A HYDRAULIC MODEL TO EXPLAIN VARIATIONS IN SUMMIT TILT RATE AT KILAUEA AND MAUNA LOA VOLCANOES

By John J. Dvorak and Arnold T. Okamura

ABSTRACT

For a period of several months to a few years preceding eruptive activity, magma accumulates in shallow reservoirs centered 3 to 5 km beneath the summits of Kilauea and Mauna Loa Volcanoes. During flank eruptions, magma movement from the summit reservoir to the rift system is indicated by subsidence of the summit. Progress of the subsidence is monitored by teleremetered tiltmeters, by 3-m-long watertube tiltmeters, and by releving 40-m-long triangles.

During large summit-subsidence events at Kilauea, the tilt pattern indicates that the subsidence center may initially migrate 1 to 2 km to the south. Following the initial southward migration, the tilt direction remains constant for the remainder of the subsidence event, suggesting no further horizontal migration of the subsidence center during this period. No variation in tilt direction was measured in the Mauna Loa summit region during the March 25–April 15, 1984, northeast-rift eruption of this volcano.

The amount of vertical migration of the subsidence center is estimated from relative tilt rates recorded at different horizontal distances from the subsidence center. During the January 18–May 10, 1980, event at Kilauea, the subsidence center migrated upward 1 km in mid-February 1980. Vertical migration of the March 25–April 15, 1984, summit subsidence at Mauna Loa was less than 0.7 km.

During most subsidence events at both Kilauea and Mauna Loa, the tilt curve follows an exponential decay rate. Since 1955, the time constants for the exponential decay in tilt rate at Kilauea have varied from 0.25 days to 21 days; longer time constants are associated with eruptions that occur at greater distances from the summit. An exception to the exponential decay was the January 2–8, 1983, summit subsidence at Kilauea, which consisted of three separate subsidence episodes.

We propose a hydraulic model to explain the observed variation in tilt rate recorded during summit-subsidence events. Magma flow rate is assumed proportional to the pressure difference between the summit reservoir and a separate reservoir within the rift system. We further assume that magmatic pressure is linearly related to volumetric strain. On the basis of these assumptions, the magma flow rate and, hence, the summit tilt curve will follow the exponential decay observed during summit subsidences.

INTRODUCTION

Frequent periods of eruptive activity at two Hawaiian volcanoes, Kilauea and Mauna Loa (fig. 46.1), make these volcanoes especially well suited for studying the accumulation and movement of magma within the crust. Processes that control the volume and rate of flow of magma within Hawaiian volcanoes may also influence magma migration from the mantle to the surface; so, results derived from monitoring near-surface magma movement at Hawaiian volcanoes should be applicable to magma movement deeper in the crust and upper mantle, at depths too remote for current studies.

Several months to a few years preceding eruptive activity, magma accumulates in shallow reservoirs beneath the summits of both Kilauea and Mauna Loa (Fiske and Kinoshita, 1969; Decker and others, 1983). The accumulation and withdrawal of magma from the summit region produce a pattern of surface displacement that is recorded as change in ground tilt (fig. 46.2). The intersection of tilt vectors locates the average center of uplift or subsidence for the time interval between surveys.

The variation in tilt magnitude as a function of distance from the uplift or subsidence center can be modeled by a single point source embedded in an elastic half-space. The tilt magnitude, \( \Phi \), is given by (Eaton, 1962)

\[
\Phi = \frac{3V}{2\pi} \frac{dh}{(d^2 + h^2)^{3/2}},
\]

(1)

FIGURE 46.1.—Structural features associated with summit regions and rift systems of Mauna Loa and Kilauea on Island of Hawaii. Places along east rift of Kilauea mentioned in text are identified.

1281
where \( h \) is the horizontal distance, \( d \) is the depth to the source, and \( V \) is the volume of elastic uplift. No tilt change is recorded directly above \( (h = 0) \) or at large distances from the point source; the maximum tilt change occurs at \( h = d/2 \) (fig. 46.3). Comparison of theoretical curves and measured tilt changes show that the summit reservoirs at Kilauea and Mauna Loa are centered 3 to 5 km beneath the summits of both volcanoes (fig. 46.4; table 46.1). A similar depth range to the summit reservoirs has been estimated from level data, using a point-source elastic model, by many earlier investigators (Mogi, 1958; Fiske and Kinoshita, 1969; Decker and others, 1983; Dvorak and others, 1983).

