Techniques of Water-Resources Investigations of the United States Geological Survey

Chapter A20

SIMULATION OF SOLUBLE WASTE TRANSPORT AND BUILDUP IN SURFACE WATERS USING TRACERS

By F.A. Kilpatrick

Book 3

APPLICATION OF HYDRAULICS

U.S. DEPARTMENT OF THE INTERIOR
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[1]This manual is a revision of "Measurement of Time of Travel and Dispersion in Streams by Dye Tracing," by E.F. Hubbard, F.A. Kilpatrick, L.A. Martens, and J.F. Wilson, Jr., Book 3, Chapter A9, published in 1982.
[2]Spanish translation also available.

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[1]This manual is a revision of TWRI 5–A3, "Methods of Analysis of Organic Substances in Water," by Donald F. Goerlitz and Eugene Brown, published in 1972.

[2]This manual supersedes TWRI 5–A4, "Methods for collection and analysis of aquatic biological and microbiological samples," edited by P.E. Greeson and others, published in 1977.

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CONVERSION FACTORS

Multiply inch-pound unit	*By*	*To obtain metric unit*
mile (mi)	1.609	kilometer (km)
pound (lb)	453.6	gram (g)
gallon (gal)	3.785	liter (L)
cubic foot (ft^3)	0.0283	cubic meter (m^3)
cubic foot per second (ft^3/s)	0.0283	cubic meter per second (m^3/s)

Sea level: In this report, "sea level" refers to the National Geodetic Vertical Datum of 1929 (NGVD of 1929)—a geodetic datum derived from a general adjustment of the first-order level nets of both the United States and Canada, formerly called Sea Level Datum of 1929.

SYMBOLS AND UNITS

Symbol	*Explanation*	*Unit*
$a, b, \ldots, g$	Designation of streamlines and response curves laterally in channel	—
A_c	Area of observed response curve	(µg/L)(min) or (µg/L)(h)
A_u	Area of unit-response curve	[(µg/L)/lb] (ft^3/L)
C_{cp}	Conservative peak concentrations	µg/L
C_d	Observed dye concentration	µg/L
C_p	Observed peak concentration	µg/L
C_s	Concentration of stock dye solution, generally as obtained from the manufacturer	µg/L or percent
C_t	Observed tracer concentration	µg/L
C_u	Unit concentration	[(µg/L)/lb] (ft^3/s)
C_{up}	Unit-peak concentration	[(µg/L)/lb] (ft^3/s)
c_w	Diluted waste concentration in stream	µg/L
C_w	Waste concentration injected	µg/L
e	A constant, equal to 2.72	—
HHSW	High high slack water; the time near high high tide when zero or minimum current exists	
HHT	High high tide; highest of the two high tidal stages occurring in a tidal day	
HLSW	High low slack water; the time near high low tide when zero or minimum current exists	
HLT	High low tide; highest of the two low tidal stages occurring in a tidal day	
k	Decay rate constant to the base 10	
K	Decay rate constant to the base e	
LHSW	Low high slack water; the time near low high tide when zero or minimum current exists	
LHT	Low high tide; lowest of the two high tidal stages occurring in a tidal day	
LLSW	Low low slack water; the time near low low tide when zero or minimum current exists	

Symbols	*Explanation*	*Unit*
LLT	Low low tide; lowest of the two low tidal stages occurring in a tidal day	
n	Slope of the unit-peak-attenuation curve	
q	Rate of constant injection of dye tracer	mL/min
q_w	Waste injection rate	lbs/day
Q	Total stream discharge	ft^3/s
R_r	Dye recovery ratio	—
ΔT	Quasi-steady-state period in an estuary during which tracer is injected at a constant rate; also the numerical integration interval in applying the superposition principle	hours
t	Elapsed time from start of injection	hours, days
t_b	Period when concentrations are building up to the peak	hours
t_r	Period when concentrations are receding from peak to when $C_t = 0.1C_p$	hours
T	Time in tidal day; normally 24.8 hours	
$T_{c,L,t,p}$	Elapsed time to centroid, leading edge, trailing edge, and to peak, respectively, of dye-response curve	hours or min
T_d	Duration in time for dye cloud to pass any one point in a section	hours or min
$T_{d_{10}}$	Duration of abbreviated response curve to point where $C_t = 0.1C_p$	hours
T_D	Duration of longest response curve in a section	hours or min
UPA	Unit-peak attenuation	
UR	Unit-concentration response	
V_s	Volume of stock dye solution	L or mL
W	Weight of soluble waste	lbs
W_d	Weight of dye injected	g or lbs
W_r	Weight of dye recovered	g or lbs
W_{rt}	Weight of waste remaining	lbs
W_t	Weight of tracer injected	lbs

SIMULATION OF SOLUBLE WASTE TRANSPORT AND BUILDUP IN SURFACE WATERS USING TRACERS

By F.A. Kilpatrick

Abstract

Soluble tracers can be used to simulate the transport and dispersion of soluble wastes that might have been introduced or are planned for introduction into surface waters. Measured tracer-response curves produced from the injection of a known quantity of soluble tracer can be used in conjunction with the superposition principle to simulate potential waste buildup in streams, lakes, and estuaries. Such information is particularly valuable to environmental and water-resource planners in determining the effects of proposed waste discharges.

The theory, techniques, analysis, and presentation of results of tracer-waste simulation tests in rivers, lakes, and estuaries are described. This manual builds on other manuals dealing with dye tracing by emphasizing the expanded use of data from time-of-travel studies.

Introduction

The extensive use of fluorescent dyes as water tracers began in the early to mid-1960's. Prior to that time, floats, chemical salts, and actual contaminants had been used as tracers. After World War II, radioisotopes such as tritium (heavy hydrogen) gained favor as tracers, but their use was severely limited by problems in handling, the special training required, and a general lack of understanding by the public. A search for a suitable substitute for radioisotopes led to the rediscovery of fluorescent dyes for tracing.

Within the U.S. Geological Survey, feasibility tests of dyes and fluorometers were made in 1961–62 and were reported by Wright and Collings (1964). The initial application of fluorometry, and by far the application most used to date, was for the measurement of time of travel of solutes in streams (Buchanan, 1964; Kilpatrick and others, 1989). The procedures also were adapted to the measurement of stream discharge by dye-dilution methods (Kilpatrick and Cobb, 1985). Fluorometry has also been applied to studies to determine reaeration rates of streams (Rathbun and others, 1977; Kilpatrick and others, 1989).

In addition to time-of-travel, dispersion, reaeration, and discharge measurements, hydrologic applications have included studies to simulate waste buildup and flushing in estuaries and streams (Bailey and others, 1966; Kilpatrick and Cummings, 1972; Yotsukura and Kilpatrick, 1973; Kilpatrick and Taylor, 1986). The subject of this manual is the application of tracers to simulate the movement, transport, and buildup of wastes in streams, lakes, and estuaries. Emphasis is on the use of dye tracers, but the principles can be applied to any water tracer.

A soluble tracer can be used to simulate a soluble waste by duplicating its movement in *any* hydrologic system, be it a steady flowing river or the unsteady oscillatory stage and flow of a tidal estuary. In the first case, a typical time-of-travel study will be expanded to provide more than just the predicted speed of a slug of soluble waste; the same tracer that simulated the speed of a river can be used to simulate and, hence, predict waste concentrations for different conditions of flow, time, and location. Similarly, a soluble tracer will replicate the movement characteristics of a soluble waste when injected in an estuary in a like manner and location, a tremendous advantage being that few if any of the complex geometries or flows of the estuary waterway need be measured.

It will be assumed that the reader is familiar with the several manuals on fluorometry (Wilson and others, 1986), time of travel (Kilpatrick and Wilson, 1989), measurement of discharge (Kilpatrick and Cobb, 1985), and measurement of reaeration (Kilpatrick and others, 1989). For simplicity, these reports will be referred to as the fluorometry, time-of-travel, dilution discharge, and reaeration manuals, respectively.

