Techniques of Water-Resources Investigations
of the United States Geological Survey

Chapter A15

COMPUTATION OF WATER-SURFACE PROFILES IN OPEN CHANNELS

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Book 3
APPLICATIONS OF HYDRAULICS

will have the value selected for that key depth. At all flows for which the mean depth corresponding to the water-surface elevation is equal to or greater than the higher key mean depth, the subsection roughness coefficient will have the value selected for that key depth. For a flow whose mean depth corresponding to the water-surface elevation lies between the two key mean depths, the value of the roughness coefficient is interpolated. The coefficient of the larger key mean depth can be set equal to, larger than, or smaller than that at the smaller key mean depth, thus providing for considerable flexibility in defining the roughness characteristics of the subsection.

Before any water-surface profiles are computed in some regions, a decision must be made as to whether the profile should be for a summer flood or for a winter flood, because of seasonal changes in vegetation. A summer flood, when vegetation is at its peak, will require larger values of roughness coefficients, which in turn will raise the elevation of the computed profile.

Special Field Conditions

Verified reaches

Where high-water marks can be found to define flood elevations at several locations for known or estimated discharges, profiles for these events should be computed. When the computed profiles match the high-water marks, the computations can be used to evaluate roughness coefficients selected, number and locations of cross sections, and adequacy of subdivisions. Then the final profiles for the selected discharges should be computed, and they should be more reliable.

Short reaches

The part of the total surveyed reach that is used in the "convergence" phase of backwater-profile computations is generally not used to establish the normal water-surface elevation within that part of the reach. The interest is usually in the profile at a point upstream or in a reach upstream from the point of convergence. Sometimes, however, the water-surface profile is desired for a reach that is short and that cannot be extended farther downstream for physical reasons. If the reach is long enough to enable any two curves from among the M1-M2 family to converge at the normal depth at the upper end of the surveyed reach, a closer estimate of the elevation of normal depth at the downstream end is possible (see figure 22). A new pair of M curves, closer to y_n, can be computed. These will converge in a shorter distance and will verify the previously computed normal depth at the upper end. In this way the normal-depth profile is established for a greater part of the reach, and more benefit accrues from the data collected.

A manual computation of the profile in the downstream end of a short reach is also possible. The individual steps in the solution of the energy equation by the standard step-backwater method are described in the section entitled "Subcritical Flows." Many of the otherwise tedious trial-and-error operations of a manual computation are reduced by the information from the initial computer run that has established the normal depth at the upstream end of the reach. All necessary cross-section properties will be available. Although step-backwater computations on a mild slope should progress in an upstream direction, if the normal depth is known at the upstream end of a reach, the solution for the normal-depth profile can progress in a downstream direction. Once the normal depth is established at the upper end of a subreach, the elevation computed at the downstream end of it will be for the normal depth. The reach must be reasonably uniform, however; otherwise, the solution will be erroneous.

Crossing profiles

Occasionally the profiles for several M1 or several M2 curves for a given discharge will cross each other in the reach in which they are being computed to establish convergence with the normal-depth profile. This occurs particularly where the cross-sectional area and α at one elevation in the cross section are considerably different from those at another elevation within a foot or two. For the same discharge, the velocity and, therefore, the velocity head

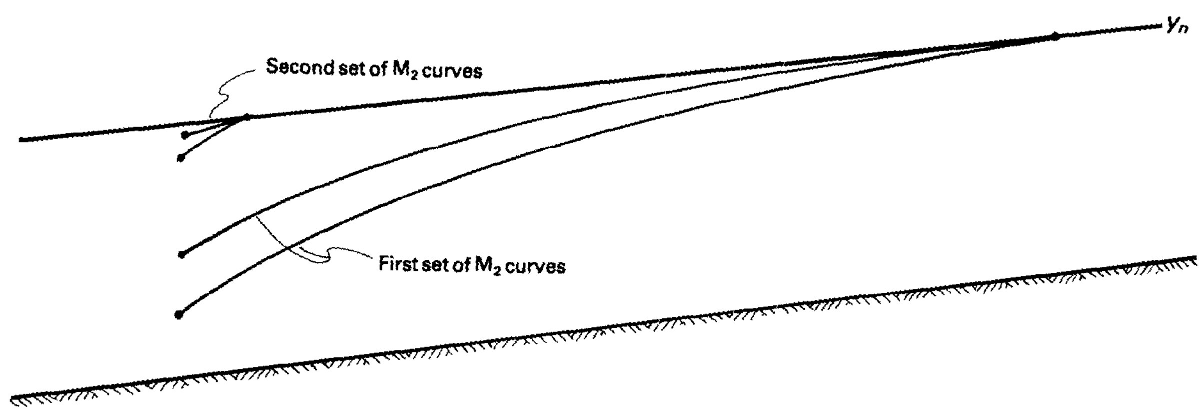


Figure 22.—Establishment of the normal-depth profile in a short reach.

may be sufficiently higher for the lower of two profiles such that the water-surface elevation computed for the next section upstream will be higher for what was the lower profile than for what was the higher profile.

Profiles will also cross in the first subreach if the starting elevation for one profile is less than the elevation of critical depth at the first cross section and the other starting elevation is above the critical-depth line. Because there is no Froude number check at the first cross section, care must be taken to ensure that starting elevations for M2 profiles are never below the critical-depth line.

Profiles can cross elsewhere in a reach if the Froude number limit is set so high as to accept otherwise supercritical solutions. Solutions involving Froude numbers larger than 1.5 should not be accepted, and computed profiles for any reaches with Froude numbers larger than unity should be closely examined.

Profiles that cross need not be more than a disconcerting problem if they occur in a steeply sloped stream, or if they occur on any M2 curves near the elevation of critical depth, where the M2 curve itself is naturally steep. Ordinarily, on steep bed slopes or on steep parts of M2 curves, the phenomenon shows up as a local perturbation that is quickly "righted" within a few subreaches. On flat slopes, however, the effects of such crossed profiles could extend far upstream. Unless crossed profiles either quickly converge or recross to their original relative positions, such a solution should be examined closely.

Transitions between inbank and overbank flow conditions

Solutions of water-surface profiles are rather straightforward either if the flow is confined to the main channel throughout the reach or if flow is a combination of main-channel and overbank flow throughout the reach. If flow conditions change from one of these to the other, there could be an interruption of the solution or an anomaly in computed values. Recognition of the circumstances under which these problems occur is essential to the proper handling or interpretation of the computations.

When flow breaks out over the banks or returns to the main channel between two cross sections, the very small change in elevation is associated with comparatively small changes in cross-sectional area and conveyance. There is a sudden change, however, in cross-sectional shape and in velocity-head coefficient, α. If there is a sudden, otherwise inexplicable jump in computed elevations both in manual and in machine computations, an abrupt change in α between the two cross sections may be the cause.

A sudden change in cross sectional shape might not create difficulties in manual computations of water-surface profiles. In machine computations, however, the solution might abort because of a Froude number problem. The explanation is that one of the overbank subsections may be larger in conveyance than the main-channel subsection; therefore, the Froude number is for the shallower depths of the overbank subsection.

If overbank subsections are further subdivided to avoid a Froude number problem, the increased number of subsections will increase the magnitude of the differences in α and the velocity-head term, $\alpha V^2/2g$. This will in turn create or compound the problem of a sudden, unreasonable change in computed water-surface elevations. On the other hand, a reduction in the amount of subdivision (and α) might induce the Froude-number problem.

Additional cross sections in the vicinity of the transition would improve the profile, but such a costly step might not be the ideal solution nor would it wholly solve the problem. Additional cross sections might be satisfactory for one discharge, but higher or lower discharges will simply translate the same problem to other points in the reach.

Either or both of the following methods should give satisfactory results for the determination of water-surface profiles in the region of transition:

A. Method of interrupting the computed profile.
   1. If the flow in the downstream reach is within the banks (over the banks) and if in the upstream reach it is overbanks (within the banks), stop the computation at the last cross section at which the flow is still inbank (overbank).
   2. Project the computed water-surface profile upstream to the next cross section where flow is out of banks (within banks) on the bases of the computed profile up to the downstream cross section and the local geometry and bed slope.
   3. Start a new profile computation at this upstream cross section, using the projected water-surface elevation in step 2 as the starting elevation.

B. Method of averaging computed profiles.
   1. Compute the water-surface profile for a discharge larger than the one under consideration so that the flow will be overbank throughout the transition reach.
   2. Compute the water-surface profile for a discharge smaller than the one under consideration so that the flow will be completely within banks in the transition reach.
   3. Estimate the profile for the given discharge through the transition reach from the profiles of steps 1 and 2.

Additional complications and uncertainties further compound the problem of sudden transitions between inbank and overbank conditions. These result from a lack of experience. For example, the sudden expansion of flow onto the flood plain from a completely inbank-flow situation is associated with tremendous expansion losses for which normally used computation guidelines may be inadequate; only 50 percent loss of energy is accounted for in an expanding reach. Conversely, the sudden drainage of overbank flows back into the main channel could be likened to a contracted opening—one for which present methods, and coefficients of contraction or of discharge for bridges, would not quite be applicable.

Flow at tributaries

As the computation of water-surface elevations progresses along the stream channel, the discharge must be known at each cross section so that the appropriate velocity heads and friction losses can be properly evaluated. At the mouth of a tributary, therefore, three discharges must be known:

1. Q_u, the main stem discharge, upstream from the confluence,
2. Q_t, the discharge in the tributary, and
3. Q_d, the main stem discharge, downstream from the confluence (sum of Q_u and Q_t).

