U.S. DEPARTMENT OF THE INTERIOR
March 3, 1849

Techniques of Water-Resources Investigations
of the United States Geological Survey

Chapter A10

DISCHARGE RATINGS AT GAGING STATIONS

By E. J. Kennedy

Book 3

APPLICATIONS OF HYDRAULICS

fect, can be moved vertically on a rectangular grid without error by use of the shifting-control method.

Assumptions 1 and 4 are approximations, and there is no way to verify how closely they may apply at a particular site. However, any related errors affect only the discharge for days on which interpolated shift adjustments were made and are minor in comparison with errors caused by grossly misshaped rating curves.

Figure 13 illustrates a low-water rating analysis for a stream whose section control of alluvial material over the remnants of a beaver dam is unstable but whose medium- and high-water ratings are relatively stable. The basic data are tabulated in figure 13*A* and include frequent *GZF* determinations. The depth column is $Ght-GZF$. The measured discharges, plotted against corresponding gage heights in figure 13*B* on a rectangular grid, give no reliable information as to the shape of the rating. The same discharges, plotted against depth at the control on a logarithmic grid in figure 13*C*, give a well-defined curve because no measurement plots farther above or below the curve than the expected error in the *GZF* determination. The heavy curve on the rectangular grid in figure 13*D* is the depth-discharge curve raised by 3.04 ft (any other value within the range of shifts would do about as well) to match the rating position on October 4. The light curves illustrate the effective rating location on other days. Shift adjustments listed in figure 13*A* are distances between the curve positions at the times of discharge measurement and the heavy base curve. If *GZF*'s had not been measured and if the October and May discharge measurements had not been made, the other measurements would have led to a differently shaped base curve, the shift adjustment variation between measurements would have been erratic, and the computed record would have been less reliable.

Complex ratings

A complex rating is used for a site where the water-surface slope is variable and where no simple relation exists between stage and discharge. Discharge must be related to stage and some other variable. Rate of change in stage is the additional variable for rating streams where storage causes the stage-discharge relation to loop (figure 14*A*). A slope rating is used, along with an auxiliary gage to measure fall in a reach, where tributaries, dams, or the return of overbank flow to the channel causes variable backwater. Index-velocity ratings, which involve special mechanical or electronic devices to measure velocity, are used where special rating problems exist.

A complex rating requires more discharge measurements for adequate definition than a simple stage-discharge rating, and the type of complex rating that will apply usually cannot be predicted before the measurements are made. A prudent procedure to follow at a newly established site where a complex rating is anticipated is to assume that a slope rating will be needed, establish temporary gages at potential auxiliary sites so that readings can be made during all discharge measurements, and measure a few rises over the entire flood hydrograph. Then the loop ratings can be plotted as one indicator of the appropriate rating type. The simplest analysis can be tried first. If it is not satisfactory, various slope ratings can be tried until an adequate rating is developed or until the need for an index-velocity rating is apparent.

A loop rating can be drawn by connecting plotted consecutive discharge measurements made during a single rise. If a rating has been developed, the loop for each major rise can be plotted without discharge measurements by connecting the successive plots of recorded instantaneous gage heights and the corresponding adjusted discharges. Typical single-storm storage loops are shown in figure 14*A*. This type of loop is distinctive in that one occurs on every rise and is roughly symmetrical about the stage-discharge curve for constant-stage conditions. Such loops are related to channel storage between the gage and the control and indicate the applicability of a rate of change in stage rating. Figure 14*D* shows typical backwater loops of the type caused by the return of overbank flow to the main channel. A backwater loop occurs only after an overbank rise—the greater the overbank depth, the wider the loop. An overbank return loop is always to the left of the free-fall rating (the rating defined

Soldier Creek near Circleville, Kansas

A BASIC DATA

Meas. No.	Date	*Ght* (ft)	*Disc* (ft3/s)	*GZF* (ft)	Depth at control (ft)	Shift Adjust. (ft)
139	10-04-72	3.36	2.35	3.04	0.32	0
140	11-15	4.22	51.4	3.31	.91	-.31
141	12-27	3.87	28.0	3.22	.65	-.12
142	1-30-73	3.95	34.2	--	--	-.15
143	2-22	3.67	19.6	3.02	.65	0
144	3-26	4.43	110	--	--	0
145	4-26	3.70	22.7	3.06	.64	0
146	5-23	3.22	13.3	--	--	+.38
147	6-20	2.98	5.11	2.50	.48	+.47
148	7-20	5.66	298	--	--	0
149	8-15	3.42	13.6	2.82	.60	+.18
150	9-11	3.15	6.43	2.74	.41	+.33

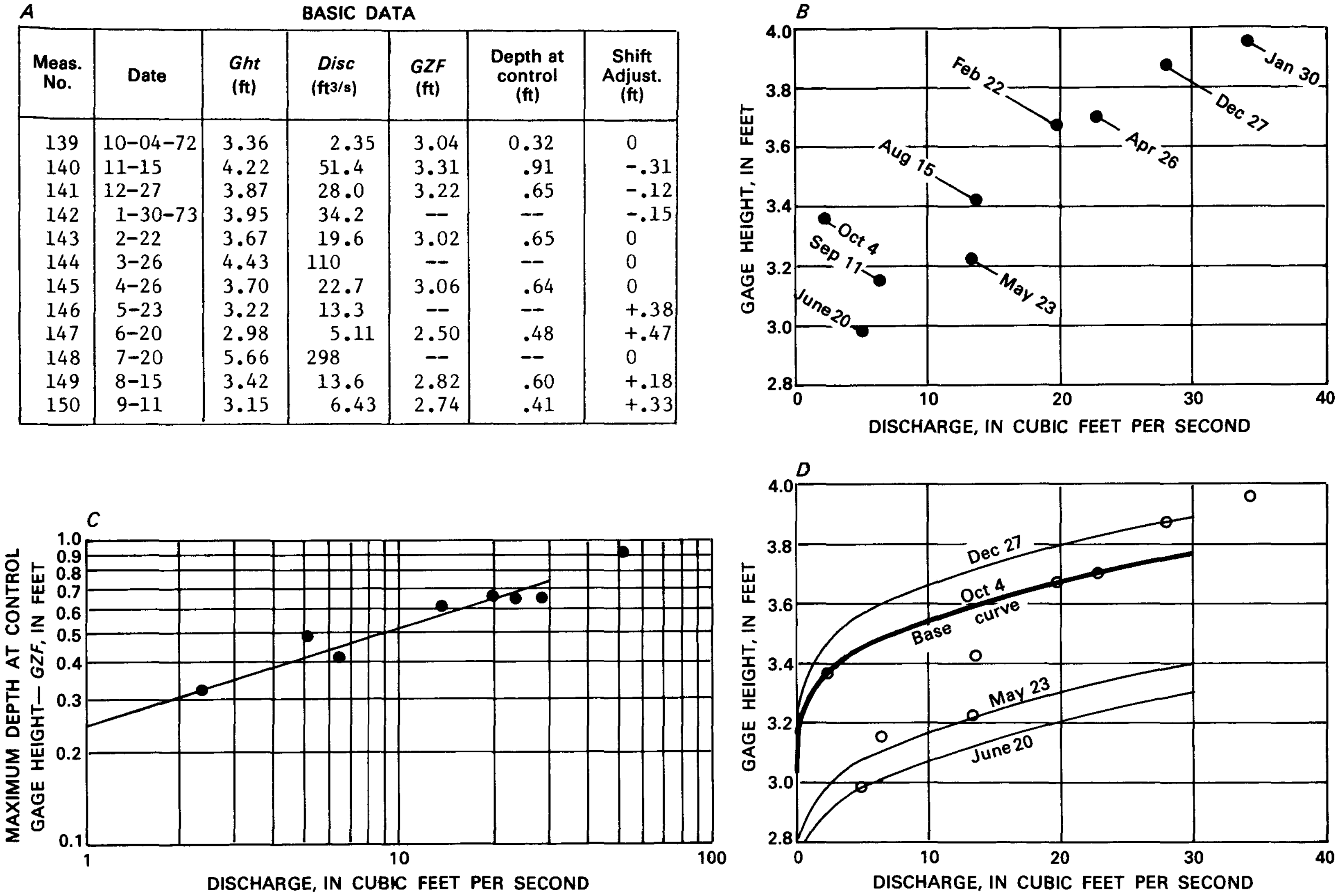
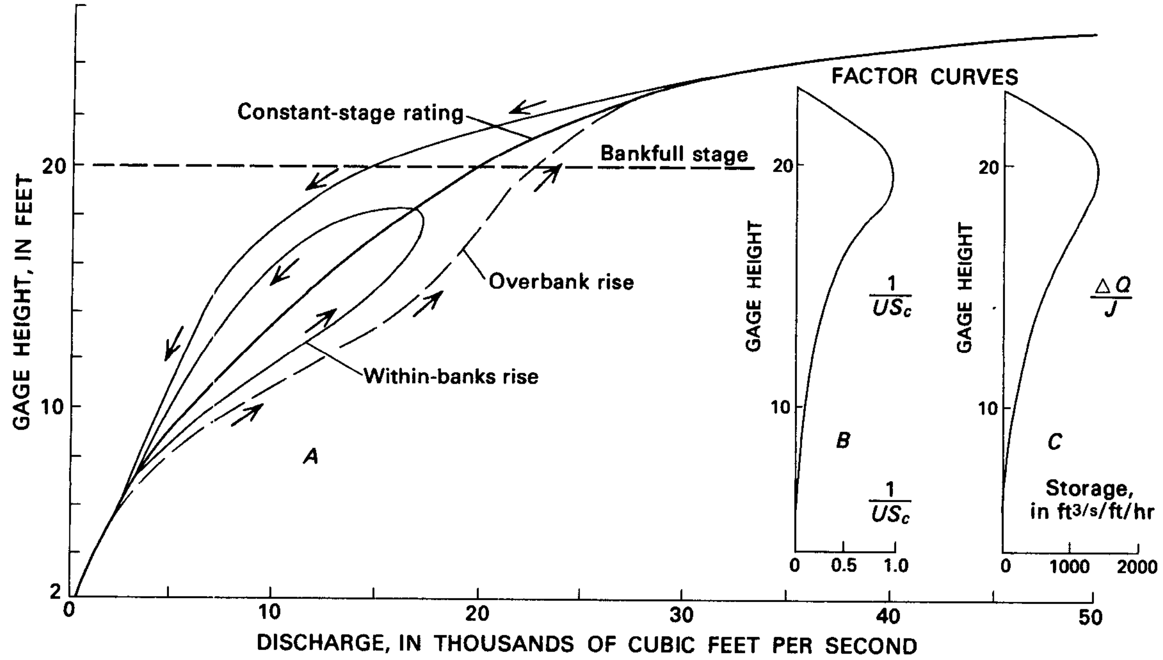


FIGURE 13.—Example of a low-water rating analysis with periodic *GZF* observations used to define unstable low-water ratings.

RATING LOOPS FROM STORAGE



RATING LOOPS FROM RETURN OF OVERBANK FLOW

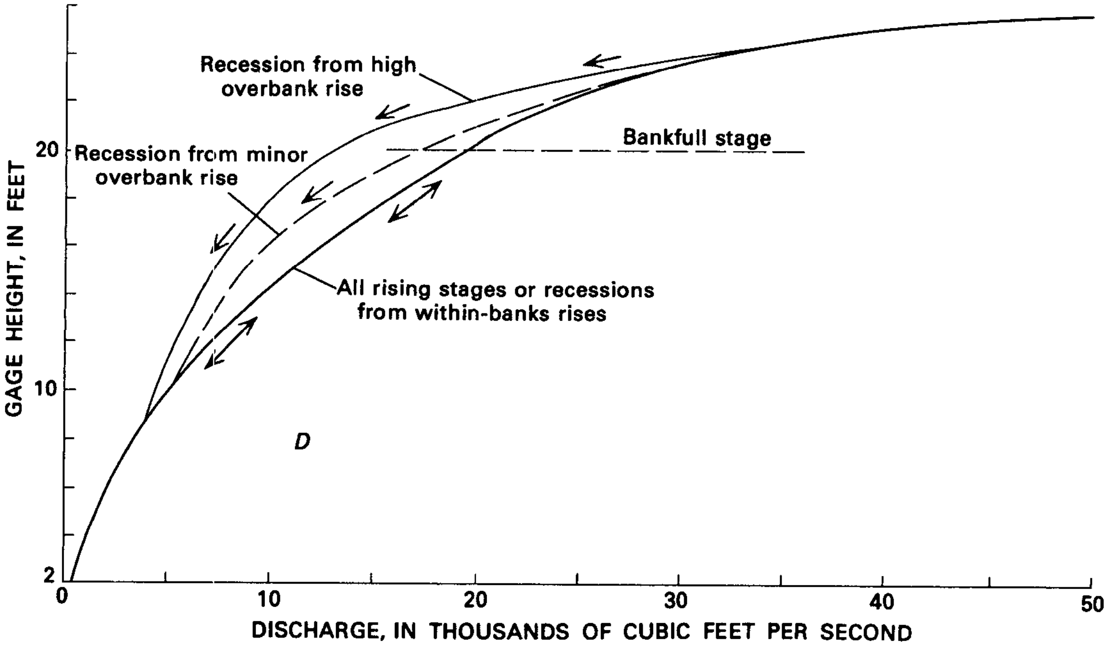


FIGURE 14.—Typical shapes of single-storm loop ratings and factor curves.

by rising-stage measurements and those falling-stage measurements that follow a within-banks rise). Loops of this type are rarely as clearcut as the illustration. They are often superimposed on storage loops and may be impossible to identify. The presence of backwater loops, alone or in combination with storage loops, rules out the use of a rate of change in stage rating and requires a slope or index-velocity rating.

The ordinary types of complex ratings (rate of change in stage, slope, or index velocity) are explained and illustrated by actual examples in this manual. The WATSTORE User's Guide (Hutchison and others, 1975, 1980) contains the instructions necessary for preparing data so that trial-and-error solutions for most of the complex rating types can be made by using a computer.

Rate of change in stage ratings

Two types of rate of change in stage ratings are in general use: (1) $\Delta Q/J$ (storage effect per unit of rate of change in stage), which treats a rating loop as a simple storage phenomenon, and (2) $1/US_c$, which relates the magnitude of the rating loop to the velocity of flood waves (U) and to the water-surface slope at constant discharge (S_c). Either method can be used at most sites that have rating loops similar to those in figure 14*A*, but one method may be clearly superior to the other at a site where the rating loop is wide. The best practice is to try both methods and select the one that best fits the discharge measurements.

A rate of change in stage rating is subject to subtle errors that are not apparent until the rating is tested by using actual data. Serious irregularities occur most often when an auxiliary curve (figs. 14*B*, *C*) is bent too sharply in the stage range where rates of change are most rapid. The sharp bends can cause false peaks and troughs in the hydrograph. Other causes of erratic record include stilling-well surge, manometer stepping, and sluggish intakes that suddenly plug or clear. Much of the gage-height surge present in some wells or bubble gages can be removed during ADP processing by using a smoothing option covered in the WATSTORE User's Guide (Hutchison and others 1975, 1980). Some errors can be prevented by checking a rating through a major rise (see figs. 16, 19) before it is used and by drawing the hydrograph and loop ratings for all subsequent major rises from ADP-generated gage heights and discharges. If the hydrographs and loop ratings are always reasonable, the rating probably is accurate and is the correct type for the site. If the hydrographs and loops are unreasonable and if adjustments to the rating do not correct them, the rating type is probably inapplicable, and a slope rating should be tried.

Daily discharges computed by using both the constant-stage discharge and the factor curves of a rate of change in stage rating are called adjusted discharges. Those computed by using only the constant-stage discharge curve as a simple rating are called unadjusted discharges. The choice of methods depends on the use of the records and the definition of the rating. The adjusted discharges from a rate of change in stage rating represent flow at the gage. Unadjusted discharges from the constant-stage curve can be considered to represent flow at the control, wherever the control happens to be at the time. If unadjusted discharges are used, the peak discharge usually will be slightly below the adjusted peak discharge, and the unadjusted discharge hydrograph will be similar in shape to and a few hours later than the adjusted hydrograph. Adjusted daily discharges for the rising and falling high-water days will differ substantially from unadjusted discharges, but the total flow for each rise will be about the same. If the rating tests are favorable, adjusted daily discharges are always preferable. If water samples collected at the gage are involved and if the constituents analyzed are related to the quantity of flow, adjusted daily discharges should always be used. If no water samples are involved and if the rating's auxiliary curve is poorly defined, unadjusted daily discharges computed from the constant-stage rating may be the best choice.

