ELASTIC AND INELASTIC MAGMA STORAGE AT KILAUEA VOLCANO

By Daniel J. Johnson

ABSTRACT

Gravity, leveling, and trilateration data recently obtained at Kilauea Volcano indicate that magma is accommodated within the summit reservoir by a combination of elastic magma compression, elastic chamber expansion, and gradual inelastic edifice widening in a south-southeast direction.

Elastic (reversible) magma storage is modeled using data from 15 brief summit-deflation episodes in 1984-85 and assuming a spherical geometry for the magma reservoir. Changes in pressure in the reservoir produce the deformation; changes in mass in the reservoir produce the gravity changes. The ratio of residual gravity change (corrected for the free-air effect) to elevation change, measured near the apex of maximum subsidence, averaged 0.28 ± 0.025 µGal/mm. If magma density is assumed to be 2.75 g/cm³, the volume of magma withdrawn from the reservoir averaged 2.4 ± 0.21 times the associated volume of subsidence. This volume difference is dependent on both the magma bulk modulus (which constrains bulk compression of the reservoir contents) and the edifice shear modulus (which constrains distortion of the surrounding edifice). If deformation is assumed to be from a point dilatational source in an elastic half-space, the observed volume difference implies that the shear modulus of the edifice is 2.0 ± 0.24 times the bulk modulus of the magma.

Trilateration data show that Kilauea's edifice widens inelastically in time intervals of months to years, primarily in a northwest direction. Over these long time periods, gravity and elevation changes indicate both surface subsidence and mass addition. These observations imply that horizontal extension of Kilauea's edifice decreases lateral support of the fluid magma reservoir, thus lowering the pressure in the reservoir and allowing more magma to be injected without vertical uplift.

Increased summit collapse occurred during the November 1975 earthquake, when new reservoir space created by inelastic seaward displacement of the volcano's south flank was not immediately filled by new magma from depth. For 16 months after the earthquake, additional reservoir capacity created by continued edifice dilation was filled with new magma. From January 1984 to July 1985, Kilauea's edifice subsided by 200 mm and widened by as much as 250 mm. No net subsurface mass changes were measured during this period, implying that the subsidence was entirely the result of horizontal extension.

A complete magma-budget estimate for Kilauea must therefore consider both elastic and inelastic magma storage. This requires a synthesis of leveling, trilateration, and gravity data.

INTRODUCTION

Kilauea Volcano grows by the accumulation of mantle-derived magma that is initially stored in a shallow summit magma reservoir (Eaton and Murata, 1960). The location of this storage region, indicated by an aseismic zone (Koyanagi and others, 1976) and by the inversion of measured surface displacements (Fiske and Kinoshita, 1969; Dvorak and others, 1983), is in a zone below the south rim of the caldera about 3 km in diameter and from 2 to 6 km below the surface. Slow uplift of the volcano's summit over several weeks to a few months is caused by the gradual addition of new magma from a deep source region. Occasional rapid surface subsidence, over periods of hours to days, occurs as magma is withdrawn from the reservoir, either migrating upward or laterally through a conduit system into one of two rift zones (Ryan and others, 1981). Though a part of the mantle-derived magma supply may eventually reach the surface, some magma remains in the summit reservoir and rift systems (Dzurisin and others, 1984).

Previous estimates of magma-volume changes in the subsurface reservoir have been based on geodetic measurements (Duffield and others, 1982; Dvorak and others, 1983; Dzurisin and others, 1984). Because the elastic properties of both magma and volcanic edifice are poorly known, subsurface volume changes cannot be determined from geodetic data alone. Rather, the volume change of the edifice, as determined by integrating vertical displacements measured at the surface, is typically used to approximate the subsurface volume change.

Subsurface volume changes can be estimated more accurately if measurements of changes in both gravity and elevation are available. Gravity and elevation measurements obtained at Kilauea by Jachens and Eaton (1980) and Dzurisin and others (1980) demonstrated that between November 1975 and September 1977 changes in the volume of magma in the reservoir exceeded changes in the volume of the edifice. This was attributed to the effect of void spaces created during periods of magma withdrawal (Jachens and Eaton, 1980), later refilled during periods of magma accumulation (Dzurisin and others, 1980). Gravity studies by Johnsen and others (1980) at Kilauea, Iceland, by Sanderson and others (1983) at Mount Etna, Italy, and by Eggers (1983) at Pacaya Volcano, Guatemala, also indicate differences between edifice and subsurface volume changes. Sanderson and others (1983) explained the differences by bulk compression of the magmatic fluid and changes in volume of exsolved gas enabling subsurface magma accumulation to exceed the volume of edifice expansion. Eggers (1983) proposed density changes of about 0.4 g/cm³ in shallow magma bodies caused by changes in the magmatic water content and vesicularity.

