A Method for Simulating Transient Ground-Water Recharge in Deep Water-Table Settings in Central Florida by Using a Simple Water-Balance/ Transfer-Function Model

By Andrew M. O'Reilly

Prepared in cooperation with the St. Johns River Water Management District

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Conversion Factors and Datum

	By	To obtain
	Length	
centimeter (cm)	0.3937	inch (in.)
millimeter (mm)	0.03937	inch (in.)
meter (m)	3.281	foot (ft)
	Flux	
millimeter per day (mm/d)	0.03937	inch per day (in/d)
	Hydraulic conductivity	
meter per day (m/d)	3.281	foot per day (ft/d)

Temperature in degrees Celsius (°C) may be converted to degrees Fahrenheit (°F) as
°F = (1.8 x °C) + 32

Vertical coordinate information is referenced to the National Geodetic Vertical Datum of 1929 (NGVD 29); horizontal coordinate information is referenced to the North American Datum of 1927 (NAD 27).

Acronyms and Additional Abbreviations

ET	evapotranspiration
PDF	probability density function
r^2	coefficient of determination
UCODE	universal inverse modeling program
USGS	U.S. Geological Survey
VS2DT	U.S. Geological Survey two-dimensional variably saturated flow and solute transport model
WBTF model	water-balance/transfer-function model

List of Symbols

Roman	
D_{rz}	depth of the root zone, [L]
E^i	average evapotranspiration rate over time interval $((i-1)\Delta t_{pe}, i\Delta t_{pe})$, [L/T]
$e(t)$	evapotranspiration rate, [L/T]
i	discretization index representing time $i\Delta t_{pe}$ or $i\Delta t_u$
I_e^i	average effective infiltration rate over time interval $((i-1)\Delta t_{pe}, i\Delta t_{pe})$, [L/T]
I_e^l	average effective infiltration rate over time interval $((l-1)\Delta t_u, l\Delta t_u)$, [L/T], which is assumed to be 0 for l<0
$i_e(t)$	effective infiltration rate, [L/T]
j	discretization index representing time lag $j\Delta\tau$ of the transfer function
k	scale parameter of gamma PDF, [T]
l	discretization index representing the time defined by $(t-\tau)$;
m	number of time-lag intervals, $\Delta\tau$, spanned by the transfer function
n	shape parameter of gamma PDF, [dimensionless]
P^i	difference between average precipitation and average surface-runoff rates over time interval $((i-1)\Delta t_{pe}, i\Delta t_{pe})$, [L/T]
$p(t)$	difference between precipitation and surface runoff rates, [L/T]
Q	outflow rate from the reservoir, [L^3/T]
Q_n	outflow rate from the nth reservoir, [L^3/T]
R^i	average recharge rate over time interval $((i-1)\Delta t_u, i\Delta t_u)$, [L/T]
$r(t)$	recharge rate, [L/T]
S	total storage (water held in the vegetative canopy and root zone), [L]
S_b	storage in vegetative canopy and root zone at beginning of simulation period, [L]
S_c	vegetative canopy storage, [L]
S^i	total storage at time step i, [L]
S^{i-1}	total storage at previous time step i-1, [L]
S_{max}	maximum storage capacity of vegetative canopy and root zone (total bucket capacity), [L]
t	time, [T]
truncate(x)	truncation of x to an integer
v	volume of water instantaneously added to the reservoir, [L^3]

List of Symbols—Continued

Greek	
$\Gamma(n)$	the gamma function, [dimensionless]
Δt_{pe}	time interval for discretization of $p(t)$ and $e(t)$, [T]
Δt_u	unit-event length defining the time interval for discretization of effective infiltration, [T]
$\Delta\tau$	time-lag interval for discretization of the transfer function, [T]
θ	average volumetric moisture content of the root zone, [L^3/L^3]
θ_{fc}	average field capacity of the root zone, [L^3/L^3]
θ_{wp}	average permanent wilting point of the root zone, [L^3/L^3]
τ	time lag of the transfer function, [T]
τ_i	initial time lag of gamma PDF, [T]
$\phi(\tau)$	a linear-transfer function, [LT^{-1}/L]
$\phi^{j-\frac{1}{2}}$	a linear-transfer function evaluated at time lag $\left(j-\frac{1}{2}\right)\Delta\tau$, [LT^{-1}/L]

PREFACE

This report presents a computer program for simulating transient ground-water recharge in deep water-table settings. The performance of this computer program has been compared to field-based data as well as models of hypothetical variably saturated flow systems; however, future applications of the program could reveal errors that were not detected in the test simulations. Users are requested to notify the USGS if errors are found in the report or in the computer program. Correspondence regarding the report or program should be sent to:

U.S. Geological Survey
224 West Central Parkway, Suite 1006
Altamonte Springs, Florida 32714

The computer program documented in this report is available from the USGS at the following World Wide Web address:

http://pubs.water.usgs.gov/sir2004-5195/

A Method for Simulating Transient Ground-Water Recharge in Deep Water-Table Settings in Central Florida by Using a Simple Water-Balance/Transfer-Function Model

By Andrew M. O'Reilly

Abstract

A relatively simple method is needed that provides estimates of transient ground-water recharge in deep water-table settings that can be incorporated into other hydrologic models. Deep water-table settings are areas where the water table is below the reach of plant roots and virtually all water that is not lost to surface runoff, evaporation at land surface, or evapotranspiration in the root zone eventually becomes ground-water recharge. Areas in central Florida with a deep water table generally are high recharge areas; consequently, simulation of recharge in these areas is of particular interest to water-resource managers. Yet the complexities of meteorological variations and unsaturated flow processes make it difficult to estimate short-term recharge rates, thereby confounding calibration and predictive use of transient hydrologic models.

A simple water-balance/transfer-function (WBTF) model was developed for simulating transient ground-water recharge in deep water-table settings. The WBTF model represents a one-dimensional column from the top of the vegetative canopy to the water table and consists of two components: (1) a water-balance module that simulates the water storage capacity of the vegetative canopy and root zone; and (2) a transfer-function module that simulates the traveltime of water as it percolates from the bottom of the root zone to the water table. Data requirements include two time series for the period of interest—precipitation (or precipitation minus surface runoff, if surface runoff is not negligible) and evapotranspiration—and values for five parameters that represent water storage capacity or soil-drainage characteristics.

A limiting assumption of the WBTF model is that the percolation of water below the root zone is a linear process. That is, percolating water is assumed to have the same traveltime characteristics, experiencing the same delay and attenuation, as it moves through the unsaturated zone. This assumption is more accurate if the moisture content, and consequently the unsaturated hydraulic conductivity, below the root zone does not vary substantially with time.

Results of the WBTF model were compared to those of the U.S. Geological Survey variably saturated flow model, VS2DT, and to field-based estimates of recharge to demonstrate the applicability of the WBTF model for a range of conditions relevant to deep water-table settings in central Florida. The WBTF model reproduced independently obtained estimates of recharge reasonably well for different soil types and water-table depths.

Introduction

Ground-water recharge from precipitation is the primary source of water to the aquifer system in central Florida. The aquifer system is recharged when sufficient precipitation overcomes evapotranspirative losses and hydrostatic capillary retention in the unsaturated zone and remaining water percolates downward to the water table. When precipitation rates exceed the infiltration capacity of the soil, excess water is rejected and becomes surface runoff, while some water continues to move downward. Downward moving water that reaches the water table and enters the saturated ground-water flow system becomes recharge (Meinzer, 1923, p. 46). In deep water-table settings, where the water table is below the reach of plant

roots, the water table is too deep for evapotranspiration (ET) to extract water from the saturated zone—virtually all water that is not lost to surface runoff or evaporation at land surface or to ET in the root zone becomes recharge. In central Florida, previous investigators indicated that areas with a deep water table generally are high recharge areas (Sumner, 1996, p. 31; Yobbi, 1996, p. 22; Knowles and others, 2002, p. 87; and McGurk and Presley, 2002, p. 27); consequently, simulation of recharge in these areas is of particular interest to water-resource managers.

Temporal variations in precipitation and ET, which produce temporal variations in recharge, can be substantial in central Florida. In addition, the unsaturated zone can serve as a filter (in a mathematical sense), effectively transforming the effects of surface meteorological processes into the subsurface expression of these processes as recharge at the water table. Recharge is delayed relative to precipitation because of the transmission time of the infiltrating water through the unsaturated zone, especially where the water table is deep. Also, the infiltrating water is subject to capture within unsaturated-zone storage. These complexities make it difficult to estimate short-term recharge rates, thereby confounding calibration and predictive use of transient hydrologic models.

A method that provides estimates of transient recharge in deep water-table settings that can be incorporated into other hydrologic models, such as regional ground-water flow models, would facilitate calibration and predictive use of such models. Ideally, the method should be relatively simple so that it is not too computationally or data intensive to preclude its practical use in regional-scale models. Such a method, described in this report, was developed by the U.S Geological Survey (USGS), in cooperation with the St. Johns River Water Management District, during a $3^{1}/_{2}$-year study, which began in 2000.

The model described herein represents a one-dimensional column from the top of the vegetative canopy to the water table and consists of two components: (1) a water-balance module that simulates the water storage capacity of the vegetative canopy and root zone; and (2) a transfer-function module that simulates the traveltime of water as it percolates from the bottom of the root zone to the water table. Data requirements include two time series for the period of interest—precipitation (or precipitation minus surface runoff, if surface runoff is not negligible) and evapotranspiration—and values for five parameters that represent water storage capacity or soil-drainage characteristics.

Purpose and Scope

This report describes the development and use of a simple water-balance/transfer-function (WBTF) model for simulating transient ground-water recharge in deep water-table settings. Application of the WBTF model is demonstrated by comparing simulated recharge to field-based estimates of recharge at a site on the Lake Wales Ridge in west Orange County (fig. 1). Utility of the WBTF model for a greater range of conditions is demonstrated at hypothetical field sites by comparing results of the WBTF model to those of the USGS variably saturated flow model VS2DT (Lappala and others, 1987; and Healy, 1990). Hypothetical field sites are simulated by using assigned values of precipitation, ET, soil type, and water-table depth representative of deep water-table settings in central Florida. Descriptions of the file structures and data formats required by the model and an example problem demonstrating use of the model are provided in appendixes 1 and 2, respectively. The FORTRAN source code, a compiled version of the program suitable for use on most computers running the Microsoft DOS or Windows operating system, and all input and output files for the example problem are available at the following World Wide Web address: http://pubs.water.usgs.gov/sir2004-5195/.

The WBTF model was developed to fill a need in central Florida. The model is useful in other areas with hydrologic characteristics similar to those in central Florida.

Hydrologic Conditions in Central Florida

The climate of central Florida is humid subtropical and characterized by warm, rainy summers and temperate, relatively dry winters. Long-term average annual precipitation is about 1,300 millimeters (mm) (51 inches), with 55-60 percent falling during the wet season (June through September) and 40-45 percent falling during the dry season (October through May) (Knowles and others, 2002, p. 30). During the wet season, daily thunderstorms are common and yield large quantities of precipitation, whereas during the dry season, precipitation generally is associated with frontal systems. Summer daily maximum air temperatures typically exceed 32 degrees Celsius; winter daily maximum air temperatures generally are mild with occasional freezes (Knowles and others, 2002, p. 9).

Precipitation provides the largest input of water to the hydrologic system in central Florida, and, on an annual basis, the largest water loss is through ET. Published data on ET in central Florida estimated using the eddy-correlation method include the following average annual values:

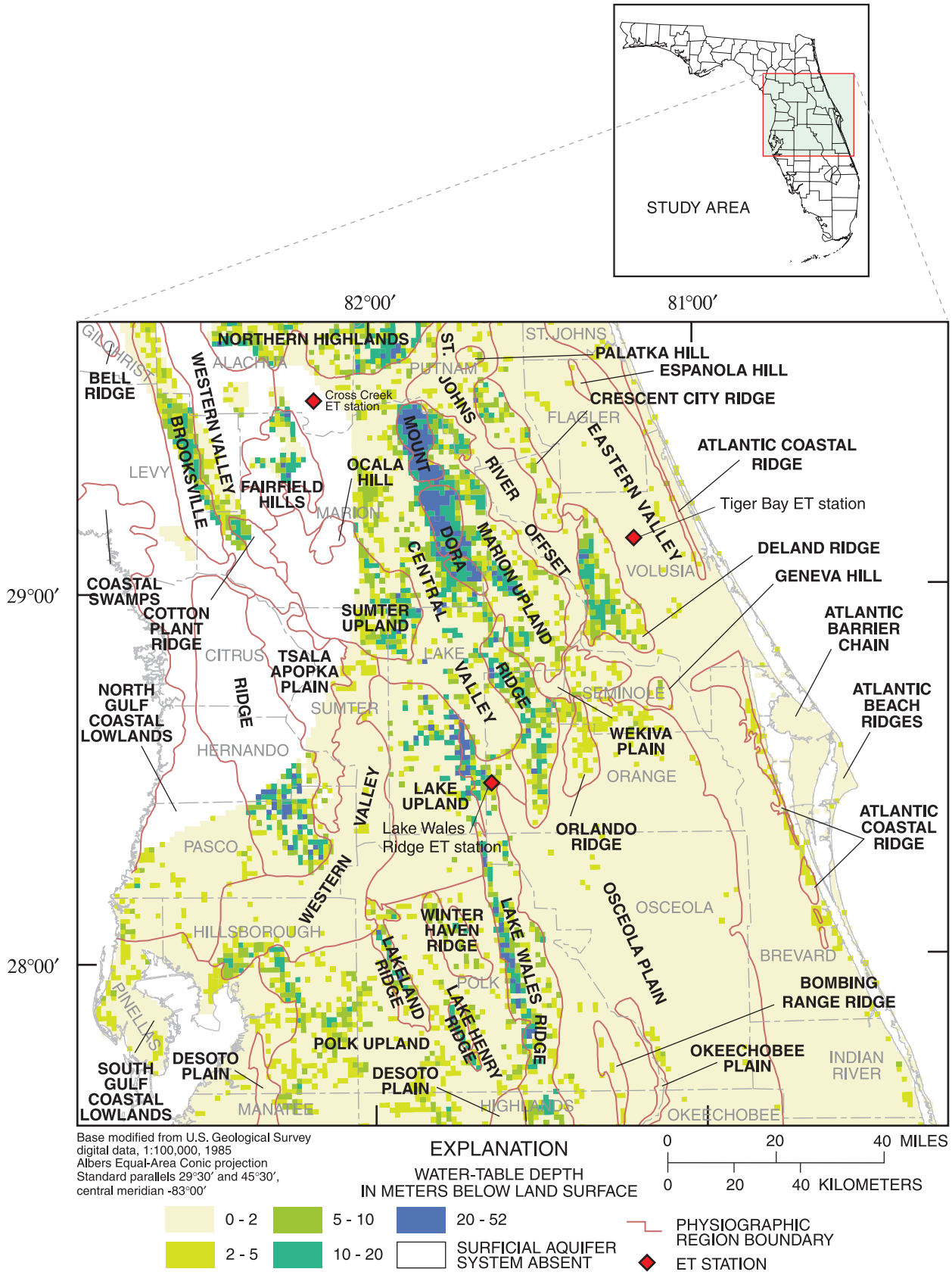

Figure 1. Water-table depth based on estimated average surficial aquifer system water level for August 1993 through July 1994 (Sepúlveda, 2002, p. 23), physiographic regions (modified from White, 1970, pl. 1), and locations of data-collection stations, central Florida.

680 mm (27 inches) for the period September 15, 1993, to September 15, 1994, for a site with herbaceous vegetation, well-drained soils, and a relatively deep water table (Sumner, 1996, p. 30; Lake Wales Ridge ET station, fig. 1); 810 mm (32 inches) for the period October 1993 through September 1994 for a site with immature slash pine, poorly drained soils, and a shallow water table (Knowles, 1996, p. 27; Cross Creek ET Station, fig. 1); and 916 mm (36 inches) for 1998 and 1,070 mm (42 inches) for 1999 for a site with cypress and pine forest (subjected to natural fires in 1998), poorly drained soils, and a shallow water table (Sumner, 2001, p. 38; Tiger Bay ET station, fig. 1). A strong temporal variation in ET is due primarily to meteorological variables (such as precipitation, solar radiation, windspeed, and humidity) and the plant/hydrologic system response to these variables. For example, in central Florida the wet season (June through September) mostly coincides with the plant growing season (largely dictated by seasonal variations in solar radiation), resulting in increased ET during summer months. The temporal variability of ET in central Florida, however, is considerably less than that of precipitation over a wide range of timescales (from daily to annual) (D.M. Sumner, U.S. Geological Survey, oral commun., 2003).

Central Florida is underlain by unconsolidated sand and clay sediments that generally range in thickness from 0 to 60 meters (m), forming the surficial aquifer system (Knowles and others, 2002, p. 15). Underlying the surficial aquifer system is the intermediate confining unit, which separates the surficial aquifer system from the deeper carbonate Floridan aquifer system. The water table generally is near land surface and occurs in the surficial aquifer system. In many areas, however, the water table is 2 m or more deep, exceeding 20 m in some areas (fig. 1). These deep water-table settings generally exist in the ridge physiographic regions of central Florida, especially the Lake Wales Ridge, Mount Dora Ridge, and Deland Ridge (fig. 1). In addition to a deep water table, the ridge regions are characterized by karst topography, with relatively high altitudes (exceeding 50 m in some areas), large hills, and numerous sinkholes and well-drained soils that preclude the development of substantial surface-drainage networks in many areas.

Estimates of recharge based on field measurements are sparse in central Florida. German (1990, p. 17) analyzed water-table fluctuations at a surficial aquifer system well in southwest Orange County and estimated that average annual recharge ranges 220-599 mm (8.5-23.6 inches), or 20-48 percent of precipitation for 6 separate years during 1975-84. Sumner (1996, p. 30) estimated that recharge ranges 570-700 mm (22-28 inches) or 43-53 percent of precipitation for the period September 15, 1993, to September 15, 1994, at a site with negligible surface runoff in west Orange County by computing the difference between measured values of precipitation and ET. About 50 percent of precipitation may be a good estimate of the maximum recharge in central Florida because the well-drained soils, shallow-rooted vegetation, and relatively deep water table at the site tend to minimize ET and maximize recharge (Sumner, 1996, p. 31).

Development of the Water-Balance/ Transfer-Function Model

The WBTF model described herein is a simple model for the transient simulation of ground-water recharge resulting from water that moves from the land surface through the unsaturated zone. Precipitation is the source of recharge simulated by the WBTF model, but other sources, such as overhead irrigation, could be simulated by using this model. The WBTF model, however, has not been used to simulate recharge involving surface flooding, such as rapid-infiltration basins or ponded infiltration, and the applicability of the model for these conditions is unknown.

Inputs for the model include two time series for the period of interest—precipitation (or precipitation minus surface runoff, if surface runoff is not negligible) and evapotranspiration—and values for five parameters that represent water storage capacity or soil-drainage characteristics. The five parameters are listed in table 1. Outputs from the model include time series of the following for the period of interest: change in water storage in the vegetative canopy and root zone, flux of water leaving the root zone, instantaneous recharge, and average recharge for a user-defined timestep.

Conceptual Model

The WBTF model represents a one-dimensional column from the top of the vegetative canopy to the water table and consists of two components: (1) a water-balance module that simulates the water storage capacity of the vegetative canopy and root zone; and (2) a transfer-function module that simulates the traveltime of water as it percolates from the bottom of the root zone to the water table. The vegetative canopy consists of all plant materials, both living and dead, that exist above land surface. The WBTF model simulates the recharge process as

Table 1. Parameters used in the water-balance/transfer-function model.

[L, length unit; T, time unit; --, dimensionless; ≤, less than or equal to; ≥, greater than or equal to; >, greater than; PDF, probability density function]

Parameter symbol	Parameter Description	Dimensions	Valid values
S_b	Storage in vegetative canopy and root zone at beginning of simulation period	L	$0 \le S_b \le S_{max}$
S_{max}	Maximum storage capacity of vegetative canopy and root zone; represents amount of water intercepted and retained by the vegetative canopy plus available soil moisture in the root zone	L	≥ 0
n	Shape parameter of gamma PDF; characterizes number of linear reservoirs necessary to represent the unsaturated zone; fractional values do not have a physical analogy, but allow greater flexibility in the shape of the gamma PDF	--	> 0
τ_i	Initial time lag of gamma PDF; represents delay time between beginning of effective infiltration and first arrival of recharge	T	≥ 0
k	Scale parameter of gamma PDF; the expression $nk + \tau_i$ represents the average delay time imposed on effective infiltration by the unsaturated zone	T	> 0

follows (fig. 2): water enters the hydrologic system as precipitation. Surface runoff is not simulated and, if not negligible, must be independently estimated and subtracted from precipitation. A fraction of the precipitation that is not lost to surface runoff is captured by the vegetative canopy; the remainder infiltrates into the soil. Water held in storage in the vegetative canopy or in the root zone as soil moisture is subject to ET. After satisfying the ET demand, water that exceeds the storage capacity of the vegetative canopy and root zone, if any, exits the bottom of the root zone where it becomes "effective" (or "net") infiltration. Effective infiltration percolates through the deeper unsaturated zone where it is not subject to extraction or upward movement. Accordingly, the bottom of the root zone is assumed to be the depth below which water is no longer subject to ET, which can extend below the depth of plant roots. Eventually, effective infiltration reaches the water table, entering the saturated ground-water flow system, where it becomes recharge (Meinzer, 1923, p. 46).

