Geology and Mineralogy

UNITED STATES DEPARTMENT OF THE INTERIOR

GEOLOGICAL SURVEY

THE EQUATION OF CONTINUITY IN GEOLOGY WITH APPLICATIONS TO THE TRANSPORT OF RADIOACTIVE GAS *

By

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May 1956

Trace Elements Investigations Report 396

*This report concerns work done on behalf of the Division of Research of the U. S. Atomic Energy Commission.

USGS - TEI-396

GEOLOGY AND MINERALOGY

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THE EQUATION OF CONTINUITY IN GEOLOGY WITH APPLICATIONS TO THE TRANSPORT OF RADIOACTIVE GAS

By A. Y. Sakakura, Carolyn Lindberg, and Henry Faul

ABSTRACT

The transport of matter by fluids percolating through a porous medium is described by setting up a mass conservation equation, analogous to that of hydrodynamics, for each of the substances in question. The sources and sinks of these equations serve to take into account the contribution from the medium and the interactions with other transported substances. The transport of radon by natural gases is treated in detail for the case of two important sources: the extended source and the cylindrical shell source. From the solutions of the steady-state cases, the two source types can be distinguished. From the solution of the transient case, the boundary of the source nearest to the gas well can be established. The results are applied to some selected data from the Texas Panhandle gas field.

INTRODUCTION

By transport phenomena, we mean those situations in which the materials of interest are present in such small quantities that their individual diffusive properties are overwhelmed by the motion of the carrier. Such situations are common in geology. There are many instances where fluids flow, picking up, carrying, and depositing substances. Waters may transport and deposit various minerals. Gases and liquids may be tagged artificially with stable or radioactive tracers so that their flow may be observed. Sometimes gas may pick up radon, the gaseous decay product of radium contained in the pore space, and carry it as a natural tracer.

Under these conditions we can solve for the behavior of the carrier separately, and then set up an equation (or equations) of continuity for the substance of interest. These problems are greatly simplified by the availability of a large number of solutions to the various flow problems (Muskat, 1946). In the first part of this paper, we shall derive the equation for a general situation, and the remainder of the paper will be devoted to the specific problem of radon transport.

Let A = the substance carried

$\vec{F}$ = mass flux of carrier (gram per unit area per second)

η_A = concentration of substance A per gram of carrier

f = porosity of the medium

γ = density of the carrier (gram per unit volume)

S_A = rate of production of substance A by the porous medium per unit volume of the medium

L_A = rate of production (or loss) of substance A per unit volume of the porous medium through chemical interaction or radioactive decay

λ_A = decay constant of A (per second),

then the equation of continuity (more conservation) becomes

$$\nabla\cdot(\vec{F}\eta_A) + f\frac{\partial}{\partial t}(\gamma\eta_A) = S_A + L_A,$$

$$\nabla\cdot(\vec{F}\eta_B) + f\frac{\partial}{\partial t}(\gamma\eta_B) = S_B + L_B, \text{ etc.} \qquad (1.1)$$

In chemical reactions of the form

$$aA + bB + cC + \ldots\ldots \rightleftarrows dD + eE \;\ldots\ldots,$$

the L's can be shown easily to be

$$\begin{aligned} L_A &= -f\left[k_1 (\gamma\eta_A)^a (\gamma\eta_B)^b (\gamma\eta_C)^c \ldots\ldots - k_2 (\gamma\eta_D)^d (\gamma\eta_E)^e \ldots\ldots\right], \\ L_B &= \frac{b}{a} L_A, \\ L_D &= -\frac{d}{a} L_A, \quad \text{etc.} \end{aligned} \tag{1.2}$$

In the radioactive chain

$$\begin{tikzcd} A \arrow[r, "\lambda_1"] \arrow[rr, bend right, "\lambda_3"'] & B \arrow[r, "\lambda_2"] & C \arrow[r, "\lambda_4"] & {} \end{tikzcd} \qquad \text{etc.}$$

the L's become

$$\begin{aligned} L_A &= -f\gamma(\lambda_1 + \lambda_3)\eta_A \\ L_B &= f\gamma(\lambda_1\eta_A - \lambda_2\eta_B) \\ L_C &= f\gamma(\lambda_3\eta_A - \lambda_4\eta_C), \quad \text{etc.} \end{aligned} \tag{1.3}$$

We may well have a combination of both processes. The flux $\vec{F}$ and the density γ of the carrier (in the case of gases) are well known from solutions of Muskat's phenomenological equations of flow through porous media. The equations in general are nonlinear and will present great difficulty in solution except by approximate methods. Of course, if the importance of the problem warrants it, they may be solved by modern high speed computers.

It should be noted that the reaction constants k_1 and k_2 in equation (1.2) are not necessarily the ones encountered in the laboratory for the surface effects of the porous medium might be considerable.

Acknowledgments

The writers wish to acknowledge the friendly cooperation of the personnel of various gas companies operating in the Panhandle area, Colorado Interstate Gas Company, Panhandle Eastern Pipeline Company, and Phillips Petroleum Company. This paper concerns work done by the U. S. Geological Survey on behalf of the Division of Research of the U. S. Atomic Energy Commission.

STEADY STATE TRANSPORT OF RADON BY GASES

Cylindrical source

We shall consider only sources possessing cylindrical symmetry about a gas well and extending throughout the vertical dimension of the gas pay zone. This problem is of paramount interest in a study of the radioactivity associated with some natural gases (Faul and others, 1954).

The gas sample tapped at the well will represent the source averaged over the surface of a cylinder centered about the well. The equation of continuity then becomes

$$\nabla \cdot (\vec{F}\eta) = -\lambda f \gamma \eta + \lambda \sigma \delta (r - r_0) \tag{2.1}$$

where (Muskat, 1946):

$\vec{F} = \gamma \vec{v}$

$\gamma = \gamma' p$

$\vec{v} = -\frac{k}{\mu} \nabla p = -\frac{Q}{2\pi D}\left(\frac{p_w}{p}\right)\frac{1}{r}\left(\frac{\vec{r}}{r}\right)$

η = radon concentration (micromicrocuries per gram of carrier)

$p = \sqrt{\frac{\Delta p^2}{\log r_e / r_w} \log r/r_w + p_w^2}$ (pressure)

$\Delta p^2 = p_e^2 - p_w^2$

p_e = reservoir pressure

p_w = well-head pressure

r_e = reservoir radius

r_w = well radius

Q = volume production at well-head conditions

σ = micromicrocuries of radon per unit area emitted by a cylindrical source of radius r_o, height D. The total source strength is $2\pi r_o D\sigma$.

D = pay zone thickness.

