Appendix 7. Example Application

The general procedure for estimating the magnitude of a flood discharge at a selected recurrence interval by using the methods described in the report is illustrated in the following example. A random location on Wells River, Vermont, was selected for this example. The drainage area, the areal percentage of wetlands in the basin, and the basin-wide mean of the average annual precipitation were determined to be 71.6 square miles (mi^2), 3.87 percent, and 44.05 inches, respectively.

An estimate of the 1-percent annual exceedance probability peak discharge (Q_1) is needed for this rural, ungaged stream in Vermont. To determine the magnitude of the peak discharge having a 1-percent annual exceedance probability in cubic feet per second, equation 8 would be solved as

$$Q_1 = 0.251A^{0.854}W^{-0.297}P^{1.809} \quad (8)$$
$$Q_1 = 0.251(71.6)^{0.854}(3.87+1)^{-0.297}(44.05)^{1.809}$$
$$Q_1 = 5{,}670 \text{ ft}^3/\text{s} .$$

The variance of prediction for this estimate would be computed by using equation 11. The row vector, x_i, in equation 11 is based on the basin characteristics and is as follows:

$$x_i = [1 \ \log_{10}(71.6) \ \ \log_{10}(3.87+1) \ \ \log_{10}(44.05)] \text{ or}$$
$$x_i = [1 \ \ 1.855 \ \ 0.688 \ \ 1.644] .$$

From table 5, the model error variance, γ^2, is 0.0352, and the $(X^t\Lambda^{-1}X)^{-1}$ matrix is

$$(X^{tr}\Lambda^{-1}X)^{-1} =$$

$$\begin{bmatrix} 4.80981e-01 & 3.00081e-03 & -1.39098e-02 & -2.84463e-01 \\ 3.00081e-03 & 8.08573e-04 & -6.16775e-04 & -2.43038e-03 \\ -1.39098e-02 & -6.16775e-04 & 3.98077e-03 & 7.43891e-03 \\ -2.84463e-01 & -2.43038e-03 & 7.43891e-03 & 1.69693e-01 \end{bmatrix} .$$

After inserting x_i, γ^2, and $(X^t\Lambda^{-1}X)^{-1}$ into equation 11, the variance of prediction is

$$V_{pred} = g^2 + x_i(X^{tr}\Lambda^{-1}X)^{-1}x_i^{tr} \quad (11)$$
$$V_{pred} = 0.0352 + 0.00139 \text{ or}$$
$$V_{pred} = 0.0366 .$$

The standard error of prediction is

$$SE_{pred} = V_{pred}^{\ 1/2} \quad (12)$$
$$SE_{pred} = 0.0366^{1/2} \text{ or}$$
$$SE_{pred} = 0.191 .$$

The standard error of prediction for the estimate in percent is then computed by using equations 13 and 14:

$$S_{pos} = 100(10^{SEpred}-1) \text{ and} \quad (13)$$
$$S_{neg} = 100(10^{-SEpred}-1) \quad (14)$$

or

$$S_{pos} = 100(10^{0.191}-1) \text{ and}$$
$$S_{neg} = 100(10^{-0.191}-1) .$$

Thus, S_{pos} = 55.2 percent, and S_{neg} = -35.6 percent.

The confidence intervals for this estimate can be computed by using equations 15 and 16. For example, if the 90-percent confidence interval were required, the critical value from a Student's t-distribution for the appropriate confidence interval would be found by using any standard statistics textbook. The value for $t_{0.05,141}$ is 1.66, and the standard error of prediction was found earlier to be 0.191. Equations 15 and 16 are solved as follows:

$$CI_{upper} = Q_{pred}10^{\left(t_{\alpha/2,n-p}SE_{pred}\right)} \quad (15)$$

$$CI_{upper} = 5{,}670 * 10^{(1.66*0.191)}$$

$$CI_{upper} = 11{,}800 \ \textit{cubic feet per second (ft}^3 / s)$$

$$CI_{lower} = \frac{Q_{pred}}{10^{\left(t_{\alpha/2,n-p}SE_{pred}\right)}} \quad (16)$$

$$CI_{lower} = \frac{5{,}670}{10^{(1.66*0.191)}}$$

$$CI_{lower} = 2{,}730 \ ft^3 / s$$

Because the site is relatively close to a streamgage, Wells River at Wells River, Vt., the regression equation result of the 100-year discharge shown can be improved by weighting the result with the 1-percent annual exceedance probability flood discharge determined from the frequency analysis of the streamgage data. To weight the result at the ungaged location, a weighted result at the gaged location is first determined. A weighted average (Q_w) of the regression estimate ($Q_{r(g)}$) and the flood discharge estimate from a frequency analysis of the streamgage record (Q_s) can be computed by using equation 17.

$$log_{10}Q_w = \frac{log_{10}Q_s\left(V_{pred}\right) + log_{10}Q_{r(g)}\left(V_s\right)}{V_{pred} + V_s} \quad (17)$$

From the results of frequency analysis of the streamgage data shown in appendix 3, the 1-percent annual exceedance probability discharge (Q_s) is 5,570 ft^3/s. The variance of estimate, V_s, for this streamgage at the 1-percent annual exceedance probability is 0.0040 and can be found in appendix 8. At the streamgage location, the drainage area is 98.9 mi^2, the percentage of the basin covered by wetlands is 3.31 percent, and the basin-wide mean of the average annual precipitation is 43.3 inches. When these values are put into equation 8, the regression equation estimate of the 1-percent annual exceedance probability discharge at the gaged location ($Q_{r(g)}$) is 7,510 ft^3/s. The variance of prediction (V_{pred}) is then computed by using equation 11 with the basin characteristics at the gaged location and the appropriate model error variance, g^2, and $(X^t\Lambda^{-1}X)^{-1}$ matrix from table 5. These values are put into equation 17 as follows:

$$log_{10}Q_w = \frac{log_{10}5,570(0.0368) + log_{10}7,520(0.0040)}{0.0368 + 0.0040}$$

$$Q_w = 5,740\ ft^3/s.$$

The weighted flood discharge, Q_w, is for the gaged location. Because a weighted estimate is desired at an ungaged site near a streamgage, the weighted estimate, Q_u, can be determined by using equations 19 and 20.

$$c = \frac{log_{10}\left(\frac{Q_{r(u)}}{Q_{r(g)}}\right)}{log_{10}\left(\frac{A_u}{A_g}\right)} + \frac{log_{10}\left(\frac{Q_{r(g)}}{Q_w}\right)}{log_{10}(a)} \qquad (19)$$

$$c = \frac{log_{10}\left(\frac{5,670}{7,520}\right)}{log_{10}\left(\frac{71.6}{98.9}\right)} + \frac{log_{10}\left(\frac{7,520}{5,740}\right)}{log_{10}(0.5)}$$

$$c = 0.485$$

$$Q_u = Q_w\left(\frac{A_u}{A_g}\right)^c \qquad (20)$$

$$Q_u = 5,740\left(\frac{71.6}{98.9}\right)^{0.485}$$

$$Q_u = 4,910\ ft^3/s$$