Water-Balance Module

The objective of the water-balance module of the WBTF model is to compute effective infiltration by simulating the water storage capacity of the vegetative canopy and root zone (fig. 2). Water is assumed to be held in storage in the vegetative canopy or in the root zone as soil moisture until a predefined maximum storage capacity is exceeded. Water storage is simulated by applying the following volume-balance equation:

$$\frac{dS}{dt} = p(t) - e(t), \tag{1}$$

where

S is total storage (water held in the vegetative canopy and root zone), [L];

$p(t)$ is difference between precipitation and surface runoff rates, [L/T];

$e(t)$ is evapotranspiration rate, [L/T];

t is time, [T]; and

L and T denote length and time units, respectively.

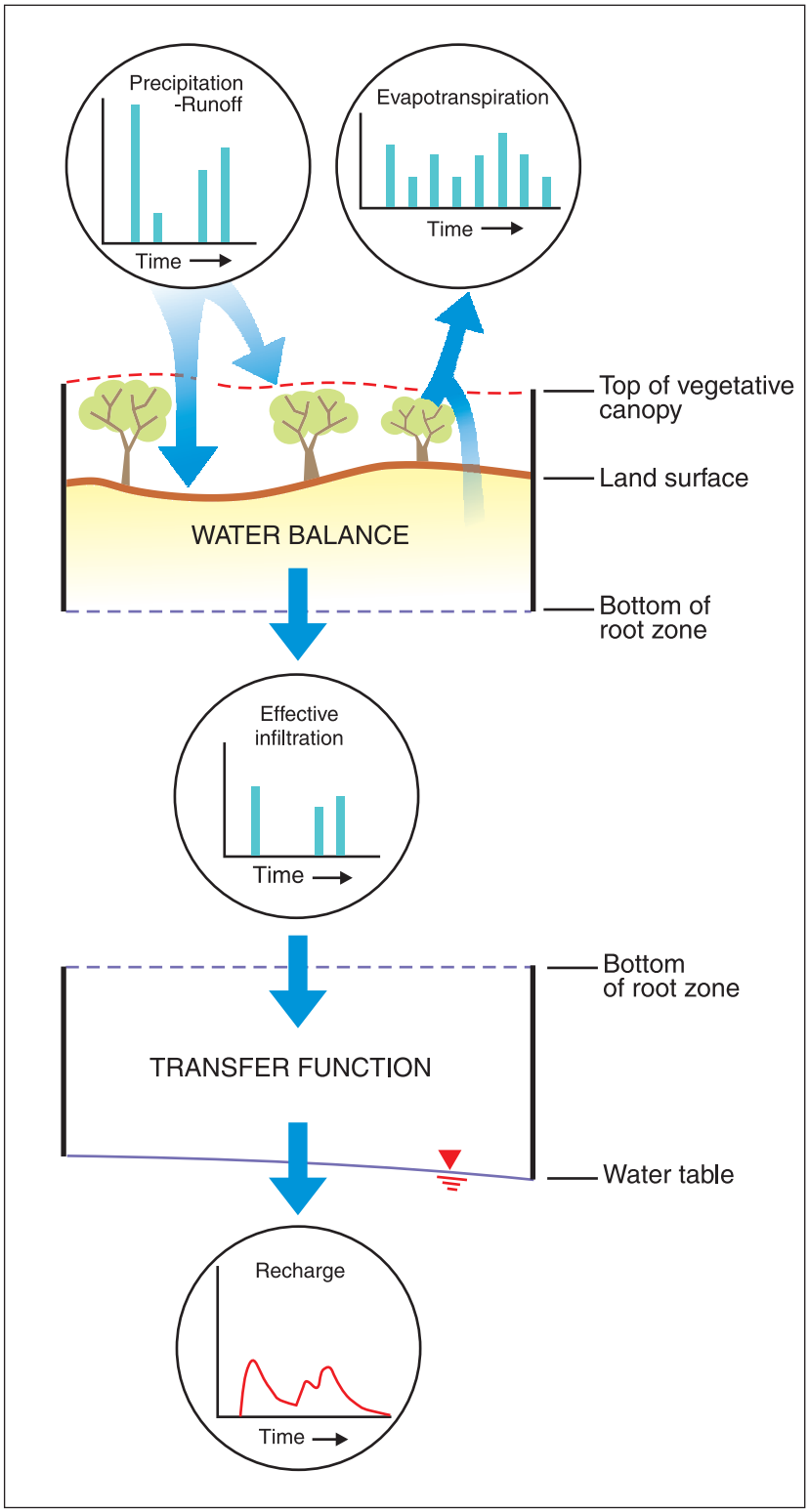

Figure 2. Conceptual model of the water-balance/transfer-function model.

In equation 1 (and throughout the WBTF model) length units are used to represent a volume of water per unit of bulk area perpendicular to the vertical direction of flow. The discretized form of equation 1 is used in the WBTF model:

$$S^i = S^{i-1} + (P^i - E^i)\Delta t_{pe}, \qquad (2)$$

where

S^i is total storage at timestep i, [L];

S^{i-1} is total storage at previous timestep i-1, [L];

P^i is difference between average precipitation and average surface-runoff rates over time interval $((i-1)\Delta t_{pe}, i\Delta t_{pe})$, [L/T];

E^i is average evapotranspiration rate over time interval $((i-1)\Delta t_{pe}, i\Delta t_{pe})$, [L/T];

i is discretization index representing time $i\Delta t_{pe}$; and

Δt_{pe} is time interval for discretization of $p(t)$ and $e(t)$, [T], which is equal to or greater than the measurement time interval.

The water-balance module is an example of a type of simple mass- or volume-balancing model commonly called a bucket model (Guswa and others, 2002, p. 2; Walker and others, 2002, p. 74), because its execution is analogous to the filling, draining, and overflowing of a bucket. In the current application, the bucket is filled by the difference between precipitation and surface runoff and drained by ET; once filled to its maximum capacity, the bucket overflow represents effective infiltration (fig. 3). Effective infiltration is computed as follows:

$$I_e^i = \begin{cases} \dfrac{S^i - S_{max}}{\Delta t_{pe}} & \text{for } S^i > S_{max} \\ 0 & \text{for } S^i \le S_{max} \end{cases}, \qquad (3)$$

where

I_e^i is average effective infiltration rate over time interval $((i-1)\,\Delta t_{pe}, i\Delta t_{pe})$, [L/T]; and

S_{max} is maximum total storage (total bucket capacity), [L].

The bucket model is nonlinear because effective infiltration is linearly proportional to storage when the bucket is full but not when the bucket is partially full or empty (eq. 3).

Two user-defined parameters are required for execution of the water-balance module: S_b and S_{max} (table 1). S_b is the initial condition for the bucket model (represents S^{i-1} when equation 2 is used to compute S^i for the initial timestep), but it is herein called a parameter for convenience. Storage (S, eq. 1) conceptually represents the

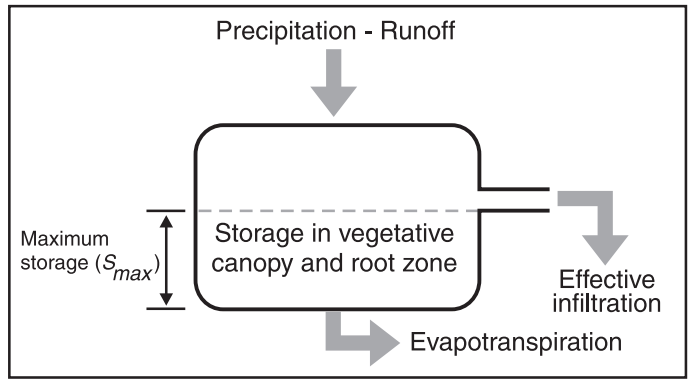

Figure 3. Bucket model used to compute effective infiltration for the water-balance module of the water-balance/transfer-function model.

amount of water intercepted and retained by the vegetative canopy plus the available soil moisture in the root zone; it can be quantified by:

$$S = S_c + (\theta - \theta_{wp})D_{rz} \quad \text{for } \theta_{wp} \le \theta \le \theta_{fc}, \qquad (4)$$

where

S_c is vegetative canopy storage, [L];

θ is average volumetric moisture content of the root zone, [L^3/L^3];

θ_{wp} is average permanent wilting point of the root zone, [L^3/L^3]; permanent wilting point represents the largest volumetric moisture content at which indicator plants, growing in a particular soil, wilt and fail to recover when placed in a humid chamber, which often is estimated by the moisture content at a pressure head of -160 m (Soil Science Society of America, 2001);

D_{rz} is depth of the root zone, [L]; and

θ_{fc} is average field capacity of the root zone, [L^3/L^3]; field capacity represents the volumetric moisture content at the time when gravity drainage becomes negligible, often considered to be 2-3 days for well-drained (sand) soil (Soil Science Society of America, 2001; Healy and Cook, 2002, p. 100).

For estimation of S_b, θ represents the average moisture content in the root zone at the beginning of the model simulation period (θ_b); for estimation of S_{max}, θ represents θ_{fc}. The minimum total storage occurs when both S_c and the available soil-moisture storage in the root zone [$(\theta - \theta_{wp})D_{rz}$] equal zero. Although soil moisture exists in the root zone under these conditions (at a moisture content equal to θ_{wp}), it is assumed to be tightly bound to soil particles and not extractable by ET, hence storage is assigned a value of zero (bucket is empty) by the water-balance module. Moisture content can drop below θ_{wp} as a result of direct evaporation, for example, during

extended dry periods. Storage data often are not available and can be difficult to estimate reliably, but equation 4 can be a useful context in which to determine the reasonableness of storage computations.

The bucket model is a simplified representation of a complex natural system, and several assumptions are important to its application in the simulation of transient recharge:

- Surface runoff is negligible or can be adequately estimated and subtracted from precipitation by using separate methods. Surface runoff depends partly on infiltration capacity, which varies with soil-moisture conditions (Fetter, 1988, p. 88). For example, an increase in moisture content of the surface soil will cause a reduction in infiltration capacity, which could result in increased surface runoff during intense precipitation. Consequently, inconsistencies could exist between storage computed by the water-balance module (eq. 2) and soil-moisture conditions estimated or assumed for the method used to determine surface runoff.
- Little water is held in detention storage in land-surface depressions. That is, any water that is not lost to surface runoff rapidly evaporates or infiltrates after precipitation.
- Average moisture content of the root zone never drops below the permanent wilting point (θ_{wp}). That is, total storage is zero when both vegetative canopy storage (S_c) and available soil-moisture storage in the root zone $[(\theta-\theta_{wp})D_{rz}]$ equal zero.
- Horizontal flow in the root zone is negligible.
- The traveltime of water through the root zone is negligible. That is, the root zone is relatively thin and consists of high-permeability soils.
- The flux of water at the bottom of the root zone is always downward or zero.

The user is advised to examine output from the water-balance module for each model simulation to ascertain its reasonableness. Negative values of storage could be computed with equation 2 if ET exceeds precipitation for an extended period. If this occurs, then S^i (eq. 2) is set equal to zero and the amount of "excess" or "unaccounted" ET is not extracted anywhere in the soil profile. This could be problematic because the resulting simulated water balance is at odds with the measured (or assumed) values of ET. Possible causes of this condition include (1) S_{max} is too small—a larger bucket capacity could sustain ET during extended dry periods (when θ might drop below θ_{wp}), although this also would reduce effective infiltration (eq. 3); (2) specified values of precipitation or ET are incorrect; or (3) the bucket model is inadequate for simulating the problem in question, because, for example, one of the assumptions listed above is violated.

Transfer-Function Module

The objective of the transfer-function module of the WBTF model is to compute recharge by simulating the traveltime of effective infiltration from the bottom of the root zone to the water table (fig. 2). In keeping with the need to develop a relatively simple model, solution of the commonly used (Richards, 1931) equation describing variably saturated water flow was deemed too computationally and data intensive. Instead, a "black box" approach using a transfer-function model was adopted. Transfer-function models are a common time-series modeling technique. A comprehensive treatment of time-series analysis, including transfer-function models, is presented by Box and Jenkins (1976) and Hipel and McLeod (1994). Transfer-function models have been applied frequently in hydrology, for example, for the analysis of base flow and water-level variations in stream-aquifer systems (Hall and Moench, 1972), solute transport in the unsaturated zone (Jury, 1982; Stewart and Loague, 2003), ground-water levels (Tankersley and others, 1993; Gehrels and others, 1994), stream discharge (Sepúlveda and others, 1996), stream water quality (Pinault and others, 2001), and karst spring discharge (Denic-Jukic and Jukic, 2003). Much of the theory and practical application used in the development of the transfer-function module of the WBTF model is based on research presented by Besbes and de Marsily (1984), Morel-Seytoux (1984), and Wu and others (1997), all of whom applied transfer functions to the time-series analysis of ground-water recharge.

The key premise for application of a transfer function for simulation of recharge is that water from each effective infiltration event has the same traveltime characteristics, experiencing the same delay and attenuation, as it moves through the unsaturated zone. Consequently, the percolation of effective infiltration through the unsaturated interval between the bottom of the root zone and the water table is a linear process. This is a good approximation if the hydraulic conductivity of the unsaturated sediments does not vary substantially with time (Besbes and de Marsily, 1984, p. 272). Hydraulic conductivity of unsaturated sediments, however, is a nonlinear function of moisture content (Koorevaar and others, 1983, p. 141). Therefore, the percolation of effective infiltration more closely approximates a linear process as temporal

variations in moisture content below the root zone become smaller. The validity of this linearity assumption is discussed in later sections.

Recharge can be simulated by using a convolution integral between effective infiltration and a linear-transfer function (Besbes and de Marsily, 1984, p. 273):

$$r(t) = \int_0^t i_e(t-\tau)\phi(\tau)d\tau, \tag{5}$$

where

$r(t)$ is recharge rate, [L/T];

$i_e(t)$ is effective infiltration rate, [L/T], which is assumed to be 0 for $t < 0$;

$\phi(\tau)$ is a linear-transfer function, [LT $^{-1}$/L];

t is time, [T], which is measured forward from an initial time assumed to be 0; and

τ is time lag of the transfer function, [T], which is time before present, measured backwards from current time.

Equation 5 is based on the theory of the instantaneous unit hydrograph (Dooge, 1959), commonly used in surface-water hydrology, where stream runoff is simulated based on the convolution of excess precipitation and a unit hydrograph (transfer function) appropriate to the watershed in question. In the present context, the transfer function in equation 5 represents the recharge resulting from the application of a unit amount of effective infiltration during an infinitesimally small (instantaneous) period of time. Because the transfer function represents a unit amount of effective infiltration, equation 5 conserves mass in that, given enough time, all effective infiltration will become recharge. Figure 4 shows that simulated recharge can be delayed and attenuated relative to effective infiltration by a transfer function. The independent variable of the transfer function (τ, eq. 5 and fig. 4) is called a time lag because the transfer function has the effect of lagging the response time series (recharge) relative to the input time series (effective infiltration). The discretized form of equation 5 is used in the WBTF model:

$$R^i = \sum_{j=1}^{m} I_e^{\,l}\,\phi^{j-\frac{1}{2}}\Delta\tau \text{, and} \tag{6}$$

$$l = i - truncate\left(\frac{\Delta\tau}{\Delta t_u}j\right), \tag{7}$$

where

R^i is average recharge rate over time interval $((i-1)\,\Delta t_u,\ i\Delta t_u)$, [L/T];

$I_e^{\,l}$ is average effective infiltration rate over time interval $((l-1)\,\Delta t_u,\ l\Delta t_u)$, [L/T], which is assumed to be 0 for $l < 0$;

$\phi^{j-\frac{1}{2}}$ is a linear-transfer function evaluated at time lag $\left(j-\frac{1}{2}\right)\Delta\tau$, [LT $^{-1}$/L]; evaluation of the transfer function at the midpoint of the time-lag interval, rather than at j-1 or j, results in a more accurate discrete representation of the continuous function;

$\Delta\tau$ is time-lag interval for discretization of the transfer function, [T], which is less than or equal to Δt_u;

i is discretization index representing time $i\Delta\tau_u$;

j is discretization index representing time lag $j\Delta\tau$ of the transfer function;

m is number of time-lag intervals, $\Delta\tau$, spanned by the transfer function;

l is discretization index representing the time defined by $(t-\tau)$;

Δt_u is unit-event length defining the time interval for discretization of effective infiltration, [T]; and

truncate (x) denotes the truncation of x to an integer.

The unit-event length (Δt_u, eq. 7) should be much shorter than the duration of a recharge event so that effective infiltration is approximately instantaneous (Wu and others, 1997, p. 135). In addition, Δt_u should be less than or equal to the discretization of input time series (Δt_{pe}, eq. 2) for the water-balance module. The time interval for discretization of the transfer function ($\Delta\tau$, eq. 6) is chosen automatically by the model and is always less than or equal to Δt_u.

Application of equation 6 requires selection of an appropriate mathematical expression for the transfer function. Besbes and de Marsily (1984) demonstrated success at conceptualizing the movement of water through the unsaturated zone between the bottom of the root zone and the water table as a succession of routings through a series of linear reservoirs. A linear reservoir is one in which storage is directly proportional to outflow rate, and the outflow rate resulting from an instantaneous inflow is described by an exponential decay (Dooge, 1959, p. 243):

$$Q = \frac{v}{k}e^{-t/k}, \tag{8}$$

where

Q is outflow rate from the reservoir, [L^3/T];

v is volume of water instantaneously added to the reservoir, [L^3]; and

k is proportionality constant between storage and outflow rate for the linear reservoir, representing a characteristic delay time imposed on the inflow by the reservoir, [T].

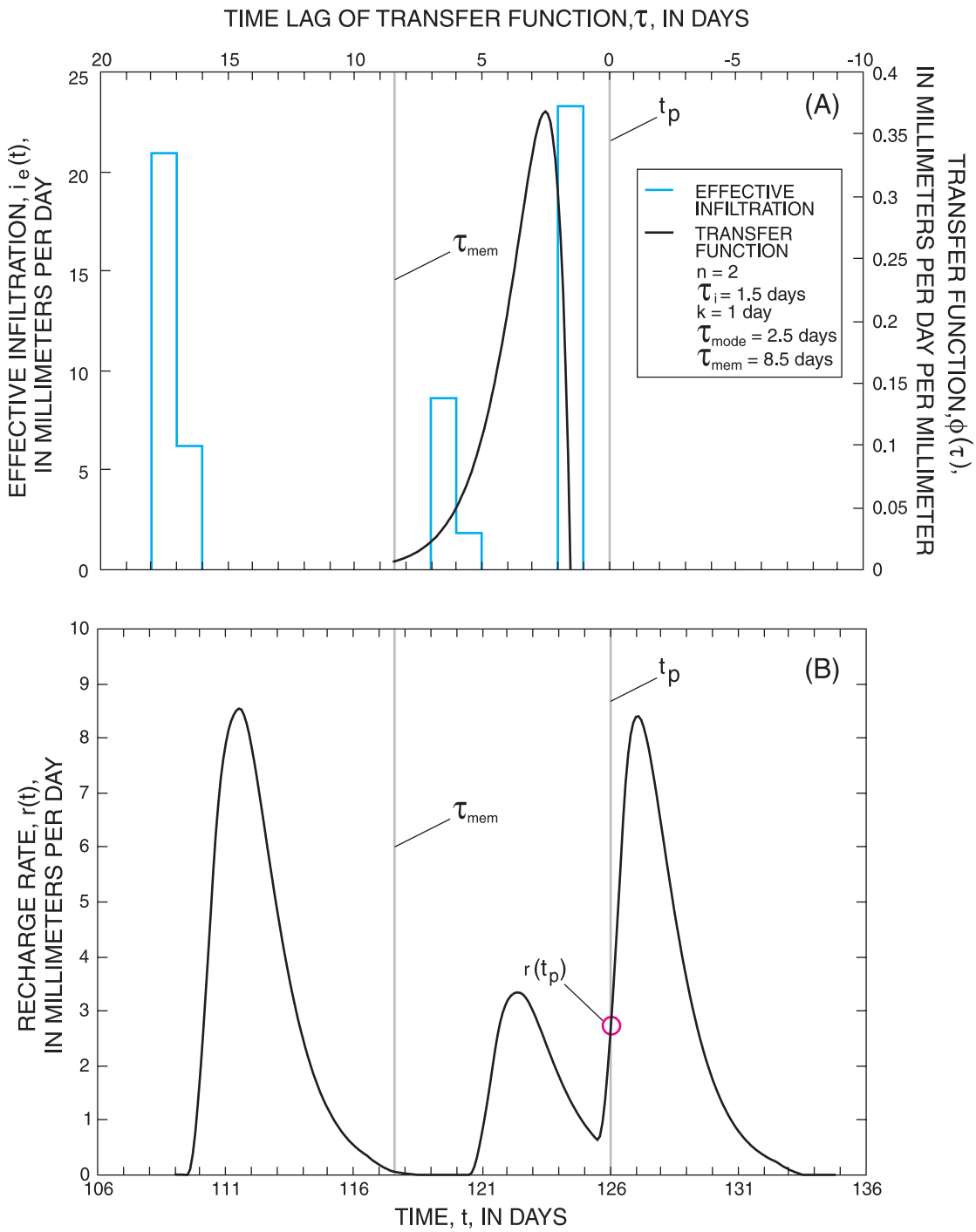

Figure 4. Example of (A) the convolution of effective infiltration with a transfer function for the computation of (B) recharge at present time t_p.

Nash (1958) described the outflow from a series of identical linear reservoirs as

$$\frac{Q_n}{v} = \frac{1}{k(n-1)!} e^{-t/k} (t/k)^{n-1}, \qquad (9)$$

where

Q_n is outflow rate from the nth reservoir, [L^3/T]; and

n is number of linear reservoirs, [dimensionless].