$\Delta Q/J$ ratings

The $\Delta Q/J$ type of rating is a logical first-trial choice if the rate of change of stage is the likely cause of loop ratings. The principal components are a constant-stage discharge curve (central curve in fig. 14*A*) and a storage curve (fig. 14*C*). Actual discharge is computed by adding a storage correction to the discharge obtained from the constant-stage rating. The storage correction is the value from the storage curve multiplied by the rate of change in stage. If the symbols as defined in figure 15*B* are used, the relation can be written:

$$Q_m = Q_r + \left[\left(\frac{\Delta Q}{J}\right)\right] \times J$$

The rating is developed by trial and error, starting with a trial constant, a stage rating curve drawn close to the measurements made during near-steady stages. The difference between each measured discharge and the constant-stage discharge is divided by the rate of change in stage and plotted against stage on a separate graph. The storage curve, which represents the storage correction per foot-per-hour change in stage, is based on these plotted points. Each discharge measurement is adjusted to constant-stage conditions (corrected for storage) by using the storage curve. The constant-stage rating curve is refined by using the adjusted measurement values. The process is repeated, usually about three times, until further refinement of rating or storage curve is unlikely. The sequence of steps used is listed in figure 15*B*. The WATSTORE User's Guide (Hutchison and others, 1975, 1980) outlines a similar procedure to be used with an appropriate computer facility.

The $\Delta Q/J$ curve must be drawn with due regard to the unequal weights of the plotted points $[(Q_m - Q_r)/J]$. Those based on discharge measurements whose rates of change in stage were high have considerable weight, and the storage curve should be drawn close to them. Measuring error is a large part of the variance between the discharge curve and the measured flow when the rate of change in stage is less than about 0.10 ft/hr. Storage values are not usually computed for those measurements, and only a little weight is given to values based on discharge measurements whose rates of change in stage are less than 0.20 ft/hr. A large departure of the storage curve from a point based on a slowly changing stage measurement has little effect on that measurement's percentage difference.

The general shape of the $\Delta Q/J$ curve is predictable. Most storage curves go through zero at the stage where the low-water control becomes submerged and again where the overbank contains more than about half the flow. The maximum storage value usually occurs at about bankfull stage. The curve should bend as gently as the data will allow.

Figure 15*A* illustrates a typical $\Delta Q/J$ method application. A $1/US_c$ analysis (not shown) also was made, and the resulting rating was not significantly different from the $\Delta Q/J$ rating. The gage is just downstream from a long highway embankment with a relatively short bridge that spans all flow. A riffle near the gage is the section control for low water and is drowned out above about the 7-ft stage. The location of the channel-controlling reach for medium stages is not apparent in the field or on a map, but its centroid is probably several miles downstream. Above bankfull stage (27 ft), the flow fans out into the relatively shallow flood plain just downstream from the gage. The flood plain that acts as control and the gage are so close together at very high stage that storage is negligible, and there is no changing-stage effect. The discharge and storage curves are typically shaped, the computations in figure 15*C* indicate only one outlier, and the testing by manual computation shown in figure 16 is favorable. The rating is sufficiently well defined to justify the use of adjusted daily discharges for the published record.

Most $\Delta Q/J$ ratings are used where the medium- and high-water ratings loop owing to storage change between the gage and a high-water control whose location depends on the stage. The process also can be used where changing-stage effect is caused by a section control far downstream and is present only at low water. Figure 17*A* illustrates this type of rating. At high stages, backwater from a downstream dam makes a slope rating necessary. All discharge measurements are made from a cableway at the gage. A rock riffle section control just downstream is submerged at a very low stage, and a series of shoals about 2 mi downstream becomes the low-water control. A storage curve was developed by the trial-and-error procedure used in the previous example, and the results of the final trial are shown in figure 17*B*.

$1/US_c$ ratings

The $1/US_c$ rating type, also called the Boyer method, is generally used if changing-stage effect cannot be related to simple storage. The method is based on the Boyer equation:

$$\frac{Q_m}{Q_r} = \sqrt{1+\left(\frac{1}{US_c}\right) \times J}$$

This equation, whose symbols are defined in figure 18, evolved from two earlier, similar equations that were used to adjust individual discharge measurements for changing-stage effect. In the early equations, the variables U and S_c were evaluated separately. The Boyer method treats the entire term $1/US_c$ as one empirical variable, and its relation to stage is defined by discharge measurements made during periods of rapidly changing stage. The rating components are a constant-stage rating (central curve, fig. 14*A*) and a stage versus $1/US_c$ curve (fig. 14*B*).

The rating is developed by trial and error, starting with a trial constant-stage rating drawn close to the measurements that were made during near-steady stages. Then the ratio of each changing-stage measured discharge to the constant-stage discharge and the rate of change in stage are entered in the Boyer equation. The equation is solved for $1/US_c$, and the result is plotted against the stage of the discharge measurement. A $1/US_c$ curve (factor curve) is drawn next on the basis of the plotted points. Each discharge measurement is then adjusted to constant-stage conditions by using the factor curve and the Boyer equation. The constant-stage curve is refined by using the adjusted measurements. The process is repeated, usually about three times, until further refinement of either curve is unlikely. The sequence of steps for manual computation is listed in figure 18*B*. The WATSTORE User's Guide (Hutchison and others, 1975, 1980) outlines the procedure to be used with appropriate computer equipment for the rating analysis.

The factor curve must be drawn so that the $1/US_c$ values computed from discharge measurements whose rate of change in stage is high are given more weight than those calculated from measurements made while stage changed slowly. Values of $1/US_c$ for measurements whose rate of change in stage is less than about 0.10 ft/hr are not usually computed because their variation from the constant-stage curve is greatly affected by normal measuring error. A large departure of the factor curve from a $1/US_c$ value based on a nearly constant stage discharge measurement has little effect on that measurement's percentage difference.

The shape of the factor curve is similar to that of a $\Delta Q/J$ curve. A typical $1/US_c$ curve goes through zero at the stage where the section control is submerged, reaches its maximum value at about bankfull stage, and approaches zero at the stage where the overbank area of the channel contains about half the total flow. The factor curve should bend as gently as the data will allow. If the value of $1/US_c$ at any stage is too great because of an erroneously drawn curve, the value under the radical in the Boyer equation may become negative for periods of rapidly falling stage in that range.The computed factor then would be the square root of a negative number, and a meaningful value could not be determined. Correcting this condition may require revision of both the constant-stage curve and the factor curve.

Figure 18*A* illustrates a typical $1/US_c$ rating. A $\Delta Q/J$ analysis (not shown) was tried for this site, and the resulting rating was essentially the same as the $1/US_c$ rating. The stream has a flat, narrow, uniform main channel and a flood plain 1 mi wide. Rating loops occur only at stages between 3 and 16 ft and rarely vary from the constant-stage rating by more than 15 percent. The rating was analyzed by using the procedure outlined in figure 18*B*, which is designed for either manual or minicomputer computation. A similar outline to be used for computing the trial curves on an appropriate terminal is contained in the WATSTORE User's Guide (Hutchison and others, 1975, 1980).

The final trial computations, tabulated in figure 18*C*, indicate a close fit of data to the rating, and the testing by manual computations shown in figure 19 is favorable. Daily discharge

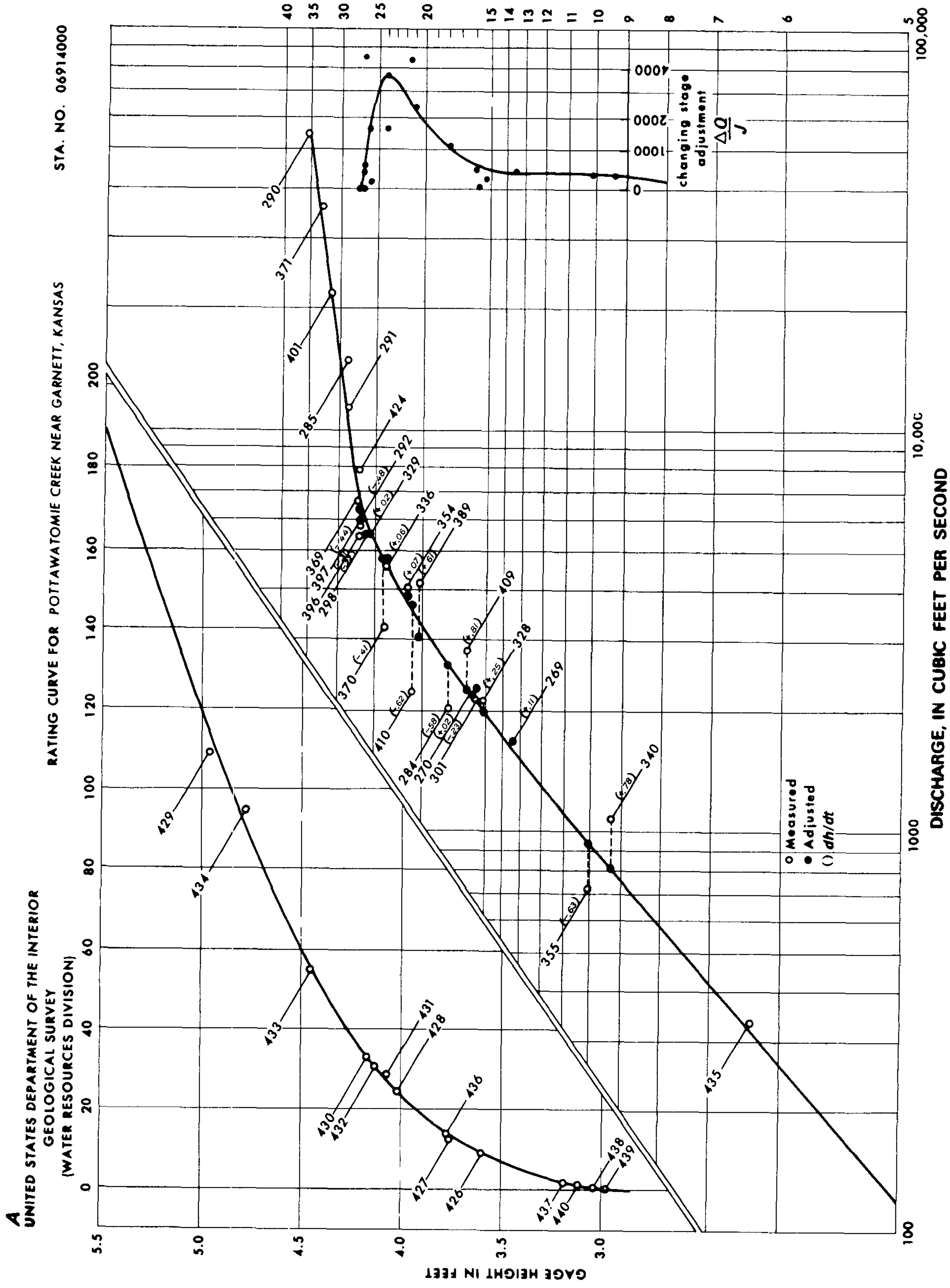


FIGURE 15.—Typical ΔQ/J discharge rating.

B

ANALYSIS PROCEDURE

STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets and table	Needed: Sheet 1, a log-log rating grid for the constant-stage discharge curve; sheet 2, a rectangular grid for the $\Delta Q/J$ curve; and a computation sheet with columns titled and numbered ① to ⑪ as in the example below.
2	Enter data	Fill in ① to ④ with data from the discharge measurements.
3	Plot Q curve data	Using sheet 1, with the appropriate *Ght* scale offset, plot *Ght* (②) vs. Q_m (③). Flag each print with J (④).
4	Draw first trial	*This trial curve should be close to constant-stage measurements, left of rising-stage measurements and to the right of falling-stage measurements. Skip to step 7 for the first trial computation.*
5	Plot Q curve data	On sheet *1, plot *Ght* (②) vs. Q_{adj} (⑩).
6	Draw refined Q curve	This discharge curve should average the step 5 points as well as possible.
7	List Q_r values	Fill in * ⑤ from step 4 curve (first trial), step 6 curve (subsequent trials), or from the curve's descriptors for the final trial.
8	Compute and list ΔQ	In ⑥ , if J (④) is between +0.1 and −0.1 enter a dash. Otherwise, * ⑥ = ③ − ⑤
9	Compute and list $\Delta Q/J$	If a dash is entered in ⑥ , enter a dash in ⑦ . Otherwise, * ⑦ = ⑥ ÷ ④
10	Plot storage curve data	Plot $\Delta Q/J$ (⑦) vs. *Ght* (②) on sheet *2. Use a distinctive symbol for rapid-change points.
11	Draw storage curve	The storage curve should resemble figure 14*C* and be closest to those step 10 points defined by rapid-change measurements. Maximum $\Delta Q/J$ is usually just above bankfull stage. $\Delta Q/J$ is zero when section control is effective and again when the flood plain contains most of the total discharge.
12	List of curve values of $\Delta Q/J$	Fill in * ⑧ from the sheet 2 curve values for the early trials. For the final trial, use values interpolated from the curve's descriptors.
13	List computed ΔQ	Fill in * ⑨ for all measurements, regardless of magnitude of J. ⑨ =− ⑧ x ④
14	List Q_{adj}	Fill in * ⑩ = ③ + ⑨
15	Next trial (step 5)	If both the Q and $\Delta Q/J$ curves are unlikely to improve with further trials, proceed to step 16. For an additional trial, return to step 5.
16	Finalize	Prepare descriptors or tables for both curves.
17	Finalize	*Recompute* * ⑤ *to* ⑩ *and compute* ⑪ *using the step 16 materials;* ⑪ =100x(⑩ − ⑤)− ⑤ . If ⑪ values are satisfactory, proceed to step 18. Otherwise, return to step 5.
18	Test	See text and figure 16. If test is satisfactory, proceed to step 19. Otherwise, return to step 5 and adjust curve shapes as necessary.
19	Finalize	Prepare master curve sheet.

*Erase any entries or plotting from previous trials.

C

COMPUTATIONS

Pottawatomie Creek near Garnett, Kansas

Meas. No. ①	Ght ②	Q_m ③	J ④	Q_r ⑤	$Q_m - Q_r = \Delta Q$ ⑥	Comp. $\frac{\Delta Q}{J}$ ⑦	Curve $\frac{\Delta Q}{J}$ ⑧	$-\frac{\Delta Q}{J} \times J = \Delta Q$ ⑨	$Q_m + \Delta Q = Q_{adj}$ ⑩	%Diff. ⑪
269	13.49	1690	0.11	1620	70	636	475	-52	1638	1.1
284	17.86	2030	-0.58	2680	-650	1121	1087	630	2660	-0.7
285	26.80	14900	-0.21	12800	2100	0	0	0	14900	16.4
290	35.27	54200	-0.19	57600	-3400	17895	0	0	54200	-5.9
291	28.60	11300	-0.33	12200	-900	2727	0	0	11300	-7.4
340	9.34	1080	0.78	801	279	358	311	-243	837	4.5
355	10.07	720	-0.63	936	-216	343	336	211	931	-0.5
370	23.97	3240	-0.41	4880	-1640	4000	3941	1616	4856	-0.5
371	32.83	35900	0.30	33900	2000	0	0	0	35900	5.9
401	31.11	23000	-0.10	22800	200	0	0	0	23000	0.9
409	16.42	2820	0.81	2310	510	630	763	-618	2202	-4.7
410	20.93	2210	-0.62	3600	-1390	2242	2076	1287	3497	-2.9
424	26.98	7940	0.30	7880	60	200	2030	-609	7331	-7.0

SYMBOLS

Comp.	Computed value	Q_m	Measured discharge (ft^3/s)
Curve	Value from curve	Q_r	Discharge from ratings (ft^3/s)
Ght	Gage height (ft)	ΔQ	Storage correction (ft^3/s)
J=**dh/dt*	Rate of change in stage (ft/hr)	%Diff.	Variation of Q_{adj} from Q_r
Q_{adj}	Adjusted discharge (ft^3/s)		

**J* and *dh/dt* are both conventional symbols for rate of change in stage. *J* is more convenient to use in an equation, especially as part of the numerator or denominator of a fraction.

FIGURE 15.—Continued.

