Estimation of Natural Historical Flows for the Manitowish River near Manitowish Waters, Wisconsin
Cover. Rest Lake Dam as viewed from Rest Lake, part of the Manitowish Chain of Lakes. (Photograph by James M. Klosiewski, Wisconsin Department of Natural Resources, September 22, 2002.)
Acknowledgments

Local Project Coordinator
James Kreitlow, Water Resources Management Specialist, Wisconsin Department of Natural Resources, Rhinelander, Wis.

Field Support

Publishing Support
Laura Nelson, Office Assistant, U.S. Geological Survey Wisconsin Water Science Center, Middleton, Wis.

Technical Reviewers

Editorial Reviewer

Approving Official
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Conversion Factors and Datum

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<td>cubic meter (m³)</td>
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<tr>
<td><strong>Flow rate</strong></td>
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<tr>
<td>cubic foot per second (ft³/s)</td>
<td>0.02832</td>
<td>cubic meter per second (m³/s)</td>
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<td>cubic foot per second per square mile [(ft³/s)/mi²]</td>
<td>0.01093</td>
<td>cubic meter per second per square kilometer [(m³/s)/km²]</td>
</tr>
<tr>
<td>inch per year (in/yr)</td>
<td>25.4</td>
<td>millimeter per year (mm/yr)</td>
</tr>
</tbody>
</table>

Vertical coordinate information is referenced to the North American Vertical Datum of 1988 (NAVD 88).

Altitude, as used in this report, refers to distance above the vertical datum.
Abstract

The Wisconsin Department of Natural Resources is charged with oversight of dam operations throughout Wisconsin and is considering modifications to the operating orders for the Rest Lake Dam in Vilas County, Wisconsin. State law requires that the operation orders be tied to natural low flows at the dam. Because the presence of the dam confounds measurement of natural flows, the U.S. Geological Survey, in cooperation with the Wisconsin Department of Natural Resources, installed streamflow-gaging stations and developed two statistical methods to improve estimates of natural flows at the Rest Lake Dam.

Two independent methods were used to estimate daily natural flow for the Manitowish River approximately 1 mile downstream of the Rest Lake Dam. The first method was an adjusted drainage-area ratio method, which used a regression analysis that related measured water yield (flow divided by watershed area) from short-term (2009–11) gaging stations upstream of the Manitowish Chain of Lakes to the water yield from two nearby long-term gaging stations in order to extend the flow record (1991–2011). In this approach, the computed flows into the Chain of Lakes at the upstream gaging stations were multiplied by a coefficient to account for the monthly hydrologic contributions (precipitation, evaporation, groundwater, and runoff) associated with the additional watershed area between the upstream gaging stations and the dam at the outlet of the Chain of Lakes (Rest Lake Dam). The second method used to estimate daily natural flow at the Rest Lake Dam was a water-budget approach, which used lake stage and dam outflow data provided by the dam operator. A water-budget model was constructed and then calibrated with an automated parameter-estimation program by matching simulated flow-duration statistics with measured flow-duration statistics at the upstream gaging stations. After calibration of the water-budget model, the model was used to compute natural flow at the dam from 1973 to 2011.

Daily natural flows at the dam, as computed by the adjusted drainage-area ratio method and the water-budget method, were used to compute monthly flow-duration values for the period of historical data available for each method. Monthly flow-durations provide a means for evaluating the frequency and range in flows that have been observed for each month over the course of many years. Both methods described the pattern and timing of measured high-flow and low-flow events at the upstream gaging stations. The adjusted drainage-area ratio method generally had smaller residual errors across the full range of observed flows and had smaller monthly biases than the water-budget method. Although it is not possible to evaluate which method may be more “correct” for estimating monthly natural flows at the dam, comparisons between the results of each method indicate that the adjusted drainage-area ratio method may be susceptible to biases at high flows due to isolated storms outside of the Manitowish River watershed. Conversely, it appears that the water-budget method may be susceptible to biases at low flows because of its sensitivity to the accuracy of reported lake stage and outflows, as well as effects of upstream diversions that could not be fully compensated for with this method.

Results from both methods are useful for understanding the natural flow patterns at the dam. Flows for both methods have similar patterns, with high median flows in spring and low median flows in late summer. Similarly, the range from monthly high-flow durations to low-flow durations increases during spring, decreases during summer, and increases again during fall. These seasonal patterns illustrate a challenge with interpreting a single value of natural low flow. That is, a natural low flow computed for September is not representative of a natural low flow in April. Moreover, alteration of natural flows caused by storing water in the Chain of Lakes during spring and releasing it in fall causes a change in the timing of high and low flows compared with natural conditions. That is, the lowest reported dam outflows occurred in spring and highest reported outflows occurred in fall, which is opposite the natural patterns.
Introduction

In 1887, a dam was constructed on the Manitowish River at the outlet of Rest Lake (the Rest Lake Dam) and formed the Manitowish Chain of Lakes. The Manitowish Chain of Lakes includes ten lakes with a total area of 4,266 acres in Vilas County, Wis. The Wisconsin Department of Natural Resources (WDNR) is charged with oversight of the dam operation, as outlined in a 1939 Public Service Commission hearing (Public Service Commission, 1939). As part of the 1939 ruling, the Public Service Commission aimed to balance the multiple needs and uses for water both upstream and downstream of the dam, including storage of water for downstream hydropower generation.

The current (2012) operation orders for Rest Lake Dam state that the minimum lake stage should be 5.0 feet (ft) above a specified local gage datum (1,597.9 ft above North American Vertical Datum of 1988 (NAVD 88)) and the maximum stage should be 8.5 ft (1,601.4 ft above NAVD 88). In addition, chap. 31 of Wisconsin State Statutes specifies that the minimum flow out of every dam in the State should be no less than 25 percent of the natural low flow (Wisconsin Statutes and Annotations, 2012). Dams alter the system from natural conditions, however, often leaving 25 percent of the natural low flow difficult to determine. In light of this uncertainty, the WDNR has equated 25 percent of natural low flow with the minimum 7-day mean flow that occurs once every 10 years (Q_{10}) as described on page 3 of the “Current operation of Rest Lake Dam – frequently asked questions” (Wisconsin Department of Natural Resources, 2007). The WDNR has considered the Q_{10} for the Rest Lake Dam to be 40 cubic feet per second (ft³/s) because this is the flow that the dam operator has used in documentation of dam operation. Implementation of these operation orders and State statutes has resulted in management of the reservoir stage and minimum released flow during most years according to table 1.

Table 1. Generalized operation of the Rest Lake Dam.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Lake stage, in feet, on dam staff gage</th>
<th>Required flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 8 to spring ice break-up (when ice is 75 percent off of Manitowish Chain of Lakes; approximately April 20)</td>
<td>Minimum of 5 ft 0 in.</td>
<td>Run of river</td>
</tr>
<tr>
<td>Spring ice break-up (approximately April 20) to July 1</td>
<td>5 ft 0 in. to 8 ft 6 in.</td>
<td>40 ft³/s or more</td>
</tr>
<tr>
<td>July 1 to September 28</td>
<td>No lower than 8 ft 4 in.</td>
<td>40 ft³/s or more</td>
</tr>
<tr>
<td>September 28 to November 8</td>
<td>No lower than 5 ft 0 in.</td>
<td>40 ft³/s of more</td>
</tr>
</tbody>
</table>

Following a 2001 Federal Energy Regulatory Commission finding that “management of the water levels and flows [from the Chain of Lakes] was neither used and useful nor necessary or appropriate to maintain or operate for hydropower generation,” the Federal Energy Regulatory Commission relinquished its authority over the dam operation (Wisconsin Department of Natural Resources, 2011). In recognition of this change to the balance in competing uses of water in the Manitowish Chain of Lakes, and the WDNR’s obligations under the Public Trust Doctrine (Wisconsin Legislative Reference Bureau, 2004), the WDNR is proposing “to issue a new operating order to the owners of the Rest Lake Dam to specify water levels and flows that balance and protect public water resource rights as well as life, health, and property, both upstream and downstream of the dam” (Wisconsin Department of Natural Resources, 2011). As part of establishing new operating orders, the WDNR partnered with the U.S. Geological Survey (USGS) to install streamflow-gaging stations on the Manitowish River system and improve estimates of natural flow at the Rest Lake Dam.

