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Dr. Phillip L. Merritt, Assistant Director
Division of Raw Materials
U. S. Atomic Energy Commission
P. O. Box 30, Ansonia Station
New York 23, New York

Dear Phil:

Transmitted herewith are six copies of TEI-333, "Tables for the calculation of radioactive equilibrium from Bateman's equation," by F. J. Flanagan and F. E. Senftle, July 1953.

We are asking Mr. Hosted to approve our plan to publish this report in Nucleonics.

Sincerely yours,

Andrew Brown

for W. H. Bradley
Chief Geologist

(200)
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Physics

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Series A.

UNITED STATES DEPARTMENT OF THE INTERIOR

GEOLOGICAL SURVEY

TABLES FOR THE CALCULATION OF RADIOACTIVE EQUILIBRIUM FROM BATEMAN'S EQUATION*

By

F. J. Flanagan and F. E. Senftle

July 1953

Trace Elements Investigations Report 333

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*This report concerns work done on behalf of the Division of Raw Materials of the U. S. Atomic Energy Commission

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Reports Processing Section	2

CONTENTS

TABLES

TABLES FOR THE CALCULATION OF RADIOACTIVE EQUILIBRIUM FROM BATEMAN'S EQUATION

By F. J. Flanagan and F. E. Senftle

ABSTRACT

Tables of decay constants and functions thereof are presented to simplify the problem of calculating the constants involved in the Bateman equation. An alternate method of solving for the amounts of the nth member of a radioactive series is given in the form of a power series.

INTRODUCTION

Chemical treatment in the laboratory of naturally radioactive ores may result in the disturbance of equilibrium in the thorium^{232}, uranium^{235}, and uranium^{238} series. As a result of such a break in the radioactive chain, it is important to know the variation with time of the quantity or activity of a particular decay product. In connection with a program undertaken by the Geological Survey on behalf of the Atomic Energy Commission, a number of tables have been compiled to facilitate calculation involving radioactive equilibrium. The half-life data are taken from Way and others (1950), Fleming (1952), and Ginnings and others (1953).

Calculations involving radioactive equilibrium were simplified by Bateman (1910), and his method is standard procedure (Rutherford and others, 1930). Bateman has given the method of solution of the general case of n products in a symmetrical form; for instance, the number of atoms N of a given species formed from N' atoms of a parent species in time t is given by

$$N = N'(c_1e^{-\lambda_1 t} + c_2e^{-\lambda_2 t} \ldots + c_ne^{-\lambda_n t}) \ldots (1)$$

where λ is the decay constant, e the Naperian base, and c_n is a constant having the general form

$$c_i = \frac{\prod_{j=1}^{n-1} (\lambda_j)}{\prod_{\substack{j=1 \\ j \neq i}}^{n} (\lambda_j - \lambda_i)}$$

For example, a typical constant is

$$c_2 = \frac{\lambda_1 \lambda_2 \ldots \lambda_{n-1}}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2) \ldots (\lambda_n - \lambda_2)}$$

These constants, although simple in themselves, are tedious to calculate especially where there are a large number of determinations. The sum of the constants is always identical to zero.

A serious disadvantage of Bateman's equation is that the results may be dependent upon the difference of two numbers. If there is a small difference between two large numbers having only a few significant figures, the results can have little or no meaning. This, a common situation, reduces the usefulness of Bateman's equation in its usual form.

POWER SERIES EXPANSION

As Ott (1952) has pointed out, a power series expansion can be used which obviates the necessity of using exponentials and gives sufficient accuracy to allow the use of slide-rule calculations. In practice it has been found that the inaccuracies incurred by using the Bateman equation can be circumvented by expanding those terms that have very large coefficients, that is, numbers which are not significant to at least the units place. The exponential terms with small coefficients are simply carried and worked out in the regular manner. As the sum of the coefficients is identical to zero, the term representing the sum of the coefficients of the expanded exponentials is more accurately represented by the sum of the coefficients of the carried exponential terms with the sign changed. Thus, if in the Bateman equation

$$N = ae^{-\lambda_1 t} + be^{-\lambda_2 t} - ce^{-\lambda_3 t}$$

the coefficients a and c were very large, then the sum of the coefficients (a+c) in the expansion of $ae^{-\lambda_1 t}$ and $-ce^{-\lambda_3 t}$ would numerically be more accurately represented by -b.

The power series expansion of $e^{-\lambda t}$ converges rapidly only for small values of λt, such as 0.1 or less. Where λt is large the exponential cannot be conveniently expanded. In practice it is found that for periods of 500 hours or less the terms that have large coefficients have small values of λt, and hence are expandable.

To facilitate the calculation of the Bateman coefficients and expansion of the exponential terms a number of tables have been computed.

CALCULATION OF TABLES

Tables 1 through 6 show the half-lives, the decay constants, and functions of decay constants in sec^{-1} required for each of the three natural radioactive series. The entries in each table have been carried to two digits beyond the last significant figure, and each entry has been calculated from eight-digit numbers to reduce the round-off error carried through the table. Although the error in each entry has been calculated, it has not been entered in the table for the sake of clarity and has been used only to indicate the number of significant figures of each entry. The following conventions have been adopted for the calculations: (1) the individual errors have been arbitrarily assumed to be $\pm$ 1 in the last digit of the half-lives; (2) where the first number in the calculated error is equal to or greater than 7, the error has been rounded off to 10 in order to assign significant figures.

The products of differences of decay constants are calculated according to the formula

$$(\lambda_1-\lambda_i)(\lambda_2-\lambda_i)\ \ldots\ (\lambda_n-\lambda_i) \pm R =$$

$$\prod_{i=1}^{n-1}(\lambda_n-\lambda_i) \pm \left[\prod_{i=1}^{n-1}(\lambda_n-\lambda_i)\right]\sqrt{\sum_{i=1}^{n=1}\frac{r_i^2 + r_n^2}{(\lambda_n-\lambda_i)^2}}$$

where R is the propagated error and r the error in the individual decay constant.

Values for simple products of decay constants are given by

$$\lambda_1\lambda_2\ \ldots\ \lambda_n \pm R = \prod_{i=1}^{n}(\lambda_i) \pm \left[\prod_{i=1}^{n}(\lambda_i)\right]\sqrt{\sum_{i=1}^{n}\frac{r_i^2}{\lambda_i^2}}$$

and values for powers of decay constants by

$$\lambda^n \pm R = \lambda^n \pm \lambda^{n-1}nr$$

The user of these tables should note that the error in accumulated products of differences, $\prod(\lambda_n-\lambda_i)$, is generally <u>greater</u> than that of partial products obtained by dividing one of these by another. It is, however, impractical to give the errors of all such possible quotients; all that could be done was to compromise by carrying in the table two figures beyond the last significant figure in each entry so that correct figures may be restored in cancelling out less accurate differences.

