

Techniques of Water-Resources Investigations
of the United States Geological Survey

**MEASUREMENT OF DISCHARGE
USING TRACERS**

Book 3
CHAPTER A16



4. Plot the fluorometer calibration on rectangular graph paper, and reconcile any data points that do not conform to the curve by reanalysis of fresh samples on the fluorometer and, if necessary, by preparation and analysis of new samples.

Performance

A schematic diagram of the step-by-step performance of a slug-injection type dye-dilution measurement is shown in figure 8. The steps in performing such a test use the example shown.

Selection of measurement reach (1)

The stream discharge and the other stream characteristics listed in figure 8, are estimated; using equation 4, compute an optimum stream reach length of about 700 ft. It is desirable to avoid any gain or loss in flow relative to that at the gage, so injection is to be made upstream with sampling close to the gage.

Background samples (2)

Samples of the stream water—at least one, preferably two—are needed before dye injection in order to obtain a reading of the background. The samples should be obtained just before the tracer is injected into the stream, preferably before the dye is handled at all (see sample number 202 on the note form in fig. 8).

Dye injection (3)

For the channel and flow conditions shown, it was decided to try for a peak concentration of about 20 $\mu\text{g/L}$, using rhodamine WT 20 percent. From figure 4, a volume of 0.5 mL is obtained for a peak of 1 $\mu\text{g/L}$; thus, for a peak of 20 $\mu\text{g/L}$, 10 mL of rhodamine WT dye is required.

Next, 10 L of river water is carefully measured into a clean bucket to which the 10 mL of dye is added and mixed. A 50-mL sample of this mixture is retained for future use. The entire contents (9,960 mL) are dumped as a slug into what is judged to be the centroid of the flow.

The time is noted, and the person at the sampling site is notified. A good practice is to start a stopwatch at the instant of injection so that all samples are recorded with respect to lapsed time from injection (see the notes and data plots of fig. 8).

Sampling the dye cloud (4)

The passage of the entire tracer cloud must be monitored for the slug-injection method of discharge measurement. Sampling should begin before the tracer arrives at the section and continue until it has entirely passed. Large numbers of samples are normally required to make certain the entire cloud is measured. Samples should be taken from at least three points in the section in what is estimated to be equal increments of flow. For example, if three points in the section are to be sampled, the samples should be taken at the 1/6, 3/6, and 5/6 points of cumulative flow across the section. Samples 203 through 241 are those taken to define the dye cloud at points a, b, and c across the channel.

Because the samples are not usually analyzed in the field, the length of time for complete passage is an estimate. As a rough approximation, sampling past the peak should continue for two to four times as long as it takes for the peak to arrive. In some instances, the arrival of the peak can be ascertained visually. Hence, if the peak was estimated to have arrived 5 minutes after injection, sampling should continue for a minimum of an additional 10 minutes and preferably 20 to 25 minutes. On this small stream, samples must be taken at very short time intervals; therefore two people are needed, one sampling and one noting the exact time and recording the data. Sampling may be less frequent as the dye cloud recedes. All samples are sealed and stored out of direct sunlight. Notes describing the measurement accompany the samples to the laboratory.

Fluorometric analysis (5)

The single scale that will accommodate the maximum concentrations expected in the field samples should be set on the fluorometer. The quantity of dye injected in step (3) was predicated on obtaining a peak of about 20 $\mu\text{g/L}$.

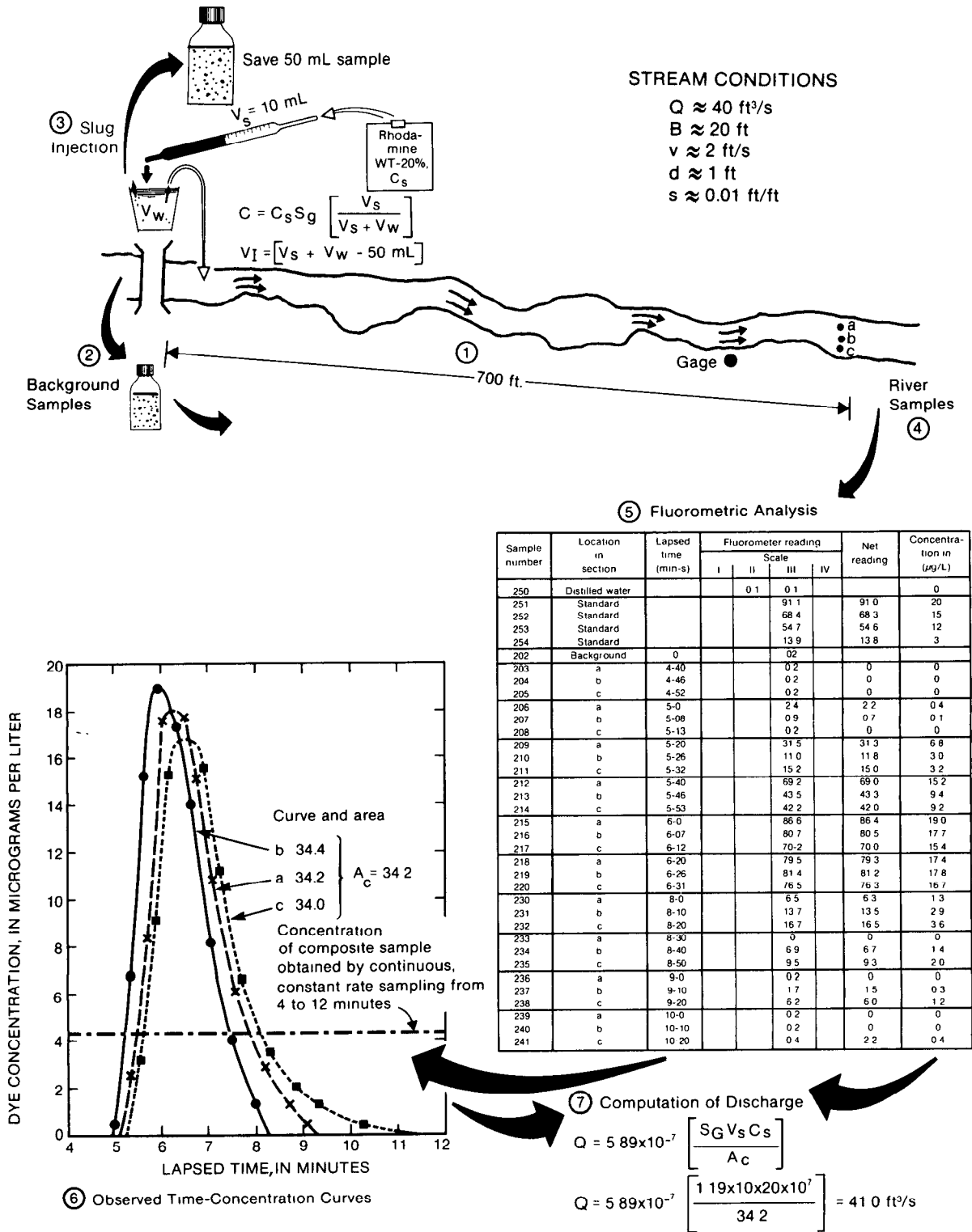


Figure 8.—Schematic diagram depicting the performance of slug-injection type dye-dilution discharge measurement.

Examination of the fluorometer calibration in figure 7 shows that it should be possible to measure that peak and all lesser sample concentrations on scale III. The use of more than one fluorometer scale should be avoided if possible.

Scale III must be recalibrated up to about 20 $\mu\text{g/L}$. Examining table 2B, four standards—20, 15, 12, and 3 $\mu\text{g/L}$ —are prepared from the 100- $\mu\text{g/L}$ working solution retained for this purpose. Those standards and all river and background samples are brought to a single temperature of 68°F. (Any temperature can be used, provided that all samples are analyzed at the same temperature.) The calibration for scale III is plotted as shown in figure 7. Note that net readings are used, the background for distilled water being subtracted first.

All river samples including the background sample, 202, are analyzed on the fluorometer, and, using the above calibration, absolute concentrations are determined for each; the stream-background reading is subtracted from each stream sample before entering the calibration curve.

Data analysis and computation of discharge (6 and 7)

The time-concentration curves for the three sample points, a, b, and c, are plotted and their areas determined. Comparison of the individual curve areas indicates that adequate mixing has been achieved. An average curve area, A_c , of 34.2 $\mu\text{g/L}$ times minutes is obtained. Using equation 3, with V_s substituted for V_1 and C_s substituted for C , the discharge may be computed as

$$Q = 5.89 \times 10^{-7} \left[\frac{1.19 \times 10.0 \times 20 \times 10^7}{34.2} \right] \\ = 41.0 \text{ ft}^3/\text{s}^7 .$$

Alternative method of analysis and computation

During the preparation of the solution for injection, 10 mL of dye was mixed with 10 L of river water, and a 50-mL sample of the mixture was retained (step 3, fig. 8). The concentration used in the earlier computation was for the manufacturer's stock solutions: $20 \times 10^{-7} \mu\text{g/L}$

for rhodamine WT. Using equation 7, it can be seen that the 10 L had a concentration of

$$C = 1.19 \times 20 \times 10^7 \left[\frac{10.0}{10,000 + 10.0} \right] \\ = 23.8 \times 10^4 \mu\text{g/L} .$$

Hence, a major dilution was made at this point. Thus, the 50 mL sample may be used in preparing standards for this particular measurement instead of starting with the stock solution.

