

UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

CHEMICAL COMPOSITION AS A GUIDE TO THE SIZE
OF URANIUM DEPOSITS IN THE SALT WASH MEMBER
OF THE MORRISON FORMATION, COLORADO PLATEAU*

By

A. T. Miesch, E. M. Shoemaker, W. L. Newman, and W. I. Finch

June 1959

Trace Elements Investigations Report 511

This preliminary report is distributed
without editorial and technical review
for conformity with official standards
and nomenclature. It is not for
public inspection or quotation.

*This report concerns work done on behalf of the Division of
Raw Materials, U. S. Atomic Energy Commission.

USGS - TEI-511

GEOLOGY AND MINERALOGY

<u>Distribution</u>	<u>No. of copies</u>
Division of Raw Materials, Austin	1
Division of Raw Materials, Casper	4
Division of Raw Materials, Denver	1
Division of Raw Materials, Grants	2
Division of Raw Materials, Lima, Peru	1
Division of Raw Materials, Salt Lake City	3
Division of Raw Materials, Washington	3
Grand Junction Operations Office	6
Technical Information Service Extension, Oak Ridge	6
U. S. Geological Survey:	
Alaskan Geology Branch, Menlo Park.	1
Foreign Geology Branch, Washington.	1
Fuels Branch, Denver.	4
Geochemistry and Petrology Branch, Washington	1
Geophysics Branch, Washington	1
Mineral Deposits Branch, Washington	6
TEPCO, Denver	2
TEPCO, Washington	2
	<u>45</u>

CONTENTS

	Page
Abstract	7
Introduction	10
Geology of the uranium deposits.	14
Selection of deposits for study.	15
Methods of sampling and sample preparation	15
Analytical method.	16
Classification of elements in the uranium deposits	19
Iron.	22
Zirconium	24
Calcium, sodium, and manganese.	24
Uranium	27
Nickel.	28
Yttrium	28
Sizes of uranium deposits.	30
Correlation of elements with sizes of the deposits	33
Estimation of sizes of deposits.	38
Simple linear regression method	40
Long multiple regression method	46
Short multiple regression method.	49
Summary of procedures for estimating sizes of deposits.	52
Size estimation procedure for the simple linear regression method 1.	52
Size estimation procedure for the long multiple regression method 2.	53

	Page
Size estimation procedure for the short multiple regression method 3.	53
Summary and conclusions.	54
References	58
Appendix I Tests of the methods	50
Appendix II Comparison of the methods	65
Appendix III Semiquantitative spectrographic analyses of samples of uranium deposits in the Morrison formation on the Colorado Plateau	In Envelope

ILLUSTRATIONS

	Page
Figure 1. Map showing locations of 75 uranium deposits.	11
2. Histograms showing frequency distributions of iron.	23
3. Histograms showing frequency distributions of zirconium	23
4. Histograms showing frequency distributions of calcium	25
5. Histograms showing frequency distributions of manganese	25
6. Histograms showing frequency distributions of sodium.	25a
7. Histograms showing frequency distributions of uranium	25a
8. Histograms showing frequency distributions of nickel.	29
9. Histograms showing frequency distributions of yttrium	29
10. Histogram showing frequency distribution of log sizes of 75 uranium deposits.	31
11. Scatter diagrams of minor element concentrations in 75 uranium deposits and sizes of the deposits	34
12. Scatter diagram of log weighted average size estimates and log "true" sizes.	42

TABLES

	Page
Table 1. Classification of elements in sandstone-type uranium deposits in the Salt Wash member of the Morrison formation on the Colorado Plateau.	In Envelope
2. Correlation coefficients of elements in 75 uranium deposits in the Salt Wash member of the Morrison formation with sizes of the deposits	In Envelope
3. "Weighted average size" (upper) and "expanded weighted average size" (lower) estimation factors (method 1). . .	In Envelope
4. Size estimation factors (method 2)	In Envelope
5. Values of the matrices C_8 and C_2 for equations 2a and 3a	In Envelope
6. Estimated log sizes and sizes (in tons) of uranium deposits based on uranium and yttrium contents (equation 3)	In Envelope
7. Comparison of log sizes (in tons) of 40 uranium deposits estimated by equations 1, 2, and 3 with range of "true" log size. Ranges of error of estimated log size are given for 95 percent (upper) and 80 percent (lower) confidence levels.	In Envelope
8. Analysis of variance of log size of uranium deposits (y) for comparing three methods of predicting sizes of deposits	In Envelope

CHEMICAL COMPOSITION AS A GUIDE TO THE SIZE
OF URANIUM DEPOSITS IN THE SALT WASH MEMBER OF THE MORRISON FORMATION,
COLORADO PLATEAU

by A. T. Miesch, E. M. Shoemaker, W. L. Newman, and W. I. Finch

ABSTRACT

The concentrations of uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel in 75 mill-pulp samples of uranium deposits in the Salt Wash member of the Morrison formation on the Colorado Plateau have been found, by statistical tests, to be significantly related to the size of the deposits represented by the samples. Linear correlation coefficients between log-element concentrations and log sizes in tons range from $\left| 0.37 \right|$ for uranium and yttrium to $\left| 0.24 \right|$ for nickel. The lowest significant correlation coefficient for 75 pairs at the 95 percent level of confidence is 0.228.

The elements listed above have played a variety of roles in the formation of the deposits. Zirconium is an intrinsic element, contained principally in the detrital syngenetic fraction of the host sandstone. Calcium, manganese, and sodium are intrinsic elements contained principally in epigenetic or diagenetic carbonate in the host sandstone. Uranium, yttrium, and nickel are principally extrinsic elements, introduced into the host sandstone by processes of or related to uranium mineralization. Somewhat more than half the iron is probably intrinsic and the remainder is extrinsic.

Three methods can be used to estimate sizes of uranium deposits in the Salt Wash member within broad limits. One is based on simple linear regression theory (method 1); and two are based on multiple regression theory, long method (2) and short method (3).

For methods 1 and 2 the estimated log sizes of deposits may be computed from tables of their known concentrations of U, Y, Na, Fe, Zr, Mn, Ca, and Ni as determined by semiquantitative spectrographic analysis. For the third method the estimated size or log size may be read directly from a table with known concentrations of U and Y only.

About 80 percent of the tonnage size estimates from method 1, will be within a factor of 13 (12-14) of the true sizes. The precision of size estimates from method 2, the long multiple regression method, is highly variable. Some estimates from method 2 will be within a factor of 12 of the true size at the 80 percent confidence level; others within a factor of 40 at the 80 percent confidence level. About 80 percent of the tonnage size estimates from method 3, the short multiple regression method, will be within a factor of 15 (13-16) of the true size.

A group of 40 deposits of known size was used to test the theoretical derivation of the confidence intervals given above. It was concluded from the test that the confidence intervals describe the precisions of the methods correctly.

The methods of size estimation will be useful in estimating sizes of deposits where exposures of the ore are limited or where an estimate is desired that is independent of other estimates. The error of the estimates may be quite large, as is indicated by the confidence limits given above, but they may at least distinguish very large from very small deposits. They may serve to encourage or discourage further expenditure in development exploration. The methods may be of particular usefulness in attempts to appraise or compare groups of deposits or mining districts, inasmuch as the average estimate of size for the group is more precise than any single estimate.

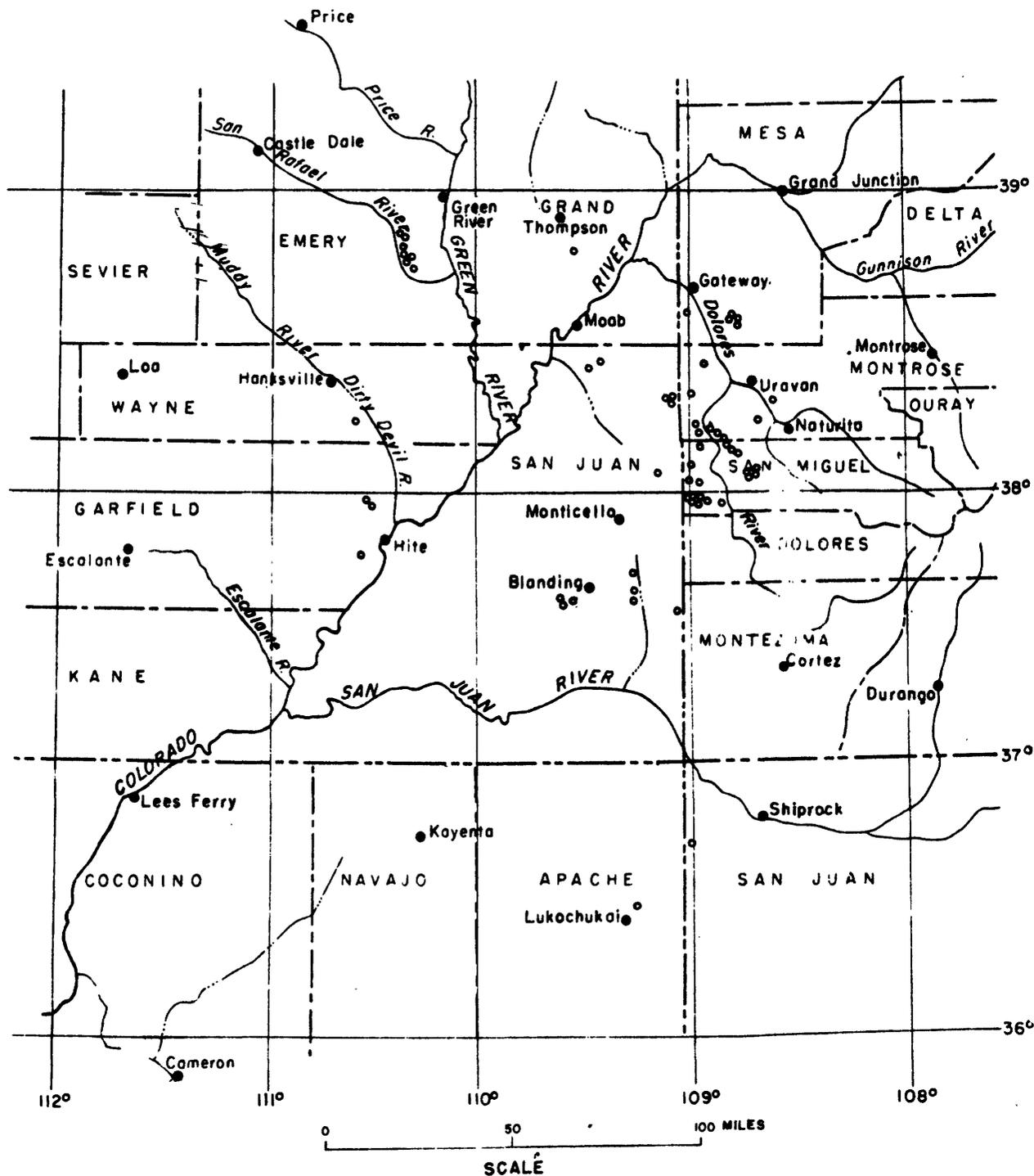
The methods of estimating size are established for deposits in the Salt Wash member of the Morrison formation. Tests indicate that the equations calculated for deposits in the Salt Wash fail completely to estimate sizes of deposits in other stratigraphic units, such as the Moss Back and Shinarump members of the Chinle formation. A further restriction on the methods, not completely evaluated at present, is that semiquantitative spectrographic or comparable analyses of mill-pulp samples be used. However, no tests have been made to determine whether other types of samples, such as drill core, will greatly alter the precision and accuracy of the methods.

INTRODUCTION

This study is part of a broader investigation of the distribution of elements in rocks and ores of the Colorado Plateau, conducted by the Geological Survey on behalf of the Division of Raw Materials of the U. S. Atomic Energy Commission. In the course of the investigation significant correlations were discovered between the concentrations of certain elements in the uranium deposits and the calculated $\sqrt{\quad}$ sizes of

$\sqrt{\quad}$ Calculated size of a uranium deposit is determined by consideration of past production and total reserves and is assumed to be the true size. Estimated or predicted size is determined by the methods described in this report.

the deposits in tons. The uranium deposits discussed in this report are all in sandstone of the Salt Wash member of the Morrison formation of Jurassic age and are located in widely separated areas on the central part of the Colorado Plateau (fig. 1).



