Chapter B7

ANALYTICAL SOLUTIONS FOR ONE-, TWO-, AND THREE-DIMENSIONAL SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

By Eliezer J. Wexler

Book 3
APPLICATIONS OF HYDRAULICS
Attachment 2.—Program source-code listings

FINITE
SEMINF
POINT2
STRIPF
STRIP1
GAUSS
POINT3
PATCHF
PATCH1
C
C ******************************************************
* ONE-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL *
* FOR A FINITE SYSTEM WITH A FIRST- OR THIRD-TYPE *
* BOUNDARY CONDITION AT X=0 *
* VERSION CURRENT AS OF 04/01/90 *
C
C ******************************************************

THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
GREATER THAN THOSE GIVEN HERE.
MAXX = MAXIMUM NUMBER OF X-VALUES
MAXT = MAXIMUM NUMBER OF TIME VALUES
MAXRT = MAXIMUM NUMBER OF ROOTS USED IN THE SERIES SUMMATION
PARAMETER MAXX=100,MAXT=20,MAXRT=1000

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
REAL XP(MAXX),CP(MAXX),TP,XSCLP
CHARACTER*10 CUNITS,WNITS,DUNITS,KUNITS,LUNITS,TUNITS
CHARACTER*1 IERR(MAXX,MAXT)
DIMENSION CXT(MAXX,MAXT),X(MAXX),T(MAXT)
DIMENSION ROOT(MAXRT)
COMMON /IOUNIT/ IN,10

PROGRAM VARIABLES
NOTE: ANY CONSISTENT SET OF UNITS MAY BE USED IN THE
MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS
LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.

CO SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
DK FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
T TIME AT WHICH CONCENTRATION IS EVALUATED [T]
CN NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
CXT SOLUTE CONCENTRATION C(X,T) [M/L**3]
XL LENGTH OF THE FLOW SYSTEM [L]
ROOT(N) ROOTS OF EQ. USED IN INFINITE SERIES SUMMATION
NBC SOURCE BOUNDARY CONDITION TYPE (1 OR 3)
NX NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
NT NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
NROOT NUMBER OF ROOTS USED IN INFINITE SERIES SUMMATION
IPLT PLOT CONTROL. IF IPLT>0, CONCENTRATION PROFILES ARE PLOTT
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS

C UNITS UNITS OF CONCENTRATION (M/L**3)
C VUNITS UNITS OF GROUND-WATER VELOCITY (L/T)
C DUNITS UNITS OF DISPERSION COEFFICIENT (L**2/T)
C KUNITS UNITS OF SOLUTE DECAY CONSTANT (1/T)
C LUNITS UNITS OF LENGTH (L)
C TUNITS UNITS OF TIME (T)

DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE

CALL QFILE
CALL TITLE
WRITE(IO,201)

READ IN MODEL PARAMETERS

READ(IN,101) NBC,NX,NT,NROOT,IPLT
IF(NBC.EQ.1) WRITE(IO,202)
IF(NBC.EQ.3) WRITE(IO,203)
WRITE(IO,205) NX,NT,NROOT
READ(IN,105) CUNITS,WNITS,DUNITS,KUNITS,LUNITS,TUNITS
READ(IN,110) CO,VX,DX,DK,XL,XSCLP
WRITE(IO,210) CO,CUNITS,VX,WNITS,DX,DUNITS,DK,KUNITS,XL,LUNITS,
1 XSCLP
READ(IN,110) (X(I),I=1,NX)
WRITE(IO,215) LUNITS
WRITE(IO,220) (X(I),I=1,NX)
READ(IN,110) (T(I),I=1,NT)
WRITE(IO,225) TUNITS
WRITE(IO,220) (T(I),I=1,NT)

GET EIGENVALUES (BETA) USED IN SERIES SUMMATION BY SOLVING FOR
THE POSITIVE ROOTS OF: BETA*COTAN(BETA)+VX*XL/(2*DX)=0.0
FOR A FIRST-TYPE SOURCE BOUNDARY CONDITION,
OR: BETA*COTAN(BETA)-BETA**2*DX/(VX*XL)+VX*XL/(4*DX)=0.0
FOR A THIRD-TYPE SOURCE BOUNDARY CONDITION.

IF (NBC.EQ.1) THEN
C=VX*XL/(2.0DO*DX)
CALL ROOT1(C,ROOT,NROOT)
ELSE
A=0.250DO*VX*XL/DX
C=DX/(XL*VX)
CALL ROOT3(A,C,ROOT,NROOT)
END IF

BEGIN TIME LOOP
DO 40 IT=1,NT

BEGIN X-COORDINATE LOOP
DO 50 IX=1,NX

CALL ROUTINE TO CALCULATE NORMALIZED CONCENTRATION
BASED ON TYPE OF BOUNDARY CONDITION SPECIFIED
IF(NBC.EQ.1) CALL CNRML1(XL,T(IT),X(IX),DX,VX,DK,ROOT,CN,NROOT,
1 IERR(IX,IT))
IF(NBG.EQ.3) CALL CNRML3(XL,T(IT),X(IX),DX,VX,DX,ROOT,CN,NROOT, I08
1 IERR(IX,IT))
CXT(IX,IT)=CN*CO
50 CONTINUE
110
C
C CONVERT X AND C TO SINGLE PRECISION AND DIVIDE BY CO TO
C PLOT NORMALIZED CONCENTRATION PROFILE FOR EACH TIME VALUE.
TF(IPLT,T,1) GO TO 40
DO 60 I=1,NX
XP(I)=SNGL(X(I))
60 CP(I)=SNGL(CXT(I,IT)/CO)
TP=SNGL(T(IT))
CALL PLOT1D(XP,CP,NX,TP,IT,NT,TUNITS,LUNITS,XSCLP)
120 CONTINUE
40
C
C PRINT OUT TABLES OF CONCENTRATION VALUES
NPAGE=1+(NT-1)/9
DO 80 NP=1,NPAG
IF(NP.EQ.1) WRITE(IO,230) TUNITS
IF(NP.NE.1) WRITE(IO,231) TUNITS
NP1=(NP-1)*9
NP2=9
IF((NP1+NP2).CT,NT) NP2=NT-NP1
WRITE(IO,235) (T(NP1+J),J=1,NP2) TUNITS
WRITE(IO,236) CUNITS,LUNITS
70 IF(MOD(NP,5).EQ.0 .AND. MOD(NP,45).NE.O) WRITE(IO,241)
80 CONTINUE
C
CLOSE (IN)
CLOSE (IO)
STOP
C
C FORMAT STATEMENTS
101 FORMAT(2014)
105 FORMAT(8A10)
110 FORMAT(8F10.0)
201 FORMAT(///1H ,30X,'ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL'/ 1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE-TRANSPORT EQUATION'/ 2 1H ,36X,'FOR A SYSTEM OF INFINITE LENGTH'///1H0,40X,'INPUT DATA'/ 3 1H ,40X,10(1H-))
202 FORMAT(1H0,25X,'FIRST-TYPE BOUNDARY CONDITION AT X = 0.0')
203 FORMAT(1H0,25X,'THIRD-TYPE BOUNDARY CONDITION AT X = 0.0')
205 FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X, 1 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,'NUMBER OF ROOTS ', 2 'USED IN INFINITE SERIES SUMMATION (NROOT) = ',I4)
210 FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (CO) =',
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND WATER SYSTEMS WITH UNIFORM FLOW

SOLUTION FOR THE ONE-DIMENSIONAL SOLUTE-TRANSPORT EQUATION
FOR A SYSTEM OF FINITE LENGTH WITH A FIRST-TYPE SOURCE
BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE
CONCENTRATION AT A GIVEN X-COORDINATE AND TIME VALUE.
FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED.

1 1PE13.6,1X,A10/1H ,25X.
2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =' .1PE13.6,1X,A10/
3 1H ,25X, 'DISPERSTION IN THE X-DIRECTION (DX) =' .1PE13.6,1X,A10/
4 1H ,25X, 'FIRST-ORDER SOLUTE-DECAY RATE (DK) =' .1PE13.6,1X,A10/
5 1H ,25X, 'LENGTH OF FINITE FLOW SYSTEM (XL) =' .1PE13.6,1X,A10/
6 1H ,25X, 'PLOT SCALING FACTOR (XSCLP) =' .1PE13.6)

FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS ',
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME',
1 15X, 'INDICATES SOLUTION DID NOT CONVERGE'/
2 1H0,25X,'TIME VALUES, IN ',A10)
FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME =',
1 15X, 'CONTINUED')/
2 1H0,25X,'TIME VALUES, IN ',A10)
FORMAT(1H ,20X,9F12.4)
FORMAT(1H0,25X,'TIME VALUES, IN ',A10)
FORMAT(lH ,19X,'*',108(1H-)/
1 1H ,4X,'X-COORDINATE,',2X,
2 AlO/lH ,4X, 'IN ',1H0/1H ,19X,','!
3 1H ,4X,'IN ',A10/1H ,4X, '1H ,1H /1H ,19X,','!'
4 1H ,4X,'IN ',A10,2X,1H/1H ,19X,','!
5 1H ,5X,'P12.4,2X,','! ',9(F11.5,Al))
FORMAT(lH ,20X,9F12.4)
FORMAT(lH0,25X,'TIME VALUES, IN ',A10)
FORMAT(lH ,19X,'*',108(1H-)/
1 1H ,4X,'X-COORDINATE,',2X,
2 AlO/lH ,4X, 'IN ',1H0/1H ,19X,','!
3 1H ,4X,'IN ',A10/1H ,4X, '1H ,1H /1H ,19X,','!
4 1H ,4X,'IN ',A10,2X,1H/1H ,19X,','!
5 1H ,5X,'P12.4,2X,','! ',9(F11.5,Al))

IERR=''
XL2=XL*XL
V2D-V/(2.0D0*D)
VX2D=V2D*X
VL2D-V2D*XL
VL2D2=VL2D*VL2D
DKL2D=DK*XL*XL/D
VSQT4D=V*V*T/(4.0D0*D)
TF(DK EQ. 0.0D0) GO TO 20

BEGIN SERIES SUMMATION FOR SOLUTE WITH DECAY
SIGMA=0.0
DO 10 N=1,NROOT
BETA=ROOT(N)
BETA2=BETA*BETA

TERM 1
X1=(BETA2+VL2D2)*DEXP(-BETA2*D*T/XL2)

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SOLUTION FOR THE ONE DIMENSIONAL SOLUTE TRANSPORT EQUATION FOR A SYSTEM OF FINITE LENGTH WITH A THIRD-TYPE SOURCE BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE CONCENTRATION AT A GIVEN X-COORDINATE AND TIME VALUE. FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED.
**ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW**

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IERR=' ', 266
XL2=XL*XL  267
V2D=V/(2.0DO*D)  268
VLD=V*XL/D  269
VX2D=V2D*X  270
VL2D=V2D*XL  271
VL2D2=V2D*VL2D  272
DKL2D=DK*XL*XL/D  273
VSQT4D=V*V*T/(4.0DO*D)  274
IF(DK.EQ.0.0DO) GO TO 10 20

BEGIN SERIES SUMMATION FOR SOLUTE WITH DECAY 275
SIGMA=0.0 276
DO 10 N=1,NROOT 277
BETA=ROOT(N) 278
BETA2=BETA*BETA 279

TERM 1 280
BETAXL=BETA*X/XL 281
XL=BETA*(BETA*DCOS(BETAXL)+VL2D*DSIN(BETAXL)) 282

TERM 2 283
DENOM=(BETA2+VL2D2+VLD)*(BETA2+VL2D2+DKL2D) 284
X2=DEXP(-BETA2*D*T/XL2)/DENOM 285
SIGMA=SIGMA+XL*X2 286

CHECK FOR CONVERGENCE OF SERIES 287
IF(N.GT.25 .AND. DABS(XL*X2).LT.1.0D-14) GO TO 15 288
CONTINUE 289
IERR='* ' 290
CONTINUE 291

TERM 3 292
U=DSQRT(V*V+4.0DO*DK*D) 293
VMU=V-U 294
VPU=V+U 295
VUPM=(U-V)/VPU 296
D2=D*2.0DO 297
X3=DEXP(VMU*X/D2)+VUPM*DEXP((VPU*X-2.0DO*U*XL)/D2) 298
X3=2.0DO*X*X3/(VPU+VMU*VUPM*DEXP(-U*XL/D)) 299
CN=X3-2.0DO*VL2D*DEXP(VX2D-VSQT4D-DK*T)*SIGMA 300
RETURN 301

BEGIN SERIES SUMMATION FOR SOLUTE WITH NO DECAY 302
SIGMA=0.0 303
DO 30 N=1,NROOT 304
BETA=ROOT(N) 305
BETA2=BETA*BETA 306

