

# **Documentation of a Daily Mean Stream Temperature Module—An Enhancement to the Precipitation-Runoff Modeling System**

Chapter 4 of  
Section D, Groundwater/Surface-Water Interactions, of  
**Book 6, Modeling Techniques**

Techniques and Methods 6–D4



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By Michael J. Sanders, Steven L. Markstrom, R. Steven Regan,  
and R. Dwight Atkinson

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**U.S. Department of the Interior**  
**U.S. Geological Survey**

**U.S. Department of the Interior**

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## Preface

This report is a supplement to the Markstrom and others (2015) report that documents the Precipitation Runoff Modeling System Version IV (PRMS-IV) hydrologic simulation code and relies heavily upon information contained in that report. This supplemental report describes a new stream temperature simulation module (stream\_temp). See the link labeled “Changes in the specification of user inputs are reported as updates to tables in the PRMS-IV Users’ Manual” (tables 2, 1-2, 1-3, 1-5) on the PRMS software distribution page ([https://wwwbrr.cr.usgs.gov/projects/SW\\_MoWS/PRMS.html](https://wwwbrr.cr.usgs.gov/projects/SW_MoWS/PRMS.html)) for summaries of changes to input and output values for PRMS-IV.

The performance of this software has been tested on several different computer systems and configurations. Future use, however, might reveal errors that were not detected during testing. Users are requested to notify the U.S. Geological Survey (USGS) of any errors in this report or the code and submit questions by using the “MoWS Contact Form” link found at the bottom of the internet address [https://wwwbrr.cr.usgs.gov/projects/SW\\_MoWS/](https://wwwbrr.cr.usgs.gov/projects/SW_MoWS/).

Although this software has been developed and used by the USGS, no warranty, expressed or implied, is made by the USGS or the U.S. Government as to its accuracy, functionality, and related material, nor shall the fact of distribution constitute any such warranty, and no responsibility is assumed by the USGS in connection therewith. The latest version of the source code, compiled executable, updated information, and this report can be obtained by using the internet address [https://wwwbrr.cr.usgs.gov/projects/SW\\_MoWS/Software.html](https://wwwbrr.cr.usgs.gov/projects/SW_MoWS/Software.html).

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## Conversion Factors

SI to Inch/Pound

Multiply	By	To obtain
	Length	
centimeter (cm)	0.3937	inch (in)
meter (m)	3.281	foot (ft)
kilometer (km)	0.6214	mile (mi)
	Flow rate	
cubic meter per second (m <sup>3</sup> /s)	35.31	cubic foot per second (ft <sup>3</sup> /s)
cubic meter per day (m <sup>3</sup> /d)	35.31	cubic foot per day (ft <sup>3</sup> /d)
	Energy	
joule (J)	0.000002	kilowatt-hour (kWh)
Langley (Ly)	41840.0	joule per square meter (J/m <sup>2</sup> )
	Mass	
kilogram (kg)	2.205	pound avoirdupois (lb)

Temperature in degrees Celsius (°C) may be converted to degrees Fahrenheit (°F) as follows:

$$^{\circ}\text{F}=(1.8\times^{\circ}\text{C})+32$$

Temperature in degrees Fahrenheit (°F) may be converted to degrees Celsius (°C) as follows:

$$^{\circ}\text{C}=(^{\circ}\text{F}-32)/1.8$$

## Terminology and Font Styles

The following terminology and font styles are used to reference components of PRMS: See appendix 1 of Markstrom and others (2015) for descriptions of modules, parameters, simulation algorithms, and computed.

- Modules, filenames, and user input are identified by the use of *Courier New* font.
- Input parameters and dimensions are identified by the use of bold, **Times New Roman** font.
- State and flux variables are identified by the use of italic, *Times New Roman* font.



# Documentation of a Daily Mean Stream Temperature Module—An Enhancement to the Precipitation-Runoff Modeling System

By Michael J. Sanders,<sup>1</sup> Steven L. Markstrom,<sup>2</sup> R. Steven Regan,<sup>2</sup> and R. Dwight Atkinson<sup>3</sup>

## Abstract

A module for simulation of daily mean water temperature in a network of stream segments has been developed as an enhancement to the U.S. Geological Survey Precipitation Runoff Modeling System (PRMS). This new module is based on the U.S. Fish and Wildlife Service Stream Network Temperature model, a mechanistic, one-dimensional heat transport model. The new module is integrated in PRMS. Stream-water temperature simulation is activated by selection of the appropriate input flags in the PRMS Control File and by providing the necessary additional inputs in standard PRMS input files. This report includes a comprehensive discussion of the methods relevant to the stream temperature calculations and detailed instructions for model input preparation.

## Introduction

Stream temperature is an important factor that can have a variety of effects on water quality and aquatic organisms (Perry and others, 2011, Gaffield and others, 2005, and Anderson and Carpenter, 1998). Water temperature can affect the solubility and mobility of toxic metals (Norton and Bradford, 2009). The geographic distribution of water temperature is also of particular importance in maintaining healthy populations of cold water fish species (Bartholow, 1991). Consequently, stream temperature simulation, based on flow and climate regime dynamics, can yield important information for the management of natural watershed systems.

The Stream Temperature module (`stream_temp`) was developed to provide a method to simulate daily mean stream-water temperatures in models for use with the U.S. Geological Survey (USGS) Precipitation Runoff Modeling System (PRMS) watershed hydrologic simulation code. PRMS (Markstrom and

others, 2015) is a modular, deterministic, distributed-parameter, physical-process based hydrologic-simulation code developed to evaluate the effects of various combinations of climate, physical characteristics, and simulation options on hydrologic response and water distribution at the watershed scale. The computational methods in the Stream Temperature module are based on the Stream Network Temperature Model (SNTEMP) originally developed by Theurer and others (1984) for the U.S. Fish and Wildlife Service and subsequently described by Bartholow (2000). The `stream_temp` module is an enhanced version of the SNTEMP source code that has been updated and integrated with PRMS.

The `stream_temp` module computes daily mean and maximum water temperatures at the outflow of each stream segment defined in a PRMS model. To activate stream-temperature computations within a PRMS simulation, parameters must be specified in the Control and Parameter Files, as described in the “Daily mean Stream Temperature Module Operation in the Precipitation-Runoff Modeling System” section of this report. See Markstrom and others (2015) for descriptions of the Control and Parameter Files.

This report describes the methods used to develop this module for simulating daily mean water temperature, and detailed instructions are given for the preparation of model input. Parameters used in the model are discussed and are presented in tables.

## Methods

Streamwater-temperature computations in the `stream_temp` module are based on an equilibrium temperature concept developed by Edinger and others (1968). Equilibrium temperature is defined as the water temperature at which the net energy fluxes across the interfaces of the water with the air and the streambed are zero. For a given set of daily meteorological energy inputs, surface water will tend toward an equilibrium temperature. The equilibrium temperature has been shown to be directly related to mean stream temperatures on the daily to monthly time scale (Bogan and others, 2003); however,

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secondary heat sources, such as stormwater and industrial outflows, can alter this relation (Bogan and others, 2004). The net heat flux can be defined in terms of thermal exchange coefficients and this equilibrium temperature (Edinger and others, 1968; Theurer and others, 1984). The `stream_temp` module calculates daily mean stream temperatures using this method, relying on the calculation of two thermal exchange coefficients, as well as the daily equilibrium temperature, in each stream segment. The final stream-temperature solution depends on the volumetric rate and temperature of all inflows of water, as well as the net heat flux.

### Energy Flux Models and Equilibrium Temperature

The equilibrium temperature calculation in the `stream_temp` module is based on the full stream channel energy balance as shown in figure 1. The energy balance is computed for each time step according to

