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Appendix A

Construction of Boxplots

The upper and lower limits of the central box are defined using either quartiles or hinges. These definitions are clarified below. Then the influence of each definition on the position of the whiskers is demonstrated. Definitions used by commercial software packages are listed, including one non-conventional form called a "box graph".

Quartiles

Quartiles are the 25th, 50th and 75th percentiles of a data set, as defined in chapter 1. Consider a data set $X_i, i=1, \dots, n$. Computation of percentiles follows the equation

$$p_j = X_{(n+1) \cdot j}$$

where n is the sample size of X_i ,

j is the fraction of data less than or equal to the percentile value (for the 3 quartiles, $j = .25, .50, \text{ and } .75$).

Non-integer values of $(n+1) \cdot j$ imply linear interpolation between adjacent values of X .

Computation of quartiles for two small example data sets is illustrated in Table 1.

Hinges

Tukey (1977) used values for the ends of the box which, along with the median, divided the data into four equal parts. These "fourths" or "hinges" are defined as:

Lower hinge $h_L =$ median of all observations less than or equal to the sample median.

Upper hinge $h_U =$ median of all observations equal to or greater than the overall sample median.

They may also be defined as:

$$\text{Lower hinge } h_L = X_L, \text{ where } L = \frac{\text{integer } \lceil (n+3)/2 \rceil}{2}, \text{ and}$$

$$\text{Upper hinge } h_U = X_U, \text{ where } U = (n+1) - L.$$

where "integer $\lceil \cdot \rceil$ " is the integer portion of the number in brackets. For example, integer $\lceil 5.7 \rceil = 5$. Again, non-integer values of L and U imply interpolation. With hinges, however, this will always

be halfway between adjacent data points. Therefore, hinges are always either data values themselves, or averages of two data points, and so are easier to compute by hand than are percentiles. Hinges will generally be similar to quartiles for large ($n > 30$) sample sizes. For smaller data sets, differences will be more apparent. For example, when $n=12$ the lower hinge is halfway between the 3rd and 4th data points, while the lower quartile is one-quarter of the way between the two points (see Table 1) . Both measures split the data into one-fourth below and three fourths above their value. Either are acceptable for use in boxplots.

Table A1

A. For the following data X_i of sample size $n=11$:

2 3 5 45 46 47 48 50 90 151 208

$$\begin{aligned}
 p_{.25} &= \text{lower quartile} = X_{(n+1) \cdot .25} = X_3 = 5. \\
 p_{.75} &= \text{upper quartile} = X_{(n+1) \cdot .75} = X_9 = 90. \\
 p_{.50} &= \text{median} = X_{(n+1) \cdot .50} = X_6 = 47. \\
 h_l &= \text{lower hinge} = \text{median} [2 \ 3 \ 5 \ 45 \ 46 \ 47] = 25. \\
 h_u &= \text{upper hinge} = \text{median} [47 \ 48 \ 50 \ 90 \ 151 \ 208] = 70.
 \end{aligned}$$

B. For sample size $n=12$, and data $X_i, i=1, \dots, n$ equal to:

2 3 5 45 46 47 48 49 50 90 151 208

$$\begin{aligned}
 p_{.25} &= \text{lower quartile} = X_{(n+1) \cdot .25} = X_{3.25} = X_3 + 0.25 \cdot (X_4 - X_3) = 15. \\
 p_{.75} &= \text{upper quartile} = X_{(n+1) \cdot .75} = X_{9.75} = X_9 + 0.75 \cdot (X_{10} - X_9) = 80. \\
 p_{.50} &= \text{median} = X_{(n+1) \cdot .50} = X_{6.5} = X_6 + 0.50 \cdot (X_7 - X_6) = 47.5. \\
 h_l &= \text{lower hinge} = \text{median} [2 \ 3 \ 5 \ 45 \ 46 \ 47] = 25. \\
 h_u &= \text{upper hinge} = \text{median} [48 \ 49 \ 50 \ 90 \ 151 \ 208] = 70.
 \end{aligned}$$

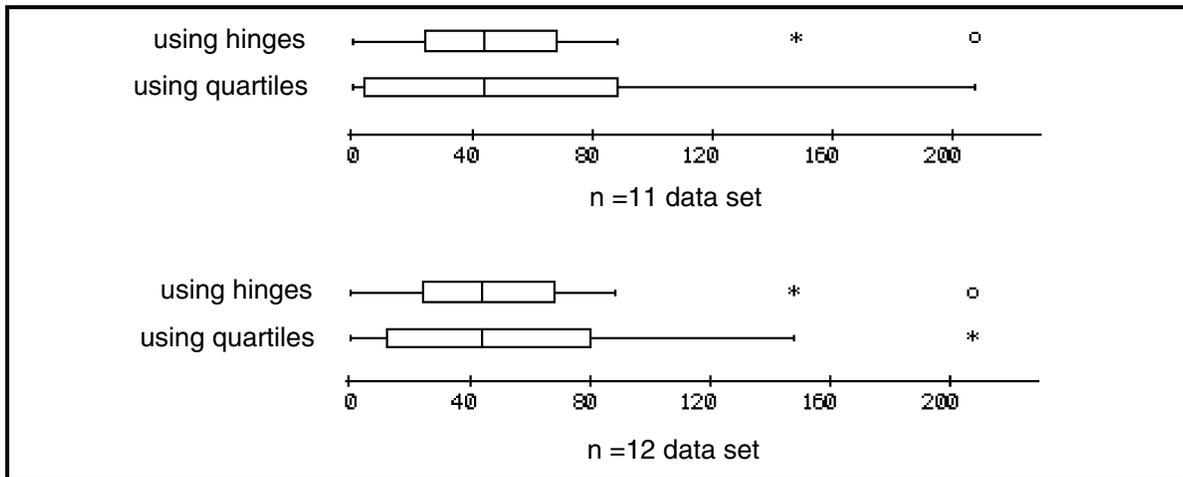


Figure A1. Boxplots for the Table A1 data

Figure A1 shows standard boxplots for the Table 1 data using both percentiles and hinges. Data in Table 1 were designed to maximize differences between the two measures. Real data, and larger sample sizes, will evidence much smaller differences. Note that the definitions of the box boundaries directly affect whisker lengths, and also determines which data are plotted as "outside" values.

It would be ideal if all software used the same conventions for drawing boxplots. However, that has not happened. Software written by developers who stick to the original definitions prefer hinges; those who want box boundaries to agree with tabled percentiles use quartiles. The Table 1 data can be used to determine which convention is used to produce boxplots.

Non-conventional definitions

Other statistical software use another (non-conventional) value for the box boundaries (Frigge and others, 1989). They use the next highest data value for the lower box boundary whenever $n/4$ is not an integer. This avoids all interpolation. Note that n , not $n+1$, is used.

StatView uses a percentile-type boxplot similar to the truncated boxplot, except that the upper and lower 10 percent of data are plotted as individual points. The weakness of this scheme is that 10 percent of the data will always be plotted individually at each end of the plot, and so it is less effective for defining and emphasizing unusual values. Also important is that StatView uses yet another definition for the box boundaries, $X_{(n+2) \cdot j}$, in calculating the quartiles. This non-conventional boxplot was called a "box graph" by Cleveland (1985).

Therefore some statistical software will produce boxes differing from conventional boxplots, particularly for small data sets.

Boxplots for Censored Data

Data sets whose values include some observations known only to be below (or above) a limit or threshold can also be effectively displayed by boxplots. First set all values below the threshold to some value less than (not equal to) the reporting limit. The actual value is not important, and could be 0, one-half the reporting limit, etc. Produce the boxplot. Then draw a line across the graph at the value of the threshold, and erase all lines below this value from the graph.

This procedure was used for data in figure A2. If less than 25 percent of the data are below the threshold, this procedure will affect at most only the lower whisker (as in the Hoover Dam through Morelos Dam boxplots). If between 25 and 75 percent are below the threshold, the box will be partially hidden below the threshold (as in the CO-UT Line and Cisco boxes). If more than 75 percent of the data are below the threshold, part of the upper whisker and outside values will be visible above the threshold, as in the Lees Ferry box. In each case, these boxplots accurately and fairly illustrate both the distribution of data above the threshold, and the percentage of data below the threshold.

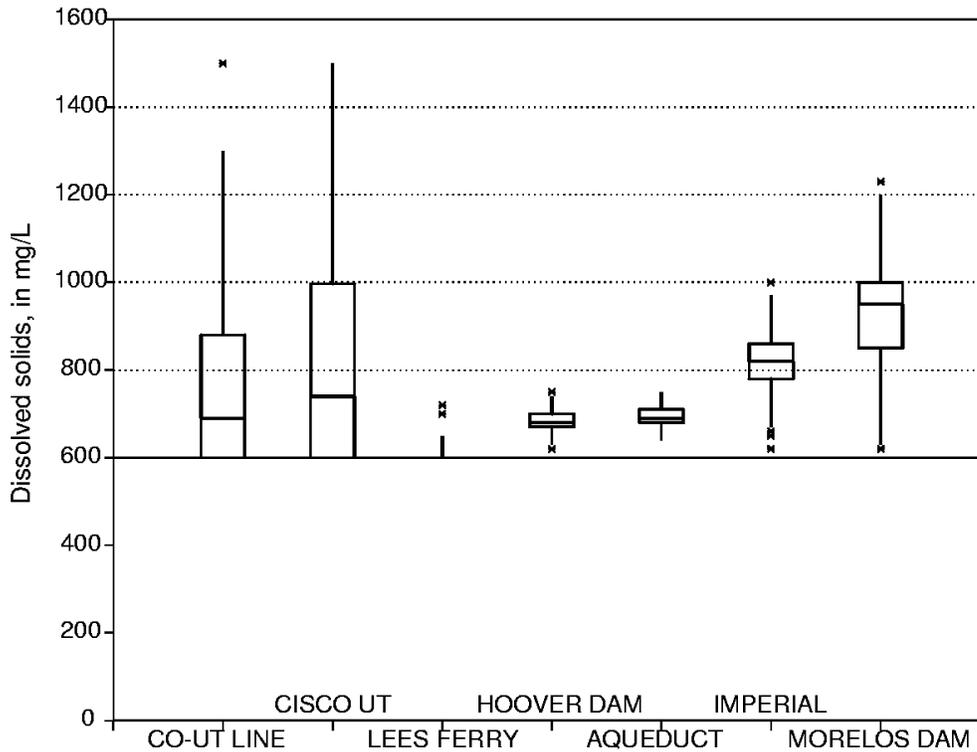


Figure A2. Dissolved solids concentrations along the Colorado River, artificially censored at a threshold of 600 mg/L.

A second alternative for boxplots of censored data is to estimate the percentiles falling below the threshold, and drawing dashed portions of the box below the threshold using these estimates. Helsel and Cohn (1988) have compared methods for estimating these percentiles. When multiple thresholds occur, such as thresholds which have changed over time or between laboratories, a solid line can be drawn across the plot at the highest threshold. Portions of the boxes above the highest threshold will be correct as long as each censored observation is assigned some value below its threshold. Quartiles falling below the highest threshold should be determined by using the methods recommended by Helsel and Cohn (1988). All lines below the highest threshold are only estimates, and should be drawn as dashed lines on the plot.

Displaying confidence intervals

As an aid for displaying whether two groups of data have different medians, confidence intervals for the median as defined in chapter 3 can be added to boxplots. When boxplots are placed side by side, their medians are significantly different if the confidence intervals do not overlap. Three methods of displaying these intervals are shown in figure A3. In the first method (A), the box is "notched" at both upper and lower limits, making the box narrower for all values within the interval. In the second (B), parentheses are drawn within the box at each limit. Shading is used in (C) to illustrate interval width. If displaying differences in medians is not of primary interest, these

methods add visual confusion to boxplots and are probably best avoided. Confusion is compounded when the interval width falls beyond the 25th or 75th percentiles. Of the three, shading seems the easiest to visualize and least confusing.

