UNIQUENESS OF VOLCANIC SYSTEMS

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ABSTRACT

The cyclical behavior of Hawaiian volcanoes resembles patterns predicted on the basis of graphical or numerical attractor dynamics. This is because the recognizable styles of volcanic behavior are outgrowths of contrasts in positive and negative rates of magma transport that control the volumetric states of a growing volcanic edifice. Attractor patterns are likewise the resultants of some specified set of positive-negative rate balances. The distributions of volcanic states (intrusive and extrusive) in space and time constitute sets of complex patterns. Attractor dynamics simulates this complexity. The specification of how unique a given subset of a growth history may be relative to other subsets, or the whole relative to other histories (volcanoes or attractors), involves questions concerning pattern recognition. A totally unique event, pattern, or style of volcanism is one that has never happened before, hence is not recognizable. Volcanism, because it recurs, must represent a hierarchy of simultaneously unique-nonunique patterns. Even though no two growth histories, nor even any two eruptive events, are identical, there are strong resemblances between various volcanic styles that can be classified. Numerical and graphical attractor patterns add insight to such classifications.

The problems of discrimination between like and unlike patterns of evolution in volcanic processes are examined on a fourfold basis involving (1) volumes and rates of magma supply and eruption during the evolution of the Hawaiian-Emperor volcanic chain, (2) factors influencing thermomechanical feedback that imply oscillatory changes in conditions of magma generation, storage, and transport, (3) theoretical examples of positive-negative feedback that illustrate methods of constructing attractor diagrams which imply oscillatory and potentially periodic behavior, and (4) comparisons with the observed patterns of Kilauea's volume and rate balances of intrusion and eruption expressed in the same form as the attractor diagrams.

These four aspects of volcanic description are mutually consistent. The rate history is dimensionally consistent with thermomechanical feedback processes, and feedback processes imply attractor-like dynamics in which conditions tend to repeat in the neighborhoods of characteristic patterns. Kilauea's eruptions are associated with intrusive inflation-deflation cycles that occur as closed loops in diagrams where present states are plotted against future states of magma supply and supply rates. Large earthquakes like the 1975 Kalapana event seem to occur during characteristic intervals of such loops, at states involving both high intrusive inflation and high inflation rates of both summit and rift storage systems.

Volcanic eruptions, intrusive events, and earthquakes are individually unique, but volcanism as a whole is a nonunique process in which repeated combinations of rate balances give rise to categorically similar patterns worldwide. Involution and convolutions of these common pattern-generating mechanisms are the sources of unpredictable uniqueness. Prediction, therefore, involves locating one sort of cyclical loop, temporally periodic or not, within both larger and smaller contexts of recurring loops. Thus, recognition is the act of predicting several different scales of temporal and spatial action simultaneously. Given sufficiently redundant information, pattern recognition becomes automatic, as in everyday experience, rather than analytic. On this basis, specific attractor-like styles are potentially recognizable on local to global scales of volcanism. Expansion of these ideas to include chemically evolved systems (including silicic cratonic systems) implies that the criteria of pattern recognition also involve information describing chemical rates of change.

INTRODUCTION

The purpose of this paper is to examine the general problem of predictability in volcanic processes in terms of the sources of possible periodic and nonperiodic influences. Recurring themes are the issue of scale ratios of relevant properties and processes and the question of self-similarity. As has been my emphasis in other papers, I rely on the documentary history of Hawaiian volcanism to test ideas. In that context, there are two simple tests of self-similarity: first is the extent to which the differing time-scale patterns of day-to-day, historical, and prehistoric activity of the active volcanoes Kilauea and Mauna Loa are distinguishable in regard to the durations and amplitudes of episodic oscillations (whether periodic or nonperiodic). A second test is the degree to which these specific signatures differ from those at the larger scales of the Hawaiian Archipelago, the Hawaiian Ridge, and the Hawaiian-Emperor system as a whole. This paper considers the historical and secular Hawaiian scales quantitatively, explores physical mechanisms that influence the timing of volcanic processes, and points out some conceptual parallels between volcanic phenomena and patterns found in numerical computer experiments in nonlinear dynamics.

The term "nonlinear" is here used loosely to include systems described by equations that are of high order and (or) transcendental, sometimes with nonconstant coefficients, and which involve feedback coupling with other systems, functions, or coefficients. Such a definition is tantamount to saying that the mathematical discussion emphasizes comparisons of patterns without attempting to model sets of descriptive parameters in the form of analytic equations. Undeniably, many basic mechanisms can be described in terms of equations. They are therefore useful in predicting the kinds of mechanisms that may be aspects of a complex system. Such studies are clearly important, even in the absence of any direct applications in forecasting behavior. This is the rationale for including a section on mechanisms of rheological feedback as an important adjunct to studies of pattern recognition (see also Hardee, chapter 54).

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The paper consists of five main parts: (1) the remainder of the introduction, which considers some generally relevant concepts from an interdisciplinary perspective; (2) concepts of self-similarity examined in terms of volumetric time-series data for Hawaiian volcanism; (3) rheological concepts of feedback processes; (4) concepts of stationary states expressed from the point of view of mass balance; and (5) Kiluaea as a natural example of attractor patterns. Although (5) refers to concepts introduced in (4), the patterns illustrated there do not depend on any theoretical considerations; they are simply plots of the data on Kiluaean rate histories recast in a form compatible with attractor patterns and concepts of autocatalytic systems (systems that contain rate-determining steps that catalyze, as well as take part in, subsequent steps in a generally cyclic process; the summit chamber of Kiluaea is shown to represent such a controlling step).

I have attempted to make each section of the paper more or less self-contained, so readers who are interested only in specific aspects of the discussion can skip the other parts (if so, however, it may still be helpful to read the introductory and concluding paragraphs in each section). The rheological discussion in (3) is included as an important step in my personal recognition that there is a need for generalized methods of quantitative description of complex processes, as introduced in (4) and (5); otherwise, it can stand alone as an updated commentary on the thermal-feedback processes discussed by Shaw (1969) and as a connection between the present paper and the paper by Hardee (chapter 34).

PREDICTION AND SELF-SIMILARITY

Concepts of predictability in natural processes are currently undergoing what eventually may be recognized as a revolutionary change or paradigm shift. Traditionally, there have been two major approaches to scientific prediction, based on (1) documentation of a clear dependence on a precisely known and inexorably persistent forcing function and (2) documentation of statistical correlations among complex sets of parameters in the absence of any dominating influence by known forcing functions. The former is often called deterministic forecasting, and the latter probabilistic or stochastic forecasting. Most discussions of natural processes involve some mixture of these methods, because even the most deterministic results involve probabilistic estimates of uncertainties, and probabilistic results usually assume that some aspects of an analysis can be expressed deterministically (for example, patterns of seismicity and ground deformation at an active volcano, even though they may be expressed in terms of probabilistic models, may be determined by known volume changes accompanying the transport of magma; in other instances deformation may be expressed in terms of constant rates of motion determined by the data of plate tectonics).

An example of deterministic forecasting is given by the problem of ocean tides, where the forcing frequency and disturbing potentials are known with high precision and accuracy relative to many other kinds of Earth processes. Examples of probabilistic predictions are given by actuarial tables and some forms of weather forecasting, where results are based on analyses of trends in average behaviors involving numerous complicated, unknown, and (or) chance events. Klein (1982) discusses problems of eruption forecasting at Kiluaea with emphasis on probabilistic methods. What is called "chance" often reflects ignorance concerning relevant processes, even though at some level there may be processes that are unknowable by instrumental methods of measurement (for example, the Heisenberg uncertainty). Unless there are such sources of random noise dominating our perceptions of a process, however, the process is potentially knowable from the standpoint of predictable recognition, whether or not it can be described in any specific analytical format.

Numerical forecasting represents the implementation of a scheme to quantify the criteria of recognition, which may simply be a statement of the number of times a given "effect" follows a given "cause." For example, I might get a rash one out of every hundred times I recognize poison ivy, and occasionally I may get a rash when I wasn't aware that I was in its proximity. Obviously, if my criteria of recognition are improved (not only of the leaf itself, but of all the many and subtle ecological indications of its presence), the incidence of rash could be reduced nearly to zero, but I can never be sure I'm infallible. So, if the consequences are bad enough, I am continually motivated to improve my powers of observation.

This example is more complex than it might seem. Its complexity could be demonstrated by attempting a quantitative statistical analysis. The analysis and its forecast of effects might be expressed in terms of numerous and complicated statistical cross-correlations of the factors influencing visibility, individual prowess at classifying plants, numbers of vines per unit area, leaf density and toxicity, humidity, wind direction, allergic predispositions, and so on, resulting in a statement of numerical probability that I will break out in a rash within varying windows of space and time, expressed in terms of calculated significances within stipulated confidence limits. At least in the case of rashes caused by poison oak, subjective experience in pattern recognition is probably the better method of forecasting risk.

Some of the above distinctions between deterministic and probabilistic methods of prediction can be matters of the relative degrees of resolution in comparing patterns of repetitive processes and their interactions, and this is the aspect with which this paper will be primarily concerned. Hence, the scale dimensions in time and space of those processes responsible for significant effects are an important concern. For example, weather becomes deterministically predictable if we are mainly concerned with seasonal behavior. On the other hand, tidal fluctuations become less and less obviously deterministic the more closely they are examined, to the extent that they may appear essentially probabilistic when they are examined at the level of microsecond variations in the length of day. (This problem is particularly instructive because it involves the interactions of numerous and subtle cyclic phenomena, as do patterns of volcanic cyclic. In this tidal example, influences involve the dynamics of the solid Earth, oceans, atmosphere, ionosphere, magnetosphere, and solar processes responsible for the solar wind. Clearly, the problem of pattern recognition in such a case is highly interdisciplinary. This paper supports the same conclusion with regard to future advances in the recognition of volcanic patterns that may be describable in deterministic as well as in probabilistic terms.) Basically, then, we...
are attempting to evaluate the quantitative relations among natural processes over many different scales of size and time.

One approach to the search for characteristic patterns that may have systematic correlations with other patterns of deterministic origins is to examine similarities in behavior at different scales of observation. If a general process, such as global ocean and atmosphere circulations or global volcanism, shows internally consistent patterns that are indistinguishable at different dimensional scales of behavior, the process is said to have elements of self-similarity. The simplest test for self-similarity is to examine any set of data (such as a map of a geometric pattern or a time series of a varying quantity like wind velocity) over domains of different size and (or) over different durations. If the unlabeled graphs are indistinguishable at different scales, the data are self-similar. In an absolute sense, all variations (such as length, velocity, and energy) would retain the same proportionality with respect to relative amplitudes, waveforms, and periodicities. A less stringent criterion of self-similarity is that the general ranges of these parameters are statistically indistinguishable, even if there are no point-for-point correspondences of the reduced data. In the latter case the patterns are self-similar from the standpoint of having the same complexity of information content at all scales.

THE CONCEPT OF FRACTAL GEOMETRY

The idea of self-similarity is sometimes considered to be essentially synonymous with the newly emerging research tool called fractal geometry. Self-similarity is a separate idea, and its documentation depends on the system of measurement by which it is described. A fractal structure, as described below, may or may not be self-similar, as in any other system of measurement. The way in which a fractal measurement is made, however, automatically involves consideration of those conditions that give rise to geometric self-similarity, particularly of the statistical variety, and departures from it (see Mandelbrot, 1982). It is therefore worth considering the general implications of fractal geometry, although the data described in this paper have not been formulated in these terms (the attractor diagrams described later produce structures that are fractal in character, but there I have restricted the discussion to the terminology of attractor theory).

A measure called the fractal dimension represents the relation of pattern lengths to an independent and unchanging (nonmagnified) unit of length as the pattern as a whole is progressively magnified without limit. Mandelbrot (1982, p. 25) calls such a length "the yardstick length," and I think of it as any ruler chosen as the standard of a particular measurement. Equivalently, the fractal dimension is determined by the ratio of the logarithms of the pattern lengths and standard lengths as the latter are changed. Any pattern, whether continuous or discontinuous, can be measured in such a way that it can be described in terms of fractal dimensions. Such patterns can therefore be referred to as fractal structures. If a pattern can be described by a single fractal dimension, it is said to be a self-similar fractal structure.

Evidently many familiar objects (trees, for example) are fractal structures. The idea extends directly to any ramifying patterns, including lava flows and paths of intrusive magma transport. It remains to be demonstrated whether any one or several such objects, or any subset thereof, are self-similar. The idea of a fractal dimension may seem strange because it describes forms that are not limited to the usual topological constructions based on lines, planes, and "solid" geometrical objects like cubes or spheres. That is to say, fractal structures can have fractional dimensions between those of the integers describing Euclidean topologies. Therefore, fractal geometry is very useful as a measure that allows us to expand our abilities to quantitatively classify complex structures showing dendritic, feathery, dusty, and other seemingly irregular forms. In this respect, it sometimes turns out that what is considered very irregular in the traditional geometric sense is actually quite regular and simple from the fractal perspective. This possibility applies to seemingly noisy time-series data as well as to complex spatial patterns. The study of coastline lengths, tracks of Brownian motion, stream-drainage networks, morphologies of fracture sets, and temporal patterns of economic trends are some common applications. Applications to geology are too numerous to mention and have only just begun.

Although fractal geometry is closely related to issues of self-similarity examined in this paper, my principal concern is with the patterns of magma-transport processes in time as they may relate to attractor diagrams. This implies a number of complex dynamic effects not explicitly measured (the distributions of magma transport in both space and time are not adequately sampled to permit a complete discussion of their fractal properties). The fractal concept is, however, implicit to generalizations of the temporal patterns illustrated later and, in principle, can be applied in dimensional contexts that include time and energy as the quantities of measurement.

APPLICABILITY OF EVOLUTIONARY CONCEPTS TO PATTERNS OF VOLCANISM

A new approach to the study of periodic patterns and complexity in physical processes is arising out of computer studies of the numerical patterns produced by various kinds of recursive algorithms. Numerical recursion refers to some computational scheme (an algorithm)—whether done by hand, graphically, or by computer—wherein repetitive calculations are performed in which the current result, or state, is used to compute the next state, and so on. Except for the initial state, and a set of rules of repetitive computation (which may, for instance, be in the form of a reference equation or a set of "if this, do that" statements), the computation is allowed to guide itself indefinitely. Such programs are sometimes called computational automata, cellular automata, evolutionary strategies, or other names that emphasize the open-ended, self-guiding principle of computation. John von Neumann, famous in the development of computers, influenced terminology and modern computer studies in this area by laying out (in lectures during December 1949 at the University of Illinois) a set of logical criteria by which a machine could reproduce a copy of itself, including a copy of the instructions (these lectures were later completed and published by A.W. Burks; see von Neumann, 1966; Burks, 1970). Resem-
balances to the biological genetic code, which was being discovered almost simultaneously, were noticed later (Judson, 1979, p. 244).

An alternative approach to the development of a system that can reconstruct itself in all details is a computational scheme that is permitted to wander wherever it will without any preconceived goals, subject to a simple set of rules. Such a scheme, invented by John Conway in 1970 (see Poundstone, 1985), is known by the evocative name “Life.” Popularized by Martin Gardner in Scientific American (his column titled “Mathematical Games”; particularly see issues during the 1970’s, or Gardner, 1983), it became the subject of extensive exploration by amateur and professional researchers (and by players of computer games). An extensive description of Life behaviors is given by Poundstone (1985), and a technical note concerning cosmological implications is given by Gosper (1984). A logical extension of computer schemes that would come closer to natural systems is an algorithm that not only reconstructs itself but also invents the rules of evolution, which it subsequently copies, reconstructs, and modifies repeatedly and indefinitely. Because the study of such systems has the potential complexity of natural evolution, a simplifying approach is clearly needed.

The approach taken in this paper is to treat pattern evolution found by means of numerical algorithms in a manner similar to that used in the study of natural systems. I search for resemblances among the results of computer experiments using particular types of recursion algorithms suggested by natural processes, and I look for those properties that have some degree of universality of behavior with natural systems. The recursion strategy I use is derived from concepts of attractor theory, in which a specific governing equation involving aspects of both positive and negative feedback guides the repetitive computation (see May, 1976). Sample computations are discussed later. An important discovery of this approach is that whatever recursive algorithm is used, the resulting numerical patterns generated have some simple properties in common. Among these properties is the observation that numerical patterns (trajectories of plotted points in space and time) often reproduce according to regular geometric progressions in self-similar sets (smaller subsets have the same relative spacings of plotted points as larger subsets; see previous comments on self-similarity). Such patterns are sensitive to the rates of change of a controlling parameter in several different ways: the result may produce one or more stable fixed points (as in the approach to a set of stable thermodynamic equilibrium points, or thermometer readings in a system of steady-state heat flow), or it may produce quasi-periodic, aperiodic, and seemingly chaotic patterns. All of these complex patterns can also be shown to be fractal structures. Some of them are of the fractally self-similar type (sometimes called strange attractors), and these may grow or appear surprisingly out of highly dissipative, random-looking structures (in nature we would describe analogous macroscopic structures as representing states far from equilibrium). In this regard, they are numerical analogs of living systems and of physical devices that do in fact operate far from thermodynamic equilibrium (examples are found in such diverse phenomena as turbulent convection, socioeconomic structures, landform evolution, electronics, laser operation, and many others).