In preparing standards from C (above), note that, according to equation 7, 20 mL of dye solution added to 1,980 mL of water is a dilution of 10^{-2} . If repeated, a total dilution of 10^{-4} is obtained, because the final dilution in any serial dilution is the product of the individual dilutions. For any dilution after the first, the specific gravity term in equation 5 may be ignored. Thus, if C (above) was diluted twice, as described, for a total dilution of 10^{-4} , the resulting concentration would be $23.8 \times 10^4 \times 10^{-4} = 23.8 \mu\text{g/L}$. While this would be off scale III in figure 7, other standards may be produced by judicious selection of dye and water volumes. The analysis in figure 8 would then be based on the fluorometer calibration thus obtained, and computations would be based on the new volume and concentration injected such that the discharge (using eq. 3) would be

$$Q = 5.89 \times 10^{-7} \left[\frac{(10,000 + 10 - 50) \times 23.8 \times 10^4}{34.2} \right] \\ = 40.8 \text{ ft}^3/\text{s} .$$

This alternative method of analysis is more accurate than the first, because any errors that occur when measuring the 10 mL of dye for use in the solution to be injected are canceled. Similarly, in preparing standards, only two dilutions are necessary, instead of four as in the previous example. Furthermore, if the exact concentration of the solution injected is unknown, a measurement is still possible if a sample of the solution that was injected is retained and used in the laboratory to develop a calibration in the manner previously described. Such a trial-and-error calibration would be more work in the laboratory but would be accurate if correctly done. That is true because the

concentrations that would be plotted for the time-concentration curves of figure 8 are based on a calibration which is, in turn, based on the injected concentration, C ; hence, the exact concentration of C is not needed. Knowledge of the absolute value of C does simplify the preparation of standards. The disadvantage of the alternative method is that more standards must be prepared, as well as new ones for each measurement. Obviously, it is important that the field person always retain a sample of the actual dye solution injected.

Alternative method of measurement

As the data in figure 8 show, 39 samples were required to define the three time-concentration curves. An alternative method of sampling greatly reduces the sampling requirements but may obscure the quality of the data and the accuracy of the measurement. This alternative method consists of withdrawing a sample continuously and at a constant rate from the stream during the passage of the dye cloud. The time period of sampling, ΔT_c , should be greater than the actual time, T_D , the dye cloud is present. Each sample obtained for the one or more points of sampling is composited, and one sample is analyzed for concentration. This concentration, less the background, is the average concentration, C_a , for the entire dye cloud for the time, ΔT_c . Therefore, $A_c = \Delta T_c C_a$, and equation 3 may be written as

$$Q = 5.89 \times 10^{-7} \frac{S_G V_I C}{\Delta T_c C_a} \quad (8)$$

Assume for the example used above that a sample was pumped at a constant rate from the center of the channel into a clean container for an 8-minute period. If this composite sample yielded a concentration of $4.30 \mu\text{g/L}$ (see fig. 8), the discharge may be computed as

$$Q = 5.89 \times 10^{-7} \left[\frac{(10,000 + 10 - 50) \times 23.8 \times 10^4}{8 \times 4.30} \right] \\ = 40.6 \text{ ft}^3/\text{s}$$

For this example, the assumption was made that mixing was complete. An alternative to pumping a sample would be to collect manually, individual samples of a given volume at uniform time intervals to form a composite sample. Note that for the above example, any time interval greater than 10 seconds might not adequately sample the peak concentrations and thus might not yield a representative sample. From a practical standpoint, it would be difficult to sample manually at such frequencies at more than one point in the section. Where this method is used, sampling just at one point in the main flow at a greater length (say 1,000 ft in the above example) might be advisable. Not only would mixing be improved (and therefore samples at just one point more likely to yield accurate results), but sampling at uniform 15-second intervals would probably be sufficient.

Note that in the example illustrated in figure 8, adequate and complete measurement of all three of the time-concentration curves requires very frequent sampling and accurate recording of time of sampling. In fact, the task would be virtually impossible for one man to perform; one man is needed to sample and one to record the exact times of sampling. In this case, a reach length of 1,000 ft or more might have been advisable to elongate the dye cloud for greater ease and accuracy of sampling. On larger streams with correspondingly greater mixing lengths, this would not be as serious a problem.

Constant-Rate Injection Method

Overview

The constant-rate injection method generally is preferred to the slug-injection method in spite of the additional requirement for a device that will release the tracer at small, constant rates. Various devices that will achieve the required small, constant rates of injection are available commercially or can be fabricated. They are described in detail later in this

manual. An advantage of the constant-rate injection method is that significantly fewer samples are needed, compared with the slug-injection method.

Equation 2 is in a more usable form as

$$Q = 5.89 \times 10^{-7} q \frac{C}{\bar{c}} \quad (9)$$

where Q is the discharge of the stream, in cubic feet per second;
 q is the rate of injection of the tracer, in milliliters per minute;
 C is the concentration of the dye solution injected into the stream, in micrograms per liter; and
 \bar{c} is the equilibrium, or plateau concentration, in micrograms per liter, averaged or weighted across the sampling section.

This equation not only is used to compute the discharge measurement, but also, by solving for q for a given c , may be used to estimate dye quantities for preparing injection solutions. Mixing distances and reach selection are determined in the same manner as for the slug-injection method.

Preparation

Selection of dye concentrations and injection rates

The curves in figure 9 were computed from equation 9 for a plateau concentration, \bar{c} , of 10 $\mu\text{g/L}$. A dye concentration of 10 $\mu\text{g/L}$ in the stream is well above background levels commonly experienced. In most cases, a good dye-dilution discharge measurement can be made

NOTE: Curves are based on equation $Q = 5.89 \times 10^{-7} q \frac{C}{\bar{c}}$ for $\bar{c} = 10 \mu\text{g/L}$;
 greater discharges may be measured by allowing \bar{c} to be less than 10 $\mu\text{g/L}$.

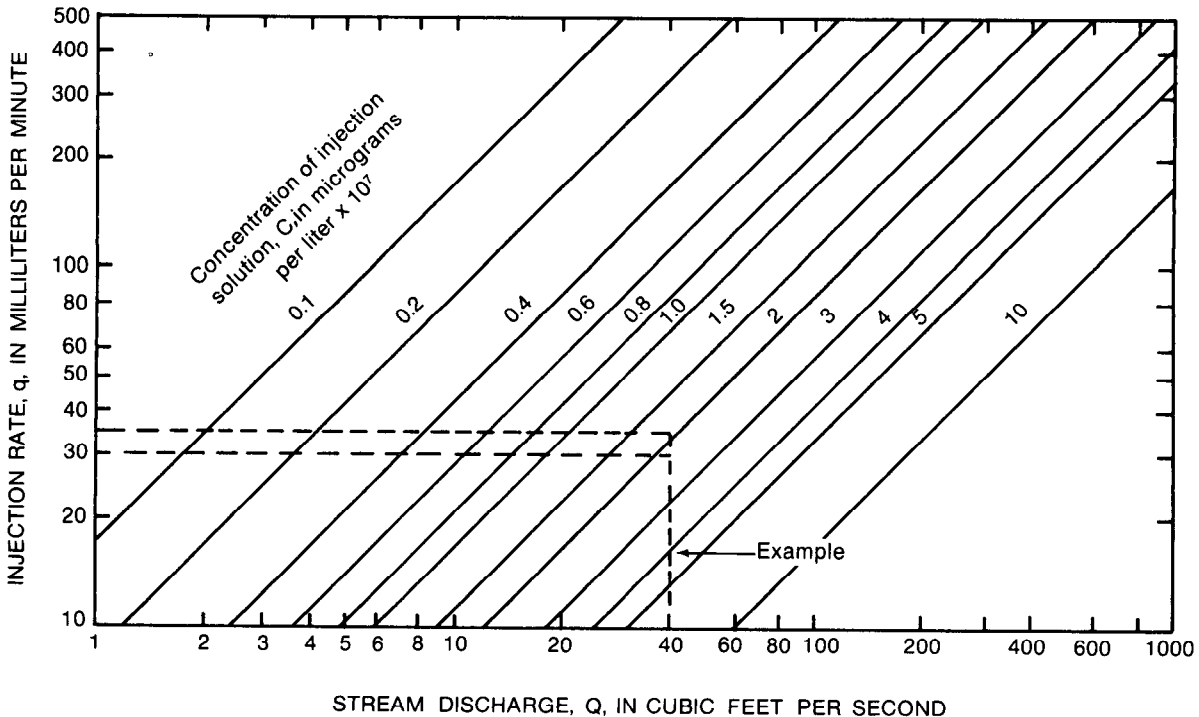


Figure 9.—Graph used for estimating rates of dye injections for different stream discharges and concentrations of injection solutions.

Table 3.—Convenient volumes of water and dye to mix for the preparation of bulk solutions of selected dye concentrations.

	Desired concentration C in $\mu\text{g/L} \times 10^7$ (1)	Volumes to mix together		Dilution factor $D_C \times 10^{-2}$ (4)	Volume of mixture in gallons (5)
		V_w Water, in milliliters (2)	V_d 20% Rhodamine WT dye, milliliters (3)		
1	10	10,700	7,750	42.0	4.87
2	10	11,000	8,000	42.1	5.02*
3	5	14,100	3,750	21.0	4.72
4	5	15,000	4,000	21.1	5.02*
5	4	14,850	3,000	16.8	4.72
6	3	15,600	2,250	12.6	4.72
7	2	16,350	1,500	8.40	4.72**
8	1.5	16,350	1,100	6.30	4.61
9	1.5	17,850	1,200	6.30	5.03*
10	1.0	17,100	750	4.20	4.72
11	0.8	17,250	600	3.36	4.72
12	0.6	17,400	450	2.52	4.72
13	0.4	17,550	300	1.68	4.72
14	0.2	17,700	150	0.80	4.72
15	0.1	17,775	75	0.42	4.72

* In many instances, commercial 5-gallon containers are slightly greater in actual volume; the smaller volumes allow for easier mixing.