USE OF THE TABLES

Powers and products ($\prod\lambda$) of the decay constants for the three series are shown in tables 1, 3, and 5 and the products of the differences between decay constants in tables 2, 4, and 6. Where the desired product is not shown in the tables, it is easily calculated by dividing out the undesired part. Thus, if the product $\lambda_3\lambda_4\lambda_5$ is desired, $\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5$ is simply divided by $\lambda_1\lambda_2$, the latter values both being read directly from the table. The "product of the difference" tables (tables 2, 4, and 6) must be read from top to bottom and the unwanted part is divided out as in the simple products ($\prod\lambda$). For example, in table 6 of the thorium series,

$$(\lambda_{Ra^{224}}-\lambda_{Th^{228}})(\lambda_{Ra^{220}}-\lambda_{Th^{228}})(\lambda_{Po^{216}}-\lambda_{Th^{228}})$$

can be obtained by dividing the entry in the seventh row of column 4 by the third entry in the same column. Thus,

$$\frac{(\lambda_{Th^{232}}-\lambda_{Th^{228}})(\lambda_{Ra^{228}}-\lambda_{Th^{228}})\cdots(\lambda_{Po^{216}}-\lambda_{Th^{228}})}{(\lambda_{Th^{232}}-\lambda_{Th^{228}})(\lambda_{Ra^{228}}-\lambda_{Th^{228}})(\lambda_{Ac^{228}}-\lambda_{Th^{228}})} =$$

$$\frac{3.67_{74} \times 10^{-28}}{3.00_{62} \times 10^{-21}} = 1.22 \times 10^{-7}$$

The use of the tables, a power series solution, and the problem of significant figures are best illustrated by an example. Consider the radioactive chain from Pb^{210} to stable lead (Pb^{206}).

$$Pb^{210} \xrightarrow{\lambda=9.9 \times 10^{-10}} Bi^{210} \xrightarrow{\lambda=1.654 \times 10^{-6}} Po^{210} \xrightarrow{\lambda=5.800 \times 10^{-8}} Pb^{206} \text{ (stable).}$$

Assume that the values of the decay constants given here are significant only to the number of places given. The number of lead atoms, $N_{Pb^{206}}$, formed in time "t" from a given number of Pb^{210} atoms is given by the Bateman equation:

$$N_{Pb^{206}}=N_{Pb^{210}}(pe^{-\lambda_{Pb^{210}}t} + qe^{-\lambda_{Bi^{210}}t} + re^{-\lambda_{Po^{210}}t} + se^{-\lambda_{Pb^{206}}t})$$

where

$$p = \frac{\lambda_{Pb^{210}}\lambda_{Bi^{210}}\lambda_{Po^{210}}}{(\lambda_{Bi^{210}}-\lambda_{Pb^{210}})(\lambda_{Po^{210}}-\lambda_{Pb^{210}})(0-\lambda_{Pb^{210}})}$$

The values for both the numerator and the difference between decay constants shown in the denominator can be determined from tables 1 and 2. Thus, the numerical value for the constant p above is:

$$p = \frac{4.4_{19} \times 10^{-86}/4.6_{16} \times 10^{-74}}{-2.7_{42} \times 10^{-78}/2.9_{16} \times 10^{-56}} = -1.0_{181}$$

Similarily

$$q = 2.1_{933} \times 10^{-5}$$

$$r = 1.8_{160} \times 10^{-2}$$

$$s = 1.000$$

If $N_{Pb^{210}}$ is the number of the Pb^{210} atoms $(1.23_7 \times 10^7)$ in equilibrium with 1 microgram of uranium, the Bateman equation for the number of Pb^{206} atoms formed will be

$$N_{Pb^{206}} = -1.25_9 \times 10^{-7} e^{-\lambda_{Pb^{210}} t} - 2.71_2 \times 10^{2} e^{-\lambda_{Bi^{210}} t} + 2.24_8 \times 10^5 e^{-\lambda_{Po^{210}} t} + 1.23_7 \times 10^7 e^{-\lambda_{Pb^{206}} t}$$

It is obvious that, if the exponentials are large (~ 1) the difference of such large numbers with but a few significant figures will result in a large error. By retaining the Bi^{210} term, expanding the rest of the terms, and letting the sum of the coefficients of the expanded terms be $+ 2.71_2 \times 10^2$ a more accurate form can be obtained. The powers have been calculated and are shown in tables 1, 3, and 5. The more accurate form will thus be

$$N_{Pb^{206}} = 2.71_2 \times 10^2 - 4.48_8 \times 10^{-4} t + 3.71_1 \times 10^{-10} t^2 - 7.29_0 \times 10^{-18} t^3 - 2.71_2 \times 10^2 e^{-\lambda_{Bi^{210}} t}$$

By substitution of numerical values into the conventional Bateman equation one obtains a value of several hundred atoms formed for t = 100 hours. Careful consideration of the decay constants and the number of Pb^{210} atoms shows that this number is much too large. The expanded form of the equation, however, gives a value of about 8 atoms which is what one would expect from the decay constants shown.

Where there are two or more large coefficients, the usual Bateman equation cannot be relied on to give accurate results in view of the inaccuracies in the decay constants. The power series form, on the other hand, is accurate enough for most applications. By the use of the accompanying tables and the approximate form of the equation, considerable time can be saved, especially where a large number of calculations are involved.

The authors are indebted to William Schlecht of the Geological Survey for his help in calculating the number of significant figures in the tables. We also appreciate the stimulating discussions of Thomas Farley and Lorin Stieff of the Survey and their interest in this work.

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Table 1.--Uranium^{238} series, half lives, powers, and products of decay constants.