The quotient Q_n/v is the outflow that would result from a unit inflow event; accordingly, equation 9 is the instantaneous unit hydrograph for n linear reservoirs arranged in series. The right-hand side of equation 9 is equivalent to the equation for the gamma probability density function (PDF) and was modified slightly for use as the transfer function in equation 6:

$$\phi(\tau-\tau_i) = \begin{cases} \frac{1}{k\Gamma(n)} e^{-\frac{(\tau-\tau_i)}{k}} \left(\frac{\tau-\tau_i}{k}\right)^{n-1} & \text{for } \tau > \tau_i \\ 0 & \text{for } \tau \le \tau_i \end{cases}, \qquad (10)$$

where

n is a shape parameter, characterizing the number of linear reservoirs necessary to represent the unsaturated zone (equivalent to n in equation 9), [dimensionless]; fractional values of n do not have a physical analogy, but allow greater flexibility in the shape of the gamma PDF;

τ_i is initial time lag, representing delay time between beginning of effective infiltration and first arrival of recharge, [T]; τ_i is graphically depicted in figure 4;

k is a scale parameter, [T]; the expression $nk + \tau_i$ represents the average delay time (mean of the gamma PDF) imposed on effective infiltration by the unsaturated zone; and

$\Gamma(n)$ is the gamma function (Potter and Goldberg, 1987, p. 111), [dimensionless]. $\Gamma(n)$ is equivalent to $(n\text{-}1)!$ for integer values of n; it replaces $(n\text{-}1)!$ in equation 9 and allows for fractional values of n.

The shape of the gamma PDF is intuitively reasonable, as recent effective infiltration can be weighted more heavily than that which occurred in the distant past. The shape of the gamma PDF is quite flexible (fig. 5) and determines how similar a time series of recharge is to a time series of effective infiltration. Recharge closely resembles effective infiltration in both magnitude and timing for small values of n and k. Large values of n and k cause significant attenuation of the effective infiltration signal, resulting in a smoother time series of recharge. Parameter τ_i effects only a constant lag and does not attenuate the input signal. Note that the gamma PDF has a shape similar to an exponential decay for n values less than or equal to 1, but is unimodal (peaks at a finite τ value greater than 0) for n values greater than 1 (fig. 5A). The mode of a gamma PDF (τ_{mode}) is

$$\tau_{mode} = \begin{cases} \tau_i & \text{for } n \le 1 \\ (n-1)\,k + \tau_i & \text{for } n > 1 \end{cases}. \qquad (11)$$

The mode of a gamma PDF is important because simulated recharge is proportionally most influenced by effective infiltration that occurred τ_{mode} time units ago. Recharge computed by the convolution (eq. 6) of a gamma PDF ($n = 2$, $\tau_i = 1.5$ days, and $k = 1$ day) with effective

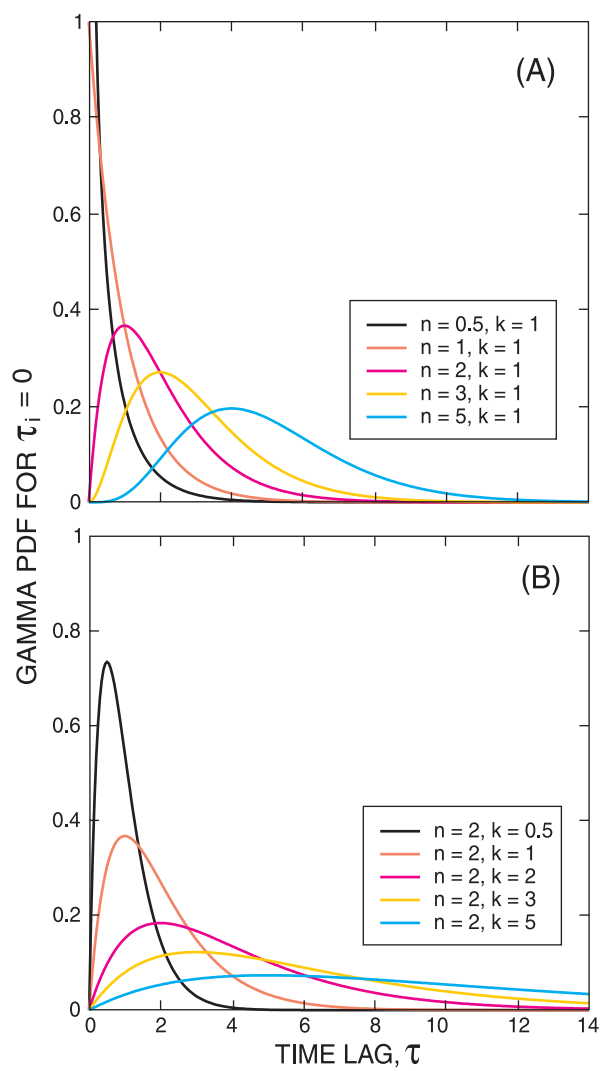

Figure 5. Gamma probability density functions (PDF) for τ_i equal to 0 and for (A) various values of n with k equal to 1 and (B) various values of k with n equal to 2.

infiltration is shown in figure 4. The mode of this gamma PDF is 2.5 days, but for computation of recharge at time t_p (time 126), effective infiltration is 0 at a time lag of 2.5 days; recharge at time t_p is influenced by effective infiltration during times 124.0-124.5 and during times 119-121. For computation of recharge at time 127, however, τ_{mode} would coincide with the large effective infiltration event during times 124-125, resulting in the high recharge rate simulated at time 127. A physical property related to the n parameter is not apparent; however, the time-delay parameters, τ_i and k, are likely related to water-table depth and soil hydraulic properties, as will be discussed in later sections. Incorporation of τ_i into the gamma PDF, as suggested by Wu and others (1997, p. 127), is an improvement of the physical basis of the model, because a finite amount of time is required for infiltrating water to travel through the unsaturated zone, regardless of water-table depth or soil properties. For the example shown in figure 4, effective infiltration that occurred from 0 to 1.5 days ago is in transit to the water table.

Memory is a critical characteristic of any time-dependent system. Skoien and others (2003) investigated characteristic timescales of various hydrologic processes, such as precipitation, soil moisture, and ground-water levels. They noted that the characteristic timescale increases as water moves along a flowpath from land surface into the subsurface and attributed this to the removal of short-term fluctuations along the flow path, consequently imposing a longer memory on the time variations (Skoien and others, 2003, p. 14). In the present application, memory is the amount of time that recharge "remembers" past effective infiltration; similarly, memory can be interpreted as the amount of time elapsed before recharge has "forgotten" past effective infiltration. A reasonable assumption is that recharge at a deep water table has a longer memory than recharge at a shallower water table because of the greater traveltime of the water as it percolates through the thicker unsaturated zone. Likewise, recharge resulting from percolation through sandy more permeable soils has a shorter memory than recharge in clayey less permeable soils. For a transfer-function model that conserves mass, memory is implicitly defined as follows:

$$\int_0^{\tau_{mem}} \phi(\tau)d\tau = 1 , \qquad (12)$$

where

τ_{mem} is the memory of the transfer function, [T].

As the values of the parameters n and k decrease, the memory of the gamma PDF decreases; likewise, as the values of these parameters increase, memory increases. This relation is apparent by examining the various gamma PDFs shown in figure 5. Because the gamma PDF asymptotically approaches zero for large values of τ (fig. 5), the area under the ϕ curve (eq. 12) will always be less than 1, even for very large values of τ. Therefore, a finite discretization of ϕ is used in the WBTF model, based on an area under the ϕ curve equal to 0.99 and the approximate discretized form of equation 12:

$$\sum_{j=1}^{m} \phi^{j-\frac{1}{2}} \Delta\tau = 0.99 \text{ , and} \qquad (13)$$

$$\tau_{mem} = m\Delta\tau . \qquad (14)$$

The value of m that satisfies equation 13 is used to compute τ_{mem} (eq. 14) and in the computation of the discretized convolution integral (eq. 6). A criterion smaller than 0.99 could be used in equation 13, but errors in mass conservation increase as this criterion decreases—total recharge will more closely equal total effective infiltration as this criterion approaches a value of 1. Effective infiltration that occurred more than τ_{mem} time units ago does not contribute significantly to recharge today or in the future. Figure 4A shows a gamma PDF with τ_{mem} equal to 8.5 days—the area under the ϕ curve between time lags 0 and 8.5, or times 117.5 and 126.0, is 0.99. The remaining 1 percent of the area under the ϕ curve extends to times before 117.5, but is not used in the WBTF model.

The transfer-function model is a simplified representation of a complex natural system and several assumptions are important to its application in the simulation of transient recharge:

- Horizontal flow in the unsaturated zone is negligible.
- The flux of water below the root zone is always downward or zero.
- No sources or sinks of water exist below the root zone; therefore, long-term average effective infiltration equals long-term average recharge.
- The percolation of water below the root zone is a linear process. That is, water from each effective infiltration event has the same traveltime characteristics, experiencing the same delay and attenuation, as it moves through the unsaturated zone.
- Effective infiltration is assumed to be 0 for times less than 0 (eqs. 5 and 6).

The user is advised to examine output from the transfer-function module for each model simulation to ascertain its reasonableness. Erroneous values of recharge (R^i, eq. 6) can be computed for times from 0 to τ_{mem} if effective infiltration is not 0 for times from $-\tau_{mem}$ to 0. This possible error is a consequence of the last assumption listed above and the memory of recharge, because some history of effective infiltration must be known in order to simulate recharge. Historical effective infiltration rates generally are not available for data near the beginning of the period of interest. For times greater than τ_{mem}, however, a sufficient history of effective infiltration is computed by the water-balance module; recharge can be simulated without concern about this assumption.

Application of the Water-Balance/Transfer-Function Model

The applicability of the WBTF model is demonstrated by comparing its results to field-based estimates of recharge at a site on the Lake Wales Ridge in west Orange County (fig. 1). Important features and limitations of the WBTF model are further demonstrated by comparing its results to those of the USGS variably saturated flow model VS2DT (Lappala and others, 1987; Healy, 1990). Detailed instructions for the use of the WBTF model are provided in appendix 1, including descriptions of the file structures and data formats necessary for running the model.

Comparing WBTF model results to field-based estimates of recharge provides a more objective evaluation of the WBTF model by imposing a minimum of preconceived ideas about unsaturated flow processes. For example, whether recharge is enhanced by macropore flow or inhibited by soil layering, effects of soil structure are reflected in field-based recharge estimates; the ability or inability of the WBTF model to replicate these effects can be tested. Data for field-based comparisons of recharge, however, are limited. Therefore, WBTF model results are compared to estimates of recharge derived from VS2DT, which is based on Richards (1931) equation describing variably saturated water flow. The Richards (1931) equation is limited to diffuse flow in a relatively homogeneous soil that is represented as a continuum, which may not exist for many field conditions such as macropore flow. Nevertheless, the use of synthesized data with VS2DT enables a greater variety of hydrologic conditions to be simulated than is available with field-based data.

Field Site in West Orange County

A micrometeorological instrumentation site was operated by Sumner (1996) in the Lake Wales Ridge area of western Orange County (fig. 1). The field site contained mostly herbaceous, successional vegetation, with a root depth that rarely exceeded 0.3 m, a typical occurrence of cleared areas in central Florida (Sumner, 1996, p. 2). The site also had well-drained soils, a relatively deep water table (2-3.5 m below land surface, fig. 6B), and negligible surface runoff. The micrometeorological data were used with measured values of ET obtained by the eddy correlation method to develop a model of actual ET for the entire period of data collection (September 15, 1993, to August 27, 1994; Sumner, 1996). Precipitation, volumetric moisture content at several depths in the unsaturated zone, and water-table altitude in a surficial aquifer system well also were measured at the site. The Lake Wales Ridge in west Orange County generally is a high recharge area and has many of the same characteristics as other high recharge areas in central Florida. A more regional investigation of the hydrogeology of west Orange and southeast Lake Counties is provided by O'Reilly (1998). This site is well suited for application of the WBTF model and investigation of recharge processes because of the hydrologic setting (high recharge area) and the complete suite of field data.

Estimation of Recharge by Analysis of Water-Table Fluctuations

Because recharge cannot be measured directly, it must be inferred from other measured data. The water-table fluctuation method (Healy and Cook, 2002) can be used to estimate transient recharge; this method is based on the assumption that a rise in the water table is caused solely by recharge. Therefore, after removing any background trend, recharge is computed as the product of water-table rise and specific yield.

The water-table hydrograph at the west Orange County site was analyzed by using the water-table fluctuation method. The water-table fluctuation method is appropriate for estimation of recharge at this site because (1) the water table was always below the root zone; therefore, water-table fluctuations do not represent ET extracted directly from the saturated zone; (2) precipitation in central Florida commonly is dominated by short-duration, high intensity rainfall to which the water table responds relatively rapidly, which is important because the rate of recharge must be significantly greater than the rate of ground-water flow away from the water table

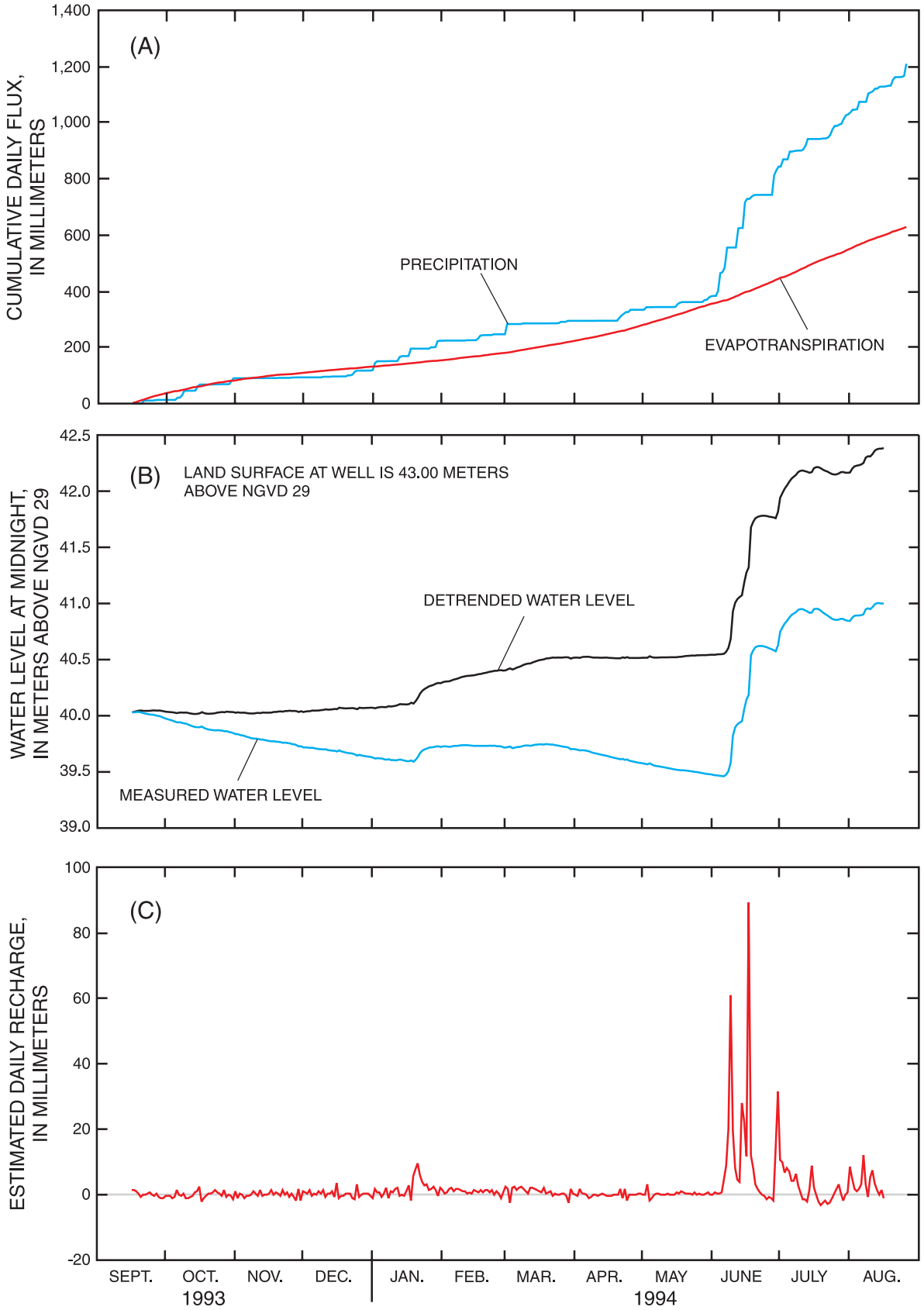

Figure 6. Data collected at a meteorological station in west Orange County (Sumner, 1996): (A) cumulative precipitation and evapotranspiration, (B) water-table altitude, and (C) ground-water recharge estimated by using the water-table fluctuation method.

for the water-table fluctuation method to be applicable (Healy and Cook, 2002, p. 93); and (3) the site is believed to be relatively unaffected by anthropogenic factors, such as artificial recharge or ground-water withdrawals, which might cause short-term water-table fluctuations. A noticeable background trend exists in the water-table hydrograph (fig. 6B). The trend probably was caused by downward leakage to the Floridan aquifer system and lateral flow in the surficial aquifer system toward surface-water features or areas where the intermediate confining unit is thinner or more permeable. The true trend probably varied in time; however, in the present analysis it was assumed to be constant. A trend of -4.1 millimeters per day (mm/d) was estimated by visually fitting a straight line to the recession period of the water-table hydrograph caused by low precipitation (2.9 mm) during November 1993 (fig. 6A and 6B). This background trend was removed from the water-table hydrograph to obtain the detrended water level (fig. 6B). Estimated daily recharge (fig. 6C) was computed by multiplying the daily change in water level (computed from the detrended hydrograph based on water levels measured at midnight) by a constant specific yield of 0.25. An interesting observation is the similarity between cumulative precipitation and the detrended hydrograph (figs. 6A and 6B, respectively), which suggests that the temporal variability of recharge is primarily dependent on the temporal variability of precipitation.

Specific yield and the time interval of recharge calculations are important factors when using the water-table fluctuation method. Laboratory measurements of moisture-characteristic curves (J.A. Tindall, U.S. Geological Survey, written commun., 1996) for soil cores collected at the site, as part of an unrelated study, indicate a specific yield of about 0.3 (based on the difference between saturated moisture content and moisture content at a negative pressure head equal to the average water-table depth). Specific yield also can be estimated by comparing the change in the detrended water level (2.35 m for the period September 16, 1993 - August 16, 1994; fig. 6B) with the measured recharge (precipitation (1,127 mm) minus ET (596 mm), fig. 6A) for the same time period, resulting in an estimate of 0.23. This difference in specific yield might be caused by gas bubble entrapment. Gas bubbles trapped below a rising water table or generated by microorganisms are common and can effectively reduce specific yield (or specific moisture capacity) (Fayer and Hillel, 1986a, b; Flühler and others, 1986, p. 1162-1163; Faybishenko, 1995, p. 2421-2423). The effects of entrapped gas have been observed in central Florida—during flooded conditions in a rapid infiltration basin (Sumner and Bradner, 1996, p. 12), during laboratory wetting of sand and clayey-sand cores (Sumner and Bradner, 1996, p. 18), and during natural water-table fluctuations (Nachabe and others, 2004). The volumetric air content that results from gas bubble entrapment and the reduction in specific yield is difficult to quantify; the specific yield of 0.25 used herein is believed to be representative of the field site. The computation of recharge at daily time intervals effectively filters the data set, removing high frequency fluctuations. Hourly water-level data indicate both periodic (period of 1 day) and random water-level fluctuations, which generally are less than 20 mm in magnitude. Based on visual comparison with precipitation and ET measured at the site, these small water-level fluctuations do not appear to be related to recharge and may be the result of atmospheric pressure effects (Freeze and Cherry, 1979, p. 234) or instrumentation error.

The water-table fluctuation method is applied in a simplified manner by assuming a constant background trend and specific yield. Negative estimated recharge rates (for example, during July 1994, fig. 6C) are likely the result of a background trend that is greater (in absolute value) than that assumed. Healy and Cook (2002) describe the effects of variable specific yield and some of the other difficulties encountered in application of the water-table fluctuation method. Nevertheless, the estimates of recharge (fig. 6C) are believed to be reasonable approximations.