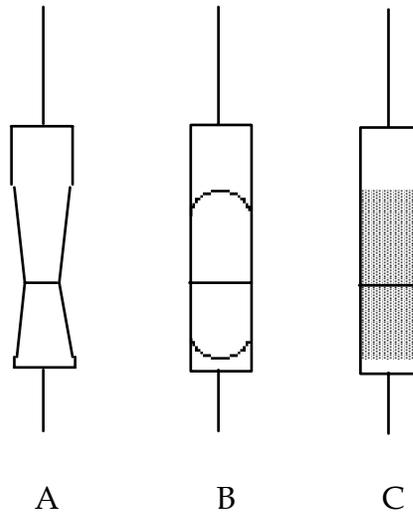


Figure A3. Methods for displaying confidence interval of median using a boxplot.
A. Notched boxplots B. Parentheses C. Shaded boxplot

Appendix B

Tables

Table B1	Cunnane plotting positions for $n = 1$ to 20
Table B2	Normal quantiles for Cunnane plotting positions of Table B1
Table B3	Critical values for the PPCC test for normality
Table B4	Quantiles (p-values) for the rank-sum test
Table B5	Quantiles (p-values) for the sign test
Table B6	Critical test statistic values for the signed-rank test
Table B7	Critical test statistic values for the Friedman test
Table B8	Quantiles (p-values) for Kendall's tau (τ)

Table B1. Cunnane plotting positions for sample sizes $n = 1$ to 20

	<u>i</u>																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
N= 5																				
	.12	.31	.50	.69	.88															
N= 6																				
	.10	.26	.42	.58	.74	.90														
N= 7																				
	.08	.22	.36	.50	.64	.78	.92													
N= 8																				
	.07	.20	.32	.44	.56	.68	.80	.93												
N= 9																				
	.07	.17	.28	.39	.50	.61	.72	.83	.93											
N= 10																				
	.06	.16	.25	.35	.45	.55	.65	.75	.84	.94										
N= 11																				
	.05	.14	.23	.32	.41	.50	.59	.68	.77	.86	.95									
N= 12																				
	.05	.13	.21	.30	.38	.46	.54	.62	.70	.79	.87	.95								
N= 13																				
	.05	.12	.20	.27	.35	.42	.50	.58	.65	.73	.80	.88	.95							
N= 14																				
	.04	.11	.18	.25	.32	.39	.46	.54	.61	.68	.75	.82	.89	.96						
N= 15																				
	.04	.11	.17	.24	.30	.37	.43	.50	.57	.63	.70	.76	.83	.89	.96					
N= 16																				
	.04	.10	.16	.22	.28	.35	.41	.47	.53	.59	.65	.72	.78	.84	.90	.96				
N= 17																				
	.03	.09	.15	.21	.27	.33	.38	.44	.50	.56	.62	.67	.73	.79	.85	.91	.97			
N= 18																				
	.03	.09	.14	.20	.25	.31	.36	.42	.47	.53	.58	.64	.69	.75	.80	.86	.91	.97		
N= 19																				
	.03	.08	.14	.19	.24	.29	.34	.40	.45	.50	.55	.60	.66	.71	.76	.81	.86	.92	.97	
N= 20																				
	.03	.08	.13	.18	.23	.28	.33	.38	.43	.48	.52	.57	.62	.67	.72	.77	.82	.87	.92	.97

Table B2. Upper tail normal quantiles for the plotting positions of Table B1
(for lower tail quantiles, multiply all nonzero quantiles by -1)

	N= 5								
0.000	0.502	1.198							
	N= 6								
0.203	0.649	1.300							
	N= 7								
0.000	0.355	0.765	1.383						
	N= 8								
0.153	0.475	0.859	1.453						
	N= 9								
0.000	0.276	0.575	0.939	1.513					
	N= 10								
0.123	0.377	0.659	1.007	1.565					
	N= 11								
0.000	0.225	0.463	0.732	1.067	1.611				
	N= 12								
0.103	0.313	0.538	0.796	1.121	1.653				
	N= 13								
0.000	0.191	0.389	0.604	0.852	1.169	1.691			
	N= 14								
0.088	0.267	0.456	0.663	0.904	1.212	1.725			
	N= 15								
0.000	0.165	0.336	0.517	0.716	0.950	1.252	1.757		
	N= 16								
0.077	0.234	0.397	0.571	0.765	0.992	1.289	1.787		
	N= 17								
0.000	0.146	0.295	0.452	0.620	0.809	1.031	1.323	1.814	
	N= 18								
0.069	0.208	0.351	0.502	0.666	0.849	1.067	1.354	1.839	
	N= 19								
0.000	0.131	0.264	0.402	0.548	0.707	0.887	1.101	1.383	1.864
	N= 20								
0.062	0.187	0.315	0.449	0.591	0.746	0.922	1.133	1.411	1.886

Table B3. Critical r^* values for the probability plot correlation coefficient test of normality (from Looney and Gullledge, 1985a)

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[reject H_0 : data are normal when PPCC $r < r^*$]

n	α -level					
	.005	.010	.025	.050	.100	.250
3	.867	.869	.872	.879	.891	.924
4	.813	.824	.846	.868	.894	.931
5	.807	.826	.856	.880	.903	.934
6	.820	.838	.866	.888	.910	.939
7	.828	.850	.877	.898	.918	.944
8	.840	.861	.887	.906	.924	.948
9	.854	.871	.894	.912	.930	.952
10	.862	.879	.901	.918	.934	.954
11	.870	.886	.907	.923	.938	.957
12	.876	.892	.912	.928	.942	.960
13	.885	.899	.918	.932	.945	.962
14	.890	.905	.923	.935	.948	.964
15	.896	.910	.927	.939	.951	.965
16	.899	.913	.929	.941	.953	.967
17	.905	.917	.932	.944	.954	.968
18	.908	.920	.935	.946	.957	.970
19	.914	.924	.938	.949	.958	.971
20	.916	.926	.940	.951	.960	.972
21	.918	.930	.943	.952	.961	.973
22	.923	.933	.945	.954	.963	.974
23	.925	.935	.947	.956	.964	.975
24	.927	.937	.949	.957	.965	.976
25	.929	.939	.951	.959	.966	.976
26	.932	.941	.952	.960	.967	.977
27	.934	.943	.953	.961	.968	.978
28	.936	.944	.955	.962	.969	.978
29	.939	.946	.956	.963	.970	.979
30	.939	.947	.957	.964	.971	.979
31	.942	.950	.958	.965	.972	.980
32	.943	.950	.959	.966	.972	.980
33	.944	.951	.961	.967	.973	.981
34	.946	.953	.962	.968	.974	.981
35	.947	.954	.962	.969	.974	.982
36	.948	.955	.963	.969	.975	.982
37	.950	.956	.964	.970	.976	.983
38	.951	.957	.965	.971	.976	.983
39	.951	.958	.966	.971	.977	.983
40	.953	.959	.966	.972	.977	.984

Table B3. Cont.

n	α-level					
	.005	.010	.025	.050	.100	.250
41	.953	.960	.967	.973	.977	.984
42	.954	.961	.968	.973	.978	.984
43	.956	.961	.968	.974	.978	.984
44	.957	.962	.969	.974	.979	.985
45	.957	.963	.969	.974	.979	.985
46	.958	.963	.970	.975	.980	.985
47	.959	.965	.971	.976	.980	.986
48	.959	.965	.971	.976	.980	.986
49	.961	.966	.972	.976	.981	.986
50	.961	.966	.972	.977	.981	.986
55	.965	.969	.974	.979	.982	.987
60	.967	.971	.976	.980	.984	.988
65	.969	.973	.978	.981	.985	.989
70	.971	.975	.979	.983	.986	.990
75	.973	.976	.981	.984	.987	.990
80	.975	.978	.982	.985	.987	.991
85	.976	.979	.983	.985	.988	.991
90	.977	.980	.984	.986	.988	.992
95	.979	.981	.984	.987	.989	.992
100	.979	.982	.985	.987	.989	.992

TABLE B4 continued

n [smaller sample size] = 7				n [smaller sample size] = 9													
m=7			m=8			m=9			m=10			m=9			m=10		
x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*	x	p	x*
61	.159	44	65	.168	47	70	.150	49	74	.15752		98	.149	73	104	.13976	
62	.130	43	66	.140	46	71	.126	48	75	.13551		99	.129	72	105	.12175	
63	.104	42	67	.116	45	72	.105	47	76	.11550		100	.111	71	106	.10674	
64	.082	41	68	.095	44	73	.087	46	77	.09749		101	.095	70	107	.09173	
65	.064	40	69	.076	43	74	.071	45	78	.08148		102	.081	69	108	.07872	
66	.049	39	70	.060	42	75	.057	44	79	.06747		103	.068	68	109	.06771	
67	.036	38	71	.047	41	76	.045	43	80	.05446		104	.057	67	110	.05670	
68	.027	37	72	.036	40	77	.036	42	81	.04445		105	.047	66	111	.04769	
69	.019	36	73	.027	39	78	.027	41	82	.03544		106	.039	65	112	.03968	
70	.013	35	74	.020	38	79	.021	40	83	.02843		107	.031	64	113	.03367	
71	.009	34	75	.014	37	80	.016	39	84	.02242		108	.025	63	114	.02766	
72	.006	33	76	.010	36	81	.011	38	85	.01741		109	.020	62	115	.02265	
			77	.007	35	82	.008	37	86	.01240		110	.016	61	116	.01764	
						83	.006	36	87	.00939		111	.012	60	117	.01463	
												112	.009	59	118	.01162	
												113	.007	58	119	.00961	

n [smaller sample size] = 8						n [smaller sample size] = 10					
m=8			m=9			m=10			m=10		
x	p	x*	x	p	x*	x	p	x*	x	p	x*
79	.13957		84	.138	60	89	.137	63	119	.157	91
80	.11756		85	.118	59	90	.118	62	120	.140	90
81	.09755		86	.100	58	91	.102	61	121	.124	89
82	.08054		87	.084	57	92	.086	60	122	.109	88
83	.06553		88	.069	56	93	.073	59	123	.095	87
84	.05252		89	.057	55	94	.061	58	124	.083	86
85	.04151		90	.046	54	95	.051	57	125	.072	85
86	.03250		91	.037	53	96	.042	56	126	.062	84
87	.02549		92	.030	52	97	.034	55	127	.053	83
88	.01948		93	.023	51	98	.027	54	128	.045	82
89	.01447		94	.018	50	99	.022	53	129	.038	81
90	.01046		95	.014	49	100	.017	52	130	.032	80
91	.00745		96	.010	48	101	.013	51	131	.026	79
92	.00544		97	.008	47	102	.010	50	132	.022	78
									133	.018	77
									134	.014	76
									135	.012	75
									136	.009	74
									137	.007	73
									138	.006	72

Table generated by D. Helsel

Table B5. -- Quantiles (p-values) for the sign test statistic S^+

Quantiles for the sign test are identical to quantiles of the binomial distribution with percentile $p=0.5$. The approximation given in chapter 6 and used by most statistical software packages can be used for $n \geq 20$. Statistics textbooks that contain a table of exact quantiles for the binomial distribution for sizes below 20 include Hollander and Wolfe (1999) and Zar (1999).

An online table of exact quantiles for the binomial distribution can be found as of 5/2002 at: <http://faculty.vassar.edu/lowry/binomial01.html>

An example of using this online table:

Enter n (the number of data pairs) and p ($=0.5$). An exact table will be printed. P-values are cumulative probabilities, or values of the cumulative distribution function (cdf). For small values of the test statistic S^+ (called k in the online table) – values below $n/2$, use the “Down” column to read off a one-sided p-value for the sign test. For S^+ larger than $n/2$, use the “Up” column. The example output below is for $n=13$. A one-sided p-value for $S^+ = 4$ (the probability of getting an $S^+ \leq 4$) is 0.133. The p-value for $S^+ = 9$ (the probability of getting an $S^+ \geq 9$) also equals 0.133. For a two-sided test, $p = 0.266$.

k	Exact Probability	Cumulative Probability	
		Down	Up
0	0.000122070313	0.000122070313	1.0
1	0.001586914063	0.001708984375	0.999877929688
2	0.009521484375	0.01123046875	0.998291015625
3	0.034912109375	0.046142578125	0.98876953125
4	0.087280273438	0.133422851563	0.953857421875
5	0.157104492188	0.29052734375	0.866577148438
6	0.20947265625	0.5	0.70947265625
7	0.20947265625	0.70947265625	0.5
8	0.157104492188	0.866577148438	0.29052734375
9	0.087280273438	0.953857421875	0.133422851563
10	0.034912109375	0.98876953125	0.046142578125
11	0.009521484375	0.998291015625	0.01123046875
12	0.001586914063	0.999877929688	0.001708984375
13	0.000122070313	1.0	
	0.000122070313		

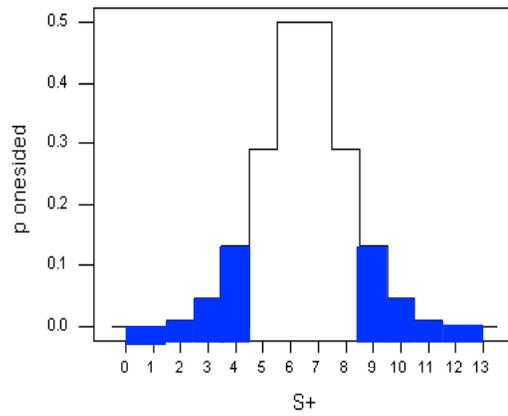


Figure B1. Two-sided critical region (p-values), shaded, for the sign test. $n=13$, $S^+=4$ or 9.

