scheme. However, if the spin axis is to be given a proper chance to fulfill its function of remaining stable in a given orientation, then it must be isolated in carefully predetermined ways from the several possible rotational motions of the overall supporting structure or base.

In reference to figure 4, note the given orthogonal set of three axes comprising the spin axis, the output (gimbal) axis, and the input (sensitive) axis, the last of which, by definition, is at right angles (in quadrature) to the other two. Compared to figure 3, the outer gimbal has been removed, and a damper added to the output axis. Next consider that if the base is moved along any arbitrary path so that its original orientation in three-dimensional space is changed, then the progressive nature of that orientation change can be fully described in terms of three component rotational rates about the foregoing set of orthogonal axes. It is appropriate now to examine individually these three possible component rates.

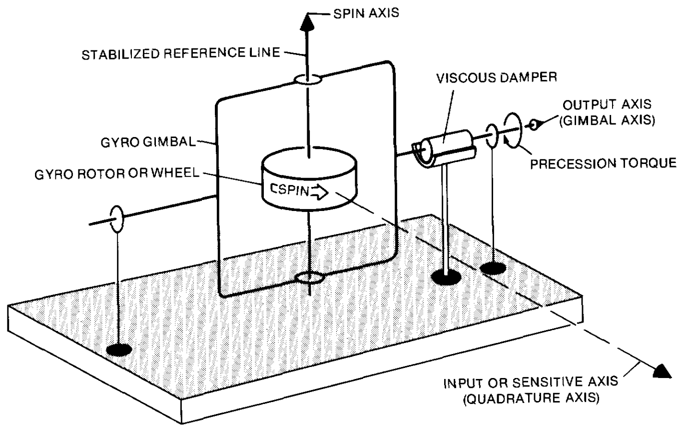


FIGURE 4.—Single-degree-of-freedom gyro.

1. Any base-rotation component around the spin axis obviously has no effect on changing the direction of that axis.
2. Any base-rotation component around the input axis (fig. 4) inevitably causes the spin axis direction to experience the same rotational rate, and the reaction, as may be predicted from the preceding discussion, is a precession torque which induces rotation of the gimbal and wheel assemblage around the gimbal (output) axis. This torque is countered by an opposing torque from the viscous damper proportional to the angular rate of the gimbal around the output axis, which, in effect, integrates the precession torque and affords a way of precisely indicating the angular rotation of the base around the input axis. An electromagnetic device known as a signal generator is used to measure the angle between gimbal and base as an output voltage.
3. Any base-rotation component around the output (gimbal) axis is countered by a resisting torque in the viscous damper. This has the effect of tending to induce rotation of the gimbal and wheel assemblage around the gimbal axis, leading to the predictable precession torque around the input axis. Due to the geometry of the gimbal structure

[3]The rotational equivalent of $F = Ma$ (which describes translational motion) is $\underline{T} = I\alpha$, where $\underline{T}$, I, and α are the torque (a pseudovector), the moment of inertia, and the angular acceleration (a pseudovector), respectively.

(see fig. 4), however, the only result is a tendency to "cock" the gimbal in its bearings.

Because the gyro spin axis is free to move in response to a torque around only one axis, the input axis, it is termed a "single-degree-of-freedom" gyro and is so labeled in figure 4. Thus, the gyro measures base rotation around the input axis (sometimes called the sensitive axis) only and quantifies this as an output voltage proportional to input angle.

The conclusion can now be made that if, for example, the input axis in figure 4 is chosen to simulate the local vertical coordinate axis and if two similar single-degree-of-freedom gyros are mounted on the same base with their input axes oriented orthogonal to each other in the local horizontal plane, to simulate stabilized reference lines in the east and true north directions, then a stable platform reference scheme in Cartesian coordinates has been instrumented, capable of sensing base rotation around any axis. The balance of the instrumentation challenge stems from the need to provide the platform with motion-isolating gimbals driven by torque motors powered by servoamplifiers receiving signals from the gyros. Each gyro signals the angular amount it has been forced to deviate because of the base motion. This information is resolved along the gimbal axes and then is fed to the respective servo-drive or torque motors to rotate the gimbals so as to restore each gyro to its required stabilized position. In effect, therefore, the gyros mutually act to isolate themselves from the three component base rotations caused by any motion of the carrying vechicle. The entire foregoing cycle of events occurs virtually instantaneously, and it is repeated continuously.

The particular gyros selected for the Survey instrument system are state-of-the-art hardware fabricated by Draper Laboratory. In the eventual production model, the gyros are to be fabricated in accord with a design that uniquely fits the Survey specifications and field-operating environment. In the APT system the gyro will carry the Draper Laboratory designation "APTG."

The gyro unit, weighing 1.0 lb (0.45 kg), is housed in a cylindrical case approximately 3 in. (76 mm) long by 2 in. (51 mm) in diameter. Its principal features are labeled in the cutaway view of figure 5. The gyro rotor, or wheel, is symmetrical and is supported by opposed hemispherical gas bearings. In operation, the rotor spins at a constant speed (24,000 rpm) which is sustained by torque from a pair of hysteresis-ring assemblies on the rotor. These ring assemblies are driven by a pair of AC motor stators. The complete motor, wheel, and bearing assembly is enclosed in a hermetically sealed cylindrical "float" (shown in red in fig. 5) which is filled with hydrogen. The hydrogen provides gas-bearing support and also serves as a thermal-conducting medium helping to produce an even temperature distribution over all inside parts of the float shell. The float is suspended in a support structure (shown in blue in fig. 5) with no mechanical contact, except for flexible power leads. The support techniques involve electromagnetic centering and a highly viscous liquid for flotation. Temperature of the liquid is maintained at a constant 139°F (59°C), to a tolerance of 0.001°F (0.0006°C), partly through the device of a thermal control heater (see fig. 5) and partly through stainless steel thermal shims used with the center-flange mount.

The power leads, which carry electrical power to the wheel-drive motor, are flexible arched metallic leads that are submerged in the flotation fluid. Semiflotation of the leads in the viscous fluid reduces the distorting effects that occur when the gyro is subjected to high linear accelerations. The leads have their outer ends mounted on a baffle plate that has small clearance grooves for each lead. These grooves are so small that solidification of the fluid at low temperature cannot produce shearing forces large enough to damage the leads.

In further reference to figure 5, an electromagnetic device with inner and outer concentric magnetic circuits is located at each end of the instrument. The concentric rotor assemblies are mounted on the ends of the float, and their respective concentric stators are mounted in the cylindrical housing. The outer circuit in one end features a signal generator that detects the rotation of the float with respect to the housing and produces a polarized voltage signal proportional to the angle of rotation. The outer circuit in the other end features a torque generator that is the means for applying precession torques to the float. The inner circuits on both ends feature magnetic suspension units to provide axial and radial centering of the float pivot within the clearance spaces and thereby eliminate mechanical pivot-to-jewel friction.

The fluid in the gyro unit supports the float so that the residual weight carried by the magnetic suspension, which acts to center the float, is no more than a small fraction of a gram under a pull equal to one times the Earth's gravitational force. Under conditions of shock, vibration, and linear acceleration, bouyant reactions proportional to the applied specific force are generated by the fluid. This means that the acceleration reaction force tending to move the float within the gyro case depends only on the residual unfloated mass and not on the total mass of the float. Furthermore, the float velocity generated by the residual unfloated mass being acted on by an inertial reaction force is so limited by the viscous reaction forces of the damping (flotation) fluid that a time period of 30 minutes to 1 hour is required for the gimbal to move radially 0.0001 in. (0.0025 mm) under a force equal to the force of gravity

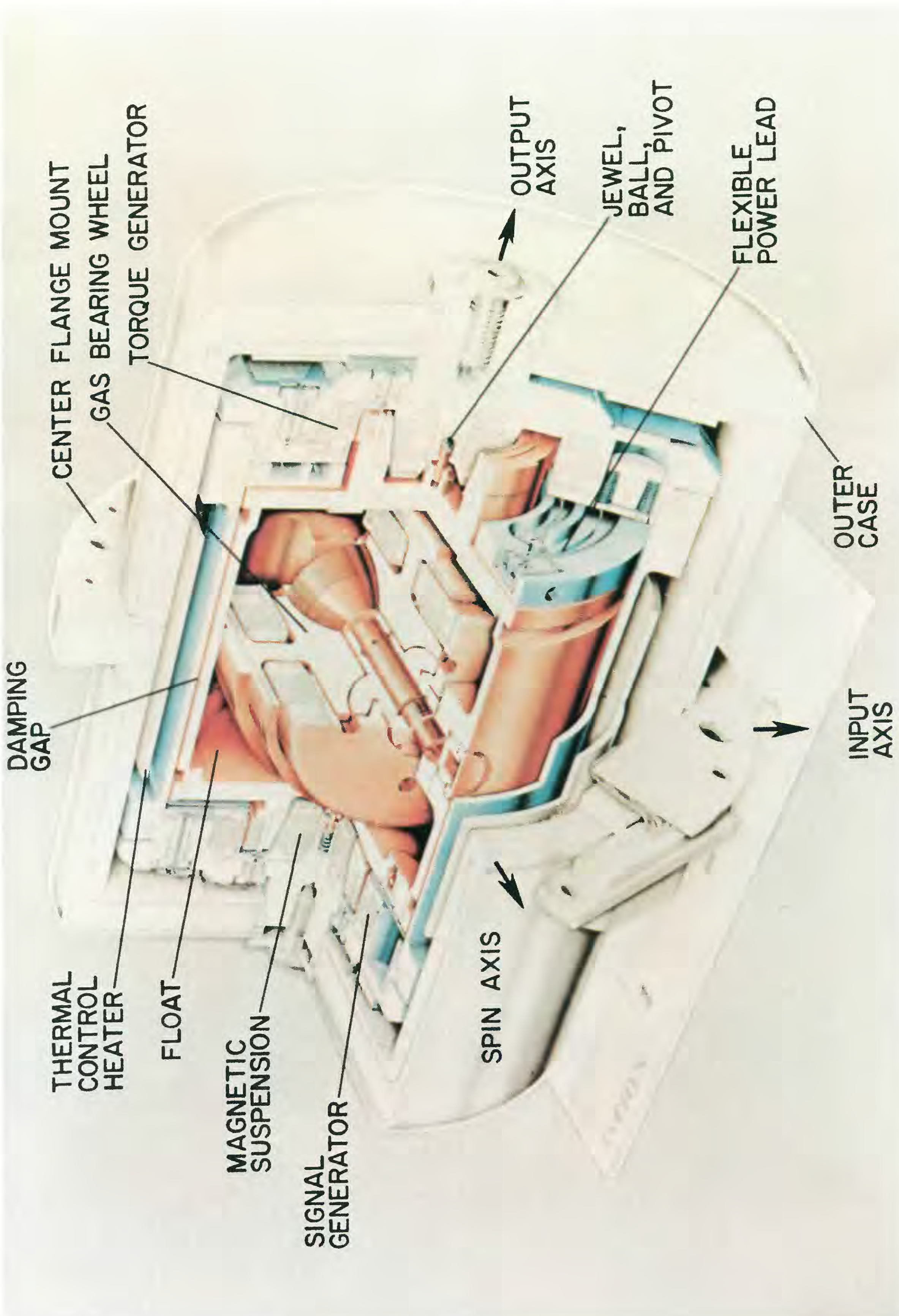


FIGURE 5.—The single-degree-of-freedom floated gyro.

near the Earth's surface. Thus, the combination of flotation, magnetic centering, and high damping makes it possible to keep the pivots from making contact with their bearings even under the most severe environmental conditions of shock, vibration, and acceleration.

Acceleration-compensated bellows are located at each end of the gyro. These permit thermal expansion of the fluid and are designed to prevent fluid-float mass motion under acceleration along the output axis.

Viscous shear forces acting across the damping gap between the case and the float provide a near-perfect velocity-proportional restraint torque to oppose the precession torque, which produces the integrating action that characterizes the gyro unit. The high viscous shear torque causes the gyro to behave as a linear, or first-order, system with a characteristic time of about 0.74 millisecond (ms). In the mathematical comparison of linear systems, the time characteristic arbitrarily represents the time interval over which the transient phase of the system response to a stimulus reaches 63-percent completion. The low value for this gyro signifies a built-in capability for rapidly and precisely tracking all stimuli of short duration in a band bounded by an upper frequency limit of about 1,400 hertz (Hz).

For a given angular momentum, the viscosity of the fluid controls the ratio of angle input to float-angle output, or sensitivity, of the gyro unit. Although a preselected sensitivity value is not usually among the design criteria for a gyro, the value must be determined once the gyro has been built to document pertinent performance characteristics. For the Draper Laboratory gyro and a wheel speed of 24,000 rpm, the sensitivity is about 0.9.

No matter how carefully a gyro is designed and constructed, it can never be an absolutely perfect device. Thus, the spin axis (stabilized reference line) is subject to stray torques brought about, for example, by minute friction in the support bearings or in the angular displacement pickoff devices. These torques, which develop randomly, cause the spin axis to precess or wander (change direction) in random amounts; this is termed "gyro drift." For a given gyro, this random drift can be observed and defined through laboratory tests and calibration procedures so that, within carefully specified limits, proper compensation can be introduced when the gyro is put into operating service.

A measure of the Draper Laboratory gyro-performance capability with respect to drift can be drawn from a practical comparison with the gyros now extant in inertial navigation units used by commercial airliners on the long transoceanic international flights. These units commonly accumulate gyro-drift errors in position of about 1 nautical mile (nmi) per flight hour. On a great circle route over the Earth's surface, 1 nmi [about 6,080 ft (exactly 1,852 m)] is equivalent to 1 minute of arc. The gyro drift, then, can be stated as 1 minute of arc per hour. The Earth, however, turns at a rate of 15°, or 900 minutes of arc, per hour. Thus, the observed drift rate for gyros now in commercial use is seen to be about one-thousandth (1×10^{-3}) of the Earth's turning rate. Draper Laboratory personnel have adopted this milli-earth-rate unit (meru) as a convenient measurement unit to describe and compare the performance characteristics of inertial navigators.

In contrast to commercial gyro performance, the design objectives for the Draper Laboratory gyro require holding the accumulation of position errors related to gyro drift to about 100 ft (30.5 m) per day or 4 ft (1.2 m) per hour. If this distance is recognized as something less than 1×10^{-3} nmi,then the gyro drift rate is obviously less than 1×10^{-3} minute of arc per hour, or less than one-millionth (1×10^{-6}) of the Earth's turning rate. In the APT system, therefore, gyro performance is projected to be several orders of magnitude better than present commercial gyros.

To give some quantitative feel for the relation between gyro drift rate and a causative torque, numbers can be substituted in the basic gyro equation (see eq 1) to reflect specific performance properties of the Draper Laboratory gyro. Equation 1 can be written more formally as

$$\underline{T}_o = \underline{L} \times \omega_I \quad , \qquad (2)$$

where $\underline{T}_o$ = torque around the gyro output axis in newton meters per radian (N·m/rad)

$\underline{L}$ = angular momentum of the gyro wheel in newton meter seconds (N·ms)

ω_I = angular velocity around the gyro input axis in radians per second (rad/s).

If the gyro spin axis changes direction (drifts) at an angular rate of 1 meru, this means $\omega_I = 7.27 \times 10^{-8}$ rad/s. For the Draper Laboratory gyro unit, $\underline{L}$ is approximately 3.75×10^{-2} m^{2}kg/s. Substituting these values in equation 2 results in a $\underline{T}_o$ value of 2.7×10^{-9} m^{2}kg/s^2 N·m for the torque created at the gyro output axis.

To reduce the effects of anisoelasticity, the gimbal (or float structure) is made of beryllium, which combines an extremely high modulus of elasticity with a low density. Also, the properties of the gas bearing have been optimized for isoelastic stiffness. To provide isolation from magnetic fields, the entire gyro is surrounded by a two-layered, 90°-overlap, metallic shield of high magnetic permeability.

The coordinate system of the stable platform is conveniently established by mounting three gyros so that their input axes are mutually orthogonal, as shown schematically in the upper part of figure 6. The orthogonal orientation is justified for two reasons: The input axes correspond to the x, y, and z axes of a Cartesian coordinate system, and this orientation permits minimizing unwanted torques on the gyro float associated with any built-in unbalance. The unwanted torques, any one of which is at a minimum when forces acting on the gyro are oriented along its output axis, are minimized for the three-gyro set by requiring two of the output axes to be alined along or near the actual gravity vector acting on the APT system.

Because their thermal sensitivity is greater than that of other associated components, the three gyros and their required electronics are arranged in their own physically separate section of the stable platform. This makes it easier to provide a benign thermal environment for the gyros which ensures achieving the highest possible performance levels. Uniform temperature distribution in the most thermally sensitive part of the gyro—the float containing the spinning wheel—is enhanced by adopting a center-flange mount (see fig. 5) for the cylindrical case. The mechanical and thermal design features provide added insurance for uniform temperature distribution by specifying a good thermal conductor from the center-flange mount to the gyro ends, and a heater and sensor for each end of the conductor so that thermal disturbances at the mount do not affect the gyro.

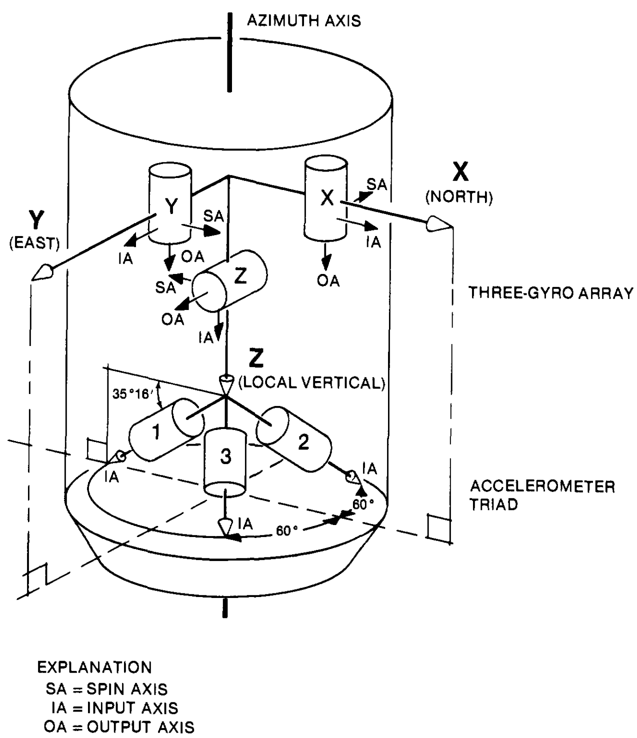


FIGURE 6.—Gyro and accelerometer arrays for aerial profiling of terrain system.

Accelerometers

In the hierarchal construction of an IMU, gyros have been described as the basic or fundamental devices needed to create a stable platform. Not only are the changes in attitude of the carrying vehicle, as it progresses along some path of motion, measured and referenced to this platform, but also other instruments can be placed on it to measure the resultant of forces acting upon that vehicle. Accelerometers are the particular instruments used in the next hierarchal construction stage for an IMU, and these precision devices accomplish the needed specific-force measurements. A quantitative detailing of selected accelerometer properties, along with related excerpts from the classical mechanics, is appropriate in this limited discussion.

The simplest variant of an accelerometer can be pictured (fig. 7*A*) as a finite mass, M, supported on a horizontal frictionless slide bearing or base and restrained horizontally by a spring that obeys Hooke's law.[4] If this accelerometer device experiences an acceleration, a, in a horizontal direction as shown, then the spring must supply a tensile force, F, to make the mass, M, move along with the base. Through precalibration, the spring deflection is a measure of that force. Thus, this simple accelerometer measures F in the general expression F = Ma which leads to a solution for "a". The foregoing solution presumes that no component of the acceleration is attributable to gravity. If held stationary, the device can be used to measure the acceleration of gravity by alining its base with the local vertical.

Among the more elegant and sophisticated variants now in accelerometer devices, the designs particularly germane to this discussion are those that capitalize, in part, on the properties of pendulums. The necessary fundamentals can be drawn from the schematic of figure 7*B* in which a simple pendulum of the indicated length and mass is suspended from a pivot point, P. If the pivot point experiences a constant acceleration, a, to the right, as shown, then inertia will cause the mass to lag behind until the pendulum arm makes the determinable angle, θ, with the vertical. Viewed from P, it appears that the pendulum has been subjected to a torque, $\underline{T}$, whose magnitude is given by the simple expression

$$a = k\underline{T} \ ,$$

where k is a constant relating to the physical characteristics of the system. The key to a rigorous mathematical analysis leading to the foregoing expression is given by Landau and Lifshitz (1976, p. 11).

[4]The extension, x, of a spring that is subjected to a tensile force, F, is directly and linearly proportional to that force; that is, F = −kx, where k is a constant property of the spring.

If an equal, but opposite, torque is applied to the pendulum in the foregoing situation, then the pendulum will be returned to its undeflected, or "null," position. This encourages, therefore, a slight modification of the simple pendulum in the manner shown schematically in figure 7*C*. The intent now is to monitor with, for example, a beam of light and the proper photocells any incipient movement of the pendulum as acceleration occurs. Then, through a correctly designed feedback loop, an electromagnetically generated countervailing torque can be applied in just the magnitude needed to keep the pendulum in its null position. Thus, a precalibrated torque magnitude becomes the unique measure of an acceleration, and the desired data can be read out electrically. Within the limits of the magnetization characteristics of the torque motor material and current supply, the torque magnitude tracks the changing values of the acceleration. The stability and linearity of such torquing devices, however, leave much to be desired.

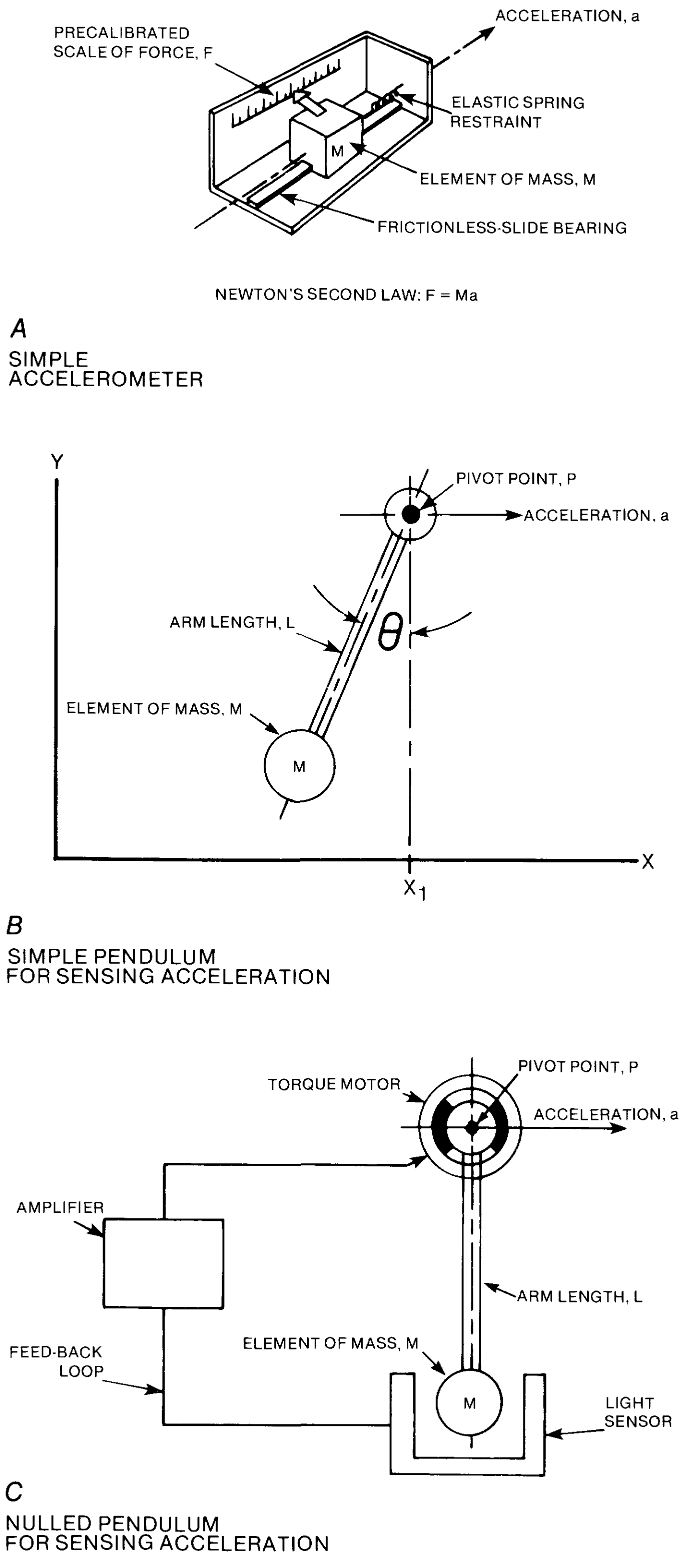


FIGURE 7.—Acceleration-measuring devices.

One significant refinement of the foregoing evolutionary review of selected acceleration-measuring devices carries the analysis to the particular type of accelerometer featured in the Survey instrument system. That refinement capitalizes on the gyro property that so precisely relates torque around one axis to angular precession rate around an orthogonal axis. In the discussion on gyros, the point that this property provides the means for very precisely generating a small torque was defined clearly. Such a torque can be used to restore the foregoing pendulum (fig. 7*C*) to null.

A cutaway and schematic view of the accelerometer selected for the APT instrument system is shown in figure 8. Note that it features a floated gyro similar to that shown in figure 5 but with one important difference—a pendulous mass built into the positive end of the spin axis. This type of accelerometer may thus be regarded as a floated pendulum with a gyro inside, magnetically suspended in the radial and axial directions with an electromagnetic device which can generate an electrical signal. As the pendulous mass (fig. 8) attempts to move downward under the influence of gravity, it tilts the float and, thereby, invokes an output from the signal generator proportional to the angle through which the float turns.

Note also that the red-shaded structure, including the float component, is free to turn around a vertical axis which is also the input axis of the gyro. If rotation should occur around that axis, then it will produce a consequent gyro precession torque that will counteract the tilt of the float. An external servoamplifier and a servomotor built into the accelerometer housing or case are the means for providing this rotation. By feeding the output of the signal generator, with the proper phase relation, into the servoamplifier, a closed-loop system is brought into operation wherein the precession torque of the gyro restores the pendulum float to the null position and keeps it there under varying conditions of acceleration. The formal name given to this measuring instrument is "pendulous integrating gyro accelerometer."

From the preceding discussion, note that, by observing the rotation rate of the red-shaded structure in figure 8 (servo-driven member), the consequent precession torque that is created around the gyro output axis is precisely determinable and is related directly to the specific force that was imposed along the gyro input axis. If, however, the total accumulated angle of rotation is observed instead of the rotation rate, then this is equivalent to performing the first time-integration of specific force, and, therefore, a precise measurement of integrated specific force is obtained, which has the units of velocity. (Recall that the time rate of change of velocity is equivalent to acceleration.) The resolver shown in the figure measures that angle, and the measurement data, which comprise the accelerometer output, are fed to the airborne computer. By using three of these accelerometers, oriented so that their input axes are mutually orthogonal, it becomes possible to measure the specific force on the vehicle carrying the IMU along any conceivable three-dimensional path of motion.

For the Survey APT instrument system, the accelerometer will carry the Draper Laboratory designation "APTA." The actual accelerometer unit is as shown in the artist's cutaway view of figure 9, and the

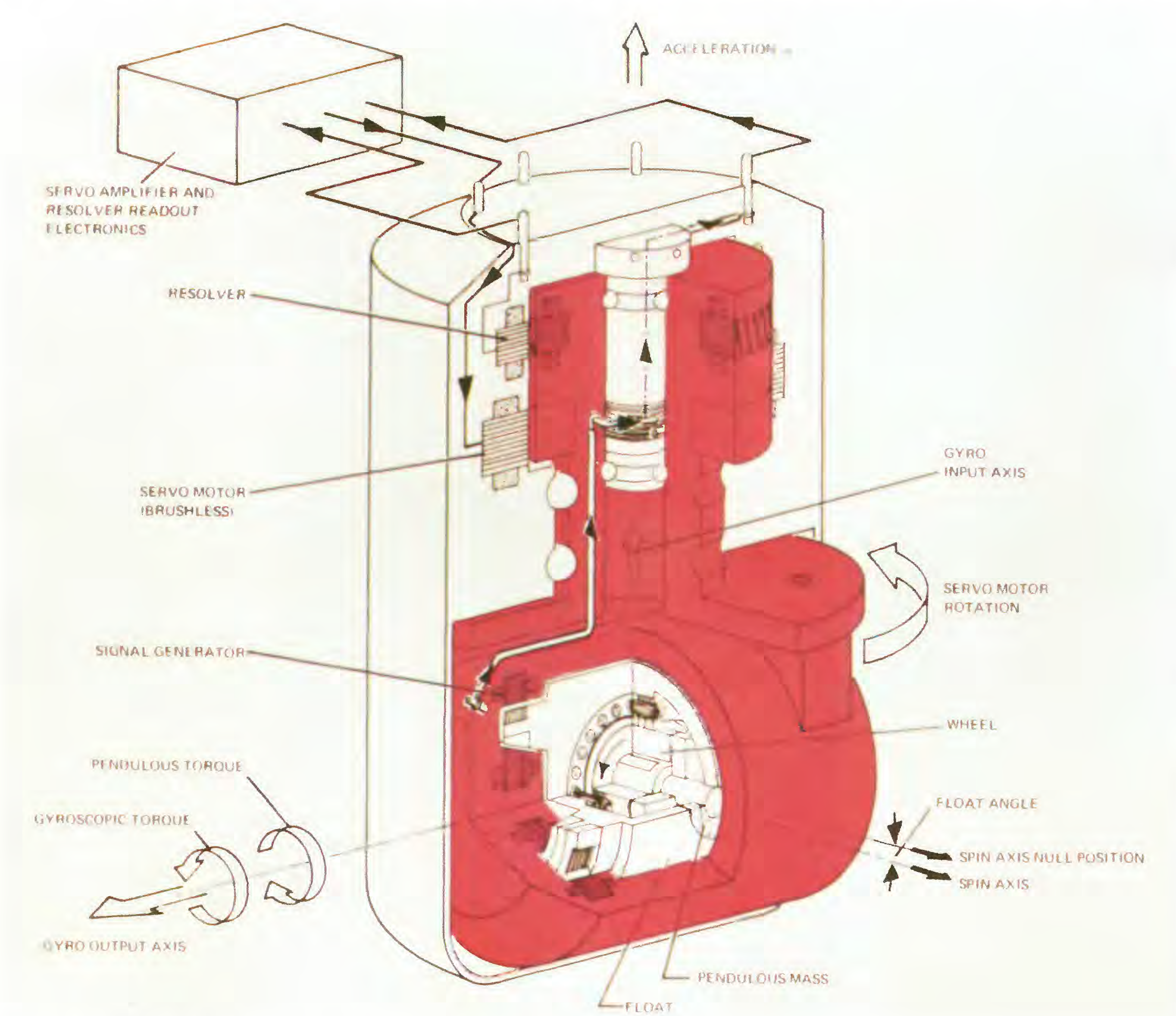


FIGURE 8.—The single-axis pendulous integrating gyro accelerometer.

principal features are as labeled. The pendulous integrating gyro component is shown in red, and the servo-driven member is in blue.

The unit features a gyro wheel driven at 24,000 rpm and supported by hydrogen-filled gas bearings. This provides an angular momentum of 2×10^{-4} N·ms under a constant [(±0.1°F)(±0.06°C)] operating temperature of 139°F (59°C). The neutrally bouyant and symmetrical float which houses the gyro has a built-in pendulosity of 5×10^{-2} N·m/g. The outermost cylindrical housing is approximately 3¼ in. (82 mm) long by nearly 2½ in. (62 mm) in diameter. The overall unit weighs 1.8 lb (0.82 kg), and its life expectancy, under continuous operation, is more than 5 years (44,000 hours). The design objective on sensitivity is to hold it to a small fraction of one-millionth of the force due to gravity.

In the mechanical design of the IMU, the three accelerometers are to be arrayed as shown schematically in the lower part of figure 6. This is an obvious departure from the more conventional arrangement in which the three sensitive, or input, axes of the accelerometers would have been oriented along the three basic Cartesian coordinate reference axes. In that conventional arrangement, the input axes of two accelerometers would be alined with the north- and east-pointing axes

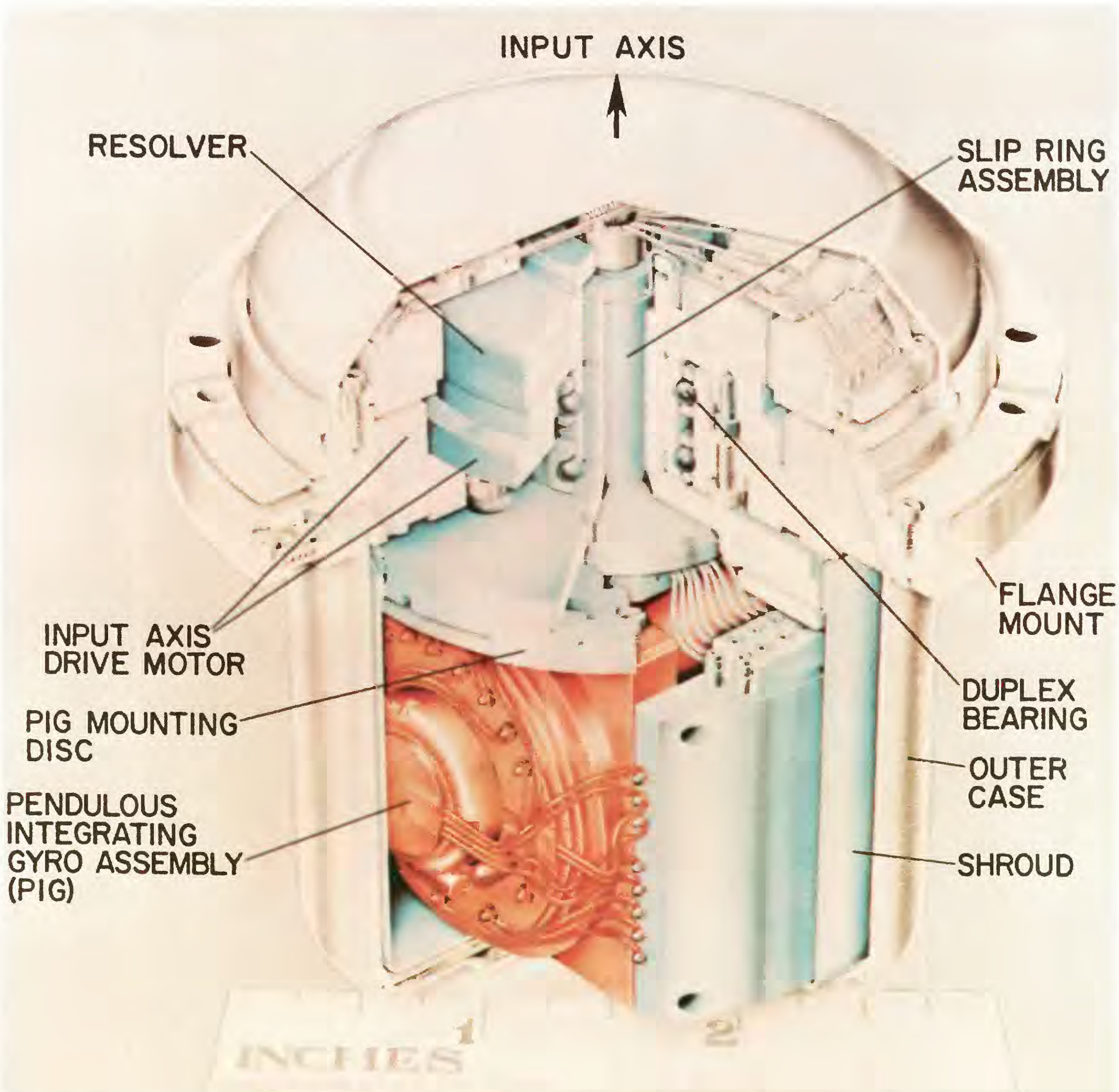


FIGURE 9.—The pendulous integrating gyro accelerometer.

of the IMU and, thus, would lie in the horizontal plane established by the stable platform. This would mean that the corresponding orthogonal unbalanced spin axes of the two accelerometers would sense no acceleration due to gravity, and, so, during much of the total operating period for the IMU, the respective two output-axis shafts would rotate very little. Yet for the accelerometer, the best performance is obtained when the output shaft experiences some rotational movement—enough to overcome the static friction of the bearing. Hence, the deliberate choice to orient the triad of accelerometers so that, in addition to the required orthogonality of the three input axes, each would continuously sense an equal share of the local gravity vector; that is, 0.58 g. If the origin of the three orthogonal reference axes is imagined as the corner of a cube, then, by placing in a vertical position the longest cube diagonal from that corner point, the correct orientational geometry is achieved. This means that each accelerometer input axis is at an angle of 54°44′ with the local vertical (gravity vector) or, as shown in figure 6, at an angle of 35°16′ with the horizontal plane of the stable platform.

The accelerometer triad occupies its own physically separate section of the stable platform and, by adopting a conical shape for this section, the thermal paths from accelerometers to enveloping heat exchanger (see discussion under "Temperature Control") are made short. For the same reason, shortness of thermal paths, the three required electronics modules are mounted between the accelerometer units; this also serves to minimize lengths of electrical leads.

Gimbal System

In the discussion on gyros, the frame that supports a gyro was identified as a gimbal. In the IMU, the function of the gimbals transcends that of simple support and extends to the more basic purpose of decoupling or isolating the gyros from the angular motion of the overall enclosing case mounted on some rigid base. For the APT system, the base is to be shock mounted on the cabin floor of a relatively light twin-engine aircraft. Nothing in the intended flight missions will require any violent maneuvers of the aircraft. Thus, the IMU gimbal structure can be simply that which is needed to isolate the stable platform, instrumented by the three gyros, from the standard operating three-dimensional angular motion (roll, pitch, and yaw) of the aircraft. As a matter of fact, slip rings provide unlimited freedom for the aircraft to yaw (turn), and freedom of plus or minus 190° is provided for it to roll and pitch.

The foregoing description of the operating environment for the APT system led to the decision to isolate the stable platform with a three-axis gimbal structure, the arrangement of which is shown schematically in figure 10. Starting at the inside and proceeding outward, the three axes in sequence are the vertical or azimuth axis (color coded in black), the elevation axis (color coded in red), and the roll axis (color coded in blue); the termini of these axes are seen to be in the elevation gimbal, the roll gimbal, and the enclosing case (also color coded in black), respectively. The arrowheads denote the positive-direction conventions. The two ends of each axis fit into a pair of preloaded ball bearings which, in turn, are mounted in steel sleeves to minimize change in preload with temperature. As an added precaution against change in preload through temperature differentials between gimbals, each gimbal is designed with a diaphragm at one end.

Mounted in the appropriate gimbal, or case, at one end of each of the three axes is a torque motor that provides a driving torque around the axis (fig. 10). In the APT system, the motor for the azimuth axis has a torque capability of 200 oz-in. (1.41 N·m), and the motors for the elevation and roll axes each have torque capabilities of 30 and 60 oz-in. (or 0.21 and 0.42 N·m), respectively. Each motor has a friction torque of less than 1.5 oz-in. (0.01 N·m). Modules containing the torque motors are shielded magnetically from other components in the system.

Mounted in the appropriate gimbal at the opposite end (from the torque motor) of each of the three axes is a precision multispeed resolver (fig. 10) to measure the angular position of the gimbal or case referenced to the three-coordinate-axis system (x, y, and z) being carried by the stable platform.

The gyro section, the accelerometer section, and their required electronics modules are all assembled, as part of the stable platform, into the elevation gimbal. The conical shape of the accelerometer section provides added stiffness for the stable platform. For maintenance service to the stable platform, access may be gained simply by separating the gyro and accelerometer sections. Gimbal and stable platform material is "6061" aluminum, which was chosen for its low density, good thermal conductivity, high mechanical stability attainable through heat treatment, and high resistance to corrosion. The natural resonant frequency of each gimbal is in the range of 75 to 100 Hz. Total weight of the stable platform is 36 lb (16.3 kg).

Resolvers

In the APT system, the sophistication of the IMU as a high-precision measuring device is enhanced through the ability of resolver components in the gimbal support structure to sense and read out with extreme accuracy the angles linking orientation (attitude) of the carrying vehicle (aircraft) with reference to the x, y, and z coordinate axes of the stable platform. This angle information is vital to the operation of the laser tracker component in the APT system, which is described in the section titled "Laser Tracker."

Among electrical devices, resolvers are really variable transformers that are descendants of earlier (World War II) computing units in gun directors that generated pointing data from radar moving-target de-

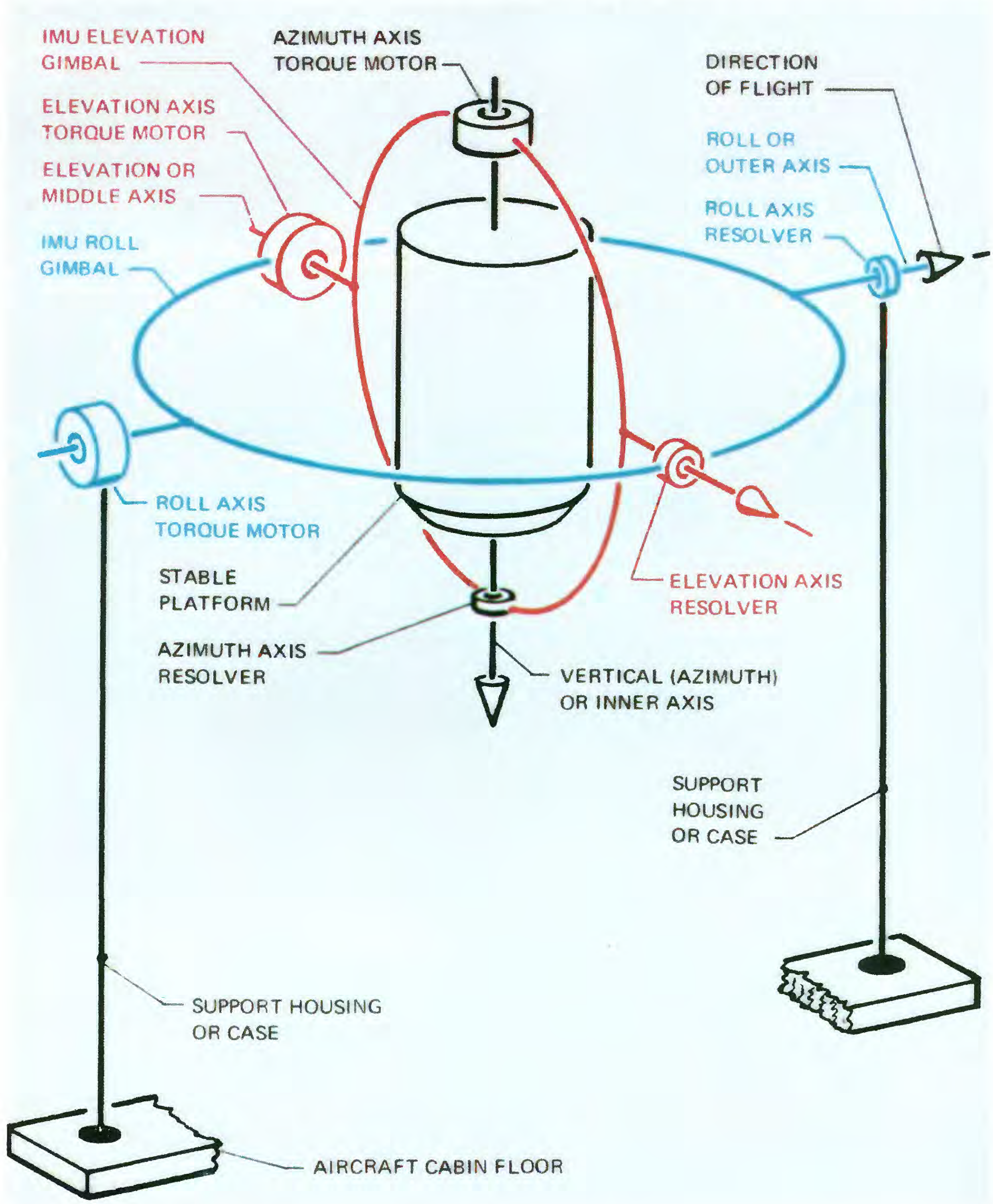
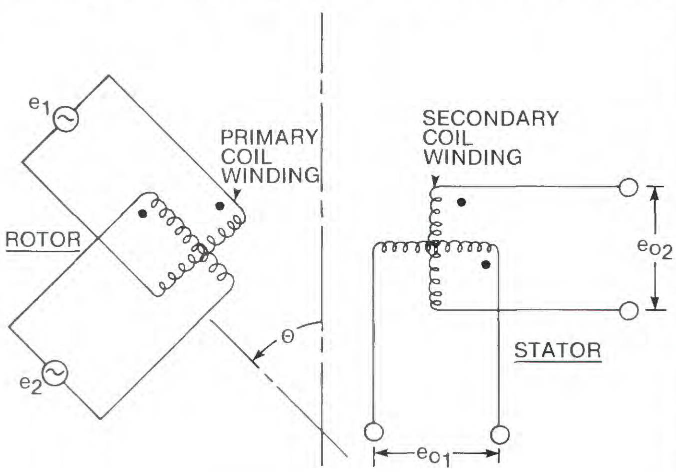


FIGURE 10.—Support structure for inertial measurement unit.

tection equipment. In essence, each resolver in the APT system comprises two concentric rings; the inner is called the rotor, and the outer, the stator. The rotor is mounted on a movable axis of the gimbal-support structure, and the stator is mounted in the companion gimbal where that particular axis terminates. The term "resolver" originated from its function of resolving electrical signals into components in accordance with the angular rotational movement of rotor relative to stator.