The equation of continuity,

$$\frac{1}{r}\frac{d}{dr}\left[r\left(-\frac{Q}{2\pi D}\frac{p_w}{p}\frac{1}{r}\right)\gamma' p \eta\right] = -\lambda f \gamma' p \eta + \lambda \sigma \delta(r - r_0) \tag{2.2}$$

is integrated to

$$e^{-\lambda\Omega(r)}\eta(r) - \eta(r_w) = -\frac{2\pi D}{Q}\frac{\sigma\lambda}{\gamma' p_w} r_0\, e^{-\lambda\Omega(r_0)} st(r - r_0) \tag{2.3}$$

where

$$\Omega(r) = f\frac{2\pi D}{Q}\frac{1}{p_w}\int_{r_w}^{r} r p(r)\, dr \tag{2.4}$$

As no radon can flow backwards, $\eta = 0$ when $r > r_o$. Thus, we find

$$\eta(r_w) = \frac{2\pi D r_0}{Q}\,\frac{\lambda\sigma}{\gamma' p_w}\, e^{-\lambda\Omega(r_0)} \tag{2.5}$$

The significance of the exponential term can be seen if we rewrite (2.4),

$$\Omega(r_0) = \int_{r_w}^{r_0} \frac{f}{v(r)}\, dr \tag{2.6}$$

which is merely the time elapsed while the radon travels from r_o to r_w. Thus, the exponential factor is the decay factor as radon travels from r_o to r_w. Ω can be calculated through integration by parts, and we find

$$\Omega(r_0) = \frac{f 2\pi D}{Q p_w}\sum_{n=0}^{n}\frac{p r^2}{2}\left(-\frac{\Delta p^2}{2\log r_e/r_w}\right)^n \frac{\Gamma(3/2)}{\Gamma(3/2 - n)}\frac{1}{p^{2n}}\Bigg|_{r_w}^{r_0} \tag{2.7}$$

which is a semiconvergent series when $\dfrac{\Delta p^2}{2(p)^2 \log r_e/r_w} < 1$. We find that

$$\Omega(r_o) \approx \left\{ p(r_o)\, r_o^2 \left[1 - \frac{\Delta p^2}{4p^2(r_o) \log r_e/r_w} \right] - p_w r_w^2 \left[1 - \frac{\Delta p^2}{4p_w^2 \log r_e/r_w} \right] \right\} \frac{\pi D f}{Q p_w} \tag{2.8}$$

It is most convenient to measure the radon in terms of unit volume at standard conditions (STP), so we find M_o the radon density in micro-microcuries per unit volume at STP tapped at the well-head originating from a cylindrical source of radius r_o and effective surface source density of σ, as

$$M_o = \gamma(T_o, p_o)\, \eta(r_w)$$

$$= \frac{2\pi D r_o}{Q_o} \lambda \sigma e^{-\lambda \Omega(r_o)} \tag{2.9}$$

where Q_o = production at STP

p_o = atmospheric pressure

T_o = standard temperature

T_w = well-head temperature

It should be noted that σ differs from the actual surface source density by a factor due to the incomplete sweeping by the gas. The degree of sweeping is a function of the emanating power of the radium or uranium mineral present in the pore space. In the Panhandle gas field, the radioactive parents are probably present in what is described as "asphaltic petroleum residues." The emanating power of this material is estimated at 10 percent (F. J. Davis, Oak Ridge National Laboratory, and J. N. Rosholt, Jr., U. S. Geological Survey, oral communications).

We shall now examine the behavior of solution (2.9). An examination of equation (2.8) reveals that for $r_o \gtrless 100$ feet, the first term in square brackets can be neglected, while for all cases the second term is essentially zero. Thus, we rewrite equation (2.9) as

$$M_o \approx \frac{2\pi D r_o}{Q} \lambda\sigma \exp\left[\frac{-\lambda f \pi D}{Q p_w} p(r_o) r_o^2 \left(1 - \frac{\Delta p^2}{4p^2(r_o) \log r_e/r_w}\right)\right] \tag{2.10}$$

This can be rewritten using the relation

$$p^3 = p_e^2 - \left(\frac{\mu}{k}\right)\left(\frac{T_w}{T_o}\right)\frac{\log r_e/r_w}{\pi D} p_o Q_o \tag{2.11}$$

and upon taking the square root and retaining the first two terms

$$p \approx p_e \left(1 - \frac{1}{2}\frac{\mu}{k}\frac{T_w}{T_o}\frac{p_o Q_o}{p_e^2}\frac{\log r_e/r_w}{\pi D}\right). \tag{2.12}$$

Equation (2.9) finally becomes

$$M_o \approx \frac{2\pi D r_o}{Q_o} \lambda\sigma \exp\left[\frac{-\lambda f \pi D}{Q_o}\frac{T_o}{T_w}\frac{p_e}{p_o} r_o^2\right] \exp\left[\frac{\lambda f}{2}\frac{\mu}{k}\frac{r_o^2}{p_e}\left(\frac{1}{2} + \log\frac{r_e}{r}\right)\right]. \tag{2.13}$$

Several points are now immediately evident. If we multiply M_o (measured at series of steady states) by corresponding values of Q_o and plot against $1/Q_o$ on a semilogarithmic paper, the result will be a straight line. If the measured values yield a straight line, when treated in the foregoing manner, we have assurance that the source is of the cylindrical type. Moreover, as $\sigma 2\pi r_o D$ is the total effective source strength, we can immediately calculate it from the measured value of M_o, as the exponential is nearly equal to one.

Extended source

We are also interested in radon distribution originating from a source extending from R to the edge of the reservoir. We then replace σ by ζ dr_o (in micromicrocuries per unit volume) and integrate M_o from R to r_e,

$$
\begin{aligned}
M_o' &= \int_R^{r_e} \frac{\zeta}{\sigma} M_o \, dr_o \\
&= \frac{2\pi D}{Q_o} \lambda \zeta \int_R^{r_e} e^{-\lambda \Omega(r_o)} r_o \, dr_o \quad .
\end{aligned}
\tag{2.14}
$$

We expand Ω in inverse powers of r_o about R and find

$$
\begin{aligned}
\lambda \Omega(r_o) &\approx - \frac{\lambda f \pi D}{Q} \; 1 - \frac{1}{2} \omega(r_w) \Big] r_w^2 \\
&\quad + \frac{\lambda f \pi D}{Q} \frac{r_o^2 p(R)}{p_w} \left\{ \left(1 - \frac{\omega}{2}\right) - \omega \left(1 + \frac{\omega}{2}\right)\left(\frac{R}{r_o} - 1\right) \right. \\
&\qquad\qquad \left. + \frac{\omega}{2} (1 - \omega) \left(\frac{R}{r_o} - 1\right)^2 \right\} \\
&= \quad r_o^2 - B r_o - C
\end{aligned}
\tag{2.15}
$$

where $A = \dfrac{\lambda f \pi D}{Q} \dfrac{p(R)}{p_w} (1 + \omega)$

$$B = \frac{\lambda f \pi D}{Q} \frac{p(R)}{p_w} \left(2\omega - \frac{\omega^2}{2}\right) R$$

$$C = \frac{\lambda f \pi D}{Q} \left[\left(1 - \frac{\omega(r_w)}{2}\right) r_w^2 - \frac{p(R)}{p_w} \frac{\omega}{2} (1 - \omega) R^2 \right]$$

$$\omega = \frac{\Delta p^2}{2 p^2(R) \log r_e / r_w}$$

$$\omega(r_w) = \frac{\Delta p^2}{2 P_w^2 \log r_e/r_w} .$$

Thus,

$$M_0' = \frac{2\pi D\lambda}{Q}\, \xi \int_R^{r_e} e^{-A r_0^2 + B r_0 + C}\, r_0\, dr_0 . \tag{2.16}$$

Changing the variable,

$$r_0 = \frac{u(r_0) + \frac{B}{2\sqrt{A}}}{\sqrt{A}} \text{ , etc.} \tag{2.17}$$

we have

$$M_0' = \frac{2\pi D}{Q_0} \lambda \xi \int_{u(R)}^{u(r_e)} e^{-[u(r_0)]^2} e^{\frac{B^2}{4A}+C} \left[u(r_0) + \frac{B}{2\sqrt{A}}\right] du(r_0)$$

$$= \frac{2\pi D}{Q_0} \frac{\lambda\xi}{A} \exp\left(\frac{B^2}{4A} + C\right) \left\{ \frac{B}{2\sqrt{A}} \int_{u(R)}^{u(r_e)} e^{-u^2} du \right.$$

$$\left. + \frac{1}{2} \exp\left[-\left(R\sqrt{A} - \frac{B}{2\sqrt{A}}\right)^2\right] - \frac{1}{2} \exp\left[-\left(r_e\sqrt{A} - \frac{B}{2\sqrt{A}}\right)^2\right] \right\}$$

$$= \frac{1}{(1+\omega)} \frac{P_0}{f p(R)} \frac{T_w}{T_0} \xi \left\{ \exp\left(-AR^2 + BR + C\right) - \exp\left(-A r_e^2 + B r_e + C\right) \right.$$

$$\left. + \exp\left(\frac{B^2}{4A} + C\right) \frac{B}{\sqrt{A}} \int_{u(R)}^{u(r_e)} e^{-u^2} du \right\} \tag{2.18}$$