$$H_n = H_a + H_v + H_w + H_s + H_e + H_c + H_d + H_f, \quad (1)$$

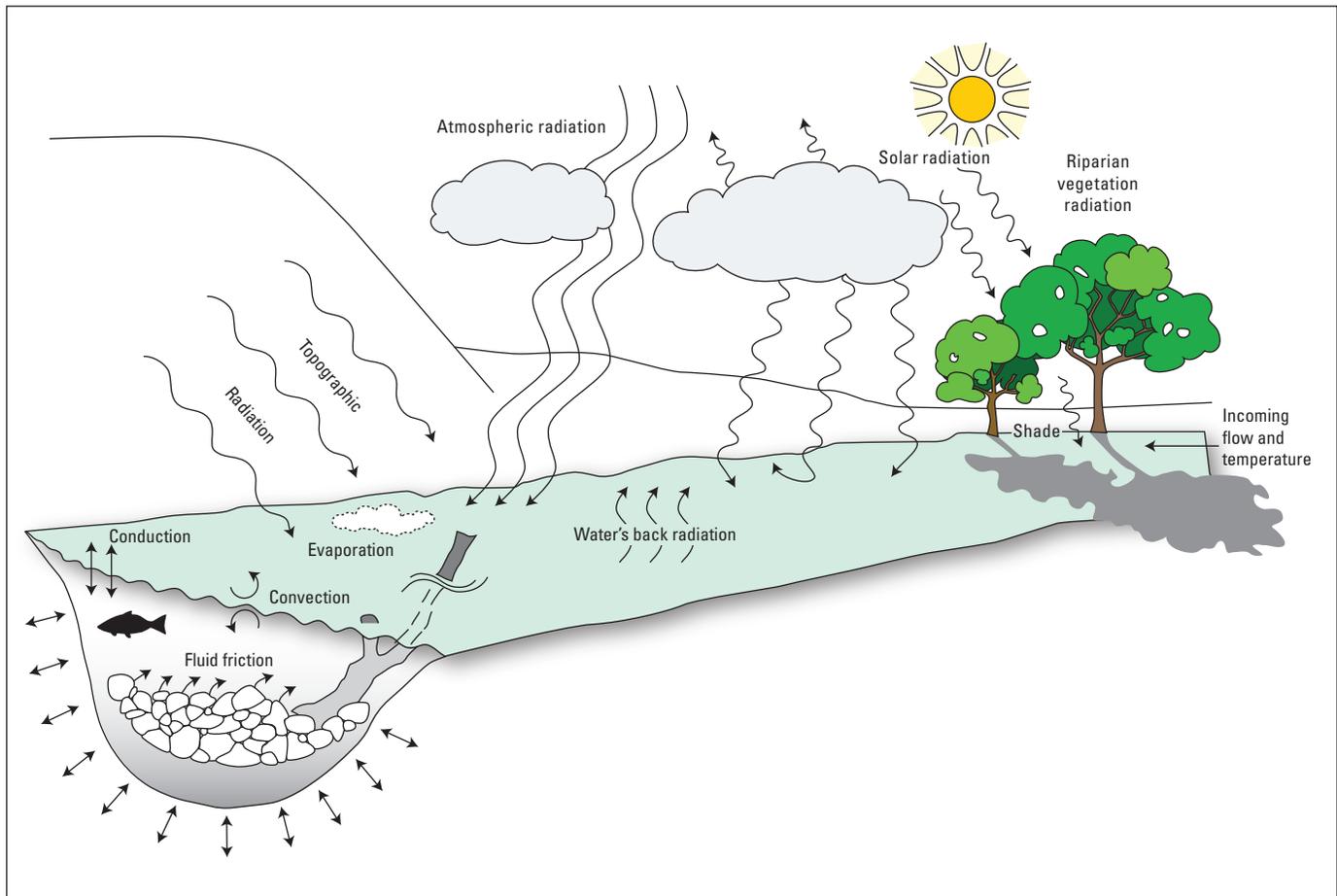
(from equation II(77); Theurer and others, 1984)

where

- $H_n$  is the net heat flux into the stream,
- $H_a$  is the flux from atmospheric-emitted longwave radiation,
- $H_v$  is the longwave radiation emitted by riparian vegetation,
- $H_w$  is the longwave radiation emitted by the water,
- $H_s$  is the net flux from shortwave solar radiation,
- $H_e$  is the evaporation heat flux from the latent heat of vaporization,
- $H_c$  is the conductive and convective heat transfer at the air–water interface,
- $H_d$  is the heat flux from conduction at the water–streambed interface, and
- $H_f$  is the heat dissipated from potential energy by friction.

All terms in equation 1 are in units of watts per square meter ( $\text{W}/\text{m}^2$ ).

Each energy flux term in equation 1 can be an important factor for the energy balance of a stream channel, though their relative importance is often site specific (Webb and others, 2008). The `stream_temp` module computes values for each of these terms for every stream segment for every day of the



**Figure 1.** Schematic design of the possible energy fluxes at the interfaces of a stream channel (from Bovee and others, 1998, fig. 4–9).

simulation period. Some computational methods are adapted directly from the original SNTMP source code (Theurer and others, 1984), whereas most important meteorological values, such as potential evaporation (PET), air temperature, and clear-sky shortwave solar radiation, are computed by other PRMS modules. The meteorological values are aggregated for use in `stream_temp` computations as the area-weighted average value for all hydrologic response units (HRUs) that contribute flow to a stream segment. Relative humidity must be specified. See appendix 1 of Markstrom and others (2015) for a description of the module selection options for PET, air temperature, and solar radiation. The computations for each of the energy-flux terms on the right-hand side of equation 1 are discussed below.

### Atmospheric Longwave Radiation ( $H_a$ )

The Earth's atmosphere scatters incoming solar radiation and consequently emits longwave radiation according to the atmospheric temperature and composition. Daily average atmospheric longwave radiation in watts per square meter is computed according to

$$H_a = (1 - r_i)(1 - Sh)(1 + 0.17C_i^2) \varepsilon_a \sigma (T_a + 273.16)^4, \quad (2)$$

(from equation II(61); Theurer and others, 1984)  
where

- $r_i$  is the longwave reflection fraction (assumed to be 0.0), dimensionless;
- $Sh$  is the shade fraction (PRMS parameters `seg_shade_sum` and `seg_shade_win`), dimensionless;
- $C_i$  is the decimal cloud fraction (PRMS variable `cloud_cover_hru`), dimensionless;
- $\varepsilon_a$  is the emissivity of the air, dimensionless;
- $\sigma$  is the Stefan-Boltzmann constant equal to  $5.670373E-08$ , in Joules (J) per second per meter squared per Kelvin to the fourth power ( $\frac{J}{s \times m^2 \times K^4}$ ); and
- $T_a$  is the air temperature (variable `tavgc`), in degrees Celsius.

The emissivity of air depends on relative humidity according to

$$\varepsilon_a = 0.61 + 0.05\sqrt{e_a}, \quad (3)$$

(from equation II(62); Theurer and others, 1984)  
where

- $e_a$  is the vapor pressure at the current air temperature (variable `tavgc`) and humidity (variable `humidity_hru`), in millibars (mb).

The decimal cloud fraction,  $C_i$ , represents the portion of daily solar radiation scattered by cloud cover and is computed on the basis of a linear function of the diurnal fluctuation in air temperature. This linear function is parameterized with a slope and intercept value for each month of the year (see eq. 1–47, appendix 1 in Markstrom and others, 2015).

### Riparian Vegetative Longwave Radiation ( $H_v$ )

Incoming shortwave solar radiation is often intercepted by surrounding riparian vegetation. However, the vegetation will emit some longwave radiation as a black body. Daily average longwave radiation of the vegetation, in  $W/m^2$ , is computed as a simple formulation of the Stefan-Boltzmann law according to

$$H_v = \varepsilon_v \sigma Sh_v (T_a + 273.16)^4, \quad (4)$$

(from equation II(66); Theurer and others, 1984)  
where

- $\varepsilon_v$  is the emissivity of the vegetation, dimensionless; and
- $Sh_v$  is the shade fraction from riparian vegetation, dimensionless.

The emissivity of riparian vegetation is a decimal fraction and assumed to have a value of 0.9526 (Theurer and others, 1984).  $\sigma$  and  $T_a$  are the same quantities as defined in equation 2.

### Water-Emitted Longwave Radiation ( $H_w$ )

The stream water itself emits longwave radiation as a black body and represents a negative flux or energy leaving the stream, depending on the water temperature. Daily average longwave radiation of the water, in watts per square meter, is computed as a simple formulation of the Stefan-Boltzmann law:

$$H_w = \varepsilon_w \sigma (T_w + 273.16)^4, \quad (5)$$

(from equation II(67); Theurer and others, 1984)  
where

- $\varepsilon_w$  is the emissivity of water, dimensionless, which is assumed to have a value of 0.9526 (Theurer and others, 1984); and
- $T_w$  is the water temperature (variable `temp_avg`), in degrees Celsius.

### Shortwave Solar Radiation ( $H_s$ )

Daily average clear-sky, shortwave solar radiation is calculated for each stream segment in the PRMS Solar Table module (see pages 78–79 in Markstrom and others, 2015), but not all of this solar radiation reaches or is absorbed by the stream. Shortwave solar radiation absorbed by the stream segment, in  $W/m^2$ , is computed according to

$$H_s = (1 - \alpha)(1 - Sh) H_{sw}, \quad (6)$$

(from equations II(28) and II(60); Theurer and others, 1984)  
where

- $\alpha$  is the albedo or fraction reflected by the stream (parameter `albedo`), dimensionless; and
- $H_{sw}$  is the clear sky solar radiation (variable `seginc_swrad`), in watts per square meter.

## Evaporation Heat Flux ( $H_e$ )

A large amount of energy is required for the phase change of water from the liquid to the gas phase. This energy is stored in the vapor as a result of the relative energy in that phase and is not used to increase the water temperature. This energy leaves the stream channel through the process of stream-water evaporation and is a negative energy flux. Daily average PET rate is calculated by the selected PRMS PET module (see pages 90–93 in Markstrom and others, 2015) and applied directly to the free-water surface of the stream segments in the `stream_temp` module. The latent heat of vaporization ( $\lambda$ ) is temperature dependent and can be approximated in the normal range of stream-water temperatures in units of joules per kilogram (kg) using

$$\lambda = 2495 \cdot 10^3 - 2360T_w, \quad (7)$$

(from Dingman, 2008). In order to get the mass flux, the latent heat of vaporization is multiplied by the density of water, which is 1,000 kg per cubic meter. The daily average evaporation heat flux, in watts per square meter, is then computed according to

$$H_e = \lambda E \cdot 10^3, \quad (8)$$

where

$E$  is the free-water surface-evaporation rate (assumed to be the PET rate, variable *potet*), in meters per second.