In a fundamental and elemental way, the rotor and stator may be visualized as carrying two distinct and separate coil windings apiece. The windings in a given pair are at right angles to each other as shown in the schematic of figure 11. In this schematic, the rotor and stator are drawn separately and side by side, but, in the actual resolver, they are concentric. If the rotor is imagined as the primary of an electrical transformer and its pair of windings energized with alternating electrical currents (at voltages e_1 and e_2 in fig. 11), then two magnetic fields are produced at right angles to each other. Field strength is related directly to number of turns in the winding and amperage of the impressed electrical current. These two magnetic fields of the rotor induce two corresponding output voltages (e_{o_1} and e_{o_2} in fig. 11) in the pair of windings carried by the stator which can now be imagined as fulfilling the role of the transformer secondary. The highest voltage a primary winding can induce in a secondary winding is



$$e_{o1}=k(e_1 \cos \Theta - e_2 \sin \Theta)$$

$$e_{o2}=k(e_1 \sin \Theta + e_2 \cos \Theta)$$

WHERE

k= ELECTRICAL COUPLING COEFFICIENT BETWEEN ROTOR AND STATOR

•= "IN PHASE"

FIGURE 11.—Resolver schematic and nomenclature.

when the two are parallel to each other; the induced strength dwindles to zero when the two orientations are mutually perpendicular. Thus, with particular reference to figure 11, if the rotor turns through an angle, θ, with respect to the stator, then the induced (output) voltages, e_{o_1} and e_{o_2}, may be computed precisely from the equations shown.

To meet the overall performance specifications for the APT system, Θ must be related to the foregoing determinations of voltage to a precision of 4 arc-seconds. To comprehend the real significance of this precision visualize a circle of 1-in. (25-mm) radius. A 1-in. chord will obviously subtend a central angle of 60° and, through simple proportion, a central angle of 4 arc-seconds, therefore, represents a chord length of only 18-millionths of an inch (4.6×10^{-4} mm). For the APT system, the resolver radius, given as a convenient round number, is 2 in. (51 mm) and, thus, an angular displacement, Θ, of 4 arc-seconds signifies a chord length of only 36-millionths of an inch (9.1×10^{-4} mm).

As a way of ensuring that the indicated level of precision is reached, the angular movement of the rotor is multiplied electromechanically in a way somewhat analogous to use of a suitable gear train. Pursuing that analogy, a gear ratio of, for example, 36 to 1 can be effectively obtained by adding a second set of windings in the design of the resolver such that a full 360° of electrical rotation, and, hence, signal resolution, takes place every 10° of rotor rotation. The result is described as a single-speed coarse and 36-speed fine output capable of providing the required accuracy for the APT system.

The 4-arc-second angular displacement referred to above is actually sensed as a rate of change in magnetic flux in the 36-speed resolver windings, which is observed as a proportional change in voltage of about 0.5 millivolt (mv) at the terminals of those windings. Electronic amplifiers are readily capable of precisely detecting and faithfully processing and relaying signals originating at these fractional millivolt levels.

Added insurance for maintaining high precision is gained by mounting rotor and stator in a steel ring to minimize thermal stress in their magnetic iron parts. Thermal tests have shown that the ambient temperature can range from 0°F to 120°F (−18°C to 49°C) with an error of less than 3 arc-seconds appearing in the resolver readout. Magnetic shielding also is designed around each resolver to eliminate or minimize errors caused by external magnetic fields. Finally, the structure and geometry of each resolver are shaped very precisely and are virtually distortion free. Similarly, the electromagnetic circuits are built very precisely for low power loss, symmetry, and stability. Principal features of a resolver are as shown in figure 12.

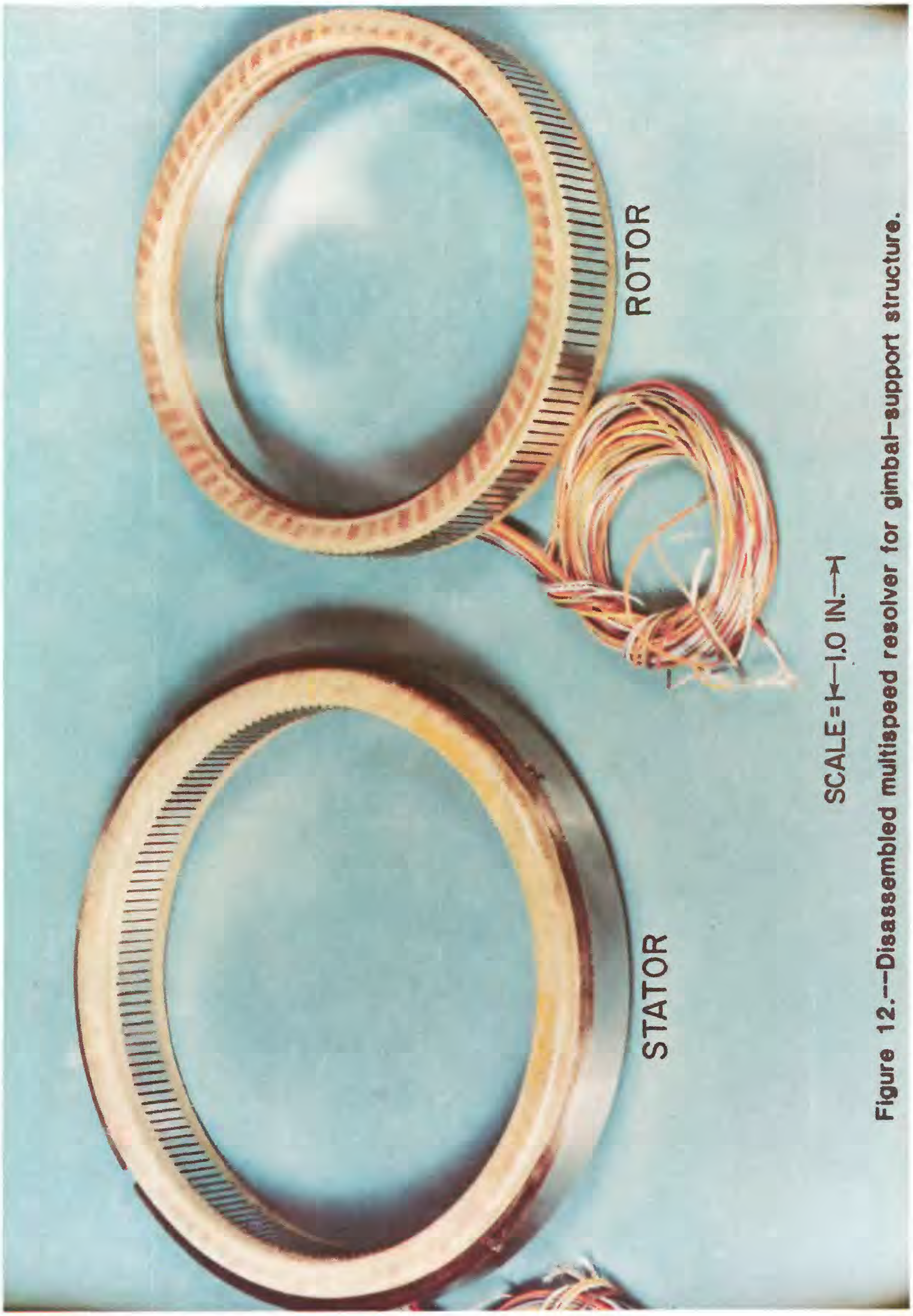


FIGURE 12.—Disassembled multispeed resolver for gimbal-support structure.

Feedback Loops

In preceding discussions, particularly those concerning resolvers, accelerometers, and gyros, references have been made to electromechanical elements whose function it is to measure angular displacements very precisely. Little has been said, however, as to the disposition of such information. An important ingredient in the ability of a stable platform to remain stable is not only a capability for sensing the magnitude and direction of any tilt (angular deviation) away from the desired stable orientation in three-dimensional space but also a capability for followthrough with appropriate corrective action. The followthrough is accomplished by way of servoamplifiers and drive motors that deliver electrical signals and mechanical torques or, as the entire process may be labeled, by way of feedback loops.

Basically, a feedback loop is a form of automatic control intended not only to sense the difference between an input, or reference, signal and an output, or status, signal but also to reduce this difference, or deviation, as it may be called, to near zero or acceptable limits. In examining the simplified electromechanical feedback loop, shown schematically in figure 13*A*, start at the electrical input signal, V_i, which is passed through the deviation sensor, a device that algebraically sums or compares it with the electrical signal, V_f, being fed back through the loop. Now, proceeding clockwise around the loop, the resultant deviation signal, V_d, from the deviation sensor is fed electrically into a servoamplifier from which suitable energy flows to a motor which must provide the mechanical muscle output to operate a controlled element in some desired way. As this element is moved, an electrical pickoff device generates load movement data which, in effect, is the feedback signal, V_f, which is relayed to the deviation sensor, thereby closing the loop. Thus, the forward part of the loop performs the control function, and the back part,the status-report function. The loop system works continuously to bring the deviation toward zero; that is, to reach and maintain a null position or balance so that $V_i \simeq V_f$. This action of the loop not only overcomes internal forces, such as friction, but also opposes external disturbances, all of which tend to force the controlled element into undesired movements or positions.

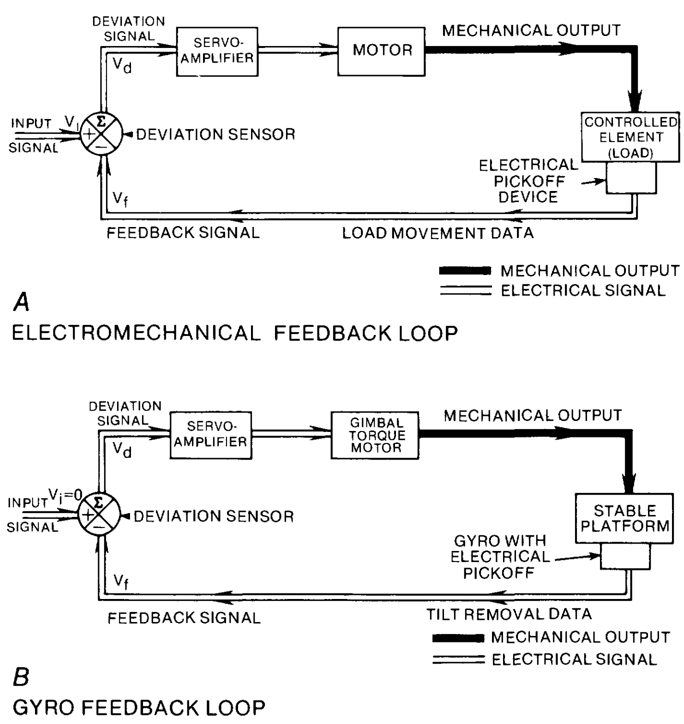


FIGURE 13.—Schematized feedback loops.

Two characteristics are significant in describing feedback loops. First, the controlling signal is not only a measure of a deviation, but it triggers a response in the direction needed to diminish that deviation. Second, design art must be applied to a determination of the rate at which the deviation is to be diminished so that the controlled element or load does not overshoot the desired position. Commonly, the disturbances acting on the controlled element are changing continuously, and, ideally, the feedback loop will follow those changes closely. However, as the loop response is quickened, the entire system bcomes less stable, and a point is reached at which it can be made to break into oscillation, or jitter. Thus, the real art is in designing the loop to be fast enough to perform its assigned task while properly limiting overshoot so as to remain stable. In general, stability is assured by incorporating compensation elements which precisely set the phase and amplitude relations throughout the loop.

In the IMU for the APT system, feedback loops play a vital role in enabling the stable platform to maintain continuously its proper orientation in three-dimensional space. A convenient example of one such loop may be taken arbitrarily from the stabilization servo loop designed and constructed to serve one of the three gyros on the stable platform. In this case, the desired position is given by the gyro null ($V_i = 0$). Any incipient tilt of the stable platform in any particular direction may be subdivided into three orthogonal component tilts (rotational movements) around the three orthogonal input (sensitive) axes of the gyros. Now, consider just one given gyro (fig. 13*B*). It will detect the incipient or small component tilt around its input axis and feed this information, as an angular deviation signal, V_f, back to the deviation sensor. Because V_i is zero, the output of the deviation sensor is simply $(-V_f)$, the deviation signal, which is fed to the servoamplifier. Continuing clockwise around the schematic (fig. 13*B*), the amplifier feeds electrically into a gimbal torque motor which provides the amount of power needed to rotate the platform and to begin to offset the component of tilt the gyro has detected. The electrical pickoff in the gyro monitors the restorative movement and relays these data on around the loop as a new and diminished feedback signal, V_f, into and through the deviation sensor for repetition of the cycle, if necessary. The system works continously to eliminate any angular deviation and to reach and maintain the desired null position. In doing this, the loop must oppose the variety of extraneous factors, such as internal friction or air turbulence on or motion of the carrying aircraft, that materialize as disturbance torques on the stable platform, tending to force it into undesired movements or positions.

Although figure 13*B* may be oversimplified, the principles are valid, and, if three such individual feedback loops are envisioned to serve the three respective gyros, then their combined performance provides the capability for holding the stable platform continuously in the desired orientation. This design is more complicated, however, than immediately meets the eye. Because the three loops act on a common element, the stable platform, and,because some nonorthogonality may have been built inadvertently into the platform mechanization, the loops could interact in the sense of cross coupling, which could then lead to oscillatory instabilities. This does not happen, however, simply because the problem has been recognized, studied, and solved by matching the dynamic characteristics of the respective loops against each other. Thus, the design choices have been made from a broad background of experimental data and experience.

Electronics

In the IMU, the electronic design concepts emphasize nonlinear techniques to minimize electrical power losses and standardization (insofar as possible) through diligence in recognizing common requirements. The electrical circuits are partitioned so that only the precision control portions associated with the inertial instruments are located on the stable platform. This includes circuits for a telemetry system to provide state-of-health information about the inertial components. Other circuits, such as those involving power supplies and resolver encoders, are kept external to the stable platform and gimbal system. The external circuits also include the capability for torquing the gyros.

The electronic design embraces electrical circuits that are the means for creating the stabilization subsystem, which must isolate the stable platform from aircraft angular motion as has been described functionally in the preceding section. Three individual stabilization circuits or loops (azimuth, elevation, and roll) are needed to separate out all possible angular motion of the aircraft and to maintain the stable platform at the desired orientation. The three individual loops are shown schematically in figure 14, but their explanation should be followed with frequent reference to the schematic for the IMU support structure (see fig. 10).

Note first (fig. 14) that the deviation signals for instructing the gimbal torque motors come from the gyro signal preamplifiers located on the stable platform. Now, with respect to the azimuth stabilization loop, obviously any "yaw" motion of the aircraft simply tends, because of slight amounts of friction in the gimbals, to drag the stable platform around its azimuth axis, and the deviation signal from the z gyro is, therefore, fed (fig. 14) directly around the loop to the azimuth axis torque motor to null out the incipient gyro motion.

Next consider the elevation stabilization loop which obviously concerns aircraft "pitching" motion. Such motion tends, because of friction, to drag the elevation gimbal around the elevation axis which can prompt deviation signals from either one or both of the x and y gyros (depending on aircraft heading) as the stable platform begins to tilt away from its locally level position. These signals (proportional to the x and y components of the pitching motion) are fed to the primary windings of the gyro signal resolver (fig. 14). One secondary winding is alined with the elevation axis to supply the elevation component signal that traverses the elevation loop to the elevation axis torque motor. The object is to null out the incipient pitching motion as measured by either one or both of the x and y gyros as appropriate.

Finally, consider the roll stabilization loop which obviously concerns aircraft "rolling" motion. Such motions tends to drag the roll gimbal around the roll axis which, again, can prompt deviation signals from either one or both of the x and y gyros (depending on aircraft heading) as the stable platform beings to tilt off level. The roll signal is supplied by the other secondary winding of the gyro signal resolver and traverses the roll loop to the roll torque motor. And, again, the object is to null out the incipient roll motion as measured by either one or both of the x and y gyros as appropriate. The reduction in gain in the roll loop, due to aircraft pitch angle, has no practical effect on servo performance in isolating the stable platform from aircraft motion.

Just before the start of a flight mission, the IMU must be leveled approximately and oriented with respect to geographic north and east in preparation for its self-alinement procedures. Thus, provision is made for operating the three gimbal servoamplifiers and their torque motors independently in an angle-command mode under computer control. In this mode, the error signal for each servoamplifier is formed when the computer determines the difference between the angular position input from each two-speed resolver on the three gimbal axes (see fig. 10) and the respective command angles that the APT system operator has entered by way of the keyboard. The three error signals then are transmitted to the appropriate servoamplifiers through the digital-to-analog (D/A) output channels (see "Computer" section), and, thus, each gimbal is driven to the corrected commanded angle. Command angles also may be generated by a computer program that allows for automatic operation. For testing purposes without the computer, the gimbal angles can be commanded manually using hand-settable resolvers as control transformers to generate the error signals in a fully analog configuration.

Temperature Control

Work began very early in this project to shape thermal design concepts that would assure control of the stable platform temperature environment to a high degree of precision. Alinement stability of the stable platform is a function of gyro and accelerometer performance, which, in turn, is extremely sensitive to temperature change. The concepts that evolved resulted in the thermal control system shown schematically in figure 15.

Immediately evident is the fact that a high operating temperature in the compartment occupied by the stable platform (fig. 15) allows aircraft cabin air to be the ultimate heat sink over a wide range of ambient temperatures [up to, for example, 115°F (46°C)]. Recall that the gyro and accelerometer rotors are operated in floats suspended in a viscous fluid held at a constant temperature of 139°F (59°C). The progressive downward steps in environmental temperature, proceeding outward from rotors to aircraft cabin (fig. 15), are as labeled or can be approximated. Strategic placement of small electrical heaters along the thermal gradient guarantees the needed precision in temperature control regardless of gimbal orientation. Thus, for much of the expected range of field-operating conditions, heat will be supplied to the instrument system, and the need for a cooling system is obviated.

The inertial instruments (gyros and accelerometers) and their internal electronics are mounted on the sta-

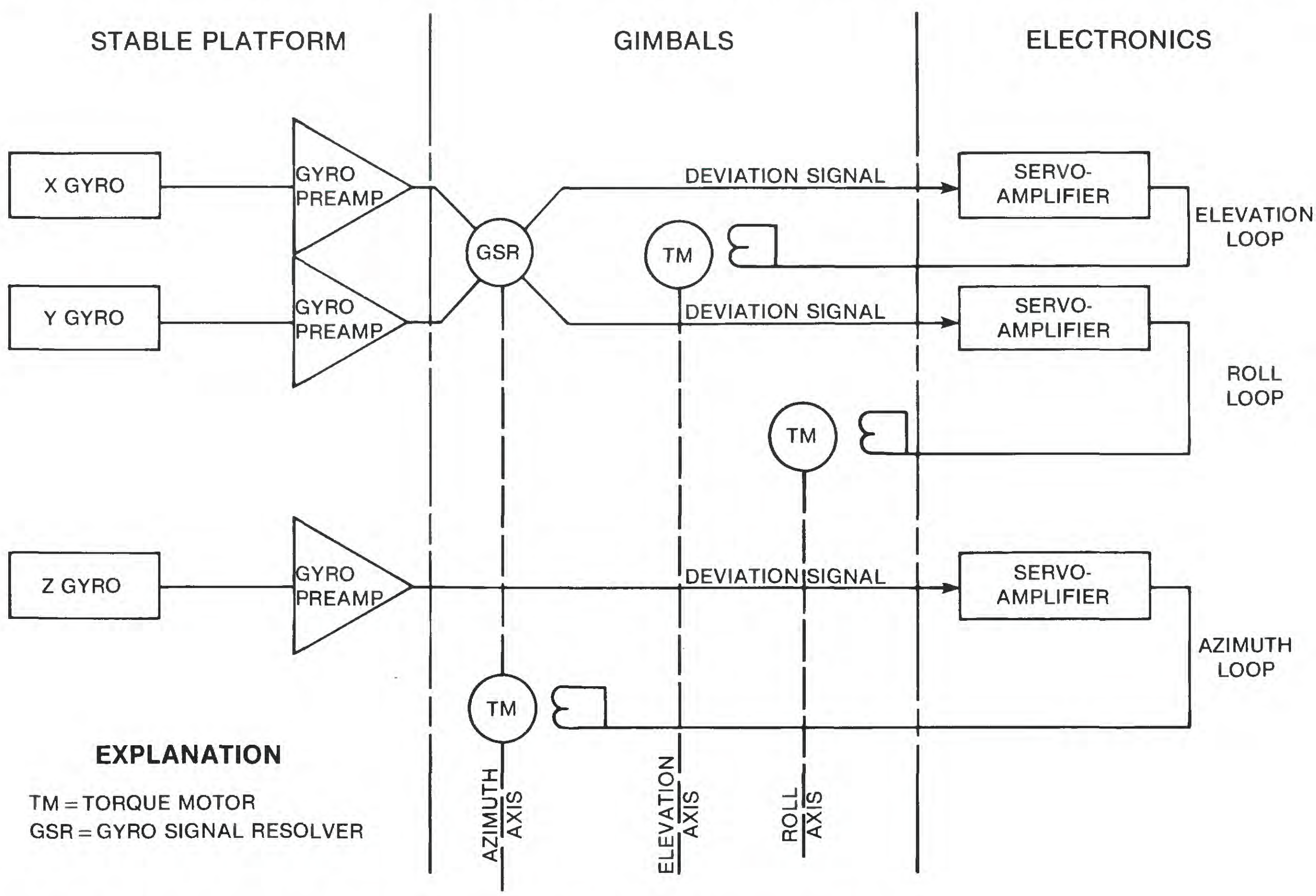


FIGURE 14.—Electrical circuits for stabilization subsystem of inertial measurement unit.

ble platform in a way that permits all the heat they generate to be conducted to the stable platform heat exchanger, which completely surrounds and is integral with the platform. High-wattage dissipators in the electronics package are placed closest to the heat exchanger to shorten thermal paths and to minimize thermal disturbances. Voids in the entire enclosure are filled with insulation to suppress convective air currents around the platform.

A constant-speed fan, mounted on the elevation gimbal, provides steady air flow at a constant temperature through the stable platform heat exchanger (fig. 15). As it transits the exchanger, the air collects and carries away heat from the platform. The air then passes through the inner chamber of the heat exchanger that is wrapped around the elevation gimbal, losing heat by conduction to the outer chamber of the exchanger during this passage. This inner air loop is completed when the cooled air moves back to the intake side of the constant-speed fan to repeat the cycle.

The rate of movement through the outer air loop will vary, depending on the temperature differential between the outer chamber of the heat exchanger on the elevation gimbal and the ambient air in the aircraft

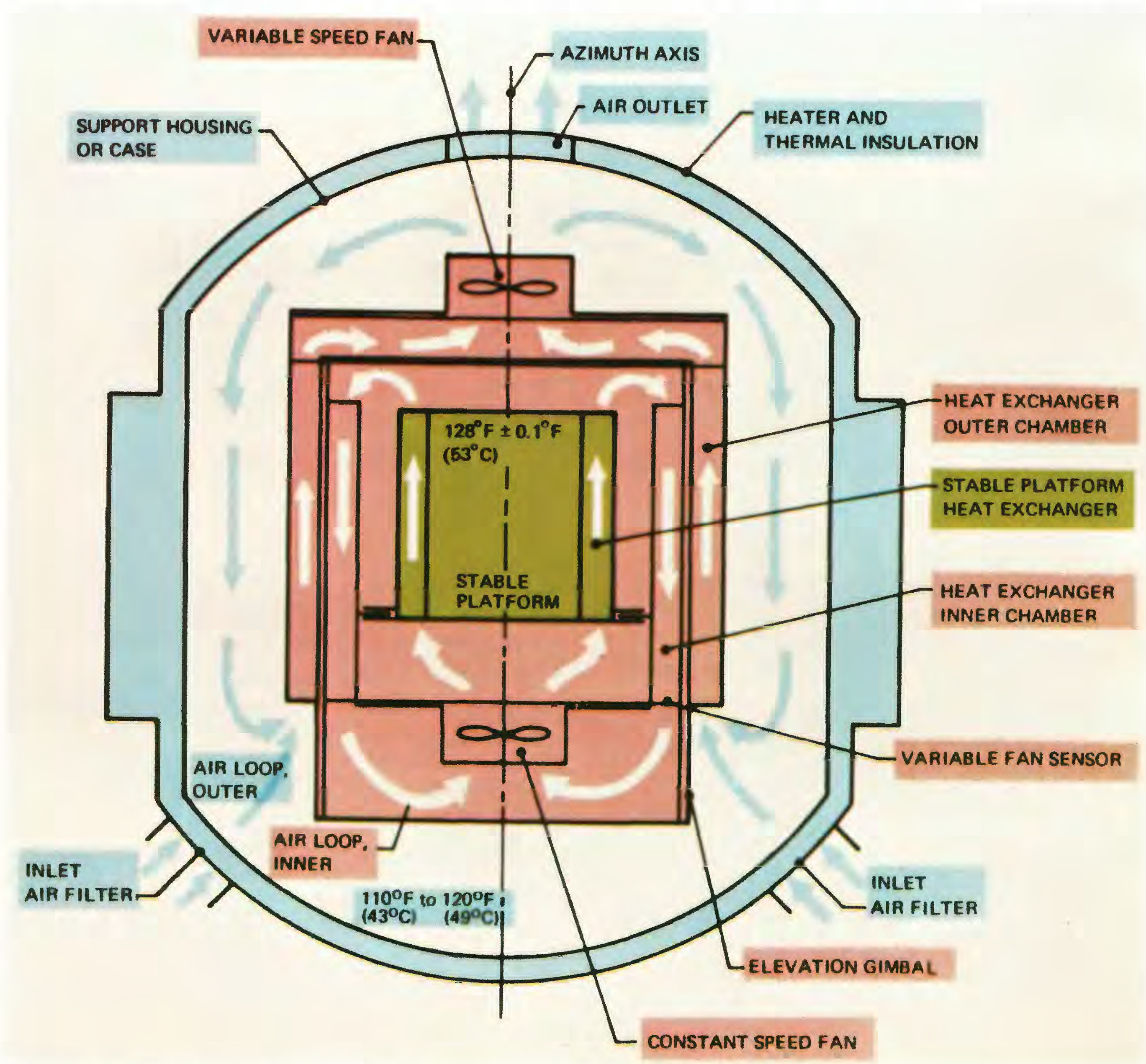


FIGURE 15.—Thermal control schematic for inertial measurement unit.

cabin. Thus, a variable-speed fan, also mounted on the elevation gimbal, is required (fig. 15) to circulate the air in this outer loop. Suitable control of the variable speed is achieved through a thermistor sensor placed in the inner air loop. By choosing its location (fig. 15) in the inner chamber of the dual-chambered heat exchanger, it serves as an early-warning device to forestall any change in air temperature that might alter the steady and known rate of heat dissipation required for the stable-platform enclosure.

Evolution of the preceding thermal control concepts had considerable impact on the design shapes for the stable platform and the internal electronic modules that are mounted thereon. Figure 15 is highly schematic to illustrate the thermal concepts. A more detailed and realistic view is shown in figure 16, which also identifies the inner and outer air loops.

In brief, air moves around the inner loop from the output of the constant-speed fan (at top center of the elevation gimbal) downward through the inner chamber of the dual-chambered heat exchanger, then upward through the heat exchanger that encloses the stable platform, and finally back to the intake side of the fan.

Air moves around the outer loop from the output of variable-speed fans (near the top of the elevation gimbal) downward through the enclosure formed by the support housing, or case, then upward through the outer chamber of the dual-chambered heat exchanger, and finally back to the intake sides of the fans. Although two variable-speed fans are shown in the vertical section of figure 16, some tests have suggested that a three-fan array may be preferable, spaced 120° apart in plan view, for better symmetry of air flow. When a prototype of the APT system is built entirely to the basic design specifications, it will feature the most appropriate array.

In the realm of temperature control, the outer air loop is really a coarsely controlled buffer zone between the outside world (aircraft cabin), where the virtually uncontrolled ambient temperature range is large, and the inner air loop, where control is precise and tight. The outer air loop connects with cabin air through an annular plenum chamber, which is protected by the annular inlet air filter shown in the lower part of figure 16. The fan shown passing through this chamber and filter can exhaust air to the cabin from the outer air loop. The coarse control, whereby the temperature range is held between 110°F (43°C) and 120°F (49°C), is simply the manipulation, as needed, of this fan and electrical heaters in the outer loop.

The foregoing details concerning temperature control constitute a fairly rigorous portrayal of what was envisioned in the original design for the APT system. The changes wrought by the decision to build the first prototype with surplus Air Force gyros and accelerometers are summarized most significantly by noting the following temperature regimen that is to be maintained:

1. Viscous fluid which buoys floats for gyro and accelerometer rotors, now to be held constant at 135°F (57°C).
2. Inside stable-member package (see figs. 15, 16), now to be held at 180°F (42°C).
3. Outer air loop (figs. 15, 16), which is the space between the elevation gimbal and the support housing, or case, now to be held at or near 66°F (19°C). Cooling coils, built into the housing, will circulate fluid at 59°F (15°C), as needed.
4. Aircraft cabin air, around the outside of the housing, will now be conditioned (heated or cooled), to be held in the range 70°F (21°C) to 80°F (27°C).

Performance Dynamics

The preceding sections have described the principal individual components, structures, and subsystems that comprise the IMU, the heart of the inertial navigator. In the functioning airborne APT system, the inertial navigator must be supplemented by other elements, such as the laser tracker and the onboard data processing unit. Before venturing into descriptions of these other key elements, however, it is appropriate to introduce and explain some of the performance dynamics that will be required of the inertial navigator. The intent is to explain by relying primarily on physical and geometric concepts that appeal to the intuition rather than on rigorous mathematical proofs. The latter are available in a number of excellent textbooks, several of which are in the list of references at the end of this paper (see especially Britting, 1971; Broxmeyer, 1964; Pitman, 1962).

A variety and substantial number of inertial navigation systems have been conceived, designed, and built over the past three decades. Most system types have been demonstrated successfully in flight, and some are now in regular production and use. The choice of a system depends on the particular flight application intended, and, as stated earlier (see section titled "Inertial Navigator"), the IMU for the APT system was chosen to be mechanized in a tangent-plane orientation. This means that, to a very good approximation, two axes (x and y) out of the set of three orthogonal axes, created by the stable platform in the IMU, will at all times define a plane perpendicular to the actual gravity direction at the position where the alinement took place.

All inertial systems require measurement of the magnitude and direction (referenced to the stable-platform x-y-z axis orientation) of the specific force vector acting on the carrying vehicle, and all must cope with the fact that the force of gravity is indistinguishable from the force associated with acceleration of the vehicle when it is in motion; that is, only their vector difference can be measured.

Because the inertial navigator in the APT system will always operate in a near-Earth environment, with emphasis on high-precision measurements in a relatively localized survey area, it is especially important to develop first a clear understanding of how the inertial navigator "feels" or senses that environment before any motion of the carrying vehicle is considered. This is tantamount to defining the initial or starting conditions for the inertial navigator because it is basically a dead-reckoning device, and these initial data must be given to at least the precision required during subsequent operation of the system as a navigator.

Every point on Earth has a gravity vector associated with it; that is, the acceleration due to gravity has a specific magnitude and direction. When the vehicle

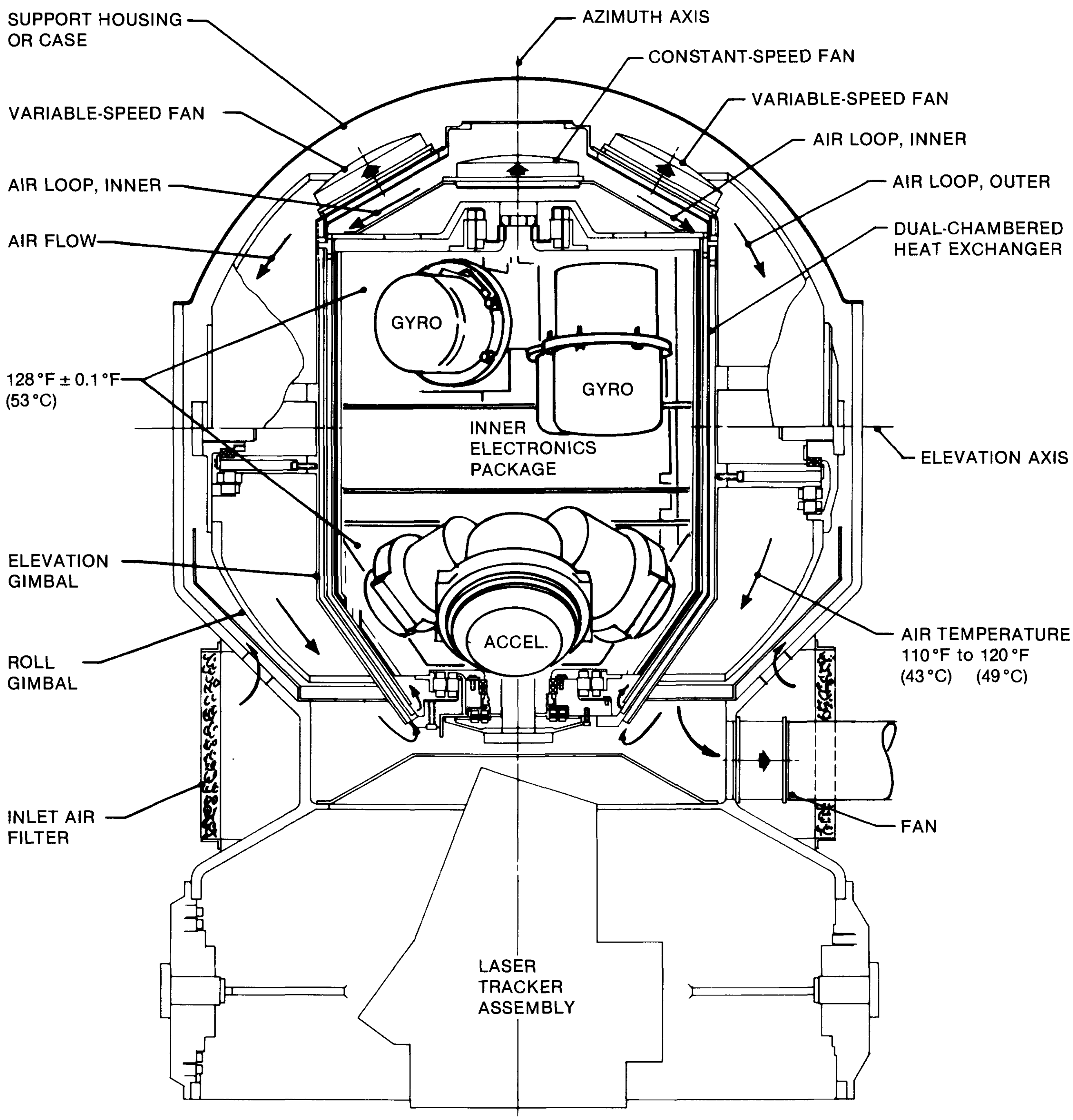


FIGURE 16.—Support and thermal control structure for inertial measurement unit.

carrying the inertial navigator is not in motion, the accelerometers will sense and measure only the gravity vector, and, in fact, this information is used initially to level the stable platform. In a tangent-plane orientation, the x–y plane of the stable platform senses no component of acceleration due to gravity. In the APT system, because of the geometry of the chosen accelerometer array (see fig. 6), the stable platform is tangent-plane level when the three accelerometers sense equal components of the specific force due to gravity, and the z axis, thereby, is alined with the actual gravity vector.

The rate of the Earth's rotation about its polar axis is a known quantity that is everywhere the same, and the motion of any point on the surface is easterly. Thus, it can be said that every point on Earth has the same angular rotation vector associated with it.

The aforementioned two vectors, gravity and Earth rotation, dominate in characterizing the dynamic Earth environment. Further discussion is needed, however, to explain more adequately how the inertial navigator is affected by and senses them and how the stable platform must be commanded to maintain continuously the desired orientation—x axis pointing north, y axis pointing east, and z axis pointing downward along the local vertical at the alinement point—regardless of the local area where the instrument is operated on or above the Earth. If it were not so commanded, then its gyros would know nothing better than to hold the x-y-z axis orientation of the stable platform inertially fixed in whatever startup orientation was given, despite the steady and continuing rotation of the Earth. If, in that situation, the vehicle carrying the inertial navigator stood still for 12 hours (one-half a revolution of the Earth), this would provoke the graphic discovery that the stable platform was by then virtually upside down—it would not have changed its orientation in space to keep up with the rotating Earth. As the following discussion will reveal, all the information needed for the initial self-alinement of the x, y, and z axes is contained in the accelerometer signals. The stable platform is simply turned to and held in the correct orientation by applying, around the appropriate axes, gyro torques, the magnitudes of which are calculated by the onboard computer in accord with the functional relations that are based on the accelerometer signals.

In seeking further enlightenment, through the following discussion, reference to a cutaway perspective view of an idealized and homogeneous ellipsoidal Earth (fig. 17) is useful. Note first how the stable-platform set of three orthogonal axes (in dark blue) must be drawn at each of the several points P_1, P_2, and P_3, shown in figure 17, to maintain correctly the stipulated orientation. For convenience, P_1, P_2, and P_3 are chosen on the Earth's surface in the northern hemisphere.

Several significant facts stand out in figure 17; for example, observe how the direction of the local vertical, the z axis, which is normal to the ellipsoid surface, changes with the location of P_1, P_2, and P_3. As a matter of fact, the definition of astronomical latitude, of special importance in geodetic surveying, is the angle, Φ, that the local vertical (z axis) makes with the equatorial plane. Thus, Φ ranges between the limits of 0° at the equator, where the z axis lies in the equatorial plane (see fig. 17), to 90°N at the north geographical pole, where the z axis is coincident with the polar axis or perpendicular to the equatorial plane. Note also that, in the definition of latitude, the local vertical need not, and, in the real world usually does not, pass through the Earth's center of mass, C, except at the equator and poles.

Observe, also (fig. 17), that P_1, P_2, and P_3, are chosen along a single meridian at locations, in the same respective order, at the equator, at an arbitrary middle latitude, and at a short distance from the north geographical pole. If the set of three orthogonal axes is studied at each point, then the difference in orientation between any two points is seen to be described as a rotation of the set around the y axis through an angle equal to the change in latitude between those points. Here, then, is a fundamental piece of orientation knowledge for the set of three axes in assessing the effect of "northward" or "southward" components of changes in location of the inertial navigator on or over the Earth's surface.

Now, if a similar reasoning process can be developed for the eastward or westward components of change in inertial navigator location, then the general base will have been completed for understanding any conceivable change in location. Immediately, however, eastward and westward components are seen to connote some sequence of points P_1, P_2, and P_3 on a single parallel of latitude. The reasoning then becomes complicated by the subtleties of the convergence of the meridians as successive northerly parallels of latitude are involved. The situation can be neatly bracketed, however, by describing two limiting cases.

First, consider P_1, P_2, and P_3 as lying on the equator ($\Phi = 0°$). Imagine further that, at each point, a set of three axes (x, y, and z) is drawn in the previously specified orientation. Now it can be seen (fig. 17) that, as the orientations at P_1, P_2, and P_3 are examined, the z axis at each location lies in the equatorial plane and is perpendicular to the equator (locally vertical). For the z axis to maintain this perpendicularity, with a change in location along the equator, requires simply a rotation of the set of axes "around the x axis" an amount equal to the angular change in longitude.

Second, consider the case where P_1, P_2, and P_3 are lying on a parallel of latitude just below the north geographical pole, near a latitude, for example, between 85°N and 90°N (85°N < Φ < 90°N). Again, imagine a set of three axes drawn at each point. Now, it can be seen that, as the orientation of the set is examined successively at P_1, P_2, and P_3, the z axis moves very little to remain locally vertical; the principal rotation for the set of axes must be "around the z axis" so that the x axis can be kept pointing toward the north pole. In the limit, as the latitude approaches 90°N, this angular amount would equal the angular change in longitude, and, at the pole, the convergence of the meridians equals the angular change in longitude.

For any parallel of latitude intermediate between the two foregoing limits, as the orientation is examined at P_1, P_2, and P_3, the required rotation of the set of three axes will involve some combination of two angular components around just the x and z axes. Quantification of these two particular components for the respective two axes now merits interpretation.