TERM 1 307
BETAXL=BETA*X/XL 308
XL=BETA*(BETA*DCOS(BETAXL)+VL2D*DSIN(BETAXL)) 309
```
TERM 2
DENOM=(BETA2+VL2D2+VLD)*(BETA2+VL2D2)
X2-EXP(-BETA2*DL/T/2)/DENOM
SIGMA=SIGMA+X1*X2
IF(N.GT.25 .AND. DABS(X1*X2).LT.1.0D-14) GO TO 35
CONTINUE
IERR='**'
CONTINUE
CN-1.0D0-2.0D0*VLD*EXP(VX2D-VSQ4D)*SIGMA
RETURN

SUBROUTINE ROOT1 (C,ROOT,NROOT)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION ROOT(NROOT)
COMMON /IOUNIT/ IN,10
DATA MAXIT,EPS/SO,l.OD-10/
THIS ROUTINE CALCULATES ROOTS OF THE EQUATION: B*COTAN(B)+C-0
USING NEWTON'S SECOND-ORDER METHOD.

PROGRAM VARIABLES
MAXIT MAXIMUM NUMBER OF ITERATIONS ALLOWED IN ROOT SEARCH
EPS CONVERGENCE CRITERION
F1,F2 1ST AND 2ND DERIVATIVES OF THE EQUATION
H SECOND-ORDER CORRECTION FACTOR

FIRST ROOT LIES BETWEEN PI/2 AND PI. START WITH .75*PI
PI-3.14159265359DO
ROOT(1)-0.750DO*PI
START LOOP FOR EACH ROOT SEARCH
DO 10 N=1,NROOT

BEGIN ITERATIVE LOOP
DO 20 I=1,MAXIT
X=ROOT(N)
SINX2=DSIN(X)*DSIN(X)
COTX=1.0D0/DTAN(X)
F=X*COTX+C
IF F IS 0.0, EXACT ROOT HAS BEEN FOUND
IF(F.EQ.0.0) GO TO 30
F1=COTX-X/SINX2
F2=-1.0D0/SINX2-(SINX2-X*DSIN(X)*2.0D0)/(SINX2*SINX2)
H=(F2/2.0D0)/F1-F1/F
H=1.0D0/H
ROOT(N)=X+H

CHECK FOR CONVERGENCE. IF NOT ACHIEVED, RE-ITERATE
IF(DABS(H).LT.EPS) GO TO 30
CONTINUE
WRITE(IO,201) MAXIT,N
STOP

C  NEXT ROOT IS ABOUT PI GREATER THAN LAST ROOT
30  IF(N.NE.NROOT) ROOT(N+1)=ROOT(N)+PI
10 CONTINUE
RETURN

C  FORMAT STATEMENTS
201  FORMAT(1H ,5X,'**** WARNING **** ROOT SEARCH ROUTINE DID NOT',
1  'CONVERGE AFTER ',I5,' ITERATIONS WHILE SEARCHING FOR ROOT',I5)
END
SUBROUTINE ROOT3 (A,C,ROOT,NROOT)
IMPLICIT DOUBLE PRECISION
DIMENSION ROOT(NROOT)
COMMON /IOUNIT/ IN,10
DATA MAXIT,EPS/50,1.OD-10/

C  THIS ROUTINE CALCULATES ROOTS OF THE EQ: B*COTAN(B)-C*B**2+A=0
USING NEWTON’S SECOND-ORDER METHOD.

C  PROGRAM VARIABLES
C  MAXIT  MAXIMUM NUMBER OF ITERATIONS ALLOWED IN ROOT SEARCH
C  EPS  CONVERGENCE CRITERION
C  F1,F2  1ST AND 2ND DERIVATIVES OF THE EQUATION
C  H  SECOND-ORDER CORRECTION FACTOR
C
C  FIRST ROOT LIES BETWEEN 0.0 AND PI. START WITH 0.5*PI
PI=3.14159265359D0
ROOT(1)=0.50D0*PI

C  START LOOP FOR EACH ROOT SEARCH
DO 10 N=1,NROOT

C  BEGIN ITERATIVE LOOP
DO 20 I=1,MAXIT
X=ROOT(N)
SINX2=DSIN(X)*DSIN(X)
COTX=1.0D0/DTAN(X)
F=X*COTX-C*X*X+A
IF F IS 0.0, EXACT ROOT HAS BEEN FOUND
IF(F.EQ.0.0) GO TO 30
F1=COTX-X/SINX2-(2.0D0*C*X)
F2=-1.0D0/SINX2-(SINX2-X*DSIN(X*2.0D0))/(SINX2*SINX2)-2.0D0*C
H=(F2/2.0D0)/F1-F1/F
H=1.0D0/H
ROOT(N)=X+H
CHECK FOR CONVERGENCE. IF NOT ACHIEVED, RE-ITERATE
IF(DABS(H).LT.EPS) GO TO 30
20 CONTINUE
WRITE(IO,201) MAXIT,N
STOP
C NEXT ROOT IS ABOUT PI GREATER THAN LAST ROOT
30 IF(N.NE.NROOT) ROOT(N+1)=ROOT(N)+PI
10 CONTINUE
RETURN

C FORMAT STATEMENTS
201 FORMAT(1H,5X,'**** WARNING **** ROOT SEARCH ROUTINE DID NOT',
1 'CONVERGE AFTER ',I4,' ITERATIONS WHILE SEARCHING FOR ROOT',I5)
END
THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
GREATER THAN THOSE GIVEN HERE.
MAXX = MAXIMUM NUMBER OF X-VALUES
MAXT = MAXIMUM NUMBER OF TIME VALUES
PARAMETER MAXX=100,MAXT=20

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
REAL XP(MAXX),CP(MAXX),TP,XSCLP
CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
DIMENSION CXT(MAXX,MAXT),X(MAXX),T(MAXT)
COMMON /IOUNIT/ IN,10
PROGRAM VARIABLES
NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS
LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.

CO SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
DK FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
T TIME AT WHICH CONCENTRATION IS EVALUATED [T]
CN NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
CXT SOLUTE CONCENTRATION C(X,T) [M/L**3]

NBC SOURCE BOUNDARY CONDITION TYPE (1 OR 3)
NX NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
NT NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
IPLT PLOT CONTROL. IF IPLT>0, CONCENTRATION PROFILES ARE PLOTTED

CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
CUNITS UNITS OF CONCENTRATION (M/L**3)
VUNITS UNITS OF GROUND WATER VELOCITY (L/T)
DUNITS UNITS OF DISPERSION COEFFICIENT (L**2/T)
KUNITS UNITS OF SOLUTE DECAY CONSTANT (1/T)
LUNITS UNITS OF LENGTH (L)
DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
CALL OFILE
CALL TITLE
WRITE(IO, 201)

READ IN MODEL PARAMETERS
READ(IN, 101) NBC, NX, NT, IPLT
IF(NBC.EQ.1) WRITE(IO, 202)
IF(NBC.EQ.3) WRITE(IO, 203)
WRITE(IO, 205) NX, NT
READ(IN, 105) CUNITS, VUNITS, DUNITS, KUNITS, LUNITS, TUNITS
READ(IN, 110) CO, VX, DX, DK, XSCLP
WRITE(IO, 210) CO, CUNITS, VX, WNITS, DX, DUNITS, DK, KUNITS, XSCLP
READ(IN, 110) (X(I), I=1, NX)
WRITE(IO, 215) LUNITS
WRITE(IO, 220) (X(I), I=1, NX)
READ(IN, 110) (T(I), I=1, NT)
WRITE(IO, 225) TUNITS
WRITE(IO, 220) (T(I), I=1, NT)

BEGIN TIME LOOP
DO 40 IT=1, NT
BEGIN X-COORDINATE LOOP
DO 50 IX=1, NX
CALL ROUTINE TO CALCULATE NORMALIZED CONCENTRATION
BASED ON TYPE OF BOUNDARY CONDITION SPECIFIED
IF(NBC.EQ.1) CALL CNRML1(DK, T(IT), X(IX), DX, VX, CN)
IF(NBC.EQ.3) CALL CNRML3(DK, T(IT), X(IX), DX, VX, CN)
CXT(IX, IT)=CN*CO
CONTINUE

CONVERT X AND C TO SINGLE PRECISION AND DIVIDE BY CO TO
PLOT NORMALIZED CONCENTRATION PROFILE FOR EACH TIME VALUE.
IF(IPLT.LT.1) GO TO 40
DO 60 I=1, NX
XP(I)=SNGL(X(I))
CP(I)=SNGL(CXT(I, IT)/CO)
TP=SNGL(T(IT))
CALL PLOTID(XP, CP, NX, TP, IT, NT, TUNITS, LUNITS, XSCLP)
CONTINUE

PRINT OUT TABLES OF CONCENTRATION VALUES
NPAGE=1+(NT-1)/9
DO 80 NP=1, NPAGE
IF(NP.EQ.1) WRITE(IO, 230) TUNITS
IF(NP.NE.1) WRITE(IO, 231) TUNITS
NP1=(NP-1)*9
NP2=9
IF((NP1+NP2).GT.NT) NP2=NT-NP1
WRITE(I0,235) (T(NP1+J),J=1,NP2) 107
WRITE(I0,236) CUNITS,LUNITS 108
DO 70 1X=1,NX 109
WRITE(I0,240) X(IX),(CXT(IX,NP1+J),J=1,NP2) 110
IF(MOD(IX,45).NE.0) GO TO 70 111
WRITE(I0,231) TUNITS 112
WRITE(I0,235) (T(NP1+J),J=1,NP2) 113
WRITE(I0,236) CUNITS,LUNITS 114
70 IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(I0,241) 115
80 CONTINUE 116
C 117
CLOSE (IN) 118
CLOSE (IO) 119
STOP 120
C 121
FORMAT STATEMENTS 122
101 FORMAT(2014) 123
105 FORMAT(8A10) 124
110 FORMAT(8F10.0) 125
201 FORMAT(///,1H,30X,'ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL'/ 126
1H,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION '/ 127
2H,38X,'FOR A SEMI-INFINITE SYSTEM'///,I0H,40X,'INPUT DATA'/ 128
3H,40X,10(1H-)) 129
202 FORMAT(1H0,25X,'FIRST-TYPE BOUNDARY CONDITION AT X = 0.0') 130
203 FORMAT(1H0,25X,'THIRD-TYPE BOUNDARY CONDITION AT X = 0.0') 131
205 FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H,25X, 132
1H,'NUMBER OF TIME VALUES (NT) = ',I4) 133
210 FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (CO) =', 134
1P1E13.6,1X,A10/1H,25X, 135
2H,'GROUND-WATER VELOCITY IN X-DIRECTION (VX) = ',1P1E13.6,1X,A10/ 136
3H,.25X,'DISPERSION IN THE X-DIRECTION (DX) = ',1P1E13.6,1X,A10/ 137
4H,.25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) = ',1P1E13.6,1X,A10/ 138
5H,.25X,'PLOT SCALING FACTOR (XSCLP) = ',1P1E13.6) 139
215 FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', 140
1H,'WILL BE CALCULATED, IN ',A10/1H,25X/1H,78(1H-)/) 141
220 FORMAT(1H0,5X,8F12.4/) 142
225 FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS ', 143
1H,'WILL BE CALCULATED, IN ',A10/1H,25X/1H,70(1H-)/) 144
230 FORMAT(1H1/I0H,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME'/ 145
2H,10H,.25X,'TIME VALUES, IN ',A10/) 146
231 FORMAT(1H1/I0H,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME =', 147
1H,5X,'(CONTINUED)'/ 148
2H,10H,'TIME VALUES, IN ',A10/) 149
235 FORMAT(1H,.20X,9F12.4) 150
236 FORMAT(1H,.19X,'*',108(1H-)/ 151
1H,.4X,'X-COORDINATE, ',2X,'!',44X,'SOLUTE CONCENTRATION, IN ' 152
2A10,.4X,'IN ',A10,.2X,.1H,.1H,.19X,'!'/) 153
240 FORMAT(1H,.5X,F12.4,2X,'!',.9F12.5) 154
241 FORMAT(1H,.19X,'!') 155
END 156
SUBROUTINE CNRMLl(DK,T,X,D,V,CN) 157
IMPLICIT DOUBLE PRECISION (A-H,O-Z) 158
C 159
SOLUTION FOR THE ONE-DIMENSIONAL SOLUTE TRANSPORT EQUATION
FOR A SEMI-INFINITE SYSTEM WITH A FIRST-TYPE SOURCE
BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE
CONCENTRATION AT A GIVEN X-COORDINATE AND TIME.
FOR NO SOLUTE DECAY. A SIMPLIFIED SOLUTION IS USED.

ALPHA=2.0D0*DSQRT(D*T)
U=DSQRT(V*V+4.0D0*D*DK)
X2D=X/(2.0D0*D)

SOLUTION WITH SOLUTE DECAY
IF(DK.EQ.0.0) GO TO 10

TERM 1
X1=X2D*(V-U)
Y1=(X-U*T)/ALPHA
CALL EXERFC(X1,Y1,Z1)

TERM 2
X2=X2D*(V+U)
Y2=(X+U*T)/ALPHA
CALL EXERFC(X2,Y2,Z2)
CN=(Z1+Z2)/(2.0D0)
RETURN

SOLUTION WITH NO SOLUTE DECAY
TERM 1
Y1=(X-V*T)/ALPHA
CALL EXERFC(0.0D0,Y1,Z1)

TERM 2
X2=X*V/D
Y2=(X+V*T)/ALPHA
CALL EXERFC(X2,Y2,Z2)
CN=(Z1+Z2)/2.0D0
RETURN

END

SUBROUTINE CNRML3(DK,T,X,D,V,CN)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
SOLUTION FOR THE ONE-DIMENSIONAL SOLUTE TRANSPORT EQUATION
FOR A SEMI-INFINITE SYSTEM WITH A THIRD-TYPE SOURCE
BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE
CONCENTRATION AT A GIVEN X-COORDINATE AND TIME.
FOR NO SOLUTE DECAY. A SIMPLIFIED SOLUTION IS USED.

ALPHA=P.0D0*DSQRT(D*T)
U=DSQRT(V*V+4.0D0*D*DK)
X2D=-X/(2.0D0*D)

SOLUTION WITH SOLUTE DECAY
IF(DK.EQ.0.0) GO TO 10

TERM 1
X1=X2D*(V-U)
Y1=(X-U*T)/ALPHA
CALL EXERFC(X1,Y1,Z1)

TERM 2
X2=X2D*(V+U)
Y2=(X+U*T)/ALPHA
CALL EXERFC(X2,Y2,Z2)
CN=(Z1+Z2)/(2.0D0)
RETURN

END

SUBROUTINE CNRML3(DK,T,X,D,V,CN)
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

C
C TERM 1
X1=VXD-DK*T
Y1=(X+V*T)/ALPHA
CALL EXERFC(X1,Y1,Z1)
Z1=Z1*2.0D0
C
C TERM 2
X2=X2D*(V-U)
Y2=(X-U*T)/ALPHA
CALL EXERFC(X2,Y2,Z2)
Z2=Z2*(U/V-1.0D0)
C
C TERM 3
X3=X2D*(V+U)
Y3=(X+U*T)/ALPHA
CALL EXERFC(X3,Y3,Z3)
Z3=Z3*(U/V+1.0D0)
CN=V*V*(Z1+Z2-Z3)/(4.0D0*D*DK)
RETURN
C
SOLUTION FOR NO SOLUTE DECAY
PI=3.14159265358979D0
C
C TERM 1
Y1=(X-V*T)/ALPHA
CALL EXERFC(0.0D0,Y1,Z1)
Z1=0.50D0*Z1
C
C TERM 2
X2=VXD
Y2=(X+V*T)/ALPHA
CALL EXERFC(X2,Y2,Z2)
Z2=Z2*0.50D0*(1.0D0+V*(X+V*T)/D)
C
C TERM 3
Z3=DEXP(-1.0D0*Y1*Y1)
Z3=Z3*V*DSQRT(T/(PI*D))
CN=Z1-Z2+Z3
RETURN
END
*** TWO-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL ***

FOR AN AQUIFER OF INFINITE AREAL EXTENT WITH A CONTINUOUS POINT SOURCE LOCATED AT X=XC AND Y=YC