Calibration of Water-Balance/Transfer-Function Model

Time series of measured daily precipitation and ET (fig. 6A) were used with the WBTF model, and values of the five model parameters (table 1) were adjusted in an effort to match daily recharge rates estimated by using the water-table fluctuation method (fig. 6C). A unit-event length (Δt_u, eq. 7) of 0.1 day was used; accordingly, recharge was computed at a time interval of 0.1 day and daily average recharge rates were computed as arithmetic means. The 0.1-day unit-event length is small relative to the length of a recharge event (3.3 days as simulated by the WBTF model) and approximates an instantaneous event. Therefore, recharge rates computed using the 0.1-day unit-event length are herein called instantaneous recharge (fig. 7), even though they actually represent an average rate for this short time period. Reduction of the unit-event length to 0.01 day resulted in negligible differences in instantaneous recharge. Precipitation and ET rates were assumed constant over each 1-day period (Δt_{pe} = 1 day, eq. 2). Some loss of accuracy results from

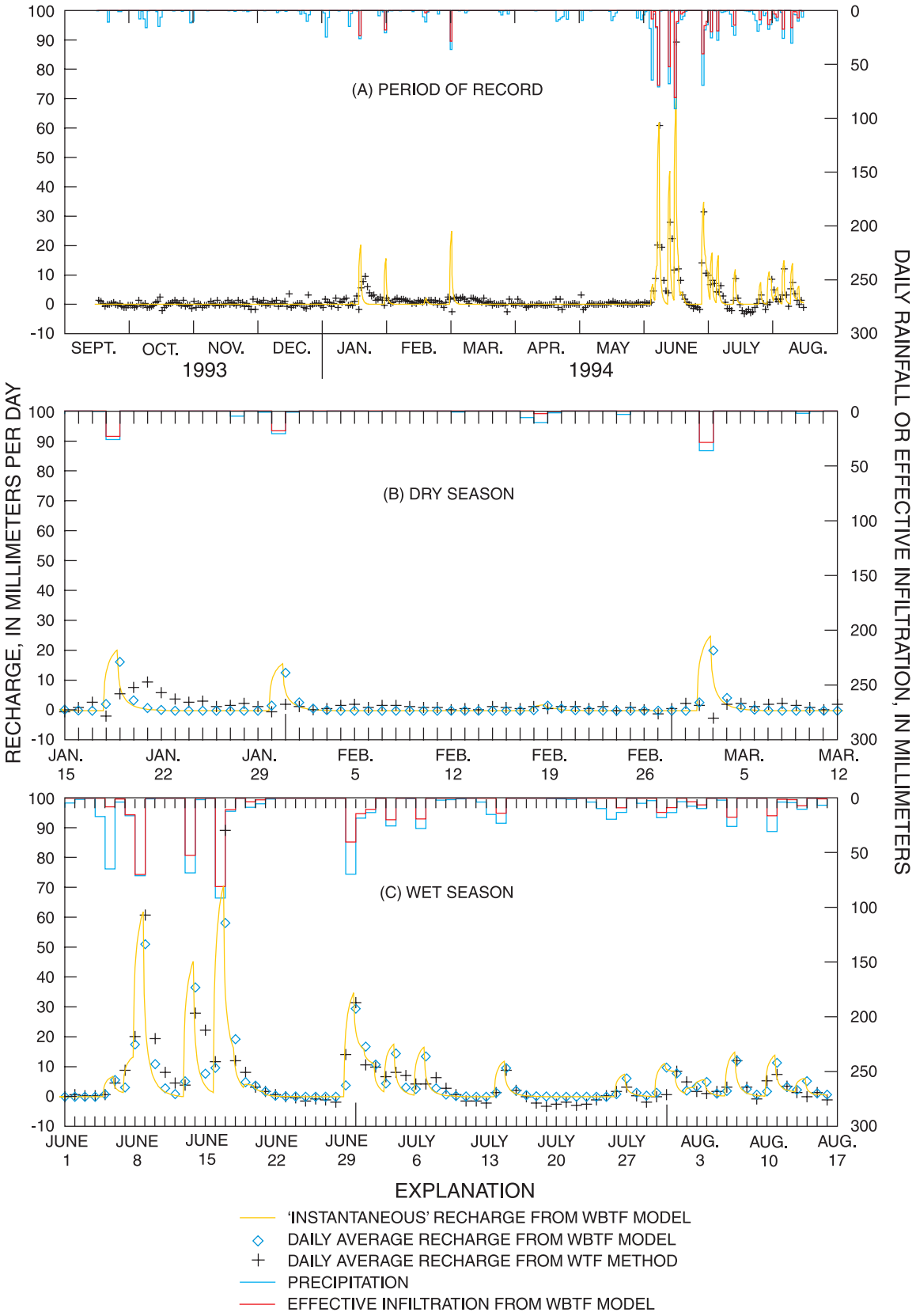

Figure 7. Comparison of recharge at meteorological station in west Orange County estimated by using the water-table fluctuation (WTF) method and by using the WBTF model for the (A) period of record, (B) dry season, and (C) wet season.

the use of a Δt_{pe}value greater than the measurement interval of precipitation and ET, but the difference is small in this application. A linear regression between simulated, daily average recharge rates calculated for Δt_{pe}values equal to both 1 hour and 1 day yielded a standard error of 1.3 mm/d, which is small compared to most recharge (fig. 7), and coefficient of determination (r^2) of 0.96. In addition, the total amount of recharge for the simulation period was 5 percent less when $\Delta t_{pe}$was equal to 1 hour rather than 1 day, which probably is within the measurement error of precipitation and ET.

The nonlinear regression code UCODE (Poeter and Hill, 1998) was used to obtain the best-fit values for each parameter except S_b. The final parameter values were S_b = 49.0 mm, S_{max} = 77.0 mm, n = 0.369, τ_i = 0.824 day, and k = 1.12 days. The model was insensitive to S_b; consequently, S_b was specified as a reasonable fraction of S_{max}. Measured moisture contents indicate that the average moisture content in the root zone at the beginning of the simulation period (fig. 8A) was greater than the permanent wilting point (0.02 cubic centimeters per cubic centimeter (cm^3/cm^3) estimated from moisture-characteristic curves for soil cores collected in the root zone), but probably less than field capacity. The root-zone depth likely extends below the 300-mm depth of plant roots observed by Sumner (1996, p. 2). For a root-zone depth of 300 mm, a permanent wilting point of 0.02 cm^3/cm^3, and a canopy storage of 0 mm, an S_{max} value of 77 mm is equivalent to a field capacity of 0.28 cm^3/cm^3 based on equation 4. A field capacity of 0.28 cm^3/cm^3 is unrealistically high for the sandy, well-drained soil at the field site; Fetter (1988, p. 95) reports that an approximate field capacity of 0.1 cm^3/cm^3 might be expected for sand. Applying equation 4 as above with a field capacity of 0.1 cm^3/cm^3 yields a root zone depth of 960 mm. Because the bottom of the root zone is defined in the WBTF model as the depth below which water is no longer subject to ET, which can extend below the depth of plant roots, a deeper root zone could be explained by upward movement of water induced by the overlying plant roots and by sparsely rooted soil layers below the more densely rooted layers. Li and others (2001) demonstrated that when available soil moisture in upper soil layers is depleted, transpiration can be met by uptake from deeper, wetter soil layers. The parameters of the gamma PDF (eq. 10) indicate that recharge from an effective infiltration event first reaches the water table in about 0.8 day (τ_i) and is delayed another 0.4 day (nk) on average. All recharge from an effective infiltration event has occurred within 4.1 days, which is equal to the memory of the gamma PDF (τ_{mem}, eqs. 13 and 14). The short memory of recharge at this site is indicative of sandy, well-drained soil and a water table that, although well below the root zone, is still relatively shallow (2-3.5 m deep, fig. 6B). It should be noted, however, that the ability to attach quantitative physical meaning to WBTF model parameters is limited, because it is a simple model that treats physical processes using a lumped parameter, “black box” approach rather than a rigorous physics-based approach.

Calibration of the model produced simulated recharge rates that matched estimated recharge rates fairly well (fig. 7). The standard error of regression was 3.2 mm/d and r^2 was 0.80; however, some notable discrepancies exist. A better match generally exists during the summer wet season (fig. 7C) than during the relatively dry winter (fig. 7B). This might be the result of nonlinearity caused by a variable moisture content induced by infrequent precipitation during winter, as opposed to a probably more constant moisture content resulting from more frequent precipitation during summer. Soil-moisture data confirm that moisture contents at depths equal to or below the bottom of the root zone vary over a greater range (0.01-0.09 cm^3/cm^3) during the drier part of the year (September 1993 through May 1994) than moisture contents (0.04-0.09 cm^3/cm^3) during the wet season (June 1994 through August 1994) (fig. 8B). The model also does not replicate the sharp peaks in estimated recharge on June 8 and 16 (fig. 7C). These peaks might result from the use of a constant specific yield when applying the water-table fluctuation method. Specific yield is a function of the moisture-characteristic curve, water-table depth, the rate of rise or decline of the water table, and gas bubble entrapment (Duke, 1972; Sophocleous, 1985; Fayer and Hillel, 1986a, b). These factors result in a smaller specific yield than the value traditionally estimated as the difference of saturated and residual moisture contents. In particular, gas bubble entrapment can cause rapid and large water-table rises during recharge (Fayer and Hillel, 1986a, b; Nachabe and others, 2004). Consequently, use of a constant specific yield with the water-table fluctuation method probably overestimates recharge during high recharge.

The effect of the water-balance module of the WBTF model is evident by comparison of precipitation and effective infiltration (fig. 7). From September 1993 through May 1994, little recharge was estimated by using the water-table fluctuation method. During this same time period, the great majority of the small amount of precipitation that occurred did not produce any effective infiltration. As simulated by the water-balance module, water from this precipitation is removed by ET or held in storage in the vegetative canopy or as soil moisture in the root zone. Not until the maximum

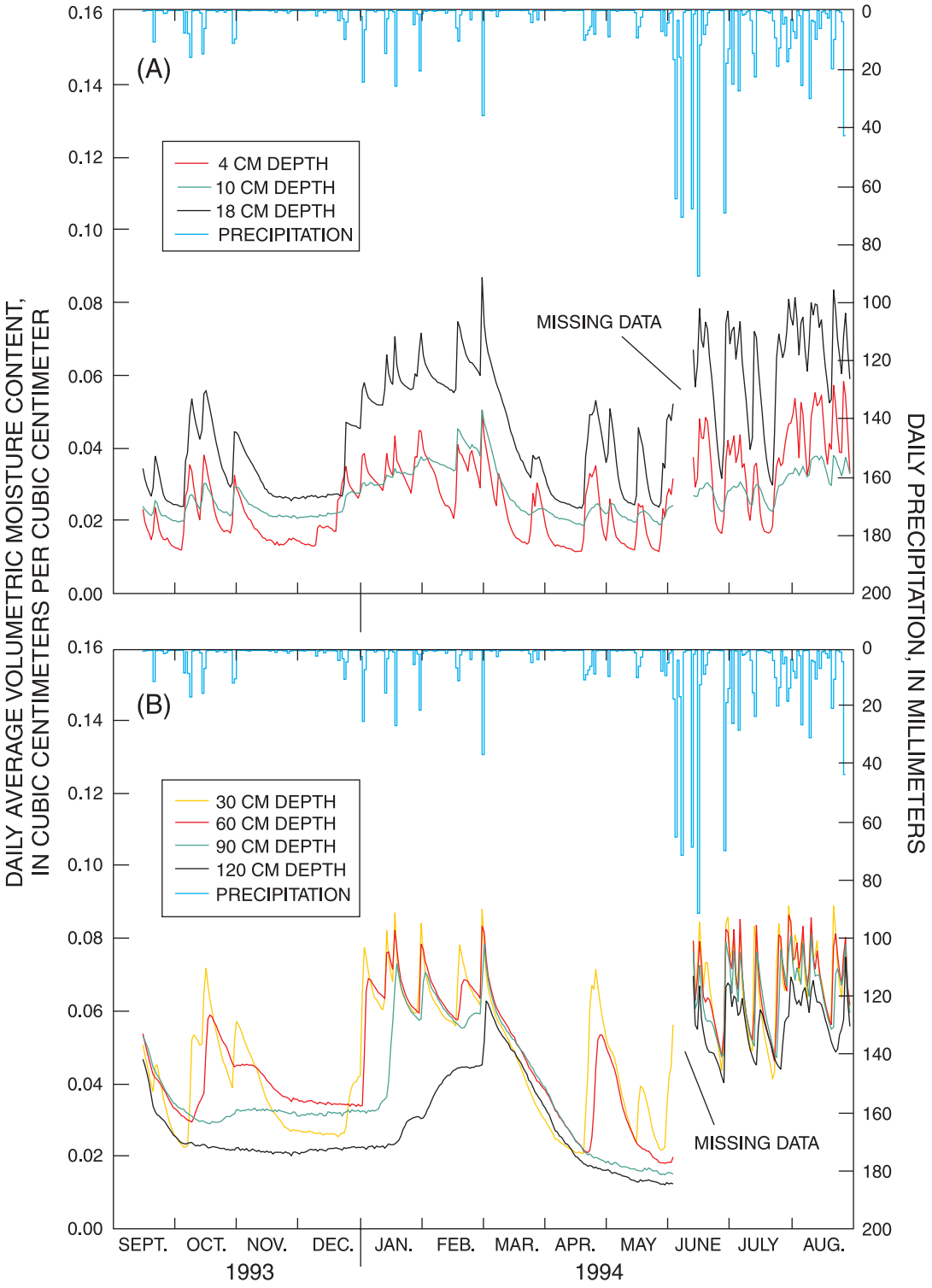

Figure 8. Moisture contents measured by Sumner (1996) using Time Domain Reflectometry at a meteorological station in west Orange County at depths (A) within the estimated 30-cm-thick root zone and (B) in the deeper unsaturated zone equal to or below the root zone.

storage capacity of the vegetative canopy and root zone (S_{max}) is exceeded, does any precipitation result in effective infiltration. For example, the combined precipitation on June 4 and 5 (fig. 7C) exceeds S_{max}, producing a small amount of effective infiltration. Subsequent precipitation on June 7 and 8 is directed exclusively to effective infiltration, with the exception of about 1 mm/d of ET, because storage is at capacity from the precipitation immediately preceding. Simulated and estimated recharge rates match closely during these times. Even though the bucket model employed by the water-balance module is simple, it is quite effective at simulating the major factors influencing effective infiltration.

Evaluation of the Transfer-Function Module by Comparison with VS2DT

Concurrent measurements of precipitation, ET, and water-table response in deep water-table settings that can be used to make comparisons between simulated and field-based estimates of recharge are limited. Comparison of results of the WBTF model with those of a commonly used variably saturated flow model provides further insight into the features and limitations of the WBTF model. In particular, the traveltime of effective infiltration from the bottom of the root zone to the water table is simulated by the USGS variably saturated flow model VS2DT (Lappala and others, 1987; Healy, 1990); the ability of the transfer-function module of the WBTF model to replicate these results is evaluated. The water-balance module was excluded from these simulations by setting storage parameters to zero (S_b and S_{max}, table 1).

Application of the Variably Saturated Flow Model VS2DT

VS2DT is a finite-difference model that simulates variably saturated transient water flow and solute transport in one or two dimensions. Only the water-flow capability was used in the present application. VS2DT solves the Richards (1931) equation describing variably saturated water flow. Fluid flux and storage changes are described in terms of the following nonlinear functional relations: (1) volumetric moisture content as a function of pressure head (commonly called the moisture-characteristic curve), (2) specific moisture capacity (slope of the moisture-characteristic curve), and (3) unsaturated hydraulic conductivity as a function of pressure head. Hydraulic head serves as the dependent variable. Recharge was computed from the output of VS2DT by using simulated values of pressure and elevation heads. Darcy's law was applied across the model nodes located directly above and below the simulated water table (where pressure head is zero). Saturated hydraulic conductivity was used because the vertical discretization was such that the node above the water table was within the capillary fringe, where saturation was 100 percent. Recharge computed in this way was in agreement with the water budget computed by VS2DT.

VS2DT was used to construct a model for each of four hypothetical sites consisting of a one-dimensional vertical column extending from land surface to 0.5 m below the water table; uniform soil properties; and a water-table depth of 2.5, 5, 10, or 20 m below land surface (fig. 9). Because the water-balance module was excluded from these simulations, the root zone was not simulated. The van Genuchten (1980) equations describing the moisture-characteristic curve, specific moisture capacity, and unsaturated hydraulic conductivity of the soil were used. The upper boundary consisted of a specified flux of 100 mm/d for 1 day followed by 19 days of no flow. A specified head equivalent to the water-table depth served as the lower boundary condition. An equilibrium profile (where the pressure head is equal to the negative of the elevation above the water table) was used as the initial condition for each VS2DT simulation. Therefore, initial moisture contents were equivalent to those indicated by the moisture-characteristic curve.

Truncation error resulting from the finite-difference approximation was investigated and appropriate spatial and temporal discretizations were selected. A uniform spatial discretization of 0.1 m was used. A variable temporal discretization was used with an initial timestep of 0.001 day, minimum timestep of 0.0001 day, and maximum timestep of 0.01 day. VS2DT automatically adjusts the timestep so that a user-specified maximum change in pressure head is not exceeded at any node between successive timesteps (Lappala and others, 1987, p. 35). Reducing the spatial discretization to 0.05 m had a negligible effect on the simulated recharge. Recharge rates were very close (standard error of 0.16 mm) to those computed with a 0.1-m discretization; total recharge for the 20-day simulation period was 92.7 mm, only 0.13 percent less than that for the 0.1-m discretization. Likewise, reducing the temporal discretization to a 0.001-day maximum timestep had only a small effect on the simulated recharge. Recharge rates were very close (standard error of 0.20 mm) to those computed with a 0.01-day maximum timestep; total recharge for the 20-day simulation period was 93.4 mm, only 0.60 percent more than that for the 0.01-day maximum timestep.

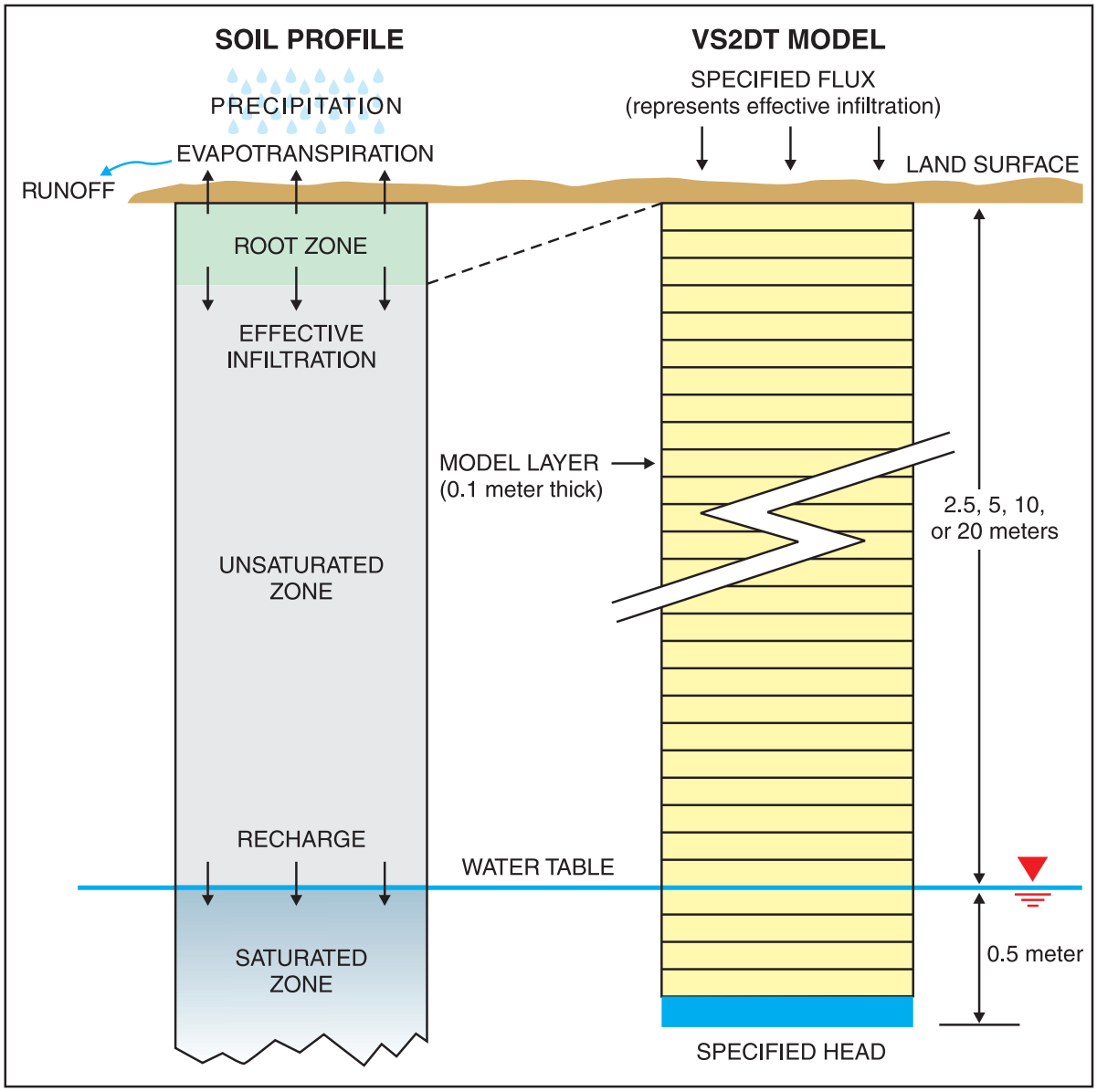

Figure 9. A typical soil profile and the equivalent spatial discretization and boundary conditions used in the one-dimensional VS2DT models.

Comparison of the Transfer-Function Module with VS2DT

Because VS2DT solves the commonly used Richards (1931) equation, the ability of the transfer-function module to replicate VS2DT results would support the applicability of transfer-function models for the simulation of recharge, at least for relatively homogeneous soil and diffuse-flow conditions. UCODE was used to estimate the values of n, τ_i , and k that yielded recharge rates that best matched VS2DT results. Table 2 lists the test problems addressed: (1) recharge resulting from a single infiltration event for three soil types and a single water-table depth (cases FS1, SL1, and ST1); (2) recharge resulting from a single infiltration event for a single soil type and four water-table depths (cases FS1, FS2, FS3, and FS4); and (3) recharge resulting from two infiltration events for a single soil type and water-table depth (case SL2).

