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of the United States Geological Survey

Chapter A4

**MEASUREMENT OF PEAK DISCHARGE
AT WIDTH CONTRACTIONS
BY INDIRECT METHODS**

By Howard F. Matthai

Book 3

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PREFACE

The series of manuals on techniques describes procedures for planning and executing specialized work in water-resources investigations. The material is grouped under major subject headings called books and further subdivided into sections and chapters; Section A of Book 3 is on surface water.

The unit of publication, the chapter, is limited to a narrow field of subject matter. This format permits flexibility in revision and publication as the need arises.

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SYMBOLS AND UNITS

<i>Symbol</i>	<i>Definition</i>	<i>Unit</i>
<i>A</i>	Area.	ft ²
<i>A_f</i>	Submerged cross-sectional area of piers or piles projected to sections.	ft ²
<i>A_s</i>	Gross area of section 3 below the lower bridge chord.	ft ²
<i>A_T</i>	Area of total cross section.	ft ²
<i>A_i</i>	Area of subsection.	ft ²
<i>b</i>	Width of contracted-flow section.	ft
<i>b_d</i>	Offset distance for straight dikes.	ft
<i>b_i</i>	Width of flow section at water surface.	ft
<i>C</i>	Coefficient of discharge.	
<i>C'</i>	Coefficient of discharge for base condition.	
<i>E</i>	Slope of the embankments, horizontal distance/vertical distance.	
<i>e</i>	Eccentricity ratio; equal to K_a/K_b .	
<i>F</i>	Froude number.	
<i>f</i>	Function of.	
<i>g</i>	Gravitational constant (acceleration).	ft/sec ²
<i>h</i>	Static or piezometric head above an arbitrary datum.	ft
<i>h_f</i>	Head loss due to friction.	ft
<i>h_s</i>	Stagnation-surface level at embankment face.	ft
<i>j</i>	Ratio of the projected area of submerged portions of piers or piles in a bridge opening to the gross area of the bridge opening.	ft
<i>K</i>	Conveyance of a section.	ft ³ /sec
<i>K_a, K_b</i> and <i>K_c</i>	Conveyance of parts of the approach section for contracted opening.	ft ³ /sec
<i>K_d</i>	Conveyance at the upstream end of dikes.	ft ³ /sec
<i>K_i</i>	Conveyance of subsection.	ft ³ /sec
<i>K_T</i>	Conveyance of entire cross section.	ft ³ /sec
<i>k</i>	Adjustment factor; subscripts refer to specific items, as <i>a</i> for skewed abutments with dikes, <i>b</i> for offset dikes, <i>c</i> for side contraction, <i>d</i> for length of dikes, <i>e</i> for eccentricity, <i>F</i> for Froude number, <i>j</i> for piers and piles, <i>L</i> for length, <i>r</i> and <i>R</i> for radius, <i>t</i> for submergence, <i>w</i> for length of wingwalls, <i>x</i> for <i>x/b</i> ratio, <i>y</i> for $y_a + y_b/2b$ ratio, <i>z</i> for projecting abutments, ϕ for angularity, and θ , wingwall angle.	
<i>L</i>	Length of bridge abutment in direction of flow.	ft
<i>L_d</i>	Length of dikes.	ft
<i>L_w</i>	Distance from approach section to upstream side of contraction.	ft
<i>m</i>	Channel-contraction ratio.	
<i>n</i>	Manning roughness coefficient.	ft ^{1/6}
<i>P</i>	Wetted perimeter of cross section of flow.	ft
<i>Q</i>	Total discharge.	ft ³ /sec
<i>q</i>	The part of the discharge that is not contracted to enter a single contracted opening.	ft ³ /sec
<i>R</i>	Hydraulic radius.	ft
<i>r</i>	Radius of entrance rounding.	ft
<i>S</i>	Friction slope.	
<i>t</i>	Vertical distance between water level at approach section and lowest horizontal member of a partially submerged bridge.	ft
<i>V</i>	Mean velocity of flow in a section.	ft/sec
<i>w</i>	Measure of the length of a wingwall or chamfer.	ft
<i>x</i>	Horizontal distance from the intersection of the abutment and embankment slopes to the location on upstream embankment having the same elevation as the water surface at section 1.	ft
<i>y</i>	Depth of flow in a contracted opening.	ft
<i>z</i>	Length of projection of abutment beyond wingwall junction.	ft
<i>1, 2</i>	Subscripts which denote the location of cross sections or section properties in downstream order.	
α	Velocity-head coefficient.	

<i>Symbol</i>	<i>Definition</i>
Δ	Difference in values, as Δh is the difference in head.
θ	Acute angle between a wingwall and the plane of contraction.
Σ	Summation of values.
ϕ	Acute angle between the plane of contraction and a line normal to the thread of the stream.
$<$	Less than.
\leq	Equal to or less than.
$>$	Greater than.
\geq	Equal to or greater than.

MEASUREMENT OF PEAK DISCHARGE AT WIDTH CONTRACTIONS BY INDIRECT METHODS

By Howard F. Matthai

Abstract

This chapter describes procedures for measuring peak discharges using open-channel width contractions. Field and office procedures limited to this method are described. The discharge equation based on the continuity and energy equations between an approach cross section and the contracted section under a bridge or contraction is given. Contractions are classified into four geometric types. Discharge coefficients and computation procedures are given with a complete facsimile example of computation of a contracted-opening measurement. Additional procedures are given for multiple-opening contractions.

Introduction

The contraction of a stream channel by a roadway crossing creates an abrupt drop in water-surface elevation between an approach section and the contracted section under the bridge. The contracted section framed by the bridge abutments and the channel bed is in a sense a discharge meter which can be utilized to compute floodflows. The head on the contracted section is defined by high-water marks; the geometry of the channel and bridge is defined by field surveys. Extensive laboratory investigations have been conducted by the U.S. Geological Survey to define discharge coefficients for different abutment geometries.

The methods presented in this chapter are based on laboratory studies. The methods have been verified at 30 different field sites where the discharge was known as a result of current-meter measurements.

The Discharge Formula

When computing peak discharge at a contraction, the drop in water-surface level between an upstream section and a contracted section is related to the corresponding change in velocity.

The discharge equation results from writing the energy and continuity equations for the reach between these two sections, designated sections 1 and 3 on figure 1:

$$Q = CA_3 \sqrt{2g \left(\Delta h + \alpha_1 \frac{V_1^2}{2g} - h_f \right)} \quad (1)$$

in which

Q = discharge,

C = coefficient of discharge,

A_3 = gross area of section 3; this is the minimum section parallel to the constriction between the abutments and is not necessarily at the downstream side of the bridge,

Δh = difference in elevation of the water surface between sections 1 and 3,

$\alpha_1 \frac{V_1^2}{2g}$ = weighted average velocity head at section 1, where V_1 is the average velocity, Q/A_1 , and α_1 is a coefficient which takes into account the variation in velocity in that section, and

h_f = the head loss due to friction between sections 1 and 3.

The procedure recommended for evaluating each term in the discharge equation is described in the following sections.

Approach Section

The approach section, section 1, is a cross section of the natural, uncontracted channel upstream from the beginning of drawdown. As shown in figure 1, locate section 1 one bridge-opening width, b , upstream from the contraction to be sure it is above the drawdown zone. Section 1 includes the entire width of the valley perpendicular to the line of flow.

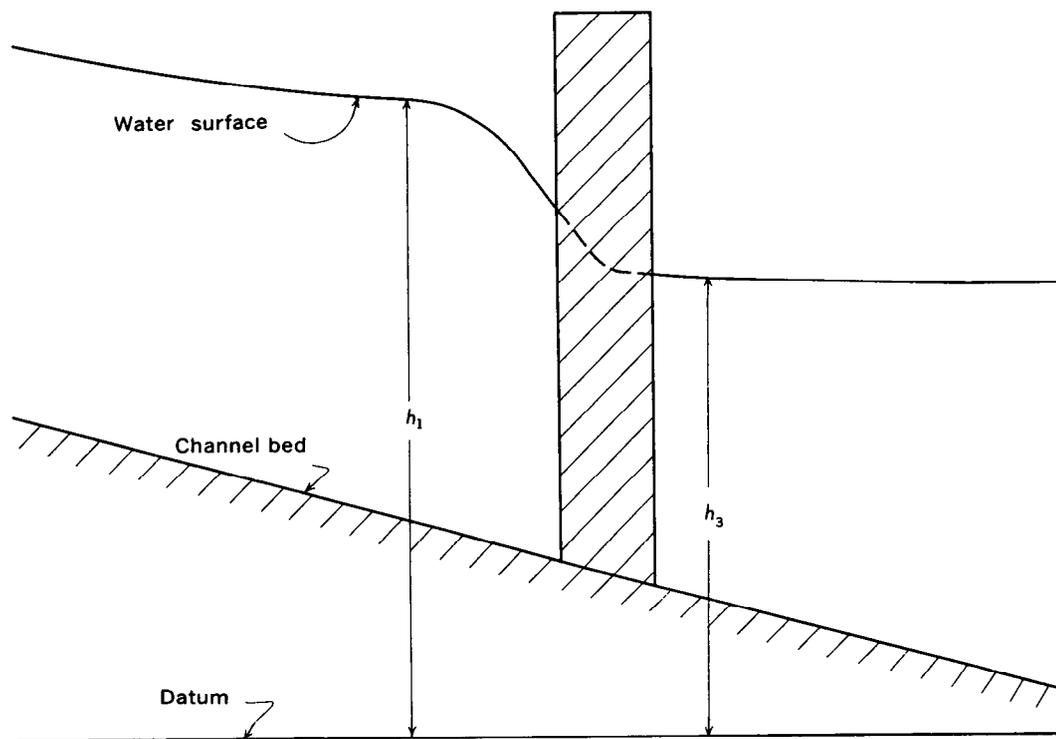
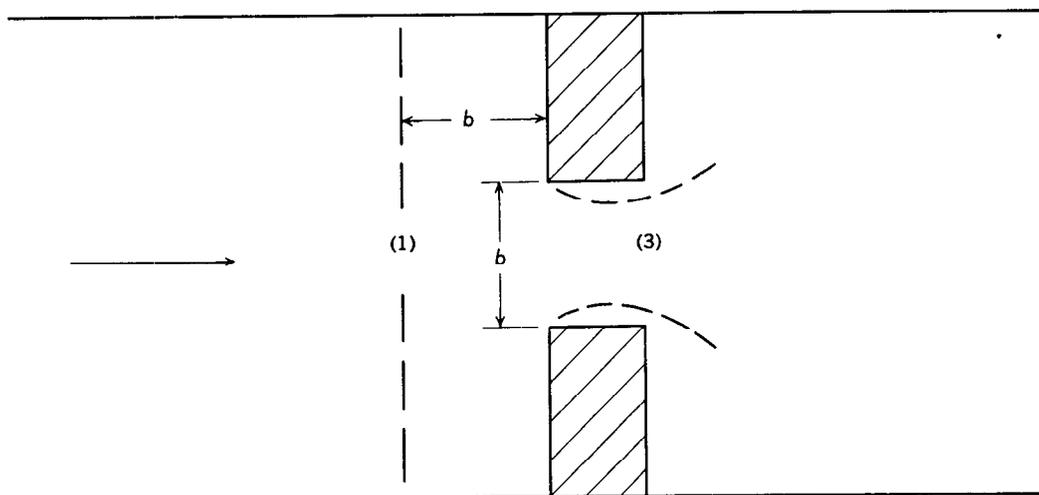


Figure 1.—Definition sketch of an open-channel contraction.

Avoid sites where the main channel meanders severely in the approach to the contraction. An extreme example is where the approach section, one b -width upstream and perpendicular

to the flow on the flood plain, intersects the main channel more than once. The energy losses cannot be correctly evaluated for this condition.

A large degree of contraction is associated with ponded approach conditions which are indicated when the water-surface profiles are level for some distance along the embankment or upstream from the contraction. However, a level profile may exist when the approach velocity head just balances the friction loss. The latter items can be large or small. Even when ponded approach conditions are indicated, obtain an approach section.

When the approach reach lies in a zone of heavy vegetation, even though the degree of contraction is large, the friction loss from section 1 to section 3 will be a large part of the total fall. Surveying an approach section through a heavily wooded zone can be a formidable task. Often a cross section along the edge of the right-of-way will be representative of a section one b -width upstream insofar as ground elevations are concerned, and the ground elevation of the approach section may be approximated in this manner. The value of the roughness coefficient, n , selected for the approach section under these conditions should represent the heavy vegetation in the reach from the assumed location of the approach section one b -width upstream to the bridge opening.

Contracted Section

The contracted section, section 3, defines the minimum area on a line parallel to the contraction. Generally this section is between the bridge abutments. When the abutments of a skewed bridge are parallel to the flow, section 3 is still parallel to the contraction even though the minimum section is perpendicular to the abutments.

When a scour hole occurs under a bridge, a section between the bridge abutments may not be the minimum section. Under extreme conditions of scour, the upstream lip of the scour hole may be the physical condition, rather than bridge geometry, that determines the head-water elevation. For such conditions the coefficients presented herein are not applicable. Avoid such sites.

The area, A_3 , to be used in the discharge equation is always the gross area of the section and is the area below the level of the free water surface as determined by the methods described

in a subsequent section. No deductions are made for the areas occupied by piles, piers, or submerged parts of the bridge if they lie in the plane of the contracted section. However, the conveyance, K_3 , is computed with the area of piles, piers, or submerged parts deducted from the gross area of section 3. Also, the wetted perimeter used to compute the hydraulic radius, R , will include the lengths of the sides of the piles, piers, or bridge surfaces in contact with the water. The mean velocity, V_3 , is computed using gross area, A_3 .

Water-Surface Levels

Determine water-surface levels for sections 1 and 3 as described below. Otherwise the coefficients derived from laboratory studies will not be applicable.

Obtain the elevation of the water surface at section 1 from a profile defined by high-water marks along each bank of the stream. If high-water marks cannot be located in this area and a large degree of contraction exists, draw a profile of the high-water marks along the upstream face of the embankment. If this profile is level for an appreciable part of the distance along the embankment, assume that this elevation is the same as that of section 1.

Obtain the water-surface elevation for section 3 on the downstream side of the embankment adjacent to the abutment regardless of the location of section 3. Water-surface profiles along the downstream face of the embankment are useful in establishing this elevation.

Compute the water-surface elevations of sections 1 and 3 as the average of the elevations on each bank. The one exception is an opening with a high degree of eccentricity. Here, determine the elevation of section 3 on the contracted side only and use this elevation to compute both the area of section 3 and the change in water-surface elevation between sections 1 and 3.

Velocity Head

The velocity head at section 1 is expressed by the term $\alpha_1 V_1^2/2g$, where V is the mean velocity in the section and α is the velocity head adjustment factor. The value of α is approximated

from the relative conveyance and area of the subsection into which a cross section is normally divided by

$$\alpha = \frac{\Sigma(K_i^3/a_i^2)}{K_T^3/A_T^2}, \quad (2)$$

where the subscript i refers to the subsections and T refers to the entire cross section.

Friction Loss

The friction loss in the general discharge equation (equation 1) is the friction loss between sections 1 and 3. The distance between the two sections is divided into two reaches; the approach reach from section 1 to the upstream side of the bridge opening and the bridge-opening reach. The conveyance at the upstream side of the bridge opening is assumed to be the same as at section 3. The total head loss due to friction can be obtained from the equation

$$h_f = L_w(Q^2/K_1K_3) + L(Q/K_3)^2, \quad (3)$$

where L_w is the length of the approach reach, L is the length of the bridge opening, K_1 and K_3 are the total conveyances of sections 1 and 3. When the approach reach has heavy brush and the reach under the bridge is relatively clear, the weighted conveyance computed from the conveyances of sections 1 and 3 will not be correct. A better approximation of the friction loss may be obtained if $L_w(Q^2/K_1K_q)$ is substituted for the first term in equation 3. K_q is that part of the approach section conveyance corresponding to the projected width b . The computation of K_q is discussed further under the heading "Channel-Contraction Ratio."

The friction loss computed from equation 3 is only an approximation of the actual loss because of the rapid change in velocity from sections 1 to 3. Satisfactory results cannot be obtained by the contraction method if the term h_f is large relative to the difference in head Δh .

Froude Number

The contraction method assumes tranquil flow at section 3. The Froude number, F , is an index of the state of flow. In rectangular channels the flow is tranquil if the Froude

number is less than 1. In irregular sections the Froude number computed as $V/\sqrt{gy_3}$, where y_3 is the average depth at section 3, is not an exact index of the state of flow. If the computed Froude number exceeds 0.8, the computed discharge may not be reliable.

Classification of Contractions

Single-opening contractions

The geometric properties of the various types of bridge openings affect the discharge coefficient. Most bridge openings can be classified as one of four types representing the distinctive features of their major geometric characteristics. Laboratory studies have defined certain ratios for each type of contraction and their effect on the discharge coefficient.

Type 1

A type 1 contraction as shown in figure 2 has vertical embankments and vertical abutments with or without wingwalls. The entrance rounding or the wingwall angle, the angularity of the contraction with respect to the direction of flow, and the Froude number affect the discharge coefficient.

Type 2

A type 2 contraction as shown in figure 3 has sloping embankments and vertical abutments. The depth of water at the abutments and the angularity of the contraction with respect to the direction of flow affect the discharge coefficient.

Type 3

A type 3 contraction as shown in figure 4 has sloping embankments and sloping spill-through abutments. The entrance geometry and the angularity of the contraction with respect to the direction of flow affect the discharge coefficient.

Type 4

A type 4 contraction as shown in figure 5 has sloping embankments, vertical abutments, and wingwalls. The wingwall angle, the angularity of the contraction with respect to the

direction of flow, and, for some embankment slopes, the Froude number affect the discharge coefficient. Note that the addition of wingwalls does not necessarily make a type 4 contraction. A type 1 contraction may have wingwalls.

If the flow passes around a vertical edge at the upstream corner of the wingwall, the contraction is type 1; if the flow passes around a sloping edge at the top of the wingwall, the contraction is type 4.

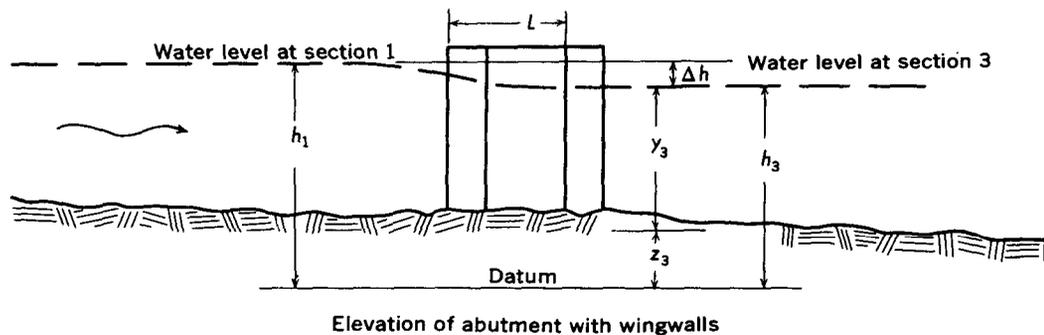
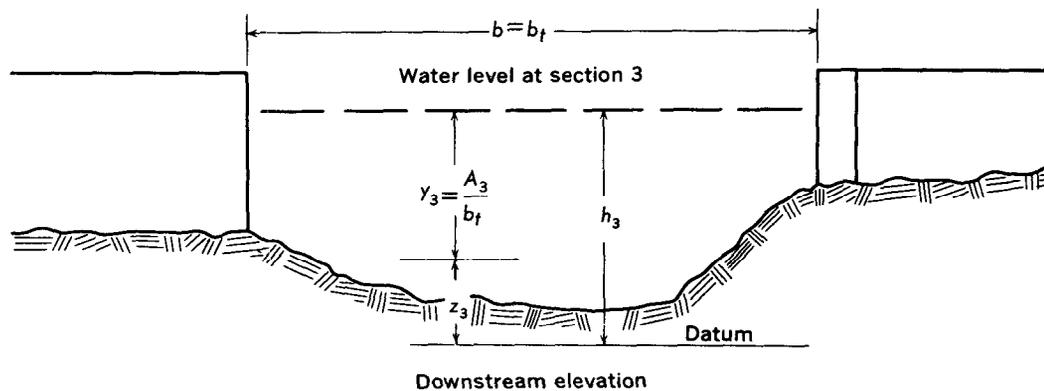
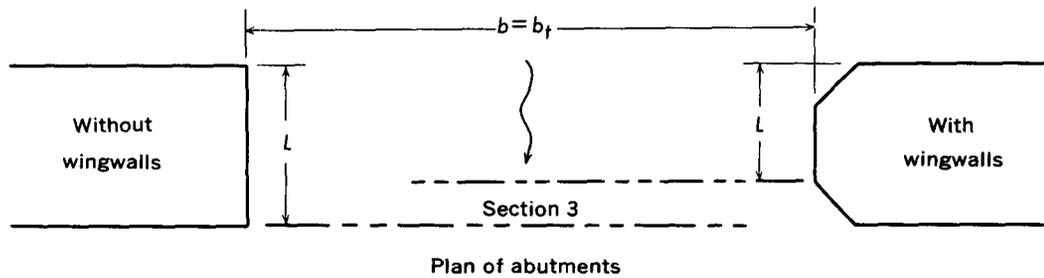


Figure 2.—Definition sketch of type 1 opening, vertical embankments and vertical abutments with or without wingwalls.