Nuclide	Half Life $T_{1/2}$	Powers & Products of Decay Constants				
		λ (sec^{-1})	λ (sec^{-1})	λ (sec^{-1})	λ (sec^{-1})	$\Pi\lambda$ (sec^{-1})
U^{238}	4.498×10^{9} y	$4.883_{28} \times 10^{-18}$	$2.384_{64} \times 10^{-35}$	$1.164_{49} \times 10^{-52}$	$5.686_{50} \times 10^{-70}$	$4.883_{28} \times 10^{-18}$
Th^{234}	24.10 d	3.32885×10^{-7}	$1.108_{13} \times 10^{-13}$	$3.688_{79} \times 10^{-20}$	$1.227_{94} \times 10^{-26}$	$1.625_{57} \times 10^{-24}$
Pa^{234}	1.22 m	$9.4_{69} \times 10^{-3}$	$8.9_{66} \times 10^{-5}$	$8.4_{91} \times 10^{-7}$	8.040×10^{-9}	$1.53_{98} \times 10^{-26}$
U^{234}	2.475×10^{5} y	$8.874_{74} \times 10^{-14}$	$7.876_{10} \times 10^{-27}$	$6.98_{98} \times 10^{-40}$	$6.20_{33} \times 10^{-53}$	$1.36_{61} \times 10^{-39}$
Th^{230}	8.0×10^{4} y	$2.74_{56} \times 10^{-13}$	$7.53_{84} \times 10^{-26}$	$2.06_{98} \times 10^{-38}$	$5.68_{28} \times 10^{-51}$	$3.75_{07} \times 10^{-52}$
Ra^{226}	1690 y	$1.29_{97} \times 10^{-11}$	$1.68_{92} \times 10^{-22}$	$2.19_{55} \times 10^{-33}$	$2.85_{35} \times 10^{-44}$	$4.8_{75} \times 10^{-63}$
Rn^{222}	3.825 d	$2.0973_{95} \times 10^{-6}$	$4.399_{07} \times 10^{-12}$	$9.22_{66} \times 10^{-18}$	$1.935_{18} \times 10^{-23}$	$1.02_{24} \times 10^{-68}$
Po^{218}	3.05 m	$3.78_{77} \times 10^{-3}$	$1.43_{46} \times 10^{-5}$	$5.43_{40} \times 10^{-8}$	$2.05_{82} \times 10^{-10}$	$3.87_{27} \times 10^{-71}$
Pb^{214}	26.8 m	$4.31_{06} \times 10^{-4}$	$1.85_{81} \times 10^{-7}$	$8.0_{10} \times 10^{-11}$	$3.45_{27} \times 10^{-14}$	$1.66_{94} \times 10^{-74}$
Bi^{214}	19.7 m	$5.86_{42} \times 10^{-4}$	$3.43_{89} \times 10^{-7}$	$2.01_{66} \times 10^{-10}$	$1.18_{26} \times 10^{-13}$	$9.7_{90} \times 10^{-78}$
Branch Via Po^{214}						
Po^{214}	1.47×10^{-4} s	$4.71_{53} \times 10^{3}$	$2.22_{34} \times 10^{7}$	$1.04_{84} \times 10^{11}$	$4.9_{43} \times 10^{14}$	$4.6_{16} \times 10^{-74}$
Pb^{210}	22 y	$9.9_{84} \times 10^{-10}$	$9.9_{68} \times 10^{-19}$	$9.9_{52} \times 10^{-28}$	9.94×10^{-37}	$4.6_{09} \times 10^{-83}$
Bi^{210}	4.85 d	$1.654_{13} \times 10^{-6}$	$2.73_{61} \times 10^{-12}$	$4.52_{59} \times 10^{-18}$	$7.48_{65} \times 10^{-24}$	$7.6_{23} \times 10^{-89}$
Po^{210}	138.39 d	$5.797_{05} \times 10^{-8}$	$3.360_{58} \times 10^{-15}$	$1.948_{14} \times 10^{-22}$	$1.129_{35} \times 10^{-29}$	$4.4_{19} \times 10^{-96}$
Pb^{206}	Stable					
Branch Via Tl^{210}						
Tl^{210}	1.32 m	$8.75_{19} \times 10^{-3}$	$7.6_{60} \times 10^{-5}$	$6.7_{03} \times 10^{-7}$	$5.8_{68} \times 10^{-9}$	$8.5_{68} \times 10^{-80}$
Pb^{210}	22 y	$9.9_{84} \times 10^{-10}$	$9.9_{68} \times 10^{-19}$	$9.9_{52} \times 10^{-28}$	9.94×10^{-37}	$8.5_{54} \times 10^{-89}$
Bi^{210}	4.85 d	$1.654_{13} \times 10^{-6}$	$2.73_{61} \times 10^{-12}$	$4.52_{59} \times 10^{-18}$	$7.48_{65} \times 10^{-24}$	$1.41_{50} \times 10^{-94}$
Po^{210}	138.39 d	$5.797_{05} \times 10^{-8}$	$3.360_{58} \times 10^{-15}$	$1.948_{14} \times 10^{-22}$	$1.129_{35} \times 10^{-29}$	$8.2_{03} \times 10^{-102}$
Pb^{206}	Stable					

Table 2.--Uranium[238] series, products of differences of decay constants (in sec^{-1})