During periods of magma accumulation in the Kilauea summit region, the uplift center usually begins in the central caldera region 1 km northeast of Halemaumau and migrates as much as 2 km south (Fiske and Kinoshita, 1969). A similar migration of the uplift center has not been recognized for Mauna Loa. Magma that accumulates in the summit reservoir may erupt in the summit region or move laterally into either rift system, where it may eventually be erupted. During periods of magma withdrawal from the summit reservoir, the subsidence center may also migrate a few kilometers within the caldera region (Jackson and others, 1975; Swanson and others, 1976).

The accumulation and withdrawal of magma in the Kilauea summit region is easily recorded by making frequent tilt measurements (fig. 46.5). Higher tilt values in figure 46.5 correspond to uplift of the summit region caused by increased magmatic pressure and accumulation of additional magma within the summit reservoir. Prolonged periods of summit uplift are interrupted by much shorter

**Table 46.1.** Point source solutions to Kilauea summit tilt data

<table>
<thead>
<tr>
<th></th>
<th>August 12-14, 1981</th>
<th>September 27-October 14, 1982</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>March 29-40, 1982</td>
<td>February 7-11, 1983</td>
</tr>
<tr>
<td>Longitude</td>
<td>155°17.7' (±0.36 km)</td>
<td>155°17.0' (±0.50 km)</td>
</tr>
<tr>
<td>Latitude</td>
<td>19°23.5' (±0.34 km)</td>
<td>19°23.8' (±0.28 km)</td>
</tr>
<tr>
<td>Depth</td>
<td>4.31 km (±0.33 km)</td>
<td>4.38 km (±0.58 km)</td>
</tr>
<tr>
<td>Volume of elastic uplift or subsidence</td>
<td>53×10⁶ m³ (±26×10⁶ m³)</td>
<td>72×10⁶ m³ (±26×10⁶ m³)</td>
</tr>
</tbody>
</table>

**Figure 46.2.** Tilt vectors measured in Kilauea summit region. A, Uplift of August 12-14, 1981, to March 3-April 5, 1982. B, Subsidence of September 27-October 14, 1982, to February 2-11, 1983. Vectors were determined by measuring changes in relative elevation along 40-m-long triangles. Station locations, triangles. Tilt vectors point in direction of downward deflection of surface. General radial pattern of tilt vectors indicates that most active part of summit magma reservoir lies beneath southern part of caldera.

**Figure 46.3.** Theoretical tilt changes as a function of horizontal distance from a point source embedded in an elastic half-space. Curves, normalized to maximum tilt change, are normalized to depth to source.
periods of summit subsidence, indicated by sudden decreases in tilt value. These summit-subsidence events are caused by the movement of magma from the summit reservoir to either rift system and are usually associated with eruptive activity along the rift (Eaton and Murata, 1960; Jackson and others, 1975; Swanson and others, 1976). A similar pattern of slow accumulation and rapid withdrawal of magma from the summit reservoir at Mauna Loa is also indicated by summit tilt measurements (fig. 46.5B).

We have selected data from 14 subsidence events at Kilauea, indicated by vertical dashed lines in figure 46.5A, to determine what factors may control variations in summit tilt rate. Data from the summit subsidence associated with the 1984 Mauna Loa eruption are also included. Based on these data, a simple hydraulic model is proposed to explain magma flow rate in the conduit systems within Kilauea and Mauna Loa.

**ACKNOWLEDGMENTS**

This research has built upon measurements made over the past 30 years by the staff of the Hawaiian Volcano Observatory. R. Decker, J. Dietrich, D. Dzurisin, and T. Wright provided useful comments and suggestions to this manuscript.

**MEASUREMENTS**

Tilt measurements were begun in the summit region of Kilauea a few years after the establishment of the Hawaiian Volcano Observatory (HVO) in 1912; however, instruments specifically designed to record tilt changes were not in use until the 1950s. Tilt measurements in the summit region of Mauna Loa were begun 5 months after the July 1975 summit eruption.

Kilauea summit tilt measurements have been made at Uwekahuna since at least 1950 and at Outlet since October 1955 (fig. 46.6A). Between 1950 and June 1956, tilt changes at Uwekahuna were recorded by measuring the trace offset from a horizontal-pendulum seismograph. This technique could probably record tilt changes of 10–20 microradians (μrad). In 1955 and 1956, 3-m-long waterubbe tiltmeters, capable of resolving tilt changes of 2 μrad, were installed at Uwekahuna and Outlet (Eaton, 1959).