Particular attention should be given to the last two manuals because an understanding of the superposition principle and of the various means of performing slug and constant rate tracer injections is vital to understanding and performing waste-simulation studies. For continuity and convenience, some of the information available in these manuals will be pre-

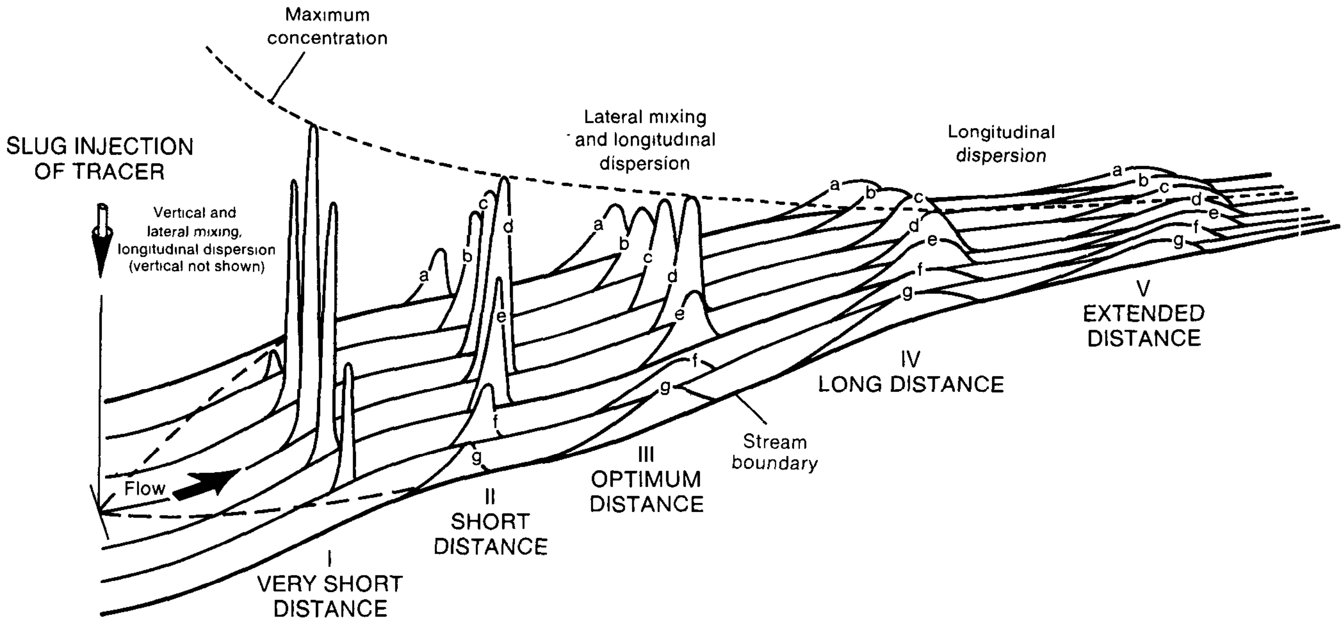


Figure 1.—Lateral mixing and longitudinal dispersion patterns and changes in distribution of concentration downstream from a single, center slug injection of tracer.

sented again. For consistency, when feasible, symbols and definitions will be the same as those presented in these manuals.

Theory

Characteristics of tracer movement in streams

Dispersion and mixing

Tracers used successfully in hydrologic studies are water soluble and have virtually the same physical characteristics as water (Feuerstein and Selleck, 1963; Smart and Laidlaw, 1977). Thus, when introduced into a flowing stream, they undergo the same movement, dispersion, and dilution as does the element of water tagged or the soluble waste introduced. An understanding of how tracers mix and disperse in a stream is essential to an understanding of their application in simulating a soluble waste.

The downstream characteristic of a tracer injected as a slug in the center of flow is shown in figure 1. Note that in figure 1, the response curves are shown as a function of longitudinal distance and not as a function of time. For clarity, the stream has been arbitrarily divided laterally into six stream tubes.

The dispersion and mixing of a tracer in a receiving stream take place in all three dimensions of the channel (fig. 1). In this manual, vertical and lateral dispersion will be referred to in a general way as "mixing." The elongation of a tracer-response cloud longitudinally will be referred to as "longitudinal dispersion." Vertical mixing is normally completed first and lateral later, depending upon the characteristics of the stream and velocity variations. Longitudinal dispersion, having no boundaries, continues indefinitely. Thus, at section I (fig. 1), vertical mixing could be complete, meaning that at any one streamline and time, the tracer concentration is the same throughout the water column, even though it varies drastically laterally. At a short distance, lateral mixing is still taking place, and the tracer mass in transport along the different streamlines is not equal because the response curves do not have equal areas. Mixing and dispersion in two dimensions, therefore, exist between sections I and III.

An optimum mixing distance (section III) is reached when the tracer-response curves *a*, *b*, *c*,..., *g*, as observed laterally, have about the same areas, even though the individual response curves can vary considerably in shape and dimensions; dispersion is approaching the one-dimensional state. Nevertheless, the peak concentrations in the center of the channel could be considerably greater than peak concentrations along the banks, while the latter response curves are longer both physically and in time of passage. Also, the tracer cloud is skewed, advancing faster in the center of the flow compared with the channel boundaries. Furthermore, as shown in figure 1, the tracer cloud might not be uniformly skewed. Sampling of the tracer cloud at several points laterally is advisable.

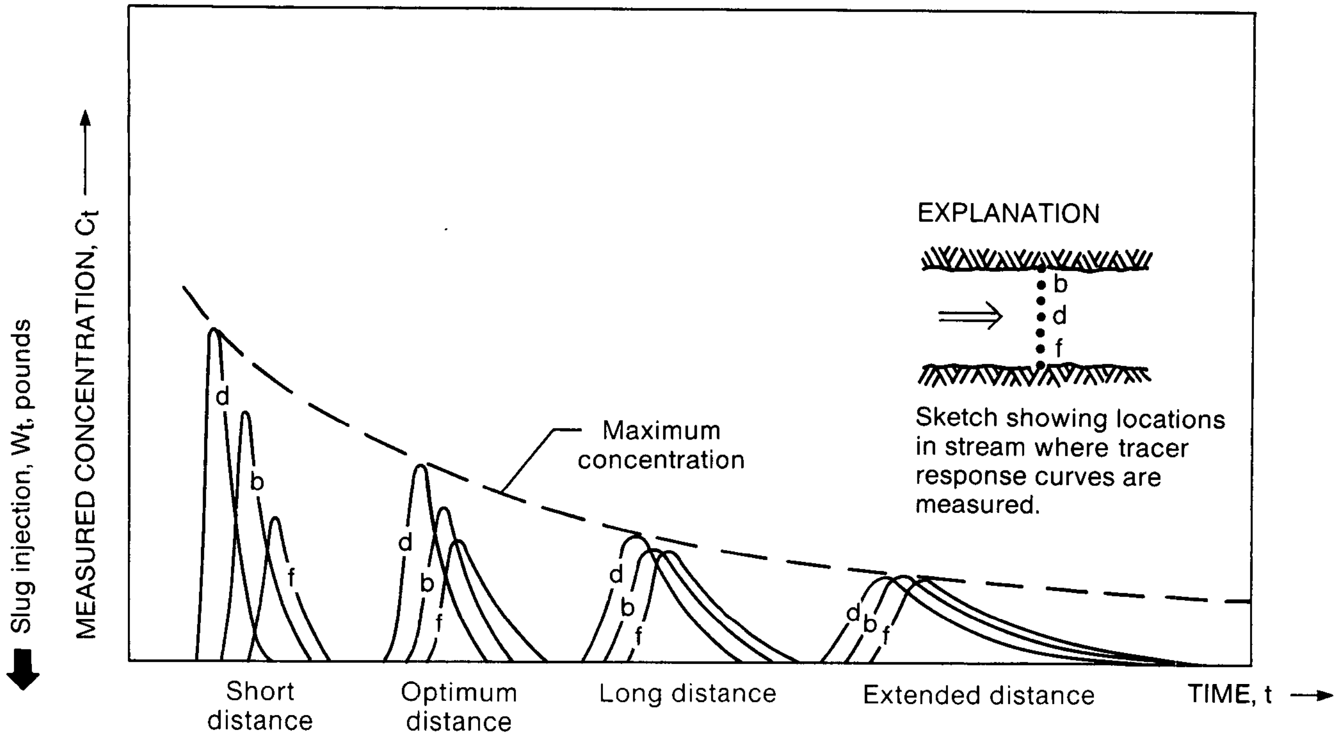


Figure 2.—Time-concentration response curves for slug injection observed at three points laterally across a channel at four different distances downstream from the injection point.

If the tracer-response curves are next examined at a long distance (section IV), the curve areas will be found to be nearly identical and peak concentrations more nearly the same laterally. Thus, a dispersion state that is nearly one-dimensional exists between sections III and IV, where longitudinal dispersion dominates and continues indefinitely downstream. Ultimately, at extended distances, peak concentrations become virtually the same laterally, and longitudinal dispersion affects the shape and dimensions of the response curves exclusively. With time and distance, peak concentrations become attenuated and the cloud lengths get longer and longer (Kilpatrick and Taylor, 1986).