GROUND-WATER FLOW IN X-DIRECTION ONLY

VERSION CURRENT AS OF 04/01/90

PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)
COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
1 IFLAG(MAXXY2)
COMMON /IOUNIT/ IN,10

PROGRAM VARIABLES

NOTE: ANY CONSISTENT SET OF UNITS MAY BE USED IN THE MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.

CO  SOLUTE CONCENTRATION IN INJECTED FLUID [M/L**3]
DX  LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
DY  TRANSVERSE DISPERSION COEFFICIENT [L**2/T]
VX  GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
DK  FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
X   X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
Y   Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
T   TIME AT WHICH CONCENTRATION IS EVALUATED [T]
CN  NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
CXY SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]
XC  X-COORDINATE OF POINT SOURCE [L]
YC  Y-COORDINATE OF POINT SOURCE [L]
QM  FLUID INJECTION RATE PER UNIT THICKNESS OF AQUIFER [L**2/
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

(UNITS MUST BE SAME AS DISPERSION COEFFICIENT)

POR AQUIFER PORISITY [DIMENSIONLESS]

NX NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED

NY NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED

NT NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED

NMAX NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)

IPLT PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED

XSCLP SCALING FACTOR TO CONVERT X TO PLOTTER INCHES

YSCLP SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES

DELTA CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)

CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS

CUNITS UNITS OF CONCENTRATION (M/L**3)

VUNITS UNITS OF GROUND-WATER VELOCITY (L/T)

DUNITS UNITS OF DISPERSION COEFFICIENT (L**2/T)

KUNITS UNITS OF SOLUTE DECAY CONSTANT (l/T)

LUNITS UNITS OF LENGTH (L)

TUNITS UNITS OF TIME (T)

DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE

CALL OFILE

CALL TITLE

WRITE(IO,201)

READ IN MODEL PARAMETERS

READ(IN,101) NX,NY,NT,NMAX,IPLT

WRITE(IO,205) NX,NY,NT,NMAX

READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS

READ(IN,110) CO,VX,DX,DY,DK

WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS

READ(IN,110) XC,YC,QM,POR

WRITE(IO,212) XC,LUNITS,YC,LUNITS,QM,DUNITS,POR

READ(IN,110) (X(I),I=1,NX)

WRITE(IO,215) LUNITS

WRITE(IO,220) (X(I),I=1,NX)

READ(IN,110) (Y(I),I=1,NY)

WRITE(IO,216) LUNITS

WRITE(IO,220) (Y(I),I=1,NY)

READ(IN,110) (T(I),I=1,NT)

WRITE(IO,225) TUNITS

WRITE(IO,220) (T(I),I=1,NT)

IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA

IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS

READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS

CALL GLQPTS (NMAX)

BEGIN TIME LOOP

DO 20 IT=1,NT
C    BEGIN X LOOP
     DO 40 IX=1,NX
     XX=X(IX)-XC
     C    CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)
     DO 50 IY=1,NY
     YY=Y(IY)-YC
     CALL CNRML2(QM, POR, DK, T(IT), XX, YY, DX, DY, VX, CN, NMAX)
     CXY(IX,IY)=CO*CN
     50 CONTINUE
     40 CONTINUE
     C    PRINT OUT TABLES OF CONCENTRATION VALUES
     NPAGE=1+(NY-1)/9
     DO 60 NP=1,NPAGE
     IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS
     IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS
     NP1=(NP-1)*9
     NP2=9
     IF((NP1+NP2).GT.NY) NP2=NY-NP1
     WRITE(IO,235) (Y(NP1+J),J=1,NP2)
     WRITE(IO,236) CUNITS,LUNITS
     DO 70 IX=1,NX
     WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)
     IF(MOD(IX,45).NE.O) GO TO 70
     WRITE(IO,231) T(IT),TUNITS,LUNITS
     WRITE(IO,235) (Y(NP1+J),J=1,NP2)
     WRITE(IO,236) CUNITS,LUNITS
     70 IF(MOD(IX,5).EQ.O .AND. MOD(IX,45).NE.O) WRITE(IO,241)
     60 CONTINUE
     C    CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
     C    PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY CO TO PLOT
     C    CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
     IF(IPLT.LT.1) GO TO 20
     NXY=NX*NY
     DO 80 I=1,NX
     IP=(I-1)*NY
     XP(I)=SNGL(X(I))
     IF(I.EQ.1) YP(J)=SNGL(Y(J))
     IF(I.EQ.1) YP(J)=SNGL(Y(J))
     DO 80 J=1,NY
     IF(J.EQ.1) YP(J)=SNGL(Y(J))
     CP(IP+J)=SNGL(CXY(I,J)/CO)
     80 CONTINUE
     TP=SNGL(T(IT))
     NXY2=NXY*2
     CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,1PLT,TUNITS,
     1 LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
     20 CONTINUE
     CLOSE (IN)
     CLOSE (IO)
     STOP
     C    FORMAT STATEMENTS
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

FORMAT(2014)
105  FORMAT(8A10)
110  FORMAT(8F10.0)
201  FORMAT(///1H ,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/
1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
2 1H ,28X,'FOR AN AQUIFER OF INFINITE AREAL EXTENT WITH A'/
3 1H ,31X,'CONTINUOUS POINT SOURCE AT X=0 AND Y=YC'/
4 ///1H ,40X,'INPUT DATA'/1H ,40X,10(1H-))
205  FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',14/1H ,25X,
1 'NUMBER OF Y-COORDINATES (NY) = ',14/1H ,25X,
2 'NUMBER OF TIME VALUES (NT) = ',14/1H ,25X,
3 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',14)
210  FORMAT(1H0,25X,'SOLUTE CONCENTRATION IN INJECTED FLUID (CO) =',
1 1P1E13.6,1X,A10/1H ,25X,
2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/1H ,25X,
3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/1H ,25X,
4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/1H ,25X,
5 1H ,25X,'FIRST-ORDER SOLUTE DEGAY RATE (DK) =',1P1E13.6,1X,A10)
212  FORMAT(1H0,25X,'AQUIFER IS OF INFINITE AREAL EXTENT'/1H ,25X,
1 'CONTINUOUS SOURCE IS LOCATED AT X =',1P1E13.6,1X,A10/1H ,25X,
2 'FLUID INJECTION RATE PER UNIT THICKNESS OF AQUIFER (QM) =',1P1E13.6/1H ,25X,
3 1P1E13.6,1X,A10/1H ,25X,'AQUIFER POROSITY (POR) =',1P1E13.6)
215  FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
1 'WILL BE CALCULATED, IN ',A10/1H ,25X,
2 'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
1 'WILL BE CALCULATED, IN ',A10/1H ,25X)
220  FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS ',
1 'WILL BE CALCULATED, IN ',A10/1H ,25X)
225  FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) = ',1P1E13.6/1H ,25X,
1 'PLOT SCALING FACTOR FOR Y (YSCLP) = ',1P1E13.6)
227  FORMAT(1H0,25X,'SOLUTE CONCENTRATION AT TIME =',
1 1F12.4,1X,A10/1H ,25X)
231  FORMAT(1H1/1H1,15X,'SOLUTE CONCENTRATION AT TIME = ',
1 1F12.4,1X,A10,5X,'(CONTINUED)'/
2 1H0,25X,'Y-COORDINATE, IN ',A10)
235  FORMAT(1H0,25X,'SOLUTE CONCENTRATION AT TIME = ',
1 1F12.4,1X,A10,5X,'(CONTINUED)'/
2 1H0,25X,'Y-COORDINATE, IN ',A10)
236  FORMAT(1H0/1H ,19X,9F12.4)
240  FORMAT(1H0/1H ,19X,9F12.5)
241  FORMAT(1H0/1H ,19X,9F12.5)

END

SUBROUTINE CNF2ML2(QM,POR,DK,T,X,Y,DX,DY,VX,CN,NMAX)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
COMMON /IOUNIT/ IN,10
COMMON /GLPTS/ WN(256),ZN(256)

THIS ROUTINE CALCULATES SOLUTE CONCENTRATION AT X,Y BASED ON
THE ANALYTIC SOLUTION TO THE TWO-DIMENSIONAL ADVECTIVE-
DISPERSE SOLUTE TRANSPORT EQUATION FOR AN AQUIFER OF INFINITE AREAL EXTENT WITH A CONTINUOUS POINT SOURCE LOCATED AT X=Xc AND Y=Yc. THE INTEGRAL FROM 0 TO T IS EVALUATED USING A GAUSS-LEGENDRE QUADRATURE INTEGRATION TECHNIQUE.

PI=3.14159265358979D0
CN=0.0D0

FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
IF(T.LE.0.0D0) RETURN

START NUMERICAL INTEGRATION LOOP
ALPHA=X*X/(4.0D0*DX)+Y*Y/(4.0D0*DY)
BETA=VX*VX/(4.0D0*DX)+DK
VX2D=VX*X/(2.0D0*DX)
SUM=0.0D0
DO 20 I=1,NMAX
SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE NON-NORMALIZED LIMITS OF INTEGRATION
WI=WN(I)
ZI=T*(ZN(I)+1.0D0)/2.0D0

TERM 1
X1=-ALPHA/ZI-BETA*ZI
X1=DEXP(X1)/ZI
SUM=SUM+X1*WI
CONTINUE
SUM=SUM*T/2.0D0
CN=QM*SUM*DEXP(VX2D)/(4.0D0*POR*PI*DSQRT(DX*DY))
RETURN
END
THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
GREATER THAN THOSE GIVEN HERE.
MAXX = MAXIMUM NUMBER OF X-VALUES
MAXY = MAXIMUM NUMBER OF Y-VALUES
MAXT = MAXIMUM NUMBER OF TIME VALUES
MAXXY = MAXX * MAXY
MAXXY2 = 2 * MAXX * MAXY

PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000

IMPlicit DOUBLE PRECISION (A-H,O-Z)
CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
CHARACTER*1 IERR(MAXX,MAXY)
REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)
COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
1 IFLAG(MAXXY2)
COMMON /IOUNIT/ IN,IO

PROGRAM VARIABLES
NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS
LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.

C0 SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
DY TRANSVERSE DISPERSION COEFFICIENT [L**2/T]
VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
DK FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
Y Y POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
T TIME AT WHICH CONCENTRATION IS EVALUATED [T]
CN NORMALIZED CONCENTRATION C/C0 [DIMENSIONLESS]
CXY SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]
W AQUIFER WIDTH (AQUIFER EXTENDS FROM Y=0 TO Y=W) [L]
Y1 Y-COORDINATE OF LOWER LIMIT OF STRIP SOLUTE SOURCE [L]
Y2  Y-COORDINATE OF UPPER LIMIT OF STRIP SOLUTE SOURCE [L]  
NX  NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED  
NY  NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED  
NT  NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED  
NMAX  NUMBER OF TERMS USED IN INFINITE SERIES SUMMATION  
IPLT  PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED  
XSCLP  SCALING FACTOR TO CONVERT X TO PLOTTER INCHES  
YSCLP  SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES  
DELTA  CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)  
CUNITS  UNITS OF CONCENTRATION (M/L**3)  
VUNITS  UNITS OF GROUND-WATER VELOCITY (L/T)  
DUNITS  UNITS OF DISPERSION COEFFICIENT (L**2/T)  
KUNITS  UNITS OF SOLUTE DECAY CONSTANT (1/T)  
LUNITS  UNITS OF LENGTH (L)  
TUNITS  UNITS OF TIME (T)  
DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE  
CALL OFILE  
CALL TITLE  
WRITE(IO,201)  
READ IN MODEL PARAMETERS  
READ(IN,101) NX,NY,NT,NMAX,IPLT  
WRITE(IO,205) NX,NY,NT,NMAX  
READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS  
READ(IN,110) CO,VX,DX,DY,DK  
WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS  
READ(IN,110) W,Y1,Y2  
WRITE(IO,212) W,LUNITS,Y1,LUNITS,Y2,LUNITS  
READ(IN,110) (X(I),I=1,NX)  
WRITE(IO,215) LUNITS  
WRITE(IO,220) (X(I),I=1,NX)  
READ(IN,110) (Y(I),I=1,NY)  
WRITE(IO,216) LUNITS  
WRITE(IO,220) (Y(I),I=1,NY)  
READ(IN,110) (T(I),I=1,NT)  
WRITE(IO,225) TUNITS  
WRITE(IO,220) (T(I),I=1,NT)  
IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA  
IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS  
BEGIN TIME LOOP  
DO 20 IT=1,NT  
BEGIN X LOOP  
DO 40 IX=1,NX  
CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)  
DO 50 IY=1,NY  


CALL CNRMLF(DK,T(IT),X(IX),Y(IY),W,Y1,Y2,DX,DY,
1 VX,CN,NMAX,IERR(IX,IY))
CXY(IX,IY)=CO*CN
CONTINUE
CONTINUE
C PRINT OUT TABLES OF CONCENTRATION VALUES
NPAGE=1+(NY-1)/9
DO 60 NP=1,NPAGE
IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS
IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS
NP1=(NP-1)*9
NP2=9
IF((NP1+NP2).GT.NY) NP2=NY-NP1
WRITE(IO,235) (Y(NP1+J),J=1,NP2)
WRITE(IO,236) CUNITS,LUNITS
DO 70 IX=1,NX
WRITE(IO,240) X(IX),(CXY(IX,NP1+J),IERR(IX,NP1+J),J=1,NP2)
70 IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
CONTINUE
C CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY CO TO PLOT
C CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
IF(IPLT.LT.1) GO TO 20
NXY=NX*NY
DO 80 I=1,NX
IP=(I-1)*NY
XP(I)=SNGL(X(I))
DO 80 J=1,NY
IF(I.EQ.1) YP(J)=SNGL(Y(J))
CP(IP+J)=SNGL(CXY(I,J)/CO)
80 CONTINUE
TP=SNGL(T(IT))
NXY2=NXY*2
CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,TUNITS,
1 LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
20 CONTINUE
CLOSE (IN)
CLOSE (IO)
STOP
C FORMAT STATEMENTS
101 FORMAT(20I4)
105 FORMAT(8A10)
110 FORMAT(8F10.0)
201 FORMAT(///1H ,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/
1 1H .28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
2 1H ,30X,'FOR A SEMI-INFINITE AQUIFER OF Finite WIDTH'/
This routine calculates the normalized concentration at X, Y.

Based on the analytic solution to the two-dimensional
advection-dispersive solute transport equation for a semi-
infinite aquifer with a finite width. A finite-width (strip)
solute source extends from Y=1 to Y=2. The solution
contains an infinite series summation which may take a large
number of terms to converge for small values of X.

END

COMMON /IOUNIT/ IN,I0

PI=3.14159265358979D0
CN=0.0D0
IERR=' ' 213
C 214
C FOR T = 0, ALL CONCENTRATIONS EQUAL 0.0 215
IF(T.LE.0.0DO) RETURN 216
C 217
C FOR X = 0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS 218
IF(X.GT.0.0DO) GO TO 10 219
IF(Y.GT.Y1 .AND. Y.LT.Y2) CN=1.0DO 220
IF(Y.EQ.Y1) CN=0.50DO 221
IF(Y.EQ.Y2) CN=0.50DO 222
RETURN 223
C 224
C BEGIN SUMMATION OF TERMS IN INFINITE SERIES 225
10 RTDXT=2.0DO*DSQRT(DX*T) 226
SIGMA=0.0DO 227
SUBTOT=0.0DO 228
NMAX1=NMAX+1 229
DO 20 NN=1,NMAX1 230
N=NN-1 231
ETA=N*PI/W 232
PN=(Y2-Y1)/(2.0DO*W) 233
IF(N.NE.0) PN=(DSIN(ETA*Y2)-DSIN(ETA*Y1))/(N*PI) 234
COSRY=DCOS(ETA*Y) 235
ALPHA=4.0DO*DX*(ETA*ETA*DY+DK) 236
BETA=DSQRT(VX*VX+ALPHA) 237
BETAT=BETA*T 238
C 239
C CALCULATE TERM 1 240
A1=X*(VX-BETA)/(2.0DO*DX) 241
B1=(X-BETAT)/RTDXT 242
CALL EXERFC(A1,B1,C1) 243
C 244
C CALCULATE TERM 2 245
A2=X*(VX+BETA)/(2.0DO*DX) 246
B2=(X+BETAT)/RTDXT 247
CALL EXERFC(A2,B2,C2) 248
C 249
C ADD TERMS TO SUMMATION 250
TERM=PN*COSRY*(C1+C2) 251
SIGMA=SIGMA+TERM 252
C 253
C CHECK FOR CONVERGENCE. BECAUSE SERIES OSCILLATES, CHECK 254
C SUBTOTAL OF LAST 10 TERMS. 255
SUBTOT=SUBTOT+TERM 256
IF(MOD(NN,10).NE.0) GO TO 20 257
IF(DABS(SUBTOT).LT.1.0D-12) GO TO 30 258
SUBTOT=0.0DO 259
20 CONTINUE 260
IERR='*' 261
30 CN=SIGMA 262
RETURN 263
END 264
STRIP1

TWO-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL
FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH
A STRIP SOURCE EXTENDS FROM Y1 TO Y2 AT X=0
GROUND-WATER FLOW IN X-DIRECTION ONLY
VERSION CURRENT AS OF 04/01/90

THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE GREATER THAN THOSE GIVEN HERE.

PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)
COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),IFLAG(MAXXY2)
COMMON /IOUNIT/ IN,10

PROGRAM VARIABLES

NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.

CO SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
DY TRANSVERSE DISPERSION COEFFICIENT [L**2/T]
VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
DK FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
Y Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
T TIME AT WHICH CONCENTRATION IS EVALUATED [T]
CN NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
CXY SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]
Y1 Y-COORDINATE OF LOWER LIMIT OF STRIP SOLUTE SOURCE [L]
Y2 Y-COORDINATE OF UPPER LIMIT OF STRIP SOLUTE SOURCE [L]
NX NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW 107

C NY    NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED 54
C NT    NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED 55
C NMAX  NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL 56
C INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256) 57
C IPLT  PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED 59
C XSCLP SCALING FACTOR TO CONVERT X TO PLOTTER INCHES 60
C YSCLP SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES 61
C DELTA CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0) 62
C C CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS 64
C CUNITS UNITS OF CONCENTRATION (M/L**3) 65
C VUNITS UNITS OF GROUND-WATER VELOCITY (L/T) 66
C DUNITS UNITS OF DISPERSION COEFFICIENT (L**2/T) 67
C KUNITS UNITS OF SOLUTE DECAY CONSTANT (1/T) 68
C LUNITS UNITS OF LENGTH (L) 69
C TUNITS UNITS OF TIME (T) 70
C C DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE 72
C CALL OPFILE 73
C CALL TITLE 74
C WRITE(IO,201) 75
C C READ IN MODEL PARAMETERS 77
C READ(IN,101) NX,NY,NT,NMAX,IPLT 78
C WRITE(IO,205) NX,NY,NT,NMAX 79
C READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS 80
C READ(IN,110) CO,VX,DX,DY,DK 81
C WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS 82
C READ(IN,110) Y1,Y2 83
C WRITE(IO,212) Y1,LUNITS,Y2,LUNITS 84
C READ(IN,110) (X(I),I=1,NX) 85
C WRITE(IO,215) LUNITS 86
C WRITE(IO,220) (X(I),I=1,NX) 87
C READ(IN,110) (Y(I),I=1,NT) 88
C WRITE(IO,216) LUNITS 89
C WRITE(IO,220) (Y(I),I=1,NT) 90
C READ(IN,110) (T(I),I=1,NT) 91
C WRITE(IO,225) TUNITS 92
C WRITE(IO,220) (T(I),I=1,NT) 93
C IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA 94
C IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS 95
C C READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS 97
C CALL GLQPTS (NMAX) 98
C C BEGIN TIME LOOP 100
C DO 20 IT=1,NT 101
C C BEGIN X LOOP 103
C DO 40 IX=1,NX 104
C C CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) 105
DO 50 IY=1,NY
CALL CNRMIL(DK,T(IT),X(IX),Y(IY),Y1,Y2,DY,DX,VX,CN,NMAX)
CXY(IX,IY)=CO*CN
CONTINUE
CONTINUE
CONTINUE
CONTINUE
PRINT OUT TABLES OF CONCENTRATION VALUES
NPAGE=1+(NY-1)/9
DO 60 NP=1,NPAGE
IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS
IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS
NP1=(NP-1)*9
NP2=9
IF((NP1+NP2).GT.NY) NP2=NY-NP1
WRITE(IO,235) (Y(NP1+J),J=1,NP2)
WRITE(IO,236) CUNITS,LUNITS
DO 70 IX=1,NX
WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)
IF(MOD(IX,45).NE.O) GO TO 70
WRITE(IO,231)
WRITE(IO,235) (Y(NP1+J),J=1,NP2)
WRITE(IO,236) CUNITS,LUNITS
IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
CONTINUE
CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY CO TO PLOT
CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
IF(IPLT.LT.1) GO TO 20
NX=NX*NY
DO 80 I=1,NX
IP=(I-1)*NY
XP(I)=SNGL(X(I))
DO 80 J=1,NY
IF(I.EQ.1) YP(J)=SNGL(Y(J))
CP(IP+J)=SNGL(CXY(I,J)/CO)
CONTINUE
TP=SNGL(T(IT))
NXY=NX*2
CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2(IT,NT,IPLT,TUNITS,
LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
CONTINUE
CLOSE (IN)
CLOSE (IO)
STOP

FORMAT STATEMENTS
101 FORMAT(204)
102 FORMAT(205)
103 FORMAT(206)
104 FORMAT(207)
105 FORMAT(208)
106 FORMAT(209)
107 FORMAT(210)
108 FORMAT(211)
109 FORMAT(212)
110 FORMAT(213)
111 FORMAT(214)
112 FORMAT(215)
113 FORMAT(216)
114 FORMAT(217)
115 FORMAT(218)
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117 FORMAT(220)
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151 FORMAT(254)
152 FORMAT(255)
153 FORMAT(256)
154 FORMAT(257)
155 FORMAT(258)
156 FORMAT(259)
157 FORMAT(260)
158 FORMAT(261)
159 FORMAT(262)
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

0

\[ \text{FORMAT} \{1H} ,25X, 'WITH A FINITE-WIDTH (STRIP) SOLUTE SOURCE AT X=0.0' \]

\[ 4 \text{ ///1H0,40X, 'INPUT DATA' ///1H ,40X,10(1H-)} \]

\[ 205 \text{ FORMAT} \{1H} ,25X, 'NUMBER OF X-COORDINATES (NX) = ',14/1H ,25X, \]

\[ 1 \text{ 'NUMBER OF Y-COORDINATES (NY) = ',14/1H ,25X,} \]

\[ 2 \text{ 'NUMBER OF TIME VALUES (NT) = ',14/1H ,25X,} \]

\[ 3 \text{ 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',14} \]

\[ 210 \text{ FORMAT} \{1H} ,25X, 'SOLUTE CONCENTRATION ON MODEL BOUNDARY (CO) =', \]

\[ 1 1\text{P1E13.6,1X,1A10/1H ,25X,} \]

\[ 2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,1A10/ \]

\[ 3 1H ,25X, 'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,1A10/ \]

\[ 4 1H ,25X, 'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,1A10/ \]

\[ 5 1H ,25X, 'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,1A10/ \]

\[ 212 \text{ FORMAT} \{1H} ,25X, 'AQUIFER WIDTH (W) IS INFINITE'///1H ,25X, \]

\[ 1 'SOLUTE SOURCE IS LOCATED BETWEEN Y1 =',1P1E13.6,1X,1A10/1H ,54X, \]

\[ 2 'AND Y2 =',1P1E13.6,1X,1A10/ \]

