Connectivity Equation and Shaly-Sand Correction for Electrical Resistivity
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Connectivity Equation and Shaly-Sand Correction for Electrical Resistivity

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Abstract

Estimating the amount of conductive and nonconductive constituents in the pore space of sediments by using electrical resistivity logs generally loses accuracy where clays are present in the reservoir. Many different methods and clay models have been proposed to account for the conductivity of clay (termed the shaly-sand correction). In this study, the connectivity equation (CE), which is a new approach to model non-Archie rocks, is used to correct for the clay effect and is compared with results using the Waxman and Smits method. The CE presented here requires no parameters other than an adjustable constant, which can be derived from the resistivity of water-saturated sediments. The new approach was applied to estimate water saturation of laboratory data and to estimate gas hydrate saturations at the Mount Elbert well on the Alaska North Slope. Although not as accurate as the Waxman and Smits method to estimate water saturations for the laboratory measurements, gas hydrate saturations estimated at the Mount Elbert well using the proposed CE are comparable to estimates from the Waxman and Smits method. Considering its simplicity, it has high potential to be used to account for the clay effect on electrical resistivity measurement in other systems.

Introduction

Most electrical-resistivity interpretation methods are based on Archie’s empirical law (Archie, 1942), which works well for the estimation of water and nonconducting constituent saturations in the pore space for clean (clay-free) sands, where formation matrices are poor conductors. However, Archie’s law is not accurate in estimating water saturation in shaly sands, where conductive clay minerals are present in the formation matrices. (Note: “shaly sand” is a commonly used term for a clay-bearing sand; see Worthington, 1985). Because clay minerals have high conductivities, in some cases higher than the conductivity of water in the formation, the effect of clay on electrical resistivity can be significant. Thus, the estimation of water saturation in the pore space using electrical resistivity log data is inaccurate if the resistivity from the conducting clay material is not accounted for.

In order to correct for clay conductivity on formation resistivity, a number of clay models have been proposed; Worthington (1985) summarized all available shaly-sand models. One of the earlier models was based on the assumption that the conductivity of an aggregate of conductive particles saturated with conducting fluid can be represented by resistors in parallel (Wyllie and Southwick, 1954). Simandoux (1963) used this concept and proposed a shaly-sand model that shows the conductivity of the formation to be the sum of the conductivity through the water and the conductivity through the clay minerals. Other models include

1. A shaly-sand model by Waxman and Smits (1968) based on the fact that clay particles contribute exchange cations to the electrolyte, thereby increasing the conductivity of the formation; and

2. A dual-water model by Clavier and others (1977) based on the assumption that the exchange cations contribute to the conductivity of claybound water that is spatially separated from the bulk water.

All shaly-sand correction methods presented in Worthington (1985) require additional input parameters such as the clay resistivity for the Simandoux method (1963) or cation exchange capacity and equivalent conductance of the clay counterions for the Waxman and Smits model (1968).

Recently, Montaron (2009) introduced a connectivity theory (or connectivity equation), which is a new approach to model “non-Archie rocks,” and applied the theory to correct the effect of clay conductivity on the basis of the dual water model (Clavier and others, 1977). The purpose of this study is to propose a modification of the connectivity equation of Montaron (2009) that accounts for the shale effect on electrical resistivity. In contrast to conventional shaly-sand methods, however, the method presented here requires no additional parameters other than an adjustable parameter that can be derived from the resistivity of water-saturated sediments.
Theory

Archie Equation

The electrical resistivity of water-saturated sediments \( R_w \) can be expressed using the well-known Archie equation (Archie, 1942) in the following way:

\[
R_w = \frac{aR_o}{\phi^n} \tag{1}
\]

where \( R_o \) is the formation resistivity of water-saturated sediment, \( R_w \) is the resistivity of connate water, \( a \) and \( m \) are Archie constants, and \( \phi \) is porosity. Archie constants \( a \) and \( m \) can be derived empirically; \( m \) is commonly called the cementation factor. Equation 1 indicates that a plot of \( \log R_w \) relative to \( \log \phi \) is linear and the slope is given by \( m \) if \( R_o \) is constant throughout the sedimentary interval being examined.

The water saturation \( (S_w) \) in a formation from resistivity log values for hydrocarbon-bearing sediments is given by Archie (1942) as:

\[
S_w = \left( \frac{aR_w}{\phi^n R_o} \right)^{\frac{1}{m}} \tag{2}
\]

where \( n \) is an empirically derived parameter close to 2 and \( R_o \) is the formation resistivity with gas hydrate or other hydrocarbons. The parameter \( n \) varies between 1.715 (unconsolidated sediment) and 2.1661 (sandstone); although somewhat dependent on the lithology of the reservoir, \( n \) is typically 1.9386 (Pearson and others, 1983) and is taken as 2 in this study.