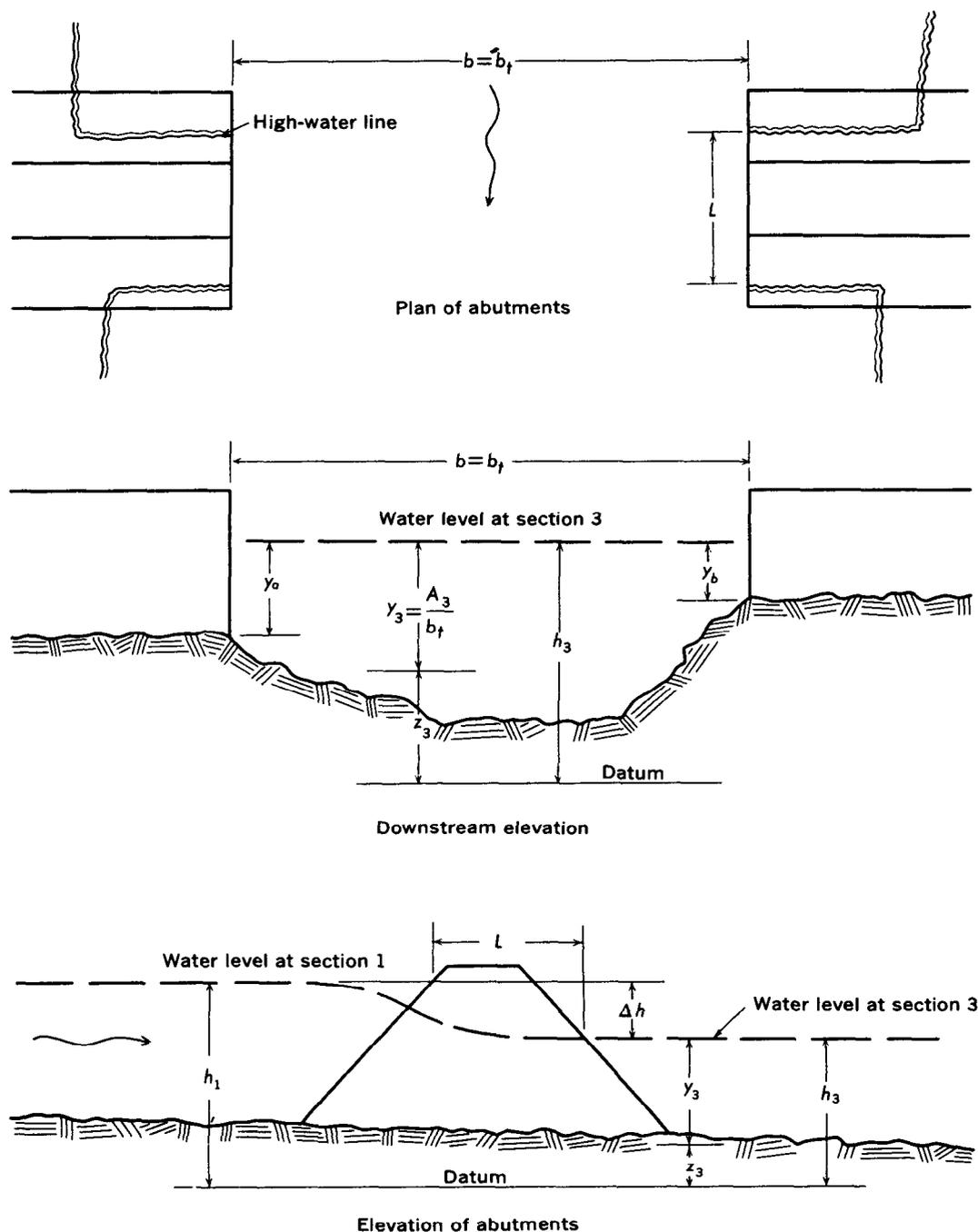


Figure 3.—Definition sketch of type 2 opening, sloping embankments and vertical abutments.

Modified standard types

Spur dikes

Spur dikes are added to some bridge abutments to modify the flow pattern and reduce scour at the abutments. The effect of elliptical and straight dikes on the discharge coefficient has been defined by laboratory study.

Dual bridges

The construction of divided highways has introduced dual lane bridges. For the special case where the abutments are continuous between the two bridges, the geometry may still be classified as one of the standard types.

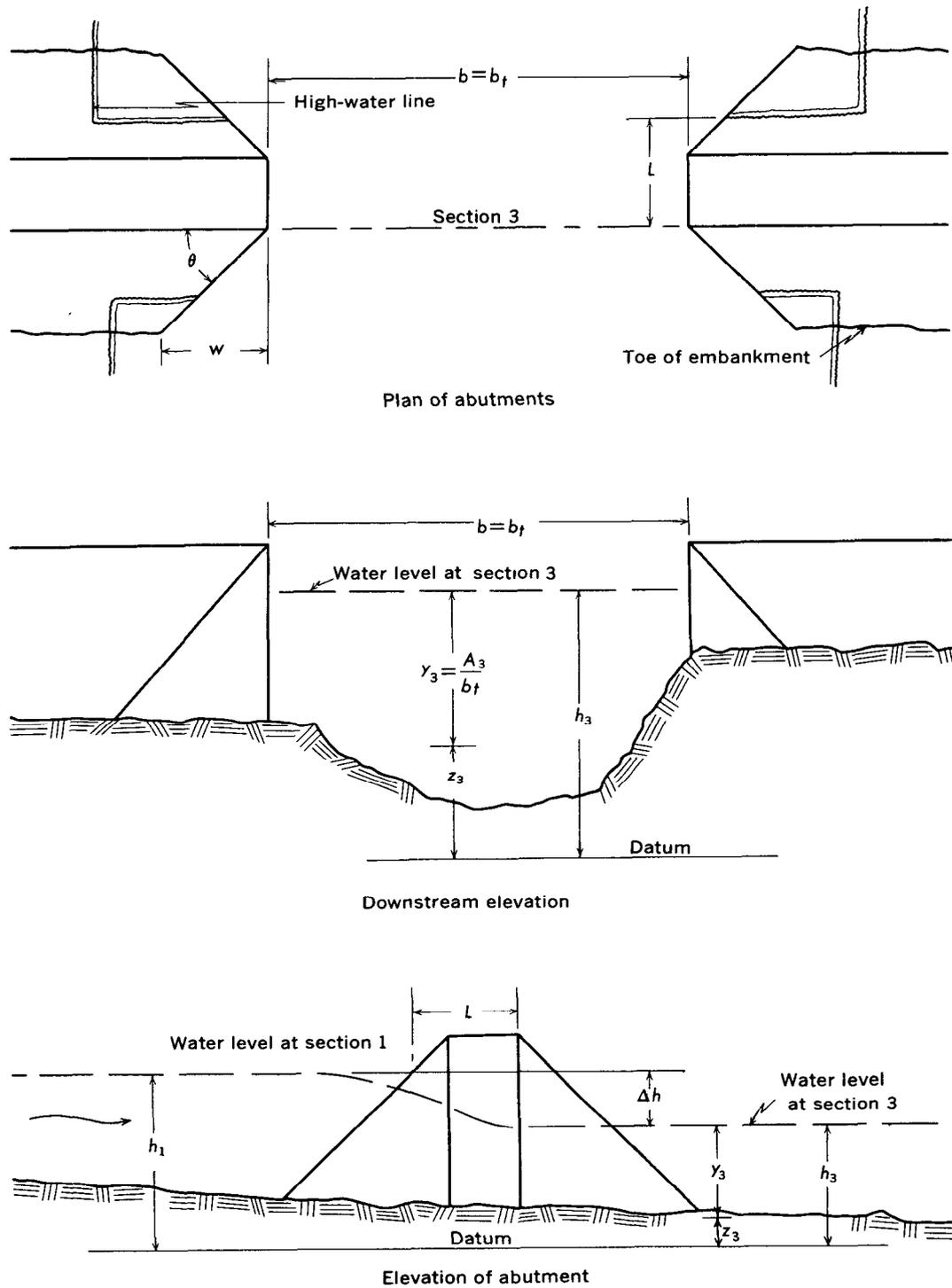


Figure 5.—Definition sketch of a type 4 opening, sloping embankments and vertical abutments with wingwalls.

conditions if the angle ϕ is less than 20° . The effect of this change in geometry on the discharge coefficient for angles greater than 20°

has not been adequately defined, and this geometry should not be used in computing peak discharge.

Nonstandard types

Some bridges do not fit any of the four types described. Bridges with type 1 abutments set on type 3 embankments, or type 4 wingwalls with vertical upstream ends for part of their height, or unique construction will require engineering judgment to select the type to be used. When there is a choice between two types, the discharge coefficient can be computed for each type; if the difference is less than 5 percent, either type can be selected; if the difference is over 5 percent, the two coefficients can be averaged.

Arch bridges often approximate a type 1 contraction; but if much of the arch is submerged a reliable answer cannot be obtained by using type 1 coefficients.

Combination sites

Floods often flow over the road near a bridge in addition to flowing through the bridge opening. This is not a desirable condition for computing peak discharge; but if such a site must be used, a combination of the contraction method and flow-over-embankment method may be used. See page 33.

Multiple-opening contractions

Roadway crossings on large streams may include more than one bridge. Procedures for computing peak discharge through multiple-opening contractions have been defined by laboratory study. In general, the procedures used for single openings are applicable, but some of the geometric ratios and terms in the discharge equation are computed in a different manner.

Field Procedures

Selection of a site

The characteristics of a site considered for computation of discharge by the contraction method should be carefully considered. If a desirable site cannot be found, other methods of computing the peak discharge, such as the slope-area method, may yield better results. In addition to the general criteria given in a previous section of the report, use the following guidelines in selecting a site for computation of peak discharge by the contraction method.

1. The channel under the bridge should be relatively stable. Because the amount of scour at the time of the peak cannot be determined, do not use the method on streams with an unlimited amount of sand in the channel bed (sand-channel streams).
2. The fall in water surface, as defined by high-water marks, between sections 1 and 3 should not be less than 0.5 foot.
3. The fall should be at least four times the friction loss between sections 1 and 3. Thus, avoid long bridges downstream from heavily wooded flood plains.
4. The bridge geometry should be reasonably close to one of the standard types or modified types described in the previous section on "Classification of contractions."

High-water marks

The fall through the constriction is the difference in high-water elevations at the approach section and at the contracted section. Survey high-water marks along the channel a reasonable distance upstream from the approach section and along the upstream face of the embankment. The high-water elevation at the contracted section is determined from marks along the downstream face of the embankment. Therefore, obtain marks along the downstream side of the embankment adjacent to the abutments and on down the channel if more marks are required to define the downstream elevation.

Cross sections

Locate the approach section upstream from the beginning of drawdown, because there is an appreciable variation in elevation of the water surface across the channel within the drawdown zone. It is recommended that the approach section be located one bridge-opening width upstream from the contraction so that it will be upstream from the drawdown zone for any given condition. For a completely eccentric contraction with no contraction on one bank, locate section 1 two bridge-opening widths upstream, because such a contraction is considered as half a normal contraction. The approach section is considered to include the entire width of the valley perpendicular to the line of flow.

A large degree of channel contraction will often cause ponded conditions upstream from the bridge. Even under these conditions, an approach section is necessary.

The smallest cross section between the abutments on a line parallel to the constriction is also obtained. This procedure should always be used even when the abutments of a skewed bridge are parallel to the flow. The minimum section is usually located along the downstream side of the bridge.

Bridge geometry

Include in the field data complete details of the bridge geometry so that both plan and elevation drawings can be made. Determine wingwall angles and lengths, lengths of abutments, positions and slopes of the embankments and abutments, elevation of roadway, top width of embankment, details of piers or piles, and elevations of the bottom of girders or beams spanning the contraction. Use a steel tape for most lineal measurements rather than scaling distances from a plan. Stereopictures of the upstream corners of both abutments are essential for review purposes.

Channel conditions

Provide stereopictures, sketches, and word descriptions of the approach reach and the channel under the bridge to substantiate the values of n assigned to the approach section and the contracted section.

Discharge Coefficient

The discharge equation (equation 1), derived from the energy equation and the continuity equation, contains a coefficient, C , which represents a combination of (1) a coefficient of contraction, (2) a coefficient for the eddy losses caused by the contraction, and (3) the velocity-head coefficient, α_3 , for the contracted section.

Dimensional analysis of the factors that influence the flow pattern through a bridge shows that C can be expressed as a function of certain geometric and flow parameters. In functional notation

$$C = f \left(m, \frac{L}{b}, \frac{r}{b}, \frac{w}{b}, \theta, \frac{x}{b}, \frac{y_a + y_b}{2b}, \frac{t}{y_3 + \Delta h}, j, F, \phi, e, E, L_d, L_d/b_d \right) \quad (4)$$

The terms on the right-hand side of equation 4 are defined as follows and illustrated where noted:

- m Channel-contraction ratio.
- b Width of bridge opening; the distance between abutment faces (figs. 2-5).
- L Length of abutment, variously defined for different types of bridge openings (figs. 2-5).
- r Radius of rounding of entrance corner of abutment for vertical-faced contractions (fig. 6).
- w A measure of the length of a wingwall or chamfer (figs. 5, 7, 14, 15).
- θ The acute angle between a wingwall and the plane of the contraction (figs. 5, 7, 14, 15).
- x Horizontal distance from the intersection of the abutment and embankment slopes to the location on upstream embankment having the same elevation as the water surface at section 1 (fig. 4).
- y_a, y_b Depths of water at the toe of each abutment at section 3 (figs. 3, 4).
- y_3 Average depth of water in section 3;

$$y_3 = \frac{A_3}{b_t}$$

- A_3 The gross area of section 3.
- t Vertical distance between water level at section 1 and lowest horizontal member of a partly submerged bridge (fig. 17).
- A_j The submerged cross-sectional area of piers or piles projected to section 3.
- j Ratio of the projected area of the submerged parts of piers or piles in the bridge opening; $j = A_j/A_3$.
- F Froude number of the contracted section.
- ϕ The acute angle between the plane of the contraction and a line normal to the thread of the stream (fig. 20).
- e An eccentricity ratio;

$$e = K_a/K_b \leq 1.00 \text{ (fig. 18).}$$

- E The slope of the embankments; the ratio of the horizontal distance to the vertical distance, as 2:1.
- Δh The difference in elevation of the water surfaces at sections 1 and 3.
- L_d Length of dikes (fig. 23).
- b_d Offset distance for straight dikes (fig. 23).

The only terms in equation 4 that are common to all types of bridge openings are m , L/b , and F . Laboratory studies have shown that, of these, m is the most important, and F has the least general significance. Therefore, m and L/b have been selected as primary variables for determining the base discharge coefficient, C' . The other terms in equation 4 are descriptive of the geometric properties of various types of bridge openings. Adjustment factors to the base coefficients have been determined for these terms and the Froude number where applicable.

Channel-contraction ratio

The channel-contraction ratio, m , describes the degree of contraction imposed by the restriction on the normal stream channel and is one of the primary variables governing the magnitude of the discharge coefficient. The channel-contraction ratio is a measure of the proportion of the total flow that enters the contraction from the sides of the channel. It can be computed from the equation

$$m = (Q - q) / Q = 1 - q / Q,$$

where Q is the total discharge, and q is the discharge that could pass through the opening without contraction. The total discharge is assumed to be distributed across the approach section in proportion to the conveyances of arbitrarily defined subsections. This assumption can be made because the energy slope is approximately constant across the section. K_0 is the conveyance of the subsection occupied by q , and K_1 is the total conveyance of the approach section (fig. 18); therefore,

$$m = (1 - K_0 / K_1). \quad (5)$$

Determination of the discharge coefficient

Base coefficient

One or more steps are required to determine the discharge coefficient. A base coefficient, C' corresponding to given values of the two primary variables, m and L/b , is determined from one of the base curves of figures 6-15 for the type of bridge opening under consideration. The base coefficient, C' , is the final discharge

coefficient if all the six or seven standard conditions shown on the base curves are met. The secondary variables given under standard conditions are the only variables to be considered for that particular type of bridge opening. For example, if a certain bridge departs from the standard conditions only with respect to the Froude number and angularity, then the discharge coefficient will be obtained from the equation

$$C = C' \times k_F \times k_\phi,$$

where

C' = the standard value of the coefficient of discharge corresponding to the given values of m and L/b and the standard values of each of the remaining variables, including F and ϕ ;

k_F = a coefficient that adjusts C' for the influence of a nonstandard value of F ;

k_ϕ = a coefficient that adjusts C' for the influence of angularity.

Adjustment factors

If the standard conditions are not satisfied, then C' must be adjusted for the effect of each condition that is not standard. These adjustment factors are shown in figures 6-17; the product of all adjustment factors and the base coefficient is the discharge coefficient.

Adjustment factors for the effects of eccentricity, piles or piers, submergence, and skewed embankments with abutments parallel to the flow are applicable to all four types of bridge openings and are discussed in the following sections.

Eccentricity

The eccentricity, e , of a bridge opening (fig. 18) is the ratio of the conveyances K_a / K_b . K_a and K_b are the conveyances of the parts of the approach section to either side of the projected b width, or to either side of the part of the section carrying the discharge, q , that passes through the opening without contraction. K_a is always the smaller of the two. These conveyances are proportional to the flow that has to deviate from its natural course to enter the bridge opening. For ratios greater than 0.12,

no correction is necessary for eccentricity. For fully eccentric conditions, K_a is zero; therefore $e=K_a/K_b=0$ (fig. 19), and the following procedures should be used:

1. Locate the approach section, section 1, a distance $L_w=2b$ upstream from the bridge.
2. Determine the base coefficient, C' , and the adjustment factors by using the ratio $L/2b$. Use abutment on contracted side only to determine C' .
3. The water-surface elevation at section 1 is average of the elevations at each end of the section.
4. The downstream elevation is determined on the contracted side only (point B , fig. 19). Use this elevation to compute both the area of section 3 and the fall.

A fully eccentric contraction is considered as half a normal contraction; therefore the effective bridge width, for computing L_w and C' , is equal to $2b$.

The adjustment factor, k_e , is obtained from the following table:

e	0	0.02	0.04	0.06	0.08	0.10	0.12
k_e	0.953	0.966	0.976	0.984	0.990	0.995	1.00

Piers or piles

Many bridge openings contain piers or piles, and their effect on the discharge coefficient must be evaluated. The total submerged area of the piers or piles projected on the plane defined by section 3 is designated A_j . The ratio of the area of piers or piles to the gross area of section 3, A_j/A_3 , is represented by the letter j . The relation of the channel-contraction ratio m to j determines the adjustment factor, k_j , for piers (fig. 16), and the ratios m , j , and L/b determine k_j for piles (fig. 16).

The dashed lines on figure 16 illustrate its use. In this example, enter the right-hand plot at the m value of 0.41; move vertically to an L/b value of 0.69; move horizontally to the line marked $j=0.10$ in the left-hand plot; then upward to the value of j , 0.04 in this example; and finally move horizontally to the k_j scale on the left to obtain a value of 0.967. For values

of j greater than 0.10, move downward from the line marked $j=0.10$.