Since 1966, a continuously recording, one-component tiltmeter has operated in the Uwekahuna vault. This instrument is a mercury capacitance instrument that records tilt changes along the east-west direction to a sensitivity of 0.1 μrad. In recent years, additional electronic tiltmeters installed in the Kilauea summit region transmit readings at approximately 10-min intervals to a recording system located at HVO.

Tilt measurements in the Mauna Loa summit region (fig. 46.6B) are made by the dry-tilt method, which consists of relieving 40-m-long triangles (Yamashita, 1981). The use of 3-m-long wooden rods with permanently installed Invar strips allows tilt changes at MOK (tilt station at Mokuaweoweo) to be measured to 3–4 μrad. At other tilt sites, an accuracy of 8–12 μrad is achieved by the use of 1.5-m-long stainless-steel rods with removable Invar strips.

Repeated tilt measurements at Uwekahuna were made during 14 summit-subsidence events at Kilauea (fig. 46.5A) and during the 1984 northeast-rift eruption of Mauna Loa. The beginning dates of summit subsidence and the locations of the related rift intrusion or eruption are shown in table 46.2. Parts of the east rift from the summit to Napau Crater, to Heheiahu, and to Kapoho are referred to as the upper, middle, and lower east rift, respectively (fig. 46.1). The distance from the summit-subsidence center to the rift eruption is measured along the rift axis to the major lava-producing eruptive vent. For the two events not accompanied by eruptive activity, in August 1981 and June 1982, the distance to the rift intrusion was assumed to correspond to the central region of shallow rift earthquakes. The beginning times and the durations of rift eruptions associated with each subsidence event are also listed in table 46.2. The occurrence of later eruptive phases, which may follow a large summit subsidence and initial eruptive phase, such as

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**Figure 46.4.**—Measured changes in radial tilt component and theoretical curves based on a point source embedded in an elastic half-space. A. Uplift. B. Subsidence. Data are same as shown in figure 46.2A, B. Model parameters are listed in table 46.1.
Figure 46.5.—Long-term tilt patterns recorded in summit regions of Kilauea and Mauna Loa. A greater tilt value generally corresponds to summit uplift and magma accumulation. A, West-east component of daily tilt recorded in Kilauea summit region at Uwekahuna (see figure 46.6A for location). The 300-μrad tilt increase on November 16, 1983, is related to a magnitude 6.5 earthquake located 13 km west-northwest of Uwekahuna. Sudden tilt change probably did not involve magma movement in summit region. Summit subsidence examined in this paper (see table 46.2) are identified by vertical dashed-line segments. B, North-south tilt component recorded in Mauna Loa summit region at MOK (see figure 46.6B for location). Sudden tilt decrease in March 1984 corresponds to recent Mauna Loa northeast-rift eruption.
in March–April 1955, September 20–23, 1977, and the succession of eruptive phases that began in February 1983, are not considered in determining eruption duration because these events are associated with smaller, individual summit subsidences.

**HORIZONTAL AND VERTICAL MIGRATION OF SUBSIDENCE CENTERS**

Jackson and others (1975) and Swanson and others (1976) noted a horizontal migration of the subsidence center associated with some subsidence events. We show in this section that, though there is a systematic migration, most of the tilt change occurs while the subsidence center is stationary.

Horizontal migration of the subsidence center is indicated by variations in tilt direction, which is determined by examining the amount of north-south and west-east tilt. A constant direction is indicated by a linear relation between north-south and west-east tilt values.

The variation in tilt direction recorded by the Uwekahuna watertube instrument during the December 31, 1974–January 3, 1975, summit subsidence is typical of most events at Kilauea (fig. 46.7). The subsidence center was initially located directly east of Uwekahuna. As the subsidence proceeded, the center migrated southward, indicated by a change in tilt direction at Uwekahuna from east to southeast. The dominant southeastern tilt recorded at Uwekahuna suggests that the subsidence center remained in the same horizontal position throughout most of the subsidence. A second, relatively minor migration of the subsidence center to a position south of Uwekahuna may have occurred near the end of the event.

The net tilt change recorded at Uwekahuna for each subsidence event is listed in table 46.2. Tilt directions listed in this table are for the period following the initial southward migration of the subsidence center. Since 1977, this direction from Uwekahuna has been remarkably constant. The only subsidence that did not display a dominant tilt direction was the February 21–23, 1969, event, which consisted of two different directions equal in tilt change, initially to the southeast and then to the south of Uwekahuna.