We are generally interested in values of R that are small compared to r_e (in fact, we often set R equal to r_w). It can be shown that, under normal well conditions and when R is small compared to r_e, equation (2.18) becomes:

$$M_o' \approx \frac{1}{1+\omega} \frac{P_o}{P(R)} \frac{T_w}{T_o} \frac{\xi}{f} \cdot e^{-\lambda\Omega(R)} \tag{2.19}$$

since $\frac{B}{\sqrt{A}} \int_{\mu(R)}^{\mu(r_e)} e^{-u^2} du \approx \frac{B}{\sqrt{A}} \frac{\sqrt{\pi}}{2} \approx 0$

and $\exp\left[-Ar_e^2 + Br_e + C\right] \approx 0$.

A semilogarithmic plot of measured values of M_o' times Q_o versus $1/Q_o$ will not yield a straight line as with the finite source. Therefore, in practice, it may be determined easily whether the radon can be attributed to a finite cylindrical or an extended source.

TRANSIENT STATE TRANSPORT OF RADON BY GASES

Considerable geologic information can be obtained from artificially induced transient motion of a gas. For example a well may be shut down for two or three weeks, or long enough to allow dynamic and radioactive equilibria to establish themselves. The well is then opened and produced at a known (and fairly constant) rate. Radon content is measured at suitable intervals for several days (fig. 2).

The time dependent transport equation takes the following form:

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(r|v|\gamma\eta\right)+f\frac{\partial}{\partial t}\left(\gamma\eta\right)=-\lambda f\gamma\eta+\lambda\sigma\delta(r-r_0) \qquad (3.1)$$

where the quantities have the same meaning as before, except that the gas properties are now functions of time. We shall solve the problem formally to show the general procedure for solving equations of this type. Let

$$r\gamma\eta=\lambda\sigma r_0\ st(r_0-r)\ st(t-t_0)\,e^{-\lambda(t-t')}\phi \qquad (3.2)$$

where if

$$\frac{dr}{dt}=-\frac{1}{f}|v| \qquad \text{with solution } G(r,t) = \text{constant},$$

then t_0 and t' satisfy the following boundary conditions,

$$G(r,t_0) = G(r_0,\ 0), \qquad G(r,t) = G(r_0,\ t') . \qquad (3.4)$$

t_0 is the time at which the radon, originating at r_0 at $t = 0$, first appears at r; the step function states the physical condition that the radon concentration is zero up to that time. t' is the time at which radon must start from r_0 in order to arrive at r at time t. Then the exponential merely denotes the decay of radon enroute. The step function in r_0 denotes the fact that radon cannot drift backwards. Inserting (3.2) now into equation (3.1) we find,

$$\delta(t-t_o)\, st(r_o-r)\, e^{-\lambda(t-t')}\phi\left[f+v\frac{\delta t_o}{\delta r}\right]$$
$$+\lambda\, st(r_o-r)\, st(t-t_o)\, e^{-\lambda(t-t')}\phi\left[-v\frac{\delta t'}{\delta r}+f\frac{\delta t'}{\delta t}\right]$$
$$+\delta(r-r_o)\left[v\, st(t-t_o)\, e^{-\lambda(t-t')}\phi-1\right]$$
$$+st(r_o-r)\, st(t-t_o)\, e^{-\lambda(t-t')}\left[-\frac{\delta}{\delta r}(v\phi)+f\frac{\delta\phi}{\delta t}\right]=0 \qquad (3.5)$$

The first term is zero because

$$\frac{\delta t}{\delta r} = -\frac{f}{v(r,t_o)}$$

The second term also can be shown to be zero by the following argument,

$$G(r,t) = G(r_o,\ t') \text{ or } t' = H\left[r_o,\ G(r,t)\right]$$

$$-v\frac{\delta t'}{\delta r}+f\frac{\delta t'}{\delta t} = \frac{\delta H}{\delta G}\left[-v\frac{\delta G}{\delta r}+f\frac{\delta G}{\delta t}\right]$$

but, $$\frac{\delta G}{\delta t} = \frac{\delta G}{\delta t}+\frac{\delta G}{\delta r}\frac{\delta r}{\delta t} = \frac{\delta G}{\delta t}-\frac{\delta G}{\delta r}\frac{v}{f} = 0$$

therefore, $$\left[-v\frac{\delta t'}{\delta r}+f\frac{\delta t'}{\delta t}\right]=0$$

The remaining terms contain singularities of different orders, so they must vanish separately. The vanishing of the third term gives us the boundary condition on ϕ, as when $r = r_o$, $t_o = 0$ and $t' = t$. Thus,

$$\phi = \frac{1}{v(r_o,t)} \qquad \text{when } r = r_o. \qquad (3.6)$$

The remaining term finally becomes the differential equation for ϕ.

$$\frac{\delta}{\delta r}(v\phi) = f\,\frac{\delta\phi}{\delta t} \quad . \tag{3.7}$$

We solve (3.7) by successive substitution. On the right hand side of equation (3.7) we set ϕ equal to $\frac{1}{v(r,t)}$, which satisfies the boundary condition (3.6). Then we find, upon solving equation (3.7),

$$\phi = \frac{1}{v(r,t)}\left[1 - f\int_{r_0}^{r} \frac{1}{v^2(r,t)}\,\frac{\delta v}{\delta t}\,dr\right] . \tag{3.8}$$

This process, of course, can be repeated indefinitely, that is, substitute (3.8) into the right hand side of equation (3.7) and integrate. However, if the second term in equation (3.8) is small compared to the first, then we can conclude that equation (3.8) furnishes a sufficiently good solution. It may be pointed out that the first approximation yields an exact answer when t approaches infinity, as the steady state is approached wherein the velocity is no longer time dependent. We shall now show that the equation (3.8) is an adequate approximation. It has been shown previously (Ritchie and Sakakura, 1956) that

$$|v(r,t)| = \frac{\frac{1}{2}\,\frac{k}{\mu r_w}\left(-\frac{\delta\psi}{\delta\rho}\right)}{\sqrt{p_e^2 - \psi}} \tag{3.9}$$

where

$$\psi = \frac{\Delta p^2}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{K_0(\sqrt{z}\,\rho)}{K_0(\sqrt{z})}\,\frac{e^{z\tau}}{z}\,dz$$

$$\tau = \frac{k}{\mu f}\,\frac{p_e}{r_w^2}\,t$$

$$\rho = \frac{r}{r_w}$$

and τ is large for almost all values of t. It further has been shown that the resulting expressions for the quantities in question are semiconvergent series in τ, and cannot be differentiated term by term. Thus, the contour integrals must be differentiated first, then integrated. If we retain only the dominant terms, we find the second term in the brackets of equation (3.8) to be $\frac{1}{2}\frac{(\rho^2 - \rho_o^2)}{\tau}$ which is small, as the validity of the semiconvergent series requires. Thus, we can state that equation (3.8) yields an adequate solution. The radon density (at STP), from equations (3.2) and (3.8), becomes

$$M_o(r_w,t) = \gamma(r_o,T_o)\,\eta(r_w,t)$$

$$= \lambda\sigma \frac{r_o}{r_w}\frac{T_w}{T_o}\frac{p_o}{p_w}\, st(r_o - r)\, st(t - t_o)\exp\left[-\lambda(t-t')\right]$$

$$\times \frac{1}{v(r_w,t)}\left[1 + f\int_{r_w}^{r_o}\frac{1}{v^2(r,t)}\frac{\delta v}{\delta t}\,dr\right] . \tag{3.10}$$

Transient gas flow phenomena are strongly dependent on the characteristics of the reservoir. These parameters are never sufficiently well known to warrant numerical evaluation of equation (3.10). The principal purpose of this derivation is to determine the nearest boundary of the radioactive source from which the radon may come.