\[ 1 215 \text{ FORMAT} \{1H} ,25X, 'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', \]

\[ 1 'WILL BE CALCULATED IN ',1A10/1H ,25X,78(1H-)/ \]

\[ 1 216 \text{ FORMAT} \{1H} ,25X, 'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ', \]

\[ 1 'WILL BE CALCULATED IN ',1A10/1H ,25X,78(1H-)/ \]

\[ 1 220 \text{ FORMAT} \{1H} ,5X,8F12.4 \]

\[ 1 225 \text{ FORMAT} \{1H} ,25X, 'TIMES AT WHICH SOLUTE CONCENTRATIONS ' \]

\[ 1 'WILL BE CALCULATED IN ',1A10/1H ,25X,78(1H-)/ \]

\[ 1 227 \text{ FORMAT} \{1H} ,25X, 'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/ \]

\[ 1 229 \text{ FORMAT} \{1H} ,25X, 'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/ \]

\[ 1 220 \text{ FORMAT} \{1H} ,15X, 'SOLUTE CONCENTRATION AT TIME =', \]

\[ 1 F12.4,1X,1A10/ \]

\[ 1 231 \text{ FORMAT} \{1H} ,1H,15X, 'SOLUTE CONCENTRATION AT TIME = ', \]

\[ 1 F12.4,1X,1A10,5X, '(CONTINUED)'///1H ,15X,1A10/ \]

\[ 1 235 \text{ FORMAT} \{1H} ,20X,9F12.4 \]

\[ 1 236 \text{ FORMAT} \{1H} ,19X, '*,108(1H-)/ \]

\[ 1 240 \text{ FORMAT} \{1H} ,5X,9F12.4,2X, '*'///9F12.5 \]

\[ 1 241 \text{ FORMAT} \{1H} ,19X, '!'/// \]

\[ \text{END} \]

\[ \text{SUBROUTINE CNRMLI(DK,T,X,Y,Y1,Y2,DX,DY,VX,CN,NMAX)} \]

\[ \text{COMMON /IOUNIT/ IN,10} \]

\[ \text{COMMON /GLPTS/ WN(256),ZN(256)} \]

\[ \text{THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y} \]

\[ \text{BASED ON THE ANALYTIC SOLUTION TO THE TWO-DIMENSIONAL} \]

\[ \text{ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-} \]

\[ \text{INFINITE AQUIFER OF INFINITE WIDTH. A FINITE-WIDTH (STRIP)} \]

\[ \text{SOLUTE SOURCE EXTENDS FROM Y-Y1 TO Y-Y2. THE SOLUTION CONTAINS} \]

\[ \text{AN INTEGRAL FROM 0 TO T**.25 WHICH IS EVALUATED USING A GAUSS-} \]

\[ \text{LEGENDRE QUADRATURE NUMERICAL INTEGRATION TECHNIQUE.} \]

\[ \text{CN}=3.141592653589797D0 \]

\[ \text{PI}=3.141592653589797D0 \]
FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
IF(T.LE.0.0D0) RETURN

FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS
IF(X.GT.0.0D0) GO TO 10
IF(Y.GT.Y1 .AND. Y.LT.Y2) CN=1.0D0
IF(Y.EQ.Y1) CN=0.50D0
IF(Y.EQ.Y2) CN=0.50D0
RETURN

START NUMERICAL INTEGRATION LOOP
SUM=0.0D0
DO 20 I=1,NMAX

SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE
NON-NORMALIZED LIMITS OF INTEGRATION
LIMITS OF INTEGRATION ARE FROM 0 TO T**0.25
TT=T**0.25D0
WI=WN(I)
ZI=TT*(ZN(I)+1.0D0)/2.0D0
ZSQ=ZI*ZI
Z4=ZSQ*ZSQ

TERM 1
XVT=X-VX*Z4
EXP1=-XVT*XVT/(4.0D0*DX*Z4)-DK*Z4
ERFCl=(Y1-Y)/(2.0D0*ZSQ*DSQRT(DY))
CALL EXERFC(EXP1,ERFCl,Z1)

TERM 2
ERFC2=(Y2-Y)/(2.0D0*ZSQ*DSQRT(DY))
CALL EXERFC(EXP1,ERFCl,Z1)
TERM=(Z1-Z2)*WI/(ZI*ZSQ)
SUM=SUM+TERM
CONTINUE
SUM=SUM*TT/2.0D0
CN=SUM*X/DSQRT(PI*DX))
RETURN
END
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

***GAUSS***

TWO-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL

FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH. A

SOURCE HAVING A GAUSSIAN-SHAPED CONCENTRATION DIS-

TRIBUTION IS LOCATED AT X=0 AND CENTERED ABOUT Y=YC

GROUND-WATER FLOW IN X-DIRECTION ONLY

VERSION CURRENT AS OF 04/01/90

THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE

GREATER THAN THOSE GIVEN HERE.

MAXX = MAXIMUM NUMBER OF X-VALUES

MAXY = MAXIMUM NUMBER OF Y-VALUES

MAXT = MAXIMUM NUMBER OF TIME VALUES

MAXXY = MAXX * MAXY

MAXXY2 = 2 * MAXX * MAXY

PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS

REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP

DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)

COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),

1 IFLAG(MAXXY2)

COMMON /IOUNIT/ IN,10

PROGRAM VARIABLES

NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE

MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS

LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.

CM MAXIMUM SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L*]

DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]

DY TRANSVERSE DISPERSION COEFFICIENT [L**2/T]

VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T]

DK FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]

X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]

Y Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]

T TIME AT WHICH CONCENTRATION IS EVALUATED [T]

CN NORMALIZED CONCENTRATION C/CM [DIMENSIONLESS]

CXY SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]

YC Y-COORDINATE OF THE CENTER OF SOLUTE SOURCE AT X=0 [L]
TECHNIQUES OF WATER-RESOURCES INVESTIGATIONS

SIGMA STANDARD DEVIATION OF GAUSSIAN CONCENTRATION DISTRIBUTION FOR THE SOLUTE SOURCE [L]

NX NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED

NY NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED

NT NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED

NMAX NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)

IPLT PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED

XSCLP SCALING FACTOR TO CONVERT X TO PLOTTER INCHES

YSCLP SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES

DELTA CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)

CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS

CUNITS UNITS OF CONCENTRATION (M/L**3)

VUNITS UNITS OF GROUND-WATER VELOCITY (L/T)

DUNITS UNITS OF DISPERSION COEFFICIENT (L**2/T)

KUNITS UNITS OF SOLUTE DECAY CONSTANT (1/T)

LUNITS UNITS OF LENGTH (L)

TUNITS UNITS OF TIME (T)

DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE

CALL OFILE

CALL TITLE

WRITE(IO,201)

READ IN MODEL PARAMETERS

READ(IN,101) NX,NY,NT,NMAX,IPLT

WRITE(IO,205) NX,NY,NT,NMAX

READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS

READ(IN,110) CM,VX,DX,DY,DK

WRITE(IO,210) CM,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS

READ(IN,110) YC,SIGMA

WRITE(IO,212) YC,LUNITS,SIGMA,LUNITS

READ(IN,110) (X(I),I=1,NX)

WRITE(IO,215) LUNITS

READ(IN,120) (Y(I),I=1,NY)

WRITE(IO,216) LUNITS

READ(IN,120) (Y(I),I=1,NY)

WRITE(IO,216) LUNITS

READ(IN,120) (T(I),I=1,NT)

WRITE(IO,225) TUNITS

WRITE(IO,220) (T(I),I=1,NT)

IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA

IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS

READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS

CALL GLQPTS (NMAX)

BEGIN TIME LOOP

DO 20 IT=1,NT


C BEGIN X LOOP
    DO 40 IX=1,NX
C
C CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)
    DO 50 IY=1,NY
    CALL CNRMLG(DK,T(IT),X(IX),Y(IY),YC,SIGMA,DX,DY,VX,CN,NMAX)
    CXY(IX,IY)=CM*CN
    CONTINUE
C
C PRINT OUT TABLES OF CONCENTRATION VALUES
    NG=1+(NY-1)/9
    DO 60 NP=1,NG
       IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS
       IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS
       NP1=(NP-1)*9
C
C CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY CM TO PLOT
C CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
    IF(IPLT.LT.1) GO TO 20
    NX=NX*NY
    IF=(IX-I)*NY
    XP(I)=SNGL(X(I))
    DO 80 J=1,NY
       IF(I.EQ.1) YP(J)=SNGL(Y(J))
       CP(IP+J)=SNGL(CXY(I,J)/CM)
       CONTINUE
   TP=SNGL(T(IT))
    NXY2=NXY*2
    CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,TUNITS,
     LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
C
C FORMAT STATEMENTS
101 FORMAT(2014)
105 FORMAT(8A10)
TECHNIQUES OF WATER-RESOURCES INVESTIGATIONS

110 FORMAT(8F10.0)
111
112 FORMAT(////1H,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/
113 1H ,28X,'ADVOLUTE DISPERSIVE SOLUTE TRANSPORT EQUATION'/
114 1H ,29X,'FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH'/
115 1H ,25X,'WITH A SOLUTE SOURCE HAVING A GAUSSIAN CONCENTRATION'/
116 4H ,24X,'DISTRIBUTION LOCATED AT X=0.0 AND CENTERED ABOUT Y=YC' /
117 ////1H0,40X,'INPUT DATA'/1H ,40X,10(1H-))

205 FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
206 1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
207 2 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,
208 3 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4)

219 FORMAT(1H0,25X,'AQUIFER WIDTH (W) IS INFINITE'/1H ,25X,
219 1 'SOLUTE SOURCE IS CENTERED AT Y = ',1P1E13.6,1X,A10/1H ,25X,
220 2 'STANDARD DEVIATION OF GAUSSIAN DISTRIBUTION (SIGMA) = ',
221 3 1P1E13.6,1X,A10)

255 FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
255 1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)

265 FORMAT(1H0,25X,'Y COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
265 1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)

295 FORMAT(1H,5X,8F12.4) 0

114 THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y
114 BASED ON THE ANALYTIC SOLUTION TO THE TWO-DIMENSIONAL
114 ADVOLUTE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-
114 INFINITE AQUIFER OF INFINITE WIDTH. THE SOLUTE SOURCE, LOCATED
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

AT X=0.0 AND CENTERED ABOUT Y=YC, HAS A GAUSSIAN CONCENTRATION DISTRIBUTION WITH A STANDARD DEVIATION OF SIGMA. THE SOLUTION CONTAINS AN INTEGRAL FROM 0 TO T**0.25 WHICH IS EVALUATED USING A GAUSS-LEGENDRE QUADRATURE INTEGRATION TECHNIQUE.

PI=3.14159265358979D0
Y1=Y-YC
SIGSQ=SIGMA*SIGMA
BETA=DK+VX*VX/(4.0D0*DX)
VX2D=VX*X/(2.0D0*DX)
CN=0.0D0

FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
IF(T.LE.0.0D0) RETURN

FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS
IF(X.GT.0.0D0) GOTO 10
CN=DEXP(-Y1*Y1/(2.0D0*SIGSQ))
RETURN

START NUMERICAL INTEGRATION LOOP

SUM=0.0D0
DO 20 I=1,NMAX

SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE NON-NORMALIZED LIMITS OF INTEGRATION LIMITS OF INTEGRATION ARE FROM 0 TO T**0.25

TT=T**0.25D0
WI=WN(I)
ZI=TT*(ZN(I)+1.0D0)/2.0D0

TERM 1
Z4=ZI**4
ALPHA=Z4*DY + SIGSQ/2.0D0
X1=DEXP(-Z4*BETA -X*X/(4.0D0*X**4) -Y1*Y1/(4.0D0*A1.PHA))
X1=X1/((ZI*ZI*ZI)*DSQRT(ALPHA))
SUM=SUM+X1*WI
CONTINUE

SUM=SUM*TT/2.0D0

TERM 2
X2=SUM*2.0D0*DEXP(VX2D)
CN=X*SIGMA*X2/(DSQRT(2.0D0*PI*DX))
RETURN
END
THREE-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL FOR AN AQUIFER OF INFINITE EXTENT WITH A CONTINUOUS POINT SOURCE AT X=XC, Y=YC, AND Z=ZC

