

# A Theoretical Framework for Interpreting and Quantifying the Sampling Time Dependence of Gravel Bedload Transport Rates

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## Abstract

Field studies have documented that the average bedload transport rate in gravel bed streams depends on the sampling time (or integration time) over which this average is computed. In this paper we use sediment transport data from a controlled laboratory experiment to document and quantify the dependence on sampling time not only of the mean but of the whole probability density function (pdf) of sediment transport rates in a gravel bed stream. We demonstrate that the higher moments (variance and skewness) scale differently than the mean and we provide a concise parameterization of this statistical scale-dependence. The results indicate that the mean sediment transport rate in moderate flows decreases with increasing sampling time, in agreement with results reported in field studies. The proposed methodology provides a framework within which to seek universal scaling relationships, compare results of different samplers, and also interpret extremes.

## 1. Introduction

In a recent study Bunte and Abt (2005) have provided an excellent account of the problem of sampling time dependence of gravel bedload transport rate and its importance for interpreting estimates from different samplers. They also provided a detailed quantification of this dependence for a range of flows in a gravel-bedded stream. Their results indicated that in moderate to high flows (50% bankfull to almost bankfull conditions) 2 min sampling led to an average transport rate 2 to 5 times lower than that found with 10 min sampling. However, at lower flows (close to the incipient gravel motion) 2 min sampling overestimated the transport rates at 10 min sampling by a factor of almost 3. The fact that bedload discharge measurements depend on both sampling time and mean flow rate was also shown by the field measurements at different time scales documented by Ergenzinger et al. (1994). In this study we restrict our analysis to the sampling time dependence in the low flow case, with results from a higher flow experiment to be presented by the authors in a follow-up work (Singh et al., 2008). The explanation of the sampling interval dependence at low flow has generally been that the instantaneous sediment transport rates exhibit rare but very high fluctuations (due to the irregular and

stochastic nature of particle movement on the bed) and thus integrating over variable time intervals changes the chance of sampling these high fluctuations. Translating this to a statistical interpretation, at very small sampling intervals the chance of sampling very high fluctuations is small and thus the pdf of sediment transport rates is highly skewed (only a small number of very high values is present); as the sampling interval increases however, more of these high fluctuations are likely to be sampled. Although most studies have been concerned only with how the mean changes with the sampling interval, it is also of interest to study the whole pdf and quantify how the shape (or the higher order statistical moments) of this pdf change as the scale (sampling interval) changes. This is important as often it is not only the mean but the variability of the sediment transport rates, including extreme quantiles, that is important in many ecological studies, e.g., studies that incorporate the effect of bedload sediment on stream habitat.

The sampling-time dependence of bedload transport rates has important practical implications for bedload measurements in the field. Several researchers (e.g. Gomez et al. 1989, Kuhnle 1996, Wilcock 2001) have argued for sampling durations that are long enough to smooth out fluctuations and measure the “true mean” transport rate for a particular level of flow. But determining the time period that fits this criterion requires knowledge of the way in which the statistics of the transport rate vary with sampling interval. In addition, practical limitations can restrict sampling time to be less than this ideal: the sampling instrument used may have a limited capacity (such as a Helley-Smith type sampler), or variations in flow rate may mean that the system can not be considered stable over such a long period. In such situations, interpreting results and comparing measurements with different sampling times requires a quantitative understanding of the dependence of bedload transport on temporal scale.

The first objective of this study is to use the results of a controlled and well-instrumented experimental set-up, described in Section 2, to reproduce some of the effects of sampling interval that have been observed in the field. By continuously measuring the mass of transported gravel bedload through an experimental flume at high temporal resolution, and then averaging over different time intervals, it is possible to simulate sampling the transport rate over different time periods. The second objective is to develop a theoretical framework for quantifying the variations in the statistical properties of bedload transport with changing sampling interval. This framework is outlined in section 3, and is focused on the ideas of scale-invariance, or the search for statistical quantities that remain the same with change of scale or can be transformed from one scale to another in a straightforward way. Of course, in this work the scale to which we refer is the temporal scale, i.e. the sampling time of the bedload transport. The results of applying this framework to the experimental data are presented in section 4, and the implications are discussed in section 5.

