

U.S. Department of the Interior
U.S. Geological Survey

Mineral Deposit Density—An Update

U.S. GEOLOGICAL SURVEY PROFESSIONAL PAPER 1640—A



CONTRIBUTIONS TO GLOBAL MINERAL RESOURCE ASSESSMENT RESEARCH

Mine photograph on cover. The Bingham Canyon porphyry copper-gold-molybdenum mine in the Oquirrh Mountains, southwest of Salt Lake City, Utah, was opened in 1904 as the first open-pit copper mine in the world. The pit is about 3 kilometers across and has produced nearly 12 million tons of copper, an unequaled record. This report provides mineral deposit densities for 13 deposit types, including porphyry copper deposits like that at Bingham Canyon. Photograph by Charles Cunningham, U.S. Geological Survey.

Satellite image on cover. View of North and South America from space. Image courtesy of the National Aeronautics and Space Administration.

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**By Donald A. Singer, W. David Menzie, David M. Sutphin,
Dan L. Mosier, and James D. Bliss**

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A description of a method to estimate the numbers of undiscovered deposits and a compilation of 27 mineral deposit density estimates for 13 deposit types

CONTRIBUTIONS TO GLOBAL MINERAL RESOURCE ASSESSMENT RESEARCH

Edited by Klaus J. Schulz

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EDITOR'S PREFACE

Global demand for mineral resources will continue to increase for the foreseeable future because of the continuing increase in global population and the desire to improve living standards worldwide. Although no global shortages of mineral resources are expected in the near future, the growing demand requires continued exploration for undiscovered mineral deposits as identified resources are depleted. However, competing land uses and growing concerns over the possible environmental degradation associated with mining are increasingly affecting mineral exploration and development worldwide.

Informed planning and decisions about biological sustainability and resource development require a long-term perspective and an integrated approach to land-use, resource, and environmental management. This approach needs unbiased information on the global distribution of identified and especially undiscovered mineral resources; the economic, social, and political factors influencing their development; and the environmental consequences of, and requirements for, their utilization.

The U.S. Geological Survey (USGS) is authorized by Congress to collect, analyze, and disseminate data on the domestic and international supply of and demand for minerals essential to the U.S. economy and national security. In response to the growing concern about global sustainability of mineral production and environmental quality, and the simultaneous increase in demand for global mineral resource information, the USGS has initiated a project to begin assessing selected mineral commodities globally. Along with providing estimates of the quantity, quality, and regional distribution of undiscovered mineral resources, these global assessments will provide consistent, systematic data bases of current geologic and mineral-resource information at continental and global scales. The assessments will provide information necessary for evaluating the potential effects of land-use, environmental, and resource-development decisions on global minerals supply and sustainable development.

A vital component of the USGS global mineral resource assessment project is new research to test and improve our data, geologic and mineral deposit models, and understanding of the fundamental processes of mineral formation, preservation, and environmental response. USGS Professional Paper 1640, "Contributions to Global Mineral Resource Assessment Research," presents significant results of this research to advance the state-of-the-art in mineral deposit modeling and resource assessment.

Klaus J. Schulz

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CONVERSION FACTORS

Multiply	By	To obtain
meter (m)	3.281	foot
kilometer (km)	0.6214	mile
square kilometer (km ²)	0.3861	square mile

Mineral Deposit Density—An Update

By Donald A. Singer,¹ W. David Menzie,² David M. Sutphin,²
Dan L. Mosier,¹ and James D. Bliss³

ABSTRACT

A key function of many quantitative mineral resource assessments is estimation of the number of undiscovered deposits. Numerous techniques can be used directly or as guidelines to make these estimates. Most robust of these methods is a form of mineral deposit model wherein for a deposit type, the numbers of deposits per unit area from well-explored regions are counted and the resulting frequency distribution is used either directly for an estimate or indirectly as a guideline in some other method. Ratios of the number of deposits per unit area can be used in histograms to show variation of deposit densities by type. In three-part quantitative resource assessments, prevention of bias requires that deposits of each type be defined the same way as in delineation of permissive tracts and as in construction of grade and tonnage models; these same definitions and rules must also be applied to deposit density models.

This paper provides mineral deposit densities for 13 selected deposit types from previous publications and this study. Many of the specially selected areas reported here provide standards to identify what should be high estimates of the number of undiscovered deposits in most situations.

Previously reported densities follow:

1. Low-sulfide gold quartz vein deposits in four regions have deposit densities ranging from 0.0043 to 0.0054 deposit per square kilometer.
2. Bedded barite deposits in Nevada have a deposit density of 0.074 deposit per square kilometer.
3. Diamond kimberlite pipe deposits in southern Africa have a deposit density of 0.000012 deposit per square kilometer.

4. Podiform chromite deposits in California have a deposit density of 0.115 deposit per square kilometer.
5. Franciscan volcanogenic manganese deposits in California and Japan have deposit densities of 0.019 and 0.03 deposit per square kilometer, respectively.
6. Cuban volcanogenic manganese deposits in Cuba and Fiji have deposit densities of 0.1 and 0.0066 deposit per square kilometer, respectively.
7. Cyprus volcanogenic manganese deposits in Cyprus have a deposit density of 0.046 deposit per square kilometer.

Densities from this study follow:

1. Placer gold deposits in two locations in Alaska have deposit densities of 0.0054 and 0.0061 deposit per square kilometer.
2. Cyprus massive sulfide deposits in the Troodos ophiolite have a deposit density of 0.0082 deposit per square kilometer.
3. Kuroko massive sulfide deposits in five localities have deposit densities ranging from 0.0033 to 0.03 deposit per square kilometer.
4. Porphyry copper deposits in Nevada and Arizona have deposit densities of 0.00015 and 0.0012 deposit per square kilometer, respectively.
5. Climax porphyry molybdenum deposits in New Mexico and Colorado have deposit densities of 0.00031 and 0.00033, respectively.
6. Wolframite (tungsten) quartz vein deposits in Xihuashan, China, have a deposit density of 0.04 deposit per square kilometer.

Frequency analysis of 39 podiform chromite deposit densities demonstrates a highly skewed distribution, suggesting that mean estimates can be misleading; however, probabilistic estimates are possible using log-transformed data and the normal distribution. The same data show deposit density varying with size of permissive area—a regression equation is required to provide unbiased estimates of deposit densities for all but median sized areas.

