# HEAT AND MASS TRANSPORT IN THE EAST-RIFT-ZONE MAGMA CONDUIT OF KILAUEA VOLCANO

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## ABSTRACT

The flow of magma in the rift conduit system of Kilauea Volcano is examined by comparing thermal and fluid-flow conduit models with field observations. A repetitive thermal-intrusion model predicts that the conduit will remain relatively open to flow for cyclic intrusion rates above a certain critical level. This agrees well with field observations for the upper east rift zone. This portion of the conduit tends to have aseismic intrusions, implying that fracturing is not required to open the conduit for flow. Observed intrusion rates and times between eruptive cycles for other portions of the Kilauea rift system and for conduits of other volcanoes agree well with the repetitive thermal-intrusion model.

Two forced-convection fluid flow models are developed for the Kilauea rift conduit system. One model is based on non-Newtonian rheology effects, and the other on viscous-dissipation effects. Both fluid flow models are compatible with field observations of summit-reservoir pressure changes, intrusive flow rates, eruption temperatures, and eruptive cycles. The nonNewtonian flow model is applicable to conduits on the order of 0.5 m thick or thicker, and this appears to be the situation for the upper portion of the east rift zone. The viscous-dissipation flow model appears to be applicable to thin conduits on the order of 0.5 m or less thick and particularly to those on the order of 0.1 m or less. Thin conduits tend to cool rapidly, and a source of internal heating such as viscous dissipation is needed to keep these conduits open for flow.

## INTRODUCTION

The model of the magmatic plumbing system of Kilauea Volcano has evolved over the years as that system has been studied by numerous investigators. Magma generated in the upper mantle rises and is stored temporarily in a summit reservoir system at a depth of 2–6 km beneath the summit of the volcano (Duffield and others, 1982). The magma appears to rise from the mantle to the summit-reservoir storage system at a more or less steady volume rate (Swanson, 1972; Dzurisin and others, 1980; Dvorak and others, 1983). As storage in the summit reservoir increases, inflation of the volcano occurs, and eventually sufficient stresses accumulate to cause part of the magma to intrude into a shallow conduit system (Fiske and Jackson, 1972; Ryan and others, 1981). Frequently, intrusion into the conduit system results in the eruption of lava. The shallow magma of the east rift zone exists at a depth of 2–6 km and extends

from the summit of Kilauea to a submarine position about 70 km east of Cape Kumukahi (Moore, 1971), a total distance of 120 km. A part of the rift is shown in figure 54.1 in a semiactive stage between eruptive phases during the Kilauea eruption that began in 1983.

Flow or intrusion in the conduit of the east rift zone is an intermittent forced-convection (pressure-driven) process. The seismic models of Ryan and others (1981) suggest that the intrusive flow occurs in a pipelike conduit. Later stress calculations and field observations by Ryan and others (1983), however, indicate that typical conduits may be sill or dike structures with thicknesses on the order of 1 m (0.15–3.38 m). The intermittent intrusions in the east rift zone appear to be of two basic types. Magma movement at large volume rates of 100–1,000 $m^3/s$ are accompanied by shallow seismic swarms and are apparently a result of the formation of new dikes by sudden fracturing (Epp and others, 1983). This process is common in the lower part of the east rift zone and in other areas, such as the southwest rift zone. Slower intrusions at volume rates of 1–10 $m^3/s$ are usually aseismic, implying that the conduit is in an open or nearly open (plastic or near-molten) state and that fracturing is not required for flow to be initiated (Epp and others, 1983). Intrusions of this latter type are common in the upper 20–30 km of the east rift zone nearest to the summit caldera. After an initial slow, aseismic intrusion, additional intrusions may occur at faster rates in later eruptive phases because the conduit remains molten or open and even increases in effective flow area. The slow, aseismic intrusions and the more rapid intrusions in later eruptive phases have been well studied, especially during the recent eruption of Kilauea that began in 1983. Decker and others (1983) have been able to correlate the extensive inflation-deflation records of the 1983–84 eruption phases with volume intrusion rates in the conduit and with estimated dynamic pressure values in the summit reservoir.

This paper concentrates on the low-volume-rate aseismic intrusions that frequently occur in the upper part of the east rift zone. This type of intrusive process repeats through a number of eruptive or intrusive phases, and therefore a large set of observations and data has accumulated. This repetition and the abundant data allow us to learn much about this type of conduit and about conditions in the summit reservoir. A repetitive-intrusion thermal model has been developed which explains how a conduit can remain relatively open to successive intrusions (Hardee, 1982). This model is described and the results applied to intrusion data for Kilauea Volcano. As expected, pressure-driven forced convection is the mechanism that

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FIGURE 54.1.—Aerial view (looking southeast) of upper east rift zone of Kilauea Volcano in a semiactive stage between eruptive episodes during 1983–84 eruptive activity.
Photograph by R.P. Striker, January 15, 1983.

drives intrusion in the plastic or near-molten conduit. Flow models based on both simple forced convection and forced convection with non-Newtonian-fluid and viscous-dissipation effects are considered. The non-Newtonian convection model is based on earlier work by Shaw and others (1968), who found that Kilauea lava could be modeled as a Bingham-plastic fluid. The viscous-dissipation model builds on the work of Gruntfest (Gruntfest, 1963; Gruntfest and others, 1964) and Shaw (1969), who were the first to introduce the concept of dissipative feedback into igneous petrology. Magma viscosity in the subliquidus temperature range depends on shear stress, shear rate, temperature, time, and previous thermal history. The Bingham-plastic flow model and the viscous-dissipation flow model are simply ways of looking at major magma-viscosity effects one at a time. The models are used to predict conduit size and dynamic pressure in the summit magma reservoir. The models are also used to explain the cyclic nature of eruptive phases. The models correlate well with field observations of the 1983–84 Kilauea eruption episodes.

A few fundamental points need to be emphasized. First, this paper primarily deals with the conduit, 20–30 km long and 2–6 km deep, that carries magma from the summit chamber downrift, where the magma may or may not rise the short remaining distance to a surface vent. To avoid confusion in this paper, the main deep part of the conduit is called the east rift conduit, and the shallow (1 km) final part at the end of the conduit is called the vent conduit. There are several fundamental differences between these two parts of the conduit system. In many intrusions in the east rift zone, eruption does not occur, and the vent conduit does not exist. If an eruption does occur, the vent conduit exists only briefly for that particular eruption. The deeper east-rift conduit has an existence much longer than, and independent of, particular eruptive vents. The east-rift conduit has existed as a fairly open conduit system for several hundred years and will likely exist for many years to come. This paper primarily considers the nature and evolution of the east-rift conduit system on the basis of thermomechanical constraints and balances. Some observations at the erupting vent bear directly on the

nature of the deeper conduit system. Other observations at the erupting vent are important only for processes occurring locally at the vent and are not directly applicable to processes occurring in the deeper east-rift conduit. For instance, at shallow depths in the vicinity of the vent, pressure in the magma is lowered to the point where gases are released. These gases affect geysering at the vent and the viscosity of the magma or lava as it exits the vent. The theoretical models discussed in this paper ignore the effect of such gases because little or no gas is present at the depths (1 km and greater) and corresponding pressures where the east-rift conduit exists.

## ACKNOWLEDGMENTS

## REPETITIVE-INTRUSION CONDUIT MODEL

Magma intrusions are of two types, as noted earlier. In the first type, the intrusion is an isolated event; repeated or successive pulses of magma through the same conduit are unlikely, although intrusions may occur in nearby regions. This type of intrusive flow occurs in fractures opened by mechanical or tectonic forces. Mechanically controlled intrusions form a majority of magma intrusions and are thoroughly discussed in a recent article by Shaw (1980). The second type of magma intrusion occurs in conduits where repeated injections of magma have left sufficient residual heat to keep the conduit effectively open during the quiescent period between intrusions. The ability of a magma conduit to remain open over a long period of time depends primarily on the average mass rate of magma throughput in the conduit. If the average mass rate of intrusive flow through the conduit exceeds a certain critical value, the conduit remains molten or nearly molten and therefore open to future magma intrusions. This fact was originally recognized from field observations (Smith, 1979; Fedotov, 1981). On the basis of field data, Smith (1979) recognized that a critical magma-eruption rate of about $1 \times 10^{-3}$ $km^3/yr$ was necessary to sustain high-level silicic magma chambers. This can also be interpreted to mean that the critical value of magma intrusion rate through the conduit supplying the magma chamber or erupting vent is also on the order of $1 \times 10^{-3}$ $km^3/yr$. Fedotov (1981), on the basis of field data, observed a critical magma-intrusion rate on the order of $1 \times 10^{-3}$ $km^3/yr$ for magma conduits feeding volcanic centers in Kamchatka. Theoretical calculations by Shaw (1980) indirectly show that critical magma-intrusion rates on the order of $1 \times 10^{-3}$ $km^3/yr$ or greater are necessary to approach and maintain the liquidus temperature of mafic magma systems.
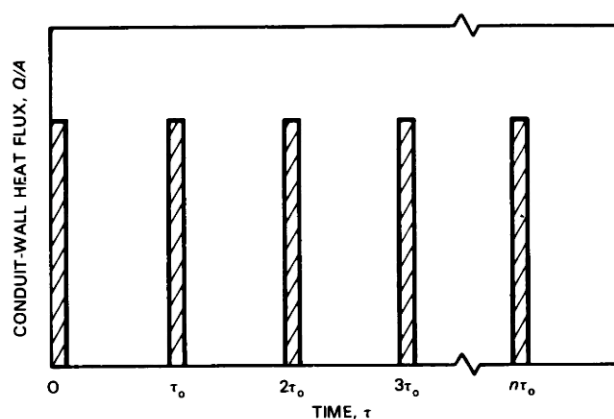


FIGURE 54.2.—Idealized repetitive sequences of thermal pulses used in repetitive-intrusion conduit model.