## Conductive and Convective Heat Flux ( $H_c$ )

At the air–water interface of the stream, heat is exchanged by means of air circulation as convective heat flux ( $H_c$ ). This flux is related to the evaporation heat flux through a dimensionless quantity called the Bowen Ratio ( $Bo$ ), which is defined as follows:

$$H_c = BoH_e, \quad (9)$$

$$Bo = B_c (T_w - T_a), \quad (10)$$

$$B_c = \frac{0.00061 \cdot P}{(e_s - e_a)}, \quad (11)$$

(from equations II(72) and II(71); Theurer and others, 1984)

where

$B_c$  is a dimensionless quantity defined by equation 11 and called the Bowen Coefficient;

$P$  is the atmospheric pressure (based on parameter `hru_elev`), in millibars;

$e_s$  is the saturation vapor pressure at the current air temperature ( $T_a$ ), in millibars;  
and

$e_a$  is the vapor pressure at the current air temperature, in millibars.

By combining equations 7, 8, 9, 10, and 11, the convective heat flux, in W/m<sup>2</sup>, is computed according to

$$H_c = 10^6 EB_c \left( -2.36T_w^2 + (2495 + 2.36T_a)T_w - 2495T_a \right), \quad (12)$$

A positive value of  $H_c$  indicates that the stream water is warmer than the air, and energy is leaving the stream. Thus, it is designated as a negative heat flux. Computation of this value depends on the water temperature,  $T_w$ .

## Streambed Conduction ( $H_d$ )

Conduction can occur at the streambed if a thermal gradient exists. This depends on the conductivity of the streambed material. The energy flux across the streambed resulting from conduction, in watts per square meter, is computed according to

$$H_d = \frac{K_g (T_g - T_w)}{\Delta Z}, \quad (13)$$

(from equation II(75); Theurer and others, 1984)

where

- $K_g$  is the thermal conductivity of the streambed (assumed to be 1.65), in watts per square meter per degrees Celsius;
- $T_g$  is the ground temperature (assumed to be the groundwater temperature), in degrees Celsius; and
- $\Delta Z$  is the equilibrium depth, in meters, from the water–streambed interface at which the temperature is  $T_g$  (assumed to be 1.0).

Conduction is defined as a positive heat flux entering the stream. This quantity also depends upon the water temperature,  $T_w$ .

## Stream Friction ( $H_f$ )

Friction can occur in streams as either internal fluid shear or as work done on the streambed boundaries. This friction converts kinetic energy from the water into heat and warms the stream. Heat flux from friction, in watts per square meter, is computed according to

$$H_f = 9805 \frac{QS_f}{W}, \quad (14)$$

(from equation II(76); Theurer and others, 1984)

where

- $S_f$  is the stream gradient (parameter **seg\_slope**), dimensionless;
- $Q$  is the discharge (variable *seg\_inflow*), in cubic meters per second (m<sup>3</sup>/s); and
- $W$  is the average top width of the stream segment (interpolated from parameters **width\_flow** and **width\_values**), in meters.

## Energy Balance in Terms of Stream-Water Temperature ( $T_w$ )

Examination of equations 5, 7, 12, and 13 indicates that the computation of the water-emitted longwave radiation ( $H_w$ ), evaporation heat flux ( $H_e$ ), conductive and convective heat flux ( $H_c$ ), and streambed conduction ( $H_d$ ) terms all depend on the temperature of the stream water,  $T_w$ . These terms can be summed for a total energy balance, in watts per square meter, as a function of water temperature. The resulting energy balance equation depends on the water temperature in degrees Kelvin raised to the fourth power and the water temperature in degrees Celsius squared from the conductive and convective heat flux term, as well as a few linear and constant terms with respect to the water temperature,  $T_w$ . Thus, equation 1 can be rewritten as a polynomial function of water temperature as follows:

$$-H_n = A(T_w + 273.16)^4 - CT_w^2 + BT_w - D, \quad (15)$$

where

$$A = 5.40 \cdot 10^{-8}, \quad (15a)$$

$$B = 10^6 E (B_c (2495 + 2.36T_a) - 2.36) + K_g / \Delta Z, \quad (15b)$$

$$C = 10^6 \cdot E \cdot B_c \cdot 2.36, \text{ and} \quad (15c)$$

$$D = H_a + H_f + H_s + H_v + 2495 \cdot E (B_c T_a - 1) + T_g \cdot K_g / \Delta Z. \quad (15d)$$

Equation 15 frames the energy balance in terms of one unknown value ( $T_w$ ). Because the equilibrium temperature ( $T_e$ ) is defined as the water temperature at which the energy flux is zero, it can be found by using a Newton's Method iteration (Hazewinkel, 2001) to calculate the root of equation 15 ( $H_n = 0$ ).

## Solar-Shade Computation

The total solar shade fraction is computed using the method described by Theurer and others (1984) and is based on the topographic and riparian vegetation shading. The fraction of shortwave radiation intercepted as a result of topographic and vegetation shading is computed for each stream segment. The total shade fraction is computed according to

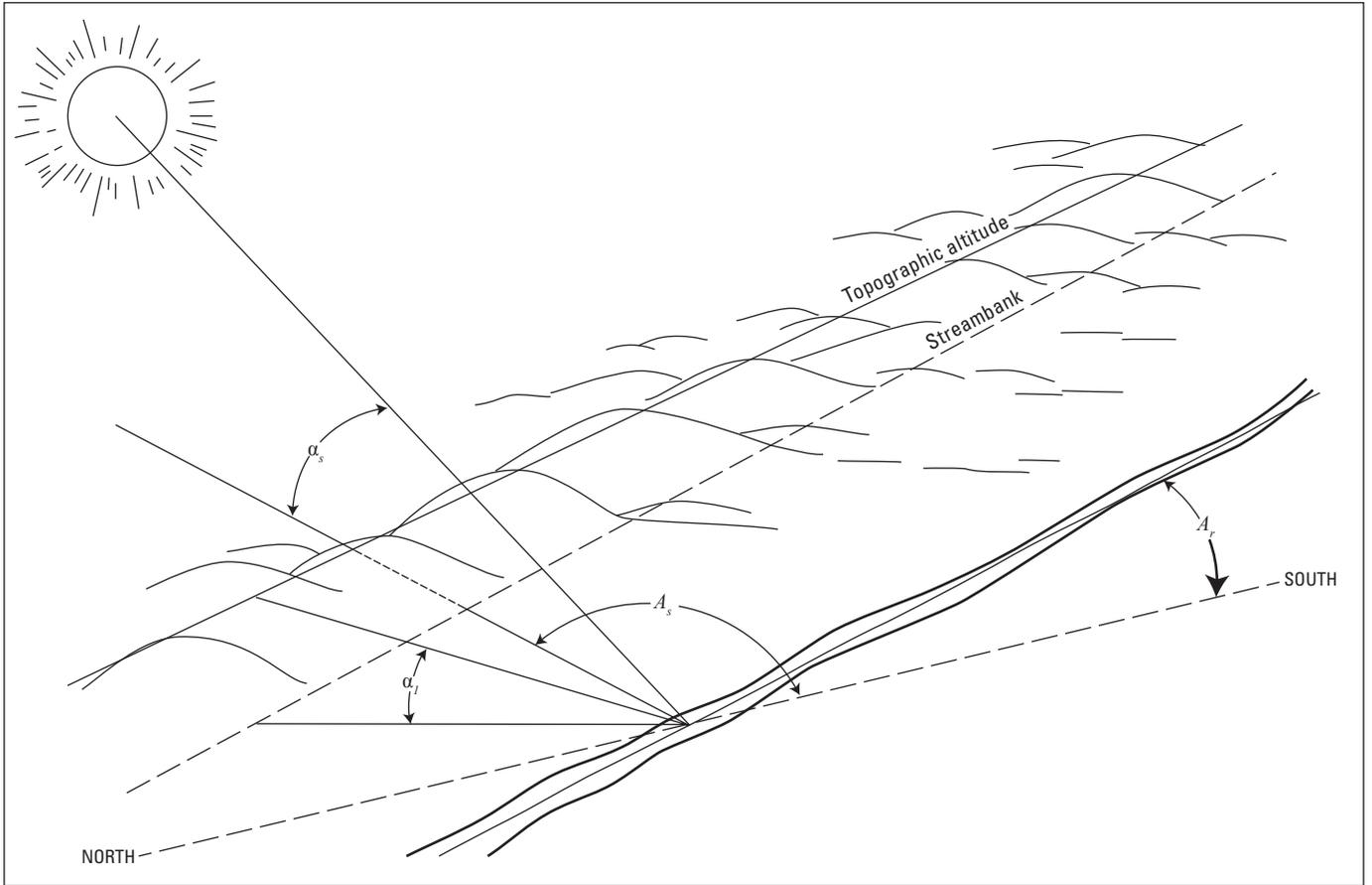
$$Sh = Sh_t + Sh_v, \quad (16)$$

(from equation II(52); Theurer and others, 1984)  
where

- $Sh$  is the total solar shade fraction,
- $Sh_t$  is the topographic shade fraction, and
- $Sh_v$  is the riparian vegetation shade fraction.

The `stream_temp` module allows the user to specify values for either the total shade fraction for each half of the year (control parameter `stream_temp_shade_flag = 1`) or to compute shade fraction values for each day of the year (control parameter `stream_temp_shade_flag = 0`). The computation of solar shade requires specification of information related to the topography and vegetation for each segment and is based on the solar azimuth angle (parameter `azrh`) for each day of the year for each segment. The option to specify summer and winter solar shade fractions is provided for models where the topography and land cover are not well known; the fractions can be estimated from solar radiation measurements or when the shade fractions are needed for calibration purposes. This computation option can be used whenever good topography and vegetation information are available and for applications where the solar shading is highly variable. In watersheds at high latitudes, where the solar azimuth angle changes considerably over the course of the year, which affects the fraction of shade over time, the daily computed shade option can be used to account for these changes.