The surprise in the above studies is caused by the complexity of patterns and an inability to follow or to visualize all steps in the evolution, which often takes on an overall form bearing little resemblance to intuition or predictions. In this regard, patterns of numerical evolution have some of the characteristics of natural systems that involve so many different determinism influences that the resulting behavior looks highly complex and chaotic. In the numerical context, however, this chaotic behavior is found to contain regions (“windows”) of structural simplicity or order. Such structures are also associated with characteristic temporal periodicities.

Thus, periodicities are produced not only by the deterministic governing algorithm, but also by the evolving autocatalytic properties of repetitive recursion (examples are shown later). Therefore, if natural periodicities are viewed in the same way, there are sources of periodic influences that may be deterministically imposed from outside the system of interest (exogenous forcing functions), and there are others that are generated by the repetitive operation of the system itself (endogenous convergences toward restricted sets of repetitive states). In systems where there are numerous possible forcing functions, it may be difficult to distinguish endogenous from exogenous periodicities, and there may be resonances between some endogenous effects and certain of the exogenous influences (a volcanic example mentioned later is the possible correspondence of self-induced response cycles and tidal cycles). Alternatively, the dynamical context within which the system is described may be expanded so that, in effect, all periodic behavior becomes endogenous. Later I suggest how and why volcanic phenomena show dynamical parallels with patterns generated by numerical algorithms.

I have intentionally avoided using the terms “random” and “nonrandom” in describing patterns in space and (or) in time. The distinction between these terms, as commonly employed, is related to concepts of uniqueness. In my opinion, use of “random” as a descriptor of natural data is unsatisfactory on at least two counts: (1) if a pattern can be recognized, it contains information that already exists in some reproducible form (otherwise, like the unique pattern, it has never been seen before), hence it is nonrandom, and (2) the random state cannot be proven mathematically (a large literature on complexity theory, not cited, concerns the fundamental question of how to quantify departures from this definable limit). Statistical tests of randomness really represent comparisons with standardized, hence artificial, concepts of complexity. As such, they are useful ways to classify categories of numerical data. In the process, however, many of the potentially recognizable characteristics of a complex pattern may be lost or overlooked. I contend that natural patterns contain much more usable information than is acknowledged in the application of standard statistical tests as a measure of randomness or nonrandomness.

ACKNOWLEDGMENTS

I thank Dan Drazin for discussions of the magma-supply budget at Kilauea, and for numerous constructive suggestions in the manuscript. I also thank Harry Hardee (Sandia National Laboratories) and Patrick Muffler for constructive reviews, Roy Bailey for discussion of the volcanic patterns at Long Valley, Calif., and Tom
SELF-SIMILARITY IN HAWAIIAN VOLCANISM

Cumulative volumes of volcanic products versus time are shown in figure 51.1 for the Hawaiian-Emperor Chain as a whole and also for the individual volcanic loci during the chain’s 74-m.y. history (data from Shaw and others, 1980, table 2). Rates of volcanism from the same database presented in figure 51.2 include a composite volume rate, rates along loci, and moving-window averages for the overall rate. Notably, the complexity of time series representing volume rates is similar on different spatial and temporal scales; rate episodes look much the same from time to time, and the ranges of rate amplitudes are similar within short as well as within longer intervals of time. If there were no labels on the graphs, therefore, it would be hard to say what general period of time or what duration in time, a given graph represents; this satisfies the qualitative definition of self-similarity mentioned earlier.

Statistical periodicities at about 12, 6, 3.6, 2.7, and 2 m.y. (fig. 51.3) were found by fast Fourier transform analysis of average volumes per time taken at equal 1-m.y. intervals of time. Similar, but not identical, periodicities are shown by the same sort of analyses for volumes per length along loci, instantaneous volume rates, linear propagation rates, and variations of the azimuths of loci with time, each set of data read at equal 1-m.y. intervals of time. Such periodicities represent averages of a variety of temporal and spatial variations among many different physical processes at the scale of plate tectonics affecting the Hawaiian-Emperor Chain. I do not imply, nor do I believe, that the above periodicities are coherent, and therefore no numerical estimates of statistical significance are assigned. Any statement of confidence limits would be misleading because the global dynamic context indicates that these apparent periodicities involve many different fundamental periods of shorter duration that can combine in numerous complicated ways. The fact that more localized sampling in space and time, to be discussed, also reveals signatures of similar proportional variability is evidence for the composite nature of such time series as those in figures 51.2 and 51.3. Existing data at the scales of observation illustrated are not sufficient to determine the values and possible varieties of fundamental periods.

The shortest resolvable variations in the above records are at time scales of the order $10^5$–$10^6$ years. The next level of extensive information is given by the historical record (<200 years), with variations resolvable at about yearly intervals. At yet finer scales is the three-decade record of modern geophysical measurements at the Hawaiian Volcano Observatory (HVO), with resolution of months to days. Finally, the east rift zone of Kilauea has had almost continuous activity since January 3, 1983, with episodes of fountaining at or near the vicinity of Puu Oo at intervals of about 1–9 weeks (George Ulrich, written commun., 1984).

Unfortunately, there are large jumps in time and size scales between the current local, historical, and Pacific-plate records. Detailed data on rate variations within the time scales of an individual edifice ($\sim 10^5$–$10^6$ yr in duration) or of an individual

Figure 51.1.—Cumulative edifice volume plotted against age for Hawaiian-Emperor Chain. Data from Shaw and others (1980). A, Total volume for all volcanic loci. B, Cumulative volume for each locus; loci identified by number, see Shaw and others (1980, table 2). Loci numbers 1 and 2 are, respectively, “Kua” and “Loa” trends of Dana (1849).
FIGURE 51.3.—Fast Fourier transform analysis of power spectrum for interpolated volumes of volcanic edifices per million years (sampling interval) for Hawaiian-Emperor Chain (Shaw and others, 1980, table 2). Dashed line gives average trend, on which are superimposed localized power maxima (power expressed as square of unit of rate samples). Analysis suggests that maxima may exist near calculated periods 2, 2.7, 3.6, 6, and 12 m.y. with generally increasing power at longer periods, but record is inadequate to demonstrate coherence. Such a trend resembles so-called 1/f frequency-dependent spectra that characterize many other kinds of complex time series in nature; see Verveen and Deeks (1968), van der Ziel (1970), and Voss and Clarke (1975).

FIGURE 51.2.—Volume rates of volcanic edifice growth for Hawaiian-Emperor Chain. A. Net rates for all loci. B. Rate along each locus (identified by number; see Shaw and others, 1980) and comparative rates for two tectonic silicic systems: Hawaiian-Emperor data are from Shaw and others (1980, table 2); data for Mt. Edziza, British Columbia, from Souther and Hickson (1984); data for Coso Range, California, from Duffield and others (1980). C. Smoothed values of net rates in A using a moving average window of 4 m.y. with 2-m.y. shifts. For comparisons of Hawaiian-Emperor and cratonic systems, see Shaw (1985) and discussions of relations among rate, size, and repose time in text (data for Long Valley, California, during latest 2 m.y., from Bailey, written commun., 1984, somewhat exceed rates for Mt. Edziza). For reference, average eruption rate for ash-flow systems is about $10^{-2}$ km$^3$/yr (Smith, 1979), and their intrusive supply rate is about $10^{-2}$ km$^3$/yr (Shaw, 1985).
island (~10⁶ yr in duration) are unavailable because of the masking effects of younger lava in undissected edifices. Thus, we are dealing with several discontinuous and widely different scale ratios when comparisons are made between data sets describing Hawaiian volcanism in a context that includes both the historical and Pacific-plate records.

Cumulative volumes of lava extruded from Kilauea and Mauna Loa are shown in figure 51.4, together with inferred volumes of the largest deflation episodes of Kilauea (those associated with earthquake activity), as compiled by Klein (1982) from HVO records. The latest data points terminating the records represent estimates for the cumulative eruptive volumes of the 1983–85 east rift activity and the 1984 Mauna Loa eruption. The approximate mean trends are indicated by dashed lines of slope 0.025 km³/yr drawn subjectively through the data; reference slopes of 0.1 and 0.01 km³/yr are shown for comparison.

The following points of interest are noted in the historical rate behavior of Kilauea and Mauna Loa, as shown in figure 51.4. (1) There is an episodic variation between rates (slopes) of about 0.01 and 0.1 km³/yr for both Kilauea and Mauna Loa. (2) These rate episodes tend to be step-like at intervals of about 10–40 yr rather than scattered uniformly about the mean trend. (3) The mean extrusion rate on the 100-yr scale is near 0.025 km³/yr at both volcanoes (the overall magma-supply rates will be discussed later). (4) The eruption frequency is higher for Kilauea than for Mauna Loa despite similar average rates, underscoring the typically larger eruptions and longer repose times at Mauna Loa (see Klein, 1982). (5) The step-like episodicity of rates resembles the variations in Hawaiian-Emperor rates in figure 51.1A, and the similarity in mean eruptive rates for Kilauea and Mauna Loa parallels the long-term similarity of slopes between loci 1 and 2 in figure 51.1B (the "Kea" and "Loa" lines of Dana, 1849; see Jackson and others, 1972; Jackson and Shaw, 1975). (6) The intrusive volumes, which are based on the larger deflation episodes listed by Klein (1982, table 1), have episodic trends similar to the eruptive trends, and they have about the same mean slope near 0.025 km³/yr. (7) The slope of the major cumulative intrusive-volume events since about 1960 is less than the slope of eruptive volumes over that time interval, so that eruptive rates intermittently exceed rates of major intrusive volumes; hence, (8) there are decade-long delays in renewed intrusive storage compensating eruptions, and (or) there are components of magma supply that are not recorded by the major intrusive events (evidence described below suggests that the latter factor, acting somewhat like a continuually leaking valve, is exceedingly important in the 100-yr scale balances of intrusive and extrusive episodes).

Volumetric variations for Kilauea between 1956 and 1983 are shown in figure 51.5 in more detail, as compiled and interpreted by Dzurisin and others (1984) in terms of magma-supply rates. I intentionally show these data in their units and format to facilitate comparisons of the two discussions and to avoid errors of transcription of the complicated history. This introduces an unavoidable difference in the convention for the orientation of the time axes compared with earlier figures, which were drawn to be compatible with the discussions of time-distance relations given by Shaw and others (1980). Their best estimate of average supply rate during this time (adjusted for vesicle porosity) is about 0.086 km³/yr (7.2 x 10⁶ m³/yr), with episodic deviations over an approximate order of magnitude (uppermost curve in fig. 51.5A). They also concluded that, during 1956–83, only 35 percent of the magma supply was extruded and 65 percent was stored in Kilauea's rift zones: 55
percent in the east rift zone and 10 percent in the southwest rift zone. Thus, their mean extrusive rate for the 1956–83 interval, calculated either from the above percentage of the total or directly from the erupted volume divided by the duration, is about 0.03 km$^3$/yr, in agreement with the average trends in figure 51.4 for the total historical record.

During an approximate 6-month interval of time (fig. 51.5B) there was a quasi-steady balance between total rates of magma supply and rates of transport between the summit and rift zones in the absence of eruptions or large intrusive episodes. The average intrusive rate during this time was about $5.0 \times 10^6$ m$^3$/mo, or 0.06 km$^3$/yr. Hence, this period seems to be representative of the 28-year intrusive average (that is, roughly two-thirds of the yearly total of 0.086 km$^3$/yr). Such increments can supply local zones of rift inflation that will become sites of eruption, or they may solidify in situ; the distinction depends on interpretations of deformation data for the east rift zone not considered here.

The fact that Kilauea’s extrusive episodes indicated by the volume record of figure 51.4 also transiently equal or exceed the mean supply rate derived by Dzurisin and others (1984) reinforces inference (8) above that either intrusive or extrusive rates may transiently exceed the other for periods of a few years, although overall the average eruption rate is significantly less than the average supply rate. That is to say, the cumulative erupted volume falls far short of the cumulative intrusive volume, at least on the 100-yr scale. This means that intrusion dominates over extrusion in the growth of the volcanic edifice. The overall volumetric rate of growth of the volcanic edifice on the 100-yr scale is about 0.1 km$^3$/yr, which is about three times the average volumetric rate of lava accumulation (about 0.03 km$^3$/yr) and about equal to the long-term rates of edifice growth over the latest 10$^6$ years shown by the data in figures 51.1 and 51.2 (also about 0.1 km$^3$/yr).

The cumulative volume of eruption of Kilauea since the beginning of 1983 is shown in figure 51.6. The mean slope of 0.15 km$^3$/yr ($12.7 \times 10^6$ m$^3$/mo), uncorrected for porosity (see below), is consistent with the above inferences concerning discrepancies between intrusive and extrusive rate histories. Even allowing for a reasonable porosity correction (for example, a 15-percent correction, as used by Swanson, 1972, gives 0.13 km$^3$/yr, or 10.8 $\times 10^6$ m$^3$/mo), the extrusion rate since the beginning of 1983 (as of this writing, June 1985) is larger than any previous estimates of supply rates and resembles the long-term edifice growth rates for locus 1 in figures 51.1 and 51.2. (Note that the higher rates indicated in fig. 51.2A represent the combined growth rates of loci 1 and 2 and of any

Figure 51.5.—Record of magma supply for Kilauea, 1957–83 (from Dzurisin and others, 1984), based on continuous records of tilt measurements at Kilauea summit. Vertical bars at bottom of diagrams indicate specific magmatic events: short bars for intrusive events, long bars for eruptions. Estimates of magma supply are independent of measurements of erupted volumes, except as they were used in making tilt calibrations; see Dzurisin and others (1984) for discussion. A, Overall record. B, Record for a six-month period in 1980.
unrecognized loci of new activity. Locus 2, which includes the Hualalai-Mauna Loa-Lohi trend, appears to represent a slightly higher rate history than the Kohala-Mauna Kea-Kilauea trend over the past $10^5$ yr, and this could also be true at the 100-yr scale, judging from the respective slopes in fig. 51.4.) An interpretation of quasi-steady balances between eruption and supply rates during May–December 1969 was the basis for Swanson’s (1972) estimate for the Kilauea magma-supply rate. His estimate of mean extrusion rate, including porosity, was $9.9 \times 10^6$ m$^3$/day, compared with $12.7 \times 10^6$ m$^3$/day in figure 51.6. He adjusted this rate downward by 15 percent to account for vesicularity, giving $8.4 \times 10^6$ m$^3$/day (0.10 km$^3$/yr). This compares with the comparable adjusted value of $10.8 \times 10^6$ m$^3$/day in figure 51.6 mentioned above (0.13 km$^3$/yr for the 1983–85 period). Although a mantle-derived magma-supply rate of either 0.10 or 0.13 km$^3$/yr is larger than the 30-yr average of about 0.09 km$^3$/yr calculated by Deurinck and others (1984), each of these values is consistent with the rate magnitudes of figures 51.1B and 51.2B in the latest 10$^2$ yr or so (see averages in fig. 51.7). If the 30-yr supply rate is typical of longer term averages for Kilauea, then we might expect to see a return to conditions where eruption rates are again lower than intrusive storage rates within future decade-length variations when the current episode of high activity wanes. By this interpretation, the longer the currently high eruption and (or) intrusion rates continue, the longer will be the next interval of subdued activity at Kilauea on the basis of existing patterns. However, if the long-term “Kea” average at Kilauea resembles the “Loa” average (see Shaw and others, 1980), then the 30-yr as well as the 100-yr averages for Kilauean supply rates are less than the potential rates for deep-seated magma generation, and the currently high values of eruption could represent a supply rate that will continue indefinitely. It is important to distinguish between intrusion and extrusion here. Subdued activity, as judged from eruptions, does not necessarily imply a low magma-supply rate, particularly if the intrusion rate is increased relative to the 30-year average because of tectonic adjustments in the structure of the volcanic edifice.

Notice that the mean rate for the large intrusive events in figure 51.4 is approximately in balance with the average extrusive rate (similar slopes). This suggests to me that eruptive events may be associated with preferred sites and paths of storage in the rift systems relative to the more pervasive and seismically quiet intrusive leakage modes (for example, as represented by fig. 51.5B). That is, there is a strong correlation between seismicity, the large intrusive pulses, and eruptive pulses. These events, then, seem to be logical candidates for transport rates in which the rate-determining step may be extensional and (or) extensional-shear fracture, according to the mechanisms discussed by Shaw (1980). This, in turn, suggests the possibility that there may be different modes of stress adjustment associated with different intrusive styles, one more localized and directly keyed to eruptions and another more distributed and keyed to the overall changes in edifice growth related to distributed intrusion. The distinction is partly one of time frame, because the overall changes in longer term edifice stresses also must govern the preferred routes of magma transport, according to the principles discussed by Shaw (1980); compare later discussions of figures 51.22 and 51.23. The coseismically more passive (as contrasted with potential for major delayed seismicity) aspects of overall edifice growth must be compensated by the smaller scale intrusive events which are not accounted for in the eruptive volumes (at least on the 100-yr scale), implying that such volumes solidify in place. Otherwise, eventually a very large magma chamber should evolve with comparably larger eruptive events. However, even if the more passive intrusive fractions do not erupt, they contribute to the high-level mass accumulations that affect the larger scale and longer term edifice stresses that, in turn, contribute to the larger and more regional earthquake events, such as the magnitude 7.2 Kilauea earthquake of 1975 (Shaw, 1980). Sites of intrusive magma accumulation and solidification, then, also contribute to edifice stresses, and therefore to frequencies of eruptions and earthquakes, and possibly to catastrophic landslide events that have occurred in the past along the Hawaiian Archipelago.