Table B6 – Critical test statistic values for the signed-rank statistic W^+
 (from McCornack, 1965)

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The approximation given in chapter 6, used by most statistics software packages, can be used for $n > 15$ and $\alpha \geq 0.025$. For $\alpha < 0.025$, see exact tables in the McCornack paper or a textbook such as Hollander and Wolfe (1999), even for large sample sizes.

[reject H_0 : at one-sided α when
 $W^+ \leq$ table entry (small W)]

[reject H_0 : at one-sided α when
 $W^+ \geq$ table entry (large W)]

n	α -level			
	.005	.010	.025	.050
5				0
6			0	2
7		0	2	3
8	0	1	3	5
9	1	3	5	8
10	3	5	8	10
11	5	7	10	13
12	7	9	13	17
13	9	12	17	21
14	12	15	21	25
15	15	19	25	30
16	19	23	29	35
17	23	27	34	41
18	27	32	40	47
19	32	37	46	53
20	37	43	52	60

n	α -level			
	.005	.010	.025	.050
5				15
6			21	19
7		28	26	25
8	36	35	33	31
9	44	42	40	37
10	52	50	47	45
11	61	59	56	53
12	71	69	65	61
13	82	79	74	70
14	93	90	84	80
15	105	101	95	90
16	117	113	107	101
17	130	126	119	112
18	144	139	131	124
19	158	153	144	137
20	173	167	158	150

Table B7 – Critical test statistic values for the Friedman statistic X_f
(from Martin, Leblanc and Toan, 1993)

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The chi-square approximation given in chapter 7 is used by most statistics software packages. For comparing 3 to 5 groups of data with sample sizes (blocks) $n < 10$ in each group, an exact table should be used.

[reject H_0 : at α when $X_f \geq$ table entry]

k = 3 groups

n	α-level				
	.005	.010	.025	.050	.10
3				6.00	6.00
4	8.00	8.00	8.00	6.50	6.00
5	10.00	8.40	7.60	6.40	5.20
6	10.33	9.00	8.33	7.00	5.33
7	10.29	8.86	7.71	7.14	5.43
8	9.75	9.00	7.75	6.25	5.25
9	10.67	8.67	8.00	6.22	5.56
10	10.40	9.60	7.80	6.20	5.00

k = 4 groups

n	α-level				
	.005	.010	.025	.050	.10
2				6.00	6.00
3	9.00	9.00	8.20	7.40	6.60
4	10.20	9.60	8.40	7.80	6.30
5	10.92	9.96	8.76	7.80	6.36
6	11.40	10.20	8.80	7.60	6.40
7	11.40	10.37	9.00	7.80	6.43
8	11.85	10.50	9.00	7.65	6.30
9	12.07	10.87	9.13	7.80	6.47
10	12.00	10.80	9.12	7.80	6.36

k = 5 groups

n	α-level				
	.005	.010	.025	.050	.10
2		8.00	8.00	7.60	7.20
3	10.67	10.13	9.60	8.53	7.47
4	12.00	11.20	9.80	8.80	7.60
5	12.48	11.68	10.24	8.96	7.68
6	13.07	11.87	10.40	9.07	7.73
7	13.26	12.11	10.51	9.14	7.77
8	13.50	12.30	10.60	9.30	7.80
9	13.69	12.44	10.67	9.24	7.73
10	13.84	12.48	10.72	9.28	7.76

Table B8 -- Quantiles (p-values) for Kendall's S statistic and tau correlation coefficient

For N>10 use the approximation given in section 8.2.2

One-sided p = Prob [S ≥ x] = Prob [S ≤ -x]

N = Number of data pairs					N = Number of data pairs				
	4	5	8	9		3	6	7	10
x					x				
0	0.625	0.592	0.548	0.540	1	0.500	0.500	0.500	0.500
2	0.375	0.408	0.452	0.460	3	0.167	0.360	0.386	0.431
4	0.167	0.242	0.360	0.381	5		0.235	0.281	0.364
6	0.042	0.117	0.274	0.306	7		0.136	0.191	0.300
8		0.042	0.199	0.238	9		0.068	0.119	0.242
10		0.0083	0.138	0.179	11		0.028	0.068	0.190
12			0.089	0.130	13		0.0083	0.035	0.146
14			0.054	0.090	15		0.0014	0.015	0.108
16			0.031	0.060	17			0.0054	0.078
18			0.0156	0.038	19			0.0014	0.054
20			0.0071	0.022	21			0.0002	0.036
22			0.0028	0.0124	23				0.023
24			0.0009	0.0063	25				0.0143
26			0.0002	0.0029	27				0.0083
28			<0.0001	0.0012	29				0.0046
30				0.0004	31				0.0023
32				0.0001	33				0.0011
34				<0.0001	35				0.0005
36				<0.0001	37				0.0002
					39				<0.0001
					41				<0.0001
					43				<0.0001
					45				<0.0001

Table generated by D. Helsel

Appendix C

Data Sets

		<u>Chapter cited</u>
Data Set C1	Annual peak discharges for the Saddle River, NJ	2, 3
Data Set C2	Annual streamflows for the Conecuh River, AL	3, 6
Data Set C3	Daily streamflow for the Potomac River, Wash. D.C.	3
Data Set C4	Atrazine concentrations	6
Data Set C5	Subset of iron concentrations at low flow	7
Data Set C6	Complete set of iron concentrations	7
Data Set C7	Specific capacities of wells in Pennsylvania	7
Data Set C8	Corbicula on the Tennessee River	7
Data Set C9	TDS concentrations for the Cuyahoga River, Ohio	9
Data Set C10	Phosphorus transport, Illinois River at Marseilles	9
Data Set C11	Grain size and permeability of alluvial aquifers	9
Data Set C12	ROE and TDS data, Rappahannock R. near Fredericksburg, Virginia	10
Data Set C13	Streamflow data used for record extension	10
Data Set C14	Mean annual runoff and basin characteristics	11
Data Set C15	Urban total nitrogen loads	11
Data Set C16	Uranium and TDS in groundwaters	11
Data Set C17	Green River, Kentucky sediment transport data	12
Data Set C18	Maumee River, Ohio total P trends data	12
Data Set C19	Water levels, P-R-M system middle aquifer, NJ	12
Data Set C20	Factors affecting contamination from impoundments	15

Data sets are available in both ASCII and MS Excel formats. See the online location from which you obtained this book for the data files HhappC.dat and HhappC.xls .

Appendix D

Answers to Selected Exercises

Chapter 1

1.1 For the well yield data:

- a) mean = 0.19
- b) trimmed mean = 0.05
- c) geometric mean = 0.04
- d) median = 0.04
- e) They differ because the data are skewed. The estimates which are more robust are similar, while the mean is larger.

1.2

- a) standard deviation = 0.31
- b) interquartile range = 0.36
- c) MAD = 0.04
- d) skew = 2.07. quartile skew = 0.83.

Because the data are asymmetric, the median difference is small, but the IQR and standard deviation are not.

1.3

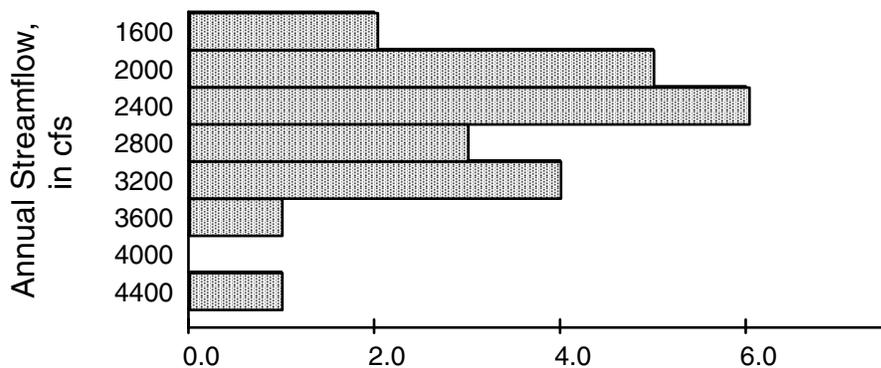
- | | |
|-----------------------|-----------------------|
| mean = 1.64 | std. dev. = 2.85 |
| median = 0.80 | IQR = 0.61 |
| geometric mean = 0.04 | MAD = 0.25 |
| skew = 3.09 | quartile skew = -0.10 |

The largest observation is an outlier. Though the skew appears to be strongly positive, and the standard deviation large, this is due only to the effect of that one point. The majority of the data are not skewed, as shown by the more resistant quartile skew coefficient.

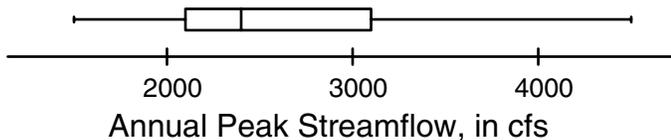
- a) assuming the outlying observation is accurate, representing some high-nitrogen location which is important to have sampled, the mean must be used to compute the mass of nitrogen falling per square mile. It would probably be computed by weighting concentrations by the surface area represented by each environment. The median would under-represent this mass loading.
- b) the median would be a better "typical" concentration, and the IQR a better "typical" variability, than the mean and standard deviation. This is due to the strong effect of the one unusual point on these traditional measures.

Chapter 2

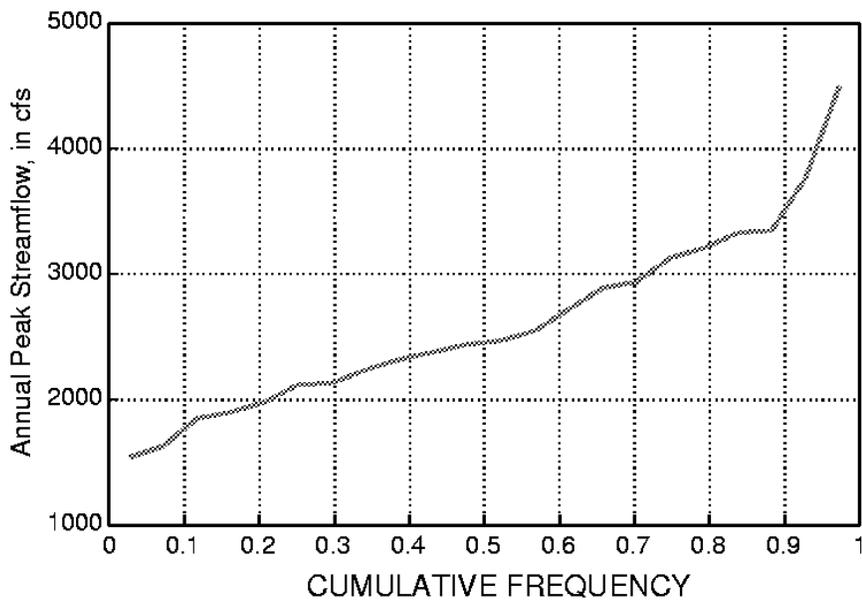
2.1 a)



b)

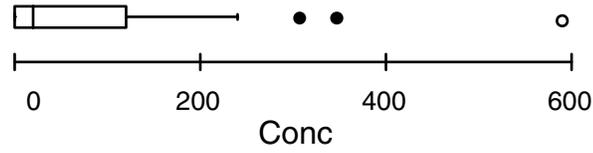


c)

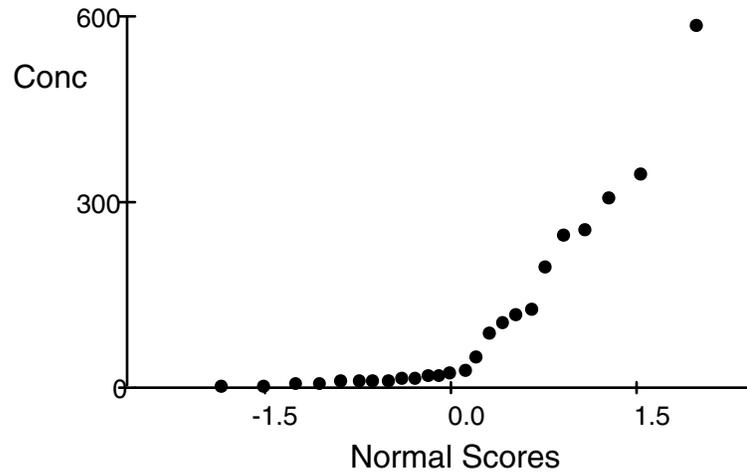


Either a cube root or logarithmic transform would increase symmetry.