There are several scientific challenges to estimating natural flow at the Rest Lake Dam, the most significant of which is the lack of historical data that were not affected by the dam operation. In addition, it is unclear if the Q_{10} is a good approximation of 25 percent of the natural low flow at the dam. As a result, this study focused on estimating flow durations that describe the frequency and range in natural flows on a monthly basis. In the Upper Wisconsin River Basin, which borders the Manitowish River watershed to the east, the Q_{10} has been estimated to approximately equal the 99.6 percent flow duration (Gebert, 1980). The 99.6 flow duration is the daily mean flow that is exceeded 99.6 percent of the time. Flow durations are typically evaluated using long-term records of daily mean flow, but can be computed for any recurring period, such as a week or month. Monthly flow-duration tables provide a means for evaluating the frequency and range in flows that are observed over the course of annual cycles that incorporate high-flow and low-flow conditions. It should be noted that the monthly 99.6 percent flow-duration values produced as part of this study are not expected to equate exactly with Q_{10} values, which are based on 7 consecutive days and, as a consequence, are computed using continuous records—not for example, on a monthly basis. In addition, flow-duration estimates are a commonly used metric to describe the frequency and range in measured flows because the flow-duration estimates have greater resistance to extreme events (Helsel and Hirsch, 2002) than, for example, the Q_{10}, which is a singular extreme event that would be difficult to estimate with indirect methods such as those described in this report.
Purpose and Scope

The purpose of this report is to describe the natural historical flow in the Manitowish River at State Highway 51 near Manitowish Waters (station 05357302), as characterized by monthly flow-duration values. Because of a lack of direct flow measurements at the dam outlet, flow measured at State Highway 51 (05357302) was assumed to represent flow at the Rest Lake Dam, located approximately 1 mile (mi) upstream of State Highway 51. In addition, the methods used in this study involved computing historical average daily flows, but the scope of work is limited to describing the computed flow patterns as represented by monthly flow-duration tables. That is, every effort was made to compute accurate historical daily flows, but daily flows are not the focus of the study – the focus of the study is to produce summary statistics that describe broader historical flow patterns. This distinction is made because individual daily flow estimates are expected to incorporate greater uncertainty than summary statistics that describe the distribution of hundreds of individual daily flow estimates.

Physical Setting

The Manitowish River watershed upstream of State Highway 51, which covers approximately 250 square miles (mi²), is part of the Northern Lakes and Forests ecoregion (Omernik and others, 2000). This area has low-to-moderate relief and contains many wetlands and headwater lakes (fig. 1). The surficial geology of the watershed consists of glacial deposits, including pitted outwash and other ice-contact deposits composed of sand, gravel, and glacial till. The depth of the surficial materials may be up to 100 ft over igneous and metamorphic bedrock (Young and Hindall, 1972; Mudrey and others, 1982). The groundwater-flow system is dominated by permeable glacial deposits, and is generally in good communication with the surface-water system (Pint, 2002; Walker and others, 2012). Soils of the rolling-to-undulating uplands are primarily sandy loam, loamy sand, and sand, with permeabilities of 2.5 to 5 in/hr or more, and muck or peat soils in the wetlands (Hole and others, 1968; Oakes and Cotter, 1975). Flow into the Manitowish Chain of Lakes is primarily from four rivers: Manitowish River, Trout River, Rice Creek, and Papoose Creek.

Data Sources

Four streamflow-gaging stations were installed in October 2009 and monitored through November 2011 to provide flow data (Rantz and others, 1982) for evaluating 2 years of flow into and out of the Chain of Lakes (fig. 1). These gaging stations were located at the Manitowish River at State Highway 51 near Manitowish Waters (station 05357302), the Trout River near Boulder Junction (station 05357259), the Manitowish River near County Highway H near Boulder Junction (station 05357157), and Rice Creek at County Highway K near Boulder Junction (station 05357182). In addition, lake stage was also measured at the Rest Lake Dam (station 460819089530500) using an automated digital recorder. Flow was measured intermittently at Papoose Creek at County Highway K near Manitowish Waters (station 05357299). All flow and lake-stage data collected as part of this study are available online from the USGS: http://waterdata.usgs.gov/wi/nwis/sw.

Historical data for lake stage and flow over the Rest Lake Dam were provided by the dam operator. Historical flow data for the Bear River near Manitowish Waters (station ID 05357335) and the Trout River at Trout Lake near Boulder Junction (station ID 05357245), which were used for regression analyses, were from the USGS National Water-Information system and are available online at http://waterdata.usgs.gov/wi/nwis/sw.

Methods

Two independent methods, the adjusted drainage-area ratio method and the water-budget method, were used to estimate historical natural flow at the Rest Lake Dam. Both methods used the two years of measured total flow into the Manitowish Chain of Lakes to calibrate equations or parameters specific to each method. Then, daily natural flows at the Rest Lake Dam were computed using historical flow data from nearby streams for the adjusted drainage-area ratio method, and historical dam outflow data from the dam operator for the water-budget method. Finally, flow-duration tables were generated from the computed daily natural flows at the Rest Lake Dam.
Figure 1. The Manitowish Chain of Lakes study area in north central Wisconsin and streamflow-gaging stations used during this study.
The first step for both methods was to compute the total natural flow into the Manitowish Chain of Lakes, as measured at the four upstream gages on the major tributaries to the Chain of Lakes during the 2-year study (fig. 1). Given the relatively small flows in Papoose Creek (05357299) compared with the Manitowish River (05357157), Trout River (05357259), and Rice Creek (05357182), Papoose Creek was not continuously gaged. Instead, flow in Papoose Creek was measured 20 times during the study, and daily flows were estimated from daily flows measured in Rice Creek, using a regression relation developed between Papoose Creek and Rice Creek (fig. 2). In addition, measured daily flows for the Trout River near Boulder Junction (05357259) were adjusted to represent natural flow by removing the effects of diversions by cranberry bog operations (fig. 3). This was done because of the interest in natural flows and was accomplished by identifying rapid changes in flows caused by pump activation, which were visible in unpublished gaging station data recorded at 15-minute intervals. Natural flows during times of these surface-water diversions were estimated by interpolating between prior and subsequent daily mean flows that appeared to be unaffected by the diversions. Except for an unknown amount of water loss due to evaporation from diverted water stored in holding ponds and transpiration of diverted water used for irrigation, the remainder of the diverted water at the Trout River gage (05357259) is expected to return to the Chain of Lakes as groundwater discharge to the Chain of Lakes, and is incorporated into estimated flows at the Rest Lake Dam as described below in “adjusted drainage-area ratio method.”

Total measured flow into the Manitowish Chain of Lakes \(Q_{\text{measured}}\) was computed on a daily basis by summing daily values of adjusted natural flow at the Trout River at Boulder Junction (05357259), measured flow at Manitowish River near County Highway H near Boulder Junction (05357157), measured flow at Rice Creek at County Highway K near Boulder Junction (05357182), and estimated flow at Papoose Creek at County Highway K near Manitowish Waters (05357299). Total measured flow into the Manitowish Chain of Lakes was then used to develop a relation with nearby long-term gaging stations (the adjusted drainage-area ratio method) and to calibrate a water-budget model that used long-term dam operation records.

**Figure 2.** Measured flows at Rice Creek at County Highway K near Boulder Junction (station 05357182) and Papoose Creek at County Highway K near Manitowish Waters (station 05357299) during the 2-year study period.
Lake stage and dam outflow reported by the dam operator were used in both the adjusted drainage-area ratio method and the water-budget method. A few adjustments were needed to make the lake stage and flow data reported by the dam operator more useful for this study. First, flows for periods with missing data or very questionable data (for example, large changes in lake stage without a corresponding change in dam outflow) were estimated, typically by linear interpolation between the preceding and subsequent values. Second, flows reported by the dam operator were known to differ from flows measured by the WDNR and USGS between July 2002 and November 2011 at the downstream gage at State Highway 51 (05357302); therefore, the results of a regression equation were used to adjust reported historical dam outflow data from 1973 to April 28, 2009, to flows measured by the WDNR from July 2002 to April 28, 2009 (fig. 4A). A second equation was applied to the reported dam outflow data from April 29, 2009, to November 30, 2011, using daily gaging station data from December 1, 2009, to November 30, 2011 (fig. 4B). Two separate equations were needed because dam improvements performed on April 29, 2009 (Dean Steines, written commun., May 26, 2009) changed the relation between estimated flows provided by the dam operator and flows measured by the WDNR and USGS (fig. 4).

Adjusted Drainage-Area Ratio Method

The adjusted drainage-area ratio method involved two steps to estimate natural historical flow at the Rest Lake Dam. The first step was to estimate total flow into the Manitowish Chain of Lakes from nearby long-term streamflow-gaging stations. The second step was to adjust this computed flow into the Chain of Lakes into natural flow at the downstream Rest Lake Dam.

Estimating total flow into the Manitowish Chain of Lakes was accomplished by relating measured flow divided by watershed area (water yield) at the up-stream gages for the monitoring period (December 2009 to December 2011) and total water yield for two nearby long-term gaging stations, the Bear River near Manitowish Waters (05357335) and Trout River at Trout Lake near Boulder Junction (05357245). Two different rivers were used instead of one river to better describe the overall patterns in runoff throughout the area. Yields were used for the regression equation rather than flows to ensure that differences in stream size were minimized so that both the Bear River and Trout River gages had equal influence on the relation with measured flow at the gages $Q_{measured}$. 

Figure 3. Estimated flow without the effects of upstream diversions and measured flow for the Trout River near Boulder Junction (station 05357259). Negative flows were measured when lake water flowed into the stream channel.
**Figure 4.** Flows measured by the Wisconsin Department of Natural Resources and U.S. Geological Survey at the Manitowish River at State Highway 51 (station 05357302) and dam outflows at the Rest Lake Dam outlet reported by the dam operator. 

One of two Line of Organic Correlation equations (equations 1 and 2) were used to compute yield at the upstream gages, dependent upon the measured lake stage:

\[ G_{\text{low stage}} = 0.3676(Y_{\text{Bear}} + Y_{\text{Trout}}) + 0.2809 \]  
\[ G_{\text{high stage}} = 0.3789(Y_{\text{Bear}} + Y_{\text{Trout}}) + 0.1662 \]

where \( Y_{\text{Bear}} \) and \( Y_{\text{Trout}} \) are the daily yields for the Bear River near Manitowish Waters (05357335) and the Trout River at Trout Lake near Boulder Junction (05357245), respectively, and \( G_{\text{low stage}} \) and \( G_{\text{high stage}} \) are the total computed yield for all of the upstream gages at low and high lake stages, respectively.