The critical magma-intrusion rate can be determined theoretically using an idealized conduit-intrusion model (Hardee, 1982). A cylindrical conduit geometry is assumed, which receives periodic intrusions of thermal magnitude $Q$ at time intervals of $\tau_0$. The heat pulses from successive intrusions are idealized as a regular series of heat-flux pulses at the conduit wall (fig. 54.2). The thermal magnitude $Q$ is determined from the latent heat or temperature of the intruding magma. The intruding magma must be at a temperature above the solidus; the conduit wall in contact with molten or flowing magma is, by definition, at the solidus temperature. The difference between these two temperatures is a measure of the energy available to overcome conduction losses at the conduit wall and into the surrounding country rock. The conduit-wall heat flux during an intrusive pulse is simply $Q/A$, where $A$ is the total surface area of the conduit. If a series of heat pulses (fig. 54.2) occur in a conduit, the wall of the conduit will gradually heat up. Even though the conduit cools between successive pulses, it will still be at a temperature above ambient just before the next intrusive pulse. If the value of $Q/A$ and the number of prior intrusive pulses are large enough, the temperature of the conduit wall and any material in the conduit will eventually reach a value on the order of the solidus temperature. At this point the conduit is assumed to be effectively open when the next intrusion occurs. Mechanical fracturing is no longer required to open the conduit for flow in this model. Forces of pressure-driven forced convection are sufficient to initiate flow in the conduit.

The critical condition for flow in an idealized cylindrical conduit with the periodic, pulsed boundary condition illustrated in figure 54.2 was solved by Hardee (1982), using a Laplace transform technique. The Laplace transform technique is well suited for problems in heat conduction with periodic boundary conditions and is uniquely suited to problems involving pulsed (delta function) boundary conditions. The critical magma-intrusion rate in the conduit, $V/\tau_0$, was shown theoretically to be about $1 \times 10^{-3}$ $km^3/$ yr, which is in agreement with related field observations (Smith, 1979; Fedotov, 1981). The critical intrusion condition for the
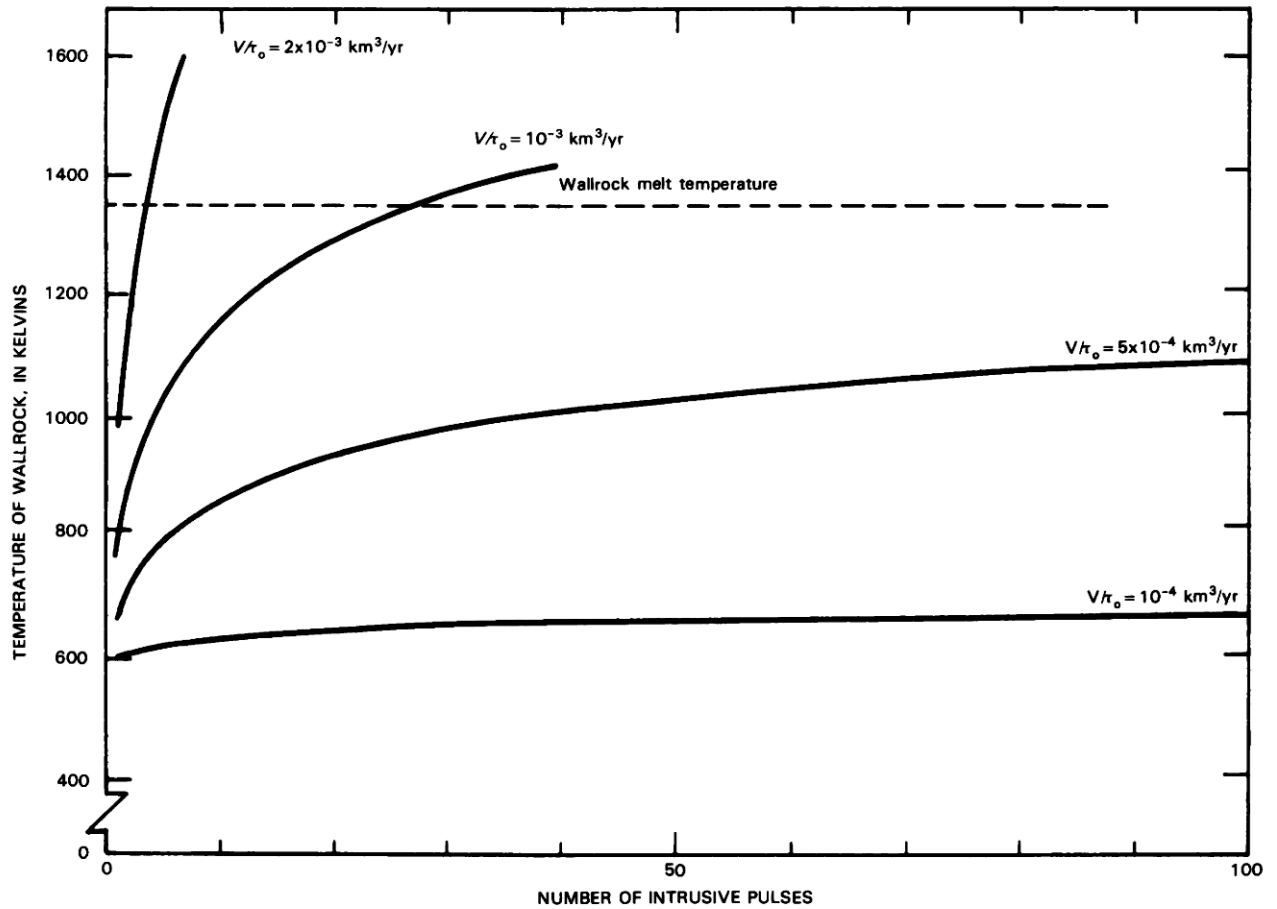
FIGURE 54.3.—Results of repetitive-intrusion model, showing conditions necessary to establish a thermally open conduit. Wallrock temperature rises as a function of increasing number of intrusive pulses; for sufficiently high volume-intrusion rates ($V/\tau_0$), it reaches and stays at or near the melting temperature.

conduit to remain open is a function of the critical magma-intrusion rate and also a function of the length of the conduit and the number of prior intrusive pulses in the conduit (Hardee, 1982). The critical intrusion parameter for a conduit such as the east-rift conduit is

$$\left[(V/\tau_0)1/l\sum_{1}^{m}(1/n)\right]_{critical} = 3\times10^{-4} \text{ km}^2/\text{yr} \qquad (1)$$

where $V/\tau_0$ is the volume rate of throughput in the conduit, $m$ is the number of prior intrusive pulses or eruptions in the conduit, $n$ is the variable number of each particular pulse from 1 to $m$, and $l$ is the length of the conduit. Equation 1 is the condition for a conduit to remain effectively open from thermal effects alone. Although this analytical result is for a cylindrical conduit, the critical intrusion parameter is independent of the diameter of the conduit. The initial intrusion obviously requires mechanical fracturing in a weakened zone of country rock to initiate the first flow of magma. Once enough flow in subsequent pulses has occurred such that the critical value of $3\times10^{-4}$ km²/yr is exceeded, the conduit will then remain open from thermal effects alone. Using typical properties for basalt and

assuming a typical conduit length, $l=10$ km, Hardee (1982) calculated the temperature of the conduit wall just before the next intrusion as a function of the volume-intrusion rate, $V/\tau_0$, and the number of intrusive pulses. The result (fig. 54.3) indicates that the critical condition for the conduit to remain open is a strong function of the average volume-intrusion rate and a weak function of the number of prior intrusive pulses. For instance, at a volume-intrusion rate, $V/\tau_0$, of $1\times10^{-3}$ km³/yr, the wallrock temperature remains at the melt temperature after about 10 intrusive pulses have occurred. At a volume-intrusion rate of only half this value ($5\times10^{-4}$ km³/yr), the wallrock has not reached the melt temperature even after more than 100 intrusive pulses. Although equation 1 gives a precise expression for the critical conduit condition, figure 54.3 shows that for many typical cases the critical conduit condition is met when the volume-intrusion rate exceeds the value $1\times10^{-3}$ km³/yr. This value is in agreement with the observations of Smith (1979), Shaw (1980), and Fedotov (1981), mentioned earlier.