With no change in latitude involved in east-west changes in location, the z axis maintains a constant angle with the equatorial plane (fig. 17). This means that no rotation is needed around the y axis and affirms the statement in the preceding paragraph that, if the set of three orthogonal axes is studied for any two points, P, at the same latitude, then the difference in

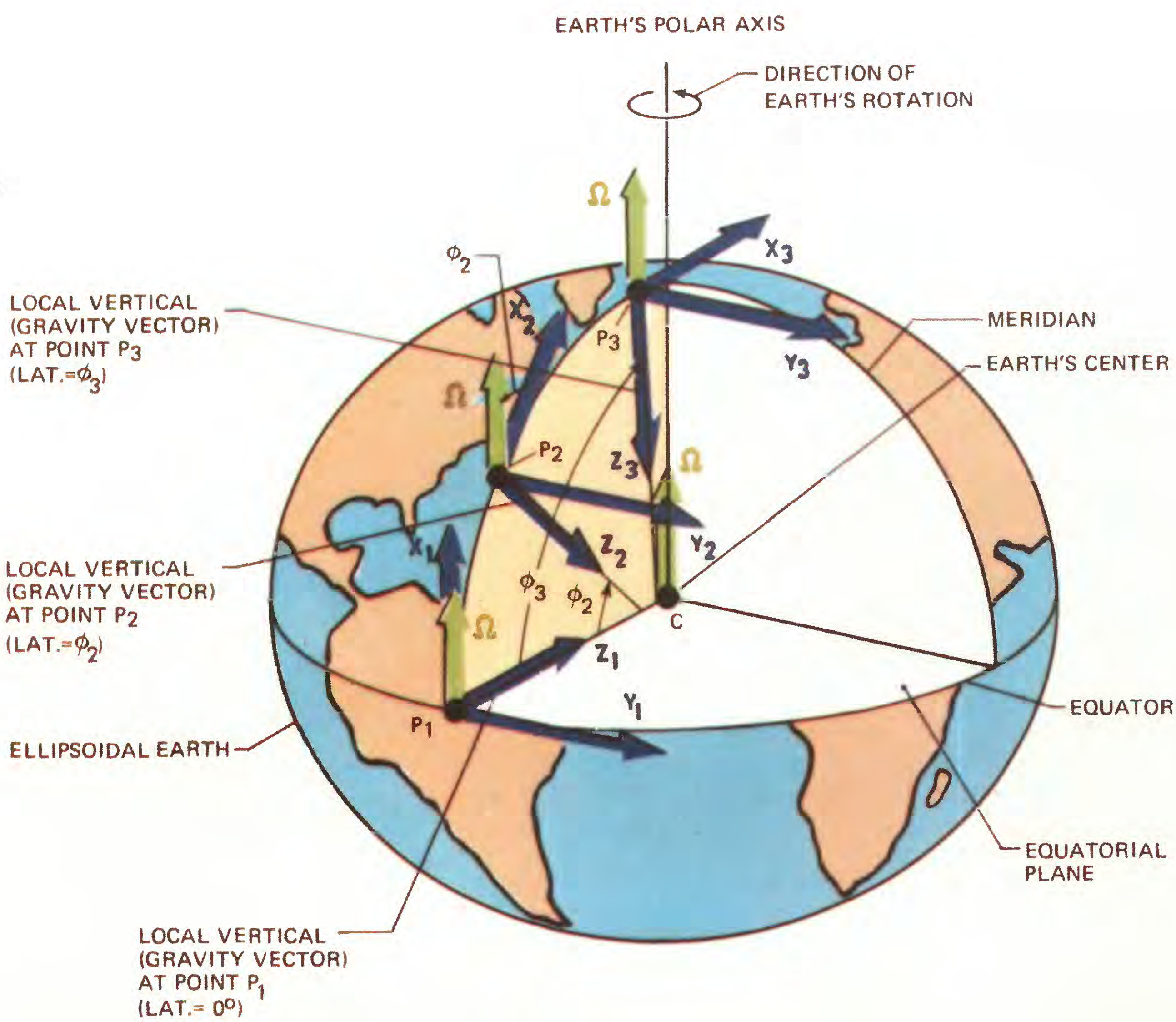


FIGURE 17.—Orientation of locally level three-coordinate reference scheme at selected points on an idealized ellipsoidal Earth.

orientation is simply accounted for as a rotation of the set by some angular amounts around the x and z axes.

The angular amount around the z axis (Rot_z) can be equated immediately to the convergence, Θ, of the meridians between the two points. This follows by visualizing the spherical geometry in figure 17 and by noting that, with no rotation allowed around the y axis, the only rotation that can keep the x axis pointing north is that which occurs around the z axis, and it must equal Θ. Geodesy texts (see, for example, Hosmer, 1948, p. 288) give the value of Θ (for a spherical Earth) as the product of the angular change in longitude, $\Delta\lambda$, and the sine of the chosen parallel of latitude. In simple mathematical form, this means

$$Rot_z = \Theta = |\Delta\lambda \sin \Phi| \ .$$

This relation can be tested immediately for agreement with the descriptions already given on how the set of three coordinate axes must be rotated for east-west changes in location of the inertial navigator along either of the two limiting parallels of latitude. Thus, at the equator ($\Phi = 0°$) obviously Θ will be zero, which signifies no rotation around the z axis as alternate locations are considered around that latitude. At the other limit, as Φ approaches 90°N, Θ closely approaches $\Delta\lambda$, which signifies that, in the limit, all the rotation will be around the z axis. Agreement, therefore is achieved.

All that remains now is to specify, in the above situation, the angular amount of the required rotational component about the x axis (Rot_x). Again, it is important to visualize the spherical geometry by studying figure 17. Note that Rot_x is the only means (with no rotation allowed around the y axis) for keeping the z axis pointing in the direction of the local vertical, and the magnitude of this rotation is measured by the length of circular arc, along the parallel of latitude, that represents the eastward or westward change in location from one point to another. This circular arc is the measure of the angle through which the z axis must be swept to stay properly oriented as a consequence of the eastward or westward change in location. For a given change in longitude, $\Delta\lambda$, the length of circular arc is obviously a function of the latitude, Φ, and, from texts on spherical geometry (see, for example, Ayres, 1954, p. 188), this is given (for a spherical Earth) as the product of $\Delta\lambda$ and the cosine of the particular parallel of latitude. Thus, in simple mathematical terms,

$$Rot_x = |\Delta\lambda \cos \Phi| \ .$$

If this relation now is tested for agreement with the earlier observations on conditions to be met for east-west differences in location of the inertial navigator along either of the two limiting parallels of latitude, then there follows:

1. At the equator ($\Phi = 0°$), $Rot_x = |\Delta\lambda|$.
2. Near the pole ($\Phi \rightarrow 90°N$), $Rot_x \rightarrow 0$.

Once again, agreement is achieved, and the foregoing developments have supplied the needed corollary piece of orientation knowledge for the set of three axes, in assessing the effect of "eastward" or "westward" components of changes in the location of the inertial navigator on or over the Earth's surface.

In a summary and generalized way, therefore, the change in orientation of the set of three orthogonal axes, with changes in location of the inertial navigator, is described in the following succinct form: For northward (+) or southward (−) components of location change,

$$Rot_x = 0,$$

$$Rot_y = -\Delta\Phi,$$

$$Rot_z = 0.$$

For eastward (+) or westward (−) components of location change,

$$Rot_x = \Delta\lambda \cos \Phi,$$

$$Rot_y = 0,$$

$$Rot_z = -\Delta\lambda \sin \Phi.$$

Note the positive and negative conventions adopted for changes in location in each of the four cardinal direction. As for angular rotational movement of the set of three axes, the positive sense is in accord with the right-hand-thread rule. Heretofore in this section, only rotational magnitudes have been shown with no regard for positive or negative directions.

To a degree, the preceding discussion has anticipated how the inertial navigator senses the Earth's rotation. Obviously, if eastward changes in location are analyzed with respect to how the orientation of the x-y-z set of axes in the stable platform must be changed, then this is very close to analyzing for Earth rotation. The picture can be enhanced, however, by observing (fig. 17) how the Earth's rotation is indicated by a vector, $\vec{\Omega}$, drawn (in light green) vertically upward along the polar axis from the Earth's center, C. The length of this vector represents the magnitude of the constant angular velocity; the direction of the vector (right-hand-thread rule) signifies that the Earth's rotation is easterly, as shown by the small circular arrow near the upper limit of the polar axis. Because every point on Earth rotates in the same direction and at the same angular velocity around the polar axis, a similar vector is drawn (in light green) vertically upward at each of the points P_1, P_2, and P_3. Thus, in the same way that the vector drawn at point C represents a complete statement on the magnitude and direction of rotation of the ellipsoidal Earth, so the vectors drawn at P_1, P_2, and P_3 represent complete statements on the same magnitude and direction of Earth rotation at these points.

It is now a relatively easy step to account properly for the Earth's rotation simply by resolving the rotation vector into its equivalent components along the x-y-z axes of the stable platform. Noting the situation at P_1, P_2, and P_3 (fig. 17), no component will be in the y-axis direction as long as the stable platform stays correctly oriented. At P_1 (on the equator), the entire vector, $\vec{\Omega}$, is in the x-axis direction. Thus, for any location on the equator, the stable platform needs only to be rotated around its x axis at the full magnitude, Ω. Similarly, at point P_3, as locations are visualized progressively nearer the pole, the limit indicates that the entire vector, $\vec{\Omega}$, is in the negative z-axis direction. Thus, the stable platform needs only to be rotated around its z axis at the full magnitude, Ω, in the negative direction.

A general statement can now be drawn, for any inertial navigator location in the northern hemisphere, describing how the stable platform must be commanded to preserve its correct orientation despite the rotating Earth. The ingredients for this statement emerge from the situation at P_2 (fig. 17). Note first that the Earth's rotation vector, $\vec{\Omega}$, can be resolved into an equivalent set of two components, Ω_{x_2} and Ω_{z_2}, along the x axis and z axis (extended), respectively. Note also that the angle between $\vec{\Omega}$ and the x axis is the latitude, Φ_2. Thus,

$$\Omega_{x_2} = \Omega \cos \Phi_2 \qquad \text{and} \qquad \Omega_{z_2} = -\Omega \sin \Phi_2.$$

For any latitude, Φ, therefore, the general statement simply requires a constant rate of rotation of the set of axes, and, by implication, the stable platform of the IMU, so that the component rotational rates are

$$\omega_x = \Omega \cos \Phi$$

$$\omega_y = 0$$

$$\omega_z = -\Omega \sin \Phi,$$

where ω_x, ω_y, and ω_z signify the component angular rates of rotation of the stable platform around the indicated axes.

The above expressions are very similar to those developed from the spherical geometry (for analyzing eastward components of change in inertial navigator location) if Ω is recognized simply as analogous to a time rate of change in the longitude, λ.

An added comment is appropriate concerning the technique for rotating the stable platform of the IMU at some stipulated angular rate about any one of its three axes; for example, suppose the stable platform is to be rotated around its x axis to satisfy the relation

$$\omega_x = \Omega \cos \Phi.$$

From the earlier discussion on gyro and from the schematic (fig. 6) showing the three-gyro array, note that the input axis of the x gyro is the axis around which the specified rotation must occur. For this to happen, a properly proportioned torque must be applied around the output axis of the x gyro, which, thereby, causes that gyro to rotate (precess) around its input axis. The IMU, then, through the appropriate servo loop, discovers that the null balance of the x gyro has been disturbed and, in seeking to reestablish that balance, commands the proper gimbal torque motors to rotate the stable platform the desired amount and in the desired direction.

For simplicity, the discussion thus far has considered only the principal environmental features of an idealized symmetrical and homogeneous Earth. Validity has not been jeopardized by this approach, however. The steps remaining to achieve a completely rigorous analysis of all environmental parameters in the real world are definable and can be taken into account in a straightforward manner. They have been painstakingly detailed in a number of texts on inertial guidance and navigation (see, for example, Britting, 1971; Draper, Wrigley, and Hovorka, 1960). Thus, much is known about the distribution of gravity disturbances and how they influence the accelerometer outputs. Also, much is known about the exact configuration of the Earth and how it departs from the idealized shape of an ellipsoid.

It is hoped that the Earth-environmental stage has now been set completely enough so that perspectives on the performance dynamics of the inertial navigator can be further enlarged. The approach here is to envision the system installed in the carrying aircraft, then to examine the basic steps needed to accomplish initial alinement, and finally to allow the aircraft to be flown and to describe how the inertial navigator can develop, in a dynamic way, a continous record of the flight path, referenced to the onboard set of three coordinate axes which, in turn, are keyed to local latitude, longitude, and elevation.

To accomplish the initial alinement of the inertial navigator, it must be presupposed that the aircraft in which it is mounted is at rest over a point on the airport parking ramp for which the coordinates of latitude, longitude, and elevation can be given. These data may be supplied to whatever accuracy is obtainable from the best available local map, as long as the uncertainty in location is no more than ±200 ft (61 m) for the horizontal coordinates and ±10 ft (3 m) for the vertical. The vertical criterion really comes from the precision with which an aircraft altimeter (a special pressure transducer) can be read while navigating enroute to the survey site. As a reference guide, the positional accuracy for locating a well-defined point on a representative Survey standard 7.5-minute quadrangle map is approximately ±40 ft (12.2 m) for the horizontal coordinates and plus or minus one-half the contour interval for the vertical. In nonmountainous terrain, the contour interval is commonly 10 ft (3.0 m), and, thus, the vertical coordinate data are accurate to ±5 ft (1.5 m).

The foregoing coordinate data constitute the necessary initial position information for the inertial navigator. The remaining requirements for describing or establishing the initial conditions are satisfied by noting that the speed of the aircraft is zero and by manually commanding the x, y, and z axes of the stable platform of the IMU to settings within a few degrees of the desired tangent-plane orientation. The proper components of Earth rotation, precomputed for the latitude given in the initial position data, now are translated into precession torques that are applied to the output axes of the x and z gyros. The y gyro need not be torqued inasmuch as its desired orientation is with its input axis perpendicular to a meridian plane of the Earth (see fig. 17) which means it then will not sense any Earth rotation.

At this point in the procedure, the stable platform probably will be out of the intended tangent-plane alinement by a few degrees and will be slowly drifting more out of alinement by whatever components of Earth rotation the misalined gyros sense. Next, the three accelerometers are read, and their outputs resolved into components along the x and y axes. The amounts by which these components disagree (ideally, the disagreements should all be zero when the stable platform is level) indicate the tilt remaining in the stable platform with respect to the horizontal plane and represent the amounts by which it must still be rotated around its x and y axes to reach the tangent-plane condition. The required amount of rotation, around, for example, the y axis is, to a first approximation, the arcsine of the x component of the gravity vector, g_x, divided by the full gravity vector, g. To accomplish the leveling, the x and y components of accelerometer outputs are fed through appropriate circuits to the torquers of the y and x gyros, respectively. Precession of the gyros, in combination with the action of the gimbal servoamplifiers and torque motors, then rotates the stable platform to the desired level condition.

Initial leveling may be accomplished in a minute or two using relatively high gains in the torquing loops. Meanwhile, azimuth misalinement (failure of x axis to point true north) will continue to cause the stable platform to rotate around the y axis as the y (east) gyro senses a component of Earth rate proportional to the misalinement angle multiplied by the latitude. The component of gravity along the x axis, measured by the accelerometers as the stable platform rotates off level, is fed to the z (azimuth) gyro with appropriate gain to null the azimuth error. The dynamics of this process, called gyrocompassing, are very much slower than for leveling. At least 15 or 20 minutes are necessary to accomplish the azimuth alinement. Final alinement in level and azimuth takes place with the leveling and the gyrocompassing loops in operation.

At this juncture, let it be assumed that the inertial navigator, as part of the APT system, has successfully completed all initial alinement steps. Although stationary in a parked aircraft for the moment, it is ready to navigate.

The aircraft engines now are started, and, as the aircraft moves along the taxiways toward the departure runway, the accelerometers sense (measure) this new movement, as well as any changes in local gravity, in terms of the equivalent three component movements along the x, y, and z axes of the stable platform. From the earlier description of the accelerometers, it should be noted that the measurement data actually recorded are in units of velocities, v, inasmuch as these instruments have automatically accomplished one integration of the specific force sensed.

The component velocities (ground speeds) v_x and v_y are for the north or south and east or west directions, respectively. These are velocities along the Earth's surface or perpendicular to its radius of curvature and to the direction of gravity (local vertical) at the alinement point.

In a dynamic way, therefore, the inertial navigator performance, as it defines changes in the two horizontal coordinates of position, can be described as a closed-loop system with two built-in stages of mathematical integration. The first stage is built into each accelerometer; the second stage is executed by the on-board computer. The mathematical designation for such a loop is a "second-order system," and this one is undamped. In the mathematical equation that describes the performance of this system, the relative values of the coefficients signify that the system is prone to oscillate. For this closed-loop system, if there should be an error in the initial conditions as, for example, a slight tilt of the stable platform off tangent-plane orientation (that is, the z axis not quite coinciding with the plumb line of the actual gravity vector as navigation begins), then the accelerometers will sense gravity components proportional to the tilt magnitude and direction. These are integrated once in the sensing process and again in the computer; they are approximated by dividing respectively by the northerly and easterly radii of Earth curvature and integrating continuously to obtain aircraft position. As successive cycles of events are compared, the change in output of the loop or the loop "gain" must be a function of g/R, where R is an appropriate value of Earth radius. This ratio of an acceleration divided by a length, coupled with the idea of an oscillating system, leads to the concept of a pendulum of arm length, R, subject to gravitational acceleration. The period at which such a pendulum would oscillate may be shown by simple computation to be 84.4 minutes. In the inertial navigation literature, this is termed the "Schuler period," and it characterizes this closed-loop system that defines changes in the horizontal coordinates of position. The important point is that, although the initial condition errors, as well as some other errors that reflect hardware imperfections, cause oscillations in the computation of position and velocity, the loop exhibits neutral stability; that is, it does not "run away" (oscillations steadily increase).

To this point, the discussion has presumed ideal or perfect inertial instruments. However, the minor asymmetries known to be present in such instruments cause errors in alining the IMU. Thus, a bias in an accelerometer output causes an error in leveling the stable platform or a drift bias in the y (east) gyro causes a misorientation in azimuth. To forestall these kinds of problems, the final stage in fabricating the IMU is to run it through an elaborate one-time program of laboratory calibration exercises. These serve to measure and document, very precisely and systematically, the magnitude, sign, and, sometimes direction in space of such parameters as scale factors, biases, gyro drifts, gravity-sensitive unbalances, and mechanical misalinements. Such data are enhanced by additional calibration exercises in the aircraft before each flight, and the combined data are stored in the onboard computer memory for subsequent operational use as appropriate.

Thus far, the aircraft has simply been taxiing along the ground, and the discussion, therefore, has focussed first on processing the accelerometer outputs to yield v_x and v_y data, inasmuch as the principal motion is in these two horizontal coordinate directions. However, the vertical position accuracy specified for the APT system, in its operational surveying mode, is of such a high order [±0.5 ft (0.15 m)] that data from a barometric altimeter will not suffice, and the system must be capable of determining vertical-coordinate (elevation) information as well as horizontal. The mechanization of this capability has certain unique characteristics and requirements.

After the IMU is initially alined, with the aircraft standing in place (zero velocity) over a known point, the accelerometers can measure continuously the actual gravity vector. Their respective measurements are mutually orthogonal components, integrated once, of the specific force vector that acts on the aircraft. These measurements are resolved continuously in the computer into components along the x, y, and z coordinate axes of the stable platform, but, because, in this case, the aircraft is not moving, the resolution yields a stream of measured values of integrated specific force representing the resultant force along the z axis only. These values, which have the dimensions of velocity, increase linearly with time because only the gravity vector is acting, and they are, thus, the "zero" reference for subsequent computations of vertical velocity when the aircraft moves. The zero reference or initial value for the vertical coordinate of position (the "elevation" coordinate) is determined from the known elevation of the point over which the aircraft is standing.

When the aircraft moves (taxis), the aforementioned accelerometer-measurement and computer-resolution processes continue, but now the stream of values that represent the changing vertical velocity must be further processed. The appropriate zero reference values are subtracted, and the results are integrated over discrete (small) time intervals to yield vertical distances that signify changes in the elevation coordinate of aircraft position. As each new elevation coordinate is determined, it must be fed back through a computer program which describes how the gravity magnitude changes with altitude. After one integration, each new or adjusted gravity value is subtracted from the next z-axis component of the integrated specific force as derived from the continuing accelerometer measurements[5]. The measurement and computation processes continuously progress, or cycle, in this closed-loop manner and include compensation for the "Coriolis acceleration" (described later in this section).

Again, the mathematical designation for such a closed loop is a second-order system, and it also is undamped. Now, however, an important difference in the mathematical equations that describe performance of the elevation-coordinate loop, as contrasted with the horizontal-coordinate loops, arises from the fact that the Earth's gravitational field changes with altitude; that is, decreases as altitude increases. The practical consequence is that, whereas in the neutrally stable horizontal-coordinate loop the errors were oscillatory

[5]In practice, two models of the gravity field are used, one in real time and the other in postprocessing. The real-time model is simply "1 g"; that is, the gravity field of an idealized, rotating, ellipsoidal Earth. This model permits the accelerometers to deliver information which, to a first approximation, is the desired inertially referenced specific force. The postprocessing model is more elaborate, designed to specify departures of the actual gravity field from the idealized one.

and described with sine and cosine functions, now the errors for the elevation-coordinate loop are described in terms of hyperbolic sine and cosine functions which grow exponentially, doubling in value every 395 seconds. Thus, the mathematical solutions for the performance equations are seen to "diverge," and the elevation-coordinate loop is not stable and tends to run away. A bias error of only one-millionth g, in determining a new or refined gravity value for a new elevation coordinate of position, therefore, can grow in 3 minutes to an error of 6 in. in the vertical position.

Because of the foregoing inherent instability problem, the inertial navigator alone is incapable of meeting the stringent positioning requirements. A source of position-related measurement data external to the inertial navigator, therefore, must be supplied at regular intervals during system operation; this external information is acquired by the "laser tracker," described in the section so titled. Enroute from the alinement and takeoff location to the field survey site, error growth in the elevation-coordinate loop is limited by using data from a very precise pressure altimeter to compute and reset the vertical coordinate of position. Earth radius also is recomputed from these data (accurate to about five parts in a million) and used in the horizontal-coordinate loops.

The Coriolis acceleration, mentioned above, deserves limited elaboration here. Succinctly stated, this effect represents an "apparent" acceleration that arises because the inertial navigator is required to keep track of its velocity and position in terms of a three-coordinate scheme of reference (latitude, longitude, and elevation) that is tied to a rotating Earth. The inertial navigator must use accelerometer measurements that are affected by that rotation. Newton's laws of motion applied to the rotating reference scheme define the Coriolis acceleration term (see, for example, Britting, 1971, p. 17–18).

A more meaningful description, couched in terms that reflect a real-world phenomenon, hinges upon careful definition of where the observer is, and his state of motion, when he witnesses the Coriolis effect. First imagine the observer to be stationed out in space so that he sees the stars as stationary or fixed and he sees the Earth, as it appears in figure 17, rotating about its polar axis. If an artillery gun at P_2 were to fire a shell horizontally (tangent to the Earth) in, for example, an easterly direction, then the space observer would view its initial path of motion as an undeviating straight line lengthening eastward. He would mathematically describe it, simply and correctly, using a straightforward application of Newton's laws in the presence of the Earth's gravity field. Now imagine the observer on the Earth at P_2 and fire a second shell in the same manner. The initial path of motion to the earthbound observer "appears" to curve steadily upward and to his right, departing more and more from the desired easterly direction which, for the rotating Earth, is defined by the parallel of latitude through P_2. The observer, with recourse only to earthbound reference schemes, finds that the complete mathematical description of the observed curving path of motion requires an added term in the application of Newton's laws. This reconciling term, which accounts for the shell's apparent acceleration upward and to the right of the easterly direction, is the Coriolis acceleration.

Although the foregoing description has, for convenience, featured a projectile in flight, the principle is equally apt for aircraft flight. Thus, the dynamic performance of the inertial navigator includes appropriate compensation inputs to offset the Coriolis acceleration.

A logical conclusion for this section on performance dynamics occurs when the aircraft reaches the departure runway and takes off. The inertial navigator simply continues to perform in the dynamic manner detailed throughout this section. As the aircraft gains altitude and normal cruising speed, however, the respective changes in elevation, latitude, and longitude rates become far more dramatic than those occurring during the taxi operation. Through the onboard computer, these data are processed as a continuous stream of position-coordinate information which is not only fed into updated computer memory, but also recorded on tape to document the aircraft flight path.

External Measurement Requirement

In the very early stages of shaping this instrumentation development project, before the first phase of contract work at Draper Laboratory was formalized, the stipulation for achieving position measurement accuracy of ±0.5 ft (0.15 m) in the vertical coordinate direction was recognized as invoking a requirement for updating the inertial navigator at 3-minute intervals. The origin of this requirement is found in the mathematical expression that describes how accelerometer measurement errors develop in the vertical direction as a function of time. It turns out that the time span, t, between updates is closely approximated by the familiar relation

$$s = (1/2)\ at^{2},$$

where s is the 0.5-ft (0.15-m) allowable position error in the vertical coordinate, a is the error in measuring vertical acceleration, which is relatable to performance characteristics of the accelerometers, and t is the time interval over which the allowable 0.5-ft (0.15-m) position error develops. A value for a that reasonably typifies the APT system accelerometer is $1 \times 10^{-6}g$, where g is the standard acceleration due to gravity, or 32.2 ft/s^2 (9.8 m/s^2). Substituting in the foregoing approximation and solving for t yields a value of 176 seconds or 3 minutes.

The update requirement is satisfied at a given instant by feeding into the computational procedures, which produce the indicated position data, very precise observations of the carrying vehicle's (aircraft's) velocity vector and position in three-dimensional space. How to make such observations from an aircraft in flight was the real challenge.

At the beginning of the 6-month engineering analysis made by Draper Laboratory, it was believed that observations of aircraft velocity could be made at the desired 3-minute intervals by the laser profiler if it were aimed at and electronically locked onto a retroreflector target at a known ground-control point. By collecting (while locked on) a stream of slant-range, azimuth-angle, and dip-angle data, all referenced to the orientation (three coordinate axes) and position of the onboard IMU stable platform, the laser profiler would yield the information needed to compute very precisely the aircraft velocity vector.

As the analysis progressed, however, it became apparent that the primary function of the laser profiler should not be interrupted to the extent represented by the inertial navigator update requirement. Furthermore, a laser-ranging device, which is obliged to operate generally with uncooperative targets (terrain features), should be designed very deliberately and exclusively for the purpose. In sharp contrast are the design requirements for a similar device scheduled to operate only with cooperative targets (retroreflectors). And, finally, it was recognized that, for those applications of the APT system where terrain profiling was not a requirement, the system should be self-sufficient without the laser profiler and be able to perform its basic function of carrying a three-coordinate reference frame for other ancillary equipment as dictated by the particular application. Thus, the idea of a laser ranging or tracking device, integral with the IMU, evolved.

Electro-optical, mechanical, and operational details of the laser tracker are given in the next section. The need for the range and angle data obtainable through the operation of such a device relates to the characteristic inherent in any inertial system whereby its indicated position and orientation slowly deviate from the true values. The deviation reflects the combined inaccuracies originating in the performance of the three gyros and three accelerometers, each one of which drifts very slowly.

The practice of aiding or updating an inertial measurement unit is well established and not unique to the APT system. An element of uniqueness, however, lies in the overall accuracy specified for the system, and this obviously can be no better than that of the updating technique and equipment.

In essence, the update procedure can be regarded as a tripartite affair that involves aiding the inertial measurement unit in tracking vertical motion of the aircraft with updates from a very sensitive baroaltimeter, while navigating to the survey site; fine calibration of the instrument system when it is first flown over the survey site; and subsequent updating adjustments to the system at no longer than 3-minute time intervals throughout the flight mission to hold performance within the stipulated precision limits.

The boroaltimetry is accomplished with a pressure transducer connected to a suitable static-air-pressure port on the aircraft. Transducer details are described in the "Laser Tracker" section of this paper.

The calibration exercise requires flying a specifically defined path over and around a sequence of three retroreflector targets prepositioned on three ground-control points, chosen so they are not in a single straight-line alinement. The x, y, and z coordinates (latitutde, longitude, and elevation) must be determined or known for each control point before the flight mission. Although not a requirement, the availability of a gravity measurement at each control point would simplify the inflight calibration procedure.

Laser Tracker

The manner in which the need for a laser tracker component evolved is described in the preceding section. The design of the laser-ranging elements for this instrument component benefited from the laboratory experiments that shaped the design for the "laser profiler," and evolutionary details on that design are given in the section titled "Laser Profiler." The present section, therefore, is limited mainly to the electro-optical, mechanical, and operational details of the laser tracker as they gelled in the final design. The term "laser" is an acronym that signifies "Light Amplification by Stimulated Emission of Radiation."

To ensure that its pointing measurements can be accurately related to the stable platform reference axes, the tracker two-axis gimbal-support structure is mounted in the same housing or case that contains the IMU. The arrangement is shown schematically in figure 18, which should be recognized as simply a more complete version of figure 10. To avoid gimbal lock for all pointing directions downward below the aircraft, the tracker outer axis (fig. 18, blue color code) is constructed parallel to the aircraft pitch axis. The tracker inner axis (fig. 18, red color code) is always at right angles to the outer axis regardless of pointing direction. Arrowheads on the two axes again denote the positive-direction conventions.

The tracker assembly can be rotated about each axis ±60° from the position shown (line-of-sight downward). The termini of the inner axis are in the gimbal; the termini of the outer axis are in the enclosing housing or case. The case structure, therefore, provides the fixed geometric reference link between the tracker and the IMU. This design aids the whole process of searching for and acquiring a retroreflector target at a ground-control point. Thus, in approaching a target, the tracker can be made to scan the ground in raster (side-to-side) fashion. The technique is to point it forward along the intended flight path by rotating its supporting gimbal around the outer axis; the desired ground-scanning motion is then obtained by simultaneously oscillating the tracker assembly itself (lower moment of inertia) from left to right around its inner axis.

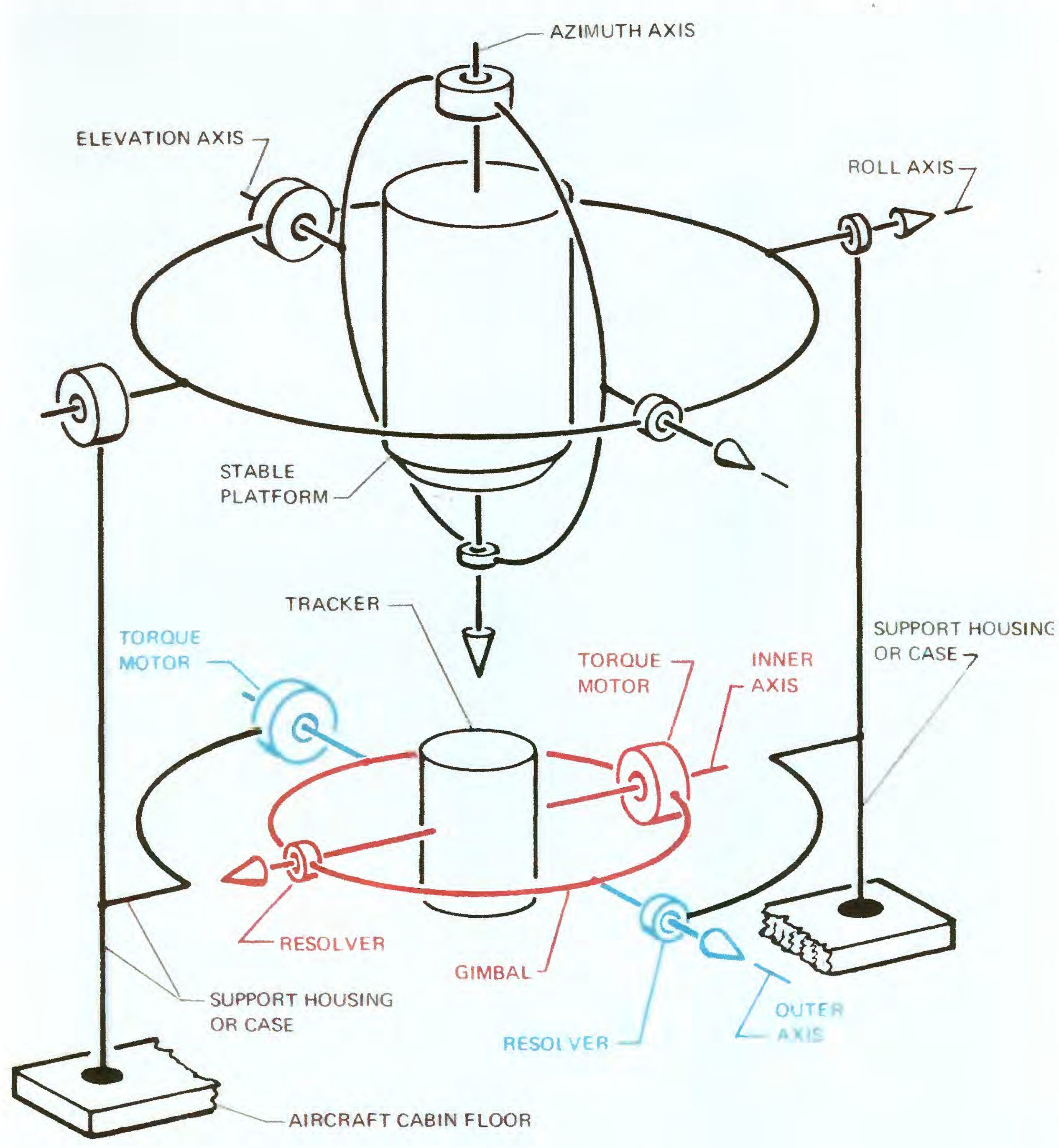


FIGURE 18.—Support structure for inertial measurement unit and tracker.

Many of the electromechanical assembly features of the IMU are repeated in the tracker assembly. Thus, each end of each axis fits into a pair of preloaded ball bearings which, in consort with the support-structure members, feature an overall structural stiffness capable of maintaining relative positions within microinch tolerances. To enhance this capability, the main support housing or case for the whole IMU and tracker assembly (fig. 19) is to be made of 6061 aluminum, which, for its low weight (density), offers good thermal conductivity, high structural and mechanical stability, and high resistance to corrosion. The enclosing jacket or housing for just the tracker unit itself (fig. 19) is to be made of a single piece of heat-treated Ti–6A1–4x titanium alloy, which, in addition to the above characteristics, has the added advantage of a thermal coefficient of expansion closely matched to that of the optical glass.

Mounted as appropriate in gimbal or case at one end of each of the abovementioned two axes is a torque motor to drive the tracker around that axis (fig. 18). Each of these two motors has a torque capability of 60 oz-in. (0.42 N·m). Mounted as appropriate in gimbal or case at the opposite end (from the torque motor) of each of the foregoing two axes is a 1:36-speed resolver (fig. 18). Each measures the angular displacement of the gimbal, and the readings are relayed to the onboard computer. The net effect of these cited electromechanical features is to assure that the overall tracker pointing accuracy is not degraded from the design accuracy of the resolver, namely, about 4 arc-seconds.

A few words are appropriate to interpret the color coding used in figure 19. Individual colors are listed and explained as follows, proceeding generally from outermost to innermost physical features:

IMU	Blue	—Main support housing or case for entire IMU and tracker assembly,
	Pink	—Elevation gimbal structure,
	Green	—Stable platform structure and internal electronics packages,
	Red	—Gyros,
	Silver	—Accelerometers,
Tracker	Gold	—Gimbal structure, and
	Lavender	—Part of tracker unit that moves on inner axis; also inset, at lower right, which shows beam splitter, range receiver, and quadrant photodiode assemblies.

The sweep angle of the tracker (±60° from the vertical) had to be obtained without letting any part of the assembly protrude beyond the bottom exterior surface of the aircraft cabin. This specification is met by constructing a floor well of appropriate dimensions, mounting the tracker as deep therein as possible, and adopting a coaxial and folded optical lens system. Layout of the tracker optics is shown in figure 20. From this layout, the laser-beam (green band, fig. 20) path is traced readily from source through transmitter lens assembly, to deflecting mirror, and to deflecting prism centered and cemented on the back side of the objective lens. At this point, the laser beam is outbound toward a retroreflector target secured on a ground-control point. After reflection, the laser beam returns inbound (light-blue band, fig. 20) through the same objective lens, to the deflecting alinement mirror, and through the beam-splitter assembly (medium-blue band, fig. 20) for delivery to a quadrant-photodiode-receiver assembly and a range-receiver assembly; the former assembly receives 90 percent of the return signal, and the latter, 10 percent. The 4.5-in. (114-mm) diameter objective lens [11.902-in. (302.3-mm) focal length] was the largest that could be used because of the 24-in. (0.61-m) diameter clear limit originally placed on the aircraft cabin-floor well.

To inhibit moisture condensation in the tracker optics, two silica-gel capsules occupy a cavity designed into the tracker unit housing. The capsules are packaged in a lint-free nondusting material, and, by choice, the gel does not feature any dye to indicate moisture content, thereby forestalling any opportunity for adversely affecting a lens coating.

The variable density optical filter, shown in the path of the outgoing laser beam, is operated by an automatic gain control loop which monitors the intensity of the return laser beam as the sum of the four quadrant pulse magnitudes. Thus, as the return signal weakens or strengthens, owing, for example, to changes in attenuation as a function of range to the retroreflector, the servo-drive motor rotates the continuously variable filter just enough to strengthen or weaken the outgoing beam the needed amount. The intent is to maintain a constant sum for the four quadrant return-signal pulses.

The range receiver assembly features a silicon avalanche photodiode detector and transimpedance preamplifier. It senses the return-signal pulse from which range to the retroreflector target is ultimately determined.

The laser beam is generated by a pulsed gallium arsenide (GaAs) transmitter that is similar to the one designed for the laser profiler. Pulses are produced at a

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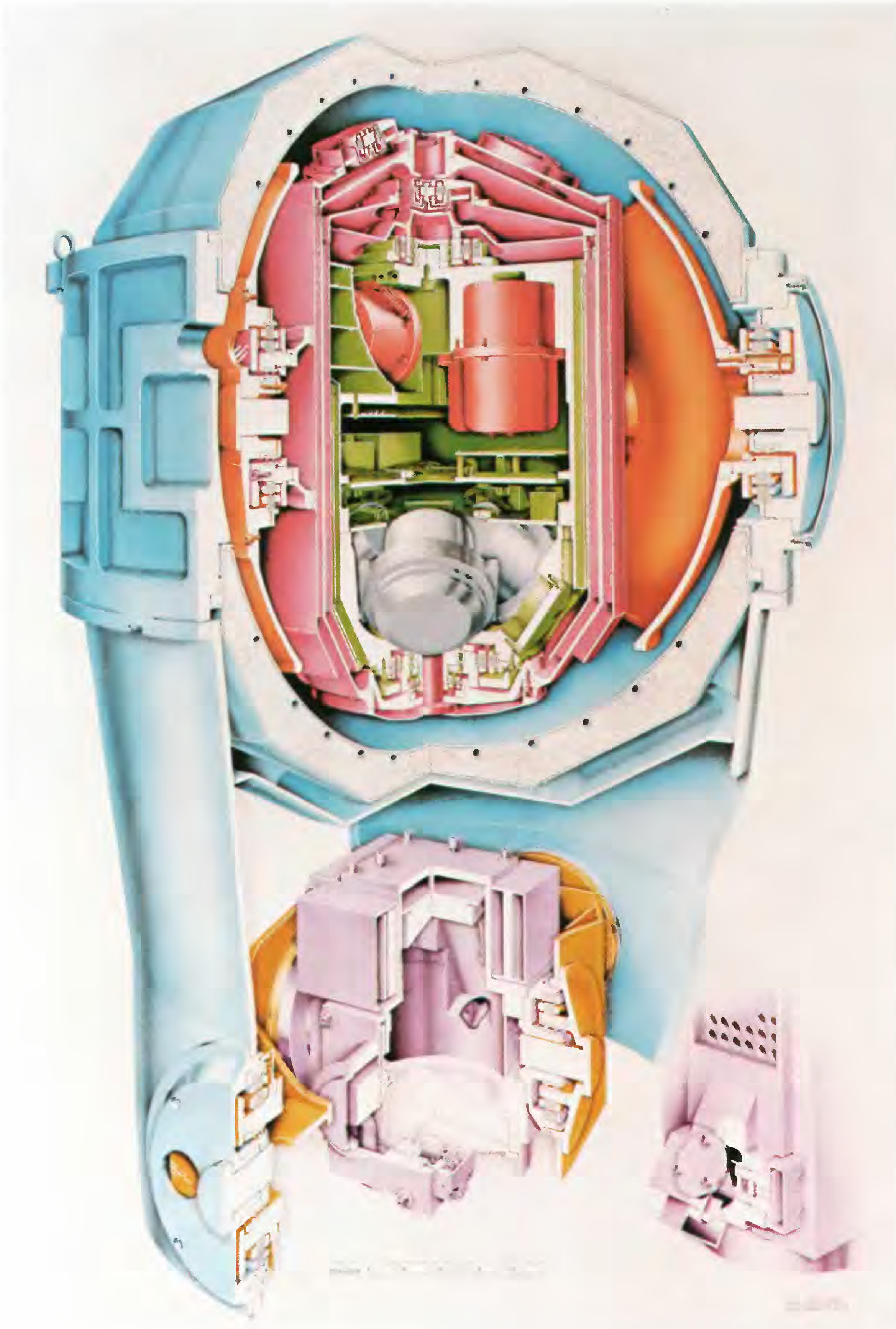
FIGURE 19.—The inertial measurement unit, laser tracker, gimbal-support structure, and housing.



rate of 3.2 kHz in the near infrared at a wavelength of 904 nanometers (nm), with 25 watts (W) peak power, subnanosecond rise times, and widths of 40 nanoseconds. These large widths (four times greater than the profiler pulses) assure that more return-signal energy will be fed to the quadrant photodiode assembly (fig. 20) and to the tracking electronics. The laser beam meets Federal safety standards in the following manner:

1. *In the laboratory*.—Eye safe for all distances greater than 18 in. (0.46 m) when using a suitable attenuation filter and limiting exposure time to a 2-hour maximum.
2. *In flight*.—Eye safe for all distances greater than 892 ft (304 m). This type of laser beam permits range determinations well within the stipulated precision of ±0.5 ft (0.15 m) over distances as great as 6,000 ft (1.8 km), which is the maximum planned operating slant range to a retroreflector target. Divergence of the laser beam is 6 to 7 milliradians (mrad) and the fields of view of the quadrant-photodiode and range-receiver assemblies, respectively, are 8.3 and 2.6 mrad.

The quadrant photodiode receiver assembly senses the return signal as four independent pulse magnitudes related to the sectors above, below, left, and right of the quadrant dividers. If the tracker boresight catches a retroreflector target off center, the target image will be unevenly distributed in the four quadrants, and the four pulse magnitudes proportional to the sighting error in each quadrant, and the corresponding control currents drive the two torque motors (see fig.