The tilt directions at Uwekahuna associated with the August 1981 southwest-rift intrusion and the January 1983 east-rift eruption

![Figure 46.6](image1)

**Figure 46.6.**—Tilt sites (triangles) and subsidence centers. *A*, Kilauea summit region. *B*, Mauna Loa summit region. MOK, tilt station at Mokuaweoweo Crater.

![Figure 46.7](image2)

**Figure 46.7.**—Tilt directions measured at Uwekahuna during December 1974 southwest-rift eruption. Linear trend indicates constant tilt direction.
TABLE 46.2.—Summit subsidence of Hawaiian volcanoes

<table>
<thead>
<tr>
<th>Location of rift intrusion</th>
<th>Duration (days)</th>
<th>Tilt change (μrad)</th>
<th>Tilt direction</th>
<th>Time constant (days)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kiluaea</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td>37</td>
<td>LER</td>
<td>2/28/55 0800</td>
<td>138</td>
<td>192</td>
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<td>1/13/60 1935</td>
<td>36</td>
<td>345</td>
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</tr>
<tr>
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<tr>
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<tr>
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<td>13</td>
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<tr>
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<td>(†)</td>
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<tr>
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<td>MER</td>
<td>1/3/83 0030</td>
<td>14</td>
<td>149</td>
</tr>
</tbody>
</table>

Mauna Loa

<table>
<thead>
<tr>
<th>Location of rift intrusion</th>
<th>Duration (days)</th>
<th>Tilt change (μrad)</th>
<th>Tilt direction</th>
<th>Time constant (days)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/25/84</td>
<td>18</td>
<td>NER</td>
<td>3/25/84 0130</td>
<td>20</td>
<td>--</td>
</tr>
</tbody>
</table>

1 Duration of initial eruptive phase, later eruptive phases had relatively small tilt changes (all <30 μrad; most <15 μrad) recorded at Uwekahuna.
2 No dominant tilt direction during the major summit subsidence.
3 Hawaiian Volcano Observatory, unpublished data.
4 No eruptive activity associated with these events.
5 Multiple subsidence event.

were almost identical. Both events began with an eastward-directed tilt and rotated to the southeast (fig. 46.8), similar to the beginning of the 1974 summit subsidence. The tilt directions measured at Uwekahuna during the August 1981 and January 1983 events were identical, suggesting the same subsidence center, even though magma moved into different rift systems. No clear relation exists between location of subsidence center, measured by tilt at Uwekahuna, and movement of magma into either rift system (table 46.2).

A constant tilt direction recorded at a single station does not eliminate the possibility that the subsidence center may migrate radially to that station. Tilt measurements made at two different sites during the January—May 1960 summit subsidence show that at least for this event the subsidence center remained in approximately the same horizontal position (fig. 46.9).

Tilt measurements made at three sites in the Mauna Loa summit region during the March 25—April 15, 1984, northeast-rift eruption showed no horizontal migration of the subsidence center (fig. 46.10). The subsidence center associated with the 1984 Mauna Loa northeast-rift eruption was located 1 km southeast of the rim of the summit caldera Mokuaweoweo (fig. 46.6B), a position identical to the uplift center determined from repeated geodetic surveys conducted since 1977 (Deckers and others, 1983).

The amount of vertical migration of the subsidence center can be estimated when simultaneous tilt measurements are made at different distances from the subsidence center. The surface displacements produced by magma withdrawal from the summit reservoir are assumed to be adequately represented by a point source embedded in an elastic half-space (Dvorak and others, 1983). The ratio of the magnitude of tilt change (Φ1 and Φ2) at different horizontal distances from the subsidence center (h1 and h2) is given by

\[ \Phi_1 / \Phi_2 = h_1 / h_2 \left( \frac{d^2 + h_2^2}{d^2 + h_1^2} \right)^{3/2} \]  

(2)

If the subsidence center does not migrate horizontally, then changes in the ratio of the magnitude of tilt change are a result of vertical migration. The amount that a small change in depth to the source, Δd, will change the ratio Δ(Φ1/Φ2) is determined by differentiating equation 2:

\[ \Delta(\Phi_1 / \Phi_2) = 5d \Phi_1 \left( \frac{1}{d^2 + h_2^2} - \frac{1}{d^2 + h_1^2} \right) \Delta d. \]  

(3)

For Kiluaea, simultaneous tilt measurements were made during the January 18—May 11, 1960, event at Outlet and Uwekahuna.
located 1 and 3 km, respectively, from the subsidence center. The ratio of tilt rates between Outlet and Uwekahuna in mid-February is not related to a change in tilt direction (fig. 46.11). The increase in ratio of tilt rates from 0.5 to 0.9 between Outlet and Uwekahuna is consistent with an upward migration of the subsidence center from 4 to 3 km that occurred in mid-February.