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INTRODUCTION

A key function of many forms of quantitative mineral resource assessments (Singer and Mosier, 1981; Harris, 1984) is the estimation of the number of undiscovered deposits. The first example of this kind of estimation is probably Allais' (1957) study in which the number of deposits per square kilometer from several explored areas was used with the Poisson distribution to estimate the number of deposits in a relatively unexplored area.

Most resource assessments based on the three-part form of assessment (see the next section and Singer and Cox, 1988; Singer, 1993) have used subjective methods to estimate the number of deposits—estimates are presented in a probabilistic form. Research in psychometrics has raised questions about the ability to make unbiased subjective estimates (Tversky and Kahneman, 1974). However, in meteorology, probabilistic forecasts made by subjective methods show little evidence of problems suggested by the psychometric results and have been quite reliable (Murphy and Winkler, 1984). Subjective methods as used in meteorology outperformed multivariate methods, but they worked best when preceded by numerical-statistical estimates (Murphy and Winkler, 1984). Experiments by W.D. Menzie (an author of this paper) in 1986 show that many geologists can make unbiased estimates of the number of porphyry copper deposits. The results in meteorology suggest that subjective estimates of the number of deposits might be improved if they are preceded by numerical-statistical estimates.

Ideally numerical-statistical estimates of the number of deposits should rely on analogies with similar well-explored areas in the same way that grades and tonnages of well-explored deposits serve as analogs of the grades and tonnages of undiscovered deposits. These numerical-statistical estimates or base rates are prepared from counts of known deposits per unit area in explored regions. Some research has been conducted on several deposit types so that these base rates can be more widely used as a guide for number-of-deposits estimates (Bliss and others, 1987; Bliss, 1992; Root and others, 1992; Bliss and Menzie, 1993). Most of these studies provide point (that is single) estimates of the number of deposits per unit area. In practice, these numerical-statistical estimates of the number of deposits have been used in only a few cases (Cox, 1993; Scott, 2000).

In this paper, we summarize earlier work and extend and update the work reported by Bliss and Menzie (1993) by providing estimates of the number of deposits for the following settings:

- Two settings for placer gold deposits
- One setting for Cyprus massive sulfide deposits
- Five settings for kuroko massive sulfide deposits
- Two settings for porphyry copper deposits

- Two settings for Climax porphyry molybdenum deposits
- One setting for wolframite (tungsten) quartz vein deposits

To explain the strengths and uses of these estimates, we first describe three-part mineral resource assessments. This description is followed by a discussion of mineral deposit densities, a review of some earlier studies, and presentation of new deposit density results for the deposit types listed in the previous paragraph. We end with a discussion of some complexities with numerical-statistical estimates of the number of deposits, scale-related issues, and conclusions.

ACKNOWLEDGMENTS

For the estimate of porphyry copper deposits in Arizona, D.P. Cox of the U.S. Geological Survey (USGS) suggested the use of Titley and Anthonys' (1989) terrane, and B. Moring (USGS) calculated areas of geologic units in this terrane.

THREE-PART ASSESSMENTS

Considerable care must be exercised in quantitative resource assessments to prevent the introduction of biased estimates of undiscovered resources. In three-part assessments (Singer, 1993), (1) areas are delineated according to the types of deposits permitted by the geology, (2) the amount of metal and some ore characteristics are estimated by means of grade and tonnage models, and (3) the number of undiscovered deposits of each type is estimated.

Part 1.—In order to be consistent, areas are delineated where geology permits the existence of deposits of one or more specified types. These areas, called permissive tracts, are based on geologic criteria derived from deposit models that are based on studies of known deposits within and, more commonly, outside the study area. Boundaries of permissive tracts are defined such that the probability of deposits of the type specified occurring outside the boundary is negligible; that is, less than 1 in 100,000 to 1,000,000.

Part 2.—A critical part of the exploration for mineral deposits and of quantitative mineral resource assessments is the estimation of the sizes of undiscovered deposits. Typically, this problem is addressed by using grade and tonnage models because a major source of variation in possible deposit sizes can be accounted for by differences among types of deposits (Singer and Kousta, 1999). In three-part assessments, previously constructed grade and tonnage models are typically used unless local deposits are significantly different from those in the general model. These models have the form of frequency distributions of tonnages and average grades of well-explored deposits of each type. They serve as models for grades and tonnages of undiscovered deposits of the same type occurring in geo-

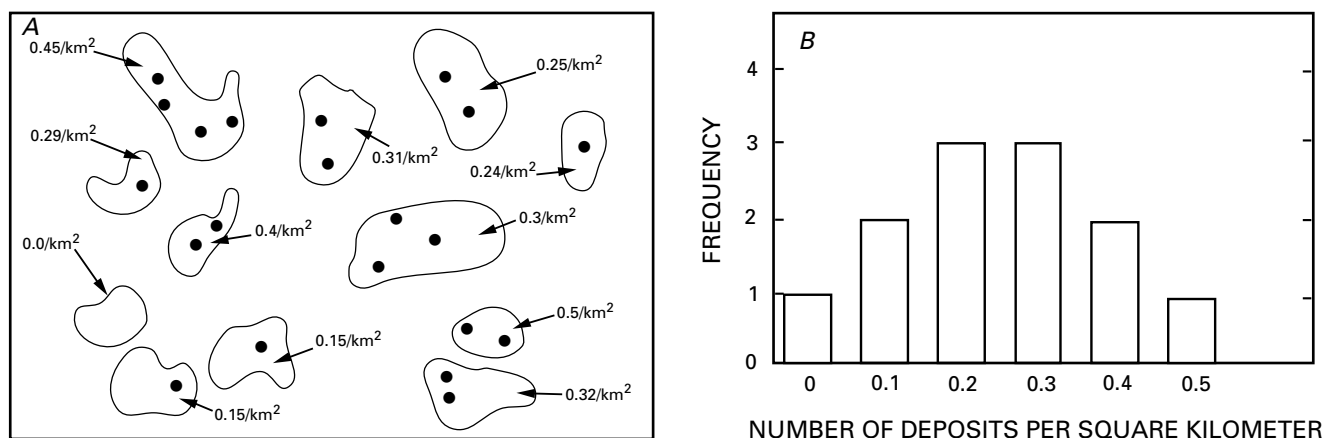


Figure 1. Plan view of known deposits (filled circles) in 12 hypothetical well-explored permissive areas (A) and histogram of derived deposit densities (B).

logically similar settings. By design, the target population is the distribution of grades and tonnages of undiscovered mineral deposits rather than mineral occurrences.