	$y \longrightarrow$	λ_{U}^{238}	λ_{Th}^{234}	λ_{Pa}^{234}	λ_{U}^{234}	λ_{Th}^{230}	λ_{Ra}^{226}	λ_{Rn}^{222}
1	$(\lambda_{U}^{238}-y)$		$-3.328_{85}\times10^{-7}$	$-9.4_{69}\times10^{-3}$	$-8.874_{25}\times10^{-14}$	$-2.74_{56}\times10^{-13}$	$-1.29_{97}\times10^{-11}$	$-2.0973_{95}\times10^{-6}$
2	$1\times(\lambda_{Th}^{234}-y)$	$3.328_{85}\times10^{-7}$		$8.9_{66}\times10^{-5}$	$-2.954_{11}\times10^{-20}$	$-9.1_{40}\times10^{-20}$	$-4.32_{63}\times10^{-18}$	$3.700_{87}\times10^{-12}$
3	$2\times(\lambda_{Pa}^{234}-y)$	$3.15_{22}\times10^{-9}$	$-3.15_{21}\times10^{-9}$		$-2.79_{73}\times10^{-22}$	$-8.6_{54}\times10^{-22}$	$-4.09_{67}\times10^{-20}$	$3.50_{37}\times10^{-14}$
4	$3\times(\lambda_{U}^{234}-y)$	$2.79_{73}\times10^{-22}$	$1.04_{93}\times10^{-15}$	$-8.4_{90}\times10^{-7}$		$1.60_{81}\times10^{-34}$	$5.28_{82}\times10^{-31}$	$-7.34_{86}\times10^{-20}$
5	$4\times(\lambda_{Th}^{230}-y)$	$7.6_{80}\times10^{-35}$	$-3.49_{29}\times10^{-22}$	$8.0_{40}\times10^{-9}$	$-5.1_{98}\times10^{-35}$		$-6.7_{28}\times10^{-42}$	$1.54_{13}\times10^{-25}$
6	$5\times(\lambda_{Ra}^{226}-y)$	$9.98_{20}\times10^{-46}$	$1.16_{27}\times10^{-28}$	$-7.6_{13}\times10^{-11}$	$-6.7_{09}\times10^{-46}$	$2.04_{59}\times10^{-45}$		$-3.23_{27}\times10^{-31}$
7	$6\times(\lambda_{Rn}^{222}-y)$	$2.09_{36}\times10^{-51}$	$2.05_{16}\times10^{-34}$	$7.2_{07}\times10^{-13}$	$-1.40_{72}\times10^{-51}$	$4.2_{91}\times10^{-51}$	$-1.41_{11}\times10^{-47}$	
8	$7\times(\lambda_{Po}^{218}-y)$	$7.9_{30}\times10^{-54}$	$7.7_{70}\times10^{-37}$	$-4.0_{95}\times10^{-15}$	$-5.3_{30}\times10^{-54}$	$1.62_{54}\times10^{-53}$	$-5.3_{45}\times10^{-50}$	$-1.22_{38}\times10^{-33}$
9	$8\times(\lambda_{Pb}^{214}-y)$	$3.41_{83}\times10^{-57}$	$3.34_{68}\times10^{-40}$	$3.7_{01}\times10^{-17}$	$-2.29_{76}\times10^{-57}$	$7.0_{06}\times10^{-67}$	$-2.30_{39}\times10^{-53}$	$-5.24_{95}\times10^{-37}$
10	$9\times(\lambda_{Bi}^{214}-y)$	$2.00_{46}\times10^{-60}$	$1.96_{15}\times10^{-43}$	$-3.2_{88}\times10^{-19}$	$-1.34_{74}\times10^{-60}$	$4.1_{09}\times10^{-60}$	$-1.35_{11}\times10^{-56}$	$-3.06_{74}\times10^{-40}$
	Branch Via Po^{214}							
11	$10\times(\lambda_{Po}^{214}-y)$	$9.4_{52}\times10^{-57}$	$9.2_{49}\times10^{-40}$	$-1.55_{02}\times10^{-15}$	$-6.3_{53}\times10^{-57}$	$1.93_{73}\times10^{-56}$	$-6.3_{71}\times10^{-53}$	$-1.44_{64}\times10^{-36}$
12	$11\times(\lambda_{Pb}^{210}-y)$	$9.4_{37}\times10^{-66}$	$-3.06_{96}\times10^{-46}$	$1.46_{79}\times10^{-17}$	$-6.3_{43}\times10^{-66}$	$1.9_{34}\times10^{-65}$	$-6.2_{78}\times10^{-62}$	$3.03_{21}\times10^{-42}$
13	$12\times(\lambda_{Bi}^{210}-y)$	$1.56_{10}\times10^{-71}$	$-4.05_{57}\times10^{-52}$	$-1.38_{97}\times10^{-19}$	$-1.0_{49}\times10^{-71}$	$3.1_{99}\times10^{-71}$	$-1.0_{38}\times10^{-67}$	$-1.34_{40}\times10^{-48}$
14	$13\times(\lambda_{Po}^{210}-y)$	$9.0_{49}\times10^{-79}$	$1.11_{50}\times10^{-58}$	$1.31_{60}\times10^{-21}$	$-6.0_{82}\times10^{-79}$	$1.8_{54}\times10^{-78}$	$-6.0_{18}\times10^{-75}$	$2.74_{11}\times10^{-54}$
15	$14\times(\lambda_{Pb}^{206}-y)$	$-4.4_{19}\times10^{-96}$	$-3.71_{16}\times10^{-65}$	$-1.24_{61}\times10^{-23}$	$5.3_{98}\times10^{-92}$	$-5.0_{91}\times10^{-91}$	$7.8_{22}\times10^{-86}$	$-5.7_{49}\times10^{-60}$
	Branch Via Tl^{210}							
16	$10\times(\lambda_{Tl}^{210}-y)$	$1.75_{44}\times10^{-62}$	$1.71_{66}\times10^{-45}$	$2.3_{58}\times10^{-22}$	$-1.17_{92}\times10^{-62}$	$3.5_{96}\times10^{-62}$	$-1.18_{24}\times10^{-58}$	$-2.68_{39}\times10^{-42}$
17	$16\times(\lambda_{Pb}^{210}-y)$	$1.75_{16}\times10^{-71}$	$-5.6_{97}\times10^{-52}$	$-2.2_{33}\times10^{-24}$	$-1.17_{72}\times10^{-71}$	$3.5_{89}\times10^{-71}$	$-1.16_{52}\times10^{-67}$	$5.6_{26}\times10^{-48}$
18	$17\times(\lambda_{Bi}^{210}-y)$	$2.8_{97}\times10^{-77}$	$-7.5_{27}\times10^{-58}$	$2.11_{43}\times10^{-26}$	$-1.9_{47}\times10^{-77}$	$5.9_{37}\times10^{-77}$	$-1.9_{27}\times10^{-73}$	$-2.49_{40}\times10^{-54}$
19	$18\times(\lambda_{Po}^{210}-y)$	$1.67_{96}\times10^{-84}$	$2.06_{94}\times10^{-64}$	$-2.00_{21}\times10^{-28}$	$-1.12_{89}\times10^{-84}$	$3.4_{42}\times10^{-84}$	$-1.1_{17}\times10^{-80}$	$5.0_{86}\times10^{-60}$
20	$19\times(\lambda_{Pb}^{206}-y)$	$-8.2_{02}\times10^{-102}$	$-6.8_{89}\times10^{-71}$	$1.89_{58}\times10^{-30}$	$1.0_{02}\times10^{-97}$	$-9.4_{49}\times10^{-97}$	$1.45_{18}\times10^{-91}$	$-1.06_{68}\times10^{-65}$

Table 2.--Continued.