Equation 1 can be used to compare magma conduit systems at Kilauea. Typical values of the critical conduit-intrusion parameter

TABLE 54.1.—*Intrusion parameters for some magma conduits*

| | Intrusion parameter $\left[(V/\tau_0)(1/l)\sum_1^m(1/n)\right]$ (km²/yr) |
|---|---|
| Kilauea—Upper east rift[1] | $45\times10^{-4}$ |
| Augustine Volcano, Alaska[2] | $40\times10^{-4}$ |
| Mount St. Helens, Washington[3] | $8.1\times10^{-4}$ |
| (critical value) | $3\times10^{-4}$ |
| Kilauea—lower east rift (Puna)[1] | $1.4\times10^{-4}$ |
| Kilauea—southwest rift[1] | $1.1\times10^{-4}$ |

[1]Based on Decker (1983) and Macdonald and Hubbard (1973). Since 1955, upper east rift $V/\tau_0=30\times10^{-3}$ km³/yr, $m=20$, and $l=25$ km. Since 1750, lower east rift $V/\tau_0=3\times10^{-3}$ km³/yr, $m=5$, and $l=50$ km. Since 1823, southwest rift $V/\tau_0=1.1\times10^{-3}$ km³/yr, $m=5$, and $l=25$ km.

[2]Based on Kienle and Swanson (1980). Four major eruptions in last 100 years; typical volume of 0.1 km³; $l=2$ km from depth of first seismicity during last (1976) eruption.

[3]Based on Decker and Decker (1981). Since 1850, $V/\tau_0=4\times10^{-3}$ km³/yr, $l=18$ km, and in the last 4,500 years $m=20$.

for conduits at Kilauea and other sites are listed in table 54.1. The average volume-intrusion rate, $V/\tau_0$, and the number of prior intrusive pulses or eruptions, $m$, are obtained from geologic and historical data. The length of the conduit, $l$, is obtained from seismic data or other estimates of the distance from the magma reservoir to the end of the conduit. Comparing the results in table 54.1 with the critical intrusion factor of $3\times10^{-4}$ km²/yr from equation 1 shows that the upper east rift zone of Kilauea (upper 25 km) and the Augustine and Mount St. Helens volcanic conduits are open conduits in the sense defined earlier, while the lower east rift zone near Puna and the southwest rift zone of Kilauea are normally closed conduits. These latter examples would require mechanical fracturing before intrusive flow could move through the conduit. The repetitive-intrusion model and the result of equation 1 can be applied to different conduits or parts of conduits at the same volcano. The only requirement is that appropriate values of volume-intrusion rate, length of conduit, and number of prior intrusive pulses are used for the particular conduit or part of conduit being studied.

## INTRUSIVE FLOW IN CONDUITS

Intrusive flow in the east-rift conduit of Kilauea can be modeled as simple forced convective flow in a conduit where the flow is driven by pressure in the summit reservoir. Epp and others (1983) and Decker and others (1983) recognized that the intrusive flow in the east-rift conduit was associated with summit deflation, which in turn was related to dynamic pressure changes in the reservoir. Epp and others (1983) noted that the volume of erupted material in many cases was equal to the volume of deflation of the summit reservoir; they further noted that the amount of magma drained from the summit reservoir was related to the elevation of the vent. This again suggests a simple reservoir-controlled, forced-convection type of conduit flow. Decker and others (1983) have made limiting calculations which indicate that dynamic pressure changes in the summit reservoir between 0.09 MPa and 400 MPa have occurred over the last 25 years at Kilauea Volcano. Recent correlations of summit tilt changes with elevations of rift eruption vents indicate pressure changes of 0.085 MPa per μrad of rapid radial summit tilt (Decker

and others, 1983). During the 1983–84 eruption episodes at Kilauea, tilt changes on the order of 10 μrad were common; these correspond to dynamic pressure changes on the order of 1 MPa. Dynamic pressure change, as used here, is the pressure change that occurs in the summit reservoir during an eruptive episode (Decker and others, 1983). This pressure is related to the pressure necessary to overcome frictional flow effects during periods of intrusive flow. The dynamic pressure change does not include a hydrostatic component, because the hydrostatic pressure is similar at the beginning and end of an eruptive episode. Hydrostatic is used here in the fluid-mechanics context to mean the static pressure in a liquid (in this case, molten magma).

Simple hydrodynamic flow relations can be used to relate observed flow volumes and flow rates to expected conduit diameters. It is obvious, however, that something more than a simple hydrodynamic conduit model is needed to explain the observed flow. Simple hydrodynamic flow relations alone predict that eruptive episodes such as at Kilauea in 1983–84 would not be observed, but rather a continuous low-level flow would occur at the expected reservoir replenishment rate of $8.5\times10^6$ m³/mo (Swanson, 1972). Instead of a continuous intrusive flow rate of 3.28 m³/s ($8.5\times10^6$ m³/mo), the flow rate in the conduit ranged from 10 to 100 m³/s during periods of eruption and was essentially zero between eruptive episodes. If the flow during eruptive phases were averaged out over a year, it would agree closely with Swanson's estimate of the reservoir replenishment rate of $8.5\times10^6$ m³/mo. An additional mechanism must somehow be included with the simple hydrodynamic flow model in order to explain why intrusive or eruptive episodes occur as isolated, repetitive events rather than as a continuous low-level flow. The analysis in the previous section of this paper explained the condition whereby repetitive intrusive pulses could occur in what I have called an open conduit without mechanical fracturing, but this analysis did not explain what controlled the cyclic nature of the intrusions or eruptive episodes. Since the magma supply rate to the summit reservoir appears to be fairly constant and continuous, the only mechanisms that might explain the observed intermittent intrusive flow in such open conduits are non-Newtonian rheology and viscous dissipation.

## NON-NEWTONIAN FORCED-CONVECTION CONDUIT MODEL

The simplest non-Newtonian flow model for intrusive flow in a magma conduit is a Bingham-plastic flow model, as used by Shaw and others (1968) in their analysis of viscosity measurements at Makaopuhi lava lake. A power-law rheology model also fits the observed viscosity data well for basaltic lava or magma (Hardee and Dunn, 1981); however, the Bingham-plastic model is simpler to use and fully adequate for the first-order calculations that follow. In the Bingham-plastic model, flow does not begin until a certain critical yield stress is exceeded; and once this occurs, the flow is similar to Newtonian flow.

If we consider a cylindrical conduit, the pressure drop in the

conduit from simple hydrodynamic theory for laminar flow is (Vennard, 1956)

$$P = 32\mu l v / D^2, \qquad (2)$$

where $P$ is the dynamic or available pressure to drive the intrusive flow, $\mu$ is the average magma viscosity, $l$ is the length of the conduit, $v$ is the average flow velocity of the fluid in the conduit, and $D$ is the diameter of the conduit. For conduits with a noncircular cross section, the concept of an equivalent diameter can be used with this same relation (Chapman, 1967). For geometries such as those of sills or dikes, an expression similar to equation 2 can be developed, or the idea of equivalent diameter can be used. The flow rate $Q$ in the conduit is simply

$$Q = \pi D^2 v / 4. \qquad (3)$$

In the absence of an internal heating mechanism, such as viscous-dissipation heating, the conduit must be sufficiently large that the fluid does not cool appreciably as it flows through. Otherwise, the magma would cool and solidify in place, thereby shutting off the flow. In the simple forced-convection flow model, the fluid magma leaves the summit reservoir at a temperature (say, 1,413 K) well above that at which the magma is considered virtually solid (1,343 K; see Shaw and others, 1968). By equating the heat lost at the conduit wall to the reduction of sensible heat in the flowing fluid, an expression can be developed for the amount of cooling that occurs in a conduit of length $l$. The heat loss of a fluid to the wall of a circular conduit is

$$q = h A_s (T_b - T_w), \qquad (4)$$

where $h$ is the convective heat-transfer coefficient, $A_s$ is the surface area of the conduit, $T_b$ is the bulk temperature of the fluid in the conduit, and $T_w$ is the temperature of the wall (1,343 K). The convective heat-transfer coefficient for fluid flow in a cylindrical conduit (Eckert and Drake, 1959) is