Part 3.—The third part is the estimate of the fixed, but unknown, number of deposits of each type that exists in the delineated tracts. Until the area being considered is thoroughly and extensively drilled, that fixed number of undiscovered deposits, which could be almost any number (including zero), will not be known with certainty. In three-part assessments, estimates of the number of deposits explicitly represent the probability (or degree of belief) that some fixed but unknown number of undiscovered deposits exists in the delineated tracts. These estimates reflect both the uncertainty of what may exist and a measure of the favorability of the existence of the deposit type. Uncertainty is shown by the spread of the number-of-deposits estimates associated with the 90- to the 10- or 1-percent quantiles—a large difference suggests great uncertainty. Favorability can be represented by the estimated number of deposits associated with a given probability level or by the expected number of deposits.

Consistency.—In these assessments, the estimates are internally consistent when the delineated tracts are consistent with descriptive models, grade and tonnage models are consistent with descriptive models, grade and tonnage models are consistent with known deposits in the area, and estimates of the number of deposits are consistent with the grade and tonnage models. Biases can be introduced into these estimates either by a flawed grade and tonnage model or by lack of consistency of the number-of-deposits estimates with the grade and tonnage model. Grade and tonnage models combined with estimates of number of deposits are the fundamental means of translating resource assessments by geologists into a language that economists can use. For these reasons, determining a mineral deposit density requires unambiguous definitions of what is a deposit and what are the rules for delineation.

MINERAL DEPOSIT DENSITIES

There are no fixed methods for making estimates of the number of undiscovered deposits. On the basis of experience and logic, however, there are a number of techniques that can be used directly or as guidelines to make these estimates. Each method represents some form of analogy. Most robust of these methods is a form of mineral deposit model wherein for a deposit type, the numbers of deposits per unit area from well-explored regions are counted and the resulting frequency distribution either is used directly for an estimate or is used indirectly as a guideline in some other method.

In figure 1A, a hypothetical situation is presented where for some deposit type, the number of discovered deposits is counted in each of 12 well-explored permissive tracts, and the areas of each tract are recorded. The ratios of number of deposits per area can then be used to make a histogram (fig. 1B) to show how commonly different deposit densities occur. It is not necessary that the base areas be explored completely, but it is necessary that the number of deposits found and the proportion of the area explored be estimated.

In some situations, it is possible to consider mineral deposit density as the probability that a deposit of a given type occurs within some standard measure of area such as a square kilometer. We do not use that approach here because it requires an assumption that there can be one and only one deposit within the area.

Examples of mineral deposit densities are shown in table 1. These examples are the main focus of this paper—they represent estimates of mineral deposit densities from various published sources and new estimates. The densities for low-sulfide gold quartz veins were discussed by Bliss and others (1987) and Bliss and Menzie (1993). These mesothermal deposits were defined in the descriptive model by Berger (1986) and are consistent with the grade and tonnage model by Bliss (1986). It is important to note that the

Table 1. Mineral deposit densities for 13 selected deposit types from this report and previous publications.

Location	Area (km ²)	Density (deposits/km ²)	Permissive rock type
Low-sulfide gold quartz vein deposit type (Bliss and others, 1987)			
Sierra Nevada, California, U.S.A.	---	0.0046	Metavolcanic, metasedimentary, and ophiolitic rocks, greenschist facies or lower.
Klamath Mountains, California/Oregon, U.S.A.	---	0.0043	Ditto.
Meguma Group, Nova Scotia, Canada	---	0.0054	Ditto.
Bendigo, Victoria, Australia	---	0.005	Ditto.
Placer gold deposit type (this report)			
Wiseman, Alaska, U.S.A.	2,760	0.0054	Greenschist-facies rocks.
Kenai, Alaska, U.S.A.	3,260	0.0061	Low-grade metamorphic rocks.
Bedded barite deposit type (Orris and Bliss, 1989)			
Nevada, U.S.A.	---	0.074	Slaven Chert.
Diamond kimberlite pipes deposit type (Bliss, 1992)			
Southern Africa	1,560,000	0.000012	Stable cratonic rocks.
Cyprus massive sulfide deposit type (this report)			
Cyprus	2,440	0.0082	Troodos ophiolite.
Cyprus	1,010	0.02	Extrusive rocks of Troodos ophiolite.
Kuroko massive sulfide deposit type (this report)			
Snow Lake, Manitoba, Canada ...	268	0.03	Volcanic rocks in basin.
Hokuroku, Japan	900	0.0088–0.013	Volcanic rocks in basin.
Western Tasmania, Australia	1,500	0.0033	Volcanic rocks at Mount Read.
Sierran subtype:			
California, U.S.A.	480	0.0083	Copper Hill Volcanics.
California, U.S.A.	1,370	0.0059	Gopher Ridge Volcanics and a western volcanics unit.
Porphyry copper deposit type (this report)			
Nevada, U.S.A.	32,800	0.00015	Exposed plutons.
Arizona, U.S.A.	34,000	0.00071	Exposed igneous rocks in southeastern Arizona that formed in the Laramide orogeny or earlier.
Arizona, U.S.A.	16,700	0.0012	Ditto.
Climax porphyry molybdenum deposit type (this report)			
Colorado, U.S.A.	12,000	0.00033	Tertiary intrusions, central Colorado mineral belt.
New Mexico, U.S.A.	3,200	0.00031	Tertiary intrusions, northern New Mexico.

Table 1. Mineral deposit densities for 13 selected deposit types from this report and previous publications—Continued.