Uranium deposit or group of deposits

Figure 1. Map of part of the Colorado Plateau showing locations of 75 uranium deposits in the Salt Wash member of the Morrison formation sampled for this investigation.

Elements whose concentrations appear to be related to the sizes of the deposits are uranium, nickel, yttrium, zirconium, iron, manganese, sodium, and calcium. Hence known concentrations of these elements in a deposit in the Salt Wash member on the Colorado Plateau may be used to estimate the size of the deposit. Though such an estimate will have a rather broad range of error, it will be a significantly better estimate than could be obtained by considering the deposit to be of mean size (i. e., equal to the mean size of all deposits in the Salt Wash member). For many deposits, a geologist with a knowledge of the geologic habit of the ores may be justifiably confident in estimating the size of a deposit within a much narrower range than can be done with the methods given in this report. For other deposits, however, in areas where the deposits have a considerable range in size and where exposures of the ore are limited, the methods suggested here may be used to advantage.

Lasky (1950) has shown that within certain porphyry copper deposits there is an inverse relationship between tonnage and grade and that this relationship may be used to predict ore reserves. That is, larger amounts of low-grade ore than of high-grade ore are present, and the amount of ore is inversely related to a specified grade. He has found similar relationships in bedded manganese deposits in the Artillery Mountains, Arizona, in the Idaho-Wyoming phosphate-vanadium deposits, in the Falconbridge nickel deposit and in the Alaska-Juneau gold deposit. The tonnage-grade or tonnage-composition relationships described in this report differ from the type described by Lasky mainly in that they are found among, rather than within, deposits.

Three methods for estimating sizes of uranium deposits in the Salt Wash member of the Morrison formation on the Colorado Plateau are given in this report. Two of the methods are based on multiple regression theory, the third on simple linear regression. The multiple regression equations were computed for the writers by statisticians of the National Bureau of Standards. Churchill Eisenhart, Chief of the Statistical Engineering Laboratory of the Bureau, suggested the methods. The computations were made by J. M. Cameron and W. S. Connor, with the assistance of Carroll Dannemiller and Marion Carson. The writers wish to express their sincere thanks to these members of the National Bureau of Standards for their very helpful consultation on the general aspect of the problem and for the great amount of time they devoted to it. The entire sections on size estimates by multiple regression and most of the analysis of variance (appendix II, table 8) are based on their work.

Mary Epling of the National Bureau of Standards read the manuscript and brought to our attention several statistical and other problems relating to the manner of presentation. We are also indebted to W. C. Krumbein of Northwestern University who kindly read the manuscript and helped us to interpret the meanings of some aspects of statistical theory in terms of this problem.

GEOLOGY OF THE URANIUM DEPOSITS

Sandstone-type uranium deposits on the Colorado Plateau consist of sandstone or conglomerate that has been impregnated, and commonly partly replaced, by uranium minerals. The deposits range in size from less than 1 ton to more than 100,000 tons. Many of them are irregularly tabular; the margins of some conform roughly to the bedding of the sandstone host rocks whereas the margins of others cut across bedding along wavy or curved structures called "rolls" (Fischer, 1942, p. 367; Shawe, 1956, p. 239-241). Some uranium deposits are highly irregular in shape. For the distribution of known uranium deposits on the Colorado Plateau areas see the map prepared by Finch (1955).

The mineralogy of uranium deposits on the Colorado Plateau has been summarized by Weeks and Thompson (1954). The uranium deposits contain two contrasting suites of ore minerals: 1) a suite of low-valent oxides and silicates, and 2) a suite of high-valent oxygen salts which may be interpreted as having been derived by oxidation of the low-valent suite. Relatively unoxidized uranium deposits in the Salt Wash member of the Morrison formation contain uraninite $[UO_2]$ and coffinite $[U(SiO_4)_{1-x}(OH)_{4x}]$. Vanadium is generally more abundant in these deposits than uranium, and occurs chiefly in montroseite $[VOOH]$, doloresite $[3V_2O_4 \cdot 4H_2O]$, and micaceous vanadium silicates. In most uranium deposits in the Salt Wash mined to date the uranium is contained chiefly in the uranyl vanadates carnotite $[K_2(UO_2)_2(VO_4)_2 \cdot 1-3H_2O]$ and tyuyamunite $[Ca(UO_2)_2(VO_4)_2 \cdot 7-10-1/2H_2O]$.

SELECTION OF DEPOSITS FOR STUDY

In the broader investigation of the distribution of elements, the mill pulp samples of some 200 deposits were analyzed by semi-quantitative spectrographic methods. Of these deposits, 75 were selected to provide a range in size from very small (less than 1 ton) to very large (more than 100,000 tons), and to provide a representative geographic sampling of the area of uranium deposits in the Salt Wash member of the Morrison formation.

METHODS OF SAMPLING AND SAMPLE PREPARATION

The samples of uranium ores were obtained from mills and buying stations and are pulverized quarters of splits or pulps that were assayed to determine payment to the shippers. Most of the samples studied were obtained from the U. S. Atomic Energy Commission's plant at Monticello, Utah, operated at the time by the American Smelting and Refining Company. The pulps were ultimately ground to 120 mesh in cast-iron disk grinders and nearly all the pulps are probably contaminated slightly with tramp iron. The amount of ore represented by the samples from each mine ranges from 1 to more than 2,000 tons, and averages about 150 tons (appendix III).

Samples of unmineralized sandstone from the Salt Wash member were collected by geologists of the U. S. Geological Survey and furnished to the writers by Robert A. Cadigan. They are hand samples selected to represent typical lithology of the Salt Wash member in widely separated areas on the Colorado Plateau. Analytical data from these samples are given in figures 2 to 9.

ANALYTICAL METHOD

The analyses were made by a rapid semiquantitative spectrographic method under the supervision of A. T. Myers of the U. S. Geological Survey. R. G. Havens is the principal analyst, but some analyses have also been made by P. R. Barnett, G. W. Boyes, Jr., and P. J. Dunton.

The analytical method is described by A. T. Myers as follows (written communication, 1957):

"In this procedure a weighed amount of the powdered sample is burned in a controlled d.c. arc and the spectrum recorded on a photographic plate. Selected lines on the resulting plate are visually compared with those of standard spectra prepared in a manner similar to that for the unknowns. The standard spectra were prepared from mixtures of materials containing 68 elements in the following concentrations--10 percent, 4.6, 2.2, 1.0, 0.46, etc. These values were chosen so that the concentrations of the elements decrease from 10 percent to about 0.0001 percent by a factor of the reciprocal of the cube root of 10. This factor provides a geometric concentration series having three members for each order of magnitude and which is consistent with the relation between the blackness of the spectral line and the amount of an element present. By means of a comparator showing enlarged adjacent images of the sample spectra and the standard spectra, visual estimates are made of concentrations of the elements in the sample which are then reported as being between two standards in the following manner: x indicating the middle portion (5-2) of an order of magnitude; x^+ the higher portion (10-5); and x^- the lower (2-1).

"The above method of reporting is used because the inherent limitations of this particular method of spectrographic analysis make the precision of the determinations less than the precision attained in preparing the standards. Major sources of error are 1) chemical and physical differences between the samples and the standards, 2) the omission of complete quantitative procedures for sample preparation, and plate calibration, and 3) lack of duplicate determinations. • Experimental work has shown that approximately 60 percent of the reported results fall within the proper portion of an order of magnitude."

Sixty elements are detectable with one exposure by the spectrographic method employed for this study. Of these elements, 20 are present in more than 70 percent of the samples in concentrations

/ Concentration is used throughout this report in the conventional chemical sense of fractional weight contained.

above the spectrographic limits of sensitivity, and 9 elements are present in less than half of the ores in concentrations above the limits of sensitivity. The remaining 31 elements looked for were not detected in either the ores or the unmineralized host sandstone. Study of the composition of the ores by the rapid spectrographic method is thus limited to about 29 elements; the only elements on which enough data are provided for detailed statistical correlation studies, are the 20 that are detected in more than 70 percent of the ores.

The limits of sensitivity for each element are in general those listed by Myers (1954, p. 195). As the analytical work has extended over a period of 3 years and some changes were made in the details of the technique during this time, and as the individual analysts followed slightly different practices in reporting elements near the limit of sensitivity, the limit of sensitivity actually attained or reported for each element has varied slightly. Histograms given in figures 6 to 9 are cut off at the highest limits of sensitivity reported.

In construction of the scatter diagrams (fig. 11) and calculation of the correlation coefficients (table 2 and fig. 11) of size of deposits against the concentrations of uranium, yttrium, nickel, and sodium some spectrographic determinations below the limits of sensitivity were used. Some of these determinations were reported as "trace," some as "looked for but not detected," and a few were reported in a concentration class immediately below the limit of sensitivity and given a questionable status. For construction of the scatter diagrams and calculation of the correlation coefficients questionable determinations were used, "trace" determinations were assigned to the second concentration class below the limit of sensitivity, and "looked for but not detected" determinations were assigned to the range of concentration immediately below that assigned to "trace" reports. As relatively few of the spectrographic determinations were given as questionable, "trace," or "looked for but not detected" this practice has not seriously biased the statistical calculations.

CLASSIFICATION OF ELEMENTS IN THE URANIUM DEPOSITS

The classification of elements in uranium deposits given in table 1 is taken from Shoemaker and others (in press) and is based

Table 1.--Classification of elements in sandstone-type uranium deposits in the Salt Wash member of the Morrison formation on the Colorado Plateau.

largely on comparison of spectrographic analyses of 211 samples of uranium ores and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation. The system of classification is based on the assumptions that the ore minerals are epigenetic, that they were formed in the sandstone host rock some time after its original deposition and that the epigenetic process that formed the ore minerals can be distinguished from other epigenetic processes that produced changes in the sandstone as, for example, cementation of the sandstone with calcite.

An element whose presence in the ore is unrelated to processes of uranium mineralization is termed intrinsic and is either syngenetic or epigenetic (diagenetic). Intrinsic syngenetic elements are contained in clastic fractions of the sandstone host rocks and in precipitates formed simultaneously with the deposition of the clastic fractions. Intrinsic epigenetic (diagenetic) elements were precipitated later by processes unrelated to uranium mineralization. They are distributed irrespectively of uranium and are much more widely distributed in the host rocks. The deposition of intrinsic epigenetic (diagenetic) elements may have either followed or preceded the deposition of uranium and no judgment of this is involved in the classification.

Extrinsic elements were introduced by processes of or related to uranium mineralization. No implication of source of the elements is made in the classification, nor is it implied that all the extrinsic elements were deposited simultaneously. Uranium and vanadium are the only ore elements in uranium deposits in the Salt Wash member of the Morrison formation, though the deposits contain a number of extrinsic "accessory" elements which are present in lower concentrations and do not add to the economic value of the ore.

Probably no single chemical element in the sandstone-type uranium ores belongs exclusively to either the intrinsic or extrinsic components of any given ore, as each element is present in some concentrations, however small, in the detrital minerals of the sandstone and a certain amount, however small, was probably carried in the solutions that introduced the uranium. The problem of classification, therefore, is to assess the relative importance of the roles played by each element in the ores. In table 1 an element is considered dominantly of one classification if the major part of the element is believed to fall in a given classification in a majority of the deposits studied.

The eight elements whose concentrations in uranium deposits in the Salt Wash member of the Morrison formation appear to be significantly related to size of the deposits fall into all categories of the classification; they have played a variety of roles in the formation of the deposits. The geochemical habits of the eight elements are only briefly summarized here; for a more detailed account the reader is referred to Shoemaker and others (in press).

Iron

The average iron content of uranium ores in the Salt Wash member of the Morrison formation is nearly four times higher than that of unmineralized sandstone of the Salt Wash (fig. 2). A part of this difference, however, is due to the fact that the ores contain, on the average, twice as much iron-bearing clay and accessory minerals as the unmineralized sandstone. Another part of the 4-fold difference is undoubtedly due to iron contamination of the samples during grinding at the mills. The average amount of iron introduced from this source (metallic iron removed from sample with hand magnet) is estimated at about 0.1 percent. The remainder of the iron in the deposits, probably slightly less than half of their total iron content, may be extrinsic, introduced by processes of or related to uranium mineralization. The iron content of an average uranium deposit is, therefore, a function of both iron originally present in the sandstone and to a lesser extent that introduced as an extrinsic component.