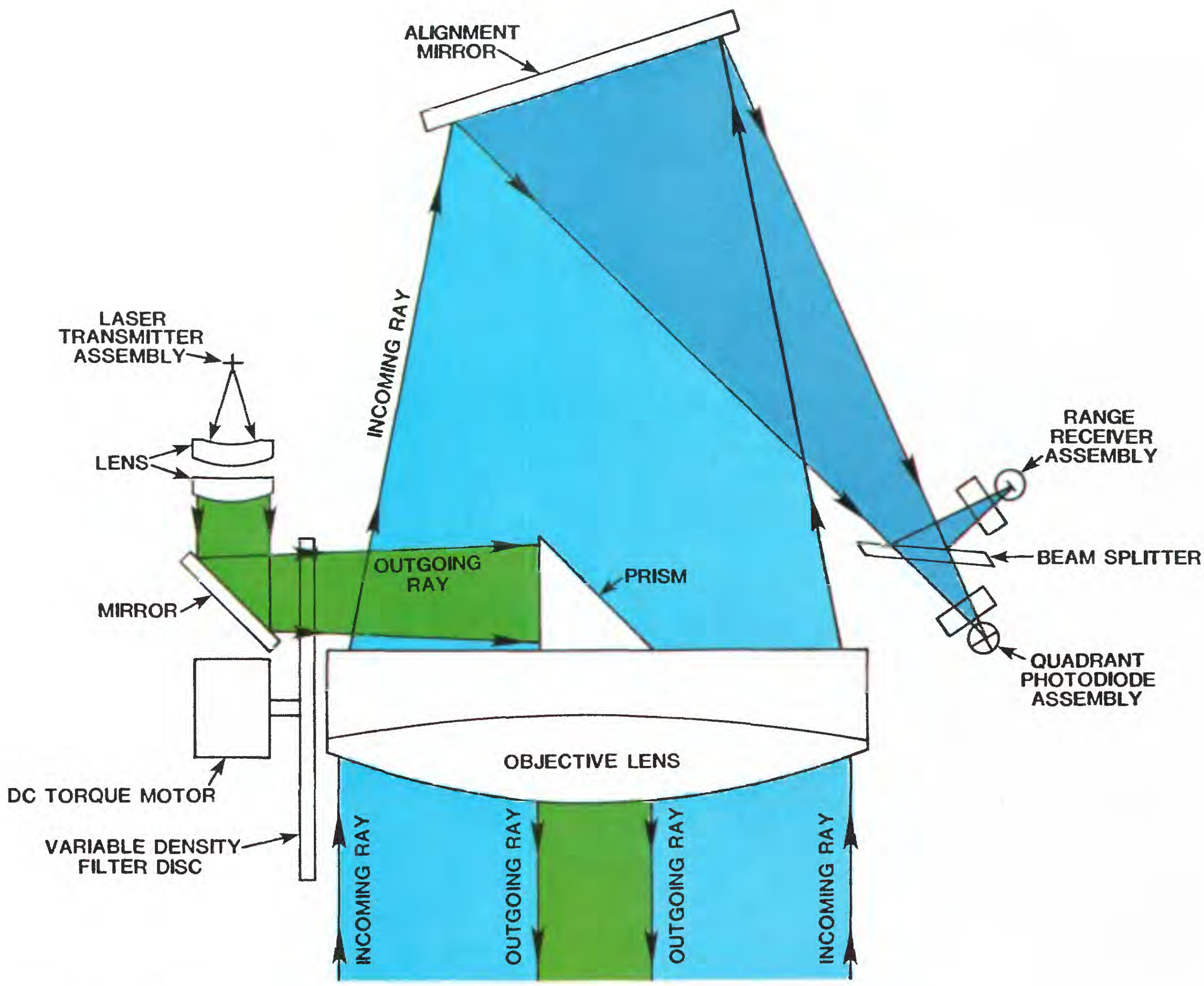


FIGURE 20.—Optical layout for laser tracker.

18) to keep the target image centered in the tracker field of view.

The discussion given in the section, "Laser Profiler," refers to a key mathematical expression termed the "range equation" which is equally important in this section in arriving at suitable design choices for the laser transceiver. Although the following discussion of the range equation closely parallels that given in the "Laser Profiler" section, there are significant differences that relate primarily to the laser beam ranging on a specially designed cooperative retroreflector target instead of the uncooperative land-surface target. Thus, for the laser tracker, the range equation has the form:

$$P_R = P_S \frac{T_1 T_2 \rho A_1 A_3 e^{-2\sigma R}}{A_2 A_4} , \qquad (3)$$

where P_R = power, receiver,
P_S = power, source,
T_1 = transmission coefficient, source optics,
T_2 = transmission coefficient, receiver optics,
ρ = reflection coefficient, retroreflector,
A_1 = area, retroreflector,
A_2 = area, light beam "footprint" at retroreflector,
A_3 = area, receiver lens,
A_4 = area, light beam return "footprint" at receiver,
σ = attenuation coefficient, atmospheric path, and
R = range.

The interrelations among the foregoing parameters are ilustrated schematically in the three parts of figure 21. These parts, studied in sequence from left to right, show (A) the significant geometric features of the laser light path from transmitter (source) to retroreflector target and back to receiver, (B) the critical points in the flow of light energy from the lasing source to the target and return, and (C) the mathematical description of the principal losses in energy flow which allows a rigorous statement to be written (eq 3) that shows the net change in energy level between laser source and receiver. Obviously figure 21*A* is schematic in the sense that, in the real tracker instrument, the field of view is coextensive (not side by side) with the field irradiated.

To arrive at the particular form of the range equation given in figure 21, note from the geometry shown in part (A) that the radius of the circular footprint of the laser beam at the retroreflector is a function of the constant beam divergence angle, α, and the range, R. Thus, the footprint area, A_2, is determined from the relation,

$$A_2 = K_2 \pi R^2,$$

where K_2 is simply the appropriate trigonometric function of α used with R to compute the footprint radius. To develop a similar expression for A_4, the area of the circular footprint of the returning laser beam at the receiver, consider first that the retroreflector is designed to have a high reflection coefficient and a shaped surface such that virtually all the laser beam that is intercepted is reflected straight back along the line-of-sight path to the source. In effect, therefore, the retroreflector becomes a new point source for the laser beam, with its own constant divergence angle, α'. (Ideally, α' should be nearly equal to, but no less than, α.) Thus, the radius of the circular footprint at the receiver is a function of α' and R, and the area, A_4, is determined from the relation,

$$A_4 = K_4 \pi R^2,$$

where K_4 is the appropriate trigonometric function of α'. By substituting the foregoing equivalences for A_2 and A_4 in equation 3, the range equation for the laser tracker has the desired form,

$$P_R = P_S \frac{T_1 T_2 \rho A_1 A_3 e^{-2\sigma R}}{K_5 \pi^2 R^4} , \qquad (4)$$

where K_5 is simply a new constant that stems from the product of K_2 and K_4.

The tracker may be operated in either a search mode or a track mode. In the search mode, when the tracker must search until it acquires a retroreflector target, the tracker gimbals are positioned automatically by computer program in much the same manner as described in the last paragraph in the section titled "Electronics." Now, however, the appropriately processed gimbal angle data are sent to the tracker gimbal servoamplifiers to drive the tracker torque motors. Included in the computer program for this pointing mode is continuing iterative calculation of the commanded tracker gimbal angles to maintain the line of sight toward the expected position of the retroreflector target. This takes into account the known target location, as well as the present aircraft position, attitude, and velocity, and then superimposes a tracker spiral scan motion that slowly opens up, in a step-by-step programmed way, around the initial, most probable target direction. At the maximum intended flight operating altitude of 3,000 ft (920 m), the spiral scan will sweep the tracker footprint over a terrain area that totals about 4.3×10^4 ft^2 (4.0×10^3 m^2) in 20 seconds. Normally, this is more than sufficient to find the retroreflector target. However, if the target is not found and is determined to be

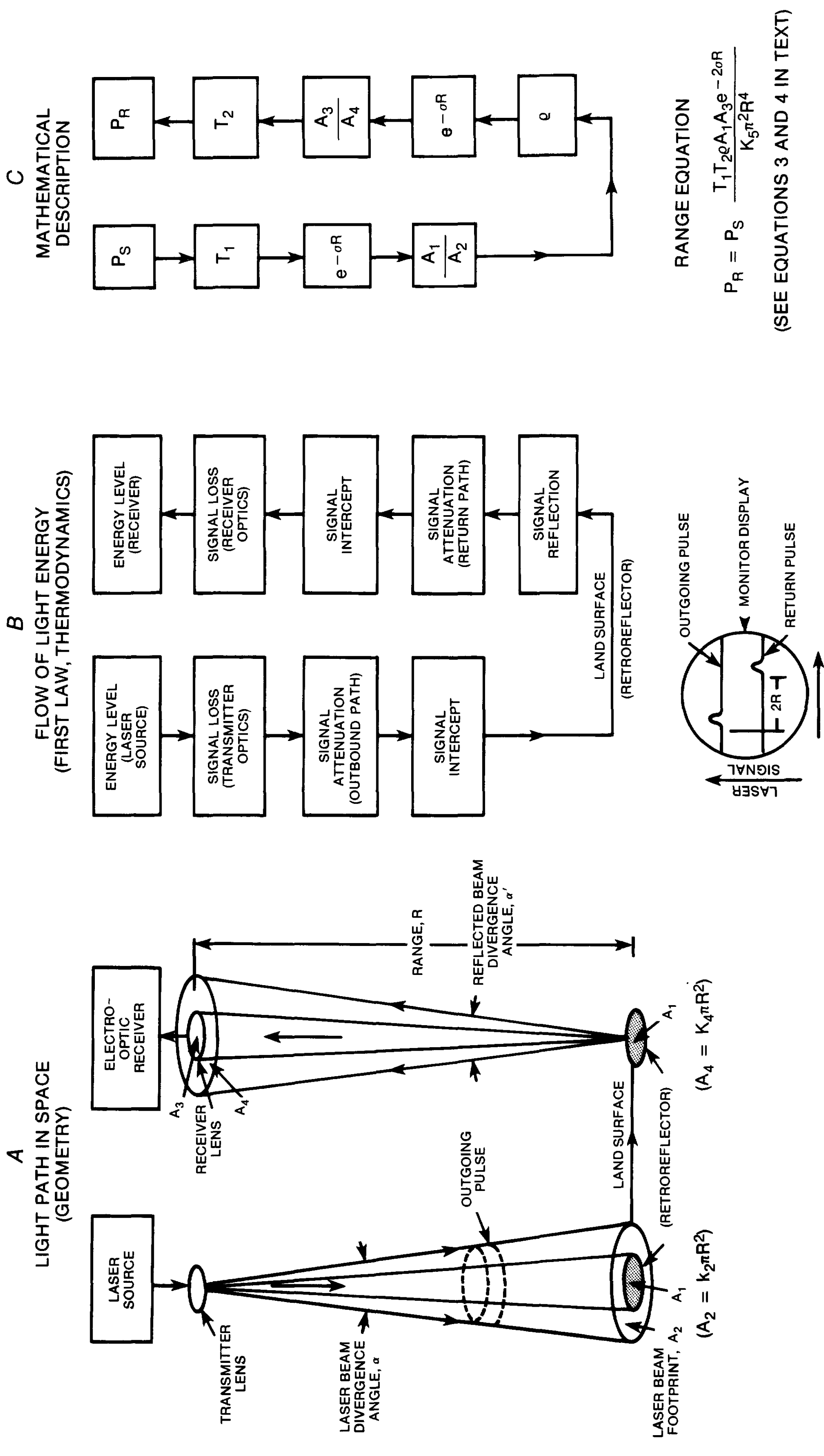


FIGURE 21.—Analysis of the range equation for the laser tracker.

still within range, then the computer program commands the tracker to enter a side-to-side oscillatory or raster scan. If this broader search mode were to persist the full program length to 40 seconds, then the tracker footprint would sweep over a terrain area that totals about 7.8×10^5 ft^2 (7.2×10^4 m^2).

When the tracker receives a return signal from the retroreflector, indicated when the total level of returned energy exceeds a preselected limit, an "acquired" flag is set by the signal processing electronics, and the tracker then is switched to the track mode. Now the outputs of the quadrant photodiode are used to generate signals that are sent through the appropriate tracker servoamplifiers to the respective torque motor (fig. 18) to keep the tracker line of sight centered on the retroreflector target. During this tracking-mode operation, readings of the IMU and tracker gimbal angles, as well as the target range, are sent to the computer at a 200-Hz rate. This stream of data provides a complete time-history of the aircraft motion vector with respect to the retroreflector while it is in view (typically 30–60 seconds). In addition, simple curve fitting programs allow the computer to extract velocity and acceleration information from the data. Should tracking contact be lost, the spiral scan submode is reinvoked automatically to reacquire the target within 5 seconds.

The tracker and the profiler operate in conjunction with a laser signal processor and time-interval counter. The former contains the master quartz-crystal oscillator (clock) for the whole APT system. Its frequency is 19.6608 megahertz (MHz), and all APT subsystem frequencies are integral submultiples thereof.

A diagram to illustrate the interrelations among the principal frequencies is given in figure 22. The master crystal is electric oven controlled to a long-term frequency stability of better than 1 in 10^8 parts. This kind of stability is more than academic because the master oscillator paces the time-interval counter which ultimately must achieve resolution of better than a nanosecond (1×10^{-9} second) if laser ranges accurate to one-half a foot (0.15 m) are to be extracted from the processed measurement data.

The laser-signal processor generates trigger pulses (under control of the time-interval counter) for the tracker and the profiler laser transmitters and delivers time-multiplexed duration pulses, which become the yardstick for determining target range, to the time-interval counter. The time-interval determinations are sent to the onboard computer for conversion to actual target ranges.

An electronic refinement in the art of measuring the round-trip traveltime of the laser pulse greatly sharpens the ultimate conversion into target range. This refinement, termed a "constant-fraction discriminator," is designed to cope with the fact that the returning laser pulses will range greatly in size. If the time-interval counter were simply set to make its measurement when the return pulse reached an arbitrary and preset voltage level, then this would ignore the known variations in pulse size and introduce unwanted errors in the range determinations. The way around this dilemma is to note that the known time constant of an electronic circuit tends to remain stable, and, thus, the instant at which the laser return pulse will reach two-thirds of its peak value is predictable. The constant fraction discriminator, therefore, is built to trigger the time-interval count always at the same fraction of pulse amplitude, and, thus, all range determinations are made from the same relative position on the curve that defines the pulse shape. In the tracker, these determinations are made on the leading-edge side of the pulse.

The time-interval-counter unit chosen for the APT system is the Digitec Model 8330. It is packaged in a metal box 5¼ in. (0.13 m) high, 17 in. (0.43 m) wide, and 16¾ in. (0.42 m) deep and can be mounted in standard light-alloy racks. The unit weighs 21.5 lb (9.8 kg), and its power requirement is 100 W, delivered as single-phase current at 113 volts (V) and 400 Hz. When it was purchased (in the late 1970's), this off-the-shelf commercial unit was unique in its capability for achieving a one-shot resolution of 0.1 ns and an accuracy of approximately 0.3 to 0.4 ns. Through time-interval averaging, however, the resolution can be increased to 1 picosecond [ps (0.001 ns)]. As used in this part of the APT system, the unit averages 128 measurements. Thus, its operating resolution will be $0.4\ \text{ns}/\sqrt{128}$, or 0.03 ns.

The time-interval counter unit has its own internal voltage-controlled oscillator which operates at 10 MHz. Inasmuch as this is synchronized with the master quartz-crystal oscillator at 9.8304 MHz (see fig. 22), the proportionality constant 10/9.8304 must be entered appropriately into the final range-measurement computations. These computations also must include adjustments for the average refractive index of air over the laser light path, based on inflight outside air temperature and static pressure measurements at the aircraft.

To collect the need measurements, a temperature transducer and a static-pressure port for a pressure transducer are installed at suitable outside points on the aircraft. The pressure transducer chosen is a Rosemount Model 1201F1, scaled to provide a DC electrical signal in the range of 0 to 10 V, over a range in atmospheric pressure from 17.72 to 32.48 in. (450–825 mm) of mercury at 0°C (60,000–110,000 Pa). A 12-bit analog-to-digital (A/D) converter transforms (digitizes) the DC signal over the 10-V range. Thus, the least significant bit corresponds to a pressure change of 0.007 in. (0.18 mm) of mercury (24.4 Pa).

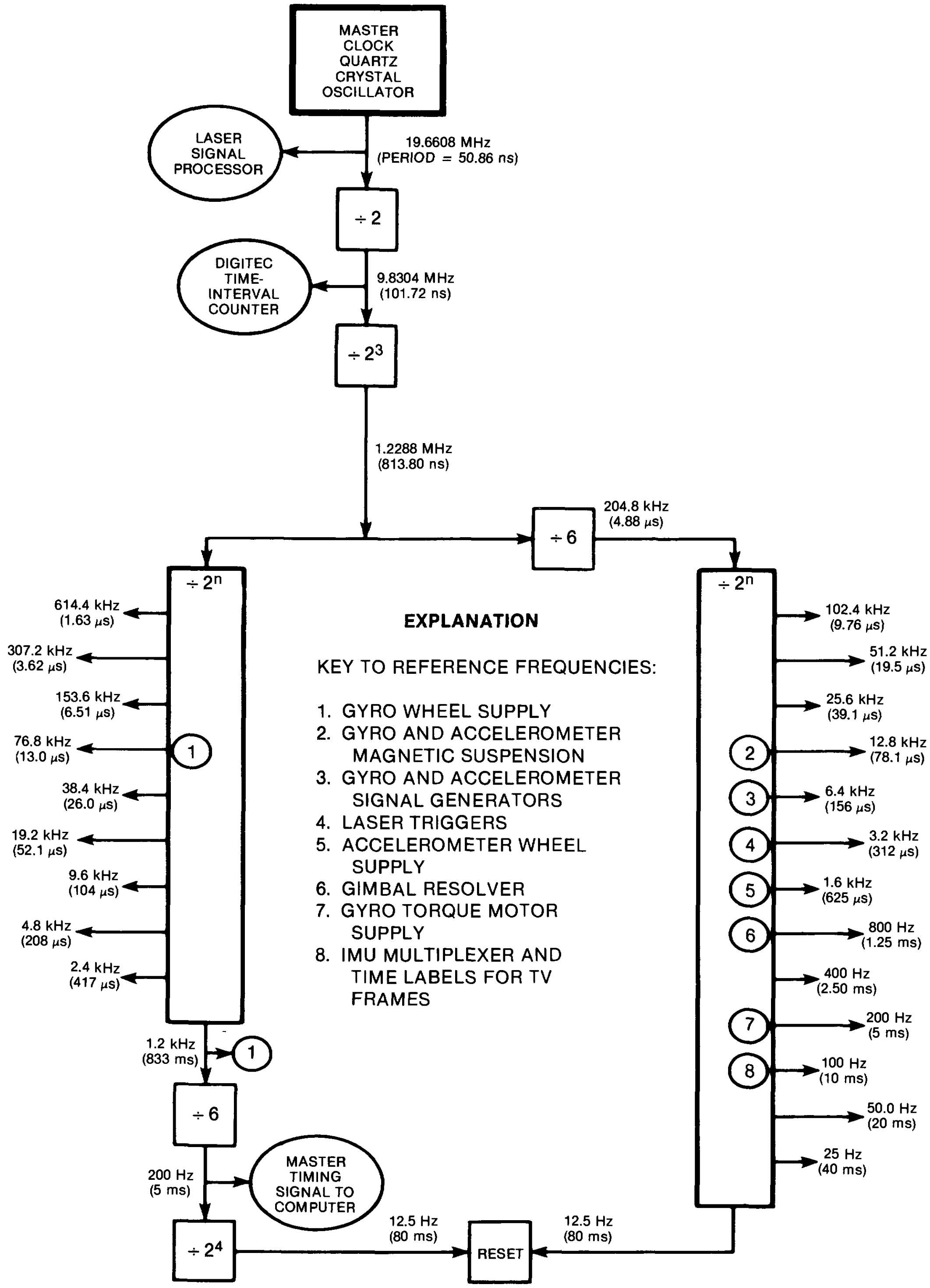


FIGURE 22.—Principal reference frequencies used in the aerial profiling of terrain system.

The temperature tranducer chosen is a Rosemount Model 101F probe and Model 510B signal-conditioning amplifier, scaled to provide a DC electrical signal in the range of 0 to 5 V over a range in air temperature from −50°F to +150°F (−46°–66°C). A 12-bit A/D converter digitizes this signal also, and the least significant bit corresponds to a temperature change of 0.195°F (0.108°C).

The capability for the tracker to be rotated ±60° from the vertical, around each of its two orthogonal axes, means that its field of view is the land-surface area embraced by the base of a right circular cone whose apex angle is 120°, whose apex is at the intersection of the two tracker axes, and whose height is the aircraft altitude above the land surface. For the tracker to capture, lock onto, and track a retroreflector target positioned on a ground-control point, imagine the preceding cone to be inverted, and its apex placed instead at the control point. As long as the aircraft carrying the tracker remains within the volume of airspace bounded by this inverted cone, it is physically possible to see the target, provided, as already stated, the retroreflector has the capability to reflect the intercepted part of the laser beam straight back along the line-of-sight path to the source. The geometry of the situation is shown in figure 23, which has been drawn for the maximum intended aircraft operating altitude of 3,000 ft (915 m). For this flight altitude, therefore, the general approximation holds that the aircraft must be sensibly within a 1-mi (1.6-km) radius of the retroreflector target for any successful tracking activity.

To be visible from all the flight directions diagrammed in figure 23, an ideal retroreflector target should be a hemisphere about 1 ft (0.3 m) in diameter. To give it the proper retroreflecting capability, the best promise for combining accuracy and low cost appears to lie in mounting individual molded-plastic retroreflectors, each one of which is about 3/4 in. (19 mm) in diameter, all over its surface.

Until the abovementioned ideas can be thoroughly researched and the prospects for commercial fabrication in quantity lots evaluated, the Survey will use its own inhouse designed and fabricated retroreflectors. The basic elements are four commercial solid-glass, corner-cube retroreflectors, each having a minimum clear aperture of 3 1/2 in. (8.9 mm) and a maximum angular deviation, between the incoming and returning light beams, of ±30 arc-seconds. The four basic elements are cemented and clamped to an aluminum circular base plate in the attitudes shown in figure 24. The in-between spaces are filled, to some convenient convex contour, with black silicone-rubber caulking, so that, when stationed in the field, little or no opportunity exists for any form of precipitation to collect. The base plate is 7 in. (18 mm) in diameter and is provided, on a smaller diameter circular plate secured on its underside, with a threaded center hole so that it can be mounted on a standard surveyor's tripod. A bullseye bubble atop the base plate provides for leveling.

The laser tracker discussion is not complete without a brief description of the physical properties of the glass window (fabricated more like a large "lens") installed in the aircraft cabin floor, through which the laser beam must pass. The principal constraints used to identify the most suitable glass are as follows:

1. Stipulated maximum rectangular clear opening, 27.0 in. (0.686 m) × 33.25 in. (0.845 m),
2. Maximum air-pressure differential that can be expected to occur between aircraft cabin and the outside world, less than 0.5 lb/in.^{2} (350 kg/m^2),
3. Maximum air-temperature differential that can be expected to occur between aircraft cabin and the outside world, about 75°F (42°C),
4. Maximum window deflection that can be tolerated due to any type of loading, less than 0.003 in. (0.08 mm),

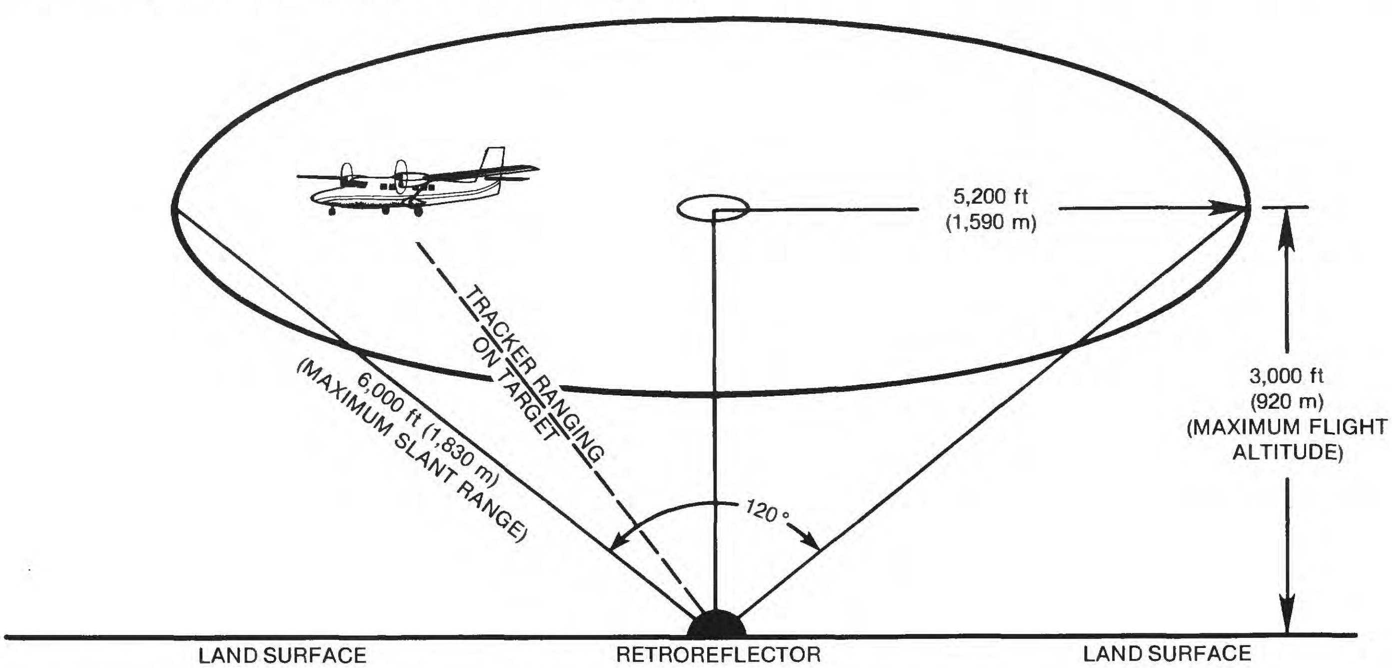


FIGURE 23.—Cone-shaped air space within which the tracker can see a retroreflector target.

5. Maximum resistance to corrosion from chemicals in the air,
6. Highest transmission coefficient for near infrared light,
7. Minimum light-beam distortions due to any residual internal stress, and
8. Minimum number of gas-bubble or striae inclusions per kilogram of mass.

These constraints led to the choice of Schott B–270 white opthalmic crown glass to be drawn (as opposed to cast) and fire polished in several sheet blanks 28 in. (0.711 m) × 34.5 in. (0.876 m) × 0.66 in. (0.017 m). The weight of a single blank is about 60 lb (27 kg). Pertinent physical properties of this glass are as follows:

Mechanical/thermal

Density is about 1.5 oz/in.^{3} (2,550 kg/m^3)

Modulus of elasticity is 10.4×10^6 lb/in.^{2} [72 gigapascals (GPa)].

Mean coefficient of expansion[6], for the range 20°C to 300°C is (93.3×10^{-7})/°C.

Optical

Coefficient of transmission is 92 percent (antireflection coating will increase this to 98- to 99-percent region).

Index of refraction for near infrared wavelengths is 1.5298.

Specifications drawn for finishing the glass blank that is installed in the aircraft typify the fact that the so-called window must really function as a lens. The finishing operations included the following:

1. Polishing to stipulated uniform thickness (tolerance ±0.5 mm and wedge angle less than 4 arc-seconds) and surface flatness (both sides),
2. Rounding the four corners of the rectangular sheet to a 1-in. (25.4-mm) radius of cuvature,
3. Beveling all edges of the sheet, and
4. Antireflection coating of the sheet (both sides) for 904-nm wavelength unpolarized light.

Computer

In the array of equipment components that constitutes the APT system, the electrical device that ties everything together into a smoothly functioning dynamic whole is the onboard data-processing digital computer. This has been partly recognized in the general definition already given for the inertial navigator. The objective in this section, however, is to detail more completely the manner in which the digital computer serves as the central communicator and controller for the flow of data, instructions, and control signals among all APT system components.

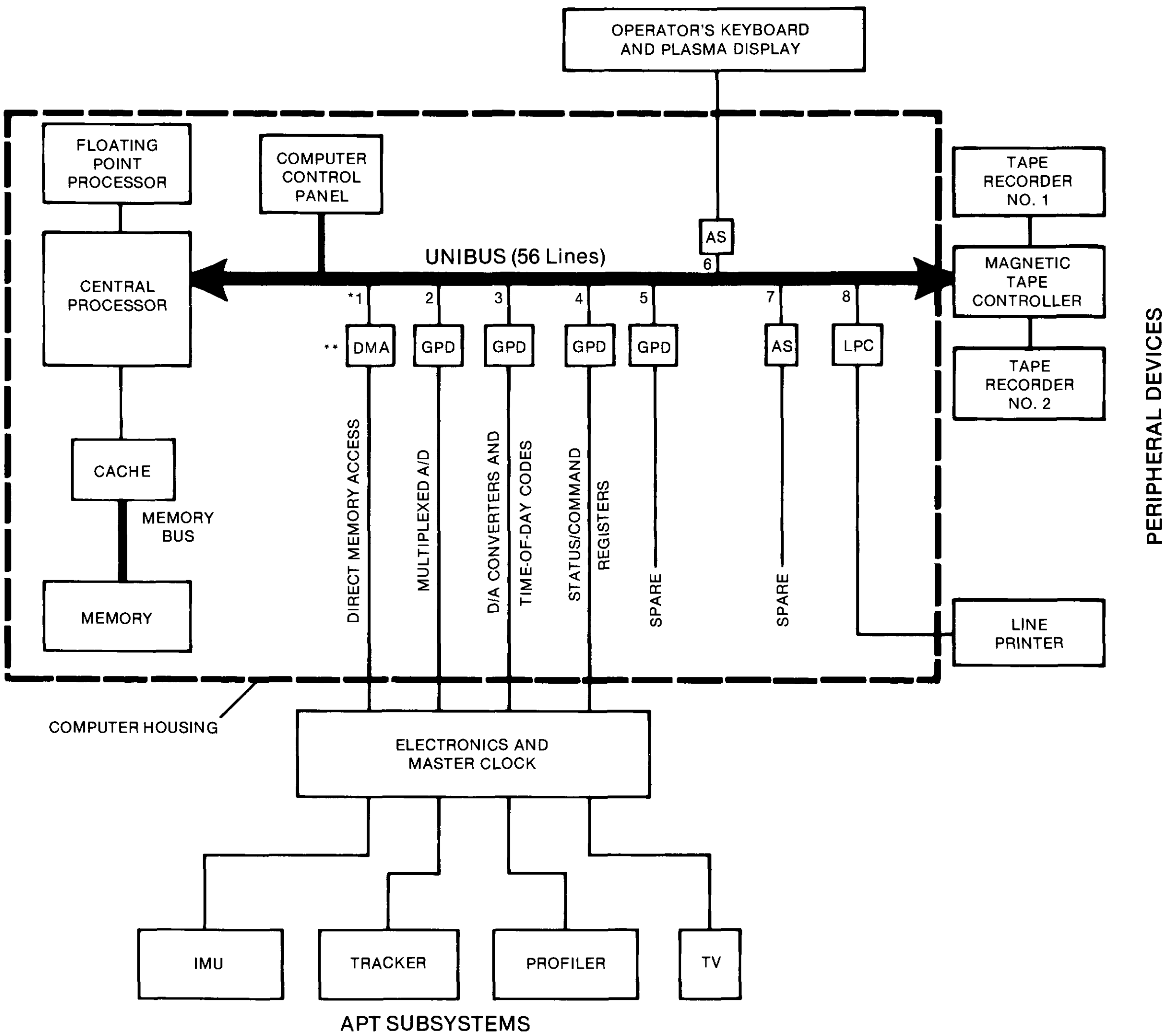
FIGURE 24.—Principal components and final assembly of the U.S. Geological Survey-designed retroreflector.

[6] A dimensionless ratio of change in length to original length, for a specified temperature change.

Physical Attributes

The following paragraphs give a description of pertinent physical characteristics and capabilities of the particular digital computer selected for the APT system, namely, the Norden PDP–11/70M. The "M" in the nomenclature signifies that it is the fully militarized, or ruggedized, model suitable for flight application. Its structure is diagrammed schematically in figure 25.

The bulk of the Norden computer is packaged in four metal boxes, whose dimensions, total weight (mass), and electric power requirements are given in table 1. The computer control panel is housed separately in a fifth box (see table 1). Figure 26 is a photograph of the several boxes which, for the desired flight applications, can be shock mounted easily in standard light-alloy racks secured inside the aircraft cabin. For its ruggedness and compact size, the Norden flight computer is



EXPLANATION

*NUMBERED INPUT/OUTPUT PORT OR CHANNEL
**CIRCUIT BOARDS
DMA, Direct Memory Access Interface
GPD, General Purpose Digital Interface
AS, Asynchronous Serial Interface
LPC, Line Printer Controller

FIGURE 25.—Functional structure of the aerial profiling of terrain system computer with connections to subsystems and peripheral devices.

TABLE 1.—Package size and power requirements for the aerial profiling of terrain system computer and peripherals

ITEM	Height in. (m)	Width in. (m)	Depth in. (m)	Mass lb (kg)	Electric power watt	Electric current type
Norden computer a/	7-3/4 (0.20)	10-1/8 (0.26)	19-1/2 (0.50)	236 (107) b/	1500 (Norden computer and computer control panel combined)	Single phase, 115^{V}, 400 Hz
Computer control panel	10-1/2 (0.27)	19 (0.48)	4 (0.10)	6 (3)		
Plasma display panel	14 (0.36)	13 (0.33)	12 (0.30)	54 (24)	275	Single phase, 115^{V}, 400 Hz
Line printer	9-1/2 (0.24)	16 (0.41)	19 (0.48)	77 (35)	750	Single phase, 115^{V}, 400 Hz
Magnetic tape controller	10-1/2 (0.27)	17 (0.43)	18 (0.46)	63 (29)	200	Single phase, 115^{V}, 400 Hz
Magnetic tape recorder c/	7 (0.18)	15-1/4 (0.39)	14-1/2 (0.37)	52.5 (24)	200	Single phase, 115^{V}, 400 Hz
TOTALS				541 (246) d/	3125 d/	

a/ Four boxes.

b/ Total values, four boxes.

c/ Two identical units.

d/ Includes two tape recorders.

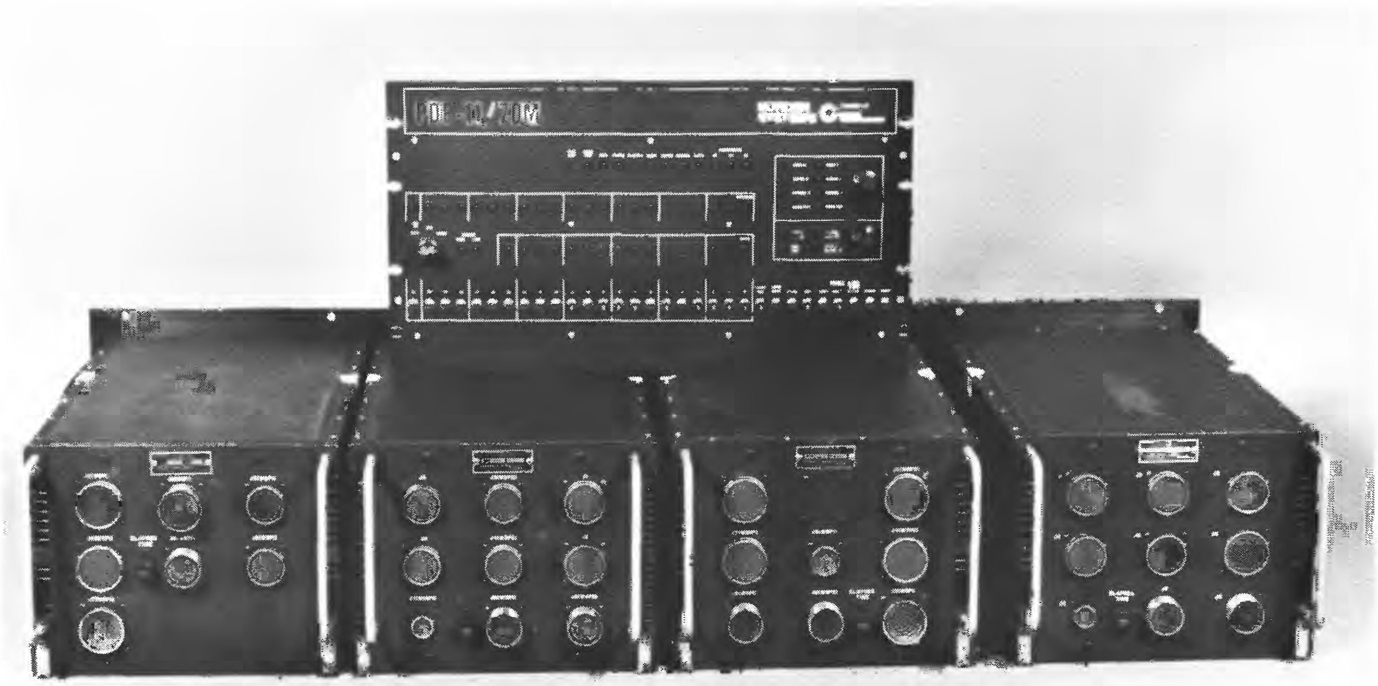


FIGURE 26.—Packaged flight computer for the aerial profiling of terrain system.

extremely versatile; only a few of its more obvious features can be described in the following paragraphs.

The structure of the Norden computer, as diagrammed schematically in figure 25, prominently features a single, high-speed, multichannel bus, termed the UNIBUS, which allows most of the computer components, peripheral devices, and APT subsystems to communicate with each other over its 56 individual (parallel) lines. The UNIBUS can be likened to a telephone party line because of its eight numbered input/output (I/O) port connections or channels and its several direct connections to other components.

Paramount among the direct-connected components is the "central processor," which occupies a key control position in the computer structure. It acts, in part, like a central telephone exchange and controls the allocation of time for the communication traffic on the UNIBUS. It also performs arithmetic and logic operations as demanded in the decoding and execution of instructions. These functions relate to manipulating and controlling the APT system through all its intended operational modes.

The central processor is built to recognize and operate at any one of eight distinct priority levels. The highest priority is given, for example, to accelerometer measurements (see IMU in fig. 25) needed in the real-time computations used to navigate the aircraft. The lowest priority is exemplified by a command to print out data on the line printer.

The central processor is a key component in other important ways that include execution of programs that have been stored in memory. Execution can entail arithmetic operations, logic operations, or manipulations of data. Arithmetic operations are self evident; logic operations involve carrying out instructions; and data manipulations involve, for example, moving specified data from one place to another in the computer.

Performance of the central processor is expedited by an internal set of 16 high-speed, general-purpose registers in which words (data or instructions) are stored temporarily. These help to speed up the computer operation by reducing the traffic of data moving into and out of the computer memory.

Operating speed of a computer is characterized by its so-called throughput rate, or the number of bits per second that it can process. In the Norden, this rate is 3×10^6 bit/s, and, thus, a basic arithmetic computation, such as the addition of two numbers, can occur in about 350 ns.

Allusions already have been made to the computer memory (see fig. 25) which provides a more permanent form of storage for data and instructions. Memory capacity is an important property, and the Norden computer was chosen partly because it could store 128K words, where K signifies 2^{10}, or 1024. This assures ample reserve capacity beyond the normal loading the APT system will put on the computer memory in the contemplated flight missions. Furthermore, this ruggedized computer model features a nonvolatile memory which means that the stored information is not lost under electrical power failure.

Positioned between the central processor and the main memory (fig. 25) is a very high-speed memory unit, termed a "cache," capable of holding 2,048 bytes at any given point in time. It augments the main memory in a very dynamic way. Whenever the processor is instructed to store words in memory these words are written to both the cache and the main memory. The premise is that the next step in the instructions may call for the processor to use some of these same words which occurs a very high percentage of the time. Storage in the cache memory is in constant flux, and the newest incoming words simply overwrite (replace) the previous words.

The central processor is aided in its arithmetic operations by the "floating-point processor" (see fig. 25), which contains six 64-bit registers. These essential elements enable calculations ranging from very large numbers (up to about 10^{38}) to very small numbers (down to about 10^{-39}). Without this capability, a register, with only a fixed number of bit positions available, would be limited to processing only those decimal numbers that fell within a relatively narrow size range of large or small.

Successful performance of the APT system, as described in previous sections, relies not only on extremely precise measurements by the gyros and accelerometers but also on real-time and very precise computations based on those measurements. The measured quantities are quite small and may involve, for example, trigonometric functions whose arguments approach zero for tiny angular displacements near 0° or 90°. The floating-point processor performs with seven-decimal-place accuracy in single-word calculations. It also can perform double-word calculations (so-called "double precision" arithmetic operations with double-legnth numbers) and, in so doing, achieve 17 decimal-place accuracy. Normally, floating-point-processor capability can be provided through either the hardware or the software. The real-time constraints imposed by the APT system, however, preclude the software approach. The floating-point processor runs independently of and concurrently with the central processor. This time-saving computer feature is an invaluable aid in coping with the overall workload.

Next among the components that are direct-connected to the UNIBUS (proceeding from left to right in fig. 25) is the computer control panel (see also fig. 26). This unique feature provides a direct link between operator and computer. Thus, the operator may preset the knobs and toggle switches, shown in figure 26, to run the computer in a completely automatic, semiautomatic, or completely manual mode. The last two modes commonly are used to test and diagnose computer operation; normal operation is fully automatic. The several rows of indicator lights display (in binary form) the contents of selected registers and memory and, thereby, allow the operator to observe, in part, the internal status of the computer. Furthermore, the operator can enter his own choices of data or instructions directly into selected registers simply by appropriately setting switches and knobs.

One particular indicator light on the control panel is worthy of special note. This light comes on when a parity check error is found. The Norden computer uses parity checks extensively to ensure that all data and instructions are moved to and from the correct addresses in memory. Such extensive checking is essential because erroneous transmission of even a single bit could change a tiny number into a huge one (10^{-9}–10^{9}, for example) or cause the instruction execution sequence to jump to some unintended address, with catastrophic consequences for the proper functioning of the computer and the APT system. The parity check involves computing, transmitting, and storing an extra check bit, the parity bit, along with each word of data. The parity bit is set to 0 or 1 in such a way that the total number of 1 bits in each word is always even. If, in subsequent processing actions, an odd number of 1 bits occurs, it labels the particular word as having been transmitted incorrectly. Special registers in the central processor hold information on the occurrence of parity errors so that corrective action can be taken automatically. The usual response is simply to reexecute the failed instruction.

Subsystems And Data Interfaces

As the review of components direct-connected to the UNIBUS continues (still proceeding from left to right in fig. 25) the eight numbered I/O ports or channels will be described, and the first four link the computer with the four principal subsystems into which the whole APT system may be divided. These subsystems must bear the brunt of whatever earth science measurements the APT system is commissioned to collect, and success will ride upon virtually continuous, high-speed, two-way communication between them and the computer. Thus, channel numbers 1 through 4 provide the data interfaces between the fundamental measuring instruments and the computer.

Of the four remaining channels, one establishes the means for the APT system operator to communicate with the computer, one provides a way to convert selected computer output into hard copy by means of a line printer peripheral device, and two will be held as spares and not used for the present.

Taken in numerical sequence, channel number 1 (fig. 25) provides direct, high-speed access to the computer memory without interrupting any computations that may be in progress. The computer circuit board through which electrical connection is made is called a Direct Memory Access (DMA) interface. In essence, any APT subsystem data or information which should be processed in virtually real time are funneled by the electronics box (fig. 25) into channel number 1. Examples of such input data are gimbal angle readings from the IMU and tracker, velocity readings from the accelerometers, range measurements from the tracker and profiler, and mode status of the entire system. These input data are processed promptly in the computer, and the needed output data sent back to the appropriate subsystem. Examples of such output data are torque commands for the x, y, and z gyros in the IMU.