$$h D / k = 3.65, \qquad (5)$$

where $k$ is the thermal conductivity of the fluid. Substituting $h$ from equation 5 into equation 4 and noting that $A_s = \pi D \, dl$ for an incremental length $dl$ gives

$$q = 3.65 \pi k (T_b - T_w) \, dl. \qquad (6)$$

The reduction in sensible heat is the energy lost from the flowing fluid

$$q = -\rho c (\pi D^2 / 4) v \, dT_b, \qquad (7)$$

where $\rho$ is the density of the fluid and $c$ is the specific heat of the fluid. Combining equations 6 and 7 and integrating the result over a conduit length $l$ yields the result

$$T_b - T_w = K \exp[-14.6\alpha l / (D^2 v)], \qquad (8)$$

where $\alpha$ is the thermal diffusivity ($\alpha = k/\rho c$) and $K$ is a constant. Assume that when $l = 0$ at the summit reservoir, the bulk temperature of the magma entering the conduit is, as noted earlier, 70 K above the temperature where the magma is virtually solid ($T_b - T_w = 70$). The constant $K$ in equation 8 then becomes 70, and

$$T_b - T_w = 70 \exp[-14.6\alpha l / (D^2 v)]. \qquad (9)$$

Decker and others (1983) note that the flow in the conduit can be estimated from summit deflation and that this volume is approximately $0.5 \times 10^6$ m$^3$ per $\mu$rad of radial tilt. As an example, episode 2 of the Kilauea eruption that started in 1983 resulted in a summit deflation of 11 $\mu$rad of tilt in 6 days. This corresponds to an intrusive-flow rate of

$$Q = (11/6)(0.5 \times 10^6) = 0.917 \times 10^6 \text{ m}^3/\text{d}$$

or

$$Q = 10.6 \text{ m}^3/\text{s}. \qquad (10)$$

Substituting this into equation 3 gives

$$D^2 v = 10.6(4/\pi), \qquad (11)$$

and substituting this along with appropriate values for magma properties and an assumed conduit length ($l$) of 20 km into equation 9 gives

$$T_b - T_w = 69.7 \text{ K}. \qquad (12)$$

This calculation is interesting in that it shows that the temperature loss as the magma flows the full length of the conduit is only 0.3 K, and this result did not depend on a knowledge of the conduit diameter. A similar result is obtained for a dike. An exact calculation for the dike is not possible because the flow rate $Q$ depends on two geometrical factors: the thickness of the dike ($a$) and the width of the dike ($b$):

$$Q = abv. \qquad (13)$$

An approximate calculation for the dike can be obtained by using the concept of equivalent diameter (Chapman, 1967). The equivalent diameter, sometimes called the hydraulic diameter, is an approximating concept used in fluid mechanics for conduits of noncircular cross section. The equivalent diameter, which attempts to preserve the flow area and the viscous shear at the wall, is simply 4 times the cross-sectional flow area divided by the wetted perimeter of the conduit. The equivalent diameter of a circular conduit is equal to the geometrical diameter. For a dike or sill, the equivalent diameter ($D_{eq}$) can be shown to be

$$D_{eq} = 2a. \qquad (14)$$

The exact temperature result for a dike or sill, similar to equation 9, can be shown to be

$$T_b - T_w = 70 \exp[-7.5\alpha l / (a^2 v)]. \qquad (15)$$

With the use of the concept of equivalent diameter from equation 14, an approximate version of equation 15 is

$$T_b - T_w = 70 \exp[-30\alpha l / (D_{eq}^2 v)]. \qquad (16)$$

Substituting the flow-rate result from equation 11 into equation 16 gives an approximate result for the dike very similar to that for the circular conduit. Specifically, for the 1983–84 Kilauea eruption episodes the temperature loss as the magma flowed in the conduit

was negligible regardless of whether the conduit was a dike or circular conduit.

As noted earlier, Decker and others (1983) estimated pressure changes in the summit reservoir of 0.085 MPa per µrad of rapid, radial summit tilt. Episode 2 of the Kilauea eruption that started in 1983 had a rapid tilt change of 11 µrad during the eruptive phase, and this corresponds to a pressure change of

$$P = 0.085 \times 11 = 0.935 \text{ MPa.} \qquad (17)$$

For the current conduit model, we can assume that the pressure driving the forced convective intrusive flow in the conduit is approximately that given by equation 17. The viscosity of the magma can be estimated by using the result of Shaw and others (1968), who found that the viscosity variation with temperature for basaltic lava could be expressed as

$$\mu = (1 \times 10^{-6}) \exp(26,170/T), \qquad (18)$$

where $\mu$ is in SI units (Pa·s) and $T$ is in kelvins. The constant 26,170 in equation 18 is an experimentally determined constant for this lava. At a magma bulk temperature of $T_b = 1,413$ K, the viscosity is approximately

$$\mu = 100 \text{ Pa·s.}$$

Substituting this and the estimate of pressure from equation 17 into equation 2 and solving for $v/D^2$ gives

$$v/D^2 = P/(32\mu l) = 0.935 \times 10^6 / [32(100)(20 \times 10^3)]$$

or

$$v/D^2 = 0.01461. \qquad (19)$$

Equations 11 and 19 are two independent equations in two variables: the flow velocity $v$ and the conduit diameter $D$. Solving these two equations for $v$ and $D$ yields

$$v = 0.444 \text{ m/s} \qquad (20)$$

and

$$D = 5.51 \text{ m} \qquad (21)$$

for conduit flow during episode 2 of the 1983–84 Kilauea eruptive activity. Values for other episodes of this eruption are listed in table 54.2. These values compare well with observed diameters (4–8 m) of conduit exit holes at eruptive vents such as those shown in figure 54.4. This similarity may be coincidental because flow in the vent conduit, with volatile phases present, is considerably different from flow in the deeper east-rift conduit.

If the conduit were a dike instead of a circular conduit, the dike thickness derived using the approximate result of equation 14 would be

$$a = D/2 = 5.51/2 = 2.76 \text{ m,} \qquad (22)$$

and the dike width from equation 13 would then be

$$b = Q/av = 10.6/(2.76 \times 0.444) = 8.67 \text{ m.} \qquad (23)$$

Estimates for the dike or sill model for this and other eruptive episodes at Kilauea in 1983–84 are listed in table 54.3.

TABLE 54.2.—*Flow rate, total intruded volume, and estimated velocity and conduit diameter (based on forced-convection flow model) for 1983–84 Kilauea eruptive episodes*

[First values of flow rate based on tilt data; second values in parentheses, supplied by E. W. Wolfe of Hawaiian Volcano Observatory, based on field observations of erupted volumes for episodes 2–15 reduced by 20 percent to account for dense-rock magma equivalent. Conduit diameter values in parentheses correspond to calculated values based on field observations of erupted volume]

| Episode | Flow rate Q (m³/s) | | Intruded volume (10⁶ m³) | Velocity (m/s) | Conduit diameter (m) | |
|---|---|---|---|---|---|---|
| 2 | 10.6 | (15) | 5.50 | 0.444 | 5.51 | (6.01) |
| 3 | 13.0 | (30) | 9.04 | .629 | 5.12 | (6.31) |
| 4 | 15.9 | (24) | 5.75 | .556 | 6.03 | (6.68) |
| 5 | 19.5 | (32) | 5.92 | .624 | 6.31 | (7.14) |
| 6 | 31.8 | (27) | 8.25 | .942 | 6.56 | (6.29) |
| 7 | 42.8 | (39) | 8.51 | 1.11 | 7.01 | (6.84) |
| 8 | 57.9 | (65) | 5.00 | .989 | 8.63 | (8.88) |
| 9 | 45.7 | (40) | 7.50 | 1.08 | 7.35 | (7.10) |
| 10 | 36.0 | (52) | 7.00 | .923 | 7.05 | (7.73) |
| 11 | 62.1 | (62) | 8.84 | 1.36 | 7.64 | (7.64) |
| 12 | 56.7 | (51) | 6.94 | 1.15 | 7.91 | (7.70) |
| 13 | 35.9 | (60) | 5.54 | .820 | 7.46 | (8.48) |
| 14 | 60.9 | (70) | 5.00 | 1.01 | 8.74 | (9.05) |
| 15 | 66.6 | (94) | 5.75 | 1.14 | 8.63 | (9.40) |
| 16 | 55.9 | | 4.83 | 1.17 | 8.00 | |
| 17 | 57.9 | | 5.00 | .990 | 8.63 | |
| 18 | 56.4 | | 4.88 | 1.36 | 7.26 | |
| 19 | 21.2 | | 1.83 | .440 | 7.80 | |
| 20 | 38.5 | | 3.33 | .807 | 7.79 | |
| 21 | 35.7 | | 3.08 | .747 | 7.80 | |
| 22 | 56.8 | | 4.91 | 1.13 | 8.00 | |
| 23 | 60.2 | | 5.20 | 1.15 | 8.16 | |
| 24 | 64.5 | | 5.57 | 1.26 | 8.08 | |
| 25 | 69.4 | | 6.00 | 1.33 | 8.16 | |