A. DAILY DISCHARGE COMPUTATION

Flood subdivision with factor
(Experimental)
(Mar. 1960)

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

Stream Pottawatomie Creek near Garnett, Kans

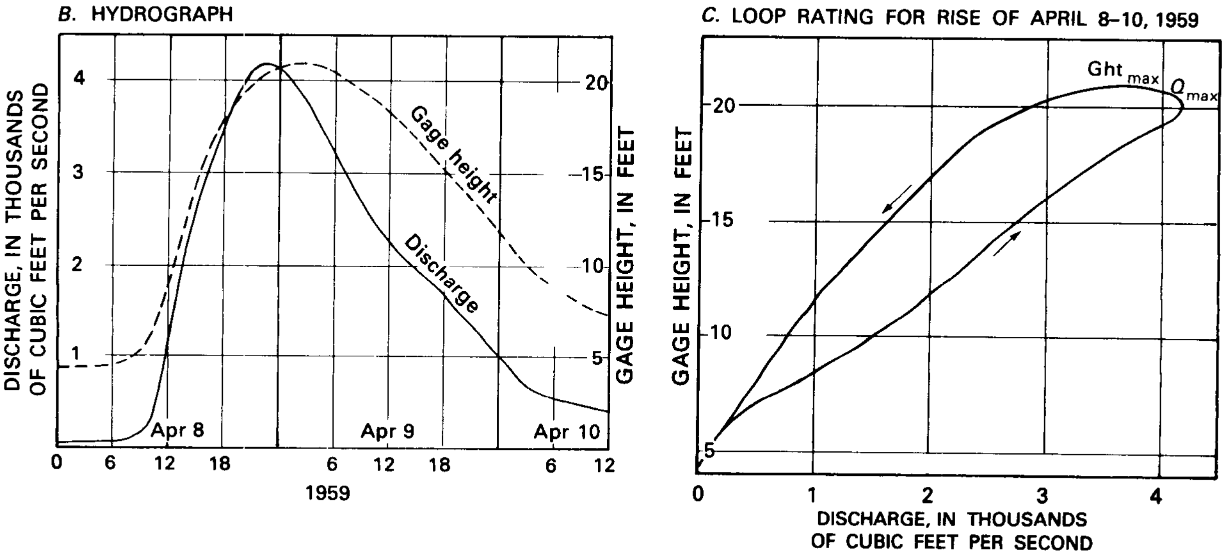
Gage height, in feet, and discharge, in cubic feet per second, at indicated time, 19 59

April 8, 1959

HOUR	GAGE HEIGHT	SHIFT ADJ.	j	$\frac{\Delta Q}{J}$		ΔQ	Q_r	DISCH.
0	4.33							42
1								
2	4.34							43
3								
4	4.38							45
5								
6	4.44							50
7								
8	4.83							92
9								
10	5.75							228
11								
12	8.85	+	1.75	290		507	674	1180
13								
14	12.55	+	1.87	400		748	1430	2180
15								
16	15.88	+	1.25	610		762	2160	2920
17								
18	17.80	+	.75	1100		825	2660	3480
19								
20	19.15	+	.58	1560		905	3060	3960
21								
22	20.12	+	.37	2050		758	3430	4190 ←
23	20.43	+	.28	2150		602	3560	4160
24	20.65	+	.20	2250		450	3650	4100
Mean	10.88							1700

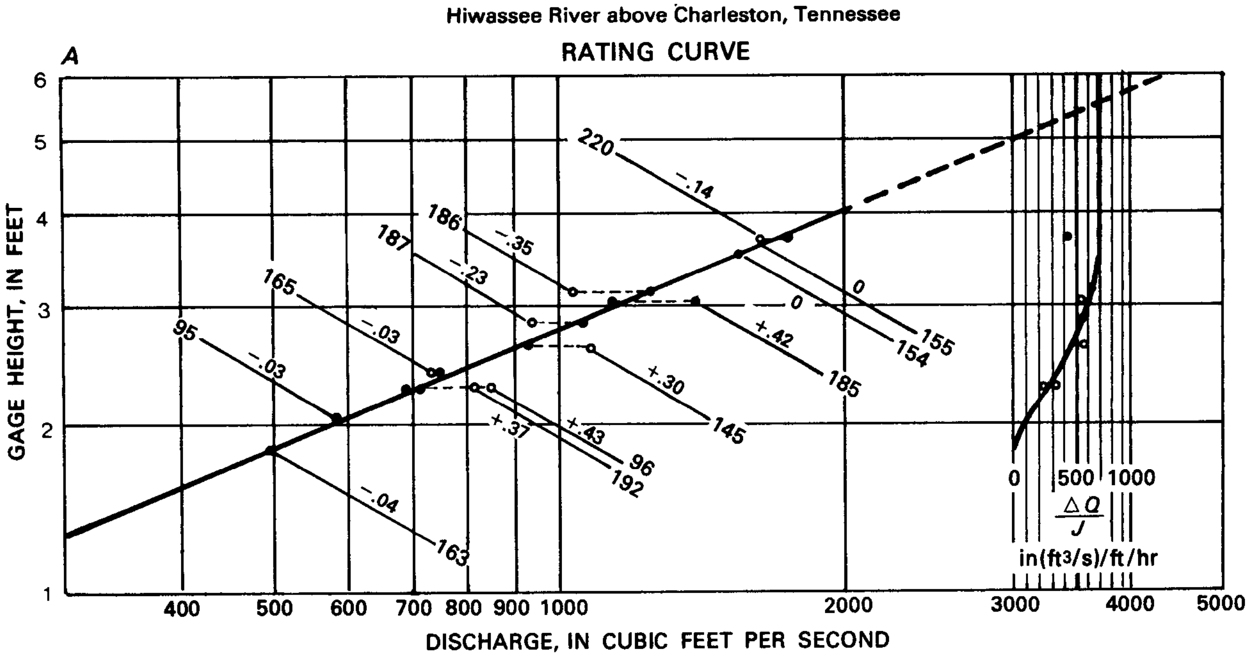
April 9, 1959

HOUR	GAGE HEIGHT	SHIFT ADJ.	j	$\frac{\Delta Q}{J}$		ΔQ	Q_r	DISCH.
0	20.65							4100
1								
2								
3	20.94	←	0					3770
4								
5								
6	20.47		-.18	2160	-	388	3580	3190
7								
8								
9								
10								
11								
12	18.36		-.44	1260	-	554	2810	2260
13								
14								
15								
16								
17								
18	15.19		-.52	550	-	286	1990	1700
19								
20								
21								
22								
23								
24	11.74		-.60	360	-	216	1240	1020
Mean	17.60							2440


B. HYDROGRAPH

C. LOOP RATING FOR RISE OF APRIL 8–10, 1959

FIGURE 16.—Formats for testing a $\Delta Q/J$ rating.



B

COMPUTATIONS

Meas. No.	*Ght*	Q_m	*J*	*Q_r	$\frac{\Delta Q}{J}$ (computed)	$\frac{\Delta Q}{J}$ (curve)	ΔQ	Q_{adj}	Percent Diff.
95	2.06	586	-.03	604	-	120	+ 5	591	-2.2
96	2.28	846	+.43	714	307	265	-114	732	+2.5
145	2.63	1080	+.30	912	560	470	-141	939	+3.0
154	3.55	1550	0	1570	-	700	0	1550	-1.3
155	3.63	1620	0	1640	-	700	0	1620	-1.2
163	1.84	492	-.04	505	-	0	0	492	-2.6
165	2.41	733	-.03	784	-	350	+ 10	743	-5.2
185	3.04	1400	+.42	1180	524	610	-256	1140	-3.3
186	3.14	1030	-.35	1250	628	635	+222	1250	0
187	2.86	934	-.23	1060	547	560	+129	1060	0
192	2.29	805	+.37	719	232	270	-100	705	-1.9
220	3.71	1650	-.14	1710	429	700	+ 98	1750	+2.3

*Scale offset −1.0, Coordinates 0.45, 100; 4.03, 2000.

SYMBOLS

Ght	Gage height (ft)	Q_r	Discharge from rating (ft^3/s)
J	Rate of stage change (ft/hr)	ΔQ	Storage correction (ft^3/s)
Q_{adj}	Adjusted discharge (ft^3/s)	% Diff.	Variation of Q_{adj} from Q_r
Q_m	Measured discharge (ft^3/s)		

FIGURE 17.—Typical storage-affected low-water discharge rating.

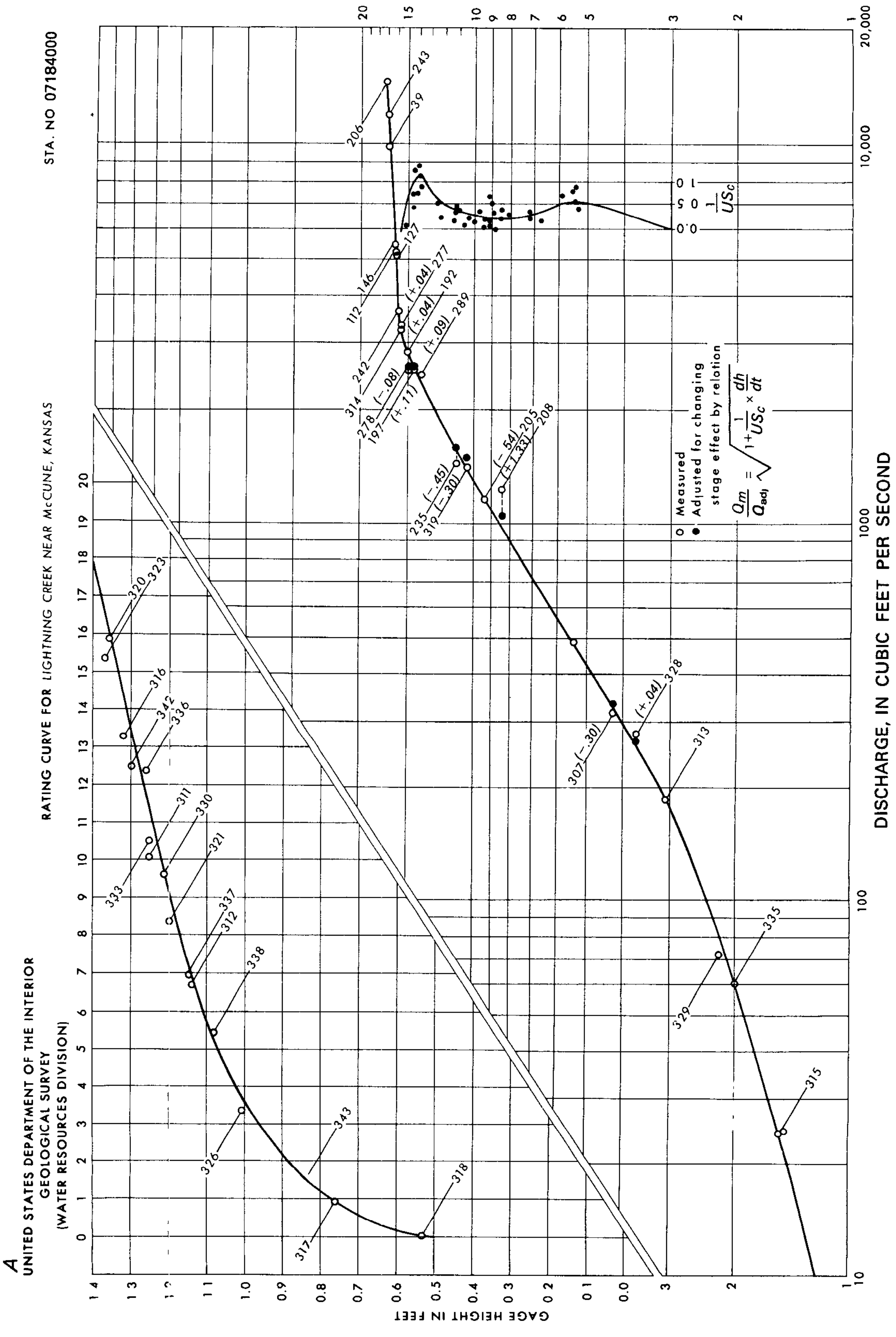


FIGURE 18.—Typical $1/US_c$ discharge rating.

B

ANALYSIS PROCEDURE

STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets and table	Needed: Sheet 1, a log-log rating grid for the constant-stage discharge curve; sheet 2, a rectangular grid for the $1/US_c$ curve; and a computation sheet with columns titled and numbered ① to ⑪ as in the example below.
2	Enter data	Fill in ① to ④ with data from the discharge measurements.
3	Plot *Q* curve data	Using sheet 1, with an appropriate *Ght* scale offset, plot *Ght* (②) vs. Q_m (③). Flag each point with *J* (④).
4	Draw first trial	This trial curve should be close to constant-stage measurements, left of rising-stage measurements, and to the right of falling-stage measurements. Skip to step 7 for the first trial computation.
5	Plot *Q* curve data	On sheet *1, plot *Ght* (②) vs. Q_{adj} (⑩).
6	Draw refined *Q* curve	This discharge curve should average the step 5 points as well as possible.
7	List Q_r values	Fill in * ⑤ from step 4 curve (first trial), step 6 curve (subsequent trials), or from the curve's descriptors for the final trial.
8	Compute and list Q_m/Q_r	In ⑥ , if *J* (④) is between +0.1 and −0.1 enter a dash. Otherwise, * ⑥ = ③ ÷ ⑤
9	Compute and list $1/US_c$	If a dash is entered in ⑥ , enter a dash in ⑦ . Otherwise, * ⑦ = (③^2 − ⑤^2) ÷ (⑤^2 x ④)
10	Plot factor curve data	Plot $1/US_c$ (⑦) vs. *Ght* (②) on sheet *2. Use a distinctive symbol for rapid-change points.
11	Draw factor curve	The factor curve should resemble figure 14*B* and be closest to those step 10 points defined by rapid-change measurements. Maximum $1/US_c$ is usually just above bankfull stage. $1/US_c$ is 0 when section control is effective and again when the flood plain contains most of the total discharge.
12	List of curve values of $1/US_c$	Fill in * ⑧ from the sheet 2 curve values for the early trials. For the final trial, use values interpolated from the curve's descriptors.
13	List computed factor	Fill in *⑨ for all measurements, regardless of magnitude of *J*. ⑨ = $\sqrt{1 + (⑧ \times ④)}$.
14	List Q_{adj}	Fill in * ⑩ = ③ ÷ ⑨
15	Next trial (step 5)	If both the *Q* and $1/US_c$ curves are unlikely to improve with further trials, proceed to step 16. For an additional trial, return to step 5.
16	Finalize	Prepare descriptors or tables for both curves.
17	Finalize	Recompute * ⑤ to ⑩ and compute ⑪ using the step 16 materials; ⑪ = 100 x (⑩ − ⑤) ÷ ⑤ If ⑪ values are satisfactory, proceed to step 18. Otherwise, return to step 5.
18	Test	See text and figure 19. If test is satisfactory, proceed to step 19. Otherwise, return to step 5 and adjust curve shapes as necessary.
19	Finalize	Prepare master curve sheet.

*Erase any entries or plotting from previous trials.

C

COMPUTATIONS

Lightning Creek near McCune, Kansas

Meas. No.	Ght	Q_m	*J*	Q_r	Q_m/Q_r	Comp. $1/US_c$	Curve $1/US_c$	Factor	Q_m /Fact = Q_{adj}	%Diff.
①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪
192	15.11	2880	0.04	2780	-	-	0.14	1.00	2872	3.3
205	9.38	1170	-0.54	1230	0.95	0.18	0.22	0.94	1246	1.3
206	17.15	14900	-0.08	14900	1.00	0.00	0.00	1.00	14900	0.0
208	8.44	1240	1.53	1040	1.19	0.28	0.24	1.17	1058	1.7
235	11.09	1450	-0.43	1610	0.90	0.44	0.41	0.91	1597	-0.8
242	15.91	3680	0.05	3770	-	-	0.00	1.00	3680	-2.4
243	16.96	12200	-0.04	12000	-	-	0.00	1.00	12200	1.7
277	15.61	3330	0.04	3280	-	-	0.00	1.00	3330	1.5
278	14.95	2560	-0.08	2680	0.96	1.09	0.40	0.98	2602	-2.9
289	13.82	2500	0.09	2350	1.06	1.46	1.18	1.05	2377	1.2
307	4.24	317	-0.30	344	0.92	0.50	0.57	0.91	348	1.2
319	10.49	1410	-0.30	1470	0.96	0.27	0.27	0.96	1471	0.0
328	3.70	280	0.04	276	-	-	0.51	1.01	277	0.4

SYMBOLS

Comp.	Computed value	Q_m	Measured discharge (ft3/s)
Curve	Value from curve	Q_r	Discharge from rating (ft3/s)
Ght	Gage height (ft)	S_c	Energy slope (ft/ft)
J=**dh/dt*	Rate of change in stage (ft/hr)	*U*	Velocity of flood wave (ft/s)
Q_{adj}	Adjusted discharge (ft3/s)	%Diff.	Variation of Q_{adj} from Q_r

****J* and *dh/dt* are both conventional symbols for rate of change in stage. *J* is more convenient to use in an equation, especially as part of the numerator or denominator of a fraction.**

FIGURE 18.—Continued.

adjustment has no apparent drawbacks with this rating and probably should be used even if no water sampling is involved.

Slope ratings

Some gaging stations, especially those on large regulated streams, are affected by variable backwater from dams almost all the time. Others, particularly those on flat gradient streams, are subject to occasional periods of backwater from downstream tributaries or from the return of overbank flow into the main channel after floods. Many such gages can be operated as slope stations by using a base gage to measure stage and an auxiliary gage some distance away to measure water-surface fall in the reach. The measured fall is an index of water-surface slope at the base gage.