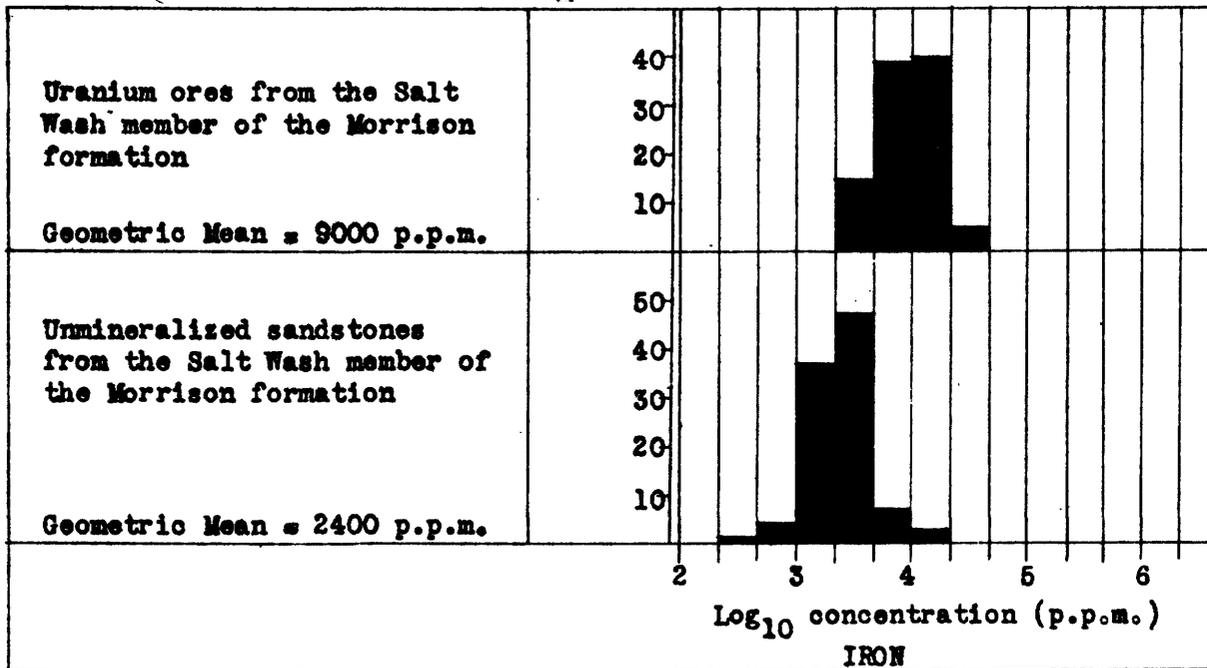


Figure 2. Histograms showing the frequency distributions of iron concentrations in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

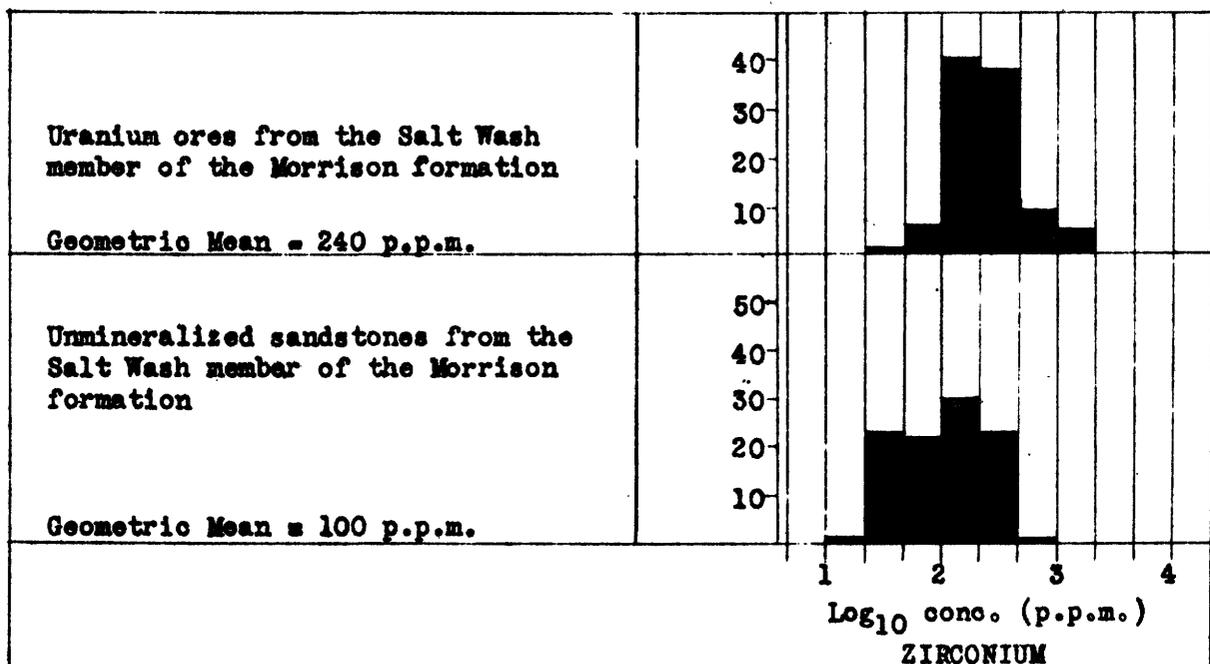


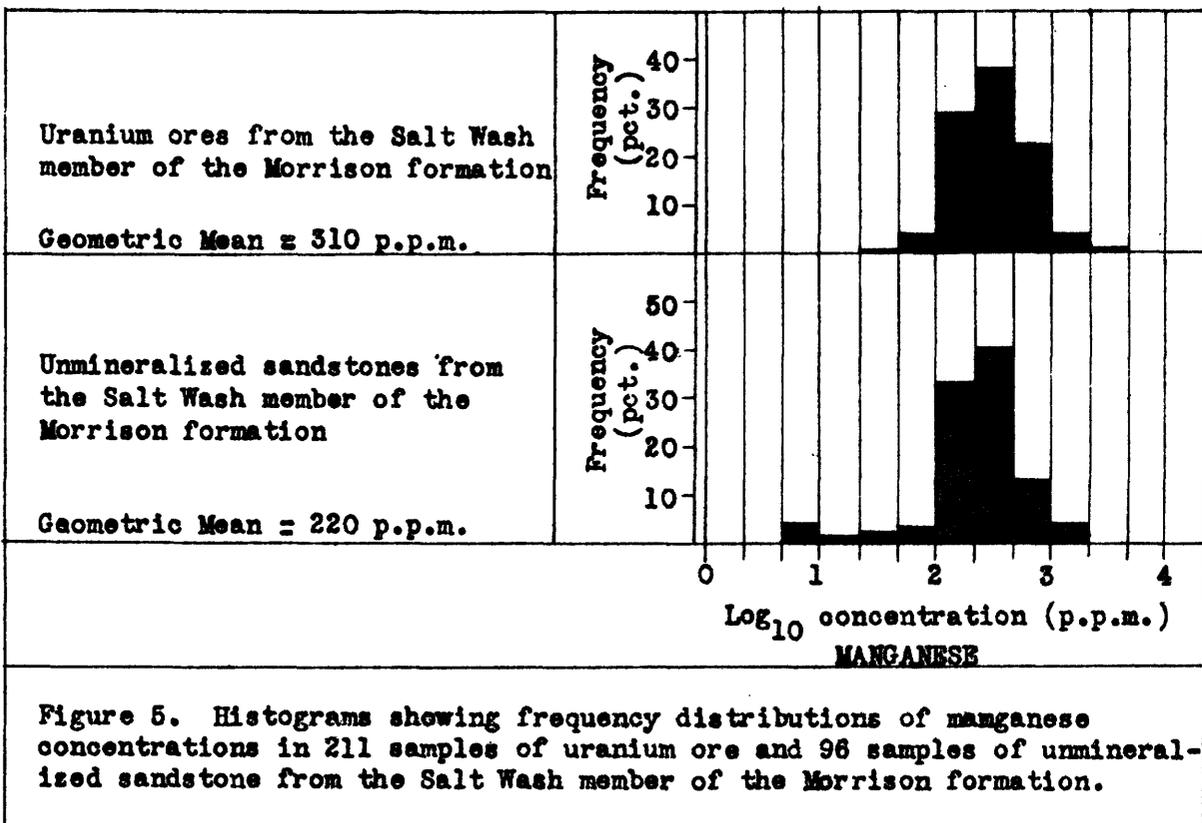
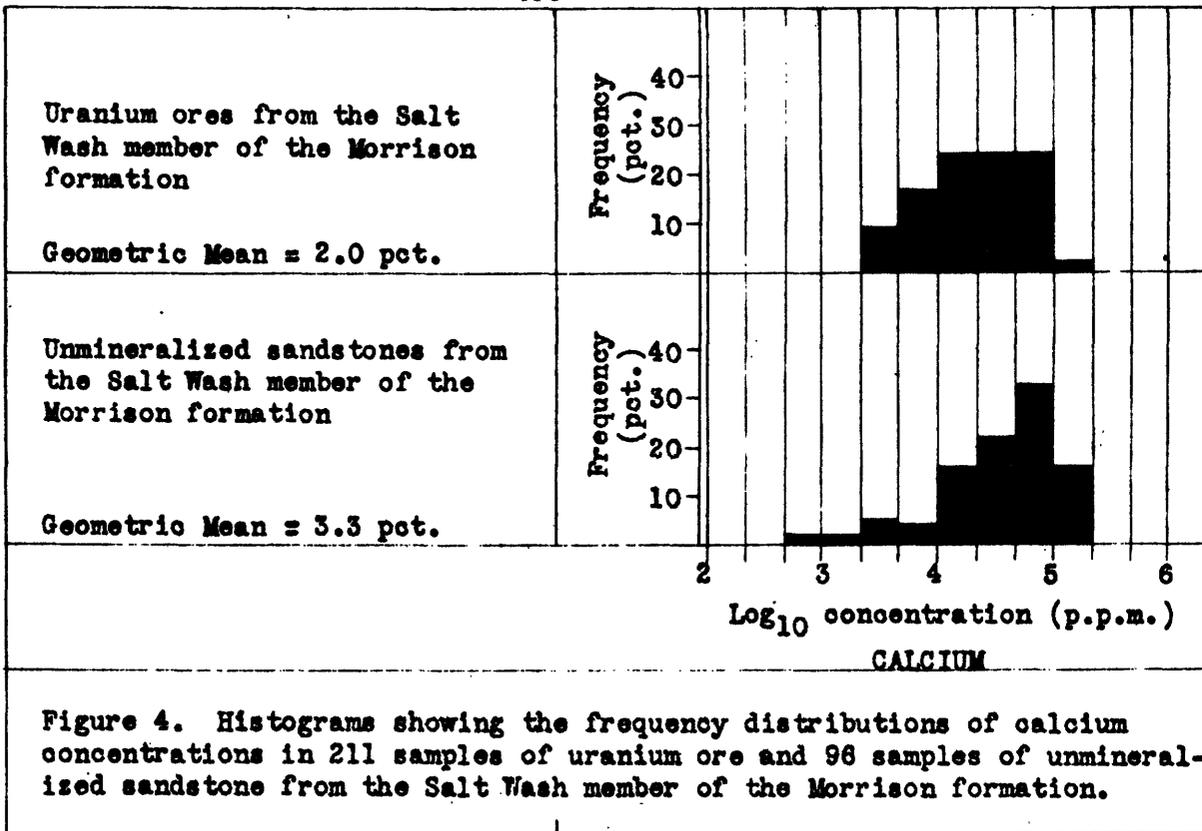
Figure 3. Histograms showing the frequency distributions of zirconium concentrations in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

Zirconium

The element zirconium is considered to be almost entirely intrinsic in the sandstone-type uranium deposits. Zirconium is not known to be an essential or major constituent of any of the ore minerals in the deposits and only rarely is it reported even as a trace constituent in qualitative spectrographic analysis of the ore minerals (Weeks and Thompson, 1954). Zirconium is more than twice as highly concentrated in uranium deposits in the Salt Wash member of the Morrison formation as in unmineralized sandstone of the Salt Wash (fig. 3), but like iron, it varies directly with aluminum in the sandstone and is more abundant in ores because of their higher clay and associated heavy mineral contents.