Waxman and Smits Equation

The effect of clay on the formation resistivity can be corrected using various shaly-sand correction methods (Worthington, 1985). The Waxman and Smits (1968) equation (WSE) for the shaly sand correction is given by:

\[
\frac{1}{R_o} = \frac{S_w^a \phi^m + S_w^{a-1} \phi^n B Q_v}{a R_w} + \frac{S_w^{a-1} V_c (1 - \phi)}{R_o} \tag{3}
\]

where \( B \) is the equivalent conductance of the clay counterions, \( Q_v \) is the cation exchange capacity per unit pore volume, \( V_c \) is the shale volume, and \( R_o \) is the resistivity of clay.

Connectivity Equation

Montaron (2009) proposed a connectivity equation (CE) to estimate the pore saturants in sediments instead of using the standard Archie equation, one advantage being that only one parameter is needed to estimate water saturation. The equation is defined as:

\[
R_c = \frac{a R_w}{(S_w \phi - \chi_w)^\mu} \tag{4}
\]

where \( \mu \) is the connectivity exponent (which is equivalent to Archie’s equation when \( n = m \)), \( a \) is the conventional Archie parameter, and \( \chi_w \) is the water connectivity correction index, which is a small number typically ranging from \(-0.02\) to \(0.02\) (Montaron, 2009). Because \( \chi_w \) is small, the effect of \( \chi_w \) in the denominator of equation 4 generally can be ignored. However, in freshwater with high conductivity due to the presence of shale, \( \chi_w \) could be large and should be retained in equation 4.

Montaron (2009) defined the CE with \( a = 1 \) in equation 4. Generally, \( a = 1 \) is accurate for clean sand or in a case where a shaly-sand correction is applied to the measured resistivity (Lee and Collett, 2006). Theoretically, therefore, CE with \( a = 1 \) appears to be more accurate because the shale effect on the resistivity is accounted for by using the parameter \( \chi_w \) in equation 4. However, connectivity equation with the parameter \( a \) is defined in this report to make the connectivity equation to be more flexible for the real data analysis.

Connectivity Equation and Shaly-Sand Correction

Equation 4 is useful in analyzing the resistivity of the shaly sands as shown in Montaron (2009), in which the dual water model by Clavier and others (1977) was investigated using the CE. The clay effect on the electrical resistivity is through \( \chi_w \) in the equation, which depends on a particular shaly-sand model. For example, Montaron (2009) used the dual water model of Clavier and others (1977) to derive \( \chi_w \), which is given by:

\[
\chi_w = -S_w \phi \left[ \frac{R_w}{(R_w)} \right]^{1/\mu} - 1 \tag{5}
\]

where \( R_w \) and \( R_o \) are the resistivity and the bulk volume of clay water, respectively.

The WSE can be written as the following, assuming \( n = m = \mu \) in equation 3:

\[
R_c = \frac{a R_w}{S_w^\mu \phi^\mu + S_w^{\mu-1} a R_w B Q_v \phi^\mu} \tag{6}
\]

Using equations 4 and 6, the connectivity correction index \( \chi_w \) can be written as:

\[
\chi_w = \frac{S_w \phi (1 - \Gamma)}{1 - S_w \phi \Gamma} \tag{7}
\]

with

\[
\Gamma = \left[ 1 + \frac{a R_w B Q_v}{S_w} \right]^{1/\mu} \tag{8}
\]

If \( n \) and \( m \) shown in equation 3 are used for \( \Gamma \) in equation 8, that is \( \Gamma = [1 + a R_w B Q_v / S_w]^{1/\mu} \), then CE is identical to WSE. However, in the case where there is no information about \( m \) and \( n \), it is useful to use equation 7 with equation 8 to derive the shaly-sand correction by calculating \( \chi_w \) using various \( \mu \); the results can be examined to assess the accuracy of the
method. Note that the effect of clay resistivity is in \( \chi_w \); thus, the CE for clean sand is equation 4 with \( \chi_w = 0 \).

Proposed Connectivity Method

The water-connectivity correction index \( \chi_w \) was originally based on the percolation threshold of the porous media, and the conductivity exponent was nearly 2 according to numerical simulations (Montaron, 2009). However, numerical simulations demonstrated that a best-fit \( \mu \) depends on the \( \chi_w \) value used, which implies the nonuniqueness of the connectivity equation unless \( \chi_w \) is fixed. Also, different connectivity equations can be developed using different value of \( \chi_w \).