The location of gages is a factor in determining the reliability of slope ratings, and, where there is a choice, several items should be considered. Both the base gage and the auxiliary gage should be stilling wells, or both should be bubble gages that compensate identically for temperature. The gages preferably should be far enough apart that minimum fall will exceed 0.5 ft, and there should be no significant tributaries or other sources of variable backwater between them. The base gage is best located at the discharge measuring section to eliminate storage adjustments. Where backwater is intermittent, the auxiliary gage should be downstream. This arrangement gives the most sensitive relation between fall and discharge and provides for positive identification of nonbackwater periods. Where backwater is always present or is caused by the return of overbank flow that has about the same magnitude upstream as it does downstream, an upstream auxiliary gage is about as good as one downstream.

Careful attention to the details of field operation (such as precise synchronization of base and auxiliary recorders, close datum control, and avoidance of current-meter measurements at velocities seriously below the limits of accurate meter registration) will improve the reliability of the lower parts of slope ratings.

Techniques that do not involve current meters can be used for low-water extensions of slope ratings at some sites. A power dam close to the gage may be a source of discharge information. Power production records usually include discharge figures, and, if all flow is through the turbines, as it generally is during low-flow periods, the discharge records during steady-flow periods may be used instead of discharge measurements. A dam downstream, where flow is cut off for long periods, may provide a reservoir that can be used as a container for volumetric measurements. The general storage equation (fig. 1) can be used to compute reservoir inflow if bank storage (underground) is not significant. Using records for other stations as a basis for extending a slope rating downward is usually a dubious practice. However, even that procedure may be more accurate than using current-meter measurements whose mean velocities are less than 0.10 ft/s.

Slope ratings fall into two broad categories: (1) constant-fall ratings in which unit fall is a special type and (2) variable-fall ratings. Unit-fall ratings are the simplest and require the fewest discharge measurements for adequate definition. Variable-fall ratings are the most complex, require more adjustments for close calibration to fit the data, and need more dischargemeasurements than the other types. the type of rating applicable to a particular site depends primarily on whether the backwater is intermittent or always present. Constant-fall ratings generally are preferred where backwater is present at all stages at all times, but they can be adapted, somewhat awkwardly, for use with intermittent backwater. Variable-fall ratings, preferable where backwater is intermittent, also can be used for full-time backwater sites but are difficult to define without free-fall discharge measurements.

Unit-fall ratings

A unit-fall rating is the relation between stage and the discharge when the fall in the reach is 1 ft. The rating is developed by plotting each measured discharge divided by the square root of its measured fall against the measurement's base gage height. The rating curve is then fitted to the plotted points. Discharge corresponding to any combination of

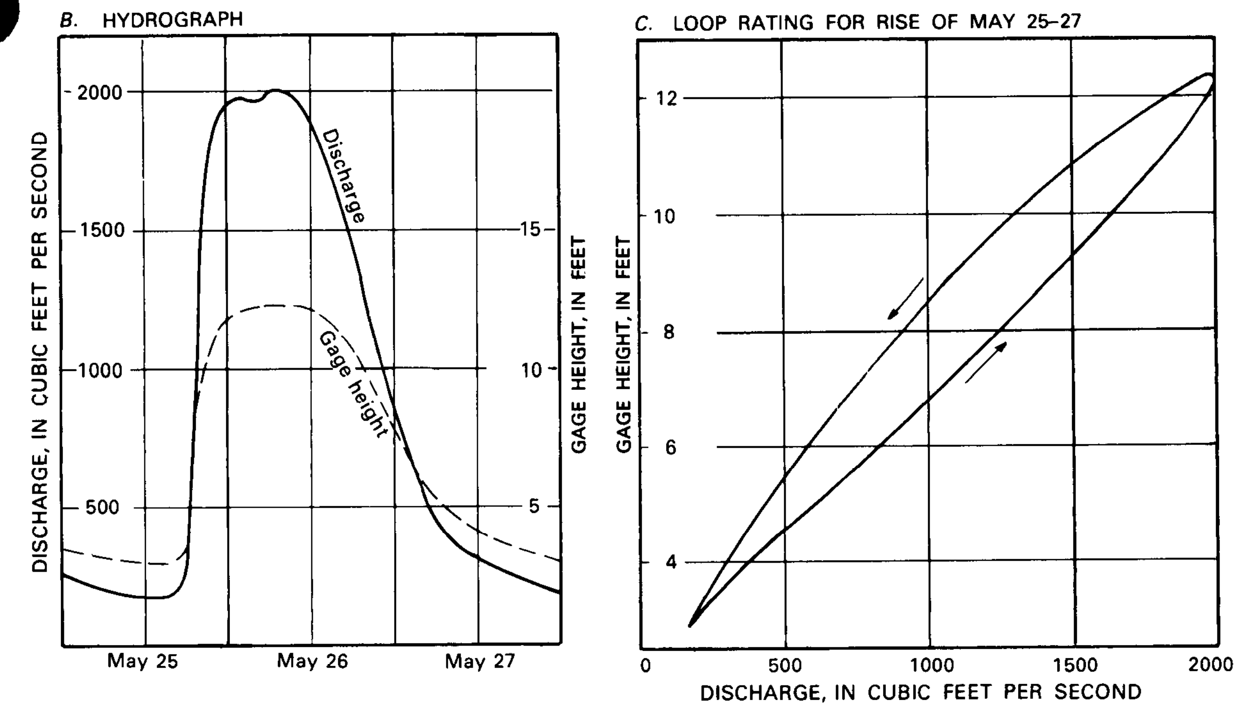
A. DAILY DISCHARGE COMPUTATION

May 25

HOUR	GAGE HEIGHT	SHIFT ADJ.	$\frac{dh}{dt}$	$\frac{1}{US_c}$		FACTOR	Q_r	DISCH.
0	3.55		-.10	.25		.98	255	250
1								
2								
3								
4	3.28		-.05	.10		1.00	220	220
5								
6								
7								
8	3.08							196
9								
10								
11								
12	2.95							178
13								
14	2.91							175
15								
16	2.92							176
17								
18	3.60		+1.50	.26		1.18	242	286
19								
20	8.90		+2.30	.22		1.23	1160	1430
21								
22	11.06		+.60	.40		1.11	1660	1840
23								
24	11.92		+.20	.46		1.04	1880	1960
Mean	4.67							520

May 26

HOUR	GAGE HEIGHT	SHIFT ADJ.	$\frac{dh}{dH}$	$\frac{1}{US_c}$		FACTOR	Q_r	DISCH.
0	11.92							1,960
1								
2	12.14		+.04	.52		1.01	1940	1960
3								
4	12.18		+.03	.49		1.01	1950	1970
5								
6	12.25		+.03	.50		1.01	1980	2000 ←
7								
8	12.29	←	0					1990
9								
10	12.23		-.05	.50		.99	1970	1950
11								
12	12.10		-.08	.48		.98	1930	1890
13								
14	11.80		-.15	.45		.96	1850	1780
15								
16	11.37		-.25	.41		.95	1740	1650
17								
18	10.65		-.35	.26		.95	1560	1480
19								
20	9.77		-.50	.26		.93	1340	1250
21								
22	8.65		-.48	.22		.95	1110	1050
23								
24	7.68		-.40	.22		.95	916	870
Mean	11.27							1700


B. HYDROGRAPH

C. LOOP RATING FOR RISE OF MAY 25–27

FIGURE 19.—Formats for testing a $1/US_c$ rating.

stage (base gage height) and fall can be computed by multiplying the discharge value corresponding to the stage by the square root of the fall. The rating applies without adjustment when the fall—and its square root—is 1.00. This type of rating usually is satisfactory where backwater is always present, fall is rarely below 0.5 ft, and the datum difference between base and auxiliary gages is known within about 0.05 ft. If these limits are exceeded, the unit-fall rating should be used only in the preliminary analysis for a more complex rating.

Figure 20*A* illustrates a unit-fall rating analysis for a site where backwater from a power dam is high at all times and stages. The same discharge measurement data were used to develop the constant-fall rating shown in figure 21. The measurement percentage differences from both analyses, listed in the last two columns of figure 20*A*, are not significantly different, an indication that the unit-fall rating is about as good as any that can be developed for this station, at least for falls greater than 0.5 ft.

Figure 20*B* illustrates a unit-fall rating analysis for a site where backwater is intermittent during floods and absent at low stages. The discharge measurement data listed also were used to develop the limiting-fall rating in figure 22*A*. The percentage differences in discharge measurements from both analyses, shown in figure 20*B*, are closely comparable. However, a factor other than the fit of the data to the rating must be considered in rating unit falls. The capacity of the channel to carry flow during backwater periods depends on the fall in the reach—the greater the fall, the greater the discharge. The carrying capacity during nonbackwater periods depends only on the geometry and roughness of the controlling reach. Fall in excess of the amount needed to assure the absence of backwater cannot indicate more discharge than the channel's capacity. Constant- or unit-fall ratings lack limiting criteria, and discharge computed by using this kind of rating during a nonbackwater period usually will be greater than the actual discharge. A limiter can be provided by using an auxiliary free-fall rating, a simple rating based only on the nonbackwater discharge measurements. The simple rating is used for a preliminary computation of records. Records for high-water periods when backwater is likely are then computed, by manual methods if only a few days are involved, as figure 20*C* illustrates. The smaller of the two discharge figures for the free-fall rating and the slope rating is accepted as the true value. This combination of free-fall rating and unit-fall auxiliary slope rating would probably be the best rating choice for the site used for the illustration if only a few discharge measurements indicating backwater had been made.

Constant-fall ratings

A constant-fall rating uses two curves: (1) the relation between stage and the discharge when the fall in the reach is some specified value, usually about 1 ft, and (2) a factor curve of fall (F_m) versus discharge ratio (Q_m/Q_r). The symbols used are defined in figure 21*C*. This rating type is similar to a unit-fall rating except that the factor curve replaces the square root relation ($Q_m/Q_r=\sqrt{F_m}$). A unique feature of the constant-fall rating is that the base gages and the auxiliary gages need not be at or adjusted to the same datum. A factor curve showing the relation of gage difference (base *Ght* less auxiliary *Ght*) to discharge ratio (Q_m/Q_r) can be used about as well as the ratio of fall to discharge. Figure 21*A* illustrates a constant-fall rating for a gaging station where backwater from a dam is always present and where slopes are highly variable owing to rapid fluctuation of discharge. The rating analysis computations in figure 21 indicate that instantaneous discharges from the rating are reliable above about 10,000 ft^3/s and satisfactory down to about 5,000 ft^3/s. Daily values are probably reliable at somewhat lower discharges. The factor curve would be close to a square-root relation of factor versus fall if the auxiliary gage datum were raised 0.03 ft. If that datum change is made, the constant-fall rating would be very close to the unit-fall rating in figure 20*A*.

Most constant-fall ratings are developed by drawing a unit-fall rating as a trial curve and using that trial rating to compute a factor (discharge ratio versus fall) curve. The factor curve is then used to improve the rating, which in turn is used to refine the factor curve. The process is continued until consequential improvement stops, usually after about three trials. The analysis can be done by using steps similar to those listed in figure 21*B*. This procedure gives a discharge curve that corresponds to a constant fall of about 1 ft. Some hydrographers prefer a discharge curve whose values approximate actual discharge during floods. Such a curve can sometimes be obtained by using a value closer to the average observed fall as the constant-fall value. If a constant-fall value other than 1.0 is wanted, the figure 21*B* procedure (step 3) provides for the conversion.

Limiting-fall ratings

A gaging station affected by intermittent backwater from tributaries or a dam may be operated for long periods as a simple rating station but needs a slope rating for some or all of the high-water periods. This type of station works best with a limiting-fall rating composed of three parts: (1) a discharge curve that represents a simple rating applicable for nonbackwater conditions and indicates the maximum possible discharge at any stage regardless of fall, (2) a fall curve that varies with stage and indicates the minimum fall in the slope reach under nonbackwater conditions, and (3) a factor curve of the relation Q_m/Q_r (ratio of measured discharge to rating discharge) versus F_m/F_r (ratio of measured fall to rating fall).

Figure 22*A* illustrates a typical limiting-fall rating for a site where backwater is intermittent. The flat-slope channel has a low-water section control, a high-water rating storage loop, and variable backwater from tributaries.

The three-curve rating analysis is much more complex than the two-curve types shown in previous examples. A limiting-fall slope rating has three interrelated component curves (discharge, fall, and factor). When two of the three components corresponding to each discharge measurement are fixed, the magnitude of the third needed to cause a perfect fit for that discharge measurement can be computed. The discharge and factor curves are tentatively drawn and "fixed" as the first step. The value of the "perfect-fit" fall for each discharge measurement is then computed and used as a plotting point to define the fall curve. Each curve is then refined in rotation by fixing the other two curves and using the perfect-fit points defined by the discharge measurements to draw or improve the unfixed or open curve.

Usually, after each curve has been refined about three times in this manner, further improvement is minimal. The computations can be made manually in steps similar to those listed in figure 22*B*, or the trial-and-error work can be facilitated by using an appropriate computer facility and the instructions contained in the WATSTORE User's Guide (Hutchison and others, 1975, 1980).

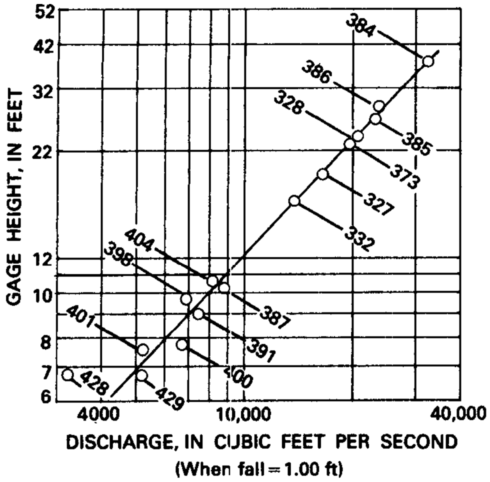
Normal-fall ratings

A normal-fall slope rating is identical to a limiting-fall rating except that the factor curve extends above the coordinates (1,1). Observed fall greater than the normal fall curve value indicates that actual discharge is greater than the discharge curve value instead of equal to it, as it would be for a limiting-fall rating. Normal-fall ratings are used sometimes where high-water measurements fail to indicate a limiting position for the discharge curve. Most such ratings are developed as limiting-fall types below a specified stage and as normal fall above. They also have some application to full-time backwater sites, where the three-component curves provide more opportunity than a two-curve constant-fall rating to achieve agreement between the discharge measurements and the rating. Three-component curves can be a disadvantage, however, because it is possible to warp the rating inadvertently into agreement with faulty data.

The analysis procedure is identical to the limiting-fall method outlined in figure 22*B* except that, in step 1, no dashes are inserted in the computation columns for high-fall measurements, and the discharge curve need not be drawn to the right of the measurement scatter. An example of a normal-fall analysis is not given because of its similarity to the much more common limiting-fall analysis.

Cumberland River at Carthage, Tennessee

A. RATING CURVE

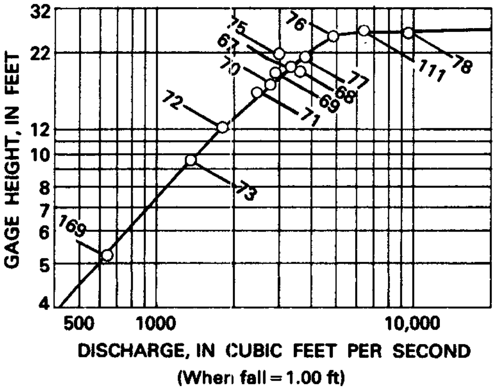


COMPUTATIONS

Meas. No.	Gage Height	F_m	Q_m	$\frac{Q_m}{\sqrt{F_m}}$	Q_r	%Diff. Unit Fall	%Diff. Const. Fall
327	19.38	6.29	41,000	16,300	16,500	- 1.2	- .6
328	23.31	7.16	53,800	20,100	20,400	- 1.5	+ .5
332	16.49	5.24	31,400	13.700	13,900	- 1.4	- .7
373	23.01	7.30	52,700	19,500	19,700	- 1.0	- .5
384	37.92	9.45	99,800	32,500	32,700	- .6	0
385	26.50	6.30	57,900	23,000	22,800	+ .9	+ 1.3
386	28.34	8.70	70,300	23,800	24,400	- 1.6	- 1.2
387	10.30	2.65	14,100	8,660	8,220	+ 5.4	+ .6
391	9.04	2.30	11,200	7,390	7,040	+ 5.0	- 1.0
398	9.72	2.02	10,200	7,180	7,680	- 6.5	-11.5
400	7.74	.67	5,520	6,740	5,810	+16.0	+12.9
401	7.50	.95	5,130	5,260	5,580	- 5.7	- 9.9
404	10.52	3.04	14,500	8,320	8,420	- 1.2	- 5.4
428	6.68	.19	1,410	3,230	4,790	-32.6	-22.5
429	6.60	.20	2,330	5,210	4,710	+10.6	-24.0

Cottonwood River near Florence, Kansas

B. RATING CURVE



COMPUTATIONS

Meas. No.	Gage Height	F_m	Q_m	$\frac{Q_m}{\sqrt{F_m}}$	Q_r	%Diff. Unit Fall	%Diff. Lim. Fall
67	19.54	5.13	7,570	3,340	3,280	+ 1.8	+ 1.4
68	19.38	3.91	6,920	3,500	3,250	+ 7.7	- 1.1
69	18.42	3.28	5,430	3,000	3,050	- 1.6	0
70	17.21	2.84	4,740	2,810	2,800	+ .4	+ 2.3
71	15.96	2.68	4,210	2,570	2,550	+ .8	+ 2.3
72	12.22	1.95	2,490	1,780	1,800	- 1.1	- 1.8
73	9.56	1.59	1,720	1,360	1,360	0	- 2.8
75	21.52	6.67	7,670	2,970	3,700	-19.7	-10.4
76	25.28	8.08	13,800	4,850	4,850	0	+ 5.3
77	21.37	5.15	8,230	3,630	3,670	- 1.1	- 1.1
78	26.50	8.14	27,100	9,500	9,480	+ .2	+ 4.2
111	26.25	8.12	18,500	6,490	6,490	0	0
169	5.20	1.79	839	627	630	- .5	- .7

SYMBOLS

F_m Measured fall (ft)
Q_{adj} Adjusted discharge (ft³/s)
Q_m Measured discharge (ft³/s)
Q_r Discharge from rating (ft³/s)

FIGURE 20.—Typical unit-fall slope ratings.