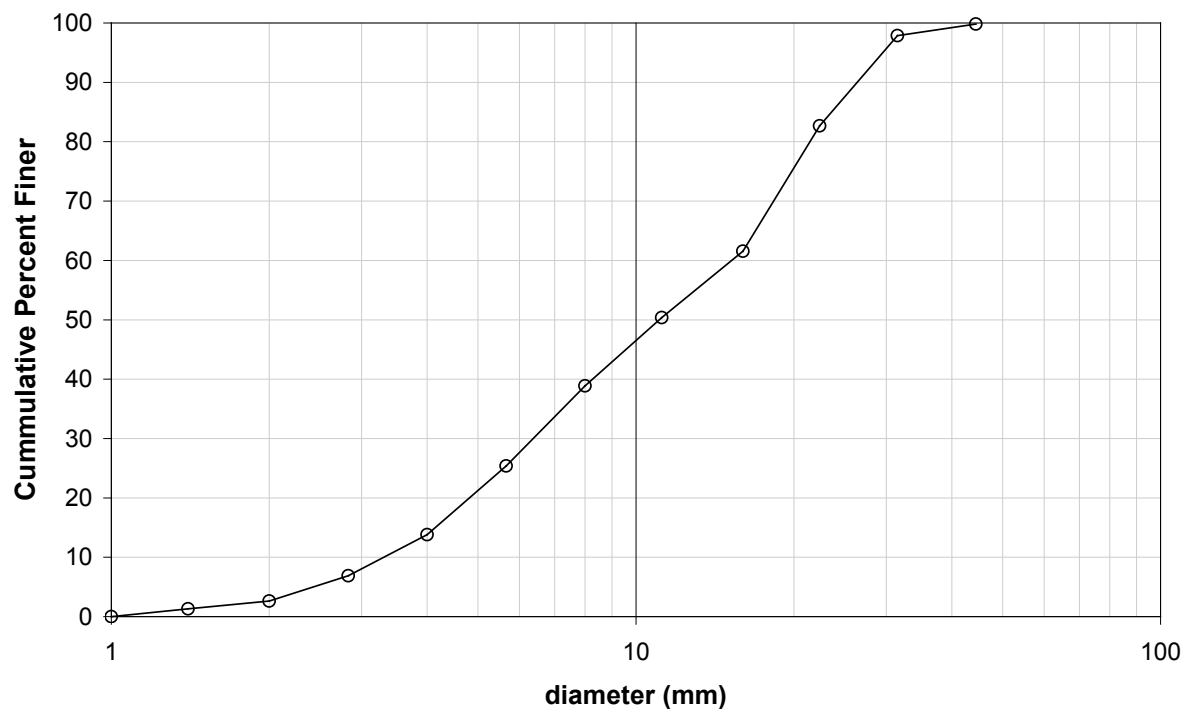
## 2. Data Used In This Study

### 2.1 Experimental set-up and data processing

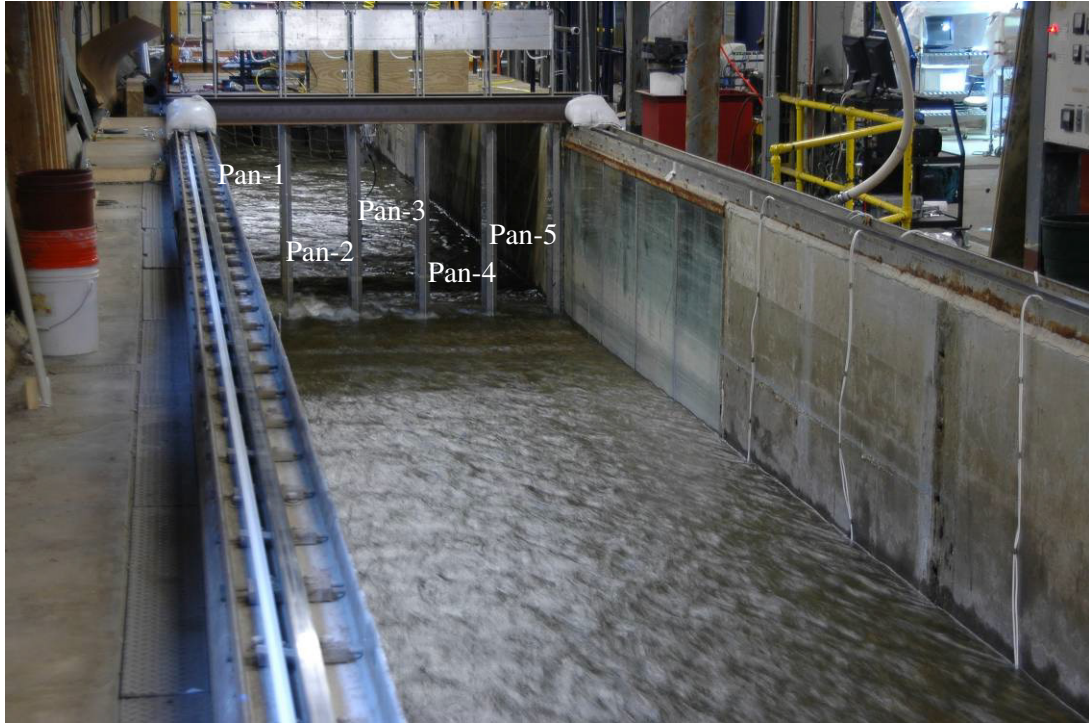
In order to investigate the dependence of bedload sediment transport on sampling time, we examine experimental data from the Main Channel facility at the St. Anthony Falls Laboratory, University of Minnesota. These experiments were conducted as part of a larger-scale experimental program (called StreamLab06) supported by the National Center for Earth-surface Dynamics (NCED) at the University of Minnesota. The flume used in this study is 2.74 m wide and 55 m long, with a maximum depth of 1.8 m. Gravel with a median particle size ( $d_{50}$ ) of 11.3 mm was placed in a 20m long mobile-bed section of the 55 m long channel. The grain size distribution for the bed material is shown in **Figure 1**. A constant discharge of water at 4300 liters per second was released into the flume. This was estimated to generate a dimensionless bed stress of about twice the critical value required to move the

bedload sediment (Shields stress = 0.085 using median gravel diameter). At the downstream end of the test section was located a bedload trap, consisting of 5 weighing pans of equal size that spanned the width of the channel, as seen in **Figure 2**. Any bedload sediment transported to the end of the test-section of the channel would fall into the weigh pans, which automatically recorded the mass they contained every 1.1 seconds. Upon filling with a maximum of 20 kg the weigh pans would tip to release the sediment into the collection hopper located below and reset the weigh pan.