A VS2DT model with a 2.5-m deep water table was used to simulate recharge from a 1-day 100-mm infiltration event for three soils: fine sand, sandy loam, and silt loam. Soil hydraulic properties were obtained from Lappala and others (1987, p. 20) using the van Genuchten (1980) parameterization of the moisture-characteristic curve, and are listed in table 2. Based on the gamma PDF parameters estimated by using UCODE, recharge

Table 2. Transfer-function parameters and model-fit statistics from the WBTF model for several soil types, water-table depths, and hypothetical infiltration events.

[n, shape parameter of gamma probability density function characterizing number of linear reservoirs necessary to represent the unsaturated zone; τ_i, initial time lag of gamma probability density function representing delay time between beginning of effective infiltration and first arrival of recharge; k, scale parameter of gamma probability density function characterizing the average delay time ($nk + \tau_i$) imposed on effective infiltration by the unsaturated zone; r^2, coefficient of determination; SE, standard error; K_{sat}, saturated hydraulic conductivity; θ_s, saturated volumetric moisture content; θ_r, residual volumetric moisture content; α, van Genuchten (1980) coefficient, reciprocal of which approximately equals the pressure head at the inflection point of the moisture-characteristic curve (for the range of β values used here); and β, van Genuchten (1980) coefficient characterizing slope of the moisture-characteristic curve]

Soil type	Test case	Number of infiltration events	Water-table depth (meters)	Parameter values			Model-fit statistics		
				n (dimension-less)	τ_i (day)	k (day)	r^2	SE (millimeter per day)	SE as percent of recharge range[a]
Fine sand[b]	FS1	1	2.5	0.393	1.21	6.44	0.95	2.2	4.3
	FS2	1	5	.745	10.3	45.7	.94	.21	4.7
	FS3	1	10	.780	108	473	.89	.035	7.9
	FS4	1	20	.800	960	4500	.85	.0052	9.9
Sandy loam[c]	SL1	1	2.5	.847	1.38	4.09	.99	.48	1.9
	SL2	2	2.5	.963	1.10	2.89	.94	2.6	6.2
Silt loam[d]	ST1	1	2.5	.705	.478	3.14	.98	1.2	2.9

[a]Recharge range is the maximum minus the minimum recharge rate simulated for each test case by the VS2DT model.

[b]Hydraulic parameters used in VS2DT: K_{sat} = 2.1 m/d, θ_s = 0.377, θ_r = 0.072, α = 1.04 m^{-1}, β = 6.9

[c]Hydraulic parameters used in VS2DT: K_{sat} = 0.7 m/d, θ_s = 0.496, θ_r = 0.15, α = 0.847 m^{-1}, β = 4.8

[d]Hydraulic parameters used in VS2DT: K_{sat} = 0.225 m/d, θ_s = 0.43, θ_r = 0.17, α = 0.505 m^{-1}, β = 7.0

simulated by the WBTF model matched the VS2DT simulations well for each soil type (cases FS1, SL1, and ST1, table 2; fig. 10). Standard error is given as a percentage of the recharge range in table 2 to provide a statistic that represents regression error relative to model response. Standard error as a percentage of recharge range is more comparable among models because, unlike standard error alone, it is not dependent on the magnitude of the recharge values. The shape of the recharge curve and, therefore, the values of the gamma PDF parameters are dependent on the soil properties as represented by VS2DT. For example, the smaller τ_i and k parameters for the silt loam compared to the fine sand are a result of the higher moisture content for a given pressure head, which is characteristic for a finer grain soil (fig. 10). The higher moisture content yields a higher unsaturated hydraulic conductivity, causing a shorter traveltime of water through the unsaturated zone and resulting in earlier arrival of the water at the water table as recharge. Because of differences in the moisture-retention characteristics of these soils, an initial condition different than the equilibrium profile used herein (for example, a constant pressure head) would result in different values of recharge simulated by VS2DT.

Four VS2DT models with different water-table depths were used to simulate recharge from a 1-day 100-mm infiltration event for a fine sand soil. As expected, VS2DT simulations show that recharge is lagged and attenuated with increasing water-table depth (fig. 11A). The WBTF model replicated the VS2DT results reasonably well, although error increased with increased water-table depths (cases FS1, FS2, FS3, and FS4, table 2). The best-fit gamma PDF parameters show interesting relations to water-table depth. A power function describes well the relations of τ_i and k to water-table depth, whereas n does not vary greatly with water-table depth (fig. 11B). Although these results are specific to this particular soil type, equilibrium profile initial condition, and infiltration event, the regularity of the parameters provide a reasonable way to extend the model to other water-table depths.

The test problems discussed above indicate that the transfer-function module of the WBTF model replicates results of a physics-based variably saturated flow model reasonably well. Wu and others (1997) also reported good results with a similar comparison of a transfer-function model with a variably saturated flow model based on the Richards (1931) equation. The linearity assumption of a transfer-function model, however, limits its applicability for the simulation of recharge, as discussed in more detail in the following section.

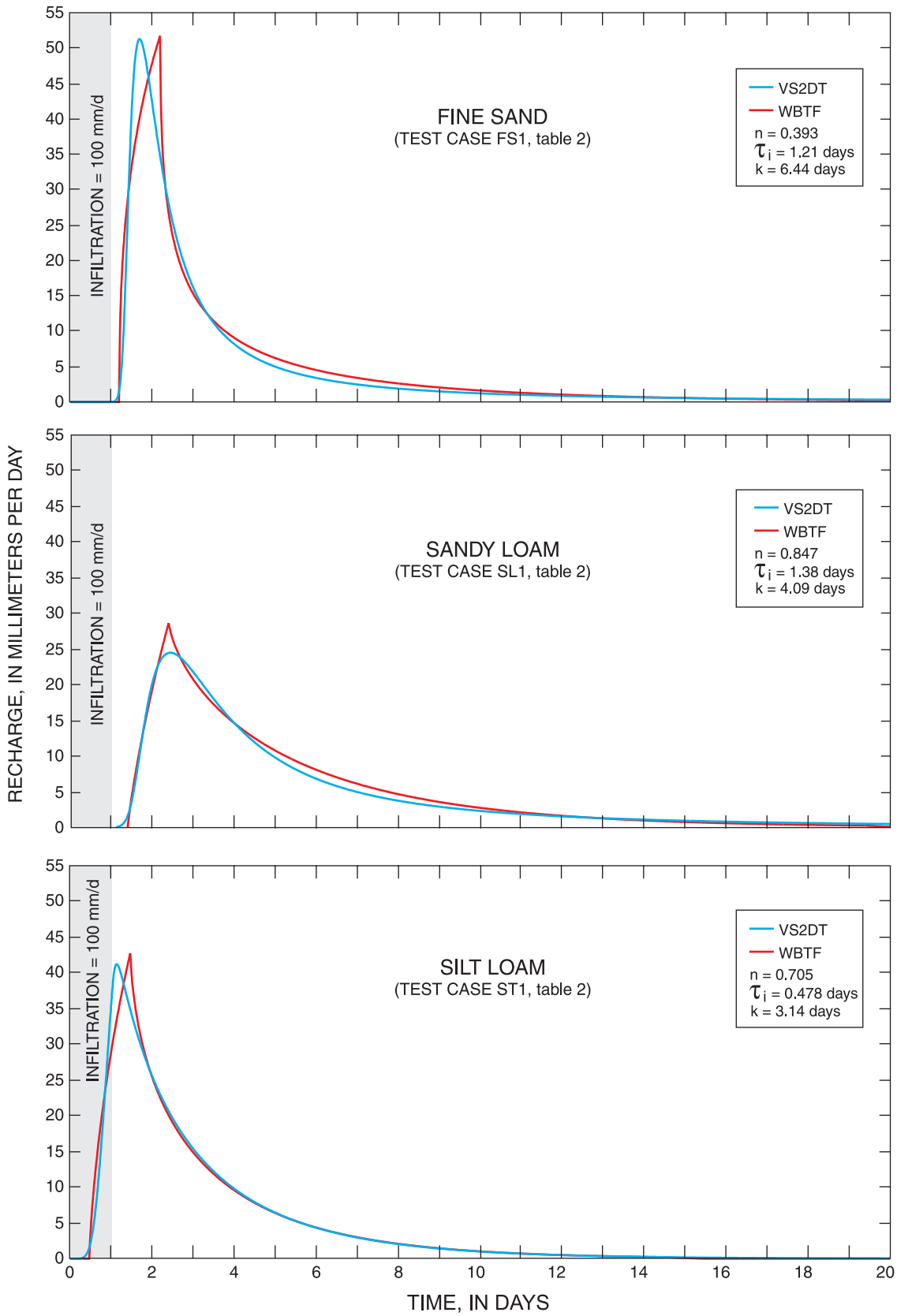

Figure 10. Comparison of ground-water recharge simulated by using the VS2DT and WBTF models for three different soil types, 2.5-meter-deep water table, and a single infiltration event.

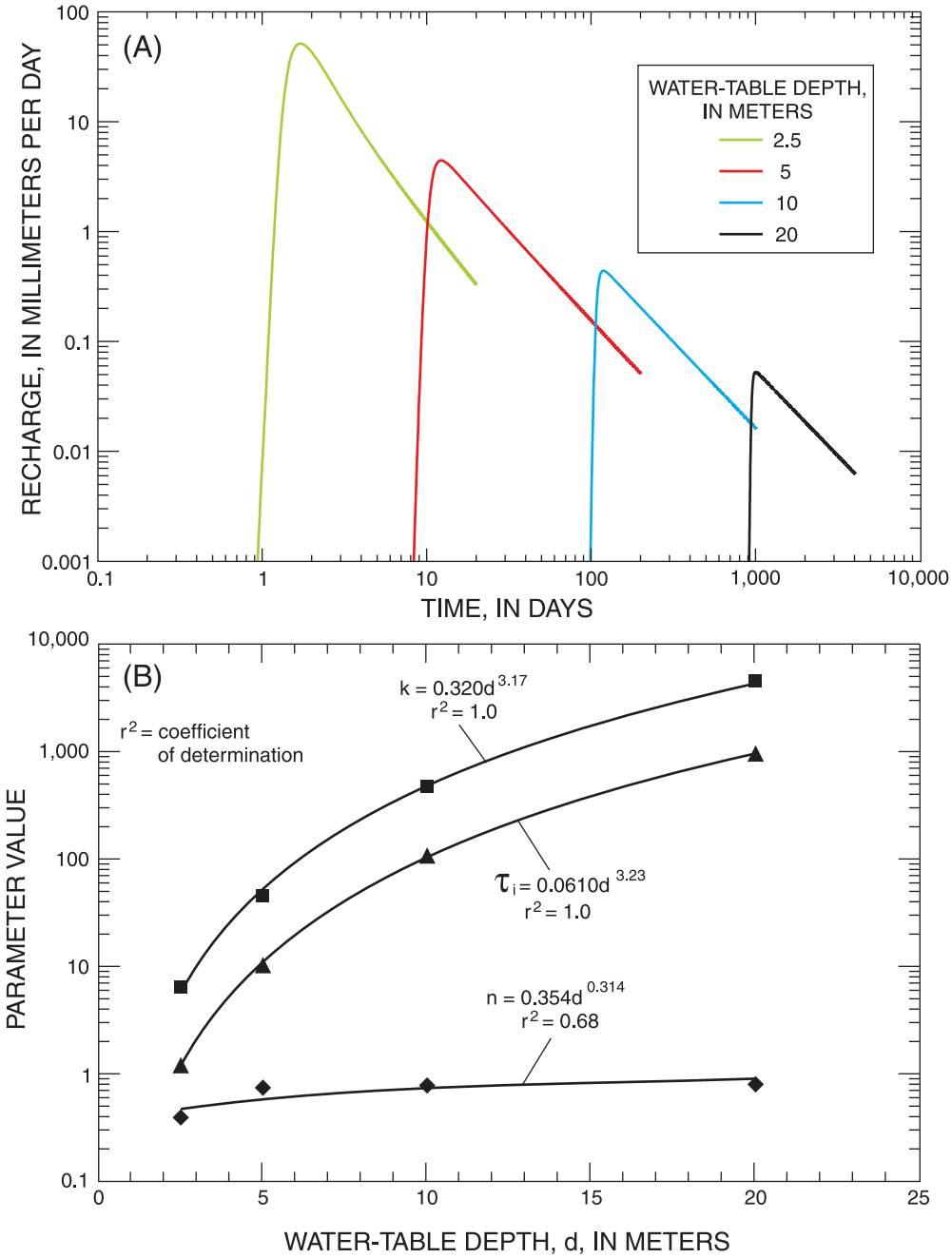

Figure 11. (A) Ground-water recharge simulated by using the VS2DT model for a fine sand soil, four water-table depths, and a single infiltration event (100 millimeters from times 0 to 1 day); and (B) corresponding best-fit transfer-function parameter values from the WBTF model for each water-table depth (see table 2 for WBTF model-fit statistics for test cases FS1, FS2, FS3, and FS4).

Linearity Assumption

The transfer-function module of the WBTF model assumes that the movement of water in the unsaturated zone is a linear process. A linear process is one in which (1) the system response is linearly proportional to the system input, and (2) individual system responses resulting from individual inputs can be summed using the principle of superposition to obtain total system response. Consider two effective infiltration events, $i_{e,1}$ and $i_{e,2}$; the total recharge resulting from these events is $r(i_{e,1}) + r(i_{e,2})$, illustrating the principle of superposition. Next, consider that the first recharge event increases to $ai_{e,1}$ and the second event to $bi_{e,2}$; the total recharge resulting from these events is $ar(i_{e,1}) + br(i_{e,2})$, illustrating both the linear proportionality of system input and response and the principle of superposition.

The Richards (1931) equation is nonlinear as a result of the dependence of unsaturated hydraulic conductivity on pressure head, which in turn is related to moisture content as described by the moisture-characteristic curve. Multiple infiltration events can cause temporal variations in moisture content that, as a result of the dependence of unsaturated hydraulic conductivity on moisture content, violate the linearity assumption of a transfer-function model. To demonstrate this point, a VS2DT model with a 2.5-m-deep water table was used to simulate recharge from two 1-day 100-mm infiltration events (separated by 3 days) for sandy loam soil (case SL2, table 2). Temporal variations in antecedent moisture content result in recharge from the second infiltration event peaking at a higher value and decreasing more quickly than recharge from the first event (fig. 12). This effect is caused by the second event starting before recharge from the first event ends and, accordingly, moisture content is greater during the second event than during the first event. Because of the different moisture contents, the velocity of the wetting front for each event is different. As a result, a single set of gamma PDF parameters cannot fit recharge rates resulting from both infiltration events as well as it can fit recharge rates from a single infiltration event (compare cases SL1 and SL2 in table 2 and figures 10 and 12, respectively).

Another effect of the nonlinearity of unsaturated flow is that recharge is dependent not only on the timing of multiple infiltration events, but also on the magnitude of an infiltration event. For example, a VS2DT simulation of a 1-day 50-mm infiltration event for a fine sand soil with a 2.5-m-deep water table indicates that the peak recharge rate is about 10 mm/d, occurring at 3.1 days; in contrast, for a 100-mm infiltration event, the peak recharge rate is

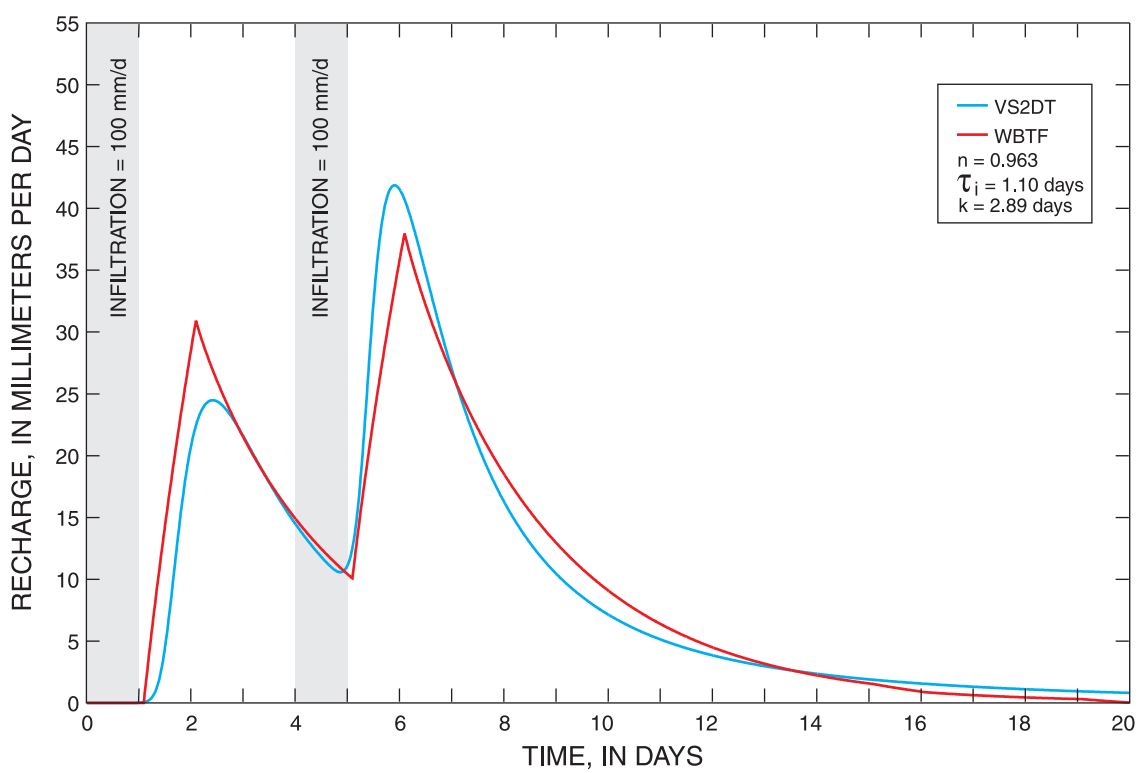

Figure 12. Ground-water recharge simulated by using the WBTF and VS2DT models for a sandy loam soil, 2.5-meter-deep water table, and two infiltration events (see table 2 for WBTF model-fit statistics for test case SL2).

about 51 mm/d, occurring at 1.7 days (fig. 10). Again, these differences result from an increase in unsaturated hydraulic conductivity, which is caused by the greater moisture contents in the soil profile induced by the larger infiltration event. Although appropriate values of gamma PDF parameters can match each set of recharge rates well, a single set of gamma PDF parameters does not exist that provides as good a match to both sets of recharge rates.

The linearity assumption of transfer-function models was problematic for Gehrels and others (1994) when using a similar model to relate water-table fluctuations to precipitation excess (equivalent to effective infiltration for the WBTF model). Many aspects of the hydrologic setting in the study area—humid climate; unconfined aquifer consisting of sandy, highly permeable sediments; and water-table depths varying from near land surface to 40 m (Gehrels and others, 1994, p. 111)—were similar to those of central Florida. The ability of the transfer-function model to replicate observed water-table fluctuations decreased noticeably for water tables deeper than about 15 m. Gehrels and others (1994, p. 130) attributed this to nonlinear processes in the unsaturated zone (temporal redistribution of soil moisture) that become increasingly important with thicker unsaturated zones. This reason likely explains the decreasing accuracy of the transfer-function module for increasing water-table depths (compare the model-fit statistics for cases FS1, FS2, FS3, and FS4 listed in table 2). Temporal variations in moisture content, whether due to redistribution of soil moisture already in the unsaturated zone or to variations in the timing and magnitude of effective infiltration (as described earlier), cause temporal variations in percolation velocity, violating the linearity assumption of transfer-function models.

In an effort to approximately address the problem of nonlinearity, Morel-Seytoux (1984, p. 1234) suggested interpreting the percolation of water through the unsaturated zone as a nonstationary linear process. By selecting time periods during which effective infiltration is approximately constant, the entire process is considered a series of linear processes. In this case, a different set of transfer-function parameter values are identified for each time period. Therefore, temporal variations in recharge caused by temporal variations in effective infiltration could be better simulated. The WBTF model is restricted to a single set of transfer function (gamma PDF) parameter values to maintain model simplicity, although at the cost of some degree of model accuracy.

Hypothetical Sites

The test problems discussed in the preceding section indicate that the transfer-function module of the WBTF model can replicate results of a physics-based variably saturated flow model reasonably well. These hypothetical problems, however, are not necessarily representative of typical field conditions in central Florida. In addition, for field-based comparisons of recharge, data from only a single site were available. Using synthesized data, hypothetical sites are developed that enable a variety of hydrologic conditions to be simulated. Application of the WBTF model is further demonstrated for eight hypothetical field sites by using data sets for precipitation, ET, and combinations of two soil types and four water-table depths typical for deep water-table settings in central Florida. These hypothetical sites help clarify the effects of the linearity assumption of a transfer-function model on the accuracy of recharge rates computed by the WBTF model. Results from the WBTF model are compared with those from a VS2DT model for each site.