When applying the CE to shaly-sand correction, \( \chi_w \) was derived based on equation 4 with the WSE. Montaron (2009) used the following approximation equation instead of equation 7:

\[
R_t = \frac{aR_w}{(S_w\phi - \chi_w)^\mu} \quad (9)
\]

However, if equation 9 with \( \chi_w \) given by

\[
\chi_w = S_w\phi(1 - \Gamma) \quad (10)
\]

is used to calculate the resistivity, then the calculated resistivities are identical to those from equation 4 with equation 7. In other words, whether using equation 4 or a simpler equation 9 is immaterial \( \chi_w \) if is calculated according to the corresponding equations.

The nonuniqueness of the CE opens a new way to apply it to the shaly-sand correction. Because an optimum \( \mu \) depends on \( \chi_w \), the following procedures are proposed to apply the CE to the shaly-sand correction.

1. For a given \( \mu \), calculate \( \chi_w \) according to the following equation using an adjustable parameter \( \alpha \):

\[
\chi_w = aC_v\phi^\alpha S_w \quad (11)
\]

2. Calculate the resistivity of water-saturated sediments using equation 9 and compare with the measured resistivity. Adjust \( \alpha \) until there is a satisfactory agreement between the calculated and measured resistivities of water-saturated sediments.

3. Estimate the water saturation using equations 9 and 10 with estimated \( \alpha \).

Because the connectivity correction index \( \chi_w \), shown in equation 11, is calculated without any particular shaly-sand models (for example, Worthington, 1985), this approach is universal for the shaly sand correction. The advantage of this approach is that no additional parameters are required other than estimating \( \alpha \) by using a trial-and-error method.

Analysis of Formation Resistivity

Laboratory Data

To test the feasibility of the CE for shaly-sand correction, data from Ohirhian (1998) were used. The data by Ohirhian (1998) in the form of values for \( B \) (equivalent conductance of the clay counterions) and for \( Q_v \) (cation exchange capacity per unit volume) were used, and these values were converted to the equivalent resistivity of clay using equation 12, assuming \( V_c = 0.2 \) for all samples. The measured and calculated values for the Ohirhian (1998) data are shown in table 1.

The relation between the resistivity of clay and \( B \) and \( Q_v \) can be derived from equation 3; it is given by:

\[
\frac{V_c(1 - \phi)}{R_c} = BQ_v\phi^m. \quad (12)
\]

The measured \( n \) value ranges from 1.6 to 2.3, and the Archie cementation parameter, \( m \), varies between 1.6 and 2.0. For all samples, \( a = 1.0 \) is assumed. Table 1 includes the saturations estimated from the connectivity equation using \( \mu = n = m = 2 \) along with those estimated using the WSE by Ohirhian (1998) with different \( n \) and \( m \) values for each sample shown in table 1. Figure 1 compares the two saturation estimates. Except for samples 2 and 5, the saturations estimated using the CE are more than 80 percent accurate. Observations that can be made from table 1 are:

- Where the value of \( n \) is close to \( \mu \), the estimated saturation is more accurate.
- As the saturation increases, the accuracy increases (compare samples 1 and 2).
- As expected, if \( \mu \) is less than \( n \), the CE overestimates, and if \( \mu \) is greater than \( n \), CE underestimates.

Well Log Analysis

In order to test the proposed connectivity method, well logs acquired at the Mount Elbert well on the Alaska North Slope were analyzed. Details of resistivity log analysis at this well are given in Lee and Collett (2011); figure 2 shows the calculated \( R_w \) from the measured salinity and temperature along with the measured electrical resistivity. Note that the average \( R_w \) for the interval between 2,000 and 2,500 ft is about 2 ohm-m except for two intervals with much higher resistivities that reflect the effect of low salinity at low temperature.
Connectivity Equation and Shaly-Sand Correction for Electrical Resistivity

Table 1. Parameters for eight samples from Ohirhian (1998) with calculated resistivity of clay and saturations estimated using the connectivity equation with the connectivity exponent $\mu = 2$.