Location	Area (km ²)	Density (deposits/km ²)	Permissive rock type
Podiform chromite deposit type (Singer, 1994)			
California, U.S.A.	6,980	0.115	Ultramafic rocks.
Franciscan volcanogenic manganese deposit type (Mosier and Page, 1988)			
California, U.S.A.	23,700	0.019	Franciscan Complex.
Japan	3,890	0.03	Sandstone, slate, and chert in the Kitakami Mountains.
Cuban volcanogenic manganese deposit type (Mosier and Page, 1988)			
Cuba	1,200	0.1	Cobre Formation.
Fiji	2,720	0.0066	Deposit-bearing volcanic conglomerates, marl, lithic tuff, lava, and andesite pyroclastics at Viti, Levu.
Cyprus volcanogenic manganese deposit type (Mosier and Page, 1988)			
Cyprus	1,960	0.046	Troodos ophiolite.
Wolframite quartz veins deposit type (this report)			
China	140	0.04	Metasedimentary rocks above granite ridge, Xihuashan-Piotang.

same proximity rule used to construct the grade and tonnage model (workings within 1.6 km of each other are treated as part of the same deposit) was used to define deposits for the deposit densities. Control area boundaries of permissive tracts, based on standards from the descriptive model, enclose areas containing metavolcanic, metasedimentary, and ophiolitic rocks of accreted terranes. Plutonic rocks (within 40 km of veins) also intrude these areas, and the metamorphic grade is greenschist facies or lower (Bliss and others, 1987). Reported densities vary over a rather narrow range from 0.0043 to 0.0054 deposit/km².

Mineral deposit densities (table 1) of bedded barite deposits (Orris and Bliss, 1989), diamond kimberlite pipes (Bliss, 1992), podiform chromite deposits (Singer, 1994), and three types of volcanogenic manganese deposits (Mosier and Page, 1988) are also based on deposits that are consistent with their respective descriptive and grade and tonnage models. The following new examples of mineral deposit densities for placer gold, Cyprus massive sulfide, kuroko massive sulfide, porphyry copper, Climax porphyry molybdenum, and wolframite quartz vein deposits further

demonstrate the variability of estimates and some consequences of assumptions about permissive areas and extent of exploration.

PLACER GOLD DEPOSITS

Two density estimates for placer gold deposits (table 1) were based on data from the areas delineated as permissible for gold placers in the mineral resource assessments of the Wiseman 1° by 3° quadrangle, Brooks Range, Alaska (Bliss, Brosgé, Dillon, Dutro, and others, 1988), and the Kenai Peninsula planning unit, Chugach National Forest, Alaska (Bliss, 1989). These regions were selected because they have well-defined placers and the delineation criteria used for both are comparable. Gold placer data from the Wiseman quadrangle were compiled by Bliss, Brosgé, Dillon, Cathrall, and others (1988), and data from the Chugach National Forest were compiled by Jansons and others (1984). All gold placers exposed on the surface in the Wiseman quadrangle are probably included; however, buried placer deposits may have been overlooked by early prospectors, who lacked the techniques and tools for finding them.

These deposits were defined in the descriptive model by Yeend (1986) and are consistent with the grade and tonnage model by Orris and Bliss (1986). It is important to note that the same proximity rule used to construct the grade and tonnage model (workings within 1.6 km of each other are treated as part of the same deposit) was used to determine the number of deposits present in the area for the deposit density estimate.

The largest (2,760 km²) of three tracts for gold placers delineated in the mineral resource assessment of the Wiseman quadrangle (Bliss, Brosigé, Dillon, Dutro, and others, 1988) was used as one of the areas for this estimate. It was delineated on the basis of the presence of low-grade metamorphic rocks (greenschist facies or equivalent). The northern boundary was delineated by the absence of significant metamorphism; the southern, by a major fault zone.

In the Kenai Peninsula planning unit, an area of low-grade metamorphic rocks was used as the other permissive area. The boundaries, however, were truncated to conform to the boundary of the national forest. The Placer River fault was used as the eastern boundary. Strictly speaking, this area is a portion of a larger area of low-grade metamorphism that should be included; although missing part of the area causes the estimate to be less robust than it might have been, it does not introduce any bias into the estimate.

CYPRUS MASSIVE SULFIDE DEPOSITS

The Troodos massif on the island of Cyprus may be the most complete and intact ophiolite exposure in the world. The ophiolite sequence ranges downward from pillow lavas through a sheeted-dike sequence and cumulate gabbros, and from peridotites to tectonized harzburgite. Deposits here have been mined for copper pyrite ore since the Bronze Age. Extensive slag dumps, surface and underground workings, and abundant artifacts are evidence of the magnitude of this early mining industry.

In the ophiolite complex, there are 20 known Cyprus volcanogenic massive sulfide deposits (Singer, 1986a) that are defined in the same way as the deposits in the Cyprus volcanogenic massive sulfide grade and tonnage model (Singer and Mosier, 1986a); several of the deposits are components of the grade and tonnage model, and all 20 deposits fit the grade and tonnage model. All known deposits are located in the extrusive pillow lavas of the ophiolite complex; some mineralization occurs in the upper levels of the Basal Group (Constantinou, 1976) of basalt lavas below the pillow lavas. Large-scale copper mineralization is not found in the diabase of the sheeted-dike complex. Most copper occurrences outside of the extrusive rocks are small copper veins and are not included in the deposit models; pyrite deposits barren of copper are also not included.

The permissive region for Cyprus volcanogenic massive sulfide deposits is defined as the ophiolite complex part

of the island where extrusive ophiolite rocks are within 1 km of the surface. Within the permissive area, the rocks of the Basal Group metabasalt, the Lower Pillow Lavas, and the Upper Pillow Lavas are the permissive rock units. Exposures of those rock types are very well explored within the permissive area; it is unlikely that any undiscovered Cyprus volcanogenic massive sulfide deposits are exposed in these rocks. Continued drilling programs may, however, locate buried deposits. The exposed permissive rocks cover an area of about 1,010 km². The Basal Group metabasalt covers 512.6 km², and the Lower Pillow Lavas and the Upper Pillow Lavas cover 299.5 km² and 198.7 km², respectively. By using descriptions of the deposits, the host rocks for each of the deposits can be determined; however, many of the deposits form at, or near, the contact between rock units. No deposits are located in the Basal Group metabasalt, 10 deposits are hosted in the Lower Pillow Lavas, another 9 deposits are found in the Upper Pillow Lavas, and for 1 deposit, the specific host pillow lava unit could not be determined.

Around the world, ophiolitic rocks may be less well explored and less well exposed than the rocks of the Troodos massif, and so the rocks in these other areas may be mapped as "ultramafic, undifferentiated" rather than as specific subunits of the ophiolite, such as mantle complex, sheeted-dike complex, and extrusive rocks. For this reason, the entire 2,440 km² of the exposed part of the ophiolite has been considered as if it were "ultramafic, undifferentiated," and a base rate of occurrence for such rocks has been calculated (table 1). We also include an estimate based on the exposed extrusive rocks for situations where these rocks are mapped.