Synthesized values of precipitation, ET, and soil properties were based on field data collected in central Florida, which were obtained from different locations or time periods. Daily precipitation for a 1-year period (annual precipitation totaled 1,310 mm) was obtained from data reported by O'Reilly (1998, p. 14) for 1995 from rain gages in west Orange and southeast Lake Counties. Daily ET for a 1-year period (annual ET totaled 680 mm) was based on daily ET data from the west Orange County site (fig. 6A; Sumner, 1996, p. 31). To generalize the data and make it less site specific, a sine function was fit to the ET data by using UCODE. The sine function provides daily values of ET as a function of Julian day, and replicates the seasonal variation in ET reasonably well (standard error of 0.47 mm/d and r^2 of 0.80). Hydraulic properties for two soil types—sand and loamy sand—were based on laboratory measurements of moisture-characteristic curves and saturated hydraulic conductivity (J.A. Tindall, U.S. Geological Survey, written commun., 2001) from soil cores collected during this study at the site of the former Tiger Bay ET station (fig. 1) (Sumner, 2001). The van Genuchten (1980) parameterization of the moisture-characteristic curve was fit to the laboratory data for sand by using UCODE (standard error of 0.008 cm^3/cm^3 and r^2 of 1.0). The Brooks and Corey (1964) parameterization of the moisture-characteristic curve was fit to the laboratory data for sandy loam by using UCODE (standard error of 0.003 cm^3/cm^3 and r^2 of 1.0). The van Genuchten (1980) and Brooks and Corey (1964) parameter values are listed in table 3.

Table 3. Transfer-function parameters and model-fit statistics from the WBTF model for sand and loamy sand soils and four water-table depths.

[WBTF and VS2DT model simulations were based on precipitation and evapotranspiration rates that are typical for central Florida; *n*, shape parameter of gamma probability density function characterizing number of linear reservoirs necessary to represent the unsaturated zone; τ_i, initial time lag of gamma probability density function representing delay time between beginning of effective infiltration and first arrival of recharge; *k*, scale parameter of gamma probability density function characterizing the average delay time ($nk + \tau_i$) imposed on effective infiltration by the unsaturated zone; r^2, coefficient of determination; SE, standard error; K_{sat}, saturated hydraulic conductivity; θ_s, saturated volumetric moisture content; θ_r, residual volumetric moisture content; α, van Genuchten (1980) parameter, reciprocal of which approximately equals the pressure head at the inflection point of the moisture-characteristic curve (for the range of β values used here); β, van Genuchten (1980) pore-size distribution parameter characterizing slope of the moisture-characteristic curve; h_b, bubbling pressure head, which is the pressure head at which the air phase first becomes continuous (Brooks and Corey, 1964); and λ, Brooks and Corey (1964) pore-size distribution parameter characterizing slope of the moisture-characteristic curve]

Soil type	Test case	Water-table depth (meter)	Parameter values			Model-fit statistics		
			n (dimension-less)	τ_i (day)	k (day)	r^2	SE (millimeter per day)	SE as percent of recharge range[a]
Sand[b]	S1	2.5	0.759	1.88	4.65	0.85	1.6	5.2
	S2	5	.588	8.99	28.0	.80	1.3	7.5
	S3	10	.771	27.4	51.9	.74	1.0	11
	S4	20	.657	87.2	152	.83	.48	8.3
Loamy sand[c]	LS1	2.5	.877	2.54	7.12	.85	1.4	5.9
	LS2	5	.818	11.0	28.8	.82	1.0	8.7
	LS3	10	.689	51.9	87.9	.73	.79	10
	LS4	20	.867	138	170	.90	.29	11

[a]Recharge range is the maximum minus the minimum recharge rate simulated for each test case by the VS2DT model.

[b]Hydraulic parameters used in VS2DT: K_{sat} = 2.2 m/d, θ_s = 0.375, θ_r = 0.0379, α = 1.62 m^{-1}, β = 3.81

[c]Hydraulic parameters used in VS2DT: K_{sat} = 0.40 m/d, θ_s = 0.397, θ_r = 0.100, h_b = -0.354 m, λ = 0.870

VS2DT was used to construct a model for each of eight hypothetical sites consisting of a one-dimensional vertical column extending from land surface to 0.5 m below the water table; uniform soil properties; and a water-table depth of 2.5, 5, 10, or 20 m below land surface. The VS2DT models were identical to that depicted in figure 9, except that the effects of water storage in the root zone were incorporated into the upper boundary condition. A specified flux represented the upper boundary, which consisted of the daily effective infiltration computed by the water-balance module of the WBTF model. The synthesized values of precipitation and ET described above, in addition to an S_b value of 30 mm and an S_{max} value of 50 mm, were used in the water-balance module. An S_{max} value of 50 mm is equivalent to a field capacity of 0.12 cm^3/cm^3 based on equation 4, and a root zone depth of 500 mm, a permanent wilting point of 0.02 cm^3/cm^3, and a canopy storage of 0. Applying the upper boundary condition in this manner resulted in a realistic, albeit hypothetical, time series of effective infiltration. A specified head equal to the water-table depth served as the lower boundary condition. An equilibrium profile (where the pressure head is equal to the negative of the elevation above the water table) was used as the initial condition for each VS2DT simulation. In the field, however, the entire soil profile probably rarely drains to hydrostatic equilibrium. To avoid the effects of this unrealistic initial condition, a simulation period of at least 1 year was executed before comparisons were made between recharge rates simulated by the VS2DT and WBTF models. Because simulations extended over multiple years, synthesized values of precipitation and ET were repeated for successive years. A unit-event length (Δt_u, eq. 7) of 0.1 day, which is much less than the length of recharge events, was used for the WBTF model. Recharge rates from the WBTF model were linearly interpolated to obtain values for the same times as those simulated by VS2DT. UCODE was used to estimate the best-fit gamma PDF parameters.

Recharge simulated by the WBTF model matched the VS2DT simulations reasonably well for each water-table depth simulated for each soil type (table 3, figs. 13-16).

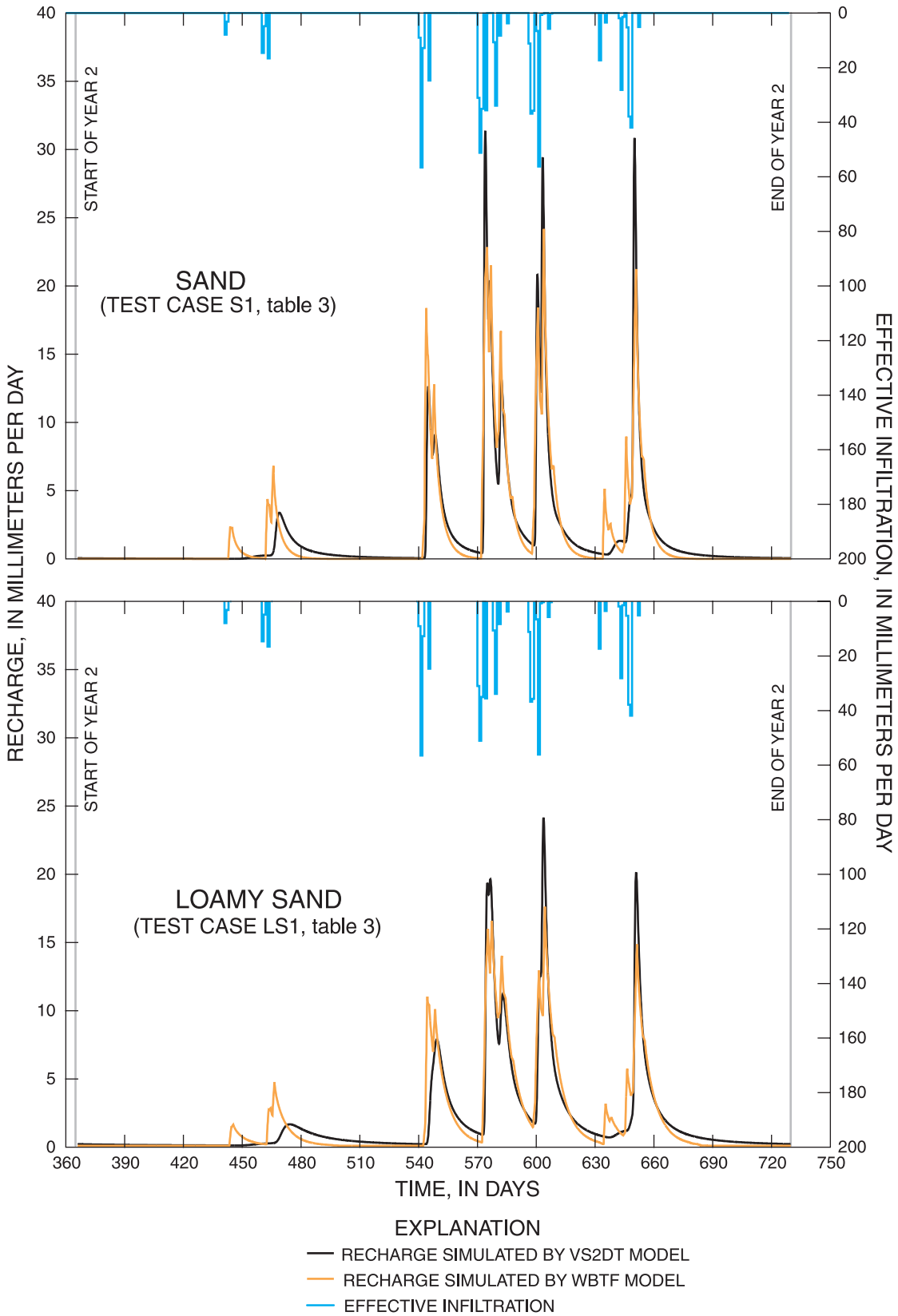

Figure 13. Comparison of ground-water recharge simulated by using the VS2DT and WBTF models for sand and loamy sand soils, 2.5-meter-deep water table, and precipitation and evapotranspiration typical for central Florida.

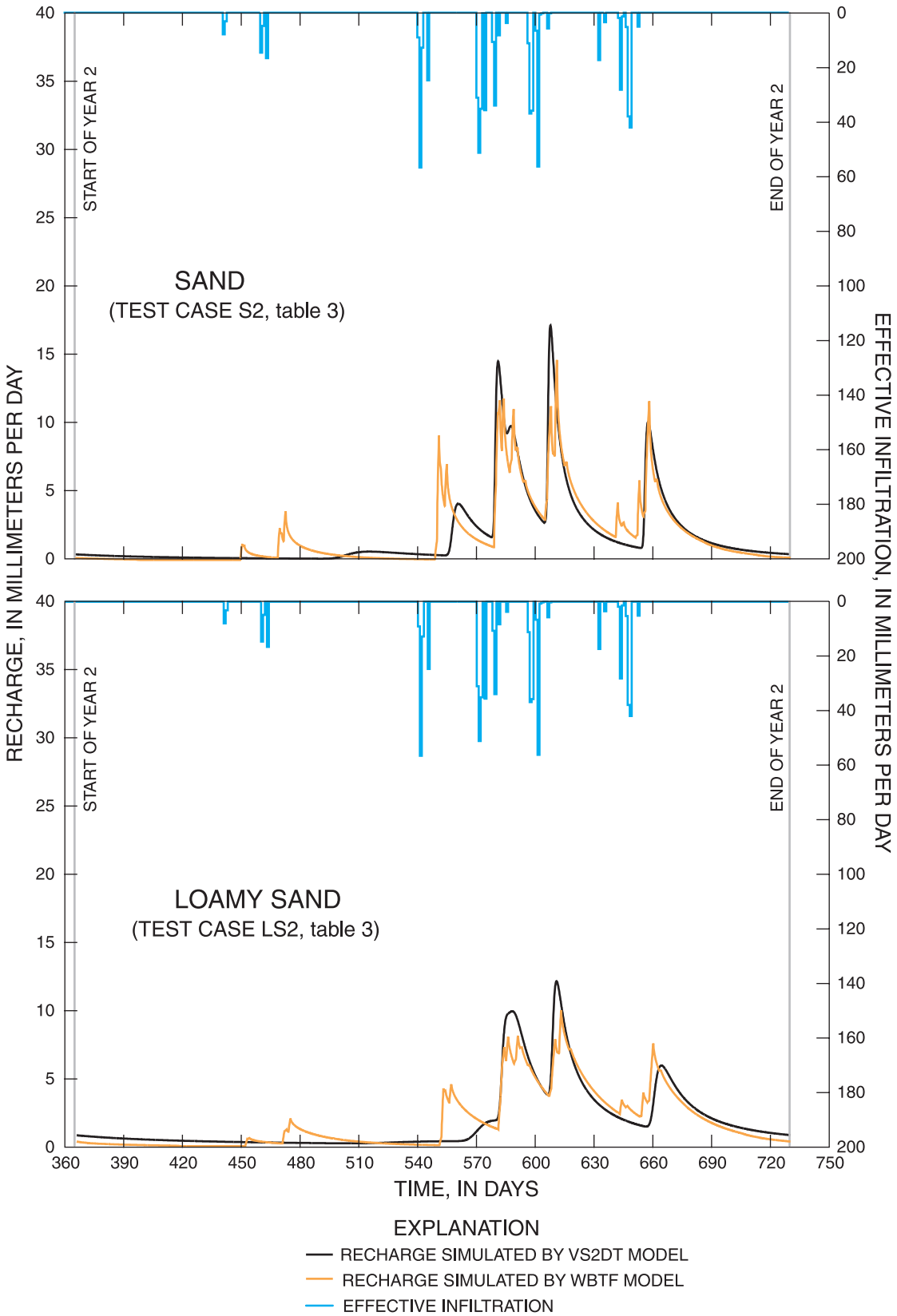

Figure 14. Comparison of ground-water recharge simulated by using the VS2DT and WBTF models for sand and loamy sand soils, 5-meter-deep water table, and precipitation and evapotranspiration typical for central Florida.

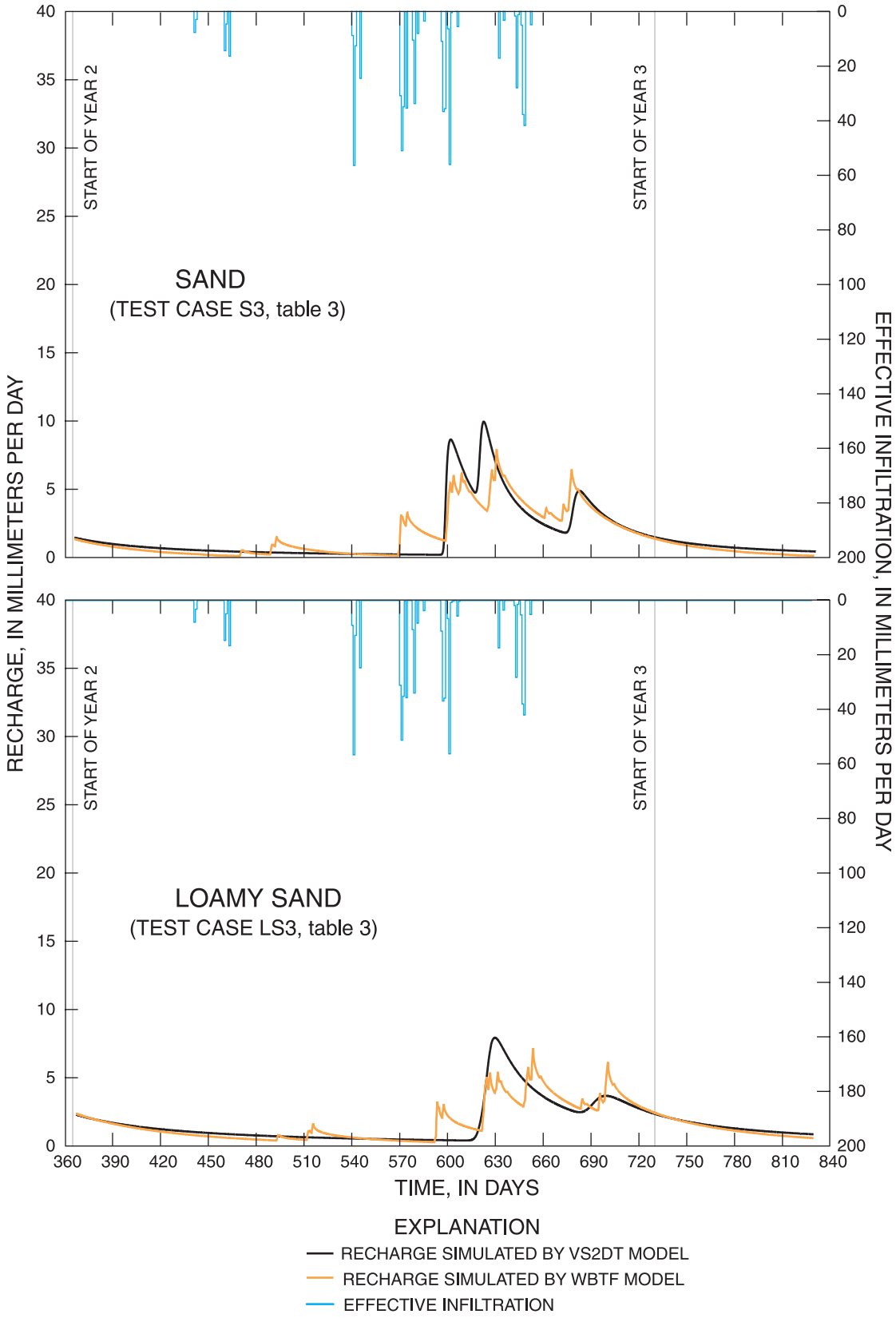

Figure 15. Comparison of ground-water recharge simulated by using the VS2DT and WBTF models for sand and loamy sand soils, 10-meter-deep water table, and precipitation and evapotranspiration typical for central Florida.

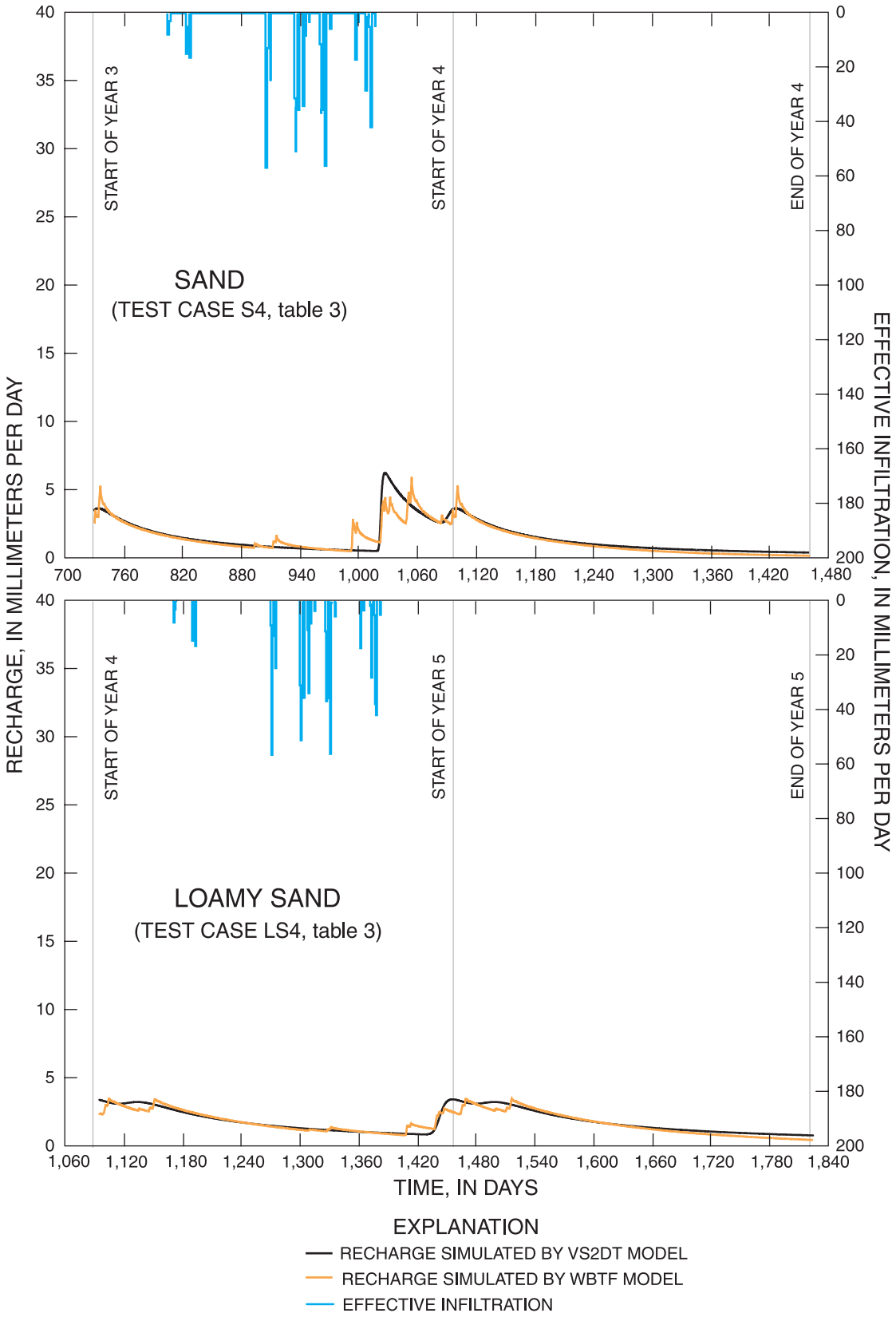

Figure 16. Comparison of ground-water recharge simulated by using the VS2DT and WBTF models for sand and loamy sand soils, 20-meter-deep water table, and precipitation and evapotranspiration typical for central Florida.