Characteristics of response curve

The conventional manner of illustrating the response in a stream to a slug injection of tracer is to plot concentration variation with elapsed time as observed at one or more points laterally in a stream cross section. This same response is observed in a like manner at other distances downstream to provide information on the time of travel or speed of a slug of solute as well as to measure its dispersion.

To illustrate this, the measured tracer-response curves for a single center-point slug injection (see fig. 1) for streamlines b, d, and f are shown in figure 2 for the short, optimum, long, and extended distances. The travel time to the leading edges, peaks, centroids, and trailing edges can be obtained from these data and related to distance and streamflow duration if tests at more than one discharge are made. The analysis and presentation of time-of-travel data are covered in the report by Kilpatrick and Wilson (1989).

Unitizing of tracer data

The shape and magnitude of the observed tracer-response curves shown in figures 1 and 2 are determined by four factors:

1. The quantity of tracer injected.
2. The degree to which the tracer is conservative.
3. The magnitude of the stream discharge.
4. Longitudinal dispersion.

Any attempt to use tracer-concentration data to predict the concentrations of contaminants that might be injected into a stream must take into consideration these four factors.

The magnitude of the tracer concentrations produced at the four downstream locations is in direct proportion to the quantity of tracer injected, W_t; doubling the amount of tracer injected would yield observed concentration values twice as large while retaining the same shape and duration. Previous investigators thus have normalized their data by dividing all observed tracer concentrations, C_t, by the weight of tracer injected. The resulting concentrations were per pound of tracer (or contaminant) used (Bailey and others, 1966; Martens and others, 1974).

It also has been found that various tracers are lost in transit due to adhesion on sediments (Scott and others, 1969), particularly where fine suspended clays are present. Rhodamine WT dye has been shown both in the field and laboratory to decay photochemically about 2 to 4 percent per day (Hetling and O'Connell, 1966; Tai and Rathbun, 1988). Kilpatrick (U.S. Geological Survey, written commun., 1988) noted decay rates for rhodamine WT dye tended to be higher in rivers, about 5 percent per day, compared with about 3 percent in estuaries. Tracer loss can vary greatly, depending on the characteristics of both the tracer and the water being tagged. In estuaries, the greater depths might reduce photochemical decay in the case of dyes, whereas in streams, larger sediment concentrations could increase dye losses. This is especially true where fine clay is present. Clays have tremendous surface areas for dye to adhere to.

To compare data and to have them simulate a conservative substance, it is desirable to eliminate the effects of tracer loss. If stream discharge, Q, is independently measured at the same time and location of the tracer-response curve, it is possible to evaluate the weight of tracer recovered, W_r, or accounted for by sampling as

$$W_r = Q\int_{T_L}^{T_t} C_t dt = QA_c , \qquad (1)$$

where

C_t is the observed concentration,
A_c is the area of the observed tracer-response curve, and
T_L and T_t are the elapsed times to the leading and trailing edges of the observed tracer-response curve.

Equation 1 is based on the assumption that mixing is complete. If mixing is incomplete, computations can be made along stream tubes and summed. When the weight of tracer injected, W_t, is known, the tracer recovery ratio, R_r, can be expressed as

$$R_r = W_r / W_t = \frac{QA_c}{W_t} . \qquad (2)$$

The factor that inversely affects the magnitude of the concentration-response curves is stream discharge. The diluting effect of tributary inflows, as well as that of natural ground-water accretion, differs from stream to stream and with location on a reach of stream. To counter the variable diluting effects of differing discharges on the same stream, as well as between streams, it is possible to adjust concentration data to a "unit" discharge, defined as "what would be observed in 1 cubic foot per second (ft^3/s)."

Unit concentrations

Observed concentrations can be adjusted for (1) the amount of tracer injected, (2) tracer loss, and (3) stream discharge to what will be termed "unit concentration," C_u, as follows:

$$C_u = Q\,\frac{(C_t/W_t)}{R_r} . \qquad (3)$$

Substituting equation 2 for the recovery ratio, adding the appropriate proportionality constant, and canceling terms, the unit concentration can be expressed as

$$C_u = 4{,}450\,\frac{C_t}{A_c} . \qquad (4)$$

When the area of the observed-response curve is in units of hours times micrograms per liter, and C_t is in units of micrograms per liter, then C_u is in units of micrograms per liter per pound times cubic feet per second [(µg/L)/lb](ft^3/s).

The unit-response curve

Equation 4 can be used to reconstruct any measured tracer-response curve to a unit-concentration-response (UR) curve. A typical measured response curve of time versus concentration is shown in figure 3*A*; if the vertical concentration ordinates are changed to unit values using equation 4, the UR for this location and dispersion time will be obtained as shown in figure 3*B*. Note that only the vertical ordinates are changed, such that the response curve now gives the concentrations that would be at that location for the injection of 1 lb of conservative tracer into 1 ft^3/s of flow. This UR curve can be used as the building block for simulating the concentrations to be expected at that location for various waste loadings.

Unitizing the tracer-response curves, in effect, fits one unit weight of tracer into one unit of flow. As such, when mixing is complete, all UR curves have the same area, 4,450 [(µg/L)/lb](ft^3/s) hours, regardless of the particular stream or the location on a stream.

Peak attenuations

Variations in dispersion on the same stream or different streams become most apparent if the unit concentrations for just the peaks, C_{up}, are plotted as a function of elapsed time to the peaks. Equation 4 can be written to apply to just the unit-peak concentration, C_{up}, as

EXPLANATION OF TERMS

A_c = Area of observed response curve
A_u = Area of unit-response curve
C_p = Observed peak concentration
C_t = Observed tracer concentration
C_{up} = Unit-peak concentration
C_u = Unit concentration
T_d = Duration of entire response curve
$T_{d_{10}}$ = Duration of abbreviated response curve to point where $C_t = 0.1C_p$
T_p = Elapsed time to peak concentration
T_L = Elapsed time to leading edge
T_t = Elapsed time to trailing edge of entire response curve
t_b = Period when concentrations are building up to the peak
t_r = Period when concentrations are receding from peak to when $C_t = 0.1C_p$