2.2 a)

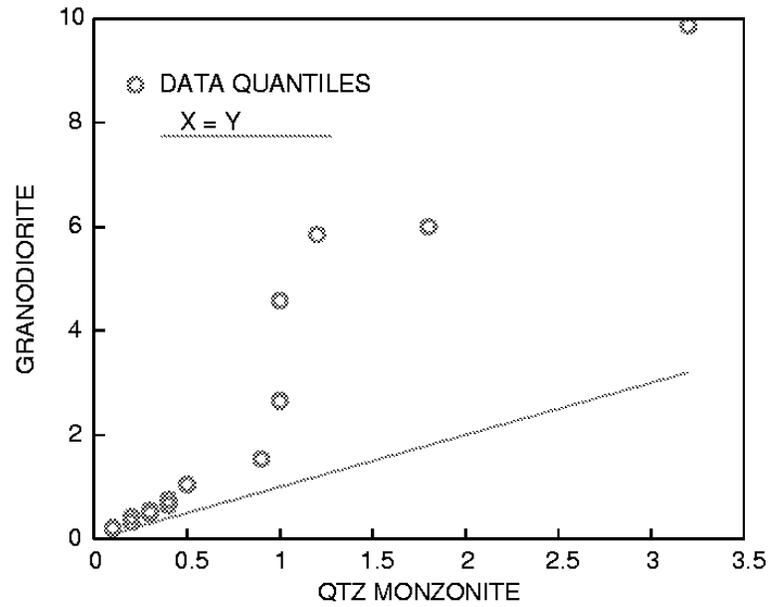


b)

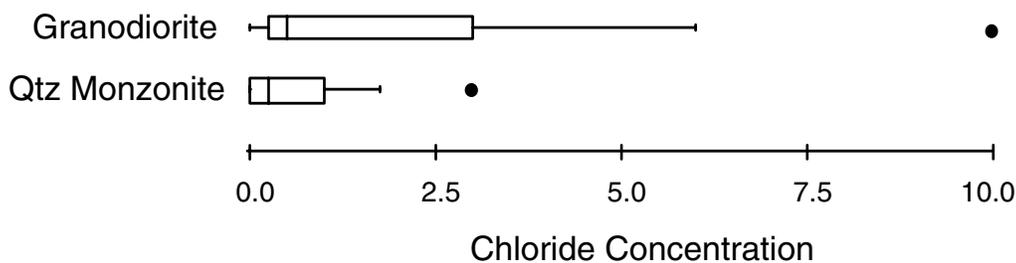


The data are strongly right-skewed. A log transformation makes these data more symmetric.

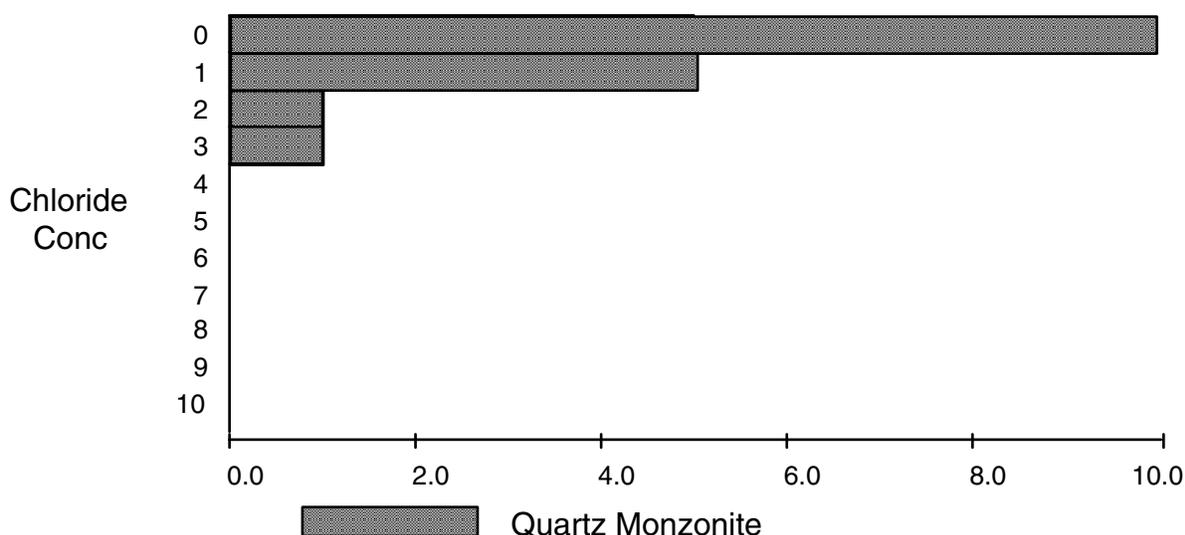
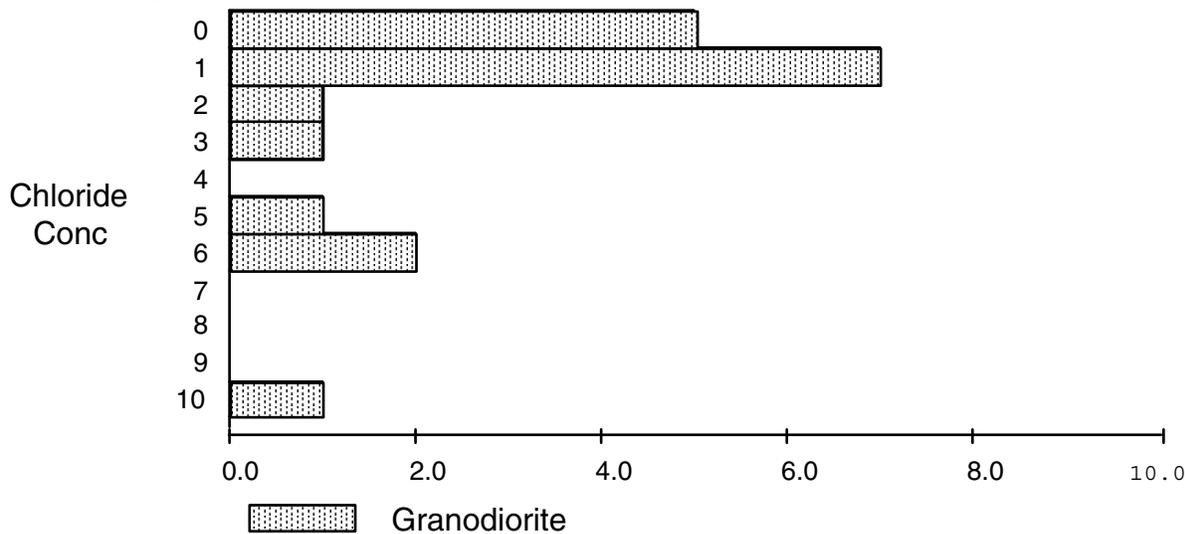
2.3 Q-Q plot.



Boxplots:



Histograms:



The granodiorite shows higher chloride concentrations than does the quartz monzonite. This can be seen with any of the three graphs, but most easily with the boxplot. From the Q-Q plot, the relationship does not look linear.

2.4 There appears to be no effect of the waste treatment plant.

Chapter 3

3.1 nonparametric: $x' = 4$ ($\alpha/2 = .0154$). $R_l = 5$, $R_u = 14$.

$$0.4 \leq Cl_{0.5} \leq 3.0 \quad \text{at } \alpha = 0.031.$$

This is as close to 0.05 as the table allows.

parametric: Using the natural logs of the data,

$$\exp(-0.045 - 2.11 \cdot \sqrt{1.63/18}) \leq GM_x \leq \exp(-0.045 + 2.11 \cdot \sqrt{1.63/18}).$$

$$0.51 \leq GM_x \leq 1.80.$$

Either of intervals is reasonable. The logs of the data still retain some skewness, so the nonparametric interval may be more realistic. The parametric interval might be preferred to obtain an alpha of 0.05. The choice would depend on whether the assumption of lognormality was believed.

3.2 symmetric: $0.706 - 2.12 \cdot \sqrt{0.639/17} \leq \mu \leq 0.706 + 2.12 \cdot \sqrt{0.639/17}$
 $0.30 \leq \mu \leq 1.12$

Point estimates: mean = 0.705 (assuming normal distribution).

$$\text{mean} = \exp(-0.849 + 0.5 \cdot 1.067)$$

$$= 0.73 \quad (\text{assuming a lognormal distribution}).$$

As the logs of the data are more symmetric than the data prior to transformation, the lognormal (2nd) estimate is preferred.

3.3 Parametric 95% prediction interval:

$$0.19 - 2.20 \cdot \sqrt{0.0975 + (0.0975/12)} \quad \text{to} \quad 0.19 + 2.20 \cdot \sqrt{0.0975 + (0.0975/12)}$$

or -0.53 to 0.91 gallons/min/foot. Includes 0.85, so same distribution.

Nonparametric 95% prediction interval:

$$X_{[0.025 \cdot 13]} \quad \text{to} \quad X_{[0.975 \cdot 13]} \quad X_{0.325} \quad \text{to} \quad X_{12.675}$$

The sample size is too small to produce such a wide (95%) nonparametric prediction interval. Therefore a parametric interval must be used. However, the negative lower end of the parametric prediction interval indicates that a symmetric interval is not appropriate. So an asymmetric interval resulting from taking logarithms should be used instead of the one above.

3.4 The data look relatively symmetric, so no logs taken.

$$\text{mean:} \quad 683 \pm 126, \quad \text{or} \quad 557 \text{--} 809 \quad \alpha = .05.$$

$$\text{median:} \quad R_l=6, R_u=15 \quad 524 \text{--} 894 \quad \alpha = .041.$$

- 3.5 The 90th percentile = 2445 cfs. A one-sided 95% confidence interval for the 90th percentile (an upper confidence limit to insure that the intake does not go dry) is found using the large-sample approximation of equation 3.17:

$$\begin{aligned} R_u &= 365 \cdot 0.1 + z_{[0.95]} \cdot \sqrt{365 \cdot 0.1 \cdot (0.9)} + 0.5 \\ &= 36.5 + 1.645 \cdot 5.73 + 0.5 = 46.4 \end{aligned}$$

The 46th ranked point is the upper CI, or 2700 cfs.

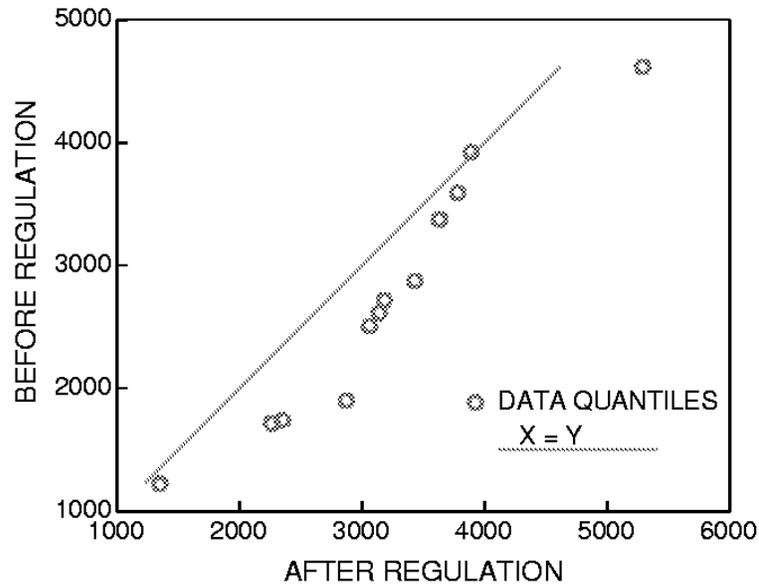
Chapter 4

- 4.1 For the before-1969 data, PPCC $r=0.986$. For the after-1969 data, PPCC $r=0.971$. Critical values of r are 0.948 and 0.929, respectively. Therefore normality cannot be rejected for either period at $\alpha = 0.05$.
- 4.2 For the arsenic data, PPCC $r=0.844$. The critical r^* from Appendix table B3 is $r^*=0.959$. Therefore reject normality. For log-transforms of the data, PPCC $r=0.973$. Normality of the transformed data is not rejected.