When both piles or piers and submergence exist, j is computed as the ratio A_j/A_s . When the upstream and downstream bridge chords are submerged, A_s is the gross area below the lower bridge chord and A_j is only the area of piles or piers. When only the upstream bridge chord is submerged, A_s equals A_3 , and A_j is only the submerged area of piles or piers.

Submergence

Many floods cause stages high enough to submerge the lower parts of bridges. The additional wetted perimeter and obstruction of the bridge members affect the flow. The vertical distance between the water level at section 1 and the lowest horizontal member of a partially submerged bridge is designated as t (fig. 17). The ratio of t to the sum of y_3 and Δh is called the bridge submergence ratio. Its effect on the discharge coefficient is indicated on figure 17 as k_t .

Whether the lower bridge members were submerged or not cannot be determined from field evidence under all conditions. Contact with the lower bridge members can be assumed when k_t is less than 1.00. The length of the bottom chord is then added to the wetted perimeter of section 3 even though the downstream bottom chord may not be submerged. The larger wetted perimeter provides a better evaluation of the true friction losses in the approach reach and through the bridge.

Where both upstream and downstream bottom chords are submerged and several adjustment factors, including k_t , are used, the discharge coefficient may be nearly 1.00; the maximum value of the discharge coefficient should be the value of k_t from figure 17 after all adjustment factors have been applied.

Some floods completely submerge the bridge floor. For this condition, the following procedure is suggested to approximate the discharge:

1. Compute A_3 as the product of b and y_3 minus the cross-sectional area of the submerged bridge.
2. Use $k_t=1.00$.
3. Compute discharge by equation 1.

4. This discharge is the total discharge for the width, b , and the flow over the floor of the bridge should not be added to it.

For a type 2 contraction affected by submergence, the depth of water at the toe of each embankment, y_a and y_b (fig. 3) are measured up to the lower bridge member rather than to the downstream water surface.

Abutments parallel to flow

When the contraction is at an angle, ϕ , to the flow, the abutments may be parallel to the flow or perpendicular to the embankment. The coefficient for both conditions is the same if the angle ϕ is less than 20° . If the angle is greater than 20° and the abutments are parallel to the flow, an additional correction factor is required. Because this factor has not been adequately defined, do not use a contraction with this geometry to compute peak discharge.

Other factors and conditions

Other adjustment factors are applicable to one or more types of contractions but not to all, or they have a different effect on different types. Adjustment factors for Froude number, angularity, entrance rounding, wingwall angle, and depth of water on the abutments are in these categories. Curves for these factors accompany the base coefficient curves for the four types of contractions in figures 6-15. Only those factors shown for each type of contraction have a significant effect upon the discharge coefficient.

Angularity

The proper interpretation of angularity (fig. 20) and the correct measurement of the distance, L_w , from section 1 to the upstream side of the contraction must be made for a contraction in a curving channel or the adjustment factor, k_ϕ , and the length of the reach will not be correct.

Angularity should not be confused with eccentricity (figs. 18, 19) or curvature of the stream (fig. 20). Both the approach section and the contracted section could be perpendicular to the flow in a channel on a curve, and no adjustment for angularity is necessary.

Angularity may be considered as the relation between the skewed contraction and the flow lines for the unobstructed channel. The ad-

justment factor, k_ϕ , for angularity is influenced by two independent variables, the channel-contraction ratio, m , and the degree of angularity, ϕ ; but one of these is not necessarily a function of the other.

The best interpretation of angularity can be made in the field if it is recognized that the influence of the angle becomes less pronounced with larger channel-contraction ratios. Thus the net direction of flow at the upstream side of the contraction, section 2, does not necessarily determine the angle ϕ , because the flow direction there is also affected by the degree of channel contraction.

For other than straight channels and parallel sections, measure the distance, L_w , from section 1 to section 2 from the centroid of the approach section to the middle of section 2. The direction of the chord between these two points is largely independent of the angularity of the contraction. In figure 20, no adjustment factor for angularity would be required, but L_w would be measured along a chord at an acute angle to section 2.

Intermediate embankment slopes

For those contractions which have sloping embankments or abutments, curves are presented for at least 1:1 and 2:1 slopes. However, a wide range of slopes is encountered in the field, and coefficients must be computed for other than 1:1 and 2:1 slopes. If the slope is between 1:1 and 2:1, straight-line interpolation is satisfactory.

Different abutments

If the abutment slope differs from the embankment slope, or if the slopes at the ends of the bridge differ, use an average slope. If the two abutments are different types, compute a C for each side, a and b , and weight the final C with respect to the conveyances, K_a and K_b , as follows:

$$C = \frac{C_a K_a + C_b K_b}{K_a + K_b} \quad (6)$$

Nonstandard geometry and flow conditions

The discharge formula cannot be applied directly to all contractions, because laboratory work has not defined base discharge coefficients and adjustment factors for all types of con-

traction geometry and flow conditions encountered in the field.

The discharge coefficient for a bridge opening that cannot be classified exactly as any of the four types described may be estimated by the engineer using his knowledge of the relative effects of the factors that influence the flow pattern and by a reasonable weighting of these factors. The most influential variables generally are m and L/b ; therefore a reasonable estimate of the adjustment factors will give results that are within the range of accuracy expected.

Maximum coefficient

Certain combinations of the adjustment factors applied to a base coefficient will yield a value of C greater than 1.00. Because this is impossible, a value of $C=1.00$ is taken as the maximum under all circumstances. If submergence of both the upstream and downstream bottom chords occurred, the maximum value of the discharge coefficient is the value of the adjustment factor, k_1 .

Special conditions

Field conditions sometimes preclude an approximation of the conditions tested in the laboratory. Some of the more common field variations have been investigated, and the modifications to the previous coefficients or new adjustment factors have been determined. The tests include the effects of embankment slopes other than those tested, timber and pile bridges, arch bridges, and spur dikes.

Other embankment slopes

The laboratory tests developed coefficients for type 2, 3, and 4 contractions with 1:1 and 2:1 embankment slopes. Coefficients for a type 3 contraction with 1½:1 slopes were computed from the original coefficients. Curves for all these conditions are included in this report. For a type 4 contraction, use the coefficient for a 2:1 slope for flatter slopes (3:1, 4:1, etc.).

For type 2 and 3 contractions, figure 21 illustrates one method of extrapolation of discharge coefficients for other slopes. The coefficient for a given embankment slope will be between that for an infinite slope and that for

zero slope. If the slope is infinite, there is no contraction and the coefficient is 1.00. The other limiting slope, 0:1, is for a type 1, vertical-faced contraction, and the coefficient can be computed. Coefficients for 1:1 and 2:1 slopes can be computed, and the four values of coefficients can be plotted as ordinates with the angle of repose, in degrees, as the abscissa. In figure 21 the desired coefficient for a 3:1 slope is obtained from a smooth curve through these four points. Note that adjustment factors k_2 , k_3 , and k_4 are not applied when computing the four coefficients; consequently they must be applied to the coefficient obtained from the curve. There will be cases, especially when the Froude number is high, when the four points will not define a smooth curve as shown in figure 21. Then, engineering judgment must be used to determine the best method of extrapolation.

Timber-and-pile bridges

Some bridges have wingwall and abutment timbers placed on the shoreward side of piles with or without a projection of the abutment timbers upstream from the junction with the wingwall (fig. 22). The projecting abutment creates a slightly reentrant condition at that junction which is not typical of the four types of contractions.

Tests of these bridges without the protruding abutment showed the effects of the piles alone. If the high-water elevation at the approach section is higher than the toe of the embankment at the upstream end of the wingwall, use the base coefficient for a type 4 contraction. If the high-water elevation at the approach section is lower than the toe of the embankment at the upstream end of the wingwall, use an average of type 2 and type 4 coefficients.

For a timber-and-pile bridge with projecting abutments, determine the base coefficient as explained in the preceding paragraph and then multiply it by an adjustment factor, k_2 . This adjustment factor is a function of the ratio z/b , where z is the length of the projection past the junction with the wingwall (fig. 22). Note that b is measured between the streamward sides of the piling, and that the wingwall angle, θ , should be measured as if the wingwall extended from the upstream end of the projecting abutment

to the streamward side of the wingwall piling, generally at the pile closest to the toe of the embankment.

Values of k_z have been determined for a limited range of z/b .

z/b	k_z
0 to 0.04	0.98
.05	.96

Spur dikes

Several types of spur dikes will be encountered in the field. Two common types of earth-embankment dikes are illustrated on figure 23. The elliptical dike is a continuation of the abutment and approximates one quarter of an ellipse. The straight dike may be a continuation of the abutment, or it may be set back a distance, b_d . The straight dike projects normal to the embankment. Adjustment factors have been defined for the addition of these dikes to type 3 contractions.

The elliptical dike is a continuation of the type 3 abutment, and its geometry is described by the length of the dike, L_d . The adjustment factor, k_a , which accounts for the influence of the dike on the discharge coefficient is a function of the ratio of the length of the dike to the width of the bridge opening, and the channel contraction ratio (fig. 13A). The addition of elliptical dikes to a constriction with skewed embankments and abutments parallel to the flow further affects the discharge coefficient. An adjustment factor, k_a , is used in addition to k_ϕ to account for this effect. The value of k_a has been defined only for an angle ϕ of 20° . The relation between k_a , L_d/b and m for this angle of skew is shown in figure 13B. Values of k_a may be determined by interpolation between angles of zero and 20° . The discharge coefficient for a constriction with an elliptical dike is thus determined as the product of C' for the type 3 constriction, the adjustment factors k_ϕ , k_z , k_a , k_a , and other factors for nonstandard conditions. The value of x used to determine k_z should be measured on the embankment slope as if the dike were not in place.

The geometry of the straight dike is defined by the length of the dike L_d , and the offset distance b_d . The functional relationships which define the adjustment factors k_a and k_b for the straight dike are shown in figure 13C, D. The discharge coefficient is determined as the product of C' for the type 3 constriction, the adjustment factors k_ϕ , k_z , k_a , k_b , and the other factors for nonstandard conditions. The value of x used to determine k_z should be measured on the embankment slope as if the dike were not in place.

For computing discharge through a contraction with spur dikes, the friction loss term

$$\left(L + \frac{L_w K_3}{K_1}\right)$$

in the denominator of the discharge equation 7 must be changed to include the friction loss due to the addition of the spur dikes. This can be accomplished by changing this term to

$$L + \frac{L_w K_3^2}{K_1 K_d} + \frac{L_d K_3}{K_d}$$

where L_d is the length of the dike, and K_d is the conveyance at the upstream end of the dikes.

Arch bridges

Coefficients for arch bridges were not determined in the laboratory study made by the U.S. Geological Survey. Most arches have virtually square corners and vertical faces and may be considered similar to type 1 contractions if the curvilinear part of the arch is not submerged to a large extent. If much of the arch is submerged, a reliable discharge cannot be expected by using methods given herein. Culvert methods may be applicable to some arch bridges.

Computations

The general discharge equation (equation 1) requires a trial-and-error solution. A direct solution is obtained if Q/A_1 is substituted for V_1 and $L_w(Q^2/K_1 K_3) + L(Q/K_3)^2$ for h_f . Thus,

$$Q = 8.02 C A_3 \sqrt{\frac{\Delta h}{1 - \alpha_1 C^2 \left(\frac{A_3}{A_1}\right)^2 + 2g C^2 \left(\frac{A_3}{K_3}\right)^2 \left(L + \frac{L_w K_3}{K_1}\right)}} \quad (7)$$

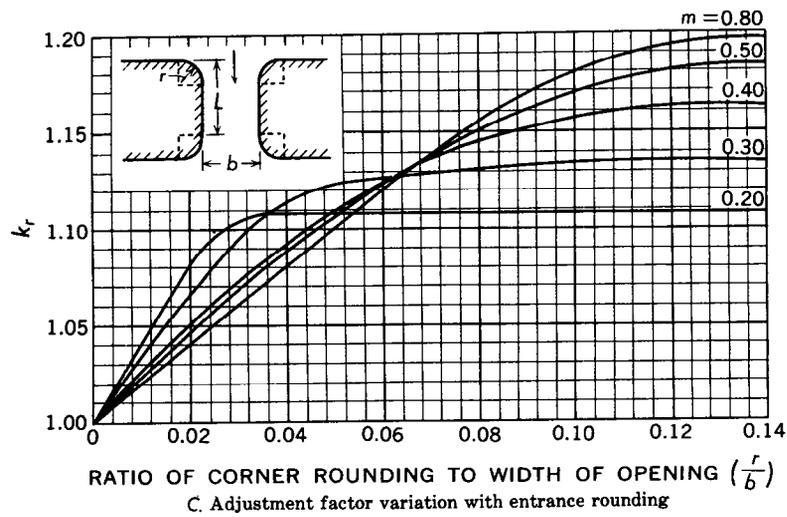
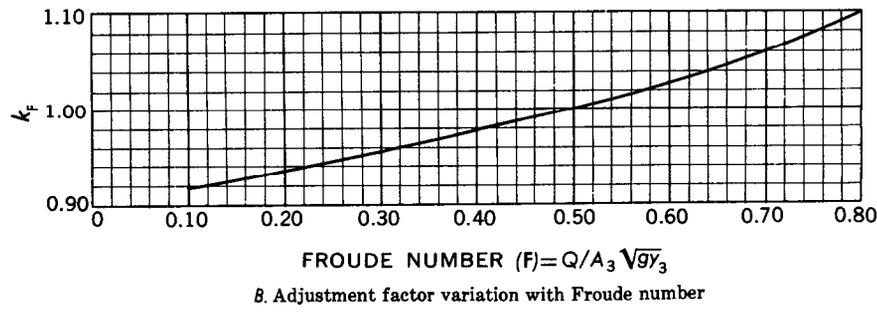
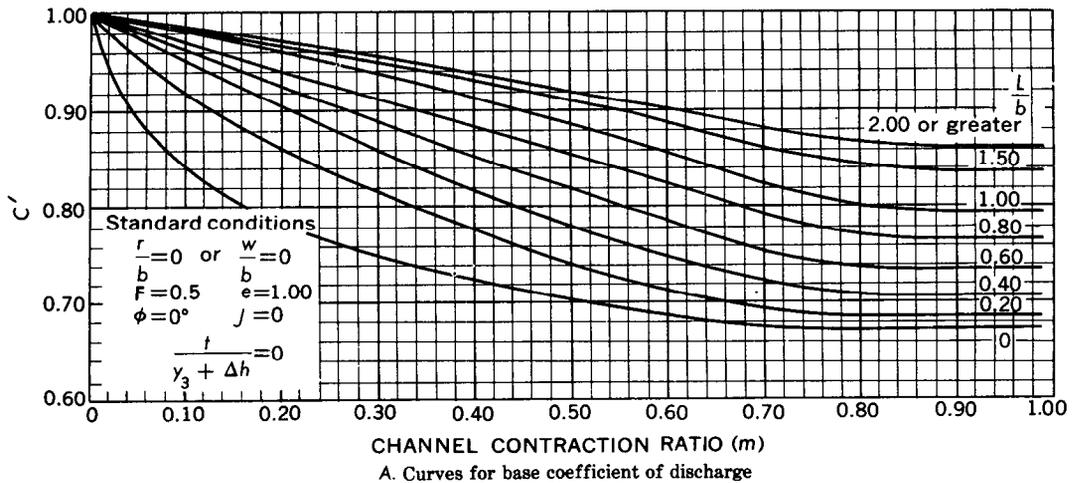
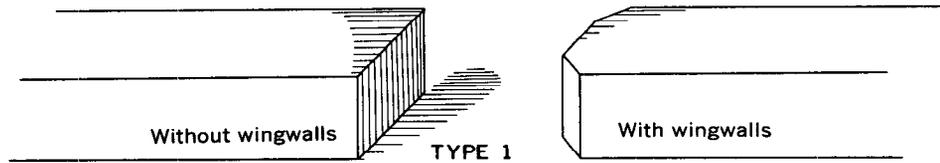


Figure 6.—Coefficients for type 1 opening, vertical embankments and vertical abutments.

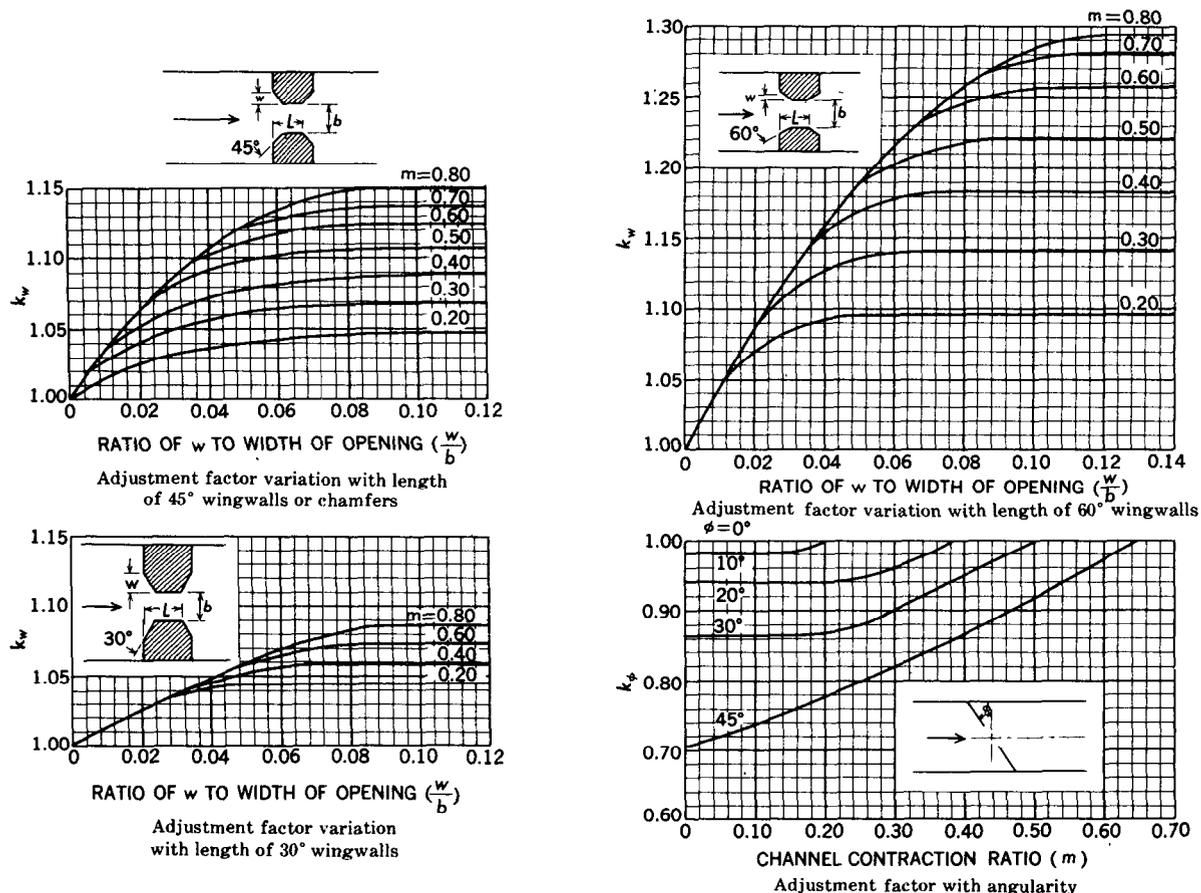


Figure 7.—Adjustment factor variation for vertical embankment and abutment of type 1 opening.

In this form, the effects of the approach velocity and the friction loss can be identified.

Outline

A step-by-step outline of the computation procedure (figs. 24–33) is given as a guide to the computer.

1. From the data of the transit-stadia field survey, plot a location sketch, list the high-water marks, plot a high-water profile for each bank, and transverse high-water profiles for upstream and downstream sides of the embankment if these will aid the definition of headwater and tailwater elevations, details of abutments and embankments in both plan and elevation, and cross sections 1 and 3 (figs. 24–28).
2. Determine from high-water profiles the water-surface elevations at sections 1 and

3 as described on page 3. See page 11 for the special case of eccentric openings. Compute the difference between the average elevation at section 1 and the average elevation at section 3. This is the fall, Δh , through the contraction.

3. The bridge width, b , for short bridges should have been measured in the field. Use this distance or scale it from the plan or the plot of section 3, and lay out an equal length on section 1. The center of the low-water channel should occupy the same relative position to b in sections 1 and 3 (fig. 33). The entire b width is used regardless of the angle of skew, ϕ .