The flume is a partial-recirculating flume; it has the ability to recirculate gravel while water flows through the flume without recirculation. A large collection hopper located underneath the weigh pans serves to collect and store gravel dumped out of the weigh drums and also serves as the material source for the recirculation system. The rate of gravel removal out of this hopper and delivered to the upstream end of the flume via a large pump is set by adjusting the rotation speed of a large helix, which serves to push gravel laterally out of the hopper and into the recirculation line. In this way, the collection hopper and helix serve to buffer small fluctuations in sediment flux out of the flume and provide a more steady “feed-type” delivery of sediment to the upstream end. Because the physical size of the collection hopper is finite, the auger speed (upstream feed rate) was adjusted to match the actual transport in the flume such that we would always observe storage of gravel in the hopper.



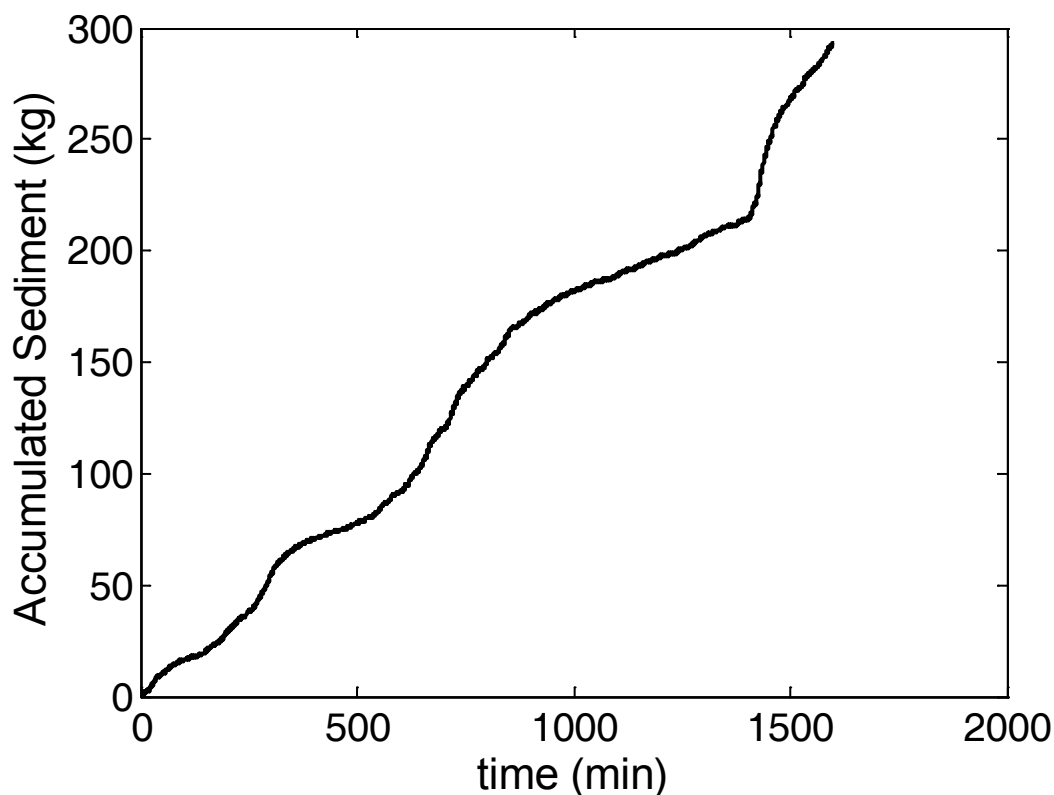
**Figure 1.** Grain Size Distribution of the bed surface.



**Figure 2.** Weighing Pans located at the downstream end of the test channel. The experiment was conducted at the St. Anthony Falls Laboratory, National Center for Earth-surface Dynamics, University of Minnesota.

Before any measurements were taken, the water supply in the flume was turned on to 4300 liters per second and was allowed to run 15 hr to develop a dynamic equilibrium in transport and slope adjustment of the water surface and bed. Determination of the dynamic equilibrium state was made by checking that the 60 min average flux was stabilized to an almost constant value during the flume run. The bedload transport data were then recorded continuously for approximately 16 hours through the rest of the experiment.

The raw sediment accumulation data was pre-processed prior to the analysis presented here. The pre-processing involves removal of weigh pan dumping events from the data and translating the data set into a continuous accumulation of sediment ( $S_c(t)$ ) time series for each weigh pan over the duration of the experiment. An example of this series can be seen in Figure 3. A single tipping event requires the removal of no more than eight data points (~8.8 seconds) of the record. On average, a single weigh pan tipped every 1.5 hr. Overall, the data affected by the weigh pan tipping constitutes less than 0.15% of the total data record and is, thus, negligible.