Channel number 2 (fig. 25) provides a General Purpose Digital (GPD) interface. A number of analog signals, emanating from the several APT subsystems, must be converted to digital format in the electronics box (fig. 25) before being multiplexed (transmitted according to a timed sequence) into the computer. Examples of such input analog data are voltage and temperature sensors inside the IMU (generating "state-of-health" signals) and certain sensors for the operating environment, such as outside air temperature and static pressure. Normally, these data simply are converted from analog to digital format, fed through the computer, and tape recorded at specified intervals. However, the electronics box has the capability to decode directly certain results of the A/D conversions and to generate an alarm if system safety is threatened. If future needs arise, this A/D channel can accommodate additional subsystem devices.

Channel number 3 (fig. 25) provides a second GPD interface. A number of digital outputs (signals) from the computer need to be routed to appropriate subsystems and at those destinations converted into analog format. Such traffic flows through this channel, and decoding for the particular destination is done in the electronics box (fig. 25). Examples of the traffic are the signals being directed to the servoamplifiers for the IMU and tracker gimbal torque motors and to the aircraft autopilot for steering. Digital data passed to the computer through this channel include the time-of-day code from the master clock (see "Laser Tracker" section for clock description). Should the need arise, this channel can accommodate even greater signal traffic if other devices that require D/A conversions are added.

Channel number 4 (fig. 25) provides the third GPD interface. This channel is the status and command communication link between the computer and the logic circuitry in the electronics box (fig. 25) that concerns the various modes in which the APT system is to operate. In effect, the electronics box contains two registers, one termed "Status Register" and the other "Command Register." At all times, the Status Register contains a 16-bit word that indicates the particular mode in which the APT system is operating. The computer can be instructed to query this Register and to read the current 16-bit word, thereby ascertaining the current APT system mode status. If a change in mode status is to be effected, the computer will transmit, through this channel, the appropriate 16-bit word to the Command Register for implementation.

Channel number 5 (fig. 25) provides a fourth GPD interface that is held as a spare in the APT system.

Channel number 6 (fig. 25) affords a two-way link between the APT system operator and the computer through what is called an Asynchronous Serial (AS) interface. The term signifies simply that digital data in serial format (a sequence of bits) can be exchanged between the computer and the keyboard-plasma-display unit in a manner that is not preprogrammed for a regular and fixed time interval. The exchange can be initiated by the operator or the computer and will occur whenever the priority of the particular message can be fitted in behind existing higher priority computer traffic. The computer circuit board, through which the electrical connection is made, is labeled AS, and it is bidirectional inasmuch as it feeds into the computer the signals from the operator's keyboard and then returns the computer's output responses which drive the operator's plasma display. Through this channel, therefore, the operator can query the computer and then view the responses on the plasma display, as to the status (state of health or readiness) of selected APT subsystems or the degree of progress through selected sets of instructions. The operator also may initiate changes in the system operating mode and enter data of his own choosing. Such keyboard entries can be viewed first on the plasma display panel before being entered into the computer through this channel.

The keyboard and plasma display combination comprises an operator terminal; the unit chosen for the APT system is the Interstate Electronics Corp. Model PD3000 (see fig. 27). The keyboard, similar in layout to a standard typewriter, is packaged separately for easy attachment to, for example, the tabletop at the APT system operator's position.

The ruggedized plasma display panel also is packaged separately in a metal box that is shock mounted in standard light-alloy racks above the keyboard at convenient eye level for the system operator. The overall dimensions, mass, and power requirement for the unit are given in table 1.

The panel display area or screen is an 8.5-in. (0.22-m) square grid of evenly spaced points or dots. The display can be operated to present point plots and graphics, and two sizes of internally generated alphanumeric characters are available. Dedicated memory in the unit eliminates the need for refreshing any display.

Channel number 7 (fig. 25) provides a second AS interface, which is held as a spare in the APT system.

Channel number 8 (fig. 25) features a Line Printer Controller (LPC) interface, through which an electrical connection is made to control the operation of a line printer. As part of its control function, the circuit board converts the selected computer output data into a format suitable for the printer.

In a generic sense, the line printer belongs with the other computer-oriented peripheral devices (see fig. 25) described in the following section. It is described here, however, primarily because its operational control is through the last of the numbered I/O channels.

The printer unit selected for the APT system is the Miltope Corp. Model HSP3609–212A (see fig. 28), which is ruggedized and packaged in a metal box that is shock mounted in standard light-alloy racks. The overall dimensions, mass, and power requirement for the unit are given in table 1.

The printer can operate at speeds as great as 500 lines per minute in 80-column [10 characters per inch (about 4 per centimeter)] or 132-column [17 characters per inch (about 7 per centimeter)] format on fanfold or friction-feed paper. The printer uses a set of 64 alphabetic, numeric, and special characters and will print, as programmed or commanded by the APT system opera-

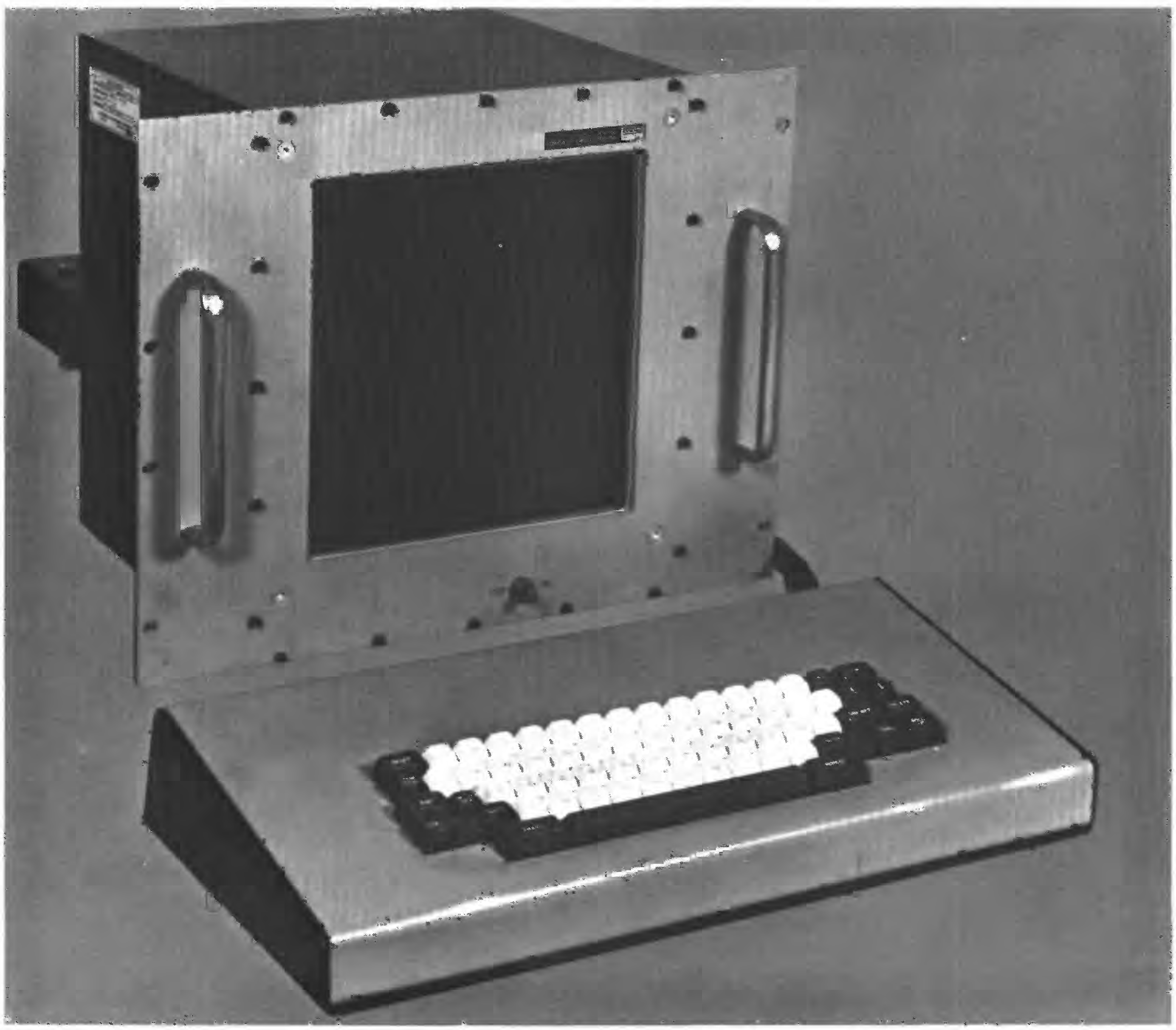


FIGURE 27.—Operator's keyboard and plasma display terminal for the aerial profiling of terrain system.

tor, selected data and computer status reports for postflight analysis. Probably, it also will be programmed to print a log of all system-operator actions.

Peripheral Devices

The only remaining component directly connected to the UNIBUS is the magnetic tape controller (see fig. 25). The unit chosen for the APT system is the Miltope Corp. Model TC–11 (see fig. 29), which is packaged in a metal box that is mounted in standard light-alloy racks. The overall dimensions, mass, and power requirement for the unit are given in table 1.

As its name implies, this peripheral device controls the flow of data (bidirectional) between the magnetic tape recorders and the computer. In performing the recording aspect of this function, the magnetic tape controller starts and stops the tape recorder and reformats the data in a way suitable for entry on the tape. When the end of a tape is reached, the controller automatically diverts the stream of data to the second recorder so that no information is lost. If the second recorder is not available, then the controller stops the data flow and signals the system operator with a warning light. The controller can accommodate tape speeds as fast as 130 in./s. At 800 bytes per inch, this represents a data rate of 104,000 bits per second.

The observations on controller capability are equally valid if a recorder is loaded with a magnetic tape cartridge containing data to be read into the computer. In

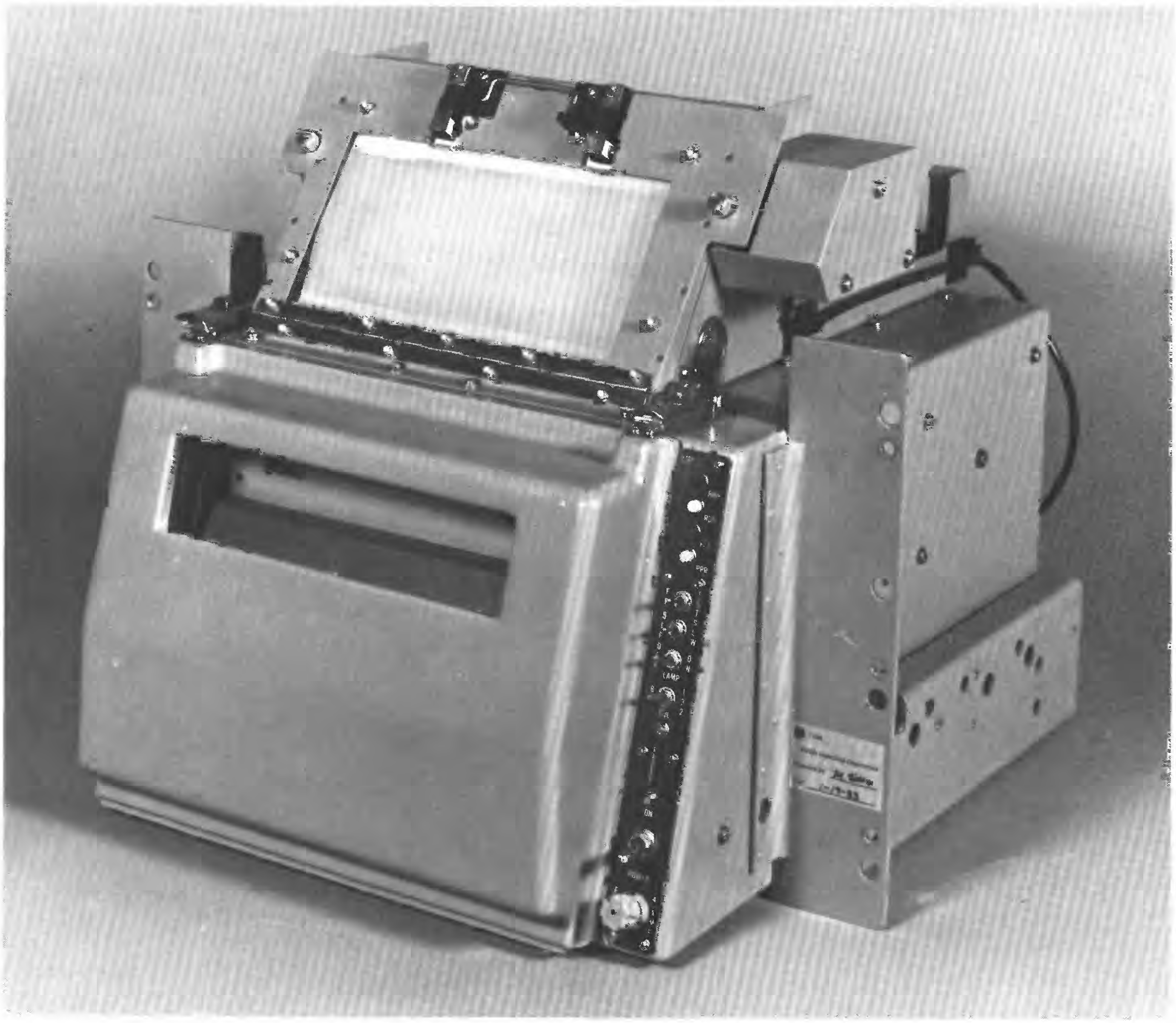


FIGURE 28.—Line printer for aerial profiling of terrain system.

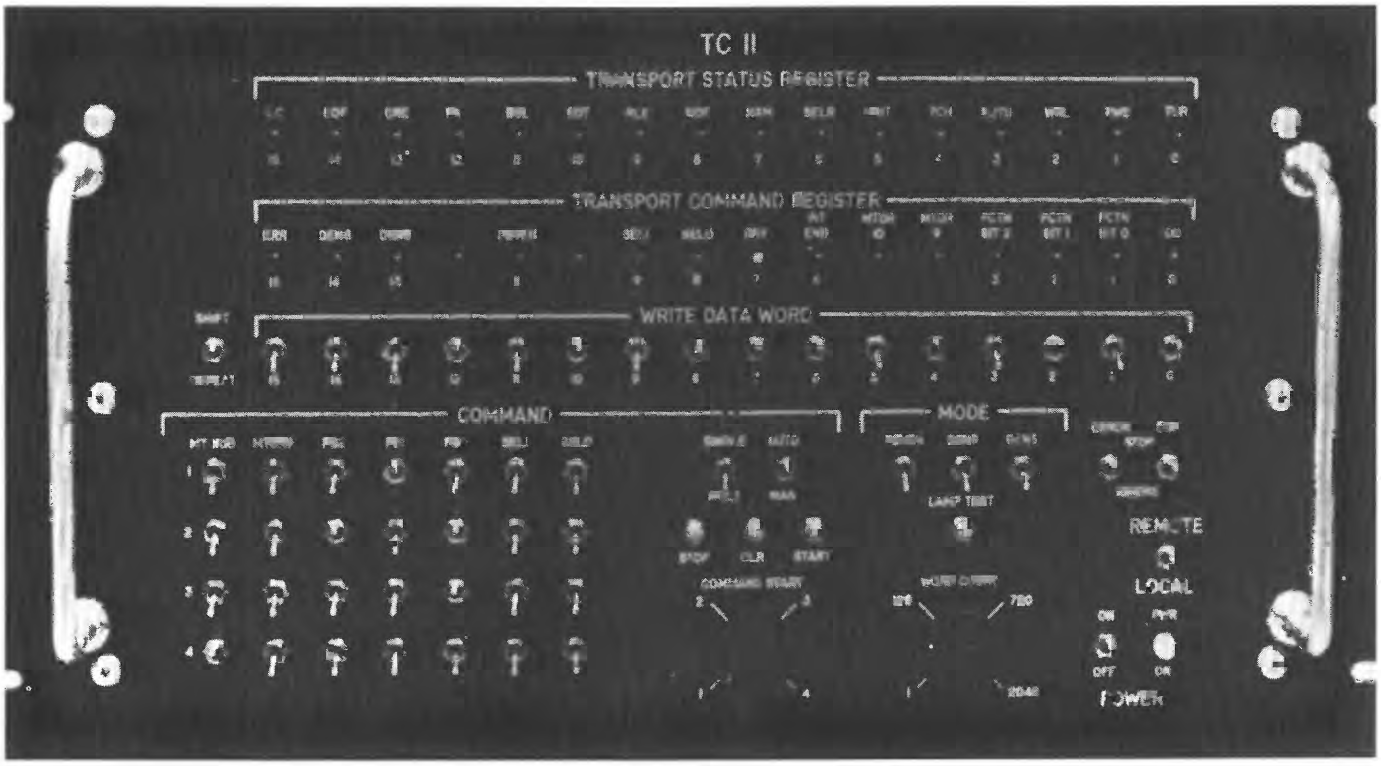


FIGURE 29.—Magnetic tape controller for the aerial profiling of terrain system.

fact, in the normal operation of the APT system, the magnetic tape devices will be used, not only to record data output from the computer (and the APT subsystems) during preflight calibration and alinement procedures and during surveying flights, but also to load data and programs into computer memory before a flight mission.

The magnetic tape recorder selected for the APT system is the Miltope Corp. Model CR600 (see fig. 30), which is a ruggedized cartridge-load type. The recorder is packaged in a metal box mounted in standard light-alloy racks. The overall dimensions, mass, and power requirement are given in table 1. Note (fig. 25) that two tape recorders will be used to forestall any break in recording data when a full cartridge must be replaced.

A cartridge contains a standard IBM-compatible 7-in. (0.18-m) diameter reel capable of storing 600 ft (183 m) of 1/2-in. (13-mm)-wide magnetic tape. Data are stored on the tape in a seven-track recording mode at a density of 800 bits per inch (315 bits per centimeter). The tape recording speed is 25 in./s (0.64 m/s) and rewind speed is 50 in./s (1.27 m/s). These specifics indicate that a single cartridge (tape) can hold 6.24 million bits of data and (or) information, which roughly translates (not a simple or direct conversion) into about a 30-minute surveying flight.

The line printer, which is the only remaining computer-oriented peripheral device shown in figure 25, has been described in the discussion on I/O channel number 8, given in the section "Subsystems and Data Interfaces."

Software

Software is, for this report, the ensemble (regardless of format) of instructions, commands, and programs needed to run the computer through all its necessary functions in all desired modes of APT system operation. A program is defined as a set of instructions or steps that tells the computer exactly and completely how to handle a given task.

If the software for the APT system is to fulfill its basic purpose, it must be keyed to a very rigorous mathematical analysis and description of how that system performs. Among the powerful analytical tools used in the mathematics that describe the APT system is the method of "state variables." This method is ideally suited for analysis of the multiple-input and multiple-output APT system, where performance details can ony be described completely if the values of, and changes in, more than 50 variables can be determined. In applying the method, it was used in a nondeterministic way; that is, it was used to get "best estimates" which closely resemble what might be obtained through the analytical technique of least-square fits. The end results of such a sophisticated mathematical analysis and the processing of such large bulks of system operating data are judged to contribute improvement in the position-coordinate computations of about two orders of magnitude.

For the APT system, a unique computer program has been written for each of eight modes of system operation. Parts of these programs, called subroutines, are common to several or all system modes. Examples include instructions for the computer to perform computations, to interpolate data, to generate mathematical functions, to accept data (input) for storage or processing or both, and to release data (output), all as required by the APT subsystems and peripheral devices described in the preceding sections.

Wherever possible, the programs have been designed to be "table driven." This is a technique whereby essential pieces of APT system control information are represented as data items or variables, organized into tables, and stored in the computer memory, rather than being built into the program structure itself. An example is the requirement for periodic computation of temperatures from thermistor outputs at dozens of widely scattered points in the APT system. The instructions for checking individual thermistors are similar but not identical. Thus, to write dozens of not-quite-identical

FIGURE 30.—Magnetic tape recorder for the aerial profiling of terrain system.

instruction sets is to invite many opportunities for errors, in addition to using up too much precious memory space. By organizing the key control information, in this case, scale factors, for all thermistors into a table (one numbered line per thermistor) then, only one set of instructions is needed. For a given thermistor, those instructions simply reference the appropriate line number in the table where all the pertinent control information resides. Thus, any subsequent requirement for making a change in that information is easily met without rewriting the master program.

The abovementioned computer programs, instructions, and related earth science and instrument-component-performance data needed to operate the APT system in all its modes are hereby labeled the "master operating program." This program is written on one magnetic tape.

The fact that the Norden computer must function in a number of ways as a real-time control device for the APT system means that a premium is placed on scheduling and timing the various program tasks the computer must execute. The message-priority features already built into the computer software greatly simplify the writing of programs to cover all tasks, but the challenge is persistent and very real to achieve maximum program efficiency so that the computer has "free" time between major program task cycles wherein the lower priority and miscellaneous program tasks can be fitted. Thus, the design and instruction coding of a particular program must reflect special attention to the completion of the desired work within the available computer time. Furthermore, the steps to be performed must be expressed as concisely as possible, so that the whole program will fit within the limited computer memory space.

Organization of the software for the APT system is better understood if prefaced by brief descriptions of the eight distinct modes in which the system will be operated. These modes encompass preflight, flight, and postflight operations and are identified by name as follows:

1. COMPUTER OFF.
2. STARTUP AND TEST.
3. CALIBRATE AND ALINE
4. STANDBY.
5. EN ROUTE. }
6. SEARCH. } Flight Operations
7. AERIAL CALIBRATE. }
8. AERIAL SURVEY. }

Once the APT system is operational, a typical flight mission might logically require it to be sequenced through the foregoing modes in the manner shown in figure 31. Frequent reference to that figure will give added meaning to the ensuing mode descriptions.

1. *COMPUTER OFF.* Mode in which no electrical power is fed to any part of the computer or its peripheral devices. The mode is characterized by a preestablished arrangement of switch settings and specified values for certain internal state-of-health sensors. In this mode, it is possible to run all analog circuitry which includes servomotors and temperature controls. Thus, the IMU can be powered and gimbal angles set.

2. *STARTUP AND TEST.* Mode used to power up the system, perform functional system testing, and enter data pertinent to operation in flight. The computer, peripheral devices, and APT subsystems are powered up through a preestablished sequence of steps, which are executed by manual reference to a checklist and subsequently by operator selection on the keyboard. A key step in this sequence includes insertion into a tape-recorder unit of the magnetic tape cartridge that contains the "master operating program." This is read into computer memory by means of hard-wired instructions invoked by operator selection on the computer control panel. When a signal shows the program to be properly loaded in the computer memory, the "executive program" is called up, and the first instructions therein are executed when the computer receives its next 200-Hz time signal from the master clock. The significance of this and other time signals, and a brief definition of the executive program, are described later in this section.

Because the keyboard-and-plasma-display terminal is the principal interface between the operator and the APT system and provides the means for shifting the system from one operating mode to another, its description needs some amplification at this point. The plasma display is divided operationally into two parts, a so-

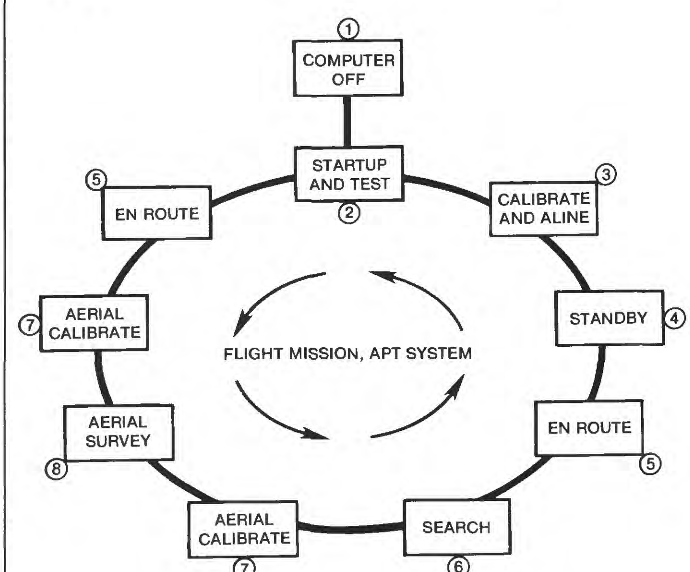


FIGURE 31.—Hypothetical sequence of the aerial profiling of terrain system modes for a flight mission.

called "fixed" upper segment and an "interactive" lower segment.

In a preestablished sequence and format, the fixed segment displays information relevant to the current operating mode and its status. This will include, for example, current readings of the IMU gimbal angles and similar current real data for the particular components of the APT system that are being functionally tested at that moment. This upper part of the display is updated by the computer every 2 seconds.

The interactive lower segment displays a "menu" of numbered alteratives from which the operator may select by means of the keyboard. Among the alternatives, for example, will be certain component test routines that can be started, other information displays that can be called up, or opportunities to enter data into the computer or to shift the APT system into another operating mode. The operator makes a selection through the keyboard, and the computer takes the commanded action when the carriage return key is depressed.

No attempt is made here to detail all I/O actions involving the computer, inasmuch as most will appear in a subsequent table and several figures. It is sufficient simply to observe that the STARTUP AND TEST mode provides for the orderly and safe automatic startup (or shutdown) of the APT system. The procedures include certain computer self-checks, various subsystem tests, and continuous alarm monitoring keyed to preestablished limits for automatically read voltage and temperature sensors at critical places in the system.

3. *CALIBRATE AND ALINE*. Mode used to fine-tune the inertial navigator while the aircraft remains standing over a known point on the airport parking ramp. Once the operator has selected this mode and commanded the computer through the previously described keyboard actions, a preestablished sequence of procedures is started. These include the deliberate precessing of the stable platform in the IMU through a prescribed schedule of attitudes by appropriately torquing the gyros. The need is to determine (calibrate) all gyro and accelerometer performance coefficients at this particular point in time and under the prevailing set of environmental conditions which include the physical location of the APT system on the Earth's surface.

The calibration exercise is an estimation process, often loosely called "Kalman filtering." It compares previously designed probability models of individual accelerometer and gyro performance against their actual physical measurement of the unchanging and known gravity and Earth-rotation vectors in the stationary aircraft. The desired performance-coefficient estimates emerge from the processing of the foregoing data over a period of from 1 to 48 hours, depending on the precision level required. The ramifications of the Kalman filtering technique for the APT system will become more apparent in the section on "Postflight Data Processing." However, the following general definition of a Kalman filter is offered as a useful preface.

The Kalman filter is a mathematical procedure (or algorithm) for combining measurement data with information about the internal configuration and performance characteristics of a system to form estimates of the current state of the system. It is named after its inventor, R. E. Kalman (1960), and is called a filter, following well-established usage, because its effect is to "purify" error-contaminated, or "dirty," data signals, thereby allowing the system state to be observed. Although the Kalman filtering procedure is conceptually akin to taking the average of replicate measurements, or fitting a regression line to a scatter-plot, the following considerations distinguish it from such elementary statistical procedures:

1. The state of the system is not directly observable but must be determined by calculation from measurable data.
2. The measured data are subject to unavoidable measurement errors, so the states calculated from them also are subject to error.
3. The true state of the system is changing at roughly the same rate as the rate at which data are being acquired, so that replicate observations are not available for averaging.
4. The complexity and volume of the necessary computations tax the ability of the computer to produce state estimates as fast as they are needed.

To cope with these problems, the Kalman filter is recursive—it computes its state estimate in the form of a weighted average of the previously computed estimate and an estimate based on the most recent set of measured variables. In addition, the filter maintains a running estimate of the statistical sampling errors in these estimates. In this way, it can compute the optimum weighting factor to minimize the error of each estimate.

The calibration exercise is complete once a current set of performance-coefficient estimates is available for all gyros and accelerometers, which the operator considers adequate to predict the performance of those instruments in the upcoming APT system flight modes. At this point in time, alinement begins, wherein the operator invokes computer functions that direct the procedures for stable-platform leveling and gyrocompassing (see "Performance Dynamics" section). The minimum time required for the alinement exercise is about 20 minutes.

The APT system operator may selectively opt to tape record or print out data produced during the calibration and alinement exercises. Normally, the operator will allow the system to continue its alinement routine until shortly before flight time. At that point the operator will command the system to shift to the STANDBY mode (see fig. 31).

4. *STANDBY.* Mode in which the APT system is maintained in powered up, calibrated, and alined configuration in anticipation of a flight mission. This program segment includes provision for inputs to computer memory from the static-air-pressure and temperature transducers mounted on the outside of the aircraft.

While the APT system is in STANDBY status, the operator is responsible for keying in the pertinent flight-mission data. These will include, for example, information on the flight path to be navigated to the survey area, position-coordinate and gravity values for the field-surveyed retroreflectors (minimum of three required), and details of the aerial survey to be accomplished. Any last-minute changes in the flight mission also would be keyed in at this time.

When all preparations for the flight mission have been completed and the flight crew is ready, but before the aircraft starts to move, the system operator will command (through the keyboard) the system to shift to the EN ROUTE mode (see fig. 31).

5. *EN ROUTE.* Mode in which the APT system will measure and record the actual path of the aircraft as it moves away from the ground-alinement location. It is also the mode in which the system will navigate the aircraft over the flight-plan route to the survey area.

When the system operator has commanded this mode, the computer functions in this program segment include processing of the measurement data from the three accelerometers and from the static-air-pressure transducer to accomplish inertial navigation and computation of steering signal outputs for the indicator on the pilot's instrument panel so the intended flight path can be flown more precisely.

Although tacitly implied, it bears repeating that the computer functions in this mode, as in all modes, continue to include the monitoring of state-of-health sensors throughout the APT system. Status readings appear at preprogrammed intervals on the operator's plasma-display terminal. A suitable alarm will be signaled if any reading is out of bounds.

At predetermined regular intervals in the EN ROUTE mode, the computer will route to the plasma-display terminal instantaneous readings of the aircraft position coordinates, heading, ground speed, and the time and distance remaining to the first presurveyed retroreflector target that is to be used in the survey. The system operator also may opt to tape record or print out any EN ROUTE data considered appropriate for use in the postflight data-processing work.

As the aircraft nears the first retroreflector, the system operator will command the system to shift to the SEARCH mode (see fig. 31). If the circumstances warrant, however, the operator may choose to command a shift to any other flight mode.

6. *SEARCH.* Mode in which the tracker will be run through its search pattern until the first presurveyed retroreflector target is found. When the APT system operator has commanded this mode, the computer functions now include, in addition to those that concern continued EN ROUTE navigation and steering, command of tracker gimbal torques sufficient to point the tracker forward toward the expected location of the retroreflector and to begin automatically to scan around the pointed direction when the retroreflector comes within range, with the intent of detecting and locking onto a pulsed-laser return signal.

The nature of the program segment, written to cover this computer role of commanding the tracker in its search patterns, can be understood through a brief account of what the computer does. The computer first selects the nearest retroreflector and computes its range. If it decides the target is out of range, then the computer simply commands the tracker to point forward at the maximum angle. If the target is determined to be in range, then the computer estimates the probable error in target position. If the estimated error is 65 ft (20 m) or less, then the computer commands the tracker to begin a spiral scan that gradually opens up around the initial most probable direction. The computer examines successive groups of return signals from the tracker quadrant photodiode and range detectors, and, when certain reasonableness criteria are met, it concludes that a valid return signal has been received. At this point, the computer commands the tracker to enter its tracking procedure, which means that the gimbal-drive servos, using the quadrant photodiode signals, maintain the pointing direction needed to keep the tracker locked onto the target. The computer simultaneously and automatically shifts the APT system to the AERIAL CALIBRATE mode (see fig. 31).

If, in the foregoing procedure, a valid return is not received within 15 seconds, then the computer terminates the spiral scan and begins a wide-angle, side-to-side, or raster, scan, around the most probable position of the retroreflector. Upon receipt of a return signal within the range-gate setting chosen for this fast raster scan, the computer points the tracker in the direction of the valid return and once again commands the spiral scan. The lock-on procedure is as described above.

If the error in retroreflector position, as first estimated by the computer, is greater than 65 ft (20 m), then the computer simply commands the fast raster scan at the outset, and the ensuing procedures to target lock-on are as described above.

7. *AERIAL CALIBRATE*. Mode in which the tracker individually tracks, according to the sequence stipulated in the flight plan, each retroreflector in the survey area. Whenever the computer has initiated this mode automatically, the pointing-error signals from the quadrant photodiode replace the computer-generated, angle-command signals feeding the tracker gimbal-drive servos, target lock-on is achieved, and tracking the retroreflector begins. Other computer functions continue as in the EN ROUTE mode. On any interruption of tracker returns (caused perhaps by obscuration of the laser-beam path to the retroreflector), as signalled to the computer by the tracker circuitry, the computer will command reentry to the SEARCH mode to reacquire the retroreflector and then continue tracking. As the tracking procedure progresses, a stream of readings of laser range, tracker pointing angles, and IMU gimbal angles accumulates in the computer storage. From these data, the computer refines the indicated aircraft position and computes preliminary position data for the retroreflectors.

The refinement process is aided by computer corrections for the tracker measurements, based on index-of-refraction calculations for the air path traveled by the laser beam. The adjusted tracker measurements are formatted and recorded for the postflight data-processing work. This work will include adjustments on the whole body of flight-mission data so that the best possible final values are determined for such key parameters as position coordinates for each unsurveyed retroreflector and for the aircraft at the succession of points that defines its flight path.

If the tracker looses lock on a retroreflector, then the computer will determine whether that target is still within range. An affirmative result will trigger automatically a slow-scan reacquisition procedure, inasmuch as the target location is already rather precisely known. When the tracking of a particular retroreflector is complete, as shown by loss of lock and failure of the range check, the computer determines the heading to the next retroreflector, and this result is relayed to the pilot's steering indicator and to the APT system operator's plasma display.

Normally, the AERIAL CALIBRATE exercise is complete when each of three specified retroreflectors has been individually tracked twice in the carefully designed flight-plan sequence. If necessary, however, the system operator can alter the preplanned exercise by adding or deleting certain tracking passes.

When the operator is satisfied that all retroreflector targets needed for the flight mission have been properly tracked, the sytem is commanded to shift to the AERIAL SURVEY mode (see fig. 31).

8. *AERIAL SURVEY*. Mode in which the APT system collects the survey data for which the flight mission was planned. When the system operator has commanded this mode, the computer functions in this program segment include, in addition to those that concern continued inertial navigation and steering, the slow-scan search and reacquisition and tracking procedures for each retroreflector that is to be used during the actual gathering of the aerial survey data. The sequence is as stipulated in the flight plan for this part of the mission. These functions draw upon the inputs made by the system operator during the STANDBY mode.

After commanding entry into the AERIAL SURVEY mode, the system operator will command activation of the laser profiler and color TV camera subsystems. Computer functions then expand to include performance monitoring of these subsystems, relaying the APT system time-code signals to the time-date generator (see "Color TV Camera" section), relaying the time-labeled video imagery to the video recorder, and formatting and tape recording the terrain-profile data that emerge from the laser-profiler range measurements. As recorded in flight, the position coordinates for the profile data are in a raw state; refinement will come through the postflight data-processing work.

When all the desired terrain-profile data have been gathered, the operator may command the system to shift back to the AERIAL CALIBRATE mode to allow the collection of a final suite of position data for the retroreflectors used in the mission. The operator then may command the system to shift to the EN ROUTE mode to navigate to the next survey site or to return to the airport of origin.

Tacit in the foregoing mode descriptions is a natural and orderly progression from one to the next, with opportunities to jump back one or more modes as operating procedures dictate.

In the foregoing decriptions of the eight distinct APT system modes, many allusions have been made to the various computer functions that make the sytem perform in the desired manner. Those functions are performed successfully, however, only as a consequence of the sophisticated structure and organization of the software. The challenge now is to give some meaningful description to that structure and organization. As this descriptive material is read, it is important to remember that all programs developed for the APT system were designed to run under the Digital Equipment Corporation's (DEC) PDP 11/70 "operating system program" featuring the Resource Sharing Executive Program, RSX–11S.

The primary purpose of the operating system program is to manage the execution of all program segments coded specifically for the APT system. For each

APT system mode, proper management requires that the program segments, hereafter called tasks, be run in the correct sequential relations with respect to timing and priority. A function of the RSX–11S, which is incorporated in the operating system program, is to establish a priority queue for the tasks that must be run and to handle a task interrupt when a higher priority need arises. An interrupt entails temporary storage of the uncompleted tasks, together with all variables and intermediate results, until it can be resumed when the higher priority work is done. In reality, several layers of interrupted tasks can be handled as the need arises.

The operating system program also includes I/O driver routines (subroutines that feed data into or out of peripherals when called), a library of mathematical and other functions used in running many tasks, and definition of common storage areas (particular portions of computer memory used for communication among various tasks).

The first noteworthy developmental milestone was reached when the PDP 11/70 operating system program, generated for the APT system software configuration, was run in the Norden computer. This milestone program served as the framework around which the balance of the program development was fitted. As the development work proceeded, program segments were written or coded in either Fortran or Macro–11 (a DEC language) assembly languages. The segments were checked by test runs on the Draper Laboratory's VAX 11/780 computer (a DEC machine), which can be configured to emulate the Norden PDP 11/70M computer. The VAX computer then produced the machine-language version of the master operating program that is loaded into the Norden.

In addition to the operating system program described above, the master operating program includes the 14 special APT system tasks that are used in various combinations to meet the performance requirements of each of the eight operating modes. The structure graph in figure 32 shows the component parts of the operating system program.

Note in figure 32 that the 14 special tasks are organized logically under six basic operational functions within the master operating program. Of the six functions, only the first, processing that must be done in real time, contains tasks that are to be run at stipulated frequencies, which are submultiples of the APT system master-clock frequency. Because of this linkage to the master frequency, the five tasks shown (fig. 32) in this category are termed "synchronous tasks"; the first four are named by their recurrrence intervals of execution, and all five are listed in their proper activation (callup) sequence.

The remaining five basic operational functions embrace the nine remaining special APT system tasks, and, because each one can be called by the APT system operator or by another task to run at any time, they are all termed "asynchronous tasks" (see fig. 32). All nine are lower in priority than the five synchronous tasks, and, hence, when one is called, it must be fitted into the priority queue (by the executive program) and await available time on the central processor.

Although the labels for the foregoing five basic operational functions (fig. 32) are reasonably self-evident, some amplification is appropriate. "Hardware mode control" is the single all-important asynchronous task of monitoring and controlling the shifting of the APT system from one mode to another. "Telemetry and alarm processing" are two asynchronous tasks that concern the monitoring of all system-health sensors, logging their readings, and alerting the system operator to any malfunctions. "Data base population and recording" are also two asynchronous tasks that allow the operator to bring in (to computer storage) programs and data preassembled on magnetic tape or permit manual entries by means of the computer keyboard and to unload (onto magnetic tape), during and after a flight mission, the complete suite of data used in the mission, as well as the survey data collected. "Hard copy command processing" includes three asynchronous tasks that relate exclusively to calls for output on the line printer. "Operator system interface" is the single and last asynchronous task that governs the APT system operator's keyboard and plasma-display terminal.

Each of the 14 special tasks (program segments) shown in figure 32 embraces a modest variety of computer processing actions. These are given in summary form in table 2, which also shows how each special task is activated (called) and the particular APT system operating mode(s) in which each computer processing action occurs. The table is quite complete and displays, to the intended degree, the range in complexity of the computer programs developed to operate the APT system. A more complete description of these programs is beyond the scope of this paper.

Just a cursory survey of table 2 shows that most of the computer processing actions relate to the five real-time synchronous tasks. Furthermore, for a given synchronous task, the variety of computer actions listed is to be recognized as a simple collection of those actions that have the same priority and required frequency (or frequency submultiple) as the task itself. Those actions that need to be accomplished at the same frequency as the task are grouped under the heading "Primary"; those actions that need only be accomplished at some specified submultiple of the task frequency are grouped under the heading "Secondary."

In the group of five synchronous tasks, the highest priority and frequency (shortest recurrence interval) are assigned to the TWENTY MS task. The other four

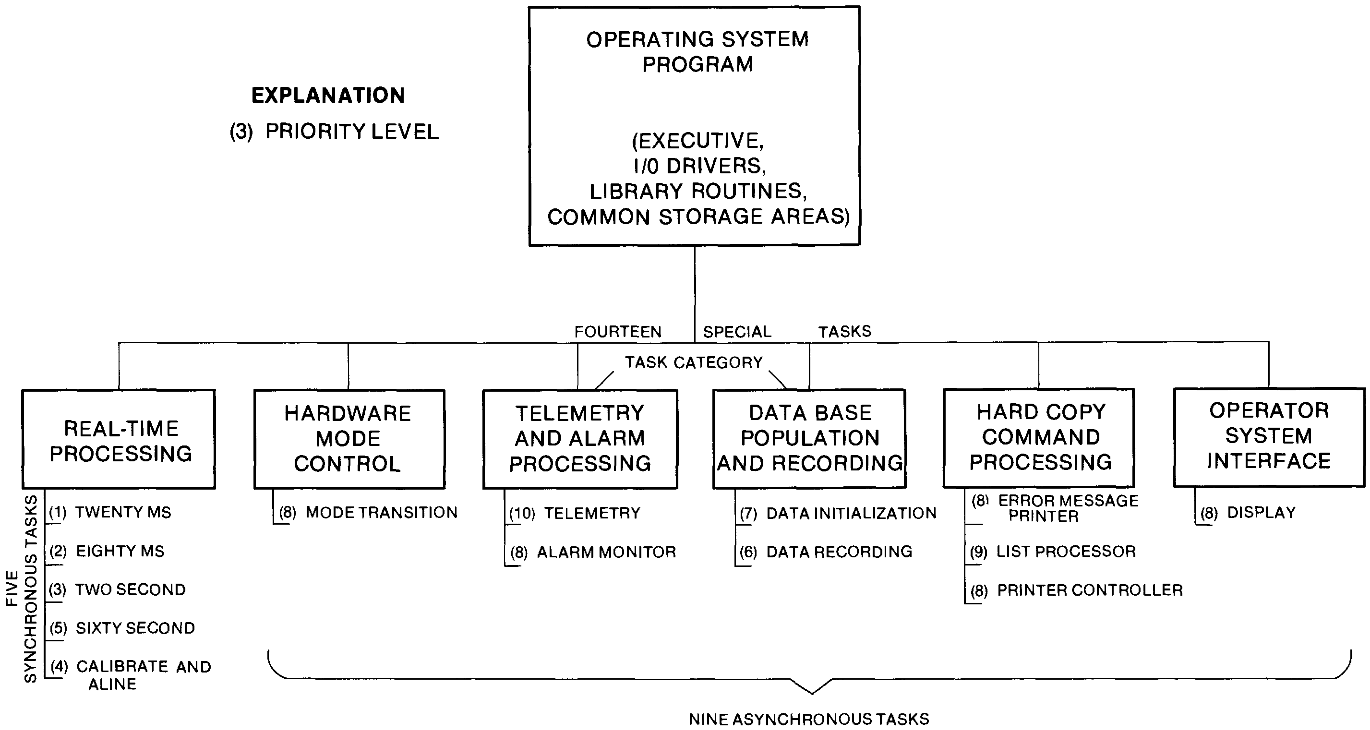


FIGURE 32.—Structure graph of the master operating program for the aerial profiling of terrain system.

tasks are assigned lower priorities and frequencies, but each of the latter is a specified submultiple of the highest frequency. This leads to a simple callup sequence for the five synchronous tasks, and a logical grouping of the primary and secondary computer processing actions thereunder, as outlined in figure 33. In effect, this illustration serves to emphasize the sequential relations that are more subtly discernible in table 2.