**Used in example.

with concentrations as low as 2 $\mu\text{g/L}$; however, higher concentrations are preferred, to lessen the significance of background levels and thus to improve accuracy. Figure 9 is used only to estimate the injection rate needed; the allowable latitude in \bar{c} virtually guarantees a successful measurement, no matter how poor the estimate may be. Furthermore, if 2 $\mu\text{g/L}$ is viewed as a lower limit for \bar{c} , discharges five times those in figure 9 may be measured, using the concentrations and injection rates shown.

Preparation of injection solutions

Bulk solutions

Frequently, the streams to be measured are readily accessible by vehicle, and several streams are to be measured by dye-dilution methods on a single field trip. An example might be a series of turbulent mountain

streams where current-meter measurements are difficult or impossible to make. In such a case, considerable field and laboratory work may be saved by preparing one or two solutions of dye in advance in bulk. The solutions should be tailored to the expected range of flows and the capabilities of the injection apparatus.

Table 3 shows convenient combinations of volumes of rhodamine WT 20-percent dye and water to yield about 5 gal of bulk solution with the concentrations shown in figure 9. Five gallons is normally sufficient for a dozen discharge measurements. The dilutions for concentrations in column 1 of table 3 were determined using equation 7. That equation permits the preparation of any smaller concentrations of dye, such as for the family of curves in figure 9, based on measured combinations of water and dye. Thus any concentration of dye desired can be obtained to suit the need, depending on the

Table 4.—Prepackaged dye quantities to produce selected concentrations of injection solutions.

	V_d , Premeasured volumes of rhodamine WT 20% dye, milliliters (1)	Number of bottles of dye (2)	$V_w + V_d$, Total volume of water and dye solution, milliliters (3)	Dilution factor $D_C \times 10^{-1}$ (4)	Resulting solution concentration, $C, \mu\text{g/L} \times 10^7$ (5)
1	50	1	2000	0.25	0.60
2	50	1	1000	0.50	1.19
3	100	2	2000	1.00	2.38
4	100	1	1000	1.00	2.38*
5	100	2	1000	2.00	4.76
6	100	4	2000	2.00	4.76
7	100	5	2000	2.50	5.95
8	100	6	2000	3.00	7.14
9	250	1	1000	2.50	5.95
10	250	2	1000	5.00	11.90

*Example

capability of the injection equipment and the stream discharge to be measured. The assumption has been made that rhodamine WT 20 percent has a specific gravity of 1.19. Tests have indicated that this may vary from 1.10 to 1.20. For a given dye lot, any such variation will not affect the accuracy of the measurement; the error is canceled, for in equations 2 and 9 the concentrations form a ratio and are relative. This will become apparent later in the discussion of the use of relative concentrations. The dye as received from the manufacturer in large containers should be well mixed before extracting dye to prepare injection solutions. Carefully roll the drums to mix any solid dye that may have been deposited on the bottom.

The advantages of bulk preparation of the dye solutions are

1. Field preparation is unnecessary.
2. Clean water may be used in solution preparation, thereby preventing impurities from causing injection stoppages.
3. The fluorometer may be calibrated in advance for the dye mixture used and only spot-checked at the time of analysis of the stream samples.

4. Many measurements can be made from one concentration of injection solution, thus greatly reducing laboratory preparation of samples and calibration of the fluorometer.

Prepackaged dye quantities

Dilution gaging is particularly well suited to measuring discharge on small cobble-strewn or rocky mountain streams. Such streams are frequently inaccessible, and carrying quantities of premixed dye solutions is onerous. An alternative is to prepackage carefully measured quantities of dye to permit the preparation of injection solutions onsite, using stream water. Table 4 provides a convenient range in premeasured dye quantities to be mixed with selected volumes of water to obtain desired concentrations of injection solutions. These solutions should be packaged in bottles of suitable size, sealed with plastic tape, and labeled. The total volumes of injection solutions have been made to be 1 or 2 L to permit use of those sizes of laboratory graduated cylinders, both for measuring the quantity of solution and as calibrated reservoirs from which injection rates can be measured.

Constant-rate injection equipment

The constant-injection dye-dilution method requires apparatus capable of injecting dye at a small, constant rate reliably and accurately. Equipment that can be purchased or fabricated that will produce constant-rate injections under field conditions includes battery-driven pumps, mariotte vessels, and various chemical-feed devices; each is discussed here.

Pumps

Figure 10 shows a small fluid-metering pump operating on a 12-volt d-c battery withdrawing a dye solution from a graduated cylinder. It is a valveless, variable, positive-displacement pump that can be set for a rate up to 48 mL per minute. Different models are available in a range of capacities and produce accuracies on the order of 1 percent. The valveless feature is desirable because it is cleaner and can handle dirt and foreign material in the dye solution. The rate settings have been found to be reproducible within a fair degree of accuracy; nevertheless, the actual injection rate should be independently measured. Figure 10

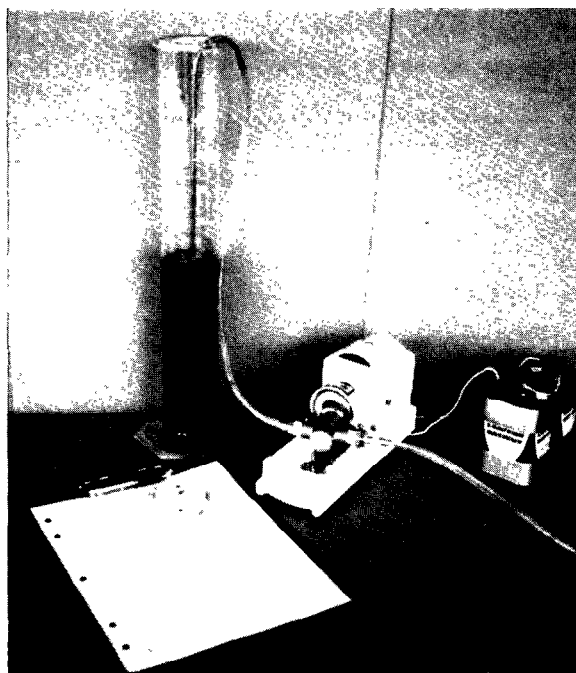


Figure 10.—Pump and graduated cylinder used for constant-rate dye injection.

shows this measurement being done volumetrically by plotting volume versus time and calculating the injection rate from the slope of the resulting line. The advantages of this method of measuring the injection rate are that

1. Any change in the injection rate during the test will be revealed by a change in the slope of the line.
2. The injection is not interrupted during the discharge measurement and may be observed throughout.
3. Sufficient data are obtained to guarantee an accurate measurement of the injection rate.
4. Separate sampling of the dye-injection rate is not necessary, thus reducing the chance of contamination of the hands; this is particularly important if only one person is performing the discharge measurement.

Thin-wall, 1/8-in-diameter brass tubing is used in the cylinder as a withdrawal line to prevent an error in the measured volumes. The pump shown in figure 10 is designed for pumping against a relatively low pressure of 5 lb/in². For pumping into pipes and under ice, and for multiple-line injections, pumps capable of injecting against greater pressures are available. Regardless of manufacturers' ratings, all pumps should be bench tested to check rates and stability of each rate.

This apparatus may be packaged into a backpack unit for use at remote sites (see fig. 11). The entire unit, shown closed in figure 12, weighs only 20 lb. It can be set up, as shown, in minutes. After each use, a water or water-and-alcohol solution should be pumped through the unit to prevent a residue of dye from drying inside and coating parts and tubes.

Peristaltic pumps have also been successfully used for the constant injection of dye solutions. They are usually less expensive than the displacement-type pump but have not proven as accurate.

Mariotte vessels

The mariotte vessel is an inexpensive, simple, and yet reliable constant-injection device. Virtually any rigid, sealable vessel can be made into a mariotte vessel (see fig. 13). The



Figure 11.—Backpack dye-injection apparatus in use.



Figure 13.—Dye injection using a 5-gal can mariotte vessel positioned over the stream by a light cable and trolley system.