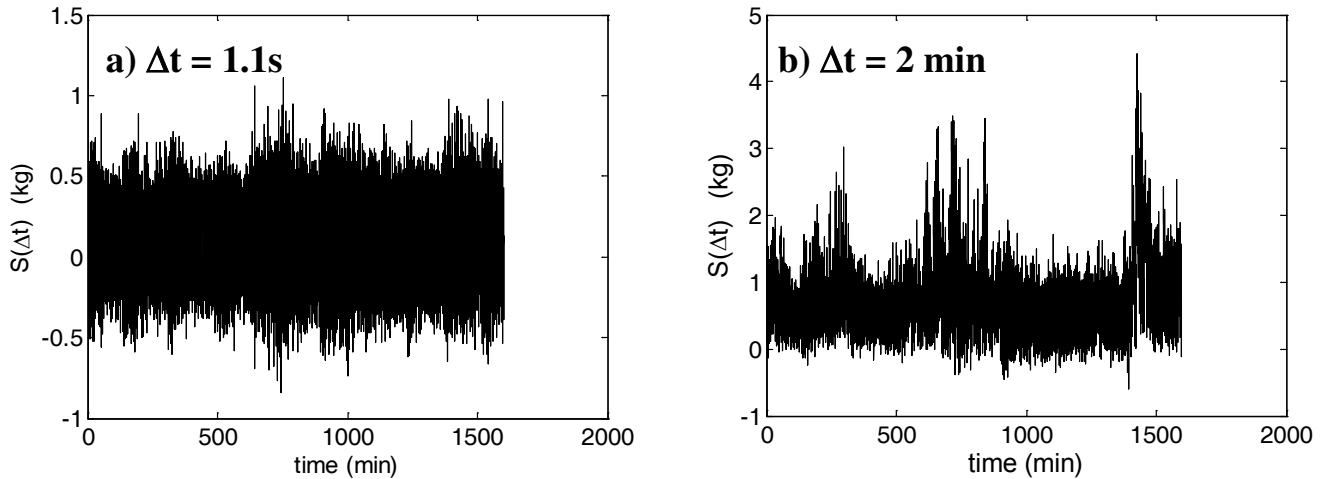


**Figure 3.** Time series of the total accumulated sediment  $S_c(t)$

The sediment accumulation  $S(t, \Delta t)$  measured over a sampling interval  $\Delta t$  is then simply

$$S(t, \Delta t) = S_c(t + \Delta t) - S_c(t) \quad (1)$$

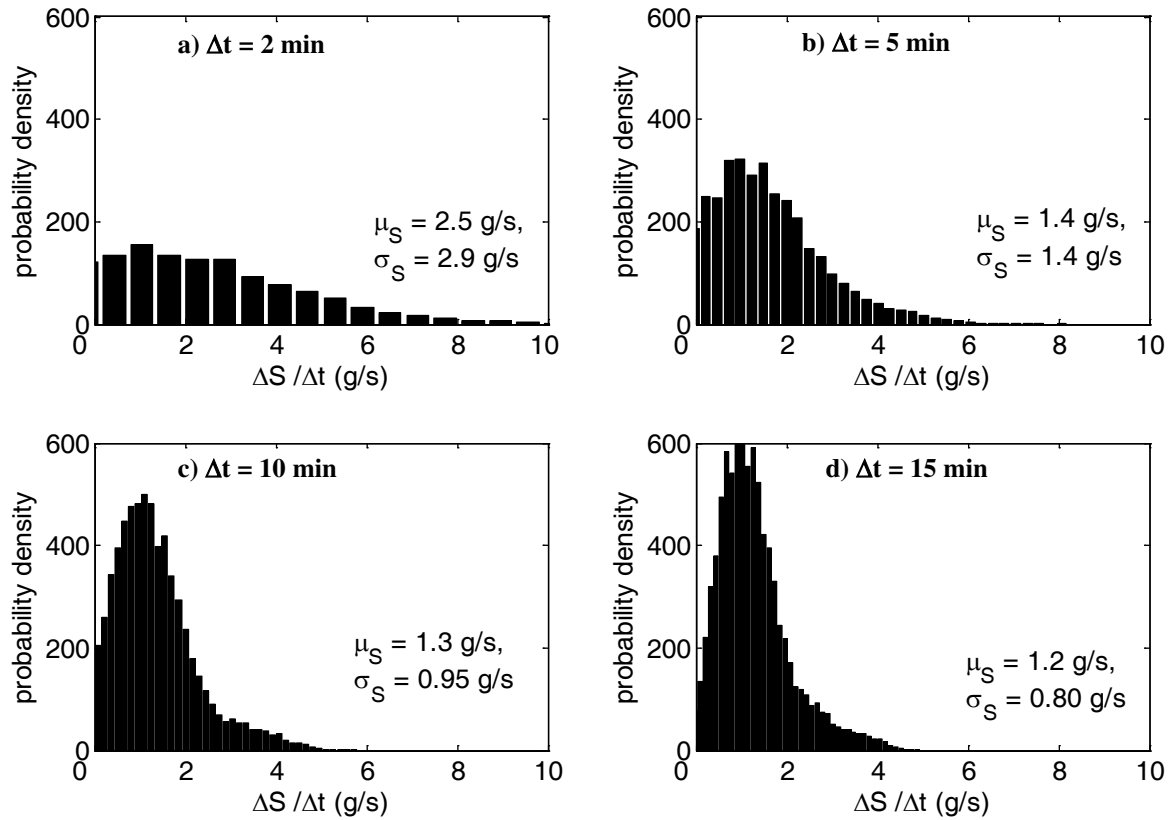
If the sampling interval  $\Delta t$  is taken to be very small, such as the original 1.1 sec resolution of the data series, one observes a large number of negative values of  $S(t, \Delta t)$ , as shown in **Figure 4a**. These negative values are clearly not physically plausible since by the experimental design (**Figure 2**) the sediment can only pass in one direction, down onto the weigh pans, and hence  $S(t, \Delta t)$  should only be positive. Thus, these negative sediment accumulations are attributed to the mechanical noise produced by: the natural oscillation of the weigh pans after being hit by the falling gravel; the fluctuating water surface over the pan; and the vibration caused by the large gravel pump which was placed near to the weigh pans (for further discussion of the errors associated with weigh pans and possible processing techniques, see Laronne et al., 2003). This noise can be smoothed out by considering longer sampling intervals  $\Delta t$ . It was found that the occurrence of negative values of  $S(t, \Delta t)$ , and hence the significance of the noise relative to the signal, was greatly reduced once the temporal scale  $\Delta t$  was increased to about 2 min. **Figure 4b** shows the time series  $S(t, \Delta t)$  for  $\Delta t = 2$  min. Hence to avoid noise distortion, we will mainly interpret results for temporal aggregation scales longer than 2 min.