The length, b , on the approach section should embrace the flow which could pass through the opening without contraction. At sites where there is a winding low-flow channel upstream

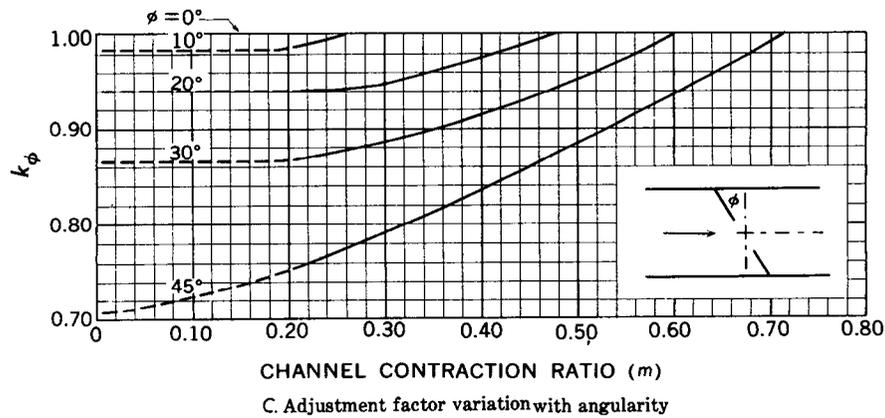
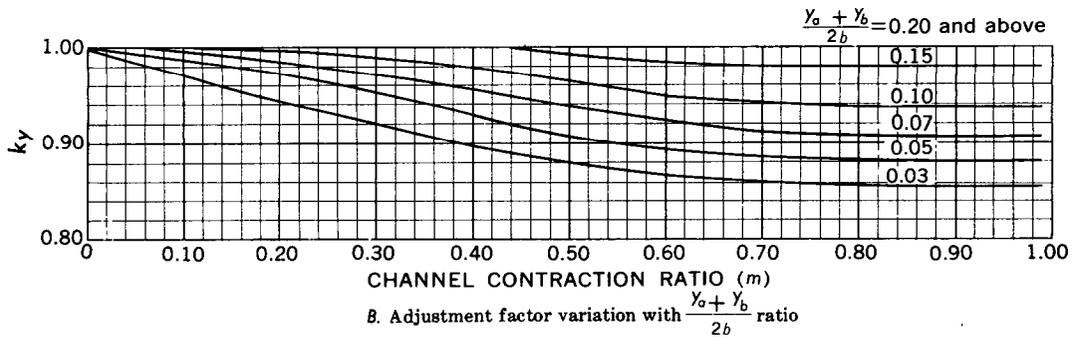
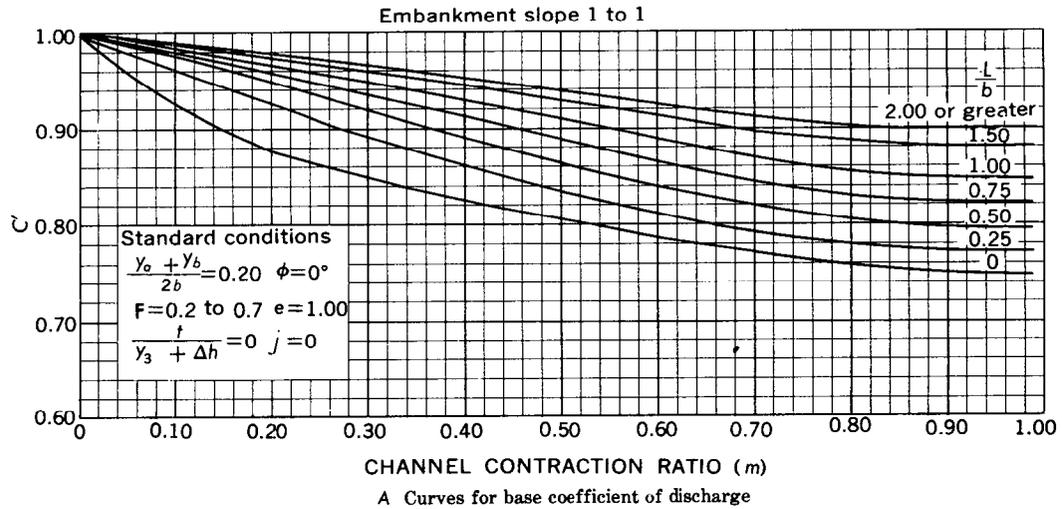
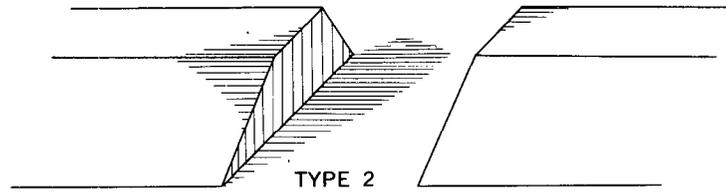


Figure 8.—Coefficients for type 2 opening, embankment slope 1 to 1, vertical abutment.

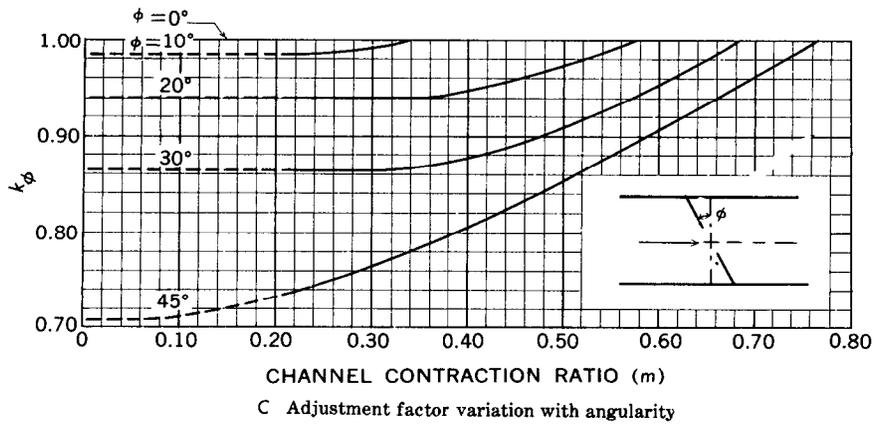
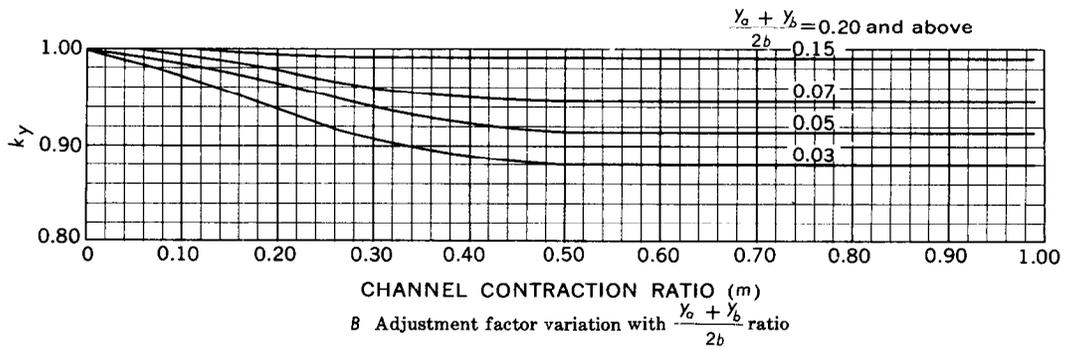
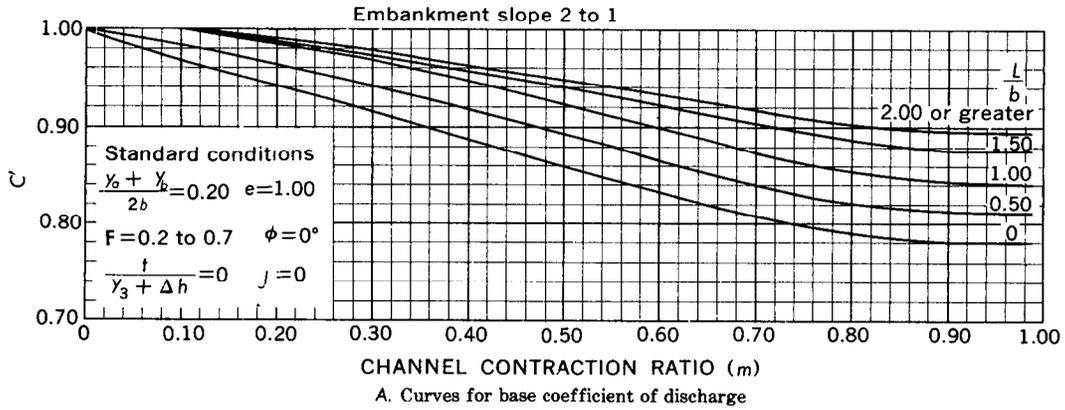
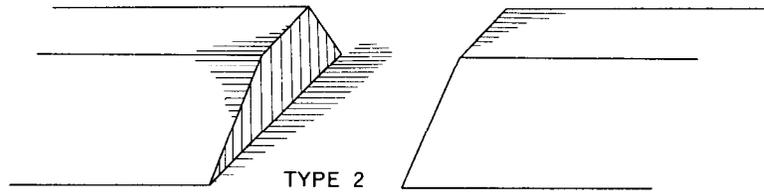


Figure 9.—Coefficients for type 2 opening, embankment slope 2 to 1, vertical abutment.

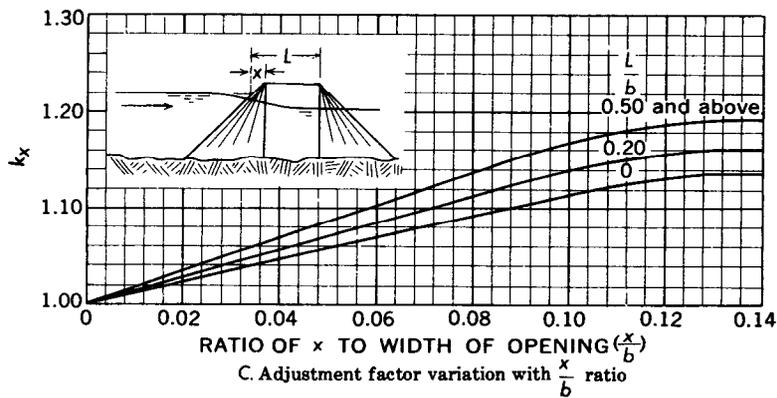
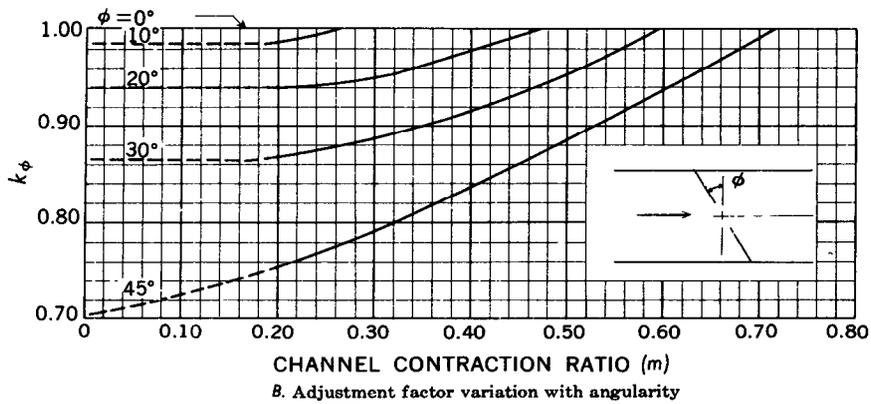
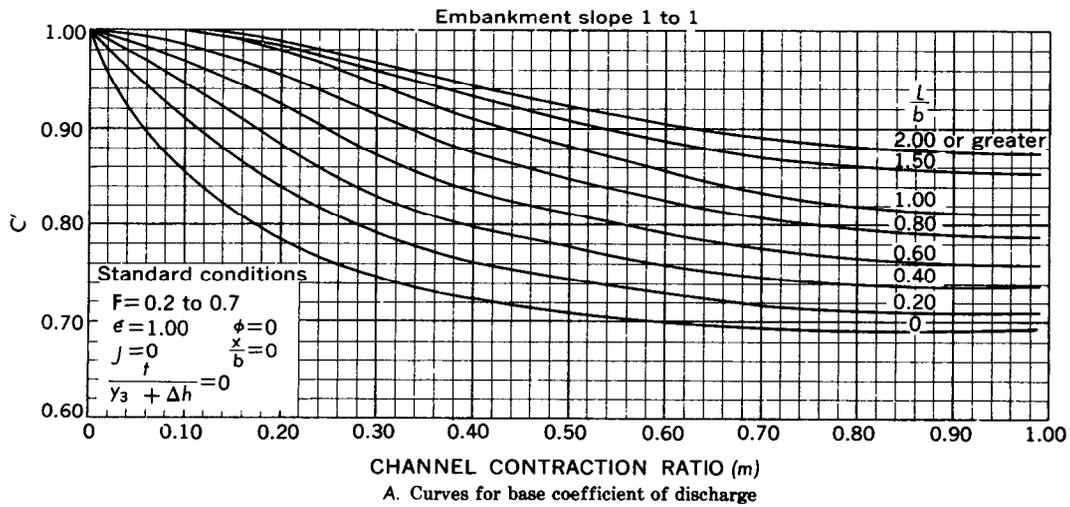
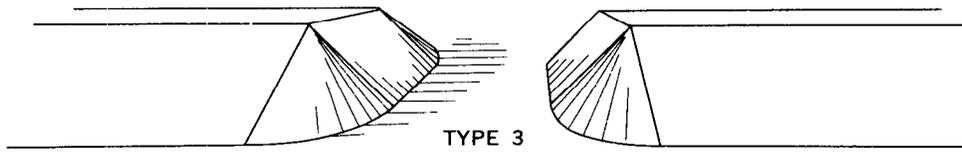


Figure 10.—Coefficients for type 3 opening, embankment and abutment slope 1 to 1.

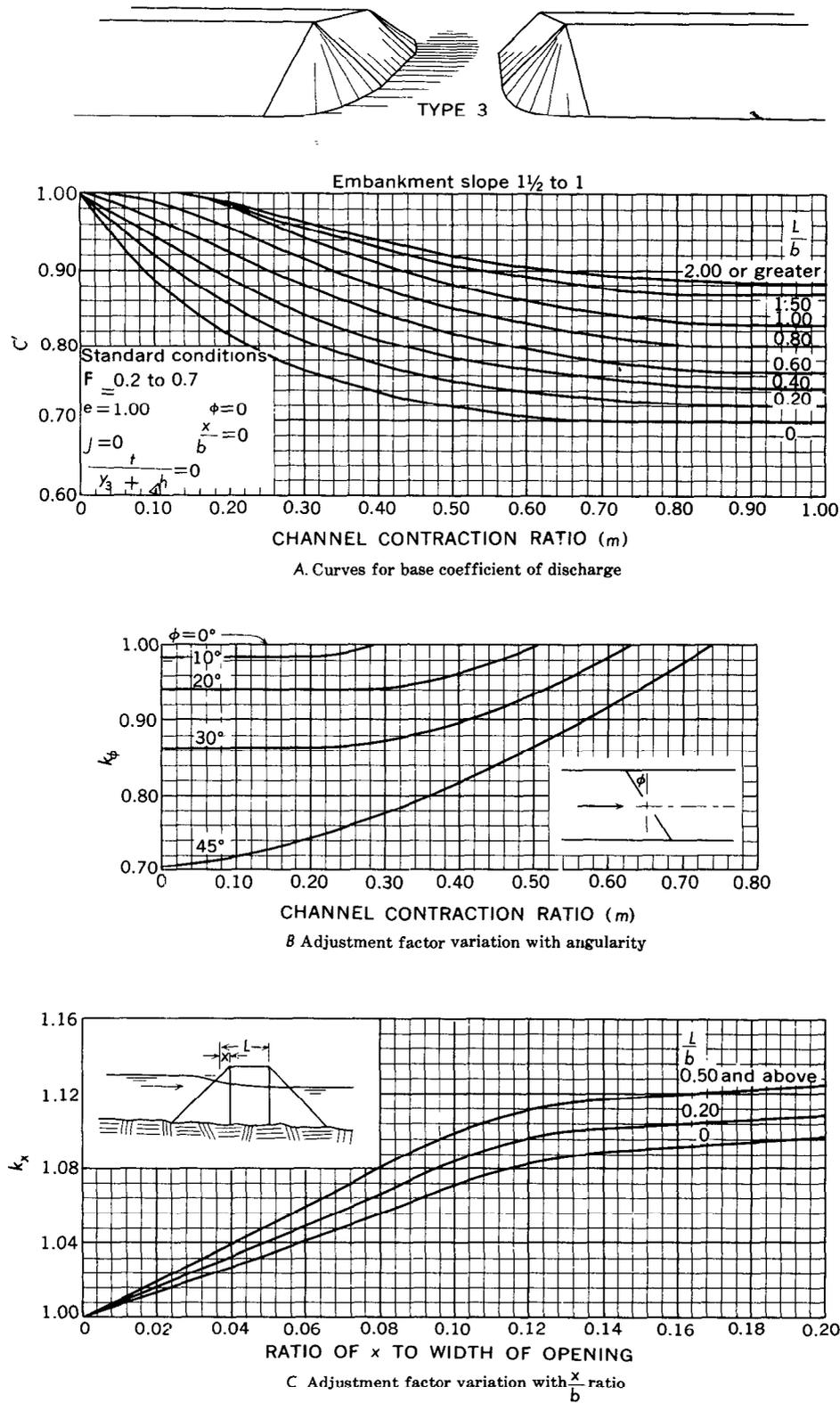
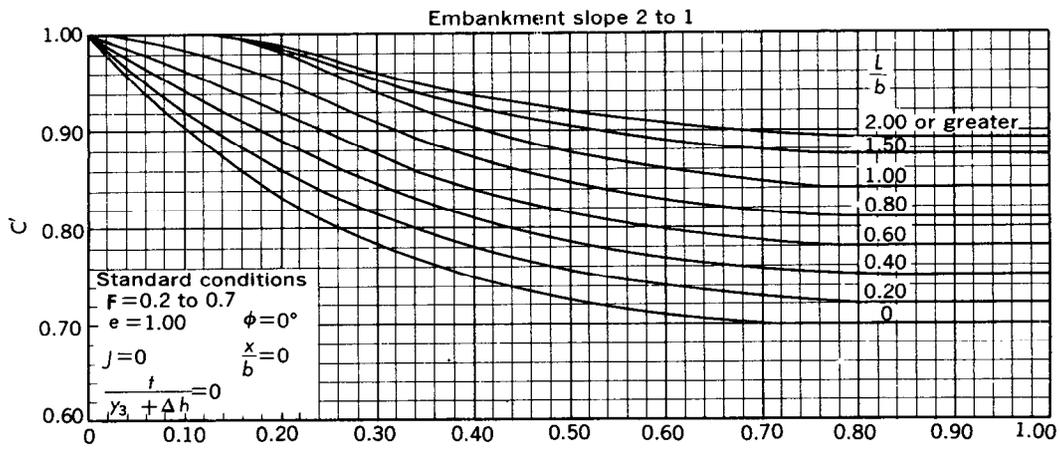
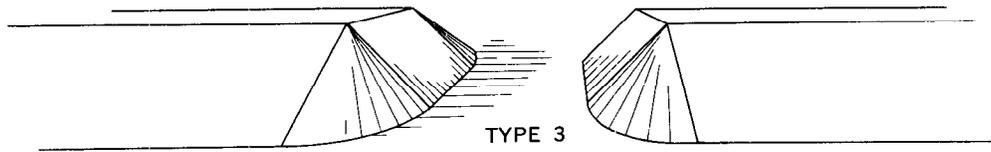
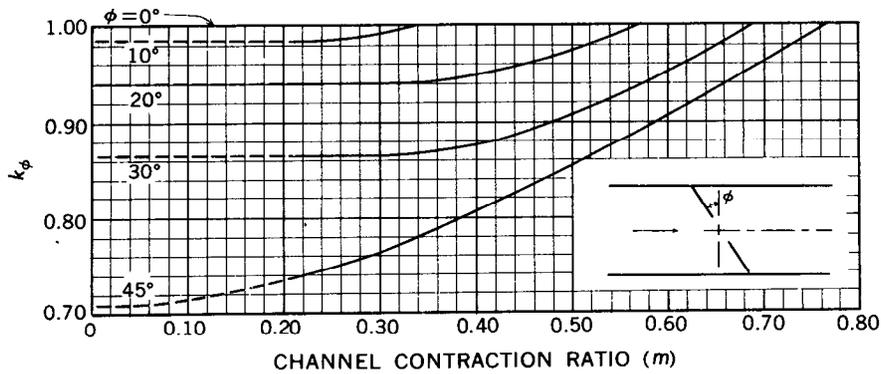


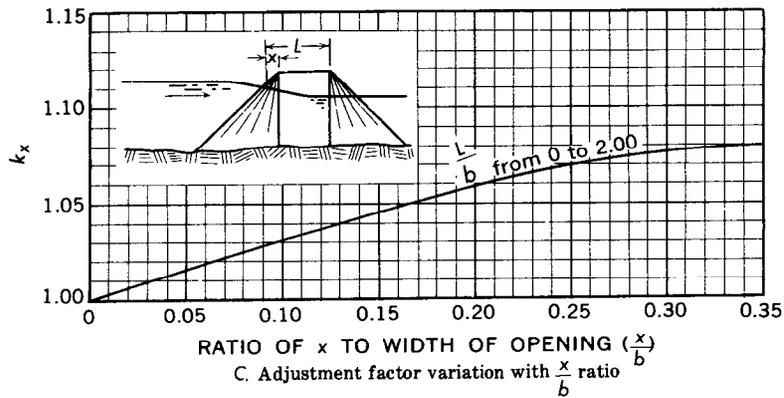
Figure 11.—Coefficients for type 3 opening, embankment and abutment slope $1\frac{1}{2}$ to 1.