The main stem discharge, Q_d, will be the one used up to the confluence. To continue the computations above the tributary, Q_u must be known. Unless the tributary discharge, Q_t, is known, some estimate of it must be made. Tice (oral communication, 1973) suggests the following approximation in the absence of a more reliable value:

$$\frac{Q_t}{Q_d}=\left(\frac{A_t}{A_d}\right)^{2/3},$$

where A_t is the drainage area of the tributary at its mouth, and A_d is the drainage area of the main stem just below the tributary mouth.

It is implicit that the arrivals of the peaks of discharges Q_u and Q_t at the confluence are simultaneous for the frequency of the main-stem discharge, Q_d. That is to say, the value of the 100-year discharge in the tributary should not necessarily be subtracted directly from the 100-year discharge on the main stem downstream from the confluence in order to determine the 100-year discharge on the main stem upstream. The engineer must adjust the discharges Q_u and Q_t for any significant lag time between the peaks along those channels.

When the discharges at the confluence are determined, the values of velocity heads and channel friction loss for the subreach into which the tributary flows, are computed as follows:

$$\text{downstream velocity head} = \alpha_d(Q_d/A_d)^2/2g,$$

$$\text{upstream velocity head} = \alpha_u(Q_u/A_u)^2/2g, \text{ and}$$

$$\text{friction loss, } h_f = \frac{L\,[1/2(Q_u+Q_d)]^2}{K_u\,K_d},$$

where subscripts d and u denote downstream and upstream cross sections, L is the subreach length, and K is conveyance.

Because of the averaging of discharges, where Q_u applies more nearly to the upstream part of the subreach, and Q_d applies to the downstream part, the cross sections should preferably be located at points equidistant from the tributary. The larger the tributary is, the more likely it is that K_u and K_d will be appreciably different, thus violating the criterion for proper evaluation of friction losses, $0.7<(K_u/K_d)<1.4$. Therefore, keep subreaches involving relatively large tributaries as short as practical, thereby confining uncertainties to the immediate locality.

The special case of a tributary in the immediate vicinity of a bridge is discussed in the section entitled "Bridges."

Flow past islands

If the channel in which water-surface profiles are being computed has an island so large that the paths around it are considerably different in length, slope, and roughness characteristics, each path around the island must be handled as a separate reach. For example, in figure 23, the total discharge, Q_t, is split into two unknown components, Q_L and Q_R. The computation progressing upstream has stopped at cross section A, at the downstream end of the island. The water-surface elevation at cross section U, just upstream of the island, must be computed to continue the profiles farther upstream. The problem is complicated because the division of the flow into components Q_L and Q_R is not known.

The junctions of the separate channels are considered to be similar to tributaries; and cross sections A, B_L, B_R, F_R, K_L, and U are located as described in the section entitled "Flow at Tributaries."

Each channel around the island is analyzed by establishing a stage-discharge relation for cross section U. For example, by beginning with cross section A, and working up the left channel, the water-surface profiles for various discharges, Q_L, are computed up to section U. To begin with, Q_L may be assumed to be equal to Q_T, and the water surface at U is computed. The same thing is done for several lower discharges, down to the other extreme, Q_L assumed to be zero. The stage-discharge relation is plotted as in figure 24 with water-surface elevation at U as ordinate, and Q_L as abscissa. A similar relation is plotted for the right channel as in figure 24. The intersection of the two curves determines the proper division of Q_T into components Q_R and Q_L, and it indicates the elevation of the water surface at cross section U. The computations would resume at cross section U with the starting elevation as determined from figure 24, and with the discharge, Q_T. The intersection of the two rating curves of figure

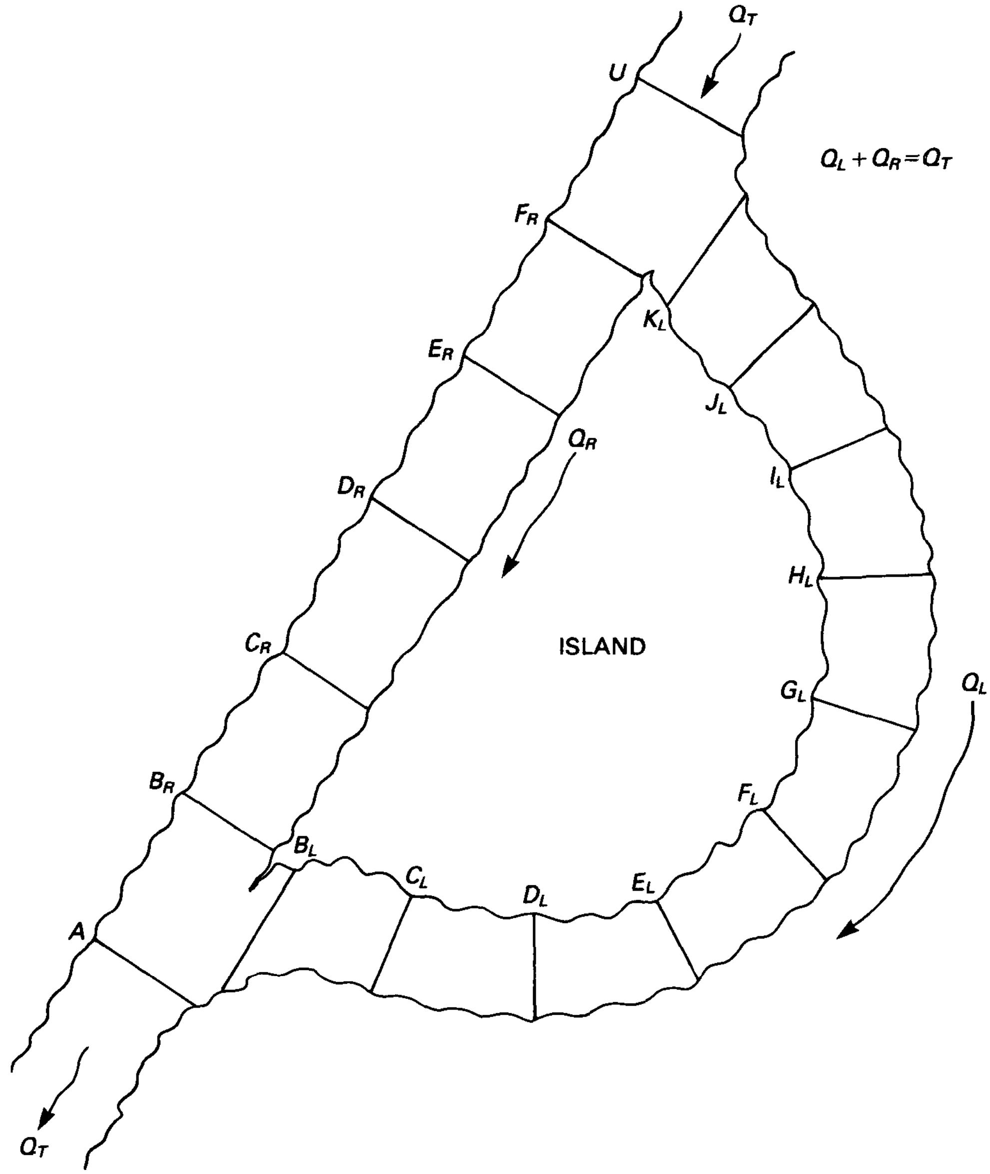


Figure 23.—Flow around an island.

24 may be defined with more precision by defining the curves with more trial runs in that vicinity.

If a known quantity of flow, Q_B, bypasses the main channel and later returns to it, the solution is greatly simplified because the division of discharge is known. There is no need to stop the computation below the point where Q_B returns to the main channel. Water-surface profile computations would progress up the main channel without interruption as follows: up to cross section A, the discharge Q_T would be used; discharge would change to $(Q_T - Q_B)$ at cross section B_R, and remain so up through

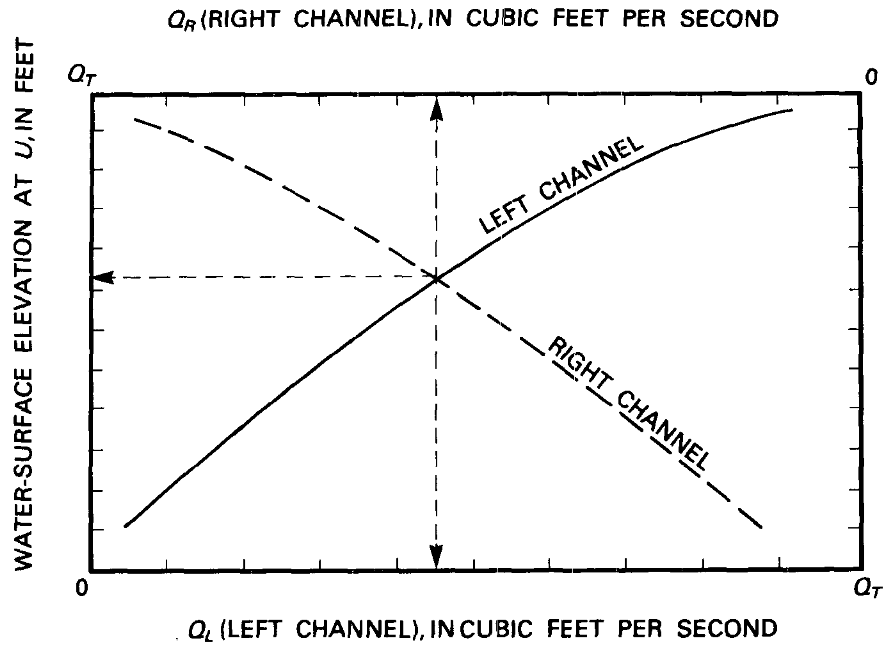


Figure 24.—Division of flow around an island.

cross section F_R; discharge would change to Q_T at cross section U, and remain so farther upstream.

