ELASTIC AND INELASTIC MAGMA STORAGE AT KILAUEA VOLCANO

By Daniel J. Johnson

ABSTRACT

Gravity, leveling, and trilateration data recently obtained at Kilauea Volcano indicate that magma is accommodated within the summit reservoir by a combination of elastic magma compression, elastic chamber expansion, and gradual inelastic edifice widening in a south-southeast direction.

Elastic (reversible) magma storage is modeled using data from 15 brief summit-deflation episodes in 1984–85 and assuming a spherical geometry for the magma reservoir. Changes in pressure in the reservoir produce the deformation; changes in mass in the reservoir produce the gravity changes. The ratio of residual gravity change (corrected for the free-air effect) to elevation change, measured near the apex of maximum subsidence, averaged 0.28 ± 0.025 μGal/mm. If magma density is assumed to be 2.75 g/cm^3, the volume of magma withdrawn from the reservoir averaged 2.4 ± 0.21 times the associated volume of subsidence. This volume difference is dependent on both the magma bulk modulus (which constrains bulk compression of the reservoir contents) and the edifice shear modulus (which constrains distortion of the surrounding edifice). If deformation is assumed to be from a point dilatational source in an elastic half-space, the observed volume difference implies that the shear modulus of the edifice is 2.0 ± 0.24 times the bulk modulus of the magma.

Trilateration data show that Kilauea's edifice widens inelastically in time intervals of months to years, primarily in a northwest direction. Over these long time periods, gravity and elevation changes indicate both surface subsidence and mass addition. These observations imply that horizontal extension of Kilauea's edifice decreases lateral support of the fluid magma reservoir, thus lowering the pressure in the reservoir and allowing more magma to be injected without vertical uplift.

Increased summit collapse occurred during the November 1975 earthquake, when new reservoir space created by inelastic seaward displacement of the volcano's south flank was not immediately filled by new magma from depth. For 16 months after the earthquake, additional reservoir capacity created by continued edifice dilation was filled with new magma. From January 1984 to July 1985, Kilauea's edifice subsided by 200 mm and widened by as much as 250 mm. No net subsurface mass changes were measured during this period, implying that the subsidence was entirely the result of horizontal extension.

A complete magma-budget estimate for Kilauea must therefore consider both elastic and inelastic magma storage. This requires a synthesis of leveling, trilateration, and gravity data.

INTRODUCTION

Kilauea Volcano grows by the accumulation of mantle-derived magma that is initially stored in a shallow summit magma reservoir (Eaton and Murata, 1960). The location of this storage region, indicated by an aseismic zone (Koyanagi and others, 1976) and by the inversion of measured surface displacements (Fiske and Kinoshita, 1969; Dvorak and others, 1983), is in a zone below the south rim of the caldera about 3 km in diameter and from 2 to 6 km below the surface. Slow uplift of the volcano's summit over several weeks to a few months is caused by the gradual addition of new magma from a deep source region. Occasional rapid surface subsidence, over periods of hours to days, occurs as magma is withdrawn from the reservoir, either migrating upward or laterally through a conduit system into one of two rift zones (Ryan and others, 1981). Though a part of the mantle-derived magma supply may eventually reach the surface, some magma remains in the summit reservoir and rift systems (Dzurisin and others, 1984).

Previous estimates of magma-volume changes in the subsurface reservoir have been based on geodetic measurements (Duffield and others, 1982; Dvorak and others, 1983; Dzurisin and others, 1984). Because the elastic properties of both magma and volcanic edifice are poorly known, subsurface volume changes cannot be determined from geodetic data alone. Rather, the volume change of the edifice, as determined by integrating vertical displacements measured at the surface, is typically used to approximate the subsurface volume change.

Subsurface volume changes can be estimated more accurately if measurements of changes in both gravity and elevation are available. Gravity and elevation measurements obtained at Kilauea by Jachens and Eaton (1980) and Dzurisin and others (1980) demonstrated that between November 1975 and September 1977 changes in the volume of magma in the reservoir exceeded changes in the volume of the edifice. This was attributed to the effect of void spaces created during periods of magma withdrawal (Jachens and Eaton, 1980), later refilled during periods of magma accumulation (Dzurisin and others, 1980). Gravity studies by Johnsen and others (1980) at Krafla, Iceland, by Sanderson and others (1983) at Mount Etna, Italy, and by Eggers (1983) at Pacaya Volcano, Guatemala, also indicate differences between edifice and subsurface volume changes. Sanderson and others (1983) explained the differences by bulk compression of the magmatic fluid and changes in volume of exsolved gas enabling subsurface magma accumulation to exceed the volume of edifice expansion. Eggers (1983) proposed density changes of about 0.4 g/cm^3 in shallow magma bodies caused by changes in the magmatic water content and vesicularity.

A series of gravity observations made in the Kilauea summit region between January 6, 1984, and July 8, 1985, have been

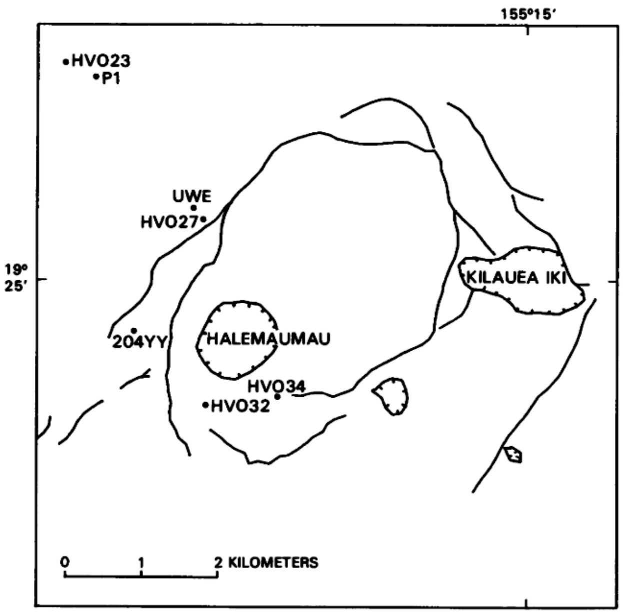

FIGURE 47.1.—Location map of Kilauea summit region showing principal fault scarps and craters, gravity monitoring sites HVO23, P1, HVO27, 204YY, HVO32, and HVO34, and east-west component electronic tiltmeter at Uwekahuna (UWE). Hachures mark inner side of crater walls.

analyzed to determine the empirical relation between magma-volume changes in the summit reservoir and edifice-volume changes as evidenced by surface uplift or subsidence. The short-term relation observed during brief episodes of elastic deflation corresponding to eruptions at the Puu Oo vent along Kilauea's middle east rift zone (Wolfe and others, chapter 17) has been used to estimate the relative elastic properties of the magma reservoir region and the surrounding volcanic edifice. The long-term relation has been used to evaluate the effect of inelastic summit widening on volume changes of the edifice and magma content of the summit reservoir. Results of this study will help improve estimates of the rate of magma supply to Kilauea.