KUROKO MASSIVE SULFIDE DEPOSITS

Snow Lake district, Manitoba, Canada.—In the Snow Lake district in Manitoba, Canada, there are eight known deposits (Chisel Lake, Lost Lake, Anderson Lake, Stall Lake, Rod, Joanne, Pot Lake, Ghost Lake) defined in the same way as deposits in the kuroko massive sulfide descriptive model (Singer, 1986b) and grade and tonnage model (Singer and Mosier, 1986b). These deposits have been thoroughly drilled, and all mineralization within 500 m is treated as part of the same deposit in order to be consistent with the rules used to construct the grade and tonnage model.

The permissive area is the basin containing felsic volcanic rocks, which occupies 268 km² according to Wright and Bonham-Carter (1996). If we assume that there are no deposits to be discovered (that is, the basin is completely explored), then the estimated mineral deposit density is 0.03 deposit/km² (8 deposits/268 km²). If the definition of a kuroko deposit is ignored and we include prospects, then the deposit density almost doubles to 0.06 (15 "deposits"/268 km²); this broadened definition of a deposit

would lead to biased estimates of the number of undiscovered kuroko deposits that are like those in the grade and tonnage model. It should be noted that the estimate of 0.03 deposit/km² (table 1) is based on a small area that is a well-studied district noted for volcanic-rock-hosted massive deposits (Sangster, 1980).

Hokuroku district, Japan.—The Hokuroku district in northern Japan is probably the most thoroughly explored and studied district for kuroko-type massive sulfide deposits in the world (Singer and Kouda, 1988). Eight known deposits (Ezuri, Fukazawa, Furutobe-Ainai, Hanaoka-Doyashiki, Hanaoka-Matsumine-Shakanai, Kosaka-Uchinotai, Kosaka-Motoyama, and Nurukawa) are defined in the same way as deposits in the kuroko grade and tonnage model (Singer and Mosier, 1986b). They are thoroughly drilled: all known mineralization within 500 m is treated as part of the same deposit in order to be consistent with the rules used to construct the grade and tonnage model. The permissive area consists of the felsic volcanic rocks associated with deposits in the middle Miocene basin, which is about 900 km² in extent. If we assume that no additional deposits are to be discovered (that is, the basin is completely explored), then the deposit density estimate is 8 deposits/900 km² = 0.0088 deposit/km².

In the Hokuroku district, we have additional information that could be used to refine this estimate. National and local governments drilled 260 exploratory holes in the district, and industry drilled a large number of holes, mostly near known deposits. The average projected surface area extent of known kuroko deposits in this district is about 0.45 km². About 10 percent of the basin has the mineralized horizon exposed (and thus, is thoroughly explored). If we assume that for the unexposed part, only the immediate neighborhoods of government drill holes are completely explored, then an additional 13 percent of the basin can be considered explored ($[260 \text{ holes} \times 0.45 \text{ km}^2/\text{hole}]/900 \text{ km}^2$). Thus, a more specific estimate of deposit density here would be 8 deposits/(900 km² \times 0.23 proportion explored) = 0.038 deposit/km². If this estimate is correct, then the expected number of undiscovered kuroko deposits in the Hokuroku district is 26, as indicated by $900 \text{ km}^2 \times (1-0.23) \times 0.038 \text{ deposit/km}^2$. From this estimated number, we can estimate that the probability that a drill hole in the covered area would hit a deposit is 0.0169, as indicated by $(26 \text{ deposits} \times 0.45 \text{ km}^2/\text{deposit})/(900 \text{ km}^2 \times 0.77 [\text{proportion of area unexplored}])$.

None of the 260 drill holes found any of these estimated 26 undiscovered kuroko deposits; some holes did penetrate local mineralization. If we assume that the placement of the drill holes was random (it wasn't), then the probability that at least one deposit would have been hit, given 260 holes and 26 deposits, is 0.99 ($1-[1-0.0169]^{260}$). Such a high probability with no deposits found is unlikely and suggests that the estimated number of undiscovered deposits is too high.

Kouda and Singer (1992a,b) estimated that there is a 10 percent chance of at least four undiscovered kuroko deposits in this district. This estimate leads to a deposit density estimate of 0.013 deposit/km² (12 deposits/900 km²), and the probability of hitting at least one deposit with 260 holes is a more reasonable 0.49 ($1-[1-0.0026]^{260}$). Thus, a reasonable estimate of the density of kuroko deposits in the well-explored Hokuroku district is between 0.0088 and 0.013 deposit/km².

Western Tasmania, Australia.—In Western Tasmania, Australia, there are five massive sulfide deposits (Mt. Lyell, Rosebery, Hercules, Que River, Hellyer) that are consistent with the kuroko descriptive and grade and tonnage models. They are all located in the Middle Cambrian volcanic rocks of Mount Read (Reid and Meares, 1981; Collins and Williams, 1986). The area, as defined by the central belt of volcanic rocks, is 150 km by 5 to 15 km, or about 1,500 km². If the area is completely explored, then the mineral deposit density is 0.0033 deposit/km².

Sierran subtype, California.—The deposit density of a subtype of kuroko deposits called Sierran kuroko massive sulfides has been estimated in California. Twelve deposits that are consistent with the grade and tonnage model (Singer, 1992) are known in the Jurassic volcanic and volcanoclastic rocks that extend about 300 km along the foothills of the western Sierra Nevada Mountains. Primarily on the basis of the geologic maps of Kemp (1982), the volcanic rocks are divided into the Copper Hill Volcanics and the Gopher Ridge Volcanics plus a western volcanics unit. The Copper Hill area (480 km²) has a mineral deposit density of 0.0083 deposit/km², whereas the Gopher Ridge area (1,370 km²) has a density of 0.0059 deposit/km². Although these rocks have been well explored on the surface, undiscovered deposits could exist at depth.