A series of gravity observations made in the Kilauea summit region between January 6, 1984, and July 8, 1985, have been
analyzed to determine the empirical relation between magma-volume changes in the summit reservoir and edifice-volume changes as evidenced by surface uplift or subsidence. The short-term relation observed during brief episodes of elastic deflation corresponding to eruptions at the Puu Oo vent along Kilauea's middle east rift zone (Wolf and others, chapter 17) has been used to estimate the relative elastic properties of the magma reservoir region and the surrounding volcanic edifice. The long-term relation has been used to evaluate the effect of inelastic summit widening on volume changes of the edifice and magma content of the summit reservoir. Results of this study will help improve estimates of the rate of magma supply to Kilauea.

ACKNOWLEDGMENTS

This study is part of a doctoral dissertation in progress at the Department of Geology and Geophysics of the University of Hawaii. The Hawaiian Volcano Observatory of the U.S. Geological Survey provided logistical support and funds for field expenses. I thank Eduard Berg, Robert Decker, Roger Denlinger, John Dvorak, Daniel Dzurisin, Albert Eggers, Robert Jackens, and Thomas Wright for helpful ideas, suggestions, and assistance given during all phases of this study. I am indebted to Thomas Wright for his continued interest and kind support. He showed this in an exceptional manner by personally doing most of the gravity surveys made after July 1984. This manuscript has benefited from reviews by Edward Brown, John Dvorak, Daniel Dzurisin, Harry Clinken, Robert Jackens, and Michael Ryan.

DATA

A total of 141 gravity surveys were conducted in the Kilauea summit region between January 6, 1984, and July 16, 1985. Initially, measurements were made at five monitoring sites referenced to a base station (P1) 5 km northwest of the point of maximum vertical movement (from earlier leveling surveys). The monitoring network was reduced to two sites after February 16, 1984, and to one site, HVO34, after July 12, 1984, because data from this site were found to be representative of the others. Only gravity data for HVO34, referenced to P1, are presented in this paper; locations of all measured stations are shown in figure 47.1.

Two LaCoste and Romberg model-G gravimeters, each equipped with galvanometer-assisted readout, were used throughout the study. Measurements were made over two (109 surveys) or three (32 surveys) closed loops between P1 and the network stations. The time to complete each loop, initially 4 h shortened to 1 h as the number of sites was reduced. Gravity readings were corrected for tidal effects (Longman, 1959) using a compliance factor of 1.16. The effects of instrumental drift and residual tide variations were removed by a least-squares solution of a second-order polynomial approximating time-dependent changes in the reading level of each gravimeter. Linear and sinusoidal gravimeter calibration functions were determined from standard calibration loops and applied to all the Kilauea data (table 47.1).

No attempt was made to correct the gravity data for water-table changes. As P1 and HVO34 are only 5 km apart, gravity variations caused by water-table changes are probably similar at both sites and do not affect the relative gravity differences.

Because data from each gravimeter were analyzed separately, agreement between the two gravimeters is a measure of the precision of the observations. A plot of the measurement discrepancy of the two gravimeters at HVO34 versus time is shown in figure 47.2 for surveys done with three loops (triangles) and with two loops (circles). The standard deviation of the intermeter difference for triple-looped surveys is 9.7 μGal and for double-looped surveys is 11.9 μGal. These figures correspond to standard errors of the mean of 3.4 μGal and 4.2 μGal, respectively.

Elevation changes were determined by optical leveling of a line from P1 to HVO34. The surveys were made in only one direction to third-order standards. East-west tilt variations, measured by a continuously recording, 1-m-base, mercury-capacitance tiltmeter at

Table 47.1.—Linear and sine-form calibration functions for LaCoste and Romberg gravimeters G-615 and G-721 as a function of dial reading

<table>
<thead>
<tr>
<th>Gravimeter</th>
<th>Wavelength = 73.33 c.u.</th>
<th>Wavelength = 56.67 c.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/2 amp (μGal)</td>
<td>Phase zero (c.u.)</td>
</tr>
<tr>
<td>G-615</td>
<td>0.01739</td>
<td>2,924.54</td>
</tr>
<tr>
<td>G-721</td>
<td>0.01415</td>
<td>2,996.05</td>
</tr>
</tbody>
</table>
Fig. 47.5.—Discrepancy between gravity measurements made by gravimeters G–615 and G–721 at site HVO34 plotted as a function of time. Triangles, surveys done with three loops; circles, surveys with two loops.