Line of Organic Correlation equations (Helsel and Hirsch, 2002, p. 276–278) were used for the regression instead of more common regression methods, such as Ordinary Least Squares, to better maintain the distributional properties (standard deviation or percentiles) of the estimated yield for the upstream gaging stations \( (G_{\text{low stage}} \) and \( G_{\text{high stage}}) \). That is, Ordinary Least Squares techniques, which are the default method for many common spreadsheet programs, slightly reduce the statistical variance (square of the standard deviation) of computed values, and therefore, their use would have slightly underestimated the predicted peak yields and overestimated the predicted minimum yields.

Two regression equations were used to compute yield at the upstream gaging stations because flow into the Chain of Lakes differs based upon the lake stage in the Chain (fig. 5).
When the lake stage is high or rising, some flow is stored in the channels upstream of the gages, resulting in less actual flow at the upstream gaging stations. It is important to note that acoustic velocity meters, which account for such “backwater” effects as part of the flow measurement, were used at the affected gages. Relatively low flows during high or rising lake stage suggest that this lake stage-dependent relation is associated with physical processes such as reduced groundwater discharge upstream of the gages and storage within the channels, and not measurement error or bias. To account for this lake stage-dependent relation with flow, equation 1 ($\text{Gage}_Y_{\text{low-stage}}$) was used for estimating historical yield into the Chain of Lakes when the lake stage was below 1,598 ft (5.1-ft gage height) or when the stage was falling between 1,600.5 ft (7.6-ft gage height) and 1,598 ft. Equation 2 ($\text{Gage}_Y_{\text{high-stage}}$) was used for estimating historical yield into the Chain of Lakes when the lake stage was above 1,600.5 ft or rising between 1,598 and 1,600.5 ft. The 1,598 ft and 1,600.5 ft levels were used to transition between equations 1 and 2 because analyses demonstrated that when the lake stage is between these elevations, flows into the Chain of Lakes differ depending upon whether the lake stage is rising or falling. Flows into the Chain of Lakes are not affected by rising or falling lake stages when lake stages are below 1,598 ft or above 1,600.5 ft. A 5-day moving average of lake stage was used to select between equations 1 and 2 in order to dampen changes in lake stage that were reported only to the nearest inch by the dam operator. The 5-day moving average minimized short-term fluctuations in lake stage and yet captured the general trend. The effect of high or low lake stages on measured flow into the Chain of Lakes is small for very large flows. Thus, both regression coefficients were computed using equation 4:

$$K_i = \frac{\text{Dam}_Q_{\text{out}} + \Delta S}{\text{Gage}_Q_{\text{in}}}$$

where $K_i$ is the coefficient for a particular month of the year ($i$), $\text{Dam}_Q_{\text{out}}$ is the monthly average measured flow at State Highway 51 (05357302) over the 2-year study period and represents dam outflow, $\text{Gage}_Q_{\text{in}}$ is the monthly average computed flow into the Chain of Lakes at the upstream gaging stations over the 2-year study period, as computed from equation 3, and $\Delta S$ is the monthly average change in water stored in the Chain of Lakes due to lake stage changes measured at station 460819089530500 over the 2-year study period.

Finally, computed yield into the Chain of Lakes was multiplied by the watershed area upstream of the gages (185.6 mi$^2$) to compute the total daily flow into the Chain of Lakes ($\text{Gage}_Q_{\text{in}}$), following equation 3:

$$\text{Gage}_Q_{\text{in}} = \text{Gage}_Y \times \text{Gage}_A,$$

where $\text{Gage}_Q_{\text{in}}$ is the total daily flow into the Manitowish Chain of Lakes at the upstream gaging stations, $\text{Gage}_Y$ is the computed daily yield at the upstream gages and represents either $\text{Gage}_Y_{\text{low-stage}}$ or $\text{Gage}_Y_{\text{high-stage}}$ from equations 1 or 2, as determined from the daily measured lake stage, and $\text{Gage}_A$ is the total watershed area that contributes to the upstream gaging stations (185.6 mi$^2$, or 5,174,231,000 square feet (ft$^2$)).

Equations 1, 2, and 3 were calibrated with measured flows from Dec. 2009 to Nov. 2011, and then applied to all historical flow data for the Bear and Trout Rivers to estimate historical natural flow into the Chain of Lakes from Dec. 1991 to Nov. 2011 (the period of record for the Bear and Trout River gages).

The second step of the adjusted drainage-area ratio method was to use estimated natural flows upstream of the Chain of Lakes ($\text{Gage}_Q_{\text{in}}$) to compute natural flow at the downstream dam ($\text{Gage}_Q_{\text{out}}$). The simplest application of the drainage-area ratio method is to use a single coefficient that is equal to the area upstream of the dam divided by the monitored area ($\text{Gage}_A$, or the area upstream of the tributary gages, fig. 1) to adjust upstream flows ($\text{Gage}_Q_{\text{in}}$) to downstream flows at the dam ($\text{Gage}_Q_{\text{out}}$). Such a constant coefficient approach assumes that the hydrologic contributions (precipitation, evaporation, groundwater, and runoff) associated with the additional watershed area are similar to those in the monitored area and are constant through time. In the adjusted drainage-area ratio method used in this study, however, the total natural flows at the gages ($\text{Gage}_Q_{\text{in}}$) were multiplied by a coefficient that differed for each month (table 2). Values for the monthly coefficients were computed using equation 4:
Estimation of Natural Historical Flows for the Manitowish River near Manitowish Waters, Wisconsin

Table 2. Monthly coefficients ($K$ values) that were multiplied by total daily flow into the Chain of Lakes ($Q_{in}$) to compute daily values of natural flow at the Rest Lake Dam ($Q_{nat}$) for the adjusted drainage-area ratio method.

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</tr>
</thead>
<tbody>
<tr>
<td>Coefficient ($K$)</td>
<td>1.155</td>
<td>1.068</td>
<td>1.237</td>
<td>1.328</td>
<td>1.081</td>
<td>1.113</td>
<td>1.130</td>
<td>1.185</td>
<td>1.276</td>
<td>1.289</td>
<td>1.420</td>
<td>1.356</td>
</tr>
</tbody>
</table>

This approach ensured that the monthly coefficients ($K$) were computed such that the product of each coefficient ($K_i$) and the monthly average computed flow into the Chain of Lakes ($Q_{in}$) minus the monthly average change in storage ($\Delta S$) equaled the monthly average measured flows at State Highway 51 ($Q_{nat}$) for each month during the 2-year study period. That is, the approach ensured proper water balance on a monthly basis. Monthly coefficients were used, rather than a single constant coefficient, because monthly coefficients account for seasonal differences in precipitation, evaporation, runoff, and groundwater discharge to the Manitowish Chain of Lakes between the upstream gages ($Q_{in}$) and the dam ($Q_{nat}$). Finally, the monthly coefficients were used to compute natural flow at the dam ($Q_{nat}$) according to equation 5:

$$Q_{nat} = K \cdot Q_{in}$$

where

- $K_i$ is the coefficient for a particular month of the year ($i$),
- $Q_{nat}$ is the historical daily natural flow at the Rest Lake Dam, and
- $Q_{in}$ is the historical daily computed flow into the Chain of Lakes at the upstream gaging stations, as computed from equation 3.

Differences among the monthly $K$ values are attributed to changes in precipitation, evaporation, near-shore runoff, and groundwater discharge over the calibration period of Dec. 2009 to Nov. 2011. All monthly values are greater than 1, indicating an increase in flow from the upstream gages to the downstream dam during all months. Much of this annual increase between the upstream gages and downstream dam is expected to derive from groundwater discharge into the Chain of Lakes, particularly during the winter when precipitation, runoff, and evaporation are expected to be near zero. The $K$ values increase during spring, likely reflecting increased precipitation, near-shore runoff and groundwater discharge due to spring recharge. The $K$ value decreases in May, likely due to a rising lake stage relative to groundwater levels, which would reduce groundwater discharge into the Chain of Lakes, as well as due to an increased rate of evaporation. The $K$ values gradually increase through summer, possibly indicating a gradual establishment of equilibrium between the lake stage and groundwater levels. The $K$ values increase again in late fall, likely due to increased groundwater discharge due to lowering of the lake stage, as well as decreased evaporation. The $K$ values steadily decrease again during the winter, likely as the lake stage and groundwater levels equilibrate and groundwater discharge becomes the primary source of water entering the Chain of Lakes between the upstream gages and the downstream dam.