The various average rates discussed above are summarized in figure 51.7, which also shows the superimposed historical variability of Kilauea and Mauna Loa eruption rates. Although the average of the historical eruption rates for the two volcanoes have been about the same (see fig. 51.4), the minimum rates for Mauna Loa hover near the overall Hawaiian-Emperor average, on the order of 0.01 km$^3$/yr, while the highest rates only occasionally reach the overall 0.3-m.y. rate of 0.18 km$^3$/yr (the value for the “Kea” locus could be lower, depending on how the total volume for Kilauea is reckoned in the 0.3-m.y. average; the 0.3-m.y. average for locus 1 excluding Kilauea is closer to the 1-m.y. average of about 0.08 km$^3$/yr). This suggests that, as with Kilauea, the average edifice growth rates for Mauna Loa (that is, rates indicated by the longer term average volume rates) are several times the average historical eruption rates and that large components of undetected magma storage and solidification are also taking place on a large scale in the Mauna Loa.
structure. This graph (fig. 51.7) also shows that, despite the same average eruption rates (see fig. 51.4), the fluctuations at Kilauea are of much higher frequencies (since about 1956 the average interval between maxima is about 2 yr for Kilauea compared with an overall average interval of about 20 yr for Mauna Loa) and oscillate over wider ranges of rate amplitudes within the time frame of Mauna Loa repose intervals.

Thus, the temporal variability of events (eruptive and intrusive) seems to relate to their size variability, because the volumes of the higher frequency Kilauea events are typically smaller than the Mauna Loa events (I infer this for the Mauna Loa intrusive events as well, based on the resemblances of eruptive trends in fig. 51.4, but they have not been documented in detail). Present knowledge of the Mauna Loa edifice structure does not allow any rigorous correlation between the periodicity and size/repose-time ratios of Kilauea and Mauna Loa. If repose times were directly proportional to size (that is, the larger the overall edifice volume, the longer the repose times between larger intrusive and extrusive events), Kilauea would be about one-tenth the volume of Mauna Loa (average repose times in fig. 51.7 for Kilauea are about one-tenth those of Mauna Loa). This has some validity on the basis that the overall edifice volume controls the potential magma-storage volume, other factors being equal (neglecting the details of edifice heights and gravitational stress distributions). But according to Shaw and others (1980, table 2) the ratio of the Kilauea to Mauna Loa edifice volume inferred from topography and bathymetry is about one-half, unless much of the volume attributed to Kilauea represents hidden fractions of the Mauna Loa edifice (this possibility is allowed by current knowledge of lava-flow stratigraphy in the region of overlap between Kilauea and Mauna Loa; T. Casadevall, oral commun., 1985; D. Dzurisin, written commun., 1985). The same discrepancy is indicated by a consideration of the relative ages of the Kilauea and Mauna Loa edifices. For example, the estimated age of Mauna Loa given by Shaw and others (1980, table 2) is about 10⁷ yr. Therefore, according to the approximate constancy of both the historical and the 0.3-m.y. average growth rates discussed above, Kilauea's age would be about half that of Mauna Loa, or 50,000 yr, if its volume is also half. So Kilauea's age would be reduced to 10,000 yr and its volume to about a tenth that of Mauna Loa if the argument based on frequency ratios were used.

Another way of looking at the proportionalities between edifice sizes and event frequencies is to consider the possibility that repose times are controlled by the ratio of summit-chamber volumes. The periodicity ratio of 10 to 1 (fig. 51.7) would then suggest that the diameter of the summit chamber of Mauna Loa is a little more than twice the diameter of the Kilauea summit chamber (compare Klein, 1982). Assuming that this ratio of chamber diameters is roughly correct, all the data favor the smaller volume and younger age limits.
for Kilauea. There is then a reasonably consistent set of constant ratios of the above parameters between Kilauea and Mauna Loa, respectively, as follows: the ratio of average edifice growth rates is about 1 to 1; the ratios of overall edifice volumes and of summit magma-chamber volumes are both about 1 to 10; the ratio of ages since inception is about 1 to 10; the ratio of repose times between distinguishable eruptive and intrusive events is about 1 to 10; and the volumes per event (both intrusive and extrusive) are in the approximate ratio 1 to 10.

If this interpretation is correct, it suggests an interesting property of volcanic evolution: volcanic systems typically evolve from smaller sizes characterized by higher frequency events to larger sizes characterized by lower frequency events at the same overall average growth rate. A similar inference has been drawn by Smith (1979) and by Shaw (1985) concerning cratonic volcanic systems. In fact, the ratio between the periods of repetitive eruptive events per order of magnitude increase in eruptive volume shown by Smith (1979, fig. 12) is also a factor of 10, as was found above for the ratio of repose times between Mauna Loa and Kilauea eruptions (fig. 51.7). Shaw (1985) demonstrates that this proportionality exists for a roughly constant average rate of intrusive magma supply which, for high-level cratonic systems, is about 0.01 km³/yr; Wadge (1982) discusses other implications of approximately steady-state magma supply in a variety of volcanic systems.

The above proportions represent a form of dynamic self-similarity that has some resemblance to fractal self-similarity. That is, if the ratios of event frequencies to average supply rates are decreasing with time as the system as a whole evolves to larger and larger sizes, then there is a great deal more “length” to the time record (more oscillations for the same measure of time) in the smaller systems relative to larger systems (see earlier discussion of fractal length relations). The fact that this tends to occur while the overall rate of magma supply is roughly constant means that the source of the higher frequency oscillations is perpetually present even if it is manifested in eruptive processes having long repose times. This means that on a graph of magma energy fluctuations (as contrasted with a record of erupted volumes of lava) a sufficiently long term record would be very dense with lines (consider the Kilauea record of figure 51.7 extended one thousand years and plotted at the same page size). The analogous map of a coastline length would be so irregular that, somewhat like an ammonite’s sutures carried to the extreme, the lines would be essentially area-filling. The area-filling limit for a plane has the fractal dimensional limit of two (it can’t be higher unless the map is plotted within a three-dimensional volume); less dense packing gives fractal dimensions between one and two for the coastline problem. By this kind of analogy the simplistic periodicity relation above resembles the fractal limit of two. Such ideas are very important to concepts of what constitutes “live,” “dormant,” and “extinct” volcanoes, and to the evaluation of possibilities for sudden changes in style (compare Shaw, 1985). The fractal viewpoint considers all such systems equally active but manifesting that activity according to differing “metabolisms” (the suggestive parallel with the relation between sizes and metabolic rates in animals would require a careful assessment of the respective energy budgets in a detail not presently possible).

There is some suggestion of antipathetic alternations of periodic phase between Kilauea and Mauna Loa eruptive activity, as shown in figure 51.7, in that over about 30-yr intervals, when one is at relatively high average rates, the other is at relatively low rates (compare 1920–50 and 1950–80). If this is true, then we might expect increasing eruptive activity at Mauna Loa and decreasing activity at Kilauea within the next few decades (see Klein, 1982). The uncertainty in the ratio of eruptive and intrusive volumes for Mauna Loa, however, precludes rigorous comparisons; we do not have information on the Mauna Loa supply budget that is comparable to the data of Dzurisin and others (1984) for Kilauea. On the time scale of figure 51.1B, there are also vague indications of alternating variations between adjacent loci, but during other time intervals adjacent loci appear to be essentially synchronous.

The overall Hawaiian-Emperor curve of cumulative volumes is shown in figure 51.8, scaled down by a factor of 10⁶ in both time and volume and superimposed on the 100-yr scale of Kilauea’s eruptive volumes. This comparison demonstrates that a crude scale invariance exists between the growth of lava volume at Kilauea and

![Figure 51.8](image_url)
the long-term growth of overall edifice volumes for the chain as a whole. This is interesting because the long-term growth history must include the intrusive volumes within the edifice, which have grown at approximately twice the extrusive growth rate during historical activity (see fig. 51.4 and earlier discussion). Thus, the overall rate of edifice growth on the 100-yr scale (intrusive plus extrusive) should be about three times the extrusive rate if the present pattern existed at all times in the past. That is, if this ratio showed up in the comparative record of figure 51.8, the average slope of the long-term record would be about three times the slope of the cumulative curve of lava volume for Kilauea. Therefore, although the youngest part of the long-term record must at present equal the total supply of both Kilauea and Mauna Loa, the fact that the cumulative curve of edifice growth as a whole over about 56 m.y. essentially parallels the 100-yr extrusive curve suggests that Kilauea's extrusive record may faithfully mimic variations in growth rates of the Hawaiian Ridge. If so, the higher intrusive-rate variations indicated by the 30-yr record of figure 51.5A might suggest that similarly large relative rate variations also apply to yet deeper components of intrusive magmasupply rates. This would imply the existence of an intrusive magmatic keel at depths greater than the ocean floor with a volume approximately twice the edifice volume. An alternative is offered by the possibility of a compensating rate of loss of volume related to subsidence; Shaw (1973, p. 1523) shows that possible subsidence rates related to the moving front of volcanic landforms could be equivalent to a substantial fraction of the observed magma-supply rate. The distinction is largely one concerning the depth range of intrusive storage in the volcanic edifice and its substructure (compare Shaw, 1980).

In either case, the 3-to-1 proportionality of total supply to extrusive volume, when applied to the edifice as a whole, implies an isostatically hidden root system representing the passively intruded magma in the crust-mantle section underlying the ridge system. By this circumstantial inference the average overall magma-supply rates over long times could be as high as 0.3 km/yr beneath individual loci, also implying transient fluctuations of edifice supply rates of comparable magnitudes over short periods of time. Such an interpretation may explain the abrupt terminations of the volumetric records along loci in figure 51.1B; that is, the volume records sometimes end while the slopes are still very steep, suggesting that unless magmatic volumes and (or) generation rates in the asthenosphere are suddenly turned off or switched to another locus, the terminations may reflect an increased rate of deep over shallow intrusive storage.

Shaw and others (1980) argued that the total volume of magma supply from the asthenosphere could be about a factor of 2 larger than their tabulated values; the above inference suggests that they may be even larger. J.G. Moore (oral commun., 1985) has also emphasized the need to account for larger isostatically hidden volumes on the basis of studies of the chronological histories of drowned terraces along the Hawaiian Archipelago; volumetric corrections suggested by his data are similar to the above estimates.

The next section explores the relativity of rate episodes and self-similarity in terms of possible physical mechanisms of feedback, and the subsequent section discusses possibilities for self-induced periodicities. Questions concerning correlations with external forcing functions and issues of possible determinism as contrasted with self-guiding processes in volcanic evolution are considered in a closing section.

**DISSIPATIVE FEEDBACK AS A SOURCE OF PERIODIC RATE VARIATIONS**

This section tries to map out in an approximate way some of the factors that influence the variability of magma production and transport. As mentioned in the introduction, the rationale for discussing dynamic and thermomechanical feedback processes is to identify the basis, and the need, for treating magmatic processes as systems of interacting and self-influencing mechanisms of transport that evolve in space and time. This section describes some of the interactions that tend to focus activity within certain ranges of a general dynamical hierarchy that is characterized by the variety of rate signatures discussed in the preceding section. In the subsequent sections, ideas of rate-controlled feedback are explored in a format that attempts to connect physical ideas of feedback with mathematical ideas of feedback according to concepts of attractors and evolutionary strategies previously outlined. The reason for such approaches is to begin to build toward concepts of cyclical magmatotectonic processes that have as their natural outcome the patterns of plutonic and volcanic behavior seen in the worldwide record of igneous provinces interacting with analogous cyclical processes of plate tectonics (and possibly also more exogenous influences; see comments at end of introduction).

The approach in this section is based on the thermomechanical relations discussed in Shaw (1969); for consistency, the same sets of physical properties are used here. Since that paper was written, however, several discussions of so-called "thermal feedback," "thermal runaways," and of the effects of variable viscosity, variable rheologies, and other dissipative phenomena in magma transport have appeared in the literature. Hardee (chapter 54) cites some of these studies that are relevant to volcanism and gives calculations demonstrating applications to concepts of magma transport in Kilauea's rift systems.

The present section focuses on comparisons between characteristic times associated with dissipative heating, cooling by thermal conduction, and variations in viscous and elastic states of stress, and on their possible implications for timing of magma transport. The aim is to delineate regimes of behavior without analyzing specific events. It is found that dynamic parameters tend to group naturally around values characteristic of the time and size scales of the observed ranges of volcanic behavior. From the viewpoint of feedback systems, such dimensional correspondences exist because they are produced by a common process of evolution. The association of characteristic dike or conduit dimensions with characteristic variabilities of flow rates is not externally imposed, but occurs as a product of the tendency for balances between competitive thermal, dynamic, and mass-transport phenomena.
The balances between viscous heating and conductive cooling of basaltic magma subjected to a system of shear can be expressed in a nomogram (fig. 51.9). It applies strictly to plane shear with constant forcing, but it also semiquantitatively identifies regimes appropriate to forced flow in tabular conduits. The lines on the nomogram identify half-thicknesses representing approximately equal times for heating by viscous dissipation and cooling by conduction. Higher stresses or lower initial viscosities relative to a given line lead to positive thermal feedback (conditions of thermal "runaway") in the manner illustrated by Gruntfest (1963) and by Shaw (1969, figs. 6-9) and as discussed by Hardee (chapter 54). The lines represent the balance \( t_c = \tau_w \), where \( t_c \) is a characteristic time for conductive cooling and \( \tau_w \) represents the time during which viscous heating would achieve very high temperatures in the absence of heat losses or decreases in shear stress. Table 51.1 summarizes the physical parameters used in the thermomechanical calculations, and

![Figure 51.9 - Dynamic range of parameters characterizing steady-unsteady thermal-feedback transition defined in terms of equal values of adiabatic viscous-heating time (\( t_w \)) and thermal-diffusion time (\( t_d \)) for planar shear layers of the indicated half-thicknesses (see tables 51.1 and 51.2 for definitions); diagonal lines represent values of shear stress and initial viscosity for a given slab thickness (and time) that satisfy equality \( t_c = \tau_w = t_d \). If \( t_c > \tau_w \), thermomechanical instability (or "thermal runaway") is likely; this occurs for values of shear stress and viscosity above and to left of each line for a given half-thickness of shear flow. Because \( t_c \) depends only on thickness, each line is associated with a characteristic time, as shown. Two areas identified by crosshatching represent examples of reasonably well delineated conditions for, respectively, (A) laboratory viscometry (Shaw, 1969) and (B) viscous conduit flow on east rift zone of Kilauea (Hardee, chapter 54; Decker, chapter 42); east-rift zone behavior was plotted on basis of a shear-layer half-thickness on the order of 1 m and an effective shear stress on the order of 10^7 dynes/cm^2 (1 MPa). This, in turn, implies an initial viscosity at onset of intermittent shear flow on the order of 10^{11}-10^{12} dynes/cm^2 (10^{10}-10^{11} Pa s); a compatible crystallinity might be in the neighborhood of 30-40 percent (Shaw, 1969). For additional discussions of thermal feedback, see Shaw (1969) and Hardee (chapter 54).

### Table 51.1. Physical parameters used in analysis

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Unit</th>
<th>Reference values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Temperature coefficient</td>
<td>cm^-1</td>
<td>1 x 10^{-2} (1-phenix)</td>
</tr>
<tr>
<td></td>
<td>of viscosity</td>
<td></td>
<td>2 x 10^{-1} (2-phenix)</td>
</tr>
<tr>
<td>B</td>
<td>Heat capacity</td>
<td>erg·cm^-3·s^-1·K^-1</td>
<td>3 x 10^7</td>
</tr>
<tr>
<td></td>
<td>(volumetric)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Effective ( \varepsilon ) for crystal + liquid</td>
<td>erg·cm^-3·s^-1·K^-1</td>
<td>1 x 10^8</td>
</tr>
<tr>
<td>D</td>
<td>Shear modulus ( \text{I} )</td>
<td>dyna·cm^-2</td>
<td>-10^11</td>
</tr>
<tr>
<td>E</td>
<td>Thermal conductivity</td>
<td>erg·sec^-1·cm^-3·K^-1</td>
<td>2 x 10^5</td>
</tr>
<tr>
<td>F</td>
<td>Permeability</td>
<td>cm²</td>
<td>10^{-11} (1 mdarcy)</td>
</tr>
<tr>
<td>G</td>
<td>Half-thickness</td>
<td>cm</td>
<td>Variable</td>
</tr>
<tr>
<td>H</td>
<td>Displacement rate</td>
<td>cm·sec^-1</td>
<td>Variable</td>
</tr>
<tr>
<td>I</td>
<td>Diffusivity of pressure</td>
<td>cm²·sec^-1</td>
<td>2 x 10^3 (water)</td>
</tr>
<tr>
<td></td>
<td>( (= k/k_w) )</td>
<td></td>
<td>2 x 10^5 (melt)</td>
</tr>
<tr>
<td>J</td>
<td>Compressibility</td>
<td>cm²·dyna^-1</td>
<td>5 x 10^{-11}</td>
</tr>
<tr>
<td>K</td>
<td>Initial viscosity</td>
<td>dyna·sec·cm^-2</td>
<td>Assigned (or derived)</td>
</tr>
<tr>
<td>L</td>
<td>Pore fluid viscosity</td>
<td>dyna·sec·cm^-2</td>
<td>10^{-3} (water)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10^{-5} (melt)</td>
</tr>
<tr>
<td>M</td>
<td>Porosity ( \varepsilon )</td>
<td>fraction</td>
<td>0.1</td>
</tr>
<tr>
<td>N</td>
<td>Shear rate</td>
<td>sec^-1</td>
<td>Variable (or assigned, or derived)</td>
</tr>
<tr>
<td>O</td>
<td>Thermal expansion ( k )</td>
<td>cm³·K^-1</td>
<td>-10^{-3}</td>
</tr>
<tr>
<td>P</td>
<td>Coefficient of pore fluid</td>
<td>cm²·dyna^-1</td>
<td>-0.5</td>
</tr>
<tr>
<td>Q</td>
<td>Dynamic friction</td>
<td>cm²·K^-1</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>Density</td>
<td>gm·cm^-3</td>
<td>Variable</td>
</tr>
<tr>
<td>S</td>
<td>Shear stress</td>
<td>dyna·cm^-2</td>
<td>Assigned or derived</td>
</tr>
</tbody>
</table>

1. The shear modulus is not a constant in mixed media such as magma and partial melt systems. In cases where variable G is possible the symbol G is used; a range of possibilities for magma is indicated in figure 51.11.