A better match generally exists during the wetter periods, when effective infiltration events are of greater magnitude and more frequent, than during drier periods, when effective infiltration events are relatively isolated. In addition, for the shallow water-table depths (2.5 and 5 m), the WBTF model generally matched the VS2DT results better than for deep water-table depths (10 and 20 m). These discrepancies may be the result of nonlinearity caused by a variable moisture content. System memory, which is represented by the memory of the best-fit transfer function (τ_{mem}, eq. 14), and the timing of effective infiltration events largely determine the magnitude of the nonlinear effect. For example, nonlinear effects generally are small during wetter periods for a sand soil with a 2.5-m-deep water table, τ_{mem} of 33 days, and effective infiltration events that generally occur in clusters separated by 10-25 days (540-660 days, see fig. 13). Physically, this indicates that previous infiltration events have nearly completely percolated through the unsaturated zone before the next set of infiltration events occurs, leading to similar profiles of soil moisture at the start of each set of infiltration events. In contrast, for the same sand soil with a 10-m-deep water table, nonlinear effects are evident. For example, note the poor match during the time period from 570 to 600 days (fig. 15). For the 10-m-deep water table, system memory (τ_{mem} = 245 days) is much longer than the time between infiltration events. These conditions indicate that previous infiltration events are percolating through the unsaturated zone when the next set of infiltration events occurs, leading to dissimilar profiles of soil moisture at the start of each set of infiltration events. As described previously for a simpler test case (fig. 12), a single set of gamma PDF parameters cannot account for nonlinear effects resulting from differences in antecedent moisture content. Accordingly, the resulting parameter values in table 3 represent the best fit for the variety of soil-moisture conditions simulated for the hypothetical field sites.

Effective saturation profiles simulated by VS2DT demonstrate the effect of water-table depth and soil type on the variability of soil moisture (fig. 17). Effective saturation represents a normalized moisture content and is equal to the difference between a given moisture content and residual moisture content divided by the difference between saturated moisture content and residual moisture content. In this context, residual moisture content is equal to θ_r in the van Genuchten (1980) and Brooks and Corey (1964) parameterizations of the moisture-characteristic curve (table 3). The profiles of effective saturation at 602 and 632 days show that sand soil with a 2.5-m-deep water table drained to nearly the initial condition in 30 days, which was the effective saturation at hydrostatic equilibrium (fig. 17B), whereas, the same soil with a 10-m-deep water table has an effective saturation profile at 632 days that is significantly wetter than that for the initial condition (fig. 17D). In addition, a greater amount of infiltration and a longer period of time are required to rewet the thicker soil profile with the 10-m-deep water table, compared to the 2.5-m-deep water table, from relatively dry conditions at 540 days to the wetter soil profile at 602 days. A total of 352 mm of recharge occurred during this time period for the 2.5-m-deep water table (case S1, fig. 13), whereas only 76 mm of recharge occurred for the 10-m-deep water table (case S3, fig. 15)—the difference exists as soil moisture in the 10-m-thick unsaturated zone (fig. 17D). Similar relations between water-table depth and the variability of soil moisture exist for the loamy sand soil (figs. 17C and 17E), but are more pronounced as a result of the different hydraulic properties of this finer-textured soil.

The concept of system memory also is demonstrated by the simulated effective saturation profiles. For example, for the sand soil with a 10-m-deep water table, effective saturation values below depths of about 7.5 m at 540 and 586 days are about equal, indicating that the percolating infiltration has not reached the water table and that soil-moisture conditions below 7.5 m "remember" effective infiltration events that occurred more than 46 days ago (fig. 17D). Likewise, for the loamy sand soil, the effective saturation profile at 602 days indicates that soil-moisture conditions below about 7.2 m "remember" effective infiltration events that occurred more than 62 days ago (fig. 17E). Because water percolating through the unsaturated zone is not a linear process, the memory of an unsaturated flow system is not constant. Variations in moisture content in the soil profile affect the traveltime of percolating water by changing the unsaturated hydraulic conductivity and the volume of unsaturated pore space available for water storage. In summary, the memory of an unsaturated flow system (relative to recharge) is longer when infiltrating water remains in the unsaturated zone for longer periods of time before reaching the water table because of a drier soil profile, lower unsaturated hydraulic conductivity, or deeper water table. It should be noted, however, that the memory of recharge simulated by the WBTF model (τ_{mem}, eq. 14) is the memory of the best-fit gamma PDF for the period of record simulated, which ignores the nonlinear effects of variations in moisture content.

The memory of an unsaturated flow system determines the degree to which the unsaturated zone serves as a low-pass filter, transforming the effects of surface meteorological processes into the subsurface expression of

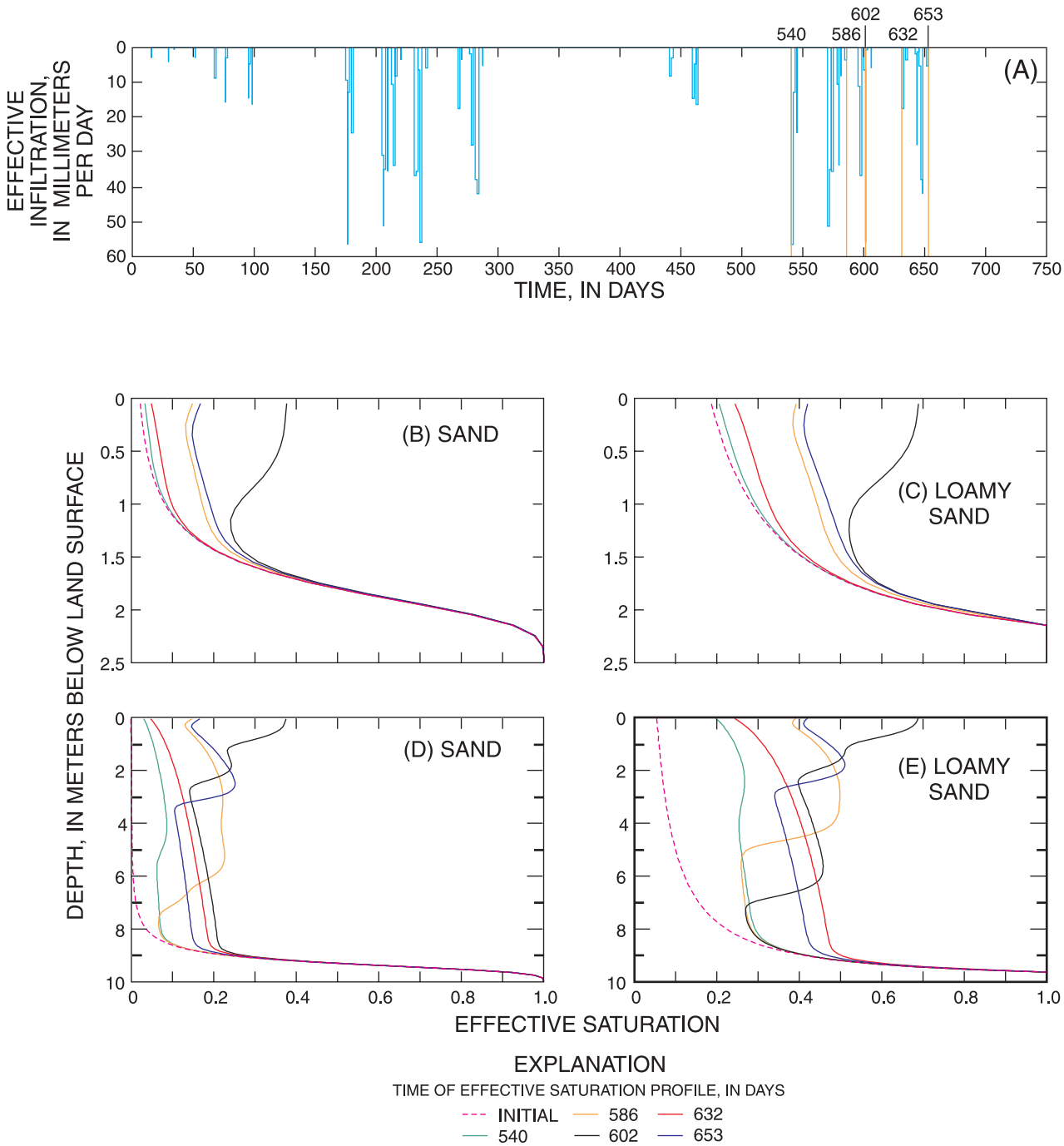

Figure 17. (A) Effective infiltration used as upper boundary condition for VS2DT model; and effective saturation profiles at indicated times simulated by using VS2DT for (B) sand with a 2.5-meter-deep water table, (C) loamy sand with a 2.5-meter-deep water table, (D) sand with a 10-meter-deep water table, and (E) loamy sand with a 10-meter-deep water table.

these processes as recharge at the water table. A low-pass filter is a mathematical tool that, when applied to a time series, preserves the low-frequency variations of the time series and removes the high-frequency variations, yielding a smoother version of the time series (Shumway, 1988, p. 87). An unsaturated flow system with a longer memory will filter higher frequency events and pass only low-frequency trends that are manifested as a relatively smoothly varying time series of recharge. The VS2DT simulations performed for the eight hypothetical field sites show that a sand soil with a relatively shallow water table (fig. 13) has a shorter memory and is a less effective low-pass filter; whereas a loamy sand with a relatively deep water table (fig. 16) has a longer memory and is a more effective low-pass filter. An unsaturated flow system with a very deep water table would be a very effective low-pass filter, yielding recharge that is essentially constant and representative of its long-term average value. Although, the water-table depth at which recharge is approximately constant is also dependent on soil-moisture conditions and unsaturated hydraulic conductivity. The frequency filtering characteristics of an unsaturated flow system can help identify the meteorological time scales of importance to ground-water recharge. These filtering characteristics will determine whether the temporal variability of recharge is dominated by daily, weekly, monthly, seasonal, or longer periods of variations in precipitation and ET.

Suggestions for Use of the Water-Balance/Transfer-Function Model

Although the WBTF model can be used alone to simulate transient ground-water recharge in deep water-table settings as demonstrated above, it is expected that the WBTF model commonly will be used to compute recharge as input to a separate hydrologic model, such as a ground-water flow model. Accordingly, some suggestions and guidelines for using the WBTF model are provided.

Relative Importance of Precipitation, Surface Runoff, and Evapotranspiration

Input for the WBTF model includes time series of precipitation minus surface runoff and ET for the period of interest. The lack of precipitation, surface runoff, or ET data at a regional scale can make it difficult to apply the WBTF model to a regional hydrologic model. Precipitation data can be obtained from regional networks of rain gages; alternatively, regional estimates of precipitation from WSR-88D radar (NEXRAD) operated by the National Weather Service have been used for hydrologic modeling (Bedient and others, 2000; Carpenter and others, 2001). In some areas of central Florida, it is reasonable to assume that surface runoff is negligible (O'Reilly, 1998, p. 41; Knowles and others, 2002, p. 7); otherwise, existing methods can be used to estimate surface runoff, such as the method used by McGurk and Presley (2002, p. 81) in east-central Florida, which is similar to the curve number method (U.S. Department of Agriculture, 1986). ET data are difficult to obtain because field measurements of ET and processing of data are time consuming and expensive, existing ET measurements are relatively sparse, and regionalization of these measurements is not readily available.

It is instructive to investigate the relative importance of precipitation, surface runoff, and ET to the estimation of recharge in order to ascertain the appropriate effort to expend in obtaining estimates of each variable. To this end, a critical observation is that the temporal variability of precipitation is considerably greater than that of ET over a wide range of timescales in central Florida (from daily to annual) (D.M. Sumner, U.S. Geological Survey, oral commun., 2003). As a result, the variability in precipitation dominates the variability in recharge at the site in west Orange County (fig. 6). In addition, surface runoff generally is much less than either precipitation or ET in deep water-table settings in central Florida. These observations indicate that a lack of detailed data for ET or surface runoff does not preclude use of the WBTF model. Sensitivity analyses using the WBTF model can help further determine the importance of surface runoff and ET for the particular hydrologic conditions of interest. For example, the WBTF model was applied at the west Orange County site using estimated, rather than observed, values of ET. Using the sine function approximation of ET discussed earlier, recharge computed by the WBTF model closely resembled that computed by using observed ET. A linear regression between the two simulated time series of recharge yielded a standard error of 0.55 mm/d and r^2 of 0.99, indicating that the model was relatively insensitive to temporal variations in ET.

Model Calibration

When using the WBTF model to compute recharge for input into a separate hydrologic model, such as a ground-water flow model, the WBTF model should be implemented in an iterative fashion during calibration of the hydrologic model. This is necessary because recharge depends not only on soil properties and water-table depth,

but also on the magnitude and timing of effective infiltration. The magnitude and timing of effective infiltration is problem dependent, and the best-fit WBTF parameter values cannot be known beforehand. To the extent that the hydrologic model is sensitive to recharge, the WBTF model parameters should be adjusted to match the observations (for example, ground-water levels and flows) used to calibrate the hydrologic model. Besbes and de Marsily (1984) applied a transfer-function model in such a manner. Estimates of recharge were computed by using the transfer-function model and were input to a regional ground-water flow model; values of the gamma PDF parameters n and k were adjusted to obtain a reasonable match between simulated and measured ground-water levels. The Besbes and de Marsily (1984) model did not incorporate a τ_i parameter. Alternatively, field-based estimates of recharge, if available (for example, by application of the water-table fluctuation method), can be used to estimate WBTF model parameters by inverse procedures.

Although WBTF model parameter values generally must be obtained by model calibration, some guidelines are provided for selection of reasonable parameter values. Equation 4 can be used to calculate reasonable parameter values for the water-balance module (S_b and S_{max}) as described previously. Results from the eight hypothetical field sites (table 3, figs. 13-16) can provide guidance for selecting parameter values for the transfer-function module. A plot of best-fit values for the gamma PDF parameters as a function of water-table depth indicates that a power function approximates this relation reasonably well for the τ_i and k parameters (fig. 18). Similar power-function relations exist for sand and loamy sand soils, both of which are common in central Florida. The n parameter shows no trend with water-table depth for either the sand or loamy sand soil (fig. 18). Interestingly, Besbes and de Marsily (1984, p. 284) reported a linear relation between n and water-table depth. For comparison, the WBTF model for the sand soil was recalibrated after excluding τ_i from the gamma PDF (that is, setting τ_i equal to 0, eq. 10); no trend with water-table depth was apparent in the n parameter best-fit values. Differences between the hydrologic settings (for example, soil properties or effective infiltration) are likely causes of the different results between the Besbes and de Marsily (1984) model and the WBTF model. Similar to the WBTF model, Wu and others (1997, p. 128) included a τ_i parameter and reported relatively constant values of n for water-table depths of 1.5-4.5 m. In summary, WBTF model parameter values initially can be based on equation 4 for the appropriate soil-moisture conditions and root-zone depth and on the parameter values shown in figure 18 for the appropriate soil type and water-table depth; parameter values can be adjusted during model calibration.

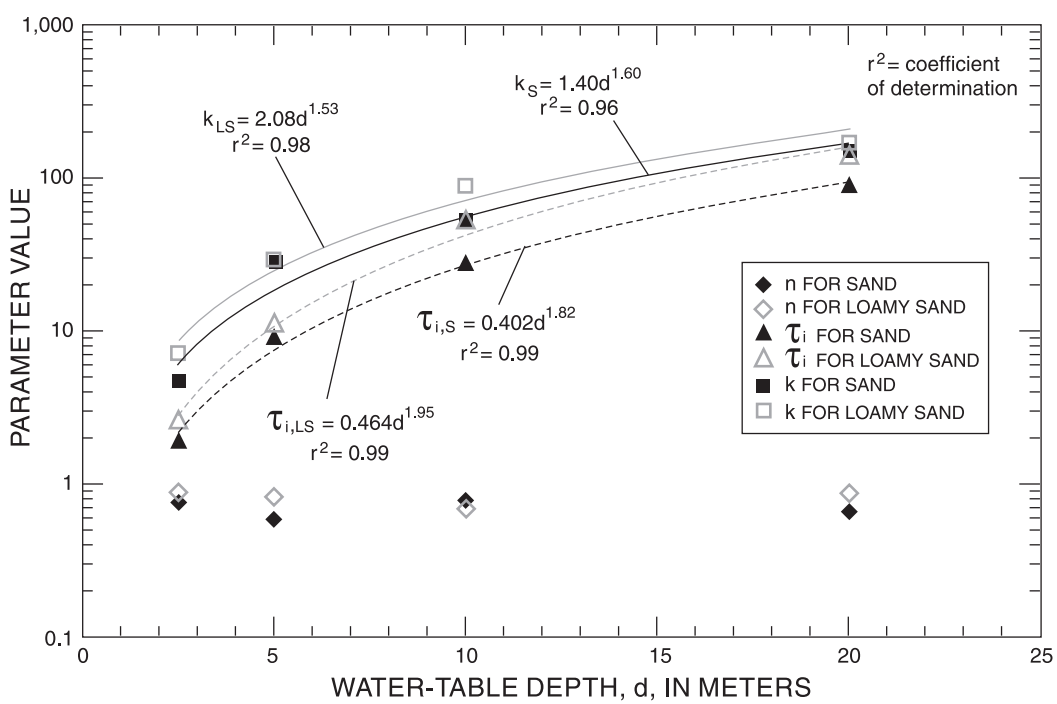

Figure 18. Best-fit transfer-function parameter values obtained by fitting ground-water recharge rates simulated by using the WBTF model to those simulated by using VS2DT for sand (S) and loamy sand (LS) soils, four water-table depths, and precipitation and evapotranspiration typical for central Florida.

Model Assumptions and Limitations

Many of the assumptions and limitations in the development and use of the WBTF model were discussed in previous sections. Following is a summary of major considerations. The order does not indicate importance in model development or application.

- The WBTF model is intended for simulation of transient ground-water recharge in deep water-table settings. A deep water-table setting is an area where the water table is below the reach of plant roots and, therefore, too deep for ET to extract water from the saturated zone.
- The WBTF model should not be used for the simulation of recharge involving surface flooding, such as rapid-infiltration basins or ponded infiltration, without appropriate testing to ensure its applicability for such conditions.
- Surface runoff is assumed to be negligible or adequately estimated and subtracted from precipitation by using methods separate from the WBTF model.
- Little water is assumed to be held in detention storage in land-surface depressions. That is, any water that is not lost to surface runoff is assumed to rapidly evaporate or infiltrate after precipitation.
- Average moisture content of the root zone is assumed to never drop below the permanent wilting point. Therefore, available soil-moisture storage simulated by the water-balance module is zero when moisture content in the root zone is equal to the permanent wilting point.
- The traveltime of water through the root zone is assumed to be negligible. That is, the root zone is relatively thin and consists of high-permeability soils.
- Horizontal flow in the unsaturated zone is assumed to be negligible.
- The flux of water at the bottom of the root zone is assumed to always be downward or zero.
- Because sources or sinks of water are assumed not to exist below the root zone, long-term average effective infiltration equals long-term average recharge.
- The percolation of water below the root zone is assumed to be a linear process. That is, water from each effective infiltration event has the same traveltime characteristics, experiencing the same delay and attenuation, as it moves through the unsaturated zone.
- Effective infiltration is assumed to be 0 for times less than 0. Therefore, it is possible for erroneous values of recharge to be computed for times from 0 to τ_{mem}.
- For the water-balance module, it is important to examine model output to ensure that a large amount of ET is not “lost” when soil-moisture storage is depleted.
- For the transfer-function module, a small value of the unit-event length (Δt_u, eq. 7) should be chosen that is much shorter than the duration of a recharge event so that effective infiltration can be treated as approximately instantaneous.
- Application of the WBTF model to conditions outside of those for which it was calibrated should be done with great caution, if at all, because the model treats physical processes using a simple, lumped parameter, “black box” approach rather than a rigorous physics-based approach.

Summary

Ground-water recharge from precipitation is the primary source of water to the aquifer system in central Florida. In deep water-table settings, where the water table is below the reach of plant roots, the water table is too deep for evapotranspiration (ET) to extract water from the saturated zone; virtually all water that is not lost to surface runoff or evaporation at land surface or to ET in the root zone eventually becomes recharge. Areas in central Florida with a deep water table generally are high recharge areas; consequently, simulation of recharge in these areas is of particular interest to water-resource managers.

Temporal variations in precipitation and ET, which produce temporal variations in recharge, can be substantial in central Florida. In addition, the unsaturated zone can serve as a filter (in a mathematical sense), effectively transforming the effects of surface meteorological processes into the subsurface expression of these processes as recharge at the water table. These complexities make it difficult to estimate short-term recharge rates, thereby confounding calibration and predictive use of transient hydrologic models. A relatively simple method is needed that provides estimates of transient recharge in

deep water-table settings, which can be incorporated into other hydrologic models, such as regional ground-water flow models.

A simple water-balance/transfer-function (WBTF) model was developed for simulating transient ground-water recharge in deep water-table settings. The WBTF model represents a one-dimensional column from the top of the vegetative canopy to the water table and consists of two components: (1) a water-balance module that simulates the water-storage capacity of the vegetative canopy and root zone; and (2) a transfer-function module that simulates the traveltime of water as it percolates from the bottom of the root zone to the water table. Data requirements include two time series for the period of interest—precipitation (or precipitation minus surface runoff, if surface runoff is not negligible) and evapotranspiration—and values for five parameters that represent water-storage capacity or soil-drainage characteristics. A simple model is desirable because it would not be too computationally or data intensive to preclude its practical use in regional-scale models.

Results of the WBTF model were compared to those of the U.S. Geological Survey variably saturated flow model, VS2DT, and to field-based estimates of recharge to demonstrate the applicability of the WBTF model for a range of conditions relevant to deep water-table settings in central Florida. The WBTF model reproduced independently obtained estimates of recharge reasonably well for different soil types and water-table depths. A limiting assumption of the model is that the percolation of water below the root zone is a linear process. That is, percolating water is assumed to have the same traveltime characteristics, experiencing the same delay and attenuation, as it moves through the unsaturated zone. This assumption is more accurate if the moisture content, and consequently the unsaturated hydraulic conductivity, below the root zone does not vary substantially with time. Large variability in the timing and magnitude of infiltration events, typical for central Florida, can cause temporal variations in moisture content that violate the linearity assumption of the transfer-function model. The implications of this assumption on the accuracy of the WBTF model are demonstrated.