As described earlier, natural flows in the Trout River were adjusted to account for diversion of water by cranberry operations near the Trout River near Boulder Junction gaging station (05357259), as shown in figure 3. Except for an unknown amount of water loss due to evaporation of diverted water stored in holding ponds and transpiration of diverted water used for irrigation, the remainder of the diverted water at the Trout River near Boulder Junction gage (05357259) is expected to return as groundwater discharge into the Chain of Lakes. This returned diversion water is accounted for by the monthly coefficients ($K$), in that the coefficient values were computed such that the total water into and out of the Chain of Lakes, as computed at the Rest Lake Dam, was balanced on a monthly basis. In other words, while the monthly coefficient ($K$) values do not identify the individual sources of water entering the Chain of Lakes between the upstream gaging stations and the downstream dam, the coefficients do account for the total amount of water entering the Chain of Lakes between the gaging stations and the dam. That is, returned diversion water is incorporated into the monthly coefficients. Moreover, because no known cranberry operations exist in the Bear River (05357335) and the upstream Trout River at Trout Lake (05357245) watersheds, the adjusted drainage-area ratio method eliminates effects of diversions within the Chain of Lakes watershed on computed natural flow at the Rest Lake Dam ($Q_{nat}$). Nonetheless, possible diversions elsewhere in the system, such as near Alder Lake (fig. 1), were not considered during this analysis due to a lack of data.

Water-Budget Method

The water-budget method also used two steps to estimate natural historical flow at the Rest Lake Dam. The first step was to calibrate the number of days (N-days) over which to average input data (lake stage, dam outflow, precipitation, and evaporation) and estimate groundwater discharge parameters by comparing computed flows with measured flows at the upstream gages for the 2-year study period. The second step was to average all historical lake stage and dam outflow data provided by the dam operator over the calibrated number of days (N-days) to compute historical natural flow at the downstream Rest Lake Dam.
The water budget for the Manitowish Chain of Lakes is described by equation 6:

\[ \Delta S = P - E + GW_{in} + Gage\ Q_{in} - Dam\ Q_{out} \]  

(6)

where

- \( \Delta S \) is change in storage of the Manitowish Chain of Lakes and is equal to the sum of water entering the Chain of Lakes minus the sum of water leaving the Chain of Lakes,
- \( P \) is precipitation on the lake surface,
- \( E \) is evaporation from the lake surface,
- \( GW_{in} \) is the net groundwater discharge into the Chain of Lakes,
- \( Gage\ Q_{in} \) is the natural flow into the Chain of Lakes at the upstream gaging stations, and
- \( Dam\ Q_{out} \) is the dam outflow.

Realizing that the change in storage (\( \Delta S \)) accounts for all flows into and out of the Chain of Lakes, natural flow at the dam can be computed directly from lake stage and dam outflow data provided by the dam operator using equation 7:

\[ Dam\ Q_{out} = Dam\ Q_{out} + \Delta S \]  

(7)

where

- \( \Delta S \) is change in storage of the Manitowish Chain of Lakes and is positive when flows entering the Chain of Lakes exceed flows leaving the Chain of Lakes (the lake stage rises), and is negative when flows leaving the Chain of Lakes exceed flows entering the Chain of Lakes (the lake stage falls),
- \( Dam\ Q_{out} \) is the dam outflow, and
- \( Dam\ Q_{out} \) is natural flow at the Rest Lake Dam.

In comparing equations 6 and 7, it is apparent that natural flow at the Rest Lake Dam \((Dam\ Q_{out})\) represents all of the natural sources and sinks of water (precipitation, evaporation, groundwater discharge, and flow at the upstream gages) to the Chain of Lakes. However, directly solving equation 7 with daily lake stage and dam outflow data results in spurious oscillations (unrealistically high and low flows) of the computed natural flow at the dam. That is, because of the large volume of the Chain of Lakes, small changes in lake stage represent a large amount of water added or removed from storage. For example, a 1-inch decrease in lake stage releases 14,800,000 cubic feet (ft\(^3\)) of water, or 171 ft\(^3\)/s, out of the Chain of Lakes, which is substantial given that the average daily flow out of the Chain of Lakes during the 2-year study period was 165 ft\(^3\)/s. To reduce the magnitude of computed oscillations, or “flashiness,” daily lake stage and dam outflow data can be averaged over many days; however, the most appropriate number of days for averaging is not known in advance. Averaging too many days would eliminate natural fluctuations, while averaging too few days would not adequately reduce spurious oscillations. For example, use of daily data (1-day averaging) does not reduce spurious oscillations, whereas averaging over 365 days eliminates natural seasonal fluctuations.

Instead of solving equation 7 directly, the water budget for the Chain of Lakes (equation 6) was rewritten into a computer model (see appendix) to solve for flow at the upstream gages \((Gage\ Q_{in})\) to facilitate comparison with measured flow at the upstream gaging stations \((Gage\ Q_{measured})\), as shown in equation 8:

\[ Gage\ Q_{in} = -P + E + GW_{in} + Dam\ Q_{out} + \Delta S \]  

(8)

where the variables are the same as those defined for equation 6. Solving for flow at the upstream gages \((Gage\ Q_{in})\) was necessary to “tune” or calibrate properties of the model (primarily the number of days over which lake stage, dam outflow, precipitation, and evaporation were averaged or smoothed) by comparing the computed flow at the upstream gages \((Gage\ Q_{in})\) with measured flow \((Gage\ Q_{measured})\) from the 2-year study period. Equation 8 was solved with the computer model using the adjusted lake stage and dam outflow reported by the dam operator, precipitation data from the nearby National Weather Service station at Rest Lake (Coop ID 477092), and biweekly evaporation rates averaged over a decade for nearby (6 mi southeast) Sparkling Lake (Lenters and others, 2005).

The model, as represented by equation 8, was calibrated with the automated calibration program, PEST (Doherty, 2011), by matching simulated flow-duration statistics with measured flow-duration statistics at the upstream gaging stations \((Gage\ Q_{measured})\) for the 2-year study period (table 3). Flow-duration statistics describe the percentage of time that a particular flow is equaled or exceeded (see Natural Historical Flow Durations, below, for additional descriptions). A detailed description of model parameter estimation and calibration is beyond the scope of this report; Aster and others (2005), Doherty and Hunt (2010), and Hill (1998) provide detailed descriptions of model calibration methods. More generally, calibration of the model results (simulated values) to match flow-duration statistics for measured flows (target values) was performed by adjusting the number of days (N-days) over which lake stage, dam outflow, precipitation, and evaporation were averaged (centered on the date being computed), in addition to estimating four groundwater discharge parameters. N-days was the primary mechanism for matching the range in magnitude (minimum and maximum flows) of simulated flows to the range in magnitude of measured flows from the 2-year study period. Inherently, the calibrated value for N-days incorporates some of the limitations of the method as well as the precision of the input data. Groundwater discharge to the Chain of Lakes was unknown and therefore estimated through the calibration process.
Groundwater discharge into the Chain of Lakes was expected to exceed the loss of lake water into the groundwater system, because the region has high groundwater recharge (Gebert and others, 2009) and the $K$ values for the Adjusted Drainage-Area Ratio method (table 2), particularly those during the winter, were all greater than 1, indicating a net gain of water from upstream to downstream that likely included a high percentage of groundwater discharge. Groundwater discharge was assumed to increase from relatively low to relatively high amounts across four conditions represented by (1) rising lake stage, (2) high lake stage, (3) low lake stage, and (4) falling lake stage. This pattern of groundwater discharge is conceptualized as being driven by relative changes in the hydraulic gradient between groundwater and the Chain of Lakes caused by operational changes in the lake stage. That is, relatively rapid increases in lake stage are expected to decrease the hydraulic gradient between groundwater and the Chain of Lakes, during which time groundwater discharge into the Chain of Lakes is at a minimum. Conversely, relatively rapid lowering of lake stage would increase the hydraulic gradient between groundwater and the Chain of Lakes, thus maximizing groundwater discharge. Periods of stable lake stage are expected to exhibit transitional hydraulic gradients, during which time intermediate rates of groundwater discharge are expected.

Groundwater discharge into the Chain of Lakes was estimated as a residual term during the calibration process, and was simulated using four adjustable parameters (table 4) based upon the measured lake stage: (1) $GW_{rising\_lake\_stage}$, (2) $GW_{high\_stage\_ratio}$, (3) $GW_{low\_stage\_ratio}$, and (4) $GW_{dropping\_stage\_ratio}$. The parameter representing the period of minimum groundwater discharge, $GW_{rising\_lake\_stage}$, was estimated directly during calibration. The remaining parameters were computed as ratios tied to $GW_{rising\_lake\_stage}$ and then summed in sequence to ensure that total groundwater discharge into the Chain of Lakes increased incrementally from periods of rising lake stage to periods of high lake stage, to periods of low lake stage, and finally to periods of dropping lake stage (table 4). A minimum value was specified for each ratio as part of the calibration process to ensure that total groundwater discharge increased in the specified sequence. Although the calibrated values are reasonable based on prior groundwater studies in the area (Pint, 2002; Hunt and others, 2005; Hunt and others, 2008; Walker and others, 2012), this relationship was not tested in a physical framework such as by an evaluation of field measurements or a model incorporating known governing laws for the physics of groundwater movement; thus, the groundwater discharge estimates should not be used beyond their application in this report. In addition, the groundwater parameters inherently account for any diverted water by cranberry operations that seeps into the ground and returns to the Chain of Lakes.
Table 4. Parameter names and descriptions, and calibrated values for the water-budget method.