2. The porosity \( \varepsilon \) appears only in the definition of \( \alpha_p \) and is not to be confused with shear rate \( \varepsilon \) which appears in the time-scale ratios of tables 51.2 and 51.3.
table 51.2 identifies the groups of these parameters that define the intrisic time scales of thermomechanical processes.

The vicinities of the crosshatched areas in figure 51.9 identify conditions resembling those of laboratory viscometry (Shaw, 1969; crosshatched area A in fig. 51.9) and the conditions for eruptions along the east rift zone of Kilauea during the 1983–84 activity as analyzed by Hardee (chapter 54; crosshatched area B in fig. 51.9). Periodic instabilities are possible at recurrence times of the indicated magnitudes. That is, in the laboratory viscometer, if shear energy were continuously supplied at approximately 10^5 dyne-cm^2 (10 kPa) at a mean viscosity varying from 10^3 to 10^4 poise (dyne·s/cm^2, or 10^2–10^3 Pa·s), the local values of shear rates and shear stresses would oscillate with an irregular periodicity of about 1–20 min. Similarly, at about 10^7 dyne/cm^2 (1 MPa), the average oscillatory periodicity would be from about one week to a few months for a system of dike-like forced shear flow (rectangular slot or pipe flow) with half-thicknesses on the order of 1 m and an initial static viscosity of 10^11 or 10^12 poise (1–10 GPa·s; roughly one-third crystallized). Such a range suggests one kind of contribution to characteristic recurrence times for repetitive eruptions.

Hardee (chapter 54) shows that equivalent arguments can be expressed in terms of either dissipative heating or nonlinear magma rheology (see Shaw, 1969). The important point for the present discussion is that there are intrinsic properties and dynamical conditions that promote crudely periodic phenomena and under some sets of conditions possibly even regular periodic phenomena.

The relations of figure 51.9 are summarized in figure 51.10 over a range of conditions that span most of those possible in terrestrial shear flows (excluding instantaneous fault-slip, explosive, or impact phenomena). The shaded fields represent broad classes of categorically similar behavior; for example, the shaded field labeled "order of 1 m" corresponds in an order-of-magnitude sense to the approximate monthly variations of the 1983–84 east-rift-zone activity, and the field labeled "5–50 m" corresponds to the historical variability of Kilauea and Mauna Loa with an event frequency in the range 3–300 yr. The large field at the highest viscosities and stress magnitudes spans the ranges of variable activity from the scale of individual volcanic systems (less than one to several million years) to the global scale of propagating volcanic systems and igneous cycles seen throughout Earth's history.

The above scale relations refer only to thermal balances for a given level of continuous forcing, as reflected in the shear-stress magnitude. Therefore, an important measure of variability of the thermal balance, in addition to the time scale of heat transfer, is the time scale of loss and recovery of stress magnitude associated with a given rate of motion. Table 51.1 indicates that a scaling factor for shear stress is given by the ratio of dynamic viscosity (η) to elastic shear modulus (G), or equivalently by the ratio of the shear stress (τ) to the product of shear rate (d) and elastic shear modulus. This latter ratio is sometimes called the Maxwell or viscoelastic relaxation time.

The physical meaning of stress relaxation can be visualized in terms of the shear rate, which is the ratio of shear displacement per unit time (R, in cm/s) to the thickness of the shear layer (L). Thus, the reciprocal 1/L represents the time associated with a 100-percent
shear strain, and the product \((\sigma/R)(1/\varepsilon)\) is that time normalized by the ratio of viscous to elastic stress components. The smaller the viscous shear stress relative to the elastic modulus, the shorter is the time for either the loss or recovery of stress equilibrium across the sheared layer. A very high shear stress at a given displacement rate, hence a given apparent viscosity of the layer, implies a relatively longer time for a large shear disturbance on one side to permeate or diffuse across the layer. For the conditions illustrated in figures 51.9 and 51.10, however, the ratio \(\sigma/R\) is less than 1, meaning that the rate of stress equilibrium is rapid relative to the time required to achieve unit shear strain in viscous flow. Nevertheless, the ratio must be taken into account because of the potentially unlimited shear strains of viscous flow.

The effect of incorporating the stress response time with the thermal relations of figures 51.9 and 51.10 is illustrated in figure 51.11; the circled points represent the formal identities \(t_\varepsilon = t_l = t_w\) (for \(\omega = t_l\), stress constant). This means that for conditions that are both above the hatched band (symbolizing elastic responses typical of rocks) and above the 45° lines (where conductive cooling is slow relative to dissipative heating), profound melting instabilities ("runaways") can persist over times longer than those indicated on the diagram or in figures 51.9 and 51.10. For conditions of smaller elastic moduli that may apply to magma (media with a liquid fraction, above an unspecified minimum threshold, have greater compliance in shear), the stress level of this regime would be lower in proportion to the scale of the effective modulus, \(G^*\), as shown at the right in figure 51.11 (see footnote 1, table 51.1); values of \(G^*\) vary with melt fraction, so numerical values are equivocal. Within regions below the hatched band but still above the 45° lines, thermal instabilities can begin at the indicated stress levels (for example, above about 10^6 dyne/cm^2 or 1 MPa for meter-scale shear couples), but they do not persist over the indicated timespans. That is, within these regions the melting instabilities are intermittent and oscillate according to the stress-recovery times; these relations are generally in accord with stress magnitudes associated with the variations described earlier.

Although the labeling of fluid and solid states on such a diagram is relative and simplistic, oscillatory fluid phenomena, possibly such as cavitating flow and tremor, are associated with low apparent viscosities, whereas shear melting of nearly solid aggregates will occur at the highest viscosities; these observations are also consistent with the great span of time scales of the respective processes (the order of a second or less in cavitating flows, the age of the Earth in mantle-wide shear flows).

The above approach to dynamic regimes can be generalized even more in terms of conditions describing fluid migration, either by percolation as a pore fluid or by percolation as injection pulses associated with extensional fracture events (Lachenbruch, 1980; Shaw, 1980). The analysis parallels that of the thermomechanical balances discussed in Shaw (1969), except that fluid pressure, effective stresses (Shaw, 1980), and fluid mobility are coupled with the thermal responses. An analysis for water as the pore fluid is given in more detail by Lachenbruch (1980) in connection with interpretations of heat-flow and stress variations associated with fault motions. A synopsis of analogous effects is repeated here in terms of ratios of response times, using an approach similar to that of figures 51.9–51.11.

Table 51.3 lists the 10 possible ratios of the time scales given in table 51.2 and gives for each of these ratios (where possible) the relation between the logarithm of shear stress \(\sigma\) and the logarithm of the product \(RI\) of displacement rate \(R\) times half-thickness \(l\) of the shear layer. Equations are written for the condition \(t_\varepsilon = t_w = t_l\). The quantity \(RI\) is a convenient way to represent deformation rate in viscous systems because it explicitly identifies the roles of both the size of the shear layer and the displacement speed. The coined term "diffusivity of action" is used here for \(RI\) because the product of displacement rate and thickness has the dimensions centimeters squared per second, as in thermal, chemical, or momentum diffusivities (its interpretation, of course, depends on the context considered, for example, faulting or magmatic flow, and the chosen

![Figure 51.11](image-url)

**Figure 51.11.** — Nomogram constructed as in figure 51.9, including stress response times (see table 51.2). Vertical lines represent values of \(t_\varepsilon = \eta/G\); circles represent locus of identities \(t_\varepsilon = t_w = t_l\). Also, \(t_\varepsilon = \eta/G\) only above horizontal interval (hatched) for value of shear modulus, \(G\), in table 51.1. Hence, stress responses (both relaxation and recovery) of melt-free rocks are rapid relative to thermal feedback responses at stress levels appropriate to all conditions shown in figures 51.9 and 51.10. Consequently, intermittent thermal instabilities automatically imply, and (or) are implied by, oscillating stress regimes, unless shear modulus is decreased significantly in the molten state (see table 51.1, footnote 1, and text for explanation of effective shear modulus \(G^*\)). When that happens, upper stress limit of combined intermittent stress and thermal-feedback regime is lowered and is indicated by scale of log \(G^*\) at right; consequently, thermal instabilities related to melting may occur at effectively constant shear stress under some conditions that permitted intermittent instabilities in solid state (see Shaw, 1969). However, conditions above horizontal limits \(t_\varepsilon > t_w\) are artificial in that thermal feedback at constant stress is necessarily transient; thermal-feedback instabilities inevitably alter material states, so that intermittent stress regimes eventually result. Shear melting events may nevertheless be of large magnitude when \(t_\varepsilon > t_w\).
VOLCANISM IN HAWAII

TABLE 51.3.—Reference equations based on possible equalities of pairs of time scales in table 51.2 using values of constants in table 51.1

[There are 10 possible combinations of the time scales defined in table 51.2; see text for discussion]

<table>
<thead>
<tr>
<th>Time-scale ratios</th>
<th>Nonmagnetic (solid + aqueous fluid)</th>
<th>Magmatic (melt + solid)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{t_e}{t_p} )</td>
<td>( \log \sigma = 12.9 + \log(P) )</td>
<td>( \log \sigma = 13.4 + \log(P) )</td>
</tr>
<tr>
<td>2. ( \frac{t_e}{t_w} )</td>
<td>( \log \sigma = 7.6 - \log(P) )</td>
<td>( \log \sigma = 6.3 - \log(P) )</td>
</tr>
<tr>
<td>3. ( \frac{t_e}{t_E} )</td>
<td>3 \times 10^5 (constant; ( t_E \neq t_p ))</td>
<td>10 (constant; ( t_E \neq t_p ))</td>
</tr>
<tr>
<td>4. ( \frac{t_E}{t_d} )</td>
<td>( \log (\frac{t_E}{t_d}) = -1.4 ) (for all ( \sigma ))</td>
<td>( \log (\frac{t_E}{t_d}) = -1.4 ) (for all ( \sigma ))</td>
</tr>
<tr>
<td>5. ( \frac{t_p}{t_w} )</td>
<td>( \log \sigma = 10.2 ) (for all ( P ))</td>
<td>( \log \sigma = 9.8 ) (for all ( P ))</td>
</tr>
<tr>
<td>6. ( \frac{t_p}{t_E} )</td>
<td>( \log \sigma = 7.4 + \log(P) )</td>
<td>( \log \sigma = 12.4 + \log(P) )</td>
</tr>
<tr>
<td>7. ( \frac{t_w}{t_p} )</td>
<td>( \log \sigma = 11.5 ) (for all ( P ))</td>
<td>( \log \sigma = 12.0 ) (for all ( P ))</td>
</tr>
<tr>
<td>8. ( \frac{t_w}{t_d} )</td>
<td>( \log \sigma = 13.1 - \log(P) )</td>
<td>( \log \sigma = 7.3 - \log(P) )</td>
</tr>
<tr>
<td>9. ( \frac{t_d}{t_w} )</td>
<td>( \log \sigma = 9.0 ) (for all ( P ))</td>
<td>( \log \sigma = 7.7 ) (for all ( P ))</td>
</tr>
<tr>
<td>10. ( \frac{t_d}{t_E} )</td>
<td>( \log (\frac{t_d}{t_E}) = 3.5 ) (for all ( \sigma ))</td>
<td>( \log (\frac{t_d}{t_E}) = -0.4 ) (for all ( \sigma ))</td>
</tr>
</tbody>
</table>

1 Equations are based on solving for either \( \log \sigma \) or \( \log (P) \) under conditions where \( \frac{t_e}{t_w} \) is possible (that is, when \( \frac{t_e}{t_w} = 1 \)); this is not possible for \( \frac{t_E}{t_w} \) with physical constants of table 51.1.

values of physical parameters in table 51.1, or values considered appropriate to other specific applications).

Among the 10 possible ratios, \( t_e/t_p \) is unique in being a constant for the parameters of table 51.1; it represents the comparative decay of a thermal pulse by conduction relative to that of a pressure pulse by pore-fluid flow, neglecting the heat-transfer effect of the flow. In this respect, then, it is a measure of the ease of permeation of fluids away from a thermal disturbance; hence, it also indicates conditions under which advective heat transfer should be important. Clearly, pressure decay is very fast for aqueous pore fluids relative to thermal conduction, but for silicate melt the two are more comparable (depending on melt viscosity, latent heat, and other factors).

Except for \( t_e/t_p \), all the other ratios in table 51.3 involve one or both of the quantities \( \log \sigma \) and \( \log (P) \). The equations that involve both terms show that, logarithmically, they are directly proportional, so that their graphs are straight lines with slopes of either +1 or -1. Equations for which shear stress (\( \sigma \)) and (or) diffusivity of action (\( P \)) are constants plot as straight lines parallel to either an axis representing \( \log \sigma \) or one representing \( \log (P) \).

Numerical values of these independent variables calculated from the equations of table 51.2, based on the reference parameters of table 51.1, are plotted in figures 51.12 and 51.13. These figures attempt to show how instabilities characterized by the range of time scales in figures 51.9–51.11 can also be expressed as functions of stress and displacement histories. This is not the place to attempt a complete description of the ramifications of such diagrams; their implications address aspects of rock deformation in a manner somewhat similar to the use of thermodynamic-facies concepts in metamorphic petrology. Instead, I focus on the regions of likely periodic variations important to volcanic phenomena, following the approach already discussed in reference to simplified thermal feedback.

The dots in figure 51.12 and the open circles in figure 51.13 mark the invariant intersections of common sets of equations in table 51.3, as listed in figure 51.12. The heavy lines in these figures represent functions based on the variables \( t_e, t_p, \) and \( t_w \), as in figure 51.11, except for the different choices of independent coordinates. Invariant points c (figure 51.12) and c' (figure 51.13) have the same implications as the circled points in figure 51.11. The hachured triangular regions to the right of c and c' represent conditions of oscillating instabilities limited by the stress–response times, as illustrated in figure 51.11. Figure 51.12 shows the response of a melt-free solid in which the instabilities would take the form of shear fractures rather than melting. If shear melting is allowed, however, the difference between c and c' in figure 51.13 indicates the likely effects of differences in physical properties arising from the inclusion of latent heat and different temperature dependence of viscous activation (see Shaw, 1969).

Points d (fig. 51.12) and d' (fig. 51.13) and the respective regions to their right are related to coupled effects of shear heating, consequent fluid–pressure responses, and tendencies for flow accord-
Figure 51.12.—All possible dynamic regimes involving thermal feedback, stress oscillations, and aqueous pore-fluid transport according to ratios in table 51.3 based on combining thermomechanical analyses from Shaw (1969) and Lachenbruch (1980). Nodal points labeled a–f represent possible invariant identities for variables shear stress, \( \sigma \), and what I call “diffusivity of action,” \( R \), where \( R \) represents displacement rate of one side of a shear layer relative to other, and \( i \) is half-thickness normal to layer. Heavy lines indicate fields delineated by thermal feedback in absence of either melting pore-fluid transport. Acute angle defined by points d, c, and e represents conditions where thermal feedback results in endothermic processes of fracture rather than pervasive melting; because such fracture would be activated intermittently, it is labeled the region of “dry stick-slip.” Analogous region involving pore-fluid weakening to right of line 9, below line 5, and above line 9 is labeled “wet stick-slip”; it occurs because thermal response is faster than both pore-pressure decay and time for weakening due to pore-pressure response to heating (termed “Darcian feedback”). Below line 9 and to right of line 8, however, Darcian response is faster than both dry thermal response and decay of pore-fluid pressure, hence that region is labeled “oscillatory Darcian feedback”; see Lachenbruch (1980) for a detailed analysis of Darcian regimes appropriate to displacements within fault zones.

Figure 51.13.—Dynamic regimes as shown in figure 51.12 but also including conditions of transport where silicate melt is pore fluid. Dashed lines, unprimed letters, and related shading from figure 51.12; arrows indicate direction in which Darcian regimes shifted when melt is pore fluid. Analogous invariant interactions reflecting properties of solid-melt interactions are indicated by primes (see parameters in table 51.1). In general, fields described in figure 51.12 are telescoped together when melt is pore fluid, and oscillatory Darcian feedback occurs at much lower shear stresses and displacement rates.

...ing to Darcy's law in response to the pressure gradient (for details, see Lachenbruch, 1980). Points f (fig. 51.12) and \( f' \) (fig. 51.13) mark the limits of regimes which I will refer to as conditions of "Darcian feedback." This phrase is used in the same context as thermal feedback, in the sense that a tendency for runaway (and eventual changes of state or decreases in effective stress) is countered by a transport rate. The components of negative feedback can be characterized by a thermally coupled pressure diffusivity in the case of Darcian feedback, and a simple thermal diffusivity (which may be an effective value that includes mass transport) in the case of thermal feedback.