λ_{Po}^{218}	λ_{Pb}^{214}	λ_{Bi}^{214}	λ_{Po}^{214}	λ_{Tl}^{210}	λ_{Pb}^{210}	λ_{Bi}^{210}	λ_{Po}^{210}	λ_{Pb}^{206}
-3.7877×10^{-3}	-4.3106×10^{-4}	-5.8642×10^{-4}	-4.7153×10^{3}	-8.7519×10^{-3}	-9.984×10^{-10}	-1.65413×10^{-6}	-5.79705×10^{-8}	4.88328×10^{-18}
1.43453×10^{-5}	1.8567×10^{-7}	3.4369×10^{-7}	2.2234×10^{7}	7.659×10^{-5}	-3.314×10^{-16}	2.1855×10^{-12}	-1.59369×10^{-14}	1.62557×10^{-24}
8.150×10^{-8}	1.6781×10^{-9}	3.0589×10^{-9}	-1.0484×10^{11}	5.49×10^{-8}	-3.138×10^{-18}	2.0692×10^{-14}	-1.5091×10^{-16}	1.5393×10^{-26}
-3.0871×10^{-10}	-7.234×10^{-13}	-1.7903×10^{-12}	4.9435×10^{14}	-4.809×10^{-10}	3.132×10^{-27}	-3.4226×10^{-20}	8.748×10^{-24}	1.3661×10^{-39}
1.1693×10^{-12}	3.1182×10^{-16}	1.0499×10^{-15}	-2.3310×10^{18}	4.208×10^{-12}	-3.127×10^{-36}	5.6615×10^{-26}	-5.0714×10^{-31}	3.7507×10^{-52}
-4.4289×10^{-15}	-1.3441×10^{-19}	-6.157×10^{-19}	1.0991×10^{22}	-3.683×10^{-14}	3.081×10^{-45}	-9.365×10^{-32}	2.9393×10^{-38}	4.875×10^{-63}
1.6766×10^{-17}	5.766×10^{-23}	3.5974×10^{-22}	-5.183×10^{25}	3.223×10^{-16}	6.459×10^{-51}	-4.1511×10^{-38}	5.9944×10^{-44}	1.0224×10^{-68}
	1.9354×10^{-25}	1.1516×10^{-24}	2.4438×10^{29}	-1.600×10^{-18}	2.446×10^{-53}	-1.5716×10^{-40}	2.2705×10^{-46}	3.8727×10^{-71}
-5.628×10^{-20}		-1.789×10^{-28}	-1.1523×10^{33}	1.331×10^{-20}	1.055×10^{-56}	-6.749×10^{-44}	9.786×10^{-50}	1.6694×10^{-74}
1.8016×10^{-22}	3.007×10^{-29}		5.434×10^{36}	-1.087×10^{-22}	6.184×10^{-60}	-3.9464×10^{-47}	5.7380×10^{-53}	9.790×10^{-78}
8.495×10^{-19}	1.4178×10^{-25}	-8.4363×10^{-25}			2.916×10^{-56}	-1.8608×10^{-43}	2.7056×10^{-49}	4.616×10^{-74}
-3.2177×10^{-21}	-6.111×10^{-29}	4.9472×10^{-28}	-2.5621×10^{40}			3.0762×10^{-49}	-1.5415×10^{-56}	4.609×10^{-83}
1.2182×10^{-23}	2.624×10^{-32}	-2.893×10^{-31}	1.2081×10^{44}		4.821×10^{-62}		-2.4604×10^{-62}	7.623×10^{-89}
-4.614×10^{-26}	-1.1311×10^{-35}	1.696×10^{-34}	-5.696×10^{47}		2.746×10^{-69}	-4.910×10^{-55}		4.419×10^{-96}
1.7477×10^{-28}	4.876×10^{-39}	-9.9475×10^{-37}	2.6860×10^{51}		-2.742×10^{-78}	8.122×10^{-61}	1.4263×10^{-69}	
8.943×10^{-25}	2.5019×10^{-31}	-1.461×10^{-30}			5.412×10^{-62}	-3.4531×10^{-49}	5.0218×10^{-55}	8.568×10^{-80}
-3.3875×10^{-27}	-1.0785×10^{-34}	8.5670×10^{-34}		9.513×10^{-25}		5.709×10^{-55}	-2.8610×10^{-62}	8.554×10^{-89}
1.2825×10^{-29}	4.631×10^{-38}	-5.0097×10^{-37}		-8.32×10^{-27}	8.947×10^{-68}		-4.5666×10^{-68}	1.4150×10^{-94}
-4.858×10^{-32}	-1.9960×10^{-41}	2.937×10^{-40}		7.28×10^{-29}	5.097×10^{-75}	-9.112×10^{-61}		8.203×10^{-102}
1.8400×10^{-34}	8.604×10^{-45}	-1.723×10^{-43}		-6.38×10^{-31}	-5.089×10^{-84}	1.5072×10^{-66}	2.6473×10^{-75}	

Table 3.--Uranium^{235} series, half lives, powers, and products of decay constants.

Nuclide	Half Life $T_{1/2}$	Powers & Products of Decay Constants λ (sec^{-1})	λ (sec^{-1})	λ (sec^{-1})	λ (sec^{-1})	$\Pi\lambda$ (sec^{-1})
U^{235}	7.13×10^{8} y	3.08064×10^{-17}	9.4904×10^{-33}	2.9236×10^{-50}	9.0067×10^{-66}	3.08064×10^{-17}
Th^{231}	25.5 h	7.5506×10^{-6}	5.7012×10^{-11}	4.3048×10^{-16}	3.2504×10^{-21}	2.3261×10^{-22}
Pa^{231}	34,300 y	6.4038×10^{-13}	4.1008×10^{-25}	2.6261×10^{-37}	1.6817×10^{-49}	1.4896×10^{-34}
Ac^{227}	21.7 y	1.01221×10^{-9}	1.0246×10^{-18}	1.0371×10^{-27}	1.0497×10^{-36}	1.5078×10^{-43}
Branch Via Th^{227}						
Th^{227}	18.6 d	4.3132×10^{-7}	1.8604×10^{-13}	8.024×10^{-20}	3.461×10^{-26}	6.5033×10^{-50}
Ra^{223}	11.2 d	7.1630×10^{-7}	5.131×10^{-13}	3.675×10^{-19}	2.633×10^{-25}	4.6583×10^{-56}
Rn^{219}	3.92 s	1.76823×10^{-1}	3.1266×10^{-2}	5.5286×10^{-3}	9.776×10^{-4}	8.237×10^{-57}
Po^{215}	1.83×10^{-3} s	3.7877×10^{2}	1.4346×10^{5}	5.434×10^{7}	2.0582×10^{10}	3.1199×10^{-54}
Pb^{211}	36.1 m	3.2001×10^{-4}	1.02408×10^{-7}	3.2772×10^{-11}	1.0487×10^{-14}	9.984×10^{-58}
Bi^{211}	2.16 m	5.3484×10^{-3}	2.8605×10^{-5}	1.5299×10^{-7}	8.182×10^{-10}	5.340×10^{-60}
Branch Via Po^{211}						
Po^{211}	5×10^{-3} s	1.386×10^{2}	1.92×10^{4}	2.66×10^{6}	3.69×10^{8}	7.40×10^{-58}
Pb^{207}	Stable					
Branch Via Tl^{207}						
Tl^{207}	4.77 m	2.42190×10^{-3}	5.8656×10^{-6}	1.4206×10^{-8}	3.440×10^{-11}	1.293×10^{-62}
Pb^{207}	Stable					
Branch Via Fr^{223}						
Fr^{223}	21 m	5.5012×10^{-4}	3.0263×10^{-7}	1.6648×10^{-10}	9.158×10^{-14}	8.2944×10^{-47}
Ra^{223}	11.2 d	7.1630×10^{-7}	5.131×10^{-13}	3.675×10^{-19}	2.633×10^{-25}	5.941×10^{-53}
Rn^{219}	3.92 s	1.76823×10^{-1}	3.1266×10^{-2}	5.5286×10^{-3}	9.776×10^{-4}	1.0506×10^{-53}
Po^{215}	1.83×10^{-3} s	3.7877×10^{2}	1.4346×10^{5}	5.434×10^{7}	2.0582×10^{10}	3.9792×10^{-51}
Pb^{211}	36.1 m	3.2001×10^{-4}	1.02408×10^{-7}	3.2772×10^{-11}	1.0457×10^{-14}	1.2734×10^{-54}
Bi^{211}	2.16 m	5.3484×10^{-3}	2.8605×10^{-5}	1.5299×10^{-7}	8.182×10^{-10}	6.811×10^{-57}
Branch Via Po^{211}						
Po^{211}	5×10^{-3} s	1.386×10^{2}	1.92×10^{4}	2.66×10^{6}	3.69×10^{8}	9.44×10^{-55}
Pb^{207}	Stable					
Branch Via Tl^{207}						
Tl^{207}	4.77 m	2.42190×10^{-3}	5.8656×10^{-6}	1.4206×10^{-8}	3.440×10^{-11}	1.6494×10^{-59}
Pb^{207}	Stable					