ACKNOWLEDGMENTS

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DATA

A total of 141 gravity surveys were conducted in the Kilauea summit region between January 6, 1984, and July 16, 1985. Initially, measurements were made at five monitoring sites referenced to a base station (P1) 5 km northwest of the point of maximum vertical movement (from earlier leveling surveys). The monitoring network was reduced to two sites after February 16, 1984, and to one site, HVO34, after July 12, 1984, because data from this site were found to be representative of the others. Only gravity data for HVO34, referenced to P1, are presented in this paper; locations of all measured stations are shown in figure 47.1.

Two LaCoste and Romberg model-G gravimeters, each equipped with galvanometer-assisted readout, were used throughout the study. Measurements were made over two (109 surveys) or three (32 surveys) closed loops between P1 and the network stations. The time to complete each loop, initally 4 h shortened to 1 h as the number of sites was reduced. Gravity readings were corrected for tidal effects (Longman, 1959) using a compliance factor of 1.16. The effects of instrumental drift and residual tide variations were removed by a least-squares solution of a second-order polynomial approximating time-dependent changes in the reading level of each gravimeter. Linear and sinusoidal gravimeter calibration functions were determined from standard calibration loops and applied to all the Kilauea data (table 47.1).

No attempt was made to correct the gravity data for water-table changes. As P1 and HVO34 are only 5 km apart, gravity variations caused by water-table changes are probably similar at both sites and do not affect the relative gravity differences.

Because data from each gravimeter were analyzed separately, agreement between the two gravimeters is a measure of the precision of the observations. A plot of the measurement discrepancy of the two gravimeters at HVO34 versus time is shown in figure 47.2 for surveys done with three loops (triangles) and with two loops (circles). The standard deviation of the intermeter difference for triple-looped surveys is 9.7 μGal and for double-looped surveys is 11.9 μGal. These figures correspond to standard errors of the mean of 3.4 μGal and 4.2 μGal, respectively.

Elevation changes were determined by optical leveling of a line from P1 to HVO34. The surveys were made in only one direction to third-order standards. East-west tilt variations, measured by a continuously recording, 1-m-base, mercury-capacitance tiltmeter at

TABLE 47.1.—*Linear and sine-form calibration functions for LaCoste and Romberg gravimeters G-615 and G-721 as a function of dial reading*

[Linear factor determined by resurveying 6 stations of calibration line established by Jachens and Eaton (1980) on Mauna Kea Volcano. Sine-form calibration functions determined in June 1984 by measuring 25 stations near Palm Desert, California, and comparing results to values obtained by Robert Jachens (written commun., 1984) with LaCoste and Romberg model-D gravimeters. Sine functions were checked and adjusted slightly by comparing values for G-615 and G-721 obtained at Ocean View Estates, Hawaii, in December 1984. c.u., dial reading in counter units; n.d., not detected]

	Wavelength = 73.33 c.u.		Wavelength = 36.67 c.u.		
Gravimeter	1/2 amp (μGal)	Phase zero (c.u.)	1/2 amp (μGal)	Phase zero (c.u.)	Linear factor
G-615 -----	0.01739	2,924.54	n.d.	n.d.	1.000573
G-721 -----	.01415	2,996.05	.02817	2,928.51	1.000296

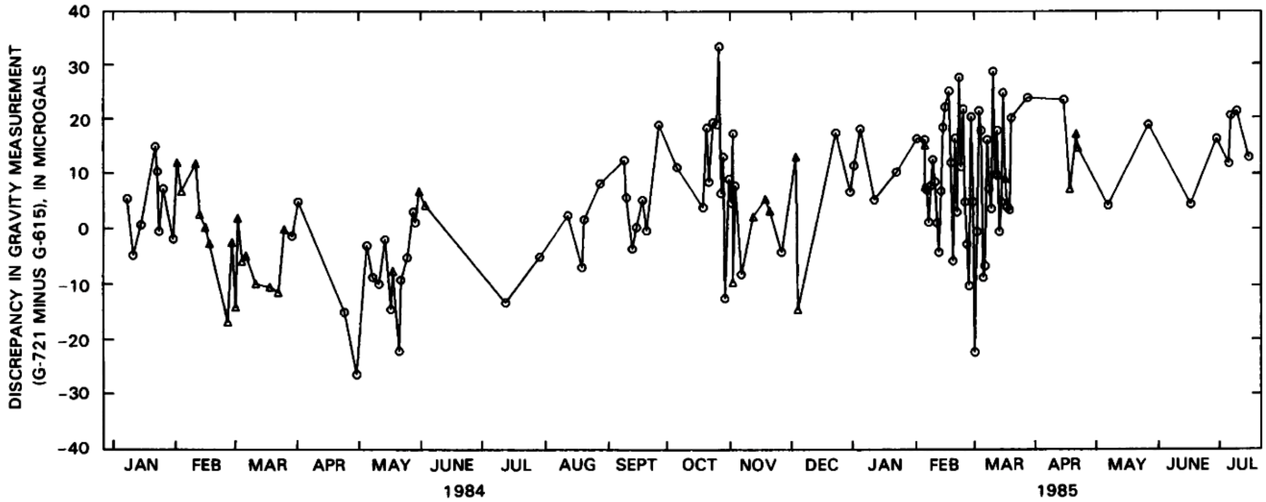

FIGURE 47.2.—Discrepancy between gravity measurements made by gravimeters G–615 and G–721 at site HVO34 plotted as a function of time. Triangles, surveys done with three loops; circles, surveys with two loops.