In the above terms, the time scale \( t_\theta \) in tables 51.2 and 51.3 represents the time for reduction of frictional resistance owing to Darcian feedback. As such, it represents the relaxation or recovery times for effective stress, hence it also represents conditions of shear failure according to some failure law. A constant value of the coefficient of friction, \( \mu \) (see table 51.1), is equivalent to a Coulomb failure law, meaning that there is a constant angle of the failure envelope on a Mohr diagram (see Shaw, 1980, figs. 6 and 7). This point is raised because magmatic extensional fracture, as discussed by Shaw (1980), employs a modified Griffith-Coulomb failure law, implying variable and potentially large values of the ratio of frictional resistance to effective normal stresses, and also implying nonconstant values of increasing dynamic friction (\( \mu \)) as conditions approach those for pure extensional failure. This point will be reiterated in discussing the shifts of points c, d, and f to points \( c' \), \( d' \), and \( f' \) from the conditions for nonmagmatic regimes (fig. 51.12) to those representing magmatic regimes (fig. 51.13).

I attempt to focus the meaning of "dynamic regime" by discussing only the regions in the vicinities of points c, d, and f, or \( c' \), \( d' \), and \( f' \) in figures 51.12 and 51.13. In figure 51.12, the region near c to the right of line 2 and below line 5 has already been mentioned as identifying oscillatory, thermally coupled shear-failure events (there is no involvement of pore fluid as part of the onset conditions, although melt or pseudotachylite could be a product of failure; if so, the validity of the regime boundaries is modified at the instant of failure). Repetitive failures could be described as episodes of reduced and accelerated displacement rates resembling laboratory
stick-slip. To the right of \( d \), below line 5 and above lines 8 and 9 (fig. 51.12), a similar situation occurs, in the sense that the thermal transient \( t_r \) is short relative to both the loss of induced pressure \( t_{ip} \) and the Darcian response time \( t_d \), meaning that analogous shear failures would occur in wet rocks for conditions of sufficiently low permeability, high stress (greater than about 10^8 dyne/cm^2 or 100 MPa), and high enough displacement rates (for example, about 1 cm/s for a shear zone 1 m in half-thickness). This regime is labeled "wet stick-slip" in figure 51.12. In figure 51.13, an analogous region to the right of \( d \) is implied with magma as the pore fluid; in this case, the magma-wetted stick-slip regime is nearly superimposed on the dry stick-slip regime, because of the changes in physical properties appropriate to magmatic environments.

Persistence of yet higher displacement rates, however, eventually creates a condition where the pressure-response time is longer than the Darcian response time. At that point, \( f \), oscillatory Darcian feedback becomes a possibility; that is, to the right of line 10, but above line 8 and below line 9. This represents the region where Darcian thermal coupling is faster than the loss of induced pressure by flow and also where the Darcian coupling is faster than simple shear heating in the absence of pore fluids. Thus, this region is labeled "oscillatory Darcian feedback" (see labeled area in fig. 51.12; in fig. 51.13 the same area is shown, and an arrow indicates the direction in which this region of oscillatory Darcian feedback is shifted when magma is the pore fluid).

The interpretation of points \( c', d' \), and \( f' \) in figure 51.13 is parallel to that of \( c, d \), and \( f \) with some qualifications. In particular, if a curved failure envelope, such as that implied by a modified Griffith-Coulomb failure law (Shaw, 1960, fig. 7) applies, the constancy of dynamic friction, \( \mu \), does not hold. In the limit of pure extensional failure, the ratio \( t_{ip}/t_d \) defining line 10 becomes indeterminate because of this fact; also, the effective permeability is governed by transport related to extensional fracture propagation. The combined effect, however, tends to reduce both the pressure-response time (related to greater ease of flow along dilatant fractures rather than through tortuous pore space) and the modified Darcian response time (because of the increasing dynamic friction associated with an increasing failure angle implied by a Griffith envelope). As a consequence of these offsetting tendencies, the more channelized flow of modified oscillatory Darcian feedback may not differ greatly from that implied by flow in porous media.

The general effect for pore fluids with magmatic properties appears to be to telescope the respective oscillatory fields of figure 51.12 to conditions near point \( f' \) in figure 51.13; that is, ordinary thermal feedback in magmatic systems implies the existence of a melt fraction. Hence, the general effect of increased transport brings the oscillatory field to lower stresses and higher diffusivities of action (higher displacement rates for the same values of half-thickness).

Values of shear stress implied by the field limited by point \( f' \) and lines 8 and 9 in figure 51.13 are within the range 10^8-10^9 dyne/cm^2 (0.1-10 MPa), which is reasonable based on other considerations for natural regimes showing oscillatory behavior (see Decker, chapter 42; Hardee, chapter 54; Shaw, 1980). If we take values of half-thickness ranging from the order of 1 m to 100 km, as in figure 51.10, values of compatible displacement rates, \( R \), can be estimated corresponding to log (RI)=0 in figure 51.13. These are, respectively, 10^-2 cm/s (~3 km/yr) and 10^-2 cm/s (3 km/yr). The former would be associated with mean effective propagation rates of dike-like injections through the lithosphere, and the latter with rates of mantle flow (the actual propagation rates for Hawaiian-Emperor lobes, however, vary as much as an order of magnitude relative to mean plate velocity, implying that the local half-thickness of associated asthenospheric flow may vary between about 10 and 100 km; see Shaw, 1973, and Shaw and others, 1980).

An analysis of magma-driven propagation of cracks by Spence and Turcotte (1985) gives a fracture-propagation velocity of 0.5 m/s for a half-thickness of 0.25 m and a fluid pressure of 6.3 x 10^8 dyne/cm^2 (63 MPa); this corresponds to a value of log (RI)=3. Although their analysis does not refer to shear-induced extensional fracture, the energy-rate balances resemble those in the field of modified oscillatory Darcian feedback. The examples plotted in figure 51.9 also plot in this general vicinity when they are converted to the independent variables of figure 51.13.

The main conclusion relevant to this paper from the considerations of dynamical time scaling is that repetitive feedback processes involving melting and magma transport as identified in figures 51.12 and 51.13 are likely to occur over a range of self-determined periodicities indicated by the estimates of time in figure 51.10. Feedback interactions of complex processes tend to be self-focusing in that the likely effects converge toward limited ranges of dynamical parameters. Over a long enough time, therefore, the temporal variability and size scales of geometric structures associated with such systems of interactive feedback are not simply imposed by external constraints on geometric states and forcing functions, as in some laboratory experiments, but become artifacts of the dynamical history. Thus, temporal, spatial, and dynamical variability interact repetitively over all the magnitude scales considered. The crude and repetitive self-similarity shown by the volcanic record as portrayed in figures 51.1-51.8 evidently records elements of dynamical similarity that have evolved according to feedback processes. Many different mechanisms exist at each scale of the general feedback process, and much can be learned through inverse reasoning about these mechanisms (and the need for more information concerning them) by considering their conceptual interactions (for example, the mechanisms and properties of magmatic percolation can be viewed in a manner parallel to the hydraulic of pore fluids as described by Lachenbruch, 1980). This generalization does not preclude interactions with external forcing functions, such as extraterrestrial energy influences mentioned later. In the next section I consider a logistical approach to the description of self-induced periodicity.

STATIONARY STATES AND ENDOGENOUS PERIODICITIES OF ATTRACTION IN THEIR NEIGHBORHOOD

This section, like the previous one, contributes to the idea that there are logical reasons for magmatic systems, and volcanic systems in particular, to be considered as feedback systems subject to the general mathematical principles of recursion outlined in the introduction. In the preceding section, it was found that the repetition of
magmatic processes acts to modify both the dynamic and geometric conditions of localized behavior in a way that leads to focusing of those conditions within limited and characteristic regimes that form a part of a hierarchy of such regimes from the local to global scales. In the present section, the feedback process is viewed as a recursion of characteristic quantities in a way that generates repetitive structures in space and time. The quantities discussed in this section are totally hypothetical, but they can be viewed as sets of data representing imaginary intrusive and (or) volcanic magmatic cycles described in terms of relations between mass, volume, and their rates of change. The purpose is to provide a description in which ideas of dynamic feedback and what may be called "logistical" feedback lead into considerations of the numerical properties of recursion that are treated elsewhere by the theory of attractors (with their implicit relations to fractal self-similarity mentioned earlier).

Whereas the previous section considered conditions consistent with feedback among physical mechanisms, this section considers aspects of logistical processes that are consequences of repetitions involving combinations of net positive-negative feedback rates. That is to say, this section examines the logistics of what I call folded feedback. First, I give working definitions of what I mean by "logistic" and "folded," because both terms are used in a variety of contexts in different disciplines. The usage here is descriptive and does not necessarily correspond to formal definitions found in the literature of economics, biology, mathematics, topology (and catastrophe theory), physics, or other fields.

I use "logistic" to indicate that the method of analysis originates in the logic of numerical sequences (as in the construction of a computer algorithm) and also in the sense of military logistics, where the responsibility entails management of processes affecting aspects of material distributions which support the strategy of operation with or without any absolute knowledge concerning the plan of action. In other words, logistical analysis is the mathematically logical manipulation of quantities, whether they are real or abstract. The quantities can be anything measurable in the sense that there is some kind of unit that can be counted. Depending on the quantities manipulated, the logistical approach in this way resembles the logic of chemical mass balances, physical kinematics, dynamics, and number theory.

"Folded feedback" is shorthand for the net effects of feedback (and feedforward) processes operating in a complex system that contains aspects of both positive (accelerative or regenerative) and negative (decelerative or degenerative) responses. These are the composite processes of the previous section considered to be characteristic of a complex system even in the absence of an ability to dissect, analyze, and describe all mechanisms. The term "feedforward" is sometimes used to describe an input designed to exercise some guiding influence (perhaps via indirect or ancillary information loops) on a following stage of an operation (examples are given by aspects of industrial plant operation or by a satellite-guidance system). It is contrasted with feedback control, which loops output information directly back into the guidance mechanism. Feedforward is subsumed within my meaning of folded feedback and is not subsequently used herein. Thermal and Darcian feedback offer examples of both types of control, depending on the nature of coupling with other processes. Strict thermal feedback in an adiabatic system represents a direct control loop of dissipative output back into the system as an input via the temperature dependence of the stress and shear-rate relations (apparent viscosity). It may be either positive (constant source of shear stress) or negative (constant displacement rate). Thermal feedback also offers a clear demonstration that negative feedback is not necessarily synonymous with stabilizing feedback (see Grunfest, 1963; Shaw, 1969, p. 531).

The positive and negative aspects of thermal feedback demonstrate some of the general aspects of folded feedback. Folded feedback can be thought of as the combined effects of coupled operations involving a net increase in a quantity followed by a net decrease, for example, an up followed by a down or an in followed by an out. The key idea is that the repetitive action of both kinds of effects as a net consequence of all operative mechanisms, however numerous and complicated they may be, is folded in as an essential aspect of feedback-system operation.

Logistical analyses of biological populations represent examples of folded feedback where there are numerous and complex sets of specific feedback mechanisms. Simplest illustrations are provided by laboratory deformation of elastic solids and viscous liquids. For an elastic solid, it is easy to maintain a constant stress (below the failure stress), whereas the attempt to maintain a constant shear rate leads eventually to a rising shear stress and failure. Continued shear may then lead to alternating conditions of sticking and slipping along the plane of fracture, with stress oscillations governed by the normal stresses and coefficients of sliding friction (thus conforming to a specific kind of folded feedback). For a liquid, on the other hand, it is usually possible to maintain a constant shear rate, at least below a critical speed; when one attempts to maintain a constant shearing stress, however, thermal feedback leads to an increasing shear rate. If the layer thickness is large enough, the adiabatic condition is approximated, and the shearing rate is limited only by stress relaxation or by the maximum shearing rate attainable with the instrument, according to the relations already discussed. Because magmatic systems involve both types of behavior and because natural deformation may involve potentially large values of both shear stress and shear rate, conditions tend to oscillate between those approximating constant stress and those approximating constant shear rate, thus implying folded feedback.

These tendencies are schematically summarized in figure 51.14, a type of diagram designed to distinguish between present and future states. The diagonal line at 45° represents those conditions in which the future state of melt concentration is the same as the present state. That is, thermal or mechanical energy is supplied in amounts just sufficient to maintain the status quo. Above the 45° line, each state represents an increase of melt fraction relative to the immediately preceding state; below the line, each state is diminished relative to the immediately preceding state by the combined effects of thermal loss associated with decreased rate of energy dissipation, conduction loss, and melt transport from the region of shear.

The dashed line in figure 51.14 signifies typical paths involving states of increasing melt fraction during episodes of shear melting or intrusive melt transport followed by states of decreasing melt fraction owing to combined effects of stress drop, limitations on shearing rate,
positive (input) rate minus the positive value of the negative (output) rate; the qualifier "apparently" is used because this convention is not free of ambiguities, as will be discussed (possibilities of ambiguity exist in any measurement made up of integrated positive and negative increments, where both are taken as positive numbers, because unless there is absolute knowledge of the exact amount and sign of every relative change, a net increase could result if there is either a greater increase in the positive than the negative term, or a greater decrease in the negative than the positive term; the balance between biologic originations and extinctions over time is a notorious example). Symbols designating positive components of the net rate can be thought of as the rates and amounts of addition or input to some general reservoir of quantity, and symbols designating negative components as the rates and amounts of the output or depletion of that reservoir (also written as a positive number).

An example illustrates that the above convention is a common one. Imagine a fuel tank with an input valve and an output valve, each equipped with a flow gauge calibrated the same way. Each gauge registers a positive reading when input and output flow are occurring simultaneously. Their differences integrated over time give the net change of fuel in the tank. An idealized magma chamber can be thought of in the same way if there is the recognition that the reservoir is of unspecified boundaries and size, and there are no predetermined values for initial and final states; also, input and output paths may be numerous, and flow rates are based on less direct evidence than a fuel gauge. This means that the volumetric states of magma chambers are relative to an arbitrary zero at some defined location and time, as is the case in figures 51.1–51.8.

To assist interpretation of the numerous stages in the construction of figure 51.15, as well as subsequent figures, it should be recognized that not only are the figures drawn approximately (don't look for numerical values to check to better than a few percent), but seemingly contradictory numerical states may be noticed (an example is the quantity "x+" in fig. 51.15D; see stepwise description below). An example of materials inventories in commerce indicates how contradictory values can occur in a series of measurements.

The strategy of logistical analysis is sketched in figures 51.15–51.18 in terms of the variety of descriptive histories that may be associated with time variation of any general quantity symbolized by the variable x. Subsequent diagrams consider the volumetric histories of Hawaiian volcanism expressed in the same way, replacing the symbol x by V, using the documentation of transport rates by Dzurisin and others (1984).

Consistent with the approach to magma-supply balances taken by Dzurisin and others (1984), figure 51.15 is based on the idea that any net transport rate of a quantity x can be decomposed into apparent rates of gain and loss, or input and output, both expressed as positive numbers. The net rate, then, apparently is given by the

melt injection into neighboring domains, and crystallization due to cooling. The foldlike trajectories involving the net effects of positive and negative feedback in figure 51.14 (dashed lines) provide a framework for subsequent diagrams describing the evolution of volcanic volume states. Henceforth, however, the discussion is shifted from dynamics to a logistical description of transport rate.

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Consistent with the approach to magma-supply balances taken by Dzurisin and others (1984), figure 51.15 is based on the idea that any net transport rate of a quantity x can be decomposed into apparent rates of gain and loss, or input and output, both expressed as positive numbers. The net rate, then, apparently is given by the
51. UNIQUENESS OF VOLCANIC SYSTEMS

Figure 51.15. - Schematic diagrams showing time-variation and x-variation of positive and negative rates of change in any extensive quantity x. A. Hypothetical information concerning positive and negative rate components of the rate dx/dt, used to construct other diagrams. B. Respective integrated positive and negative contributions and net change in x. C. Respective values (dx/dt)⁺ and (dx/dt)⁻ versus the positive cumulative values of x, read from B at same values of time in A (x or net x could be used on abscissa instead of x). Dots, values of x, that tend toward stationary values (convergent arrows). Identifying values where there is relatively slow net increase in x; circles, points of divergence identifying intervals of relatively rapid net increase in x. D. Adjusted future values (x₁₂₃) versus present values referred to x. (x or net x could be used as reference; see F). Calculated from net rates of change (dx/dt)⁺ - (dx/dt)⁻ over a specified unit of time; these two graphs and that in F are analogous to intervals of folded feedback in figure 51.14 (see text for discussion of apparent contradiction between dictated trajectories in D and monotonic curve of x, in B). Dashed trajectories with arrows (D,F) show arbitrary constructs where projections indicate that future states are defined by present states. That is, among possibilities limited by rate history, if next value is dictated equal to present value, then next value of x, in graph D, or (x₁₂₃) in graph F, is given by horizontal projections from master curve (solid curve representing trend of available states) to diagonal line as locus of intersections of dashed trajectories; along diagonal, next value on abscissa equals immediately preceding ordinate value. Area labeled "region of prediction" in D represents uncertainties associated with possible trends beyond limit of record. E. Net rate plotted versus net values of x=(x₁₂₃). F. Corresponding loci of available present/future states and corresponding idealized limits of episodic feedback limited by net x and net x plus net rates relative to diagonal line of steady-state net balances (construction as in D for net x). Regions refer to different regimes of possible episodic feedback (see figure 51.16 and discussions in text). Prescriptions of future values determined by present values based on logistics of population-dependent biological reproduction (see May, 1976) and concentration-dependent feedback melting. Dashed trajectories in D and F represent idealized histories that may: (1) converge to a fixed value, (2) oscillate over some set of regular intervals around a fixed value, (3) oscillate chaotically in vicinity of fixed value, or (4) diverge to totally different neighborhoods of higher or lower x-values.
Types of regions of attraction

A Fixed-point

Stationary point

$x_{n+1}$

$x_n$

 transient excursions

steady limit

TIME

B Period 2

Stationary twofold oscillation

$x_{n+1}$

$x_n$

TIME

C Period 4

Stationary fourfold oscillation

$x_{n+1}$

$x_n$

TIME

D Chaotic

Explosive increase

Unstable

$x_{n+1}$

$x_n$

Implosive decrease

TIME

Chaotic values
of rate. That is, an input or output record always represents some finite relation between the counts of quantity and time. Notice that the graph in figure 51.15A does not specify the units of time. Whatever the choice, there is always a smaller or larger unit possible. Thus, any such curves really represent trends of average readings at an appropriate time scale. The materials inventory above might be taken per day, per week, and so on. At any smaller scale of time the possible measures of rates might be much larger numbers if short-term averages were linearly extrapolated to longer times (for example, if all items are received during one day while the inventory is taken weekly, an extrapolation of the one-day rate overestimates the weekly rate). Inversely, knowledge of longer term averages does not identify the ranges of rates taken over short intervals of time. Examples of this have already been seen in the record of Kilauea's eruption rates (figure 51.7) and in the discussion of fractal self-similarity.