Table 4.--Uranium[235] series, products of differences of decay constants (in sec^{-1}).

	$y \longrightarrow$	$\lambda_{U^{235}}$	$\lambda_{Th^{231}}$	$\lambda_{Pa^{231}}$	$\lambda_{Ac^{227}}$	$\lambda_{Th^{227}}$	$\lambda_{Fr^{223}}$
1	$(\lambda_{U^{235}} - y)$		-7.5506×10^{-6}	-6.4035×10^{-13}	-1.01221×10^{-9}	-4.3132×10^{-7}	-5.5012×10^{-4}
2	$1 \times (\lambda_{Th^{231}} - y)$	7.5506×10^{-6}		-4.8350×10^{-18}	-7.6418×10^{-16}	-3.071×10^{-12}	2.9847×10^{-7}
3	$2 \times (\lambda_{Pa^{231}} - y)$	4.8350×10^{-18}	5.7012×10^{-11}		7.7302×10^{-24}	1.3244×10^{-18}	-1.6420×10^{-10}
4	$3 \times (\lambda_{Ac^{227}} - y)$	4.8941×10^{-27}	-4.3042×10^{-16}	-4.8910×10^{-27}		-5.699×10^{-25}	9.033×10^{-14}
	Branch Via Th^{227}						
5	$4 \times (\lambda_{Th^{227}} - y)$	2.1109×10^{-33}	3.0643×10^{-21}	-2.1096×10^{-33}	3.3264×10^{-30}		
6	$5 \times (\lambda_{Ra^{223}} - y)$	1.5120×10^{-39}	-2.0942×10^{-26}	-1.5111×10^{-39}	2.3793×10^{-36}	-1.624×10^{-31}	
7	$6 \times (\lambda_{Rn^{219}} - y)$	2.6736×10^{-40}	-3.7029×10^{-27}	-2.6719×10^{-40}	4.2072×10^{-37}	-2.872×10^{-32}	
8	$7 \times (\lambda_{Po^{215}} - y)$	1.0127×10^{-37}	-1.4026×10^{-24}	-1.0120×10^{-37}	1.5935×10^{-34}	-1.0878×10^{-29}	
9	$8 \times (\lambda_{Pb^{211}} - y)$	3.2407×10^{-41}	-4.3824×10^{-28}	-3.2387×10^{-41}	5.100×10^{-38}	-3.476×10^{-33}	
10	$9 \times (\lambda_{Bi^{211}} - y)$	1.7333×10^{-43}	-2.3406×10^{-31}	-1.7322×10^{-43}	2.7274×10^{-40}	-1.859×10^{-35}	
	Branch Via Po^{211}						
11	$10 \times (\lambda_{Po^{211}} - y)$	2.403×10^{-41}	-3.245×10^{-28}	-2.401×10^{-41}	3.781×10^{-38}	-2.577×10^{-33}	
12	$11 \times (\lambda_{Pb^{207}} - y)$	-7.40×10^{-58}	2.450×10^{-33}	1.538×10^{-53}	-3.827×10^{-47}	1.112×10^{-39}	
	Branch Via Tl^{207}						
13	$10 \times (\lambda_{Tl^{207}} - y)$	4.1978×10^{-46}	-5.651×10^{-33}	-4.1951×10^{-46}	6.605×10^{-43}	-4.502×10^{-38}	
14	$13 \times (\lambda_{Pb^{207}} - y)$	-1.2932×10^{-62}	4.2668×10^{-38}	2.6865×10^{-58}	-6.686×10^{-52}	1.942×10^{-44}	
	Branch Via Fr^{223}						
15	$4 \times (\lambda_{Fr^{223}} - y)$	2.6923×10^{-30}	-2.3358×10^{-19}	2.6906×10^{-30}	4.2525×10^{-27}		
16	$15 \times (\lambda_{Ra^{223}} - y)$	1.9285×10^{-36}	-1.5960×10^{-24}	1.9273×10^{-36}	3.0418×10^{-33}		-4.9626×10^{-17}
17	$16 \times (\lambda_{Rn^{219}} - y)$	3.4100×10^{-37}	2.8220×10^{-30}	3.4079×10^{-37}	5.3785×10^{-34}		-8.748×10^{-18}
18	$17 \times (\lambda_{Po^{215}} - y)$	1.2916×10^{-34}	1.0689×10^{-27}	1.2908×10^{-34}	2.0372×10^{-31}		-3.3133×10^{-15}
19	$18 \times (\lambda_{Pb^{211}} - y)$	4.1339×10^{-38}	3.3399×10^{-31}	4.1307×10^{-38}	6.519×10^{-35}		7.624×10^{-19}
20	$19 \times (\lambda_{Bi^{211}} - y)$	2.2106×10^{-40}	1.7838×10^{-33}	2.2092×10^{-40}	3.4868×10^{-37}		3.6582×10^{-21}
	Branch Via Po^{211}						
21	$20 \times (\lambda_{Po^{211}} - y)$	3.065×10^{-38}	2.473×10^{-31}	3.063×10^{-38}	4.83×10^{-35}		5.07×10^{-19}
22	$21 \times (\lambda_{Pb^{207}} - y)$	-9.44×10^{-55}	-1.867×10^{-36}	-1.961×10^{-50}	-4.89×10^{-44}		-2.790×10^{-22}
	Branch Via Tl^{207}						
23	$20 \times (\lambda_{Tl^{207}} - y)$	5.354×10^{-43}	4.3066×10^{-36}	5.351×10^{-43}	8.445×10^{-40}		6.847×10^{-24}
24	$23 \times (\lambda_{Pb^{207}} - y)$	-1.6494×10^{-59}	-3.2518×10^{-42}	-3.4264×10^{-55}	-8.548×10^{-49}		-3.767×10^{-27}

Table 4.--Continued.