NEAREST BOUNDARY OF SOURCE

We shall now discuss the most important aspect of the transient problem, common to the cylindrical source problem and to the extended source problem, which results from integrating the cylinder over r_o from R to r_e. When radon first appears, it must necessarily come from the point nearest to the well. Thus, we are interested in the solution to the equation

$$\frac{dr}{dt} = - \frac{v(r,t)}{f} \tag{4.1}$$

with the boundary conditions,

$t = 0 \qquad r = r_o$

$t = t_o \qquad r = r_w$

that is, we are looking for t_o. Using the substitutions for τ and ρ (equation 3.9) and letting

$$\xi = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{K_0(\sqrt{z}\,\rho)}{K_0(\sqrt{z})} \frac{e^{z\tau}}{z} dz \tag{4.2}$$

$$\alpha = \frac{\Delta p^2}{p_e^2}$$

equation (4.1) can be rewritten into a dimensionless form

$$\frac{d\rho}{d\tau} = - \frac{\delta}{\delta\rho} \sqrt{1 - \alpha\xi} = \frac{\alpha}{2} \frac{1}{\sqrt{1-\alpha\xi}} \frac{\delta\xi}{\delta\rho}$$

$$= -\frac{\alpha}{2} \frac{1}{\sqrt{1-\alpha\xi}} \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{K_1(\sqrt{z}\,\rho)}{K_0(\sqrt{z})} \frac{e^{z\tau}}{\sqrt{z}} dz \tag{4.3}$$

with $\rho = \rho_0$ when $\tau = \tau$ (eventually we shall set $\tau = 0$) and $\rho = 1$ when $\tau = \tau_0$. If we consider values of r_o sufficiently small and (or) τ sufficiently large, the radical can be set equal to its steady state value $\frac{p}{p_e}$ and only the first term of the expansion of the Bessel functions need be considered in the Laplace inversion integral. It has been shown that

$$\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\frac{K_1(\sqrt{z}\rho)}{K_0(\sqrt{z})}\frac{e^{z\tau}}{\sqrt{z}}\,dz = \frac{1}{\rho}\left[-2I_{1,-1}(\tau)+\ldots\right] \tag{4.4}$$

where

$$I_{n,-\nu} = \frac{1}{2\pi i}\int_{\infty}^{\beta+}\frac{z^{\nu}e^{z\tau}}{(\log\beta^2 z/4)^n}\,dz$$

$\beta = e^{\gamma}$, γ = Euler's constant.

Thus equation (4.3) becomes

$$\frac{d\rho}{d\tau} \approx \frac{\alpha}{\rho}\frac{\rho_e}{\rho} I_{1,-1} \tag{4.5}$$

and

$$\int_1^{\rho_0}\frac{\rho_e}{\rho}\rho\,d\rho = \alpha\int_{\tau_0}^{\tau} I_{1,-1}\,d\tau = \alpha\left[I_{1,-2}(\tau) - I_{1,-2}(\tau_0)\right] \tag{4.6}$$

We could obtain t from the above equation by fixing τ and solving for τ_0. We shall not do so for the same reason given for not evaluating equation (3.10). Returning now to finding the limiting case when t = 0, τ in equation (4.3) represents some earlier value of τ, as the series diverge for $\tau = 0$. However, it is known also that the cumulative production is

$$\int_0^t Q\,dt = Q^* = -2\pi D\,\frac{\Delta p^2}{p_e\,p_w}\,f r_w^2\,I_{1,-2}(\tau) \tag{4.7}$$

so we can write (4.6) as

$$\int_1^{\rho_0} p\rho\, d\rho = \frac{p_w}{2\pi D f r_w^2}\left[Q^*(T_0) - Q^*(T)\right]$$

$$\approx \frac{p_w}{2\pi D f r_w^2}\, Q^*(T_0) \,, \tag{4.8}$$

where we have dropped the cumulative production at T as it is zero when t is equal to 0. Furthermore, we rewrite this in terms of the function in equation (2.4),

$$\Omega(r_0) = \frac{Q^*}{Q_0} \,. \tag{4.9}$$

This must be corrected for the volume of gas standing in the well before opening. If V were the volume of the well, we replace Q^* by $Q^* - V\frac{p_e}{p_w}$, and upon correcting to STP, we find

$$\Omega(r_0) = \frac{Q_0^*}{Q_0} - V\,\frac{p_e}{p_0}\,\frac{T_0}{T_w}\,\frac{1}{Q_0} \tag{4.10}$$

as the expression which will yield the nearest boundary of the source.

APPLICATION TO THE PROBLEM OF RADON FLOW IN THE TEXAS PANHANDLE

A series of calculations were made to find the source density (equation 2.19) for five wells in the Texas Panhandle gas field. The exponential term in the equation is a function of the source radius R and of the pressure at R, and these quantities cannot be measured. If our assumption that R is small enough compared to the reservoir boundary radius r_e and that R can be set equal to the well radius r_w is valid, calculations of the source density can be made with data obtained on the surface. To test the validity of this assumption, we investigated the location of the nearest boundary of the radioactive source. Equation (4.10) yields the boundary in terms of the time $\Omega(r_o)$ it takes the radon to travel from the source to the well and can be calculated from well data. In order to determine the actual distance to the source, we made use of equation (2.8) which expresses Ω in terms of r_o. For each well various values are assigned to r_o, the corresponding Ω is calculated (table 1) and plotted against r_o giving a curve of the type shown in figure 1. The r_o corresponding to Ω, calculated from equation (4.10) is found from these curves. No ambiguity arises from using the equations to find r_o to justify setting R equal to r_e; the location of a cylindrical source will serve equally well in locating the nearest boundary of an extended source.

Table 1

Data for calculated values of nearest boundary of radon source for five Colorado Interstate gas wells in the Texas Panhandle area

Well name	Q_o^* $10^4 ft^3$	Q_o $10^6 ft^3$ per day	V ft^3	Reservoir pressure p_e psi	$\Omega(r_o)$ days	r_o ft
Thompson B-7	2.1	2.50	1300	302	-.001	r_w
Thompson A-1	3.5	2.66	1210	252	.006	7
Masterson A-2	1.7	2.17	1120	273	-.001	r_w
Kilgore A-11	1.9	1.84	1440	317	-.005	r_w
Thompson C-1	2.5	2.61	1270	313	.0002	r_w

The graphs of $\Omega(r_0)$ versus r_0 were calculated from equation (2.8). Here, p (r_0) was found from the relation

$$p^2(r_0) = p_w^2 + \Delta p^2 \frac{\log r_0/r_w}{\log r_e/r_w} \tag{5.1}$$

where p_w is measured at the well, Δp^2 is determined from $\Delta p^2 = p_e^2 - p_w^2$ where p_e was the measured shut-down pressure in October, 1951, r_e was taken equal to 500 feet, and r_w equal to 3/8 foot. The value assigned to r_0 were r_w, 1, 2, 5, 10, 25 and 50 feet. The pay zone thickness D was taken equal to 50 feet, as no exact data were available on the actual thickness. Q_0 is the production measured at STP at time of sampling, and T_w was taken as 300° K. The porosity of the pay zone f is known to vary between 8 and 13 percent in the Panhandle area; 10 percent was used in the calculations. The various values assumed are reasonable, and the error involved is negligible in the overall results.