Topographic shade occurs when topography blocks direct sunlight from the stream segment for a portion of the day following sunrise and (or) preceding sunset. This generally occurs at streams flowing through canyons or hilly terrain. Topographic shade effectively changes the sunrise and sunset time at the location of the stream segment. Shade is modeled as a function of the time of year, latitude, stream segment azimuth angle, and topographic altitude angle for each stream segment. Latitude and time of year determine the daily path and timing of the position of the sun in the sky. The stream segment azimuth angle and topographic altitude angle are used to calculate the portions of the sun path that can be blocked by the surrounding topography. The segment azimuth angle,  $A_p$ , is the angle in the horizontal plane that the stream segment makes from the north-south line (fig. 2). This angle is defined facing south along the north-south line for streams in the northern hemisphere. Values of the azimuth angle, in radians, range from  $-\pi/2$  to  $\pi/2$  (equivalent to  $-90$  degrees to  $90$  degrees). Thus, streams oriented north-south, northeast-southwest, and northwest-southeast would have azimuth angles equal to 0, less than 0, and greater than 0, respectively. The azimuth angle also defines the east bank and west bank of a stream segment, where the east bank is always on the left hand side facing southward. The topographic altitude angle,  $\alpha_p$ , is the vertical angle from the horizontal line perpendicular to the streambank to the top of the topographic features along a segment. A depiction of these angles along with the solar altitude and azimuth angles is shown in figure 2. The topographic altitude can be specified for each of the east and west banks as defined by the segment azimuth angle.



**Figure 2.** Schematic design showing altitude and azimuth angles in the topographic shade calculation.  $A_s$  and  $\alpha_s$  are the current solar azimuth and altitude, respectively.  $A_r$  and  $\alpha_l$  are the segment azimuth and topographic altitude angles, respectively. Modified from Theurer and others (1984).

The daily topographic shade fraction is computed by subtracting the portion of sunlight between the local sunrise and sunset from the total amount of sunlight between sunrise and sunset for a horizontal plane at the specified segment latitude and day of the year. The local sunlight time period is less than or equal to the total possible sunlight resulting from the topography blocking light that would otherwise hit a segment if the surrounding area were completely flat (i.e., a horizontal plane). This topographic shade fraction ( $Sh_t$ ) is computed by relating the integrals of solar altitude,  $\alpha_s$ , for the local and horizontal plane according to

$$Sh_t = 1 - \left( \left[ \int_{h_{sr}}^{h_{ss}} \sin \alpha_s dh \right] / \left[ \int_{-h_s}^{h_s} \sin \alpha_s dh \right] \right), \quad (17)$$

(from equation II(41); Theurer and others, 1984),

where

- $h_{sr}$  is the local segment sunrise hour angle, in radians;
- $h_{ss}$  is the local segment sunset hour angle, in radians;
- $-h_s$  is the horizontal plane sunrise angle, in radians;
- $h_s$  is the horizontal plane sunset angle, in radians, and
- $\alpha_s$  is the solar altitude as a function of date and time, in radians.

The fraction of the sunlight reaching the stream segment is subtracted from the total potential sunlight in the horizontal plane to yield the fraction of sunlight blocked by topography. Hour angles are defined using zero as solar noon, or midday, which is the zenith of the sun's arc on

a day. One hour of time corresponds to  $\pi/12$  radians hour angle. Equation 17 can be directly evaluated to yield

$$Sh_t = 1 - \frac{[(h_{ss} - h_{sr})(\sin\phi\sin\delta)] + [(\sin h_{ss} - \sin h_{sr})(\cos\phi\cos\delta)]}{2[(h_s \sin\phi\sin\delta) + (\sin h_s \cos\phi\cos\delta)]}, \quad (18)$$

(from equation II(42); Theurer and others, 1984)

where

$\phi$  is the segment latitude (from parameter **hru\_lat**), in radians; and  
 $\delta$  is the declination of the sun, in radians.

Solar declination is computed according to

$$\delta = 0.40928 \cos \left[ (2\pi / 365)(172 - D_j) \right], \quad (19)$$

(from equation II(29); Theurer and others, 1984)

where

$D_j$  is the Julian day of the time step.

In order to compute the topographic shade fraction, the local sunrise and sunset hour angles, as well as the horizontal plane hour angle, are required. The horizontal plane hour angle, in radians, is computed according to

$$h_s = \arccos \left[ -(\sin\phi\sin\delta) / (\cos\phi\cos\delta) \right], \quad (20)$$

(from equation II(30); Theurer and others, 1984).

Because the zero hour angle is designated as solar noon, the horizontal plane hour angle has the same magnitude for sunrise and sunset with the opposite sign. The local sunrise and sunset hour angles, in radians, are computed according to

$$h_{sr} = -\arccos \left[ \sin\alpha_{sr} - (\sin\phi\sin\delta) / (\cos\phi\cos\delta) \right], \quad (21)$$

(from equation II(33); Theurer and others, 1984)

where

$\alpha_{sr}$  is the local sunrise solar altitude, in radians.

The sunset hour angle can be found by substituting the sunset solar altitude into equation 21 for the sunrise altitude and by changing the sign so the solution is positive. The sunset or sunrise solar altitude can be computed by numerically solving a system of two equations with two unknowns. This system is given by

$$\alpha_{sr} = \arctan \left[ (\tan\alpha_t) \sin(A_{sr} - A_r) \right], \text{ and} \quad (22)$$

$$A_{sr} = -\arccos \left( \frac{[(\sin\phi\sin\alpha_{sr}) - (\sin\delta)]}{[\cos\phi\cos\alpha_{sr}]} \right), \quad (23)$$

(from equations II(34) and II(35); Theurer and others, 1984)

where

$A_{sr}$  is the sunrise solar azimuth angle, in radians;  
 $A_r$  is the segment azimuth angle (fig. 2), in radians;  
 $A_s$  is the solar azimuth angle (fig. 2), in radians; and  
 $\alpha_t$  is the topographic altitude on the sunrise-side bank (parameters **alte** and **altw**), in radians.

The value of  $\alpha_t$  is determined to be the east bank unless  $A_r$  is negative and less than the horizontal plane sunrise solar azimuth. For this case, the sun rises to the west (or more likely south) side of the stream segment. The numerical solution for  $\alpha_{sr}$  is constrained such that it has a positive value and cannot go above the maximum solar altitude for a given day. This is achieved by ensuring that

$$\sin\alpha_{sr} \leq (\sin\phi\sin\delta) + (\cos\phi\cos\delta), \quad (24)$$

which is the condition for the maximum solar altitude.

This same system of equations is applied to compute the sunset hour angle,  $h_{ss}$ , with a sign change in equation 23 to compute the sunset solar azimuth,  $A_{ss}$ . In the sunset computations,  $\alpha_i$  is determined to be the west bank, except in the case that the stream segment has a higher positive azimuth angle than the horizontal plane solar sunset azimuth angle for the day. Once the local sunrise and sunset hour angles, as well as the horizontal plane hour angle, have been computed for a segment, the topographic shade factor can be computed from equation 18.

Once topographic shade is computed, the amount of shade cast by riparian vegetation between local sunrise and sunset can be computed. The riparian vegetation shade is a function of the length of shadows cast on the water, the total width of the stream, and the density of shadow-casting riparian vegetation along the bank of the stream segment. Again, the shade fraction is a measure of the shadows cast over some width and percentage of the streambank as a fraction of the total potential radiation into the stream segment. The riparian shade fraction can be expressed as

$$Sh_v = \left[ \int_{h_{sr}}^{h_{ss}} (V_d W_s \sin\alpha_s) dh \right] / \left[ \int_{-h_s}^{h_s} (W \sin\alpha_s) dh \right], \quad (25)$$

(from equation II(50); Theurer and others, 1984)

where

- $V_d$  is the vegetation density along the bank (parameters **vde**, **vdw**, **vde\_win**, and **vdw\_win**), dimensionless; and
- $W_s$  is the length of the shadows cast by riparian vegetation at a given solar altitude  $\alpha_s$ , in meters.

Equation 25 is numerically integrated for each stream segment using Gaussian quadrature.  $W_s$  is constrained to be greater than zero and less than or equal to the width of the stream segment  $W$ . This instantaneous shadow length can be computed according to

$$W_s = \left[ (V_h \cot\alpha) \sin(|A_s - A_i|) \right] + \left[ \frac{V_c}{2} - V_o \right], \quad (26)$$

(from equation II(49); Theurer and others, 1984)

where

- $V_h$  is the vegetation height (parameters **vhe** and **vhw**), in meters;
- $V_c$  is the vegetation crown width (parameters **vce** and **vcw**), in meters; and
- $V_o$  is the offset of the riparian vegetation from the bank (parameters **voe** and **vow**), in meters.

The vegetative shade calculation also allows for different parameter values ( $V_h$ ,  $V_c$ , and  $V_o$ ) for the east and west bank. Just like the topographic shade calculation, the sunrise (before solar noon) bank parameter values are used in equations 25 and 26, then the sunset-side parameters are used with these equations. Finally, the vegetative shade factor from equation 25 is added to the topographic shade factor from equation 18 to compute the portion of potential sunlight blocked by the vegetation.