Uwekahuna (fig. 47.1, UWE), were used to interpolate elevation and edifice-volume changes to values corresponding in time with the gravity surveys. Resurveys of an extensive trilateration network in the summit area of Kilauea in January 1984 and July 1985 provided information on horizontal displacement.

RESULTS

Observed gravity differences (fig. 47.3*C*) and elevation differences (fig. 47.3*A*) between HVO34 and P1 and changes in east-west tilt (fig. 47.3*B*) at Uwekahuna have been plotted as a function of time. Episodes of the eruption at Puu Oo are sequentially numbered (fig. 47*B*). Increasing tilt values correspond to inflation of the volcano. Tilt data indicate that activity was characterized by cycles of gradual inflation followed by brief deflation episodes during eruptive pulses at the Puu Oo vent (Wolfe and others, chapter 17). Considering the uncertainty of the gravity observations (3–4 μGal), no systematic short-term gravity changes can be inferred. Apparently the free-air gravity changes caused by cycles of elevation variations are compensated by gravity changes caused by fluctuations in the mass of magma in the reservoir. Superimposed on these eruptive cycles is a net deflation of the summit indicated by a tilt change of −42 μrad and an elevation change of −200 mm. During the same period, the gravity at HVO34 gradually increased by about 65 μGal. These long-term variations follow a free-air relation, implying that there was no net subsurface mass change.

INTERPRETATION

The gravity data for HVO34 presented in figure 47.3*C* are corrected for earth tides. The effects of anomalous earth tides, ocean tides, and variations in atmospheric pressure are at least partly removed by the drift correction. The remaining processes that affect the gravity values include (1) accumulation or loss of magma from the reservoir, (2) vertical movement of the observation point in the Earth's gravity field (the free-air effect), and (3) density distribution change in the edifice as a result of deformation. These processes are usually coupled at Kilauea. For example, an intrusion of magma will increase gravity because of its mass, but it will decrease gravity because of the surface uplift that it causes. The gravity effect of (3) for a spherically symmetric source of dilatation is zero, as shown by a numerical analysis (Rundle, 1978) and by an analytical treatment (Walsh and Rice, 1979). The gravity effect of (3) for other source geometries, such as vertical and horizontal cracks, are not zero (Savage, 1984). While other geometries (consistent with the geologic structures seen in ancient volcanoes) have been suggested by Dieterich and Decker (1975) and Ryan and others (1983), the point source approach has been used in this report because of its computational convenience. In considering the gravity data presented here, the assumption was made that the gravity effect of (3) was indeed zero.

Because changes in the magma content of the subsurface reservoir are partly accommodated by slight elastic compression of the total volume of reservoir magma, the density of the reservoir magma can change slightly through time. In this analysis, the phrase magma-volume change indicates a change in the magma content of the reservoir and is expressed using units of volume calculated at a constant, arbitrary density of 2.75 g/cm^3. Although the parameter that is actually being measured by gravity is the mass change, conversion to volume change is done here to enable comparison with dense-rock equivalent volumes of lava flows and intrusions reported elsewhere. Another type of subsurface volume change discussed here is changes in the capacity of the reservoir, which refers to changes in the physical dimensions of the reservoir.