From the information given in figures 32 and 33 and table 2, a general statement naturally evolves to summarize the callup sequence for the 14 special tasks in the APT system. The computer processing actions proceed from the highest to the lowest priority tasks, which mean that all five synchronous tasks are completed before the nine asynchronous tasks. The latter then are completed in accord with their individually assigned priorities. The entire train of events is initiated by a 200-Hz (5-ms-period) signal from the master clock, which invokes the transfer of measurement data between the APT subsystems and the computer by means of the DMA channel (number 1 in fig. 25). When all system tasks are up-to-date, the computer is idle momentarily, awaiting the next 200-Hz signal.

The stage has now been set to conclude the discussion of "Software" with three functional diagrams (figs. 34–36) to illustrate how tasks are linked and how the data flow to bring about the computer processing actions needed for each operating mode of the APT system. In the three diagrams, the conventions used are to show a given computer processing action as a rectangular box in which an abbreviated action title appears. The subtitle at the base of the box is generally the special task under which the action occurs; occasionally, it will be a subroutine of the master operating program, as, for example, the DMA I/O DRIVER. Note that color coding identifies computer actions with the particular APT subsystem(s) for which they are being performed. Thus, orange indicates actions related to the IMU and inertial navigation, green relates to the tracker, and blue relates to the profiler. Note also that, in each diagram, the data flow and the action proceed generally from left to right. Three initiating or input sources, namely, DMA data transfer, operator's keyboard, and analog telemetry, are on the left; outputs to the APT subsystems and peripheral devices are near the right. Lines that connect boxes signify the transfer of data or commands. A dot where lines cross indicates a physical connection; absence of a dot indicates no connection.

The computer processing actions in the STARTUP AND TEST mode (fig. 34) use all input and output channels of the computer in powering up the system, testing for proper functioning, and entering basic data. Many of the computer actions involve formatting data and relaying information through channels used for more complex and sophisticated functions in subsequent operating modes.

The computer processing actions for the CALI-

TABLE 2.—Computer-action summary for the 14 special tasks in the aerial profiling of terrain system

TASK NAME	ACTIVATION	COMPUTER PROCESSING ACTION	APT-SYSTEM MODES*
TWENTY MS	Called by the direct memory access (DMA) input/output (I/O) driver every fourth DMA data transfer.	Primary:	
		Compute IMU and tracker gimbal angles and accelerometer head angles from DMA data transfer, and sum for 80 ms averaging.	2-8 incl.
		Compute IMU and tracker gimbal rates.	2-8 incl.
		Detect IMU and tracker gimbal movement.	2, 6, 7, 8
		Sum and save profiler and tracker time-interval data from each DMA data transfer for computation of laser ranges.	2, 6, 7, 8
		Compute IMU and tracker gimbal-drive signals.	2, 6, 7, 8
		Compute gyro torque pulse sequences to be applied each 5 ms over next 20 ms.	5, 6, 7, 8
		Digital-to-analog output signal processing.	2, 5, 6, 7, 8
		Save data for recording.	2-8 incl.
		Secondary:	
		Forty ms (Every other pass):	
		Average profiler time-interval data for later computation of laser ranges.	2, 8
		Save data for recording.	2, 8
		Eighty ms (Every fourth pass):	
		Reorder and transfer partially processed tracker-range data for EIGHTY MS task.	2, 6, 7, 8
		Save gimbal and accelerometer head-angle data for EIGHTY MS task.	2-8 incl.
		Call EIGHTY MS task.	2-8 incl.
		Save data for recording.	2-8 incl.
		Two Second (Every 100th pass):	
		Call TWO SECOND task.	2-8 incl.
		Save data for recording.	2-8 incl.
		Sixty Second (Every 3000th pass):	
		Sample and save accelerometer data for validation in SIXTY SECOND task.	2-8 incl.
		Call SIXTY SECOND task.	2-8 incl.
		Save data for recording.	2-8 incl.

*Explanation of code numbers:

2. STARTUP AND TEST.
3. CALIBRATE AND ALINE.
4. STANDBY.
5. EN ROUTE.
6. SEARCH.
7. AERIAL CALIBRATE.
8. AERIAL SURVEY.

TABLE 2.—Computer-action summary for the 14 special tasks in the aerial profiling of terrain system—Continued

TASK NAME	ACTIVATION	COMPUTER PROCESSING ACTION	APT-SYSTEM MODES*
DATA INITIAL-IZATION	Called by DISPLAY task.	Load (record) predesignated segments of data base from (to) magnetic tape, on a single call by DISPLAY task.	2-8 incl.
		Load data manually entered by DISPLAY task into predesignated segments of data base.	2-8 incl.
		Call and transfer predesignated segments of data base to PRINTER CONTROLLER task for listing, on a single call by DISPLAY task.	2-8 incl.
DATA RECORDING	Called by TWO SECOND, SIXTY SECOND, and DISPLAY tasks	Buffer data saved for recording by synchronous tasks.	2-8 incl.
		Enable/disable predetermined recording lists, on call by DISPLAY task.	2-8 incl.
		Format and transfer data from enabled recording lists to magnetic-tape recorder	2-8 incl.
		Switch to alternate tape unit, on either tape-error or end-of-tape signal, without loss of data.	
		Transfer status of tape units, on call by DISPLAY task.	2-8 incl.
ERROR MESSAGE	Called by ALARM MONITOR or any other task.	Format all system status and error information.	2-8 incl.
		On call, transfer message, date, time, and identity of calling task to PRINTER CONTROLLER task for printing.	2-8 incl.
PRINTER CONTROLLER	Called by ERROR MESSAGE and LIST PROCESSOR tasks.	Control transfer of information to the line printer on a "time available", non-interference basis.	2-8 incl.
		Monitor printer off-line status.	
		Transfer status information for display every 15 seconds, if enabled by DISPLAY task.	2-8 incl.
LIST PROCESSOR	Called by any processing (synchronous or CAL & ALINE) task or DISPLAY task.	On a single call by a processing task, format requested list from APT system data base and call PRINTER CONTROLLER (or DISPLAY) task to print out (or display) list.	2-8 incl.
		On a single call by DISPLAY task, format and call PRINTER CONTROLLER (or DISPLAY) task to print out (or display) list.	2-8 incl.
DISPLAY	Called by operator.	Format and route operator commands to initiate, control, and monitor APT system moding.	2-8 incl.
		Format and route operator commands to DATA RECORDING, DATA INITIALIZATION, and LIST PROCESSOR tasks.	2-8 incl.
		Format and transfer data entered by operator.	2-8 incl.
		Format and display data requested by operator.	2-8 incl.

*Explanation of code numbers.

2. STARTUP AND TEST.
3. CALIBRATE AND ALINE.
4. STANDBY
5. EN ROUTE.
6. SEARCH
7. AERIAL CALIBRATE.
8. AERIAL SURVEY

TABLE 2.—Computer-action summary for the 14 special tasks in the aerial profiling of terrain system—Continued

TASK NAME	ACTIVATION	COMPUTER PROCESSING ACTION	APT-SYSTEM MODES*
SIXTY SECOND	Called by TWENTY MS task.	Transform sixty-second accelerometer data to locally-level coordinate frame.	2, 3
		Compute sixty-second gyro torque pulse sequences.	2, 3, 4
		Validate accelerometer head-angle data.	3
		Compute gyro-torque-test commands.	2
		Initialize and start accelerometer coning misalinement estimate computations.	2
		Compute coefficients for accelerometer coning compensation.	2
		Perform accelerometer diagnostic calibration.	2
		Call CAL & ALINE task.	3
		Save data for recording.	2, 3, 4
		Call DATA RECORDING task.	2, 3, 4
CAL & ALINE	Called by SIXTY SECOND task.	Compute gyro torque commands to rotate IMU stable platform through a series of predetermined trajectories.	3
		Compute estimates of IMU-performance parameters in 57-state Kalman filter.	3
		Save updated estimates of IMU-performance parameters.	3
MODE TRANSITION	Called by DISPLAY, ALARM MONITOR, or EIGHTY MS task.	Monitor and control moding of IMU, tracker, and profiler.	2-8 incl.
TELEMETRY	Called by executive program (about once per second).	Sample analog-to-digital converter outputs.	2-8 incl.
		Smooth those outputs that indicate temperature	2-8 incl.
		Convert sampled and smoothed outputs to engineering units.	2-8 incl.
ALARM MONITOR	Called by executive program every minute or by DISPLAY task.	Primary: Monitor telemetry data, flagging and saving quantities crossing predetermined thresholds.	2-8 incl.
		Call MODE TRANSITION task for appropriate emergency action according to predetermined list of quantities if enabled by DISPLAY task.	2-8 incl.
		Call ERROR MESSAGE task to report quantities when flagged.	2-8 incl.
		When called by DISPLAY task, call ERROR MESSAGE task to report number and status of flagged quantities.	2-8 incl.
		Secondary: Sixty Minute (Every 60th time through): Call ERROR MESSAGE task to report number of flagged quantities.	2-8 incl.

*Explanation of code numbers:

2. STARTUP AND TEST.
3. CALIBRATE AND ALINE.
4. STANDBY.
5. EN ROUTE.
6. SEARCH.
7. AERIAL CALIBRATE.
8. AERIAL SURVEY.

TABLE 2.—Computer-action summary for the 14 special tasks in the aerial profiling of terrain system—Continued

TASK NAME	ACTIVATION	COMPUTER PROCESSING ACTION	APT-SYSTEM MODES*
EIGHTY MS	Called by TWENTY MS task.	Primary:	
		Compute accelerometer head-angle averages and rates.	2-8 incl.
		Compute IMU and tracker gimbal-angle averages.	2-8 incl.
		Compute tracker range for the previous 80 ms.	2, 6, 7, 8
		Compute accelerometer coning misalinement estimates.	2
		Compute accelerometer coning misalinement compensations.	2-8 incl.
		Transform accelerometer data to locally-level coordinate frame.	5, 6, 7, 8
		Compute position and velocity (inertial navigation).	5, 6, 7, 8
		Tracker search commanding and moding.	2, 6, 7, 8
		Compute gyro torque commands.	5, 6, 7, 8
		Save data for recording.	2-8 incl.
		Secondary:	
		Two Second (Every 25th pass):	
		Save data for recording.	2-8 incl.
		Sixty Second (Every 750th pass):	
		Save IMU gimbal-angle averages for gyro torque test.	2
		Save data for recording.	2
TWO SECOND	Called by TWENTY MS task.	Compute IMU and tracker gimbal-angle commands.	2
		Compute position and velocity update.	7, 8
		Compute heading vectors and time to reach retroreflectors.	5, 6, 7, 8
		Compute flight-indicator signals.	5, 6, 7, 8
		Format APTS date and time for displays, printer, and recorder.	2-8 incl.
		Save data for recording.	2-8 incl.
		Call DATA RECORDING task.	2-8 incl.

*Explanation of code numbers:

2. STARTUP AND TEST.
3. CALIBRATE AND ALINE.
4. STANDBY.
5. EN ROUTE.
6. SEARCH.
7. AERIAL CALIBRATE.
8. AERIAL SURVEY.

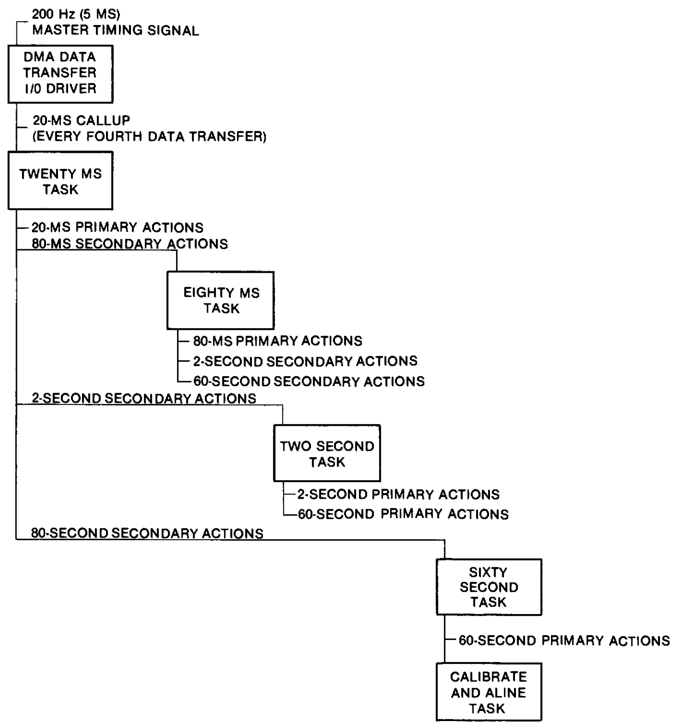


FIGURE 33.—Recurrence intervals and callup sequence for the five synchronous tasks in the aerial profiling of terrain system.

BRATE AND ALINE mode (fig. 35) show the same basic structure and organization as for the STARTUP AND TEST mode, with the important addition of the special actions needed to calibrate and aline the APT system. The STANDBY mode continues the use of the same computer-action structure to hold the system in its alined configuration ready to perform a flight survey and to allow time for entering flight-mission data that will be used by the computer in the flight modes that will follow.

Computer processing actions for all four flight modes—EN ROUTE, SEARCH, AERIAL CALIBRATE, and AERIAL SURVEY—have enough similarities in basic structure and organization so that they are shown conveniently in a single diagram (fig. 36).

Figures 34 through 36 faithfully portray all computer processing actions given in table 2, although some actions have been lumped to improve clarity. It should be emphasized, however, that the computer-action linkages and flow of data shown in these figures are, to some degree, schematic, or analogous, to "snapshots" of many actions that are commingling in apparent kaleidoscopic fashion. An infinite number of such snapshots are possible, but the several chosen serve to illustrate important features of the many necessary computer actions. To delve any deeper into the real complexities of the software development is well beyond the intended scope of this paper.

Laser Profiler

As already described in tracing the history of this instrument development project, the beginning centered on the innocent question, "Is it possible to measure terrain profiles to a specified high degree of precision from an aircraft?" The precision requirements in fixing aircraft position in the vertical and horizontal coordinate directions were what gave pause in answering the question. An immediate "yes" seemed possible, insofar as the instantaneous electromagnetic measurement of distance from aircraft to terrain was concerned. Subsequent events showed, however, that even this partial answer was brash to a degree because the design of the laser profiler encountered its own set of unique challenges.

Measurement of distance or length is so fundamental a problem that much attention is given to it in most studies of basic physics. Earth scientists in general are familiar with the platinum-iridium bar, exactly 1 m in length, which, for many years, has been maintained under precisely stipulated and controlled conditions at the International Bureau of Weights and Measures to represent a standard unit of length used throughout most of the scientific world. In 1960, the eleventh gathering of the General Conference of Weights and Measures on the International System of Units (SI) redefined the meter in the more rigorous terms of a specified number of wavelengths of electromagnetic radiation corresponding to the decay of the krypton-86 atom between two particular activity or product levels. Thus, in this part of the overall instrument development project, it was fundamentally sound to set out to measure a length with a laser beam of light.

The basic processes involved in such a measurement technique require a capability for timing the known velocity of light propagation [about 186,281 mi/s (299,790 km/s)]. If the length measurement is to be precise to ± 0.5 ft (0.15 m), this means the time-measuring capability must reach a precision of 1 ns, or the time required for the light beam to advance about 1 ft. Tremendous developments in timekeeping by electronic means now make it possible to obtain "off-the-shelf" equipment equal to the nanosecond timing task; special purpose equipment can do even better.

In essence, the measurement technique entails generation of a light beam by a laser source. The light thus produced has some very unique and desirable properties, the ones of principal interest being the spectral and temporal coherence of the beam. Thus, when the light beam is generated, successive wave forms have the same frequency, phase, amplitude, and direction. Although the particular laser chosen for the terrain-profiling task does not have as high a degree of coherence as most other lasers, it can be packaged so compactly that its small emitting area literally can be

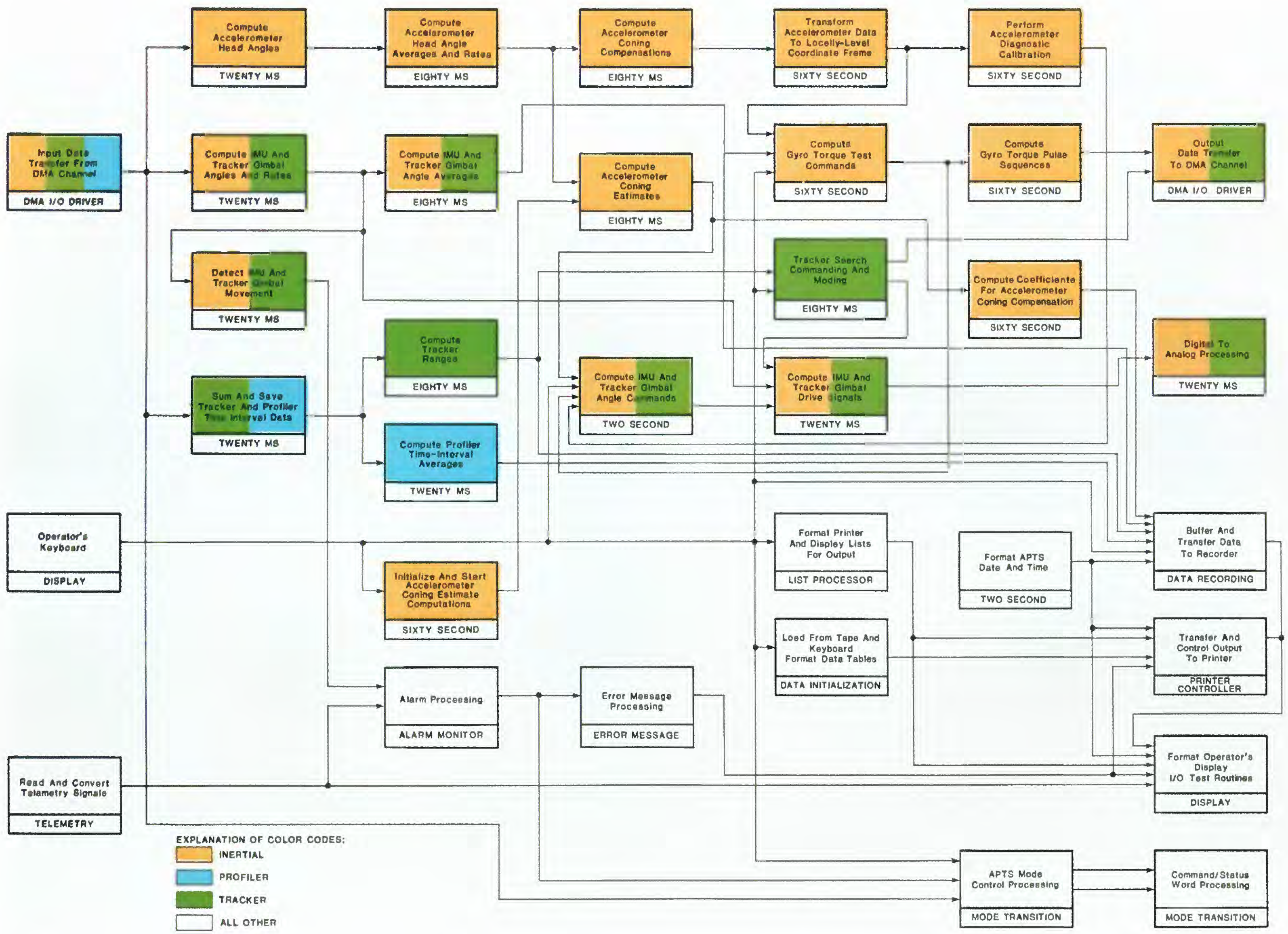


FIGURE 34.—Data flow for the STARTUP AND TEST mode of the software for the aerial profiling of terrain system.

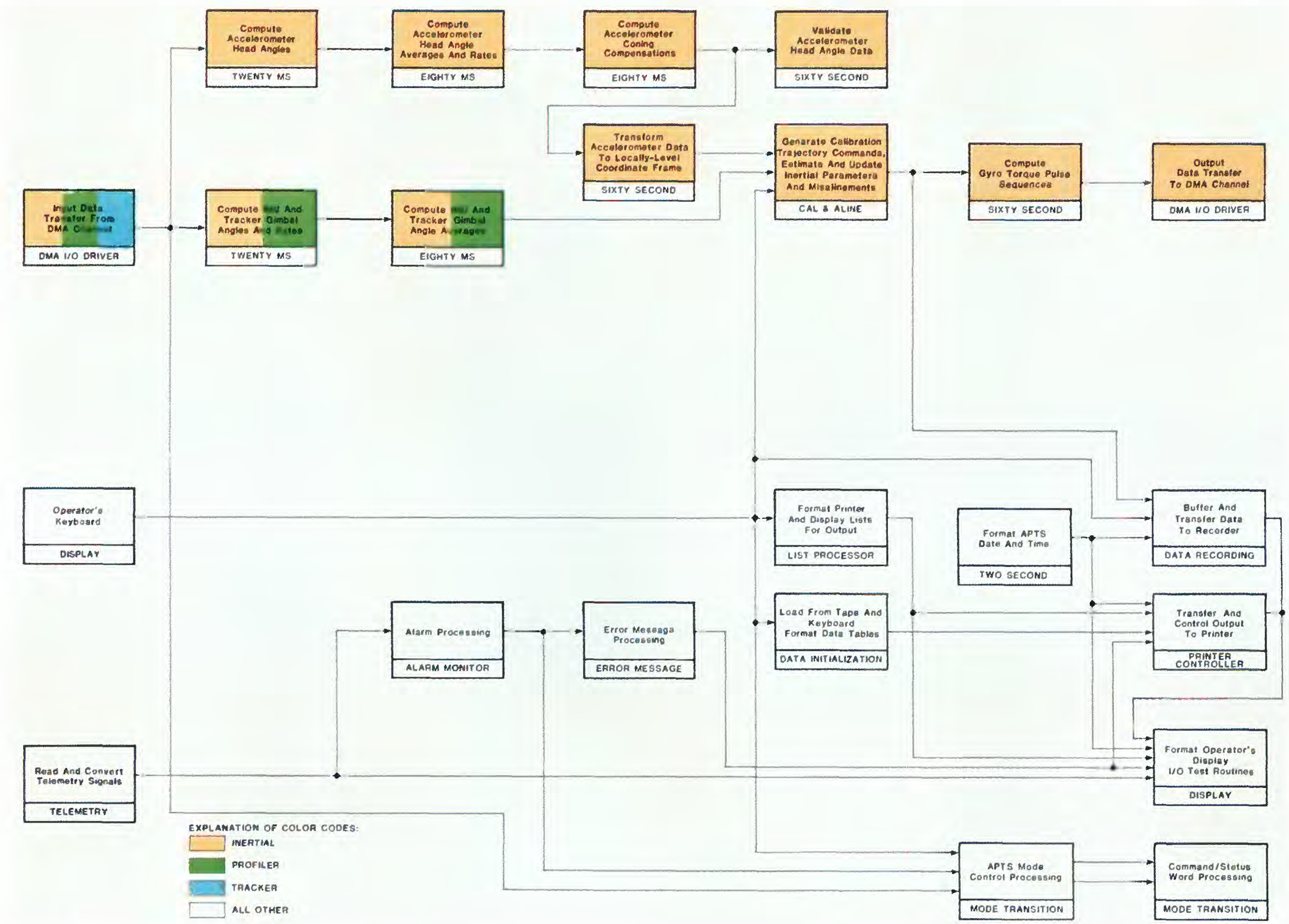


FIGURE 35.—Data flow for the CALIBRATE AND ALINE and STANDBY modes of the software for the aerial profiling of terrain system.

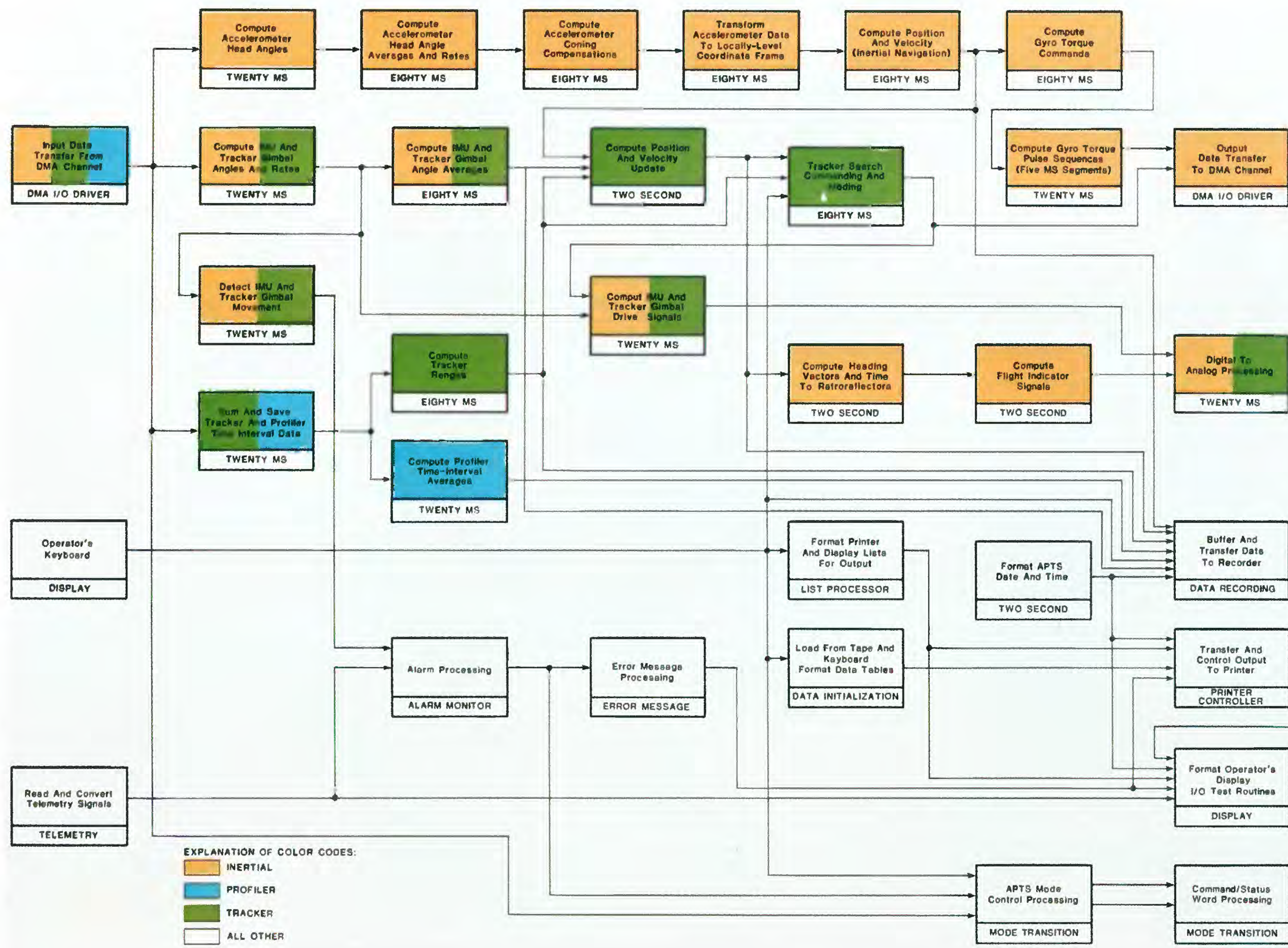


FIGURE 36.—Data flow for the four flight modes of the software for the aerial profiling of terrain system.

placed at the focal point of the transmitting optics. Thus, the divergence of the transmitted beam can be extremely small, of the order of 1 mrad, or about 3 minutes of arc. In contrast with more conventional light forms this results in very tight control of the "footprint" at the land-surface target.

The balance of the measurement process includes the deliberate labeling of selected outgoing light-wave groups by appropriate electronic modulation so they can be recognized when they return, provision of the aforementioned optical system to project the outgoing laser beam in a slightly diverging conical shape, provision of a second optical system either alongside or concentric with the first to collect and focus onto a photodiode detector part of the beam reflected back from the land-surface target, and suitable electronic amplification and processing of the returned light signals to provide the desired length or range readout data. The aggregation of components designed to perform these functions has been termed a "laser profiler" because of its planned use in collecting terrain profiles from an aircraft.

Because the laser profiler is intended as an adjunct to the APT system, its principal features must satisfy the same general body of design criteria which includes stipulations that limit bulk size, weight, and electrical power consumption. At the outset, the availability of commercial laser units was examined, but it was quickly concluded that manufacturers had done relatively little to capitalize on recent state-of-the-art developments capable of achieving much-desired reductions in size, weight, and power. Thus, the Survey, with key help from the Harry Diamond Laboratories, chose to develop its own design for the laser profiler and negotiated one phase of its contract with Draper Laboratory to accomplish the necessary laboratory experiments that led to the first "brassboard" working model. This approach served to maximize the prospects for well-designed interfaces with the APT system.

After investigating the advantages of new miniature solid-state components for the laser source, it was decided to use a development by Harry Diamond Laboratories that featured a simple pulsed GaAs-injection laser source. This particular semiconductor source is rugged, reliable, and inexpensive. It can be readily modulated at a very high rate, which is a key property in distance measurement, and, with its corollary electronic components, it offers high accuracy and nonambiguous range resolution along with small size, weight, and power consumption. It requires no water cooling or precisely controlled temperature environment.

At this point in time, the Survey and Draper Laboratory learned of successful test flights of a pulsed GaAs laser-ranging device independently developed for the Naval Research Laboratory. This was flown about 500 ft (152 m) above the sea surface to collect profile data on waveforms. Draper Laboratory obtained a working copy of the chassis, optics, mechanical components, and some electronic circuitry for the pulsed laser transceiver (see fig. 37) from the contractor.

Draper Laboratory's final design of the laser profiler reflects not only the many inputs from Harry Diamond Laboratories and the Survey, but also the experience gained from two sets of Survey-designed flight tests. The salient laser profiler features are summarized in this section. For added technical details, see especially the reports by The Charles Stark Draper Laboratory, Inc. (1977), Mamon and others (1976, 1978), and Youmans (1977).

An ultrashort-pulse laser source of GaAs permits use of a high-pulse repetition rate (3.2 kHz) and hence the integration of a large number of return pulses, which is an important feature when the target (terrain) changes rapidly with the forward movement of the aircraft. Furthermore, the pulsed laser unit, in contrast with a continuous wave unit, gives the capability for discriminating between multiple returns from the individual pulses when foliage covers some of the terrain. By providing an operational option for a last pulse selection technique in examining the return-signal data, the number of range determinations to the desired terrain surface can be maximized.

The laser transmitter module features a three-stack laser driven by an avalanche transistor in a hybrid circuit designed and described by Vanderwall and others (1974). This produces light pulses, at a wavelength in the near infrared of 904 nm, with 25-W peak power, subnanosecond rise times, and widths of less than 10 ns. Such characteristics ensure complete satisfaction of the stringent Federal safety standards (eye-safe at zero range) with ample reserve margin and handily set the stage for vertical range determinations, up to flight altitudes of about 2,500 ft (760 m), that are well within the stipulated precision of ± 0.5 ft (0.15 m).

The chassis and mechanical and optical components for the transceiver unit (fig. 37) are basically those built by Associated Controls and Communications, Inc. of Lynn, Mass., for the Naval Research Laboratory. Side-by-side optics, compensated for spherical aberrations, are featured with a 6-in. (0.15-m) diameter transmitter lens and an 8-in. (0.20-m) receiver lens. Transmitted beam divergence is about 1 mrad, and receiver field of view is about 2.6 mrad. An optical filter in the receiver has a light transmission efficiency of 0.8 and a bandwidth of 25.6 nm.

A key mathematical expression used in sharpening some design choices for the laser transceiver is the so-called range equation, which offers a convenient, rigorous way of relating light energy transmitted to light energy returned. For this situation where the range

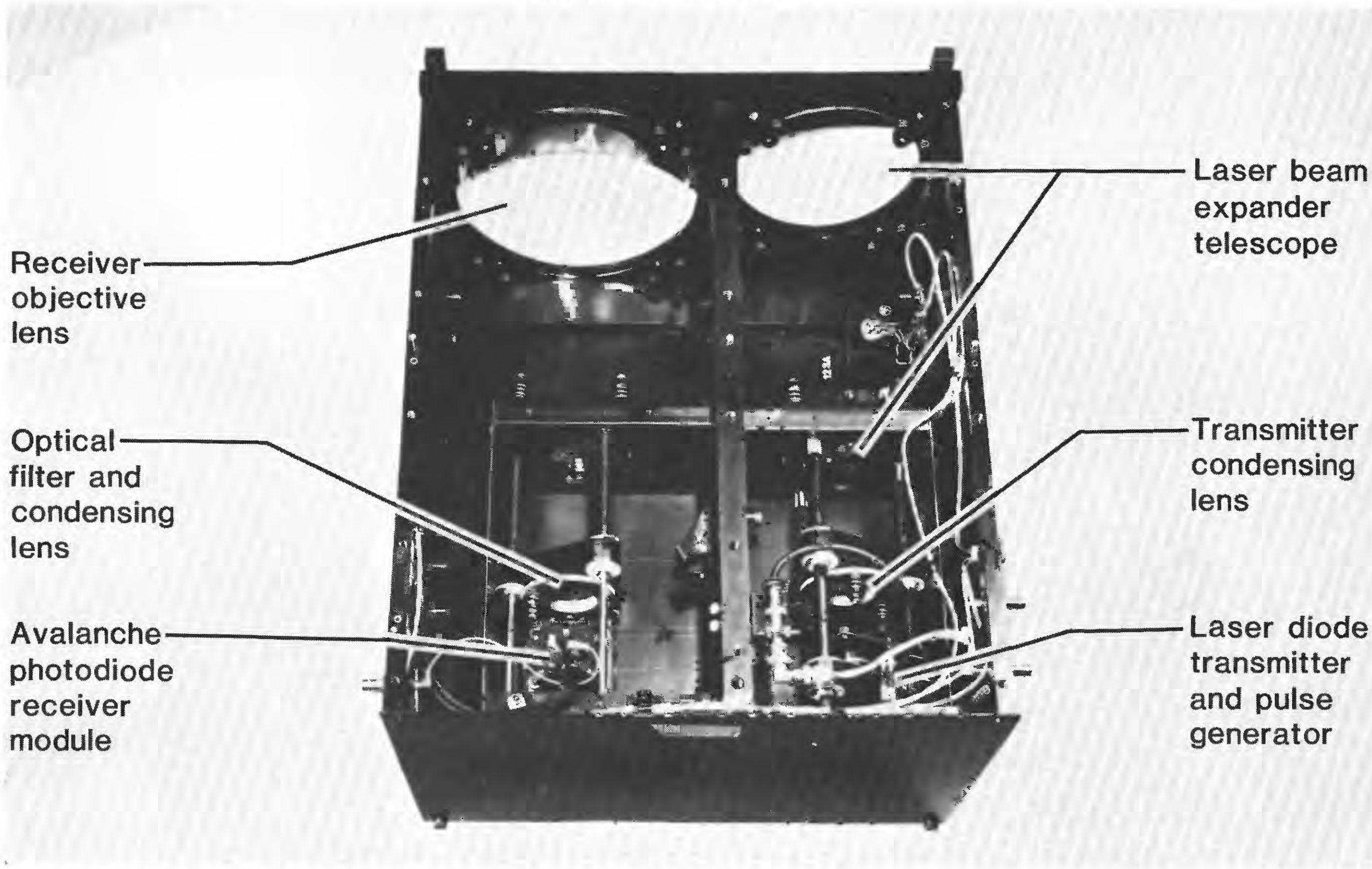


FIGURE 37.—Basic chassis with optics and some electronics for the laser profiler.

determination involves reflection of the laser beam back from an uncooperative target (that is, the land surface), the range equation has the form

$$P_R = P_S \frac{T_1 T_2 \rho A e^{-2\sigma R}}{\pi R^2}, \qquad (5)$$

where P_R = power, receiver,
P_S = power, source,
T_1 = transmission coefficient, source optics,
T_2 = transmission coefficient, receiver optics,
ρ = reflection coefficient, land surface,
A = area, receiver lens,
σ = attenuation coefficient, atmospheric path, and
R = range.

The interrelations among the parameters comprising the range equation are illustrated schematically in the three parts of figure 38. These parts, studied in sequence from left to right, show (A) the significant geometric features of the laser light path from transmitter (source) to land surface and back again, (B) the critical points in the flow of light energy from the lasing source to the land surface and back again, and (C) the mathematical description of the principal losses in energy flow which allows a rigorous statement to be written that shows the net change in energy level between laser source and receiver.

A key factor in the design choices is to ensure that the receiver optics can "see" a target area slightly larger than the laser "footprint" and thereby intercept the maximum possible amount of reflected light. Figure 38 is schematic in the sense that, in the real profiler instrument the field of view is coextensive with the field irradiated.

Paramount among other fators that are critical in the design choices is the signal-to-noise ratio (SNR) in the photodiode amplifier at the focus of the receiver optics. The level of light energy returned to the receiver must be greater than the background energy "noise" in the receiver by a factor (the SNR) that increases with the precision desired in the range measurement. The dominant, hard-to-reduce source of receiver noise is solar radiation scattered by the atmosphere and terrain within the receiver field of view. However, design characteristics of the electronics circuits in the receiver quantitatively affect the SNR, and the precision specified for the range measurements of this profiler device requires the overall assemblage of design choices to yield an SNR factor of at least 5 (see Mamon and others, 1978, p. 872).

As mentioned in the laser-tracker description, both laser devices (tracker and profiler) are served by a laser signal processor and a time-interval counter. These components manage the precision timing features inherent in the outgoing laser pulses and in the return signals, which are so critical in the subsequent translations into precision ranges. As used in this profiler part of the APT system, the cited components select a maximum range (to represent land surface as opposed to a treetop) out of each of 8 successive measurement blocks, each of which contains 16 sequential range measurements. The eight maximums are then averaged to yield a single range value that is recorded. Thus, the operating resolution of the time-interval counter will here be its accuracy (0.4 ns) divided by the square root of the number of measurements that are averaged (eight), or 0.1 ns.

For an operating flight altitude of 2,000 ft (610 m) above the terrain, and an aircraft speed of 120 mi/hr or 104 knots (193 km/hr), the foregoing profiler characteristics indicate that a vertical range measurement to the terrain will be saved and recorded at 7-ft (2.13-m) intervals along the flight path. Each such recorded measurement is the average of eight range measurements (maximums selected from 128 successive measurements) spaced along the centerline of an imaginary "illuminated" rectangle on the terrain about 2 ft (0.61 m) wide and 7 ft (2.13 m) long. The horizontal position coordinates assigned to this recorded mesurement are taken arbitrarily as the center of that imaginary rectangle.

The art of measuring the round-trip traveltime of the laser profiler pulse is electronically refined yet another step, beyond the constant-fraction-discriminator fea-

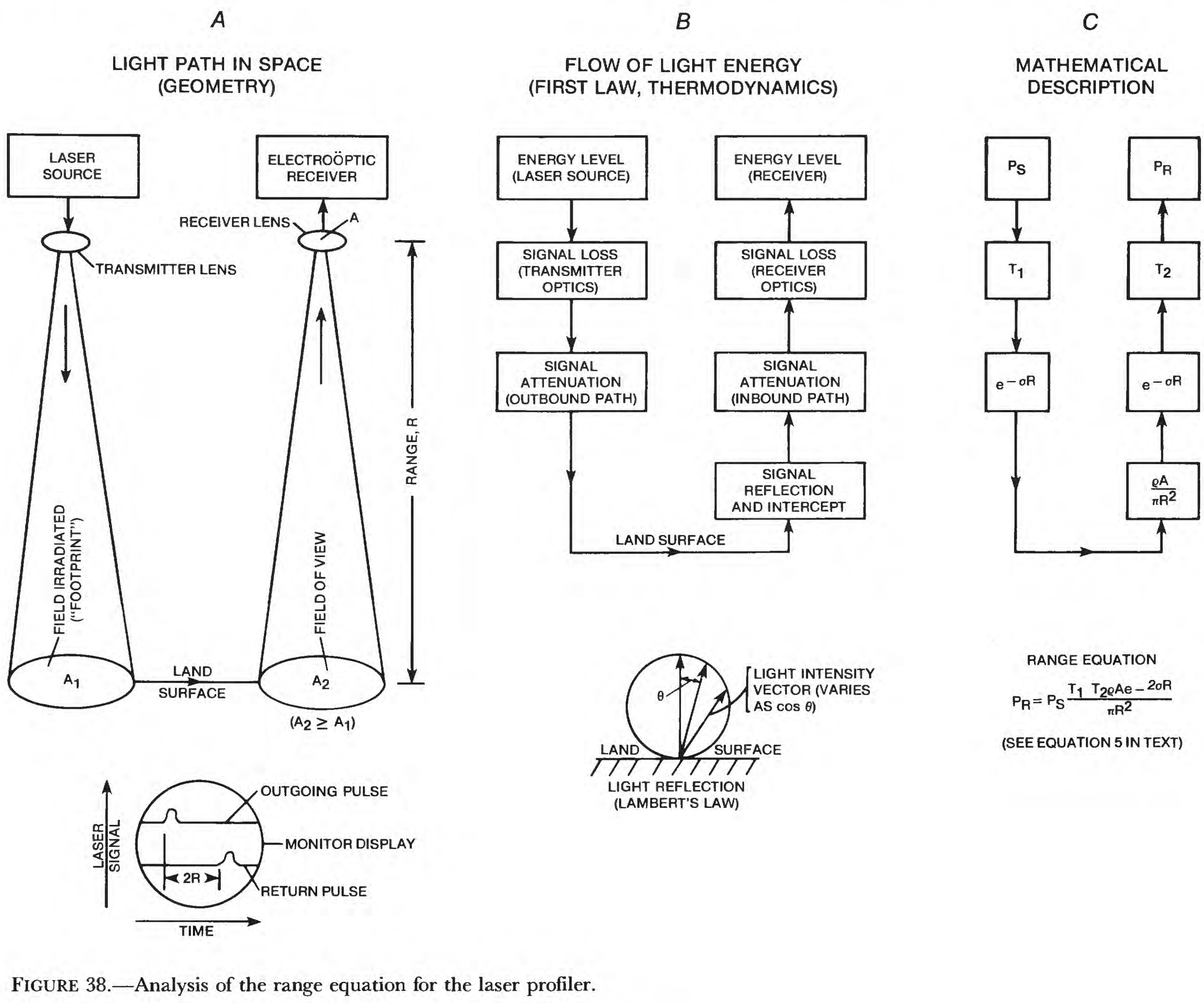


FIGURE 38.—Analysis of the range equation for the laser profiler.

ture previously described (see "Laser Tracker" section). This refinement is termed a "last-pulse discriminator" and it is designed to cope with multiple-return signals (pulses) that will occur when flying the profiler over some types of vegetation; for example, trees or heavy brush. In a multiple-return sequence, the first pulse could represent a treetop; the next one or more pulses, individual branches; and the last pulse, the land surface. If the intent is to maximize the number of valid returns from the terrain, then a capability must be provided for identifying the critical last pulse.