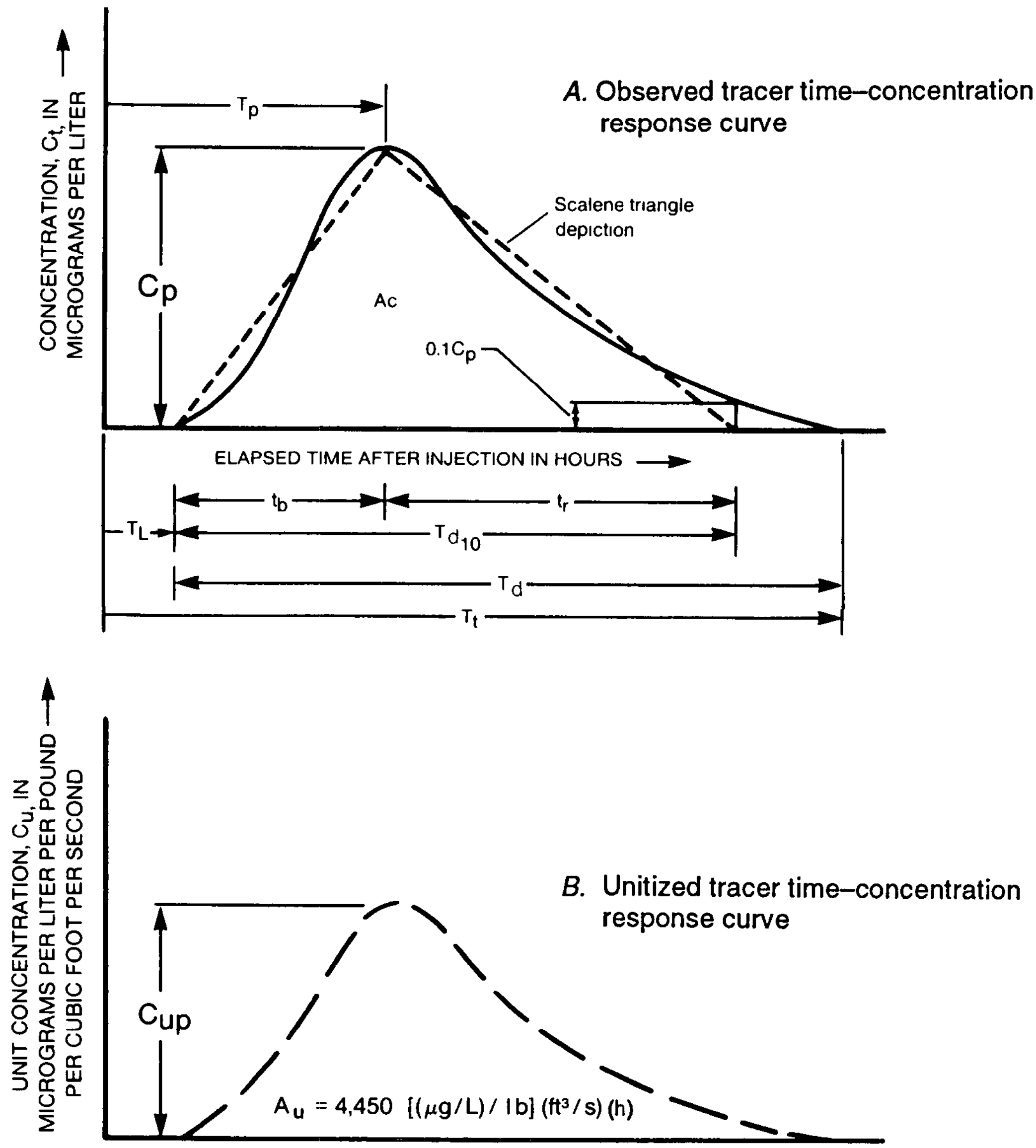


Figure 3.—Observed and unitized time-concentration response curves and scalene triangle depiction.

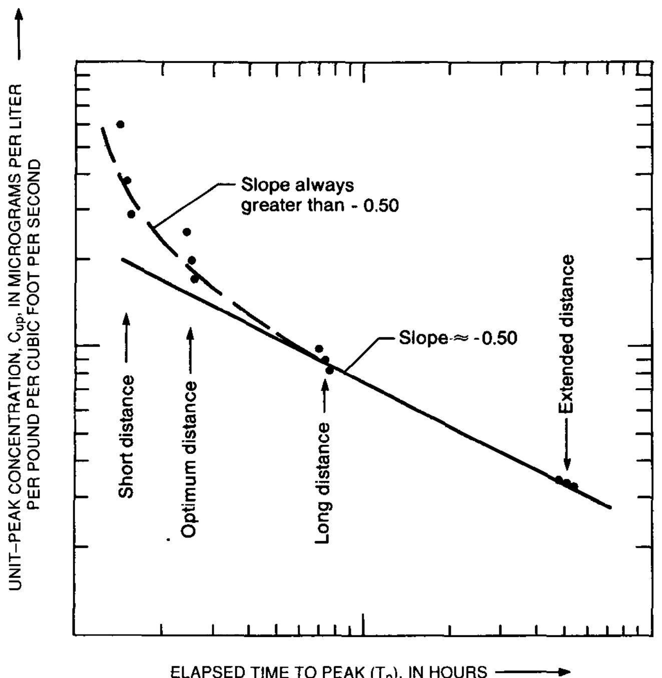


Figure 4.—Theoretical unit-peak-attenuation curve as related to different mixing times.

$$C_{up}=4{,}450\ \frac{C_p}{A_c}\ , \tag{5}$$

where C_p is the observed peak concentration in micrograms per liter.

A plot of the peak concentrations (converted to unit-peak concentrations) with elapsed time for the response curves measured along streamlines b, d, and f at the four distances (figs. 1 and 2) is shown in figure 4. The curve shown in figure 4 is referred to as the unit-peak-attenuation (UPA) curve, as it describes the longitudinal dispersion of a slug of conservative soluble tracer as it moves downstream.

It has been pointed out by Yotsukura (U.S. Geological Survey, written commun., 1976) that, based on the Fickian theory of diffusion (Fischer, 1967), C_{up} is proportional to $t^{-(n)}$, where t is the elapsed time and n is the slope of the UPA curve. Furthermore, ideally n=0.5 for the case of steady flow in a straight uniform channel after vertical and lateral mixing are complete and only longitudinal dispersion is continuing. If mixing is in an intermediate stage, where concentrations are uniform in the vertical but not in the lateral, then n=1.0; and at extremely short distances and (or) times where neither vertical nor lateral mixing is complete, n=1.5. These conditions have been depicted in figure 4; thus the slope of the UPA curve is shown as 0.5 only after nearly complete mixing exists. The ideal case seldom exists in nature, and thus, n typically will be 0.5 or greater. As shown in figure 4, unless the tracer is completely mixed, the slope of the UPA curve will always be greater than 0.5. Where initial mixing is known to be complete, the slope of the UPA curve can be viewed as a measure of the longitudinal dispersion efficiency over and above the ideal. Longitudinal dispersion efficiency refers to the rapidity with which peak concentrations are reduced. The presence of pools and riffles, bends, and other channel and reach characteristics will almost always yield a slope greater than 0.5; the larger the slope the more efficient the reach, in terms of longitudinal dispersion.

The UPA curve, along with the time-of-travel curves, provides a ready means of predicting, at any location, maximum contaminant levels that would be experienced downstream from the spill of any amount of soluble contaminant at any location in the reach after total mixing has been achieved for the range in flows tested. This is accomplished using the equation

$$C_{cp}=\frac{C_{up}W}{Q}\ , \tag{6}$$

which gives the conservative peak concentration, C_{cp}, that would result downstream in discharge Q if W pounds of soluble waste are injected; C_{up} must be obtained from a suitable UPA curve, which requires a knowledge of the elapsed time to the location in question. Elapsed time or time of travel can be thought of as dispersion time; distance is not as significant as the actual time available for dispersion.

It will often be found that if tracer tests are performed in a stream reach at more than one discharge, all the unit-peak-concentration data will define one UPA curve having the same slope. This indicates that longitudinal dispersion is the same for the reach over the range of discharges tested (Kilpatrick and Taylor, 1986). More commonly, different UPA curves are obtained, sometimes having the same slope and sometimes different slopes, but both cases allow interpolation to be made of the results of contaminant spills at other locations and discharges.

Characteristics of tracer movement in estuaries

The techniques involved in simulating the movement and buildup of wastes in an estuary using tracers and the superposition principle will be presented later. An understanding of the complex flow patterns occurring in estuaries is essential in selecting test periods to suit the objectives.

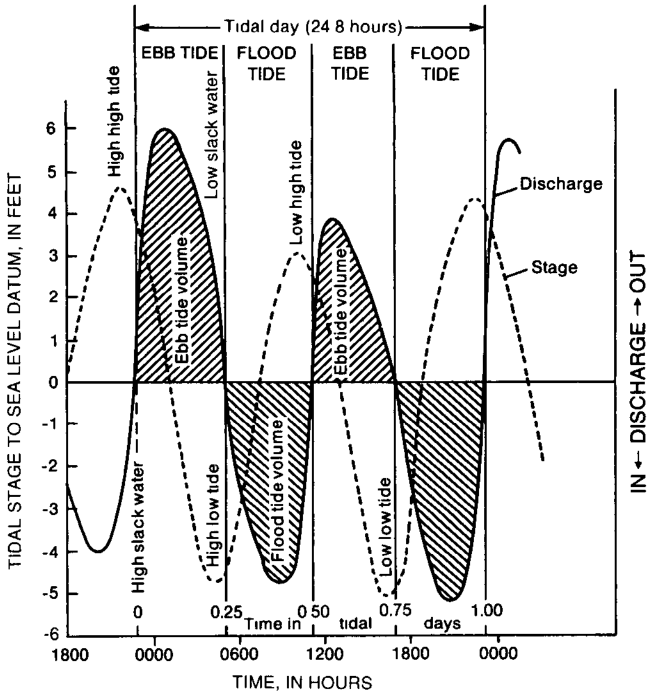


Figure 5.—Typical stage and discharge hydrographs for East Coast estuaries.

Tidal hydraulics

Flows in estuaries are some of the most complex to be encountered. Whereas flow in rivers is essentially steady (or a period may be selected when it is steady), the oscillatory tidal action present in estuaries produces unsteady flow of the most extreme case. To further complicate the hydraulic picture in estuaries, some may have significant freshwater inflow from one or more sources, both surface and ground water. Estuaries are also classified as mixed or unmixed, the latter being characterized by a saltwater wedge that moves inland with changes in freshwater flow and tidal stage. Worse still, the entire picture can change from week to week and season to season due to long-term changes in tidal action and seasonal changes in freshwater inflows.