The nearest boundary Ω (r_0) was computed from equation (4.10), where $V = r_w^2 \pi h$, where h is the total depth of the drill hole and r_w, p_e, p_0, T_w and T_0 are taken as before. Q^*_0 was taken from the graphs (Q^*_0 versus M_0') of dynamic tests on the wells (fig. 2a, b, c, d, e) at the point at which radon first appears; Q_0 was taken from the production curve at the same time as Q^*_0 was chosen. The results are summarized in table 2. Graphs, similar to figure 1, for each well were used to find r_0.

Table 2

Total calculated uranium source density for wells in the Texas Panhandle gas field

Company and well name	Well-head pressure p_w psi	M_o' $\mu\mu c$ per L	ζ' 10^{-6} g_U per g
Panhandle Eastern			
Kilgore 1-16	232	522	9.08
Colorado Interstate			
Thompson B-2*	216	475	7.54
Thompson B-7	204	226	3.46
Thompson A-1	215	163	2.64
Masterson A-2	206	71	1.10
Kilgore A-11	240	45	.81
Thompson C-1	223	23	.38

* Average of 10 samples

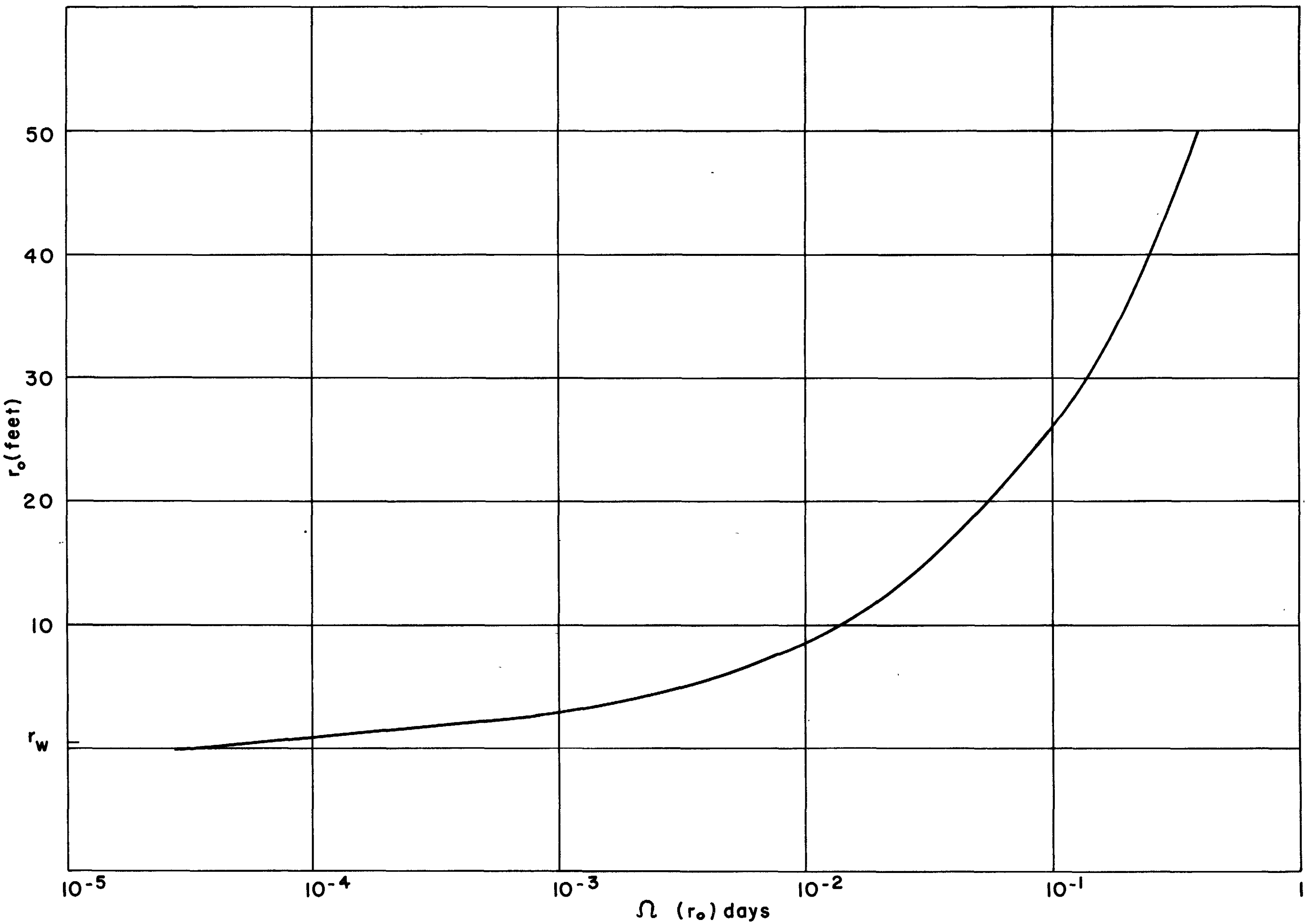

Figure 1 - Typical curve for computing distance of radon source from well

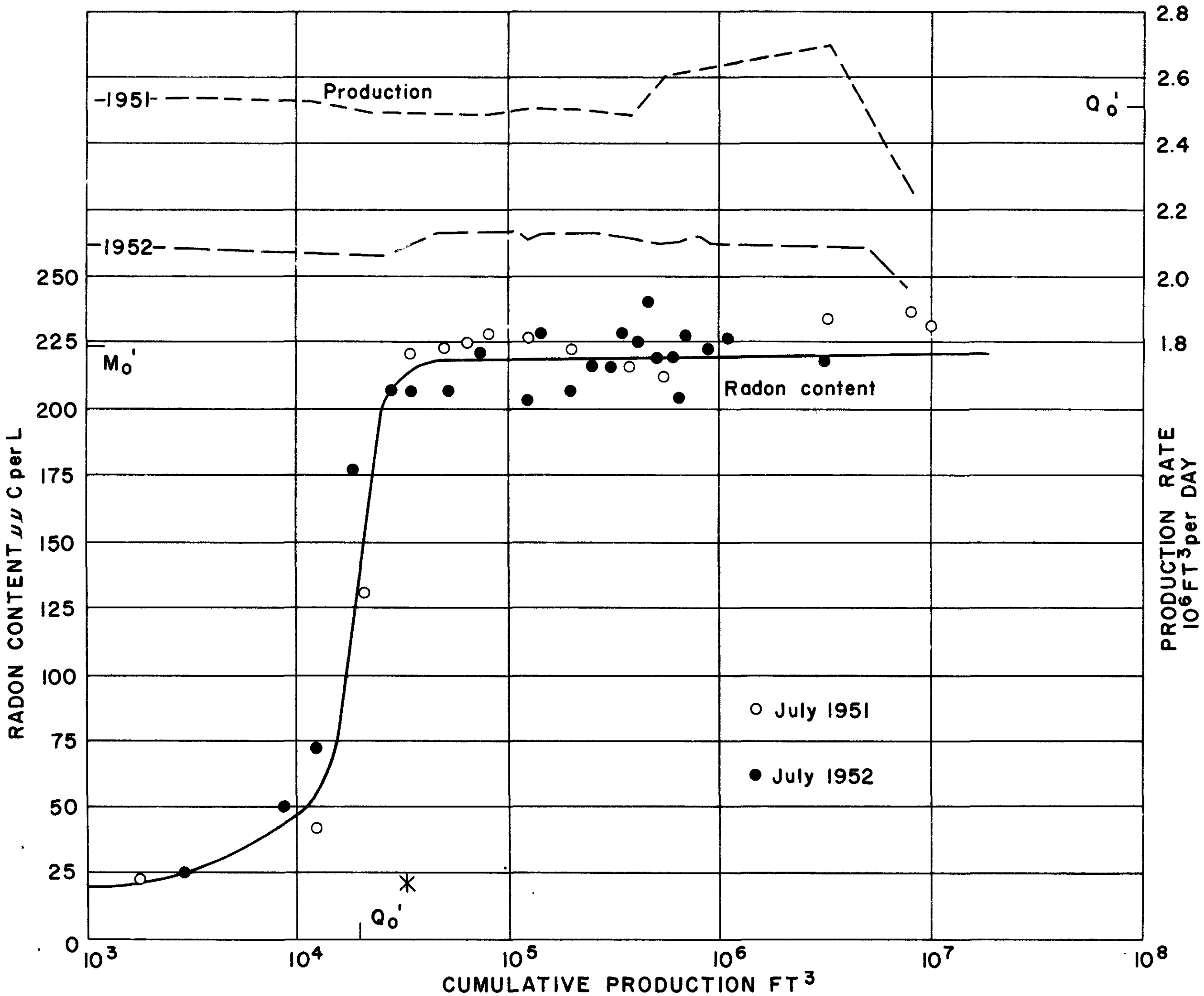

Figure 2 a - Dynamic test data for Thompson B-7 showing points at which data were taken for calculation

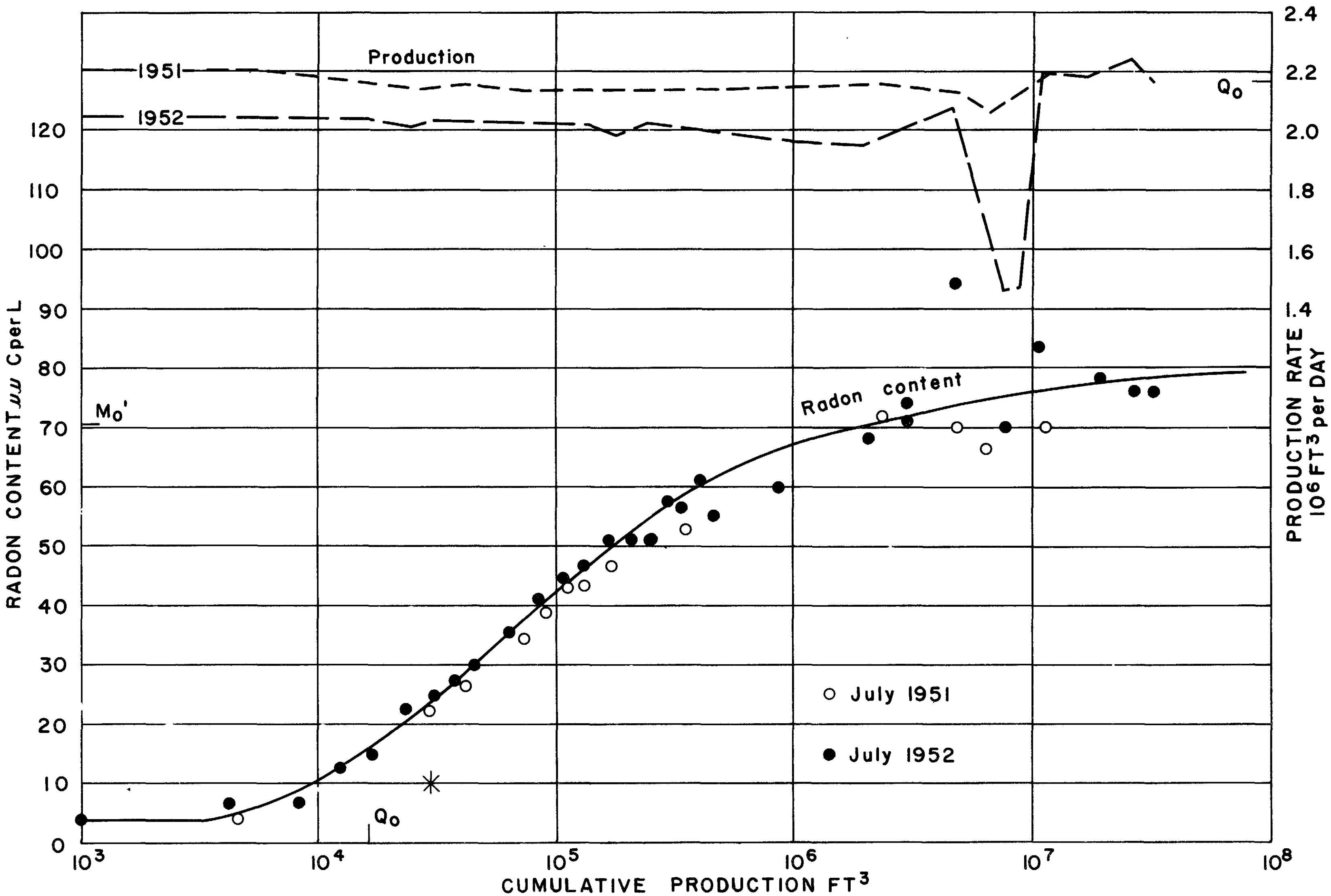

Figure 2 b - Dynamic test data for Masterson A-2 showing points at which data were taken for calculation