Simultaneous tilt measurements made at three different stations in the Mauna Loa summit region during the 1984 northeast-rift eruption showed no apparent change in the ratios of tilt rate (fig. 46.12). The MOK, HVO135, and AIAPAO tilt sites are located at horizontal distances of 3.5, 0.5, and 1.5 km, respectively, from the subsidence center (fig. 46.6B). Using a value of 4 km for the depth to the source in (3) and noting that the slopes in figure 46.12 do not vary by more than 0.3, data in figure 46.12 indicate that the subsidence center related to the 1984 Mauna Loa eruption did not migrate vertically more than 0.7 km.

**EXPONENTIAL DECAY IN SUMMIT TILT RATE**

The summit tilt rate measured during subsidence events usually decreases with time (figs. 46.13–46.15). We have fitted exponential curves to both tilt components recorded during the first 13 Kilauea subsidences listed in table 46.2 and the 1984 Mauna Loa subsidence. For each subsidence event we estimate the magnitude of the north-south and the west-east tilt change, the time constant related to the decay rate, $\tau$, and the beginning time of subsidence.

The time dependence of the summit tilt curves recorded during subsidence of both Kilauea and Mauna Loa is adequately matched by exponential curves, especially when tilt measurements are made by the 3-m-long watertube instrument (fig. 46.13) and by the dry-tilt technique (fig. 46.15). The poorer agreement between measured tilt changes and exponential curves for the March 1955 Kilauea subsidence is probably caused by less precise tilt measurements obtained from the horizontal-panulum seismograph (fig. 46.14).

The only summit subsidence listed in table 46.2 that significantly departs from an exponential decay is the January 1983 Kilauea subsidence. This subsidence was a multiple event; the summit subsided in three stages over a period of 7 days (fig. 46.16). These three stages correspond to three episodes of downrift earthquake migration, crack opening, and eruption.

The exponential decay in summit tilt rate is reminiscent of a rheological response observed in the laboratory when crustal material is subjected to sudden stress changes, such as may result from the sudden movement of magma. For example, transient creep of crustal material will exhibit an exponential decay in strain rate (Jaeger and Cook, 1969). The time constants for summit subsidence at Kilauea

![Figure 46.9](image-url)  
**Figure 46.9.**—Tilt directions measured at Outlet and Uwekahuna during January–May 1960 summit subsidence of Kilauea.
vary by a factor of more than 80. It seems unlikely that transient creep could control summit tilt rates, because rheological properties of the Kilauea summit would also have to vary by a factor of 80.

Along the east rift of Kilauea, the time constant appears to be related to the distance from the summit-subidence center to the affected part of the rift (fig. 46.17). Longer time constants are associated with eruptions located farther downrift, suggesting that conduit length from the summit reservoir to the eruptive site controls, in part, the summit tilt rate. Time constants related to eruptions along the southwest rift of Kilauea and the northeast rift of Mauna Loa are longer than at comparable distances along the east rift of Kilauea.

Tilt rates recorded during periods of magma accumulation in the summit reservoir from a deeper source may also display an exponential decay. The west-east tilt component at Uwekahuna measured by a continuously recording tiltmeter revealed a cyclic pattern during a recent series of eruptive phases along the middle east rift (fig. 46.18). Beginning in February 1983 and lasting at least two years, the Kilauea summit went through several cycles of gradual uplift and more rapid subsidence. Periods of subsidence were associated with eruptive activity along the middle east rift. The summit tilt curve during noneruptive periods approximately followed an exponential decay curve with a maximum tilt change of 32 μrad. The time constant for summit uplift from February to September 1983 was 40 days, a time constant a factor of 2 greater than the maximum value determined for periods of summit subsidence and rift eruption.

We have argued that migration of the subsidence center and rheology of the volcano are unimportant in explaining summit-subidence tilt curves. Moreover, an exponential time dependence of tilt rate observed during periods of both magma accumulation and withdrawal suggests that factors that control magma flow rate may be responsible for observed tilt rates.

**HYDRAULIC MODEL.**

We adopt a hydraulic model for magma movement that was originally applied by Machado (1974) to explain decreasing extrusive rates observed at some volcanoes. Magma is stored in reservoirs between the upper mantle and the summit and along the rift systems. Magma moves within a conduit system that connects reservoirs that are under different confining pressures.