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter description</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-days</td>
<td>Number of days over which reported lake stage, dam outflow, precipitation, and evaporation were averaged</td>
<td>23 days</td>
</tr>
<tr>
<td>GW_rising_lake_stage</td>
<td>Simulated net groundwater discharge to the Manitowish Chain of Lakes (-between the upstream gages and the dam) when the lake stage was rising</td>
<td>24.7 ft$^3$/s</td>
</tr>
<tr>
<td>GW_high_stage_ratio</td>
<td>A ratio multiplied by GW_rising_lake_stage to simulate net groundwater discharge to the Chain of Lakes when the lake stage is high ($&gt; 1,599.5$ ft)</td>
<td>1.0, translates to 24.7 ft$^3$/s at high stage</td>
</tr>
<tr>
<td>GW_low_stage_ratio</td>
<td>A ratio multiplied by GW_rising_lake_stage and added to the discharge represented by GW_high_stage_ratio (24.7 ft$^3$/s) to simulate net groundwater discharge to the Chain of Lakes when the lake stage is low ($&lt;= 1,599.5$ ft)</td>
<td>0.015, translates to 25.1 ft$^3$/s at low stage</td>
</tr>
<tr>
<td>GW_dropping_stage_ratio</td>
<td>A ratio multiplied by GW_rising_lake_stage and added to the discharge represented by GW_low_stage_ratio (25.1 ft$^3$/s) to simulate net groundwater discharge to the Chain of Lakes when the lake stage is dropping during dropping stage</td>
<td>0.62, translates to 40.4 ft$^3$/s</td>
</tr>
</tbody>
</table>

Following estimation of values for the N-days and groundwater discharge parameters through calibration of the water-budget model (equation 8) using measured flow into the Chain of Lakes ($\text{Gage} \dot{Q}_{\text{measured}}$), equation 9 was used to compute natural flow at the Rest Lake Dam ($\text{Dam} \dot{Q}_{\text{nat}}$).

\[
\text{Dam} \dot{Q}_{\text{nat}} = P_{\text{low-stage}} - E_{\text{low-stage}} + GW_{\text{in}} + G\text{age} \dot{Q}_{\text{in}}
\]  \tag{9}

where

- $\text{Dam} \dot{Q}_{\text{nat}}$ is natural flow computed at the Rest Lake Dam,
- $P_{\text{low-stage}}$ is the volume of water that falls as precipitation onto the surface area of the lake at low stage (1,598 ft),
- $E_{\text{low-stage}}$ is the volume of water that evaporates from the surface area of the lake at low stage (1,598 ft),
- $GW_{\text{in}}$ is the estimated groundwater discharge into the Chain of Lakes, as represented by one of the four calibrated groundwater discharge values tied to the measured lake stage, and
- $G\text{age} \dot{Q}_{\text{in}}$ is the computed natural flow into the Chain of Lakes at the upstream gaging stations, as estimated from equation 8.

Precipitation and evaporation were computed using the low lake-stage area because large operational increases in lake stage would not occur under natural flow conditions. Natural fluctuations of lake stage are not known and were not estimated.

Although the first step of the water-budget method (the calibration step) was limited to the 2-years for which flows at the upstream gages were measured (2009–11), the second step used all available historical lake stage and dam outflow data provided by the dam operator to compute natural flows at the Rest Lake Dam. The procedure for computing daily natural historical flow at the dam ($\text{Dam} \dot{Q}_{\text{nat}}$) consisted of first solving for flow into the Chain of Lakes ($\text{Gage} \dot{Q}_{\text{in}}$) with equation 8 by averaging daily precipitation, evaporation, and the adjusted dam outflow and lake stage data provided by the dam operator over N-days (23 days). Then, the result of equation 8 ($\text{Gage} \dot{Q}_{\text{in}}$) was combined with the appropriate stage-dependent groundwater discharge parameter value (table 4) and N-day averaged precipitation and evaporation amounts computed with the low lake stage area to solve equation 9. This procedure, as implemented in the computer code provided in the appendix, was performed for each day that adjusted dam outflow and lake stage data were provided by the dam operator (Dec. 1973–Nov. 2011).

Some components of the water budget calculation (precipitation, evaporation, and groundwater) are of limited significance to the final calculation ($\text{Dam} \dot{Q}_{\text{nat}}$). This is because precipitation, evaporation, and groundwater were primarily used as a means to determine the number of days (N-days) over which to average input data provided by the dam operator, which acts as the primary mechanism for matching the range in computed and measured flows at the upstream gages. That is, although daily values of precipitation, evaporation, and groundwater discharge are important for computing natural flow at the upstream gages ($\text{Gage} \dot{Q}_{\text{in}}$) using equation 8, these components of the water budget are of the opposite sign in equation 9, and therefore are largely removed (“cancelled out”) during the process of computing natural flow at the dam ($\text{Dam} \dot{Q}_{\text{nat}}$). The only difference in the daily volume of precipitation and evaporation used in equations 8 and 9 is the difference in the lake surface area over which precipitation and evaporation occurs on the lake. The daily groundwater discharge values applied in equations 8 and 9 are identical, thus effectively eliminating the calibrated groundwater
parameter values from direct computation of natural flow at the dam \( (\text{Dam}Q_{\text{nat}}) \). In other words, only the low-stage lake area used to compute precipitation and evaporation volumes and the number of days (23) used to average the input data directly affect the conversion of lake stage and dam outflow data provided by the dam operator into computed natural flow at the dam \( (\text{Dam}Q_{\text{nat}}) \).

### Natural Flows at the Rest Lake Dam

Comparison of the measured \( (\text{Gage}Q_{\text{measured}}) \) and computed flow at the gages \( (\text{Gage}Q_{\text{in}}) \) for the 2-year study period helps in evaluating the relative accuracy of the adjusted drainage-area ratio method and the water-budget method (figs. 6 and 7). Both methods generally described the pattern and timing of high-flow and low-flow events at the upstream gages. The adjusted drainage-area ratio method, however, generally had smaller residual errors over the full range of observed flows and had smaller monthly biases than the water-budget method. Although a close fit with the measured flows at the upstream gages is important, the focus of the study is to estimate natural flow at the dam \( (\text{Dam}Q_{\text{nat}}) \). Both methods produce similar total annual flows at the dam (table 5). Unfortunately, neither method can be directly evaluated against daily dam outflows, because dam operation alters the natural flow pattern.

Results from the two methods can be used to aid in understanding how current dam operations affect water levels in the Chain of Lakes and flows downstream of the dam. The median flow computed for each day of the year using all years with available data is shown for each method in figure 8. Alteration of dam outflows caused by storing water in the Chain of Lakes during spring and releasing water from the Chain of Lakes in fall is readily apparent, with reported dam outflows lowest in spring and highest in fall. The computed natural flow hydrographs for both methods have peak flows in spring, minimal flows in late summer or early fall, and a gradual increase in flows during fall. Natural and observed flows for particular years vary and may not match this pattern exactly, but these typical patterns are useful for understanding the relationship between capturing and releasing water for storage in the Chain of Lakes and downstream flows.

### Table 5

Annual total measured dam outflow, total computed natural flow, and difference between measured dam outflows and computed natural flows from the water-budget and adjusted drainage-area ratio methods at the Rest Lake Dam.

<table>
<thead>
<tr>
<th>Time period</th>
<th>Total measured dam outflow ( (\text{ft}^3) )</th>
<th>Total computed natural flow at the dam from the water-budget method ( (\text{ft}^3) )</th>
<th>Total computed natural flow at the dam from the adjusted drainage-area ratio method ( (\text{ft}^3) )</th>
<th>Difference between measured dam outflow and computed natural flow from the water-budget method ( (\text{percent}) )</th>
<th>Difference between measured dam outflow and computed natural flow from the adjusted drainage-area ratio method ( (\text{percent}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 2009 – Nov. 2010</td>
<td>5,063,000,000 ( \text{(ft}^3))</td>
<td>4,937,000,000</td>
<td>5,132,000,000</td>
<td>2.5 ( \text{percent})</td>
<td>−1.3 ( \text{percent})</td>
</tr>
<tr>
<td>Dec. 2010 – Nov. 2011</td>
<td>5,324,000,000 ( \text{(ft}^3))</td>
<td>5,450,000,000</td>
<td>5,275,000,000</td>
<td>−2.4 ( \text{percent})</td>
<td>.9 ( \text{percent})</td>
</tr>
<tr>
<td>Dec. 2009 – Nov. 2011</td>
<td>10,387,000,000 ( \text{(ft}^3))</td>
<td>10,387,000,000</td>
<td>10,407,000,000</td>
<td>.0 ( \text{percent})</td>
<td>−.2 ( \text{percent})</td>
</tr>
</tbody>
</table>
Figure 6. Measured total flow at gaging stations upstream of the Manitowish Chain of Lakes and computed flow into the Chain of Lakes using the adjusted drainage-area ratio and water-budget methods.
Figure 7. Monthly error in estimated flow at the gaging stations upstream of the Manitowish Chain of Lakes for the adjusted drainage-area ratio and water-budget methods.
EXPLANATION

- Median reported dam outflow (adjusted) 1973–2011
- Median computed streamflow from the water-budget 1973–2011
- Median computed streamflow from the adjusted drainage-area ratio 1991–2011
- Median reported dam outflow (adjusted) 1991–2011
- Median computed streamflow from the water-budget 1991–2011
- Median reported lake stage (adjusted) 1973–2011