Uwekahuna (fig. 47.1, UWE), were used to interpolate elevation and edifice-volume changes to values corresponding in time with the gravity surveys. Resurveys of an extensive trilateration network in the summit area of Kilauea in January 1984 and July 1985 provided information on horizontal displacement.

RESULTS

Observed gravity differences (fig. 47.3C) and elevation differences (fig. 47.3A) between HVO34 and P1 and changes in east-west tilt (fig. 47.3B) at Uwekahuna have been plotted as a function of time. Episodes of the eruption at Puu Oo are sequentially numbered. Inflation of the volcano. Tilt data indicate that activity was characterized by cycles of gradual inflation followed by brief deflation episodes during eruptive pulses at the Puu Oo vent (Wolfe and others, chapter 17). Considering the uncertainty of the gravity observations (3–4 μGal), no systematic short-term gravity changes can be inferred. Apparently the free-air gravity changes caused by cycles of elevation variations are compensated by gravity changes caused by fluctuations in the mass of magma in the reservoir. Superimposed on these eruptive cycles is a net deflation of the summit indicated by a tilt change of −42 μrad and an elevation change of −200 mm. During the same period, the gravity at HVO34 gradually increased by about 65 μGal. These long-term variations follow a free-air relation, implying that there was no net subsurface mass change.

INTERPRETATION

The gravity data for HVO34 presented in figure 47.3C are corrected for earth tides. The effects of anomalous earth tides, ocean tides, and variations in atmospheric pressure are at least partly removed by the drift correction. The remaining processes that affect the gravity values include (1) accumulation or loss of magma from the reservoir, (2) vertical movement of the observation point in the Earth's gravity field (the free-air effect), and (3) density distribution change in the edifice as a result of deformation. These processes are usually coupled at Kilauea. For example, an intrusion of magma will increase gravity because of its mass, but it will decrease gravity because of the surface uplift that it causes. The gravity effect of (3) for a spherically symmetric source of dilatation is zero, as shown by a numerical analysis (Rundle, 1978) and by an analytical treatment (Walsh and Rice, 1979). The gravity effect of (3) for other source geometries, such as vertical and horizontal cracks, are not zero (Savage, 1984). While other geometries (consistent with the geologic structures seen in ancient volcanoes) have been suggested by Dieterich and Decker (1975) and Ryan and others (1983), it is possible that the point source approach has been used in this report because of its computational convenience. In considering the gravity data presented here, the assumption was made that the gravity effect of (3) was indeed zero.

Because changes in the magma content of the subsurface reservoir are partly accommodated by slight elastic compression of the total volume of reservoir magma, the density of the reservoir magma can change slightly through time. In this analysis, the phrase magma-volume change indicates a change in the magma content of the reservoir and is expressed using units of volume calculated at a constant, arbitrary density of 2.75 g/cm³. Although the parameter that is actually being measured by gravity is the mass change, conversion to volume change is done here to enable comparison with dense-rock equivalent volumes of lava flows and intrusions reported elsewhere. Another type of subsurface volume change discussed here is changes in the capacity of the reservoir, which refers to changes in the physical dimensions of the reservoir.
RELATION BETWEEN EDIFICE AND SUBSURFACE VOLUME CHANGES

The ratio of the change in magma volume of the reservoir, ∆Vₑ, to change in volume of the edifice, ∆Vₑ, may be estimated from gravity changes, ∆g', derived from the observed gravity differences, ∆g, by correcting for vertical displacement with a free-air gradient of −0.3086 μGal/nm, and from elevation differences, ∆h. The analysis is based on a point dilatational source in an elastic half-space (see, for example Mogi, 1958). The volume change of the edifice was determined by integrating vertical surface displacement. Uplift at the surface is given (Mogi, 1958) by:

$$\Delta h = \frac{3\pi a^3 \Delta P}{4\mu_e} \frac{Z}{(X^2 + Z^2)^{3/2}}$$  \hspace{1cm} (1)

where a is the radius of the source, ∆P is the pressure change, \(\mu_e\) is Lamé's constant for the edifice, Z is depth to the source, and X is horizontal distance to the chamber. Integration of equation 1 over the Earth's surface gives an expression for the volume increase of the edifice, ∆Vₑ:

$$\Delta V_e = \frac{3\pi a^3 \Delta P}{2\mu_e}.$$  \hspace{1cm} (2)