Calcium, sodium, and manganese

Three elements contained largely in the cementing materials of sandstone of the Salt Wash member of the Morrison formation are calcium, manganese, and, to a lesser degree, sodium (figs. 4, 5, and 6). These elements are classified as dominantly intrinsic and epigenetic. Calcium, the second most abundant element in the sandstone, occurs mainly in calcite, which is the dominant cement of the sandstone of the Salt Wash member (Cadigan, in press) but a significant part is also in dolomite. Locally a large part of the calcium is in gypsum.



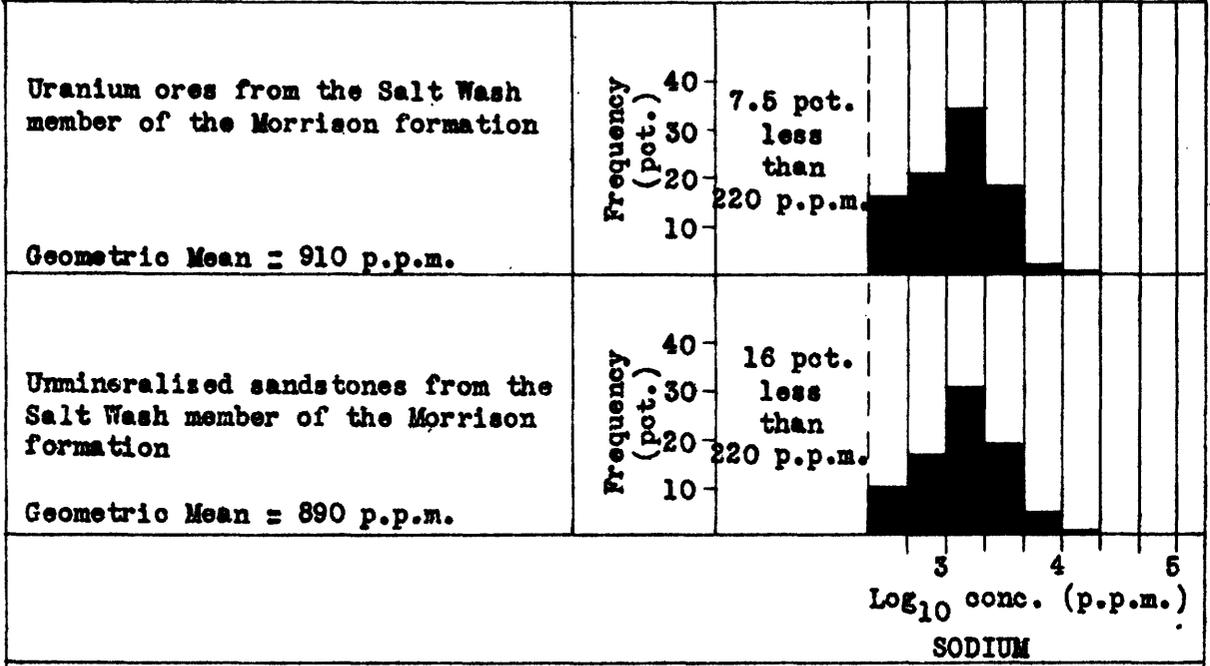


Figure 6. Histograms showing frequency distributions of sodium concentrations in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

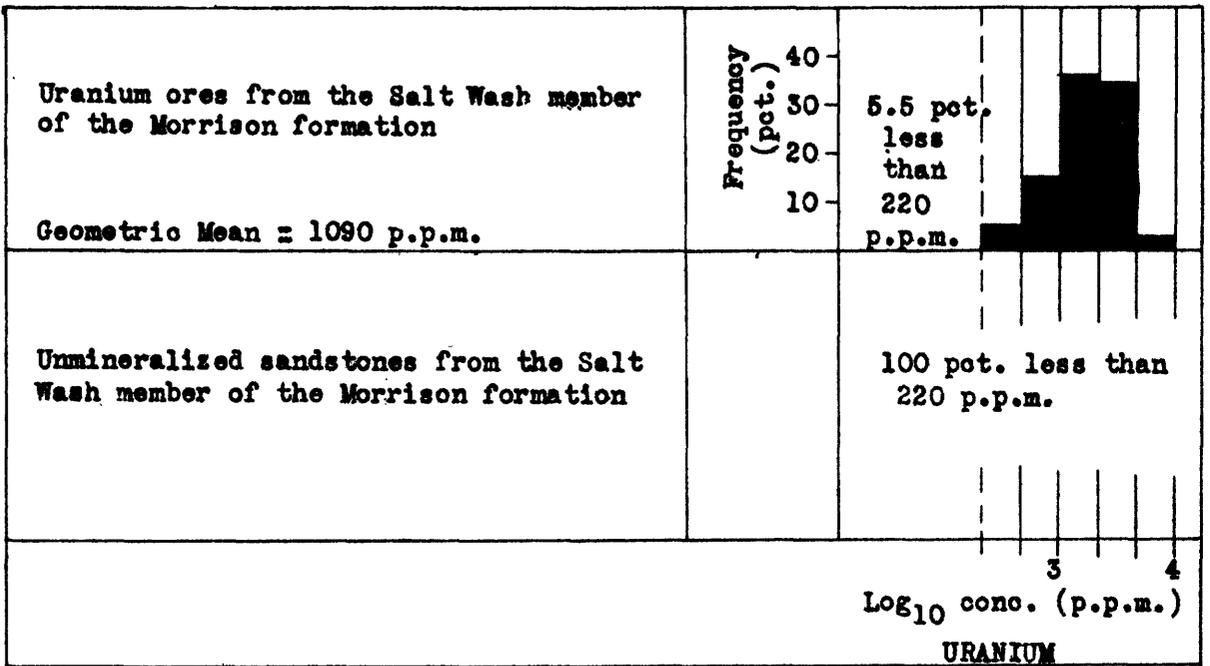


Figure 7. Histograms showing frequency distributions of uranium concentrations in 211 samples of uranium ore and 96 samples of unmineralized sandstones from the Salt Wash member of the Morrison formation.

Manganese has a very high correlation with calcium in both the unmineralized sandstone and in the uranium deposits (Shoemaker and others, in press) and is probably contained largely in calcite. A minor portion of the manganese is present as an oxide.

Sodium is contained principally in plagioclase feldspars and in nonsilicate sodium salts in the sandstone of the Salt Wash member of the Morrison formation. Flame photometric analyses of the acid leached sand-size fraction of 14 sandstones from the Salt Wash (Robert A. Cadigan, written communication, 1956) suggest that the sodium in the feldspars represents about half of the total sodium in the sandstone. The remainder of the sodium is thought to be contained mainly in soluble salts, principally sodium bicarbonate. A moderate correlation of sodium with calcium and manganese in the sandstone and high correlation in the uranium deposits (Shoemaker and others, in press) suggests the bulk of the sodium (as bicarbonate) is physically associated with the carbonates.

Judging from the ratios of abundance of the elements in uranium deposits to their abundance in unmineralized sandstone in the Salt Wash member it is very probable that essentially no calcium or sodium, and only very little, if any, manganese, is extrinsic. The abundance ratio of calcium (0.6) may be attributed to the fact that the upper sandstone strata of the Salt Wash member, in which most of the uranium deposits in the Salt Wash are located, contain about half as much calcium as the Salt Wash as a whole. The higher frequency of occurrence of uranium deposits in the upper sandstone strata of the Salt Wash member may be partly a function of a relatively low carbonate cement content and consequent higher permeability. It is not improbable, however, that some leaching of calcium from the mineralized host rock took place with precipitation of the ore metals, thereby causing the low abundance ratio.

Uranium

Uranium is not only the principal ore element in the deposits but also has the highest ratio of abundance in the deposits to abundance in unmineralized sandstone (table 1 and fig. 7). Uranium contents of the samples vary considerably (fig. 7) but no systematic or orderly regional pattern of variation is apparent, either because it is highly complex or because it was obscured by the selectivity of mining or ore mixing or both. Fortunately the uranium contents of the mill-pulp samples have no pronounced relationship to the contents of any of the other detected elements, except vanadium (Shoemaker and others, in press). Selective mining, therefore, being directed at uranium grade, has probably not appreciably biased the samples with regard to the other elements.

Nickel

The mean nickel content of uranium deposits in the Salt Wash member of the Morrison formation is about 20 times higher than that of unmineralized sandstone of the Salt Wash (table 1 and fig. 8). Nickel and iron have a moderately high correlation in uranium deposits in the Salt Wash (Shoemaker and others, in press) probably a reflection of an association of nickel with iron sulfides in low-valent ore minerals, perhaps bravoite or nickeliferous pyrite. The concentration of nickel in uranium deposits in the Salt Wash (and in deposits in the Chinle formation of Triassic age) tends to be broadly zoned across the central part of the Colorado Plateau. Nickel is higher in ores toward the west or northwest part of the region and lower in ores toward the east or southeast.

Yttrium

Yttrium is a dominantly extrinsic element in uranium deposits in the Salt Wash member of the Morrison formation. It is estimated to be about 7 times more abundant in the deposits than in unmineralized sandstone of the Salt Wash (table 1 and fig. 9). Correlation studies suggest some affinity of yttrium for the nickel-cobalt-molybdenum group of extrinsic elements (Shoemaker and others, in press) and the regional distribution of yttrium in ores in the Salt Wash (and in ores in the Chinle formation) is similar, though more erratic, to the regional distributions of these three elements.

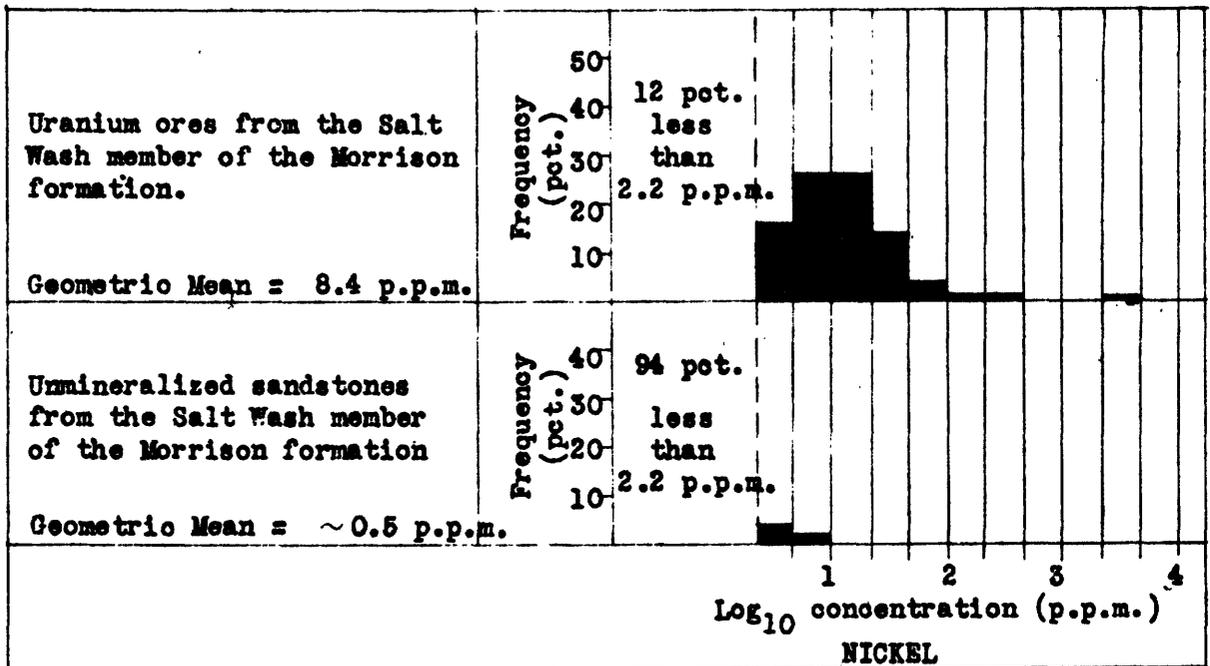


Figure 8. Histograms showing the frequency distributions of nickel concentrations in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