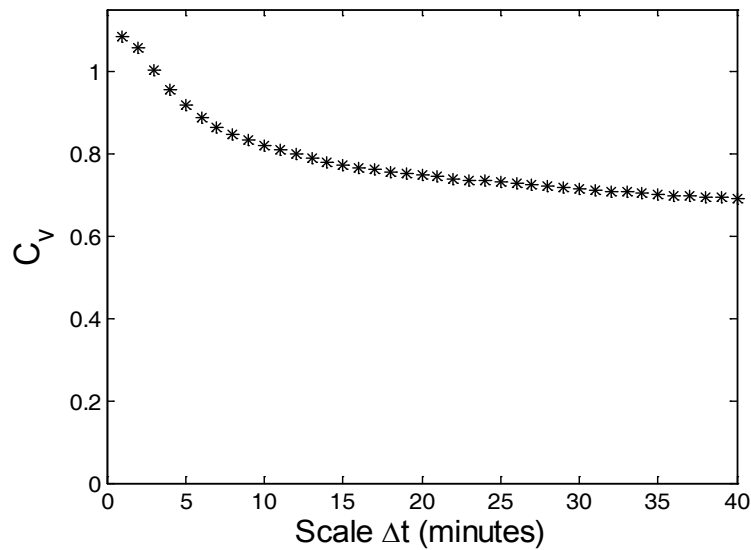
**Figure 4.** The time series of the sediment,  $S(\Delta t)$ , accumulated over sampling intervals  $\Delta t$  of: (a) 1.1 seconds and (b) 2 min

## 2.2 Statistical distribution of the data.

To begin investigating the statistical properties of bedload transport and its dependence on sampling interval, consider the sediment transport rate, given by  $(S(t, \Delta t) / \Delta t)$ . The pdfs of the sediment transport rate, calculated at sampling intervals of 2, 5, 10, and 15 min, are shown in **Figure 5**. At the smallest scale,  $\Delta t = 2$  min, the probability distribution is wider, with a higher mean and standard deviation than at the longer sampling intervals. As the sampling interval increases, the mean decreases slightly and the standard deviation decreases more significantly, as the distributions become more tightly peaked at longer sampling time. Note that the pdf remains negatively skewed at the larger temporal scales. One measure of the shape of the probability distribution is the coefficient of variation,  $C_v$ , which is the ratio of the standard deviation to the mean,  $C_v = \frac{\sigma}{\mu}$ . For the sediment transport rate,  $C_v$  decreases with increasing temporal scale, as shown in **Figure 6**. So the width of the pdf, relative to the mean, diminishes with increasing sampling time.



**Figure 5.** Probability density function of sediment transport rate for different sampling intervals (2, 5, 10, 15 minutes), with mean and standard deviation listed (units of g/s).



**Figure 6.** The coefficient of variation of sediment transport rate as a function of temporal scale  $\Delta t$  (sampling interval).

### 3. Framework of Analysis

If the sediment transport rate ( $S(t, \Delta t) / \Delta t$ ) was independent of the sampling interval, the mean of the accumulated sediment  $S(t, \Delta t)$  would depend linearly on  $\Delta t$ ; that is, in twice as large a sampling interval, on the average twice as much sediment would be accumulated,. In practice, however, it has been found that the mean of  $S(t, \Delta t)$  depends on  $\Delta t$  in a way that has not yet been statistically characterized. The purpose of this paper is to explore whether this dependence falls under any scale invariance characterization, widely found in turbulence and other geophysical processes (e.g., see Parisi and Frisch 1985; Lovejoy et al., 1993; Gupta et al. 1994; Fofoula-Georgiou 1998; Sornette and Ouillon 2005; Venugopal et al. 2006, Lashermes and Fofoula-Georgiou, 2007).