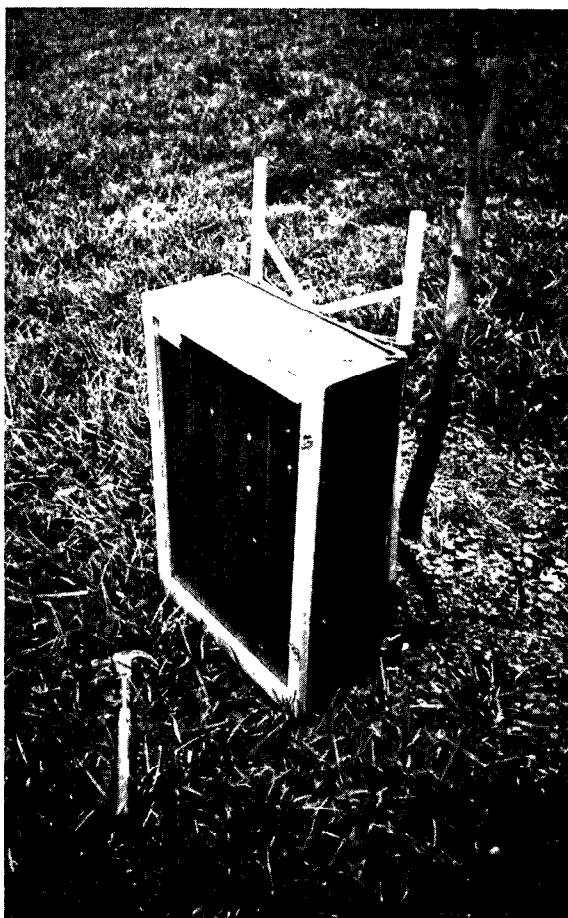
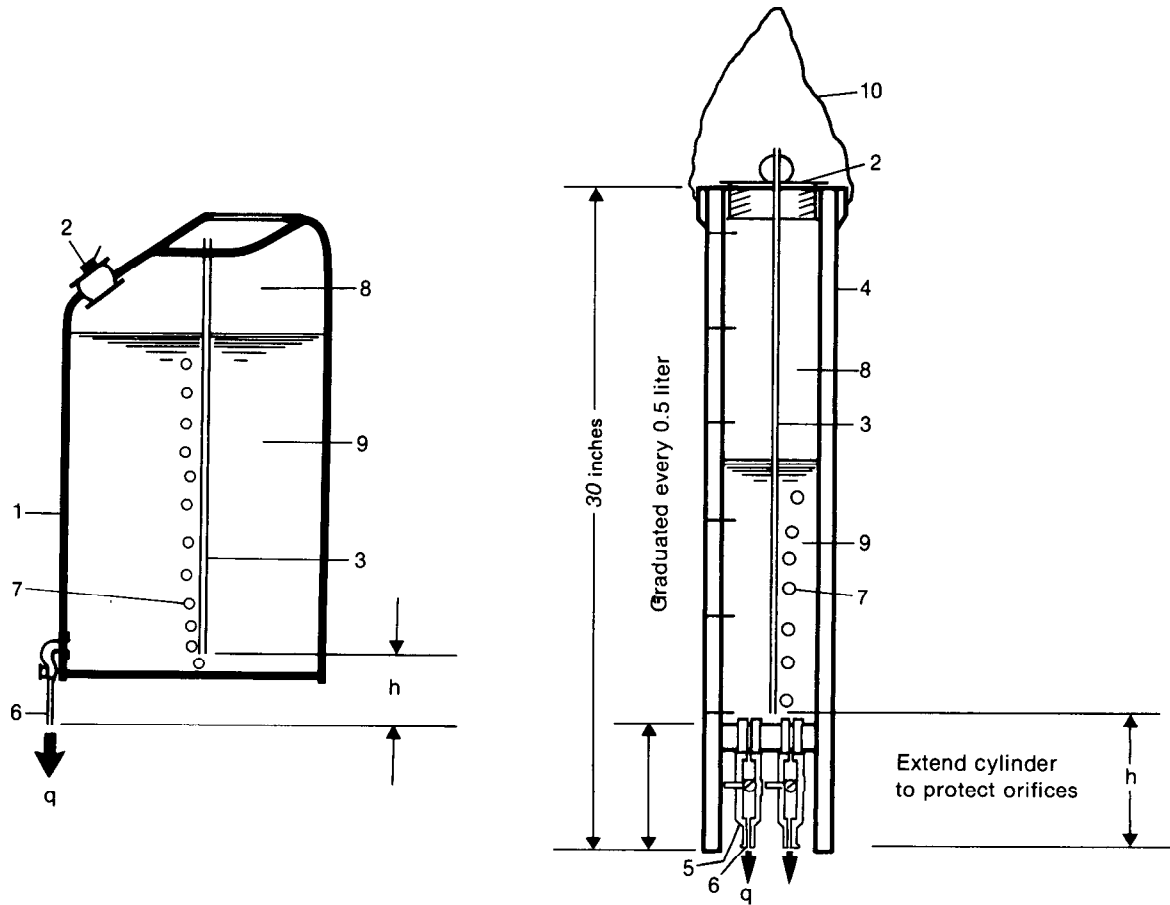


Figure 12.—Backpack dye-injection apparatus ready for transport.

device works on the principle that the air entering the tank to replace the outgoing dye solution is at all times at atmospheric pressure at the tip of the air-intake tube (see fig. 14). Thus, whatever the depth of fluid in the sealed container, constant pressure exists at the tip of the air-intake tube regardless of its depth. The dye solution discharges through a bottom tube or orifice at a rate depending on outlet orifice size and head, h , on the orifice. The head can be varied by raising or lowering the elevation of the outlet tube relative to the bottom of the air-intake tube. The mariotte vessel is a reliable constant-rate injection device if these conditions are ensured:

1. The dye solution is clean and free of dirt that might clog the orifice; it is sometimes desirable to strain the solution.



EXPLANATION

1. Water can, 5-gallon, with heavy enameled interior.
 2. Vacuum bottle top, or plumber's 4-inch pipe plug, expandable type; all openings must be sealed airtight.
 3. Air-inlet tube.
 4. Plastic cylinder, rigid, 4-inch inside diameter.
 5. Gas valve, brass.
 6. Orifice, brass; a range of sizes is used to match desired injection rate.
 7. Air emitted to replace discharging dye solution; atmospheric pressure exists at low point of air tube regardless of fluid above it.
 8. Airspace above dye solution; a partial vacuum exists, requiring that container be of rigid-wall, airtight construction.
 9. Dye solution.
 10. Sling.
- h. Head on orifice; equal to difference in elevation between tip of orifice and bottom of air tube.
- q. Injection rate; a function of the diameter, D , of the orifice, and of h . An approximate equation for the injection rate is $q(\text{mL/min}) = 50,000 D^2 \sqrt{h}$ where D and h are both in inches.

Figure 14.—Design drawing of mariotte-type, constant-rate injection apparatus.

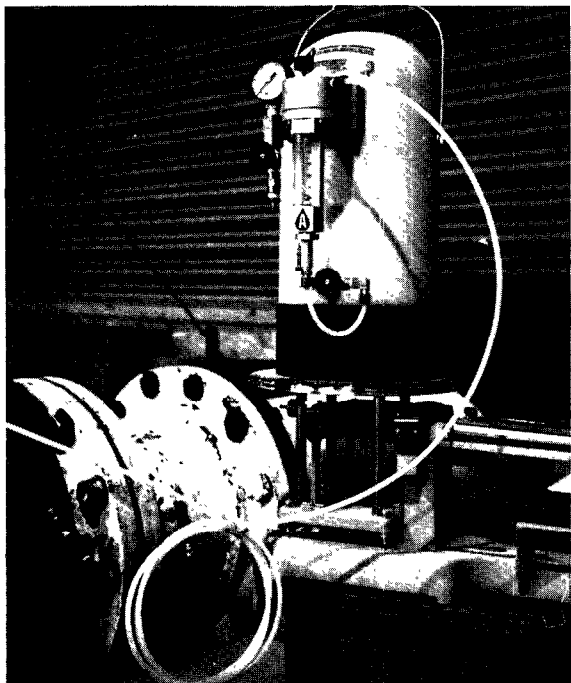


Figure 15.—Pressurized chemical-feed tank being used to inject dye into a pipeline. A laboratory balance measures the injection rate.

2. Air bubbles do not block the orifice or cause a change in rate by collecting in the discharge tubing. Air bubbles can be avoided by using warm water to prepare the dye solution and by shielding the vessel and tubing from sunlight.
3. Orifices and tubing with larger diameters are used, yielding fairly high injection rates and thus transporting dirt and bubbles with less chance of blockages. A minimum injection rate of about 60 mL/min is recommended. Figure 14 shows that, by using a uniform clear plastic cylinder to construct the mariotte vessel on the right, the cylinder can be graduated and injection rates measured by observing change in volume with time. Figure 10 shows this procedure being done with a graduated cylinder.

Chemical-feed devices

Many types of chemical-feed devices are in use to inject chemicals into water and wastewater treatment systems. Most operate on 110 volts

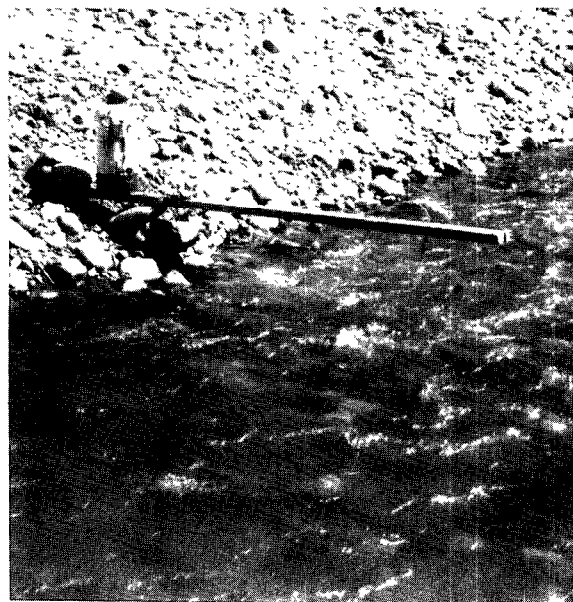


Figure 16.—Single-point, midchannel dye injection from a chemical-feed tank.

and are unsuited for field use. Figures 15 and 16 show a fiberglass chemical-feed tank powered by air pressure that was designed originally to inject chlorine into remote water supplies like those at parks. Such tanks have been used with some success for the constant-rate injection of dye solutions. The main disadvantage of the equipment has been the problem of keeping the rate controller clean. The device must be thoroughly flushed after each use and the controller unit dismantled and cleaned after several uses. The unit is powered by air pressure, and certain safety precautions must be observed. The advantages of the unit are reliable constant-rate injection and the capability of injecting under pressures up to 50 lb/in². This feature is particularly valuable when injecting into a pipeline, as illustrated in figure 15; in this figure, the injection rate is shown being measured with a laboratory balance by observing the change in weight of the dye solution with time.

Performance

A schematic diagram of the performance of a constant-rate dye-dilution measurement is

shown in figure 17. The steps in performing such a measurement use the example shown.