Ratings for regulating control structures

Dams can be used as gaging-station sites by rating the fixed spillways, gates, turbines, and locks separately. The procedures, explained and illustrated by Collins (1977), have little in common with those described in this manual.

Index-velocity ratings

An index-velocity gaging station generally is used where backwater is variable, particularly from tide, and the water-surface slope is too flat for a slope rating. The equipment consists of a stage recorder and a device that records an indicator of the stream velocity. Stage and index velocity are correlated with discharge in several ways that depend on the type and placement of the equipment. Deflection vanes are used as velocity sensors on most of the older index-velocity stations. Most new installations use electromagnetic meter probes permanently mounted at the index location. An acoustic velocity meter that records the average stream velocity along a line between two underwater transducers mounted diagonally

MERGING OF DISCHARGE VALUES FROM NON-LIMITING SLOPE RATING AND SIMPLE RATING AT END OF BACKWATER PERIOD

HOUR	GAGE HEIGHT	F_m	Q_r	$Q_{adj.}$	Q_r	DISCH.
		Slope Rating			Non-BW Rating	
0	18.61	5.16	3090	7020	7100	7100
1						
2	19.21	5.29	3220	7410	7400	7400
3						
4	19.69	5.36	3320	7690	7640	7640
5						
6	20.05	5.40	3390	7880	7820	7820
7						
8	20.27	5.42	3440	8010	7940	7940
9						
10	20.48	5.42	3480	8100	8040	8040
11						
12	20.55	5.41	3500	8140	8080	8080
13	20.56	5.36	3500	8100	8080	8080
14	20.53	5.30	3490	8030	8060	8030
15						
16	20.43	5.16	3470	7880	8020	7880
17						
18	20.27	5.01	3440	7700	7940	7700
19						
20	20.05	4.83	3390	7050	7820	7050
21						
22	19.71	4.59	3320	7110	7660	7110
23						
24	19.11	4.29	3200	6630	7360	6630
Mean	20.01					7660

FIGURE 20.—Continued.

across the stream from one another can be used, usually at a deep river site that requires instant onsite computation of highly accurate discharge records.

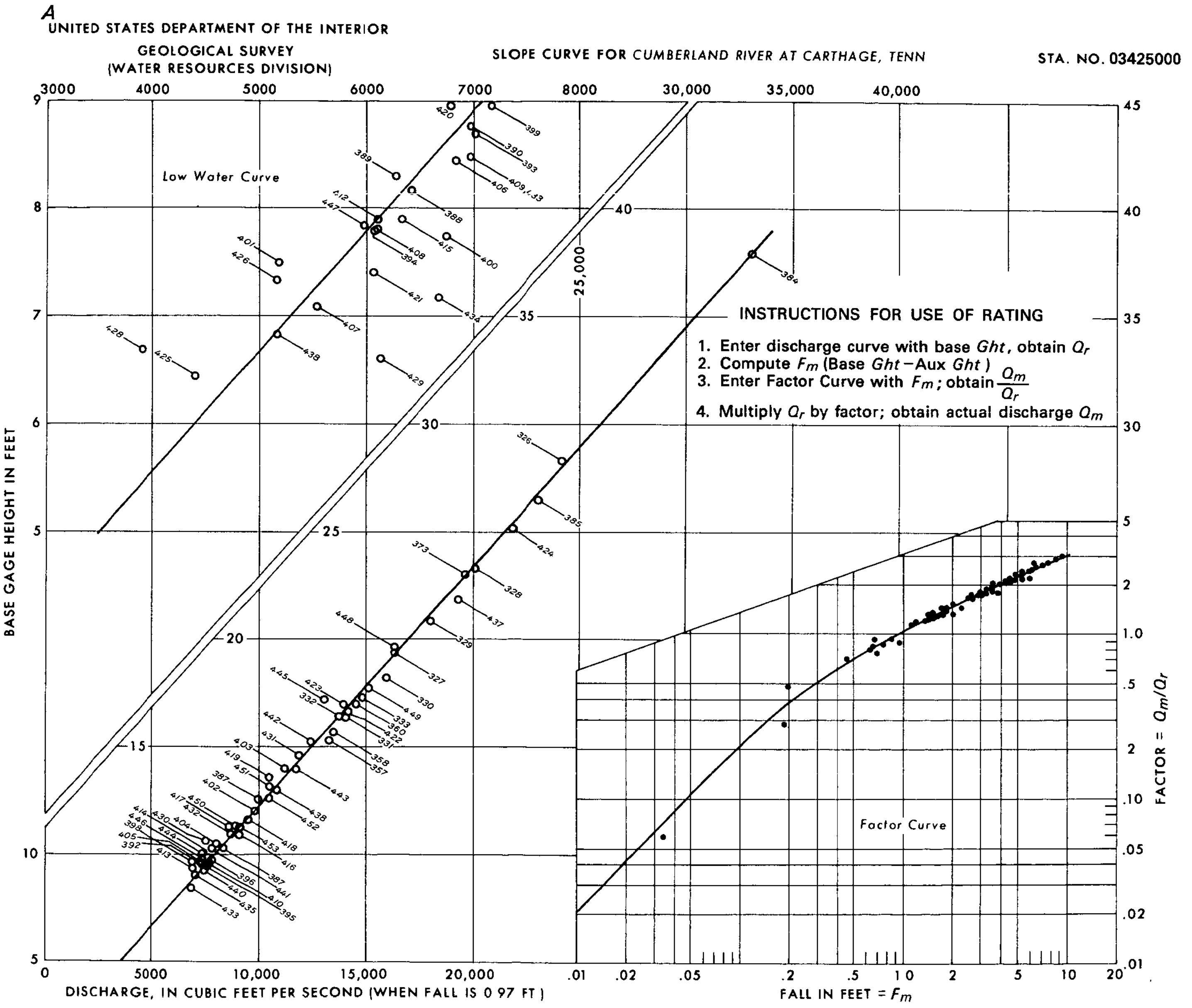
Electromagnetic meters and acoustic meters measure the index velocity directly in feet per second. Deflection-gage readings can be recorded in degrees of rotation but are usually graduated in nonlinear arbitrary units, which complicate the rating analysis considerably. Figure 23 illustrates a relation of index velocity to mean velocity in the cross section. The relation varies considerably with stage. The family of curves shown is typical for magnetic meters or acoustic gages whose sensors are high enough above the streambed to be in a live (stagnant only at zero flow) part of the cross section. Lower sensors would place the index location in a less stable part of the vertical velocity curve, and the family of curves would be less likely to be made up of nearly straight lines. The equivalent curves for a deflection vane would have complex S shapes that are particularly difficult to define.

A curve showing stage versus area represents the total of relatively dead and relatively live parts of a cross section. Cross-section scour or fill in a relatively dead area has little effect on the relation of stage and index velocity to discharge, whereas a similar change within a live area has a large effect on the rating. A change in the total area may or may not indicate a consequential rating change.

An index-velocity rating is composed of from one to three curves. One-curve ratings (stage versus effective area) can be used for most acoustic-velocity-meter installations and for some magnetic-meter stations where the discharge is directly proportional to the index velocity. Two-curve ratings (stage versus coefficient and index velocity versus adjusted discharge) can be used at all index-velocity stations. Three-curve ratings (stage versus coefficient, stage versus area, and index velocity versus adjusted mean velocity), generally more complex and less reliable than two-curve ratings, can be used where there is some special need to derive and maintain a curve showing stage versus total area.

Most of the rating relations can be expressed as equations by using the procedures shown in figure 10. Equations are the only means of entering ratings in some acoustic-meter processors and greatly simplify the use of calculators and computers in all index-velocity rating computations.

Some index-velocity ratings used on canals or estuaries apply to both upstream and downstream flow but most require separate ratings. Vane-gage and magnetic-meter ratings are most reliable in trapezoidal channels where the velocities are reasonably well distributed throughout the cross section and where the velocity sensor is located as high as the stage range allows in a live area free from obstruction. These ratings are least satisfactory where the velocity sensor is isolated from the main channel or wherever there is a combination of a wide channel and low velocities during a period of high winds. Rating problems also can be caused by a sensor that is inaccessible for regular cleaning or that is located where it can be bumped by debris or river traffic. Acoustic velocity meters are less sensitive to these conditions but may malfunction because of unusually high sediment concentration or air entrainment. A channel that is too large or complex



INSTRUCTIONS FOR USE OF RATING

1. Enter discharge curve with base *Ght*, obtain Q_r
2. Compute F_m (Base *Ght* – Aux *Ght*)
3. Enter Factor Curve with F_m; obtain $\frac{Q_m}{Q_r}$
4. Multiply Q_r by factor; obtain actual discharge Q_m

FIGURE 21.—Typical constant-fall slope rating.

B

ANALYSIS PROCEDURE

STEP OPERATION	INSTRUCTIONS
1 Prepare work sheets	Needed: Sheets 1 and 2, log-log rating grids for trial discharge curves; sheet 3, a log-log grid for the factor curve; and a computation sheet with columns titled and numbered ① to ⑩ as in the example below (a column ⑪ is needed for the first trial only).
2 Enter data	Fill in ① to ⑤ with data from the discharge measurements.
3 Compute unit-fall discharge	Fill in ⑪ = ⑤ ÷ $\sqrt{④}$(for a value of contstant fall, *n*, other than 1 foot, ⑪ = (⑤ ÷ $\sqrt{④}$) ÷ $\sqrt{n}$).
4 Draw preliminary discharge curve	Plot on sheet 1, ⑪ vs. ② . Flag all points whose ④ < 1.0. Draw the curve, using an appropriate *Ght* scale offset, giving the least weight to flagged points.
5 Fill in Q_r	Fill in * ⑥ from the sheet 1 curve (first trial), sheet 2 curve (intermediate trials), or the curve descriptors for the final trial.
6 Draw factor curve	Fill in * ⑦ (*Y* axis) vs. ④ on sheet *3. Draw the curve, giving equal weight to all points. Preferred final curve format is an equation (figure 10*B* or 10*C*).
7 Fill in factor	Fill in * ⑧ from sheet 3 curve or equation.
8 Fill in Q_{adj}	Fill in * ⑨ = ⑤ ÷ ⑧ .
9 Draw trial discharge curve	Plot, with the step 4 *Ght* scale offset, ⑨ vs. ② on sheet 2. Draw the curve, giving equal weight to all but very low velocity discharge measurements (low ④). Preferred final curve format is a set of log descriptors (figure 7).
10 Go to step 5	Repeat steps 5 to 9 about three times or until further improvement is unlikely. Then proceed to step 11.
11 Finalize	Prepare the curves in final format (descriptors, equation, or tables).
12 Finalize	Recompute * ⑥ to ⑨ and compute ⑩ with step 11 materials. ⑩ = 100 (⑨ − ⑥) ÷ ⑥ . If ⑩ values are unsatisfactory, return to step 5 giving special attention to outliers. Otherwise, proceed to step 13.
13 Finalize	Prepare the master curve sheet.

*Erase any entries or plotting from previous trials.

C

COMPUTATIONS

Cumberland River at Carthage, Tennessee

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪
Meas. No.	Gage Height		F_m	Q_m	Q_r	$\frac{Q_m}{Q_r}$	Factor (Table)	$Q_{adj} = \frac{Q_m}{\text{Factor}}$	%Diff.	1st Trial Q_r $\frac{Q_m}{\sqrt{F_m}} \div \sqrt{n}$
	Base	Aux								
327	19.38	13.09	6.29	41000	16400	2.500	2.508	16300	− .6	16300
328	23.31	16.15	7.16	53800	20000	2.690	2.674	20100	+ .5	20100
332	16.49	11.25	5.24	31400	13800	2.275	2.298	13700	− .7	13700
373	23.01	15.71	7.30	52700	19700	2.675	2.695	19600	− .5	19500
384	37.92	28.47	9.45	99800	33100	2.015	3.018	33100	0	32500
385	26.50	20.20	6.30	57900	22800	2.539	2.510	23100	+ 1.3	23100
386	28.34	19.64	8.70	70300	24500	2.869	2.905	24200	− 1.2	23800
387	10.30	7.65	2.65	14100	8270	1.705	1.695	8320	+ .6	8660
391	9.04	6.74	2.30	11200	7140	1.569	1.585	7070	− 1.0	7390
398	9.72	7.70	2.02	10200	7750	1.316	1.487	6860	−11.5	7180
400	7.74	7.07	.67	5520	5970	.925	.819	6740	+12.9	6740
401	7.50	6.55	.95	5130	5750	.892	.990	5180	− 9.9	5260
404	10.52	7.48	3.04	14500	8470	1.712	1.810	8010	− 5.4	8320

SYMBOLS

F_m	Measured fall	%Diff.	Variation of Q_{adj} from Q_r
Q_{adj}	Discharge adjusted to rating fall	ⓝ	Column number on computation sheet
Q_m	Measured discharge	n	Constant fall other than 1.00
Q_r	Discharge from rating curve		

FIGURE 21.—Continued.

or whose velocity distribution is too variable to rate with one velocity sensor can be subdivided with a separate sensor and rating for each subarea.

Vane-gage ratings

A vane gage is a mechanical velocity sensor whose components are usually arranged approximately as they are in figure 24*A*. This type of vertical axis vane is deflected by the force of the current acting against the torque from a counterweight. The linkage from the counterweight to the vane varies the resisting torque from zero at zero velocity to a maximum at about 45° deflection. A cam or some other device can vary the torque further at higher deflections. Some vanes have springs rather than counterweights, and others have horizontal axes where the weight of the pendulum vane furnishes the resistance to deflection. The recorder linkage can be arranged so that deflection is recorded in degrees or a multiple of degrees, but counterweight movement is usually recorded. Most velocity-sensor scales are in arbitrary nonlinear units, and the scale often is offset so that zero velocity gives a scale reading of 1, 5, or 10.

The rating analysis method, outlined in detail in figure 24*H*, is a trial-and-error procedure. The relation between vane deflection and discharge is a family of curves, one for each stage, that are parallel to each other on a logarithmic grid. The family of curves is roughly defined by the discharge measurements, as figure 24*C* shows. The best-defined single curve from the family (the 2.5-ft stage curve in fig. 24*C*) is used as the first trial curve for the base-stage rating (fig. 24*D*). The ratio of measured discharge to discharge from the trial base-stage rating (fig. 24*G*) defines a stage-coefficient curve. Each measured discharge is divided by its stage coefficient and used to refine the base-stage rating. The refined rating is then used to improve the stage-coefficient curve and vice versa until, usually after about three trials, further improvement is unlikely.

The base-stage rating curve is best described by logarithmic digital descriptors (fig. 24*E*). The gage-height coefficient curve can be described by an equation using the procedure shown in figure 10*B*. The entire rating is described by the equation in figure 24*F*, which combines the base-stage rating and the stage-coefficient relation.