GROUND-WATER FLOW IN X-DIRECTION ONLY

VERSION CURRENT AS OF 04/01/90

THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE GREATER THAN THOSE GIVEN HERE.

MAXX = MAXIMUM NUMBER OF X-VALUES
MAXY = MAXIMUM NUMBER OF Y-VALUES
MAXZ = MAXIMUM NUMBER OF Z-VALUES
MAXT = MAXIMUM NUMBER OF TIME VALUES
MAXXY = MAXX * MAXY

PARAMETER MAXX=100,MAXY=50,MAXZ=30,MAXT=20,MAXXY=5000,MAXXY2=10000

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,QUNITS,TUNITS
REAL XP,YP,ZP,CP,TP,DELTA,XPC,YPC,XPCL,YPC,YPCL
DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),Z(MAXZ),T(MAXT)
COMMON /PDAT/ ~P(MAXX),YP(MAXY),~P(MAXXY),~P~(~~),~P~(~~),
1 IFLAG(MAXXY2)
COMMON /IOUNIT/ IN,10 z2

PROGRAM VARIABLES

NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.

CO SOLUTE CONCENTRATION IN INJECTED FLUID [M/L**3]
DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
DY TRANSVERSE (Y-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
DZ TRANSVERSE (Z-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
DK FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
Y Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
Z Z-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
T TIME AT WHICH CONCENTRATION IS EVALUATED [T]
CN NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
CXY SOLUTE CONCENTRATION C(X,Y,Z,T) [M/L**3]
**ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW**

**C**

XC X-COORDINATE OF CONTINUOUS POINT SOURCE [L] 54
YC Y-COORDINATE OF CONTINUOUS POINT SOURCE [L] 55
ZC Z-COORDINATE OF CONTINUOUS POINT SOURCE [L] 56
QM FLUID INJECTION RATE [L**3/T] 57
POR AQUIFER POROSITY [DIMENSIONLESS] 58

NX NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED 60
NY NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED 61
NZ NUMBER OF Z-POSITIONS AT WHICH SOLUTION IS EVALUATED 62
NT NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED 63

IPLT PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED 65
XSCLP SCALING FACTOR TO CONVERT X TO PLOTTER INCHES 66
YSCLP SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES 67
DELTA CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0) 68

CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS 70
CUNITS UNITS OF CONCENTRATION (M/L**3) 71
VUNITS UNITS OF GROUND-WATER VELOCITY (L/T) 72
DUNITS UNITS OF DISPERSION COEFFICIENT (L**2/T) 73
KUNITS UNITS OF SOLUTE DECAY CONSTANT (1/T) 74
LUNITS UNITS OF LENGTH (L) 75
QUNITS UNITS OF FLUID INJECTION RATE (L**3/T) 76
TUNITS UNITS OF TIME (T) 77
80

DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE 79
CALL OFILE 80
CALL TITLE 81
WRITE(IO,201) 82

READ IN MODEL PARAMETERS 84
READ(IN,101) NX,NY,NZ,NT,IPLT 85
WRITE(IO,205) NX,NY,NZ,NT 86
READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,QUNITS,TUNITS 87
READ(IN,110) CO,VX,DX,DY,DZ,DK 88
WRITE(IO,210) CO,CUNITS,VX,WNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS, 89
1 DK,KUNITS 90
READ(IN,110) XC,YC,ZC,QM,POR 91
WRITE(IO,212) XC,LUNITS,YC,LUNITS,ZC,LUNITS,QM,QUNITS,POR 92
READ(IN,110) (X(I),I=1,NX) 93
WRITE(IO,215) LUNITS 94
WRITE(IO,220) (X(I),I=1,NX) 95
READ(IN,110) (Y(I),I=1,NY) 96
WRITE(IO,216) LUNITS 97
WRITE(IO,220) (Y(I),I=1,NY) 98
READ(IN,110) (Z(I),I=1,NZ) 99
WRITE(IO,217) LUNITS 100
WRITE(IO,220) (Z(I),I=1,NZ) 101
READ(IN,110) (T(I),I=1,NT) 102
WRITE(IO,225) TUNITS 103
WRITE(IO,220) (T(I),I=1,NT) 104
IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA 105
IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS 106
BEGIN TIME LOOP
DO 20 IT=1,NT

BEGIN Z LOOP
DO 30 IZ=1,NZ
ZZ=Z(IZ)-ZC

BEGIN X LOOP
DO 40 IX=1,NX
XX=X(IX)-XC
CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) AND Z=Z(IZ)
DO 50 IY=1,NY
YY=Y(IY)-YC
CALL CNRML3(QM,POR,DK,T(IT),XX,YY,ZZ,DX,DY,DZ,VX,CN)
CXY(IX,IY)=CO*CN
CONTINUE
CONTINUE
PRINT OUT TABLES OF CONCENTRATION VALUES
NPAGE=1+(NY-1)/9
DO 60 NP=1,NPAGE
IF(NP.EQ.1) WRITE(I0,230) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
IF(NP.NE.1) WRITE(I0,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
NP1=(NP-1)*9
NP2=9
IF((NP1+NP2).GT.NY) NP2=NY-NP1
WRITE(I0,235) (Y(NPl+J),J=1,NP2)
WRITE(IO,236) CUNITS,LUNITS
DO 70 IX=1,NX
WRITE(IO,240) X(IX),(CXY(IX,NPl+J),J=1,NP2)
IF(MOD(IX,45).NE.O) GO TO 70
WRITE(I0,231) T(IT),TUNITS,Z(IZ),LUNITS,W
WRITE(I0,235) (Y(NPl+J),J=1,NP2)
WRITE(I0,236) CUNITS,LUNITS
IF(MOD(IX,5).EQ.O .AND. MOD(IX,45).NE.O) WRITE(I0,241)
CONTINUE
CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
IF(IPLT.LT.1) GO TO 30
NPX=NX+NY
DO 80 I=1,NX
X=1-(I-1)*NY
XP(I)=SNGL(X(I))
DO 80 J=1,NY
IF(I.EQ.1) YP(J)=SNGL(Y(J))
CP(I*J)=SNGL(CXY(I,J)/CO)
CONTINUE
TP=SNGL(T(IT))
ZP=SNGL(Z(IZ))
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

119

NXY2 = NXY * 2
CALL PLOT3D (XP,YP,ZP,CP,TP,DELTA,NX,XY,NXY,NXY2,I2,NZ,IPHT,
1 TUNITL,ULNITS,XSCLP,YSCLP,XPC,YPC,IFILAC)
CONTINUE
CONTINUE
CLOSE (IN)
CLOSE (IO)
STOP

FORMAT STATEMENTS

1 FORMAT(2014)
2 FORMAT(8AlO)
3 FORMAT(8FlE13.6)
4 ///lH ,25X, 'ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL' /
5 1 /H ,28X, 'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
6 2 1H ,34X, 'FOR AN AQUIFER OF INFINITE EXTENT' /
7 3 1H ,30X, 'WITH A CONTINUOUS POINT SOURCE AT XC,YC,ZC' /
8 4 // /H0,40X, 'INPUT DATA'/H ,25X, 1O(lH-)
9 /H0,25X, 'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
10 1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
11 2 'NUMBER OF Z-COORDINATES (NZ) = ',I4/1H ,25X,
12 3 'NUMBER OF TIME VALUES (NT) = ',I4
13 /H0,25X, 'AQUIFER IS OF INFINITE EXTENT'
14 2 /H0,25X, 'CONTINUOUS POINT SOURCE IS AT X =',1PlE13.6, 1X,A10/
15 3 1H ,25X, 'Y =',1PlE13.6,1X,A10/1H ,55X, 'Z =',1PlE13.6,1X,A10/
16 5 1H ,25X, 'FLUID INJECTION RATE (QM) =',1PlE13.6,1X,A10/
17 6 1H ,25X, 'AQUIFER POROSITY (POR) =',1PlE13.6
18 /H0,25X, 'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
19 1 'WILL BE CALCULATED, IN ',A10/1H ,25X,
20 2 'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
21 1 'WILL BE CALCULATED, IN ',A10/1H ,25X,
22 1 'WILL BE CALCULATED, IN ',A10/1H ,25X,
23 1 'WILL BE CALCULATED, IN ',A10/1H ,25X,
24 /H0,25X, 'TIMES AT WHICH SOLUTE CONCENTRATIONS '
25 1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)
26 /H0,25X, 'PLOT SCALING FACTOR FOR X (XSCLP) =',1PlE13.6/
27 1 'PLOT SCALING FACTOR FOR Y (YSCLP) =',1PlE13.6/
28 2 1H ,25X, 'DISPERSION IN THE X-DIRECTION (DX) =',1PlE13.6,1X,A10/
29 3 1H ,25X, 'DISPERSION IN THE Y-DIRECTION (DY) =',1PlE13.6,1X,A10/
30 4 1H ,25X, 'DISPERSION IN THE Z-DIRECTION (DZ) =',1PlE13.6,1X,A10/
31 5 1H ,25X, 'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1PlE13.6,1X,A10
32 /H0,25X, 'SOLUTE CONCENTRATION AT TIME =',
33 1 F12.4,1X,A10/1H ,78(1H-)/
34 2 1H ,25X, 'SOLUTE CONCENTRATION AT TIME =',
35 1 F12.4,1X,A10/1H ,78(1H-)/
36 2 1H ,25X, 'SOLUTE CONCENTRATION AT TIME =',
37 1 F12.4,1X,A10/1H ,78(1H-)/
SUBROUTINE CNRML3(QM,POR,DK,T,X,Y,Z,DX,DY,DZ,VX,CN)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
COMMON /IOUNIT/ IN,10
C
C THIS ROUTINE CALCULATES SOLUTE CONCENTRATION AT X,Y,Z BASED ON
THE ANALYTIC SOLUTION TO THE THREE-DIMENSIONAL ADVECTIVE-
DISPERSE SOLUTE TRANSPORT EQUATION FOR AN AQUIFER OF
INFINITE EXTENT WITH A CONTINUOUS POINT SOURCE LOCATED AT
X=XC, Y=YC, AND Z=ZC. A CLOSED FORM SOLUTION WAS OBTAINED.
C
PI=3.14159265358979DO
CN=0.0DO
C
FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
IF(T.LE.0.0DO) RETURN
C
CHECK FOR X=Y=Z=0
IF(X.EQ.0.0DO .AND. Y.EQ.0.0DO .AND. Z.EQ.0.0DO) THEN
WRITE(IO,200)
RETURN
END IF
C
BETA=DSQRT(VX*VX+4.0DO*DX*DK)
GAMMA=DSQRT(X*X+Y*Y*DX/DY+Z*Z*DX/DZ)
RTDXT=2.0DO*DSQRT(DX*T)
TERM 1
X1=(VX*X-GAMMA*BETA)/(2.0DO*DX)
Y1=(GAMMA-BETA*T)/RTDXT
CALL EXERFC(X1,Y1,22)
TERM 2
X2=(VX*X+GAMMA*BETA)/(2.0DO*DX)
Y2=(GAMMA+BETA*T)/RTDXT
CALL EXERFC(X2,Y2,22)
TERM 3
Z3=Z1+Z2
Z4=DSQRT(DY*Z4)
CN=QM*Z3/(8.0DO*POR*PI*GAMMA*Z4)
RETURN
C
C FORMAT STATEMENTS
200 FORMAT (1HO,5X,'**** WARNING **** A SOLUTION CAN NOT BE COMPUTED'
1 ' FOR X=XC,Y=YC,Z=ZC/')
END
C
C******************************************************************************
C **** PATCHF ****
C
C THREE-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT
C MODEL FOR A SEMI-INFINITE AQUIFER WITH A FINITE WIDTH AND HEIGHT. A PATCH SOURCE EXTENDS FROM Y1 TO Y2 AND Z1 TO Z2 AT X=0
C GROUND-WATER FLOW IN X-DIRECTION ONLY
C VERSION CURRENT AS OF 04/01/90
C
C******************************************************************************
C
THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE GREATER THAN THOSE GIVEN HERE.

MAXX = MAXIMUM NUMBER OF X-VALUES
MAXY = MAXIMUM NUMBER OF Y-VALUES
MAXZ = MAXIMUM NUMBER OF Z-VALUES
MAXT = MAXIMUM NUMBER OF TIME VALUES
MAXXY = MAXX * MAXY
MAXXY2 = 2 * MAXX * MAXY

PARAMETER MAXX=100,MAXY=50,MAXZ=30,MAXT=20,MAXXY=5000,MAXXY2=10000

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
CHARACTER*1 IERR(MAXX,MAXY)
REAL XP,YP,ZP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),Z(MAXZ),T(MAXT)
COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),IFLAG(MAXXY2)
COMMON /IOUNIT/ IN,10

PROGRAM VARIABLES

NOTE: ANY CONSISTANT SET OF UNITS MAY BE USED IN THE MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.

C0 SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
DX LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
DY TRANSVERSE (Y-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
DZ TRANSVERSE (Z-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
VX GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
DK FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
X X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
Y Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
Z Z-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
TECHNIQUES OF WATER-RESOURCES INVESTIGATIONS

T
TIME AT WHICH CONCENTRATION IS EVALUATED [T]

CN
NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]