Multichannel flows

If the main-channel flow is divided into several branches rather than only two, as described for flow around an island, the following procedures are recommended. They will help to determine the discharge through each channel and the water-surface elevation upstream from the branches. The method is based on that described by Woodward and Posey (1941).

The main-channel cross section, D-D_R in figure 25*A*, is the last for which a water-surface elevation has been computed with the total discharge, Q_T. The elevation at the upstream cross section, U-U_R, must be determined as well as the division of Q_T into components Q_L, Q_M, Q_R, and any other branches, and the water-surface profiles in each branch.

To solve for the unknowns, an approximate division of flow is estimated, and each channel is analyzed by computing the profile for that channel's discharge from cross section D-D up to cross section U-U. In figure 25*B*, the elevation at U-U is plotted as ordinate, and the discharge producing it is plotted as abscissa. The steps are repeated for other estimated divisions of flow until a rating curve is defined for stage at section U-U corresponding to discharge in each channel. An additional rating curve is drawn to represent total discharge as abscissa by adding, for several elevations, the quantities Q_L, Q_M, and Q_R. This final curve gives the relation between total discharge and elevation at the upstream cross section, U-U. The known value of Q_T is used with figure 25*B* to determine the corresponding value of water-surface elevation at cross section U-U. The discharge in each branch for that elevation of U-U is determined from the individual branch rating curves. These discharges, Q_L, Q_M, and Q_R, are now used to compute water-surface elevations in each channel between cross sections D-D and U-U.

In the computations of profiles from section D-D to section U-U through any one of the branches, there will be a sudden and large change in magnitude of conveyance (a) between

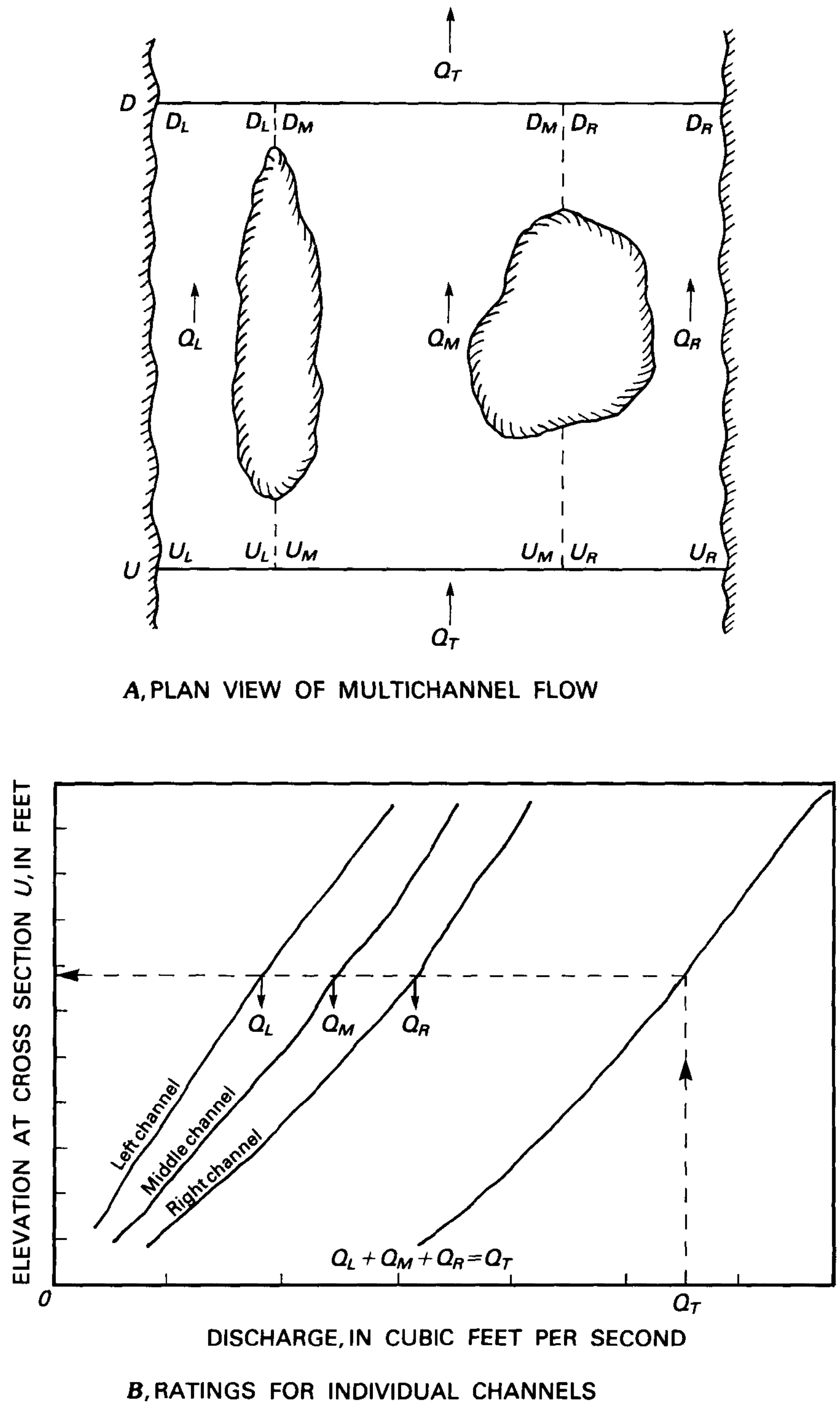


Figure 25.—Division of flow in multichannel reach.

section D-D and the first cross section in the branch, and (b) between the last cross section in the branch and section U-U. To minimize errors in the computation of head losses which are due both to expansion and contraction, and to channel friction, a logical and consistent method of subdividing sections D-D and U-U must be used. Three possible methods are:

1. The shapes of sections D-D and U-U, as determined from plots of those cross sections, may reveal some obvious geometric basis for subdivision. If so, artificially extend these boundaries to the upstream and downstream ends of the islands or embankments, thereby dividing the stream into channels.
2. On the basis of the water surface at section D-D, make a reasonable estimate of the water surface at section U-U. For these elevations, plot the cumulative conveyance versus distance from left bank for sections D-D and U-U. The total conveyances are labeled K_D and K_U. Compute conveyances, K_L, K_M, and K_R, based on water-surface elevation at section D-D for the minimum cross section in each channel. Compute positions for pseudo-boundaries in sections D-D and U-U to simulate the actual boundaries of each branch by multiplying K_D and K_U by the ratio $K_L/(K_L+K_M+K_R)$ for the division between the left and middle channels, and by $(K_L+K_M)/(K_L+K_M+K_R)$ for the division between the middle and right channels. Extend pseudo-boundaries to the upstream and downstream ends of the islands or embankments.
3. On the basis of the water-surface elevation at section D-D, determine the gross cross-sectional areas for the most constricted cross section in each channel. Project the gross width of each island or embankment to the upstream end and divide it on the basis of these gross areas in the adjacent channels, as is illustrated in figure 29. This pseudo-boundary between channels is projected upstream to section U-U, and downstream to section D-D, as is shown in figure 25A.

Some engineering judgment must be used to select the best method. Boundaries determined by the first two methods might not yield similar divisions of section D-D and U-U, or they might be neither parallel to each other nor to extensions of the general axes of the dividing islands. The third method, which is suggested for multiple bridges, is least ambiguous, and should be used if there is not good reason to favor one of the other methods.

After Q_L, Q_M, and Q_R are determined, and the water-surface elevation at U-U is computed, the velocities at U-U should be checked to make certain that they are subcritical. If the main-channel flow at U-U is not tranquil, the proportion of flow going into each channel will depend upon flow conditions upstream from the point of division.

If the flow in any one channel is not tranquil, the steps described in the sections entitled "Steep Slopes," and "Supercritical Flows" are followed. The rating curve, as shown in Figure 25B for each channel, can still be defined in terms of discharge and the elevation at U-U.

Any one or all of the individual waterways of figure 25B could be natural stream channels around islands, bypass canals, or control structures such as bridges, culverts, or dams. Indeed, each path itself could have a series of such structures and(or) stretches of natural stream channel between cross sections U-U and D-D. The methods of computing water-surface elevations at bridges and at culverts are described in the appropriately entitled sections in this manual. Regardless of whether the flow through any one of the individual channels in figure 25B is subcritical or supercritical, or even whether the flow regime successively changes between sections U-U and D-D, the final relation to be plotted is between the discharge and the water-surface elevation at section U-U for each path.

Bridges

Water-surface profile computations may be carried through bridges and other constrictions providing that tranquil open-channel flow conditions exist and that no pressure flow is involved for the discharges being considered.

The effects of bridges or other constrictions on the computed M1 and M2 backwater curves were described in the section entitled "Local Effects on Profiles." Bridges do not present a serious problem if they are located in the reach downstream from the point of convergence of M1 or M2 curves. Bridges located in a channel for which the water-surface profile is being computed, also present no serious problems if the amounts of backwater are insignificant compared to the total fall in the approach reach. This would be true in a streambed having a fairly good slope and at sites where there is not much contraction involved.