Selection of measurement reach (1)

The stream discharge, mean velocity, and geometry are estimated; using equation 4, an optimum reach length of 700 ft is computed. If there is any question of gain or loss of flow in the stream, or if stage is changing rapidly, an injection is made about 700 ft upstream of the gage and the samples taken close to it.

Background samples (2)

One or two stream background samples are taken before handling dyes and injection equipment.

Dye preparation, injection, and sampling (3)

For an estimated streamflow of 40 ft³/s, figure 9 shows that an injection rate of about 35 mL per minute of a 2.0×10^7 µg/L (2-percent) solution should yield a plateau concentration of about 10 µg/L. Referring to table 3, a 2-percent bulk solution could be prepared by mixing those quantities of water and dye shown in columns 2 and 3, line 7. For a single measurement, just the quantity of solution needed would be preferable. Referring to line 4, column 5, of table 4, it would be convenient to use a 2.38×10^7 µg/L solution with an injection rate of about 30 mL per minute. Thus 1,000 mL would allow injection for 33 minutes. That is usually sufficient on most small streams, but if more time were needed, a 2,000-mL cylinder could be used, adding two 100-mL bottles of dye rather than one, to obtain the necessary dye concentration.

The injection solution is prepared by first emptying a 100-mL bottle of dye into the 1-L cylinder and adding stream water until a total volume of 1,000 mL is obtained. Before topping off the graduated cylinder with river water, it is a good practice to rinse the bottle that had contained the dye with river water, emptying it several times into the cylinder. Then the cylinder should be carefully filled to the 1,000-mL mark. Precision of measurements at this time will make it unnecessary to prepare standards from a sample of this solution, although a sample should be retained in the event it is needed.

The dye mixture in the cylinder should be thoroughly mixed and allowed to stand for a few minutes.

The intake line to the pump is secured inside the graduated cylinder to reach nearly to the bottom. The plastic discharge line leading from the pump should be positioned and secured where the injection will enter what is judged to be the centroid of flow. The pump is turned on and set to a rate of about 30 mL/min. At the end of the injection—after steps (4) and (5)—a sample of injected solution should be bottled, labeled, and stored separately from the stream samples.

Measurement of injection rate (4)

As soon as the injection rate has stabilized, the change in the volume of dye-solution level in the cylinder should be timed with a stopwatch. The watch is not stopped, but the time and volume are observed simultaneously several times before and after stream samples are collected downstream. These volumes are plotted versus time and the rate determined from the slope of the line as 31.2 mL/min.

Sampling the plateau (5)

In contrast to the slug-injection method, immediate and complete sampling of the dye downstream is neither necessary nor desirable. The injection must be maintained at a constant rate long enough for plateau concentrations to be fully established laterally across the channel and with time. The best practice is to inject longer and sample later than is likely needed. Sampling should, of course, be done toward the end of the injection but before its termination.

Stream samples containing the diluted dye should be taken at a minimum of three points across the flow at the sampling section (labeled a, b, and c in fig. 17). Additional sampling points may be advisable if the channel is wide and shallow, or if mixing is thought to be less than optimum.

All samples should be sealed, labeled with the stream name, sampling location and time, and stored out of direct sunlight. Notes describing the measurement should be retained with the samples and sent with them to the laboratory.

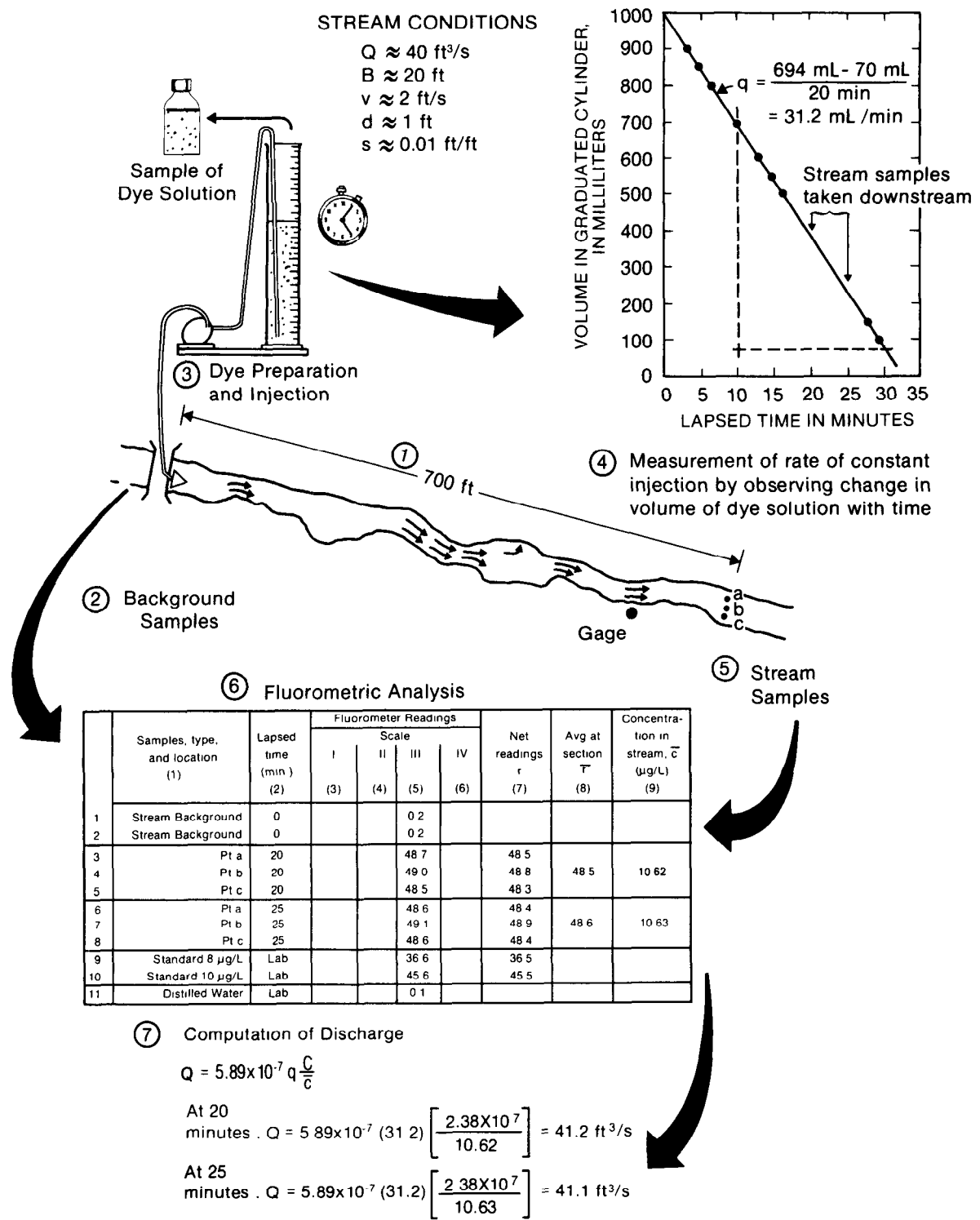


Figure 17.—Schematic diagram of the performance of a constant-rate dye-dilution discharge measurement.

Fluorometric analysis (6)

Assuming that c values at sampling points a , b , and c will be on the order of $10 \mu\text{g/L}$, standards in that concentration range are required. A working solution of $100 \mu\text{g/L}$ should be prepared from the dye lot used and stored for future use. Thus, according to lines 9c and 10c of table 2B, standards of 8 and $10 \mu\text{g/L}$ are prepared by mixing 50 mL of $100\text{-}\mu\text{g/L}$ solution in each of two containers containing 575 mL and 450 mL of distilled water, respectively. The 8- and $10\text{-}\mu\text{g/L}$ standards and a background sample of the distilled water used in their preparation are placed in sample bottles similar to those used for collecting field samples. All stream samples, stream background, standards, and distilled-water background samples are brought to a common temperature— 68°F in the example. All samples are analyzed on one fluorometer scale.

The net dial readings for the 8- and $10\text{-}\mu\text{g/L}$ standards are plotted on the existing fluorometer calibration after subtracting the dial readings for the distilled water sample. If the readings do not plot exactly on the established calibration curve, a new curve is drawn through the two points for that scale parallel to the original calibration (see fig. 7). The primary reason for the shift from the original calibration curve is the difference in sample temperature from that used in the original calibration. The net dial readings for the stream samples are averaged, and, using the new calibration curve, the net concentrations of the stream samples for each set are determined, as shown in column 9 of figure 17. In this case, an average plateau concentration, \bar{c} , of 10.62 and $10.63 \mu\text{g/L}$ was obtained for the sets taken at 20 and 25 minutes.

Computation of discharge (7)

Based on the data presented and using equation 9, the discharge may be computed as

$$Q = 5.89 \times 10^{-7} (31.2) \left[\frac{2.38 \times 10^7}{10.62} \right] = 41.2 \text{ ft}^3/\text{s}$$

for the samples taken 20 minutes after the start of injection and 41.1 cubic feet per second for samples taken after 25 minutes.

Discussion

The example of a constant-rate dye-dilution measurement has been kept rather simple. Frequently, those attempting to apply the method for the first time will overly complicate the procedures. The use of fluorometers in the field, either to analyze individual samples or in a pump-through mode such as is done in time-of-travel studies, is discouraged. Aside from the added labor involved, most fluorometers are not sufficiently accurate under field conditions. Suspended sediment and varying sample temperatures are sources of error not acceptable for a discharge measurement.

Estimation of the mixing length and selection of sampling time are critical. In practice, three samples taken laterally are a minimum; five are advisable, especially if the stream is wide and the flow is shallow. Similarly, in the example cited, sampling at 1,000 ft would be advisable if there were any question regarding the adequacy of mixing length. For a 1,000-ft distance, samples might need to be taken at 30 minutes or later, and the injection continued as necessary.