If subsequent discharge measurements indicate that a rating shift has occurred, the measurements should be used to redefine the base rating curve and to obtain a different set of descriptors. If a temporary condition, such as aqueous growth on the vane, causes the rating to change, shift adjustments varied with time only can be applied to the coefficient a_0 (in this rating, $a_0=0.5725$). For instance, if a shift to measurement 635 (fig 24*B*) is considered necessary, its amount is ⑥−⑦ (see symbols in fig. 24*B*) or $0.86-0.93=-0.07$. This shift would modify the rating equation applicable to measurement 635 ($G=2.03$, $V_g=0.60$, shift is -0.07, and $Q_b=293$) to

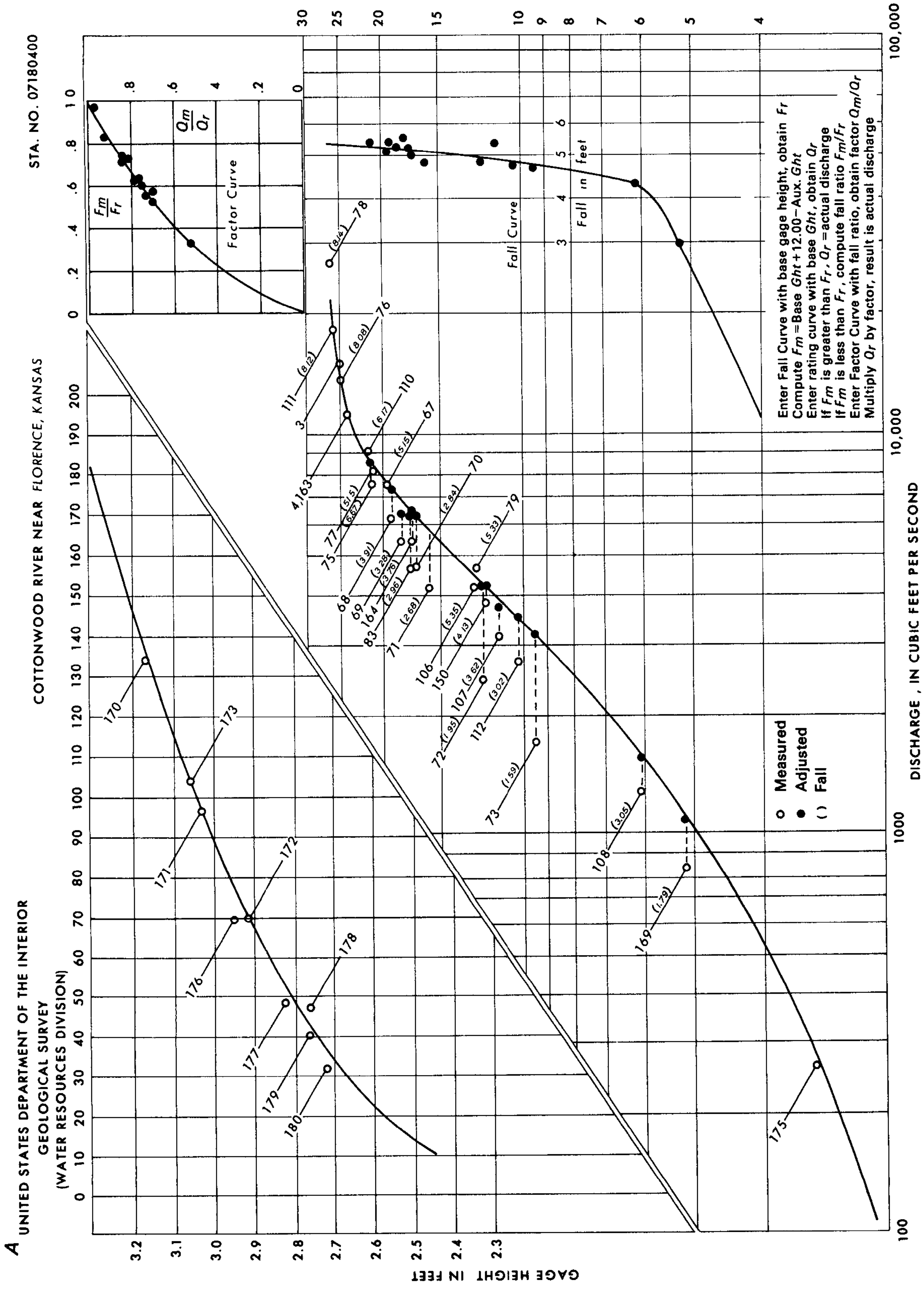
$$Q_r=293[(0.5725-0.07)+(0.187\times2.03)-(0.0047\times2.03^2)]=253 \text{ ft}^3/\text{s}$$

This shifted value changes the percentage difference for measurement 635 from −7.3 to 0, and the daily discharge computation would be changed accordingly.

Few vane-gage ratings are likely to approach the quality of the one illustrated in figure 24. The equipment is a well-designed, well-constructed Keeler deflection meter. The channel is a wooden flume 48 ft wide, and the freshwater site is free from the common, serious problems of channel shifting and heavy aqueous growth on the vane. The good equipment and conditions plus the unusually wide distribution of the discharge measurements result in an exceptionally reliable rating for a vane gage.

Magnetic-meter gage ratings

Electromagnetic meter equipment is usually arranged approximately as figure 25*A* shows. The velocity sensor, the probe of the magnetic meter, is usually attached to the end of a pipe, which generally is part of a frame that permits the probe to extend into an unobstructed area within the live part of the cross section. A typical frame is designed to permit easy removal of the probe for periodic cleaning and to facilitate its replacement in precisely the original location. Minor probe movement or rotation is likely to affect the rating. The rating analysis



Enter Fall Curve with base gage height, obtain F_r
Compute F_m = Base Ght + 12.00 − Aux. Ght
Enter rating curve with base Ght, obtain Q_r
If F_m is greater than F_r, Q_r = actual discharge
If F_m is less than F_r, compute fall ratio F_m/F_r
Enter Factor Curve with fall ratio, obtain factor Q_m/Q_r
Multiply Q_r by factor, result is actual discharge

FIGURE 22.—Typical limiting-fall slope rating.

B

ANALYSIS PROCEDURE

STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets	Needed: Sheets 1 and 2, log-log rating grids for trial discharge curves; sheet 3, a rectangular grid for trial fall curves (*Fm* along the *X* axis); sheet 4, log-log grid for trial factor curves (*Fm*/*Fr* along the *X* axis); and a computation sheet with the columns titled and numbered from ① to ⑭ as in the example (fig. 22*C*).
2	Enter data	Fill in ① to ⑤ with data from all discharge measurements made at stages above the low-water section control range. Enter dashes in ⑦ , ⑨ , ⑩ , ⑬ and ⑭ for each nonbackwater measurement.
3	Draw preliminary discharge curve	Plot ② vs. ⑤ on sheet 1. Choose *Ght* scale offset to straighten the lower end of the curve. Draw the curve close to all nonbackwater measurements and to the right of the backwater measurement scatter.
4	Fill in *Qr*	Fill in ⑥ from the sheet 1 curve, which can then be discarded.
5	Fill in *Qm*/*Qr*	Fill in ⑦ = ⑤ ÷ ⑥ .
6	Fill in *Fm*/*Fr*	For this first approximation, ⑬ = $⑦^2$.
7	Fill in F_{adj}	Fill in * ⑭ = ④ ÷ ⑬ .
8	Plot fall curve	Plot ⑭ vs. ② on sheet *3. Draw the curve, which is usually parabolic. *Fr* is 0 at *GZF* and is usually maximum at the maximum stage. The preferabale final format is a set of descriptors for linear interpolation.
9	Fill in *Fr*, *Fm*/*Fr*	Fill in * ⑧ from the sheet 3 curve or its descriptors and * ⑨ = ④ ÷ ⑧ .
10	Plot factor curve	Plot ⑨ vs. ⑦ on sheet *4. Draw the curve, which should approximate ⑨ = $⑦^2$ at its upper end. The preferable final curve format is an equation (figure 10*B* or 10*C*)
11	Fill in factor	Fill in * ⑩ from the sheet 4 curve or its equation.
12	Fill in Q_{adj}	Fill in * ⑪ = ⑤ ÷ ⑩ .
13	Plot discharge curve	Plot ⑪ vs. ② on sheet *2. Draw the curve, giving equal weight to all points except those for very low fall mesurements (low ④). Use step 3 *Ght* scale offset. The preferable final format is a set of log curve descriptors (fig. 7).
14	Fill in *Qr*, *Qm*/*Qr*	Fill in * ⑥ from the sheet 2 curve or its descriptors and * ⑦ = ⑤ ÷ ⑥ .
15	Fill in *Fm*/*Fr*	Fill in * ⑬ from the sheet 4 curve or equation. Enter the curve with ⑦ to obtain ⑬
16	Go to step 7	Repeat steps 7–15 until improvement stops, then proceed to step 17.
17	Finalize	Prepare all curves in final format (descriptors, equations, or tables).
18	Finalize	Recompute * ⑥ to ⑭ from step 17 material. ⑫ =100(⑪ − ⑥)− ⑥ . If ⑫ values are unsatisfactory, return to step 7, giving special attention to outliers. Otherwise proceed to step 19.
19	Finalize	Prepare the master curve sheet.

*Erase any entries or plotting from previous trials.

FIGURE 22.—Continued.

procedure, outlined in detail in figure 25*H*, is almost identical to that for a vane gage. However, the direct recording of index velocity in feet per second removes most of the nonlinearity from the relations, reduces the number of trial-and-error steps needed, and makes a reliable rating possible from a limited number of discharge measurements.

The discharge measurements are plotted (fig. 25*C*) in the same manner as those of a vane gage. For a magnetic meter, the family of curves for index velocity versus discharge is likely to be a series of parallel straight lines on a logarithmic grid. One curve from the family is selected as a base curve, and its corresponding stage is the base stage. A stage-coefficient curve (fig. 25*E*) is defined by the ratio of each measured discharge to the discharge from the base-stage rating plotted against stage. The coefficient curve is used to adjust the discharge measurements to the base stage (fig. 25*D*). The base stage rating and the coefficient curve are each used to refine the other until the rating is satisfactory. Both curves can be put into equation form by using the methods outlined in figure 10, and the end product can be a relatively simple equation (fig. 25*G*).

If a temporary condition, such as debris on the probe, causes the rating to shift, adjustments that are varied with time only can be applied to the coefficient a_0 of the stage-coefficient equation (in the rating illustrated, $a_0=0.387$). For instance, if a shift to measurement 11 was justified, its amount would be ⑦−⑧ (see symbols in fig. 25*C*) or

C

COMPUTATIONS

Cottonwood River near Florence, Kansas

Meas. No.	Gage Height Base	Gage Height Aux.*	F_m	Q_m	Q_r	Q_m/Q_r	F_r	F_m/F_r	Factor from curve	Q_m/Fact = Q_{adj}	% Diff.	F_m/F_r from curve	$\frac{F_m}{F_m/F_r} = F_{adj}$
①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭
67	19.54	26.39	5.15	7570	7530	1.01	5.28	0.97	0.99	7680	2.0	1.00	5.15
68	19.38	27.47	3.91	6290	7450	0.84	5.28	0.74	0.85	7410	-0.5	0.73	5.33
69	18.42	27.14	3.28	5430	6980	0.78	5.25	0.63	0.77	7020	0.6	0.63	5.20
70	17.21	26.37	2.84	4740	6400	0.74	5.21	0.55	0.72	6590	3.0	0.58	4.92
71	15.96	25.28	2.68	4210	5840	0.72	5.16	0.52	0.70	6020	3.1	0.55	4.88
72	12.22	22.27	1.95	2490	4210	0.59	5.01	0.39	0.60	4170	-1.0	0.38	5.10
73	9.56	19.97	1.59	1720	3080	0.56	4.77	0.33	0.55	3130	1.6	0.34	4.62
75	21.52	26.85	6.67	7670	8540	--	5.34	--	--	7670	-10.2	--	
76	25.28	29.20	8.68	13800	13400	--	5.40	--	--	13800	3.0	--	
77	21.37	28.22	5.15	8230	8460	0.97	5.33	0.97	0.98	8390	-0.8	0.95	5.42
78	26.50	30.36	8.14	27100	26000	--	5.40	--	--	27100	4.2	--	
79	12.69	19.36	5.33	4720	4420	--	5.04	--	--	4720	6.8	--	
83	17.77	26.81	2.96	4710	6670	0.71	5.22	0.57	0.73	6420	-3.7	0.53	5.61
105	4.96	9.75	7.21	907	939	--	2.29	--	--	907	-3.4	--	
106	12.78	19.43	5.35	4240	4460	--	5.04	--	--	4240	-4.9	--	
107	11.44	19.82	3.62	3170	3880	0.82	4.95	0.73	0.84	3760	-3.1	0.69	5.25
108	6.10	15.05	3.05	1300	1550	0.84	4.20	0.73	0.84	1550	0.0	0.72	4.21
110	21.95	27.78	6.17	9160	8770	--	5.35	--	--	9160	4.4	--	
111	26.25	30.13	8.12	18500	19400	--	5.40	--	--	18500	-4.6	--	
112	10.36	19.34	3.02	2760	3420	0.81	4.85	0.62	0.77	3570	4.4	0.68	4.47
150	12.13	20.00	4.13	3800	4180	0.93	5.01	0.82	0.90	4310	3.1	0.87	4.74
164	17.50	25.74	3.76	5500	6540	0.84	5.22	0.72	0.84	6570	0.5	0.73	5.16
165	5.96	12.37	5.59	1520	1480	--	4.06	--	--	1520	2.7	--	
169	5.20	15.41	1.79	839	1060	0.79	2.73	0.66	0.80	1060	0.0	0.65	2.75

* Datum 12 ft lower than base gage datum

SYMBOLS

F_{adj}	Adjusted fall
F_m	Measured fall
F_r	Fall from rating
Q_{adj}	Adjusted discharge
Q_m	Measured discharge
Q_r	Discharge from rating
%Diff.	Variation of Q_{adj} from Q_r
ⓝ	Column number on computation sheet

FIGURE 22.—Continued.

0.94 − 0.89 = +0.05. This shift would modify the rating equation (G=3.81, V_g=5.02, shift is +0.05) to

$$Q_r=[5511(5.02-1)^{0.832}][(0.133\times3.81)+(0.387+0.05)]=16{,}500$$

This shifted value would change the percentage difference for measurement 11 from +5.1 percent to 0, and the same degree of adjustment would be applied to the computed daily discharge.

The rating illustrated looks very good, considering that a single sensor was used in a channel more than 400 ft wide where tidal backwater was present. However, this rating gives erratic instantaneous discharge figures when flow is less than about 2,000 ft^3/s and the wind is strong. The faulty record might be eliminated by using additional velocity sensors.

Acoustic-velocity meter gage ratings

The equipment for a typical single-path version of an acoustic-velocity-meter (AVM) gaging station, described in detail by Smith and others (1971), is laid out as figure 26A illustrates. An acoustic signal consisting of a short

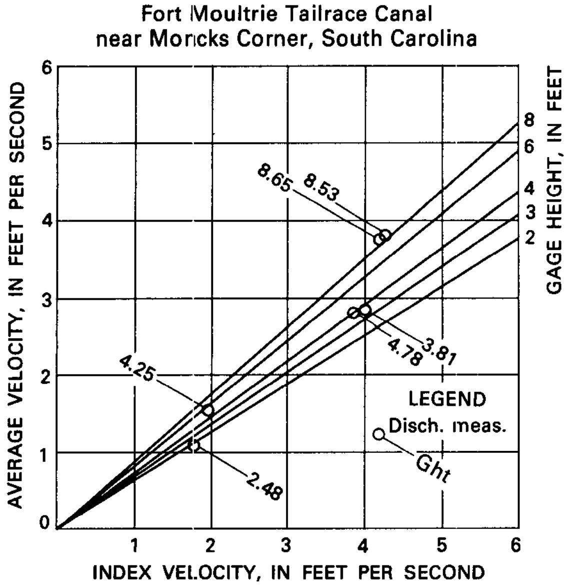


FIGURE 23.—Family of curves relating index velocity to average velocity in the cross section.

burst of energy is transmitted from point A to point B; then, either simultaneously or immediately after, another signal is transmitted from point B back to point A. The time differential between the two transmissions is proportional to the velocity of the water, which has increased the speed of the signal in one direction and decreased it in the other. The "true" velocity of sound in water is computed from the average of the two transmission-reception times. This information, along with the distance A–B and the angle θ, permits computation of the index velocity in feet per second. The index-velocity value is the average velocity of the stream parallel to the banks in the horizontal plane of the diagonal line between A and B. The maximum length of the acoustic path for reliable operation is limited by stream depth and other factors such as maximum sediment concentration and air entrainment.

The type of rating most applicable to an AVM gaging station depends on the channel size and shape and the nature of the flow. A nontidal deep river may need only one acoustic path and may have a relatively simple rating. The rating can be more complex if the acoustic path spans only part of the channel. If the channel contains a stratified mix of saltwater and freshwater at times and has periods of upstream flow, the site may require multiple acoustic paths and a very complex rating. In any event, the rating must be compatible with the program built into the processor by the equipment manufacturer. The two-curve rating analysis, illustrated in figures 24 and 25, can be modified to suit the other types of equipment used at most AVM sites.