The magma flow rate, $Q$, is assumed to be proportional to the pressure change, $\Delta P$, between reservoirs

$$Q = \alpha \Delta P,$$  \hspace{1cm} (4)

where the constant of proportionality, $\alpha$, depends on type of fluid flow.

We further assume that magma pressure is linearly related to volumetric strain, $\Delta V/V$,

$$\Delta P = -E \frac{\Delta V}{V},$$  \hspace{1cm} (5)

where $E$ is a constant of proportionality, the bulk modulus of the system. A pressure decrease results in a volume increase. When flow starts, pressure will change according to the equation (Machado, 1974, equation 48)

$$\Delta P = \frac{E}{V} \left( \Delta V + \int_0^t Q \, dt \right).$$  \hspace{1cm} (6)

Substitution of (4) into (6) yields

$$Q = Q_0 - \frac{1}{\tau} \int_0^t Q \, dt,$$  \hspace{1cm} (7)

where

$$\frac{1}{\tau} = \alpha \frac{E}{V},$$  \hspace{1cm} (8)

and

$$Q_0 = -\frac{\Delta V}{\tau}.$$  \hspace{1cm} (9)
Differentiating (7) gives a simple differential equation for the magma flow rate:

\[ \frac{dQ}{dt} + \frac{1}{\tau} Q = 0. \]  

(10)

The solution to the above equation is

\[ Q = Q_0 e^{-\frac{t}{\tau}}. \]  

(11)

Several types of fluid flow are based on flow rate proportional to the pressure difference (4). In particular, for laminar fluid flow through a circular pipe of radius \( r \), Poiseuille's law gives the constant of proportionality relating flow rate to pressure difference as:

\[ \alpha = \frac{\pi}{8\eta} f^4 \frac{\Delta P}{l} \]  

(12)

where \( \eta \) is magma viscosity, \( f \) is the frictional-loss factor, and \( l \) is the conduit length. The quantity \( \Delta P/l \) is the pressure gradient between the two reservoirs. The time constant is given by

\[ \tau = \frac{8\eta}{\pi} \frac{f^4}{f^4} \left( \frac{\Delta P}{l} \right). \]  

(13)

The change in volume, \( \Delta V \), is related to the subsidence volume. The time constant increases with conduit length in (13), a relation observed along the east rift of Kilauea (Fig. 46.17), though not linearly along the entire rift, as would be expected if \( r \), \( f \), and \( \Delta P \) remained constant.

Equation 13 emphasizes the importance of conduit width on magma flow rate. A 50-percent decrease in conduit width will decrease the flow rate by a factor of 16. Because conduit width probably does not remain constant along the rift system, a time constant derived from the summit tilt curve relates to the minimum constriction along the active part of the conduit.

**INITIAL EARTHQUAKE LOCATIONS AND SHAPE OF THE SUMMIT TILT CURVE**

Earthquakes located in the summit region and along the rift zones of Kilauea are probably directly related to the magma conduit system within the volcano (Ryan and others, 1983). The location of the initial earthquake swarm associated with a summit subsidence may indicate where magma movement was initiated. We might expect that earthquake swarms initiated at different locations within the volcano will influence the initial shape of the summit tilt curve.

Figure 46.11—Tilt magnitudes recorded at Outlet as a function of tilt magnitudes recorded at Uwekahuna during January–May 1960 summit subsidence of Kilauea.
Increased coverage of the seismic network and advances in seismic processing at HVO have improved hypocenter determinations on the Island of Hawaii (Klein and Koyanagi, 1980). In particular, earthquake swarms occurring at Kilauea over the past decade can be located to within a few kilometers on any part of the volcano with greater resolution in the summit region and along the upper rift systems.

Earthquake hypocenters associated with the onset of summit subsidence and southwest-rift intrusion frequently begin in the south caldera area of Kilauea. For example, earthquake swarms associated with the December 1974 eruption (fig. 46.19A) and the August 1981 intrusion (fig. 46.19B) began in the south caldera area and migrated along the southwest rift for several hours (Klein and others, chapter 43). The beginning of these two earthquake swarms coincided with abrupt change in slope of the summit tilt curves (fig. 46.20). On the basis of the location of the initial earthquake swarm and the initial, rapid summit tilt change, we infer that magma movement on December 31, 1974, and on August 10, 1981, was initiated in the summit reservoir.