The computation of water-surface profiles at bridges, including bridges with road overflow, has been incorporated into a computer program (Shearman, 1976). The methodology and coefficients outlined by the Bureau of Public Roads (Bradley, 1960, 1970) are used. Because of the methodology within the program and by Bradley, the computer solution is satisfactory only for the circumstances described in the preceding paragraph. At other bridges computer solutions should be stopped and backwater curves should be manually computed. Manual computations should be considered, in particular, at the following sites:

1. reaches having extremely flat streambed slopes,
2. two or more bridges in close proximity, longitudinally along the stream,
3. sites at which the flow is greatly constricted, and
4. sites at which the vegetation in the overbanks is extremely dense (n in excess of 0.10).

If the contraction causes critical- or supercritical-flow conditions, it is acting as a control section through which water-surface profiles cannot be computed without a break in computations. When the Froude number in the constricted cross section is 0.8 or greater, the manual methods of computing discharge (Matthai, 1967) or of backwater (Cragwall, 1958) are not reliable. Under such circumstances, terminate the profile at the downstream side of the bridge and attempt a manual routing of the flow through the constriction as if it were a culvert flowing as type 1, type 2, or type 5 (see Bodhaine, 1968).

Computations of water-surface profiles at constrictions having embankment or road overflows involve a trial-and-error solution. The division of flow must be estimated and the water-surface elevation at the approach cross section must be computed for each of the discharges until an acceptable approach-section elevation is found to satisfy the bridge-backwater and embankment-head requirements. Details of the iterative computer-program solution are discussed by Shearman (1976). Criteria for the hydraulics of the flow over highway embankments, including submerged-flow conditions, are discussed by Hulsing (1967).

Sometimes, the computer solutions will be disrupted if bridges have flow over very low embankments or the solution will result in an apparent discharge over the road larger than the discharge for which profiles are being computed. The main-channel subsection, as compared to the total cross-sectional area, could be quite small at such problem sites. If computations are interrupted or if the results appear to be unrealistic, the probable cause is that the bridge-with-road-overflow computation is unfeasible. Ignore the presence of the bridge and replace the bridge sections with a cross section running from the left bank along the crest of the road, down into the main channel, and up the other bank along the crest of the road. In addition to it and the approach cross section, add a third cross section across the whole valley at flood-plain level, one bridge-opening width downstream. Substitution of these three cross sections for the bridge-associated sections will generally provide satisfactory results. If differences between these cross sections are quite significant, additional full-valley cross sections may be required at the upstream and downstream faces of the embankment.

Tributaries are common in the immediate vicinity of bridges, but such flows (street runoff, drainage ditches, or very small tributaries) are generally small enough in comparison to the main-channel discharge to be ignored. If a large tributary enters the main stream immediately upstream of the bridge but below the approach cross section, manual computations of the bridge backwater present no particular problem. In the Survey's machine computation, however, the discharge cannot be changed in the subreach between the approach cross section and the constriction (Shearman, 1976). For computational purposes, therefore, such tributaries are assumed to enter immediately upstream from the approach cross section. As a consequence of this assumption, the next cross section above the approach cross section should be located at a distance equal to the width of the tributary mouth. Such a cross section need not be surveyed if channel conditions are almost identical with those at the approach cross section; it is sufficient to repeat and transpose the approach cross section and use an appropriate longitudinal station distance.

Tributaries entering the main channel immediately downstream from the bridge do not present such problems. The computer program will permit a change in discharge between the full-valley cross section at the exit of the bridge and the next downstream cross section. The full-valley bridge-exit cross section should be repeated and transposed downstream, using an appropriate longitudinal river-station distance, such that the tributary will enter the main channel at mid-subreach. In such a transposition of cross sections, consideration should be given to vertical adjustments of ground elevations if there is appreciable slope in the streambed.

Flow Through Culverts

Culvert flow has been classified into six types on the basis of the location of the control section and the relative heights of the headwater and tailwater elevations (Bodhaine, 1968). Of these types, only type 3 has tranquil flow throughout; therefore, it is only for type 3 flows that water-surface elevations may be computed by the step-backwater method through the culvert. All other types of flow through culverts involve either critical flows or pressure flows; the profile computations must be terminated at the downstream side of the culvert, and the elevation at the upstream side must be determined by other means.

If the culvert is one of the standard types described by Bodhaine, the following procedure is suggested. The U.S. Geological Survey computer program A526[1] will produce a stage-discharge relation for the culvert in terms of headwater elevation, tailwater elevation, and discharge. Inasmuch as the discharge is known and the tailwater elevation is that computed for a cross section located at the downstream end of the culvert, the headwater elevation can be determined easily. Begin the profile computations again at the approach cross section, using this headwater elevation and the total discharge.

[1] Matthai, H. F., Stull, H. E., and Davidian, Jacob, 1970, Preparation of input data for automatic computation of stage-discharge relations at culverts: unpublished data.

Road overflow at culverts

Flow of water both through a culvert and over the road is not infrequent. Because culvert flows associated with road overflow are likely to involve pressure-flow conditions, the culverts and roads must be individually rated. Much of the work, however, can be done by computer, thereby simplifying the procedure.

Figure 26 depicts a culvert with road overflow. The total discharge, Q_T, is divided into unknown quantities Q_C, flowing through the culvert, and Q_R, flowing over the road. The tailwater elevation, H_4, is known. The water-surface elevation H_1 at the approach cross section must be determined.

The flow must be divided so that the headwater elevation computed for the flow through the culvert agrees within a selected tolerance with the headwater elevation computed for the flow over the road. The culvert itself can be calibrated by means of the Survey's computer program A526. Plot the rating, headwater, H_1, versus discharge, Q_C (fig. 27). As long as $Q_C=Q_T$, there will be no road overflow. The rating will have a family of curves if tailwater elevations, H_4, become a factor. A rating curve can also be established for the flow over the road, Q_R, in terms of H_1 and H_4. Criteria for the hydraulics of the flow over embankments, including submerged-flow conditions, are discussed by Hulsing (1967).

The two ratings are plotted in the same manner as was done for flow around an island (see fig. 24). A composite rating is shown in figure 27. The point at which the embankment, or road-overflow rating crosses the culvert rating at the known tailwater elevation, H_4, is shown with a filled circle. Lines through

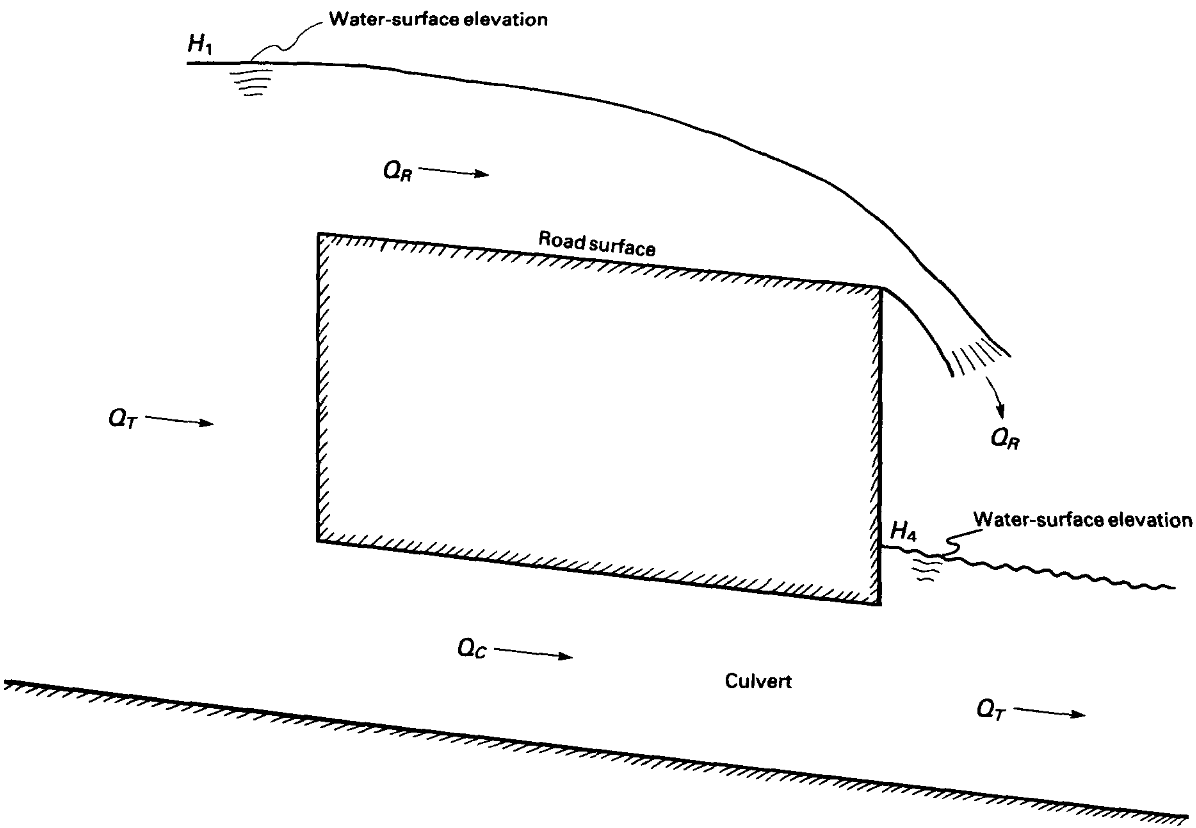


Figure 26.—Culvert with road overflow.

this point, extended to the upper and lower abscissas, and to the ordinate give the appropriate values of Q_R, Q_C, and H_1.

Once H_1 is determined, computations of water-surface elevations for Q_T can commence at the approach cross section and continue up the channel.