The previous flow calculations for the forced-convection model are valid as long as the temperature drop in the conduit is small (that is, as long as $T_b - T_w$ at the end of the conduit is approximately the same as at the entrance of the conduit). In the calculation for episode 2, this was certainly the case for the flow rate $Q = 10.6$ m³/s. From equation 9, assuming the same magma properties, this temperature condition can be shown to be satisfied as long as the intrusive flow rate is greater than about $Q = 0.5$ m³/s for a circular conduit. This condition is certainly met for all the eruptive episodes listed in table 54.2. If a dike geometry is assumed, the temperature condition can be shown to be satisfied for the eruptive phases listed in table 54.3 as long as the ratio of dike thickness to dike width ($a/b$) is greater than about 0.025, and this is true for all the episodes listed in table 54.3. In a circular conduit, if the intrusive flow rate $Q$ were very low, the magma would reside in the conduit for a long time and would cool and solidify in the conduit. In a dike, if the ratio $a/b$ were very small, then conduction losses would be large, and the magma would again tend to cool and solidify in the conduit.

Although the previous forced-convection calculations give acceptable values for velocity and conduit size, these calculations do not explain the cyclic nature of the eruptive episodes, with periods of fairly steady intrusive flow separated by quiescent periods with little or no flow. In simple pressure-driven forced convection of Newtonian liquids, we would expect the flow to slow down to the rate at which the summit reservoir is replenished. The flow would continue at this slow rate of 3.28 m³/s ($8.5 \times 10^6$ m³/mo) and would not shut off between eruptive episodes, as observed. This replenishment flow rate of 3.28 m³/s is still greater than the minimum value of 0.5 m³/s

FIGURE 54.4.—Eruptive vents during February 1983 eruptive episode at Kilauea Volcano. Photograph by W.C. Luth, February 25, 1983. Highest fountains are spraying about 50 m high.

required to keep the conduit flow from solidifying in place. A possible explanation for the on-off nature of the cyclic eruptive episodes can be found by looking at the non-Newtonian flow properties of magma at temperatures below the liquidus such as occur in the conduit flow process. Shaw and others (1968) observed that Hawaiian basaltic lava at temperatures similar to those seen in the 1983–84 Kilauea eruptive activity behaved much like a Bingham-plastic fluid with a yield stress of 7–12 kg/m² (700 to 1,200 dyne/cm²). If we assume that a pressure difference large enough to overcome the Bingham yield stress must exist before flow can begin, then this pressure can be estimated and compared with the pressure estimates of Decker and others (1983) used in the previous flow

calculations. A simple pressure balance on the conduit is:

$$PA_c = \sigma A_s \tag{24}$$

where $A_c$ is the cross-sectional area of the conduit, $A_s$ is the surface area of the conduit, and $\sigma$ is the Bingham yield stress. For a circular conduit this becomes

$$\pi D^2 P/4 = \pi D l \sigma$$

or

$$P = 4\sigma l/D. \tag{25}$$

For a yield stress of 7 kg/m², this becomes

TABLE 54.3.—*Estimate of dike size for east-rift conduit (based on forced-convection conduit model) for 1983–84 Kilauea eruptive episodes*

| Episode | Dike thickness (m) | Dike width (m) |
|---------|--------------------|----------------|
| 2  | 2.76 | 8.67 |
| 3  | 2.56 | 8.04 |
| 4  | 3.01 | 9.47 |
| 5  | 3.15 | 9.91 |
| 6  | 3.28 | 10.3 |
| 7  | 3.50 | 11.0 |
| 8  | 4.32 | 13.6 |
| 9  | 3.67 | 11.5 |
| 10 | 3.52 | 11.1 |
| 11 | 3.82 | 12.0 |
| 12 | 3.96 | 12.4 |
| 13 | 3.73 | 11.7 |
| 14 | 4.37 | 13.7 |
| 15 | 4.31 | 13.6 |
| 16 | 4.00 | 12.6 |
| 17 | 4.32 | 13.6 |
| 18 | 3.63 | 11.4 |
| 19 | 3.90 | 12.3 |
| 20 | 3.89 | 12.2 |
| 21 | 3.90 | 12.3 |
| 22 | 4.00 | 12.6 |
| 23 | 4.08 | 12.8 |
| 24 | 4.04 | 12.7 |
| 25 | 4.08 | 12.8 |

$$P = 4(7)(20 \times 10^3)/5.5 = 0.102 \times 10^6 \text{ kg/m}^2$$

or

$$P = 0.985 \text{ MPa.} \tag{26}$$

Using the upper value for yield stress of 12 kg/m$^2$ results in a predicted pressure of $P=1.69$ MPa. The predicted pressure differential driving the convective flow in the conduit, therefore, ranges from $P=0.985$ MPa to $P=1.69$ MPa. This compares favorably with the estimate of Decker and others (1983), who suggested that this pressure is 0.085 MPa per $\mu$rad of rapid radial tilt. Radial tilt changes for the early eruptive episodes in 1983 ranged from 10 to 18 $\mu$rad, corresponding to pressures of 0.85 to 1.53 MPa. These pressure estimates using the prediction of Decker and others (1983) agree well with the estimates above, 0.985 to 1.69 MPa, based on the Bingham model of Shaw and others (1968). This agreement supports the idea that the cyclic nature of the Kilauea eruptive episodes in 1983–84 may have been a result of non-Newtonian rheology effects. This calculation also supports the pressure predictions of Decker and others (1983) for the summit reservoir.

The forced-convection flow model can also be applied to other eruptions at Kilauea Volcano. For instance, the 1959–60 eruption was a major one with probably 17 eruptive phases (Richter and Moore, 1966). The conduit length $l$ was probably much shorter than in the 1983–84 eruptive episodes because the eruption site was close to the summit of Kilauea; the 1959–60 conduit length $l$ was probably on the order of 5 km (Peck and others, 1979). The principle effect of this shorter conduit length would be to reduce the time between eruptive episodes. Equation 25 shows that the summit-reservoir pressure necessary to overcome the Bingham-plastic yield stress of magma in the conduit is proportional to conduit length $l$. Since $l$ was much shorter in the 1959–60 eruption, the dynamic pressure needed to overcome the yield stress of the magma was much less. As the summit magma chamber inflated, the necessary pressure level for an eruptive episode would be reached quickly, and the eruptive phases would be expected to repeat on a time period much shorter than the approximate one-month time periods of the 1983–84 eruptive activity. As expected, the eruptive episodes for the 1959–60 eruption repeated on an average interval of about 2 days (Richter and Moore, 1966).

## VISCOUS-DISSIPATION FORCED-CONVECTION CONDUIT MODEL

Viscous dissipation may be an important mechanism in intrusive flow of magma in the rift conduits of Kilauea. Viscous dissipation has been suggested before as a key mechanism in eruptive cycles (Shaw, 1969, 1973; Anderson and Perkins, 1974; Fujii and Uyeda, 1974; Hardee and Larson, 1977; Fujii, 1981). Previous confirmation of the theory has been limited to estimates of runaway temperature and dike thickness for various types of magma. Because of the extensive amount of real-time field observations of conduit flow in the east-rift conduit during the 1983–84 Kilauea activity, it is possible to compare conduit models with a large set of field data, such as eruption temperature of lava, time between successive eruptive episodes, intrusive flow rate, and, in some cases, the conduit exit diameter at the vent.

The energy of viscous dissipation ultimately comes from conversion of pressure energy in the summit reservoir into frictional heat. The pressure in the reservoir overcomes the frictional resistance to flow, and in turn the viscous shear caused by the frictional resistance is converted to heat, which raises the temperature of the fluid. This happens in all cases of simple forced-convective flow in conduits, but usually the heating effect is small. In the case of magmatic intrusions, however, the heating effect can be significant. Without the effect of viscous dissipation, magma would have difficulty flowing in small conduits and thin dikes, and there would be a tendency for the magma to cool and solidify in place. Even where mechanical fracturing occurs and opens the conduit, viscous dissipation is likely to be important in allowing the magma to flow rather than quickly solidify. Since the energy available for viscous heating comes from the pressure or potential energy in the reservoir, field observations of the thermal state of erupting lava can be used to estimate conditions back in the reservoir. Also, it is obvious that the magnitude of the viscous-heating effect is limited ultimately by the pressure or potential energy in the reservoir.