## Average Stream-Temperature Computation

For the average temperature computation, steady-state streamflow throughout the day in a roughly prismatic (constant average width/depth) stream channel was assumed. These conditions allow use of a steady-state heat flux equation. In the steady-state condition, all energy enters and leaves the segment by either advection or heat flux. The equation for change in temperature along the segment is expressed by

$$dT_w / dx = \left[ (q_l / Q)(T_l - T_w) \right] + \left[ (WH_n) / (Q\rho C_p) \right], \quad (27)$$

(from equation II(87); Theurer and others, 1984)

where

- $x$  is the downstream distance, in meters;
- $q_l$  is the lateral flow per unit length along the segment (based on variable *seg\_inflow*), in square meters per second;
- $Q$  is the total discharge at the segment outlet (variable *seg\_outflow*), in cubic meters per second;
- $T_l$  is the temperature of the lateral flow (computed as described below), in degrees Celsius;
- $T_w$  is the temperature of the water in the channel (variable *temp\_ave*), in degrees Celsius;
- $W$  is the average top width of the channel (variable *seg\_width*), in meters;
- $H_n$  is the net heat flux into the channel, in watts per square meter;
- $\rho$  is the density of water, in kilograms per cubic meter; and
- $C_p$  is the specific heat of water, Joules per kilogram per degrees Celsius.

The first bracketed term in equation 27 is the temperature change resulting from lateral flows along the stream segment. These lateral flows are the primary hydrologic connection to the watershed that affects the stream-water temperature. Lateral flow water temperatures are calculated as the flow-weighted average of surface runoff (variable *seginc\_sroff*), subsurface (variable *seginc\_ssflow*) and groundwater (variable *seginc\_gwflow*) flow components, and temperatures for all HRUs that contribute flow to a stream segment. Temperatures of the lateral flow components are assigned as running averages of air temperature on the basis of their respective

residence times. Groundwater inflow temperatures default to a running yearly average air temperature; however, a different residence time can be specified (parameter **gw\_tau**). The residence time over which the running average is computed for subsurface interflow has a default of 30 days; likewise, a different value can be specified (parameter **ss\_tau**). Residence time for direct surface runoff is always 1 day. Equation 27 has three conditions based on the amount of lateral flows. In a losing stream (which never occurs in PRMS), where  $q_l < 0$ ,  $T_l$  is equal to  $T_w$ , and the first term of equation 27 is zero. Similarly,  $q_l$  is equal to zero in the case of zero lateral flow. The second bracketed term tracks the temperature change in a water column resulting from net heat exchange across the boundaries of the stream. In its entirety, equation 27 represents the temperature change in a column of water as it moves down the channel.  $H_n$  has a fourth-order dependence on water temperature (eq. 15), making it difficult to solve directly, so a solution based on an equilibrium temperature concept is used as described in Edinger and others (1968).

The equilibrium temperature of each segment, as well as the thermal exchange coefficients, is calculated from meteorological conditions. The modified second-order Taylor expansion between  $T_e$  (equilibrium water temperature) and the initial water temperature,  $T_o$  yields a solvable approximation for the net heat flux of a stream segment. The full derivation of the expansion is given in appendix 1. This expansion relies on thermal exchange coefficients, which relate the difference between the actual ( $T_w$ ) and equilibrium ( $T_e$ ) water temperatures to the net heat flux across the boundaries of a stream segment, as computed according to

$$H_n = K_1(T_e - T_w) + K_2(T_e - T_w)^2, \quad (28)$$

(from equation II(83); Theurer and others, 1984)

where

$K_1$  and  $K_2$  are the first- and second-order thermal exchange coefficients computed for each stream segment, respectively.

The first-order thermal exchange coefficient is the first derivative of energy balance equation 15 evaluated at the equilibrium water temperature,  $T_e$ , as computed according to

$$K_1 = 4A(T_e + 273.16)^3 - 2CT_e + B, \quad (29)$$

where  $A$ ,  $B$ , and  $C$  are the constants from equation 15. The second-order coefficient  $K_2$  calculates the actual heat flux at the initial water temperature,  $T_o$ , of the upstream inflow and corrects the first-order Taylor series expansion about  $T_e$ , giving an approximation of the heat-flux function between the two temperatures as computed according to

$$K_2 = (H_i - [K_1(T_e - T_o)]) / [(T_e - T_o)^2], \quad (30)$$

(adapted from equation II(81); Theurer and others, 1984)

where  $H_i$  is the initial net heat flux at temperature  $T_o$  found by evaluating equation 15 at this initial temperature.

The relation to the equilibrium temperature given by equation 28 can be substituted into equation 27 to create an ordinary differential equation given by

$$dT_w / dx = \left[ (q_l / Q)(T_l - T_w) \right] + \left[ W \left( K_1(T_e - T_w) + K_2(T_e - T_w)^2 \right) / (Q\rho C_p) \right], \quad (31)$$

which is the second-order Taylor series expansion. The second-order solution to this equation used in the `stream_temp` module was developed by Theurer and others (1984). The solution to equation 31 has three different cases for gaining, losing, and zero lateral flow stream segments. It is presented this way because the first term goes to zero for some flow conditions and because  $Q$  varies in terms of initial streamflow ( $Q_o$ ) and lateral flow ( $q_l$ ) between these cases. The general form is given by

$$T_w = T_e' - \frac{(T_e' - T_o)R}{1 + (K_2 / K_1)(T_e' - T_o)(1 - R)}, \quad (32)$$

(from equation II(104); Theurer and others, 1984)  
 where  $R$  is a characteristic temperature-independent constant that has different values depending on whether the segment is gaining or losing and (or) has incoming lateral flow. The effective equilibrium temperature defined by the solution,  $T_e'$ , is directly dependent on  $T_e$  in all cases but also varies from case to case. A first-order solution is used when lateral flow enters a stream segment and a second-order solution when there is no lateral flow. The first- and second-order solutions can be directly solved. For the conditions with lateral flow, the first-order solution is adjusted on the basis of the second-order zero lateral flow solution. So given

$$a = (q_l T_l) + [(K_1 W) / (\rho C_p)] T_e, \text{ and} \quad (33)$$

$$b = q_l + (K_1 W) / (\rho C_p), \quad (34)$$

(from equations II(105) and II(106); Theurer and others, 1984)  
 for case one of a gaining stream with positive  $q_p$ , the effective equilibrium temperature and characteristic temperature-independent constant are calculated according to

$$T_e' = a/b, \text{ and} \quad (35)$$

$$R = [1 + (q_l L / Q_o)]^{(-b/q_l)}, \quad (36)$$

(from equations II(107) and II(108); Theurer and others, 1984)  
 where

$Q_o$  is the initial channel flow from upstream (variable *seg\_upstream\_inflow*), in cubic meters per second; and

$L$  is the length of the segment (parameter **seg\_length**), in meters.

For case two, losing stream with negative  $q_p$ ,

$$T_e' = T_e, \text{ and} \quad (37)$$

$$R = [1 + (q_l L / Q_o)]^{[(q_l - b)/q_l]}, \quad (38)$$

(from equations II(109) and II(110); Theurer and others, 1984).

For case three with zero lateral flow,

$$T_e' = T_e, \text{ and} \quad (39)$$

$$R = \exp[-(bL) / Q_o], \quad (40)$$

(from equations II(111) and II(112); Theurer and others, 1984).

This solution approach accounts for the heat flux by perturbing from the equilibrium temperature where heat flux is zero. It also adjusts for conditions in which lateral flow affects the temperature of a stream segment. The thermal exchange coefficients allow for the equilibrium temperature relation to be calibrated to the specific energy exchange conditions of each individual stream segment. Owing to the steady-state flow and thermal exchange assumptions, this solution method is not appropriate for the prediction of diurnal fluctuations or operation on a shorter than daily time step. It also is not well suited for prediction of temperatures in stream reaches with highly heterogeneous hydraulic properties because prismatic reaches are assumed. Stream channels should be split into discrete segments where there are substantial changes in hydraulic properties, shading, or lateral flow. Additionally, all segments need to have a travel time between 7 and 24 hours.

## Daily Mean Stream Temperature Module Operation in the Precipitation-Runoff Modeling System

Computation of stream-water temperatures using the `stream_temp` module within a PRMS simulation is optional. The module is activated using a control parameter specified in the Control File and requires several input parameters specified in the Parameter File. See appendix 1 of Markstrom and others (2015) for a description of the Control and Parameter Files. Knowledge of representative ranges for input parameters and climatic and streamflow data for a given site are needed to obtain representative stream temperature simulations.

### Parameters Specified in the Control File

The `stream_temp` module is activated by specifying the **stream\_temp\_flag** control parameter equal to 1. Humidity (converted to vapor pressure by the model) is important information for the calculation of the convective and atmospheric longwave heat fluxes. Daily humidity values are required; however, humidity can be specified in the Parameter File as a constant value that is used for all time steps and segments. Time-series relative humidity values can be input in Climate-by-HRU (CBH) Files, and their use is selected by specifying control parameter **humidity\_cbh\_flag** equal to 1. The CBH File contains daily, pre-distributed relative humidity values for each HRU. See Markstrom and others (2015) for a description of CBH Files. Other meteorological data, such as air temperature, PET, and solar radiation, needed in `stream_temp` calculations can be input in the Data File and distributed by HRU using climate distribution modules within PRMS, or the data may be specified using CBH Files. Control parameters for the stream-temperature module are described in table 1.