Division of equation 2 by equation 1 and rearrangement yields:

$$\Delta V_e = \frac{2\pi (X^2 + Z^2)^{3/2} \Delta h}{Z}.$$  \hspace{1cm} (3)

The mass change of the reservoir was determined next. So that it may be compared with volumes of subsidence and of eruptive products, mass change is expressed in terms of an effective magma-volume change, ∆Vₑ, using a constant density ρ for the magma.
47. ELASTIC AND INELASTIC MAGMA STORAGE AT KILAUEA VOLCANO

The gravitational attraction of the new magma, approximated by a
point mass, is given by:

\[ \Delta g = \Delta V_m \gamma \rho \frac{Z}{(X^2 + Z^2)^{3/2}} \]  
(4)

where \( \gamma \) is the universal gravitational constant \((6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2})\) and \( \rho \) is the standard magma density. If the location of the
dilational source in equation 3 is assumed to be the same as the new
magma in equation 4, a combination of equations 3 and 4 yields an
expression relating the observed gravity and elevation change to
volume changes of the edifice and magma within the reservoir:

\[ \frac{\Delta V_m}{\Delta V_e} = \frac{\Delta g^*}{\Delta h 2\pi \gamma \rho} \]  
(5)

Jachens and Eaton (1980) and Dzurisin and others (1980) found a
good spatial correlation between gravity changes and elevation
changes at Kilauea; their results are consistent with coincident
sources for the gravity changes and the dilatation. This correlation
of gravity and elevation changes indicates that in equation 5 the ratio
of relative changes at two or more points showing differential changes
can be used as validly as the ratio of absolute changes. The present
analysis uses relative changes between two points.

Notice that when \( \Delta V_m = \Delta V_e \) (that is, when there is no
subsurface density change), equation 5 is equivalent to the Bouguer
gradient. Furthermore, when \( \Delta V_e = 1.5 \Delta V_m \) (that is, no magma
compression but dilution of the edifice as predicted by the Mogi model
with a Poisson's ratio of 0.25), equation 5 plus the free-air gradient
is equivalent to theoretical gravity gradients proposed by Dzurisin
and others (1980, equation A-11) and Hagihara (1977, equation
19). In this analysis, the ratio of magma accumulation in the reservoir
to surface uplift is considered to be an independent variable.
Equation 5 shows that the gravity-change gradient \( \Delta g^*/\Delta h \) is near
0.08 \( \mu \text{Gal} / \text{mm} \) for an incompressible magma with a density of 2.75
\( \text{g/cm}^3 \) accommodated by edifice expansion. The gradient
approaches infinity for magma compressed into an unyielding edifice.

Gravity measurements were made near the beginning and end
of 15 summit-deflation events corresponding to eruptive episodes
13-17, 24-32, and 34 at Puu Oo. Because of the brevity of each
deflation, these gravity changes should not be influenced by long-
term changes (for example, south-flank movement or magma migration
unrelated to eruption), but should reflect mass and elevation
changes related to simple elastic deflation. The changes in relative
gravity at HVO34 and in east-west tilt at Uwekahuna during the
monitored deflations are summarized in table 47.2. No real gravity
changes (\( \Delta g \)) significantly larger than the data uncertainty
were measured during these events because the gravity data reflect a
composite of free-air gravity increases owing to subsidence and
gravity decreases, \( \Delta g^* \), owing to removal of magma.

Elevation changes of HVO34 between gravity surveys were
estimated for the period from December 28, 1983, to July 16,
1985, from the calculated relation between elevation of HVO34
and tilt at Uwekahuna. A factor of 5.19 mm/\mu rad was derived by
a least-squares fit to tilt and elevation data (fig. 47.4). Elevation
changes during eruptive episode 32 were measured directly by
leveling surveys conducted on April 21 and 22, 1985. Table 47.3
summarizes the estimates of \( \Delta g^* \) (corrected for vertical displacement
with a free-air gradient of \(-0.3086 \mu \text{Gal/mm}\), \( \Delta h \), and the ratio
\( \Delta V_m/\Delta V_e \) calculated using equation 5. Because virtually no real
gravity changes (\( \Delta g \)) were measured, most of the \( \Delta g^* \) values shown
in table 47.3 result from the free-air correction. These calculations
show that the volume of magma lost from the reservoir is 1.2 to 4.5
times (average 2.4 ± 0.21, 1 standard error) greater than the volume
of subsidence at the surface during the periods of rapid deflation.