It is not possible to inject too long or to sample too late, assuming that in the latter case the rate is constant. Figure 3 shows that if samples are taken too soon, the concentrations along one or both banks will be measured prematurely, yielding low values compared to the correct plateau concentrations. Thus the computed discharge would be high. This might suggest loss of dye when in reality sampling was done too soon. Similarly, this much variation in concentration laterally might suggest poor mixing, whereas waiting longer to sample would have revealed good mixing. Thus it is emphasized that injection should be long enough and sampling late enough to ensure that what is being measured is a fully developed concentration plateau across the stream and with time.

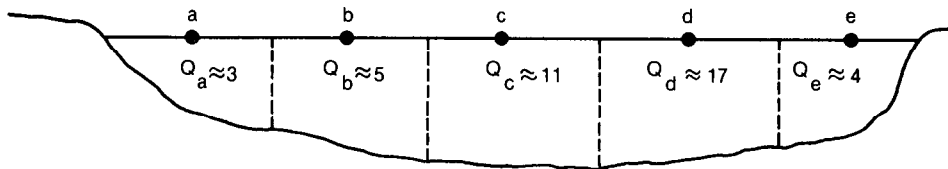
In the example, the concentration of the injection solution was chosen for a discharge estimated at $40 \text{ ft}^3/\text{s}$. Had the discharge actually been $60 \text{ ft}^3/\text{s}$, the stream samples would have had a concentration of about $7 \mu\text{g/L}$. Such concentrations are more than adequate for an accurate measurement. All samples are best ana-

lyzed on the same scale of the fluorometer; an additional standard of about 7 µg/L would have been desirable. For convenience, a 6-µg/L standard could be prepared, working from line 11c of table 2B.

Measurements with less than optimum mixing

The nearly identical concentrations observed at 700 ft in the previous example indicate that

mixing is adequate and the mixing distance probably optimum. If a reach length of 400 ft had been chosen, mixing probably would not have been as complete. As a further example, it is assumed that the flow at the 400-ft sampling section is distributed as shown in figure 18 and that dye concentrations at a through e are fully stabilized with time as shown. Equation 2 is predicated on c as a measure of the dilution throughout the flow section, which is not the case at this short distance. If data reveal that



Cross-section and approximate discharge distribution at 400 feet

Location of samples taken at section 400 feet downstream (1)	Fluorometer reading for stream samples (2)	Stream background reading (3)	Net reading (4)	Estimated portion of total discharge (5)	Product of columns 4 and 5 (6)
Pt a	32.2	0.2	32.0	3	96
Pt b	42.2	0.2	42.0	5	210
Pt c	58.6	0.2	58.4	11	642.4
Pt d	40.2	0.2	40.0	17	680
Pt e	37.4	0.2	37.2	4	148.8

Note: all readings on same fluorometer scale

Σ = 40

Σ = 1777.2

Arithmetic average = 41.9

From figure 7, \bar{c} = 9.20 µg/L

Weighted average scale III reading for \bar{c} = $\frac{1777.2}{40}$ = 44.4

From figure 7, \bar{c} = 9.80 µg/L

$$Q = 5.89 \times 10^{-7} q \frac{C}{\bar{c}}$$

Using

Arithmetic

Average: $Q = 5.89 \times 10^{-7} (31.2) \left[\frac{2.38 \times 10^7}{9.20} \right] = 47.5 \text{ ft}^3/\text{s}; 16 \text{ percent high}^*$

Using

Weighted

Data: $Q = 5.89 \times 10^{-7} (31.2) \left[\frac{2.38 \times 10^7}{9.80} \right] = 44.6 \text{ ft}^3/\text{s}, 8 \text{ percent high}^*$

*Assuming 41.1 ft³/s to be the correct discharge.

Figure 18.—Example of flow-weighting concentration data where mixing is poor.

mixing is not as good as desired (appendix A contains a computation of the percentage of mixing for this example), a more correct discharge can be computed if the concentrations at each lateral sampling point are discharge weighted. Thus it is a good practice to estimate and assign a proportion of the total flow to each sampling point and weight the data during the computation as shown in figure 18. Although the weighting procedure is recommended, it should not be used as a substitute for trying to select an adequate stream reach to produce optimum mixing.

Appendix B contains a form for the recording of field data, fluorometric analysis of samples, and computation of discharge using the constant-injection method incorporating the above weighting procedure. The data used in the figure 18 example are shown in a completed form to help the user.

As indicated, dilution-type measurements of discharge can be made where mixing is less than optimum if samples are flow weighted across the stream. This can also be accomplished by using a depth-integrating sediment sampler to obtain a number of equally spaced, vertically integrated samples across a stream, as is standard practice in sediment sampling (Guy and Norman, 1970). A single composite sample is analyzed to obtain \bar{c} . The number of verticals required will depend on the degree of mixing that has taken place at the sampling section and on the accuracy requirements for the discharge measurement. This method can be approximated by moving the integrating sediment sampler laterally at a constant speed at approximately middepth. These methods generally should not be used where mixing is less than 75 percent, as there is some subjectivity in getting depth-integrated samples.

The Use of Relative Concentrations

It is apparent from equation 9 that concentrations could be measured in any similar units, since C/c is a ratio—for example, fluorometer dial readings. Furthermore, if both river samples and standard samples are analyzed in an identical manner, certain potential

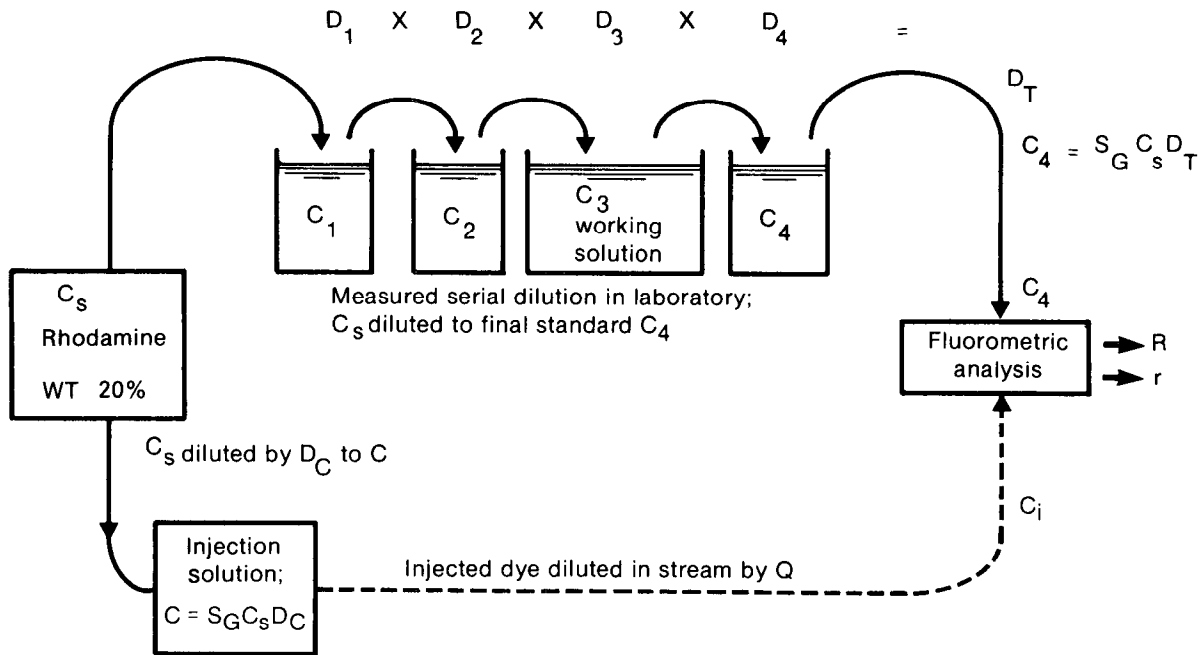
errors may be canceled or lessened. This is one of the reasons for placing river samples and standards samples in identical bottles and allowing all samples to come to a common temperature. For the same reason, it is recommended that all samples be analyzed on the same fluorometer scale. As has been noted, the 8- and 10- $\mu\text{g/L}$ standards were chosen to coincide closely with the concentrations expected for the stream samples. This was done to ensure that all samples would be analyzed on the same fluorometer scale, thereby lessening any errors that might occur by shifting from one fluorometer scale to another, as well as for simplicity. Similarly, the fluorometer calibration should be prepared from the same dye lot as used in the actual field test.

Two approaches to the use of relative concentration are presented; both have advantages and disadvantages as to simplicity and accuracy. Both use net fluorometer dial readings to evaluate the ratio C/\bar{c} in the constant-rate dye-dilution discharge relationship, equation 9. Both require the computation and use of dilution factors. The first case requires the total dilution factor, D_T , necessary in reducing the manufacturer's stock solutions, C_s , down to a final standard solution and in the second case the single dilution factor, D_C , necessary in reducing C_s down to C , the concentration of the dye solution that was injected. The second case requires that the determination start with the injected solution rather than the manufacturer's solution. The schematic diagrams in figures 19 and 20 aid in understanding the concept and use of relative concentrations for computing the discharge where the constant-injection method is used. The examples in figures 19 and 20 use the data for the same example as that in figure 17. The use of relative concentrations and dilution factors may at first appear complex but, once understood, will simplify computational procedures and laboratory work involved in such measurements.