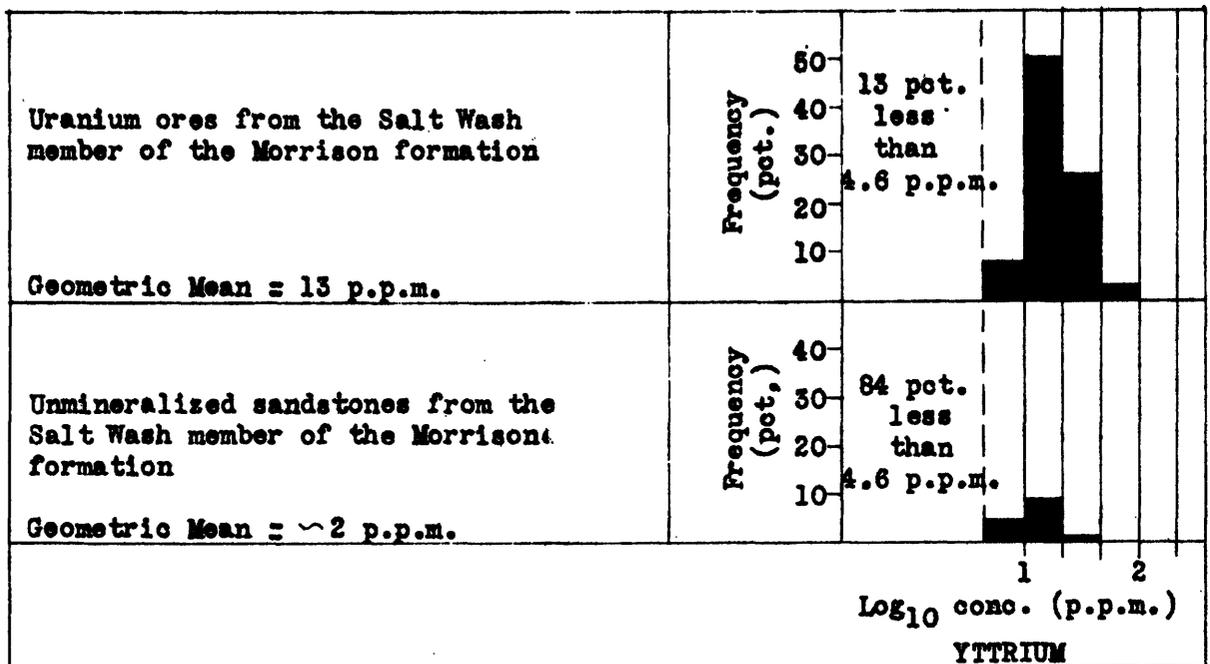


Figure 9. Histograms showing the frequency distributions of yttrium concentrations in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

SIZES OF URANIUM DEPOSITS

Sizes of 75 uranium deposits in the Salt Wash member of the Morrison formation were calculated by adding their total production to their indicated and inferred reserves. The production data were compiled from the records of the Finance Division of the Grand Junction Operations Office of the U. S. Atomic Energy Commission for the period of April 1948 through December 1953, and from the records of the U. S. Geological Survey for the time prior to 1948. The figures for the reserves were compiled from records of the Ore Reserve Section of the Grand Junction Operations Office of the U. S. Atomic Energy Commission and from numerous reports of the U. S. Geological Survey.

Each of the 75 deposits was represented by a spectrographic analysis of a mill-pulp sample of one or more ore shipments from the mine. The frequency distribution of the calculated sizes of the 75 deposits is given in figure 10. Calculated sizes of the deposits were classified in logarithmic classes with limits of integral powers of 10, as in the following table:

Tonnage	Log size
0-10	0-1
10-100	1-2
100-1,000	2-3
1,000-10,000	3-4
10,000-100,000	4-5
100,000-1,000,000	5-6

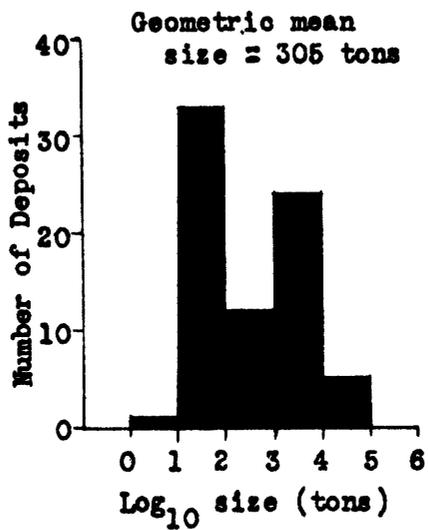


Figure 10. Histogram showing frequency distribution of log sizes of 75 uranium deposits in the Salt Wash member of the Morrison formation.

A sandstone-type uranium deposit (as the term "deposit" is used here) is not in every case a continuous body of mineralized rock but may be composed of more or less discontinuous, but closely spaced, ore bodies of varying dimensions. The specification, "closely spaced," is not rigid and is determined to some degree by mining practice. Groups of ore bodies that are or can be mined as one deposit may generally be considered as one deposit by those estimating reserves. Rarely, closely spaced ore bodies that may or may not be connected are mined by two or more companies through different adits and are considered as separate deposits. Had they been mined by one company they might have been considered as a single deposit.

Estimation of the extent of incompletely mined deposits or deposits not completely blocked out by drilling involves a considerable amount of personal judgment. Normally the third dimension of an exposure of ore in a mine or on an outcrop is considered nearly equal to the length of the exposure, unless some idea of the third dimension can be obtained in crosscuts in the mine, irregularities on the outcrop, or from drill holes.

The error involved in calculation of the size of a uranium deposit is undoubtedly large, but probably few sizes are misclassified in the broad class intervals given on figure 10.

In estimating tons of ore reserves at particular mines no consideration was given the grade of the ore except that it matched or exceeded the minimum requirements of 0.10 percent U_3O_8 or 1.0 percent V_2O_5 in continuous layers a foot or more thick.

CORRELATION OF ELEMENTS WITH SIZES OF THE DEPOSITS

In making the correlation, logs of the element concentration were determined from semiquantitative spectrographic analyses of samples from uranium deposits for which sizes could be calculated. Scatter diagrams (frequency distributions) were prepared, plotting the logs of the element concentration against the logs of the sizes of the deposits. The frequency distributions of the eight elements that correlate significantly with size are bivariate (fig. 11).

Computed correlation coefficients were used to judge the statistical significance of the relations between size and composition of the uranium deposits. These coefficients require certain assumptions regarding the form of the bivariate frequency distributions. We recognize that these assumptions may be invalid; even if invalid, however, these assumptions do not affect the completely separate regression analyses on which the size-prediction methods are actually based. The correlation studies served only to point out the elements which could be useful in regression analyses. The test of the correlation lies in the comparison of the predicted size of the deposits with the calculated size. The usefulness of these elements is verified in an analysis of variance given in Appendix II and table 8, and also in tests of the methods made on 40 "new" deposits of known size.

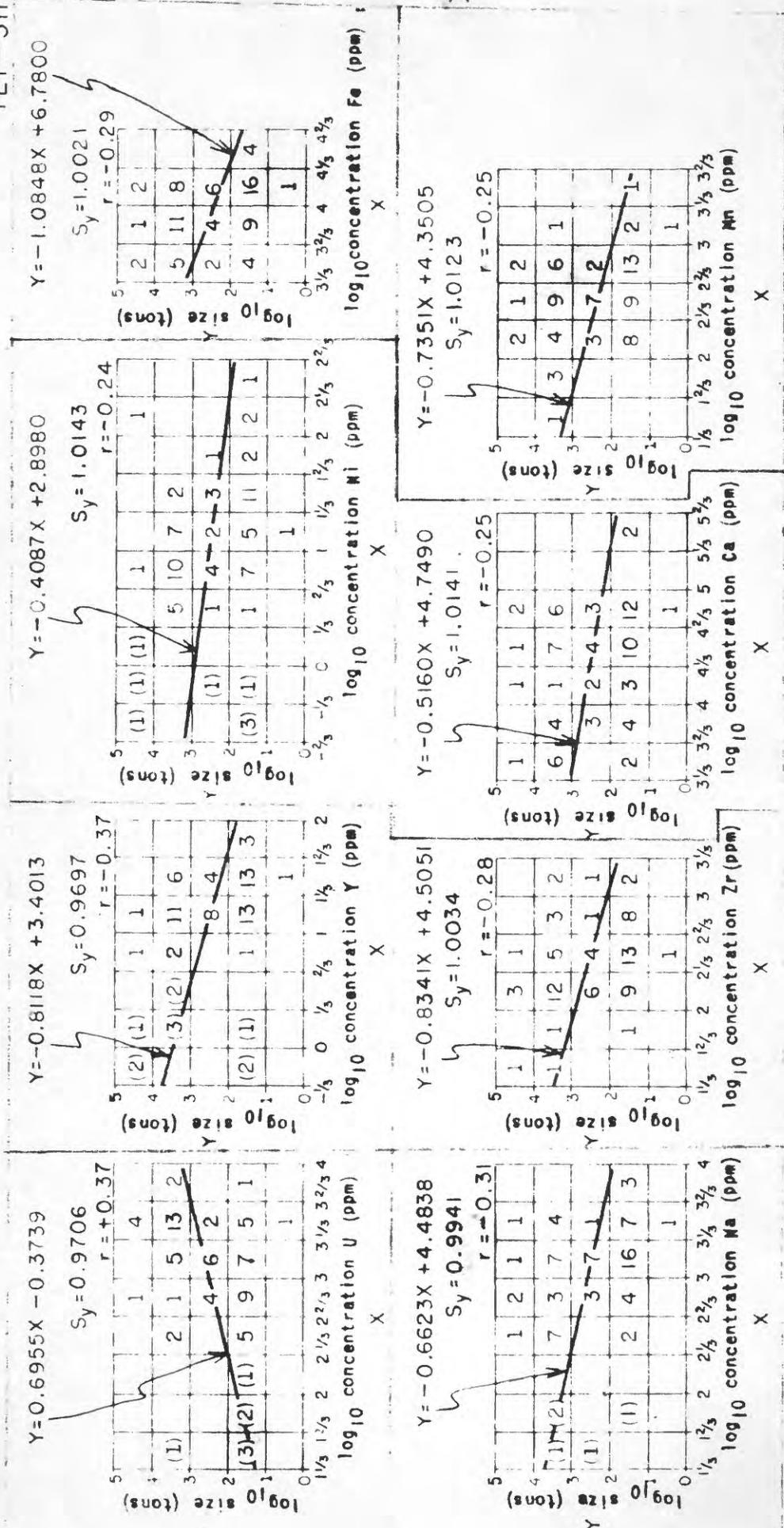


Fig. 11.--Scatter diagrams of minor element concentrations in 75 uranium deposits and sizes of the deposits (on log scales), showing regression lines and equations with standard errors, S_y , of the equations. Correlation coefficients, r , are also given. The positions of numbers in parentheses on the scatter diagrams are based on analytical reports of "trace" or "looked for, not found," or on reports indicated by the analyst as "questionable." All such reports are below the normal limit of sensitivity of the spectrographic method for the element. See text.

Simple linear correlation coefficients were computed between the logs of the element concentration and the logs of the sizes of the deposits. These are given in table 2. The computed linear correlation coefficients of the log concentrations of eight of the elements (nickel, calcium, manganese, zirconium, iron, sodium, yttrium, and uranium) against log sizes of the deposits are higher than the lowest significant coefficient for 75 pairs at the 0.95 level of confidence; that is, less than 5 times in 100 would coefficients this high be expected to occur by chance alone.

Significant correlations of size of the uranium deposits, with element concentration do not indicate, necessarily, that elemental composition controls the size or that size controls the composition. The probability values given in table 2, however, do indicate that most of the statistically significant correlation coefficients are not due to chance, and that sizes of the deposits and the concentrations of eight elements are in some way related. The low values of the coefficients indicate that the relationships are not strong.

Uranium is the only element of those studied (table 2) whose concentration has a significant positive correlation with the size of the deposits. The relationship indicates that the higher grade ore shipments tend to come from the larger ore deposits. As the analyzed samples are of ore shipments, there may be some doubt as to whether this observed relationship is a function of larger deposits containing higher concentrations of uranium or of economic practices in mining, i.e., operators of mines in larger deposits selectively mining and shipping higher grade ore. The latter reason seems unlikely to be important because mining practices are highly variable and possibly random with respect to size of the deposit. The mining practice most pertinent to the problem considered here is that of mixing ore, not to reach a high grade, but to maintain a grade that will give the highest return under the various payment procedures administered by the U. S. Atomic Energy Commission. It seems likely that operators of mines in the larger deposits shipped higher grade ore, not because they could afford to leave low-grade material in the mine, but because they were mining deposits that contained less low-grade material. A possible exception to this practice may occur in the case where a large deposit consisting of zoned high- and low-grade ore is being preferentially mined in the high-grade zones and the low-grade ore is not conveniently available for mixing or dilution of grade.