During the two most recent Kilauea subsidence listed in table 46.2, in June 1982 and January 1983, initial earthquake swarms were located along the rift and not in the south caldera area (fig. 46.21). The summit tilt responded gradually for several hours to these events (fig. 46.22). Along the east rift near the site of the initial earthquake swarm in January 1983, a tiltmeter did record an abrupt change in slope (fig. 46.23). Presumably, initial magma movement was from a reservoir within the southwest rift in June 1982 and within the east rift in January 1983 and not from the summit reservoir. Initial magma flow from the summit reservoir was slower as pressure along the conduit system from the summit to the rift was gradually reduced as magma moved within the rift system.

![Graph](image-url)

**Figure 46.12.**—Tilt magnitudes recorded at HVO 135 and AINAPO as a function of tilt magnitudes recorded at MOK during March 25–April 14, 1984 summit subsidence of Mauna Loa.

**Figure 46.13.**—Tilt changes recorded by 3-mm-long water-tube instrument at Uwekahuna. North-south tilt changes, upper curve; west-east tilt changes, lower curve; solid lines, exponential functions empirically fitted to tilt data; numerical constant in each equation is value of total tilt change in microradians. A, August 1981 southwest-rift intrusion. B, December 1974 southwest-rift eruption of Kilauea.
A gradual response of summit tilt for 1 to 3 days was also observed after the onset of eruptive activity along the lower east rift in March 1955 and January 1960. Initial earthquake swarms and ground cracking in these two cases began along the rift.

DISCUSSION

The well-establish relation between summit-subsidence events and rift intrusions and eruptions at Hawaiian volcanoes can be exploited to characterize magma movement within a long conduit system. In particular, we have examined the relation between rate of magma withdrawal from the summit reservoir and the rate and location of rift eruptions. Magma movement from the Kilauea summit reservoir to the east-rift system may be an analog to vertical magma ascent through the upper mantle and crust beneath Hawaiian volcanoes. Therefore, our results may also contribute to understanding how deeper magmatic sources influence eruptive activity at other volcanoes.

Summit tilt measurements at Kilauea and Mauna Loa record magma accumulation and withdrawal from a shallow reservoir. Magma originating in the mantle migrates through the upper mantle and crust and accumulates in the summit reservoir prior to eruption. Horizontal migration of the uplift center at Kilauea suggests that the summit reservoir consists of a plexus of dikes and sills (Fiske and Kinoshita, 1969).

Several lines of evidence indicate that magma also resides in shallow reservoirs along the east rift of Kilauea. The 4- to 5-day delay between the onset of eruptive activity along the lower east rift
FIGURE 46.15. — Tilt changes recorded by dry-tilt method at MOK during March 25-April 15, 1984, northeast-rift eruption of Mauna Loa. North-south tilt changes, upper curve; west-east tilt changes, lower curve; solid lines, exponential functions empirically fitted to tilt data; numerical constant in each equation is value of total tilt change in microradians.

FIGURE 46.16. — Tilt changes recorded by automatic tiltmeter located at Uwekahuna during January 1983 middle-east-rift eruption of Kilauea.

FIGURE 46.17. — Time constants for summit subsidence plotted as a function of distance from subsidence center to rift intrusion or eruption.

FIGURE 46.18. — West-east tilt changes recorded from February to September 1983 by tiltmeter located at Uwekahuna. Periods of generally increasing tilt values correspond to summit uplift and magma accumulation in Kilauea summit region; periods of rapid decrease in tilt values correspond to magma withdrawal from Kilauea summit region and eruptive activity along middle east rift. Same exponential curve (dashed lines) has been empirically fitted to periods of summit uplift. Exponential curve is for a maximum tilt change of 32 μrad and a time constant of 40 days.
and the onset of summit subsidence in March 1955 and January 1960 led Eaton (1962) to suggest that magma was stored within the rift system. Jackson and others (1975) noted that the volume of summit subsidence at Kilauea exceeds the volume of lava erupted along the rift: some magma withdrawn from the summit remains in the rift system. Differentiated lava initially erupted from the east rift during a new eruptive sequence suggests that rift eruptions are initially fed by magma that has resided for longer periods at shallow levels in the crust than magma erupted in the summit (Wright and others, 1968; Moore, 1983). Finally, geodetic surveys conducted along the east rift of Kilauea have identified centers of uplift presumed to be regions of magma accumulation (Jackson and others, 1975; Swanson and others, 1976; Dzurisin and others, 1984), one persistent center along the upper east rift is centered 2 to 4 km beneath the surface (Dvorak and others, 1983).