Chapter 5

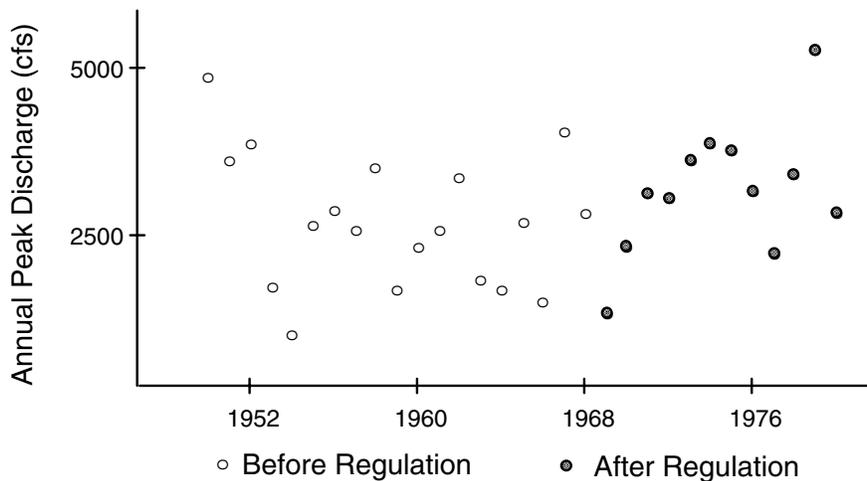
- 5.1 The p-value remains the same.
- 5.2 Given that we wish to test for a change in concentration, but the direction of the change is not specified in the problem, this should be a two-sided test. If it had stated we were looking for an increase, or a decrease, the test would have been a one-sided test.
- 5.3 a. Quantiles are the 12 "after" data, and 12 quantiles computed from the 19 "before" data :

i	j	"after"	"before"
1	1.34	1350.00	1222.13
2	2.92	2260.00	1715.25
3	4.49	2350.00	1739.84
4	6.06	2870.00	1900.82
5	7.64	3060.00	2506.23
6	9.21	3140.00	2614.92
7	10.79	3180.00	2717.21
8	12.36	3430.00	2873.61
9	13.93	3630.00	3375.24
10	15.51	3780.00	3591.15
11	17.08	3890.00	3922.29
12	18.66	5290.00	4617.37



The relationship appears additive. The Hodges-Lehmann estimate (median of all possible after-before differences) = 480 cfs.

- b. After regulation, the reservoir appears to be filling. Any test for change in flow should omit data during the transition period of 1969-1970. Plots of time series are always a good idea. They increase the investigator's understanding of the data. Low flows after regulation are not as low as those before. This produces the pattern seen in the Q-Q plot of the low quantiles being lower after regulation, while the upper quantiles appear the same, as shown by the drift closer to the $x=y$ line for the higher values.



- c. With 1969 and 70 included, $W_{rs} = 273.5$ $p=0.22$. The after flows are not significantly different. With 1969 and 70 excluded, $W_{rs} = 243.5$ $p=0.06$. The after flows are close to being significantly different -- more data after regulation is needed.

5.4 Exact test

<u>X</u>	<u>Y</u>	<u>R(Y)</u>	<u>R(X)</u>
1			1
	1.5	2	
2			3
	2.5	4	
3			5
	3.5	6	
4			7
	4.5	8	
	5.5	9	
	7.0	10	
	10.0	11	
	20.0	12	
	40.0	13	
	100.0	14	

$$n = 4 \quad m = 10$$

$$W_{rs} = \sum R_x = 16$$

From table B4, $\text{Prob}(W_{rs} \leq 16) = .027$. The two-sided exact p-value = 0.054

Large-sample approximation

$$\text{The mean is } \mu_W = \frac{n \cdot (N+1)}{2} = \frac{4 \cdot 15}{2} = 30$$

$$\text{The standard deviation is given by } \sigma_W = \sqrt{\frac{n \cdot m \cdot (N+1)}{12}} = 7.0711$$

$$Z_{rs} = \frac{16 - \mu_W + 1/2}{s_W} = -1.909$$

Using linear interpolation between -1.9110 and -1.8957 in a table of the standard normal distribution gives the one-tail probability of 0.028. So the two-sided approximate p-value is 0.056.

t-test on the ranks

Replacing variable values by ranks gives

$$\begin{array}{llll} \bar{x} = 4 & S_x = 2.582 & S_x^2 = 6.667 & n = 4 \\ \bar{y} = 8.9 & S_y = 3.928 & S_y^2 = 15.429 & m = 10 \end{array}$$

The pooled variance is :

$$S^2 = \frac{3S_x^2 + 9S_y^2}{12} = 13.2386$$

$$S = 3.639$$

$$t = \frac{\bar{x} - \bar{y}}{S\sqrt{(1/n + 1/m)}} = -2.27610$$

Linear interpolation for a student's t with 12 degrees of freedom gives

$$.975 + \frac{(2.27610 - 2.1788)}{(2.6810 - 2.1788)} \cdot .015 = .97791 \qquad 1.0 - .97791 = .022$$

The two-sided rank transform p-value is .044.

<u>Summary</u>	
Approach	p-value
Rank-Sum Exact	0.054
Rank-Sum Approx.	0.056
t test on ranks	0.044

To compute $\hat{\Delta}$, the $(n \cdot m) = 40$ differences $(X_i - Y_j = D_{ij})$ are:

(Y ₁)	0.5	1.5	2.5	3.5	4.5	6	9	19	39	99
(Y ₂)	-0.5	0.5	1.5	2.5	3.5	5	8	18	38	98
(Y ₃)	-1.5	-0.5	0.5	1.5	2.5	4	7	17	37	97
(Y ₄)	-2.5	-1.5	-0.5	0.5	1.5	3	6	16	36	96

$$\hat{\Delta} = \text{median of 40 } D_j\text{'s } (D_{\text{rank } 20} + D_{\text{rank } 21})/2 = 3.75$$

5.5

Yields with fracturing
 $r_{\text{crit}} = .932$, accept normality

Yields without
 $r_{\text{crit}} = .928$, reject normality

Because one of the groups is non-normal, the rank-sum test is performed.

$W_{\text{RS}} = \Sigma R_{\text{without}} = 121.5$. The one-sided p-value from the large-sample approximation $p = 0.032$. Reject equality. The yields from fractured rocks are higher.

5.6

The test statistic changes very little ($W_{\text{RS}} = 123$), indicating that most information contained in the data below detection limit is extracted using ranks. Results are the same (one-sided p-value = 0.039. Reject equality). A t-test could not be used without improperly substituting some contrived values for less-thans which might alter the conclusions.

Chapter 6

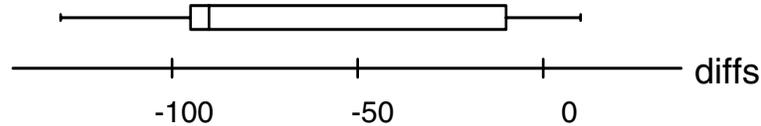
6.1

The sign test is computed on all data after 683 cfs is subtracted. $S^+ = 11$. From table B5, reject if $S^+ \geq 14$ (one-sided test). So do not reject. $p > 0.25$.

6.2 c is not a matched pair.

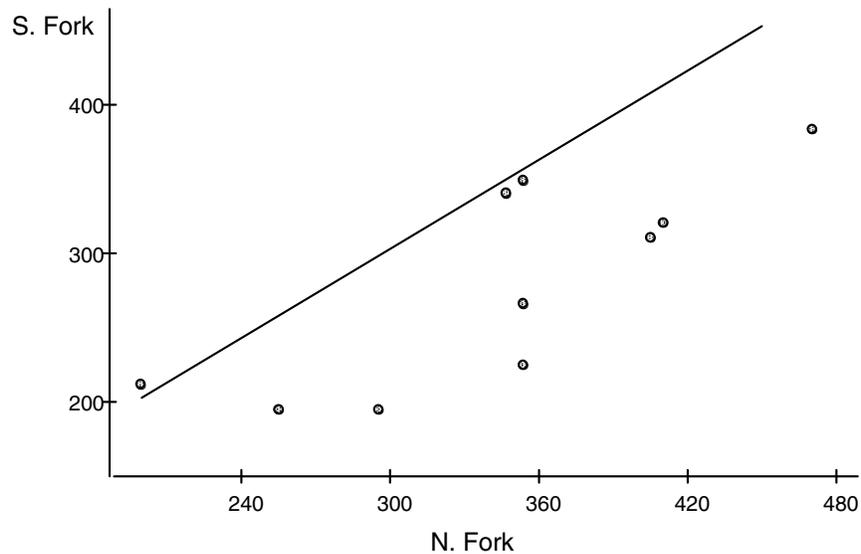
6.3 a. $H_0: \mu (\text{South Fork}) - \mu (\text{North Fork}) = 0.$
 $H_1: \mu (\text{South Fork}) - \mu (\text{North Fork}) \neq 0.$

b. A boxplot of the differences shows no outliers, but the median is low. Conductance data are usually not skewed, and the PPCC $r=0.941$, with normality not rejected. So a t-test on the differences is computed (parametric).



c. $t = -4.24$ $p = 0.002$ Reject H_0 .

d. Along with the boxplot above, a scatterplot shows that the South Fork is higher only once:



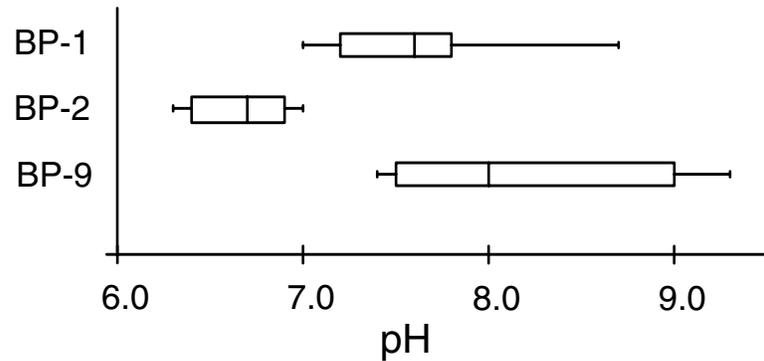
e. The mean difference is -64.7 .

6.4 Because of the data below the reporting limit, the sign test is performed on the differences Sept–June. The one-sided p -value = 0.002. Sept atrazine concentrations are significantly larger than June concs before application.

6.5 For the t-test, $t=1.07$ with a one-sided p -value of 0.15. The t-test cannot reject equality of means because one large outlier in the data produces violations of the assumptions of normality and equal variance.

Chapter 7

- 7.1 As a log-transformed variable, pH often closely follows a normal distribution. See the following boxplots:



pH for three piezometer groups (from Robertson et al., 1984)

The PPCC for the three groups (0.955 for BP-1, 0.971 for BP-2, and 0.946 for BP-9) cannot disprove this assumption. Therefore ANOVA will be used to test the similarity of the three groups.

Anova Table:

Source	df	SS	MS	F	p-value
Piez Gp	2	7.07	3.54	9.57	0.002
<u>Error</u>	<u>15</u>	<u>5.54</u>	0.37		
Total	17	12.61			

The groups are declared different. Statistics for each are:

GP	N	Mean	Std. Dev.	Pooled Std. Dev = 0.608
BP-1	6	7.65	0.596	
BP-2	6	6.68	0.279	
BP-9	6	8.20	0.822	

A Tukey's test on the data is then computed to determine which groups are different. The least significant range for Tukey's test is

$$\begin{aligned} \text{LSR} &= q_{(0.95, 2, 15)} \cdot \sqrt{0.37/6} = 3.01 \cdot 0.248 \\ &= 0.75 \end{aligned}$$

Any group mean pH which differs by more than 0.75 is significantly different by the Tukey's multiple comparison test. Therefore two piezometer groups are contaminated, significantly higher than the uncontaminated BP-2 group:

$$\text{BP-9} \cong \text{BP-1} > \text{BP-2}$$

Since the sample sizes are small (n=6 for each group) one might prefer a Kruskal-Wallis test to protect against any hidden non-normality:

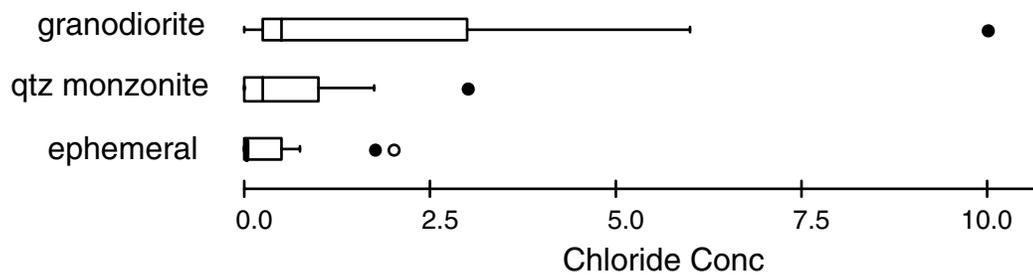
GP	N	MEDIAN	\bar{R}_j
BP-1	6	7.60	11.3
BP-2	6	6.75	3.6
BP-9	6	8.00	13.6
Overall Median = 9.5			

$K = 11.59$ $\chi^2_{0.95,(2)} = 5.99$. Reject H_0 , with $p = 0.003$.
ANOVA and Kruskal-Wallis tests give identical results.