λ_{Ra}^{223}	λ_{Rn}^{219}	λ_{Po}^{215}	λ_{Pb}^{211}	λ_{Bi}^{211}	λ_{Po}^{211}	λ_{Tl}^{207}	λ_{Pb}^{207}
-7.1630×10^{-7}	-1.76823×10^{-1}	-3.7877×10^{2}	-3.2001×10^{-4}	-5.3484×10^{-3}	-1.386×10^{2}	-2.42190×10^{-3}	3.08064×10^{-17}
-4.8954×10^{-12}	3.1265×10^{-2}	1.4347×10^{5}	9.9992×10^{-8}	2.8565×10^{-5}	1.922×10^{4}	5.8473×10^{-6}	2.3261×10^{-22}
3.5066×10^{-18}	-5.5284×10^{-3}	-5.4340×10^{7}	-3.1999×10^{-11}	-1.5277×10^{-7}	-2.66×10^{6}	-1.41616×10^{-8}	1.4896×10^{-34}
-2.5082×10^{-24}	9.7765×10^{-4}	2.0582×10^{10}	1.02399×10^{-14}	8.171×10^{-10}	3.69×10^{8}	3.4298×10^{-11}	1.5078×10^{-43}
7.148×10^{-31}	-1.7285×10^{-4}	-7.796×10^{12}	-3.2725×10^{-18}	-4.3697×10^{-12}	-5.12×10^{10}	-8.3051×10^{-14}	6.5033×10^{-50}
	3.0564×10^{-5}	2.9529×10^{15}	1.0449×10^{-21}	2.3368×10^{-14}	7.10×10^{12}	2.0108×10^{-16}	4.6583×10^{-56}
1.2639×10^{-31}		-1.1179×10^{18}	1.8443×10^{-22}	4.0070×10^{-15}	-9.83×10^{14}	3.5069×10^{-17}	8.237×10^{-57}
4.787×10^{-29}	1.1571×10^{-2}		6.9855×10^{-20}	1.5177×10^{-12}	-2.36×10^{17}	1.3283×10^{-14}	3.1199×10^{-54}
1.5286×10^{-32}	-2.0424×10^{-3}	4.2344×10^{20}		-7.631×10^{-15}	3.27×10^{19}	-2.7919×10^{-17}	9.984×10^{-58}
8.174×10^{-35}	3.5022×10^{-4}	-1.6038×10^{23}	3.5126×10^{-22}		-4.54×10^{21}	-8.170×10^{-20}	5.340×10^{-60}
1.133×10^{-32}	4.85×10^{-2}	3.851×10^{25}	4.87×10^{-20}	-1.058×10^{-12}			7.40×10^{-58}
-8.12×10^{-39}	-8.57×10^{-3}	-1.459×10^{28}	-1.558×10^{-23}	5.66×10^{-15}	6.29×10^{23}		
1.9791×10^{-37}	-6.1079×10^{-5}	6.075×10^{25}	7.383×10^{-25}	2.2333×10^{-17}			1.293×10^{-62}
-1.4176×10^{-43}	1.0800×10^{-5}	-2.3009×10^{28}	-2.3627×10^{-28}	-1.1945×10^{-19}		1.9788×10^{-22}	
-1.3780×10^{-27}	-1.7232×10^{-4}	-7.796×10^{12}	2.3563×10^{-18}	-3.9206×10^{-12}	-5.12×10^{10}	-6.4198×10^{-14}	8.2944×10^{-47}
	3.0469×10^{-5}	2.9529×10^{15}	-7.523×10^{-22}	2.0966×10^{-14}	7.10×10^{12}	1.5544×10^{-16}	5.941×10^{-53}
-2.4366×10^{-28}		-1.1179×10^{18}	-1.3279×10^{-22}	3.5951×10^{-15}	-9.83×10^{14}	2.7108×10^{-17}	1.0506×10^{-53}
-9.229×10^{-26}	1.1535×10^{-2}		-5.030×10^{-20}	1.3617×10^{-12}	-2.36×10^{17}	1.0268×10^{-14}	3.9792×10^{-51}
-2.947×10^{-29}	-2.0360×10^{-3}	4.2344×10^{20}		-6.847×10^{-15}	3.27×10^{19}	-2.1581×10^{-17}	1.2734×10^{-54}
-1.5759×10^{-31}	3.4913×10^{-4}	-1.6038×10^{23}	-2.5291×10^{-22}		-4.54×10^{21}	-6.316×10^{-20}	6.811×10^{-57}
-2.185×10^{-29}	4.83×10^{-2}	3.851×10^{25}	-3.51×10^{-20}	-9.49×10^{-13}			9.44×10^{-55}
1.565×10^{-35}	-8.55×10^{-3}	-1.459×10^{28}	1.122×10^{-23}	5.08×10^{-15}	6.29×10^{23}		
-3.815×10^{-34}	-6.0889×10^{-5}	6.075×10^{25}	-5.316×10^{-25}	2.0038×10^{-17}			1.6494×10^{-59}
2.7330×10^{-40}	1.0767×10^{-5}	-2.3009×10^{28}	1.7012×10^{-28}	-1.0717×10^{-19}		1.5296×10^{-22}	

Table 5.--Thorium^{232} series, half lives, powers, and products of decay constants.

Nuclide	Half Life $T_{1/2}$	Powers & Products of Decay Constants				
		λ (sec^{-1})	λ^2 (sec^{-1})	λ^3 (sec^{-1})	λ^4 (sec^{-1})	$\Pi\lambda$ (sec^{-1})
Th^{232}	1.39×10^{10} y	1.5802×10^{-18}	2.4971×10^{-36}	3.946×10^{-54}	6.235×10^{-72}	1.5802×10^{-18}
Ra^{228}	6.7 y	3.2784×10^{-9}	1.0748×10^{-17}	3.523×10^{-26}	1.1551×10^{-34}	5.180×10^{-27}
Ac^{228}	6.13 h	3.14096×10^{-5}	9.86556×10^{-10}	3.0976×10^{-14}	9.7331×10^{-19}	1.6272×10^{-31}
Th^{228}	1.90 y	1.15605×10^{-8}	1.3364×10^{-16}	1.5450×10^{-24}	1.7861×10^{-32}	1.8811×10^{-39}
Ra^{224}	3.64 d	2.20399×10^{-6}	4.8576×10^{-12}	1.0706×10^{-17}	2.3596×10^{-23}	4.146×10^{-45}
Rn^{220}	54.5 s	1.27183×10^{-2}	1.61755×10^{-4}	2.0572×10^{-6}	2.6165×10^{-8}	5.273×10^{-47}
Po^{216}	0.158 s	4.3870	1.9246×10^{1}	8.443×10^{1}	3.7040×10^{2}	2.3132×10^{-46}
Pb^{212}	10.6 h	1.8164×10^{-5}	3.2994×10^{-10}	5.993×10^{-15}	1.0886×10^{-19}	4.202×10^{-51}
Bi^{212}	60.5 m	1.90950×10^{-4}	3.6462×10^{-8}	6.9624×10^{-12}	1.3295×10^{-15}	8.023×10^{-55}
Branch Via Po^{212}						
Po^{212}	3.04×10^{-7} s	2.2801×10^{6}	5.1988×10^{12}	1.1854×10^{19}	2.7028×10^{25}	1.8294×10^{-48}
Pb^{208}	Stable					
Branch Via Tl^{208}						
Tl^{208}	3.1 m	3.727×10^{-3}	1.3887×10^{-5}	5.175×10^{-8}	1.929×10^{-10}	2.990×10^{-57}
Pb^{208}	Stable					