The above kinds of uncertainties are encountered in the later illustrations of magma-supply balances at Kilauea. The budget described by Dzurisin and others (1984) makes the necessary assumption (without it no quantitative inventory could be made) that the inflation of Kilauea's summit magma input from deep sources to a near-surface reservoir ("summit reservoir"), and summit deflation documents magma supply to the rift systems from the summit reservoir. In a manner analogous to the uncertainties between the wholesaler-factory and wholesaler-retailer exchanges, this inventory does not explicitly document the possibility of exchanges having more continuous or higher frequency characteristics. For example,

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**Figure 51.16.** Schematic patterns of idealized oscillations relative to possible stationary points (dots) either or controlled by interactions of locus of steady states (dashed diagonal lines at left) with present-future control curves (dotted lines of present x₀ versus future xₙ₊₁) of available states in an interval of x (vertical light lines). Trajectories of oscillations in present-future values of quantity x shown by solid lines with arrows in left-hand column; corresponding oscillations for x-values in time shown in right-hand column (solid lines indicate time-independent limits of stabilized oscillations, dashed lines represent transient or unstable short-term and long-term deviations from time-independent limits). Several regimes of behavior represented, at least two of which are implicit. A, Small positive and negative deviations (folded feedback) occur relative to diagonal locus of steady-state limits, so that trajectories (solid lines with arrows) of feedback events involving neighboring values of x within interval (vertical lines) converge toward point of intersection (dot); this regime implies divergence toward lower values of x if dotted curve is totally below dashed diagonal but crosses it at right-hand vertical limit. B, Curves showing xₙ₊₁ relative to x₀ eventually split away from point of interaction in A, which can be thought of as bifurcating or fusing in a well-organized or relatively static alternation of x on opposite sides of steady-state locus; that is, stationary point in B becomes stationary zone or region that contains two stationary points counterbalanced across diagonal locus. Note that xₙ₊₁ not proportional to x₀. If scales 1:1, twofold bifurcation sequence begins when dotted curve exactly perpendicular to dashed diagonal line. C, As folded feedback amplitudes increase, stationary region grows in size and bifurcates or fusions into doubling sequence of counterbalancing stationary points; that is, there is a 4-fold bifurcation that will split to 8-fold, 16-fold, and so on at slightly higher feedback amplitudes. D. As sufficiently high feedback amplitudes, doubling sequence 2⁴ gives way to complex fusing events that include possibly regular odd-period repeats and irregular or chaotic trajectories, depending on exact amplitudes and curve forms; if this regime occurs totally within vertical lines, stationary region persists (even though it is "fuzzy" and may contain no stationary points); but if amplitude permits excursion of xₙ₊₁ to exceed vertical limit in x₀, eventually all trajectories escape to either larger or smaller values of x (called "sensitive dependence on initial conditions"). Values of x initially very close together (for example, magma concentrations within adjacent sites of a volcanic edifice) may rapidly diverge in opposite directions (that is, one set of trajectories escapes to higher concentrations, the other to lower concentrations).

Regions of stationary behavior called "basins of attraction"; term "attractor" stems from patterns of regular to chaotic oscillations within basins.

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**Figure 51.17.** Schematic rate histories and derived curves for situation in which net value of x both increases and decreases with time. A, Schematic curves of rate against time. B, Plot of net x against time, showing both increase and decrease rather than monotonic increase as in figure 51.15B. C, Rate plotted against net x; note that rate history repeats itself over intervals of x reversals. D, Present states plotted against future states of x. Resulting loop of folded feedback here and in figure 51.17C repeat themselves in various patterns of tangled and potentially closed loops. Thus, intervals of closure automatically define regions of stationary states that occur within more inclusive intervals of stationary states in a potentially unlimited series of invocations and convolutions. Trajectories describing stepwise feedback, as in figures 51.15 and 51.16, accordingly imply even more localized sets of potential stationary points and regions within a web of available states. In a sense, central curves in D resemble cyclical trajectories of stepwise feedback in more monotonic histories, such as those in figures 51.15D, F and 51.16 (see discussion of rate uncertainties and time scales in text); compare figure 51.18.
any magma that is continually leaking through the summit chamber into a rift system may not show up as an event in the inflation-deflation record (analogous to a blackmarket operation not recorded in a wholesaler's inventory). Also, no explicit accounting is made of any magma increments that may be analogous to material returns. While errors caused by this possibility can be discounted in the intrusive magma balances on physical grounds, they are possible in principle. A magmatic example where they are explicitly included in the budget is given by the data of Eaton and others (chapter 48, tables 48.1-48.3) describing the history of filling of Kilauea Iki lava lake during the 1959 eruption of Kilauea. There, returns from the ponded lava reservoir to the subsurface reservoir were significant, and there were also losses of pyroclastic components during fountaining.
As a consequence of considerations of these kinds, any rate curves such as those in figure 51.15A (and especially any that document a natural process) are really very fuzzy lines if viewed at finer scales of resolution. For example, compare figures 51.5, 51.7, and 51.2, in that order, with attention to the relative changes of time and amplitude scales. That is, the time scales increase greatly while the amplitude scales remain similar (see averages over different durations in fig. 51.7; compare fig. 51.8). I will return to a discussion of how such uncertainties relate to an interpretation of hypothetical trajectories of changing quantities in diagrams like figure 51.15.

The schematic rate curves in figure 51.15A are drawn arbitrarily to represent the kind of irregular variations seen in the earlier illustrations of volcanic rates (for example, the derived curve of net rate in fig. 51.15E qualitatively resembles the Hawaiian-Emperor rate curve in fig. 51.2C). Figure 51.15B shows the approximate values of graphical integrations of areas under the (+) and (−) rate curves in figure 51.15A, as well as the integral defined by their difference (the net rate is not shown in fig. 51.15A). The input (+) and output (−) rates are plotted against the integrated input (x+) in figure 51.15C. This is analogous to one showing total magma supply rates and depletion rates (either by eruption or other losses from a chamber) versus total magma supply (fig. 51.5 shows these quantities plotted against time; it is also used subsequently to examine volcanic volumes in a manner like that of fig. 51.15).

Plots of future states against present states for positive or negative net rate increments relative to the total input quantity x are shown in figure 51.15D; again, the net rate is simply the difference given by the positive (+) curve less the positive (−) curve of figure 51.15A. This graph is included to illustrate the implications of applying net rates to the different choices of x-values. One might wish to plot the sum of x and the net rate against x for at least two reasons. (1) It might be desired to see where in the record of input rates there is a tendency for strong depletion effects. (2) The record of output rates might be intermittent and inadequate to permit an integration of x; hence, the net value of x that ordinarily would be the abscissa of choice is unavailable (that is, it might not be possible to plot fig. 51.15F). When the dashed trajectories of present-future states in figure 51.15D are constructed, however, a contradiction is encountered. This is shown by the reversal in the directions of trajectories and the implied oscillation in “x.”

According to figure 51.15B, x+ is a monotonically increasing function of time; there is no indication that x+ might decrease anywhere. While it is correct that the lateral reading of figures 51.15B and C indicates that such trajectories of recursion imposed artificially in figure 51.15D are impossible, the previous discussion of uncertainty indicates that the master rate curves in figure 51.15A are average trends of data that have unidentifiable fluctuations at smaller scales of time. If we were to also allow the possibility that x+ stands for an input quantity that could have components of reversals (analogous to the merchandise returns), then actual reversals in x+ would be possible. Such an effect is equivalent to saying that there are high-frequency fluctuations in at least the (+) rate curve of figure 51.15C that extend below zero on the ordinate (that is, the integral must be capable of changing sign locally). The hypothetical trajectories in figure 51.15D indicate the magnitudes and locations of where such effects might be looked for in terms of the input record, x−.

These results can then be compared with the same vicinities of figure 51.15C (dots indicate tendencies for net convergences, or stationary states if plotted against net x) to see if there are in fact possibilities for stronger fluctuations. These considerations also explain how a seemingly closed repetition, shown on the left of figure 51.15D, can escape to higher values of x or net x (see later discussion of basins of attraction and the regions in fig. 51.15F). The interval of high variability in Kilauea’s eruption record (fig. 51.7) is an example where the record of supply and depletion of the summit chamber is expected to have a highly variable rate history as well. We might suspect complex reversals of flow in such intervals (analogous to drainback; see Eaton and others, chapter 48), although they are also possible in the absence of eruptive phenomena.

The trajectories of recursion in figures 51.15D and 51.15F symbolize endogenous folded feedback; “endogenous” here refers to the net effects of mechanisms of self-regulation that are dominantly influenced by the way the system has internally evolved, rather than by being dependent on some external system. The cross-hatched regions of figures 51.9, 51.10, 51.12, and 51.13 are examples of conditions controlled endogenously, in that the delineated regions are products of the net effects of several kinds of dynamical feedback intrinsic to the system at different characteristic scales of time and size.

The central idea in figures 51.15D and 51.15F, and in possible parallels between the logic of repetitive physical processes and the logic of repetitive numerical processes performed graphically or by computer, is the extent to which the quantity portrayed is a product of endogenous folded feedback. That is, if the system states at a given time or quantity (x, x−, and their differences) are products of preceding states and in turn will in some sense dictate the nature of following states, no matter how intricate the paths may be, then the form and amplitude relations of plots such as figures 51.15D and 51.15F provide logistical information on how stable or unstable that evolution has been and might become relative to variations in the quantity and in time.

Examples such as the factory-wholesaler-retailer system are instructive in illustrating how oscillations can occur out of attempts to stabilize the inventory and maximize profits. For example, if x+ in figure 51.15D represented items from the factory less those returned, then the oscillating behavior of “x+” could be real. The trajectories of recursion might have been designed to provide negative feedback, perhaps because there is limited storage space in the warehouse. The strategy is revealed as a self-defeating one in the particular example at the left in figure 51.15D, because until x+ eventually increases, no lasting sale of goods is being made (they are just being carried back and forth between the factory and the wholesale warehouse; implicitly, in this case, returns between the consumer and retailer and the retailer and wholesaler also would have to balance). This kind of oscillation applied to the behavior of a magma chamber would correspond to a condition where magma is pumped in and out through the same source conduit without any evidence of lasting outputs in the form of eruptions or intrusive leaks.
(if this kind of behavior existed conspicuously in Kilauea's budget, summit inflation-deflation cycles would occur without eruptions or evidence of magma moving into rift systems). It may also be noted that the transport budget of a geyser basin resembles several aspects of the above kinds of oscillatory transactions.

These and other forms of oscillating material inventories essentially reflect imbalances related to time delays of incremental transport which are intrinsic to nonlinear systems (for example, a large sale of goods triggers a large order from the factory, which may be followed by a large return to the factory if subsequent sales are poorer, and so forth). Although we cannot rule out chance or systematic external influences as a part of the record (for example, thefts of merchandise), we can describe the system in terms of endogenous influences represented by patterns such as those shown in figure 51.15C (as another example, if in fig. 51.15 represented water in an open reservoir, the variations would describe the main features of the material balances, taking into account scale effects as discussed, even if they included the vagaries of rainfall as an input and the chance of external objects falling in).

The relation between positive values of input and output rates versus quantity identifies tendencies for convergence or divergence in the neighborhood of specific values of the quantity. If the positive input is greater than the positive output in some vicinity of the value \( x \), the quantity in question is increasing in that vicinity (and implicitly at a given time, or at a given set of times if the pattern is repetitive). Conversely, if the positive output is greater than the input, the quantity is decreasing in that vicinity. Thus, where the rate curves cross, there is either a convergence or a divergence with respect to that value of \( x \). Loci of convergence in figure 51.15C are identified by the dots with inward-directed arrows, whereas loci of divergence are identified by circles and outward-directed arrows. In a tangible sense, any material states symbolized by the quantity \( x \) tend toward the vicinities of the dots relative to any other nearby states. The loci of the circles are avoided, so to speak. In this respect, values of \( x \) are "attracted" to the vicinities of convergent rate balances. The circles, therefore, are analogously loci of relative "repulsion" from the standpoint that values of \( x \) persist longer in the vicinity of convergent rather than divergent states. Neither type of locus, however, is necessarily an absolute or global center of logistical attraction or repulsion if the net variation of \( x \) (as in fig. 51.15B, dotted curve) eventually increases beyond the recorded interval. Thus, the set of diagrams in figure 51.15 represents an array of transient states in which are imbedded several quasi-stationary (less transient) attracting states within a set of generally divergent (more transient) states. It may be noted that any system of merchandising that transports goods efficiently and has high sales will have both kinds of states in some optimum balance (barring a factory-outlet system in which goods are made precisely in balance with their consumption at all times).

In computer studies of numerical folded feedback, the incremental paths of convergent circulation just described are often called attractor trajectories. The word "attractor" refers descriptively to the resulting patterns of stable, quasi-stable, or unstable trajectories, much as the terms "cell," "vortex," and others are used in describing fluid motion. The usage stems literally from the circulation of trajectories in the vicinities of stationary loci within a field of numerical possibilities governed by folded feedback. The literature of attractor mathematics is now very extensive, and space does not permit rigorous development. The best introduction I know is given by May (1976); the background of his study is in the biological sciences.

In the present discussion I use the term "attractor" as a shorthand for any patterns represented by the trajectories of present and future states. This is done to distinguish the pattern of trajectories based on numerical recursion from more complete logistical descriptions of all potentially available states represented by loci such as the dots and circles in figure 51.15C. In a parallel context, every attractor pattern also implies its converse as a repellor pattern of divergent circulations; generally, these are not shown.

A brief summary of the types of attractor patterns that can be expected in the vicinities of convergent states is given in figure 51.16. The four types of behavior shown are abstracted from an array of classifiable regimes that have been studied numerically by many other workers of diverse backgrounds (assisted by P. Doheray, USGS Computer Division, I have duplicated many of the published results; we have also written algorithms for attractors of higher dimensions, as well as variants of recursive strategies that contain options for evolutionary choices by the program itself along the lines of evolutionary strategies outlined in the introduction). May (1976) gives a concise summary of the possible numerically generated attractor regimes found relative to standard models of folded feedback (feedback curves of analytically explicit forms involving parabolic and exponential functions). The analytic type of feedback equation, however, is less crucial to the attractor pattern than is the range of slope variations in the vicinities of crossing points of the master curve and the reference line, as shown in figure 51.16. The present research has confirmed to my satisfaction that the general concepts of pattern generation by means of computer experiments can be a powerful tool for the exploration of evolutionary strategies in nature.

In figure 51.16 the parallel vertical lines represent the interval of \( x \) to be investigated in terms of trajectories of attraction (solid lines with arrows) in the vicinity of a general trend of present-future states (dotted master curves) relative to a line of equal balance of positive and negative feedback (dashed diagonal lines). The large dots again represent stationary or quasi-stationary states, but in this instance such states are those implied by the convergence or divergence of the attractor trajectories themselves, which are generated from the dotted master curve that describes the average trends of states. Stationary states, as fixed points in the graphical content, represent either the intersection of the dotted curve with the diagonal (fig. 51.16A) or the intersections of a stable circulation of attractor trajectories as a limit of exactly repetitive paths (fig. 51.16B, C). Such circulations in dynamics are often called "limit cycles."

One of the major discoveries of computer experiments with numerical feedback is that there is a systematic diversity of limit cycles and other forms of repetitive or nearly repetitive attractor patterns (see, for example, May, 1976). As shown by figure 51.16C, the repetitive loop may involve cyclical multiplicities that increase in numbers of stationary states as powers of 2 and that can multiply in that fashion theoretically without limit until a critical
amplitude is exceeded. Actually, it is the combinations of amplitudes and slopes of the dotted curve that determine how many stable states are possible.

One example of divergent paths, shown in figure 51.16D, is found when the amplitude and slope attain values that imply both (1) the generation of chaotic (unstable, nonrepetitive) trajectories and (2) the likelihood that at least some trajectories will transcend the boundaries of the x-interval. In this diagram there are no stationary states, and two trajectories may begin at almost identical values of x and after one or more circulations spin off, as it were, in totally opposite directions. This is sometimes described as "sensitive dependence on initial conditions."