The WBTF parameter values appropriate to the hydrologic conditions in question must be obtained by model calibration. Recharge depends not only on soil properties and water-table depth, but also on the magnitude and timing of precipitation, surface runoff, and ET, which generally are unique to a given problem. Guidelines are provided for selection of reasonable, or initial, parameter values.

The simplicity of the WBTF model is balanced against limitations on its applicability and accuracy. Results presented in this report, however, indicate that the WBTF model is capable of providing reasonable estimates of the temporal variability of recharge that can be of sufficient accuracy for incorporation into separate, regional-scale hydrologic models.

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Appendix 1—Documentation Of Water-Balance/Transfer-Function Model Input and Output

File structures and data formats for information input to and output from the water-balance/transfer-function (WBTF) model are described in this appendix. All data are read using a free format. All variables must have a non-blank value and a comma or at least one blank space separating all adjacent values. Any consistent set of length and time units may be used.

Main Input File

When the WBTF model is started, it prompts the user to enter the name of the main input file, which is described below. Once the name of the main input file is entered, the WBTF model runs without further user interaction.

`Item 1. PREFIL (50-character field)`

PREFIL—is the name of the **input file** containing a time series of precipitation (or precipitation minus surface runoff, if surface runoff is not negligible).

`Item 2. ETFIL (50-character field)`

ETFIL—is the name of the **input file** containing a time series of evapotranspiration.

`Item 3. EIFIL (50-character field)`

EIFIL—is the name of the **output file** containing the time series of simulated effective infiltration.

`Item 4. RCHFIL (50-character field)`

RCHFIL—is the name of the **output file** containing the time series of instantaneous simulated recharge based a time step equal to DTU.

`Item 5. RCFIL2 (50-character field)`

RCFIL2—is the name of the **output file** containing the time series of average simulated recharge based on a time step equal to DTRAVG.

`Item 6. SB  SMAX (single-precision real-number fields)`

SB—is the amount of water stored in the vegetative canopy and root zone at the beginning of the simulation period.

SMAX—is the maximum storage capacity of the vegetative canopy and root zone.

`Item 7. N  TAUI  K (double-precision real-number fields)`

N—is the shape parameter of gamma probability density function, characterizing the number of linear reservoirs necessary to represent the unsaturated zone. Fractional values of N do not have a physical analogy, but allow greater flexibility in the shape of the gamma probability density function.

TAUI—is the initial time lag of gamma probability density function, representing the delay time between beginning of effective infiltration and first arrival of recharge.

K—is the scale parameter of gamma probability density function, characterizing the average delay time (N*K+TAUI) imposed on effective infiltration by the unsaturated zone.

`Item 8. DTPE  DTU (double-precision real-number fields)`

DTPE—is the input-data time-step length, which is the time interval at which precipitation and evapotranspiration data are specified in files PREFIL and ETFIL. Time unit must be consistent with that for the precipitation and evapotranspiration rates.

DTU—is the unit-event length, which is the time step at which effective infiltration is discretized and recharge is simulated. DTU must be equal to or less than DTPE and be an even multiple of DTPE. DTU should be much shorter than the duration of a recharge event so that effective infiltration can be treated as approximately instantaneous. Time unit must be consistent with that for the precipitation and evapotranspiration rates.

`Item 9. TRUC  TRI  DTRAVG (double-precision real-number fields)`

TRUC—is a unit conversion factor used to calculate the times for the data records in files PREFIL and ETFIL into time units different than DTPE for output to the files EIFIL, RCHFIL, and RCFIL2. Specify a value of 1 to use the same time units as DTPE.

TRI—is the time for the initial data record in files PREFIL and ETFIL. Time unit must be consistent with that for TRUC.

DTRAVG—is the time step over which instantaneous simulated recharge rates are averaged for output to file RCFIL2. DTRAVG must be equal to or greater than DTU and be an even multiple of DTU. Time unit must be consistent with that for TRUC.

Precipitation File

Data records representing the time series of precipitation (or precipitation minus surface runoff, if surface runoff is not negligible) are specified in the Precipitation File as described below. Explicit times are not specified for the data; the records are assumed to represent data at consecutive time steps based on DTPE.

`Item 1. #Text`

Text—is a comment line that starts in column 2. The “#” character must be in column 1. This line is for user documentation purposes and is ignored by the WBTF model. Any number of repetitions of Item 1 records may be included.

`Item 2. DUM  PRECIP (single-precision real-number fields)`

An appropriate number of repetitions of Item 2 records are required based on DTPE and desired period of simulation. No records are allowed after Item 2 because Item 2 records are assumed to extend to the end of the file.

DUM—is a dummy variable that is read but not used. DUM is for user documentation purposes, for example, it could represent the time at which data were collected.

PRECIP—is the average precipitation rate (or average precipitation minus average surface runoff rates) for time interval (DTPE*(i-1), DTPE*i), where i is a discretization index.

Evapotranspiration File

Data records representing the time series of evapotranspiration (ET) are specified in the Evapotranspiration File as described below. Explicit times are not specified for the data; the records are assumed to represent data at consecutive time steps based on DTPE.

`Item 1. #Text`

Text—is a comment line that starts in column 2. The "#" character must be in column 1. This line is for user documentation purposes and is ignored by the WBTF model. Any number of repetitions of Item 1 records may be included.

`Item 2. DUM  ET (single-precision real-number fields)`

An appropriate number of repetitions of Item 2 records are required based on DTPE and desired period of simulation. No records are allowed after Item 2 because Item 2 records are assumed to extend to the end of the file.

DUM—is a dummy variable that is read but not used. DUM is for user documentation purposes, for example, it could represent the time at which data were collected.

ET—is the average ET rate for time interval (DTPE*(i-1), DTPE*i), where i is a discretization index.

Main Output File

General output is displayed on the computer screen. The user should examine this output for error messages and to ensure that the WBTF model is operating as expected. In particular, the user should examine the water budget computed by the water-balance module. The water-budget summary for the entire simulation period lists the total amount of water for each budget component: precipitation (or precipitation minus surface runoff, if surface runoff is not negligible), ET, effective infiltration, change in storage, and "unaccounted" ET (amount of ET "lost" when ET exceeds the sum of precipitation and storage). Also, the memory of the gamma probability density function is written to the main output file.

Effective Infiltration File

Data records representing the time series of the budget components simulated by the water-balance module are written to the Effective Infiltration File as described below.

`Item 1. Header`

Header—is a single line that contains a short description of each field.

`Item 2. TR  EI  ST  PRECIP  ET`

An appropriate number of repetitions of Item 2 records are written based on DTPE and the period of simulation.

TR—is time computed as TRUC*DTPE*i + TRI, where i is a discretization index.

EI—is the average effective infiltration rate simulated for time interval (DTPE*(i-1), DTPE*i).

ST—is the amount of water stored in the vegetative canopy and root zone simulated for time interval (DTPE*(i-1), DTPE*i).

PRECIP—is the average precipitation rate specified in the Precipitation File for time interval (DTPE*(*i*-1), DTPE**i*).

ET—is the average ET rate specified in the Evapotranspiration File for time interval (DTPE*(*i*-1), DTPE**i*).

Instantaneous Recharge File

Data records representing the time series of instantaneous simulated recharge are written to the Instantaneous Recharge File as described below. Instantaneous recharge rates actually represent an average rate for the time step, DTU, but the recharge rates are approximately instantaneous because DTU is chosen by the user to be much shorter than the duration of a recharge event.

`Item 1. Header`

Header—is a single line that contains a short description of each field.

`Item 2. TR  EI  RCHIN`

An appropriate number of repetitions of Item 2 records are written based on DTU and the period of simulation.

TR—is time computed as TRUC*DTU**j* + (TRI - DTPE), where *j* is a discretization index.

EI—is the average effective infiltration rate simulated for time interval (DTU*(*j*-1), DTU**j*). EI for time steps equal to DTU are assigned the value of EI for the overlapping DTPE time step (DTU is less than DTPE), because EI is computed by the water-balance module for time steps equal to DTPE.

RCHIN—is the instantaneous recharge rate simulated for time interval (DTU*(*j*-1), DTU**j*).

Average Recharge File

Data records representing the time series of simulated recharge averaged over a time step equal to DTRAVG are written to the Average Recharge File as described below.

`Item 1. Header`

Header—is a single line that contains a short description of each field.

`Item 2. TRA  RCHAVG  TRA1  TRA2`

An appropriate number of repetitions of Item 2 records are written based on DTRAVG and the period of simulation.

TRA—is time at the middle of the averaging period computed as (TRA1 + TRA2)/2.

RCHAVG—is the average recharge rate simulated for time interval (DTRAVG*(*j*-1), DTRAVG**j*), where *j* is a discretization index.

TRA1—is time at the beginning of the averaging period DTRAVG*(*j*-1).

TRA2—is time at the end of the averaging period DTRAVG**j*.

Appendix 2—Example Problem

A listing of input and output files for an example problem are included below. Only a partial listing of some files is included; electronic versions of all files are available at the World Wide Web address http://pubs.water.usgs.gov/sir2004-5195/. The example problem consists of a sand soil with a 2.5-meter-deep water table. This hypothetical field site is discussed in more detail in the text of this report. Information on soil properties is shown in table 3; and simulated effective infiltration and recharge are shown in figure 13 in the report.

Listing of Main Input File

```
precip.txt
et.txt
ei.csv
rch_inst.csv
rch_avg.csv
3.e1   5.e1     SB, SMAX
7.59112d-001  1.87817d+000  4.64891d+000     N, TAUI, K
1.d0   1.d-1     DTPE, DTU
1.d0   1.d0   1.d0     TRUC, TRI, DTRAVG
```

Listing of Precipitation File

```
#Average of 8 rain gages (1995 data, repeat for 2nd year)
#cummul-julianday    rain-mm/d
1             0.0
2             0.0
3             0.0
4             0.2
5             0.7
6             0.1
7             7.6
8             0.9
9             0.0
10            0.0

(710 similar lines omitted)

721           0.0
722           0.0
723           0.0
724           0.0
725           0.0
726           0.0
727           0.0
```

```
728            0.0
729            0.4
730            0.2
```

Listing of Evapotranspiration File

```
#sine curve fit to daily ET from Lake Wales Ridge station
#BEST FIT: min ET = 0.5452; amplitude = 1.310; phase shift = -100.53
#cummul-julianday    ET-mm/d
1            .558
2            .555
3            .553
4            .551
5            .549
6            .547
7            .546
8            .546
9            .545
10           .545
```

(710 similar lines omitted)

```
721          .610
722          .603
723          .596
724          .590
725          .585
726          .579
727          .574
728          .570
729          .566
730          .562
```

Listing of Main Output File

```
**************************************************
** Water-Balance/Transfer-Function (WBTF) model **
*********** Version 1.0 -- 05/28/2004 ************
**** For simulating ground-water recharge in *****
************ deep water-table settings ***********
**************************************************

Enter name of input file:
wbtf.in
```

```
Time step of precip and ET data (DTPE) =       1.000000000000000
Factor to convert time units (TRUC) =       1.000000000000000
Time of first precip and ET data record (TRI) =     1.000000000000000

Number of data records in precip file=        730
Number of data records in ET file:        730


**************************************************
******* Water-Balance module of WBTF model *******
*********** Version 1.0 -- 05/28/2004 ************
**************************************************

Initial storage capacity (SB) =      30.000000
Maximum storage capacity (SMAX) =      50.000000

***************************************************
      Water budget for entire data period:
              Precipitation (P) =    2632.600000
        Evapotranspiration (ET) =    1354.303000
    Effective infiltration (EI) =    1338.432000
         Change in storage (dS) =     -26.976000
   Unaccounted ET (P + dS - ET) =     -33.159990
Error (P - ET - EI - dS - unET) =   9.791851E-04
***************************************************

Effective infiltration is in file: ei.csv


**************************************************
***** Transfer-Function module of WBTF model *****
*********** Version 1.0 -- 05/28/2004 ************
**************************************************

Gamma pdf shape parameter (N) =  7.591120000000000E-001
Gamma pdf initial time lag (TAUI) =       1.878170000000000
Gamma pdf scale parameter (K) =       4.648910000000000
Time step for driving function EI (DTPE) =       1.000000000000000
Time step for response function RCHIN (DTU)=1.000000000000000E-001
Time step for averaging period (DTRAVG) =       1.000000000000000

Number of unit-event time steps (DTU) per averaging period =      10
Number of unit-event time steps (DTU) per driving-function EI time step
(DTPE/DTU) =         10
Number of unit-even time steps (DTU) spanned by driving function EI =
7300

++++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
The following information concerns the selection of the appropriate value
for dtau...
Too large a dtau will result in discretization error.
*****************
Trying dtau =  .1000
Slope of Gamma pdf at tau =  1.000000000000000E-001 is    -1.147839
Prescribed Gamma pdf at tau = 0 is   5.522250E-01
Statistics concerning actual and estimated slope of Gamma pdf for dtau =
.1000
and for   .000     < tau <=   30.700
P(tau <=     .000) =  .00000
P(tau <=   30.700) =  .99001
Root mean square error =  .1194E-01
Average absolute slope =  .1645E-01
RMSE as % of avg abs slope =  72.61
Average residual = -.3363E-03
Maximum residual = -.1982
Gamma cdf >   9.900000E-01 but dtau probably too large to calculate Gamma
cdf >    9.990000E-01
++++++++++++++++++++++++++++++++++++++++++++++++++++++

++++++++++++++++++++++++++++++++++++++++++++++++++++++
The following information concerns the memory of the gamma pdf...
P(tau <=   53.500) =  .99069
Memory of specified gamma pdf excluding TAUI =    31
Memory is calculated as X, where X defined by P(tau <= X) =  .99000
and X rounded up to nearest integer.
Memory (not rounded) of specified gamma pdf including TAUI =    32.6
Gamma pdf is in file gam.csv
++++++++++++++++++++++++++++++++++++++++++++++++++++++

++++++++++++++++++++++++++++++++++++++++++++++++++++++
The following information concerns the convolution of EI with the gamma
pdf and a mass balance check (comparison of areas under EI and RCHIN
curves)...
Number of tau steps in gamma pdf memory =        310
Number of tau steps in initial time lag =         19
 ntsr =       7300  ;  m =       7301
m =       7301  ;  Area under rsf for t<=tf:    1325.122000
m =       7630  ;  Area under rsf for t>tf:   0.000000E+00
Area under the driving function EI curve =    1338.432000
Area under the response function RCHIN curve =    1325.122000
Area under the response function curve RCHIN is      99.005570 percent
that under the driving function curve EI.
++++++++++++++++++++++++++++++++++++++++++++++++++++++

Instantaneous response function is in file: rch_inst.csv
```

```
Averaged response function is in file: rch_avg.csv

Normal termination of WBTF Version 1.0 -- 05/28/2004
```

Listing of Effective Infiltration File

```
Time, Eff.infil: Sb=  30.00 Smax=  50.00, Storage, Precip, ET
    1.000  ,    .000  ,  29.442  ,    .000  ,    .558
    2.000  ,    .000  ,  28.887  ,    .000  ,    .555
    3.000  ,    .000  ,  28.334  ,    .000  ,    .553
    4.000  ,    .000  ,  27.983  ,    .200  ,    .551
    5.000  ,    .000  ,  28.134  ,    .700  ,    .549
    6.000  ,    .000  ,  27.687  ,    .100  ,    .547
    7.000  ,    .000  ,  34.741  ,   7.600  ,    .546
    8.000  ,    .000  ,  35.095  ,    .900  ,    .546
    9.000  ,    .000  ,  34.550  ,    .000  ,    .545
   10.000  ,    .000  ,  34.005  ,    .000  ,    .545
   11.000  ,    .000  ,  33.459  ,    .000  ,    .546
   12.000  ,    .000  ,  32.912  ,    .000  ,    .547
   13.000  ,    .000  ,  32.364  ,    .000  ,    .548
   14.000  ,    .000  ,  33.214  ,   1.400  ,    .550
   15.000  ,    .000  ,  47.662  ,  15.000  ,    .552
   16.000  ,   2.908  ,  50.000  ,   5.800  ,    .554
   17.000  ,    .000  ,  49.443  ,    .000  ,    .557
   18.000  ,    .000  ,  48.883  ,    .000  ,    .560
   19.000  ,    .000  ,  48.320  ,    .000  ,    .563
```

(706 similar lines omitted)

```
  726.000  ,    .000  ,   4.696  ,    .000  ,    .579
  727.000  ,    .000  ,   4.122  ,    .000  ,    .574
  728.000  ,    .000  ,   3.552  ,    .000  ,    .570
  729.000  ,    .000  ,   3.386  ,    .400  ,    .566
  730.000  ,    .000  ,   3.024  ,    .200  ,    .562
```

Listing of Instantaneous Recharge File

```
Time, EI, Rch-inst:n=   .76 TAUi=  1.88 k=  4.65 TAUmem=  32.6
     .100,  .00000    ,  .00000
     .200,  .00000    ,  .00000
     .300,  .00000    ,  .00000
     .400,  .00000    ,  .00000
     .500,  .00000    ,  .00000
     .600,  .00000    ,  .00000
     .700,  .00000    ,  .00000
     .800,  .00000    ,  .00000
     .900,  .00000    ,  .00000
    1.000,  .00000    ,  .00000
```

(139 similar lines omitted)

```
   15.000,  .00000    ,  .00000
   15.100,  2.9080    ,  .00000
   15.200,  2.9080    ,  .00000
   15.300,  2.9080    ,  .00000
   15.400,  2.9080    ,  .00000
   15.500,  2.9080    ,  .00000
   15.600,  2.9080    ,  .00000
   15.700,  2.9080    ,  .00000
   15.800,  2.9080    ,  .00000
   15.900,  2.9080    ,  .00000
   16.000,  2.9080    ,  .00000
   16.100,  .00000    ,  .00000
   16.200,  .00000    ,  .00000
   16.300,  .00000    ,  .00000
   16.400,  .00000    ,  .00000
   16.500,  .00000    ,  .00000
   16.600,  .00000    ,  .00000
   16.700,  .00000    ,  .00000
   16.800,  .00000    ,  .00000
   16.900,  .00000    ,  .00000
   17.000,  .00000    ,  .14390
   17.100,  .00000    ,  .25803
   17.200,  .00000    ,  .35681
   17.300,  .00000    ,  .44595
   17.400,  .00000    ,  .52808
   17.500,  .00000    ,  .60466
   17.600,  .00000    ,  .67666
   17.700,  .00000    ,  .74473
   17.800,  .00000    ,  .80938
   17.900,  .00000    ,  .87098
   18.000,  .00000    ,  .78594
   18.100,  .00000    ,  .72816
   18.200,  .00000    ,  .68344
   18.300,  .00000    ,  .64623
   18.400,  .00000    ,  .61407
   18.500,  .00000    ,  .58560
   18.600,  .00000    ,  .56000
   18.700,  .00000    ,  .53669
   18.800,  .00000    ,  .51527
   18.900,  .00000    ,  .49544
   19.000,  .00000    ,  .47699
```

(7100 similar lines omitted)

```
  729.100,  .00000    ,  .00000
  729.200,  .00000    ,  .00000
  729.300,  .00000    ,  .00000
  729.400,  .00000    ,  .00000
  729.500,  .00000    ,  .00000
  729.600,  .00000    ,  .00000
  729.700,  .00000    ,  .00000
  729.800,  .00000    ,  .00000
  729.900,  .00000    ,  .00000
  730.000,  .00000    ,  .00000
```

Listing of Average Recharge File

```
Time, Rch-avg:n=   .76 TAUi=  1.88 k=  4.65 TAUmem=  32.6, T-s, T-e
     .500 ,  .0000000     ,      .000 ,     1.000
    1.500 ,  .0000000     ,     1.000 ,     2.000
    2.500 ,  .0000000     ,     2.000 ,     3.000
    3.500 ,  .0000000     ,     3.000 ,     4.000
    4.500 ,  .0000000     ,     4.000 ,     5.000
    5.500 ,  .0000000     ,     5.000 ,     6.000
    6.500 ,  .0000000     ,     6.000 ,     7.000
    7.500 ,  .0000000     ,     7.000 ,     8.000
    8.500 ,  .0000000     ,     8.000 ,     9.000
    9.500 ,  .0000000     ,     9.000 ,    10.000
   10.500 ,  .0000000     ,    10.000 ,    11.000
   11.500 ,  .0000000     ,    11.000 ,    12.000
   12.500 ,  .0000000     ,    12.000 ,    13.000
   13.500 ,  .0000000     ,    13.000 ,    14.000
   14.500 ,  .0000000     ,    14.000 ,    15.000
   15.500 ,  .0000000     ,    15.000 ,    16.000
   16.500 ,  .1438973E-01 ,    16.000 ,    17.000
   17.500 ,  .6081222     ,    17.000 ,    18.000
   18.500 ,  .5841868     ,    18.000 ,    19.000
```

(706 similar lines omitted)

```
  725.500 ,  .0000000     ,   725.000 ,   726.000
  726.500 ,  .0000000     ,   726.000 ,   727.000
  727.500 ,  .0000000     ,   727.000 ,   728.000
  728.500 ,  .0000000     ,   728.000 ,   729.000
  729.500 ,  .0000000     ,   729.000 ,   730.000
```