7.2 Boxplots of the data indicate skewness. Therefore the Kruskal-Wallis test is computed:

$K = 7.24$ Corrected for ties, $K = 7.31$. $p = 0.027$

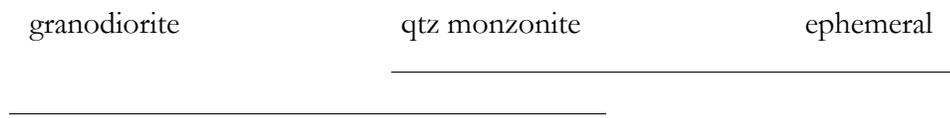
Reject that all groups have the same median chloride concentration.



The medians are ranked as granodiorite > qtz monzonite > ephemeral. Individual K-W tests are computed for adjacent pairs at $\alpha = 0.05$:

granodiorite \cong qtz monzonite ($p = 0.086$)

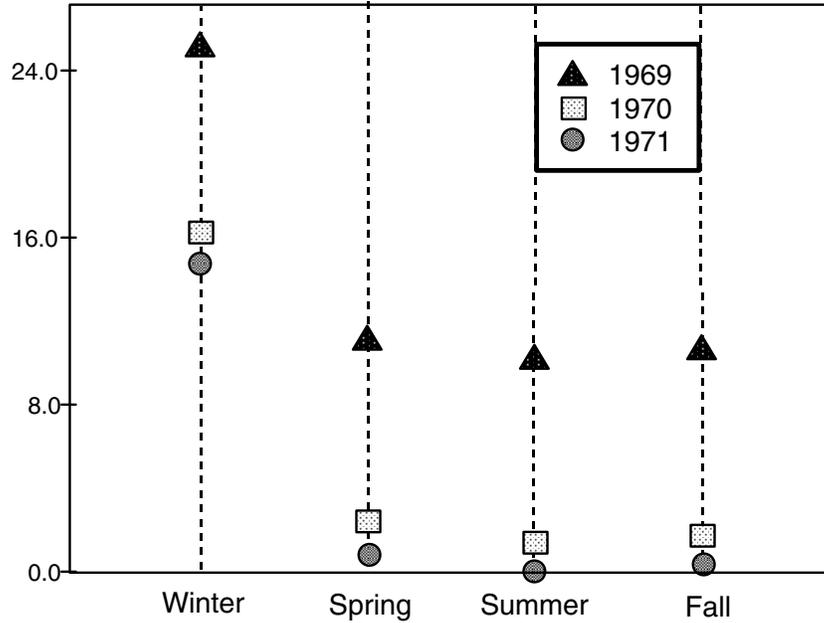
qtz monzonite \cong ephemeral ($p = 0.27$). So:



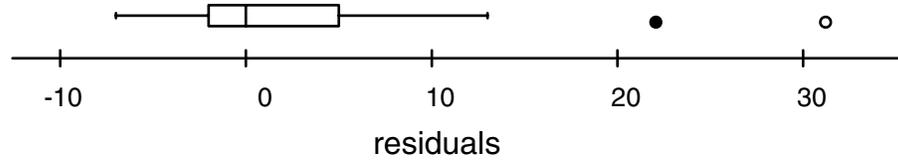
7.3 Median polish for the data of strata 1:

	Winter	Spring	Summer	Fall	Year median
1969	25.25	11.25	10.25	10.75	8.75
1970	16.5	2.5	1.5	2	0.00
1971	15	1	0	0.5	-1.50
Season median	14.25	0.25	-0.75	-0.25	2.25

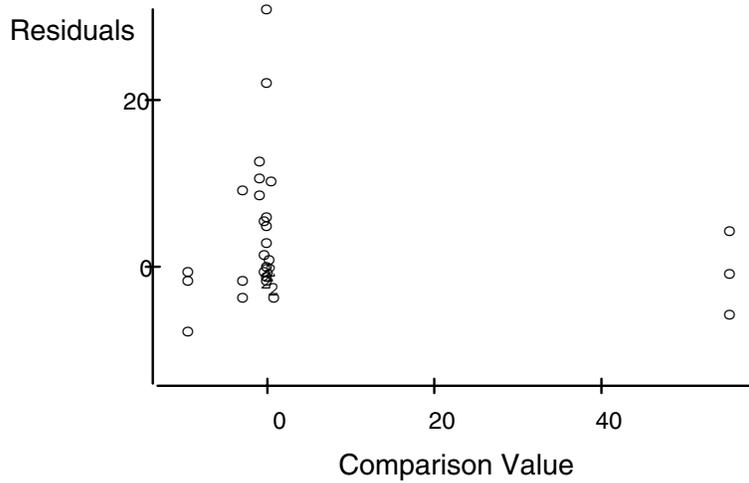
Corbicula densities were 14 units higher in winter than in other seasons, and 9 to 10 units higher in 1969 than 1970 or 1971. Those effects dominated all others. This is shown by a plot of the two-way polished medians:



The residuals are skewed, as shown in a boxplot:



However, a residuals plot of cell residuals versus the comparison value shows outliers, but an overall slope of zero, stating that no power transformation will improve the situation very much.



- 7.4 Due to the outliers noted above, ranks of the Corbicula data were used to test the effects of season and year on the number of organisms.

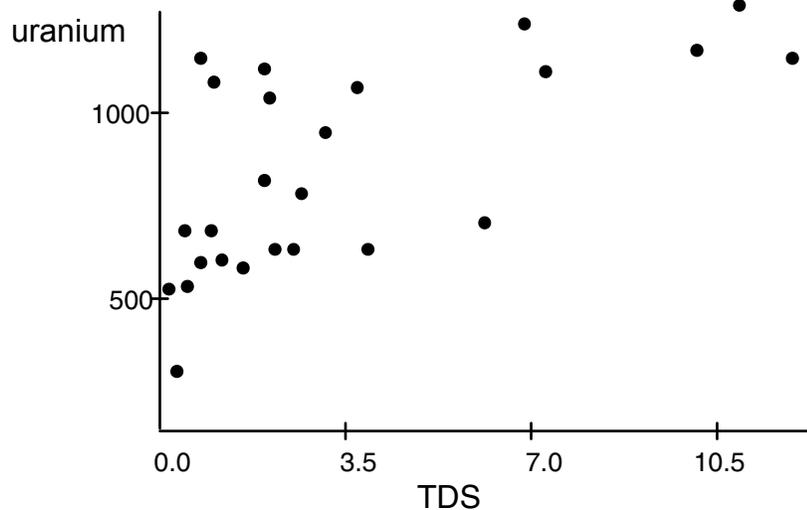
Source	df	SS	MS	F	p-value
Year	2	1064.7	532.33	13.78	0.000
Season	3	1300.7	433.57	11.23	0.000
Year*Season	6	560.8	93.46	2.42	0.057
<u>Error</u>	<u>24</u>	<u>926.8</u>	38.62		
Total	35	3853.0			

A two-way ANOVA on the ranks indicates that both season and year are significant influences on the density of Corbicula, and that there is no interaction. This is illustrated well by the plot of polished medians above.

- 7.5 Not answered.

Chapter 8

- 8.1 The plot of uranium versus total dissolved solids looks like it could be nonlinear near the 0 TDS boundary. So Spearman's rho was computed, and $\rho = 0.72$ with $t_r = 4.75$ and $p < 0.001$.



- 8.2 Pearson's $r = 0.637$ with $t_r = 3.79$ and $p < 0.001$. Kendall's tau = 0.53 with $p < 0.001$. The suggestion of nonlinearity would favor either rho or tau, though the nonlinearity is not serious in this case.
- 8.3 Not answered.

Chapter 9

- 9.1 A residuals plot for the untransformed variables shows strong curvature. A log-log regression gives an acceptable plot, with one outlier not influencing the line:

$$\log(\text{Yield}) = 6.74 + 1.39 \cdot \log(\text{Grain Size}) \quad t = 8.14 \quad p < 0.001$$

- 9.2 The overall mean yield will be the average of estimates of mean yield for the four wells from the regression equation. Applying the $1/2 s^2$ correction factor to obtain the mean yield rather than the median, the estimated mean yields are:

46.104 120.830 316.669 556.380 with overall mean = 260 gal/day/ft².

- 9.3 Here are some possible transformations, including the log. Can logQ be improved on?

<u>explanatory variable</u>	<u>R²</u>
Q	40.8%
Q ^{0.5}	51.1%
log Q	57.3%
Q ^{-0.25}	57.4%
Q ^{-0.5}	55.4%
Q ⁻¹	47.9%
1/(1+0.00001Q)	41.8%
1/(1+0.0001Q)	47.6%
1/(1+0.001Q)	58.5%
1/(1+0.01Q)	52.4%
1/(1+0.1Q)	48.5%

There are perhaps two other good candidate explanatory variables on this list, Q^{-0.25}, and 1/(1+0.001Q). Neither improve significantly over logQ, based on R² or on residuals plots. A residuals plot and probability plot of residuals for the hyperbolic transformation having b=0.001 are below.

When b=0.00001 or smaller, the model is virtually identical to the linear model TDS = b₀+b₁Q [a power transformation with $\theta = 1$]. When b=0.1 or larger, the model is virtually identical to the inverse model TDS = b₀+b₁(1/Q) [a power transformation with $\theta = -1$]. Values of b in between these provide functions similar to moving down the ladder of powers from $\theta = 1$ to $\theta = -1$. The advantage of using the hyperbolic function is its interpretability as a mixing of ground and surface waters (Johnson et al., 1969).

- 9.4 If the objective is to predict LOAD, then that (or its transform) should be the dependent variable. The regression statistics (especially PRESS) will then tell how well the predictions will do. If ln(C) is used as the dependent variable, the standard error s = 0.3394, exactly

the same as in the equation for $\ln(\text{LOAD})$, but $R^2=17.3\%$ rather than 67.9% for $\ln(\text{LOAD})$. The t statistic on β_1 is -4.43 , also significant but not as much as when $y = \ln(\text{LOAD})$. In other words, the error of the $\ln(C)$ values is exactly the same magnitude as the errors of $\ln(\text{LOAD})$. The percent variation explained drops from 67.9% to 17.3% , the difference being the strong effect of Q on variation in LOAD . Note the changes in regression coefficients. The previous model was $\ln(\text{LOAD}) = 0.789 + 0.761 \ln(Q)$. This one is $\ln(C) = -0.194 - 0.239 \ln(Q)$. The intercept decreased by an amount equal to $\ln(2.7)$ (the log of the unit conversion coefficient) and the slope decreased by exactly 1 because Q is removed from both sides. The standard errors of the coefficients are both unchanged.

If LOAD were computed by using the regression for $\ln(C)$ and then multiplying that result by $2.7Q$, exactly the same estimates would result as when using the equation for $\ln(\text{LOAD})$. This is true regardless of which estimation method is employed (median, MLE, or Smearing), and will always be true for log-log regression estimation. The moral of the story is: if your boss thinks that you shouldn't use $\ln(\text{LOAD})$ as the dependent variable and you can't convince him or her otherwise, go ahead and predict $\ln(C)$, and from that $\ln(\text{LOAD})$, and you will still get the results you got doing it the simple way.