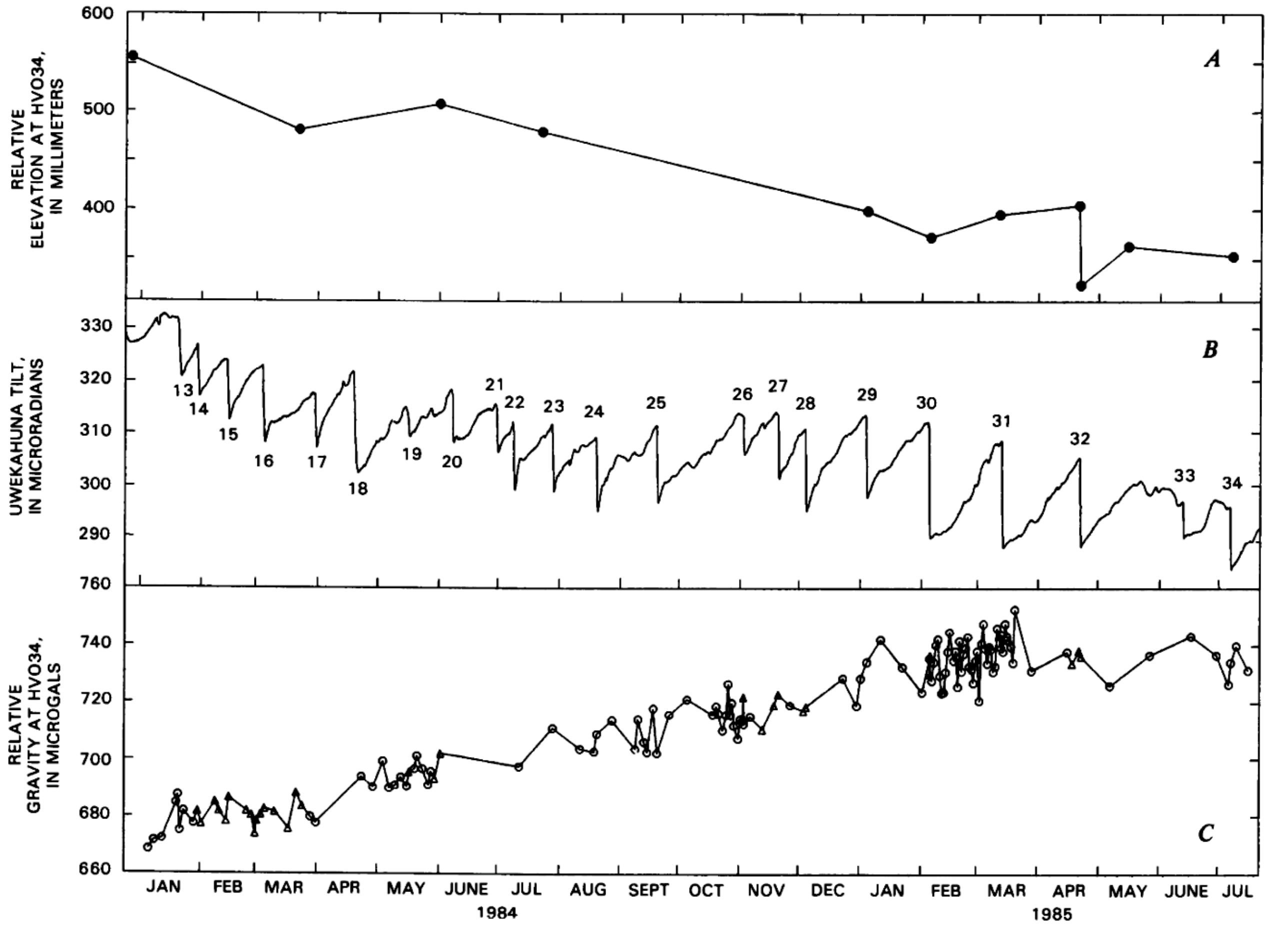

FIGURE 47.3.—Time plots of relative elevation and gravity at HVO34, and east-west tilt at Uwekahuna. ***A***, Relative elevation of site HVO34. ***B***, East-west tilt at Uwekahuna; eruptive episodes at Puu Oo vent indicated by numbers. ***C***, Relative gravity at site HVO34, uncorrected for vertical uplift. Triangles, gravity surveys done with three loops; circles, surveys done with two loops.

RELATION BETWEEN EDIFICE AND SUBSURFACE VOLUME CHANGES

The ratio of the change in magma volume of the reservoir, ΔV_m, to change in volume of the edifice, ΔV_e, may be estimated from gravity changes, $\Delta g'$, derived from the observed gravity differences, Δg, by correcting for vertical displacement with a free-air gradient of -0.3086 μGal/mm, and from elevation differences, Δh. The analysis is based on a point dilatational source in an elastic half-space (see, for example Mogi, 1958). The volume change of the edifice was determined by integrating vertical surface displacement. Uplift at the surface is given (Mogi, 1958) by:

$$\Delta h = \frac{3a^3\Delta P}{4\mu_e} \cdot \frac{Z}{(X^2+Z^2)^{3/2}}, \quad (1)$$

where a is the radius of the source, ΔP is the pressure change, μ_e is Lame's constant for the edifice, Z is depth to the source, and X is horizontal distance to the chamber. Integration of equation 1 over the Earth's surface gives an expression for the volume increase of the edifice, ΔV_e:

$$\Delta V_e = \frac{3\pi a^3 \Delta P}{2\mu_e}. \quad (2)$$

Division of equation 2 by equation 1 and rearrangement yields:

$$\Delta V_e = \frac{2\pi (X^2+Z^2)^{3/2}\Delta h}{Z}. \quad (3)$$

The mass change of the reservoir was determined next. So that it may be compared with volumes of subsidence and of eruptive products, mass change is expressed in terms of an effective magma-volume change, ΔV_m, using a constant density ρ for the magma.

The graviational attraction of the new magma, approximated by a point mass, is given by:

$$\Delta g = \Delta V_m \gamma \rho \frac{Z}{(X^2+Z^2)^{3/2}}, \qquad (4)$$

where γ is the universal gravitational constant (6.67×10^{-11} nm^2/kg^2) and ρ is the standard magma density. If the location of the dilational source in equation 3 is assumed to be the same as the new magma in equation 4, a combination of equations 3 and 4 yields an expression relating the observed gravity and elevation change to volume changes of the edifice and magma within the reservoir:

$$\frac{\Delta V_m}{\Delta V_e} = \frac{\Delta g'}{\Delta h 2\pi\gamma\rho}. \qquad (5)$$

Jachens and Eaton (1980) and Dzurisin and others (1980) found a good spatial correlation between gravity changes and elevation changes at Kilauea; their results are consistent with coincident sources for the gravity changes and the dilatation. This correlation of gravity and elevation changes indicates that in equation 5 the ratio of relative changes at two or more points showing differential changes can be used as validly as the ratio of absolute changes. The present analysis uses relative changes between two points.

Notice that when $\Delta V_m = \Delta V_e$ (that is, when there is no subsurface density change), equation 5 is equivalent to the Bouguer gradient. Furthermore, when $\Delta V_e = 1.5 \Delta V_m$ (that is, no magma compression but dilation of the edifice as predicted by the Mogi model with a Poisson's ratio of 0.25), equation 5 plus the free-air gradient is equivalent to theoretical gravity gradients proposed by Dzurisin and others (1980, equation A-11) and Hagiwara (1977, equation 19). In this analysis, the ratio of magma accumulation in the reservoir to surface uplift is considered to be an independent variable. Equation 5 shows that the gravity-change gradient $\Delta g'/\Delta h$ is near 0.08 μGal/mm for an incompressible magma with a density of 2.75 g/cm^3 accommodated by edifice expansion. The gradient approaches infinity for magma compressed into an unyielding edifice.

Gravity measurements were made near the beginning and end of 15 summit-deflation events corresponding to eruptive episodes 13–17, 24–32, and 34 at Puu Oo. Because of the brevity of each deflation, these gravity changes should not be influenced by long-term changes (for example, south-flank movement or magma migration unrelated to eruption), but should reflect mass and elevation changes related to simple elastic deflation. The changes in relative gravity at HVO34 and in east-west tilt at Uwekahuna during the monitored deflations are summarized in table 47.2. No real gravity changes (Δg) significantly larger than the data uncertainty were measured during these events because the gravity data reflect a composite of free-air gravity increases owing to subsidence and gravity decreases, $\Delta g'$, owing to removal of magma.

Elevation changes of HVO34 between gravity surveys were estimated for the period from December 28, 1983, to July 16, 1985, from the calculated relation between elevation of HVO34 and tilt at Uwekahuna. A factor of 5.19 mm/μrad was derived by a least-squares fit to tilt and elevation data (fig. 47.4). Elevation changes during eruptive episode 32 were measured directly by leveling surveys conducted on April 21 and 22, 1985. Table 47.3 summarizes the estimates of $\Delta g'$ (corrected for vertical displacement with a free-air gradient of -0.3086 μGal/mm), Δh, and the ratio $\Delta V_m/\Delta V_e$ calculated using equation 5. Because virtually no real gravity changes (Δg) were measured, most of the $\Delta g'$ values shown in table 47.3 result from the free-air correction. These calculations show that the volume of magma lost from the reservoir is 1.2 to 4.5 times (average 2.4 ± 0.21, 1 standard error) greater than the volume of subsidence at the surface during the periods of rapid deflation.