SUBSURFACE VOLUME CHANGES DURING DEFLATION
EVENTS

The empirical relation between volume changes of magma
within the summit reservoir and volume changes of the volcanic
edifice can be used to estimate the former from measurements of
the latter. This means of estimating magma-volume changes at depth is

| Table 47.2.—Nominal relative gravity change at HVO34 and tilt changes at Uwekahuna for deflation intervals corresponding to 15 eruptions at Puu Oo, 1984–85
| [Times given are Hawaiian standard time; figures for loops give number of closed loops between HVO34 and Pl for each gravity survey, s.e., standard error] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Eruptive episode | First survey | | | | | | | | | | | | | |
| | Date | Start time | Duration (h) | Loops | | Date | Start time | Duration (h) | Loops | | |
| 13 | Jan. 20, 1984 | 1445 | 4.00 | 2 | | Jan. 22, 1984 | 0900 | 4.75 | 2 | -4 (5.8) | -10.4 |
| 14 | Jan. 29, 1984 | 1200 | 5.00 | 2 | | Jan. 31, 1984 | 0830 | 7.50 | 3 | 4 (5.1) | -7.3 |
| 16 | Mar. 1, 1984 | 0815 | 6.50 | 3 | | Mar. 5, 1984 | 0745 | 6.50 | 3 | 2 (4.4) | -14.1 |
| 17 | Apr. 28, 1984 | 1250 | 3.75 | 2 | | Mar. 31, 1984 | 1245 | 3.75 | 2 | -5 (5.8) | -9.5 |
| 18 | Aug. 19, 1984 | 1130 | 2.00 | 2 | | Aug. 20, 1984 | 1845 | 2.00 | 2 | 6 (5.8) | -14.1 |
| 19 | Sept. 18, 1984 | 1000 | 1.50 | 2 | | Sept. 20, 1984 | 0645 | 1.75 | 2 | -16 (5.8) | -14.4 |
| 20 | Nov. 2, 1984 | 0845 | 1.75 | 2 | | Nov. 3, 1984 | 0500 | 2.00 | 2 | -1 (5.8) | -7.5 |
| 21 | Nov. 18, 1984 | 0200 | 3.00 | 3 | | Nov. 20, 1984 | 1415 | 2.00 | 3 | 4 (4.4) | -12.5 |
| 22 | Dec. 3, 1984 | 0815 | 2.75 | 3 | | Dec. 4, 1984 | 1315 | 3.00 | 3 | 2 (4.4) | -15.6 |
| 23 | Jan. 1, 1985 | 0645 | 1.50 | 2 | | Jan. 4, 1985 | 0730 | 1.50 | 2 | 6 (5.8) | -15.2 |
| 24 | Feb. 2, 1985 | 0645 | 1.75 | 2 | | Feb. 5, 1985 | 0500 | 2.00 | 2 | 12 (5.8) | -21.5 |
| 25 | Mar. 12, 1985 | 0930 | 2.00 | 3 | | Mar. 14, 1985 | 0615 | 1.50 | 2 | 6 (5.1) | -19.9 |
| 26 | Apr. 21, 1985 | 1515 | 2.25 | 3 | | Apr. 22, 1985 | 1445 | 4.25 | 6 | -8 (5.8) | -16.9 |
| 27 | July 6, 1985 | 1110 | 1.25 | 3 | | July 7, 1985 | 1245 | 1.25 | 2 | 8 (5.8) | -12.2 |

1First gravity survey done shortly after onset of deflation.
2Second gravity survey done shortly before deflation was complete.
Gravity and elevation data for eruptive episode 32 were obtained over a timespan of 24 h bracketing the deflation. Because independent calculations of the volume change of the edifice and of the subsurface magma were made from these data, without requiring tiltmeter interpolation, the results for this period provide a good check of the more general procedure. While the edifice subsided by about $6 \times 10^6$ m$^3$ during eruptive episode 32, nearly $18 \times 10^6$ m$^3$ (calculated to be 2.75 g/cm$^3$) of magma was removed from the reservoir (table 47.5). This agrees well with the estimated $12 \times 10^6$–$13 \times 10^6$ m$^3$ (table 47.5, corrected for 20–25 percent vescularity) of lava erupted at Puu Oo.

The general agreement between reservoir magma-volume change and lava output supports the idea of a volume-for-volume link between the summit reservoir and Puu Oo, with magma transfer from summit to rift zone taking place principally during episodes of summit deflation. In other words, during eruptions at Puu Oo, summit magma enters the rift-zone conduit system and forces out an equal volume of rift-zone magma at the Puu Oo vent.