PORPHYRY COPPER DEPOSITS

Nevada.—Seven known deposits in Nevada (Yerington, SFS [Luning], Macarthur, Bear, Ely, Ann Mason, and Copper Canyon) are defined in the same way as deposits in the porphyry copper descriptive model (Cox, 1986) and the grade and tonnage model (Singer, Mosier, and Cox, 1986). The tract permissive for all pluton related deposits, including porphyry copper, is defined as an area extending 10 km outward from the outcrop of a pluton, or, in cases where the pluton has a geophysical expression, from the subsurface boundary of the pluton inferred from its geophysical expression (Cox and others, 1996). The permissive tract occupies about 117,300 km², which is about 41 percent of the area of the State. The exposed permissive rocks in Nevada cover an area of about 32,800 km², which is well explored. This terrane hosts five of the seven known porphyry copper deposits. About 72 percent of the permissive tract (84,500 km²) is covered by 1 km or less of upper Tertiary and Qua-

ternary rocks and sedimentary deposits. Areas covered by more than 1 km are excluded from consideration. Two of the known porphyry copper deposits (Bear and Ann Mason) are completely covered by younger materials and thus cannot be considered to belong to the population of deposits that are well explored and exposed.

If we assume that there are no additional porphyry copper deposits to be discovered in the exposed plutons in Nevada, then 5 deposits/32,800 km² (exposed permissive area) equals 0.00015 porphyry copper deposit/km². We can use this density of deposits to estimate the expected number of undiscovered porphyry copper deposits in Nevada. Thus, 0.00015 porphyry copper deposit/km² times 84,500 km² of covered permissive area equals an expected 12.9 concealed deposits; subtracting the 2 discovered deposits leaves 11 undiscovered concealed deposits that are defined in the same way as the deposits in the porphyry copper grade and tonnage model. For comparison purposes, the subjective estimate of the number of undiscovered porphyry copper deposits in Nevada by Cox and others (1996) is 8.7 deposits.

Arizona.—In Arizona, there are 36 known porphyry copper deposits like those in the descriptive and grade and tonnage models. The permissive area is the part of southeastern Arizona that may have igneous rocks that formed in the Laramide orogeny. In the permissive area, exposed rocks are very well explored: it is unlikely that undiscovered porphyry copper deposits are exposed. Deposits with mineralized systems greater than 50 percent covered are counted in the covered population. Thus, 12 of the 36 deposits are counted as part of the covered population, and the remaining 24 deposits are from the exposed population. If we assume that there are no additional porphyry copper deposits in exposed rocks of the permissive area, then 24 deposits/34,000 km² equals a mineral deposit density of 0.00071 porphyry copper deposit/km².

A particularly favorable part of the permissive area is defined in the southeastern corner of Arizona. This area, called the Pinal-Paleozoic Terrane of Titley and Anthony (1989), consists of 59,700 km² of covered rocks and 16,700 km² of exposed rocks. This favorable part hosts 32 known porphyry copper deposits, is characterized by Basin and Range Province and Central Mountains landforms, and contains most Paleozoic outcrops in the southern part of Arizona (Reynolds, 1988). In this favorable area, 12 of the 32 deposits are in the covered population, and 20 are from the exposed population. If we assume that there are no more porphyry copper deposits to be discovered in exposed rocks of the favorable part of the permissive area, then 20 deposits/16,700 km² equals 0.0012 porphyry copper deposit/km². The estimate of 0.0012 deposit/km² is about 50 percent higher than the estimate for all of the permissive area.

CLIMAX PORPHYRY MOLYBDENUM DEPOSITS

Delineating the boundaries of tracts that contain intrusion-related mineral deposits can be difficult because some types of deposits are emplaced in the upper parts of intrusive complexes or in overlying rocks. Climax-type porphyry molybdenum deposits are related to high-silica granites (Ludington, 1986). These granites are thought to have formed by partial melting of lower crustal rocks due to the emplacement of mafic to intermediate-composition magmas following cessation of subduction and related calc-alkaline magmatism (White and others, 1981). Melting of Precambrian crust that had been subjected to repeated crustal heating is thought to be an important regional control for the generation of high-silica granites associated with Climax-type molybdenum deposits. However, the nature of the lower crust is not a suitable operational criterion for delineating ground that might contain undiscovered mineral deposits. An operational criteria used for estimating mineral deposit density of Climax-type porphyry molybdenum deposits is the distribution of Tertiary intrusive rocks and thickened continental crust, as indicated by Bouguer gravity anomalies (see figure 35 in White and others, 1981).

Colorado.—The area of Tertiary intrusive rocks in the central Colorado mineral belt is approximately 12,000 km². There are four known Climax-type porphyry molybdenum deposits in the tract (Climax, Henderson, Mount Edmonds, and Redwell Basin) that are consistent with the grade and tonnage model (Singer, Theodore, and Mosier, 1986). Thus, the mineral deposit density is 0.00033 deposit/km².

New Mexico.—Another area of Tertiary subduction-related calc-alkaline intrusions and later high-silica granites occurs around the Questa molybdenum deposit in northern New Mexico. The area of Tertiary intrusive rocks is 3,200 km². The area contains one Climax-type porphyry molybdenum deposit. Thus, the mineral deposit density is 0.00031 deposit/km².

WOLFRAMITE QUARTZ VEIN DEPOSITS

Wolframite (tungsten) quartz vein deposits, described as W vein deposits by Cox and Bagby (1986), are numerous in South China. Kang Yongfu and others (1990) reported that in the Zhuguang Mountains (Chongyi Dayu-Shangyou region), 185 deposits occur in an area of 7,800 km² (0.02 deposit/km²) and, in the Pangushan region, 111 deposits occur in an area of 11,000 km² (0.01 deposit/km²). Yang and Lu (1982) reported more than 20 deposits occurring within an area of 300 km² (0.067 deposit/km²) in the Xihuashan-Piotang district. In these cases, the deposits are probably not consistent with the grade and tonnage model (Jones and Menzie, 1986).

By using figures 1 and 3 from Yang and Lu (1982), it is possible to calculate the area above the Xihuashan-Piotang granite ridge. The area is broken into two parts, one northeast of a northwest-trending inferred fault southwest of Piotang and the other southwest of the fault. The northeast area is 30 km², and the area to the southwest is 110 km². The host rocks are metasediments. Although many of the deposits may be too small to be consistent with the grade and tonnage model (Jones and Menzie, 1986), the whole area (140 km²) contains up to 10 known deposits: Xihuashan, Dangping, Luokeng, Shenlongkou, Xialuogushan, Niuzishi, Muziyuan, Dalongshan, Piotang, and Zongshukeng. At least 4 of the deposits (Xihuashan, Dangping, Muziyuan, and Piotang) are believed to fit existing grade and tonnage models (Jones and Menzie, 1986); 2 more (Dalongshan and Zongshukeng) are thought likely to fit the models; and all 10 deposits might fit the models. These numbers of deposits would yield estimated deposit densities of 0.03, 0.04, and 0.08 deposit/km², respectively; the middle estimate seems most robust and is provided in table 1. The area can be considered well explored.