Storage at culverts

If headwater elevations are very high with respect to the elevation of the top of the culvert and if the size of the opening is very small with respect to the size of the approach cross section, reservoir-type storage effects are possible. The transition from an inflow hydrograph to an outflow hydrograph may be accompanied by attenuation in the peak rate and a time lag in the centroid. Figure 28 illustrates the effect of embankment-storage attenuation for a hypothetical hydrograph routed through so-called "linear" storage. The peak rates of discharge for inflow and outflow hydrographs and the pond elevations upstream of the culvert can be influenced considerably. The culvert peak attenuation problem has been discussed by Young (1971) and Bodhaine (1968). Jennings (1977) describes culvert hydrograph analysis by a reverse routing method. Mitchell (1962) developed techniques for correcting the outflow peak for the effects of embankment storage. His work is useful for culvert sites where only outflow peak is observed.

Multiple-Opening Constrictions

Multiple constrictions may be combinations of bridges or other constrictions spaced so that the embankments or even a small island be-

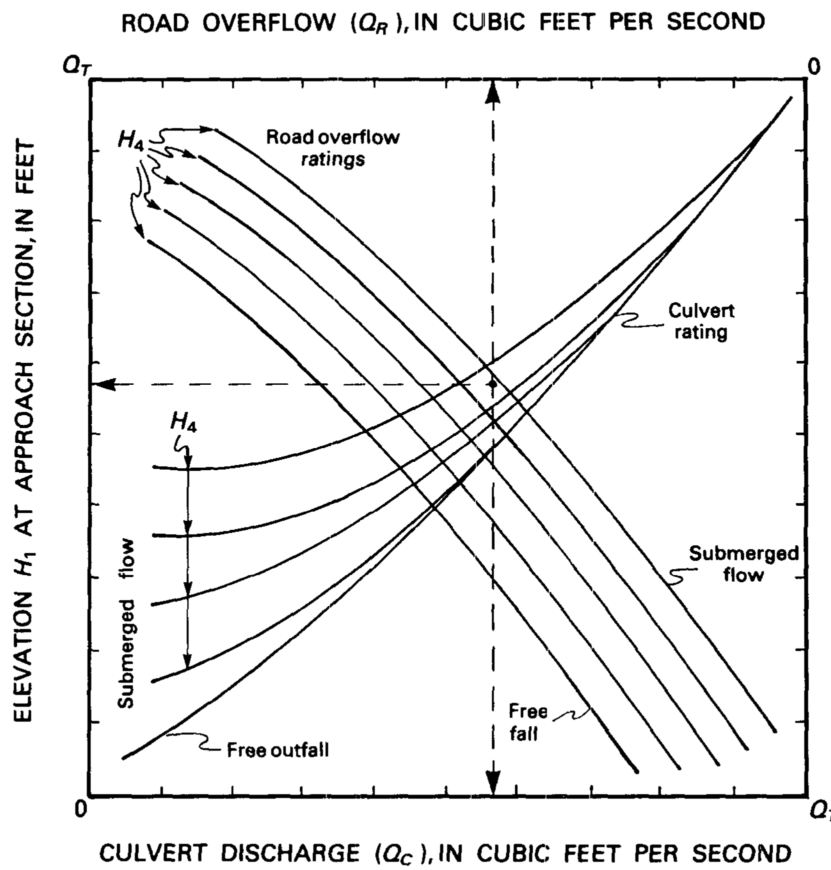


Figure 27.—Composite rating curve for culvert with road overflow.

tween them cannot be considered webs or piers in one very long bridge. The multiple-opening constriction is assumed to be a series of independent, single-opening constrictions, each geometrically and hydraulically distinct from the other (Davidian, Carrigan, and Shen, 1962). The discharge characteristics of the individual openings may then be defined in terms of those for single openings. This method requires that pseudo-boundaries be located in the reach upstream from each of the openings to simulate the actual upstream boundaries of a single-opening constriction. The boundaries may be extended downstream from each opening, also. The techniques are similar to those described in the sections entitled "Flow Past Islands" and "Multichannel Flows."

Division into single-opening units

The upstream flow boundaries may be located by first apportioning the width of each embankment in direct proportion to the gross flow areas of the openings on either side, the larger part of the embankment being assigned to the larger opening. The sketch in figure 29 illustrates the division of an embankment of length W_T into components W_L and W_R. The areas should be computed on the basis of the depths appropriate to the water-surface elevation at the downstream side of the embankment for the full-valley cross section.

After division of each embankment between two openings has been determined, lines parallel to the mean direction of flow are projected upstream from the points on the embankments thus determined. For computation, the lines are assumed to represent the fixed, solid upstream boundaries of an equivalent single-opening constriction. At the constriction embankments, they are reasonably close to the points at which the flow separates; elsewhere, they rarely coincide with the actual limits of the separate flow regions. They do, however, provide an adequate and unambiguous means of dividing the constriction into independent single-opening units.

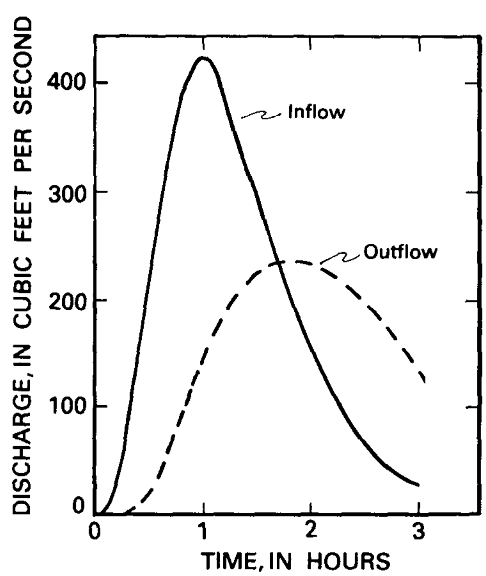
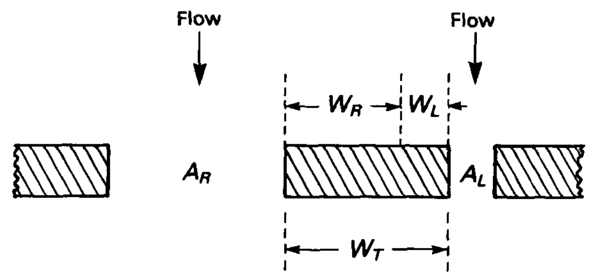


Figure 28.—Hypothetical culvert hydrographs illustrating the effects of embankment storage.

Two-bridge openings

Figure 30 is a sketch of two bridge openings in the main channel. The water-surface elevations will have been computed for the total discharge, Q_T, at a cross section D-D downstream from the constrictions and at V_L-V_R at the downstream face of the embankments. The latter elevation is used to determine the cross-sectional areas in the openings, and the center embankment is divided as shown in figure 29. The pseudo-boundary between the two openings, the dashed line in figure 30, is projected upstream to a full-valley cross section U-U, at



$$W_L = W_T \left(\frac{A_L}{A_L + A_R} \right)$$

$$W_R = W_T \left(\frac{A_R}{A_L + A_R} \right)$$

W —Embankment width
A —Gross area of cross section
L/R —Subscripts denoting left and right openings
T —Denotes total width

Figure 29.—Apportionment of width of embankment between two bridge openings.

the approach section to the larger opening. It is also projected downstream to full-valley cross section D-D.

Just as for flow around an island, the computations involve the determination of the water-surface elevation at section U-U and the proper subdivision of the total discharge into components Q_L and Q_R (see figure 24). In these computations for flow through each opening, an approach section is taken at one bridge-opening width upstream from each opening: section A_L-A_L for the left opening, and section A_R-A_R, which is part of section U-U, for the right opening. Section A_L-A_L can be estimated from sections U-U, V-V, and D-D, providing it is adequately representative of actual conditions one bridge width upstream from the smaller opening.

The computation of backwater through any one opening entails large changes in discharge and probably conveyance between adjacent sections. These sudden and large changes in magnitude are associated with improperly computed friction losses and large changes in velocity heads. To minimize errors and confine

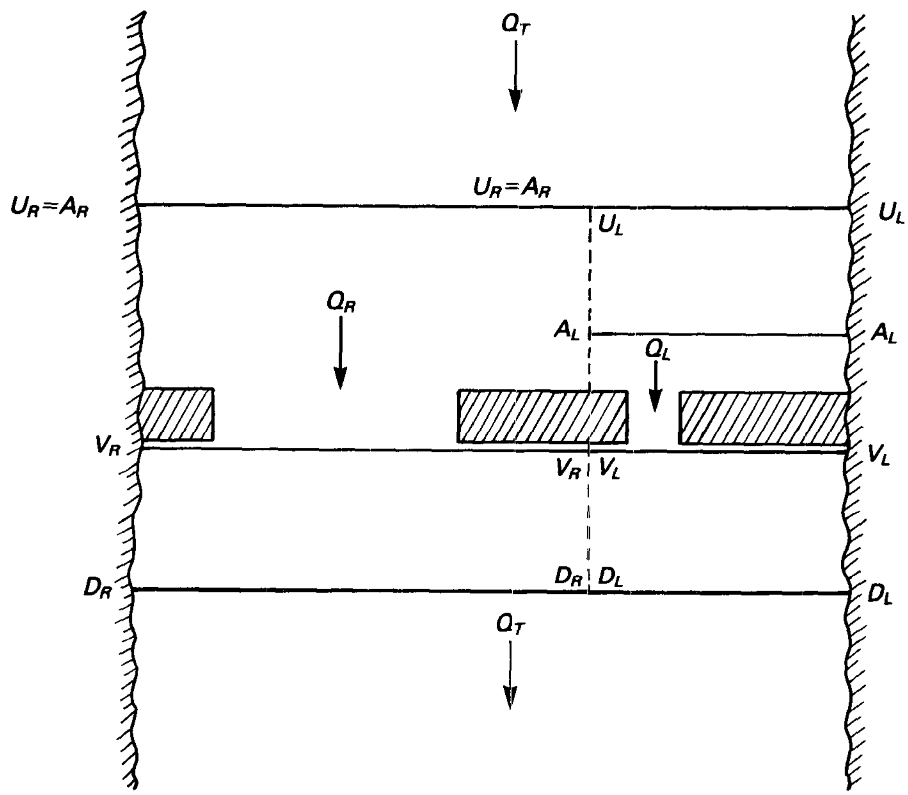


Figure 30.—Division of flow at multiple-bridge openings.