A. Curves for base coefficient of discharge

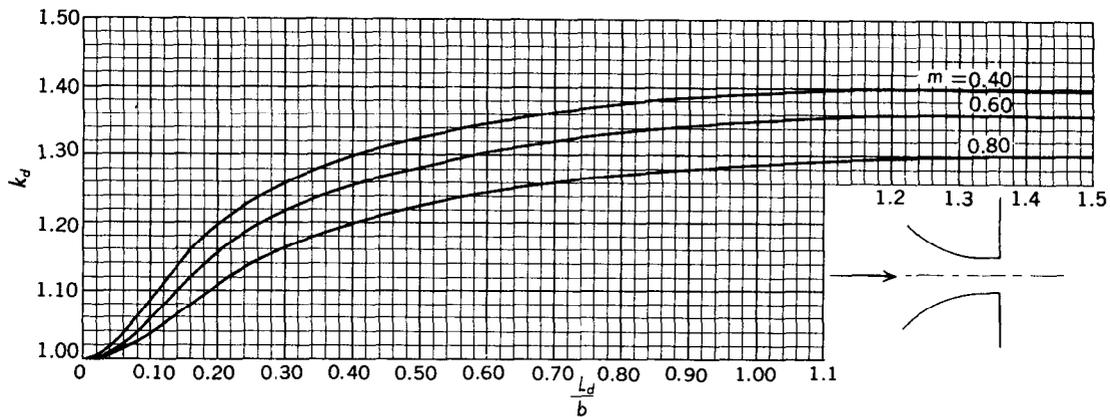


B. Adjustment factor variation with angularity

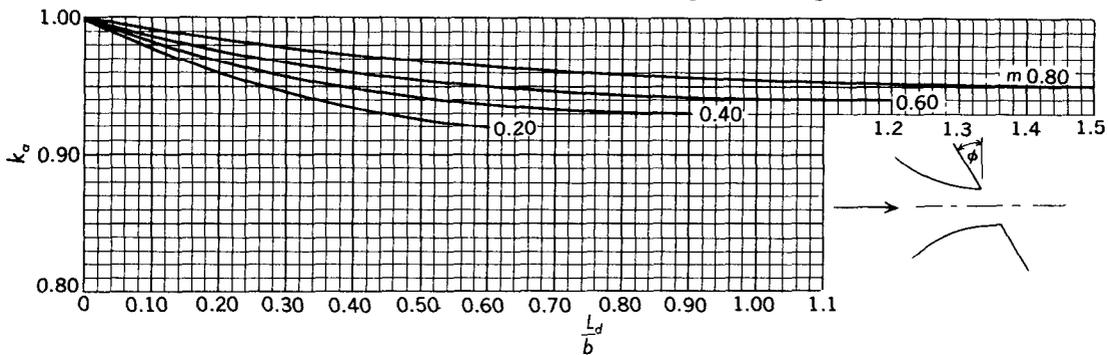


C. Adjustment factor variation with $\frac{x}{b}$ ratio

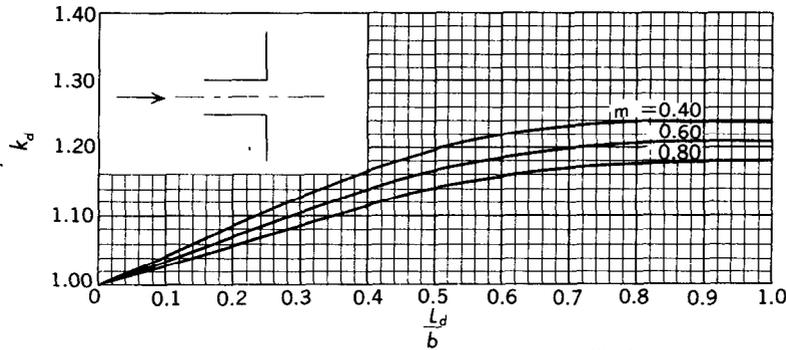
Figure 12.—Coefficients for type 3 opening, embankment and abutment slope 2 to 1.



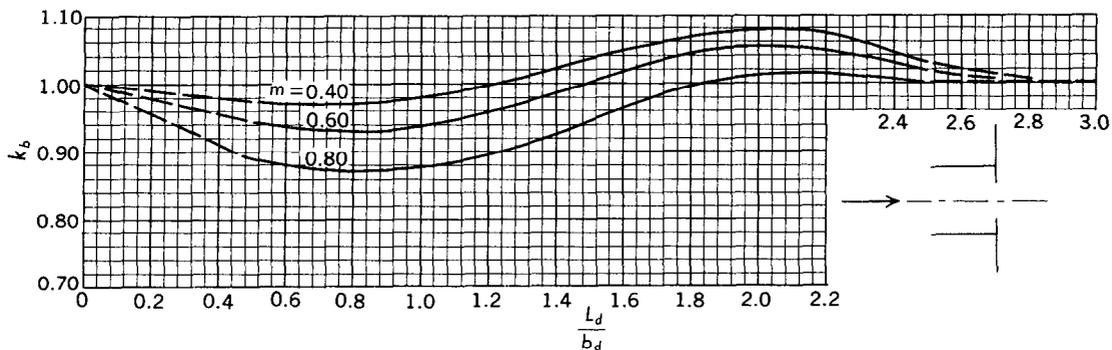
A. Adjustment factor variation with elliptical dike length



B. Adjustment factor variation with elliptical dike angularity



C. Adjustment factor variation with straight dike length



D. Adjustment factor variation with straight dike offset

Figure 13.—Adjustment factor variation for spur dikes.

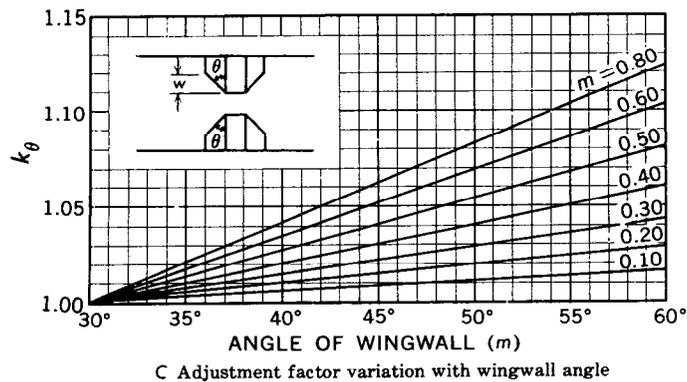
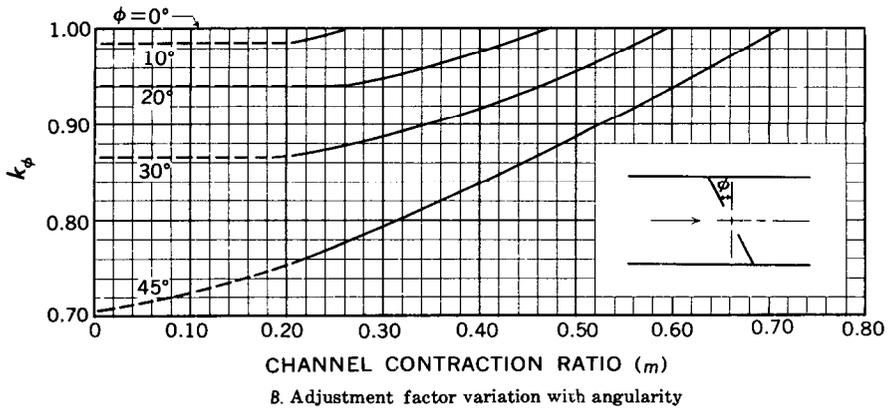
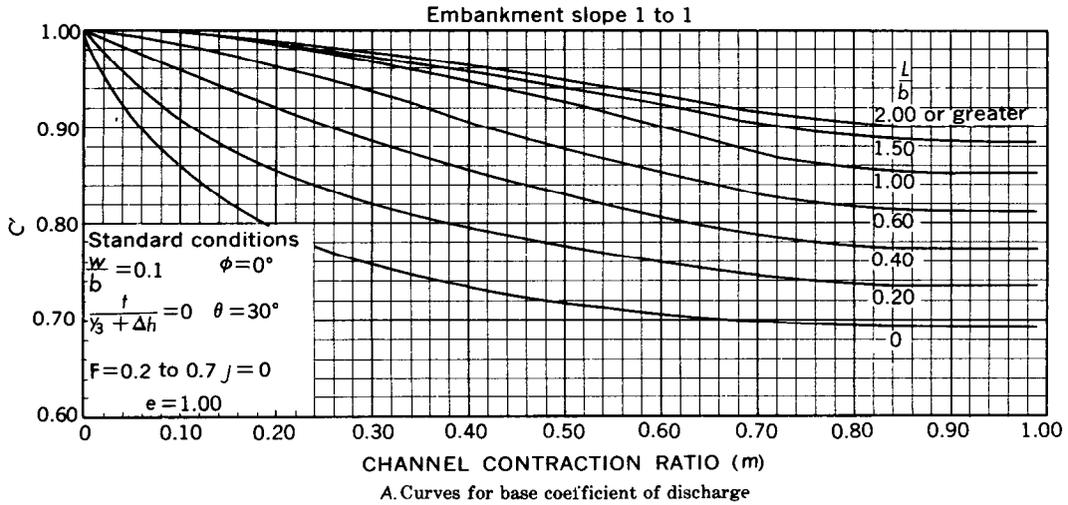
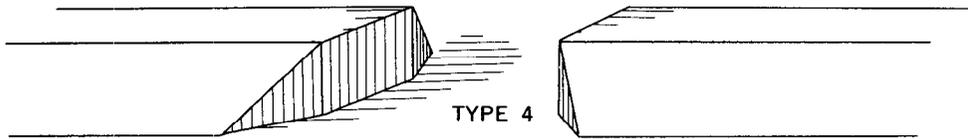


Figure 14.—Coefficients for type 4 opening, embankment slope 1 to 1, vertical abutments with wingwalls.

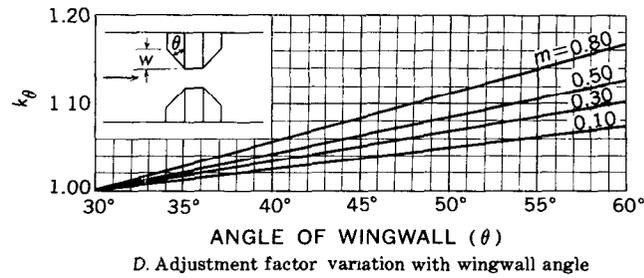
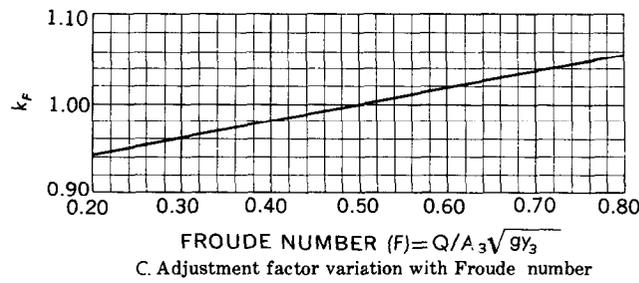
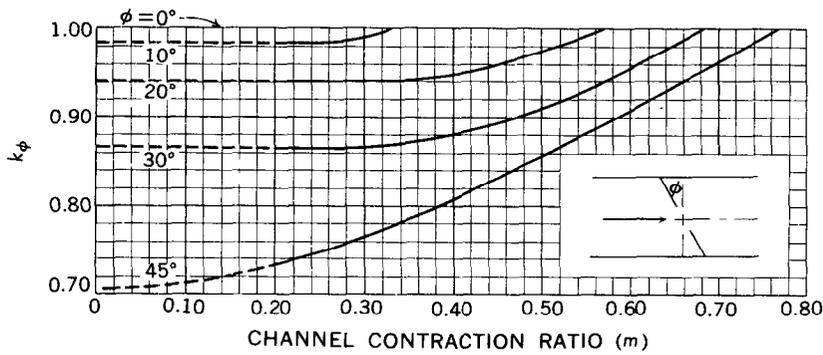
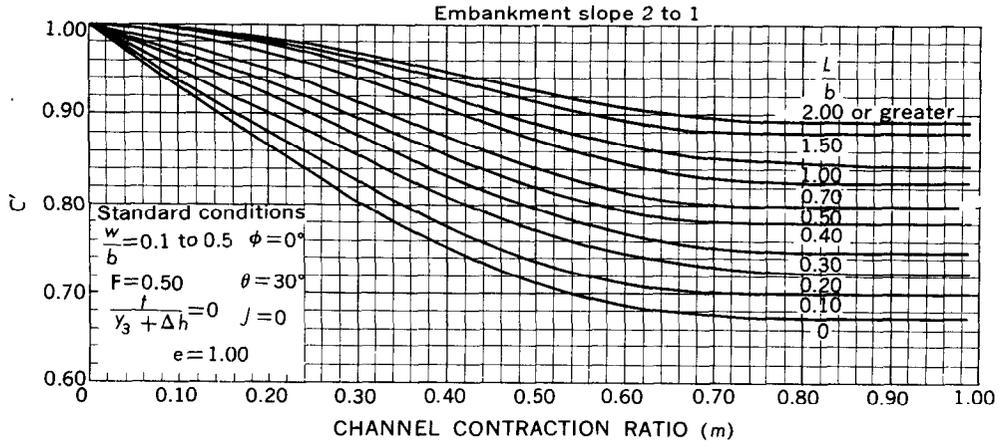
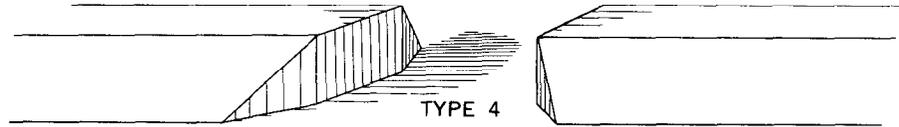


Figure 15.—Coefficients for type 4 opening, embankment slope 2 to 1, vertical abutments with wingwalls.

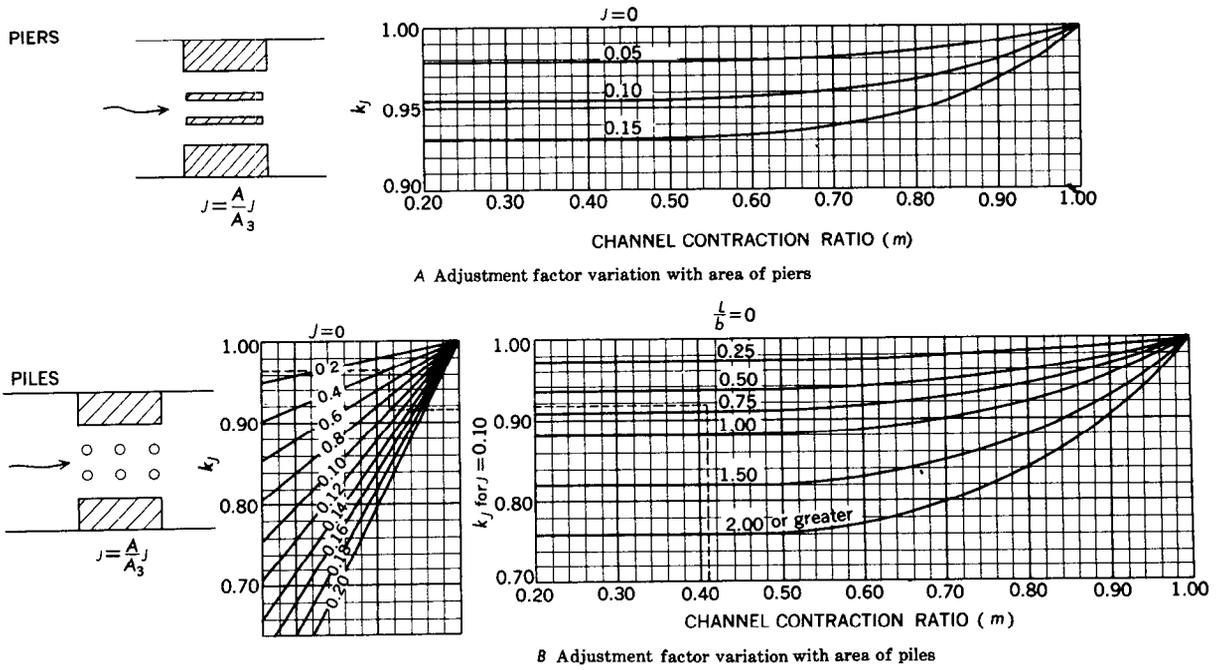


Figure 16.—Adjustment factor variation for piers and piles.

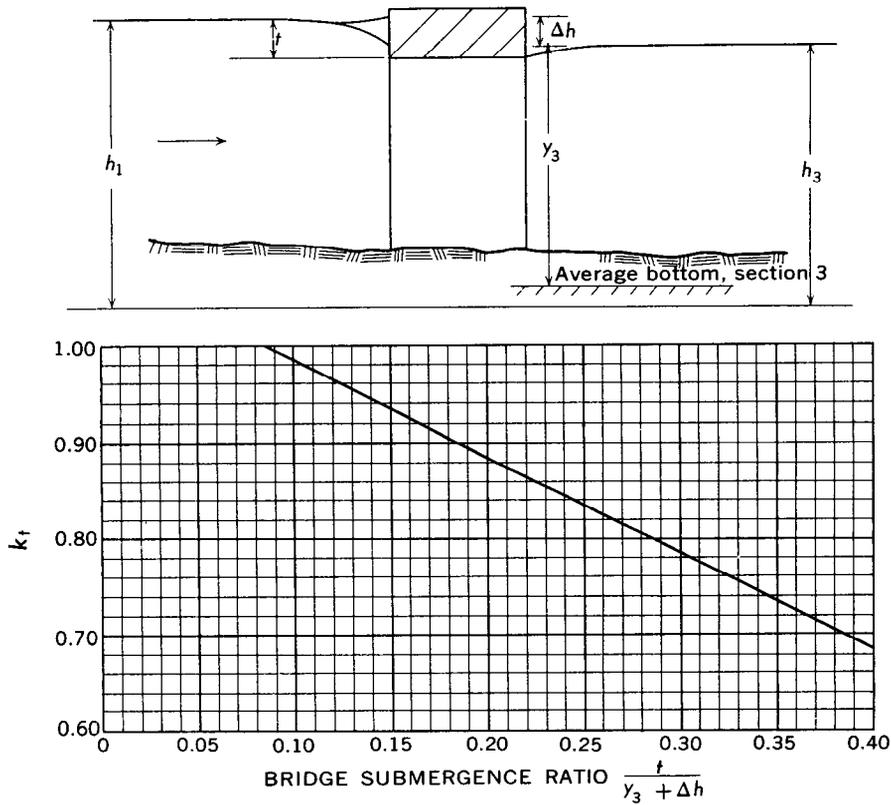


Figure 17.—Adjustment factor variation with bridge submergence ratio.