The reason why larger uranium deposits contain higher average grades of uranium is not clear, but it may be partly due to the distribution of a uranium precipitating agent, such as organic carbon, in the host rock (McKelvey, Everhart, and Garrels, 1955, p. 506-507).

Nickel and yttrium tend to be more highly concentrated in uranium deposits in the Salt Wash member of the Morrison formation located in the west and northwest parts of the Colorado Plateau where the deposits tend to be of smaller size. This fact alone might account for the significant negative correlation between nickel and yttrium concentrations and the sizes of the deposits. It should be noted, however, that molybdenum shows a similar regional variation within uranium deposits in the Salt Wash to that of nickel and yttrium, but does not exhibit a significant correlation with sizes of the deposits.

Calcium, manganese, and sodium are present mainly in carbonate cements of the host rock sandstone and their significant negative correlations in uranium deposits with size of the deposits might be explained by the hypothesis that higher cementation in the host rock is unfavorable to the development of large ore deposits.

The reasons for the significant negative correlations between sizes of the deposits and their concentrations of iron and zirconium are completely unknown.

ESTIMATION OF SIZES OF DEPOSITS

Regardless of the cause of the correlations between element concentrations in uranium deposits and the sizes of the deposits, the relationships provide a basis for estimation or prediction of sizes of deposits. In the absence of any other geologic information the best estimate[/] of the size of any single deposit would be the geometric

[/] "Best estimate" is used here in the sense of that estimate which has the smallest standard error.

mean size of all the deposits in the group--in this case the geometric means size of all deposits in the Salt Wash member of the Morrison formation. The indices of prediction[/] (Davies and Yoder, 1941, p. 348)

[/] Index of prediction = $1 - \sqrt{1-r^2}$, where r is the correlation coefficient.

reveal that estimation or prediction of size based on the concentration of any one of the eight elements discussed in preceeding sections will improve the estimate about 3 to 7 percent. This gain is tangible, but small. Estimates based on the concentrations of one of these elements would have about 97 to 93 percent of the standard error that would have been obtained if every deposit were estimated to be of mean size.

Estimation of size can be improved considerably, however, if it is based on the concentrations of all of the eight elements--uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel--simultaneously.

Three methods of estimating sizes of deposits from elemental composition will be given. The first method is based on simple linear regression; the statistics were computed by the writers. The other two methods are based on multiple regression; these statistics were computed at the Statistical Engineering Laboratory, National Bureau of Standards.

Simple linear regression method

Regression lines and equations expressing the variations of size as a function of element concentration are given in figure 11. The standard errors of estimate (Waugh, 1952, p. 445) for each equation are also given. When the concentrations of uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel in a uranium deposit are known, it is possible to estimate eight different sizes for any given deposit. If the eight estimates are each weighted by the \underline{z} value/ corresponding to the

/ The term \underline{z} is a function of the correlation coefficient (r), but, unlike r , \underline{z} is approximately normally distributed. (Fisher, 1950, p. 197-201)

$$\underline{z} = 1.1513 \log_{10} \frac{1+r}{1-r}$$

correlation coefficient of the element with size of the deposits, a "weighted average size" estimate can be computed. In other words, an average estimate is computed in which the estimates are weighted according to the degree of correlation on which they are based. Thus, the estimate of size based on the concentration of uranium would receive more weight than that based on manganese. Table 3 has been prepared to facilitate calculation of the "weighted average size estimate." The constants on the table have been weighted by the proper \underline{z} values and it is necessary only to sum the eight constants that correspond to the proper elements and their concentrations to obtain the log of the weighted estimate of the size. It is important that all eight elements be considered. The sum of the uppermost constants in table 3 is the \log_{10} of the "weighted average size estimate."

The logs of the "weighted average size" estimates of each of the 75 deposits on which the study is based are compared with the logs of their calculated or true sizes in figure 12. The correlation coefficient of these two variables is 0.61. The index of prediction calculated from this coefficient indicates that the size estimates using all eight elements can be improved about 21 percent in comparison to the approximate 3 to 7 percent improvement obtained using any single element. The regression line of y on x is,

$$y = 3.85x - 7.0963, \quad (1)$$

where y is a second estimate of \log_{10} of the size and x is \log_{10} of the weighted average size estimate. Thus, if the log of the weighted average size estimate is computed from table 3 and inserted in the above equation another weighted average size estimate is obtained. This second estimate is here called the "expanded weighted average size estimate," as the regression equation, $y = 3.85x - 7.0963$, expands the range of sizes that may be estimated. For example, 109 weighted average sizes of 2 and 3 computed from table 3, which correspond to tonnages of 100 and 1,000, will be expanded with the equation to 0.6037 and 4.4537 respectively, corresponding to tonnages of 4 and 28,430. The log of the "expanded weighted average size estimate" is computed by summing the eight appropriate lower values from table 3, which is constructed to include all computational steps given above. For example if a spectrographic

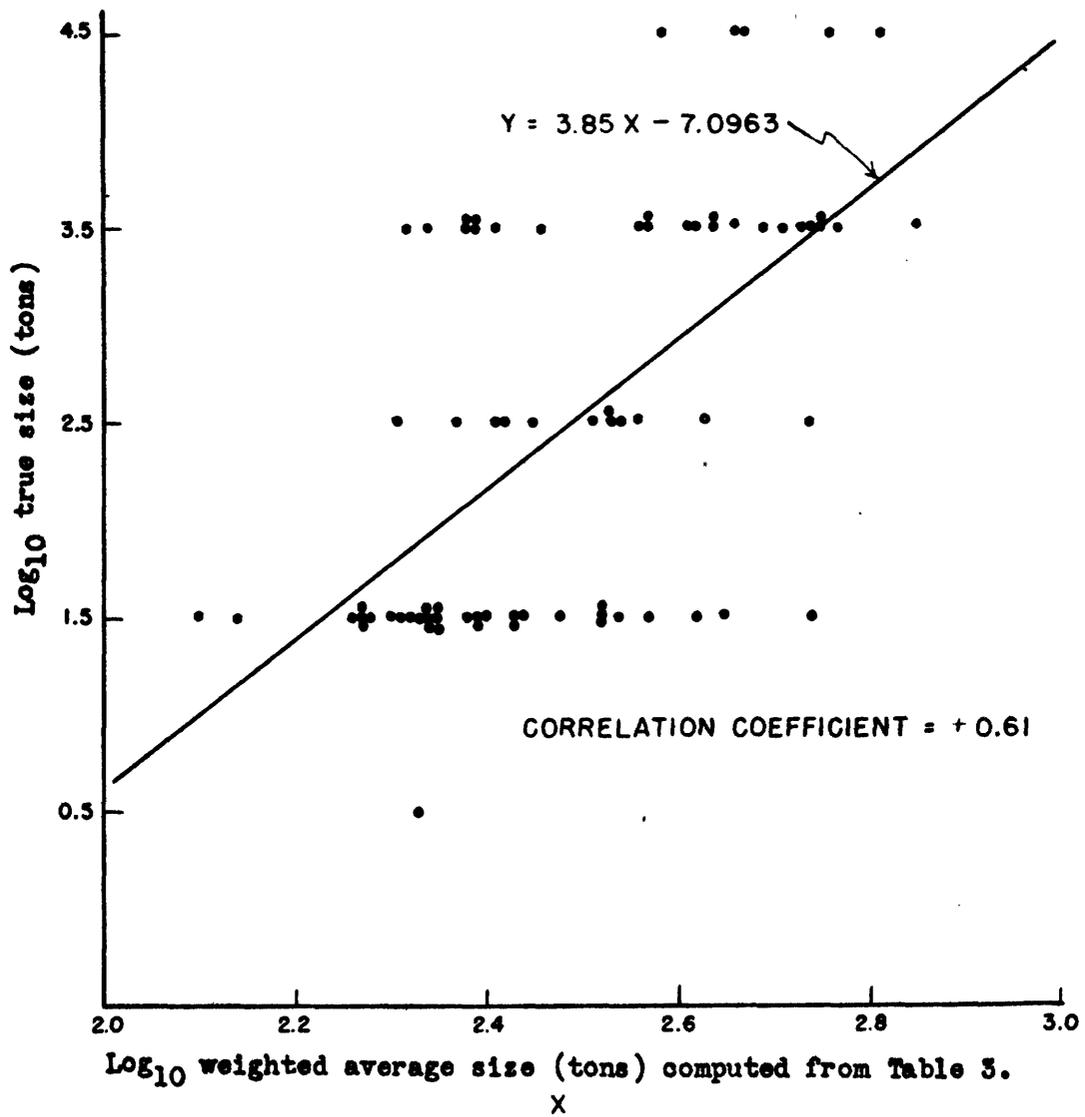


Figure 12. Scatter diagram of log weighted average size estimates of 75 uranium deposits and the log true sizes of the deposits.

analysis of a mill-pulp sample from a mine in the Salt Wash member of the Morrison formation gives $U = 0.X-$, $Y = 0.00X-$, $Na = 0.X-$, $Fe = 0.X+$, $Zr = 0.0X$, $Mn = 0.0X$, $Ca = X.$, and $Ni = 0.00X-$ the appropriate constants are 0.6919, 0.6272, 0.3127, 0.3485, 0.2160, 0.1394, 0.0659, and 0.0640. The sum of these factors (2.4556) is the estimated log size of the deposit in tons.

The standard error (s_y/x) of the regression line (equation 1) is 0.836 (appendix II, table 8). The standard error measures the deviation of points from the fitted regression line in terms of y (fig. 12).

The error involved in the prediction of y from any specific value of x may be obtained by the following expression:

$$\pm \left[\frac{t^2}{n} \cdot s_{y/\hat{x}}^2 \left\{ 1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{(x-\bar{x})^2} \right\} \right]^{\frac{1}{2}} \quad (1a)$$

(Anderson and Bancroft, 1952, p. 156-164) where t is taken from a table of t for a given confidence level, $s_{y/\hat{x}}$ is the standard error of estimate given above (0.836), and n is the number of pairs of values of x and y (75). To estimate the error of a size prediction from equation 1 at the 95 percent confidence level constants are inserted and the expression becomes where x is the log weighted

$$\pm 1.99 (0.836) \left\{ 1.0133 + \frac{(x-2.489)^2}{2.09} \right\}^{\frac{1}{2}} \quad (1b)$$

average size obtained by summing the appropriate upper values from table 3. Thus the error of a prediction varies with the difference between the mean x on figure 12 and the value of x with which we wish to predict. This is understandable when it is considered that an error in the slope of the fitted regression line (fig. 12) is most serious in terms of the dependent variable at its extremities and least serious near the mean.

Size predictions and errors for two cases are given below, one where all the element concentrations are in the same spectrographic group or frequency class as the mean of the element/ in the 75

/ For the mean case (example A) $U = 0.X-$, $Y = 0.00X-$, $Na = 0.X-$, $Fe = 0.X+$, $Zr = 0.0X$, $Mn = 0.0X$, $Ca = X.$, $Ni = 0.00X-$.

deposits (example A) and another where they are in extreme classes/

/ For the extreme case (example B) $U = 0.X+$, $Y = 0.000X+$, $Na = 0.00X$, $Fe = 0.X$, $Zr = 0.00X-$, $Mn = 0.X+$, $Ca = XX.-$, and $Ni = 0.0000X$. The values for Y, Na, and Ni for the extreme case are below the spectrographic limits of sensitivity for these elements; however, spectrographic reports of "looked for, not detected" for these elements were assigned to these respective classes for all calculations. See section on statistical treatment of data.

(example B). The error of size estimates at the 95 percent level of confidence will practically always be intermediate between these two cases.