### Table 47.3

Changes in residual gravity ($\Delta g'$) and elevation ($\Delta h$), ratio between them, and ratio of subsurface ($\Delta V_s$) to edifice volume ($\Delta V_e$) changes for 15 Kiluaea deflation episodes, 1964–85

<table>
<thead>
<tr>
<th>Eruptive episode</th>
<th>$\Delta g'$ (µGal)</th>
<th>$\Delta h$ (mm)</th>
<th>$\Delta g'/\Delta h$ (µGal/mm)</th>
<th>$\Delta V_s/\Delta V_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>$-21$</td>
<td>$-54$</td>
<td>0.38</td>
<td>3.3</td>
</tr>
<tr>
<td>14</td>
<td>$-8$</td>
<td>$-38$</td>
<td>0.2</td>
<td>1.8</td>
</tr>
<tr>
<td>15</td>
<td>$-6$</td>
<td>$-46$</td>
<td>0.13</td>
<td>1.2</td>
</tr>
<tr>
<td>16</td>
<td>$-20$</td>
<td>$-73$</td>
<td>0.28</td>
<td>2.4</td>
</tr>
<tr>
<td>17</td>
<td>$-17$</td>
<td>$-49$</td>
<td>0.35</td>
<td>3.0</td>
</tr>
<tr>
<td>18</td>
<td>$-16$</td>
<td>$-75$</td>
<td>0.23</td>
<td>2.0</td>
</tr>
<tr>
<td>25</td>
<td>$-39$</td>
<td>$-75$</td>
<td>0.52</td>
<td>4.5</td>
</tr>
<tr>
<td>26</td>
<td>$-13$</td>
<td>$-39$</td>
<td>0.33</td>
<td>2.9</td>
</tr>
<tr>
<td>27</td>
<td>$-16$</td>
<td>$-65$</td>
<td>0.25</td>
<td>2.1</td>
</tr>
<tr>
<td>28</td>
<td>$-23$</td>
<td>$-81$</td>
<td>0.28</td>
<td>3.5</td>
</tr>
<tr>
<td>29</td>
<td>$-18$</td>
<td>$-79$</td>
<td>0.23</td>
<td>3.0</td>
</tr>
<tr>
<td>30</td>
<td>$-22$</td>
<td>$-111$</td>
<td>0.20</td>
<td>1.7</td>
</tr>
<tr>
<td>31</td>
<td>$-25$</td>
<td>$-103$</td>
<td>0.25</td>
<td>3.2</td>
</tr>
<tr>
<td>32</td>
<td>$-27$</td>
<td>$-80$</td>
<td>0.33</td>
<td>2.9</td>
</tr>
<tr>
<td>34</td>
<td>$-11$</td>
<td>$-63$</td>
<td>0.18</td>
<td>1.6</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>2.4</strong></td>
</tr>
</tbody>
</table>
Table 47.4.—Point-source solutions to level data for selected time intervals in Kilauea summit-area deflation events

<table>
<thead>
<tr>
<th>Time interval</th>
<th>Longitude (W.)</th>
<th>Latitude (N.)</th>
<th>Depth (km)</th>
<th>Edifice volume change (10^9 m^3)</th>
<th>Vertical movement of HVHZ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 28, 1983–Mar. 22, 1984</td>
<td>155.2848° (±0.06 km)</td>
<td>19.4000° (±0.13 km)</td>
<td>2.92 (±0.19)</td>
<td>-4.42 (±0.70)</td>
<td>-9 (±2)</td>
</tr>
<tr>
<td>July 23, 1984–Jan. 4, 1985</td>
<td>155.2903° (±0.58 km)</td>
<td>19.4804° (±0.62 km)</td>
<td>2.46 (±0.78)</td>
<td>-3.26 (±0.13)</td>
<td>-0.5 (±2)</td>
</tr>
<tr>
<td>Apr. 21, 1985–Apr. 23, 1985</td>
<td>155.2752° (±0.30 km)</td>
<td>19.4101° (±0.11 km)</td>
<td>2.87 (±0.37)</td>
<td>-6.09 (±2.04)</td>
<td>-15 (±7)</td>
</tr>
<tr>
<td>Dec. 28, 1983–July 8, 1985</td>
<td>155.2822° (±0.13 km)</td>
<td>19.3960° (±0.26 km)</td>
<td>3.06 (±0.26)</td>
<td>-13.68 (±3.38)</td>
<td>-24 (±6)</td>
</tr>
</tbody>
</table>

The volume change of magma, \( \Delta V_m \), can be estimated by

\[
\Delta V_m = \frac{4}{3} \pi a^3 \Delta P \left( \frac{1}{K_m} + \frac{3}{4 \mu_v} \right).
\]

(9)

The combination of equations 9 and 2 gives:

\[
\frac{\Delta V_m}{\Delta V_e} = \frac{8}{9} \frac{\mu_v}{K_m} + \frac{2}{3},
\]

(10)

which can be related directly to the gravity gradient by equation 5.