A forced-convection transport model of the east-rift conduit can also be developed in which viscous-dissipation effects are important. Viscous dissipation can affect both the temperature of erupting lava and the cyclic episodes of the eruption. The time required for thermal effects to build to thermal runaway, coupled with the summit-reservoir pressure history, determines the repose period between dissipatively controlled eruptive cycles. The rapid increase of viscosity of the magma as partial crystallization occurs on cooling probably causes the magma to jam up and cease flowing in a reverse type of thermal runaway. Calculations to demonstrate this jam-up phenomenon or reverse thermal runaway are difficult because

the viscosity of the liquid/crystal magma composition at near-solidus temperatures is not well known (Shaw and others, 1968). Such an effect, once started, would rapidly accelerate and cause the flow to shut off suddenly. As pressure in the summit reservoir increased again, a point would be reached at which viscous-dissipation effects would again produce a thermal-runaway situation, and a new eruptive episode would begin. The observed cyclic nature of the eruptive episodes, the observed eruptive temperatures and volumes, and the highly temperature-dependent nature of the magma viscosity are all consistent with a forced-convection magma-conduit model that includes viscous-dissipation effects. The calculations that follow here are for this transport model.

First, the viscous-dissipation thermal-runaway temperature is determined and shown to be consistent with observed eruption temperatures. The pressure drop in the conduit is determined, and this in turn can be related back to the pressure in the summit reservoir. The shape of the conduit is then determined. Next, an approximate calculation shows that the runaway time is of the right order of magnitude in comparison with observed periods of repose between eruptive episodes.

For a cylindrical conduit, the pressure drop in the conduit from simple hydrodynamic theory for laminar flow (equation 2) is (Vennard, 1956) $P = 32 \mu l v / D^2$, where $P$ is the dynamic or available pressure to drive the intrusive flow, $\mu$ is the average magma viscosity, $l$ is the length of the conduit, and $D$ is the diameter of the conduit. For conduits with a noncircular cross section, the concept of an equivalent diameter can again be used. From the energy equation and the expression for viscous dissipation in cylindrical coordinates (Batchelor, 1970), it can be shown that for steady flow in a conduit where the viscous-dissipation heating in the conduit is in balance with conduction loss at the wall, the temperature of the fluid in the conduit is

$$T_b - T_w = (8/9)\mu v^2/k, \tag{27}$$

where $T_b$ is the average or bulk temperature of the fluid in the conduit, $T_w$ is the temperature of the wall (again, 1,313 K), $v$ is the average flow velocity of the fluid, and $k$ is the thermal conductivity of the fluid. The flow rate $Q$ (equation 3) is $Q = \pi D^2 v/4$. Between eruptions or eruptive episodes, the pressure in the summit reservoir builds in magnitude, and the pressure acts on material in the conduit that is hot but solidified and resistant to flow. The viscosity is high at this point, and the flow velocity is near zero. Flow will begin to occur at an imperceptibly slow rate, producing a small amount of viscous-dissipation heating. Gradually at first and then rapidly, the process will accelerate as viscous heating reduces the viscosity of the fluid. Most of the noticeable activity occurs in the latter part of the thermal-runaway process, giving it the appearance of suddenness. Equations 2 and 18 can be used to calculate the condition under which appreciable flow begins. Once initiated, the flow will tend to persist at the conditions of temperature and velocity in which the viscous heat generation in the fluid is exactly balanced by the conduction loss at the wall. At slower flow rates the fluid will cool too much by conduction loss to the wall, and some of the magma will tend to solidify in place. At higher flow rates the excessive viscous

heat generation will produce melting at the walls.

The energy absorbed by latent heat at the wall and the reduced flow velocity as the conduit diameter increases will both tend to restrict the flow and prevent the velocity and temperature from increasing indefinitely. The natural tendency is for the intrusive flow to assume a steady flow condition at the critical temperature at which viscous-heating effects predominate. This critical viscous-dissipation temperature or "runaway" temperature has been studied by Fujii and Uyeda (1974), Hardee and Larson (1977), and others. The runaway temperature, which corresponds roughly with the expected value of the lava-eruption temperature, can be determined by using equations 2, 18, and 27. First, equations 2 and 27 are combined to eliminate $v$:

$$P^2 = (9/8)k(32l/D^2)^2 (T_b - T_w)\mu. \tag{28}$$

Assuming a wallrock or melt temperature of 1,343 K (Shaw and others, 1968) for basalt and using equation 18 for viscosity yields:

$$\mu = (1 \times 10^{-6})\exp[26,170/(\Phi + 1,343)] \tag{29}$$

where $\Phi = T_b - T_w$. Substituting equation 29 into equation 28 gives:

$$P^2 = [(9/8)k(32l/D^2)^2(1 \times 10^{-6})]\Phi \cdot$$
$$\exp[26,170/(\Phi + 1,343)]. \tag{30}$$

The condition of thermal runaway occurs when a small change in reservoir pressure $P$ results in a large change in temperature $\Phi$ or, in other words, when $dP/d\Phi = 0$. Differentiating equation 30 with respect to $\Phi$, setting $dP/d\Phi = 0$ and then solving for $\Phi$ results in

$$[26,170\Phi/(\Phi + 1,343)^2] - 1 = 0$$

or

$$\Phi = 77 \text{ K}. \tag{31}$$

The average temperature of the magma during intrusive flow and the corresponding expected lava-eruption temperature is then

$$T_b = 1,343 + 77 = 1,420 \text{ K}. \tag{32}$$

This compares well with typical lava-eruption temperatures from measurements such as those shown being made in figure 54.5 near the vent during the 1983–84 eruptive activity of Kilauea (Hardee, 1983).

The dynamic pressure in the reservoir or the pressure driving the flow in the conduit can be determined from equation 2. First, equations 3 and 27 are used to eliminate $D$ and $v$ from equation 2 to give

$$P = 9\pi kl(T_b - T_w)/Q. \tag{33}$$

Substituting the flow rate from equation 10 into equation 33 gives

$$P = 9\pi(2.09)(20 \times 10^3)(1,420 - 1,343)/10.6 = 8.59 \text{ MPa.} \tag{34}$$

This pressure is within the range (0.09–400 MPa) estimated by Decker and others (1983), although it is higher than their estimates

FIGURE 54.5.—Thermal measurements being made in molten lava during February 1983 eruptive episode at Kilauea Volcano. Photograph by W.C. Luth, February 25, 1983.

for dynamic pressure changes in the summit reservoir during Kilauea's 1983–84 activity. The expected flow velocity in the conduit can be found from equation 27:

$$v = [(9/8)k(T_b - T_w)/\mu]^{0.5}. \tag{35}$$

The viscosity to be used in equation 35 is found from equation 18 at the fluid runaway temperature of equation 32:

$$\mu = (1 \times 10^{-6})\exp(26,170/1,420) = 100 \ \text{Pa·s}. \tag{36}$$

Substituting this and other values into equation 35 gives

$$v = [9(2.09)(1,420-1,343)/8(100)]^{0.5} = 1.33 \ \text{m/s} \tag{37}$$

for the average intrusive flow velocity in the conduit during eruptive episode 2. The effective diameter of the conduit can then be found from equation 3:

$$D = (4Q/\pi v)^{0.5} = [4(10.6)/\pi(1.33)]^{0.5} = 3.18 \ \text{m}. \tag{38}$$

This value compares reasonably well with observed diameters of conduit exit holes, typically around 4–8 m in diameter, at eruptive vents such as those shown in figure 54.4 during the 1983–84 Kilauea activity. The predicted diameter is not too different from the value for the non-Newtonian forced-convection conduit model in the previous section (table 54.4).

TABLE 54.4.—*Comparison of estimated conduit diameter for east-rift conduit during 1983–84 Kilauea eruptive episodes (based on forced-convection conduit model and forced-convection model with viscous dissipation)*

| Episode | Conduit diameter from table 54.2 (m) | Conduit diameter with viscous dissipation (m) |
|---------|--------------------------------------|-----------------------------------------------|
| 2  | 5.51 | 3.18 |
| 3  | 5.12 | 3.52 |
| 4  | 6.03 | 3.90 |
| 5  | 6.31 | 4.32 |
| 6  | 6.56 | 5.52 |
| 7  | 7.01 | 6.40 |
| 8  | 8.63 | 7.44 |
| 9  | 7.35 | 6.61 |
| 10 | 7.05 | 5.87 |
| 11 | 7.64 | 7.71 |
| 12 | 7.91 | 7.36 |
| 13 | 7.46 | 5.86 |
| 14 | 8.74 | 7.63 |
| 15 | 8.63 | 7.98 |
| 16 | 8.00 | 7.31 |
| 17 | 8.63 | 7.44 |
| 18 | 7.26 | 7.34 |
| 19 | 7.80 | 4.50 |
| 20 | 7.79 | 6.07 |
| 21 | 7.80 | 5.84 |
| 22 | 8.00 | 7.37 |
| 23 | 8.16 | 7.59 |
| 24 | 8.08 | 7.85 |
| 25 | 8.16 | 8.14 |

The assumption of laminar flow used in the selection of equation 2 can be checked. The Reynolds number ($R$) for flow in a conduit is

$$R = vD\rho/\mu. \tag{39}$$

Inserting values of velocity and conduit diameter from equations 37 and 38 gives

$$R = (1.33)(3.18)(2,700)/100 = 113. \tag{40}$$

This value of $R$ is well below the critical value of 2,700 for transition to turbulent flow in conduits, and therefore the original assumption of laminar flow in the conduit is valid.