SUBSURFACE VOLUME CHANGES DURING DEFLATION EVENTS

The empirical relation between volume changes of magma within the summit reservoir and volume changes of the volcanic edifice can be used to estimate the former from measurements of the latter. This means of estimating magma-volume changes at depth is

TABLE 47.2.—*Nominal relative gravity change at HVO34 and tilt changes at Uwekahuna for deflation intervals corresponding to 15 eruptions at Puu Oo, 1984–85*

[Times given are Hawaii standard time; figures for loops give number of closed loops between HVO34 and P1 for each gravity survey. s.e., standard error]

	First survey				Second survey					
Eruptive episode	Date	Start time	Duration (h)	Loops	Date	Start time	Duration (h)	Loops	Δg (±1 s.e.) (μGal)	Δ tilt (μrad)
13 ----------	Jan. 20, 1984	1445	4.00	2	Jan. 22, 1984[2]	0900	4.75	2	−4 (5.8)	−10.4
14 ----------	Jan. 29, 1984	1200	5.00	2	Jan. 31, 1984[2]	0800	7.50	3	4 (5.1)	−7.3
15 ----------	Feb. 14, 1984[1]	2145	4.25	3	Feb. 16, 1984	0845	6.25	3	8 (4.4)	−8.9
16 ----------	Mar. 3, 1984	0815	6.00	3	Mar. 5, 1984	0745	6.50	3	2 (4.4)	−14.1
17 ----------	Mar. 28, 1984	1230	3.75	2	Mar. 31, 1984	1245	3.75	2	−2 (5.8)	−9.5
24 ----------	Aug. 19, 1984	1130	2.00	2	Aug. 20, 1984	1845	2.00	2	6 (5.8)	−14.1
25 ----------	Sept. 18, 1984	1000	1.50	2	Sept. 20, 1984	0645	1.75	2	−16 (5.8)	−14.4
26 ----------	Nov. 2, 1984	0845	1.75	2	Nov. 3, 1984	0500	2.00	2	−1 (5.8)	−7.5
27 ----------	Nov. 18, 1984	1045	3.00	3	Nov. 20, 1984	1415	2.00	3	4 (4.4)	−12.5
28 ----------	Dec. 3, 1984	0815	2.75	3	Dec. 4, 1984	1315	3.00	3	2 (4.4)	−15.6
29 ----------	Jan. 1, 1985	0645	1.50	2	Jan. 4, 1985	0730	1.50	2	6 (5.8)	−15.2
30 ----------	Feb. 2, 1985	0645	1.75	2	Feb. 5, 1985	0400	2.00	2	12 (5.8)	−21.5
31 ----------	Mar. 12, 1985	0930	2.00	3	Mar. 14, 1985	0615	1.50	2	6 (5.1)	−19.9
32 ----------	Apr. 21, 1985	1515	2.25	3	Apr. 22, 1985	1445	4.25	6	−2 (3.8)	−16.9
34 ----------	July 6, 1985	1130	1.25	2	July 7, 1985	1245	1.25	2	8 (5.8)	−12.2

[1]First gravity survey done shortly after onset of deflation.

[2]Second gravity survey done shortly before deflation was complete.

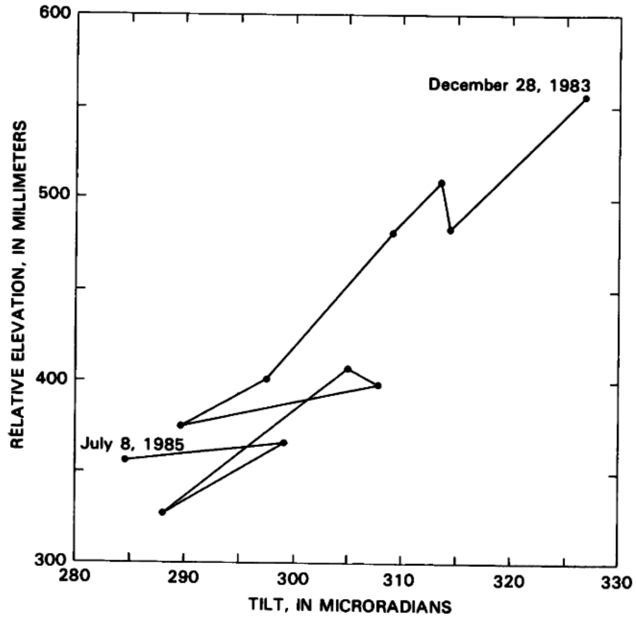

FIGURE 47.4.—Relative elevation of site HVO34 plotted against east-west tilt measured at Uwekahuna between December 28, 1983, and July 8, 1985.

particularly useful for studying intrusions, in which no lava reaches the surface.

The change in volume of the edifice (ΔV_e) was computed by fitting a point-source model (Dvorak and others, 1983) to measured elevation changes for periods when sufficient elevation change and a uniform pattern of surface displacement enabled a stable solution. These periods were: December 28, 1983, to March 22, 1984; July 23, 1984, to January 4, 1985; and April 21 to 22, 1985. The solutions obtained are listed in table 47.4.

Edifice-volume changes (table 47.4) were interpolated to the time of the gravity surveys by means of the continuous record of tilt change at Uwekahuna. From the estimated volume changes of the edifice and the calculated ratios of subsurface-magma to edifice volume change given in table 47.3, the changes of magma volume in the reservoir were calculated. These volume changes are listed in table 47.5, together with estimated lava-flow volumes from the Puu Oo vent for comparison (Wolfe and others, chapter 17; George Ulrich, written commun., 1985).