The statements concerning figure 51.16D are qualified in two important respects. (1) Chaotic trajectories lacking any quasi-stationary values can exist between figures 51.16C and 51.16D for values limited to the interval between the vertical boundary lines; these limits define the region within which a stationary or quasi-stationary attractor pattern can evolve as a distinct, though possibly chaotic, spatial analog of a stationary or quasi-stationary point (within this pattern, discrete odd-period stable attractors are also possible at special values of amplitudes and slopes; they are often called "windows"); (2) If the extrema of the dotted master curve exceed the limits of the intersection with the vertical interval boundaries, and the slope of the master curve at the crossing point with the diagonal is in the neighborhood of \(-1\) (perpendicular for equal units on the ordinate and abscissa), some trajectories of stable fixed point and period-doubling sets of attractors may also escape the interval toward either higher or lower values.

The circulatory regions in figure 51.16 are sometimes called "basins of attraction," and the limits just described indicate the tendencies toward either quasi-stationary or unstable behavior of potential attractor patterns relative to these basins. Because the master curve represents a locus of average feedback balances, however, statistical fluctuations around the overall growth trend, as in figure 51.15, imply a net drift toward higher values and what may be termed "leakage" from any particular basin of attraction.

Such leakages of attractor trajectories refer to the numerical persistence of any particular attractor pattern (including fixed points, period-doubling and odd-period stable sets of points, and chaotic regions) and are not to be confused with physical leakage from an actual physical reservoir. Some deliberation of rate balances should show that the concepts of physical leakage relative to attractor leakage are determined by the definitions of positive-negative transport rates relative to some physical local. That is to say, a leaky basin of attraction could refer to either a progressively filling or emptying mode of an actual physical reservoir. These considerations are the basis for subdividing figure 51.15F into three regions: In region a, subintervals like those of figure 51.16 might exist even where the overall trend is below the diagonal line; that is, the attractor leakage might be in a negative direction there, whereas it is in the positive direction at the transition to region b, and so forth.

The time variations shown at the right in figure 51.16 mimic the above discussion of attractor trajectories. The solid curves show the tendencies toward stable waveforms identified by circulations between stationary or quasi-stationary states. The dashed lines indicate transient trajectories related to deviations from the master curve of average feedback balances. If there is a one-to-one correspondence or a constant proportionality between x-values and the timing of the attractor oscillations, then the numerical repetitions of x-values are associated with regular time periods of comparable multiplicities; for example, the twofold attractor would have one frequency of oscillation, the fourfold attractor would have a different and phase-shifted frequency of two different amplitudes, and so on.

Because the potential time variations of figure 51.16 are produced by other time variations like those of figure 51.15 that govern the feedback relations of the master curve, it is evident that the overall oscillatory behavior may take place at more than one scale of description in space and time. That is, regular oscillations like those of figure 51.16 may exist at smaller time scales within the irregular oscillations of figure 51.15. By the same token, however, the patterns of figure 51.15 may relate to other scales of variation, which are also describable by relations like those of figure 51.16. Such uncertainties of scale are intrinsic to the behavior of natural open systems and underscore the need to examine rate relations of the sorts just discussed at as many scales of time and size as possible. They are also possible sources of the fractal characteristics and self-similarity in time discussed earlier relative to figures 51.5, 51.7, and 51.8.

These uncertainties of scale are emphasized by figure 51.17, which also draws attention to issues of implicit versus explicit master control curves. This figure is drawn arbitrarily, much like figure 51.15, except that the net rate dips below zero more persistently, rather than being generally positive. As a consequence, the net values of x alternate increase and decrease in magnitudes rather than increasing almost monotonically. Therefore, when the net rate is plotted versus the net quantity, the overall trend loops back on itself over oscillating intervals of x. Thus, the stationarity of patterns in certain vicinities of net x is automatically constrained by this alternation, without consideration of possible attractor trajectories of subsidiary oscillations.

It may be asked, however, what happens if figure 51.17D is also considered to represent the average path of a master curve that governs more localized tendencies for folded feedback. Each point on the curve implies a feedback influence on a future value, just as before, except that such values are now potentially multivalued; that is to say, there is a considerable redundancy concerning the overall effects on future states. A graphical or computer algorithm derived for single-valued recursions, such as those shown by the trajectories in figure 51.16, could not distinguish between curves except in a specific temporal sequence along the path of the arrows in figure 51.17. Since this path is the average locus of complex variations and may repeat itself again beyond the range of values shown, no unique trajectories of attraction can be specified. Instead, several kinds of attractor patterns may grow and disappear episodically within the time frame of several overall circulations, as rate values oscillate over the repetitive net values of x.

Simultaneously evolving trajectories of recurring attractor patterns on a redundantly looping master curve are shown schematically in figure 51.18. Each attractor locus represents one possible unique path of graphical recursions (computer-like repetitions of present-
future value decisions). In figure 51.18A most of the loops intersect the diagonal line at slopes permitting fixed-point attractors, except at the upper right, where the trajectories progressively diverge from the crossing point. This may mean that the looping system represents a transient stage of growth toward larger values or that the control loop closes back on the origin. In the latter case, the divergent locus will continually feed attractor trajectories back toward lower values of rates and quantities that will endlessly tend toward the vicinities of the stable attractors.

In figure 51.18B–E, however, the slopes of the loops at crossing points are less stabilizing (steeper, or more negative than -1). Fixed-point attractors do not exist, and most trajectories generate twofold or higher order circulations. Because these circulations interact in many combinations, particular trajectories are sensitive to the starting values and to the parts of the loop that exercise dominant control during any particular history. Thus, if the overall control loop is in any way repetitive (for example, by an intermittent repeat of the same general rate relations or by closure of the outermost loop back to the origin without termination), then many possible combinations may recur for twofold to chaotic attractor patterns that will grow and shift in timing and ranges of x as different parts of the curve exercise their influence.

If the control functions alone were rigorously precise and repetitive, eventually a steady distribution of attractor circulations would evolve. Such a pattern is deterministically predictable in the sense that the structure is generated by combinations of many possible trajectories that lead to the same final states. However, the master curves are fuzzy, as already discussed, or have finite widths because they represent averaging of processes at smaller scales of time and size (for example, a thermal-feedback melting event in polycrystalline aggregate is a composite of many localized events that relate to more localized energy balances). Therefore, there is a continual shifting of trajectories relative to any theoretically exact stable-limit cycles. An expectable consequence of such uncertainties within the context of an overall loop that closes on the origin in figure 51.18 is that attractor patterns will themselves migrate back and forth over an unspecifiably large number of possible configurations. In detail, such a pattern is unpredictable, in that there are intervals of chaotic behavior of individual trajectories. At the same time, however, characteristic regimes of transient twofold and higher order attractors repetitively come and go within the same general ranges of x. Thus, even though the overall pattern is unpredictable, there are recurring intervals of subpatterns that show regular temporal periodicities from time to time in a manner like that illustrated in figure 51.16. Unpredictability only refers to the fact that the regularities aren't determined once and for all on the basis of the knowledge symbolized by a master curve that has a finite line width (representing a range of uncertainty in feedback control).

KILAUEA AS A NATURAL SYSTEM OF RECURRING ATTRACTOR PATTERNS

In this section I attempt to show that many of the generalized styles of attractor circulations discussed in the preceding section actually exist in the circulatory patterns of behavior of Kilauea Volcano. In fact, Kilauea and volcanic systems in general appear to be remarkably good analog computers in terms of demonstrating the effects of repetitive recursions in the vicinities of characteristic basins of attraction. These patterns, however, involve multiple trajectories analogous to the schematic trajectories of figure 51.18. Attractor patterns generated by folded feedback evidently are chaotic if rate amplitudes exceed a critical value (figure 51.16). Furthermore, even the generalized basins of attraction, stable or chaotic, are unpredictable if there is a significant impression of rate control in the vicinities of potential stationary states of cyclical control loops (figs. 51.17, 51.18). Nonetheless, the circulatory pattern itself may be predictable, in that it retains a characteristic overall form. Before attempting to summarize the implications concerning general patterns of volcanic evolution, I compare the rate histories of Kilauea's activity with the above schematic examples.

The model of magma supply proposed and documented by Dzurisin and others (1984) provides for the first time a quantitative reference frame for these comparisons. It considers the balances of an overall history of supply between about 1957 and 1983 relative to a distribution between generalized summit and rift loci of storage. Although the erupted volumes are also tabulated in their study, they are not specifically included in the magma-supply budget that refers to intrusive gains to and losses from the summit chamber. In the present discussion of positive-negative rate balances, however, several different combinations among the possible types of data exist; these data include the following categories: (1) Total magma-supply rates, (2) total integrated volumes of magma supply, (3) localized rates of summit supply, (4) localized summit volumes, (5) nonsummit supply rates and volumes (representing rates and volumes of supply to rift systems), and (6) net rates and volumes of stored magma at summit and rift localities, taking into account eruptive losses.

From the previous logistical discussion of positive and negative rate contributions to the history of any net quantity, it is evident that in volcanic systems there are many ambiguities as to relations between the algebraic signs of rates and appropriate loci of accumulation. To simplify the discussion, therefore, the balances will be referred to a generalized summit reservoir and a generalized rift reservoir. A similar approach was used by Dzurisin and others (1980) to estimate the magma-supply rate to Kilauea from November 1975 to September 1977; the longer term budget discussed by Dzurisin and others (1984) is based on analogous considerations. Erupted volumes can be considered to represent possible losses from either reservoir or from their combination. Ignoring the eruptive loss, supply to the generalized rift reservoir can also be considered to represent loss from the potential volumetric states of the generalized summit reservoir. Thus, in figure 51.5 (from Dzurisin and others, 1984) the total supply minus the summit supply (volume or rate) defines the positive supply to the generalized rift reservoir or the loss (negative supply) relative to the hypothetical growth of the generalized summit reservoir.

The above relations are summarized in the form of rate diagrams as in the preceding section; figure 51.19 shows the total supply and eruption rates versus the total volume of supply (fig. 51.19A), and also the balances of summit and rift supply, ignoring
eruptions (fig. 51.19B). The positions of convergent and divergent tendencies are indicated by dots and circles, as in figures 51.15. Various combinations of rate balances are shown in figures 51.20–51.23, as identified in the captions.

Although the patterns are complicated, there are resemblances in all these diagrams to the hypothetical examples discussed above, as follows. (1) The total and rift supply balances in figure 51.20, and the net-supply-minus-eruption balances of figure 51.21, resemble the patterns in figure 51.15; there is a general history of increasing quantities, with localized circulations related to intervals of large negative rates (eruptive losses). (2) The total and net rates of gain and loss from the summit reservoir, as portrayed in figures 51.22 and 51.23, resemble the cyclical behaviors of figures 51.17 and 51.18, whether or not eruptive losses are taken into account.

These patterns suggest that the generalized rift reservoir behaves as a system of almost monotonically increasing magma storage, with intervals of quasi-stationary states and attractor-like circulations. Eruptive episodes accentuate the stationary states, but they do not control the pattern except over intervals near their occurrences, as in the circulations associated with the negative excursions in figure 51.21. The pattern of overall balances in figure 51.20 is not conspicuously different over times of eruption or noneruption, as shown by the fact that maxima that might be expected to correlate with attractor-like periodicities of gain and loss are not coincident with times of eruption.

The most conspicuous attractor-like behavior is shown by the circulatory states of the generalized summit reservoir. The patterns of repetitive control loops shown in figure 51.22 and 51.23 are analogous to those of figures 51.17 and 51.18, regardless of the way in which the present-future balances are plotted. That is, the patterns seem to oscillate relative to two (or possibly three) general volumetric states, whether they are viewed in terms of total supply rates, net total-minus-rift supply rates, or net total-minus-eruption supply rates.

Perhaps the most informative plots from a pattern-predictive viewpoint are figures 51.22A and 51.23, which do not depend on information concerning eruptive volumes; they are determined solely by the volumetric states deduced from summit-tilt data (see Dzurisin and others, 1984). Figure 51.22A is hypothetical in the sense that the future states, \( V_{n+1} \), are those that would be implied if each increment of total supply actually inflated the chamber by that amount before the next state is encountered (that is, volumes are predicted by extrapolating rates based on short intervals of time). Figure 51.23, on the other hand, more nearly represents the rate increments associated with the actual changes in net volume of the generalized summit chamber as the remaining magma is entering the generalized rift reservoir. The fact that the two patterns are similar suggests that the oscillatory states of the summit reservoir are “controlling” in the logistical sense of figures 51.17 and 51.18.

The summit-reservoir control loop may operate physically somewhat as follows: When the volume of the generalized summit reservoir exceeds some fairly high value, it returns to the vicinity of the stationary state because of higher-than-average rates of loss to the generalized rift reservoir; conversely, when it is at low volumetric states, it returns to higher volumetric levels because of lower-than-
average rates of loss to the rift reservoir. These rate excursions do not seem to be dominantly eruption controlled; eruptions occur at times of either higher- or lower-than-average rates of return to the stationary states. That is, if eruptions represented the only rate control, presumably they would occur predominately at times of higher-than-average volumetric states.

Another way of describing the pattern is paraphrased from Dzurisin (written commun., 1985); the statement resembles balances described earlier for the factory-wholesaler-retailer system: An important point is that the summit reservoir tends to buffer the rate of shallow magma supply by adding to the inertia of the magma system. Fluctuations in the rate of magma supply from depth are damped at the surface by the effect of the summit reservoir. When the rate of magma supply from depth is low, eruptions can still be fed by magma stored in the reservoir. When the magma-supply rate is high, eruptions can be avoided by increased subsurface storage.

An addition to this concise summary by Dzurisin is implied by figure 51.23. Whereas eruptions evidently can occur at almost any volumetric stage of the summit chamber, large eruptions occur at times near the locus of stationary states. This fact indicates that the eruption mechanisms themselves may be controlled by the summit-rift reservoir balances. A high net inflation state of the summit reservoir seems to require adjustments by increased net transfer to the rift reservoir from the summit reservoir before optimal conditions for eruption can recur. That is, if the values of magma-supply volume and rate are both too high, the resistance to eruption also seems to increase until summit storage returns toward the stationary states by increased rift storage. If both reservoirs increase together, rift earthquakes may be a more likely outcome than eruption, as suggested by the volumetric excursions during 1974–75 (compare figs. 51.20B and 51.22A). Conversely, if the summit reservoir is operating at much lower volumes than the stationary states, the eruptive head may be too low to overcome the net viscous dissipation associated with inflation. Consequently, eruptions seem to be most likely at values near those of the stationary states, and perhaps at slightly higher volumes, regardless of the net transport rates. Thus, intrusion is the favored mode of growth of Hawaiian volcanoes.

Eruptions are special events that occur when certain preconditions are satisfied—that is, near the stationary states. Such balances also seem to be consistent with the earlier observations on the existence of self-similar relations between system size, chamber size, and repose time (see earlier discussion of fig. 51.7 and the relations between Kilauea and Mauna Loa compared with crustal eruptive periodicities discussed by Smith, 1979, and Shaw, 1985).

There are many interesting analogies to the above patterns from studies of mechanical systems of forced vibrations with damping. Such systems have been studied extensively, in the form of both physical and numerical experiments. In both types of study, damped
51. UNIQUENESS OF VOLCANIC SYSTEMS

![Figure 51.21](image)

**Figure 51.21.**—Diagrams of future state plotted against present state as in figure 51.20, but taking into account volumes and volume rates of eruption. Eruptions are lumped as representing rift events without regard to location. Points are plotted at 6-month intervals. Strong negative feedback and attractor-like circulations are clearly associated with eruptive events, but these are abrupt excursions that do not control in the sense of forming master control loops. That is, pattern resembles figure 51.20 except for localized circulations. **A,** Net storage volume (total volume minus erupted volume) versus net volume plus net storage-rate increment per year (total rate minus eruption rate). **B,** Rift storage volume (rift supply as in fig. 51.20 minus total erupted volume) versus rift storage plus rift storage-rate increment per year (rift supply rate minus eruption rate).

Oscillations have been shown to be highly nonlinear and to conform generally to regimes described by mathematical attractors (see Campbell and others, 1985; Shaw, 1984). The physical and numerical forms of behavior of leaky faucets studied by Shaw (1984) show strong analogies to the description of volcanic behavior. The events of drop formation are analogous to eruptions, and the regimes of overall flow rates supplied at the faucet are analogous to magma supply (except that magma leakage is complicated by many other factors than the setting of a single calibrated valve).

Circulatory patterns and stationary states illustrated in the present paper are analogous to factors influencing oscillations in total rates of leakage in the water-faucet model. At high rates of leakage, drops do not form because flow is continuous; at very low rates of leakage, they do not form because evaporation can keep pace with supply (in the magmatic mode, the analogous process is solidification). The behavior described numerically by Shaw (1984) refers only to the periodicity regimes of drop distributions. These show all the attractor structures mentioned earlier (see fig. 51.16), including stable to chaotic patterns, as documented in terms of the timing between drop-forming events. Eruptions could be illustrated similarly by plotting times between extrusive events. It is evident, from the wide range of extrusive repose intervals, that the eruptive attractor analogous to the faucet attractor is typically much more chaotic. In both cases, however, special regimes of precisely tuned flow can result in stable periodic repetitions. In the faucet model, these regimes can be easily isolated for study. In the volcanic case there is additional feedback among all the other processes mentioned earlier. Therefore, these other systems of oscillation (for example, the oscillation of stress states in the edifice) vitiate any obviously simple interpretation of the possible tuning effects of the faucet model, possibly excepting special intervals of flow (for example, fountain episodes could represent a subregime in which valvedike control might be sufficiently coordinated that an independent analysis may reveal attractor patterns characteristic of the local process; in this respect there may also be a resemblance to patterns of greyer eruptions described as attractors).