The simplest analysis, a one-curve rating, is illustrated in figure 26*A* and can be used only where conditions approach the ideal, as they did at the site used for this illustration. The equipment, a single-path installation, is laid out as figure 26*A* shows. The stream is 80 ft deep at low water, and its stage range is only 15 ft. There is no overbank flow, and reliable discharge measurements are made from a specially designed boat. The acoustic path is located in the upper, relatively straight part of the vertical velocity profile, and the discharge at a given stage is directly proportional to the index velocity.

The one curve used is effective area (measured discharge divided by the index velocity) versus stage. Each measured discharge is divided by its index velocity and plotted against the stage (fig. 26*B*). This relation is fitted to a parabolic curve by using the procedure shown in figure 10*B*. Discharge is computed by using the equation in figure 26*D*. The analysis steps for this type of rating are listed in figure 26*E*, and the computations are tabulated in figure 26*C*. The percentage differences are impressively small.

The rating actually used at The Dalles site is almost the same as the one illustrated except that two curves (stage versus actual area and stage versus stage coefficient) are used. The product of the area and the coefficient is the effective area, which is multiplied by the index velocity to compute the discharge. The rating has not changed during 12 years of AVM operation.

The coefficient a_0 (fig. 26*D*) is −596,500. This coefficient can be varied if necessary and used as a shift adjustment. For instance, if the variance of measurement 309 (fig. 26*C*) had been due to a channel change and confirmed by subsequent measurements, the rating could have been shifted to fit the measurements by using −572,000 (C_c) for a_0 in the rating equation.

Rating analysis by computer

Minicomputers and desktop programmable calculators that have adequate storage and peripheral equipment, which may include a printer, a plotter, diskette storage, a CRT viewer, and a digital-tape translator, are used in some field offices to process the daily records locally. The programs that fit the available equipment are complex and often include a discharge rating analysis.

The minicomputer or calculator can be programmed to store all the discharge measurements that were made at a gaging station and to select and plot the relevant ones so that the hydrographer can draw the rating curve and select its descriptors. The computer then tabulates the rating data and computations and prints the rating tables. Table 3 illustrates a computation printed from a typical semiautomatic rating analysis program.

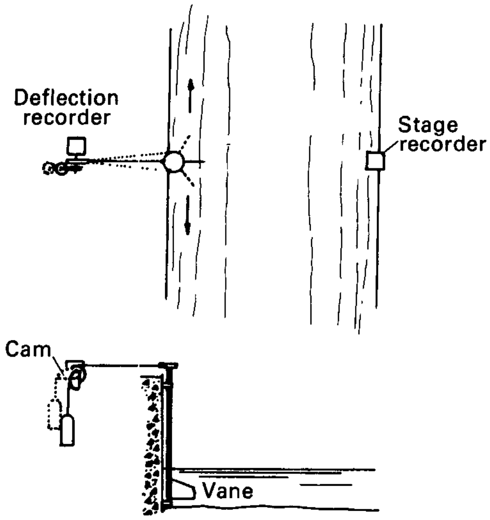
Completely automatic rating analysis using the curve-fitting programs available for each calculator or computer is technically practical but is emphatically discouraged for stage-discharge relations. The programs use a least-squares fitting technique. However, the fitting is done without benefit of human judgment as to the quality of individual measurements, especially outliers, and the hydraulic factors

TABLE 3.

```
                  RATING ANALYSIS,PLOT,AND TABULATION

                JACK DANIEL SPRING AT LYNCHBERG, TENN.
                             03580990

                         MEASUREMENTS USED
                  ALL AFTER 40170   AND BEFORE 122570

                        RATING COORDINATES

GHT         1.07      1.30      1.40      1.85      1.90
DISC        0.006     0.65      1.4       13.5      15

GHT         2.00      2.30      2.40      2.50      2.70
DISC        17        21.5      23.5      28        41

GHT         2.90      3.00
DISC        69        84

GHT SCALE OFFSET=1.04

MST NO     DATE       GHT        DISC        RATING Q       %DIFF      SHIFT

   1       42670      2.70        39.5         41.0         -3.8       -0.02
   2       42670      2.86        61.9         62.5         -1.0        0.00
   3       42670      2.92        67.2         71.8         -6.8    .  -0.03
   4       42670      2.82        60.3         56.4          6.5        0.03
   5       51070      1.12         0.049        0.0504      -2.9        0.00

   6       51070      1.18         0.168        0.17        -1.2        0.00
   7       51070      1.23         0.331        0.329        0.6        0.00
   8       51070      1.25         0.403        0.409       -1.5        0.00
   9       51070      1.30         0.627        0.65        -3.7        0.00
  10       51070      1.34         0.902        0.911       -1.0        0.00

  11      122370      1.81        11.3         11.7         -3.5       -0.01
  12      122370      1.79        11.4         10.9          4.4        0.01
  13      122470      1.58         4.26         4.35        -2.1        0.00
  14      122470      1.58         4.44         4.35         2.0        0.00
```

LAKE WINNIPESAUKEE OUTLET AT LAKEPORT, NEW HAMPSHIRE

A. TYPICAL DEFLECTION VANE EQUIPMENT ARRANGEMENT

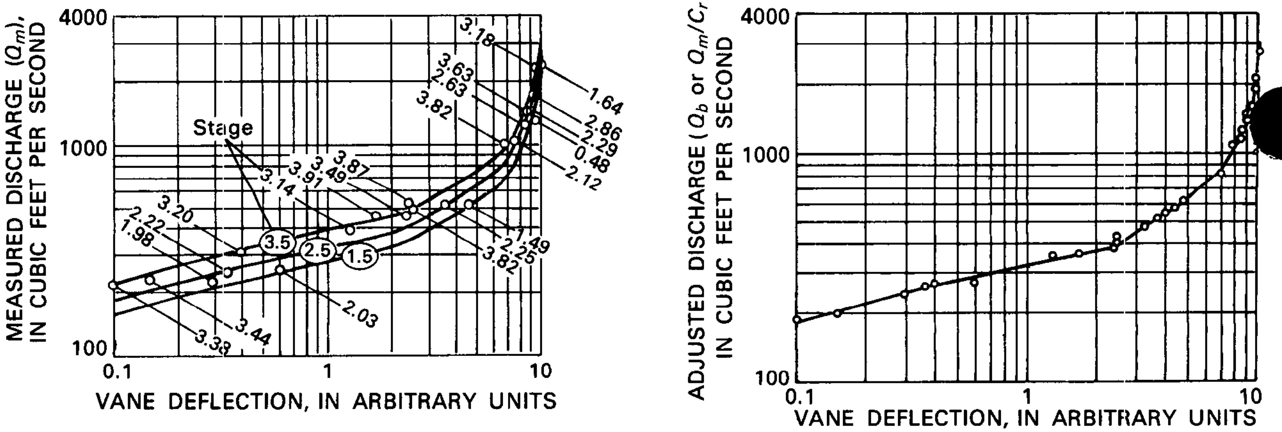


B. COMPUTATIONS

NO ①	GHT ②	VG ③	QM ④	QB ⑤	QM/QB ⑥	CP ⑦	QM/CP ⑧	QR ⑨	% ⑩
596	2.18	0.40	257	270	0.95	0.96	268	259	-0.8
597	2.22	0.35	249	261	0.95	0.96	258	252	-1.2
598	2.00	4.40	537	576	0.93	0.93	579	534	0.6
599	1.66	9.40	1440	1640	0.88	0.87	1650	1430	0.7
600	3.82	7.00	1010	863	1.17	1.22	829	1050	-3.8
604	2.75	3.30	510	478	1.07	1.05	485	503	1.4
605	3.03	0.40	310	270	1.15	1.10	283	296	4.7
606	1.70	9.00	1340	1430	0.94	0.88	1530	1250	7.2
607	0.48	9.80	1320	2110	0.63	0.66	2000	1400	-5.7
608	1.98	0.30	226	250	0.90	0.92	244	231	-2.2
609	3.82	2.50	495	404	1.23	1.22	406	492	0.6
610	3.63	8.60	1490	1230	1.20	1.19	1240	1460	1.4
611	2.86	9.50	1710	1700	1.01	1.07	1600	1320	-6.0
612	3.18	9.80	2350	2110	1.11	1.12	2100	2360	-0.4
613	3.14	1.30	393	342	1.15	1.11	353	381	3.1
614	2.25	3.70	509	515	0.99	0.97	525	499	2.0
616	2.30	9.00	1360	1430	0.97	0.98	1410	1400	-1.4
617	2.39	3.70	495	515	0.96	0.99	499	511	-3.1
618	3.49	2.40	453	398	1.14	1.17	388	465	-2.6
619	3.20	8.80	1400	1330	1.05	1.12	1250	1490	-6.0
624	2.12	7.80	1040	1040	1.00	0.95	1100	986	5.5
625	1.47	4.70	511	601	0.85	0.84	610	503	1.6
626	2.63	8.60	1230	1230	1.00	1.03	1190	1270	-3.1
628	3.87	2.50	526	404	1.30	1.23	429	495	6.3
629	3.20	3.80	584	524	1.11	1.12	520	588	-0.7
630	2.29	9.20	1480	1530	0.97	0.98	1520	1490	-0.7
631	2.68	8.90	1550	1390	1.12	1.04	1490	1440	7.6
632	3.91	1.70	455	361	1.26	1.23	369	445	2.2
635	2.03	0.60	253	293	0.86	0.93	271	273	-7.[illegible]
636	1.64	10.10	2370	2610	0.91	0.87	2730	2260	4.9
637	3.44	0.15	230	208	1.11	1.16	198	241	-4.6
638	3.38	0.10	216	187	1.16	1.15	188	215	0.5
639	3.20	0.40	301	270	1.11	1.12	268	303	-0.7

C. RATING CURVE FAMILY

D. * BASE STAGE RATING CURVE



*Base curve for 2.5 ft stage selected from *C.*

E. DESCRIPTORS FOR BASE RATING

Log scale offset=0

Deflection	Q_b	Deflection	Q_b
0.01	100	5.0	625
.10	187	6.7	800
.40	270	8.6	1230
2.05	375	9.5	1700
2.60	410	10.3	3000

F. RATING EQUATION

$$Q_r = Q_b \times (0.5725 + 0.187G - 0.0047G^2)$$

G. STAGE COEFFICIENT CURVE

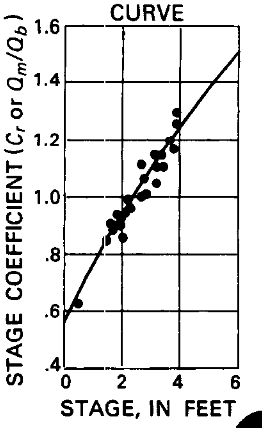


FIGURE 24.—Two-curve index-velocity rating for a vane-gage station.

H. ANALYSIS PROCEDURE

STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets	Needed: Sheets 1 and 2, log-log grids for rating curves; sheet 3, a rectangular grid for the stage coefficient; and a computation sheet with columns titled and numbered from ① to ⑩ as in figure 24*B*. An additional column, unnumbered, may be needed if the deflection scale is offset for negative velocities.
2	Enter data	Fill in ① to ④ with the data from the discharge measurements.
3	Define family of ratings	Plot V_g (③) along X axis vs. Q_m (④) on sheet 1. Flag each point with its stage (figure 24*C*). Draw a family of curves, based on the plotted points, as completely as the data allow. The curve for each stage should ordinarily be above all points that are flagged with values less than that stage and below the points flagged with higher values. Select the best defined curve from the family as the base rating, and its corresponding stage will be the base stage.
4	First trial curve	Plot the base curve from step 3 on sheet 2 and fill in Q_b (⑤).
5	List Q_m/Q_b	Fill in * ⑥ = ④ ÷ ⑤ .
6	Stage coefficient curve	Plot ② along X axis vs. ⑥ on sheet *3. Draw a curve based on the points. The preferred final format is an equation (figure 10*B*).
7	List C_r	Fill in * ⑦ from the step 6 curve or equation.
8	List $Q_m/C_r = Q_{adj}$	Fill in * ⑧ = ④ ÷ ⑦ .
9	Plot base rating curve	Plot V_g (③) along X axis vs. Q_{adj} (⑧) on sheet *2. Draw the base stage rating curve based on the points. The preferred final format is a set of logarithmic curve descriptors (figure 7).
10	List Q_b	Fill in * ⑤ from step 9 curve or descriptors.
11	Go to step 5	Repeat steps 5–10 until further improvement becomes unlikely, then proceed to step 12.
12	Finalize	Prepare descriptors for sheet 2 curve and equation for sheet 3 curve.
13	Recompute final	Recompute * ⑤ to ⑧ and compute ⑨, ⑩ . ⑩ = 100 x (④ − ⑨) ÷ ⑨ using step 12 material. If ⑩ values are unsatisfactory, return to step 5, giving special attention to outliers. If ⑩ values are satisfactory, proceed to step 14.
14	Finalize	Prepare the master curve sheet.

*Erase any entries or plotting from previous trials.

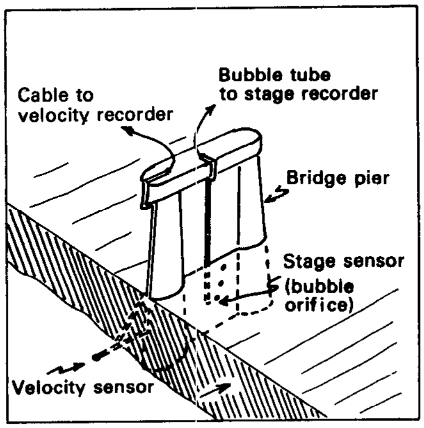
SYMBOLS

$CR = C_r$	Stage coefficient
G	Stage or gage height
NO	Serial number of measurement
$Q_{adj} = QM/CR$	Discharge adjusted to base stage
$QB = Q_b$	Discharge from base stage rating
$QM = Q_m$	Measured discharge
$QR = Q_r$	Discharge computed from rating = $QB \times C_r$
$VG = V_g$	Vane deflection reading
% Diff.	Variation of Q_m from Q_r
ⓝ	Column number on computation sheet

FIGURE 24.—Continued.

LAKE MOULTRIE TAILRACE CANAL NEAR MONCKS CORNER, SOUTH CAROLINA

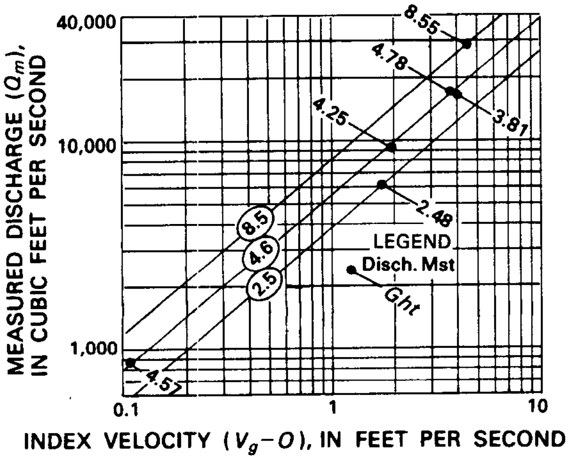
A. TYPICAL MAGNETIC METER EQUIPMENT ARRANGEMENT



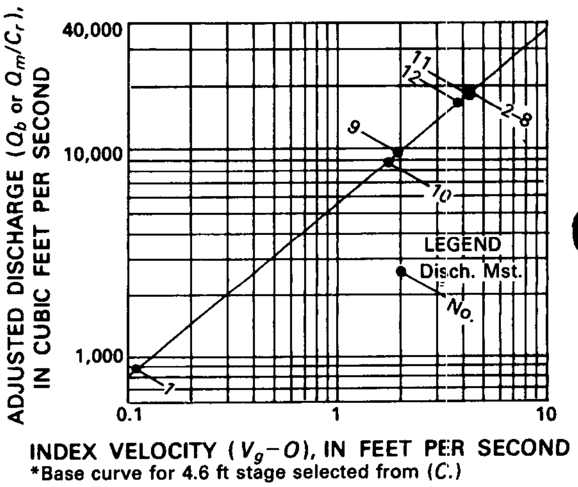
B. COMPUTATIONS

```
NO  GHT    VG   VG-O      QM      QB  QM QB  CR   QM/CR     QR    %
①   ②      ③    ④         ⑤       ⑥    ⑦    ⑧      ⑨      ⑩    ⑪
 1  4.57  0.89 -0.11    -870    -877  0.99 0.99    875   -872 -0.2
 2  8.53  5.38  4.38   28300   18900  1.50 1.52  18600  28800 -1.7
 3  8.50  5.40  4.40   28800   18900  1.52 1.52  19000  28700  0.3
 4  8.50  5.33  4.33   28600   18700  1.53 1.52  18800  28400  0.7
 5  8.49  5.29  4.29   28300   18500  1.53 1.52  18700  28000  1.1

 6  8.55  5.38  4.38   28300   18900  1.50 1.52  18600  28800 -1.7
 7  8.62  5.24  4.24   27900   18400  1.52 1.53  18200  28200 -1.1
 8  8.65  5.29  4.29   27900   18500  1.51 1.54  18100  28400 -1.8
 9  4.25  2.99  1.98    9140    9730  0.94 0.95   9600   9270 -1.4
10  2.48  2.80  1.80    6190    8990  0.69 0.72   8640   6440 -3.9

11  3.81  5.02  4.02   16500   17600  0.94 0.89  18500  15700  5.1
12  4.78  4.89  3.89   17200   17100  1.01 1.02  16800  17500 -1.7
```

C. RATING CURVE FAMILY

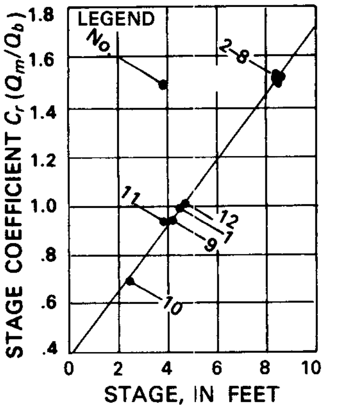


D. BASE STAGE RATING CURVE



*Base curve for 4.6 ft stage selected from (*C.*)

E. STAGE COEFFICIENT CURVE



F. DESCRIPTORS FOR BASE RATING

Log scale offset=0

Index velocity	Q_b
0.10	810
10.00	37,500

G. RATING EQUATION

$$Q_r = Q_b\ (0.133G+0.387)$$

or

$$Q_r = [5511\ (V_g-1)^{.832}]\ [0.133G+0.387]$$

SYMBOLS

$CR=C_r$	Stage coefficient
$GHT=G$	Stage or gage height
NO	Serial number of measurement
O	Magnetic-meter reading at zero velocity
$Q_{adj}=Q_m/C_r$	Discharge adjusted to base stage
$QB=Q_b$	Discharge from base stage rating
$QM=Q_m$	Measured discharge
$QR=Q_r$	Discharge computed from rating
$VG=V_g$	Magnetic-meter reading
% Diff.	Variation of Q_m from Q_r
ⓝ	Column number on computation sheet

FIGURE 25.—Two-curve index-velocity rating for a magnetic-meter station.