**Table 1.** Description of stream temperature module control parameters.

[HRU, hydrologic response unit]

Parameter name	Description	Number of values	Data type	Default
<b>humidity_cbh_flag</b>	Flag to indicate whether humidity is input using a Climate-by-HRU file (0=no; 1=yes)	1	Integer	0
<b>humidity_day</b>	Pathname of the Climate-by-HRU file of pre-processed relative humidity input data	1	Character	humidity.day
<b>strmflow_module</b>	Module name for streamflow routing simulation method (must be muskingum, strmflow_in_out, or strmflow_lake)	1	Character	strmflow_in_out,
<b>stream_temp_flag</b>	Flag to indicate whether stream temperature module is active (0=no; 1=yes)	1	Integer	0
<b>stream_temp_shade_flag</b>	Flag to indicate total shade computation (0=computed; 1=user-specified)	1	Integer	0

## Dimensions and Parameters Specified in the Parameter File

Dimensions and parameters required for input to the stream temperature module are described in table 2. Most parameter values are specified for each stream segment and thus are one-dimensional arrays with dimension **nsegment**. The parameter **width\_values** is a two-dimensional array of dimensions **nsegment** and **width\_dim**, where **width\_dim** is the number of channel widths in the width to flow rating table as specified using parameters **width\_flow** and **width\_values**. Parameter **width\_flow** specifies three values—the minimum and maximum flow rate (the range) and the flow interval used to divide the flow range into equal intervals. These three values are used for all segments. The values of **width\_flow** are in units of cubic meters per second ( $\text{m}^3/\text{s}$ ). The flow intervals are determined from the range of flow divided by **width\_dim**. Specifying the three values of **width\_flow** as 0, 200, and 100 and **width\_dim** equal to 2 means that the rating table for each segment has the minimum flow rate equal to 0  $\text{m}^3/\text{s}$ , maximum flow rate equal to 200  $\text{m}^3/\text{s}$ , and flow-rate interval equal to 100  $\text{m}^3/\text{s}$ . For this case, **width\_values** specifies two stream top widths. One corresponds to the segment width for flow rates from 0 to 100  $\text{m}^3/\text{s}$ , and the second is the segment width for flow rates from 100 to 200  $\text{m}^3/\text{s}$ , inclusive. If a flow rate in any segment exceeds the maximum value, it is set to the last value specified for **width\_values**. This rating table approach allows the user to specify different stream top widths for each segment to account for changes in width at different flow rates, including changes in top width when overbank flow occurs.

## Variables

All declared module variables are listed in table 3. Most are area-weighted average values of variables computed in other modules from the contributing (and therefore connecting) HRUs, such as air temperature, PET, snowmelt, precipitation, latitude, and elevation. Others are instream variables, such as top width, flow rate, and mean and maximum water temperature.

**Table 2.** Description of dimensions and parameters specified in the Parameter File for the Stream Temperature module -stream\_temp.

[HRU, hydrologic response unit; m<sup>3</sup>/s, cubic meters per second; elev\_units, 0=feet, 1=meters]

Parameter name	Description	Dimension	Units	Data type	Range	Default
Dimensions						
<b>nhru</b>	Number of HRUs	<b>one</b>	None	Integer	user defined	1
<b>nsegment</b>	Number of stream-channel segments	<b>one</b>	None	Integer	user defined	0
<b>one</b>	The constant one	<b>one</b>	None	Integer	fixed	1
<b>three</b>	The constant three	<b>one</b>	None	Integer	fixed	3
<b>width_dim</b>	Number of stream-channel width intervals of segment outflow	<b>one</b>	None	Integer	user defined	1
Parameters						
<b>albedo</b>	Shortwave solar radiation reflected by the stream	<b>one</b>	Fraction	Real	0 to 1.0	0.1
<b>alte</b>	East bank topographic altitude	<b>nsegment</b>	Radians	Real	0.0 to 1.5708	0.0
<b>altw</b>	West bank topographic altitude	<b>nsegment</b>	Radians	Real	0.0 to 1.5708	0.0
<b>azrh</b>	Azimuth angle for each segment	<b>nsegment</b>	Radians	Real	-1.5708 to 1.5708	0.0
<b>gw_init</b>	Initial temperature of groundwater entering each segment; recommended value equal to long-term average air temperature	<b>nsegment</b>	Degrees Celsius	Real	0.0 to 45.0	15.0
<b>gw_tau</b>	Average residence time for groundwater flow	<b>one</b>	Days	Integer	10 to 365	365
<b>hru_elev</b>	Mean elevation for each HRU	<b>nhru</b>	<b>elev_units</b>	Integer	-1,000.0 to 30,000.0	0.0
<b>hru_lat</b>	Latitude of each HRU	<b>nhru</b>	Angular degrees	Real	-90.0 to 90.0	40.0
<b>seg_length</b>	Length of each segment	<b>nsegment</b>	meters	Real	1.0 to 1,000.0	1,000.0
<b>seg_shade_sum</b>	Total shade fraction; required when <b>stream_temp_shade_flag=1</b>	<b>nsegment</b>	Fraction	Real	0.0 to 1.0	0.0
<b>seg_shade_win</b>	Total winter shade fraction; required when <b>stream_temp_shade_flag=1</b>	<b>nsegment</b>	Fraction	Real	0.0 to 1.0	0.0
<b>seg_slope</b>	Bed slope of each segment	<b>nsegment</b>	Fraction	Real	0.0001 to 2.0	0.015
<b>ss_init</b>	Initial temperature of interflow entering each segment; recommended value equal to long-term average air temperature	<b>nsegment</b>	Degrees Celsius	Real	0.0 to 45.0	15.0
<b>ss_tau</b>	Average residence time for subsurface interflow	<b>one</b>	Days	Integer	0 to 300	30
<b>vce</b>	East bank average vegetation crown width for each segment	<b>nsegment</b>	Meters	Real	0.0 to user defined	0.0
<b>vcw</b>	West side average vegetation crown width for each segment	<b>nsegment</b>	Meters	Real	0.0 to user defined	0.0
<b>vde</b>	East bank vegetation density for each segment	<b>nsegment</b>	Fraction	Real	0.0 to 1.0	0.0
<b>vde_win</b>	East bank average winter vegetation density for each segment	<b>nsegment</b>	Fraction	Real	0.0 to 1.0	0.0
<b>vdw</b>	West bank average vegetation density for each segment	<b>nsegment</b>	Fraction	Real	0.0 to 1.0	0
<b>vdw_win</b>	West bank average winter vegetation density for each segment	<b>nsegment</b>	Fraction	Real	0.0 to 1.0	0.0
<b>vhe</b>	East bank average vegetation height for each segment	<b>nsegment</b>	Meters	Real	0.0 to user defined	0.0
<b>vhw</b>	West bank average vegetation height for each segment	<b>nsegment</b>	Meters	Real	0.0 to user defined	0.0
<b>voe</b>	East bank vegetation offset for each segment	<b>nsegment</b>	Meters	Real	0.0 to user defined	0.0
<b>vow</b>	West bank vegetation offset for each segment	<b>nsegment</b>	Meters	Real	0.0 to user defined	0.0
<b>width_flow</b>	Minimum flow, maximum flow, and flow interval for the width values array	<b>three</b>	m <sup>3</sup> /s	Real	0.0 to user defined	50
<b>width_values</b>	Width at flow interval for each segment	<b>nsegment, width_dim</b>	Meters	Real	0.0 to user defined	10

## 14 Daily Mean Stream Temperature Module—An Enhancement to the Precipitation-Runoff Modeling System

**Table 3.** Stream temperature module declared variables and internally calculated parameters.

[HRU, hydrologic response unit; nhru, number of HRUs; nsegment, number of segments, ft<sup>3</sup>/s]

Variable name	Description	Dimension	Units	Data type
Input				
<i>cloud_cover_hru</i>	Cloud cover proportion of each HRU	<b>nhru</b>	Fraction	Real
<i>hru_humidity</i>	Relative humidity for each HRU; specified in the humidity Climate-by-HRU File when control parameter <b>humidity_cbh_flag=1</b>	<b>nhru</b>	Fraction	Real
<i>hru_rain</i>	Rainfall distributed to each HRU	<b>nhru</b>	Inches	Real
<i>potet</i>	Potential evapotranspiration for each HRU	<b>nhru</b>	Inches	Real
<i>seg_inflow</i>	Total flow entering a segment	<b>nsegment</b>	ft <sup>3</sup> /s	Real
<i>seg_outflow</i>	Streamflow leaving a segment	<b>nsegment</b>	ft <sup>3</sup> /s	Real
<i>seg_upstream_inflow</i>	Sum of inflow to a segment from all connecting segments	<b>nsegment</b>	ft <sup>3</sup> /s	Real
<i>seginc_gwflow</i>	Area-weighted average groundwater discharge for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	ft <sup>3</sup> /s	Real
<i>seginc_sroff</i>	Area-weighted average surface runoff for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	ft <sup>3</sup> /s	Real
<i>seginc_sflow</i>	Area-weighted average interflow for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	ft <sup>3</sup> /s	Real
<i>seginc_swrad</i>	Area-weighted average solar radiation for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	Langleys	Real
<i>snowmelt</i>	Snowmelt from snowpack on each HRU	<b>nhru</b>	Inches	Real
<i>tavgc</i>	Average air temperature distributed to each HRU	<b>nhru</b>	Degrees Celsius	Real
<i>tmaxc</i>	Maximum air temperature distributed to each HRU	<b>nhru</b>	Degrees Celsius	Real
Output				
<i>temp_avg</i>	Module computed average stream temperature for each segment	<b>nsegment</b>	Degrees Celsius	Real
<i>seg_ccov</i>	Area-weighted average cloud cover fraction for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	Fraction	Real
<i>seg_humid</i>	Area-weighted average relative humidity for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	Fraction	Real
<i>seg_maxtemp</i>	Area-weighted average maximum air temperature for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	Degrees Celsius	Real
<i>seg_melt</i>	Area-weighted average snowmelt for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	Inches	Real
<i>seg_potet</i>	Area-weighted average potential evapotranspiration for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	Inches	Real
<i>seg_rain</i>	Area-weighted average rainfall for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	inches	Real
<i>seg_width</i>	Width of each segment	<b>nsegment</b>		
<i>t_roff</i>	Area-weighted average air temperature for each segment from HRUs contributing flow to the segment	<b>nsegment</b>	Degrees Celsius	Real
<i>upstrm_temp</i>	Temperature of streamflow entering each segment	<b>nsegment</b>	Degrees Celsius	Real