Absence of significant geodetic change along the rift zones during 1984 and early 1985 indicates that the cumulative volume of lava erupted was equal to the volume of magma drained from the summit reservoir. This inference is supported by the gravity and surface-displacement data. A total of 125×10^6 m^3 of lava was extruded during the monitored eruptive episodes (table 47.5). Corrected for 20 percent (Swanson, 1972) to 25 percent (Wolfe and others, chapter 17) vesicularity, the total volume of lava flows is 94×10^6–100×10^6 m^3. For comparison, the cumulative volume of reservoir magma lost during the deflations, based on gravity and geodetic measurements, is an estimated 120×10^6 m^3 (table 47.4).

Gravity and elevation data for eruptive episode 32 were obtained over a timespan of 24 h bracketing the deflation. Because independent calculations of the volume change of the edifice and of the subsurface magma were made from these data, without requiring tiltmeter interpolation, the results for this period provide a good check of the more general procedure. While the edifice subsided by about 6×10^6 m^3 during eruptive episode 32, nearly 18×10^6 m^3 (calculated to 2.75 g/cm^3) of magma was removed from the reservoir (table 47.5). This agrees well with the estimated 12×10^6–13×10^6 m^3 (table 47.5, corrected for 20–25 percent vesicularity) of lava erupted at Puu Oo.

The general agreement between reservoir magma-volume change and lava output supports the idea of a volume-for-volume link between the summit reservoir and Puu Oo, with magma transfer from summit to rift zone taking place principally during episodes of summit deflation. In other words, during eruptions at Puu Oo, summit magma enters the rift-zone conduit system and forces out an equal volume of rift-zone magma at the Puu Oo vent.

ELASTIC PROPERTIES OF VOLCANO AND MELT

Data for the ratio of edifice-volume to subsurface magma-volume changes for summit deflations accompanying eruptions of Puu Oo may be used to model the relative strength of the volcanic edifice and the magma-reservoir region. For this purpose, I have derived an expression that relates the ratio of subsurface-magma and edifice-volume changes to the ratio of the rigidity of the edifice, μ_e, and the compressibility of the reservoir melt, K_m. Assumptions are that Poisson's ratio for the edifice is 0.25, that the source region can be approximated by a spherical volume of fluid, that the injected magma has a density of 2.75 g/cm^3, and that the response of the volcanic edifice and contained fluid to subsurface pressure changes is elastic. Although the source region most likely contains subsolidus rock in addition to a melt phase, the aggregate bulk modulus of the

TABLE 47.3.—*Changes in residual gravity ($\Delta g'$) and elevation (Δh), ratio between them, and ratio of subsurface (ΔV_m) to edifice volume (ΔV_e) changes for 15 Kilauea deflation episodes, 1984–85*

[Elevation changes interpolated from tilt changes using factor of 5.19 mm/μrad. Subsidence during eruptive episode 32 was determined by leveling surveys before and after period of deflation]

Eruptive episode	$\Delta g'$ (μGal)	Δh (mm)	$\Delta g'/\Delta h$ (μGal/mm)	$\Delta V_m/\Delta V_e$
13	−21	−54	0.38	3.3
14	−8	−38	.20	1.8
15	−6	−46	.13	1.2
16	−20	−73	.28	2.4
17	−17	−49	.35	3.0
24	−16	−73	.23	2.0
25	−39	−75	.52	4.5
26	−13	−39	.33	2.9
27	−16	−65	.25	2.1
28	−23	−81	.28	2.5
29	−18	−79	.23	2.0
30	−22	−111	.20	1.7
31	−26	−103	.25	2.2
32	−27	−80	.33	2.9
34	−11	−63	.18	1.6
Average				2.4

TABLE 47.4.—*Point-source solutions to level data for selected time intervals in Kilauea summit-area deflation events*

[Uncertainties given are 1 standard error]

Time interval	Longitude (W.)	Latitude (N.)	Depth (km)	Edifice volume change (10^6 m^3)	Vertical movement of HVO23 (mm)
Dec. 28, 1983–Mar. 22, 1984	155.2848°(±0.06 km)	19.4003°(±0.13 km)	2.92(±0.19)	−4.42(±0.70)	−9(±2)
July 23, 1984–Jan. 4, 1985	155.2801°(±0.58 km)	19.4014°(±0.62 km)	2.46(±0.78)	−3.26(±0.13)	−0.5(±2)
Apr. 21, 1985–Apr. 22, 1985	155.2753°(±0.30 km)	19.4101°(±0.11 km)	2.87(±0.37)	−6.09(±2.04)	−15(±7)
Dec. 28, 1983–July 8, 1985	155.2822°(±0.13 km)	19.3960°(±0.26 km)	3.06(±0.26)	−13.68(±3.38)	−24(±6)

source region is assumed equal to the bulk modulus of the melt phase. This assumption is valid because the effect of the melt phase is to reduce the aggregate bulk modulus to close to that of melt (Ryan, 1980). First, the change in capacity of the spherical source region was determined. A pressure change within the reservoir displaces the reservoir boundary radially outward. The change in radius, Δa, of a spherical source region of radius a is given by (Hagiwara, 1977):

$$\Delta a = \frac{a\Delta P}{4\mu_e}. \tag{6}$$

If the radius of the source is large relative to the change in radius, the volume of the the newly added shell, ΔV_r, is approximately:

$$\Delta V_r = \frac{\pi a^3 \Delta P}{\mu_e}. \tag{7}$$

The volume of magma, ΔV_c, accommodated by compression of the fluid within the spherical reservoir region of volume $4\pi a^3/3$ and within the newly added shell of magma, ΔV_r, is dependent on the bulk modulus of the magma, K_m:

$$\Delta V_c = \frac{\Delta P}{K_m}\left(\frac{4\pi a^3}{3} + \Delta V_r\right). \tag{8}$$

For simplicity, the relatively small term ΔV_r in equation 8 can be ignored. The total amount of magma injected is the sum of magma accommodated by enlargement of the reservoir (equation 7) and by compression of the material in the reservoir (equation 8):

$$\Delta V_m = \frac{4}{3}\pi a^3 \Delta P\left(\frac{1}{K_m} + \frac{3}{4\mu_e}\right). \tag{9}$$

The combination of equations 9 and 2 gives:

$$\frac{\Delta V_m}{\Delta V_e} = \frac{8}{9}\cdot\frac{\mu_e}{K_m} + \frac{2}{3}, \tag{10}$$

which can be related directly to the gravity gradient by equation 5.