In an over-simplified way, the last-pulse discriminator can be regarded as a split electronic circuit, one side of which features a pulse counter, and the other features a delay line created by 100 ft (30.5 m) of coaxial cable. The multiple-return signals are fed simultaneously through both sides of the circuit, and, thus, when the entire sequence emerges from the delay line, the counter already knows how many pulses there are. It is no great electronic feat to tell the signal processor which numbered pulse to act upon.

Foreseeable variations in environmental conditions during flight, as well as variations in data-collection objectives, prompted the deliberate inclusion of the following signal-processing options for range determinations with the laser profiler:

1. Last pulse, leading edge,
2. Last pulse, alternate between trailing edge and leading edge,
3. Last pulse, trailing edge, and
4. Last pulse, leading edge, alternate between first pulse, leading edge.

The APT system operator selects the particular option desired by setting a rotary switch.

The original intent in designing the APT system was to mount the profiler on a gimbaled platform that could be slaved to the stable platform. In this way, the profiler could be held oblivious to the three-dimensional motion of the aircraft, so that its laser beam path would remain continuously oriented with the local vertical. When the need arose to invoke reasonable cost-saving measures, however, this feature was scrapped, and, in effect, the profiler now is hard mounted to the aircraft cabin floor. Within reasonable limits, corrections can and will be made for "off-vertical" laser ranges, inasmuch as the onboard computer will continuously store aircraft attitude data referenced to the stable platform.

Color Television Camera

When the APT system is used to collect terrain profile data, an important companion to the profiler subsystem is the color television (TV) camera subsystem (see fig. 25). Principal units in this subsystem are the camera, a video recorder, a control box for the recorder, and a time-date generator. The original design for the first prototype APT system stipulated that the TV camera was to be bore sighted with the profiler and that both units were to be mounted on a gimbaled platform, which would be slaved to the stable platform of the IMU. Like the profiler, however, the TV camera fell victim to the cost-saving measures and is now hard-mounted to the same rigid frame that supports the housing structure for the IMU and tracker.

The color TV camera chosen for the APT system is the Sony Model DXC–1640 (see fig. 39). The camera is housed in a metal case with suitable padding and handholds for operation and portability. Overall dimensions of the basic unit are about 9 1/4 in. (0.24 m) high, 7 1/2 in. (0.19 m) wide, and 17 in. (0.43 m) long, and it weighs nearly 10 lb (4.5 kg). The power requirement is 11 W delivered as DC current at 12 V. Significant camera features include automatic exposure and gain controls for stable operation throughout a range in illumination ("illuminance") from about 9 to 9,3000 ft-c (100–100,000 lux), normal operation throughout a range in environmental air temperature from 32°F to 104°F (0°C–40°C); National Television System Committee standard color reproduction through a 2/3-in. (26-mm) MF Trinicon tube pickup, and an individual picture (frame) comprised of a conventional raster of 525 lines, two-to-one interlace, and 30 frames per second, with 300-line horizontal resolution.

The video recorder chosen for the APT system is the TEAC Video Corp. Model V–1000 AB–R (see fig. 40), which is a ruggedized cassette-load type. It can record simultaneously imagery from one camera and two audio sources, if desired. The unit is packaged in a metal box 6 1/4 in. (0.16 m) high, 9 1/2 in. (0.24 m) wide, and 13 in. (0.33 m) deep and is shock mounted in standard light-alloy racks. The unit weighs 23 lb (10.5 kg), and the power requirement is 30 W delivered as unregulated DC current at 28 V. For normal operation, the recommended environmental temperature should be in the range from 5°F to 131°F (−15°C–55°C). Acutal powering, starting, and stopping of the recorder is through a separate control box, TEAC Video Corp. Model VS–100G, which also contains certain appropriate test and check functions. The box is 4 in. (0.10 m) high, 9 7/8 in. (0.25 m) wide, and 6 1/2 in. (0.16 m) deep and weighs about 10 lb (4 1/2 kg).

A standard video cassette, 7 3/16 in. (0.18 m) wide, 4 3/4 in. (0.12 m) deep, and 1 1/4 in. (0.03 m) thick accommodates magnetic tape 1/2 in. (13 mm) wide and 562 ft (171 m) long. Camera imagery is recorded on the tape by a two-headed device; its rotary motion, combined with the linear tape speed of 3 3/4 in./s (95 mm/s), results in a "helical" scan. Total recording time available per tape is 30 minutes; rewind time is about 2 minutes. Because the recording heads should be replaced after about 500 hours of use, the recorder unit has an elapsed-time indicator that is direct reading to that limit.

The need to correlate camera imagery very precisely with profiler data means that time data, synchronized with the APT system master clock, must be superimposed on each image (frame). The time-date generator device selected to do this is a Laird Telemedia Inc. Model 3201 (see fig. 41*A*). It is packaged in a metal case 3 in. (0.07 m) high, 17 in. (0.43 m) wide, and 13$^{1}/_{2}$ in. (0.34 m) deep and is mounted in standard light-alloy racks. The unit weighs about 10 lb (4$^{1}/_{2}$ kg), and its power requirement is 22 W delivered as single-phase current at 115 V and 400 Hz.

The electronic stratagem used to accomplish the desired time labeling of each camera frame involved modification of the time-date generator unit so that it could accept time-signal data from an external source, rather than rely on its own internal clock. These external time-signal data arrive as seven-decade binary-coded-decimal numbers, with a resolution of 0.01 second. The camera imagery passes through the time-date generator so that each frame has the appropriate time-code number superimposed before the combination is fed to the tape recorder.

As the design of data-editing equipment evolved, it became apparent that it would be advantageous to have the abovementioned time-code labels on the audio track of the videotape. To accomplish this, an EECO Model MTG–551 time-code generator is connected in series with the Laird Telemedia unit. The EECO unit (see fig. 41*B*) is packaged in a metal case 1$^{3}/_{4}$ in. (0.04 m) high, 19 in. (0.48 m) wide, and 14 in. (0.36 m) deep and weighs 10 lb (4$^{1}/_{2}$ kg). Its power requirement is 10 W delivered as single-phase current at 115 V and 60 Hz.

SPECIAL ROLE OF GRAVITY

Because the APT system, by virtue of its IMU, needs gravity information to perform its vital navigation task, it seems paradoxical to note that the APT system

FIGURE 39.—Color television camera for the aerial profiling of terrain system.

FIGURE 40.—Video recorder and control box for the aerial profiling of terrain system.

also provides gravity information if certain conditions are satisfied. In view of this paradox and the importance of gravity information to studies of geophysics and geodesy, it seems appropriate to focus on the role of gravity in a single discussion here.

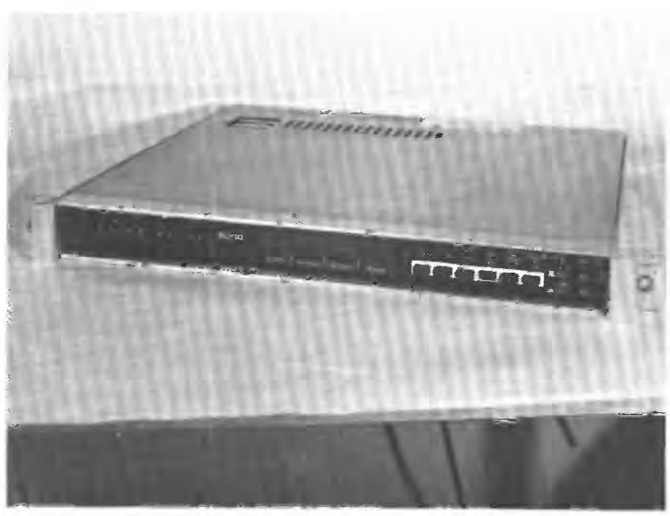
Perhaps the most fundamental principle of inertial technology is the fact that an accelerometer detects along the direction of its sensitive axis, two types of specific force or acceleration, each of which is indistinguishable from the other because they are manifestations of the same physical process: inertially referenced acceleration, associated with Newton's laws of motion, and gravitational acceleration, associated with mass attraction. The gravitational acceleration detected by a near-Earth IMU, such as the one in the APT system, is associated almost entirely with the mass of the Earth, although perceptible tidal effects are associated with the masses of the Moon and Sun. In terms of gravity, which is the sum of the Earth's gravitational acceleration and the centripetal acceleration linked to Earth rotation, such an IMU is said to detect a combination of Earth-referenced acceleration, whereby the inertial coordinate system is Earth centered and rotating at the Earth rate, and gravity. Thus, Earth-referenced acceleration can be obtained if gravity is known or estimated. Conversely, gravity can be obtained if Earth-referenced acceleration is known or estimated.

To extract Earth-referenced acceleration of the IMU for purposes of navigation, gravity can be estimated in two parts: a mathematically idealized normal part, associated with a reference ellipsoid having a size, shape, mass, and rotation rate similar to those of the Earth, and a disturbance part, associated with departures of the real Earth from the ellipsoid, especially those caused by irregular distributions of masses within the Earth. The normal part is represented by a geodetic mathematical formula; the disturbance part commonly is modeled using methods of probability and statistics.

To extract gravity sensed by the IMU, for purposes of geophysics and geodesy, Earth-referenced acceleration can be estimated using data from the laser tracker, which can serve as an independent navigator not affected by gravity. The tracker provides a time history of aircraft position while a retroreflector is in view and thus provides velocity and acceleration data, given sufficient viewing time or a sufficient number of successive passes within viewing range of the retroreflector. By estimating and removing a linear combination of error sources tied to the IMU and tracker, aircraft motion can be effectively eliminated. Thereby, the redundant navigation permits extraction of the gravity-disturbance vector, the magnitude and direction of which correspond closely to gravity anomaly and deflection of the vertical.

Gravity also plays a special role in error analysis of the APT system. In the design engineering phase of the APT system development, it was recognized that errors associated with accelerometer and gyro drift cannot be distinguished from horizontal rates of change of gravity magnitude and deflection of the vertical. Further analysis indicated that, because accelerometer and gyro drift are time correlated in contrast to the gravity effects which are space correlated, it would be possible to separate the inertial and gravity effects by flying the same path in opposite directions during short time periods. Such a procedure would permit the APT system to develop a gravity gradient map to the extent that the retroreflectors serve as points between which straight lines in gravity gradients can be drawn.

A TIME—DATE GENERATOR



B TIME—CODE GENERATOR

FIGURE 41.—Time-date and time-code generators for the aerial profiling of terrain system.

From a broader perspective, it is recognized that gravity irregularities induce acceleration errors in the IMU which are a major source of position error. These errors are defined as the difference between the true gravity vector at the aircraft and the gravity vector which is computed on board using the postulated gravity model as a function of the IMU-indicated aircraft position. The total gravity-induced error can be expressed as a sum of two terms: Schuler feedback, which gives rise to oscillatory error change in the horizontal channels of the accelerometer triad and exponential error growth in the vertical channel of the triad, and the error in modeling true gravity. The second term, which is a challenging subject of ongoing research, can be considered the sum of an acceleration bias, a linear combination of gravity-disturbance states, and a noisy residual. The success of gravity modeling depends largely upon how accurately gravity disturbance can be estimated by applying principles of probability and statistics to the term involving gravity-disturbance states.

Finally, it should be noted that it is possible to separate the effects of gravitation and inertia on the basis of the structure of their respective fields. Such a separation is possible for the second and higher derivatives of the gravity potential but not for the gravity vector itself. In practice, the separation may be made by simultaneously measuring the first and second derivatives of the gravity potential, using an accelerometer in combination with an airborne gravity gradiometer. For an aircraft with an inertial stable platform, the second derivatives of the force field do not contain inertial disturbances, and this enables measurement of purely gravitational second-order gradients. The gradients are integrated along the flight path to yield the gravitational force vector. It is of interest that the first airborne gravity gradiometer is currently (1985) under development, which offers hope that deployment of the APT system with such a gradiometer may soon be feasible.

AIRCRAFT FOR DEPLOYMENT OF INSTRUMENT SYSTEM

Among the operational specifications is the stipulation that the APT system be deployable in a relatively light fixed- or rotary-wing aircraft. "Relatively light" is intended to mean below the Federal Aviation Administration's (FAA) gross-weight classification limit of 12,500 lb (5,670 kg). Because deployment in a fixed-wing aircraft presents the greater design and operating challenge and because most field applications envisioned for the APT system will favor using that type aircraft, the following discussion ignores rotary-wing aircraft. Nothing in the design effort has precluded in any way, however, deployment in rotary-wing aircraft should the field situation so dictate.

To help in identifying the most suitable aircraft makes and models, the list of selection factors shown in table 3 was developed. Some data in the table that concern the APT system, crew, and aircraft modifications are shown for the originally intended system design and for the modified prototype design that will be flown first. Comparison of these particular data reveals the extent to which some of the original design objectives were necessarily compromised to accommodate budget constraints.

TABLE 3.—Factors bearing upon aircraft selection

Flight mission factors

Arbitrary ground speed—120 mi/hr or 104 knots (193 km/hr) = 175 ft/s (52 m/s).
Maximum round trip flying time to field-survey site—2 hours.
Calibration and terrain-profiling time—2-1/4 hours.
Loiter time and reserve—3/4 hour.
Maximum total flight time—5 hours.
Standard rate turns during profiling time—3°/s.
Normal operating flight altitudes above terrain—less than 3,000 ft (914 m).

APT system and crew factors

APT system size—15.5 ft^3 (0.43 m^3) ---------- 64 ft^3 (1.81 m^3)*.
APT system weight—400 lb (182 kg) --------1,470 lb (667 kg)*.
APT system power load—1,000 W-------------------4,800 W*.
Maximum crew of four persons---------------- 680 lb (309 kg).

Aircraft modification factors

Window in bottom of fuselage for tracker, profiler, and TV camera operation, 33 in. (0.84 m) × 53 in. (1.35 m)------- 65 lb (29 kg)

Base structure, secured to cabin floor beams to mount APT system components --------------------------------- 45 lb (20 kg)

Installation of two alternators to meet electrical power load—100-amp capacity each.

*Installation of gas-turbine auxiliary power unit, inverters, battery, and cabling---------------------------------690 lb (313 kg)

*Installation of cabin air-conditioning system ------- 75 lb (34 kg)

Aircraft performance factors

Good short-field takeoff and landing characteristics.
Good slow-flight characteristics.
Good margin on payload capability.
Economical operating and maintenance characteristics.
No cabin pressurization or air conditioning.

*Modified design, APT system.

The aircraft review task was narrowed quickly by the particular mission factors of relatively low flight speeds and altitudes. These factors, coupled with the desired low initial, operating, and maintenance costs, immediately favored piston-engine type aircraft rather than gas-turbine or turboprop types. The latter conclusion holds, however, only if the intent is to field an APT system built in accordance with the original design. An added selection factor concerned the requirement for openings through the cabin floor. This inclined the review task toward high-wing-type aircraft to avoid the need for complicated modifications in wing-root structure.

With the range of possible aircraft choices effectively limited through application of the foregoing choice factors, Survey and Draper Laboratory personnel now could introduce such secondary factors as cabin size and available FAA-approved designs for cabin-floor openings. In due course, this round of evaluation led to final choices of the following twin-engine aircraft:

1. North American Rockwell Commander, Model 680FL, gross weight 8,500 lb (3,860 kg), with two Lycoming Model IGSO–540–BIA piston engines, six cylinder, supercharged, fuel injected, 360 horsepower (hp) each (maximum continuous) at 3,200 rpm. (Suitable for fielding the APT system in its "original design.")
2. DeHavilland Twin Otter 300, Model DHC-6, gross weight 12,500 lb (5,670 kg), with two Pratt and Whitney Model PT6A–27 gas-turbine engines, 620 shaft hp each. (Suitable for fielding the APT system in its reduced-cost or "modified" design.)

Inasmuch as the first prototype model of the APT system to be flown has been built to the modified design, the remaining discussions in this paper concern only the DeHavilland Twin Otter aircraft.

The principal performance characteristics of the Twin Otter aircraft are assembled in table 4, and its overall appearance is as shown in figure 42. The general arrangement of APT system components, as mounted in the aircraft cabin, is in the artist's cutaway perspective (fig. 43).

OPERATION OF THE INSTRUMENT SYSTEM

During a flight mission, the APT system can be used for scientific measurement in three principal ways that relate to the profiling of terrain, establishment of horizontal and vertical control, and control or monitoring of the position and orientation of a companion remote sensing instrument. Regardless of the way in which the APT system is to be used, its operation requires that three retroreflectors, centered over three presurveyed ground-control points, be tracked individually during

TABLE 4.—Performance characteristics, DeHavilland Twin Otter aircraft

POWER PLANTS

Pratt & Whitney PT6A–27, 620 shaft horsepower each.

PERFORMANCE CONDITIONS

Performance estimations are based upon U.S. Standard (1962) atmospheric conditions and performance is contingent upon engine manufacturer's guaranteed performance as indicated in FAA Type Certificate.

OPERATING SPEEDS AT 12,5000 POUNDS

	Miles per hour	*Knots*	*Kilometers per hour*
Maximum speed	207	182	333
Cruise [10,000 ft (3,050 m)], 70-percent power	170	150	274
Stall (clean configuration)	84	74	135
Stall (landing configuration)	66	58	106
Minimum single-engine control speed	68	60	109

MAXIMUM CRUISING RANGE

(Includes fuel for taxi, takeoff, climb, normal cruise, descent, and landing, and 45-minute reserve at optimum altitude.)

A. With maximum fuel—full tanks (2,470 lb) (1,120 kg)	[1]680 statute mi (1,090 km) (590 nmi)
B. With special equipment and operating crew, and fuel available up to gross weight	[1]450 statute mi (720 km) (390 nmi)
Twin-engine initial rate of climb (10° flaps)	1,600 ft/min (490 m/min)
Single-engine initial rate of climb (0° flaps)	340 ft/min (100 m/min)
Twin-engine service ceiling	26,700 ft (8,140 m)
Single-engine service ceiling	11,600 ft (3,540 m)
Takeoff distance to clear 50-ft (15-m) obstacle	1,500 ft (460 m)

Maximum gross weight 12,500 lb (5,670 kg).

Empty (operating) weight to be determined after installation of special surveying equipment.

[1]Approximate only.

the flight mission. Additional retroreflector-equipped ground-control points may be needed in the project area to satisfy the requirement that a target be visible for tracking at 3-minute intervals during the survey flight. In this event, the added unsurveyed retroreflectors also are tracked individually during what might be called an initialization flight, before the collection of the intended survey data. Approximate coordinates are needed for the unsurveyed retroreflectors to ensure successful tracker search. They are obtained by measurements on such large-scale maps as the Survey standard $7^1/_2$-minute quadrangle sheets (scale 1:24,000).

For the terrain-profiling use, the APT system is flown along a course that overlies those lines on the ground for which profile data are desired. As it traverses that course, the APT system maintains a three-coordinate scheme of reference (latitude, longitude, and elevation), with which the laser profiler range measurements to the land surface are combined vectorially. The combination is vectorial in the sense that aircraft roll and pitch adjustments are made to convert range measurements to the desired vertical direction. This produces a profile of the terrain along a line that ideally lies vertically beneath the aircraft flight path. Moderate roll motion of the aircraft, however, will cause moderate departures from this ideal line, inasmuch as the profiler is hard mounted to the cabin floor. The video subsystem, which operates concurrently with the profiler, scans and records an aerial view (in color) of the terrain path that is being profiled. Such video imagery serves an important postmission role in the editing of the profile data to remove unwanted laser signal returns from foliage and manmade structures and to verify that the data have indeed been taken along the correct path. It also serves effectively in the precise location or placement of the terrain profiles on aerial photographs or large-scale maps.

For the establishment of horizontal and vertical control, the APT system determines the locations of desired unsurveyed ground-control points, as marked by retroreflectors. Normally, in this type of field use, the laser profiler and video subsystem are not operated. The fight technique simply entails an attempt to follow the preplanned sequence of individually tracking all retroreflectors mounted on the presurveyed and the unsurveyed ground-control points. The tracking sequence might conceivably feature a crisscrossing of the entire project area. In the postmission processing of the flight data, adjustments to the inertial navigator data are founded on tracker measurements to the retroreflectors at the presurveyed ground-control points. From this foundation, the further processing and adjusting of the data yield the position coordinates for the unsurveyed ground-control points. High-accuracy results can be expected, inasmuch as the unsurveyed sites are exclusively for tracked retroreflector targets.

For the control or monitoring of the position and orientation of a companion remote sensing instrument, the APT system normally is operated without the laser profiler and video subsystem. As with all other uses, it maintains a three-coordinate scheme of reference, as it controls or monitors the orientation of the companion instrument with reference to the stable platform in the inertial navigator. The companion instrument can be any one of a number of devices, such as an aerial camera, an infrared scanner, magnetometer, or side-looking radar. The usefulness of the remotely sensed data collected by such instruments is greatly enhanced by continuously knowing the instrument position and, for certain instruments, the orientation of their receiving optics.

Ground Control

Field operation of the APT system presupposes some beginning knowledge of where things are in the project area; that is, the latitude, longitude, and elevation coordinates for at least three ground-control points. These coordinates are to be determined and given in terms of the national control networks; for the United States, these are the NAD of 1927 (for latitude and longitude) and the NGVD of 1929 (for elevation). The obvious intent is to ensure that data collected by the APT system are compatible with existing maps and other geographic information systems. The ground-control points also serve to orient and scale the flight survey. These attributes emerge from the postmission data processing and adjustment when, in the final steps, the survey is fitted to the ground control. As a sound and practical operating technique, the combined array of presurveyed and unsurveyed ground-control points should be designed to embrace the entire survey area so that the APT system always is used as an interpolative rather than extrapolative position-determining device.

FIGURE 42.—DeHavilland Twin Otter aircraft.

The national control networks consist of bench marks and triangulation stations; the former are points whose elevations are accurately known, and the latter are points whose latitudes and longitudes are accurately known. Over 1 million bench marks dot the 48 contiguous States, and most are along highways and railroads at intervals of about 2 mi. On standard Survey quadrangle maps, a bench mark is shown as a small black cross, and just a brief inspection of a given map shows that most are found in the valleys and places where the roads are concentrated. Triangulation stations are not as plentiful and number only about 200,000. They are shown as black triangles on standard Survey maps and generally appear on high ground, such as hill and mountain tops.

Typically, a field project that is to be surveyed by the APT system will be in relatively flat country or along a stream. Thus, because the odds favor the preexistence of a number of bench marks in the project area, the desired elevation control is rather easily established. Triangulation stations in the project area are likely to be scarce to nonexistent, however, and a field-survey party must then establish a minimum of three that appropriately bracket the area.

The amount and type of local vegetation and topographic relief will strongly influence the type of surveying instruments used to establish triangulation stations. If the intervisibility among the proposed station locations is good, then the least costly surveying technique is the traverse method whereby plane angles are measured with theodolites and distances with electronic-distance-measuring equipment. If station intervisibility is poor, then semiautomatic satellite-tracking instruments will be used; for example, the JMR–1, the Magnavox 1502, or the Motorola Mini-Ranger. The ideal technique in that event is to operate several instruments simultaneously at several stations, one of which must be the nearest preexisting triangulation station. In this manner, appropriate "relative" position data are measured, and the necessary transformations can be made to the desired NAD. A day of satellite tracking will yield horizontal position

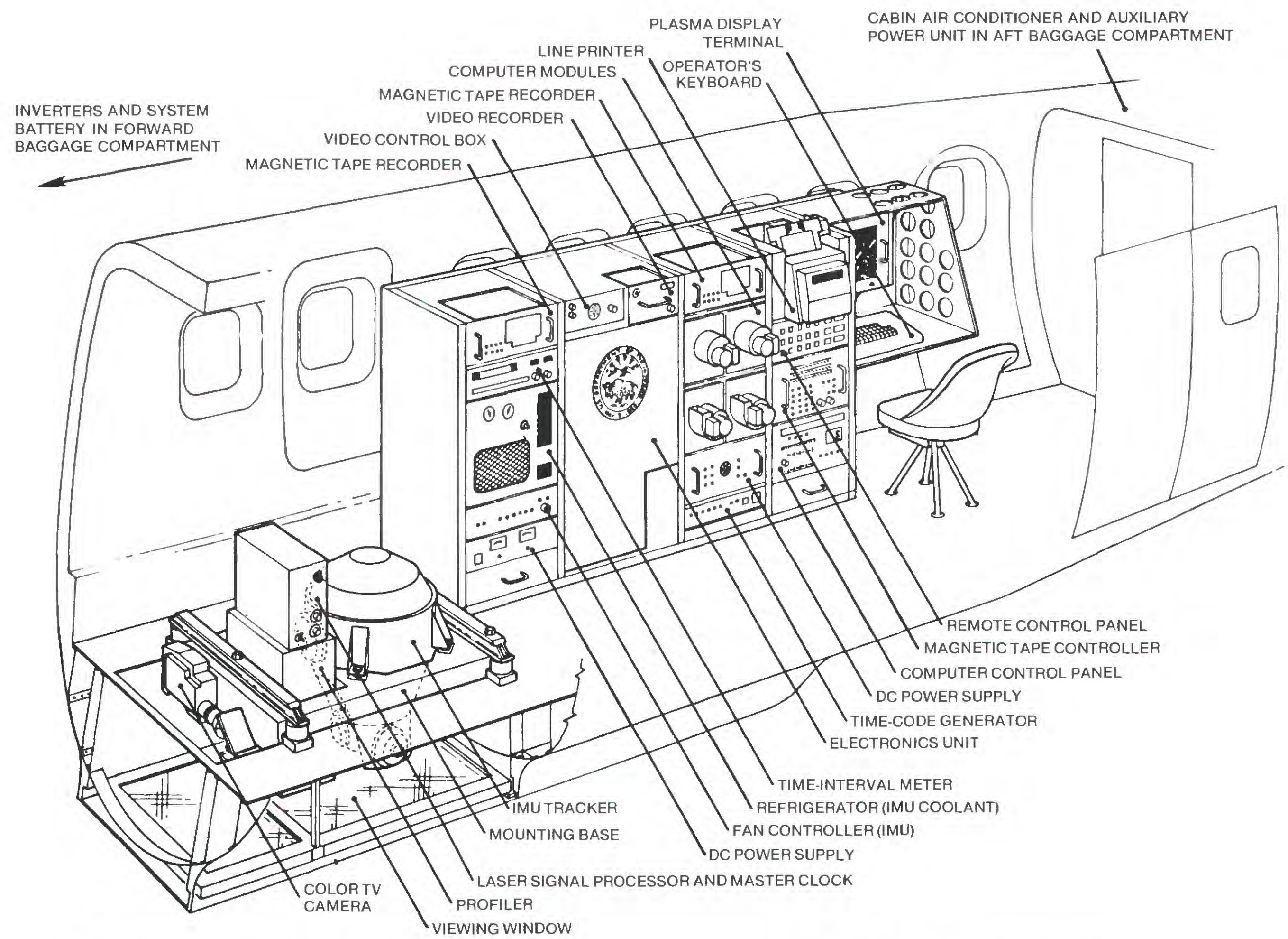


FIGURE 43.—Arrangement of aerial profiling of terrain system components in DeHavilland Twin Otter aircraft.

coordinates accurate to within 1 m. When the NAVSTAR satellite navigation system, now being developed by the Defense Department, becomes operational, even greater accuracy will be possible with a tracking time of only 2 hours. To be compatible with most uses for the APT system, the field surveys for triangulation stations should be held to third-order standards. However, when the APT system itself is to be used to establish horizontal geodetic control, the initial field surveys should be held to second-order standards.

Despite the general profusion of bench marks, they will seldom be found at the exact sites chosen for ground-control points. Thus, short lines of levels must be run to transfer elevations from the nearest bench marks to all the triangulation stations established as ground-control points and to as many intermediate control points (when used) as practical. Normally, such lines will be run to third-order standards. For especially precise uses of the APT system, however, level instruments and procedures appropriate for second-order standards should be used.

Flight Planning

Thorough advance planning for field work is instinctive in the Geological Survey. Beyond the innate incentives, however, the use of the APT system has an added incentive because its operating cost (aircraft, instrument maintenance, crew salaries and per diem, data editing, and computer processing) will be high, about $5,000 per flight hour. Success of a flight mission, as well as its most efficient execution, will hinge on thorough and astute flight planning.

A well-designed plan for a survey mission with the APT system literally means the detailed layout of the entire flight path—from the moment of departure from the principal airport nearest the field-survey area until the moment of return. That path must include, therefore, the route to be navigated to the survey area, the precise pattern to be flown initially over all retroreflectors mounted on ground-control points in the area, the precise pattern to be flown to collect the desired aerial survey data, and the route to be navigated back to the airport of origin. Layout of the flight path on a suitably scaled map will graphically illustrate the sequence of flight maneuvers needed to accomplish the mission. From the map layout, the horizontal and vertical coordinates of each retroreflector and the horizontal coordinates for the beginning and ending points of each discrete segment of the flight path (for example, each terrain profile) will be scaled and listed. That list also will include the scaled position coordinates for a convenient point on the aircraft parking ramp at the departure airport. Advance study of these listed data and the map layout by the flight crew must be sufficient to ensure mission performance without hesitation and preliminary familiarization flights.

Design of the most suitable flight path is dominated by the following criteria:

1. At 3-minute intervals during the aerial survey part of the mission, a retroreflector must be tracked by the laser tracker to update the APT system consistent with a 1/2-ft vertical accuracy. For a flight speed of about 120 mi/hr or 104 knots (193 km/hr), this suggests a retroreflector spacing of about 6 mi (10 km) along the flight path. If vertical accuracy is relaxed, then fewer updates are needed and retroreflector spacing can be increased; for example, 1-ft (0.30-m) vertical accuracy requires updates at about 4-minute intervals, or a retroreflector spacing of about 8 mi (13 km); 2-ft (0.61-m) accuracy equates to 6-minute updates and a spacing of about 12 mi (19 km).
2. Aircraft turning maneuvers should not exceed so-called standard-rate turns, in which the limiting bank (roll) is 15°. At that limit, the turning rate (change in azimuth) is 3°/s, or 2 minutes for a complete circle. This protects the stable member of the IMU from undue centrifugal acceleration. Flight experience ultimately may allow some relaxation in this criterion.

The salient features of a flight plan are shown in figure 44, for an aerial survey related to a flood-plain mapping problem. The illustration covers what might be termed one module of the flight plan, inasmuch as it spans only a 3.5-mi (5.6-km) portion of the much longer reach of the Framington River that is to be surveyed. This flight-plan module typifies, however, the kinds of maneuvers involved in collecting terrain-profile data along a stream valley. The modular layout can be progressively repeated, with minor local variations, as many times as needed until the entire stipulated reach of the river is covered.

The flight-path layout in figure 44 is based on a constant aircraft speed of 115 mi/hr or 100 knots (185 km/hr), a flight altitude of 2,000 ft (610 m) above the terrain, and all turns made at standard rate. Each loop in the path is sized so that it represents a flying time of about 3 minutes. This can be verified by following the consecutively numbered path segments. For convenience in layout, a segment length unit was arbitrarily chosen as the distance traveled in a 45-second flight. This means that four consecutive units represent a 3-minute interval. As the illustration shows, the retroreflectors are logically positioned near the convergence of flight paths to insure that tracking can be done at the required 3-minute intervals throughout the flight. However, at this flight altitude, the tracker can only see a retroreflector when the aircraft is within a horizontal range of about 3,200 ft (975 m). A circle of this

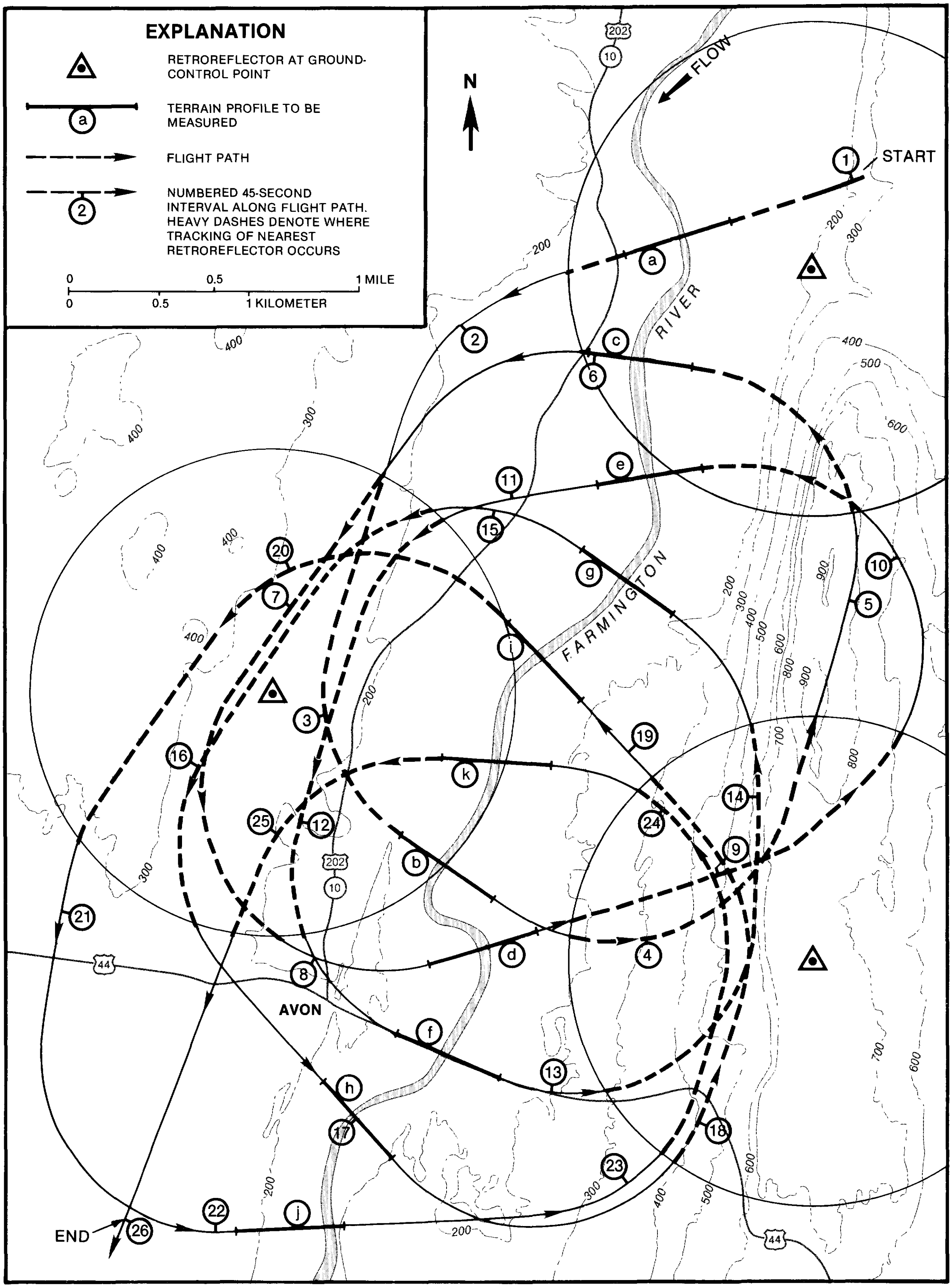


FIGURE 44.—Flight path layout for profiling the Farmington River flood plain near Avon, Conn.

radius is shown (fig. 44) around each retroreflector, and the flight-path segments that fall within any circle represent tracking opportunities.

The location and orientation of the desired stream-valley profiles are shown in figure 44, and the order in which they actually will be flown is given by the sequence of circled lower-cased letters a through k.

FLIGHT MISSION

Presentation of an abbreviated script for a hypothetical flight mission with the APT system is a convenient way to highlight the principal operating techniques. For this mission, it is assumed that the aerial survey is to collect terrain-profile data along specified paths in the broad flood plain of a fairly sizable stream valley. The sequence of operating steps is as follows:

1. On a convenient date before the flight, an advance ground party mounts a retroreflector on each ground-control point that is to be used by the APT system. The party also reaffirms the availability of appropriate ground-power equipment at the selected principal airport nearest the field-survey area. This equipment must support the APT system in all the preflight and postflight ground phases of the mission.

2. On the day before the flight mission, the APT system is flown to the selected airport near the field-survey area. The aircraft is taxied to and parked over the preselected point on the ramp, whose position coordinates already have been scaled from a map and listed. The APT system is connected to the ground-power equipment, and, at the appropriate number of hours before the flight is due to begin, the operator sequences the system through STARTUP AND TEST and into CALIBRATE AND ALINE. While the system is in this latter mode, the operator loads his list of position coordinates for all retroreflectors and the start and end points for each discrete flight-path segment into the computer. He also verifies that suitable gravity-model and reference-ellipsoid data are available for the survey area, which will be used in postmission data processing. A few minutes before flight time, the APT system is switched to STANDBY, the aircraft auxiliary power unit (APU) is started, and the ground-power equipment is disconnected.

3. Just before the aircraft is ready to taxi as the flight time nears, the APT system is switched to EN ROUTE. The inertial navigator now will measure and record the actual path of the aircraft as it moves into and through the beginning phases of the flight mission. Approximately 30 minutes may elapse in this mode before the APT system arrives within range of the first retroreflector in the survey area.

4. As the aircraft nears a point where the tracker will be within range of the first listed presurveyed retroreflector, the APT system is switched to SEARCH. When the tracker has found and locked onto the retroreflector, the system is switched automatically to AERIAL CALIBRATE. The preplanned calibration segments of the flight path are now flown so that each retroreflector in its proper turn can be tracked individually. The usual pattern is to track each presurveyed retroreflector in one continuous pass, followed by a return pass in which each is tracked again. At some convenient time during this round-trip calibration exercise, each unsurveyed retroreflector is tracked, and its measured and computed position-coordinate data stored for subsequent use in the aerial surveying part of the flight mission as well as in postmission data processing.

5. When the calibration exercise is complete, the APT system is switched to AERIAL SURVEY, and the preplanned segments of the flight path needed to collect the specified terrain profiles are flown. During this part of the flight mission, individual retroreflectors of opportunity are tracked so that gaps in tracker data do not exceed 3 minutes. The tracking procedure is automatic and in keeping with the flight plan. All tracker data are recorded, together with the terrain-profile data, for use in postmission data processing to update the inertial-navigator position and velocity information. The only such updating during the flight mission itself occurs whenever the position error is determined to have grown to more than 200 ft (61 m). The time and magnitude of each inflight update are also recorded for use in postmission data processing.

6. When the desired terrain-profile data have been collected, the APT system is switched once more to AERIAL CALIBRATE, and a final round-trip calibration exercise is flown. This will allow the subsequent (postmission) prorating of any changes in APT system performance parameters that may have occurred during the flight mission. When the calibration exercise is complete, the APT system is switched to EN ROUTE, and the aircraft returns to its home-base airport.

The products from the flight mission are two sets of magnetic tapes, one of which holds the tracker and terrain-profile data and the other, the color video imagery of the terrain-profile paths.

MAINTENANCE AND SERVICE

The inertial instruments in the APT system must be powered continuously to achieve their peak high-accuracy performance. Whenever a shutdown occurs, several days of warmup and retesting are necessary to restabilize the internal temperature gradients at the proper levels and to remeasure the performance parameters of all inertial sensors. Unexpected breakdowns will cause substantial delays, interrupt carefully drawn flight schedules, and, thus, be intolerably costly. For these reasons, periodic preventive maintenance shutdowns are deliberately built into the overall field scheduling for the APT system.

During a maintenance shutdown, the IMU and all other instruments will be opened and inspected; components that have been performing marginally will be replaced. Routinely, the poorest performing gyro and accelerometer will be replaced with better performing units from the supply of spares. The units that are pulled from the IMU will be reconditioned, recalibrated, and returned to the spare supply. The ideal frequency of such maintenance shutdowns can be determined only from experience. At the outset, once every few months will be tried.

Records will be maintained of APT system performance parameters and operating temperatures as measured during normal field operations. Sudden changes in any parameter will indicate a potential for an imminent problem that should be promptly investigated and overcome. Gradual changes in any parameter will point toward a longer range problem and might, therefore, serve to gage the time for a maintenance shutdown.

POSTFLIGHT DATA PROCESSING

The basic purpose of data processing after the flight mission is to reconstruct as accurately as possible the three-coordinate position information for the aircraft at specified time intervals along the flight path. If profiler data also were collected, then these can be combined with the processed position information to produce terrain profiles.

The kinds of "raw" data brought back (on magnetic tape) from a terrain-profiling flight mission include color video imagery of the profile paths, profiler ranges, tracker ranges, tracker gimbal angles, IMU gimbal angles, and accelerometer readouts. For a 3-hour mission, this could total as much as 10 megabytes of data, to which there would be added about 1 megabyte of lower-frequency data comprised of system initialization, calibration, and related information.

A schematic of how these flight data would flow through the postmission data-processing software is shown in figure 45. Principal steps in the process are numbered in the appropriate boxes in the figure and are briefly described as follows:

1. *Data editing.*—A program that transfers magnetic tape contents (the flight data) onto a disc in a way that reformats from DEC-compatible to IBM-compatible format. The program sorts, prints, and plots raw data; removes or replaces "outriders" in the data; and reformats as appropriate for the next steps.

2. *Renavigating.*—A program that solves the appropriate navigation equations, using the recorded system initialization, calibration, and accelerometer data to reconstruct, at 80-ms intervals, inertial-navigator-indicated values of aircraft velocity (relative to the rotating Earth) and position coordinates. Concurrently, the program computes tracker-indicated values of aircraft velocity and position coordinates using the recorded measurements of tracker range and IMU and tracker gimbal angles. These computed values are subtracted from the corresponding inertial-navigator-derived values to obtain "difference" data.

3. *Data compressing.*—A program that accepts the renavigation difference data (step 2) as input and performs a polynomial curve fit on each successive group of 25 values, which is equivalent to a 2-second time

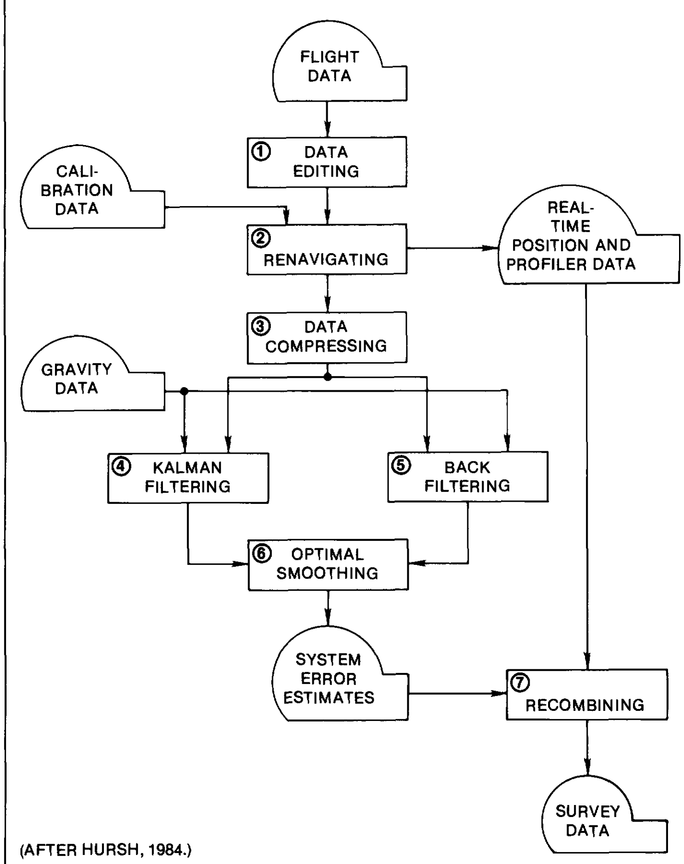


FIGURE 45.—Information flow through postmission data-processing software.

interval. At each 2-second time boundary on the fitted curve, the polynomial is evaluated for output to the Kalman filter. The program also selects the corresponding values of inertial-navigator-indicated aircraft acceleration, velocity, and position coordinates, tracker-indicated range and position coordinates, and IMU and tracker gimbal angles for output to the Kalman filter. The net results of compressing the data are to reduce its volume by a factor of 25, average out errors, and synchronize, in 2-second time steps, all parameter values that are to be fed to the Kalman filter.