[Resistivity of clay ($R_c$), resistivity of water ($R_w$), and resistivity of formation ($R_t$) are in ohm meters. $S_{w}$, saturation estimated by Ohirhian using the Waxman and Smits (1968) equation; $S_{c}$, saturation estimated using the connectivity equation $\phi$, porosity; $m$, Archie cementation parameter; $n$, Archie saturation exponent]

<table>
<thead>
<tr>
<th>Sample</th>
<th>$R_w$</th>
<th>$\phi$</th>
<th>$R_c$</th>
<th>$n$</th>
<th>$m$</th>
<th>$R_t$</th>
<th>$S_{w}$</th>
<th>$S_{c}$</th>
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<td>1</td>
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<td>50</td>
<td>0.80</td>
<td>0.64</td>
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<tr>
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<td>6</td>
<td>0.27</td>
<td>3</td>
<td>1.6</td>
<td>1.6</td>
<td>20</td>
<td>0.32</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
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<td>0.25</td>
<td>2.1</td>
<td>2.2</td>
<td>1.9</td>
<td>30</td>
<td>0.51</td>
<td>0.59</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>0.25</td>
<td>2.1</td>
<td>1.9</td>
<td>1.9</td>
<td>30</td>
<td>0.60</td>
<td>0.59</td>
</tr>
<tr>
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<td>2.85</td>
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<td>2.0</td>
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<td>0.25</td>
<td>0.36</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>0.25</td>
<td>3.0</td>
<td>2.0</td>
<td>1.9</td>
<td>50</td>
<td>0.65</td>
<td>0.67</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>0.3</td>
<td>3.9</td>
<td>2.1</td>
<td>1.8</td>
<td>35</td>
<td>0.32</td>
<td>0.35</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>0.25</td>
<td>5.9</td>
<td>2.1</td>
<td>1.8</td>
<td>70</td>
<td>0.49</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Figure 1. Saturations estimated from the Waxman and Smits equation by Ohirhian (1998) are compared with those estimated from the connectivity equation with the connectivity exponent $\mu = 2$, which is equivalent to using Archie parameter $a = 1$, $m = 2$, and $n = 2$.

Figure 2. Plot showing measured electrical resistivity with shale volume calculated from gamma ray log, resistivity of pore water calculated using salinity and temperature, and baseline resistivity calculated from porosity and resistivity of connate water ($R_w$) with the Archie parameters $a = 1.7$ and $m = 1$. $R_o$, resistivity of water-saturated sediments.
Considerations for the Application of Connectivity Equation

Archie Parameters and Connectivity Exponent

Theoretically, the Archie parameters for clean sands are \( a = 1 \) and \( m = 2 \). However, many core measurements require \( a \neq 1 \) to fit the observations (Carothers, 1968; Porter and Carothers, 1970). If \( a = 1 \), then equation 1 yields inconsistent \( R_w \) at \( \phi = 1 \). However, Lee and Collett (2006) suggested that \( a \neq 1 \) is due to the effect of shale on the measured resistivity.

The value of \( m \) is estimated from the cross plot between porosity and the formation factor of shaly sands. Therefore, if the shale effect is not accounted for, the estimated \( m \) may be different from \( m \) for clean sands. Laboratory measurement of \( m \) for shaly sands with clay correction ranges from \( m = 1.8 \) to \( m = 2.1 \) (Waxman and Thomas, 1974), whereas \( m \) can be as
low as \( m = 1.4 \) without clay correction for the same dataset. Smaller \( m \) for shaly sand compared to \( m \) for clean sand agrees with the prediction of the shaly-sand model by Lee and Collett (2006). The \( m \) values for the shaly sand by Ohirhian (1998) range from 1.6 to 2.0. Therefore, \( a = 1, m = 2, \) and \( n = 2 \) are reasonable parameters to use where there is no information for these parameters.

The application of the CE is intended for use in the case that there is no information for the shaly-sand model such as the resistivity of clay, clay counterions, the cation exchange capacity, and others. Therefore, the proposed CE can be considered as a kind of blind application of the shaly-sand model based solely on the measured resistivity. Because \( a = 1, \ m = 2, \) and \( n = 2 \) represent a reasonable model for sand, \( \mu = 2 \) is appropriate to use unless more information for the shaly-sand model is available.

### Saturation Differences

The saturation difference estimated from the connectivity and Archie equations can be assessed from the following clean (clay-free) sand equations. For clean sand, the WSE \( (R'^{w}) \) can be written as

\[
R'^{w} = \frac{aR_w}{S_w \phi^m}
\]  

(13)

and the CE \( (R'^{ce}) \) as

\[
R'^{ce} = \frac{aR_w}{S_w \phi^\mu}
\]

(14)

From equations 13 and 14, the relation between the conductivity index (\( \mu \)) and the saturation exponent (\( n \)) can be written as:

\[
\mu = n + \frac{(m - n) \ln \phi}{\ln S_w}
\]

(15)

Equation 15 indicates that:

1. Saturations estimated from the CE differ from those from the Archie equation unless \( m = n \).

2. The difference between \( n \) and \( \mu \) increases as \( \phi \) decreases and \( S_w \) increases. Consequently, the saturations estimated from the CE are similar to those from the Archie equation for sediments with high porosity and low water saturation.