A COMPLEXITY

In an assessment of Alaska (MacKevett and others, 1978), some of the estimates for the number of undiscovered podiform chromite deposits (Albers, 1986) were based on a regression of the area of ultramafic rock on the number of known podiform chromite deposits (Singer and Page, 1986) in well-explored California and Oregon. The same regression was used to estimate the number of chromite deposits in Costa Rica (Singer and others, 1987) and in wilderness areas of the U.S. Pacific mountain system (Drew and others, 1986).

Singer (1994) presented the regression of the area of ultramafic rock on the number of known podiform chromite deposits referred to above and compared this form of estimation with deposit density estimates derived from the same areas. Podiform chromite deposits occur in the ultramafic parts of ophiolite assemblages. Although lithologic variations of ultramafic rocks are useful in estimating the number of deposits (N.J. Page, USGS, oral commun., 1983), most geologic maps simply report undifferentiated ultramafic rocks; this study is therefore restricted to undifferentiated ultramafic rocks. The area of ultramafic rock in each county of California was estimated by point counting using an effective grid spacing of 1 mile (1.6 km) on 1:250,000-scale maps (Singer, 1971). In southwestern Oregon, Page and Johnson (1977) estimated the areal extent of ultramafic rock for 12 subareas based on geologic boundaries. The areas of 12 ultramafic masses in Oregon, the areas of ultramafic rock in each of the 27 counties of California that contain podiform chromite deposits, and their associated number of deposits represent the samples in this analysis.

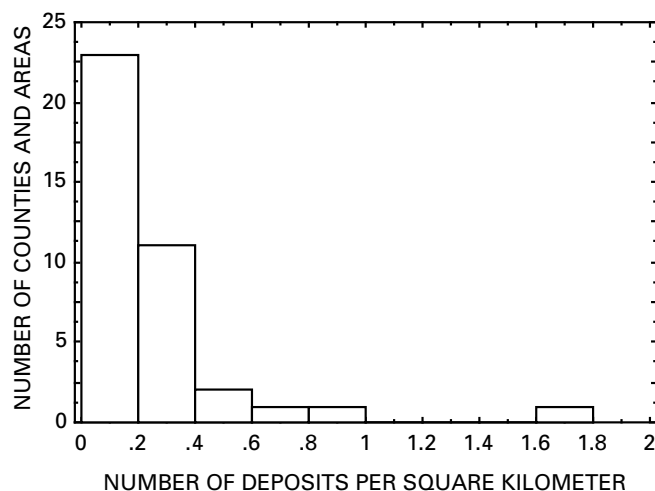


Figure 2. The number of podiform chromite deposits per square kilometer of ultramafic rock in 27 California counties and in 12 Oregon areas (from Singer, 1994).

It is important to note that the chromite deposits reported reflect very thorough exploration of the surface, but deposits not exposed were rarely discovered. Thus, the statistics presented here apply to the surface only and are not reliable for three-dimensional estimates (see Menzie and Singer, 1980). A second point of consideration is the use of county data for California and subareas for Oregon, which leads to a grouping of individual ultramafic bodies that has the effect of an undetermined, but probably small, reduction in the variability of estimates. A common way to estimate the deposit density of occurrence is to average the number of deposits per permissive area over a number of permissive areas. Calculated with these data, the base rate of occurrence, averaged over the 39 sample areas, is 0.2248 podiform chromite deposit per square kilometer of ultramafic rock. This estimate is of questionable value, however, because the frequency distribution of the untransformed variable (deposits/area) is significantly skewed and peaked (fig. 2). Thus, a few high values have a very large influence on the estimate, and probabilistic estimates of the number of undiscovered deposits are very difficult to calculate.

A reasonable way to make probabilistic estimates in this situation would be to use the mean and standard deviation of the transformed data and the normal distribution. The frequency distributions of the logarithms of the area of ultramafic rock, the number of podiform chromite deposits, and the number of podiform deposits per square kilometer of ultramafic rock are not significantly different from normal distributions. It is more appropriate to model the distribution of the number of deposits with a discrete distribution such as the negative binomial (Agterberg, 1977). As pointed out by Agterberg (1984), the continuous lognormal distribution is equivalent to the discrete negative binomial distribution, but use of the lognormal distribution can lead to discrepancies for small frequencies.

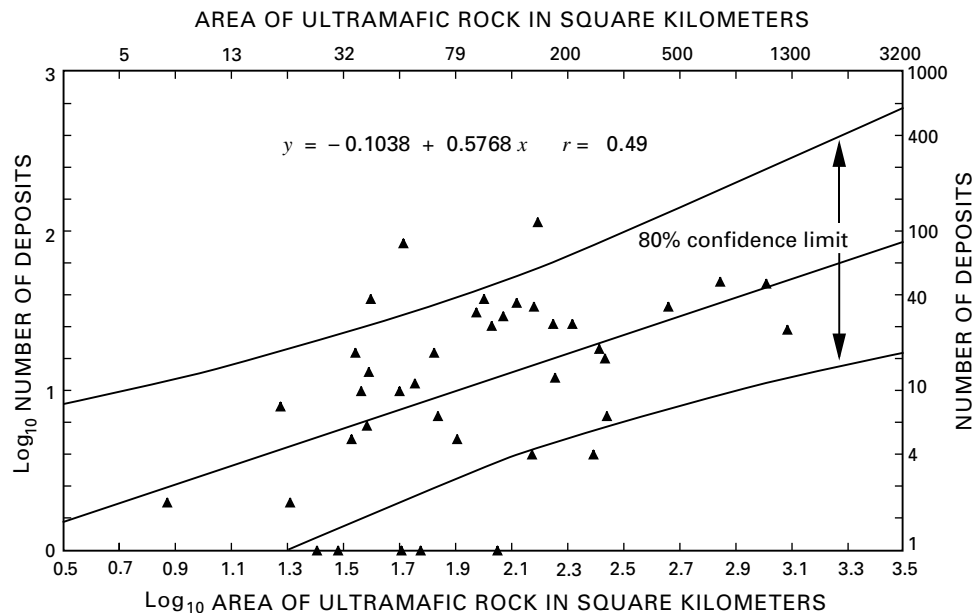


Figure 3. Number of podiform chromite deposits in 27 California counties versus area of ultramafic rock with regression line and 80-percent confidence limits for individual points (from Singer, 1994).