Chapter 10

10.1	<u>X</u>	<u>Y</u>	<u>Slopes</u>				
	1	10	30	10	15	13	9.2
	2	40	-10	7.5	7.33	4	
	3	30	25	16	8.67		
	4	55	7	0.5			
	5	62	-6				
	6	56					

Ranked slopes: $-10, -6, 0.5, 4, 7, 7.33, 7.55, 8.67, 9.2, 10, 13, 15, 16, 25, 30$

$$\begin{aligned} \text{a) Median slope} &= 8.67 = \text{Theil slope estimator } \hat{b}_1 \\ \text{Median X} &= 3.5 \\ \text{Median Y} &= 47.5 \\ S = P - M &= 13 - 2 = 11 \end{aligned}$$

$$\text{b) } \tau = \frac{S}{n(n-1)/2} = \frac{11}{6 \cdot 5 / 2} = 0.73$$

$$\begin{aligned} \text{c) intercept } \hat{b}_0 &= Y_{\text{med}} - \hat{b}_1 \cdot X_{\text{med}} = 47.5 - 8.67 \cdot 3.5 = 17.17 \\ Y &= 17.17 + 8.67 \cdot X && \text{is the Kendall-Theil equation} \\ (Y &= 10.07 + 9.17 \cdot X && \text{is the OLS equation for the same data)} \end{aligned}$$

d) from table B8, for S=11 and n=6, two-sided p-value = $2 \cdot 0.028 = 0.056$.

10.2	X	Y	Slopes				
	1	10	30	10	15	47.5	9.2
	2	40	-10	7.5	53.33	4	
	3	30	25	85	8.67		
	4	55	145	0.5			
	5	200	-144				
	6	56					

Ranked slopes -144, -10, 0.5, 4, 7.5, 8.67, 9.2, 10, 15, 25, 30, 47.5, 53.33, 85, 145

a) Median slope = 10 = Theil slope estimator \hat{b}_1
 Median X = 3.5
 Median Y = 47.5

b) S and τ are unchanged

c) $\hat{b}_0 = 47.5 - 10 \cdot 3.5 = 12.5$
 $Y = 12.5 + 10 \cdot X$ the Kendall-Theil equation is similar to ex. 10.1.
 $(Y = -8.33 + 21 \cdot X$ the OLS slope has changed a lot from ex. 10.1)

d) the p-value is unchanged.

e) for a 95% confidence interval, the closest entry in table B8 to $\alpha/2=0.025$ is $p=0.028$ for $X_u=11$. From eqs. 10.3 and 10.4,

$$R_u = \frac{(15 + 11)}{2} = 13 \text{ for } N=15 \text{ and } X_u=11.$$

The rank R_l of the pairwise slope corresponding to the lower confidence limit is

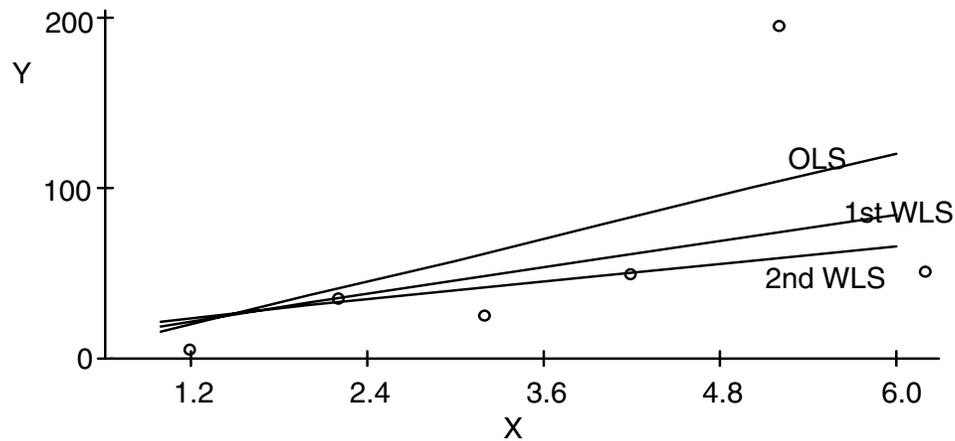
$$R_l = \frac{(15 - 11)}{2} + 1 = 3.$$

So an $\alpha = 0.054$ confidence limit for $\hat{\beta}_1$ is the interval between the 3rd and 13th ranked pairwise slope (the 3rd slope in from either end), or

$$0.5 \leq \hat{\beta}_1 \leq 53.3.$$

10.3 The unweighted OLS regression equation is

$$Y = -8.3 + 21.0 \cdot X \quad t = 1.41 \quad p = 0.23$$



The residuals are then divided by $6 \cdot (\text{MAD})$, where the MAD is the median of the absolute values of the residuals. Bisquare weights are computed for each data point:

pt #	1	2	3	4	5	6
weight	0.999	0.996	0.935	0.954	0.179	0.631

A first weighted least squares is then computed:

$$Y = 3.1 + 13.1 X \quad t = 1.49 \quad p = 0.21$$

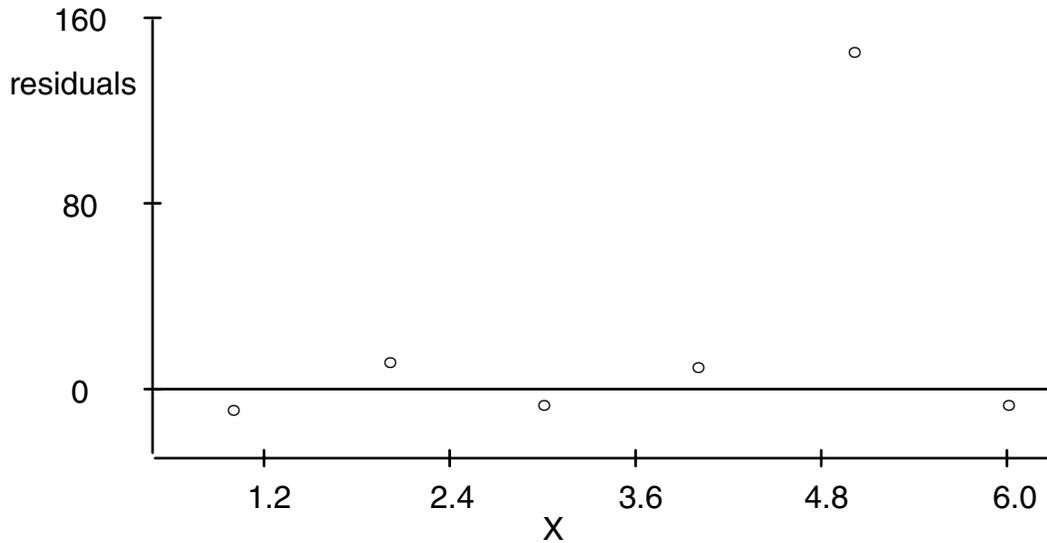
Bisquare weights are again computed for each data point, using residuals from the first WLS:

pt #	1	2	3	4	5	6
weight	0.984	0.952	0.938	1.000	0.000	0.746

A second WLS is then computed:

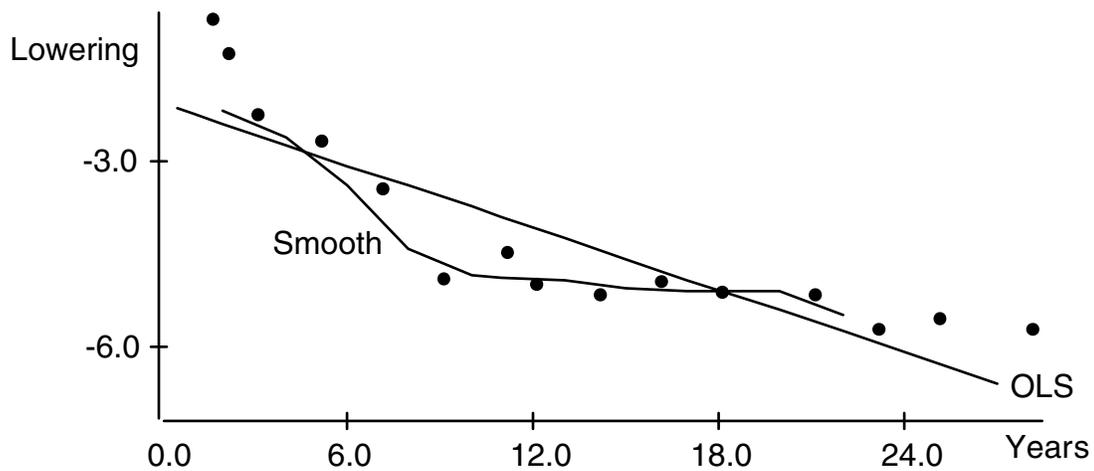
$$Y = 10.4 + 8.80 X \quad t = 2.73 \quad p = 0.07$$

Though the slope has diminished from the OLS line, the significance has greatly increased due to the lower weight of the outlier. Note the similarity between this WLS and the Kendall's robust line. A residuals plot shows that the WLS line fits most of the data much better than with OLS. The outlier's influence on the slope has diminished, and its residual remains large.



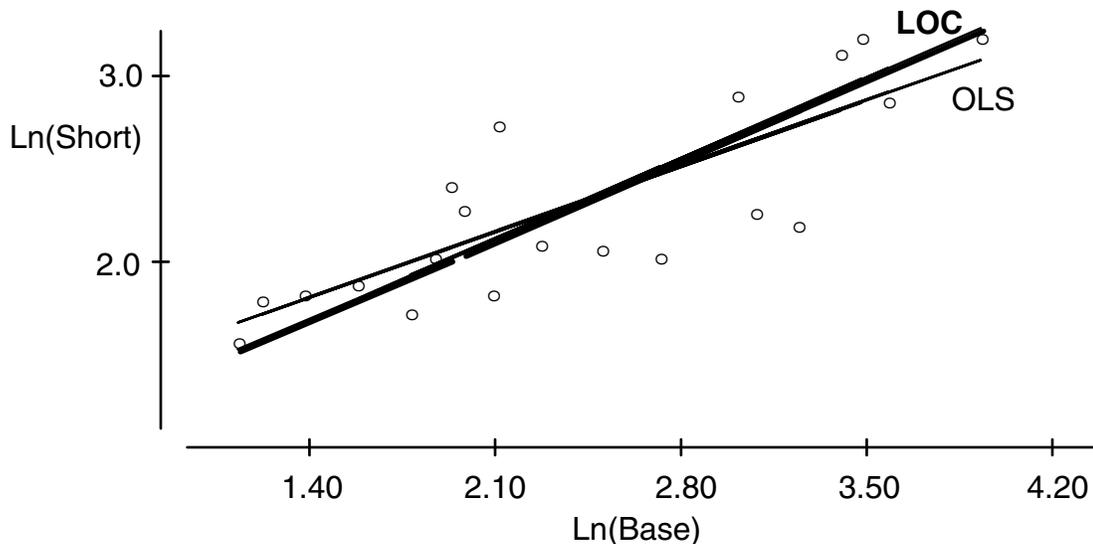
Some object: "Isn't this WLS line the same as throwing away the outlier -- it has a weight of zero?" The difference is that the outlier was determined to be downweighted to zero by the data itself, not an arbitrary decision by the data analyst. Weighted least squares also allows outliers to have partial weights, not simply a zero or one weight as with discarding the outlier. So WLS is far less arbitrary and far more consistent in its assignment of weights to all data points than is throwing away outliers.

10.4 Lowering = $-2.07 - 0.167 \cdot \text{Years}$ $r^2 = 0.76$



OLS does not follow the data as well as the smooth because the data are nonlinear.

10.5 Plotting the 20 years of joint data shows that curvature and heteroscedasticity exist, and transformation is required before regression. Thus the natural logs of both are taken. A linear relation results, as shown in the following plot.