3. If \( m < n \), then the CE overestimates the saturation in comparison to the Archie equation irrespective of water saturation and porosity.

4. Depending on the sign of the second term of equation 14, the CE overestimates or underestimates the saturation in comparison to the Archie equation.

### Original and Proposed Connectivity Methods

As shown previously and in Montaron (2009), the water-connectivity correction index for shaly sand correction depends on shaly-sand models such as those represented by equation 5 for the dual water model and by equation 7 for the Waxman and Smits (1968) model. According to Lee and Collett (2011), the Archie parameters appropriate for the electrical log at the Mount Elbert well are \( a = 1, m = 1.6, \) and \( n = 2 \). Because \( n \) differs from \( m \), there are two options for \( \mu: \mu = m = 1.6 \) or \( \mu = n = 2 \). Figure 4A shows the calculated resistivity for water-saturated sediments (WSS) using \( \mu = 1.6 \) and \( m = 2 \); the CE with \( \mu = 1.6 \) closely predicts the resistivity, whereas the CE with \( \mu = 2 \) overestimates the resistivity. On the other hand, the gas hydrate saturation estimated, shown in figure 4B, indicates that the CE with \( \mu = 1.6 \) highly overestimates the gas hydrate saturations, and the CE with \( \mu = 2 \) slightly underestimates the saturation. If there is no saturation estimated from the NMR log, it would be concluded that the CE with \( \mu = 1.6 \) is more accurate than the CE with \( \mu = 2 \) because the calculated resistivity of water-saturated sediments with \( \mu = 1.6 \) agrees more closely with the measured resistivity even though the saturations are actually highly overestimated.
Figure 4. A, measured and calculated resistivity at the Mount Elbert well on the Alaska North Slope, using the connectivity equation (CE) (Montaron, 2009) with Archie constant $a = 1$, and two values for connectivity exponent $\mu = 1.6$ and $\mu = 2$. B, gas hydrate saturations estimated from the nuclear magnetic resonance (NMR) log and using the proposed CE.
The results from the proposed CE with $\mu = 2$ indicate that calculated resistivities of water-saturated sediments and gas hydrate saturations agree well with those measured and those estimated from the NMR log, respectively, as shown in figure 3. This comparison indicates a great potential for the proposed connectivity method for the shaly-sand correction, and further investigations are warranted.

**Conclusions**

Based on the nonuniqueness of the connectivity equation (CE), a simple CE is proposed to account for the effect of conductive clay on the electrical resistivity of sediments. The proposed CE was applied to account for the effect of shale on electrical resistivity by using laboratory data and well logs acquired at the Mount Elbert well on the Alaska North Slope. By incorporating gas hydrate saturations estimated from the nuclear magnetic resonance (NMR) porosity log, it can be concluded that if all parameters such as the Archie constants and characteristics of clay are available, then a specific shaly-sand model (for example, that of Waxman and Smits [1968]) is optimal in estimating saturations (for example, hydrocarbon or gas hydrate). However, this approach requires additional information such as the resistivity of clay or cation exchange capacity. In cases where there is no available information for the electrical properties of shale, the proposed CE can be used to model the resistivity of the sediments.

It is demonstrated that the proposed CE can be used for shaly-sand correction without any parameters, assuming that the Archie parameters for clean sand are $a = 1$ and $m = 2$ and the connectivity exponent $\mu = n = m = 2$. The water connectivity-correction index is derived independently from any particular shaly-sand models, and the proposed CE requires one adjustable parameter $\alpha$. The parameter $\alpha$ can be estimated by a trial-and-error method by fitting resistivity of water-saturated sediments calculated from the CE to the measured resistivity. The gas hydrate saturations estimated from the proposed CE are comparable to those from the NMR porosity log acquired at the Mount Elbert well on the Alaska North Slope. Saturations estimated from the proposed CE become more accurate as the porosity increases and water saturation decreases.

Because the proposed CE is simple and requires no additional parameters except an adjustable parameter, more investigations involving the application of the proposed CE to shaly-sand correction are warranted for estimating gas hydrate saturations or any other hydrocarbons from the resistivity logs.

**References Cited**