Based on the lognormal distribution and the values, in 90 percent of the cases, the base rate would be 0.0238 deposit or more per square kilometer, in 50 percent of the cases, it would be 0.1152 deposit or more per square kilometer, and in 10 percent of the cases, it would be 0.5581 deposit or more per square kilometer. The median estimate of 0.1152 deposit/km² is almost exactly the same as an estimate made by dividing the total number of deposits (805) by the total area of ultramafic rock (6,982 km²); that is, 0.1153 deposit/km². This result suggests that the usual method of calculating the base rate of occurrence with untransformed data probably yields reliable estimates of the median density when the permissive area is quite large. However, unless many areas are used, there is no way to estimate variability and, consequently, no way to make probabilistic estimates of the number of deposits without some other information. A further improvement in the estimates can be made by examining the relation between the area of ultramafic rocks and the number of podiform chromite deposits.

A plot of the area of ultramafic rock versus the number of podiform chromite deposits in 27 California counties shows a clear positive relation in figure 3. The linear regression line and the 80-percent-confidence limits to the regression estimates are also provided in figure 3. Estimates of the number of podiform chromite deposits can be made from figure 3 by using the logarithm of ultramafic rock area on the x axis projected to the lower confidence limit for the 90-percent estimate of the number of deposits, to the regression line for the 50-percent estimate, and to the upper confidence limit for the 10-percent estimate.

Although the correlation coefficient is not particularly high ($r=0.49$), it and the associated regression slope are significantly different from zero at the 1-percent level. The slope of the regression line ($b=0.5768$) is also significantly different from 1.0. A slope of 1.0 would mean that a doubling of permissive area would result in a doubling of the estimated number of deposits; that is, the ratio of the number of deposits to the size of the permissive area would be independent of the size of the permissive area. Thus, if the slope equaled 1.0, then the ratio of the number of deposits to the size of the permissive area would provide an unbiased estimate for any sized area. The fact that the slope is significantly lower than 1.0 means that the ratio of the number of deposits to the size of the permissive area is a biased estimator in many cases. This conclusion is reinforced by the observation that the correlation between deposits per area of ultramafic rock and area of ultramafic rock ($r=-0.38$) is significant at the 5-percent level; that is, the base rate of occurrence decreases as the size of the permissive area increases.

The observed frequencies of the area of ultramafic rock, the number of podiform chromite deposits in ultramafic areas or groups of areas (counties), and the rate of occurrence of podiform chromite deposits (deposits per permissive area) are all represented better by lognormal distributions than by normal distributions. Because the frequency distribution of each variable is highly skewed, use of the mean of the untransformed data is of questionable value. The normal distribution with the mean and standard deviation of the log-transformed rate of occurrence is rec-

ommended over the simple rate of occurrence for probabilistic estimates. Because of the significant correlation between the number of deposits and the area of ultramafic rock, even this improvement is useful only for ultramafic bodies that are near the median area of 93 km². The fact that the base rate of occurrence for podiform chromite deposits is dependent on the size of the permissive area suggests that more research is needed to see if this relation exists for other mineral deposit types. Unbiased estimates of the number of podiform chromite deposits can be made by using the regression and 80-percent confidence limits represented in figure 3.

SCALE ISSUES

When we take samples that represent small areas, such as the two well-explored kuroko massive sulfide districts discussed in this paper (Snow Lake district, Canada, and Hokuroku district, Japan), we expect that there will be large differences in deposit density from sample to sample based on sampling theory. If we had a number of well-explored regions that each represented a large area, we would expect much lower variability in the estimates of deposit density. Thus, even in situations where we do not have the complexity observed for podiform chromite deposits, the scale of the observations affects the variability of mineral deposit densities. For example, if samples representing larger areas had been used to make the histogram of deposit densities in figure 1, we would expect that there would be fewer observations at the higher and lower densities because there would be less variability of the estimates.

The relation of sample size affecting deposit densities was recognized by Agterberg (1977) in his studies of the volcanogenic massive sulfide deposits of the Abitibi region of Canada and was extensively discussed by Bliss and Menzie (1993) in terms of the distributions and spatial correlations of several deposit types. These studies of frequency distributions and spatial correlations are typically concerned with variability within mineral deposit districts, whereas the present study is concerned with variability among districts or other larger areas.

Consideration of the effects of scale on the variability of deposit densities is meaningful only where the samples are a random selection of all possible (that is, permissive) areas. The two well-explored kuroko massive sulfide districts discussed here are not random selections, but purposeful selections of some of the best-endowed districts in the world. The same could be said of the estimates of the density of porphyry copper deposits in Arizona, of Climax porphyry molybdenum deposits in Colorado, and of wolframite veins from Xihuashan, China. The proper use of each of these estimates is as a guide to what might be considered a very good density of deposits. If a kuroko massive sulfide deposit density estimate made for some region is as

high as those in table 1, then an exceptionally good district is being predicted; commonly, this prediction would require some explanation. Thus, these specially selected areas provide standards to identify what might be unreasonably high estimates of the number of undiscovered deposits in most situations.

CONCLUSIONS

A key function of many forms of quantitative mineral resource assessments is the estimation of the number of undiscovered deposits. Numerous techniques can be used directly or as guidelines to make these estimates. Most robust of these methods is a form of mineral deposit model wherein for a deposit type, the numbers of deposits per unit area from well-explored regions are counted and the resulting frequency distribution is used either directly for an estimate or indirectly as a guideline in some other method. Ratios of the number of deposits per unit area can be used in histograms to show variation of deposit densities by type. In three-part quantitative resource assessments, prevention of bias requires that deposits of each type be defined the same way as in delineation of permissive tracts and as in construction of grade and tonnage models; these same definitions and rules must also be applied to deposit density models.

The 27 mineral deposit density estimates reported here representing 13 different deposit types should be considered only a start at compiling the estimates necessary to guide assessments. In addition, many of the specially selected areas reported here provide standards to identify what should be high estimates of the number of undiscovered deposits in most situations. Many of the issues that should be addressed in developing mineral deposit density estimates are identified. Not only should care be used in defining mineral deposits and their permissive areas, but skewed distributions of densities and possible correlations between densities and areas should be considered.

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