CX Y
SOLUTE CONCENTRATION C(X,Y,Z,T) [M/L**3]

W
AQUIFER WIDTH (AQUIFER EXTENDS FROM Y=0 TO Y=W) [L]

H
AQUIFER HEIGHT (AQUIFER EXTENDS FROM Z=0 TO Z=H) [L]

Y 1
Y-COORDINATE OF LOWER LIMIT OF PATCH SOLUTE SOURCE [L]

Y 2
Y-COORDINATE OF UPPER LIMIT OF PATCH SOLUTE SOURCE [L]

Z 1
Z-COORDINATE OF LOWER LIMIT OF PATCH SOLUTE SOURCE [L]

Z 2
Z-COORDINATE OF UPPER LIMIT OF PATCH SOLUTE SOURCE [L]

NX
NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED

NY
NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED

NZ
NUMBER OF Z-POSITIONS AT WHICH SOLUTION IS EVALUATED

NT
NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED

NMAX
NUMBER OF TERMS USED IN INNER INFINITE SERIES SUMMATION

MMAX
NUMBER OF TERMS USED IN OUTER INFINITE SERIES SUMMATION

IPLT
PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED

XSCLP
SCALING FACTOR TO CONVERT X TO PLOTTER INCHES

YSCLP
SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES

DELTA
CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)

CUNITS
UNITS OF CONCENTRATION (M/L**3)

VUNITS
UNITS OF GROUND-WATER VELOCITY (L/T)

DUNITS
UNITS OF DISPERSION COEFFICIENT (L**2/T)

KUNITS
UNITS OF SOLUTE DECAY CONSTANT (L/T)

LUNITS
UNITS OF LENGTH (L)

TUNITS
UNITS OF TIME (T)

DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE

CALL OFILE
CALL TITLE
WRITE(IO,201)

READ IN MODEL PARAMETERS

READ(IO,101) NX,NY,NZ,NT,NMAX,MMAX,IPLT
WRITE(IO,205) NX,NY,NZ,NT,NMAX
READ(IO,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
READ(IO,110) CO,VX,DX,DY,DZ,DK
WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS,
1 DK, KUNITS
READ(IO,110) W,H,Y 1,Y 2,Z 1,Z 2
WRITE(IO,212) W,LUNITS,H,LUNITS,Y 1,LUNITS,Y 2,LUNITS,Z 1,LUNITS,
1 Z 2, LUNITS
READ(IO,110) (X(I),I=1,NX)
WRITE(IO,215) LUNITS
WRITE(IO,220) (X(I),I=1,NX)
READ(IO,110) (Y(I),I=1,NY)
WRITE(IO,216) LUNITS
WRITE(IO,220) (Y(I),I=1,NY)
READ(IO,110) (Z(I),I=1,NZ)
WRITE(IO,217) LUNITS

DELTA
CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)

IPLT
PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED

XSCLP
SCALING FACTOR TO CONVERT X TO PLOTTER INCHES

YSCLP
SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES

CUNITS
UNITS OF CONCENTRATION (M/L**3)

VUNITS
UNITS OF GROUND-WATER VELOCITY (L/T)

DUNITS
UNITS OF DISPERSION COEFFICIENT (L**2/T)

KUNITS
UNITS OF SOLUTE DECAY CONSTANT (L/T)

LUNITS
UNITS OF LENGTH (L)

TUNITS
UNITS OF TIME (T)

DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE

CALL OFILE
CALL TITLE
WRITE(IO,201)

READ IN MODEL PARAMETERS

READ(IO,101) NX,NY,NZ,NT,NMAX,MMAX,IPLT
WRITE(IO,205) NX,NY,NZ,NT,NMAX
READ(IO,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
READ(IO,110) CO,VX,DX,DY,DZ,DK
WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS,
1 DK, KUNITS
READ(IO,110) W,H,Y 1,Y 2,Z 1,Z 2
WRITE(IO,212) W,LUNITS,H,LUNITS,Y 1,LUNITS,Y 2,LUNITS,Z 1,LUNITS,
1 Z 2, LUNITS
READ(IO,110) (X(I),I=1,NX)
WRITE(IO,215) LUNITS
WRITE(IO,220) (X(I),I=1,NX)
READ(IO,110) (Y(I),I=1,NY)
WRITE(IO,216) LUNITS
WRITE(IO,220) (Y(I),I=1,NY)
READ(IO,110) (Z(I),I=1,NZ)
WRITE(IO,217) LUNITS

DELTA
CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)

IPLT
PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED

XSCLP
SCALING FACTOR TO CONVERT X TO PLOTTER INCHES

YSCLP
SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES

CUNITS
UNITS OF CONCENTRATION (M/L**3)

VUNITS
UNITS OF GROUND-WATER VELOCITY (L/T)

DUNITS
UNITS OF DISPERSION COEFFICIENT (L**2/T)

KUNITS
UNITS OF SOLUTE DECAY CONSTANT (L/T)

LUNITS
UNITS OF LENGTH (L)

TUNITS
UNITS OF TIME (T)

DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE

CALL OFILE
CALL TITLE
WRITE(IO,201)

READ IN MODEL PARAMETERS

READ(IO,101) NX,NY,NZ,NT,NMAX,MMAX,IPLT
WRITE(IO,205) NX,NY,NZ,NT,NMAX
READ(IO,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
READ(IO,110) CO,VX,DX,DY,DZ,DK
WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS,
1 DK, KUNITS
READ(IO,110) W,H,Y 1,Y 2,Z 1,Z 2
WRITE(IO,212) W,LUNITS,H,LUNITS,Y 1,LUNITS,Y 2,LUNITS,Z 1,LUNITS,
1 Z 2, LUNITS
READ(IO,110) (X(I),I=1,NX)
WRITE(IO,215) LUNITS
WRITE(IO,220) (X(I),I=1,NX)
READ(IO,110) (Y(I),I=1,NY)
WRITE(IO,216) LUNITS
WRITE(IO,220) (Y(I),I=1,NY)
READ(IO,110) (Z(I),I=1,NZ)
WRITE(IO,217) LUNITS

DELTA
CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)

IPLT
PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED

XSCLP
SCALING FACTOR TO CONVERT X TO PLOTTER INCHES

YSCLP
SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES

CUNITS
UNITS OF CONCENTRATION (M/L**3)

VUNITS
UNITS OF GROUND-WATER VELOCITY (L/T)

DUNITS
UNITS OF DISPERSION COEFFICIENT (L**2/T)

KUNITS
UNITS OF SOLUTE DECAY CONSTANT (L/T)

LUNITS
UNITS OF LENGTH (L)

TUNITS
UNITS OF TIME (T)

DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE

CALL OFILE
CALL TITLE
WRITE(IO,201)
WRITE(IO,220) (Z(I),I-1,NZ)
READ(IN,110) (T(I),I-1,NT)
WRITE(IO,225) TUNITS
WRITE(IO,220) (T(I),I-1,NT)
IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA
IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS

C BEGIN TIME LOOP
DO 20 IT-1,NT
C BEGIN Z LOOP
DO 30 IZ-1,NZ
C BEGIN X LOOP
DO 40 IX-1,NX
C CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) AND Z=Z(IZ)
DO 50 IY-1,NY
CALL CNRMLP(DK,T(IT),X(IX),Y(IY),Z(IZ),W,H,Y1,Y2,Z1,Z2,DX,
1 DY,DZ,VX,CN,NMAX,MMAX,IERR(IX,IY))
CXY(IX,IY)=CO*CN
CONTINUE
CONTINUE

C PRINT OUT TABLES OF CONCENTRATION VALUES
NPAGE=1+(NY-1)/9
DO 60 NP-1,NPAGE
IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
NP1=(NP-1)*9
NP2=9
IF((NP1+NP2).GT.NY) NP2=NY-NP1
WRITE(IO,235) (Y(NP1+J),J-1,NP2)
WRITE(IO,236) CUNITS,LUNITS
DO 70 IX-1,NX
WRITE(IO,240) X(IX),(CXY(IX,NP1+J),IERR(IX,NP1+J),J-1,NP2)
IF(MOD(IX,45).NE.0) GO TO 70
WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
WRITE(IO,235) (Y(NP1+J),J-1,NP2)
WRITE(IO,236) CUNITS,LUNITS
70 IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
CONTINUE

C CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY CO TO PLOT
C CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
IF(IPLT.LT.1) GO TO 30
NXY=NX*NY
DO 80 I-1,NX
IP=(I-1)*NY
XP(I)=SNGL(X(I))
DO 80 J-1,NY
IF(J.EQ.1) YP(J)=SNGL(Y(J))
CP(IP+J)-SNGL(CXY(I,J)/CO)
CONTINUE
CP=SNGL(T(IT))
ZP=SNGL(Z(IZ))
NXY2=NXY*2
CALL PLOT3D(XP,YP,ZP,CP,TP,DELTA,NX,NY,NXY2,IZ,NZ,IPLT,
1 TUNITS,LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
CONTINUE
CONTINUE
CLOSE(IN)
CLOSE(IO)
STOP
C
C FORMAT STATEMENTS

205 FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',14/1H ,25X,
1 'NUMBER OF Y-COORDINATES (NY) = ',14/1H ,25X,
2 'NUMBER OF Z-COORDINATES (NZ) = ',14/1H ,25X,
3 'NUMBER OF TIME VALUES (NT) = ',14/1H ,25X,
4 'NUMBER OF TERMS IN INNER INFINITE SERIES SUMMATION (NMAX) = ',
5 14/1H,25X,'NUMBER OF TERMS IN OUTER INFINITE SERIES SUMMATION (MMAX) = ','
6 'AQUIFER WIDTH (W) = ',1PE13.6,1X,A10/1H ,25X,
7 'AQUIFER HEIGHT (H) = ',1PE13.6,1X,A10/1H ,25X,
8 'SOLUTE SOURCE IS LOCATED BETWEEN Y1 = ',1PE13.6,1X,A10/1H ,58X,
9 'Y2 = ',1PE13.6,1X,A10/1H ,58X,
10 'Z1 = ',1PE13.6,1X,A10/1H ,54X,
11 'AND Z2 = ',1PE13.6,1X,A10)

225 FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
1 'WILL BE CALCULATED, IN ','A10/1H ,25X,78(1H-)/)

227 FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) = ',1PE13.6/
1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) = ',1PE13.6/
ANALYTICAL SOLUTIONS FOR SOLUTE TRANSPORT IN GROUND WATER SYSTEMS WITH UNIFORM FLOW

2 1H,25X,'CONTOUR INCREMENT (DELTA) =', 1PE13.6, 1X, A10)

230 FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =', I12, 1X, A10)
  1 F12.4, 1X, A10/1H, 35X,'AND AT Z =', F12.4, 1X, A10,
  1 15X,'* INDICATES SOLUTION DID NOT CONVERGE'/
  2 1H0, 25X,'Y-COORDINATE, IN ', A10)

231 FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =', I12, 1X, A10)
  1 F12.4, 1X, A10/5X,'(CONTINUED)'/1H, 35X,'AND AT Z ='. F12.4, 1X,
  2 A10/1H0, 25X,'Y-COORDINATE, IN ', A10)

235 FORMAT(1H1, 20X, 9F12.4)

236 FORMAT(1H1, 19X,'*',108(1H-)/
  1 1H,4X,'X-COORDINATE,' ,2X,'!',44X,'SOLUTE CONCENTRATION, IN ')
  2 A10/1H,4X,'IN ', A10, 2X, 1H!/1H,19X,'!')

240 FORMAT(1H1, 5X, F12.4, 2X, '! ', 9(F11.5, A1))

241 FORMAT(1H1, 19X,'!')

END

SUBROUTINE CNRMLP(DK, T, X, Y, Z, W, H, Y1, Y2, Z1, Z2, DX, DY, DZ, VX, CN, NMAX, IERR)

IMPLICIT DOUBLE PRECISION(A-H,O-Z)

COMMON /IOUNIT/ IN, IO

THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y,Z
BASED ON THE ANALYTIC SOLUTION TO THE THREE-DIMENSIONAL
ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-INFINITE AQUIFER WITH A FINITE WIDTH AND HEIGHT. THE SOLUTE
SOURCE HAS A FINITE WIDTH AND HEIGHT, EXTENDING FROM Y=Y1 TO
Y=Y2 AND Z=Z1 TO Z=Z2. SOLUTE MAY BE SUBJECT TO FIRST-ORDER
CHEMICAL TRANSFORMATION. THE SOLUTION CONTAINS TWO INFINITE
SERIES SUMMATIONS WHICH MAY CONVERGE SLOWLY.

PI=3.14159265358979

CN=0.0DO

IERR=' '

FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
IF(T.LE.0.0DO) RETURN

FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS
IF(X.GT.0.0DO) GO TO 10
IF(Y.EQ.Y1.OR.Y.EQ.Y2) THEN
  IF(Z.GT.Z1.AND.Z.LT.Z2) CN=0.50DO
  IF(Z.EQ.Z1.OR.Z.EQ.Z2) CN=0.25DO
END IF
IF(Z.EQ.Z1.OR.Z.EQ.Z2) THEN
  IF(Y.GT.Y1.AND.Y.LT.Y2) CN=0.50DO
END IF
IF(Y.GT.Y1.AND.Y.LT.Y2.AND.Z.GT.Z1.AND.Z.LT.Z2) CN=1.0DO
RETURN

RTDXT=2.0DO*DSQRT(DX*T)

BEGIN SUMMATION OF TERMS IN INFINITE SERIES (OUTER SERIES)
NMAX1=NMAX+1
MMAX1=MMAX+1
TECHNIQUES OF WATER-RESOURCES INVESTIGATIONS

SIGMAN=0.0D0
SUBTM=0.0D0
DO 20 MM=1,MMAX1
M=MM-1
ZETA=M*PI/H
OM=(Z2-Z1)/H
IF(M.NE.0) OM=(DSIN(ZETA*Z2)-DSIN(ZETA*Z1))/(M*PI)
COSSZ=DCOS(ZETA*Z)
C
BEGIN SUMMATION OF TERMS IN INFINITE SERIES (INNER SERIES)
SIGMAN=0.0D0
SUBTN=0.0D0
DO 30 NN=1,NMAX1
N=NN-1
ETA=N*PI/W
PN=(Y2-Y1)/W
IF(N.NE.0) PN=(DSIN(ETA*Y2)-DSIN(ETA*Y1))/(N*PI)
COSRY=DCOS(ETA*Y)
ALPHA=4.0D0*DX*(ETA*ETA*DY+ZETA*ZETA*DZ+DK)
BETA=DSQRT(VX*VX+ALPHA)
BETAT=BETA*T
IF M>O AND N>0, USE GENERAL FORM
TERM 1
A1=X*(VX-BETA)/(2.0D0*DX)
B1=(X-BETAT)/RTDXT