Relative concentrations based on stock solutions

All measurements start with the stock dye solution as supplied by the manufacturer, having a concentration C_s (see fig. 19). For

CONCEPTUAL DIAGRAM



EXAMPLE COMPUTATION

$$\frac{C}{C_4} = \frac{S_G C_s D_C}{S_G C_s D_T} = \frac{D_C}{D_T} \quad C = C_4 \frac{D_C}{D_T}$$

Equation 9: $Q = 5.89 \times 10^{-7} \ q \frac{C}{\bar{c}}$

By substitution:

$$Q = 5.89 \times 10^{-7} \ q \left[\frac{C_4}{\bar{c}} \times \frac{D_C}{D_T} \right]$$

Since samples of C_4 and \bar{c} are to be read on the same scale

$C_4 = kR$ and $\bar{c} = kr$, where k is a scale constant.

$$\text{Therefore } Q = 5.89 \times 10^{-7} \ q \left[\frac{R}{r} \times \frac{D_C}{D_T} \right]$$

From data in figure 17 and dilution factors in table 4, line 3, and table 2B, line 9:

$$Q = 5.89 \times 10^{-7} (31.2) \left[\frac{45.5}{48.6} \times \frac{10^{-1}}{4.20 \times 10^{-8}} \right] = 41.0 \text{ ft}^3/\text{s}$$

Figure 19.—Conceptual diagram and example of computations using relative concentrations and dilution factors originating with the manufacturer's stock dye solution.

rhodamine WT, this is 20 percent by weight. As seen in figure 19, C_s is usually reduced to some lesser concentration C for injection into the stream. As seen in column 4 of tables 3 and 4, based on the volumes of water and dye mixed, a precise dilution factor, D_C , may be computed using equation 7:

$$C_n = C_i S_G \left[\frac{V_d}{V_w + V_d} \right] = C_s S_G D_C$$

where

$$C_i = C_s \text{ and } \left[\frac{V_d}{V_w + V_d} \right] = D_C .$$

In the previous examples, a stock solution C_s of rhodamine WT 20 percent was reduced before injection to a C of $2.38 \times 10^7 \mu\text{g/L}$ by combining 100 mL of dye with 900 mL of water (see table 4, line 4). Thus C_s was diluted by a factor, D_C , of 100/1,000 or 10^{-1} . The concentration in the stream \bar{c} is then a function of the rate of injection of C and of its dilution by Q .

In preparing laboratory standards, a sample of C_s is reduced until a final concentration, C_4 , is reached after a four-step serial dilution (see fig. 19). Thus, by this serial-dilution process, the standard C_4 , the one tailored to closely match that expected in the stream, \bar{c} , is prepared from C_s as

$$C_4 = S_G C_s D_T$$

where D_T is the total dilution required and the product of the individual dilutions. In table 2B, the total dilution factor after four dilutions is shown in column 5. The total dilution factor for the $10\text{-}\mu\text{g/L}$ standard used in the previous example is

$$D_T = D_1 \times D_2 \times D_3 \times D_4$$

$$D_T =$$

$$\left[\frac{30}{50+3792} \right] \left[\frac{20}{20+3500} \right] \left[\frac{20}{20+3500} \right] \left[\frac{50}{50+450} \right]$$

$$D_T = 4.20 \times 10^{-8}$$

and the final concentration, C_4 , starting with rhodamine WT 20 percent is

$$C_4 = C_s S_G D_T = 20 \times 10^7 \times 1.19 \times 4.20 \times 10^{-8} \\ = 10 \mu\text{g/L}$$

as desired.

Using the net dial readings R and \bar{r} in place of C and \bar{c} in equation 9 and the appropriate dilution factors, the discharge may now be computed as

$$Q = 5.89 \times 10^{-7} q \left[\frac{R}{\bar{r}} \times \frac{D_C}{D_T} \right] \quad (10)$$

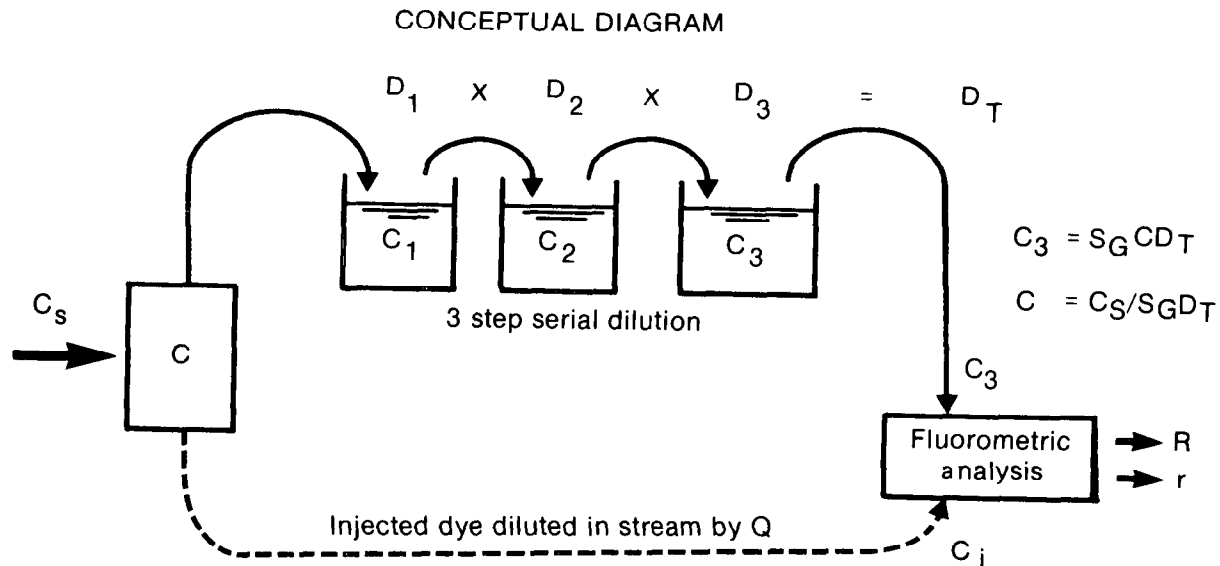
and, using the example data provided in figure 17 and tables 2B and 4, the discharge may be computed as

$$Q = 5.89 \times 10^{-7} \times 31.2 \left[\frac{45.5}{48.6} \times \frac{10^{-1}}{4.20 \times 10^{-8}} \right] \\ = 41.0 \text{ ft}^3/\text{s} .$$

Note that if C is selected as a concentration shown in column 1 or 5 of table 3 or 4, respectively, and if C_4 is chosen as one of those in column 4 in table 2B, the determination of the discharge is essentially a matter of determining the ratio of the fluorometer dial readings for the standard and the stream samples. The dilution factors, D_C and D_T , may be taken from tables 2B, 3, and 4 or any convenient tables the user might wish to develop. Certain precautionary measures must be taken in using this technique. All measurements and computations start with C_s and assume that the manufacturer's solution is well mixed and the same whenever quantities are taken from the container of a particular dye lot.

Relative concentrations based on injected solutions

A schematic diagram and an example of computations of discharge, using relative concentrations and dilution factors starting with the concentration, C , of the dye injected rather than C_s , are shown in figure 20. This method has the advantage of avoiding errors that might occur if the manufacturer's solution was not mixed each time dye was extracted, or if an



COMPUTATIONS

Equation: $Q = 5.89 \times 10^{-7} q \frac{C}{\bar{c}}$

By substitution:

$$Q = 5.89 \times 10^{-7} q \left[\frac{C_3}{\bar{c}} \times \frac{1}{S_G D_T} \right]$$

Since samples of C_3 and \bar{c} are to be read on the same scale.

$C_3 = kR$ and $\bar{c} = k\bar{r}$, where k is a scale constant.

Therefore

$$Q = 5.89 \times 10^{-7} q \left[\frac{R}{\bar{r}} \times \frac{1}{S_G D_T} \right]$$

Note: S_G is the specific gravity of C , not of rhodamine WT 20-percent.

If C is $1 \times 10^7 \mu\text{g/L}$ or less, S_G will be less than 1.01 and can be ignored.

EXAMPLE: As in the example in figure 17: $C = 2.38 \times 10^7 \mu\text{g/L}$
 $q = 31.2 \text{ mL/min}$
 $\bar{r} = 48.6$ net dial reading on
 scale III of the fluorometer
 $S_G = 1.02$

Using table 5A, prepare a standard, $C_3 = 10.34 \mu\text{g/L}$ (line 3c)
 This standard yielded a net dial reading on scale III of 47.0
 Thus the discharge is

$$Q = 5.89 \times 10^{-7} (31.2) \left[\frac{47.0}{48.6} \times \frac{1}{(1.02) (4.34 \times 10^{-7})} \right] = 40.1 \text{ ft}^3 \text{ 3/s}$$

Figure 20.—Conceptual diagram and example of computations using relative concentrations and dilution factors originating with the injected solution.

Table 5.—Convenient three-step serial-dilution combinations for the preparation of selected standards for selected dye-injection solutions; prepackaged solutions.