Let us define the  $q$ -th statistical moment  $\langle S(\Delta t)^q \rangle$  as the expectation value of the  $q$ -th power of  $S(t, \Delta t)$ , which is estimated by

$$\langle S(\Delta t)^q \rangle = \frac{1}{N} \sum_{i=1}^N (S(t, \Delta t))^q, \quad (2)$$

where  $N$  is the total number of data points available at the scale  $\Delta t$ . The 1<sup>st</sup> statistical moment is the mean and the 2<sup>nd</sup> statistical moment is a measure of the variability about the origin. Combined, the statistical moments  $\langle S(\Delta t)^q \rangle$  for all  $q$  completely capture the shape of the pdf. Statistical scaling, or scale invariance, requires that  $\langle S(\Delta t)^q \rangle$  is a power law function of the scale, that is

$$\langle S(\Delta t)^q \rangle \propto (\Delta t)^{\tau(q)}, \quad (3)$$

where  $\tau(q)$  is the scaling exponent, which is independent of  $\Delta t$  and depends only on the order of the moment  $q$ . For a scale-invariant variable, the function  $\tau(q)$  therefore completely determines how the pdf of the variable changes with scale. For example, the mean will vary as sampling interval to the power of  $\tau(1)$ .

The simplest form of scaling, known as simple scaling or monoscaling, is when the scaling exponents are a linear function of the moment order i.e. when  $\tau(q) = Hq$ . In this case the single parameter  $H$  characterizes how the whole pdf changes over scales, so that if  $P(S(t, \Delta t))$  is the pdf of sediment transport at scale  $\Delta t$ , the pdf at a second scale  $\Delta t'$  is given by (e.g., Kumar and Fofoula-Georgiou 1993)

$$P(S(t, \Delta t')) = \left( \frac{\Delta t}{\Delta t'} \right)^{-H} P \left( \left( \frac{\Delta t}{\Delta t'} \right)^{-H} S(t, \Delta t) \right) \quad (4)$$

That is, the pdf at sampling interval  $\Delta t'$  is simply the original pdf normalized by a factor  $\left( \frac{\Delta t}{\Delta t'} \right)^{-H}$ .

If  $\tau(q)$  is nonlinear, known as multiscaling, more than one parameter is required to define the behavior of the probability distribution change over scales. In fact, equation (4) takes on a more complicated form and involves a convolution of the pdf at the previous scales with a kernel that depends on the ratio of scales (e.g., Castaing and Dubrulle 1995; see also Venugopal et al. 2006). Concentrating on the second order moments, one can quantify the relative change in shape of the pdf with the coefficient of variation,  $C_v$ . Note that  $C_v$  is expressed in terms of the first and second moments as

$$C_v = \left[ \frac{\langle S(\Delta t)^2 \rangle}{\langle S(\Delta t)^1 \rangle^2} - 1 \right]^{1/2}. \quad (5)$$



In the presence of scaling (equation 3) this results in

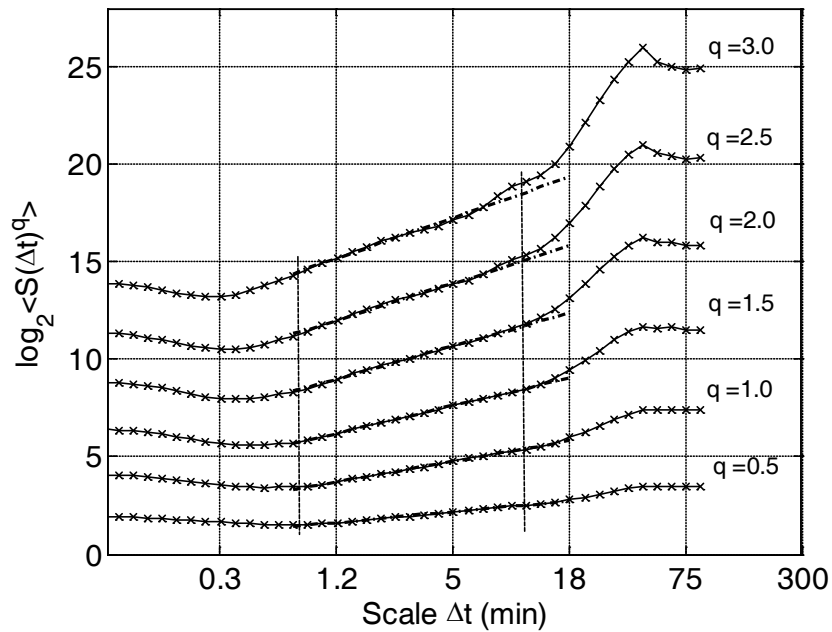
$$C_v^2 + 1 \propto (\Delta t)^{\tau(2)-2\tau(1)}, \quad (6)$$

which, for the multiscaling case, indicates that the coefficient of variation changes across scales as a function of  $\tau(2)-2\tau(1)$ . On the contrary, for simple scaling  $\tau(q)$  is linear and hence  $\tau(2) = 2\tau(1)$ , so that equation (6) means that  $C_v$  will be a constant across scales. The variation in the shape of the pdf with scale could also be quantified in more detail by higher order dimensionless moments, such as