Along the east rift of Kilauea, the volume of summit subsidence increases with distance to the rift eruption (Epp and others, 1983). The subsidence rate also increases with distance, though not linearly, along the entire east rift (fig. 46.17). Time constants for summit subsidence range from 0.25 to 1.5 days for southwest- and east-rift eruptions that occur within 35 km of the summit; time constants for lower-east-rift eruptions were considerably longer, from 16 to 21 days. The abrupt increase in time constant for rift eruptions downrift of Heiheiahu (fig. 46.1) may be caused by constriction of the magma conduit between the middle and the lower parts of the east rift.

The time constant for the 1984 Mauna Loa summit subsidence and northeast-rift eruption is close to the linear trend extrapolated from time constants for the Kilauea lower-east-rift eruptions. This similarity may be related to the several-year interval between rift
intrusions into the lower east rift of Kilauea and the northeast rift of Mauna Loa. By comparison, more frequent episodes of magma movement in the upper east rift result in less flow resistance and, hence, much shorter time constants (Dvorak and Okamura, 1985).

Changes in summit tilt rate recorded during subsidence events usually follow an exponential decay. Periods of magma accumulation in the summit reservoir may also follow an exponential decay in tilt rate, a result previously recognized by Shimozuru (1981), who proposed an electrical analog for the summit magma reservoir.

If we assume that summit tilt changes are related to summit magma accumulation or withdrawal, then the rate of magma accumulation in the summit reservoirs of Kilauea and Mauna Loa may be as much as a factor of 100 less than the rate of magma withdrawal (figs. 46.5, 46.17, 46.18). The different rates of magma accumulation and withdrawal for an eruptive cycle may reflect a basic difference between the vertical conduit feeding the summit reservoir and the horizontal conduit along the rift system. In the context of our model, an increase by a factor of 100 in magma flow rate could correspond to an increase by a factor of 10 in cross-sectional area or of 100 in pressure gradient (equation 13).

If the east rift of Kilauea is an appropriate analog for the vertical conduit beneath Hawaiian volcanoes, then we may have a better understanding of the possible role the upper mantle has on magma supply rate. The 40- to 60-km length of the vertical conduit from the mantle source to the summit, based on maximum earthquake depths beneath Hawaiian volcanoes and on a hydraulic model (Eaton and Murata, 1960), is comparable to the distance from the Kilauea summit to the 1960 lower-east-rift eruption. Because individual magma reservoirs exist along the east rift, magma reservoirs may also be expected to exist along the vertical conduit. Conduit width probably varies with length along both the vertical conduit and the rift system; the conduit width in equation 13 is the width at a constriction that limits the flow rate in the remainder of the conduit.
Magma flow may be initiated in a reservoir that lies along the rift system; however, the summit reservoir strongly influences the total volume of magma moved, as evidenced by continued summit subsidence several weeks after the end of eruptive activity in 1955 and 1960. Similarly, magma flow may be initiated in a reservoir within the vertical conduit, although the mantle source region may control the magma flow rate and total volume of magma involved. Recently, Dzurisin and others (1984) have suggested a close link between bursts of deep harmonic tremor, an indicator of magma movement, and magma supply rate to the Kilauea summit. Recognition of a relation between magma volume and flow rate from the Kilauea summit reservoir to east-rift eruptions supports their suggestion.

SUMMARY

Summit magma reservoirs exist within a few kilometers of the surface at both Kilauea and Mauna Loa. The accumulation and withdrawal of magma in the reservoir is monitored by tilt measurements that can easily be made over periods of a few minutes to several weeks.

Summit tilt curves recorded during subsidence events frequently begin with an abrupt change in slope. The tilt rate is usually greatest at the beginning of the event and steadily decreases with time.

Horizontal or vertical migration of the subsidence center are unimportant in explaining variations in summit tilt rate at Kilauea and Mauna Loa. The summit-subsidence pattern at Kilauea is independent of the rift system involved in magma movement.

Most summit tilt curves associated with subsidence events follow an exponential decay. At Kilauea, the decay time constant has varied by more than a factor of 80, which suggests that rheological properties of the volcano do not control subsidence rate.

The rate of magma accumulation in the summit reservoir may also follow an exponential decay.

A simple hydraulic model may explain the observed exponential decay in summit subsidence rate. The model assumes that magma flow rate is proportional to the pressure difference along the magma conduit from the summit reservoir to a second reservoir or eruptive site located along the rift system. Furthermore, magmatic pressure is assumed to be linearly proportional to volumetric strain. One type of fluid flow consistent with these assumptions is laminar flow through a circular pipe.

REFERENCES


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