Some edifice deformation and magma compression are expected during magma injection. Equation 10 shows that, for a given value of \( \Delta V_e/\Delta V_m \), a relatively strong edifice implies more magma compression. Conversely, a relatively strong magma implies more edifice deformation. Solution of equation 10, incorporating the average \( \Delta V_m/\Delta V_e \) factor of 2.4 ± 0.21 (1 standard error) from the 15 deflations listed in table 47.3, yields \( \mu_v = 2.0 ± 0.24 \text{ K}_m \). Additional error stems from uncertainty of the magma density, here assumed to be 2.75 g/cm^3.

The effective shear modulus of Kilauea's edifice, \( \mu_v \), is difficult to determine independently because Kilauea is built of many vesicular, commonly rubbly lava flows with abundant void space (Ryan and others, 1983). The intrinsic shear modulus of Hawaiian basalt was found by pulse transmission to average 25 GPa (Manghnani and Wollard, 1968) with a range of 4.7–40.5 GPa. Samples with greater porosity had a lower shear modulus. Presence of voids in the volcanic pile results in an effective shear modulus of the volcano that is less than the intrinsic shear modulus of hard samples.

Seismic velocities, and hence rigidity, determined from P-wave arrival data from earthquakes and explosions increase with depth and vary laterally (Thurber, chapter 38; Hill and Zucca, chapter 37). This reflects partly the compressive reduction of porosity with increasing depth, and the solidified intrusive zones surrounding the cores of the summit reservoir and rift zones. Any estimate of \( \mu_v \) from seismic velocities would be strongly dependent on the depth and horizontal position of the source, and size of the region chosen to represent the edifice. The bulk modulus of magma, \( \text{K}_m \), in Kilauea's reservoir should be uniform because the size of the summit magma reservoir is perhaps 1,000 times the yearly magma supply rate (Swanson, 1972), so short-term fluctuations in the gas content and chemical composition of the supplied magma are distributed over a large volume and do not affect the overall bulk modulus. Laboratory
determinations of the bulk modulus made from lava erupted from Kilauea, therefore, should be representative of the bulk modulus of magma in the reservoir. The value of 11.5 GPa determined for $\mu_w$ of molten Kilauea 1921 olivine tholeiite by Murase and others (1977) was used to calculate the shear modulus of Kilauea's edifice from the relation $\mu_s = (2.0 \pm 0.24) \cdot \mu_w$ determined above. The value for $\mu_s$, so determined, $23 \pm 3$ GPa, is close to the value for the average shear modulus of Hawaiian basalt given above. Note that this value does not apply to Kilauea's edifice as a whole, but only to the region surrounding the magma reservoir. This is because it was determined by modeling in-place melt-volume changes in the reservoir and resulting surface displacements.

Presence of a CO$_2$ gas phase in the magma would lower the aggregate bulk modulus of the magma. This may not be important for Kilauea, because, as Greenland and others (1985) assert, as CO$_2$ is lost from the magma as soon as it reaches the shallow magma reservoir. However, if some CO$_2$ is retained in the reservoir, then both the magma bulk modulus and the predicted edifice shear modulus would be reduced.

**RELATION BETWEEN LONG-TERM INELASTIC EDIFICE WIDENING AND SUBSURFACE VOLUME CHANGES**

The ratio of the long-term change of magma volume within Kilauea's summit reservoir to the edifice volume change between January 6, 1984, and July 16, 1985, may be determined from equation 5 and from the long-term elevation and gravity changes. A gradient of $\Delta g / \Delta h = -0.29 \mu$Gal/mm was determined for the interval by a least-squares fit to the tilt and gravity data, adjusted by the elevation/tilt correlation of 5.19 mm/μrad given above. This is essentially the free-air gradient, so $\Delta g^\prime$ is zero and equation 5 shows that the ratio of $\Delta V_e / \Delta V_r$ is zero. That is, there was no net change in magma storage in the summit reservoir during this interval.