Example	x log weighted average size (from table 3)	y log expanded weighted average size (from table 3)	Error of y (95 percent confidence)
A.	2.4809	2.4556	+ 1.6746 ($\frac{1}{2}$ 47.3) ✓
B.	2.9780	4.3690	+ 1.7646 ($\frac{1}{2}$ 58.2)

✓ The range $y-1.6746$ to $y+1.6746$ is equivalent to the range antilog $y \div 47.3$ to antilog $y \times 47.3$, where antilog y = predicted size of the deposit in tons, rather than log tons.

If one wishes to obtain the 80, rather than the 95, percent confidence intervals the expression becomes

$$\pm 1.29 (0.836) \left\{ \frac{1.0133 + (x-2.489)^2}{2.09} \right\}^{\frac{1}{2}}, \quad (1c)$$

and for examples A and B given above the errors would be ± 1.0856 ($\frac{1}{2}$ 12.2) and ± 1.1439 ($\frac{1}{2}$ 13.9) respectively.

Long multiple regression method

The multiple correlation coefficient between log sizes of the 75 deposits and the logs of their concentrations of U, Y, Na, Fe, Zr, Mn, Ca, and Ni as determined by semiquantitative spectrographic analysis is 0.67. The index of prediction calculated from this value indicates that size estimates can be improved about 26 percent in comparison to about 21 percent improvement using the simple linear regression method and 3 to 7 percent improvement using a single element.

The multiple regression equation calculated at the Bureau of Standards is:

$$y = -0.0170 + 0.1579x'_U - 0.0908x'_Y - 0.0709x'_{Na} - 0.0494x'_{Fe} \quad (2)$$

$$-0.0854x'_{Zr} + 0.0139x'_{Mn} + 0.0098x'_{Ca} - 0.0400x'_{Ni}$$

where y is the estimated \log_{10} size of the deposit in tons and $x'_U, x'_Y, \dots, x'_{Ni}$ are respectively six times the \log_{10} of the U, Y, Na, Fe, Zr, Mn, Ca, and Ni spectrographic determinations in percent /.

/ As the \log_{10} of the midpoints (in percent) of the semiquantitative spectrographic ranges in which the analyses are reported form a series such as 8 1/6-10, 8 3/6-10, 8 5/6-10, 9 1/6-10. . ., multiplication of each value by a factor of 6 reduces the series to one of simple whole numbers such as -11, -9, -7, -5. . ., which are easier to handle in calculation.

The same value of estimated log size may be obtained with less computation by summing the eight appropriate values from table 4.

The standard error of equation 2 is 0.820 (table 8). The range of error involved in the prediction of y from any specific combination of $x_U^i, x_Y^i, \dots, x_{Ni}^i$ is given by plus or minus the square root of:

$$\pm 0.672049 \left\{ 1 + \left[1, x_U^i, x_Y^i, \dots, x_{Ni}^i \right] C_8 \begin{bmatrix} 1 \\ x_U^i \\ x_Y^i \\ \vdots \\ x_{Ni}^i \end{bmatrix} \right\} \quad (2a)$$

The matrices C_8 are given in table 5. Expansion of this expression is a tedious process, and is impractical for the amount of information gained. The error in prediction will generally be within the ranges

Procedure for expansion of equations 2a and 3a is given under short multiple regression method in the following section.

given below for the 95 and 80 percent confidence levels.

	Example A	Example B
95 percent confidence	± 1.6532 ($\frac{\$}{\$} 45.0$)	to ± 2.4654 ($\frac{\$}{\$} 292.0$)
80 percent confidence	± 1.0717 ($\frac{\$}{\$} 11.8$)	to ± 1.5982 ($\frac{\$}{\$} 39.7$)

Examples A and B were calculated using values of $x_U^i, x_Y^i, \dots, x_{Ni}^i$ corresponding to the same spectrographic classes used in the sample calculations with equation 1b in the previous section.

Short multiple regression method

Inasmuch as logs of the uranium and yttrium contents of the deposits show the highest linear correlation with logs of the deposit sizes (table 2) a multiple regression equation was calculated at the Bureau of Standards which includes only uranium and yttrium contents as the dependent variables. The equation is:

$$y = 0.5074 + 0.1326x_U^1 - 0.1594x_Y^1, \quad (3)$$

where x_U^1 is 6 times the log of the percent uranium, x_Y^1 is 6 times the log of the percent yttrium, and y is the \log_{10} of the estimated size of the deposit in tons. To eliminate calculation, table 6 is given from which the estimated log size or size may be read directly using semiquantitative spectrographic determinations of uranium and yttrium.

The standard error of equation 3 is 0.866 (appendix II, table 8). The range of error involved in the prediction of y from any specific combination of x_U^i and x_Y^i is given by plus or minus the square root of:

$$\underline{t}^2 \cdot 0.749634 \left\{ 1 + \begin{bmatrix} 1, x_U^i, x_Y^i \end{bmatrix} C_2 \begin{bmatrix} 1 \\ x_U^i \\ x_Y^i \end{bmatrix} \right\} \quad (3a)$$

The matrices C_2 are given in table 5. Expanded, the square root of equation 3a becomes $\pm \underline{t} \sqrt{0.749634 (1 + 0.495495 x_U^i + 0.002528 x_Y^i + 0.027117 x_U^i x_Y^i + 0.002528 x_U^i + 0.001140 x_U^i{}^2 - 0.000237 x_U^i x_Y^i + 0.027117 x_Y^i - 0.000237 x_U^i + 0.001653 x_Y^i{}^2)}$, when like terms are combined the expression is

$$\pm \underline{t} \sqrt{1.121374 + 0.003790 x_U^i + 0.040656 x_Y^i - 0.000355 x_U^i x_Y^i + 0.000855 x_U^i{}^2 + 0.001239 x_Y^i{}^2}$$

The error in prediction will generally be within the ranges given below for the 95 and 80 percent confidence levels.

	Example A	Example B
95 percent confidence	± 1.7351 ($\frac{\%}{54.1}$) to ± 1.8533 ($\frac{\%}{71.3}$)	
80 percent confidence	± 1.1248 ($\frac{\%}{13.3}$) to ± 1.2014 ($\frac{\%}{15.3}$)	

Example A was computed from equation 3a using values of x_U^i and x_Y^i which correspond to the respective log midpoints of the classes in which the mean log concentrations of the elements fall ($U = 0.X^-$, $Y = 0.00X^-$). Calculated errors for example A are about the minimum that will be obtained from equation 3a at the 95 and 80 percent confidence levels with any combination of uranium and yttrium concentrations. Example B was computed from equation 3a using values of x_U^i and x_Y^i which correspond to the respective log midpoints of extreme classes ($U = 0.X^+$, $Y = 0.000X^+$). Unless higher concentrations of uranium or lower concentrations of yttrium are detected in the sample, the ranges of error of a size estimate at the given confidence levels cannot be larger than those for example B.

Summary of procedures
for estimating sizes of deposits

The type of sample and analysis required for use of the equations and tables given in this report are as follows:

- 1) The uranium deposit must be in the Salt Wash member of the Morrison formation on the Colorado Plateau.
- 2) The sample of the deposit must be a mill-pulp sample representing several tons or more if it is desired to estimate the error of the prediction. Other types of samples, such as drill core, could be used but the precision and accuracy of estimation for such samples is indeterminate.
- 3) The analytical determinations of U, Y, Na, Fe, Zr, Mn, Ca, and Ni must be semiquantitative spectrographic and made in the Denver laboratory of the U. S. Geological Survey or by essentially identical methods as used by the Denver laboratory.

Size estimation procedure for the simple linear regression method 1

For each element a factor is obtained from table 3 corresponding to the semiquantitative spectrographic determination of the concentration of that element. The eight constants from table 3 are totalled, using the appropriate constants for expanded weighted average size. Use lower values. The antilog of this sum is the estimated size in tons. About 95 percent of such estimates will be within a factor of 47 to 58 of the true sizes of the deposits. About 80 percent of the estimates will be within a factor of 12 to 14 of the true sizes.

Size estimation procedure for the long multiple regression method 2

For each element a factor is obtained from table 4 corresponding to the semiquantitative spectrographic determination of the concentration of that element. The eight appropriate constants from table 4 are summed. The antilog of this sum is the estimated size in tons. About 95 percent of these estimates will be within a factor of 45 to 292 of the true sizes of the deposits. About 80 percent of the estimates will be within a factor of 12 to 40 of the true sizes.

Size estimation procedure for the short multiple regression method 3

The log size or size of the deposit in tons is read directly from table 6 for spectrographically concentrations of uranium and yttrium determined by semiquantitative spectrographic analysis. About 95 percent of the estimated sizes in tons will be within a factor of 54 to 71 of the true size of the deposit. About 80 percent of the estimated sizes will be within a factor of 13 to 16 of the true sizes.

SUMMARY AND CONCLUSIONS

Three methods have been given for estimating the size of a uranium deposit in the Salt Wash member of the Morrison formation on the Colorado Plateau. Two of the methods are based on the concentrations of U, Y, Na, Fe, Zr, Mn, Ca, and Ni in the deposit and the third only on concentrations of U and Y. The element concentrations are determined by semiquantitative spectrographic analyses of mill-pulp samples obtained from uranium ore mills and buying stations. The samples are splits of those assayed to determine payment to the ore shipper; with one exception each represents several tons or more of ore. The spectrographic analyses are made by the U. S. Geological Survey.

None of the methods of size estimation given in this report is very precise; each gives an estimate of size in tons that is good only within a factor of 12.2 or more at the 80 percent confidence level. For example, if the size of a deposit were estimated from equation 1 to be 285 tons (as in example A for method 1) one could be only 80 percent confident that the true size lies somewhere between 23 and 3,477 tons. For the person concerned with appraisal of a single ore deposit this appears to be little help. However, 285 tons is the best estimate of the size of the deposit if no other information regarding the size of the deposit is available, as may often be the case where mine development or exploratory drilling is meager and the deposits in the general area have a large range in size. If the size of a deposit were estimated with method 1 to be 23,400 tons (as in example B) one could be only 80 percent confident that the true size lies somewhere between 1,700 and 325,000 tons. However, the best estimate of the size of the deposit is still 23,400 tons and the miner could be encouraged toward further expenditure in exploration development. The method has indicated that he has a large deposit. When the estimated size of a deposit is small the range of error of the estimate is also small in terms of tons because the error is a function of the estimated size. For example, if the size of a deposit were estimated with equation 1 to be 43 tons, one could be 80 percent confident that the true size is less than 600 tons. Known production or reserves may serve to place a lower limit on the size of the deposit.

As applied to individual deposits the methods may be useful in appraisal of old mining property. A deposit thought to have been "mined out" may be tested to judge the likelihood that undiscovered ore is present. For example, a deposit may have produced 2,000 tons of ore and is then believed to have been exhausted. If its composition suggests that the deposit is 20,000 tons, further exploration may be encouraged. However, if the deposit is estimated to be near 2,000 tons or less the methods would tend to confirm the belief that the deposit is "mined out."

For individual deposits the methods of size estimation given here are useful in distinguishing only the general range of size, i. e., small versus intermediate versus large. The precision of estimation is greater, however, for the average of a number of deposits than for a single deposit. If the estimate of a single deposit has a frequency distribution with a standard deviation of σ , then the mean of N independent estimates for this deposit has a frequency distribution with a standard deviation of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$. Thus, if the precision of estimation does not vary too widely for deposits of different estimated size, the confidence interval (expressed in log values) for the estimated mean of 10 deposits, for example, is about one-third the confidence interval for a single estimate. The precision of the mean in terms of tons of ore, in other words, is greatly increased. The methods of size estimation given in this report, therefore, may be most useful in appraising groups of deposits or ore districts.

The precision and accuracy of the methods of size estimation have not been determined for ore samples other than mill pulps. However, the principles underlying the methods may serve to establish similar methods for other types of samples, such as drill core. In drilling exploration such methods may be extremely useful in extracting maximum information from costly drill data. It is anticipated that estimation of the size of ore deposits from the composition of drill core samples of the deposits will be considerably less precise than estimation from mill pulps.

The controls of the relations between size and composition of uranium deposits are largely unknown, though some speculation on this is given in a previous section. More thorough understanding of these controls may lead to more precise methods of estimating sizes of uranium deposits in the Salt Wash member of the Morrison formation.

Similar relations may also be found among other types of ore deposits and may be useful in estimating their size. A search for such relationships may not only be rewarding from an economic point of view but may certainly add to our knowledge of the nature of ore deposits and the controls of their localization.