H. ANALYSIS PROCEDURE

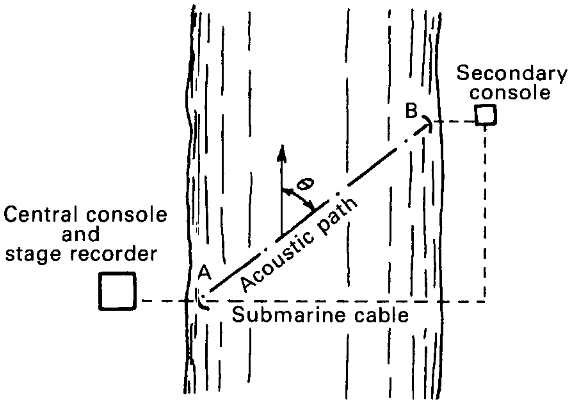
STEP	OPERATION	INSTRUCTIONS
1	Prepare work sheets	Needed: Sheets 1 and 2, log-log grids for rating curves; sheet 3, a rectangular grid for the stage coefficient; and a computation sheet with columns titled and numbered ① to ⑪ as in *B*. Columns ③ and ④ are identical if the magnetic-meter scale is zero at zero velocity.
2	Enter data	Fill in ① to ⑤ with data from the discharge measurements.
3	Define family of ratings	Plot V_g–0 (④) along *X* axis vs. Qm (⑤) on sheet 1. Flag each point with its stage (fig. 25*C*). Draw a family of curves, based on the plotted points, as completely as the data allow. The family should be a series of parallel and nearly straight lines (fig. 25*C*). The spread between curves depends largely on the height of the velocity sensor above the streambed. Select the best-defined curve from the family as the base rating, and its corresponding stage will be the base stage.
4	First trial curve	Use the sheet 1 base curve from step 3 as the first trial curve and fill in Qb (⑥).
5	List $QmQb$	Fill in * ⑦ = ⑤ ÷ ⑥ .
6	Stage coefficient curve	Plot ② along the *X* axis vs. ⑦ on sheet *3. Draw a curve based on the points. The preferred final format is an equation (fig. 10*A* or 10*B*).
7	List Cr	Fill in * ⑧ from the step 6 curve or equation.
8	List Q_{adj} = $QmCr$	Fill in * ⑨ = ⑤ − ⑧ .
9	Plot base rating curve	Plot Vg–0 (④) along the *X* axis vs. Q_{adj} (⑨) on sheet *2. Draw the base stage rating curve based on the plotted points. The preferred final format is an equation (fig. 10*B* or 10*C*) or a set of logarithmic curve descriptors (fig. 7).
10	List Qb	Fill in * ⑥ from step 9 curve, equation, or descriptors.
11	Go to step 5	Repeat steps 5–10 until further improvement becomes unlikely, then proceed to step 12.
12	Finalize	Prepare the final equations, descriptors, or tables.
13	Recompute final figures	Recompute * ⑥ to ⑨ and compute ⑩ , ⑪ . ⑪ = 100 x (⑤ − ⑩)÷⑩. If the ⑪ values are unsatisfactory, return to step 5, giving special attention to the outliers. If ⑪ values are satisfactory, proceed to step 14.
14	Finalize	Prepare the master curve sheet.

*Erase any entries or plotting.

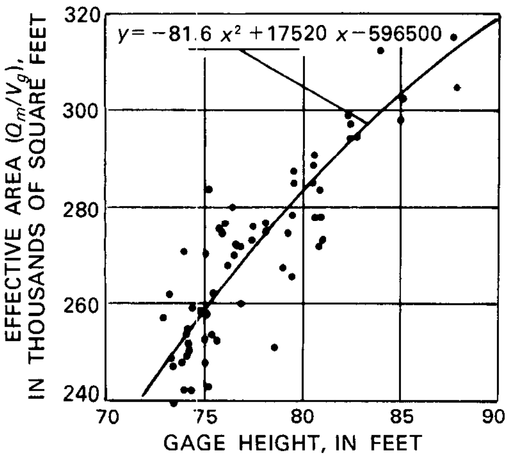
FIGURE 25.—Continued.

COLUMBIA RIVER AT THE DALLES, OREGON

A. TYPICAL ACOUSTIC VELOCITY EQUIPMENT ARRANGEMENT



B. EFFECTIVE AREA CURVE



C. COMPUTATIONS

```
 NO  GHT    VG      QM    QM/VG       CC        CU        QR     %
 ①    ②     ③       ④      ⑤         ⑥         ⑦         ⑧     ⑨
270 79.54 1.219  347000  284659  -592695  -596500   342000  1.5
271 79.49 1.186  341000  287521  -589606  -596500   333000  2.4
272 82.37 1.510  444000  294039  -595500  -596500   442000  0.5
273 82.28 1.485  443000  298316  -590856  -596500   435000  1.8
274 78.02 1.063  292000  274694  -595575  -596500   291000  0.3

275 78.04 1.071  296000  276377  -593988  -596500   293000  1.0
276 76.38 0.754  211000  279840  -582363  -596500   200000  5.5
277 75.28 0.680  177000  260294  -596252  -596500   177000  0.0
278 75.61 0.694  175000  252161  -606103  -596500   182000 -3.8
279 74.16 0.460  111000  241304  -609279  -596500   117000 -5.1

280 74.04 0.484  123000  254132  -595800  -596500   123000  0.0
281 74.11 0.485  122000  251546  -598766  -596500   123000 -0.8
282 73.82 0.481  130000  270270  -578463  -596500   121000  7.4
283 74.76 0.674  174000  258160  -595643  -596500   173000  0.6
284 74.98 0.649  164000  252696  -602273  -596500   168000 -2.4

285 75.27 0.826  215000  260290  -596204  -596500   215000  0.0
286 75.36 0.816  213000  261029  -595935  -596500   213000  0.0
287 80.77 1.409  399000  283179  -599632  -596500   403000 -1.0
288 80.67 1.366  397000  290629  -591747  -596500   391000  1.5
289 81.00 1.416  393000  277542  -606261  -596500   407000 -3.4

290 81.00 1.422  388000  272855  -610948  -596500   409000 -5.1
293 79.35 1.203  334000  277639  -598850  -596500   337000 -0.9
294 79.43 1.232  327000  265422  -611432  -596500   345000 -5.2
295 79.20 1.195  328000  274476  -601326  -596500   334000 -1.8
297 79.00 1.157  309000  267070  -607810  -596500   322000 -4.0

298 77.35 0.967  264000  273009  -594018  -596500   262000  0.8
299 77.31 0.955  263000  275392  -591439  -596500   258000  1.9
300 75.83 0.769  211000  274382  -585017  -596500   202000  4.5
301 74.17 0.576  144000  250000  -600638  -596500   146000 -1.4
302 74.17 0.576  144000  250000  -600638  -596500   146000 -1.4

303 74.09 0.550  137000  249090  -601114  -596500   140000 -2.1
304 73.41 0.528  131000  248106  -598370  -596500   132000 -0.8
305 73.45 0.548  131000  239051  -607646  -596500   137000 -4.4
306 76.81 0.844  219000  259479  -604882  -596500   226000 -3.1
307 75.05 0.796  215000  270100  -585238  -596500   206000  4.4

308 75.06 0.849  210000  247349  -608042  -596500   220000 -4.5
309 75.08 0.790  224000  283544  -571952  -596500   205000  9.3
310 75.18 0.837  203000  242532  -613490  -596500   217000 -6.5
311 85.13 1.709  517000  302516  -597647  -596500   519000 -0.4
312 85.13 1.713  511000  298307  -601856  -596500   520000 -1.7

313 82.94 1.623  509000  313616  -583119  -596500   486000  4.7
314 84.00 1.634  510000  312117  -583847  -596500   489000  4.3
315 82.41 1.570  466000  296815  -592887  -596500   460000  1.3
316 82.62 1.622  477000  294081  -596471  -596500   477000  0.0
317 76.60 0.970  263000  271134  -592176  -596500   259000  1.5

318 76.56 0.963  260000  269989  -593121  -596500   257000  1.2
319 75.35 0.585  148000  252991  -603921  -596500   152000 -2.6
320 87.82 1.958  617000  315117  -594204  -596500   613000  0.7
321 87.87 1.946  593000  304727  -604753  -596500   609000 -2.6
322 74.29 0.642  166000  258566  -592720  -596500   164000  1.2

323 74.00 0.662  160000  241691  -608024  -596500   168000 -4.8
324 73.89 0.302   74700  247350  -601765  -596500    76300 -2.1
325 73.34 0.596  147000  246644  -599444  -596500   149000 -1.3
326 80.60 1.460  416000  284931  -597140  -596500   417000 -0.2
327 80.71 1.490  414000  277852  -604698  -596500   426000 -2.8

328 73.87 0.588  151000  256802  -586660  -596500   145000  4.1
329 73.14 0.578  151000  261245  -583730  -596500   144000  4.9
330 76.07 1.029  284000  275996  -584632  -596500   272000  4.4
331 76.34 1.034  277000  267891  -594109  -596500   275000  0.7
333 75.10 0.473  122000  257928  -597673  -596500   123000 -0.8

334 80.59 1.089  316000  290174  -591853  -596500   311000  1.6
335 80.57 1.085  313000  288479  -593286  -596500   310000  1.0
336 75.80 0.793  219000  274905  -584340  -596500   208000  4.8
```

FIGURE 26.—One-curve index-velocity rating for an acoustic-velocity-meter station.

D. RATING EQUATION

$Q_r = V_g(a_2G^2 + a_1G + a_0)$ or

$Q_r = V_g(-81.6G^2 + 17{,}520G - 596{,}500)$

E. ANALYSIS PROCEDURE

STEP OPERATION	INSTRUCTIONS
1 Prepare work sheets	Needed: a rectangular grid for the effective area curve and a computation sheet with columns titled and numbered ① to ⑨ as in *C*. Columns ⑥ and ⑦ are unnecessary for sites whose ratings do not shift.
2 Enter data	Fill in ① to ④ with data from the discharge measurements.
3 Effective area	Fill in * ⑤ $=Qm \div Vg=$ ④ ÷ ③ .
4 Plot effective area curve	Plot *Ght* (②) vs effective area ⑤ on rectangular grid. Draw a curve and fit an equation to it (fig. 10*B*). If rating does not shift, skip to step 7.
5 Compute C_c	Fill in ⑥ = ⑤ $-a_2$ ②^2 $-a_1$ ② .
6 List C_u	Fill in ⑦ =value of a_0 applicable at time of measurement. a_0 is used as a shift adjustment.
7 List Q_r	Fill in ⑧ $=Vg$ x (value from step 4 equation).
8 List percentage	Fill in ⑨ =100 (④ − ⑧)÷ ⑧ .
9 Finalize	Prepare the master curve sheet.

SYMBOLS

a_n	Equation coefficient for a second-degree polynomial
CC	Value of a_0 that makes $Q_m = Q_r$
CU	Value of a_0 applicable at time of measurement. *CU* can be varied and used as a shift adjustment.
$GHT = G$	Gage height
NO	Serial number of measurement
$QM = Q_m$	Measured discharge
$QR = Q_r$	Discharge computed from rating equation
$VG = V_g$	Acoustic-velocity meter reading
% Diff.	Variation of Q_m from Q_r
ⓝ	Column number on computation sheet

FIGURE 26.—Continued.

that are related to bends and breaks in rating curves. Extrapolation of an automatically fitted curve is particularly unsatisfactory. Fitting an equation to a manually drawn curve by inputting selected points from that curve rather than from the observed data to a fitting program avoids the problem and is encouraged wherever the equation format is needed.

References

Bailey, J. F., and Ray, H. A., 1966, Definition of stage-discharge relation in natural channels by step-backwater analysis: U.S. Geological Survey Water-Supply Paper 1869-A, 23 p.

Brater, E. F., and King, H. W., 1976, Handbook of hydraulics (6th ed., revised): New York, McGraw Hill, 329 p.

Collins, D. L., 1977, Computation of records of streamflow at control structures: U.S. Geological Survey Water-Resources Investigations 77-8, 57 p.

Dawdy, D. R., 1961, Depth-discharge relations of alluvial streams-discontinuous rating curves: U.S. Geological Survey Water-Supply Paper 1498-C, 16 p.

Hutchison, N. E., and others, 1975, WATSTORE user's guide, v.1: U.S. Geological Survey Open-File Report 75-426, 532 p.

———WATSTORE user's guide, v.5: U.S. Geological Survey Open-File Report 77-729-I, 230 p.

Johnson, L. H., 1952, Nomography and empirical equations: New York, John Wiley, 150 p.

Rantz, S. E., and others, 1982, Computation of discharge, v.2: U.S. Geological Survey Water-Supply Paper 2175, 241 p.

Shearman, J. O., 1976, Computer applications for step-backwater and floodway analysis: U.S. Geological Survey Open-File Report 76-499, 103 p.

Simons, D. B., and Richardson, E. V., 1962, The effect of bed roughness on depth-discharge relations in alluvial channels: U.S. Geological Survey Water-Supply Paper 1498-E, 26 p.

Smith, W., Hubbard, L. L., and Laenen, A., 1971, The acoustic streamflow-measuring system on the Columbia River at The Dalles, Oregon: U.S. Geological Survey Open-File Report, 60 p.

INTERNATIONAL SYSTEM OF UNITS (SI) AND INCH-POUND SYSTEM EQUIVALENTS

SI unit	Inch-pound equivalent
Length	
centimeter (cm) =	0.3937 inch (in)
meter (m) =	3.281 feet (ft)
kilometer (km) =	0.6214 mile (mi)
Area	
centimeter^2 (cm^2) =	0.1550 inch^2 (in^2)
meter^2 (m^2) =	10.76 feet^2 (ft^2)
kilometer^2 (km^2) =	0.3861 mile^2 (mi^2)
Volume	
centimeter^3 (cm^3) =	0.06102 inch^3 (in^3)
meter^3 (m^3) =	35.31 feet^3 (ft^3)
=	8.107×10^{-4} acre-foot (acre-ft)
Volume per unit time	
meter^3 per second (m^3/s) =	35.31 feet^3 per second (ft^3/s)
=	1.585×10^4 gallons per minute (gal/min)
Mass per unit volume	
kilogram per meter^3 (kg/m^3) =	0.06243 pound per foot^3 (lb/ft^3)
gram per centimeter^3 (g/cm^3) =	6.243×10^{-5} pound per foot^3 (lb/ft^3)
Temperature	
degree Celsius (°C) =	(degree Fahrenheit – 32)/1.8 (°F)