them to short subreaches, the following procedure is recommended. Cross sections *D-D* and *U-U* are divided on the basis of the pseudo-boundaries and each segment is considered to be a cross section. The computation of backwater through the left opening proceeds as follows:

1. Let D_L-D_L be the first cross section. Use the elevation already determined for section *D-D* with Q_T as the starting elevation. Assume any value of Q_L for computations.
2. Continue step-backwater computations at the valley cross section, V_L-V_L; the constriction; the approach section, A_L-A_L; and the left segment of the upstream cross section, U_L-U_L.
3. Plot the computed elevation at U_L-U_L versus Q_L as in figure 24.
4. Repeat steps 1 through 3 for other trial values of Q_L until a rating is developed for the left opening.

Perform the same operations for the right opening, up to approach section A_R-A_R, which is segment U_R-U_R of section *U-U*. After the elevation at *U-U* has been determined, make certain that flow conditions are subcritical at all cross sections in both the left and right channels for the appropriate values of Q_L and Q_R as determined from the composite stage-discharge relation. Flow conditions for the entire cross section, *U-U*, must also be subcritical for the elevation from the stage-discharge relation, and for Q_T. If these conditions are satisfied, full-valley computations can be resumed. Begin at section *U-U*, using Q_T, and start with the water-surface elevation for *U-U* as chosen from the composite bridge ratings.

Three or more bridges

Should there be three or more bridges, or combinations of bridges, culverts, and bypass channels, the computation procedures would be similar to those described earlier under "Multichannel Flows" and "Two-Bridge Openings." Pseudo-boundaries are located using the concepts shown in figure 29 and discussed in the sections mentioned above. Each bridge opening is considered to be a single opening and a rating is established for it in terms of elevation of water surface at an upstream cross section and discharge. The ratings are plotted as in figure 25*B*. The individual ratings are added horizontally to establish an additional rating representing the upstream stage versus the sum of the discharges passing through the individual openings, which correspond to that upstream discharge. Inasmuch as the true value of Q_T is known, the values of the individual discharges and of the water-surface elevation of the full-valley cross section upstream are all easily determined.

Before further computations are resumed, a check should be made of flow conditions at each cross section in the individual channels and at the full-valley section upstream. Supercritical-flow conditions at any one of them will require special consideration; refer to the section entitled "Bridges."

Alluvial Channels

The hydraulics of alluvial streams is complicated and not yet fully understood. The discharge, bed load, bed-material size, bed form, depth, and roughness coefficient are all interrelated in manners that are difficult to evaluate reliably. Scour, fill, and changes in configuration of the channel bed are continuous processes; therefore, the shape and position of the stage-discharge relation change with time and with changes in flow. The computation of water-surface profiles in such channels is, therefore, affected by such uncertainties. Even water temperature has been determined to be a factor in triggering a change in bed form in some streams and in laboratory studies. Familiarity with the results of research studies, such as Simons and Richardson, 1966, and the many references cited by Simons and Richardson, will assist the analyst with studies in alluvial streams.

Flow and bed forms in alluvial channels are classified into three major regimes:

A. Lower flow regime
   1. Ripples
   2. Dunes with ripples superposed
   3. Dunes
B. Transition zone (bed roughness ranges from dunes to plane bed or antidunes)
C. Upper flow regime
   1. Plane bed
   2. Antidunes
      a. Standing waves
      b. Breaking antidunes
   3. Chutes and pools

A relation which defines bed forms as a function of hydraulic radius, *R*, in feet, slope, *S*, mean velocity, *V*, in feet per second, and grain size, has been proposed by Simons and Richardson (1966). It is shown in figure 31. Another useful criterion for the classification of flow regimes is the ratio

$$\frac{V^4}{g^2\, d_m^{1/2}\, d_{50}^{3/2}},$$

in which g is the acceleration of gravity in feet per second per second, d_m is the mean depth in feet, and d_{50} is the median grain size in feet. For values of this ratio less than 1×10^3, the lower regime of bed forms will occur, and for values greater than 4×10^3, the upper regime will occur. Between these two values, the bed will be in the transition zone.

To compute depths or water-surface profiles in alluvial streams, the bed elevations and the channel roughness must be known. The bed forms and roughness coefficients for the bed depend on the regime of the flow, which in turn requires knowledge of the velocity and depth. Because water-surface elevations are more likely to be computed for high flows, it is probable that such computations will be for upper-regime flow conditions.

Flows in the higher ranges of the transition zone, and in the upper regime, frequently, but not necessarily, are critical or supercritical. In antidune flow, the fact that the water and bed surfaces are inphase is a positive indication that the flow is rapid ($\mathbf{F} > 1$). In many alluvial channels, the natural banks cannot withstand prolonged high-velocity flow without eroding. The erosion increases the cross-sectional area, and this reduces the average velocity and Froude number. Rarely does a Froude number,

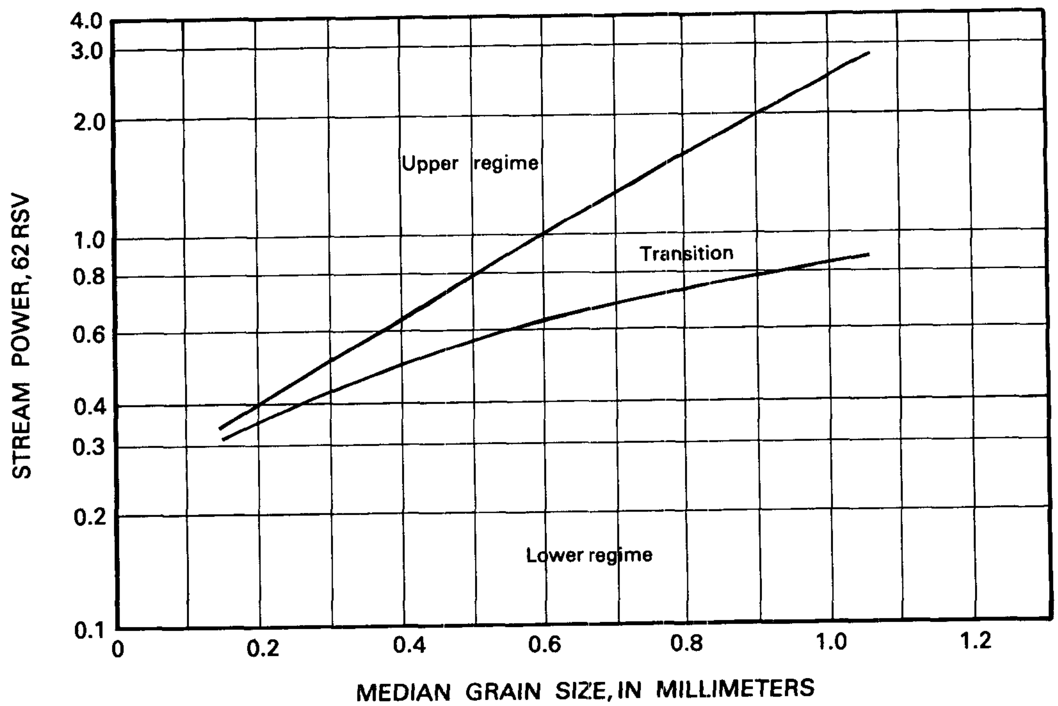


Figure 31.—Relation of form of bed roughness to stream power and median grain size (modified from Simons and Richardson, 1966).

based on average velocity and depth, exceed unity for any extended time period in a natural stream with erodible banks.

Values of Manning's n, for a hydraulic radius of 1.0 foot, were computed from the values of f given by Simons and Richardson (1966, p. 56) by the equation

$$n=\left[\frac{(1.486)^2 f (R^{1/6})^2}{8g}\right]^{1/2} = (0.00858\, f)^{1/2}$$

	Lower flow regime
1. Ripples	$0.021 \leqslant n \leqslant 0.033$
2. Dunes	$.019 \leqslant n \leqslant .037$

	Upper flow regime
1. Plane bed	$0.013 \leqslant n \leqslant 0.016$
2. Antidunes	
Standing waves	$.013 \leqslant n \leqslant .017$
Breaking waves	$.016 \leqslant n \leqslant .024$
3. Chutes and pools	$.024 \leqslant n \leqslant .028$

NOTE.—Multiply values tabulated by $R^{1/6}$ for correct value of n.

The smaller value of n for a given bed form goes with smaller sizes of bed material. For example, for antidunes-standing waves, the range of n is given as 0.013 to 0.017 times $R^{1/6}$. These data are based on laboratory tests for grain sizes (d_{50}) of 0.19 mm, 0.27–0.28 mm, 0.45–0.47 mm, and 0.93 mm.

Nordin (1964) reports on resistance coefficients measured in a reach of the Rio Grande near Bernalillo, New Mexico. For upper-regime flows, largely plane bed, and an average bed material size, d_{50}, of 0.29 mm, values of n range from 0.012 to 0.018 for mean depths ranging approximately between 2.0 and 4.5 feet.