This qualitative explanation of the logistical circulations seems to be in general accord with the earlier discussion of dynamical feedback. That is, the optimal conditions of magma accumulation represent some compromise, between melting and transport, that
compensates oscillations of stress related to flow rates, rates of solidification, and so on (see Hardee, chapter 54). A logistical interpretation of such balances leads to the analogous inference that there are characteristic regimes of oscillatory behavior that may be products of the path of evolution without the intervention of any externally applied forcing frequency. The inclusion of external influences, however, may also be subsumed by the pattern in a way that is then indistinguishable from the endogenous periodicities. Correlations with characteristic frequencies of external signals may be observable only during times when there are reinforcements of the endogenous periodicities or when the external influence is so large that itoverrides the previous patterns.

UNIQUENESS IN PATTERNS OF VOLCANIC EVOLUTION

The words “unique,” “pattern,” and “evolution” used together raise issues fundamental to prediction as a goal of the natural sciences. I will attempt to summarize a viewpoint concerning their application to volcanology, suggested by the foregoing analysis, as the basis for my concluding comments.

First, I realize after some deliberation, that any rigorous statement concerning this topic is likely to contradict itself before it proceeds very far. Therefore, I limit conclusions to some questioning remarks concerning ways in which the present results may differ from or resemble other viewpoints concerning natural processes. If there is an accepted theory or body of knowledge concerning volcanic evolution (or the meaning of “natural evolution” in general), I do not know where it is stated, so I cannot say that I am commenting on any particular form of conventional wisdom.

Unique means literally “being without a like or equal.” That much seems easy, until such questions are asked: How does one know there is no like or equal? Just how much alike do things have to be to be called “like”? What numerical tolerance is applied to

**Figure 51.22**—Diagrams of future state plotted against present state as in figures 51.20 and 51.21 but based only on volume of supply or storage in summit chamber as defined by Drusin and others (1984). Here again, effect of eruption is to superpose large negative feedback events on more general pattern, based on intrusive balances, which circulates in vicinities of two characteristic stationary regions of summit volume, even though future states are expressed in terms of total supply rates (see similar patterns in fig. 51.23 restricted to summit rates). Dates are given for chronological perspective. Major eruptions (of the order 0.1 km$^3$) are indicated by asterisks; they apparently have occurred within a fairly narrow interval of attractor-like circulation, implying that eruption may reflect an intrusive feedback circulation within a characteristic bandwidth of rate-volume relations (see discussion of fig. 51.18). Note that Kalapana earthquake of 1975 occurred during a time of extreme positive divergence from localised circulations, while eruptions occurred in vicinities of median states; both correlate with summit-rift intrusive balances. A, Summit supply volume versus summit supply volume plus total supply-rate increment per year (extrapolated 6-mo rate); this diagram, like figure 51.23, is generated totally on the basis of summit tilt data (see discussion in Drusin and others, 1984). B, Summit storage volume versus summit storage volume plus net rate increment (supply minus eruption) per year. Circed point, beginning of record; x, end of record.
quantitative measures of likeness? What does “equal” mean in any natural context? What are the tolerance limits of numerical equality? (Numerical dynamics makes the latter problem vivid. In some regimes, changes by several digits in the first decimal place of a normalized unit quantity do not change the likeness or equality of a pattern, expressed in terms of periodicity. However, near conditions of critical change between periodicity regimes, such as from period-doubling to chaotic, the likeness of patterns can change drastically with numerical changes at the limit of computer precision. That is to say, in this respect, one part in $10^6$ is a large change, in that periodic equality and likeness are drastically changed. This point is illustrated qualitatively in figure 51.16; see May, 1976, for some numerical examples.)

The word “pattern” can be applied to a crystal structure or to a bomb blast. Can one or the other be said to be either unique or nonunique? “Evolution,” at least to some protagonists, implies uniqueness in the sense of a path-dependent process involving so many variables that it seems inconceivable that any given structural stage will ever repeat itself exactly. By this view, natural evolution never has nor ever will construct two identical volcanoes. That is, no complex form, including life, ever evolves in exactly the same way twice in the natural record. Yet, ideas of model building, fabrication, cloning, and even the genetic engineering of arbitrarily designed living forms are now almost commonplace and raise questions such as those mentioned above concerning what is meant by the words “unique,” “pattern,” and “evolution.” Although no fundamental discussion of predictability and forecasting can really avoid such conundrums of science, I will attempt to restrict my conclusions to issues of necessarily rather vague criteria of pattern recognition.

If one were to digitize the coordinates of every piece of shrapnel and debris from every bomb blast in the history of explosives technology, it is highly unlikely that any two lists of numbers would be identical, except for individual or short sequences of numbers which may be similar because of the limits of resolution. Yet people who study such patterns, or patterns of gunshot blasts, can classify different types of shells and explosive materials simply in terms of learned recognitions of pattern morphologies.

Uniqueness in any applied sense of recognition is apparently judged in terms of some, usually unspecified, classification of relative complexity. In that context, the question of forecasting complex behavior is partly one of recognizing a class of complexity as distinct from any other class of complexity (implicitly, this also means that there exists some matrix or background system of states that is simple to some recognizable degree relative to what is called complex). If the recognition of distinguishable complexity is accurate, however, it subsumes the issue of forecasting, because the behavior it is desired to forecast is not separable from the criteria of recognition. One can test this idea by contemplating the recognition of any dangerous animal or of poison ivy. Their recognition and the forecasting of consequences are based on a complex system of evolved information; it is a rare person who would choose to attempt prediction solely on the basis of statistical study of the behavior of the particular specimen or situation confronting him. Yet, that is the way some investigations seem to approach volcano and earthquake hazards. Similar processes are operative even when the species is unknown; behavioral forecasting operates within a context of what is presumed will be a recognizable complexity. That recognition, however, obviously must include its own evolution (the reading of the complexity, the learning of the behavior, and the formulation of the forecast).

These comments may sound familiar to anyone who has spent some time studying the eruptions of Kilauea Volcano, or anyone who has studied the patterns of ocean breakers over a period of variable tides and weather on a particular stretch of beach. The behavior of breaking waves offers a particularly apt comparison to issues of volcanic prediction in the light of patterns studied in this paper.

A breaking wave represents a locus of familiar form and generally predictable action which is constructed in detail by the operation of progressive folded feedback from a more general state of chaotic motion. The trajectories of individual water molecules are statistically turbulent; no individual particle trajectory can be predicted in detail, except in the sense of a generalized involvement in a wave cycle. Yet, at stages of growth just prior to and during the course, say, of a surfer’s ride, the waveform is highly familiar and even predictable. From the viewpoint of particle trajectories, no two ocean waves are demonstrably identical, hence each is unique. From the standpoint of recognition and regimes of behavior they are familiar and predictable, hence nonunique, except for the unexpected event (for example, a tsunami).

For ocean breakers, the demarcation between the familiar and the unfamiliar and (or) unexpected event (from a fisherman’s or surfer’s viewpoint) represents a practical criterion for distinguishing between endogenous and exogenous processes in the sense used earlier in this paper. Yet, on a global basis the general states of ocean circulation are net effects of all such processes. Therefore, the terms endogenous and exogenous, too, are relative to the goals of pattern recognition.

If length permitted, it would be possible to derive a series of logistical diagrams that would describe the relation between, say, water level and time along a stretch of coast during a period of variable barometric pressure (for example, during the passage of a storm front). Expressed as maps of present versus future heights, such diagrams (as I have drawn them from published storm-wave data) resemble the cyclical control loops outlined in figures 51.17 and 51.18. In this case, however, the meaning of the feedback loops is fairly obvious in terms of the incidence, amplitudes, and repetitive frequency ranges of rising storm waves.

From the logistical viewpoint, figures 51.20–51.23 indicate that the recognizable behaviors of Kilauea Volcano may operate in a fashion somewhat parallel to that of storm fronts and breaking waves. The circulatory excursions of volumetric states relative to quasi-stationary states (for example, figs. 51.22 and 51.23) are analogous to frontogenesis associated with an interval of evolution. The implications for long-range forecasting are also parallel. A forecast of wave heights on a given stretch of beach depends critically on a knowledge of frontogenesis on at least a regional scale, as well as on such background information as what are considered normal tidal heights. The familiarity of the resulting local pattern is therefore contingent on a familiarity with other contributing patterns on both smaller and larger scales of time and size. Thus, a forecast on a given scale is ultimately conditioned by
Figure 51.23.—Present-future volume states of summit reservoir of Kilauea. Numbers are chronological at half-year intervals beginning with (1) in January 1957. These plots resemble figure 51.22A, except that they reflect only summit rate variations rather than total rate variations (that is, they reflect inflation input minus deflation output of summit chamber based on interpretation of tilt data by Dzurisin and others, 1984). Asterisks again mark vicinities of major eruptions, and location of trajectory during time of 1975 earthquake is also indicated. These diagrams and figure 51.22A offer an exceedingly simple technique of logistic mapping of future eruption and earthquake patterns based on existing measurements of summit tilt without requiring other information. A. Plot of present value versus next value, based on figure 51.5A, in which rate variations are implicit. B. Plot of summit volume versus summit volume plus summit supply rate increment per year (yearly extrapolation of 6-mo rate shown at each 6-mo interval). This graph amplifies effects of rate amplitudes but illustrates same pattern as in figure 51.23A.

Both an even more local and a more global forecast; that is to say, the confidence in our recognition of familiarity is conditioned by our confidence in recognizing some more universal pattern of familiarity. A probabilistic weather forecast evidently represents the measure of nonrecognition of what is, at least potentially, quite familiar. In order to be as candid as possible in the forecast, therefore, it would
Figure 51.23—Continued
have to be concatenated some way according to the possible arrays of logistical folded feedback encompassing the consequences of recognition of all potentially active control loops. The potential diversity and complexity in predicting reasonable variants of diagrams like figures 51.17, 51.18, 51.22, and 51.23 obviously is large. Nevertheless, these alternative histories are integral aspects of the complexity that must be learned if we are to improve the quality of prediction.

The problems of weather forecasting and eruption forecasting are basically little different from everyday processes of pattern recognition. The long timeframe involved in geologic processes and the possibility that there may be future events of unprecedented and hence unrecognizable form, however, are not encompassed by our usual scales of automatic recognition (an example was the May 18, 1980, eruption of Mount St. Helens, Washington; on the other hand, at least several individuals clearly did recognize the potential for that eruption on the basis of familiarity with multiple lines of long-term evidence). The utility of logistical analysis for volcano-hazards forecasting stems from our ability to construct descriptions of folded feedback representing a variety of different scales of volcanic processes. Indications of self-similarity, such as those pointed out earlier in this paper, are important clues to the validity of this approach.

For example, examination of logistical feedback on the scale of monthly variations during the one-year record of figure 51.5B also reveals a pattern that shows scaled-down circulations resembling the 28-year pattern of figure 51.23. Another diagram (not shown) is based on the timing of the secular evolution of the Hawaii-Emperor Chain scaled down by a factor of 10^6. The resulting synthetic record also resembles figure 51.23, suggesting self-similarity in the form of circulations about characteristic stationary states at the larger scale. A literal interpretation of this construction would be that the asthenosphere-lithosphere section has relative properties resembling the interactions of the generalized summit and rift reservoirs. The compatible volume scale of magma in the region of storage, relative to the summit reservoir (that is, 10^6 times larger), would be on the order of 10^8 km^3, which is appropriate to the size scale of individual islands (closely coordinated groups of volcanic edifices) along the chain.

The logistical approach can be applied as well to positive-negative rate balances during the growth of fractionated magma reservoirs. Preliminary graphical and numerical experiments along these lines suggest that the structures and periodicity scales in time and size of caldera-forming sillicic ash-flow systems may behave in principle much like the diagrammatic processes of folded feedback described in this paper. Obviously, the problem is more intricate because of the multiplicity and scale ranges of control loops. In principle, however, conditions of storage, states of fractionation, and eruption represent circulations about nested sets of stationary basins of attraction much like those described in figures 51.15–51.18. The parallels between conditions of charge and discharge (and eruptibility) of a magma chamber with those describing the conditions of the breaking wave apply as well to nested systems of volcanoes. The only differences are in the varieties of timing and size scales of processes analogous to frontogenesis. In silicic cratonic systems, this aspect of the problem also involves variations of supply rate of primitive magma much like the variations shown by the Hawaiian systems (Shaw, 1985). Therefore, in this sense, there are important elements of self-similarity between patterns of oceanic and continental volcanism (also, compare earlier discussions of fig. 51.7 concerning size and repose time relations).

In the global context, logistical methods may be useful in studying correlations of volcanic periodicities with those of external forcing frequencies such as the earth tides. It seems evident from the earlier discussion that endogenous attractor-like periodicities having some regularity are possible in volcanic systems; basaltic systems apparently involve significant variability at the daily to yearly scale as well as at the more secular scales of variations. Analyses by Klein (1976) and by Dzurisin (1980) have confirmed the existence of significant fortnightly tidal correlations with volcanic behavior during certain intervals of time and location. If a crude endogenous periodicity exists in the vicinity of a more regular external (or exogenous) forcing frequency, entrainment of that frequency is likely. By implication, however, this could apply to any of a variety of sources of similar periodic effects (for example, there may be a 27-day solar geomagnetic signal that, according to numerous recent discussions, conceivably becomes involved even in mantle processes by way of momentum coupling between the atmosphere and the solid Earth affecting microsecond fluctuations in the Earth's rotation).

Forecasting terrestrial events evidently implies eventual familiarity with periodic behavior involving a very broad spectrum of cosmological phenomena. Our (at present) low probabilities of accurate geologic forecasting, aside from issues of humanly unknowable hidden variables, can be viewed either as a measure of the uniqueness of terrestrial evolution or as a measure of our reluctance to seek familiarity with a broader context (including major cosmological processes) that may make Earth processes more familiar. If familiarity with processes of change in the Earth is the aim of volcanological research, it will necessarily involve increased familiarity with methods for documenting the evolution and recognition of complex patterns along lines at least analogous to those proposed in this paper. Obviously, these broader goals will require increased emphasis on analysis of interdisciplinary sets of data with an aim toward discovery of common factors in their evolution.

I conclude with some suggestions concerning what might be done within the existing research efforts at the Hawaiian Volcano Observatory to test and (or) to implement the ideas outlined here. There is no doubt in my mind that the methods of documentation and analysis already in progress are of the highest quality and utility for purposes of day-to-day monitoring of hazards potentials. I recommend, however, that these day-to-day records also be plotted in terms of logistic diagrams such as those constructed in this paper. The same approach can be applied to a variety of different scales, ranging from the records of individual episodic of fountaining behavior to long-term studies of the timing patterns in lava-flow stratigraphy. Similar remarks concern the analysis of seismicity and deformation patterns as data accrue. For example, seismic moments can be summed within specified domains, from scales of time and size associated with Kilauea to scales associated with the Hawaiian Archipelago, and their variations plotted in exactly the same format.
as the variations in magma supply (see Shaw, 1980). In an analogous way, times between earthquake events and eruptive events (either separately, or in combination) can be plotted at different scales of resolution to create what can be called attractor maps of possible repetitive patterns and trajectories of change. Such an approach is analogous to the method used by Shaw (1984) to document and classify dynamical regimes of nonlinear drop formation in the leaky-facuets experiments mentioned earlier. Also, volume-time data for deformations of the east rift zone of Kilauea could be plotted in much the same way as the summit-tilt data. Trajectories of circulation could be updated incrementally, as are the present plots of summit tilt versus time.

All these additional maps of the dynamical states of the volcano will provide an additional pictorial image and a greater feeling of confidence in describing patterns of waxing and waning activity. In the process, it may become more evident that eruptions, earthquakes, and other events (for example, large-scale landslides), occur where they do in the cycles of repetitive states as those illustrated in this paper. Needless to say, we are sorely in need of the same sets of data for parallel analyses of Mauna Loa. In view of the current signs of a waxing period of activity there (see figs. 51.4, 51.7), this goal would appear to be of high priority.

Even though the patterns illustrated here would be formally classed in the chaotic regime of attractor dynamics, this does not mean that they have no predictive value. There are clear indications of characteristic patterns of repetition, even if they are not periodically exact. That is to say, prediction, in this sense, stems from an ability to identify multiple sequences of indications of trajectoryal trends toward conditions of imminent instability (that is, toward conditions typical of various types of intrusive episodes, eruption, changing edifice-deformation states, or earthquakes). In the chaotic mode, individual sets of trajectories do not repeat themselves exactly, but some composite set of multiple aperiodic trajectories may be diagnostic (that is, a chaotic attractor is very much a recognizable entity, even though any subset of points may be totally unique and mystifying when examined out of context). In principle, the same criteria are automatically activated when we recognize and predict the behavior of any human individual. Prediction is far from infallible, but if I know a person's habits well, it is often obvious what the behavior will be in sufficiently familiar circumstances. The same could be said of predictive abilities concerning traffic flow in a large metropolitan area. Although no individual trajectories of motion are likely to repeat exactly in daily, weekly, monthly, and yearly cycles, most of us who live in such environments can say, even during a nonperiodic episode (that is, patterns within those that clearly correlate with time of day), what will happen next. At that level of familiarity, statistical probabilities are very useful ways to document that what we sense happening is, in truth, likely.

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