4. *Kalman filtering.*—A program that uses an optimal estimation algorithm (mathematical procedure) to produce a 2-second time sequence of best estimates of the state vector of the APT system (see "Software" section, Calibrate and Aline mode, for a general description of Kalman filtering). Inputs to the Kalman filter include data from step 3 and available gravity data. A detailed and rigorous filter description is beyond the scope of this paper. The interested reader may consult Soltz and others (1981, p. 12–15) for a mathematical description. Suffice it to say that the filter output embodies error estimates that propagate forward in time for principal elements of the APT system state vector which include:

Elements	*Description*
3	Errors in indicated IMU position coordinates
3	Errors in indicated IMU velocity
3	Stable platform misalinement angles
3	Accelerometer bias errors
3	Gyro drift-rate bias errors
4	Gimbal angle readout bias errors
1	Tracker range bias error
3	Survey errors at each retroreflector
3	Gravity disturbance errors at each retroreflector

5. *Back filtering.*—A program that uses an optimal estimation algorithm similar to that used in the Kalman filter. The program processes the same data (as in step 4) in reverse time sequence to produce error estimates that propagate backward in time.

6. *Optimal smoothing.*—A program that optimally combines the two sets of time-sequenced error estimates from steps 4 and 5. This program produces a time history of the best possible estimates of all errors in the system at 2-second intervals.

7. *Recombining.*—A program that accepts the time history of errors from step 6, interpolates the error values to yield a 40-ms time sequence, and combines these results with the appropriate inertial-navigator-indicated position data from the renavigation program (step 2) to produce a 40-ms time sequence of best estimates for the aircraft position coordinates. For stipulated time periods, these results now may be combined with the profiler range data, appropriately adjusted for pitch and roll of the aircraft, to yield the desired survey data; that is, the terrain profiles.

PERFORMANCE EVALUATION FLIGHTS

OVERALL OBJECTIVES

The logical and necessary conclusion for the long gestation period for the prototype APT system was the successful accomplishment of certain specifically defined proof-of-concept, or performance-evaluation, flights. The Survey drew these specifications to ensure the collection of flight data that would permit a quantitative measure of the following:

1. Terrain-profile accuracy under selected spacings and geometries of retroreflector arrays.
2. Terrain-profile accuracy under selected aircraft maneuvers, such as standard-rate versus half-standard-rate turns.
3. Terrain-profile accuracy for a variety of topographic features that includes ponds, streams, densely wooded tracts, urban areas, and farmland.
4. Accuracy in determining the positions of unsurveyed retroreflectors under selected spacings for the initial presurveyed or fixed control points.

CALIBRATION RANGE

To meet the stated objectives for the performance-evaluation flights, a large amount of precisely measured ground truth was needed. No land area featuring such ground truth existed within easy flight range of the aircraft operations base at Hanscom Field, Bedford, Mass. Thus, the Survey chose to design and prepare its own "calibration range" (see fig. 46) in an area roughly 10 mi (16 km) wide (east and west) and 30 mi (48 km) long (north and south). Hanscom Field lies virtually at the midpoint of the eastern boundary, and the range area embraces the Massachusetts' cities of Framingham and Lowell, near the southern and northern boundaries, respectively. Western fingers of the greater Boston metropolitan area reach a few miles into the eastern part of the area.

The calibration range design was based on standard Survey $7^1/_2$-minute quadrangle maps (outlined and labeled in fig. 46), quadrangle-centered aerial photography, and geodetic control diagrams. The sites for 15 control points or stations were selected in locations scattered throughout the range area from Lowell to Framingham (fig. 46). Precise field surveys established the three-coordinate position data for each point, and a retroreflector was mounted over each before the evaluation flights. The point named "Haystack" (fig. 46), which was established some years ago by the National Geodetic Survey, was included in the survey net and provided the corrections to the NAD of 1927.

A Doppler satellite relative positioning technique was used to establish field positions. Four Magnavox Model 1502 receivers were set up at the first four control points to be surveyed, which were chosen to form a quadrilateral. The receivers simultaneously observed the required number of satellite passes over a period of 2 to 3 days. When sufficient data were recorded, two receivers were moved to two new points, which were chosen to form a new quadrilateral, and a new data set collected. This receiver leap-frogging process was repeated until data for a series of interlocking and overlapping quadrilaterals covered the entire area. From the lines drawn to connect the points shown in figure 46, the general scheme of quadrilaterals can be visualized. Because of radio interference, the point named "Lab" was not occupied with a Magnavox receiver. The point named "Lincoln" was occupied as an alternative, and, once its position coordinates had been determined, they were transferred to the Lab site by standard surveying techniques.

Second-order leveling was used to determine elevations at all control points, relative to the NGVD of

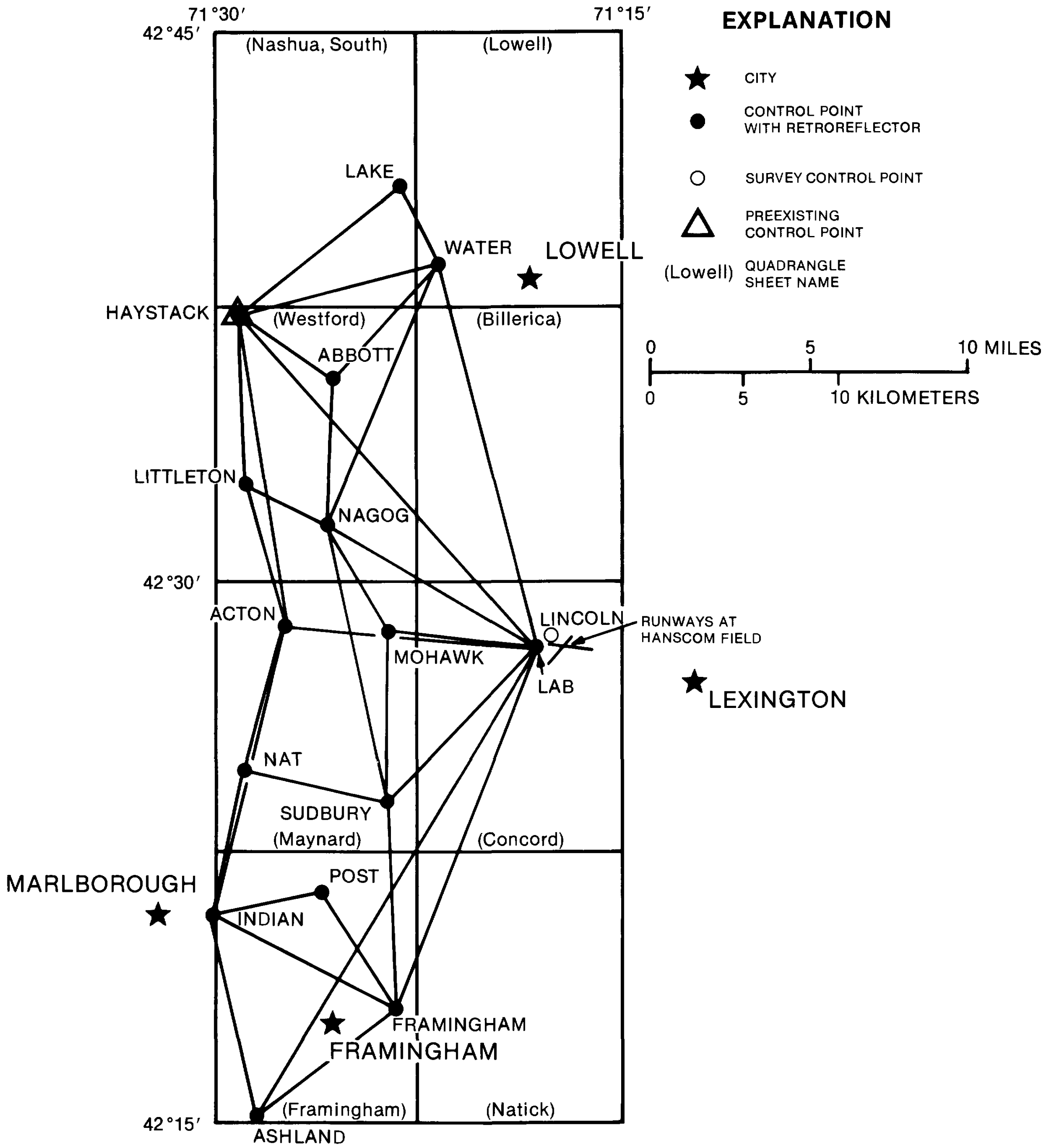


FIGURE 46.—Control diagram and retroreflector sites for calibration range.

1929. Elevations also were determined for the local control needed for large-scale aerial photography flown at six of the retroreflector sites—Acton, Ashland, Framingham, Littleton, Nagog, and Sudbury. From this photography, large-scale (1:800) photomaps with a 1-ft contour interval were prepared for use in evaluating the accuracy of profiles measured by the APT system. Individual map coverage is a square approximately 1,570 ft (480 m) on a side.

Each of the 15 retroreflector sites is described in detail in station descriptions compiled by the field survey crew. Ultimate life and recoverability are further protected by a bronze bench mark tablet, installed in a permanent location at each site.

Gravity measurements were made throughout the calibration range area at 161 stations, that included 14 of the retroreflector sites. The field party used a La Coste and Romberg geodetic gravity meter, and the resultant 161 site-measurement values are to be combined with 185 values extracted from gravity files maintained by the National Oceanographic and Atmospheric Adminstration in Boulder, Colo., to compile a gravity map for the range area.

Results of the foregoing survey and gravity-measurement work are summarized in table 5 for the 15 retroreflector sites.

SPECIFIED FLIGHTS

Four distinct flight plans were designed to meet the specified performance-evaluation objectives. The flights were to be made at an altitude of 2,000 ft (610 m) above mean ground level and at a constant speed of 115 mi/hr or 100 knots (185 km/hr). The intent was to fly directly over the retroreflectors. Details of each flight are summarized as follows:

Flight 1

Purpose: To determine accuracy of the APT system for point positioning surveys.

Flight plan: Layout as shown in figure 47. Retroreflectors are to be tracked in the indicated sequences, after departing Hanscom Field. The APT system is to be initialized by tracking, in sequence, the Mohawk, Acton, Littleton, and Haystack retroreflectors. Thereafter, a southbound flight is to be flown with a northbound return first over the retroreflectors connected (fig. 47) by the solid line (loop 1) and then over those connected by the dashed line (loop 2). The return to Hanscom Field is over the same retroreflectors (in reverse order) used to initialize the APT system.

Aircraft turns: All are to be made at one-half the standard rate; that is, $1\frac{1}{2}°$/s.

Flight 2

Purpose: To repeat flight 1 on a different day, after a new preflight calibration and alinement procedure for the APT system. This should illustrate the accuracy with which the APT system can repeat surveys or detect changes in control-station position (subsidence).

Flight 3

Purpose: To determine accuracy of the APT system in terrain profiling, with tracker updates at about 5-minute intervals.

Flight plan: Layout as shown in figure 48. Retroreflectors are to be tracked in the indicated sequences after departing Hanscom Field. The APT system is to be initialized by tracking, in sequence,

TABLE 5.—Position coordinates and gravity values for retroreflector sites in the calibration range

Station Name	Latitude[a]	Longitude[a]	Elevation[b] (meters)	Gravity Value[c] (milligals)
Abbot	42°35′17.8072″ N	71°25′57.6360″ W	116.16	980,380.988
Acton	42°28′46.5206″ N	71°27′27.3357″ W	75.35	980,373.153
Ashland	42°15′ 6.7190″ N	71°28′29.7035″ W	84.77	980,333.761
Framingham	42°18′ 9.6600″ N	71°23′47.1111″ W	59.64	980,360.025
Haystack	42°37′21.4841″ N	71°29′19.2579″ W	122.06	980,373.30
Indian	42°21′ 0.5948″ N	71°30′ 7.6261″ W	97.13	980,352.555
Lab	42°27′52.5270″ N	71°17′58.8542″ W	47.22	---
Lincoln*	42°28′23.5882″ N	71°17′54.1945″ W	38.73	980,380.574
Lake	42°40′59.1785″ N	71°23′20.4719″ W	63.90	980,391.069
Littleton	42°32′42.1234″ N	71°29′ 6.4914″ W	76.32	980,383.650
Mohawk	42°28′22.3676″ N	71°23′58.7138″ W	60.67	980,374.194
Nagog	42°31′20.8391″ N	71°26′ 2.9862″ W	79.86	980,379.565
Nat	42°24′48.2871″ N	71°29′ 7.4322″ W	114.19	980,362.398
Post	42°21′33.8249″ N	71°25′37.2286″ W	49.15	980,365.562
Sudbury	42°23′50.6621″ N	71°24′ 0.7526″ W	59.55	980,367.476
Water	42°38′26.5968″ N	71°21′51.2507″ W	40.33	980,397.790

[a]Position of retroreflector, referenced to the North American Datum of 1927.
[b]Elevation of retroreflector, referenced to the National Geodetic Vertical Datum of 1929.
[c]Measured at Survey bench mark tablet, and corrected for instrument drift and tidal effects.
*Control station only, without retroreflector.

the Mohawk, Nagog, and Littleton retroreflectors. Thereafter, four "counterclockwise" passes are to flown around the indicated (fig. 48) near-rectangular loop, a round-trip excursion to the Indian retroreflector, four "clockwise" passes around the foregoing loop, and return to Hanscom Field over the same retroreflectors (in reverse order) used to initialize the APT system.

Aircraft turns: All are to be made at standard rate.

Profiler and video camera: Operate while over the Acton and Sudbury orthophoto map areas.

Flight 4

Purpose: To determine accuracy of the APT system in terrain profiling, with tracker updates at about 4-minute intervals.

Flight plan: Layout as shown in figure 49. Retroreflectors are to be tracked in the indicated sequences after departing Hanscom Field. The APT system is to be initialized by tracking first the Mohawk and then the Acton retroreflectors. Thereafter, four "clockwise" passes are to be flown around the indicated (fig. 49) loop, a round-trip excursion to the Abbott retroreflector, four "counterclockwise" passes around the foregoing loop, and return to Hanscom Field over the same retroreflectors (in reverse order) used to initialize the APT system.

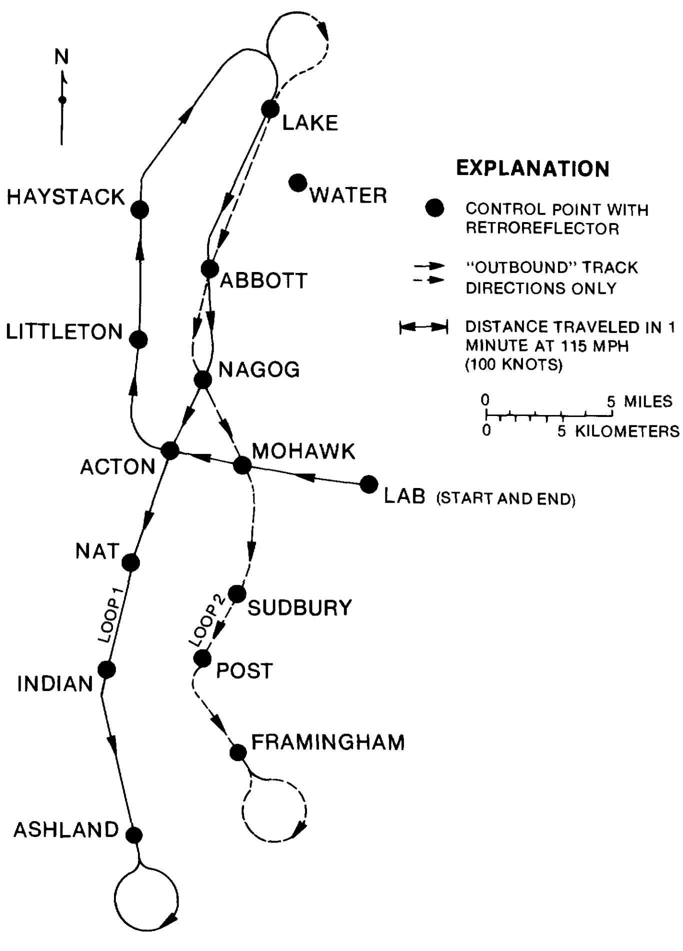


FIGURE 47.—Flight plan for performance-evaluation flights 1 and 2.

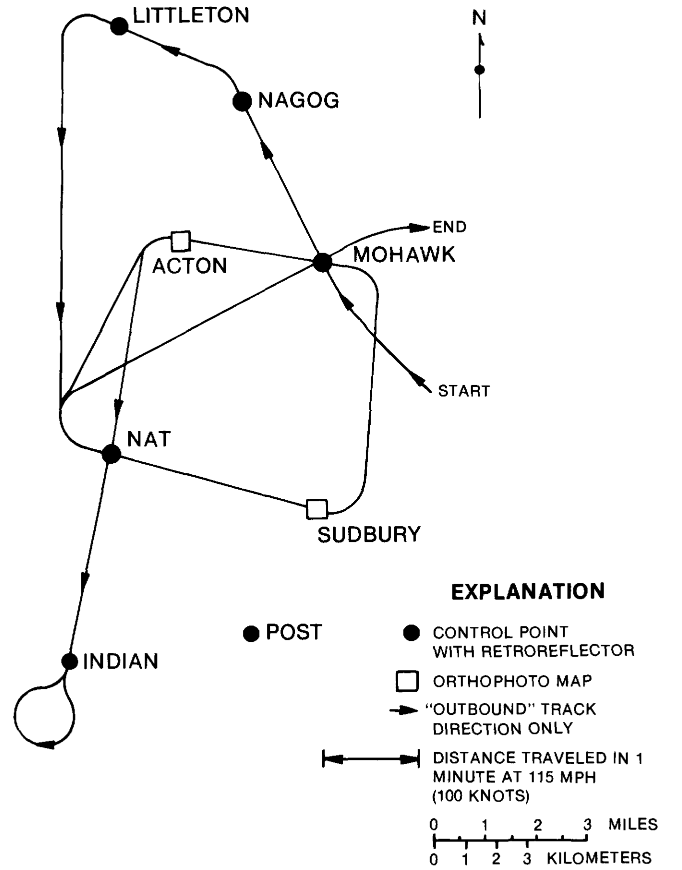


FIGURE 48.—Flight plan for performance-evaluation flight 3.

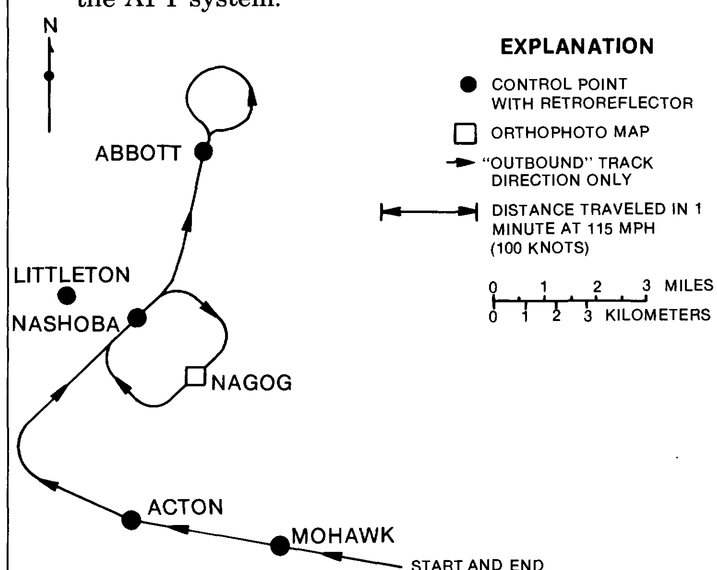


FIGURE 49.—Flight plan for performance-evaluation flight 4.

Aircraft turns: All are to be made at standard rate.
Profiler and video camera: Operate while over the Nagog orthophoto map area.

Flight Results

In the following discussion, it will be apparent that some departures from the detailed flight specifications were necessary. These resulted from the recurring need to compromise with such disruptive factors as the threat of intolerable weather delays, unpredictable malfunctions of key instrument components, uncertainties in visual navigation, and proximity of other aircraft entering the calibration range airspace.

Flights 1 and 2

Although intended to be at least 1 day apart, both flights were accomplished on January 26, 1984—one in the morning and one in the afternoon. All retroreflectors were tracked successfully in accordance with the specified order, flight altitude, and speed. The data were processed after the flights in a way that allowed surveying accuracy to be shown for three different spacings of control (retroreflectors). These spacings can be described, for a given flight line, in terms of the following retroreflectors for which it was assumed that position coordinates were known:

a. Alternate (every other) retroreflectors.
b. The two end and the midpoint retroreflectors.
c. The two end retroreflectors only.

In each of the foregoing cases, the position coordinates were computed from the flight data for all other retroreflectors. The values thus computed then were compared to the corresponding values determined in the original field surveys to ascertain the APT system surveying errors. Results of these two flights, and the subsequent data processing, are given in table 6. Parts A, B, and C of the table correspond to the foregoing retroreflector-spacing cases a, b, and c.

In part A, the position-coordinate errors are seen to range, in centimeters, from -23 to 38 for latitude, -31 to 80 for longitude, and -27 to 10 for elevation. The corresponding standard errors, in centimeters, are ±25, ±38, and ±13. Some of the error in the horizontal coordinates can be attributed to noise in the Doppler equipment used in the original field surveys. This is estimated to be ±10 to ±20 cm. The changes in errors, when the morning and afternoon flight data are compared, are seen to range, in centimeters, from -1 to 12 for latitude, -43 to 15 for longitude, and -5 to 8 for elevation. The corresponding standard errors, in centimeters, are ±7, ±22, and ±6. Thus, the agreement between the morning and afternoon flight surveys is excellent and gives a good measure of the precision with which the APT system is performing; for example, the smallness of the standard-error values indicates capability of the system to monitor, through periodic repeat flight surveys, land surfaces that are subject to long-term changes due to erosion or subsidence.

Inspection of the results given in parts B and C of table 6 shows that, even with substantial relaxation in the frequency with which the IMU is updated (through tracking of retroreflectors at known locations), overall accuracy and precision degrades little. Conversely, it can be concluded that even greater accuracy and precision might be achieved if the frequency of updates was increased through the use of additional retroreflectors.

Analysis of all data in table 6 supports the conclusion that overall performance capability for determining position and elevation coordinates will indeed be the specified ±60 and ±15 cm, respectively, 90 percent of the time, if the IMU is updated at 3-minute instead of 5-minute intervals.

Flights 3 and 4

These flights were first flown, primarily as planned, on January 22, 23, 27, and 30, 1984. However, frost and snow cover required almost daily clearing of the retroreflectors, and the fine-detail imagery (painted lines on parking lots, playgrounds, and tennis courts) needed to assess horizontal accuracy of the terrain-profile data generally was obscured. All retroreflectors were tracked successfully in accordance with the specified order, flight altitude, and speed. It was noted, however, that laser return signal dropouts occurred as the profiler passed over the few snow-cleared areas, which were commonly asphalt parking lots and roads. It was concluded that, at this flight altitude [2000 ft (610 m)], the black asphalt absorbed most of the laser energy. Thus, when all data were processed after the flights, the best terrain-profile points available for comparison with ground truth were those on the frozen surface of Nagog Pond and on the football fields at Sudbury and Acton High Schools, but even these comparisons suffered from the unknown vagaries of depth of snow cover.

Because of the foregoing uncertainties, flight 3 was repeated on May 10, 1984, and flight 4 on June 15, 1984. Retroreflectors were tracked in accordance with the specified order and flight speed, but the flight altitude was lowered slightly to 1,650 ft (503 m) to minimize laser return-signal dropouts caused by asphalt surfaces. Results of only the May and June flights are given in table 7. The intent was simply to identify on each profile leg a few (four to eight) points for which a reliable and accurate elevation was interpolated on the orthophoto map. The average values for each set of points were then tabulated.

The average vertical error is the average difference between the profiler elevations and the orthophoto map

TABLE 6.—Results of performance-evaluation flights 1 and 2

A. Alternate retroreflector positions known (5.2 min. between IMU updates)

Retro-reflector	Flight	Error (cm) Lat. φ	Long. λ	Elev. E	Change (cm) Δφ	Δλ	ΔE
ABBOT	am	37	-19	-19			
	pm	38	-21	-27	-1	2	8
ACTON	am	- 1	-13	3			
	pm	- 7	-28	- 4	6	15	7
MOHAWK	am	-23	-22	- 2			
	pm	-23	-31	3	0	9	- 5
INDIAN	am	34	37	-17			
	pm	28	80	-12	6	-43	- 5
POST	am	- 1	31	10			
	pm	-13	49	5	12	-18	5
Standard Error		±25	±38	±13	± 7	±22	± 6

B. End and midpoint retroreflector positions known (7.8 min. between IMU updates)

Retro-reflector	Flight	Error (cm) Lat. φ	Long. λ	Elev. E	Change (cm) Δφ	Δλ	ΔE
ABBOT	am	49	4	-16			
	pm	63	35	-23	-14	-31	7
NAGOG	am	11	33	4			
	pm	26	66	3	-15	-33	1
NATICK	am	-58	26	-18			
	pm	-38	-72	- 2	-20	98	-16
SUDBURY	am	- 2	5	-14			
	pm	23	-55	1	-25	60	-15
INDIAN	am	- 9	58	-29			
	pm	- 3	12	-16	- 6	46	-13
POST	am	- 3	36	- 2			
	pm	6	6	7	- 9	30	- 9
Standard Error		±32	±41	±14	±16	±55	±11

C. End retroreflector positions known (15.7 min. between IMU updates)

Retro-reflector	Flight	Error (cm) Lat. φ	Long. λ	Elev. E	Change (cm) Δφ	Δλ	ΔE
ABBOT	am	41	37	-12			
	pm	73	50	-24	-32	-13	12
NAGOG	am	3	68	8			
	pm	37	79	2	-34	-11	6
ACTON	am	- 4	40	6			
	pm	20	11	- 6	-24	29	12
MOHAWK	am	-22	27	2			
	pm	0	9	6	-22	18	- 4
NATICK	am	-61	60	-11			
	pm	-23	-65	- 8	38	125	- 3
SUDBURY	am	- 7	15	-15			
	pm	26	-51	2	-33	-66	-17
INDIAN	am	- 9	83	-23			
	pm	11	16	-23	-20	67	0
POST	am	- 5	43	- 1			
	pm	11	9	7	-16	34	- 8
Standard Error		±30	±48	±12	±28	±58	± 9

(After Cyran and Chapman, 1984).

TABLE 7.—Results of performance-evaluation flights 3 and 4

Flight date	Orthophoto map site	IMU update interval (min)	Average Errors (cm)		
			Latitude	Longitude	Elevation
May 10, 1984	SUDBURY	6.6	−30	−90	10
	ACTON	6.4	90	−40	22
	SUDBURY	7.4	−190	20	−3
	ACTON	6.3	No suitable point		7
	SUDBURY	7.5	−80	80	−15
	ACTON	8.0	50	−80	15
	--- do ----	4.8	−40	180	0
	SUDBURY	7.0	30	130	−20
	ACTON	4.8	−70	−200	16
	SUDBURY	7.1	0	−160	−8
	ACTON	4.9	90	110	3
June 15, 1984	NAGOG	4.6	80	−70	−2
	--- do ---	5.0	−60	50	11
	--- do ---	5.3	0	60	7
	--- do ---	3.9	No suitable point		−15
	--- do ---	3.9	No suitable point		3
	--- do ---	9.3	−30	60	−15
	--- do ---	9.3	110	−100	1
	--- do ---	3.8	120	−10	4

elevations. The average horizontal errors are the average differences between the latitudes and longitudes as determined by the APT system and as scaled from the orthophoto map. The ranges in position coordinate errors, in centimeters, as shown in table 7, are from -20 to 22 for elevation, -190 to 120 for latitude, and -200 to 180 for longitude. The vertical errors are reasonably representative and tolerable; the horizontal errors are prejudiced to some degree by the small-scale video image and its lack of crisp definition.

Processing of all profiler data from all flights (January, May, and June) yielded a standard vertical error of ±6 cm when the IMU is updated at 3-minute intervals. If this is combined with the noise value for the profiler itself of ±6 cm, then an overall performance capability results in an elevation position coordinate of ±14 cm 90 percent of the time, which is within the ±15 cm given in the specifications for the APT system.

USES FOR THE INSTRUMENT SYSTEM

The stimuli for this instrument-development project have persistently arisen from field-investigational problems for which more precise, faster, and less costly (with less manpower) data-gathering techniques are perennially sought. The list of potential uses for the instrument system appears endless as field programs of the many agencies engaged in earth science studies are reviewed. In the following sections, descriptions are given for a few selected uses drawn from the more obvious and pressing requirements in field programs of the Geological Survey.

HYDROLOGIC STUDIES

As part of its Congressional mandate to evaluate the Nation's water resources, the Survey continuously maintains a nationwide network of stream-gaging stations, sediment-sampling stations, ground-water-level observation wells, and water-quality sampling and observation stations. The network is in constant selective revision as some stations are discontinued and new ones are introduced.

Although the data regularly collected from the foregoing station networks are used for many purposes, one particular use is of special interest here. That use relates to the large variety of mathematical models being designed to describe and predict such everyday phenomena as surface runoff from watersheds of many sizes and types, which include pristine as well as man-altered states (examples of the latter are strip-mined or urbanized watersheds); open-channel flow characteristics in selected reaches of streams in various geologic settings; the spread and migration of contaminants introduced in various ways into the flow regimes in watersheds and stream channels; the land subsidence in and around heavily pumped ground-water reservoirs and producing geothermal reservoirs; and erosional processes along beaches and shorelines.

In any given modeling problem, the data from the nearest available network station or stations constitute only a small fraction of the total data requirement. A large percentage of what is lacking can best be described as precise geometric details (three-dimensional shape) on the surrounding land surface or stream channel or beach or shoreline. Commonly, such details cannot be obtained with the needed precision from any available maps. Thus, recourse, heretofore, could only be through arrangements for special surveys of the desired land or channel area using field parties and conventional surveying or mapping techniques.

The advent of the APT system brings into focus the following immediate and specific uses related solely to ongoing nationwide hydrologic studies:

1. To measure stream-valley cross sections at 1/4-mi (0.4-km) spacings, along all streams that are represented in the station networks.
2. To measure stream-valley cross sections at designated spacings along stream reaches that are encompassed in specific one- and two-dimensional mathematical models that are to be used for describing open-channel flow.
3. To measure terrain profiles at designated sites in watershed areas modified by man that are encompassed in specific mathematical models which are to be used for describing changes in surface runoff.
4. To measure precise positions for ground-control points at designated locations in areas where ob-

servation-well positions in a local network must be accurately fixed.

5. To measure terrain profiles periodically at designated sites in areas where innumerable natural ponds are expressions of the local water table. These data are to augment and enhance the conventional water-level observation-well data and will allow preparation of more detailed water-table contour maps for use by water managers.
6. To measure precise positions periodically for ground-control points at designated locations in subsidence areas. Initially, the area choices will be limited arbitrarily to those that have a maximum diameter of less than 10 mi (16 km).
7. To measure longitudinal profiles of streams in flood, as opportunities occur, to enhance and extend the conventional flood-stage data that commonly are collected only at selected and, perhaps widely spaced, points.

Map Production Tasks

The National Mapping Division conducts the National Mapping Program, which provides graphic and digital cartographic products and information for the United States, Territories, and U.S. possessions. The products include several series of topographic maps, land use and land cover maps and associated data, geographic names information, geodetic control data, and remote sensing data. The $7^1/_2$-minute 1:24,000- and 1:25,000-scale topographic maps in the conterminous United States and Hawaii and the 15-minute 1:63,360-scale maps in Alaska provide basic coverage of these areas and are the building blocks used for developing almost all other cartographic products. Over 46,000 maps in the series have been completed, and the total national coverage of about 55,000 maps is planned for this decade. Present work includes new mapping to complete the coverage and maintenance of the existing maps. The APT system can provide support to this program in the following ways:

1. In the construction of topographic maps, map details and contour lines are compiled using stereoscopic models, which are formed by merging pairs of aerial photographs. Basic control to orient, scale, and position the models generally is acquired by field survey and aerotriangulation methods. The APT system can provide model control by establishing terrain profiles in the side-lap areas midway between the aerial photograph flight lines. The APT system video imagery provides the connection to the aerial photography for control point identification and selection.
2. Terrain-profile data also can assist in the map compilation process by providing ground elevations in areas of tall timber. The operator of a photogrammetric instrument seldom sees the ground under tall timber conditions and must estimate tree heights to position the contours. The APT system profiles can provide a sampling of tree height data, as well as elevations, so that contour estimates can be more accurate.
3. Some maps may not meet National Map Accuracy Standards. From 1944 to 1958, the Survey produced $7^1/_2$-minute maps using photogrammetric methods that were in their early phases of development. In some instances, the technology and methodology may not have produced maps that meet the high-accuracy standards of today. Many of the maps produced during this period are now candidates for revision or replacement. The APT system has the capability to determine the absolute accuracy of the maps in question and, based on these findings, to assist in determining the most cost-effective means of updating. Accuracy testing will consist of a series of terrain profiles spaced across the map and following along roads and other linear features. Points common to the map and the APT video system will be selected, and positions and elevations from both sources obtained. Over 20 points will need to be compared to determine the horizontal accuracy of the map and another 20 points to determine the vertical accuracy.
4. The video imagery as recorded by the APT system has a potential application in the map revision program. The images will be collected at a flight height of several thousand feet over an area to be revised. The images can be computer processed to correct for tip, tilt, and scale; then oriented to north, and displayed as an overlay to the map image. New buildings, roads, reservoirs, shopping centers, and other features then can be identified quickly and captured for placement on the revised map.

The APT system has the capability of developing into a versatile mapping tool. The video image provides a record of the planimetry, and suitable scale can be selected by varying the flight height or changing the camera zoom setting. Because the image will be digitally processed, geometric corrections can be made to account for variations in flight height and orientation. The profile data provide ground elevations that can be used to develop contours or digital elevation models. If a scanning capability ultimately is added to the profiler, a swath of elevation data can be obtained in a single pass. The area to be mapped can be covered with many parallel or, for more accuracy, crisscrossing flight lines. Flight-line spacing, as well as flight height, can be changed to meet the specified scale and accuracy of the desired map product. Because the video image and the elevation data will be processed in a computer, it appears that further research in this direction will produce a nearly automatic mapping system.

Geologic Studies

The APT system has the capability for improving the quality of routine airborne or ground-based surveys and also can serve as an experimental device from which three-component, gravity-disturbance information can be extracted.

Routine airborne surveys include those that use electromagnetic, gas or vapor detection, magnetic, photographic, and radioactive techniques. All have a critical dependence upon accurate knowledge of aircraft position as a function of time of measurement. The APT system can determine aircraft position more accurately than existing radar or radio transponders.

Routine ground-based surveys include those designed for measuring gravity anomalies and determining the longitudinal profiles of stream valleys. Both types of survey are critically dependent on the accuracy of elevation (vertical coordinate) control. The APT system can determine the vertical coordinate to within 6 in. (0.15 m) and, thereby, offers elevation control sufficient for gravity-meter profiling or for correlating changes in stream gradient with geologic contacts between rock types of contrasting erosional resistance.

Extraction of three-component, gravity-disturbance information from the APT system depends, in part, on future contract work to refabricate the system more closely in accordance with the original design specifications. In that format, the system offers promise as an airborne gravity mapper, with a capability for achieving a gravity-anomaly accuracy to within 0.2 milligal, and vertical-deflection accuracy to within 0.1 arc-second. An airborne gravity mapper suffers, relative to land-based gravimetry, from attenuation of high-amplitude, short-wavelength anomalies that relate to locally concentrated, shallow-depth sources. However, such a mapper benefits enormously from the standpoints of unlimited accessibility, speed and density of coverage, and spatial consistency of all simultaneously measured data. The principal use of the APT system as a gravity mapper is envisioned for nonrepeat surveys over terrains of low-to-moderate relief characterized by low-to-moderate gravity gradients.

Economic feasibility eventually will prompt the development of the next and even more accurate generation of the APT system. When this occurs, the system can be applied more widely to the important realm of precise repeat surveys that involve the measurement of differential ground movements over selected time intervals. Such repeat surveys are vital to a variety of geologic studies that include monitoring of active soil creep, movement along major strike-slip fault zones, and landsliding; volcano inflation and deflation; ongoing beach and slope erosion; the melting and growth of glaciers; and land subsidence in and around producing oil fields, geothermal reservoirs, and heavily pumped ground-water reservoirs.

FUTURE IMPROVEMENTS IN THE INSTRUMENT SYSTEM

Some clues already have been given on highly desired uses for the APT system when it can be built to conform as precisely as possible to the full set of original design specifications. Important elements in those specifications concern the need to minimize overall system size, weight, and power requirements so that the logistics for and cost effectiveness of regular field operation will be minimized and maximized, respectively. Of equal importance is the need to provide a stable platform for the profiler slaved to the stable platform of the inertial navigator. The present technology in all pertinent scientific fields is advanced beyond the points needed to meet the original specifications. Thus, an efficient field-operational APT system can be built whenever the administrative decision dictates. Such a step is one viable way of improving the present system.

Other improvements are foreseeable, however, that would allow the original specifications to be refined in several desirable ways. Three leading-candidate improvements, out of a longer list of possibilities, relate to a scanning-laser device, a gravity-gradiometer device, and a global positioning system (GPS) receiver.

An expanding variety of engineering and environmental problems has created a large demand for digital models of terrain that can then be analyzed by computer. As an adjunct to the APT system, a laser device capable of side-to-side scanning in a direction across the aircraft track would provide a wealth of terrain elevation data for digital-modeling purposes. Furthermore, the accuracy of such data would greatly exceed that obtainable through such existing techniques as digitizing available topographic maps or application of photogrammetric methods. Ideally, a large scan angle would maximize the amount of data obtained along the flight path; practically, the scan angle is usually limited to about 10° or 15° either side of the vertical to obtain reasonable penetration of vegetation.

In the Federal establishment, interest is building for the development of an airborne gravity gradiometer. Over the past decade, the Navy Department and the Air Force have supported developmental work by such contractors as Draper Laboratory, Bell Aerospace–Textron Laboratories, and Hughes Research Laboratories. Working models should become available in the near future, and one such device would be an ideal companion for the present and the improved versions of the APT system. The combination would permit simultaneous extraction of the highest quality gravity data and highest quality positional data yet available from an aircraft. This would immediately extend aerial surveying operations to terrains of higher relief, where gravity-anomaly gradients commonly are severe and the spacing between ground-surveyed retroreflectors may need to be increased greatly because of difficult ground access.

The idea of combining a GPS receiver with the APT system in the same aircraft is especially attractive because it appears that the individual position-error models for the two instruments will complement each other. This means that a given flight mission could be flown with significantly fewer retroreflectors without any degradation in accuracy. An added benefit would accrue from the much slower growth in the real-time navigation error, which would ensure greater success in tracker search for and lock-on to each retroreflector programmed for use during the mission.

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SYMBOLS AND DIMENSIONS*

Symbol	Conventional Units	Dimensions	Description
kg	kg	M	kilogram, the unit of mass
m	m	L	meter, the unit of length
s	s	T	second, the unit of time
N	$m \cdot kg \cdot s^{-2}$	LMT^{-2}	newton, the unit of force
G		(1)	giga, a multiplying factor of 1×10^9
M		(1)	mega, a multiplying factor of 1×10^6
k		(1)	kilo, a multiplying factor of 1×10^3
m		(1)	milli, a multiplying factor of 1×10^{-3}
μ		(1)	micro, a multiplying factor of 1×10^{-6}
n		(1)	nano, a multiplying factor of 1×10^{-9}
p		(1)	pico, a multiplying factor of 1×10^{-12}
rad	rad	(1)	radian, the unit of measure for a plane angle (360° = 2π radians). It is the plane angle between two radii of a circle which cut off a circumferential arc whose length equals the radius.
Hz	s^{-1}	T^{-1}	hertz, the unit of frequency, usually expressed as number of cycles per second
Pa	$N \cdot m^{-2}$	$L^{-1}MT^{-2}$	pascal, the unit of pressure, usually expressed in newtons per square meter
$\underline{T}$	$N \cdot m$ or $m^2 \cdot kg \cdot s^{-2}$	L^2MT^{-2}	torque, the moment of force, usually expressed in newton meters
ω	$rad \cdot s^{-1}$	T^{-1}	angular velocity, usually expressed in radians per second
$\underline{L}$	$N \cdot m \cdot s$ or $m^2 \cdot kg \cdot s^{-1}$	L^2MT^{-1}	angular momentum, usually expressed in newton meter seconds
v	$m \cdot s^{-1}$	LT^{-1}	velocity, usually expressed in meters per second
g	$m \cdot s^{-2}$	LT^{-2}	acceleration due to gravity, usually expressed in meters per second squared. The standard value is 9.80665
$\vec{\Omega}$	s^{-1}	T^{-1}	Earth rate of rotation or angular velocity vector, usually expressed in radians per second
R_M	m	L	Earth radius of curvature in the meridian plane, usually expressed in meters
R_N	m	L	Earth radius of curvature normal to the meridian plane, usually expressed in meters
Φ			latitude, the angular distance, in degrees, of a point on the Earth's surface north or south of the equator, measured along a meridian (the equator is lat. 0°, the North Pole lat. 90°N, and the South Pole lat. 90°S)
λ			longitude, the angular distance, in degrees, west or east, measured along the equator or a parallel of latitude, between the meridian through a point on the Earth's surface and the reference meridian (long. 0°) through Greenwich, England (longitude values increase to a maximum of 180°W or 180°E)
x	m	L	position coordinate (north-south direction) in the horizontal (locally level) plane

Symbol	Conventional Units	Dimensions	Description
y	m	L	position coordinate (east-west direction) in the horizontal (locally level) plane
z	m	L	position coordinate (local vertical direction) in reference to the NGVD of 1929
α	rad	(1)	angle of divergence, usually expressed in radians
bar	$N \cdot m^{-2}$	$L^{-1}MT^{-2}$	bar, a unit of pressure, usually restricted to meteorology, equal to 10^5 pascals
cd			candela, the base unit of luminous intensity (optics), formerly known as candle and new candle
sr			steradian, a unit solid angle with its vertex at the center of a sphere, and of a size such that it subtends a spherical surface area equal to a square, each of whose sides equals the sphere radius
lm	cd·sr		lumen, the unit luminous flux, which is the flux emitted within a unit solid angle (1 steradian) from a point source having an intensity of 1 candela
lx	$lm \cdot m^{-2}$ or $cd \cdot sr \cdot m^{-2}$		lux, the unit of illumination—also called the *illuminance*, which is the illumination on a surface area of 1 square meter, over which there is a uniformly distributed luminous flux of 1 lumen

*International System of Units (SI).
(1)Dimensionless.

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