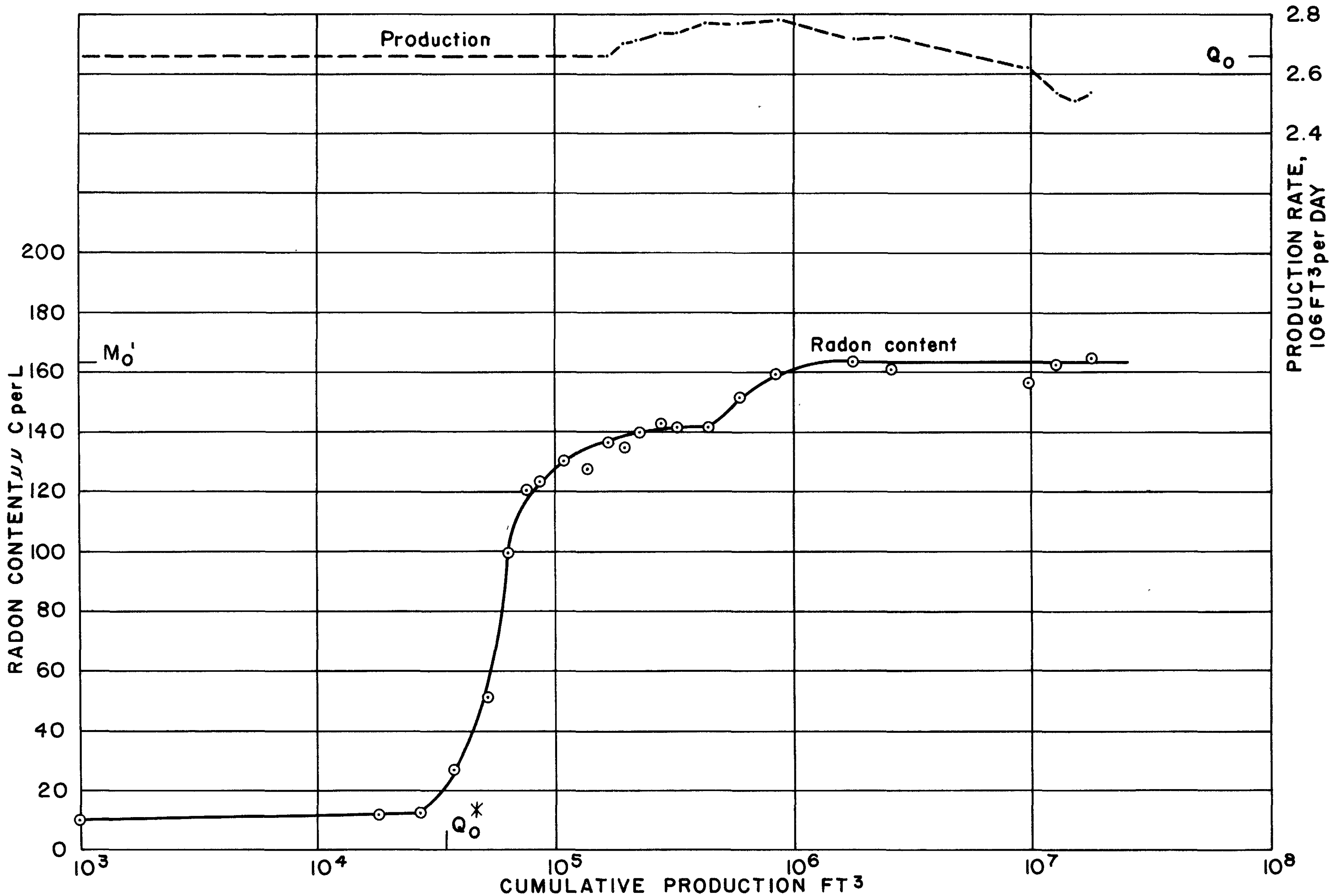

Figure 2c-Dynamic test data for Thompson A-1 showing points at which data were taken for calculation

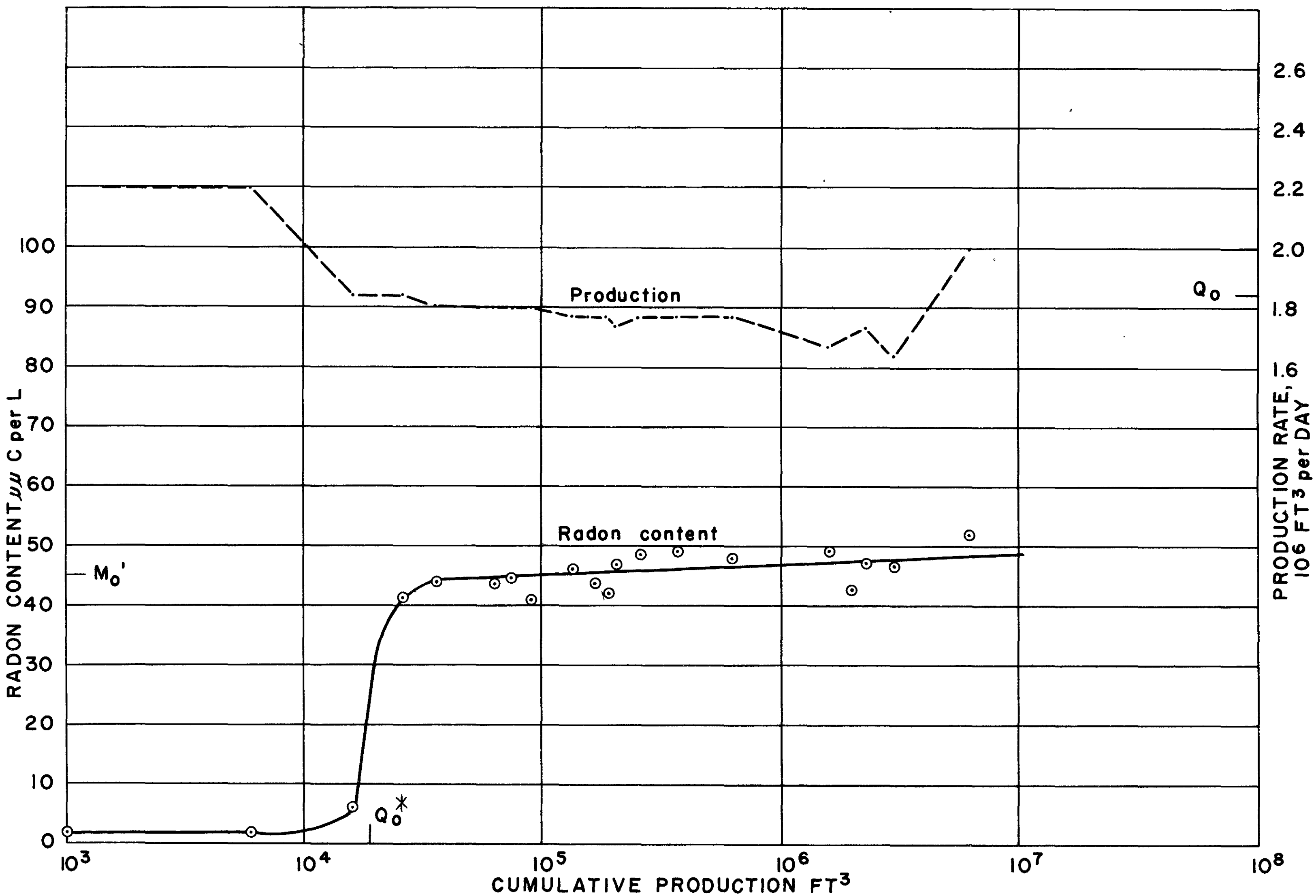

Figure 2 d-Dynamic test data for Kilgore A-11 showing points at which data were taken for calculation