Figure 8. Median daily flow at the Rest Lake Dam for each calendar day over the period of record available for each method. (Dashed lines represent the median values for only 1991–2011 for the water-budget method and reported dam outflows, and are included for direct comparison with the adjusted drainage-area ratio method.)
One impetus for this study was to improve understanding of natural flow at the dam during extreme conditions, which cannot be evaluated using average annual values such as those in table 5. Instead, a comparison of the computed long-term natural flow at the dam for both methods (fig. 9) allows for qualitative evaluation of historical extreme flows. Although the comparison does not allow one to evaluate which method may be more accurate, it is useful for identifying potentially problematic anomalies associated with each method. For example, the adjusted drainage-area ratio method indicated a large runoff event during the fall of 1994. Closer evaluation of this event, which was not computed by the water-budget method, indicated that the event was associated with high flows measured in the Bear River that were not, however, measured in the Trout River. This indicates that the assumption that weather patterns driving flows in the Bear, Trout, and Manitowish Rivers are the same may not hold at all times. Violation of this assumption is expected to be more important for high flows than low flows, because droughts are typically regional, whereas summer storms are often local. Conversely, several negative and near-negative natural flows at the dam were computed with the water-budget method for Oct. 1974, Aug.–Sept. 1976, Aug.–Oct. 1989, July–Sept. 2005, and July 2006. Some of the near-negative natural flows that occurred in fall (for example, Oct. 1974) may be attributed to the precision of reported dam outflow data and the sensitivity of the method to large releases of water from the Chain of Lakes associated with lowering lake levels. That is, imprecision in the reported data can result in reported dam outflows during periods of falling stage that were small in comparison to the estimated amount of water released from the Chain of Lakes, resulting in a very low computed natural flow at the dam. Some near-negative natural flows were also computed during summer for years with below average dam outflow (for example, Aug.–Sept. 1976). During such instances, both the lake stage and reported dam outflow were declining, indicating that natural flow into the Chain of Lakes was likely affected by drought conditions. In addition, it is possible that diversions associated with cranberry operations in the Chain of Lakes Watershed could contribute to the near-negative computed flows for these periods (as evident in fig. 3). Natural flows at the dam calculated by the adjusted drainage-area ratio method are not expected to be affected by diversions from cranberry operations because no such operations are known to exist upstream of the Bear or Trout River gages. However, the water-budget method lacks adequate information to systematically remove possible effects from this type of diversion. This is because daily reported dam outflows inherently incorporate reductions in daily flow into the Chain of Lakes due to diversions, which, because the diversions were not measured cannot be systematically removed. Moreover, calibrating the water-budget method with natural flow (fig. 3) into the Chain of Lakes \( Q_{nat} \) only accounts for effects of diversions in terms of the number of days (N-days) over which daily reported dam outflow values are averaged and in terms of the groundwater discharge parameters, which as described earlier, are effectively removed from direct computation of daily natural flow at the Rest Lake Dam \( Q_{nat} \). As a result, to the extent that near-negative flows computed by the water-budget method are affected by diversions, the flows should not be considered natural.

### Natural Historical Flow Durations

Daily natural flows at the dam \( Q_{nat} \) computed by equation 5 for the adjusted drainage-area ratio method and by the combination of equations 8 and 9, which used 23-day-averaged values for the water-budget method (fig. 9), were used to compute monthly flow-duration values for the period of available historical data. Historical data for the Bear River near Manitowish Waters (05357335) and the Trout River at Trout Lake near Boulder Junction (05357245), which were used for the adjusted drainage-area ratio method, spanned from December 1, 1991, to November 30, 2011. Historical lake stage and dam outflow reported by the dam operator, which were used for the water-budget method, spanned from December 1, 1973, to November 30, 2011. Monthly flow-duration statistics were computed as described by Helsel and Hirsch (2002) by ranking all computed \( Q_{nat} \) values within a given month for the duration of data available from each method (fig. 10, table 6).

A single flow-duration value, in cubic feet per second, represents the flow rate that is equaled or exceeded on the associated percentage of days over the period of calculation. For example, the Flow Duration 90 (FD90), or 90-percent flow duration, is the flow rate that is equaled or exceeded on 90 percent of the days being evaluated. For this study, flow-duration values were computed on a monthly basis to evaluate seasonal changes in flow. Thus, a FD90 value of 116 ft\(^3\)/s for January, as computed with the adjusted drainage-area ratio method (table 6), means that 90 percent of all daily flows during January over the years 1992–2011 were equal to or greater than 116 ft\(^3\)/s. Flow-duration values computed in this way are useful for describing natural flow patterns estimated using historical records, because flow-duration calculations are not overly influenced by extreme values, as opposed to an average value that may be strongly influenced by a single extreme runoff event. That is, flow-duration values, particularly the FD50 or median, are generally resistant to outliers or infrequent events and, therefore, are well suited for evaluating a range of typical conditions. Nonetheless, flow-duration values at the high and low range (FD2, 10, or FD 90, 98, 99.6) are more susceptible to extreme flows, because by definition these flows are observed infrequently (0.4 percent of days for the FD99.6). These characteristics for flow-duration values are evident in the results shown in table 6 and figure 10.
Figure 9. Dam-operator reported stage and dam outflow for the Manitowish Chain of Lakes and computed natural flow at the Rest Lake Dam ($Q_{nat}^{Dam}$) from the adjusted drainage-area ratio and water-budget methods, 1973–2011.
Table 6. Monthly flow-duration values for computed natural flow at the Rest Lake Dam based on the adjusted drainage-area ratio and water-budget methods.

[FD, flow duration, is followed by a number indicating the percentage of time that the computed value is equaled or exceeded]

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Results from both methods are useful for understanding the natural flow patterns at the dam. The monthly FD50 (median) values in figure 10 have similar patterns to the median daily flows shown in figure 8, with high flows in spring and low flows in late summer. This seasonal pattern is evident in both high (FD10) and low flows (FD90) as well. For example, the FD10 increases from about 200–230 ft$^3$/s in February (from the adjusted drainage-area ratio and water-budget methods, respectively) to about 420–390 ft$^3$/s in April, decreases to about 220–200 ft$^3$/s in August, and ends the year at about 280 ft$^3$/s (both methods) in December (table 6, fig. 10). Similarly, the FD90 increases from about 110–100 ft$^3$/s in February to about 160–180 ft$^3$/s in April, decreases to about 70–40 ft$^3$/s in August, and ends the year at about 120–100 ft$^3$/s in December. Similarly, the range from high to low flows widens during spring, narrows during summer, and widens again during fall.

Differences among the results of each method demonstrate the difficulty in computing natural flows from surrogate records. For example, although the median values (FD50) from each method tend to be relatively similar, values for the less frequent flows (for example, the FD10 and FD90) tend to be more variable, as would be expected given the greater uncertainty associated with estimating infrequent flows, regardless of the method.

Identification of a representative natural low flow at the dam, as required by Chapter 31 of the Wisconsin State Statutes (Wisconsin Statutes and Annotations, 2012), is subject to interpretation. What is considered a low flow by one criterion may differ from what is considered a low flow by another criterion. For example, the effect of a particular low-flow value on the health of an aquatic organism can differ from the effect of that same flow on the utility of a water resource for economic or recreational gain. In addition, the results illustrate a challenge with interpreting a single value of natural low flow, because natural low flow varies on a seasonal basis. That is, a natural low flow computed for September is not representative of a natural low flow in April. The lowest natural flow computed by either method is −8.3 ft$^3$/s for the FD99.6 during October based on the water-budget method (table 6), and that flow is physically impossible under natural conditions. This value is undoubtedly influenced by the sensitivity of the water-budget method to small changes in lake stage, the precision of reported lake stage and dam outflow, and upstream diversions. As stated previously, natural flows computed by the adjusted drainage-area ratio method are expected to be less susceptible to biases at low flows than those computed by the water-budget method. The lowest natural flow computed by the adjusted drainage-area ratio method is 51.3 ft$^3$/s for the FD99.6 during August. It is important to reiterate that a FD99.6 computed on a monthly basis is not equal to the FD99.6 computed on an annual basis, which Gebert (1980) equated with the $Q_{7,10}$ in the nearby Wisconsin River Basin. The FD99.6 computed on an annual basis for natural flows estimated by the adjusted drainage-area ratio and water-budget methods are 57 ft$^3$/s and 10 ft$^3$/s, respectively, with the water-budget method likely biased low.

Assumptions and Limitations

The methods used to estimate the monthly flow-duration values in table 6 for natural flow at the Rest Lake Dam incorporate many assumptions and limitations. It was assumed that the relation between reported dam outflow and measured flow by WDNR staff at State Highway 51 (05357302) prior to April 28, 2009 (fig. 4A), was stationary over time. That is, despite knowledge that the amount of leakage between stoplogs varied depending upon the stoplog conditions (use) and configuration (orientation and shape), no data were available for time-dependent adjustments to the reported dam outflows, and, therefore, the relation in figure 4A was assumed to be appropriate for all reported data from December 1973 to April 28, 2009. A second assumption was that the relation between the yield at the upstream gaging stations and the reference gaging stations on the Bear and Trout Rivers (stations 05357335 and 05357245, respectively) was stationary over time. A third assumption was that the estimated daily amount of water diverted by cranberry operations that subsequently returned to the Chain of Lakes through the groundwater system was accounted for on a monthly basis through the coefficients (K) used in the adjusted drainage-area ratio method. A fourth assumption was that averaging reported lake stage and dam outflow over 23 days, as estimated during calibration of the water-budget method for the 2-year study period, was appropriate for the entire period of record.