REFERENCES

- Anderson, R. L., and Bancroft, T. A., 1952, Statistical theory in research: New York, McGraw-Hill Book Co., Inc., 339 p.
- Cadigan, R. A., in press, Characteristics of the host rocks, in Garrels, R. M., and Larsen, E. S., III, Geochemistry and mineralogy of Colorado Plateau uranium deposits: U. S. Geol. Survey Prof. Paper 320, pt. 2, p. ____.
- Davies, G. R., and Yoder, Dale, 1941, Business statistics: New York, John Wiley and Sons, Inc., 2d ed., 616 p.
- Dixon, W. J., and Massey, F. J., Jr., 1957, Introduction to statistical analysis: New York, McGraw-Hill Book Co., Inc., 3rd ed., 370 p.
- Finch, W. I., 1955, Preliminary geologic map showing the distribution of uranium deposits and principal ore-bearing formations of the Colorado Plateau region: U. S. Geol. Survey Min. Inv. Field Studies Map MF-16.
- Fischer, R. P., 1942, Vanadium deposits of Colorado and Utah, a preliminary report: U. S. Geol. Survey Bull. 936-P, p. 363-394.
- Fisher, R. A., 1950, Statistical methods for research workers: New York, Hafner Publishing Co., Inc., 11th ed., 354 p.
- Lasky, S. G., 1950, How tonnage and grade relations help predict ore reserves: Eng. Mining Jour., v. 151, p. 81-85.
- McKelvey, V. E., Everhart, D. L., and Garrels, R. M., 1955, Origin of uranium deposits: Econ. Geology Fiftieth Anniv. volume, p. 464-533.

- Myers, A. T., 1954, Spectrographic methods, Denver, in Geologic investigations of radioactive deposits, semiannual progress report, December 1, 1953, to May 31, 1954: U. S. Geol. Survey TEI-440, p. 194-195, issued by U. S. Atomic Energy Comm., Tech. Inf. Service, Oak Ridge, Tenn.
- Shawe, D. R., 1956, Significance of roll ore bodies in genesis of uranium-vanadium deposits on the Colorado Plateau, in Page, L. R., Stocking, H. E., and Smith, H. B., compilers, Contributions to the geology of uranium and thorium by the United States Geological Survey and Atomic Energy Commission for the United Nations International Conference on Peaceful Uses of Atomic Energy, Geneva, Switzerland, 1955: U. S. Geol. Survey Prof. Paper 300, p. 239-241.
- Shoemaker, E. M., Miesch, A. T., Newman, W. L., and Riley, L. B., in press, Elemental composition of the sandstone-type deposits, in Garrels, R. M., and Larsen, E. S., III, Geochemistry and mineralogy of Colorado Plateau uranium deposits: U. S. Geol. Survey Prof. Paper 320, pt. 3, p. ____.
- Waugh, A. E., 1952, Elements of statistical method: New York, McGraw-Hill Book Co., Inc., 3d ed., 531 p.
- Weeks, A. D., and Thompson, M. E., 1954, Identification and occurrence of uranium and vanadium minerals from the Colorado Plateaus: U. S. Geol. Survey Bull. 1009-B, p. 13-62.

APPENDIX I

TESTS OF THE METHODS

The logs of the sizes of 40 uranium deposits in the Salt Wash member of the Morrison formation were estimated from semiquantitative spectrographic determinations of the eight elements in mill-pulp samples using equations 1, 2, and 3. These estimated log sizes are given in table 7 with the range of the log of the calculated or "true" size for each deposit. The range of error of each estimate from equations 1 and 3 at the 95 and 80 percent confidence levels is also given. None of the 40 deposits listed in table 7 is part of the original 75 deposits on which the methods of size estimate are based.

As was the case for the original 75 deposits, the "true" log sizes of the 40 deposits listed in table 7 are known only within broad ranges. For purposes of comparison the "true" log size of each deposit may be taken as the midpoint of the range of the "true" log size. Errors arising from this assumption are probably compensating.

The purpose in testing the methods of size estimation is not to determine the precision of the estimates; the precision may be calculated from statistical theory (equations 1a, 2a, and 3a). The purpose here is to determine whether the precision of size predictions for a particular group of deposits is similar to or greatly different from that calculated for the method. Theoretically, about 38 estimates out of 40 for each equation should be within the 95 percent confidence intervals and about 32 out of 40 should be within the 80 percent confidence intervals. For equation 1, 39 of the estimates out of 40 (97.5 percent) proved to be within the calculated 95 percent confidence intervals and 28 out of 40 (70 percent) proved to be within the 80 percent confidence intervals. For equation 3, 37 of the estimates out of 40 (92.5 percent) are within the 95 percent confidence intervals and 31 out of 40 (77.5 percent) are within the 80 percent confidence intervals. Confidence intervals for estimates made with equation 2 were not calculated except for two extreme cases. Had the confidence intervals for each estimate been calculated, 38 to 40 of the 40 size estimates made with equation 2 (95 to 100 percent) would be within the 95 percent confidence intervals and 29 to 38 (72.5 to 95 percent) would be within the 80 percent confidence intervals. Thus, it is apparent that the calculated confidence intervals describe fairly well the precision of size estimates made for this group of 40 deposits.

The accuracy of the 40 size estimates may be judged by observing the deviation of the estimates from the "true" log sizes or by comparing the mean of the estimates with the mean "true" log size of the 40 deposits. Deviations of the estimates from the "true" log sizes may be expressed in terms of

$$\left[\frac{\sum (y-y')^2}{n-1} \right]^{\frac{1}{2}}$$

which is equivalent to the standard errors of the regression equations (table 8). The term y is the "true" log size of a deposit, y' is the estimated log size and n is the number of estimates. For the 40 estimates made with each of the three equations this value is as follows:

Equation	$\left[\frac{\sum (y-y')^2}{n-1} \right]^{\frac{1}{2}}$	Standard error
1	0.905	0.836
2	0.944	0.822
3	1.017	0.866

For each equation the value $\left[\frac{\sum (y-y')^2}{n-1} \right]^{\frac{1}{2}}$ is somewhat higher than the calculated standard error. This may be due primarily to the fact that the mean log size of the 40 deposits (3.05) is greater than the mean log size of the 75 deposits on which the calculations of the regression equations and their standard errors are based (2.49). Log size estimates that average nearly 2.49 will have a value $\left[\frac{\sum (y-y')^2}{n-1} \right]^{\frac{1}{2}}$ nearer the standard error. Log size estimates greater or less than 2.49 will have a value $\left[\frac{\sum (y-y')^2}{n-1} \right]^{\frac{1}{2}}$ greater than the standard error because the errors in the regression equations are more serious away from the mean when the error is measured in terms of y .

Although the standard error of equation 2 is less than those of equation 1 and 3 the deviation of log sizes estimated with equation 2 from the "true" log sizes is greater than that for equation 1. However, this was only found to be true for this group of 40 deposits and cannot be considered representative of what might be found with other tests.

The "true" mean log size of the 40 deposits listed on Table 7 is 3.05. The mean of the estimates is 2.73 for equation 1, 2.80 for equation 2, and 2.70 for equation 3. Thus, the estimates from all three equations appear to be too low on the average with the bias increasingly more serious with equations 2, 1, and 3, in that order.

If we wish to consider the accuracy of the equations in estimating the log sizes of particularly large or small deposits, or deposits of near average size, the following data are of interest:

"True" mean log size of group	4.50	3.50	2.50	1.50
Mean estimated log size of group:				
From equation 1	3.37	2.78	2.58	2.05
From equation 2	3.40	2.76	2.71	2.24
From equation 3	3.07	2.64	2.67	2.34
Number of deposits in group	8	12	15	5

Here the 40 deposits are grouped according to log size classes; the 8 deposits with log size in the range from 4 to 5 are considered to have a "true" mean size of 4.50, for example. The means of the estimated log sizes from equations 1, 2, and 3 are 3.37, 3.40, and 3.07, respectively. It may be seen that the mean log sizes estimated from equations 1 and 2 are somewhat nearer the "true" mean log sizes for each class than those from equation 3. For the test group of 40 deposits equation 3 is considerably less accurate in estimating log sizes of large deposits; equation 1 is more accurate than either equation 2 or 3 for estimating log sizes of small deposits.

For the 40 deposits listed on table 7, equation 1 based on the simple linear regression method of size estimation and equation 2 from the long multiple regression method are slightly more accurate than equation 3.

APPENDIX II

COMPARISON OF THE METHODS

The precision with which estimates of size may be made with each of the methods is dependent on two factors: 1) the degree of relationship found between size and the independent variable or variables used in the method, 2) the error or uncertainty of the equation used to express the relationship between size and the independent variable or variables. The first factor, the actual degree of relationship between size and x or $x'_U \dots x'_{N1}$, may be examined for the three methods by a comparison of the correlation coefficients and an analysis of variance which is based on these coefficients (table 8). The highest correlation coefficient given in table 8 is that between log size and logs of the eight element concentrations ($R_2 = 0.67$). Equation 2 (page 47) ignoring the second factor, the uncertainty of the equation, reduces the sum of squares of the "true" log size more than either of the other two methods and has a smaller standard error. The reduction in the sum of squares due to equation 1 is somewhat less than that obtained with equation 2 and the standard error of equation 1 is slightly greater. Equation 3 causes the least reduction in the sum of squares and has the greatest standard error. The F-ratios for all three methods indicate that each of the equations reduce the variance by statistically significant amounts.

With the long multiple regression method (equation 2) most of the reduction of sum of squares is due to the variables x_1^1 and x_2^1 (uranium and yttrium). However, the reduction in variance due to inclusion of the variables $x_2^1 . . . x_8^1$ in equation 2 is significant at the 0.95 probability level.

It may be concluded from the analysis of variance in Table 8 that significant reductions in variance of logs of the sizes of uranium deposits are caused by each of the methods and so each method will be useful in estimating sizes.

Equations 1a, 2a, and 3a give the ranges of error of individual size estimates at specified confidence levels. These equations consider both the degree of relationship found between log size and the independent variable or variables and the uncertainty of the equation used to express this relationship. The ranges of error at the 95 and 80 percent confidence levels have been computed for each of the three methods for the case where all elements are at their mean concentration (example A) and for the case where all elements are at extreme concentrations (example B). These are given in the appropriate sections of the text but may be brought together here for comparison.

It is suggested on the basis of these comparisons and the tests of the methods described in the previous section that the simple linear regression method (equation 1) and the long multiple regression method (equation 2) are more accurate for estimating sizes of deposits than the short multiple regression method (equation 3). In most cases (except where the deposit is estimated to be of near average size) equation 1 will give a more precise estimate than equation 2. Equation 1 has the additional advantage that confidence intervals of estimates can be calculated more easily.

Method Estimated log 95 percent confi- 80 percent confi-
 size (in tons) dence level dence level

	Example A	Example B	Example A	Example B	Example A	Example B
Equation 1	2.4556	4.3690	± 1.6746	± 1.7646	± 1.0856	± 1.1439
Equation 2	2.4939	5.8933	± 1.6532	± 2.4654	± 1.0717	± 1.5982
Equation 3	2.5542	4.3598	± 1.7353	± 1.8533	± 1.1248	± 1.2014

Example A; $U = 0.X-$, $Y = 0.00X-$, $Na = 0.X-$, $Fe = 0.X+$, $Zr = 0.0X$,
 $Mn = 0.0X$, $Ca = X.$, $Ni = 0.00X-$, $x = 2.4809$;
 $x_U^i \dots x_{Ni}^i = -5, -17, -5, -1, -9, -9, 3, -17.$

Example B: $U = 0.X+$, $Y = 0.0000Y+$, $Na = 0.00X$, $Fe = 0.X$, $Zr = 0.00X-$,
 $Mn = 0.Y+$, $Ca = XX.-$, $Ni = 0.0000X$; $x = 2.9780$;
 $x_U^i \dots x_{Ni}^i = -1, -25, -15, -3, -17, -1, 7, -27.$

It appears from these sample calculations of the range of error of individual size estimates, wherein the uncertainty of the regression equation is considered, that the range of error for estimated sizes is generally smaller for equation 1 than for equations 2 and 3. The range of error for estimated log-sizes from equation 2 may be especially large if the eight elements are at extreme concentrations (the deposit is extremely large or small).

U. S. Geological Survey Library
Denver Branch

12