The main portion of the east-rift conduit from the summit reservoir down to the vicinity of the erupting vent may be a tabular dike rather than a circular conduit. Flow equations for the geometry of a dike or sill can be developed that are similar to the previous equations for the circular conduit. Flow in a dike or sill can be modeled as flow between parallel plates. The pressure drop for this flow geometry is (Gebhart, 1961)

$$P = 12\mu lv/a^2, \tag{41}$$

where $a$ is the thickness of the dike. With viscous-dissipation effects considered, the temperature of the fluid flowing in the dike is

$$T_b - T_w = (3/5)\mu v^2/k. \tag{42}$$

Equations 41 and 42 are similar to equations 2 and 27 for the cylindrical conduit model. The volume flow rate in the dike (equation 13) is again $Q = abv$, where $b$ is the width of the dike. If the calculation that led to equation 31 is carried out for the dike geometry using equations 41 and 42, the thermal runaway temperature can be shown to be the same as in equation 31: $\Phi = 77$ K.

The different constant in equation 42 does not affect the result for runaway temperature. The flow velocity at runaway is found from equation 42:

$$v = [(5/3)\Phi k/\mu]^{0.5} = [(5/3)(77)(2.09)/100]^{0.5} \tag{44}$$

or

$$v = 1.63 \text{ m/s}. \tag{45}$$

Although velocity and flow rate are known, it is not possible to determine the thickness of the dike or sill from equation 13 because of the additional geometric parameter $b$, the width of the dike, that now occurs in equation 13. In the absence of additional information, the best procedure for dike and sill flow calculations is to use again the concept of equivalent diameter (Chapman, 1967).

Referring to the previous calculation for conduit diameter in equation 38, where the conduit diameter was 3.18 m, and substituting this into equation 14 gives

$$a = D_{eq}/2 = 3.18/2 = 1.59 \text{ m}. \tag{46}$$

This implies that if the flow during episode 2 of the 1983–84 Kilauea activity passed through a dikelike conduit rather than a circular conduit, the dike was on the order of 1.59 m thick, the dynamic pressure was similar to that determined in equation 34, and the flow velocity for the dike was similar to that determined in equation 45, assuming that viscous-dissipation effects were a major factor. The dike thickness in equation 46 is not thin enough for sufficient cooling and solidification to occur to cause the conduit to easily jam up and go into a period of repose. This can be shown by applying equation 8. The value of dike thickness in equation 46, however, is consistent with observations of fossil dikes in Hawaii. The thickness of the dike or conduit in episode 2 of Kilauea's 1983 eruptive activity can be determined using equation 13, the value of dike thickness in equation 46, the flow velocity from equation 45, and a typical value of intrusive flow rate such as that in equation 10:

$$b = Q/(a/v) = 10.6/[(1.59)(1.63)] = 4.09 \text{ m}. \tag{47}$$

Viscous dissipation can explain the intermittent intrusive flow observed in the various eruptive episodes. When a certain critical pressure and temperature (see, for example, equation 32) are reached, flow begins, and the pressure starts to drop. Eventually the flow stops, and magma in the conduit cools. A period of repose then occurs, during which the pressure builds up again, and the magma in the conduit is heated to the runaway condition once more by viscous dissipation. The period of repose or time between eruptive episodes can be determined by a simple calculation in which the viscous-dissipation energy is equated to the increase in internal energy necessary to raise the magma from its solidification temperature back up to its runaway temperature. In the interest of keeping the calculation simple, I assume that the magma and surroundings are hot and near the solidification temperature, as suggested by the earlier analysis leading to equation 1. Conduction losses would then be small during the period of repose between eruptive episodes. This is probably a good assumption; in thermal-runaway problems of this type, the temperature does not increase much until nearly the

moment of thermal runaway (Hardee and others, 1977). The rate of viscous heat dissipation per unit volume in a cylindrical conduit is given by Batchelor (1970) as $(P/l)^2 R^2/4\mu$ at any radius $R$. The volume-average viscous heat dissipation for a cylindrical conduit of radius $R$ is found by integrating the local value over the whole volume and is $(P/l)^2 R^2/(8\mu)$. The repose time between eruptive episodes (thermal-runaway time) is found by equating the volume-average viscous heat dissipation to the change in internal energy necessary to raise the magma in the conduit from the solidification temperature to the runaway temperature,

$$\rho c \ dT/d\tau = P^2 R^2/8\mu l^2, \tag{48}$$

and then integrating this result

$$\int_{1343}^{1420} \exp(26{,}170/T) \ dT = \int_0^{\tau_r} [P^2 R^2/(8l^2 \rho c \times 10^{-6})] \ d\tau.$$

The integration is performed after making use of the substitution $Z = 26{,}170/T$ where $T = 1{,}343$ K, $Z_1 = 26{,}170/1{,}343$, $T_2 = 1{,}420$ K, and $Z_2 = 26{,}170/1{,}420$. The solution for the repose time is then

$$\tau_r = [8l^2 \rho c \times 10^{-6}/(P^2 R^2)] \ [(1/T)\exp(26{,}170/T)$$
$$- 26{,}170 Ei(26{,}170/T)] \Big]_{1343}^{1420}. \tag{49}$$

$Ei(x)$ is the exponential integral function (Mathews and Walker, 1965), which can be evaluated using tabulated values (Gautschi and Cahill, 1965) and equation 49. The expression for the repose time then becomes

$$\tau_r = [8l^2 \rho c \times 10^{-6}/(P^2 R^2)](1.370 \times 10^{10}). \tag{50}$$

Equations 2 and 27 can be combined to eliminate $v$, and the result can be substituted into equation 50 to eliminate $l^2/P^2$:

$$\tau_r = (1.370 \times 10^4) D^2 \rho c /[36k\mu_r(T_b - T_w)]. \tag{51}$$

Substituting into equation 51 the appropriate values and the diameter determined for the conduit in episode 2 gives

$$\tau_r = (1.370 \times 10^4)(3.18)^2(2{,}700)(4{,}186)/[36(2.09)(100)(77)]$$

or

$$\tau_r = 2.70 \times 10^6 \ s = 31.3 \ d. \tag{52}$$

This value of repose time compares well with observed values for the 1983–84 Kilauea eruptive episodes. Repose times for the various episodes listed in table 54.5 were determined either by inflation of the summit or by absence of lava production. A repose period of 41 d based on inflation data or 26 d based on the absence of lava production was observed between the first and second episodes (table 54.5). The average of these two values, 33.5 d, compares well with the estimated value of 31.3 d in equation 52. The repose time of 31.3 d and the average magma-supply estimate of Swanson (1972) imply that each eruptive episode should produce a total magma volume of about $9 \times 10^6$ m$^3$; the observed values in table 54.3 are not far from this, ranging from $5 \times 10^6$ to $9 \times 10^6$ m$^3$.

Equation 51 and the result from equation 52 give an alternative viewpoint to the yield-stress calculations of equations 24–26 for observed cyclic eruptive behavior. Equation 24 is based on the assumption that the non-Newtonian or Bingham yield stress combined with an increasing reservoir pressure is the mechanism behind cyclic eruptive behavior. Equation 51 assumes that viscous dissipation is the mechanism behind cyclic eruptive behavior. Although the agreement of the viscous-dissipation conduit model with observed data is good during early episodes of the eruption, the agreement is not as good in the later episodes. For instance, the observed flow rate, $Q$, increases and the observed repose time, $\tau_r$, decreases in later eruptive episodes (see tables 54.2, 54.5). Substituting conduit diameter, $D$, from equation 3 into equation 51, however, would predict that repose time is proportional to $Q$. Although this may be true in some cases, it is not the pattern of the later episodes of the 1983–84 Kilauea eruptive activity.