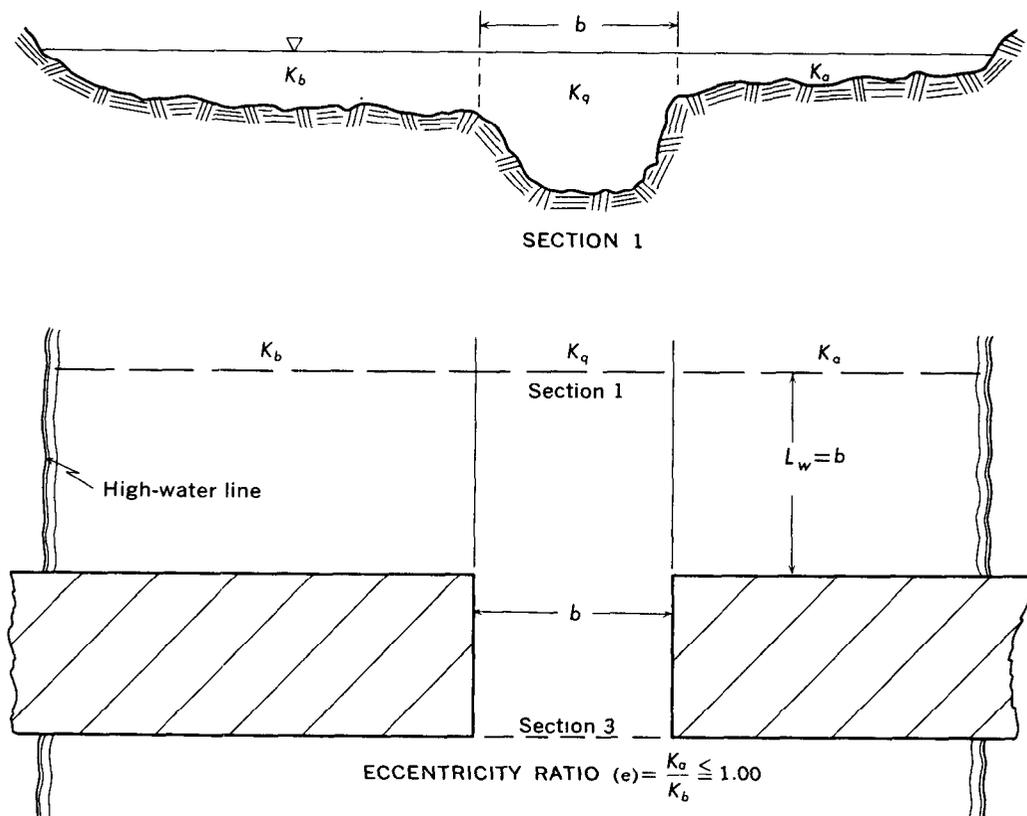


Figure 18.—Definition sketch of an eccentric contraction.

from the contraction, judgment will be required to lay out the b width in the proper position.

4. Subdivide the approach section if abrupt changes in the hydraulic radius occur. Subdivision for abrupt changes in roughness are not made unless they occur on the relative shallow overbank parts (fig. 28).
5. List the properties of section 1 (fig. 29) and compute the areas and wetted perimeters of each part. Compute the total area.
6. Compute the conveyance for each subsection (fig. 32). Sum the partial conveyances to obtain the total conveyance. Compute α_1 .
7. Plot cumulative conveyance and cumulative areas versus stationing from left bank for section 1 (fig. 30). Cumulative conveyance can be computed at each station where a subdivision was made in step 4. Plot a supplemental diagram of the lineal distances between the area curve and the conveyance at end points and subdivision

points. Interpolate between these plotted points. Shape the conveyance curve by (1) obtaining the lineal difference between the curves at as many stations as required, (2) add this difference algebraically to the area curve, and (3) connect the plotted points in a smooth curve to represent the cumulative conveyance.

8. Indicate stationing of ends of the b width, as determined in step 3, on the cumulative conveyance curve (fig. 30). Compute conveyance for the b width, K_q , and the conveyances K_a and K_b to the left and right of the b width. K_a is the smaller of the two and must be zero for fully eccentric conditions.
9. List the properties of section 3 on form 9-191 (fig. 31) and compute areas and wetted perimeters. Subdivide the section at the edge of each pier or pile bent and at abrupt changes in hydraulic radius.
10. Compute the conveyance of each subsection (fig. 32). Use net areas in conveyance

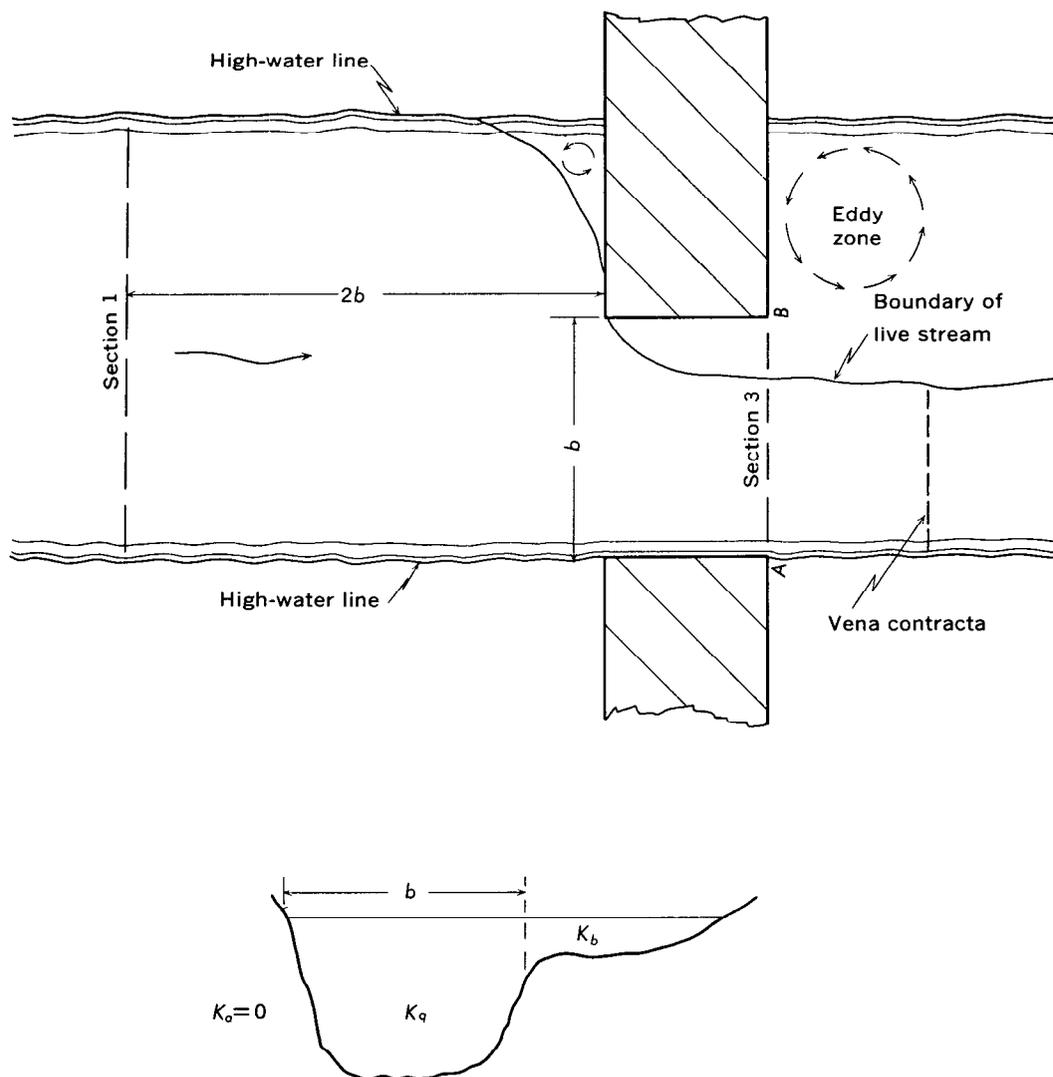
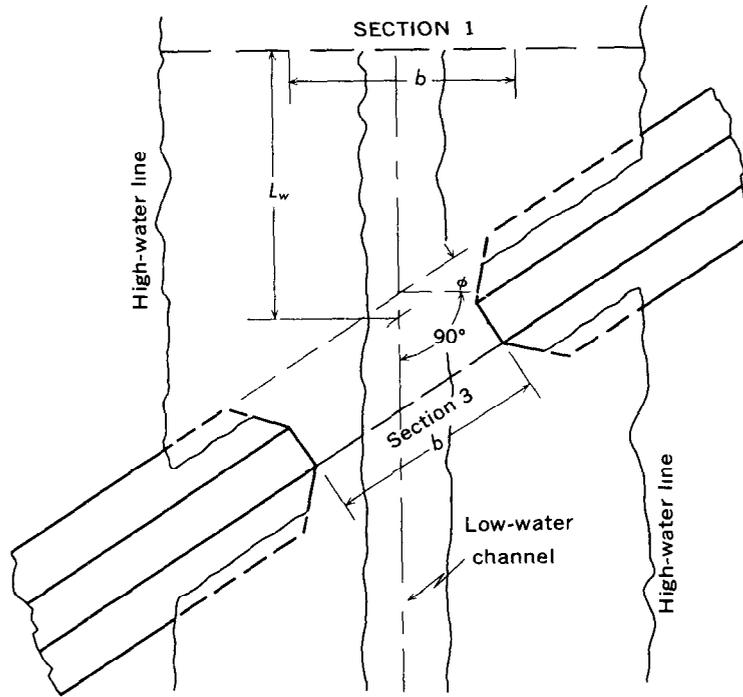


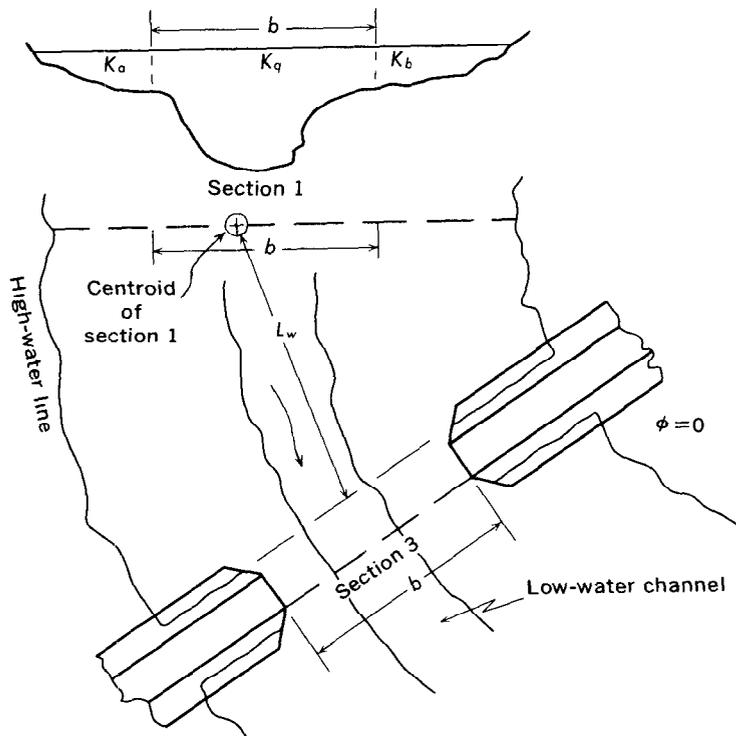
Figure 19.—Definition sketch of a fully eccentric contraction.

- computations. Even if section 3 is subdivided, α_3 is considered to be 1.00.
11. Measure and list the slope of the embankment and the abutments (fig. 32).
 12. Classify the abutments as to type (p. 4). If the abutments are different types, the discharge coefficient should be computed for each abutment as explained on page 13. For compound abutments, use judgment in weighting the relative effects of each.
 13. List the value of the items necessary to compute the basic ratios, m and L/b .
 14. Find the base curve of the discharge coefficient (figs. 6–15) corresponding to the type and slope of the embankments' as determined in steps 11 and 12. Obtain the base coefficient, C' , from this curve. Interpolate between the final values of C' if the slope is between the slopes for which base curves are shown. For flatter or steeper slopes, see method explained under special conditions.
 15. List (fig. 32) the values of the items necessary to compute the ratios shown under "Standard conditions" (figs. 6–15). Compute these ratios.
 16. Determine the adjustment factors, k 's, from the secondary curves. Indicate values



ϕ is a measure of angularity

A Condition of angularity



B Condition of channel curvature

Figure 20.—Definition sketch comparing conditions of angularity and channel curvature.

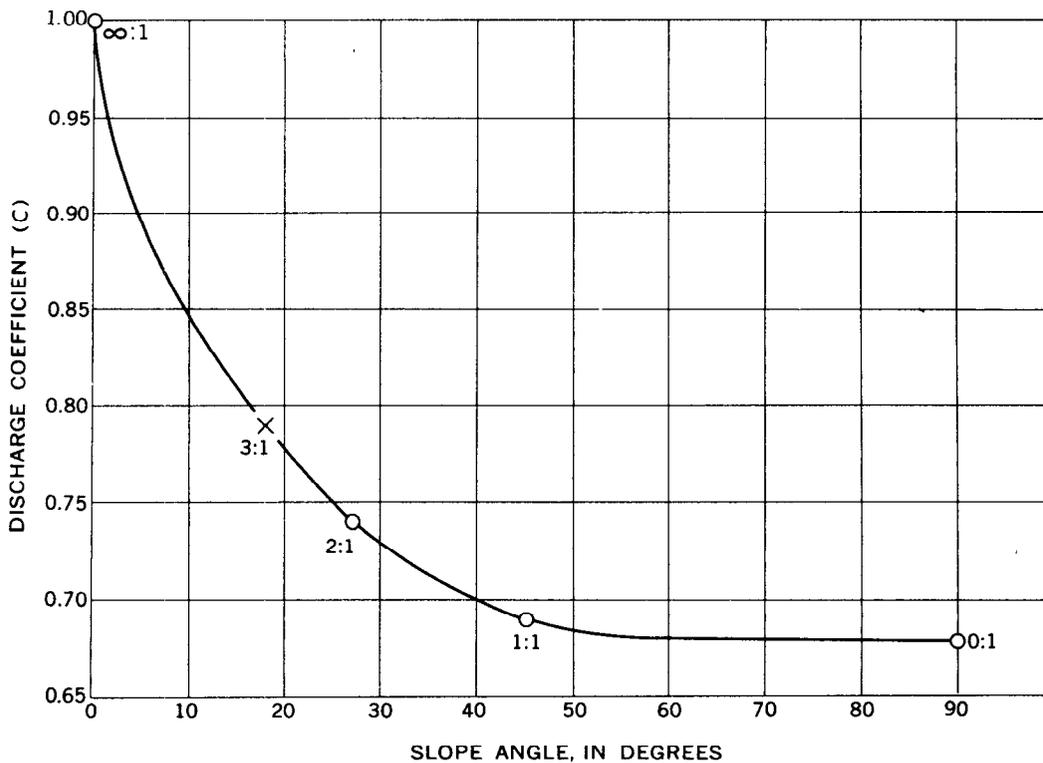


Figure 21.—Method to determine discharge coefficient for embankment slope other than 1 to 1 or 2 to 1.

- | | |
|--------------------------------------------------------------------|--------------------------------------------------------------------|
| 1. Type 1. Slope angle=90°
$C = C' \times k \times k = 0.68$ | 3. Type 2 or 3. 2:1 slope=27°
$C = C' \times k \times k = 0.74$ |
| 2. Type 2 or 3. 1:1 slope=45°
$C = C' \times k \times k = 0.69$ | 4. No embankment. ∞:1 slope=0°
Assume $C = 1.00$ |

Example.—For 3:1 slope, or 18°, C from curve is 0.79.

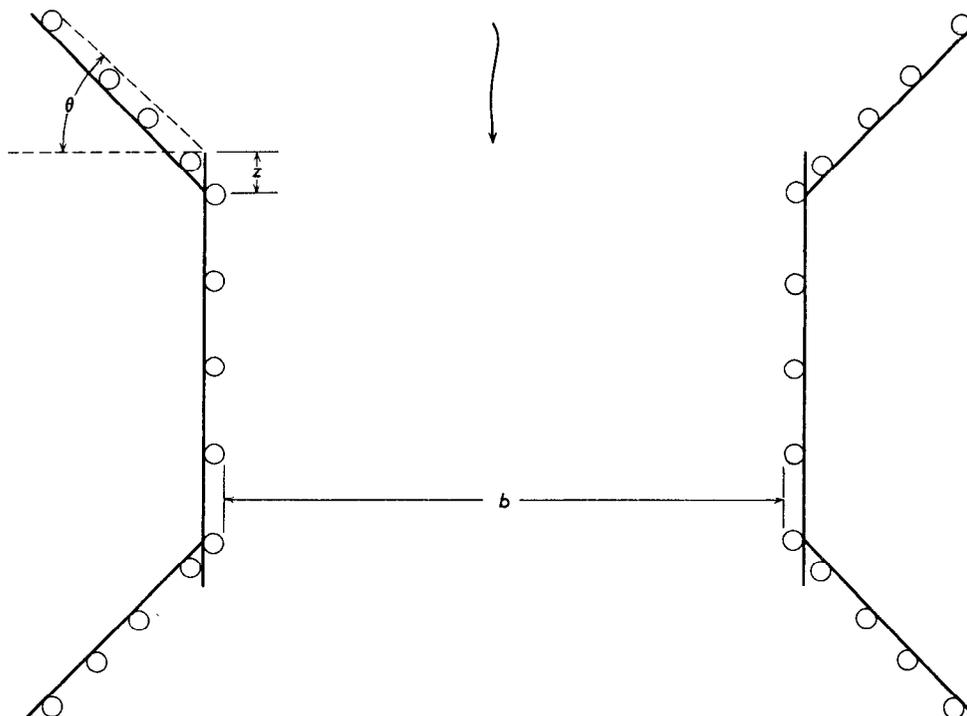


Figure 22.—Definition sketch of timber-and-pile bridge with projecting abutments.