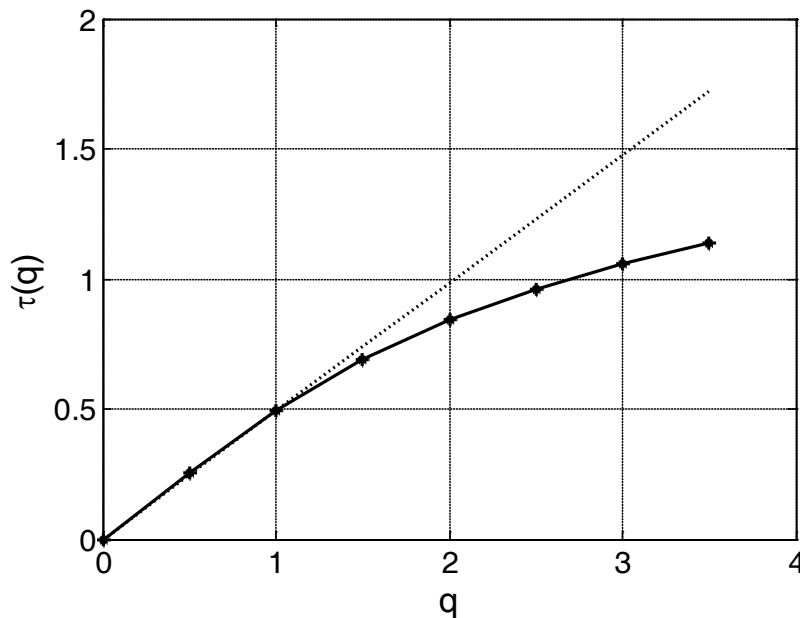
$$\frac{\langle S(\Delta t)^4 \rangle}{\langle S(\Delta t)^2 \rangle^2} \text{ (e.g., Mahrt 1988).}$$

## 4. Results

To quantify the scale-dependence of bedload transport, this multiscale analysis methodology was applied to the experimental sediment transport series described in section 2. The statistical moments  $\langle S(\Delta t)^q \rangle$  are displayed as a function of  $\Delta t$  on a log-log plot in **Figure 7**. If the sediment transport series are scale invariant, we would expect to see a linear relationship in this figure, since the power law relationship between  $\langle S(\Delta t)^q \rangle$  and  $\Delta t$  expressed in equation (3) is linear on a log-log plot. Figure 7 indeed shows a linear relationship between the statistical moments and temporal scale over the range of approximately 1 min to 15 min (indicated by the dashed lines on the figure), and hence implies a scale invariant regime within this range. At sampling times shorter than 1 min, the statistical moments do not decrease as would be implied by the scale invariant system, and in fact increase at short enough scales. This behavior of the statistics at short time-scales is interpreted as being dominated by the mechanical noise, discussed in section 2.1, which can be identified by the high frequency of occurrence of negative values in the sediment accumulation rate, which by experimental design should only be positive. At sampling times  $\Delta t > 15$  min, the statistical moments also deviate from the log-log linear relationship, eventually leveling out to be relatively constant with sampling time. This is seen as reaching a critical scale, around 15-30 min, at which the largest fluctuations of the series are sampled regularly and above which the statistics of the flow are stable.



**Figure 7.** Statistical moments  $\langle S(\Delta t)^q \rangle$  of the sediment flux as a function of temporal scale (sampling interval), for the range of moment orders  $q = 0.5, 1, 1.5, 2.0, 2.5, 3.0$ . The vertical dashed lines mark the limits of the scaling range.



**Figure 8.** The scaling exponents  $\tau(q)$  as a function of the moment order  $q$  (computed values from the sediment transport series are shown as points for  $q=0$  to 3 in increments of 0.5 and the solid line is the fitted quadratic approximation). Deviation from monofractality is depicted by the deviation from the straight line.

Within the scaling regime, it can be observed from **Figure 7** that the statistical moments have different slopes. Estimating these slopes by least squares fitting gives the scaling exponents  $\tau(q)$  for all moment orders  $q$ , which are plotted in **Figure 8**. Concentrating on first order ( $q = 1$ ) statistical moment, which is in fact the mean of  $S(t, \Delta t)$ , we see that  $\tau(1) \approx 0.5$ . This implies that within the scaling range the mean amount of sediment accumulated increases as approximately  $\sqrt{\Delta t}$ , so for example, if one doubles the sampling interval the amount of mean sediment accumulated does not double but only increases by a factor of about 1.41. When considering the mean sediment transport *rate*,  $(S(t, \Delta t) / \Delta t)$ , this implies that

$$\frac{S(t, \Delta t)}{\Delta t} \propto \frac{\sqrt{\Delta t}}{\Delta t} = \Delta t^{-0.5}, \quad (7)$$

or that the bedload transport rate decreases with increasing sampling interval  $\Delta t$ . In other words, doubling the sampling interval results in a transport rate which is approximately  $0.7 (=1/\sqrt{2})$  times smaller.

If one then considers the statistical moments of order  $q$  higher than 1, **Figure 8** indicates that their scaling exponents  $\tau(q)$  do not increase as a linear power of  $q$  (the theoretical linear relationship is shown as a dashed line in the figure for comparison). So  $\tau(2)$  is slightly less than twice  $\tau(1)$ , etc. Therefore, the simple scaling described by equation (4) does not hold, and a multiscaling framework is required. This is consistent with the fact that we saw in section 2.2 that  $C_v$  decreased with scale, corresponding to the pdf narrowing with increasing sampling time: using equation (6), a decreasing  $C_v$  implies that  $\tau(2)$  is less than  $2\tau(1)$ .