Table 6.--Thorium²³² series, products of differences of decay constants (in sec^{-1})

	$y \longrightarrow$	λ_{Th}^{232}	λ_{Ra}^{228}	λ_{Ac}^{228}	λ_{Th}^{228}	λ_{Ra}^{224}	λ_{Rn}^{220}	λ_{Po}^{216}	λ_{Pb}^{212}	λ_{Bi}^{212}	λ_{Po}^{212}	λ_{Tl}^{208}	λ_{Pb}^{208}
1	$(\lambda_{Th}^{232} - y)$		-3.2783×10^{-9}	-3.14096×10^{-5}	-1.15605×10^{-8}	-2.20399×10^{-6}	-1.27183×10^{-2}	-4.3870	-1.8164×10^{-5}	-1.90950×10^{-4}	-2.2801×10^{6}	-3.726×10^{-3}	1.5802×10^{-18}
2	$1 \times (\lambda_{Ra}^{228} - y)$	3.2784×10^{-9}		9.8646×10^{-10}	9.574×10^{-17}	4.8504×10^{-12}	1.61755×10^{-4}	1.9246×10^{1}	3.2988×10^{-10}	3.6461×10^{-8}	5.1988×10^{12}	1.3888×10^{-5}	5.180×10^{-27}
3	$2 \times (\lambda_{Ac}^{228} - y)$	1.0297×10^{-13}	-1.0296×10^{-13}		3.0062×10^{-21}	1.41657×10^{-16}	-2.05216×10^{-6}	-8.443×10^{1}	4.369×10^{-15}	-5.8170×10^{-12}	-1.18537×10^{19}	-5.132×10^{-8}	1.6272×10^{-31}
4	$3 \times (\lambda_{Th}^{228} - y)$	1.1904×10^{-21}	-8.527×10^{-22}	-3.0973×10^{-14}		-3.1057×10^{-22}	2.6100×10^{-8}	3.7040×10^{2}	-7.932×10^{-20}	1.11069×10^{-15}	2.7028×10^{25}	1.912×10^{-10}	1.8811×10^{-39}
5	$4 \times (\lambda_{Ra}^{224} - y)$	2.6236×10^{-27}	-1.8766×10^{-27}	9.0458×10^{-19}	6.590×10^{-27}		-3.3189×10^{-10}	-1.6250×10^{3}	1.2659×10^{-24}	-2.0964×10^{-19}	-6.1625×10^{31}	-7.122×10^{-13}	4.146×10^{-45}
6	$5 \times (\lambda_{Rn}^{220} - y)$	3.3368×10^{-29}	-2.3867×10^{-29}	1.14763×10^{-20}	8.382×10^{-29}	-3.9493×10^{-24}		7.108×10^{3}	1.6077×10^{-26}	-2.6262×10^{-21}	1.4051×10^{38}	-6.404×10^{-15}	5.273×10^{-47}
7	$6 \times (\lambda_{Po}^{216} - y)$	1.4639×10^{-28}	-1.0471×10^{-28}	5.0347×10^{-20}	3.6774×10^{-28}	-1.7326×10^{-23}	-1.45179×10^{-9}		7.055×10^{-26}	-1.1521×10^{-20}	-3.2038×10^{44}	-2.807×10^{-14}	2.3182×10^{-46}
8	$7 \times (\lambda_{Pb}^{212} - y)$	2.6590×10^{-33}	-1.9016×10^{-33}	-6.6686×10^{-25}	6.676×10^{-33}	-2.7652×10^{-28}	1.8438×10^{-11}	-3.1183×10^{4}		1.9906×10^{-24}	7.3049×10^{50}	1.041×10^{-16}	4.202×10^{-51}
9	$8 \times (\lambda_{Bi}^{212} - y)$	5.077×10^{-37}	-3.631×10^{-37}	-1.0639×10^{-28}	1.2746×10^{-36}	-5.2192×10^{-32}	-2.3098×10^{-13}	1.3679×10^{5}	1.2186×10^{-29}		-1.6656×10^{57}	-3.681×10^{-19}	8.023×10^{-55}
	Branch Via Po^{212}												
10	$9 \times (\lambda_{Po}^{212} - y)$	1.1577×10^{-30}	-8.279×10^{-31}	-2.4258×10^{-22}	2.9062×10^{-30}	-1.1900×10^{-26}	-5.2665×10^{-7}	3.1189×10^{11}	2.7786×10^{-23}	4.5388×10^{-18}			1.8294×10^{-48}
11	$10 \times (\lambda_{Pb}^{208} - y)$	-1.8294×10^{-48}	2.714×10^{-39}	7.619×10^{-27}	-3.3598×10^{-38}	2.6228×10^{-31}	6.6981×10^{-9}	-1.3683×10^{12}	-5.047×10^{-28}	-8.667×10^{-22}	3.7977×10^{63}		
	Branch Via Tl^{208}												
12	$9 \times (\lambda_{Tl}^{208} - y)$	1.892×10^{-39}	-1.3531×10^{-39}	-3.931×10^{-31}	4.750×10^{-39}	-1.9438×10^{-34}	2.0769×10^{-15}	-5.996×10^{5}	4.519×10^{-32}	7.038×10^{-27}			2.990×10^{-57}
13	$12 \times (\lambda_{Pb}^{208} - y)$	-2.990×10^{-57}	4.436×10^{-48}	1.2348×10^{-35}	-5.491×10^{-47}	4.284×10^{-40}	-2.6414×10^{-17}	2.6304×10^{6}	-8.209×10^{-37}	-1.3439×10^{-30}		1.372×10^{-21}	