Some edifice deformation and magma compression are expected during magma injection. Equation 10 shows that, for a given value of $\Delta V_m/\Delta V_e$, a relatively strong edifice implies more magma compression. Conversely, a relatively strong magma implies more edifice deformation. Solution of equation 10, incorporating the average $\Delta V_m/\Delta V_e$ factor of 2.4±0.21 (1 standard error) from the 15 deflations listed in table 47.3, yields $\mu_e = 2.0 \pm 0.24$ K_m. Additional error stems from uncertainty of the magma density, here assumed to be 2.75 g/cm^3.

The effective shear modulus of Kilauea's edifice, μ_e, is difficult to determine independently because Kilauea is built of many vesicular, commonly rubbly lava flows with abundant void space (Ryan and others, 1983). The intrinsic shear modulus of Hawaiian basalt was found by pulse transmission to average 25 GPa (Manghnani and Wollard, 1968) with a range of 4.7–40.5 GPa. Samples with greater porosity had a lower shear modulus. Presence of voids in the volcanic pile results in an effective shear modulus of the volcano that is less than the intrinsic shear modulus of hand samples.

Seismic velocities, and hence rigidity, determined from *P*-wave arrival data from earthquakes and explosions increase with depth and vary laterally (Thurber, chapter 38; Hill and Zucca, chapter 37). This reflects partly the compressive reduction of porosity with increasing depth, and the solidified intrusive zones surrounding the cores of the summit reservoir and rift zones. Any estimate of μ_e from seismic velocities would be strongly dependent on the depth and horizontal position of the source, and size of the region chosen to represent the edifice.

The bulk modulus of magma, K_m, in Kilauea's reservoir should be uniform because the size of the summit magma reservoir is perhaps 1,000 times the yearly magma supply rate (Swanson, 1972), so short-term fluctuations in the gas content and chemical composition of the supplied magma are distributed over a large volume and do not affect the overall bulk modulus. Laboratory

TABLE 47.5.—*Edifice volume change* (ΔV_e), *subsurface volume change* (ΔV_m), *and lava flow volume for 12 Kilauea eruptive episodes, 1984–85*

[All figures in millions of cubic meters. Edifice volume changes, from level data inversion, interpolated to time of gravity surveys by tilt variations. Subsurface mass changes, expressed as volume change at standard density of 2.75 g/cm^3, estimated using equation 5 and changes of edifice volume, relative gravity, and relative elevation. Measured lava-flow volumes (Wolfe and others, chaper 17; George Ulrich, written commun., 1985) not adjusted for flow porosity]

Eruptive episode	ΔV_e	ΔV_m	*Lava volume*
13	−3.7	−12.3	10
14	−2.6	−4.7	6
15	−3.2	−3.7	8
16	−5.1	−12.3	12
17	−3.4	−10.2	10
24	−4.0	−7.9	12
25	−4.0	−18.2	11
26	−2.1	−6.1	7
27	−3.5	−7.3	8
28	−4.4	−11.0	12
29	−4.3	−8.6	13
32	−6.1	−17.7	16
Total	−46.4	−120.0	125

determinations of the bulk modulus made from lava erupted from Kilauea, therefore, should be representative of the bulk modulus of magma in the reservoir. The value of 11.5 GPa determined for K_m of molten Kilauea 1921 olivine tholeiite by Murase and others (1977) was used to calculate the shear modulus of Kilauea's edifice from the relation $\mu_e=(2.0\pm0.24)\cdot K_m$ determined above. The value for μ_e so determined, 23 ± 3 GPa, is close to the value for the average shear modulus of Hawaiian basalt given above. Note that this value does not apply to Kilauea's edifice as a whole, but only to the region surrounding the magma reservoir. This is because it was determined by modeling in-place melt-volume changes in the reservoir and resulting surface displacements.

Presence of a CO_2 gas phase in the magma would lower the aggregate bulk modulus of the magma. This may not be important for Kilauea, because, as Greenland and others (1985) assert, as CO_2 is lost from the magma as soon as it reaches the shallow magma reservoir. However, if some CO_2 is retained in the reservoir, then both the magma bulk modulus and the predicted edifice shear modulus would be reduced.

RELATION BETWEEN LONG-TERM INELASTIC EDIFICE WIDENING AND SUBSURFACE VOLUME CHANGES

The ratio of the long-term change of magma volume within Kilauea's summit reservoir to the edifice volume change between January 6, 1984, and July 16, 1985, may be determined from equation 5 and from the long-term elevation and gravity changes. A gradient of $\Delta g/\Delta h=-0.29$ µGal/mm was determined for the interval by a least-squares fit to the tilt and gravity data, adjusted by the elevation/tilt correlation of 5.19 mm/µrad given above. This is essentially the free-air gradient, so $\Delta g'$ is zero and equation 5 shows that the ratio of $\Delta V_m/\Delta V_e$ is zero. That is, there was no net change in magma storage in the summit reservoir during this interval.

The long-term ratio of volume changes in edifice and subsurface magma is different from that in short-term deflations accompanying Puu Oo eruptions (table 47.3). This implies that different processes affect the volume changes over longer time intervals. Earlier, the short-term deflations of Kilauea were explained using an elastic model, with inflow and outflow of magma accompanied by compression and decompression of the melt. Because short-term and long-term processes occurred concurrently, variations in the physical properties of the magma cannot explain differences in the volume-change ratios. The differences between the long-term and short-term volume changes must then be due to some superimposed long-term process that affects Kilauea's edifice. That process produces subsidence at the surface without a change in the magma content of the reservoir. One possibility is that horizontal spreading of the volcano's summit removes lateral support of the fluid magma reservoir, thereby decreasing the pressure in the reservoir and causing the summit to subside. Horizontal distance measurements at Kilauea have shown that the summit region is prone to widening in a south-southeast direction (Dvorak and others, 1983). This widening was particularly notable during the 1975 magnitude 7.2 earthquake along the south flank and for several months afterward (Lipman and others, 1985).