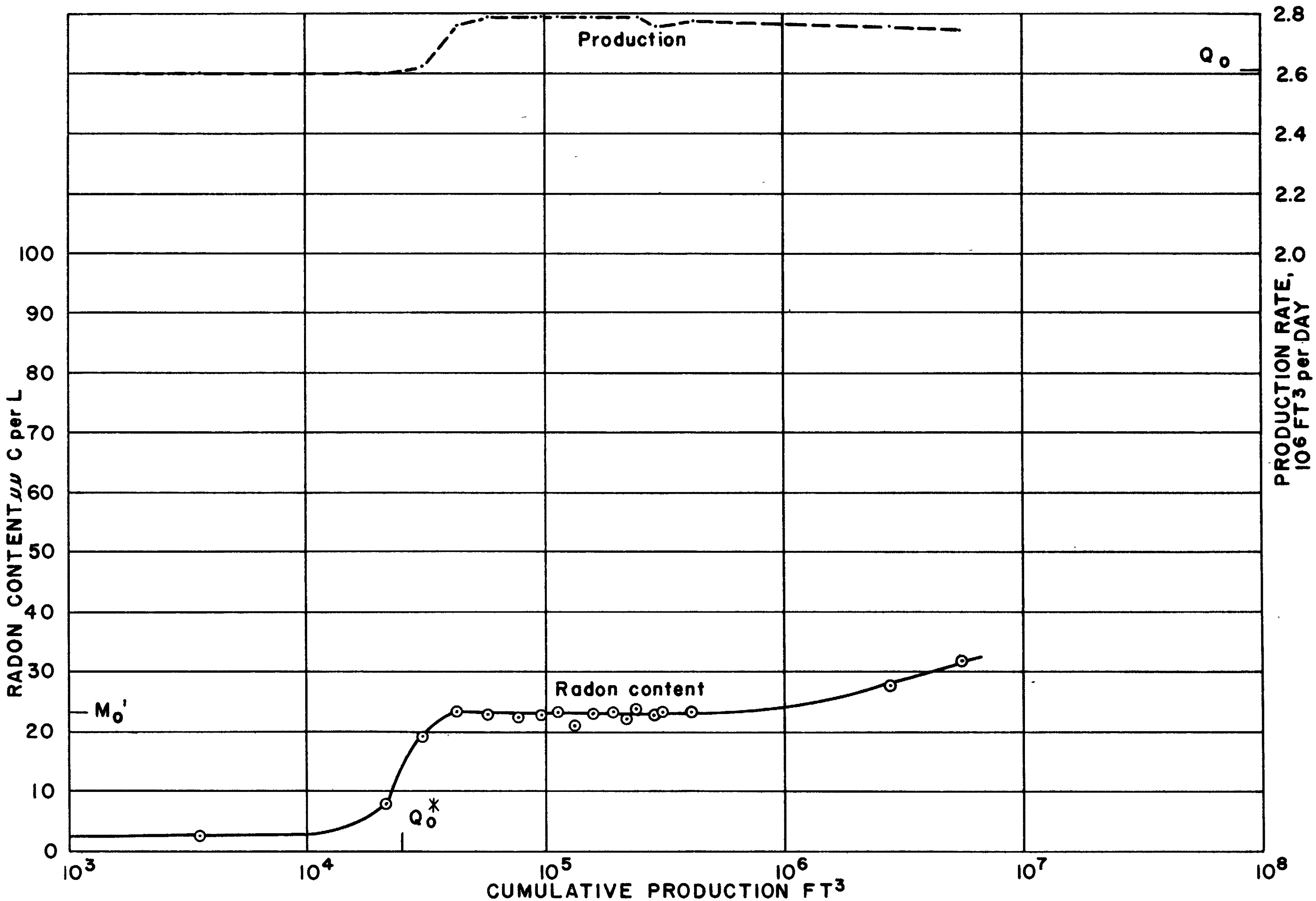

Figure 2e - Dynamic test data for Thompson C-1 showing points at which data were taken for calculation

The negative values of Ω can be attributed to various sources of error: 1) assumed values of the parameters; 2) neglect of higher order terms in the derivation of the formulas; 3) assumption that the gas in the well expands isothermally, which is not true in general; 4) the uncertain effect of acidizing on the well radius and volume. (Caliper logs in the Panhandle area show the well diameter increases about an inch after acidizing; it seems logical that acidizing could produce near-cavernous regions at the well bottom); and 5) experimental error involved in measuring p_w, p_e, Q_o, M_o' and h. We have taken these negative values to mean zero-time, giving $r_o = r_w$. These negative values also give us a rough measure of the error involved, that is $\pm$ 0.005. Thus, even though the Ω (r_o) versus r_o plot for Thompson A-1 gives r_o = 7 feet, it would be more appropriate to assume $r_o = r_w$ in this case.

The effective source density ζ was computed from

$$\zeta = \frac{p_w}{p_o} \frac{T_o}{T_w} f M_o' \left[1 + \frac{\Delta P^2}{2 P_w^2 \log r_e/r_w} \right]. \tag{5.2}$$

As it can be shown that the quantity in brackets is always very nearly equal to one for the wells in question, it was taken as such. M_o' the measured radon content at the well, was chosen after the radon content reaches equilibrium (fig. 2) and p_w is the pressure at the time of sampling. The remaining parameters were chosen as before. The results were converted from micromicrocouries per liter to grams of uranium per gram of the medium. As the effective source is assumed to have an emanating power of 10 percent, ζ was corrected to give total source density ζ'. These source densities are shown in table 2.

The character of the graph of radon content versus cumulative production for the Masterson A-2 well is different from the other four graphs. Although the radon first appears relatively shortly after opening, thus giving $r_o = r_w$, the intensity continues to rise slowly and the equilibrium value is not reached until about two million cubic feet have been produced. This behavior was thought anomalous and the tests were repeated one year later with the same result. The slow rise could be explained by assuming that the producing zone has several layers of varying permeability and that the less permeable rock contains more radioactive parent elements. An alternative, and possibly better, explanation would be a lack of symmetry in the radon source about the well. It is plausible that the radium concentration could increase gradually in any direction away from the hole. Similarly, the double knee in the graph for Thompson A-1 could be interpreted as evidence of a localized, sharply bounded concentration of radium. The value for ζ' shown in table 2 would then be a weighted average of the two sources, the farther (and stronger) source being less effective. It must be emphasized that all the source strengths in table 2 are minimum values. It is known from geological studies (J. W. Mytton, oral communication) in this area that the radioactive asphaltic residues generally tend to obstruct the permeability. Consequently, the more concentrated sources are likely to be swept less effectively by the gas.

DISCUSSION AND CONCLUSIONS

To recapitulate, we list here the various formulas derived in this paper:

Radon density for the steady state, cylindrical source at r :

$$M_o(r_w) \approx \frac{2\pi r_o D}{Q_o} \lambda \sigma \exp\left[\frac{-\lambda f \pi D}{Q p_w} p(r_o) r_o^2 \left(1 - \frac{\Delta P^2}{4P^2(r_o) \log r_e/r_w}\right)\right]; \tag{2.10}$$

Total radon density for the steady state, extended source from R to r_e :

$$M_o'(r_w) \approx \frac{p_o}{p(R)} \frac{1}{1+\omega} \frac{T_w}{T_o} \frac{S}{f} \exp\left[-\lambda \Omega(R)\right]; \tag{2.19}$$

Radon density for the transient state, cylindrical source:

$$M_o(r_w, t) \approx \frac{1}{v(r_w, t)} \left\{1 + f \int_{r_w}^{r_o} \frac{1}{v^2(r, t)} \frac{\delta v}{\delta t} dr\right\} \frac{\lambda \sigma r_o}{r_w} \frac{T_w}{T_o} \frac{p_o}{p_w} st(r_o - r)$$

$$\times\ st(t - t_o) \exp[-\lambda(t - t_o')]; \tag{3.10}$$

Nearest boundary of radon source:

$$\Omega(r_o) = \frac{Q_o^*}{Q_o} - V \frac{p_e}{p_o} \frac{T_o}{T_w} \frac{1}{Q_o}. \tag{4.10}$$

Geologic conditions are never so simple and clear cut as indicated here. Geologic structures certainly are not homogeneous or isotropic, nor in a convenient geometric configuration. The greatest source of error in the evaluation of source densities is the value of the emanating power. Moreover, it should be noted that the source distribution obtained will be an average, and localized high concentrations are likely to go unnoticed. The calculation of source densities is made on the basis of extended source distribution, assuming that a uniform distribution would be the most probable one. In the Texas Panhandle gas field, from which examples were selected, the edge of the source can be considered to be at the well according to the calculations on dynamic test data.

It is interesting to note that three of the wells (Masterson A-2, Kilgore A-11 and Thompson C-1) exhibit normal uranium density, whereas the others show abnormally high uranium content.

LITERATURE CITED

Faul, Henry, Gott, G. B., Manger, G. E., Mytton, J. W., and Sakakura, A. Y., 1954, Radon and helium in natural gas: Internat. Geol. Cong., 19th, Algiers 1953, Comptes Rendus, sec. 9, fasc. 9, p. 339-348.

Muskat, M., 1946, The flow of homogeneous fluids through porous media: Ann Arbor, Edwards, Inc.

Ritchie, R. H. and Sakakura, A. Y., 1956, Asymptotic expansions of solutions of the heat conduction equation in internally bounded cylindrical geometry: Jour. Applied Physics, v. 27, no. 12, p. 1453-1459.