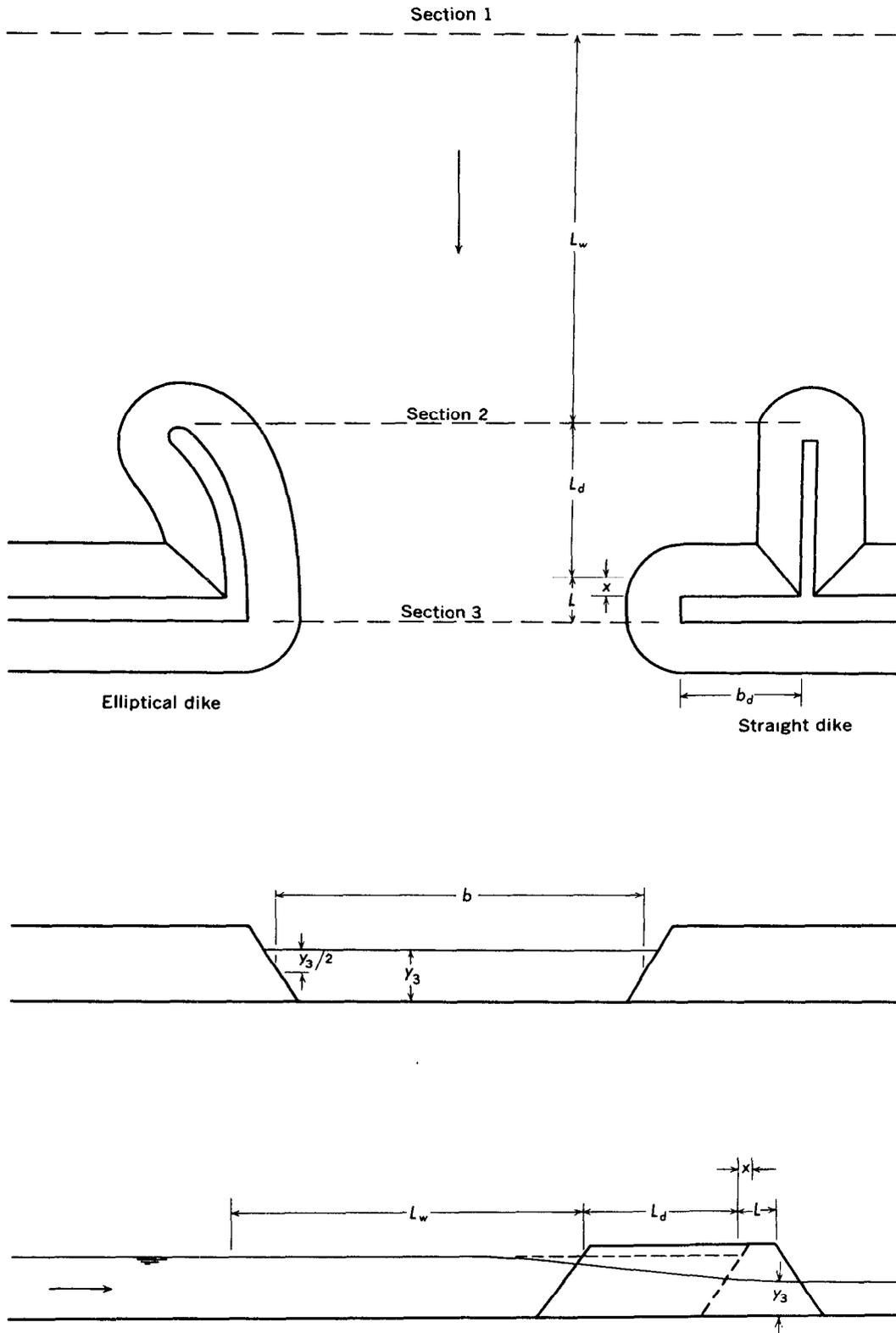


Figure 23.—Definition sketches of spur dikes.

of all factors even though they may be 1.00.

17. Compute C by multiplying C' by the applicable k 's.
18. Compute the discharge by substituting in the discharge formula (fig. 33).
19. Compute the mean velocities in sections 1 and 3. The reasonableness of these velocities is a rough check on the computed discharge. Compute Froude number for final discharge. It should be less than 0.8 for a valid computation by this method. Show data for drainage area, unit discharge, gage height, and discharge (fig. 33).

Example

The various steps in the computation of discharge outlined above are illustrated in the following example:

1. From information obtained at the time of the field survey, the location plan (fig. 24), the high-water profiles (fig. 26), cross sections 1 and 3 (fig. 28), and details of the abutments (fig. 27) were plotted and a list of high-water marks (fig. 25) was made.
2. The high-water elevations at sections 1 and 3 were obtained from the profiles (fig. 26) and entered at the top of the form (fig. 32). Then the difference in elevation, Δh , was computed.
3. The width of the bridge opening, b , was measured as 21 feet. This distance was superimposed upon the plot of section 1 (fig. 28) to satisfy the conditions mentioned in step 3 of the outline.
4. The approach section (fig. 28) was subdivided at stations 40 and 76 for the changes in hydraulic radii and for the change in roughness at station 90.
5. Partial and total areas and wetted perimeters for section 1 were computed (fig. 29). Cumulative areas were also computed.
6. The computations of the conveyance for each subsection, of the total conveyance, and of α_1 were made (fig. 32).
7. The cumulative conveyance was computed from figure 32, at stations 5, 40,

76, 90, and 112, which are the ends of the section and the subdivision points. These values and the cumulative areas were plotted on figure 30.

The lineal distance, not the scale distance, between the cumulative area curve and the cumulative conveyance computed at five stations was measured and plotted at the top of figure 30. These points were connected by straight lines. Lineal distances were determined from this diagram at stations 25, 44.5, 60, 70, and 72, and these distances were offset, in the proper direction, from the cumulative area curve to establish additional definition of the cumulative conveyance curve.

8. The stationing of the ends of the b width was determined in step 3. The cumulative conveyance was determined at these two stations from the cumulative conveyance curve as 2,100 and 8,600. K_a , K_o , and K_b were computed (fig. 30) from these figures and the total conveyance. Note that K_a is the smaller of conveyances for the side sections, and that for fully eccentric conditions, K_a must be zero.
9. The areas and wetted perimeters of section 3 were computed (fig. 31).
10. Conveyance of each subsection of section 3 and the total conveyance were computed (fig. 32). The conveyance is for the opening; therefore, areas of piers, piles, or submerged bridge members are not included in the conveyance computations. The coefficients and adjustment factors were developed using α_3 as 1.00; so it should be considered as 1.00, even if section 3 is subdivided.
11. and 12. The embankment and abutment slopes and the abutment type were recorded (fig. 32).
13. The value of the items shown on figure 32 that are applicable to the bridge geometry were listed and the ratios, m and L/b , were computed. Because the abutments are different types, several items have different values for the left and right banks.

14. The base curve for the left abutment, type 1 and vertical, is in figure 6. The right abutment is type 4 with a 2:1 embankment slope; so the base curve is found in figure 15. The base coefficient, C' , for each abutment was determined from these curves.
15. The six "Standard conditions" for a type 1 opening are listed in figure 6, and the seven for a type 4 opening with 2:1 embankment slopes are listed in figure 15. The necessary items were listed, and the ratios were computed (fig. 32).
To compute the Froude number, F , a discharge must be assumed. If the Froude number for the computed discharge does not change the adjustment factor, k_F , then the computed discharge is correct; otherwise successive discharges must be assumed until there is no change in k_F .
16. The six adjustment factors for the left abutment were determined from the secondary curves of figures 6 and 7 and from figures 16 and 17. The seven adjustment factors for the right abutment were determined from figures 15-17. All factors were listed near the bottom of form 9-193A (fig. 32).
17. The discharge coefficient, C , for each abutment was computed by multiplying the base coefficient, C' , by each adjustment factor. The coefficient for the bridge was computed by weighting the coefficients for each abutment with respect to the conveyances (p. 13).
18. The discharge was computed by substituting in the discharge formula (fig. 33).
19. The mean velocities in sections 1 and 3 were computed and entered on figure 33 as a rough check on the reasonableness of the computed discharge.

Combination Sites

Floods often cause flow over the road near the bridge in addition to the flow through the bridge, and approach section properties and most ratios must be computed differently. The following procedure is suggested:

1. Compute flow over the road, using the entire area of section 1 to calculate the velocity of approach.
2. Estimate the total discharge and divide section 1 so that the total conveyance is divided in proportion to the discharges through the bridge and over the road.
3. Use just that part of section 1 supplying flow to the bridge to compute A_1 , L_w , K_a , K_o , K_b , α_1 , m , and e .
4. Discharge through the bridge plus that over the road should check estimated discharge within 1 percent. If it does not, make new estimates until check is obtained.

When the flow over the road occurs on both sides of the bridge, divide the approach section into three parts.

Multiple-Opening Contractions

A multiple-opening contraction is defined as a series of independent single-opening contractions, all of which freely conduct water from a common approach channel. Independence of the openings is generally indicated when two or more pairs of abutments and one or more interior embankments exist. Structures in which piers or webs separate several openings between two abutments are considered single-opening contractions.

To determine the discharge, establish in the approach channel pseudochannel boundaries which divide the flow between openings. This procedure defines a separate approach channel for each individual opening. Take section 1 at a distance one opening width upstream from the embankment in the approach channel defined for each opening. Use the water-surface elevation at that point for h_1 . As shown in figure 34, the approach sections to the various openings will not be located on a continuous line across the valley unless the width of all openings is the same. Compute the discharge through each opening, using identical procedures and discharge coefficients given previously for single-opening contractions.

Division into single-opening units

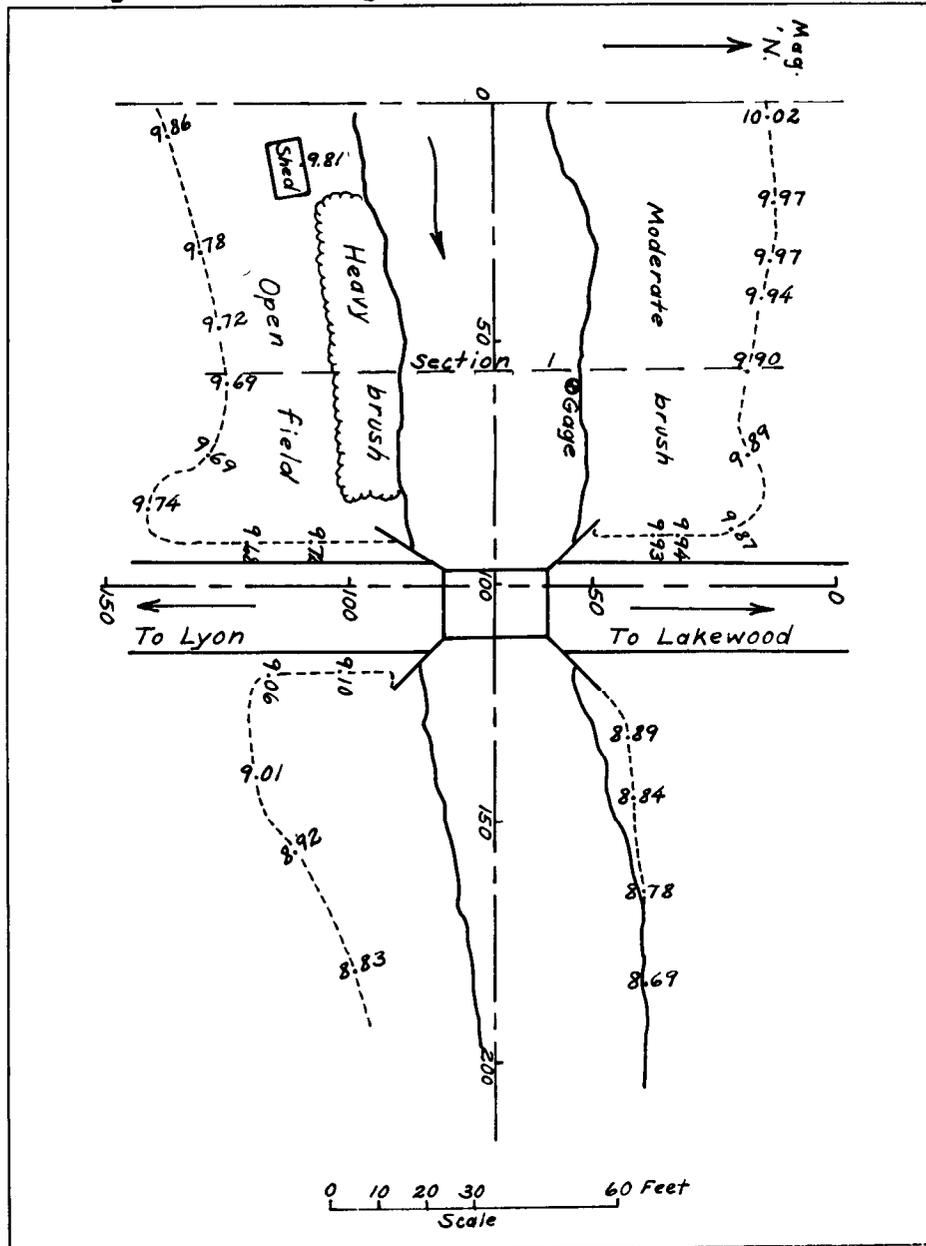
Locate the upstream flow boundaries first by apportioning the length of each embankment

9-910-C
(March 1949)
Sketch Form

UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

File

Roaring River near Lyon, Colo. - Flood of July 3, 1961



Sheet No. 1 of 10 Sheets. Prepared by H. S. C. Date 7-7-61. Checked by E. R. H. Date 7-10-61
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Figure 24.—Sample contracted-opening computation, location plan.

9-220

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

File No. { Washington _____
District _____

Roaring River near Lyon, Colo. Flood of 7-3-61

		High-water marks			
		Left bank		Right bank	
Sta.	Elev.			Sta.	Elev.
3	10.02			7	9.86
21	9.97			14	9.81 in. shed
33	9.97			31	9.78
41	9.94			48	9.72
55	(9.91)	Sec 1		56	(9.70) Sec 1
56	9.90			58	9.69
73	9.89			73	9.69
88	9.87			84	9.74
89	9.94			91	9.68
90	9.93			91	9.72
87	-	Upstr. wingwalls		87	- Upstr. wingwall
97	-	Upstr. bridge		97	- Upstr. bridge
111	(8.91)	Sec 3		111	(9.08) Sec 3
132	8.89			118	9.10
144	8.84			119	9.06
164	8.78			141	9.01
183	8.69			156	8.92
				181	8.83
Transverse profile					
Upstream			Downstream		
L.B.	17	9.89	L.B.	40	8.78
	21	9.87		41	8.84
	32	9.94		43	8.89
	37	9.93		59	- L. abut.
	49	- End L. wingwall		80	- R. abut.
	97	- End R. wingwall		101	9.10
	107	9.72		116	9.06
	121	9.68	R.B.	119	9.01
R.B.	141	9.74			

Comp. D.E.F.

checked G.H.T.

2 of 10

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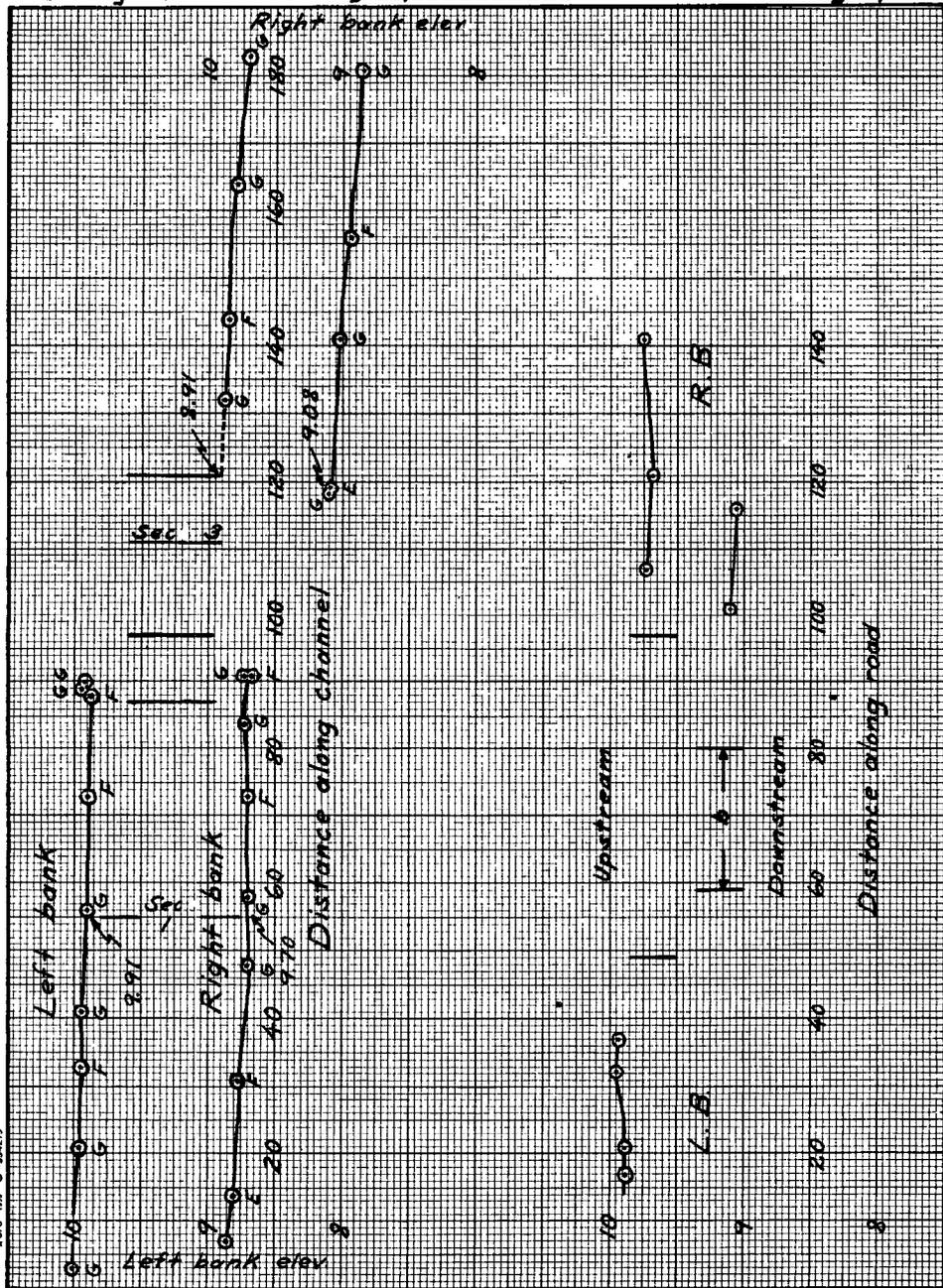
Figure 25.—Sample contracted-opening computation, list of high-water marks.

9-918-E
(March 1949)
Cross-Section

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GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

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Roaring River near Lyon, Colo. Flood of July 3, 1961



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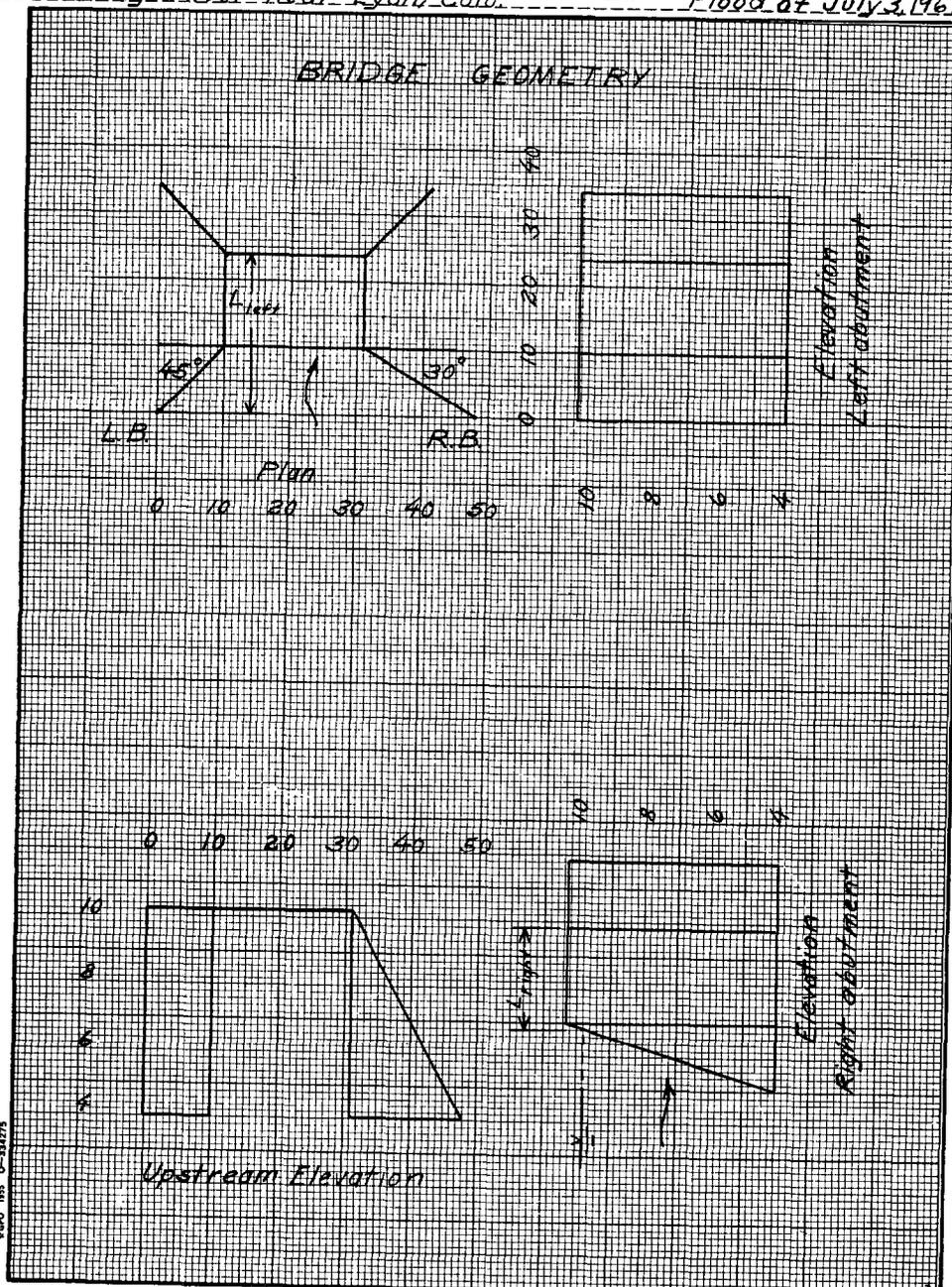
Figure 26.—Sample contracted-opening computation, high-water profiles.