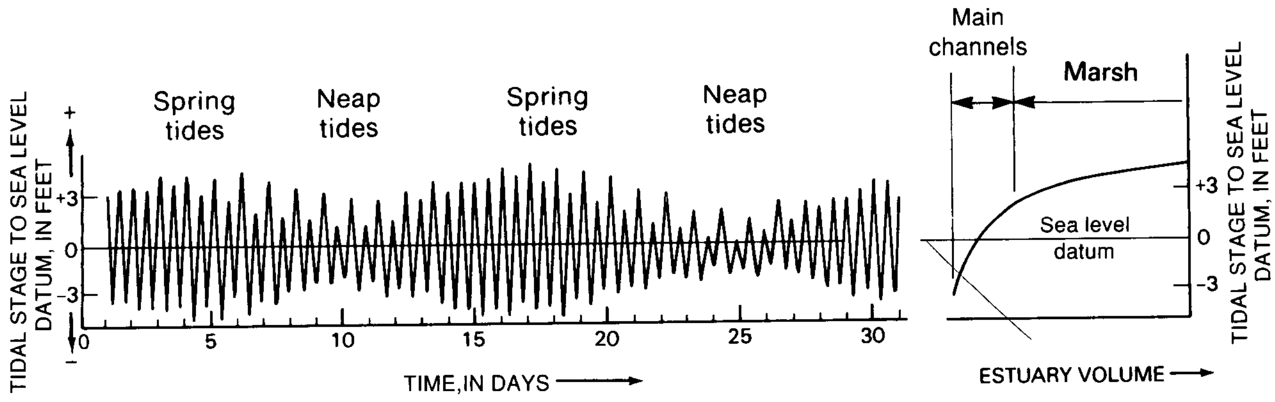
A typical graph of tidal stage, as shown in figure 5 for an East Coast estuary, features two high tides and two low tides per tidal day. These are, as might be observed, at the mouth of an estuary or at any section in an estuary. The tides are driven primarily by the alternating attraction (gravity) of the sun and moon, the moon being the dominant force. One of the high tides will usually be larger (high high tide) than the other (low high tide) within the tidal day as a result of the combined forces of both sun and moon. Note that the maximum discharge in or out of the estuary occurs approximately at zero stage datum (when referenced to mean tide level) as the maximum energy gradient exists in going from high to low or low to high. If the discharge hydrograph is measured as shown in figure 5, its area is a measure of the tidal volume of water flow in (flood tide) and out (ebb tide) of the estuary.

As seen in figure 5, high slack water and high slack tide do not necessarily occur simultaneously. The highest or lowest tidal stage may be on the order of an hour earlier than when flow ceases. As will be seen subsequently, tracer sampling will usually be during high or low slack water and not necessarily at high or low tidal stages.

The data to predict tidal stages and flows are available for selected coastal locations from the Coast and Geodetic Tide Tables. These tables also provide corrections for inland points in the estuaries, since patterns depicted in figure 5 may be significantly different at inland locations.

In general, estuaries experiencing large ranges in tidal stage and, hence, discharges are less apt to be polluted, as contaminants are more likely to be carried seaward. For example, estuaries along the Gulf Coast are less efficient in flushing due to tidal action that is subdued in comparison with that of the Atlantic and Pacific Coasts.

The tidal day is approximately 24.8 hours long, as the moon orbits the Earth in approximately 29.5 days, yielding a tide that occurs about 50 min later each day. The moon's orbit about the Earth must be distinguished from the Earth's own rotation about its axis. Typical tide variations over a month are illustrated in figure 6*A*. The approximately weekly pattern of strong spring and weak neap tides is caused by the 29.5-day orbit of the moon about the Earth. In a general way, the change in estuary volume with tidal stage resulting from flows in and out of the estuary is shown in figure 6*B*. Typically, when the tide is out, water occupies only the main channels of an estuary. At flood tide, flows move inland and laterally out of the main channels and into adjoining marshlands and tidal flats. This cyclic pattern of flooding and dewatering is believed to be critical to the biological community of an estuary (Teal and Teal, 1969). As can be seen by comparing the graphs in figures 6*A* and 6*B*, the degree of marshland flooding may vary significantly between spring and neap tides. More important to the subject of this report, the injection of a tracer into an estuary during spring tide will result in more extensive transport of the tracer (or a soluble contaminant) into the marshlands and, at the same time, the most dilution will occur. Conversely, if injected during a neap tide period, the tracer will be more confined to the main channels and will be diluted the least. Either situation may be undesirable when it comes to injecting contaminants into an estuary.



A. Typical tidal stage variation during one month.

B. Change in affected estuary volume with tidal stage.

Figure 6.—Typical long-term pattern of tidal stage and variation in marshland flooding.

Dispersion and mixing

A typical estuary is depicted in figure 7. This illustration will be used in this report to present the theory and technique of simulating waste buildup in an estuary using a tracer and the superposition principle.

As has been discussed previously, flow in a tidally affected estuary is oscillatory. The magnitude of these flows is determined by the magnitude of the tidal stage changes and the volume of both main channel and marshland areas affected. In a typical East Coast estuary, flows reverse every 6.2 hours, filling inland channels and marshlands on the flood tide and draining out on the ebb tide.

A tracer introduced into an estuary flow system is affected by these flow patterns. Where there is no freshwater inflow, a tracer slug injected inland at location A (fig. 7) will mix with the flow and oscillate back and forth, elongating with time and ultimately dispersing into adjoining lesser channels and the bordering marshlands. A tracer injected on the ebb tide will return on the flood tide, but because of dispersion and advection, will enter adjoining channels and marshland that were bypassed on the ebb tide.

The response to such a slug injection will look like that shown in figure 8*A*, assuming no freshwater inflow. Ultimately, depending on the magnitude of the tidal excursion, the tracer will be flushed into the ocean to be swept away by coastal currents. The tidal excursion, the distance an element of water will move during a tidal cycle, will change from spring tide to neap tide. Note that the tracer-response curve shown in figure 8*A* is essentially symmetrical in contrast to that for a river (fig. 2), which is typically skewed. In an estuary without freshwater inflow, the centroid of the tracer will return to the injection point but will spread as a result of longitudinal dispersion and tidal discharges.

If the point of tracer injection is close to the ocean (location B in fig. 7), the tidal excursion will transport some tracer directly to the ocean, and longitudinal dispersion will eventually move the rest into the ocean (fig. 8*B*). Of the injection made on the ebb tide, large portions, perhaps all, might be carried quickly into the ocean. The dye injected on the flood tide, however, will be carried inland, ultimately returning to the injection point. Eventually, longitudinal dispersion will transport the bulk of the tracer into the ocean. Thus, if a waste effluent was introduced continuously at point B, it would enter both flood and ebb tide flows, the result being some buildup of waste concentrations upstream but a fairly rapid flushing of wastes seaward.

Regardless of the location of the injection, if freshwater inflow exists, it will aid in transporting the tracer seaward (fig. 8*C*). In estuaries experiencing large changes in tidal stage and hence large tidal discharges, the freshwater inflow will normally be small by comparison. Thus, the predominant flushing mechanisms in many estuaries are longitudinal dispersion and tidal discharge.

The superposition principle

General

One of the most useful tools to hydrologists has been the unit-hydrograph method (Linsley and others, 1958) for predicting stream runoff from precipitation in a drainage basin. The unit-hydrograph theory assumes that the stream-runoff response is linear and that unit hydrographs can be added to synthesize the response to different rainfalls. This approach must be