## Guidelines and Considerations

The `stream_temp` module provides a method for deterministic computation of daily mean and maximum stream-water temperatures in simulations at a watershed scale. The module is based on the Stream Network Temperature Model (SNTEMP) originally developed by Theurer and others (1984). The integration of SNTEMP with PRMS allows for joint calibration of the computed streamflows and temperatures with measured streamflow and water-temperature values. With this integrated approach, computed `stream_temp` variables (table 3) are directly available for use as input to PRMS modules and available to be output separately or along with variables computed by other PRMS modules. Previously, simulations could be only loosely coupled, that is a one-way coupling, to SNTEMP. See Markstrom (2012) and Hunt and others (2013, 2016) for examples of SNTEMP with PRMS and the USGS coupled Groundwater and Surface-water FLOW (GSFLOW) model (Markstrom and others, 2008), respectively.

The `stream_temp` module runs on a daily time step, consistent with all other PRMS modules. The equilibrium temperature concept of this module is best suited for normal, that is not extreme, stream-water temperatures at daily or longer time-scales (Bogan and others, 2003). Thus, this module is not suited to the prediction of diurnal fluctuations in stream-water temperature. Because of the assumptions incorporated into the computational methods, the module is not well suited for very heterogeneous stream networks in terms of slope, roughness, stream-channel shape, and shading characteristics. The module incorporates the assumption of homogenous segments in regard to these characteristics, as well as streamflow travel times greater than 7 and less than 24 hours.

Watershed size and segment length need to be considered when discretizing a PRMS application to compute stream temperatures. Herb and Stefan (2011) analyzed the time and length scales at which stream temperature will reach equilibrium given constant climate, flow, and landscape conditions. In their study of the Vermillion River watershed in Minnesota, they found that time to equilibrium varied from about 7 to about 50 hours in stream lengths from 3 to 45 kilometers, depending on flow rates and segment properties. The computations in the `stream_temp` module do not require the water temperature to be at or very near equilibrium temperature because the module uses a perturbation solution. The representativeness of results could suffer if segments are shorter than this length and time scale (for example 100-meter segments and (or) only 1 or 2 hours of travel time). PRMS does not allow segment travel times greater than 24 hours, so segments with travel times greater than a day will need to be split into multiple segments. Very long segments can tend to make the model assumptions of steady flow in a prismatic channel with instantaneous mixing less valid owing to likelihood of increased heterogeneity in hydraulic and channel-shape properties, shading, or meteorological conditions as segment length increases. Very large watersheds may prove difficult to model accurately, as errors may propagate and magnify in

higher-order segments. When used in a watershed suited for its underlying assumptions, the `stream_temp` module can be a useful tool for watershed management decisions that require estimates of stream-water temperatures.

## References Cited

- Anderson, C.W., and Carpenter, K.D., 1998, Water quality and algal conditions in the North Umpqua River Basin, Oregon, 1992-95, and implications for resource management: U.S. Geological Survey Water-Resources Investigations Report 98-4125, 95 p., accessed June 27, 2017 at <https://pubs.er.usgs.gov/publication/wri984125>.
- Bartholow, J.M., 2000, The stream segment and stream network temperature models: A self-study course, Version 2.0: U.S. Geological Survey Open-File Report 99-112, 276 p., accessed June 27, 2017 at <https://pubs.er.usgs.gov/publication/ofr99112>.
- Bartholow, J. M., 1991, A modeling assessment of the thermal regime for an urban sport fishery: *Environmental Management*, v. 15, no. 6, p. 833-845, accessed June 27, 2017 at <https://link.springer.com/article/10.1007/BF02394821>.
- Bogan, T., Mohseni, O., and Stefan, H. G., 2003, Stream temperature-equilibrium temperature relationship: *Water Resources Research*, v. 39, no. 9, p. 1245, accessed June 27, 2017 at <http://onlinelibrary.wiley.com/doi/10.1029/2003WR002034/full>.
- Bogan, T.R., Stefan, H.G., and Mohseni, O., 2004, Imprints of secondary heat sources on the stream temperature/equilibrium temperature relationship: *Water Resources Research*, v. 40, no. 12, W12510, accessed June 27, 2017 at <http://onlinelibrary.wiley.com/doi/10.1002/hyp.6994/full>.
- Bovee, K.D., Lamb, B.L., Bartholow, J.M., Stalnaker, C.B., Taylor, J., and Henriksen, J., 1998, Stream habitat analysis using the instream flow incremental methodology: U.S. Geological Survey Information and Technology Report 1998-0004, 130 p., accessed June 27, 2017 at <https://pubs.er.usgs.gov/publication/itr19980004>.
- Dingman, S. L., 2008, *Physical hydrology* (2d ed.): Long Grove, Ill., Waveland Press Inc., 656 p.
- Edinger, J.E., Duttweiler, D.W., and Geyer, J.C., 1968, The response of water temperatures to meteorological conditions: *Water Resources Research*, v. 4, no. 5, p. 1137-1143, accessed June 27, 2017 at <http://dx.doi.org/10.1029/WR004i005p01137>.
- Gaffield, S.J., Potter, K.W., and Wang, Lizhu, 2005, Prediction the summer temperature of small streams in southwestern Wisconsin: *Journal of the American Water Resources Association*, v. 41, no. 1, p. 25-36, accessed June 27, 2017 at <http://onlinelibrary.wiley.com/doi/10.1111/j.1752-1688.2005.tb03714.x/full>.

- Hazewinkel, Michiel, ed., 2001, Newton method, *Encyclopedia of Mathematics*: New York, Springer, 732 p.
- Herb, W.R., and Stefan, H. G., 2011, Modified equilibrium temperature models for cold-water streams: *Water Resources Research*, v. 47, no. 6, 13 p., accessed June 27, 2017 at <http://onlinelibrary.wiley.com/doi/10.1029/2010WR009586/full>.
- Hunt, R.J., Westenbroek, S.M., Walker, J.F., Selbig, W.R., Regan, R.S., Leaf, A.T., and Saad, D.A., 2016, Simulation of climate change effects on streamflow, groundwater, and stream temperature using GSFLOW and SNTMP in the Black Earth Creek Watershed, Wisconsin: U.S. Geological Survey Scientific Investigations Report 2016–5091, 117 p., accessed June 27, 2017 at <https://doi.org/10.3133/sir20165091>.
- Hunt, R.J., Walker, J.F., Selbig, W.R., Westenbroek, S.M., and Regan, R.S., 2013, Simulation of climate-change effects on streamflow, lake water budgets, and stream temperature using GSFLOW and SNTMP, Trout Lake watershed, Wisconsin: U.S. Geological Survey Scientific Investigations Report 2013–5159, 118 p., accessed June 27, 2017 at <https://pubs.usgs.gov/sir/2013/5159/>.
- Markstrom, S.L., 2012, P2S-Coupled simulation with the Precipitation-Runoff Modeling System (PRMS) and the Stream Temperature Network models: U.S. Geological Survey Open-File Report 2012–1116, 19 p., accessed June 27, 2017 at <https://pubs.er.usgs.gov/publication/ofr20121116/>.
- Markstrom, S.L., Niswonger, R.G., Regan, R.S., Prudic, D.E., and Barlow, P.M., 2008, GSFLOW–Coupled Ground-water and Surface-water FLOW model based on the integration of the Precipitation-Runoff Modeling System (PRMS) and the Modular Ground-Water Flow Model (MODFLOW-2005): U.S. Geological Survey Techniques and Methods 6–D1, 240 p., accessed June 27, 2017 at <http://pubs.er.usgs.gov/publication/tm6B7>.
- Markstrom, S.L., Regan, R.S., Hay, L.E., Viger, R.J., Webb, R.M.T., and Payne, R.A., 2015, PRMS-IV, the Precipitation-Runoff Modeling System, Version 4: U.S. Geological Survey Techniques and Methods 6–B7, 158 p., accessed June 27, 2017 at <https://pubs.er.usgs.gov/publication/tm6B7>.
- Norton, G.E., and Bradford, A., 2009, Comparison of two stream temperature models and evaluation of potential management alternatives for the speed river, southern Ontario: *Journal of Environmental Management*, v. 90, no. 2, p. 866–878, accessed June 27, 2017 at <https://doi.org/10.1016/j.jenvman.2008.02.002>.
- Perry, R.W., Risley, J.C., Brewer, S.J., Jones, E.C., and Rondorf, D.W., 2011, Simulating daily water temperatures of the Klamath River under dam removal and climate change scenarios: U.S. Geological Survey Open-File Report 2011–1243, 78 p., accessed June 27, 2017 at <https://pubs.er.usgs.gov/publication/ofr20111243>.
- Theurer, F.D., Voos, K.A., and Miller, W.J., 1984, Instream water temperature model part II physical processes and math models: U.S. Fish and Wildlife Service Instream Flow Information Paper 16, 86 p., accessed June 27, 2017 at <https://www.fort.usgs.gov/products/sb/7527>.
- Webb, B.W., Hannah, D.M., Moore, R.D., Brown, L.E., and Nobilis, F., 2008, Recent advances in stream and river temperature research: *Hydrological Processes*, v. 22, no. 7, p. 902–918, accessed June 27, 2017 at <http://onlinelibrary.wiley.com/doi/10.1002/hyp.6994/full>.