Regardless of the assumptions, application of the results should recognize several limitations. First, although computed long-term median natural flows at the dam tend to be relatively similar between the two methods, extreme values (for example, the FD2, 10, 90, and 99.6) deviate substantially between methods (fig. 10 and table 6). These large differences between methods illustrate the increased uncertainty associated with estimating extreme events using secondary data rather than directly measured natural flows at the dam. This higher level of uncertainty should be considered when using the results. Additional uncertainty is also incorporated into the water-budget method in the form of the precision with which reported lake stages and dam outflows were measured or estimated. Specifically, lake stage provided by the dam operator was reported to the nearest inch and measured at approximately noon each day. As stated previously, the water-budget method is highly sensitive to small changes in lake-stage
fluctuations—even when reporting to the hundredth of an inch—and low precision in the historical stage data. Rapidly changing stages may confound this sensitivity, although averaging stages over 23 days is expected to reduce the effects of this limited precision. Conversely, averaging over 23 days limits the precision with which the timing of extreme flows can be estimated, thus potentially affecting the month during which an estimated extreme event occurs. Similarly, reported dam outflows were estimated from lake stage and an equation for flow over a weir. The extent to which the equation is affected by the configuration of the stoplogs, as described previously, is incorporated into the accuracy of the reported dam outflows. Finally, computed near-negative flows at the dam from the water-budget method may incorporate biases due to the reported data as described previously, as well as upstream diversions that were not accounted for using this method.

## Summary and Conclusions

The Wisconsin Department of Natural Resources is charged with oversight of how the dam at the outlet of Rest Lake in Vilas County, Wis., is operated. Chapter 31 of the Wisconsin State Statutes specifies that the minimum flow out of every dam in the State should be no less than 25 percent of the natural low flow (Wisconsin Statutes and Annotations, 2012). Dams alter the system from natural conditions, however, often leaving 25 percent of the natural low flow difficult to determine. As part of establishing new operating orders for the dam, the Wisconsin Department of Natural Resources partnered with the U.S. Geological Survey to improve estimates of natural flow at the dam.

Two independent methods were used to estimate historical natural flow for the Manitowish River at State Highway 51 near Manitowish Waters (station 05357302), the site of a streamflow-gaging station located approximately 1 mile downstream of the Rest Lake Dam. Because of a lack of direct flow measurements at the dam outlet, flow measured at State Highway 51 (05357302) was assumed to represent flow at the Rest Lake Dam. The first method, the adjusted drainage-area ratio method, related flow divided by watershed area (yield) for the total measured flow into the Manitowish Chain of Lakes at the upstream gages ($Q_{\text{nat, measured, in}}$) with total yield for the Bear River near Manitowish Waters (station 05357335) and the Trout River at Trout Lake near Boulder Junction (station 05357245) to develop an equation for computing historical flows into the Chain of Lakes ($Q_{\text{nat, in}}$) using all historical data for the Bear and Trout River gages (1991–2011). To convert the computed total natural flow into the Chain of Lakes at the upstream gages ($Q_{\text{nat, upstream}}$) into natural flow at the downstream Rest Lake Dam ($Q_{\text{nat, downstream}}$), computed daily flows at the upstream gages ($Q_{\text{mea, upstream}}$) were multiplied by a coefficient that varied by month. The second method, the water-budget method, used a computer model to compute natural flow at the dam using lake stage and dam outflow data provided by the dam operator. The water-budget model separated the computation into two equations. The first equation computed flows into the Chain of Lakes at the upstream gaging stations ($Q_{\text{nat, upstream}}$), and the second equation used those upstream flows to compute natural flows at the downstream Rest Lake Dam ($Q_{\text{nat, downstream}}$). Calculation of natural flows at the dam was separated into two equations so that the most appropriate number of days (23 days) over which to average input data (lake stage, dam outflow, precipitation, and evaporation) could be determined by comparing measured flows at the upstream gages ($Q_{\text{mea, upstream}}$) for the 2-year study period with computed flows ($Q_{\text{nat, upstream}}$). Averaging input data over 23 days was the primary mechanism for adjusting the “flashiness” of the computed flows. Following calibration of the water-budget model to determine the most appropriate number of days over which to average input data, the model was applied using all historical lake stage and dam outflow data from the dam operator (1973–2011) to compute historical natural flow at the Rest Lake Dam ($Q_{\text{nat, downstream}}$).

Daily natural flows at the dam ($Q_{\text{nat, downstream}}$), computed by the adjusted drainage-area ratio and the water-budget methods, were used to compute monthly flow-duration values for the period of historical data available for each method (Dec. 1991 to Nov. 2011 for the adjusted drainage-area ratio method and Dec. 1973 to Nov. 2011 for the water-budget method). Monthly flow durations provide a means for evaluating the frequency and range in flows for specific months over many years, including the frequency with which certain high and low flows occur.

Results from both methods are useful for understanding the natural flow patterns at the dam. The monthly natural flows for both methods had similar patterns, with high median flows in spring and low median flows in late summer. Similarly, the range from high flows to low flows increased during spring, decreased during summer, and increased again during fall. These seasonal patterns illustrate a challenge with interpreting a single value of natural low flow. Moreover, alteration of natural flows caused by storing water in the Chain of Lakes during spring and releasing water from the Chain of Lakes in fall causes a change in the timing of high and low flows compared with flows under natural conditions. That is, the lowest reported outflows occurred in spring and highest reported outflows occurred in fall.
Both methods captured the pattern and timing of measured high-flow and low-flow events at the upstream gaging stations. The adjusted drainage-area ratio method generally had smaller residual errors than the water-budget method over the full range in observed flows and had smaller monthly biases. Unfortunately, neither method could be directly compared against daily dam outflow, because dam operations altered the natural flow patterns. Both methods produced similar total annual flows at the dam, suggesting that the computed natural flows at the dam were correct on an annual basis for both methods. Although it was not possible to definitively determine which method was more accurate at estimating all monthly natural flows, particularly at very high or very low flows, comparisons of the results of each method indicate that the adjusted drainage-area ratio method could be more susceptible to biases at high flows because of isolated storms outside of the Manitowish River watershed. Conversely, the water-budget method may be more susceptible to biases at low flows because of the sensitivity of the method to the accuracy of reported lake stages and dam outflows, as well as potential upstream diversions that could only partially be accounted for with this method (the adjusted drainage-area ratio method should not be affected by diversions by cranberry operations, because no such operations are known to be present in the Bear and Trout River watersheds that were used as part of this method).

References Cited


Appendix.  Code for the Python-based Water-Budget Model

“Python” is a registered trademark of the Python Software Foundation.

Although this program has been used by the U.S. Geological Survey (USGS), no warranty, expressed or implied, is made by the USGS or the U.S. Government as to the accuracy and functioning of the program and related program material nor shall the fact of distribution constitute any such warranty, and no responsibility is assumed by the USGS in connection therewith.

# This program computes Rest Lake natural flows at upstream gages and at the dam given lake stage, outflow, and climate data.

import sys
import math
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime  # pull the datetime module from datetime
from datetime import timedelta  # grab the time differencing module

echo = False
TARGDATES = []
DATES = []
NEWDATES = []
INFLOWS = []
OUTFLOWS = []
FLOWINS = []
FLOWINS_CFS = []
OUTF = []
LLS = []
PPT = []
EVAP = []
LEVELS = []
SIMLEVELS = []
SIMOUTS = []
GWS = []
TF = '%m/%d/%Y'   # time format used in the files

# Error Exception Classes
# #
# -- cannot read/write/open/close file
class FileFail(Exception):
    def __init__(self, filename, filetype):
        self.filename = filename
        self.ft = filetype
    def __str__(self):
        return('

Problem with ' + self.ft + ': ' + self.filename + ' 
' +
"Either it can't be opened or closed, can't be read from or written to, or doesn't exist")

# -- wrong number of lines in cal file
class CalFail(Exception):
    def __init__(self, nlines, fn):
        self.nlines = nlines
        self.fn = fn
    def __str__(self):
        return('

Cal File: ' + self.fn + ' has wrong number of lines. 
' +
"Either it can’t be opened or closed, can’t be read from or written to, or doesn’t exist")