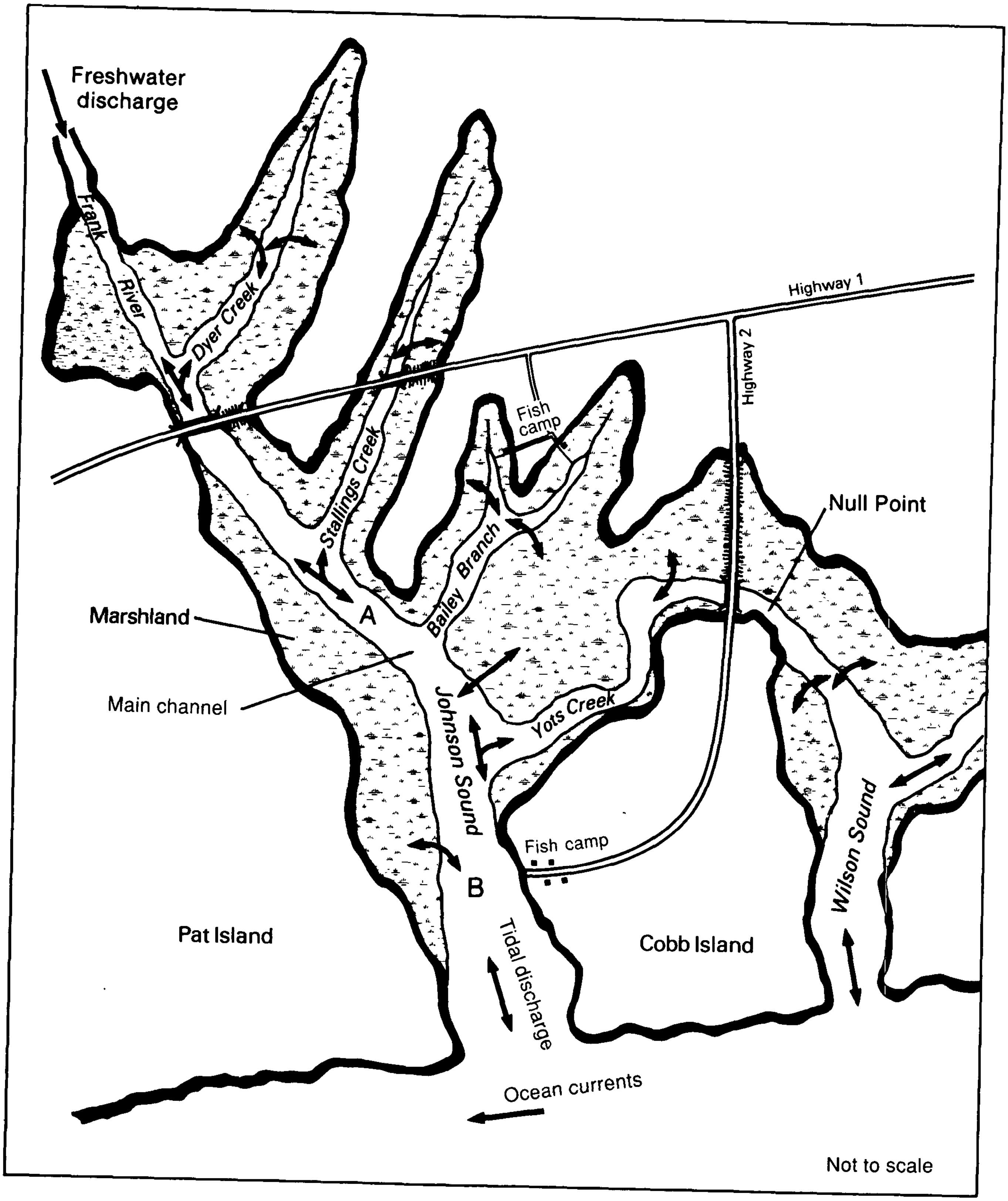
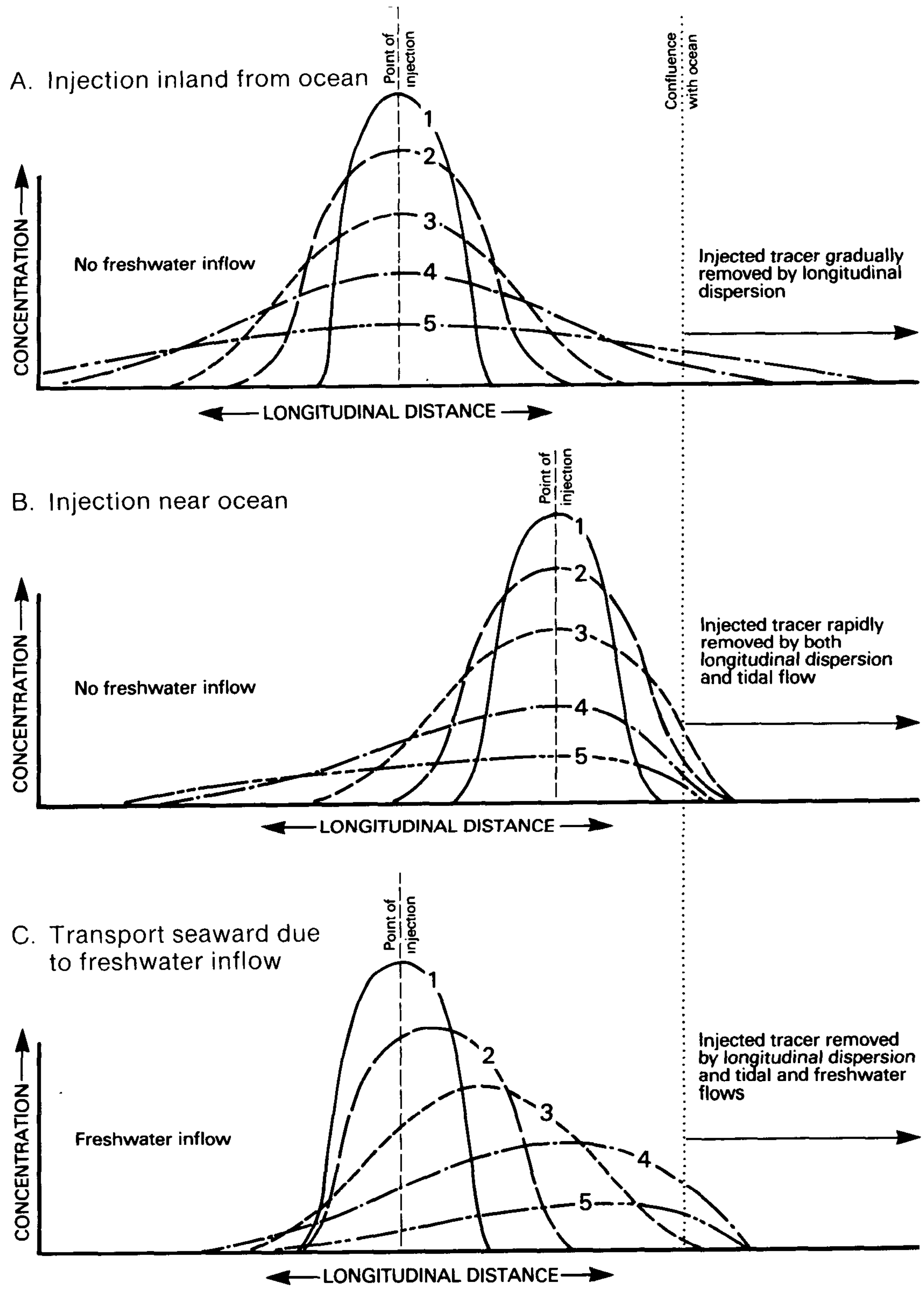


Figure 7.—Typical estuary showing physical geography and flow characteristics.



Note: Numbers refer to time in tidal days. Response curves are from an instantaneous injection observed at times of high slack water; at other times they would be displaced downstream from point of injection as well as be transported seaward by freshwater flow.

Figure 8.—Tracer-response curves in an estuary resulting from different injection points and freshwater and tidal flow conditions.