In natural streams, standing and breaking waves associated with antidunes in upper-regime flow will generally be located in the middle of the cross section. The water surface at the banks might be relatively quiet. Therefore, flow at the sides may be in the lower regime or in the transition zone while the center of the stream is in the upper flow regime. Computations of depths, velocities, Froude numbers, and water-surface elevations will, however, be based on the bulk cross-sectional values.

The choice of a reasonable roughness coefficient is still a difficult problem. Therefore, a few studies of the resistance coefficients for alluvial channels, both in natural rivers and in laboratory flumes, are summarized below to indicate the relative magnitudes of the roughness factors to be expected. A considerable amount of judgment must be exercised by the analyst in choosing appropriate values.

Simons and Richardson (1966) offer some guidelines on resistance coefficients for alluvial channels in terms of the Darcy-Weisbach resistance coefficient, f, and the Chezy discharge coefficient, C. The relation between f, C, and Manning's n is:

$$\frac{8g}{\sqrt{f}} = C = \frac{(1.486\ R^{1/6})}{n},$$

in which

g = acceleration due to gravity, ft/s^2,
R = hydraulic radius, ft,
S = channel slope,
V = mean velocity, ft/s.

Benson and Dalrymple (1967) state that values of Manning's n for upper regime flow may be selected from the following table which shows the relation between median grain size and the roughness coefficient.

Median grain size	Manning's n
0.2 mm	0.012
.3	.017
.4	.020
.5	.022
.6	.023
.8	.025
1.0	.026

Culbertson and Dawdy (1964) made a study of hydraulic variables at several sites along the Rio Grande in New Mexico. Figure 32 shows Chezy's C as a function of d_{50} for upper regime flows. The relation between hydraulic radius, velocity, and C is shown for one station, Rio Grande at Cochiti, in figure 33. The median diameter of bed material at Cochiti is approximately 0.44 mm, and the mean depth for upper regime flow is between 3.6 and 4.8 feet.

The examples cited (Simons and Richardson, 1966; Nordin, 1964; and Culbertson and Dawdy, 1964) are primarily for relatively shallow depths and for discharges that probably are not as large as the design floods for which profiles are desired. Any information from past floods, such as measured profiles, bed forms, photographs, or eyewitness accounts, would be of great value in determining the probable regime of flow as well as in choosing appropriate values for the resistance coefficients.

Use of Step-Backwater Method for Indirect Discharge Measurements

The step-backwater technique can be applied to the determination of discharge by indirect means in a long, slope-area reach. The reach may be ideal in every respect for a slope-area measurement, having a uniform shape and roughness, and steep sides, but it may lack high-water marks except for an excellent mark or two at the upstream end. A stage-discharge relation can easily be established at the upstream end where the high-water marks are located.

Cross sections (at least 8–10) can be located through the reach, and two or more M2 profiles can be computed for each of a series of assumed discharges about the magnitude of the expected discharge. The reach should be long enough for the several M2 curves for each discharge to converge. In this manner, a stage-discharge relation is established for the cross section at which the high-water mark is located, as in figure 34.

The discharge corresponding to the elevation of the high-water mark, as determined from a well-defined rating as in figure 34, should be every bit as reliable as a slope-area measurement made in that reach with good high-water marks to define the water-surface profile.

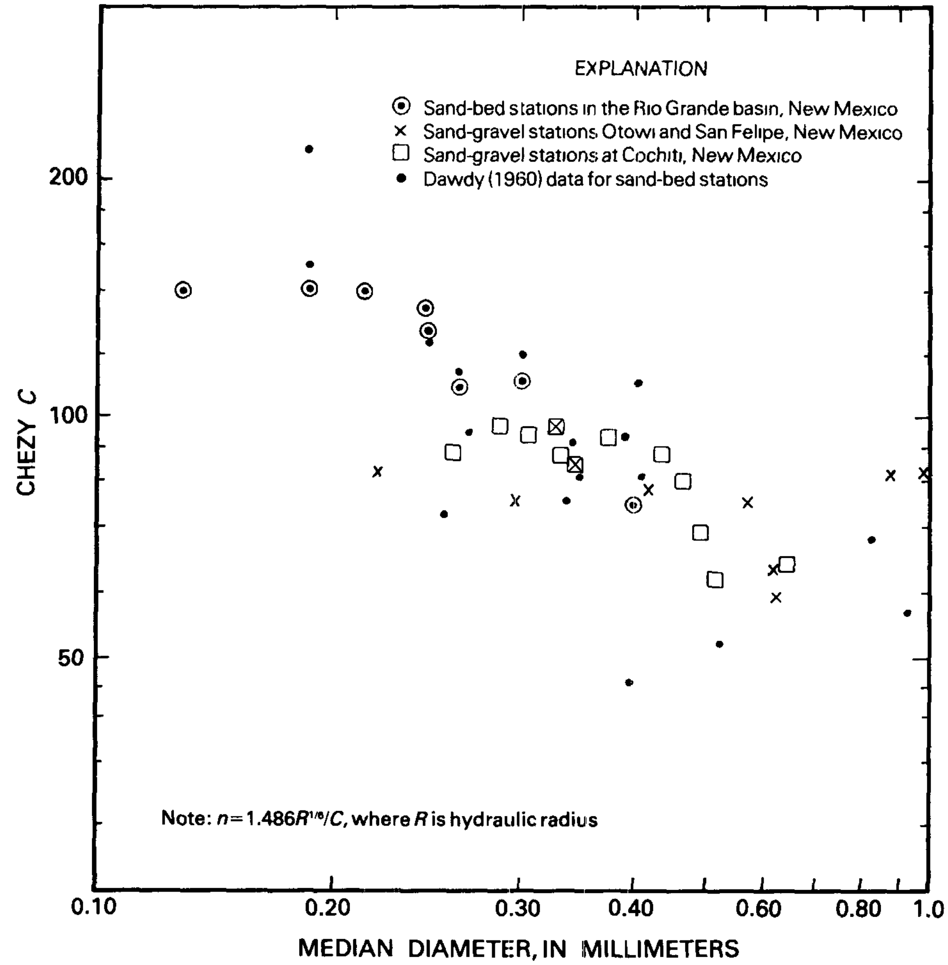


Figure 32.—Variation of Chezy C with median diameter of bed material for upper regime flows (modified from Culbertson and Dawdy, 1964).

Floodway Analysis

The material in the next two paragraphs is paraphrased from Shearman (1976). Floodway, as used in this manual, refers to a land use control measure widely used in the field of flood-plain management. In this context, a floodway may be defined as that portion of a watercourse required to convey a discharge of specified magnitude without exceeding a specified surcharge (fig. 35). The discharge magnitude and surcharge limit depend upon criteria established by the appropriate regulatory agency (which may be Federal, State, regional, or local).

Encroachment of cross sections

Ideally, floodway limits should be located such that the encroachments on both sides of the watercourse contribute equally to the surcharge. Encroachments could be based on equal area or equal horizontal distance. However, elimination of an area of open pasture on one overbank would contribute far more to the surcharge than would elimination of an area of dense forest on the other overbank. Likewise, encroachments of equal length on overbanks with unequal flow depths and(or) unequal roughness would also contribute unequally to the surcharge. Encroachments having equal

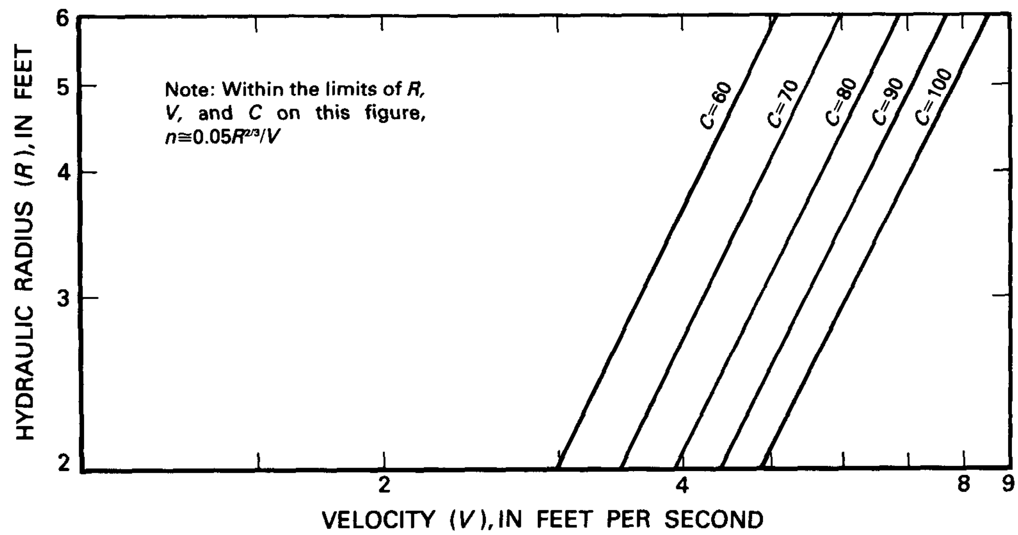


Figure 33.—Relation of roughness coefficient to hydraulic radius and velocity for Rio Grande at Cochiti, New Mexico (modified from Culbertson and Dawdy, 1964).

conveyance, which includes area (thereby length and depth) and roughness, would be more likely to contribute equally to surcharge. Therefore, conveyance is used in this manual as the basis for establishing floodway limits. Several problems may be posed as follows:

1. A surcharge, y, is acceptable if the side boundaries can be moved closer to the center. The conveyance to be removed from the left bank, K_L, is to be equal to its counterpart on the right bank, K_R, and their sum, K_L+K_R, is to be equal to the conveyance of the surcharged part, K_S, such that y is not exceeded. Where are the side walls, L and R, to be located?
2. The left boundary, L, is to be at a preselected location on the left flood plain. Where should the right boundary, R, be placed such that K_L+K_R are equal to K_S, and y is not to exceed a preselected value?
3. Move the left and right boundaries to any locations on their respective flood plains ($K_L \neq K_R$). At what depth will the discharge now flow in the constricted channel (y is not fixed)?