To investigate the possibility of summit extension during 1984–85, horizontal displacements measured by trilateration surveys conducted on January 16, 1984, and July 16, 1985, were analysed. In order to isolate the pattern of regional displacements in the vicinity of Kilauea's summit from the effects of observed deflation, the contribution of changes in the inflation level of the reservoir (approximated by use of a point source of dilatation determined from leveling data) was subtracted from the observed horizontal displacements. Residual line lengths were then used to compute horizontal displacement vectors (fig. 47.5). South-southeastward movement as great as 250 mm of points located on the south side of Kilauea caldera relative to points to the north show that the caldera widened slightly.

A $\Delta g'/\Delta h$ gradient of 0.138 µGal/mm was measured by Jachens and Eaton (1980) during the subsidence accompanying the November 29, 1975, earthquake and eruption. A value for the $\Delta V_m/V_e$ ratio of 1.2 was calculated from the gravity gradient and equation 5 using an assumed magma density of 2.75 g/cm^3. This ratio indicates a greater component of surface collapse following the 1975 eruption, compared to the 1984–85 deflations (table 47.3). This difference results from 1.8 m of south-southeast crosscaldera extension during the November 1975 event (Lipman and others, 1985), in contrast to no measurable extension during the periods of very brief (1–3 d) deflation associated with eruptions in 1984 and 1985. The increased collapse in November 1975 is consistent with horizontal dilation of the edifice and surface subsidence because of the resultant drop in reservoir pressure.

Horizontal extension of Kilauea's edifice lowers the pressure in the magma reservoir; additional magma can be injected without uplifting the surface. In this way the reservoir volume increases by lateral growth. Growth by horizontal spreading and magma intrusion occurs during rift-zone development (Swanson and others, 1976), and it might also apply to the growth of the summit reservoir. Swanson and others (1976) attributed horizontal dilation of the rift zone to forceful injection of magma into dikes. However, at Kilauea's summit area, observation of subsidence concurrent with extension indicates that magma accumulation there is a passive response to dilation. Perhaps the extension of Kilauea's summit that increases the capacity of the reservoir is an indirect response to the forceful dilation of the rift zones that bound the caldera on both the east and west sides.

Net gravity and elevation changes for the 16-mo period following the November 1975 deflation imply mass filling of the reservoir concurrent with subsidence. Dzurisin and others (1980) reported a $\Delta g'/\Delta h$ gradient of -0.294 µGal/mm for December 1975 to April 1977, which corresponds to a $\Delta V_m/\Delta V_e$ ratio of -2.5 from equation 5, using a magma density of 2.75 g/cm^3. This, combined with observed subsidence of the summit area of as much as 180 mm, indicates that the volume of magma in the reservoir actually increased despite the subsidence. The summit region widened by about 0.5 m in a south-southeast direction (Lipman and others, 1985) during this period; that extension created more reservoir space in which additional magma accumulated.

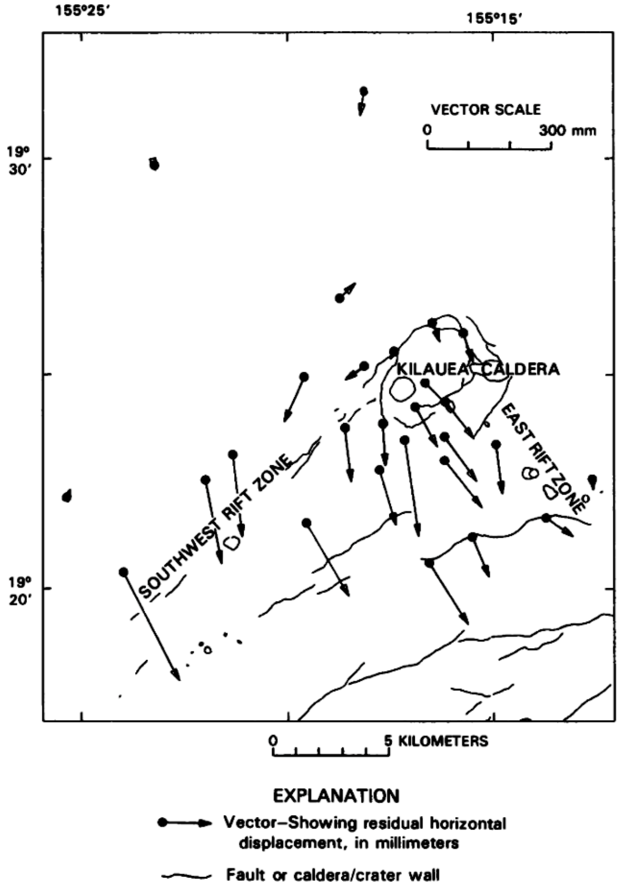

FIGURE 47.5.—Horizontal displacements in the Kilauea summit region determined from line-length changes (January 16, 1984, to July 16, 1985). To remove effects of summit subsidence, computed horizontal displacements based on inversion of level data obtained on December 28, 1983, and July 8, 1985 (table 47.4), were subtracted from measured changes in line lengths between January 16, 1984, and July 16, 1985. Computed residual displacements indicate south-southeastward widening of Kilauea summit area.

CONCLUSIONS

Gravity and surface-displacement data demonstrate that volume changes of Kilauea's edifice, as determined by integrating vertical displacements, are dependent, not only on the balance between volume of magma supplied from the mantle and volume of magma lost from the reservoir by eruption but also on the volume of summit collapse owing to horizontal extension of the summit region. During times of elastic inflow or outflow of magma from the reservoir, the ratio of subsurface-magma volume change to edifice volume change is 2.4 ± 0.21 (1 standard error). This is consistent with expected changes, calculated using a ratio of shear strength of edifice to bulk modulus of magma of 2.0 ± 0.24 (1 standard error). On the basis of a laboratory-determined magma bulk modulus of $K_m = 11.5$ GPa, the shear modulus of the edifice is $\mu_e = 23$ GPa. Inelastic horizontal dilation of the edifice in a south-southeast direction increases the capacity of the summit reservoir to accept magma without surface uplift. For the 16-mo period following the November 1975 deflation, additional magma was accommodated by this process. When insufficient magma is supplied to fill space created by summit widening, the pressure in the reservoir drops and the surface subsides as it did during and after the November 1975 earthquake and between January 1984 and February 1985.

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