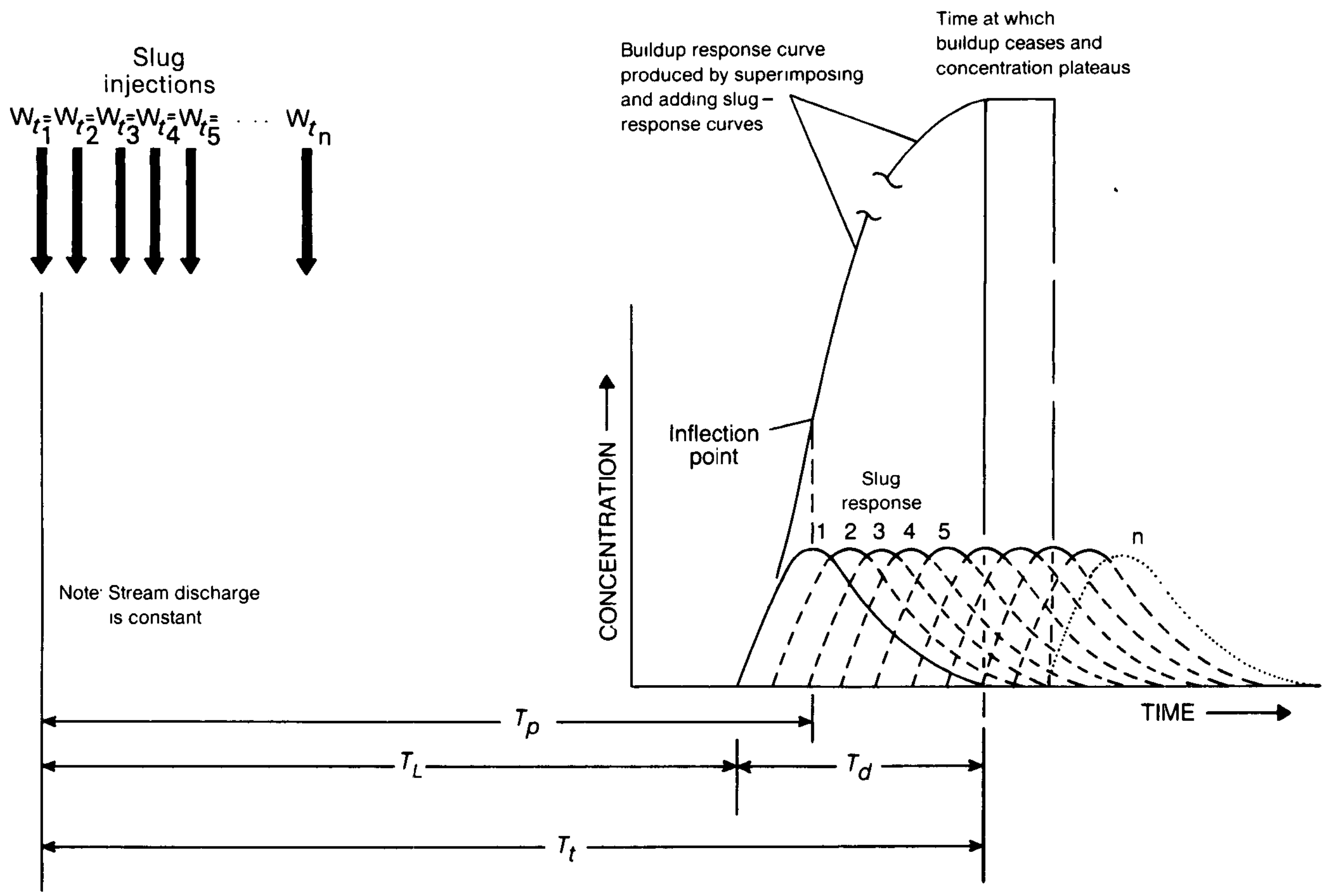


Figure 9.—Superposition of slug-response curves to simulate constant-injection buildup to a plateau at one location in stream section.

used with care, as such systems are seldom completely linear.

Another application of the linear superposition approach is for the simulation of buildup of soluble-waste concentrations in streams and estuaries using tracer tests (Bailey and others, 1966; Yotsukura and Kilpatrick, 1973). By this method, the slug injection of a soluble tracer is assumed to imitate the response of a soluble contaminant and as such can be used to simulate the contaminant. In this instance, the assumption of linearity is correct for any likely test conditions.

In rivers

It was shown by Kilpatrick and Cobb (1985) that the response to a continuous, constant-rate injection of tracer could be simulated by adding a sequence of superimposed response curves from a single slug response as measured for the same stream, location, and discharge. Thus, for example, assume that a series of slug injections of tracer (simulating a constant injection), each of weight W_t, is injected in the stream as depicted in figure 9. As can be envisioned, there would be a repetition of the same responses downstream at the different distances shown in figure 2. As an example, at streamline d for the long distance, there would be a buildup to a constant plateau of concentration as shown in figure 9. The same would occur at every streamline and distance downstream shown in figure 2, so that plateaus of concentration would ultimately exist at every location if the constant injection were continued long enough and the stream discharge remained constant.

Figure 9 shows that for a plateau to be reached at any particular location, a constant injection must be maintained for a length of time equal to the duration of the slug-response curve, T_d, at that location. Furthermore, to fully plateau across a section, injection would have to be maintained for a length of time equal to the response curve of longest duration in the cross section, T_D, probably along one bank or another. Similarly, the duration of the constant injection necessary to establish a plateau in the entire stream reach shown in figure 2 is dictated by the longest duration of slug response at the most downstream location.

The unit-response curve

It becomes apparent that the response curve produced by a slug injection of tracer may be used as a building block with the superposition principle to simulate the buildup of a given soluble waste introduced into the stream. In fact, linearity permits the superposition of varying loads of waste to simulate the resulting response downstream. For convenience, it is practical to reduce the curves of figure 2 to UR curves using equation 4. A UR curve (fig. 3*B*) is for a conservative soluble waste of 1 lb in 1 ft^3/s of discharge. If the UPA curve has been obtained as in figure 4, the peak of the UR curve may be obtained by interpolation for any lapsed peak travel time to the location where the simulation is desired. The UR curve that is being synthesized may be shaped by inspection of the measured UR curves in the reach. When mixing is complete, it will have an area of 4,450 when in units of $[(\mu g/L)/lb](ft^3/s)(h)$.

It has also been shown by Kilpatrick and Wilson (1989) that the duration of the slug-response curve from the leading edge to the point where the receding concentration reaches 10 percent of the peak, $T_{d_{10}}$, may be approximated by the equation

$$T_{d_{10}}=0.7T_p^{\,0.86}, \qquad (7)$$

where T_p is the elapsed time to the peak in hours (see fig. 3).

Furthermore, Taylor (Kilpatrick and Taylor, 1986) showed that the normal response curve produced by a slug injection could be represented as a scalene triangle (see fig. 3*A*), in which

$$t_r=0.68T_{d_{10}}-0.19. \qquad (8)$$

Thus, as an approximation, about one-third of the duration, $T_{d_{10}}$, is t_b, or the time from the leading edge until concentrations build up to the peak, and about two-thirds of the duration is t_r, or the time from the peak until receding to the trailing edge, when concentrations have reached 10 percent of the peak. A $T_{d_{10}}$ of 4 hours is a lower limit for application of this approximation. In the absence of actual test data, a scalene triangle approximation will yield useful results.

If measured time-concentration response curves are available for a stream reach, it is obviously better to use them rather than scalene triangles because they represent the actual dispersive characteristics of the river in question. It should be borne in mind that an upstream response curve may be thought of as input to the stream producing the responses observed subsequently downstream. The most downstream response to an injection of tracer reflects the cumulative dispersive characteristics of the upstream test reach.

Once the UR curve for the particular location or elapsed time has been synthesized, it may be scaled to suit the particular waste loading and stream discharge involved. Thus, any waste-loading pattern may be divided into convenient increments or "slugs" and the response curves that have been suitably scaled may be superimposed to obtain the additive result. The desired response curves are obtained by multiplying the concentration ordinates of the synthesized UR curve by the pounds of waste and dividing by the stream discharge. The length or duration T_d does not change from that of the UR or observed curves (see fig. 3).

In reservoirs and lakes

It should not be construed that the application of the superposition principle with tracer-response curves measured in the prototype is limited to cases where mixing is complete. The technique is applicable wherever the slug of tracer can be injected at the location of a known point of waste input and the resulting tracer history (response) curves can be measured. This will be best illustrated, later, by example.

Nevertheless, it is cautioned that the results of a tracer simulation test in any body of water reflect prevailing discharges, winds, currents, thermal conditions, and any other factors governing the hydraulics of the system. In rivers and estuaries, this limitation is not too severe; in reservoirs and lakes, some of these conditions may be significant in influencing results. Several tests under different conditions may be necessary, or at least test conditions must be borne in mind when applying and presenting the results.

In estuaries

The superposition principle can only be applied in the unsteady flow of an estuary to simulate the buildup of a waste introduced at a point if tracer injection procedures are tailored to resolve the cyclic nature of the tidal stages and discharges. Advantage can be taken of the repetitive nature of the typical tidal system illustrated in figures 5 and 6. Instead of an instantaneous slug injection, tracer may be continuously and constantly injected over one or more tidal cycles, so as to tag all portions of the tidal hydrograph. Thus unsteadiness can be circumvented and the superposition principle applied to the resulting response curves observed at key locations in the estuary.