Many variations of these problems are possible. It may be desirable to do either 1, 2, or 3, as described above, at each cross section in the total reach. All the new left boundaries would be connected, and their loci would define a new left edge of water. After this is repeated for the other bank, it may be desirable to go back and readjust some of the boundaries to achieve a more nearly uniform constricted channel shape and alinement throughout the reach. In doing this, the relation $K_L=K_R$ must be preserved if that constraint had been selected; and the new depths must be checked so as not to violate the surcharge limit, y, if that constraint had been selected.

In another variation, a combination of problems 1, 2, and 3 may be used, with a different one at each cross section. It could be desirable to use none of these at some places, leaving the cross section unchanged.

In the manipulation of boundaries in floodway studies, care must be taken at bridges and culverts. If there is any possibility of road overflow, the reach between the approach cross section and the road embankment, and an equal distance downstream, should be examined carefully before and after any encroachments are made on any cross sections within this subreach. Any computed road overflow must be able to reenter the live stream again on the downstream side of the embankment.

Floodway analyses are made after the normal water-surface profiles are determined as described in the section entitled "Standard

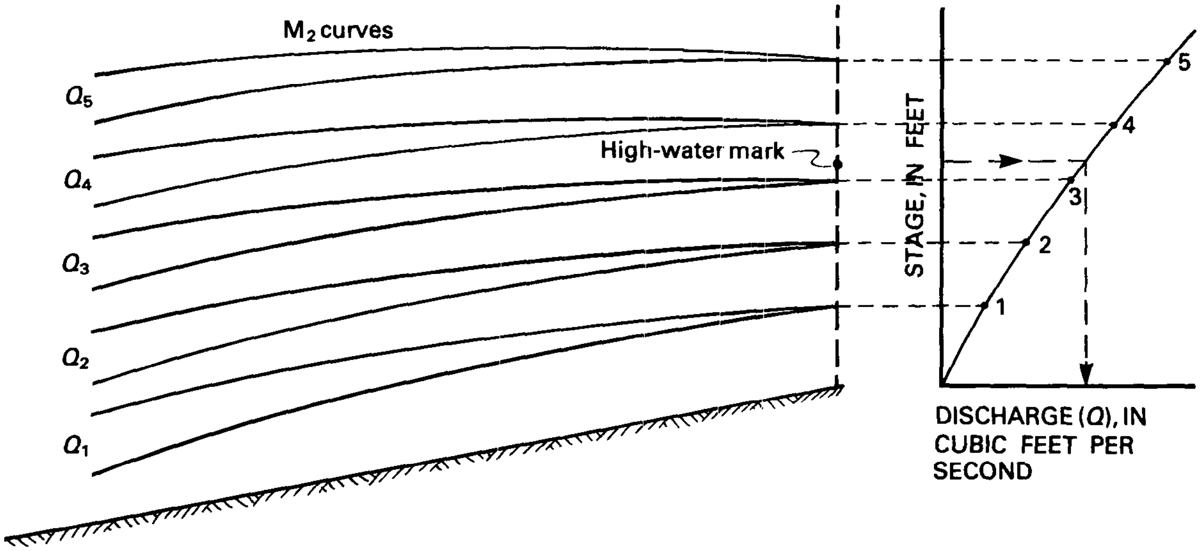


Figure 34.—Definition of a rating curve at the upper end of a long reach by means of the step-backwater method, using convergence of M2 curves.

Step Method, Subcritical Flows." Manual computations for a floodway analysis are impractical. However, a solution by computer is not difficult. Shearman (1976) describes in detail the documentation of the Geological Survey computer program, E431, and the various options available for solution of problems similar to those mentioned above (1, 2, and 3). Several of these options are described briefly below.

VER option

In this floodway option the surcharge, y, is specified. The locations of boundaries L and R are not fixed, but they are positioned so that equal conveyances are removed from each bank. With reference to figure 35, the following requirements are satisfied: (1) $K_L=K_R$; (2) $K_S=K_L+K_R$; and (3) $K_M+K_S=K_L+K_M+K_R$.

The VER option should be used preferably at cross sections having wide flood plains of roughly equal widths and(or) conveyances, and the reasonableness of the computed results should be evaluated. It is possible to obtain unsatisfactory solutions which would place both the L and the R boundaries on the same bank, or one of these boundaries in the main channel. Should either of these unacceptable solutions be obtained, some other option must be used, some constraints must be imposed, or some requirements must be relaxed. For example, it may be necessary to accept a solution from another option, one in which K_L and K_R are not necessarily equal, but their sum is still made equal to K_S by preventing either boundary from being located anywhere but on its own flood plain.

VSA option

This option specifies the surcharge limit, y, and also imposes a subsection constraint. The requirement that K_L equal K_R is removed, but their sum is still to be equal to K_S, and the quantity K_M+K_S is to be equal to $K_L+K_M+K_R$.

The subsection constraint is exercised by dedicating a certain subsection, usually the main channel, or a group of adjacent subsections including the main channel, as part of the floodway. If the main channel subsection is not to be encroached upon by boundaries L or R, the computer will manipulate locations for them from the edge of the flood plain up to the demarcation of the dedicated subsection, but will not go beyond. If the computer finds that L

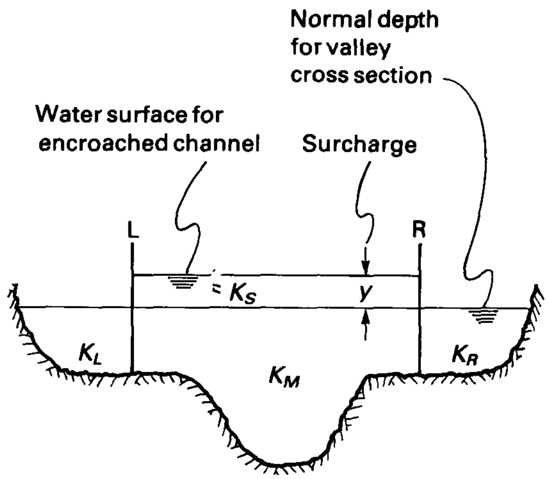


Figure 35.—Effect of encroachment of flood plains on normal valley cross section.

should stop at the edge of such a subsection, it will compute K_L up to that point. Then it will move R until it gets K_L+K_R=K_S, providing R will not encroach into the dedicated subsection(s) also. If both L and R must stop at the edges of the dedicated subsection, the criteria for the VSA option mentioned above will not be fulfilled. Because the sum of K_L and K_R is less than it would be at the surcharge limit, y, the surcharge in the floodway channel between L and R is less than the limit allowed.

VHD option

In the VHD option, the maximum allowable surcharge, y, is specified. A horizontal distance or limit constraint is also imposed on the locations of L and R, beyond which they may not be placed. It is thus possible to preserve an unencroachable part of the cross section by specifying the stationing of its edges. In all other respects this option is similar to the VSA option. K_L and K_R need not be equal, but their sum is equal to K_S, and the quantity K_M+K_S is equal to the sum of K_L+K_M+K_R. Because of the horizontal distance constraints on the locations of L and R, the sum of K_L and K_R may yield a value of K_S which corresponds to a smaller surcharge than that allowed; therefore, the constraint on the magnitude of y will not be violated.

In specifying the limiting stations for the locations of L and R, the analyst should not try to create new subdivisions of the cross section. Such a step would unnecessarily affect the velocity-head coefficient, α. The specified stations serve only as limits.

Despite the specification of limits for L and R, sometimes the computed water-surface elevation at that cross section may be so low that all the flow is confined entirely within the restricted area. The computer printout will, therefore, show the stations of the left and right edges of water not to be at the limiting values of L and R, but within the restricted area. The criteria of the VHD option will not, however, have been violated. In this case, limits for L and R are not applicable. For higher discharges, the water-surface elevation will be higher, and the left and right edges of water will coincide with the locations of L and R if these boundaries are at their limiting stations and if the surcharge, y, is not exceeded.

HOR option

The HOR option has specified locations for L and R in figure 35. These are not variable locations with limiting values for the station or distance; they are fixed locations for an encroached cross section. There is no constraint on the surcharge, y. The effects of the encroachment are, therefore, reflected in the elevation of the computed water surface.

As is described for the VHD option, the specification of limits, or the designation of specific locations, for boundaries L and R does not necessarily mean that the computed water surface will be high enough for the left and right edges of water to reach these stations for all discharges.

An example of an advantageous use of the HOR option is a study of "before and after" water-surface profiles for a given discharge in a reach that is to have a part of its flood plain removed from the available cross section. If a highway were to be placed along the flood plain at L in figure 35, and parallel to the main stream, the highway would be the effective new left bank boundary. The location of L would be known for each cross section. The right boundary would remain on the right edge of the valley. The water-surface profile throughout the reach for this encroached channel would be determined by computing the resulting surcharge, y, at each cross section.

The HOR option could also be applied to a study of levee heights and locations along the flood plains for various flood discharges.

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