9-913-E
(March 1949)
Cross-Section

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WATER RESOURCES DIVISION

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Roaring River near Lyon, Cal. — *Flood of July 3, 1961*



GPO 1955 O-34275

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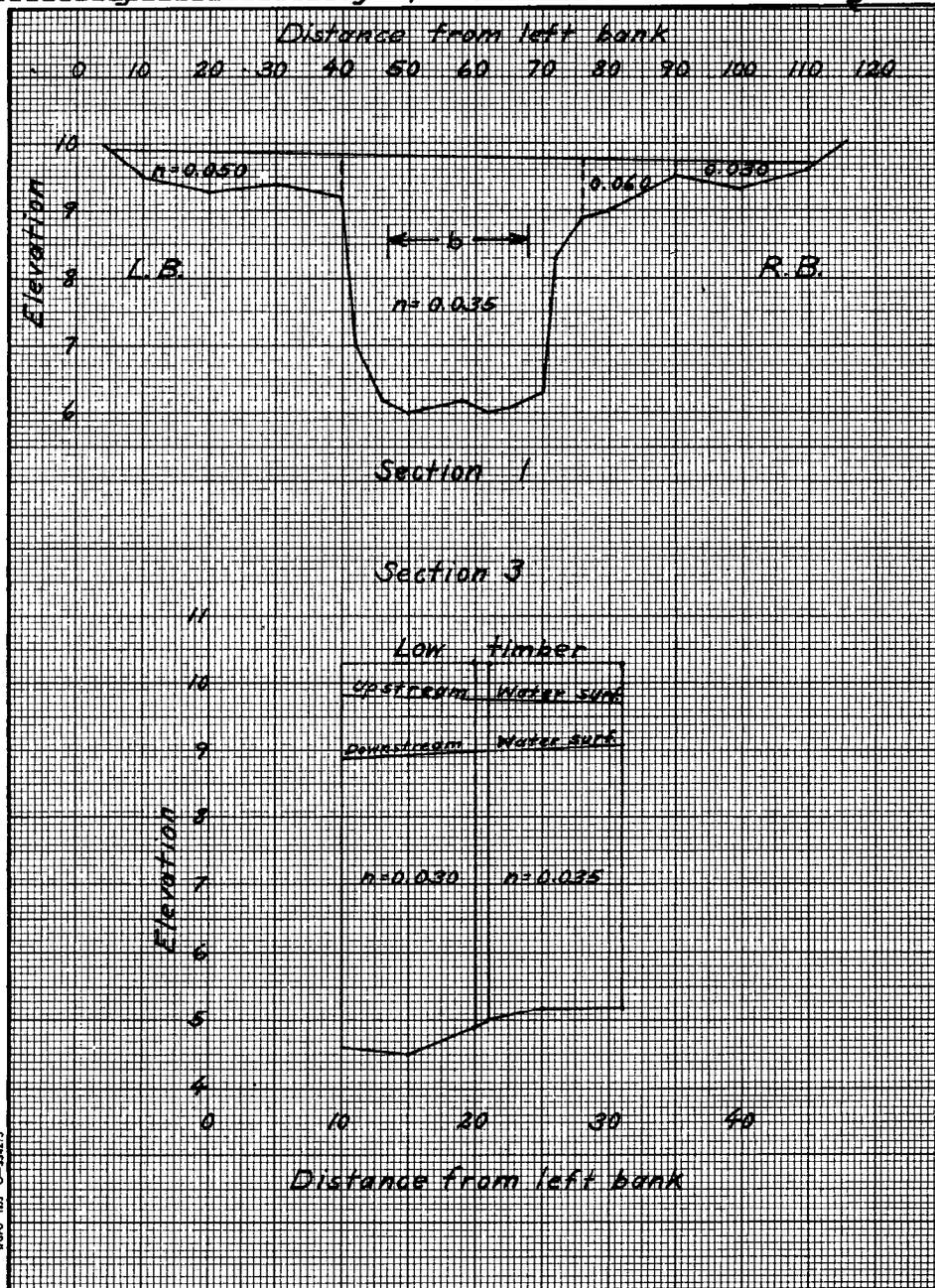
Figure 27.—Sample contracted-opening computation, bridge geometry.

9-913-E
(March 1949)
Cross-Section

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File

--- Rating River near Lyon, Colo. --- Flood of July 3, 1961



XERO 185 G-334275

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Figure 28.—Sample contracted-opening computation, cross sections.

9-101
(July 1952)
Cross-section
properties

UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

File

Roaring River near Lyon, Colo. FLOOD OF July 3, 1961

SECTION - 1 (Approach section)										
STA.	DIST.	W S. ELEV.	ELEV.	DEPTH	MEAN DEPTH	AREA	Cumul. Area	W. P.		
L. B. 4			10.0							
(5)	-	9.9	(9.9)	0	-	-				
10	5		9.5	.4	0.2	1.0	1.0	5.0		A=17.0
20	10		9.3	.6	.5	5.0	6.0	10.0		WP=35.0
30	10	9.9	9.4	.5	.55	5.5	11.5	10.0		
40	10	9.8	9.2	.6	.55	5.5	17.0	10.0		
42	2		7.0	2.8	1.7	3.4	20.4	3.0		
46	4		6.2	3.6	3.2	12.8	33.2	4.1		
50			6.0	3.8	3.7	14.8	48.0	4.0		
54			6.1	3.7	3.75	15.0	63.0	4.0		A=131.6-17.0
58			6.2	3.6	3.65	14.6	77.6	4.0		=114.6
62			6.0	3.8	3.7	14.8	92.4	4.0		WP=37.9
66			6.1	3.7	3.75	15.0	107.4	4.0		
70	4		6.3	3.5	3.6	14.4	121.8	4.0		
72	2		8.3	1.5	2.5	5.0	126.8	2.8		
76	4		8.9	.9	1.2	4.8	131.6	4.0		
80	4	9.8	9.0	.8	.85	3.4	135.0	4.0		A=140.5-131.6=8.9
90	10	9.7	9.5	.3	.55	5.5	140.5	10.0		WP=14.0
100	10		9.3	.5	.4	4.0	144.5	10.0		A=148.2-140.5
110	10		9.6	.2	.35	3.5	148.0	10.0		= 7.7
(112)	2	9.7	(9.7)	0	.1	.2	148.2	2.0		WP=22.0
116			10.0							
107	107					148.2				

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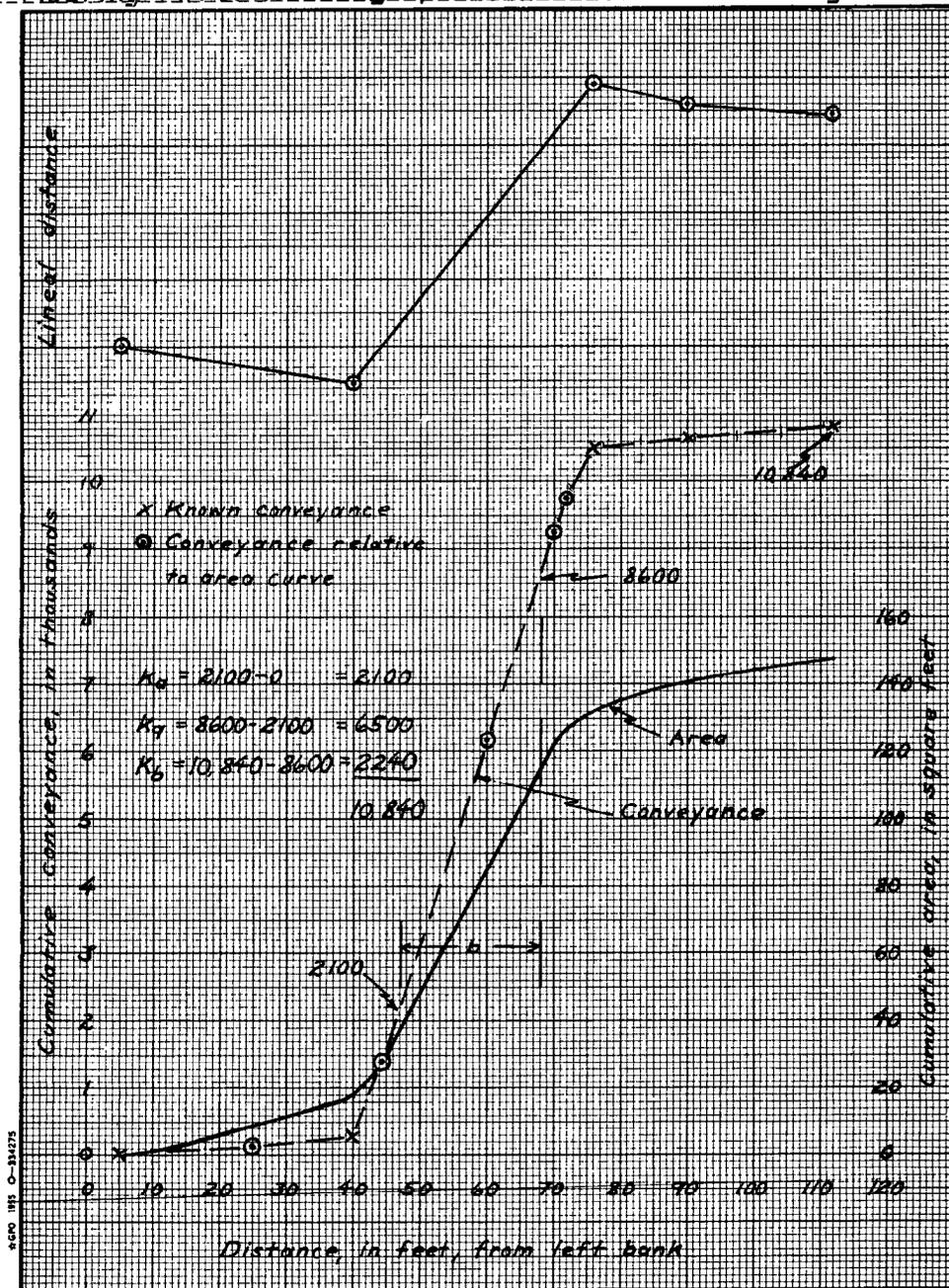
Figure 29.—Sample contracted-opening computation, cross-sectional properties of section 1.

9-213-E
(March 1949)
Cross-Section

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File

Roaring River near Lyon, Colo. Flood of July 3, 1961



Sheet No. 7 of 10 Sheets. Prepared by H. S. C. Date 7-12-61 Checked by E. R. H. Date 7-13-61

Figure 30.—Sample contracted-opening computation, cumulative conveyance curve.

9-191
(July 1952)
Cross-section
properties

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GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

File

Roaring River near Lyon, Colo. FLOOD OF July 3, 1961

SECTION - 3 (Contracted section)									
STA.	DIST.	W. S. ELEV.	ELEV.	DEPTH	MEAN DEPTH	AREA		W. P.	
L. B.									
10	—	8.91	8.9	0	—	—		—	
10	0	8.9	4.6	4.3	—	—		4.3	
15	5	9.0	4.5	4.5	4.4	22.0		5.0	A=43.5
20	5		4.9	4.1	4.3	21.5		5.0	WP=18.4
20	0		9.0	0	—	—		4.1	
Piles	1		—	—	—	—	Pile area	4.0	
21	—		9.0	0	—	—		—	
21	0		5.0	4.0	—	—		4.0	
25	4	9.0	5.2	3.8	3.9	15.6		4.0	A=38.7
31	6	9.1	5.2	3.9	3.85	23.1		6.0	WP=17.9
31	0	9.08	9.1	0	—	—		3.9	
						82.2	net area		
						4.0	pile area		
						86.2	gross area = A ₃		

Sheet No. 2 of 10 sheets. Prepared by H. S. G. Date 7-10-61 Checked by E. R. H. Date 7-12-61

U. S. GOVERNMENT PRINTING OFFICE 16-60957-1

Figure 31.—Sample contracted-opening computation, cross-sectional properties of section 3.

9-193 A
Contracted-opening
coefficients.

UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

File _____

Computation of coefficients for Roaring River near Lyon, Colo.
for flood of July 3, 1961

Water-surface elevation at approach section	Left	Right	Average	CHARACTERISTICS OF CONSTRICTION Embankment and abutment slope <u>L-vert. R-2:1</u> Type of abutment <u>Left-1 Rt.-4</u>
Water-surface elevation at contracted section	9.91	9.70	9.805	
Fall in water surface, $\Delta h =$	0.81			

APPROACH SECTION		SECTION 1								
Subsection	n	$\frac{1.486}{n}$	a	w.p.	r	$r^{2/3}$	$K = \frac{1.486}{n} a r^{2/3}$	K^3/a^2	a_1	
5-40	0.050	29.7	17.0	35.0	0.49	0.62	310	10		
40-76	.035	42.5	114.6	37.9	3.02	2.09	10,180	8030		
76-90	.060	24.8	8.9	14.0	.64	.74	160	10		
90-112	.030	49.5	7.7	22.0	.35	.50	190	10		
Total			148.2				$K_1 = 10,840$	5800	1.39	

CONTRACTED SECTION		SECTION 3							$[a_1 = \Sigma (K^3/a^2) - K_{total}^3/A_{total}^2]$
10-20	0.030	49.5	43.5	18.4	2.36	1.77		3,810	
21-31	.035	42.5	38.7	17.9	2.16	1.67		2,750	
Total			822 (net)				$K_3 =$	6,560	

Use net areas for contracted subareas

ITEMS AND RATIOS

Δh	0.81	t	0	Ratios	.60
b	21	A_3	86.2	$m = 1 - (K_a/K_1) = 1 - \frac{6500}{10840} = 0.40$	
L	left 24; rt. 15	y_3	$86.2/21 = 4.10$	L/b left 1.14; rt. 0.71	
L_w	left 32; rt. 40	A_1	4.0	t/b	
r	—	ϕ	0°	W/b left 0.48; rt. 0.81	
W	left 10; rt. 17	θ	left 45°; rt. 30°	x/b	
x	—	K_a	2100	$(y_a + y_b)/2b$	
y_a	—	K_b	2240	$t/(y_3 + \Delta h)$	0
y_b	—	K_a	6500	$F = y_3/\sqrt{W y_3} = \frac{4.10}{\sqrt{21 \times 4.10}} = 0.58$	
		K_1		$j = A_1/A_3 = \frac{4.0}{86.2} = 0.046$	
				$e = K_a/K_b = \frac{2100}{2240} = 0.94$	

$$C = \frac{C'}{C} \times \frac{K_w}{K} \times \frac{K_e}{K} \times \frac{K_j}{K} \times \frac{K_p}{K} \times \frac{K_t}{K} \times \frac{K_o}{K} \times \frac{K_F}{K} = 0.96$$

$$C = \frac{.91}{.91} \times 1.00 \times 1.00 \times .96 \times 1.00 \times 1.00 \times 1.00 \times 1.02 = .89$$

$$C = \frac{C_L K_a + C_R K_b}{K_a + K_b} = \frac{0.96 \times 2100 + 0.89 \times 2240}{2100 + 2240} = 0.92$$

Sheet No. 9 of 10 Sheets. Prepared by H. S. C. Date 7-13-61 Checked by E. R. H. Date 7-13-61

Figure 32.—Sample contracted-opening computation, discharge coefficients.

9-193 B
Contracted-opening
measurement

UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

File -----

Meas No. -----

Contracted opening measurement of Roaring River near Lyon, Colo.
for flood of July 3, 1961

Drainage area <u>7.39</u> square miles	Gage height <u>5.42</u> feet
Unit discharge <u>77.8</u> cfs/sq mi	Discharge <u>575</u> cfs

COMPUTATION OF DISCHARGE

$$Q = 8.02 C A_3 \sqrt{\frac{\Delta h}{1 - a_1 C^2 (A_3/A_1)^2 + 2gC^2 (A_3/K_3)^2 (L + L_w K_3/K_1)}}$$

$$= \frac{8.02 (0.92) (86.2) \sqrt{0.81}}{\sqrt{1 - 1.39 (0.92)^2 \left(\frac{86.2}{148.2}\right)^2 + 64.3 (0.92)^2 \left(\frac{86.2}{6560}\right)^2 (19.5 + \frac{36(6560)}{10840})}}$$

.846 (.470) .846 .0111 41.3
 .398 .458
 .388

$$= \frac{0.92 \times 622}{\sqrt{0.99}} = \frac{572}{.995} = 575 \text{ cfs}$$

Assumed $Q = 575 \text{ cfs}$ K_F factors OK

When $L_w > 1.25 b$
 $(L + L_w K_3/K_1) =$
 $(L + (K_3/K_1) b + (K_3/K_1)^2 (L_w - b))$

$V_1 = \underline{3.88}$
 $V_3 = Q/A_3(\text{gross area}) = \underline{6.67}$
 $F = V_3/\sqrt{gV_3} = \underline{0.58} \leq 0.8$

Remarks _____

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Figure 33.—Sample contracted-opening computation, discharge.

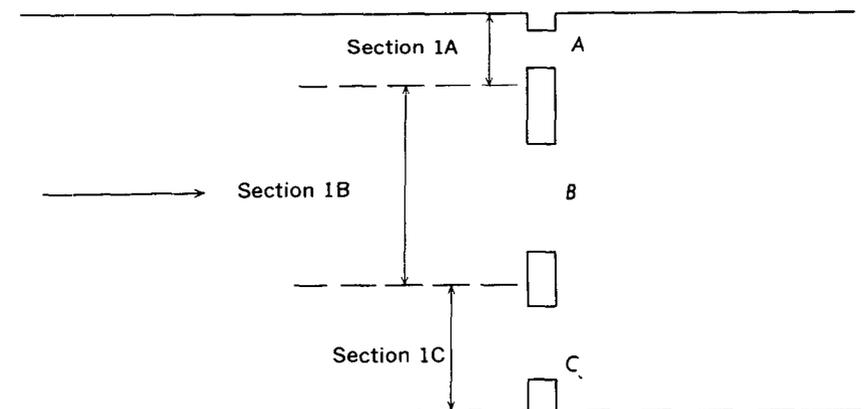


Figure 34.—Location of approach section and flow division lines for a multiple-opening constriction.

between openings in direct proportion to the gross areas, A_3 , of the openings on either side, the larger length of embankment being assigned to the larger opening. Then, from the points on the embankment thus determined, project lines upstream parallel to the mean direction of flow. For computation, these lines are assumed to represent the fixed solid upstream boundaries of an equivalent single-opening contraction.

Determination of h_1 and h_3

Determine the water-surface level, h_1 , at the location of section 1 for each opening. Because high-water marks are commonly found only along the edge of the channel and the embankment, the value of h_1 for the central openings must usually be estimated from high-water marks on the upstream side of the interior embankments. Defining h_s as the maximum water-surface elevation along an interior embankment, the value of h_1 may be determined as

$$h_1 = h_s - \frac{\alpha_1 Q_1^2}{2gA_1^2} + \frac{Q_1^2 L_w}{3K_1^2} \quad (8)$$

All quantities in the equation are for the pseudo-single-opening channel. The procedure requires the use of an assumed discharge which

must be later verified in the computation of discharge.

Determine the downstream water level h_3 as for a single opening—the average of water-surface elevations on the downstream side of the embankment on each side of the bridge opening.

The values of h_1 , h_3 , and Δh will generally be different for each opening.

Computation of discharge

The discharge is computed separately for each opening, using the procedure and discharge coefficients given previously for single-opening contractions. The individual discharges are then added to obtain the total discharge.

Selected References

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- Lane, E. W., 1920, Experiments on the flow of water through contractions in open channels: Am. Soc. Civil Engineers Trans., v. 83.
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