CALL EXERFC(A1,B1,C1)
A2=X*(VX+BETA)/(2.0D0*DX)
B2=(X+BETAT)/RTDXT
CALL EXERFC(A2,B2,C2)
TERM1=COSRY*PN*(C1+C2)
MULTIPLY TERM BY L(MN)
IF(M.EQ.0 .AND. N.EQ.0) TERM1=TERM1*0.50D0
IF(M.GT.0 .AND. N.GT.0) TERM1=TERM1*2.0D0
30 CONTINUE
SIGMAN=SIGMAN+TERM1
C
ADD TERM TO SUMMATION
SIGMAN=SIGMAN+TERM1
C
CHECK FOR CONVERGENCE OF INNER SERIES. BECAUSE SERIES
OSCILLATES, CHECK SUBTOTAL OF LAST 10 TERMS.
SUBTN=SUBTN+TERM1
IF(MOD(NN,10).NE.0) GO TO 30
IF(DABS(SUBTN).LT.1.0D-12) GO TO 25
SUBTN=0.0D0
30 CONTINUE
IERR="*"
25 SIGMAN=SIGMAN+SIGMAN*COSSZ*OM
C
CHECK FOR CONVERGENCE OF OUTER SERIES. BECAUSE SERIES
OSCILLATES, CHECK SUBTOTAL OF LAST 10 TERMS.
SUBTM=SUBTM+SIGMAN*OM*COSSZ
IF(MOD(MM,10).NE.0) GO TO 20
IF(ABS(SUBTM).LT.1.0D-12) GO TO 35
SUBTM=0.0D0
CONTINUE
IERR='*
CN=SIGMA
RETURN
END
THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE GREATER THAN THOSE GIVEN HERE.

MAXX = MAXIMUM NUMBER OF X-VALUES
MAXY = MAXIMUM NUMBER OF Y-VALUES
MAXZ = MAXIMUM NUMBER OF Z-VALUES
MAXT = MAXIMUM NUMBER OF TIME VALUES
MAXXY = MAXX * MAXY
MAXXY2 = 2 * MAXX * MAXY

PARAMETER MAXX=100, MAXY=50, MAXZ=30, MAXT=20, MAXXY=5000, MAXXY2=10000

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
CHARACTER*10 CUNITS, VUNITS, DUNITS, KUNITS, LUNITS, TUNITS
REAL XP, YP, ZP, CP, TP, DELTA, XPC, YPC, XSCLP, YSCLP
DIMENSION CXY(MAXX, MAXY), X(MAXX), Y(MAXY), Z(MAXZ), T(MAXT)

COMMON /PDAT/ XP(MAXX), YP(MAXY), CP(MAXXY), XPC(50), YPC(50), IFLAG(MAXXY2)
COMMON /IOUNIT/ IN, IO

PROGRAM VARIABLES

NOTE: ANY CONSISTENT SET OF UNITS MAY BE USED IN THE MODEL. NO FORMAT STATEMENTS NEED TO BE CHANGED AS LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.

CO
SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]

DX
LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]

DY
TRANSVERSE (Y-DIRECTION) DISPERSION COEFFICIENT [L**2/T]

DZ
TRANSVERSE (Z-DIRECTION) DISPERSION COEFFICIENT [L**2/T]

VX
GROUND-WATER VELOCITY IN X-DIRECTION [L/T]

DK
FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]

X
X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]

Y
Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]

Z
Z-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]

T
TIME AT WHICH CONCENTRATION IS EVALUATED [T]

NORMALIZED CONCENTRATION $C/CO$ [DIMENSIONLESS]

SOLUTE CONCENTRATION $C(X,Y,Z,T)$ [M/L**3]

Y-COORDINATE OF LOWER LIMIT OF PATCH SOLUTE SOURCE [L]

Y-COORDINATE OF UPPER LIMIT OF PATCH SOLUTE SOURCE [L]

Z-COORDINATE OF LOWER LIMIT OF PATCH SOLUTE SOURCE [L]

Z-COORDINATE OF UPPER LIMIT OF PATCH SOLUTE SOURCE [L]

NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED

NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED

NUMBER OF Z-POSITIONS AT WHICH SOLUTION IS EVALUATED

NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED

NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)

PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED

SCALING FACTOR TO CONVERT X TO PLOTTER INCHES

SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES

CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)

UNITS OF CONCENTRATION (M/L**3)

UNITS OF GROUND-WATER VELOCITY (L/T)

UNITS OF DISPERSION COEFFICIENT (L**2/T)

UNITS OF SOLUTE DECAY CONSTANT (1/T)

UNITS OF LENGTH (L)

UNITS OF TIME (T)

DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE

READ IN MODEL PARAMETERS

WRITE(I0,201)

READ(IN,101) NX,NY,NZ,NT,NMAX,IPLT

WRITE(I0,205) NX,NY,NZ,NT,NMAX

READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS

READ(IN,110) CO,VX,DX,DY,DZ,DK

WRITE(I0,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS,

READ(IN,110) Y1,Y2,Z1,Z2

WRITE(I0,212) Y1,LUNITS,Y2,LUNITS,Z1,LUNITS,Z2,LUNITS

READ(IN,110) (X(I),I=1,NX)

WRITE(I0,215) LUNITS

READ(IN,220) (X(I),I=1,NX)

WRITE(I0,216) LUNITS

READ(IN,110) (Y(I),I=1,NX)

WRITE(I0,217) LUNITS

READ(IN,220) (Y(I),I=1,NX)

WRITE(IN,110) (Z(I),I=1,NX)

WRITE(IO,217) UNITS

READ(IN,220) (Z(I),I=1,NX)

WRITE(IO,225) UNITS

WRITE(IO,220) (T(I),I=1,NT)
IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA
IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS

READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS
CALL GLQPTS (NMAX)

BEGIN TIME LOOP
DO 20 IT=1,NT

BEGIN Z LOOP
DO 30 IZ=1,NZ

BEGIN X LOOP
DO 40 IX=1,NX

CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) AND Z=Z(IZ)
DO 50 IY=1,NY

CALL CNRMLP(DK,T(IT),X(IX),Y(IY),Z(IZ),Y1,Y2,Z1,Z2,DX,
DY,DZ,VX,CN,NMAX)

CXY(IX,IY)=CO*CN
CONTINUE
CONTINUE

PRINT OUT TABLES OF CONCENTRATION VALUES
NPAGE=1+(NY-1)/9
DO 60 NP=1,NPAGE

IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS

NP1=(NP-1)*9
NP2=9
IF((NP1+NP2).GT.NY) NP2=NY-NP1
WRITE(IO,235) (Y(NP1+J),J=1,NP2)
WRITE(IO,236) CUNITS,LUNITS

DO 70 IX=1,NX
WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)
IF(MOD(IX,45).NE.O) GO TO 70
WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
WRITE(IO,235) (Y(NP1+J),J=1,NP2)
WRITE(IO,236) CUNITS,LUNITS

IF(MOD(IX,5).EQ.O .AND. MOD(IX,45).NE.O) WRITE(IO,241)
CONTINUE

CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY CO TO PLOT
CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH
TIME VALUE.
IF(IPLT.LT.1) GO TO 30
NXY=NX*NY
DO 80 I=1,NX
IF=(I-1)*NY
XP(I)=SNGL(X(I))

IF(T.EQ.1) YP(J)=SNGL(Y(J))
CP(IP+J)=SNGL(CXY(I,J)/CO)
CONTINUE
TP=SNGL(T(IT))
ZP=SNGL(Z(IZ))
NXY2=NXY*2
CALL PLOT3D (XP,YP,ZP,CP,TP,DELTA,NX,NY,NXY2,IZ,NZ,IPLT,
1 TUNITS,LUNITS,XSCLP,YSCP,XPC,YPC,IFLAG)
CONTINUE
CONTINUE
CLOSE (IN)
CLOSE (IO)
STOP

FORMAT STATEMENTS
1 FORMAT(2014)
2 FORMAT(8A10)
3 FORMAT(1F10.0)

201 FORMAT(///1H ,29X,'ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL'
1 /1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
2 1H ,30X,'FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH'/
3 1H ,28X,'AND HEIGHT WITH A PATCH SOLUTE SOURCE AT X=0.0'
4 //1HO,40X,'INPUT DATA'/1H ,40X,1O(lH-))

205 FORMAT(1HO,25X,'NUMBER OF X COORDINATES (NX) = ',14/1H ,25X,
1 'NUMBER OF Y-COORDINATES (NY) = ',14/1H ,25X,
2 'NUMBER OF Z-COORDINATES (NZ) = ',14/1H ,25X,
3 'NUMBER OF TIME VALUES (NT) = ',14/1H ,25X,
4 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4)

210 FORMAT(1HO,25X,'SOLUTE CONCENTRATION ON MODEL Boundary (CO) =',
1 1P1E13.6,1X,A10/1H ,25X,
2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/
5 1H ,25X,'DISPERSION IN THE Z-DIRECTION (DZ) =',1P1E13.6,1X,A10/
6 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10)

212 FORMAT(1HO,25X,'AQUIFER WIDTH (W) AND HEIGHT (H) ARE INFINITE'
1 /1H ,28X,'SOLUTE SOURCE IS LOCATED BETWEEN Y1 =',1P1E13.6,1X,A10/
3 1H ,58X,'Y2 =',1P1E13.6,1X,A10/1H ,58X,
4 'Z1 =',1P1E13.6,1X,A10/1H ,54X,
5 AND Z2 =',1P1E13.6,1X,A10)

215 FORMAT(1HO,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
1 'WILL BE CALCULATED, IN ',1A10/1H ,25X,78(1H-))

216 FORMAT(1HO,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
1 'WILL BE CALCULATED, IN ',1A10/1H ,25X,78(1H-))

217 FORMAT(1HO,25X,'Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
1 'WILL BE CALCULATED, IN ',1A10/1H ,25X,78(1H-))

220 FORMAT(1HO ,5X,8F12.4)
225 FORMAT(1HO,25X,'TIES AT WHICH SOLUTE CONCENTRATIONS '
1 'WILL BE CALCULATED, IN ',1A10/1H ,25X,70(1H-))

227 FORMAT(1HO,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/
1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/
2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10)

230 FORMAT(1HO/1HO ,15X,'SOLUTE CONCENTRATION AT TIME =',
1 F12.4,1X,A10/1H ,35X,' AND AT Z =',F12.4,1X,A10/
2 1HO,25X,'Y-COORDINATE, IN ',A10)
FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =', F12.4,1X,A10/
1 1H2.4,1X,A10,5X,'(CONTINUED)'/1H ,35X,'AND AT Z =',F12.4,1X,A10/
2 1HO,25X, 'Y-COORDINATE, IN ',A10)
235 FORMAT(1H ,20X,9F12.4)
236 FORMAT(1H ,19X,'*',108(1H-)/
1 1H ,4X,'X-COORDINATE, ',2X,'!',44X,'SOLUTE CONCENTRATION, IN ' 
2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240 FORMAT(1H ,5X,F12.4,2X,'!',9F12.6) 
241 FORMAT(1H ,19X,'!')
END
SUBROUTINE CNRMLP(DK,T,X,Y,Z,Y1,Y2,Z1,Z2,DX,DY,DZ,VX,CN,NMAX)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
COMMON /IOUNIT/ IN,10
COMMON /GLPTS/ WN(256),ZN(256)
C
C THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y,Z 
C BASED ON THE ANALYTIC SOLUTION TO THE THREE-DIMENSIONAL 
C ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-
C INFINITE AQUIFER WITH INFINITE WIDTH AND HEIGHT. THE SOLUTE 
C SOURCE HAS A FINITE WIDTH AND HEIGHT, EXTENDING FROM Y=Y1 TO 
C Y=Y2 AND Z=Z1 TO Z=Z2. THE SOLUTE MAY BE SUBJECT TO FIRST-ORDER 
C CHEMICAL TRANSFORMATION. THE SOLUTION CONTAINS AN INTEGRAL 
C FROM 0 TO T**.25 WHICH IS EVALUATED NUMERICALLY USING A GAUSS-
C LEGENDRE QUADRATURE TECHNIQUE.
C
PI=3.14159265358979DO
CN=0.0DO
FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
IF(T.LE.0.0DO) RETURN
FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS
IF(X.CT.0.0DO) CO TO 10
IF(Y.EQ.Y1.OR.Y.EQ.Y2) THEN
 IF(Z.GT.Z1.AND.Z.LT.Z2) CN=0.50DO
 IF(Z.EQ.Z1.OR.Z.EQ.Z2) CN=0.25DO
 END IF
IF(Z.EQ.Z1.OR.Z.EQ.Z2) THEN
 IF(Y.GT.Y1.AND.Y.LT.Y2) CN=0.50DO
 END IF
IF(Y.GT.Y1.AND.Y.LT.Y2.AND.Z.GT.Z1.AND.Z.LT.Z2) CN=1.0DO
RETURN
START NUMERICAL INTEGRATION LOOP
10 SUM=0.0DO 
DO 20 I=1,NMAX
 C SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE 
 C NON-NORMALIZED LIMITS OF INTEGRATION 
 C LIMITS OF INTEGRATION ARE FROM 0 TO T**0.25 
 TT=T**0.25DO
 WI=WN(I)
 ZI=TT*(ZN(I)+1.0DO)/2.0DO
ZSQ = z1*z1
Z4 = ZSQ*ZSQ

TERM 1
XVT = X - VX*Z4
EXP1 = -(XVT*XVT/(4.0D0*DX*Z4) - DK*Z4)
ERFC1 = (Y1 - Y)/(2.0D0*ZSQ*DSQRT(DY))
CALL EXERFC(EXP1, ERFC1, Q1)

TERM 2
ERFC2 = (Y2 - Y)/(2.0D0*ZSQ*DSQRT(DY))
CALL EXERFC(EXP1, ERFC2, Q2)

TERM 3
EXP2 = 0.0D0
ERFC1 = (Z1 - Z)/(2.0D0*ZSQ*DSQRT(DZ))
CALL EXERFC(EXP2, ERFC1, Q3)
ERFC2 = (Z2 - Z)/(2.0D0*ZSQ*DSQRT(DZ))
CALL EXERFC(EXP2, ERFC2, Q4)
TERM = (Q1 - Q2)*(Q3 - Q4)*W1/(Z1*ZSQ)

SUM = SUM + TERM

CONTINUE
SUM = SUM*TT/2.0D0
CN = SUM*X/(2.0D0*DSQRT(P1*DX))
RETURN
END