## Appendix 1. Net Heat Flux at Current Water Temperature as a Taylor Series Expansion.

### Taylor Series Background and Form of Expansion

The net heat flux in the Stream Temperature module is computed using a modified Taylor polynomial expansion of the zero heat flux at the equilibrium stream-water temperature. A Taylor series expansion is an expression of a function  $f(x)$  as a power series with the form

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n, \quad (1-1)$$

where  $f^{(n)}(x_0)$  is the  $n$ th derivative of the function evaluated at the point of expansion  $x_0$  (Hazewinkel, 2001). A full Taylor series or polynomial expansion can be used to represent any continuously differentiable function at any given  $x$ . However, typically the expansions use first- or second-order derivative terms as a simplifying approximation for  $x$  near  $x_0$ , as in the technique used in the `stream_temp` module.

The full heat flux ( $H_n$ ) equation (see eq. 15 in the body of this report) is a fourth-order polynomial as a function of water temperature ( $T_w$ ) and is computed according to

$$-H_n = A(T_w + 273.16)^4 - CT_w^2 + BT_w - D, \quad (1-2)$$

with  $A$ ,  $B$ ,  $C$ , and  $D$  coefficients computed for each time step. Water temperature used in equation 1-2 is in degrees Celsius with the heat flux having units of watts per square meter. In order to simplify the analytical solution to the differential equation for water temperature, the heat flux at a given water temperature is found by using a second-order Taylor polynomial expansion about the equilibrium temperature. The value of  $H_n$  at  $T_w$  is found by using this type of expansion around the equilibrium temperature  $T_e$ . Thus, from equation 1-1,  $f(x)$  is  $H_n(T_w)$ ,  $x_0$  is  $T_e$ , and  $x$  is  $T_w$ ; thus, the net heat flux is computed according to

$$-H_n(T_w) = -H_n(T_e) + \frac{dH_n(T_e)}{dT}(T_w - T_e) + \frac{d^2H_n(T_e)}{2dT^2}(T_w - T_e)^2. \quad (1-3)$$

When using  $T_e$  as the point of expansion or  $x_0$ , the zero-order term,  $H_n(T_e)$ , is assumed to equal 0. Taking this into account and computing the heat flux as a positive value, equation 1-3 becomes

$$H_n(T_w) = \frac{dH_n(T_e)}{dT}(T_e - T_w) + \frac{d^2H_n(T_e)}{2dT^2}(T_e - T_w)^2. \quad (1-4)$$

This is the general form of the approximation for net heat flux as a function of water temperature used in the `stream_temp` module.

## Modification of Expansion to Span a Specified Water Temperature Range

In the `stream_temp` module, streamflow enters a segment at an initial water temperature ( $T_o$ ) and progresses toward  $T_e$  by gaining or losing heat at a rate  $H_n$ , which is dependent on the antecedent water temperature (fig. 1-1). Because of this, it is assumed that the temperature of the water is between  $T_o$  and  $T_e$  for the purposes of `stream_temp` module calculations and evaluation of the net heat flux is valid within this range. The typical results of the expansion given by equation 1-4 give a good approximation for the net heat flux, assuming  $T_w$  is close to  $T_e$ . However, when  $T_w$  is not close to  $T_e$ , the solution to equation 1-4 could potentially deviate significantly from the solution of the full heat flux equation (eq. 1 in the body of this report). The approximation used in the `stream_temp` module modifies the second-order term of the net heat flux at  $T_o$ , as well as at  $T_e$ , using a correction factor. For this correction factor approximation approach, a constant second derivative between the two temperatures is assumed.

The solution technique in the `stream_temp` module uses thermal exchange coefficients to represent the derivative terms of the Taylor series expansion, thus equation 1-4 can be expressed as

$$H_n = K_1(T_e - T_w) + K_2(T_e - T_w)^2, \quad (1-5)$$

where  $K_1$  and  $K_2$  are the first- and second-order thermal exchange coefficients, respectively. The first-order coefficient is given by

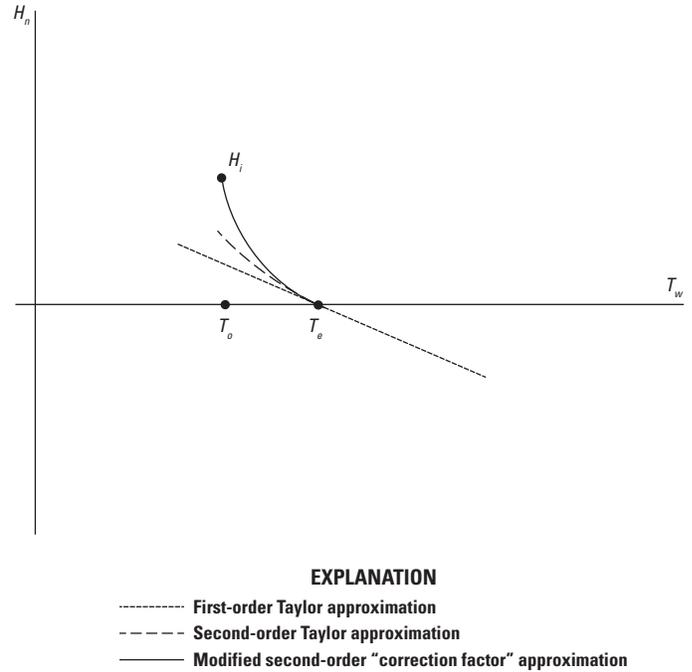
$$K_1 = \frac{dH_n(T_e)}{dT} = 4A(T_e + 273.16)^3 - 2CT_e + B, \quad (1-6)$$

which yields the first-order term in equation 1-5 using the form given by equation 1-4. This first-order term projects the first derivative out from  $T_e$  to  $T_w$ , as shown by the dotted line in figure 1-1.

For the second-order term, instead of projecting the local second derivative from  $T_e$ , as in equation 1-4 (the line in figure 1-1 with longer dashes), a finite-difference calculation of the average deviation from the first-order approximation is computed across the  $T_o$ - $T_e$  range. Thus the second-order exchange coefficient is computed according to

$$K_2 = (H_i - [K_1(T_e - T_o)]) / [(T_e - T_o)^2], \quad (1-7)$$

where  $H_i$  is the net heat flux from equation 1-2 evaluated at  $T_o$ . This definition of  $K_2$  is the second-order correction factor to the first-order solution, based on the difference from  $H_i$  at  $T_o$ , and is shown by the solid, curved line in figure 1-1. Equation 1-5 evaluated at  $T_o$  using this definition of  $K_2$  now yields  $H_i$ , which is the actual heat flux at  $T_o$ . Evaluated elsewhere in the  $T_o$ - $T_e$  range, equation 1-5 gives a fraction of this difference that changes constantly as  $T_w^2$ . This solution technique generates a smooth curve between zero at  $T_e$  and



**Figure 1-1.** Graph showing contrasting the first- and second-order Taylor expansions about  $T_e$  with the correction factor approach used in this solution.  $T_o$ ,  $T_e$ , and  $T_w$  are the initial, equilibrium, and final water temperatures in the segment, respectively.  $H_n$  (labeled on the y-axis) represents the range of the net heat flux at various water temperatures, and the point labeled  $H_i$  represents the net heat flux at temperature  $T_o$ .

$H_i$  at  $T_o$ , starting with the evaluated first derivative at  $T_e$  and using a constant second derivative throughout. This technique provides an accurate way to compute the net heat flux across this entire temperature range on the basis of a second-order approximation that assumes a smooth, continuous heat flux function. The correction factor approach can yield a value close to measured values of net heat flux at and close to both  $T_e$  and  $T_o$ , whereas the typical second-order Taylor polynomial expansion approximation may deviate more from measured values the further  $T_w$  is from  $T_e$ . This is evidenced by the slight deviation of the Taylor second-order solution underpredicting the value of  $H_i$ , as shown in figure 1-1. The heat flux equation should be monotonic in stream-water temperature, so there should not be significant local minima or maxima deviation using this modification of the Taylor series approximation. However, some sharper, more corner-like features may be smoothed by use of this type of approximation across the entire range.

## Reference Cited

Hazewinkel, Michiel, ed., 2001, Taylor series, Encyclopedia of Mathematics: New York, Springer, 732 p.

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