# This program computes Rest Lake natural flows at upstream gages and at the dam given lake stage, outflow, and climate data.
Estimation of Natural Historical Flows for the Manitowish River near Manitowish Waters, Wisconsin

```python
# -- Failure parsing the input data file
class ParseFail(Exception):
    def __init__(self, offending_line):
        self.offending_line = offending_line
    def __str__(self):
        return("There was a problem parsing a line in your data file. The offending line was:"
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for each_line in target_file[1:]:
    DATA_TARG = each_line.split(’,’ ) # split on commas
    DATE_TARG = DATA_TARG[0]  
    RICE_CFS = float(DATA_TARG[1])
    MANITOWISH_CFS = float(DATA_TARG[2])
    TROUT_CFS = float(DATA_TARG[3])
    PAPOOSE = float(DATA_TARG[4])

    INFLOW_TARG = RICE_CFS + MANITOWISH_CFS + TROUT_CFS + PAPOOSE
    INFLOWS.append(INFLOW_TARG)  # appends each INFLOW to the list INFLOWS (for plotting purposes)

try:
    MONTH, DAY , YEAR =  DATE_TARG.split("/")
except:
    raise(ParseFail,each_line)
MM = MONTH.zfill(2)
DD = DAY .zfill(2)
OBS_outstring = ('obs_inflow ' + '{0}/{1}/{2}'.format(MM,DD,YEAR) + ' 12:00:00 ' +
                 '{0:.2f}
'.format(INFLOW_TARG))

if (echo):
    print OBS_outstring

obs_file.write(OBS_outstring)

# open file with calibration parameters
try:
    calib_file = open(calfilename,'r').readlines()
except:
    raise(FileFail(calfilename,’calibration parameter file’))

# make a list of just the numbers from the file
lines = []
for each_line in calib_file:
    line_stripped = each_line.strip()  #removes leading and trailing whitespace
    line_split = line_stripped.split()  # splits a string at whitespaces
    lines.append(line_split[0])  # appends each split character to the list: “lines”

if len(lines) != 6:
    raise(CalFail(len(lines),calfilename))  #call an error if not 6 lines in the calibration file.

# now pull out the calibration coeffs.
GW_drop = float(lines[0])
GW_low = float(lines[1])
GW_rise = float(lines[2])
GW_high = float(lines[3])
FLOAT_NSTAGE = float(lines[4])
VOLDAMP = float(lines[5])

# Pest needs to work with float numbers so that derivatives can be calculated. This code needs integer days.
# Thus, the N-days over which to average all input values calibrated by PEST is rounded and then converted to an integer.
NSTAGE = int(round(FLOAT_NSTAGE))  # round first (rounds 0.5 upwards), then convert to integer.

# open file with input data on lake stage, dam outflow, precip and evap.
try:
Estimation of Natural Historical Flows for the Manitowish River near Manitowish Waters, Wisconsin

```python
data_file = open(datfilename,'r').readlines()
except:
    raise(FileFail(datfilename, 'precipitation and evaporation data file'))

for each_line in data_file[1:]:
    DATA = each_line.split(',
    DATE = DATA[0]
    PRECIP = float(DATA[1])
    OUTFLOW_CFS = float(DATA[2])
    LL = float(DATA[3])  # lake level
    DATET = datetime.strptime(DATE, TF)  # converts to a tuple
    DATES.append(DATET)
    OF = 86400.0 * OUTFLOW_CFS # convert outflow to ft3/day
    OUTF.append(OF)  # creates a running list of outflow each day
    PI = PRECIP / 12.0  # convert the daily precipitation to feet
    PPT.append(PI)  # creates a running list of precipitation each day
    LLS.append(LL)  # creates a running list of lake stage each day

    # For plotting only...
    if ((datetime.strptime(DATE, TF) >= STARTDAY) & (datetime.strptime(DATE, TF) <= ENDDAY)):
        OUTFLOWS.append(OUTFLOW_CFS)  # accumulating the measured outflow for plotting purposes
        LEVELS.append(LL * 100.0 - 159500)  # appends each measured lake level for plotting purposes
        # the equation simply offsets the lake level for better visualization in the plot
    try:
        EO = float(DATA[4])
    except:
        EO = 0.0  # assign zero evap if blank
    EVAP.append(EO)  # creates a running list of evap with zero on days for which evap is not specified, such as during
    # the winter.

# Start of calculations
# # Average stage, outflow, ppt, & evap over NSTAGE days ("N-days" in the report)
# the multiple if, elif, and else statements are used to adjust how the stage is averaged for the beginning of the record
# when there’s only 1 to NSTAGE days of data.
for i, val in enumerate(DATES):  # iterates through each date in the input file

    # for first day, sets values to first day value
    if (i == 0):
        STAGE1 = LLS[0]
        STAGE2 = STAGE1
        OUTFLOW = OUTF[0]
        PRECIP = PPT[0]
        EV = EVAP[0]

    # At the start, when “I” is less than or equal to NSTAGE (or “N-day” in the report), average around the date of interest
    # from the first day to half way to NSTAGE
    elif (i <= (NSTAGE/2.0) - 1):
        ENDAVERAGE = (2*I)
        STAGE1 = np.mean(np.array(LLS[0:ENDAVERAGE]))
        OUTFLOW = np.mean(np.array(OUTF[0:ENDAVERAGE]))
        PRECIP = np.mean(np.array(PPT[0:ENDAVERAGE]))
```

EV = np.mean(np.array(EVAP[0:ENDAVERAGE]))

# average around the date of interest from 1/2 NSTAGE back to 1/2 NSTAGE forward of the date
else:
    STARTAVERAGE = int(math.floor(I-(NSTAGE/2.0-1)))  # round first (math.floor rounds 0.9 down), then convert to
    # an integer
    ENDAVERAGE = int(math.floor(I+NSTAGE/2.0))  # round first (math.floor rounds 0.9 down)

STAGE1 = np.mean(np.array(LLS[STARTAVERAGE:ENDAVERAGE]))   # averages lake stages around the current
    # date
OUTFLOW = np.mean(np.array(OUTF[STARTAVERAGE:ENDAVERAGE])) # averages outflow around the current
data
PREcip = np.mean(np.array(PPT[STARTAVERAGE:ENDAVERAGE]))   # averages precip around the current date
EV = np.mean(np.array(EVAP[STARTAVERAGE:ENDAVERAGE]))      # averages evap around the current date

# Calculate lake area in square feet
AREA = (158065.01 * STAGE1 * STAGE1) - (495798355.44 * STAGE1) + 38881232721.9  # area based on stage
LOWAREA = (158065.01 * 1597.9 * 1597.9) - (495798355.44 * 1597.9) + 38881232721.9 # area based on normal low
    # stage only

# Determine which of the 4 groundwater parameters (calibrated with PEST) to apply to this date based on the stage and
# change in stage
if (STAGE1-STAGE2 > 0.015):
    GW = GW_rise * 86400.0  # convert to ft3/d
elif (STAGE1-STAGE2 < -0.02):
    GW = GW_drop * 86400.0
elif (STAGE1 > 1599.5):
    GW = GW_high * 86400.0
else:
    GW = GW_low * 86400.0

# Water balance and lake level calculations
DSTAGE = STAGE1 - STAGE2  # subtracts yesterday’s lake stage (STAGE2) from the current lake stage STAGE1 to
    # compute change in stage
DVOL = AREA * DSTAGE      # compute change in lake storage due to change in lake stage
FLOWIN = DVOL + OUTFLOW + (EV * AREA) - (PREcip * AREA) - GW  # Compute natural flow into the chain of
    # lakes at the gaging stations upstream of
    # the lake

FLOWIN_CFS = FLOWIN / 86400.0  # convert to ft3/s
FLOWINDAM = FLOWIN + GW + (PREcip * LOWAREA) - (EV * LOWAREA) # Compute natural flow at the dam.
    # This includes inflow at the gages
    # plus groundwater and Precip –
    # Evap for lake area at low stage.

FLOWINDAM_CFS = FLOWINDAM / 86400.0  # convert to ft3/s
STAGE2 = STAGE1   # convert the current date into “yesterday” for the next iteration of this loop

# END OF WATER BUDGET CALCULATIONS

# remove the starting and ending values that were added to simplify date averaging, and limit results to the dates of
# interest.
if ((val >= STARTDAY) & (val <= ENDDAY)):
    NEWDATES.append(val) # for plotting only
    FLOWINS.append(FLOWIN_CFS)  # for plotting only
    SIMLEVELS.append(STAGE1 * 100.0 - 159500)  # shifts lake stage for plotting and appends each stage to the list
        # SIMLEVELS for plotting
    SIMOUTS.append(OUTFLOW/86400.0) # for plotting only
    GWS.append(GW/86400.0)   # for plotting only
# format the output string for natural flow at the upstream gages and at the dam
SIM_GS_outstring = 'SimGSin ' + DATES[I].strftime(TF) + ' 12:00:00 ' + '{0:.2f}
'.format(FLOWIN_CFS)
SIM_DAM_outstring = 'SimDAMin ' + DATES[I].strftime(TF) + ' 12:00:00 ' +
' {0:.2f}
'.format(FLOWINDAM_CFS)

if (echo):
    print SIM_GS_outstring
    print SIM_DAM_outstring

output_file.write(SIM_GS_outstring)  # write to the output files
outDAM_file.write(SIM_DAM_outstring)

# close output file
try:
    output_file.close()
except:
    raise(FileFail(outfilename,'output file'))

#begin block of code to produce Matplotlib plot
plt.figure()
plt.plot(NEWDATES, LEVELS, 'y', NEWDATES, SIMLEVELS, 'c', NEWDATES, OUTFLOWS, 'g', NEWDATES, SIMOUTS, 'm', NEWDATES, INFLOWS, 'b', NEWDATES, FLOWINS, 'r', NEWDATES, GWS, '+',
    label=['Levels','Sim_Levels','Outflow','Sim_Out','Measured','Python','GW'])
plt.legend( ('Levels','Sim_Levels','Outflow','Sim_Out','Measured','Python','GW'), loc='upper left')
plt.show()