Injection solution C in $\mu\text{g}/\text{L} \times 10^7$	Dilutions						Resulting conc. C_3 in $\mu\text{g}/\text{L}$	Resulting dilution factor, $D_T \times 10^{-7}$
	First		Second		Third			
(1)	V_d	V_w	V_d	V_w	V_d	V_w	(8)	(9)
1a 0.60	50	3000	20	2000	20	3000	6.45	10.75
b	50	3000	20	2000	20	2500	7.73	12.88
c	50	3000	20	2000	20	2000	9.64	16.07
d	50	3000	20	2000	20	1500	12.82	21.36
2a 1.19	50	3000	20	3000	20	3500	7.34	6.17
b	50	3000	20	3000	20	3000	8.56	7.19
c	50	3000	20	3000	20	2500	10.25	8.62
d	50	3000	20	3000	20	2000	12.79	10.75
3a 2.38	25	3000	20	3000	20	3500	7.40	3.11
b	25	3000	20	3000	20	3000	8.63	3.62
c	25	3000	20	3000	20	2500	10.34	4.34
d	25	3000	20	3000	20	2000	12.90	5.42
4a 4.76	20	3000	20	3500	10	2000	8.91	1.87
b	20	3000	20	3500	20	3500	10.18	2.14
c	20	3000	20	3500	20	3000	11.86	2.49
d	20	3000	20	3500	20	2750	12.93	2.72
5a 5.95	20	3500	20	3500	10	2500	7.65	1.29
b	20	3500	20	3500	10	2000	9.56	1.61
c	20	3500	20	3500	10	1750	10.91	1.83
d	20	3500	20	3500	10	1500	12.72	2.14
6a 7.14	20	3500	20	3500	10	3000	7.66	1.07
b	20	3500	20	3500	10	2500	9.18	1.29
c	20	3500	20	3500	10	2000	11.47	1.61
d	20	3500	20	3500	10	1750	13.10	1.83
7a 11.90	20	3500	10	3000	10	3000	7.46	0.627
b	20	3500	10	3000	10	2250	9.94	0.835
c	20	3500	10	3000	10	2000	11.18	0.939
d	20	3500	10	3000	10	1750	12.76	1.073

error was made in measuring the quantities of dye and water in preparing the injection solution. Thus the stream discharge could be determined if the absolute value of C was unknown but a sample of C was retained from each test. This is good practice regardless of the method of analysis. The fact that C/\bar{c} is a ratio allows the comparison and measurement of \bar{c} relative to C using dial readings and measured dilutions. Unlike the previous method in which a working solution of $100 \mu\text{g}/\text{L}$ is prepared and retained for each dye lot (and, hence, only a single additional dilution, D_4 , is required for a measurement), a three-step serial dilution of C must be done for each test where a different injection solution is used. This emphasizes the

advantages of preparing a bulk quantity of dye solution of a single concentration, such as one of those in table 3, for a series of discharge measurements. The same would be true for the use of prepackaged dye quantities (such as in table 4); selection of a single dye concentration, C , for a series of measurements always lessens the laboratory work in preparing standards.

By starting with the injected solution, C , only a three-step serial dilution need be prepared to obtain a standard about equal to that expected in the stream for \bar{c} . Tables 5 and 6 provide convenient combinations of dye and water to yield a range of standards from about 7 to $13 \mu\text{g}/\text{L}$ (column 8), which should be the range obtained in the field samples. Column 9

Table 6.—Convenient three-step serial-dilution combinations for the preparation of selected standards for selected dye-injection solutions; bulk solutions.

Injection solution C in $\mu\text{g/L} \times 10^7$	Dilutions						Resulting conc. C_3 in $\mu\text{g/L}$	Resulting dilution factor, $D_T \times 10^{-7}$
	First		Second		Third			
(1)	V_d	V_w	V_d	V_w	V_d	V_w	(8)	(9)
8a 0.1	50	3000	50	1000	20	2000	7.73	77.29
b	50	3000	50	1000	20	1750	8.82	88.21
c	50	3000	50	1000	20	1500	10.27	102.71
d	50	3000	50	1000	20	1000	15.31	153.07
9a 0.2	50	3000	50	2000	20	2000	7.92	39.59
b	50	3000	50	2000	20	1750	9.04	45.18
c	50	3000	50	2000	20	1500	10.52	52.61
d	50	3000	50	2000	20	1250	12.59	62.97
10a 0.4	50	3000	50	3000	20	1500	14.14	35.36
b	50	3000	50	3000	20	1750	12.15	30.37
c	50	3000	50	3000	20	2000	10.64	26.61
d	50	3000	50	3000	20	2500	8.53	21.33
11a 0.6	50	3000	20	2000	20	3000	6.45	10.75
b	50	3000	20	2000	20	2500	7.73	12.88
c	50	3000	20	2000	20	2000	9.64	16.07
d	50	3000	20	2000	20	1500	12.82	21.36
12a 0.8	50	3000	20	2000	20	3000	8.60	10.75
b	50	3000	20	2000	20	2750	9.38	11.72
c	50	3000	20	2000	20	2500	10.31	12.88
d	50	3000	20	2000	20	2000	12.86	16.07
13a 1.0	50	3500	20	3000	20	2500	7.40	7.40
b	50	3500	20	3000	20	2000	9.24	9.24
c	50	3500	20	3000	20	1750	10.54	10.54
d	50	3500	20	3000	20	1500	12.27	12.27
14a 1.5	50	3500	20	3000	20	3500	7.95	5.30
b	50	3500	20	3000	20	3000	9.27	6.18
c	50	3500	20	3000	20	2500	11.10	7.40
d	50	3500	20	3000	20	2000	13.85	9.24
15a 2.0	50	3500	20	3500	10	2000	7.96	3.98
b	50	3500	20	3500	20	3500	9.09	4.55
c	50	3500	20	3500	20	3000	10.60	5.30
d	50	3500	20	3500	20	2500	12.70	6.35
16a 3.0	25	3500	20	3000	20	3500	8.01	2.67
b	25	3500	20	3000	20	3000	9.33	3.11
c	25	3500	20	3000	20	2500	11.18	3.73
d	25	3500	20	3000	20	2000	13.95	4.65
17a 4.0	25	3500	20	3500	10	2000	8.02	2.00
b	25	3500	20	3500	20	3500	9.16	2.29
c	25	3500	20	3500	20	3000	10.67	2.67
d	25	3500	20	3500	20	2500	12.79	3.20
18a 5.0	20	3500	20	3500	10	2000	8.03	1.61
b	20	3500	20	3500	20	3500	9.17	1.83
c	20	3500	20	3500	20	3000	10.69	2.14
d	20	3500	20	3500	20	2500	12.81	2.56
19a 10.0	20	3500	20	3500	10	3750	8.59	0.859
b	20	3500	20	3500	10	3500	9.20	0.920
c	20	3500	20	3500	10	3000	10.73	1.07
d	20	3500	20	3500	10	2500	12.86	1.29

in tables 5 and 6 provides the total dilution factors, D_T , for whatever concentration is used. Following the diagram in figure 20, it can be seen that the standard, C_3 , is equal to $S_G CD_T$. Thus, $C = C_3/S_G D_T$ and the discharge equation, substituting net dial readings for concentrations, is

$$Q = 5.89 \times 10^{-7} q \left[\frac{R}{\bar{r}} \times \frac{1}{S_G D_T} \right] \quad (11)$$

As an example, the same test conditions as those previously used and presented in figure 17 are assumed, except that now a three-step serial dilution of C must be performed. Referring to table 5, a convenient serial dilution combination for the case where $C = 2.38 \times 10^7 \mu\text{g/L}$ is found on line 3c (for $C_3 = 10.34$), since the injection rate for the estimated stream discharge was chosen to produce a plateau concentration of about $10 \mu\text{g/L}$ (fig. 9). A standard, C_3 , is analyzed on the fluorometer on scale III at the same time the field samples are analyzed. This sample gives a dial reading of 47.2 for a net of 47.0 (the stream background was 0.2). Thus for the stream samples taken at 25 minutes after the start of injection and having an average dial reading, \bar{r} , of 48.6, the discharge may be computed as

$$Q = 5.89 \times 10^{-7} \times 31.2 \left[\frac{47.0}{48.6} \times \frac{1}{(1.02)4.34 \times 10^{-7}} \right] \\ = 40.1 \text{ ft}^3/\text{s}$$

The dilution factor, D_T , was picked from column 9, line 3c of table 5 for the standard chosen. Note that in tables 5 and 6, the first two dilutions for any one concentration, C , are the same. Thus the second dilution may be treated as a working solution. If the $10.34\text{-}\mu\text{g/L}$ standard was not close to that experienced in the stream (R did not approximately equal \bar{r}), another standard could be prepared quickly, starting with the second dilution.

This method must be considered the most accurate of those presented, because only a three-step serial dilution of C is involved. Any error in preparing the injection solution is eliminated, and fluorometer dial readings are used directly without recourse to calibration curves.

Appendix B contains a form for recording field sampling, fluorometer analysis, and calculation of discharge using the constant-injection method. It incorporates the use of relative concentrations, dilution factors, and discharge weighting. Provision is made for sampling as many as five locations laterally across a flow section; however, on narrow well-confined flows, three sampling locations are usually sufficient unless the adequacy of mixing is suspect. Appendix B also contains a completed form using the data for the example measurements illustrated in figures 17 and 19 where relative concentrations are used.

Mixing Length

The mixing length equation, equation 4, can be rewritten as

$$L_m = K \frac{vB^2}{E_z} \quad (12)$$

where the variables are as previously defined, except that K is a variable with a value dependent on the degree of mixing, the location of injection and the number of injections. The degree of mixing is a measure of the degree to which a tracer is mixed in the flow; the higher the percentage, the more nearly are the concentrations the same laterally. Appendix A shows the recommended method of computing percentage of mixing.

The value of K of 0.1 given in equation 4 is for 95-percent mixing with a single-center injection at midstream. Coefficients, K , for other conditions are given in table 7.

The effect of injecting tracer at n points, where each injection is at the center of flow of each n equal-flow segment, is that the tracer has to mix throughout an equivalent width of about $(1/n)B$. Because B is squared in the mixing length equation, the value of K for a single-point injection is modified by the factor $(1/n)^2$.

Injection at the side of a channel is equivalent to an injection into a stream with a width of $2B$. Because B is squared, the coefficient K must be increased by a factor of 4.