Knowledge of the  $\tau(q)$  curve allows the complete rescaling of the pdf with changing sampling interval. It is often convenient to parameterize  $\tau(q)$  in order to describe the scaling properties of the data in a parsimonious way. Although several nonlinear parameterizations of  $\tau(q)$  are possible, a typical parameterization results from assuming that  $\tau(q)$  accepts a polynomial expansion of the form

$$\tau(q) = c_0 + c_1 q - \frac{c_2}{2} q^2 + \frac{c_3}{3!} q^3 + \dots, \quad (8)$$

with the constants  $c_i, i = 0, 1, 2, 3, \dots$  as the model parameters. In this work, due to the uncertainty in estimation of higher order moments from limited data, the polynomial is truncated at the second order i.e. it is assumed to be quadratic and all  $c_i$  are assumed to be zero for  $i > 2$ . This quadratic approximation, which is consistent with the so-called lognormal multiplicative cascade model (e.g., see Arneodo et al., 1998b), has been found adequate for modeling several geophysical processes including atmospheric boundary layer flows (e.g., Basu et al. 2006) and high resolution temporal rainfall (Venugopal et al. 2006), among others. The constant  $c_0 = \tau(0)$  is the scaling exponent of the zeroth-order moment, which will be equal to zero if the support fills the space, as is the case here. This leaves two parameters,  $c_1$  and  $c_2$ , to describe the scaling, which can be obtained by fitting a second degree polynomial to the  $\tau(q)$  curve. A least squares fitting was performed on the experimental data, and resulted in  $c_1 = 0.56$  and  $c_2 = 0.05$  (for comparison, for fully developed turbulence,  $c_1 = 1/3$  and  $c_2 = 0.025$ ). The  $\tau(q)$  curve fitted with this parameterization is shown as the solid curve in **Figure 8**, and can be seen to approximate very well the empirical curve. Assuming this model, the mean of the sediment accumulation is seen to scale as  $\langle S(\Delta t) \rangle \propto (\Delta t)^{c_1 - c_2/2}$ , which is dominated by the  $c_1$  value, while the scaling of the coefficient of variation is given by  $(C_v^2 + 1) \propto (\Delta t)^{-c_2}$ . Hence, the parameter  $c_2$  determines the widening of the pdf with decreasing scale. Scaling of higher order moments (and the whole pdf) can also be derived in terms of the two parameters  $c_1$  and  $c_2$  (e.g., see Venugopal et al., 2006 for an application to rainfall series).

## 5. Discussion and Conclusions

In this study the sampling-time dependence of the statistics of bedload transport has been examined, and the statistical moments have been found to change as power law functions of sampling time within a range of scales between 1 and 15 min. At temporal scales larger than around 30 min the statistics were observed to stabilize and become constant with sampling interval. This indicates that, at least for this system, the ideal way to measure bedload transport rates, to avoid any issues of scale-dependence, would be to use a sampling interval of 30 min or greater. While this would certainly be possible in a controlled laboratory experiment such as the one presented here, in field studies this approach may not be practical, since we expect that the critical sampling time will scale up with the dimension of the system, and so may require very long sampling times in large rivers. Such long sampling times may not be feasible if the bedload sampler has a finite capacity or if the flow conditions in the river change within this time. For this reason further research into the scale dependence of bedload transport is required, not only to determine the upper limit of variations with scale, but also to quantify the scale-dependence at shorter sampling times, in order to allow the correction of statistics in the cases when long sampling times are not feasible.

In this work we have outlined a framework to facilitate further investigation into sampling-time dependence, using the statistical moments of sediment transport to identify regimes of scale-invariance and scaling exponents  $\tau(q)$  to quantify the changes in the probability distribution with scale. The quadratic parameterization of  $\tau(q)$ , equation (7), allows description of the continuum of scaling exponents with just two parameters. This should become increasingly useful in future studies as experiments are performed in a range of differing conditions and researchers attempt to identify how the scale-dependence of bedload transport varies with parameters such as flow rate and sediment size-distribution. In this study, under low flow conditions and with a gravel bed of median particle size of 11.3mm,  $c_1$  was 0.56 and  $c_2$  was estimated to be 0.05, indicating that the mean amount of sediment transported increased as approximately the square-root of sampling time. This means that the sediment transport *rate* decreased as the inverse of the square-root of sampling time (within the scaling range). However, there is no reason to expect this behavior to be universal for sediment transport. The flow rate and geometry in this experiment produced a moderate dimensionless shear stress of approximately twice the critical values for the median grain size. Preliminary analysis of data from an experiment with a higher flow rate, and hence higher bed-stress, indicate a reversal of this scaling behavior, with the mean sediment transport rate increasing with sampling interval through a similar scaling regime. These results will be presented in a further publication (Singh et al. 2008), along with analysis of the relationship between the modes of sediment transport and the features of the bed elevation, which may explain some of the characteristic scales of the sediment transport.

Another long-term goal for further research is to understand the cause of the observed scaling, and its connection to the particle-scale dynamics. We believe that the scaling we see is not completely driven by the near bed turbulence, for the reason that the two-point statistics of the sediment transport rates (not presented here) do not show long range dependence, which would be characteristic of the multiplicative mechanism of eddy energy transfer in turbulence (Arneodo et al. 1998a). Instead, there is no long-range correlation, implying a different mechanism giving rise to the observed scaling, which we suggest might be particle interactions and emergent collective behavior. Of course, the grain size pdf and the shear stress are both important factors that influence such particle-scale dynamics and thus would affect the statistics of the resulting sediment transport rates. Exactly what type of particle-scale dynamics describes the observed statistics remains an open question which we plan to investigate in the future.

## Acknowledgments

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