The previous analysis has examined how viscous dissipation might be the mechanism responsible for intermittent intrusive flow in the east-rift conduit. As noted earlier, viscous dissipation has been proposed before as the mechanism behind recurring volcanic eruption cycles (Shaw, 1969, 1973; Anderson and Perkins, 1974; Fujii and Uyeda, 1974; Hardee and Larson, 1977; and Fujii, 1981). Another interesting possibility is that viscous dissipation may be the mechanism responsible for the fairly constant magma supply to the summit reservoir from the asthenosphere. As mentioned earlier, the summit reservoir appears to be replenished at a rate of 3.28 m$^3$/s (Swanson, 1972). If a conduit from the asthenosphere to the summit reservoir has magma flow due to viscous dissipation at the critical runaway condition, then equations 2, 3, and 27 apply. Fujii and Uyeda (1974) argued that the driving force for magma ascent in the upper mantle or lower crust is buoyancy due to density differences between the country rock and the magma and that this is approximately equal to

$$P/l = 1{,}000 \ J/m^4 \ (100 \ dyne/cm^3). \tag{53}$$

TABLE 54.5.—*Repose times between 1983–84 eruptive episodes of Kilauea*

[Data from Hawaiian Volcano Observatory (HVO) and from E.W. Wolfe]

| Episodes | Repose time ($\tau_r$) | |
|---|---|---|
| | Duration of inflation (d) | Absence of lava production (d) |
| 1–2 | 41 | 26.0 |
| 2–3 | 24 | 23.4 |
| 3–4 | 63 | 65.3 |
| 4–5 | 13 | 12.0 |
| 5–6 | 20 | 19.3 |
| 6–7 | 22 | 20.6 |
| 7–8 | 23 | 19.6 |
| 8–9 | 11 | 8.5 |
| 9–10 | 21 | 17.3 |
| 10–11 | 30 | 27.3 |
| 11–12 | 24 | 22.4 |
| 12–13 | 51 | 50.1 |
| 13–14 | 10 | 8.3 |
| 14–15 | 15 | 15.3 |
| Average | 26.3 | 24.0 |

Substituting this pressure gradient and the viscosity (100 Pa·s) at thermal runaway into equation 2 and solving gives

$$D^2/v = 3.2 \text{ m·s.} \qquad (54)$$

Substituting this result and the flow velocity at thermal runaway from equation 37 into equation 3 gives

$$Q = (\pi/4)D^2v = (\pi/4)(3.2)(1.33)^2 = 4.45 \text{ m}^3/\text{s.} \qquad (55)$$

This magma-reservoir replenishment rate (4.45 m³/s) compares well with the observed replenishment rate (3.28 m³/s), considering the simplicity of the model and assumptions.

## CONCLUSIONS

The upper part of Kilauea's east-rift conduit (upper 25 km) is relatively open to flow, and intrusions in it are usually aseismic. The lower part of the east-rift conduit is usually closed and requires fracturing to produce intrusions, which are normally associated with seismicity. A repetitive thermal-intrusion model was used to show that the difference between these two types of conduit flow is determined by the average volume-intrusion rate, length of conduit, and, to a lesser extent, the number of previous intrusions in that part of the conduit. Non-Newtonian rheology or viscous-dissipation effects are probably important in determining flow characteristics in the Kilauea conduit system.

A conduit flow model based on forced convective flow of a non-Newtonian (Bingham plastic) liquid is compatible with field observations for the recent 1983–84 Kilauea eruptive activity. Predictions of dynamic pressure based on this model (0.985–1.69 MPa) agree well with the summit pressure estimates (0.85–1.53 MPa) of Decker and others (1983). Predicted effective conduit diameters are on the order of 5–10 m. For the diameters, conduit lengths, and intrusion rates involved in the 1983–84 Kilauea eruptive episodes, the magma does not have time to cool appreciably as it flows the length of the conduit. The temperature of the magma at the vent is therefore probably close to the temperature of the summit reservoir. Although the conduit is circular at the vent, at depth it may have a shape like a dike or sill. The concept of an effective diameter was used to examine conduit shapes such as dikes or sills; predicted dike thicknesses were still large enough that heat loss at the conduit wall would not have been a major factor in the 1983–84 Kilauea activity. The field measurements of Bingham-plastic yield stress of Kilauea basalt (Shaw and others, 1968) were used to explain the cyclic nature of eruptive episodes. The summit magma-reservoir pressure must build sufficiently to overcome the Bingham-plastic yield stress of the magma in the conduit before each eruptive cycle. The resulting calculated pressure changes in the summit reservoir agree closely with pressure-change estimates of Decker and others (1983).

A conduit flow model was also developed in which viscous dissipation effects were included. This conduit model is particularly applicable to thin conduits, on the order of 0.1 m thick or less. Such conduits are sufficiently thin that thermal losses to the conduit wall

are important, and an internal heat supply is needed to keep the flowing magma from solidifying rapidly in the conduit. Viscous dissipation is a likely internal heat source for such intrusive flows. The forced-convection flow model with viscous-dissipation effects developed here is compared with field observations. The model predicts lava-eruption temperatures around 1,420 K, and this is in good agreement with field observations. The dynamic pressure driving this intrusive flow is approximately 10 times higher than corresponding pressure predictions for the flow model based on non-Newtonian rheology. The higher pressures occur because pressure is the source of energy for the internal viscous-dissipation heat. Calculation of thermal-runaway time based on the viscous-dissipation flow model predicted times between eruptive episodes on the order of a month for the 1983–84 Kilauea eruptive activity, in agreement with observations. Although the viscous-dissipation flow model gives results that are compatible with field observations, this model predicts effective conduit diameters for the 1983–84 Kilauea activity that are sufficiently large that an internal heat mechanism like viscous dissipation is not needed. This suggests that viscous-dissipation effects may not have been important in the 1983–84 Kilauea eruptive episodes. Viscous-dissipation effects may well be important in other cases of rift intrusions, particularly for rapid intrusions in thin dikes and sills. Viscous dissipation can also be used to explain how the summit reservoir is replenished at a fairly constant rate over long periods of time. Both the non-Newtonian flow model and the viscous-dissipation flow model depend primarily on the unusual rheology of magma. The Bingham-plastic model considers that viscosity depends on shear stress, and the viscous-dissipation model considers that viscosity depends on temperature. Actually, magma viscosity in the subliquidus temperature range depends on shear stress, shear rate, temperature, time, and previous thermal history (Hardee and Dunn, 1981). The Bingham-plastic flow model and the viscous-dissipation flow model are simply ways of looking at major magma-viscosity effects one at a time.

Epp and others (1983) noted that an inverse relation exists between the summit tilt deflation of Kilauea and the elevation of associated eruptive vents. Decker and others (1983) found that this relation indicated a pressure change of 0.085 MPa per μrad of rapid radial summit tilt. By correlating data from near the summit to near sea level, Epp and others (1983) showed that a general relation exists between volume of erupted lava and vent elevation: volume of erupted lava tends to increase with a decrease in elevation. These observations are consistent with the conduit and dike models developed earlier. The dynamic pressure change available to overcome frictional resistance in the conduit was calculated to be about 1.0 MPa (0.985–1.69 MPa). Sufficient pressure must also be available in the summit reservoir to overcome the hydrostatic head from the reservoir up to the vent. The difference in hydrostatic head between summit vents and those near sea level is about 30 MPa (roughly 1 km of liquid magma). The hydrostatic head favors the vents near sea level. The conduit length from the summit reservoir to the vents near sea level is 2 to 3 times longer, and the pressure required to overcome frictional losses is that much greater. However, although this pressure doubles or triples to 2–3 MPa, the 30-MPa

reduction in hydrostatic head more than offsets this. Greater reservoir pressure tends to be available to drive intrusive flows to the lower elevation vents. It is not surprising that the volume of eruption tends to be larger at lower elevation vents.

If pressure favors eruptions at lower elevation vents, the obvious question is, why do most eruptions occur at higher elevation vents? The answer is that thermal effects tend to keep the upper portion of the east rift zone more open to flow (see table 54.1). The lower elevation vents are connected to a portion of the east-rift conduit that is not normally open or available for intrusive flow except at special times when fracturing events or earthquakes have temporarily opened the conduit all the way to vents near sea level.

The east-rift conduit and its eruptions provide valuable data that can be used to construct and test models of intrusive magma flow. Considerably more data are available from east-rift eruptions than are available from other intrusive magmatic processes. Once models of intrusive magmatic flow are tested, it is possible to use them for prediction in other situations where data are meager or absent. It is possible to use the intrusive flow models to predict the thermal-runaway conditions for a shallow magma reservoir that is intersected by a drill hole. These models can also be extended to magma other than basalt. For example, using the data of Piwinskii and Weed (1980), the viscosity for a molten rhyolite sample from Nevada is found to be $\mu = (0.857 \times 10^{-6}) \exp(36,194/T)$. This is the same general form of the viscosity function as used earlier in equation 18. When the earlier calculations are repeated using the viscosity function of Piwinskii and Weed (1980), the thermal-runaway times, or times between eruptive episodes, for rhyolite are on the order of hundreds of years or more, depending on the assumptions used for conduit diameter.

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