As illustrated in figure 10, a constant-rate injection of tracer may be initiated on a high slack water and continued for either 12.4 hours or 24.8 hours until the next high slack water. As can be seen, injection over a full 24.8-hour tidal day covers a period that more nearly returns to the same hydraulic conditions as at

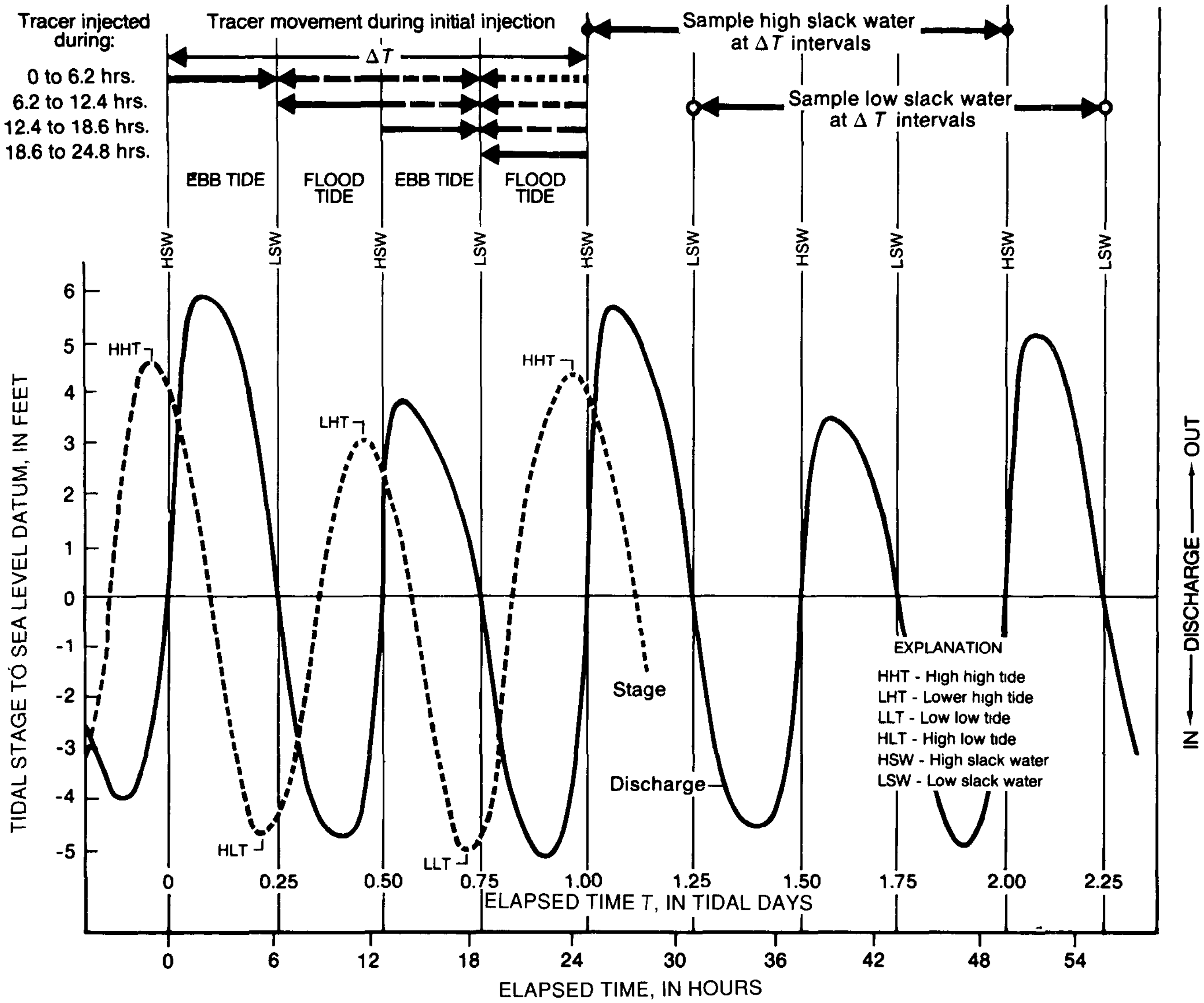


Figure 10.—Discharge and stage hydrographs in an estuary, illustrating a tracer injection over a tidal day, movement with the tides, and initial sampling schedules.

the start. This quasi-steady-state period, ΔT, also determines the integration interval when applying the superposition principle.

Conceptually, the direction of movement of the tracer as it is injected at a point is shown in figure 10. From T=0 to 0.25 and from 0.50 to 0.75 tidal days, ebb tide exists and the tracer moves seaward; from T=0.25 to 0.50 and from 0.75 to 1.00 tidal days, flood tide exists and the tracer moves upstream. Each quarter tidal day is 6.2 hours long. As the tracer injection progresses, each new 6.2-hour injection may be thought of as being superimposed on previous 6.2-hour tracer clouds that are already in various degrees of dispersion in the estuary.

As may be envisioned, tracer injection could be continued indefinitely until a steady state has been reached. Concentrations could then be measured at selected locations to provide a simulation of what might be expected if a soluble waste were continuously injected. Unfortunately, to reach steady state in most estuaries would require weeks and in some instances months of continuous tracer injection!

It is easy to see, however, that the tracer slug injected over the 24.8-hour tidal day will ultimately experience, as the days progress, the same dispersion, dilution, transport, and flushing from the estuary (see fig. 8) as would a continuously injected tracer or waste. To understand the application of the superposition principle in an estuary, picture the 24.8-hour tracer-response curve for succeeding tidal days, such as depicted in figure 8. The superposition of each new 24.8-hour response curve onto the preceding cumulative responses is the equivalent of what would be measured due to a continuous injection.

Simulation of nonconservative substances

In rivers

By definition, unit concentrations are for a totally conservative solute, hence producing the safest scenario for the prediction of waste concentrations downstream of a spill or other input. To be realistic, it may be desirable to "put back" the decay into the simulation if it is known for the substance involved. The first-order reduction of a substance by decay may be expressed by the equation

$$\frac{W_{rt}}{W}=10^{-kt}=e^{-Kt}, \tag{9}$$

where W_{rt} is the weight of waste remaining after time t, W is the initial weight of waste injected, k is a rate constant to the base 10, and K a rate constant to the base e; hence $k=K/2.3$. The base e is more commonly used, but caution is advised to ascertain which rate constant is being provided.

In application, the concentrations of the UR curve shown in figure 3*B* may be reduced by W_{rt}/W for the time t. If T_d is large, it may be advisable to use elapsed times varying from T_L to T_t and reduce UR concentrations progressively over T_d.

In estuaries

The concept of unit concentration has not been applied to an estuary or lake system because an applicable discharge is not available to determine either recoveries or dilution. The best approach perhaps is to normalize the data by dividing concentrations by the weight of tracer injected.

Since the decay rate of rhodamine WT dye in estuaries has been estimated at about 3.4 percent per day, equation 9 for this dye may be expressed as

$$\frac{W_{rt}}{W}=e^{-0.034t}, \tag{10}$$

where t is elapsed time in days.

Planning, Instrumentation, and Data-Acquisition Techniques

Much of the planning, instrumentation, and data-acquisition techniques needed in tracer studies designed to simulate waste movement and buildup are similar to those needed in performing other types of tracer studies. For example, the techniques used in performing time-of-travel tests in streams are applicable to simulation tests in streams using tracers. The time-of-travel manual (Kilpatrick and Wilson, 1989) should be used by the reader to better understand tracing techniques in rivers. Similarly, the dilution discharge (Kilpatrick and Cobb, 1985) and reaeration manuals (Kilpatrick and others, 1989) are valuable references on the superposition principle and on equipment and techniques needed for constant-rate tracer injection, respectively.

Tracer injection, quantities, and techniques

Slug injection in streams

A slug injection of tracer, usually rhodamine WT dye, is customarily used to obtain the typical series of time-concentration response curves, such as shown in figure 2. The time-of-travel manual provides equations and curves for estimating dye quantities for different stream discharges and reach lengths. For waste simulation tests it is particularly desirable to use as long a test reach as feasible and to select three or more sampling sections, so that UPA curves can be defined. Dye quantities should thus be chosen accordingly.

Slug injection in lakes and estuaries

Where the affected discharge is not readily determined, such as in a lake or estuary, less exact means must be used to estimate the volume of tracer necessary to perform a test. In a lake or estuary, it is best to estimate the volume of water that will ultimately be tagged with tracer and, from this, determine the weight of tracer to produce the desired concentration. If the estuary or lake volume likely to be affected is estimated in cubic feet, the weight of 20-percent rhodamine WT dye, in pounds, required to produce an average concentration of 1 µg/L may be computed by the equation

$$W_d=312\times10^{-9}\ \text{(volume of estuary or lake in cubic feet).} \tag{11}$$

Such estimates are for an average concentration of 1 µg/L. Much higher concentrations would exist in the vicinity of the injection and then drop off to zero at the limits of the dye cloud. Therefore, it might be advisable to design for a lesser average concentration and use about 50 to 75 percent of the amount computed by