The long-term ratio of volume changes in edifice and subsurface magma is different from that in short-term deflations accompanying Pau Oo eruptions (table 47.3). This implies that different processes affect the volume changes over longer time intervals. Earlier, the short-term deflations of Kilauea were explained using an elastic model, with inflow and outflow of magma accompanied by compression and decompression of the melt. Because short-term and long-term processes occurred concurrently, variations in the physical properties of the magma cannot explain differences in the volume-change ratios. The differences between the long-term and short-term volume changes must then be due to some superimposed long-term process that affects Kilauea's edifice. That process produces subsidence at the surface without a change in the magma content of the reservoir. One possibility is that horizontal spreading of the volcano's summit removes lateral support of the fluid magma reservoir, thereby decreasing the pressure in the reservoir and causing the summit to subside. Horizontal distance measurements at Kilauea have shown that the summit region is prone to widening in a south-southeast direction (Dvorak and others, 1983). This widening was particularly notable during the 1975 magnitude 7.2 earthquake along the south flank and for several months afterward (Lipman and others, 1985).

To investigate the possibility of summit extension during 1984–85, horizontal displacements measured by trilateration surveys conducted on January 16, 1984, and July 16, 1985, were analysed. In order to isolate the pattern of regional displacements in the vicinity of Kilauea's summit from the effects of observed deflation, the contribution of changes in the inflation level of the reservoir (approximated by use of a point source of dilatation determined from leveling data) was subtracted from the observed horizontal displacements. Residual line lengths were then used to compute horizontal displacement vectors (fig. 47.5). South-southeastward movement as great as 250 mm of points located on the south side of Kilauea caldera relative to points to the north show that the caldera widened slightly.

A $\Delta g^\prime / \Delta h$ gradient of 0.138 $\mu$Gal/mm was measured by Jacobs and Eaton (1980) during the subsidence accompanying the November 29, 1975, earthquake and eruption. A value for the $\Delta V_e / \Delta V_r$ ratio of 1.2 was calculated from the gravity gradient and equation 5 using an assumed magma density of 2.75 g/cm$^3$. This ratio indicates a greater component of surface collapse following the 1975 eruption, compared to the 1984–85 deflations (table 47.3). This difference results from 1.8 m of south-southwest crosscaldera extension during the November 1975 event (Lipman and others, 1985), in contrast to no measurable extension during the periods of very brief (1–3 d) deflation associated with eruptions in 1984 and 1985. The increased collapse in November 1975 is consistent with horizontal dilation of the edifice and surface subsidence because of the resultant drop in reservoir pressure.

Horizontal extension of Kilauea's edifice lowers the pressure in the magma reservoir; additional magma can be injected without uplifting the surface. In this way the reservoir volume increases by lateral growth. Growth by horizontal spreading and magma intrusion occurs during rift-zone development (Swanson and others, 1976), and it might also apply to the growth of the summit reservoir. Swanson and others (1976) attributed horizontal dilation of the rift zone to forceful injection of magma into dikes. However, at Kilauea's summit area, observation of subsidence concurrent with extension indicates that magma accumulation there is a passive response to dilatation. Perhaps the extension of Kilauea's summit that increases the capacity of the reservoir is an indirect response to the forceful dilation of the rift zones that bound the caldera on both the east and west sides.

Net gravity and elevation changes for the 16-mo period following the November 1975 deflation imply mass filling of the reservoir concurrent with subsidence. Dzurisin and others (1980) reported a $\Delta g^\prime / \Delta h$ gradient of $-0.294 \mu$Gal/mm for December 1975 to April 1977, which corresponds to a $\Delta V_e / \Delta V_r$ ratio of $-2.5$ from equation 5, using a magma density of 2.75 g/cm$^3$. This, combined with observed subsidence of the summit area of as much as 180 mm, indicates that the volume of magma in the reservoir actually increased despite the subsidence. The summit region widened by about 0.5 m in a south-southeast direction (Lipman and others, 1985) during this period; that extension created more reservoir space in which additional magma accumulated.
Inelastic horizontal dilation of the edifice in a south-southeast direction increases the capacity of the summit reservoir to accept magma without surface uplift. For the 16-mo period following the November 1975 deflation, additional magma was accommodated by this process. When insufficient magma is supplied to fill space created by summit widening, the pressure in the reservoir drops and the surface subsides as it did during and after the November 1975 earthquake and between January 1984 and February 1985.

REFERENCES CITED