Regression between the 20 year joint record at the two stations is:

$$\text{Ln(Short)} = 1.095 + 0.507 \cdot \text{Ln(Base)} \quad t = 6.00 \quad p < 0.001 \quad R^2 = 0.67$$

Using this equation and the 30 additional years of record at Base, 30 years of simulated flows at Short are generated. Now the LOC is used to generate estimates of the "Short" 30-year record. Summary statistics for the 20 years of joint Ln(Base) and Ln(Short) records are as follows:

	<u>n</u>	<u>Mean</u>	<u>Stdev</u>	<u>Median</u>	<u>P25</u>	<u>P75</u>
Ln(Short)	20	2.319	0.524	2.160	1.862	2.850
Ln(Base)	20	2.414	0.844	2.190	1.802	3.200

From equation 10.10,

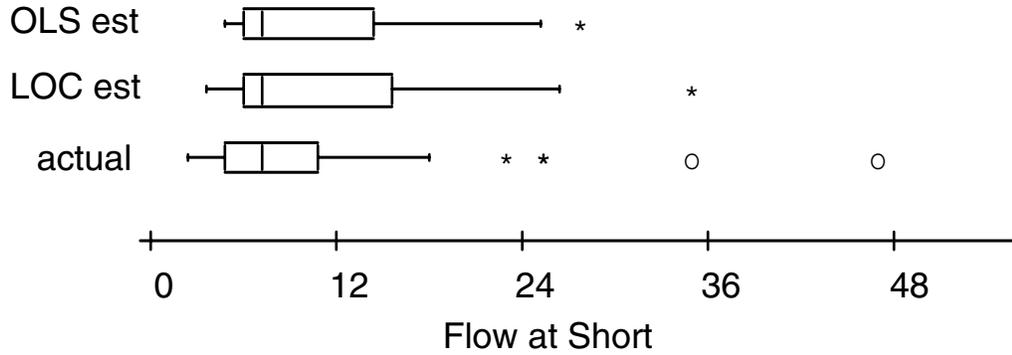
$$\begin{aligned}
 Y_i &= \bar{Y} + \text{sign}[r] \cdot \frac{s_y}{s_x} \cdot (X_i - \bar{X}), \quad \text{or} \\
 \text{Ln(Short)} &= 2.319 + (.524/.844) \cdot (\text{Ln(Base)} - 2.414) \\
 &= 0.820 + .621 \cdot \text{Ln(Base)}
 \end{aligned}$$

Note how the slope and intercept for LOC differ from the regression coefficients. Summary statistics for the estimated flows at "Short" by the two methods are compared to the true 30-year record from Appendix C13 in the following table.

	<u>n</u>	<u>Mean</u>	<u>Stdev</u>	<u>Median</u>	<u>P25</u>	<u>P75</u>
OLS est.	30	2.2087	0.4975	2.0228	1.7731	2.6249
LOC est.	30	2.184	0.609	1.956	1.651	2.694
true values	30	2.079	0.613	1.930	1.630	2.290

The standard deviation for the regression estimate is too small, as expected.

Boxplots are presented below for three groups: the 30-year estimates using regression and LOC combined with the 20-year record at Short, and the actual 50-year record. LOC comes closer to correctly estimating the lowest and highest flows. The regression estimates are too low for high flows, and too high for low flows. They "regress" toward the mean more than the actual data because the standard deviation of the estimates is too small, as $R^2 < 1$.



10.6 Not answered.

Chapter 11

11.1 The full multiple regression model contains strong multi-collinearity. The VIFs among the four percentage variables are huge:

$\text{LOGTN} = -1.3 + 0.596 \text{ LOGDA} + 0.346 \text{ LOGIMP} + 0.0314 \text{ MMJTEMP}$ $- 0.0494 \text{ MSRAIN} + 0.040 \text{ PRES} + 0.035 \text{ PNON} + 0.037 \text{ PCOMM} + 0.024 \text{ PIND}$					
$n = 42$	$s = 0.61$	$R^2 = 0.59$			
<u>Parameter</u>	<u>Estimate</u>	<u>Std.Err(β)</u>	<u>t-ratio</u>	<u>p</u>	<u>VIF</u>
Intercept β_0	-1.28	24.60	-0.05	0.959	
Slopes β_k					
LOGDA	0.596	0.121	4.94	0.000	1.8
LOGIMP	0.346	0.228	1.52	0.138	3.8
MMJTEMP	0.031	0.019	1.65	0.107	10.1
MSRAIN	-0.049	0.021	-2.32	0.026	9.1
PRES	0.040	0.245	0.16	0.873	9227.2
PNON	0.035	0.246	0.14	0.888	3062.2
PCOMM	0.037	0.245	0.15	0.882	8311.4
PIND	0.024	0.246	0.10	0.922	2026.2
Table 11.9 Regression statistics and VIF's for Exercise 11.1					

To determine why the multi-collinearity is so strong, the correlation matrix is computed.

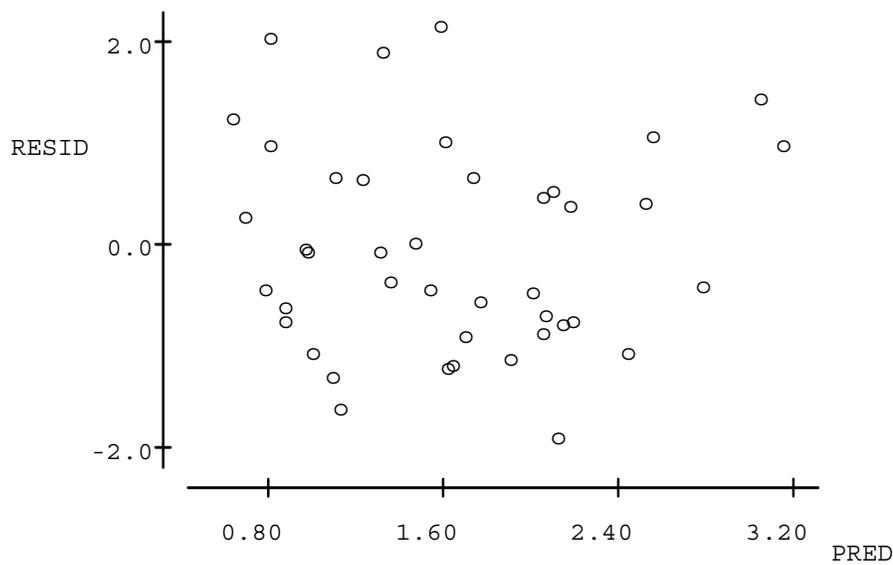
	LOG	MMJ	MS					
	LOGTN	LOGDA	IMP	TEMP	RAIN	PRES	PNON	PCOMM
LOGDA	0.565							
LOGIMP	0.058	-0.382						
MMJTEMP	-0.205	-0.188	0.094					
MSRAIN	-0.259	-0.083	0.018	0.915				
PRES	0.294	0.210	-0.246	0.040	0.003			
PNON	-0.042	0.319	-0.639	0.066	0.065	-0.321		
PCOMM	-0.218	-0.441	0.589	-0.027	0.039	-0.747	-0.206	
PIND	-0.131	0.060	0.114	-0.111	-0.164	-0.226	-0.124	-0.180

Surprisingly, the percentage terms do not have large pair-wise correlation coefficients. Instead, they are strongly related in that the four of them add to 100%, except for rounding error. This is why the VIF's are so large. Therefore at least one of them should be dropped. The variable with the smallest partial F (PIND) could be chosen. This brings the VIF down from over 9000 to 10, still large. In order to save much time the Cp and PRESS statistics can be computed for all possible models. The results below show that the best 5-variable model, containing LOGDA, LOGIMP, MMJTEMP,

MSRAIN, and PIND, is the best in terms of prediction errors (PRESS) and model bias/standard error (Cp). VIFs are below 10 ($R^2 < 0.9$) and so are acceptable.

# of Vars	R-sq	PRESS	C-p	Max VIF	<u>X: Variable is in the model</u>																	
					M	L	M	M	L	O	J	S	P	P	C	P						
					A	P	P	N	S	N	M	D										
1	32.0	22.9	17.0	----	X																	
1	8.6	30.0	35.8	----						X												
2	40.7	21.2	11.9	1.2	X	X																
2	37.5	22.2	14.5	1.1	X						X											
3	46.4	20.2	9.3	1.2	X	X			X													
3	45.2	19.8	10.3	1.2	X	X															X	
4	51.2	18.9	7.4	1.2	X	X		X													X	
4	50.9	19.8	7.7	1.2	X	X		X	X													
* 5	57.0	17.8	4.7	6.8	X	X	X	X													X	
5	56.2	18.0	5.4	7.5	X		X	X	X					X								X
6	59.1	18.2	5.1	7.1	X	X	X	X	X													X
6	58.6	18.9	5.4	7.1	X	X	X	X		X												X
7	59.1	19.4	7.0	10.0	X	X	X	X	X	X	X											
7	59.1	19.4	7.0	13.2	X	X	X	X	X					X							X	X
8	59.1	20.1	9.0	9227	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

A residuals plot from the minimum PRESS and Cp model shows no hint of curvature or increasing variance. Therefore this model is preferred.



Residuals plot for the regression of Exercise 11.1

11.2 First compare those models which have equal numbers of parameters and eliminate the ones with higher SSE.

Compare 4 to 7 , eliminate 4

Compare 3 to 6 , eliminate 3

Compare 2 to 5 , eliminate 2

Compare 6 to 8 , eliminate 6

Now, for the remaining models (1, 5, 7, 8, 9, 10) perform F tests between pairs of nested models. The order in which to proceed is arbitrary.

<u>Compare</u>	<u>F</u>	<u>df_{num}</u>	<u>df_{denom}</u>	<u>F_{crit}</u>	<u>conclusion</u>
Models 1 to 5	11.18	1	123	3.9	reject H_0 , eliminate model 1
Models 5 to 7	0.28	2	121	3.1	do not reject eliminate model 7
Models 5 to 8	1.39	1	122	3.9	do not reject eliminate model 8
Models 5 to 9	0.77	3	120	2.68	do not reject eliminate model 9
Models 5 to 10	0.88	5	118	2.29	do not reject eliminate model 10

Model 5 is the preferred model.

Another possible approach is to use either PRESS or Mallows Cp.

<u>Model</u>	<u>p</u>	<u>s²</u>	<u>Cp</u>
1	3	0.5636	14.29
2	4	0.5350	8.37
3	5	0.5343	9.17
4	6	0.5359	10.51
5	4	0.5183	4.41
6	5	0.5207	5.98
7	6	0.5245	7.84
8	5	0.5166	5.00
9	7	0.5212	8.06
10	9	0.5208	9.95

The results are interpreted as: the transport curve is quadratic with a shift in intercept for the winter months. Only two seasons (not three) can be distinguished. The slope of the curve does not change with season.

11.3 Not answered.

11.4 Not answered.

Chapter 12

12.1 Regression

$$\begin{array}{l} \text{load} = 25,250 - 12.6 \text{ year} \quad r^2 = 10.6\% \\ (t) \quad (1.53) \quad (-1.50) \quad \text{two-sided p value} = 0.134 \end{array}$$

Multiple regression

$$\begin{array}{l} \text{load} = 28,152 - 14.4 \text{ year} + 0.696 \text{ q} \quad r^2 = 88.3\% \\ (t) \quad (4.69) \quad (-4.60) \quad (10.91) \quad \text{two-sided p value} \cong 0.0001 \end{array}$$

Mann-Kendall

$$\text{load} = 11,800 - 5.8 \text{ year} \quad \text{two-sided p value} = 0.415$$

Mann-Kendall on Residuals

$$\begin{array}{l} \text{Regression model is} \quad \text{load} = -110 + 0.681 \text{ q} \quad r^2 = 74.5\% \\ (t) \quad (-1.24) \quad (7.44) \end{array}$$

$$\begin{array}{l} \text{Kendall fit:} \quad \text{residual} = 28,250 - 14.4 \text{ year} \quad \text{two-sided p value} = 0.0001 \\ \text{therefore} \quad \text{load} = -110 + 0.681 \text{ q} + \text{residual} \\ \quad \quad \quad = -110 + 0.681 \text{ q} + 28,250 - 14.4 \text{ year} \\ \quad \quad \quad = 28,140 - 14.4 \text{ year} + 0.681 \text{ q} \end{array}$$

12.2 Winter: $P = 16$, $M = 34$, $S = -18$

1 tie of 3, 2 ties of 2

$\text{Var}[S] = 159.33$

$Z = -1.347$

$p = 0.18$ very little evidence of downtrend in winter lead

Spring: $P = 27$, $M = 38$, $S = -11$

3 ties of 2, 1 tie of 5

$\text{Var}[S] = 249$

$Z = 0.633$

$p = 0.53$ no evidence of downtrend in spring lead

Summer: $P = 16$, $M = 33$, $S = -17$

1 tie of 4

$\text{Var}[S] = 156.33$

$Z = -1.28$

$p = 0.20$ very little evidence of downtrend in summer lead

Fall: $P = 11$, $M = 37$, $S = -26$

1 tie of 4, 1 tie of 2

$\text{Var}[S] = 155.33$

$Z = 2.005$

$p = 0.045$ fairly strong evidence of downtrend in fall lead

Seasonal Kendall: $S = -72$

$\text{VAR}[S] = 720$

$Z = -2.646$

p (2-sided) = 0.008

Thus, even though the evidence from no individual season was highly conclusive, the data from all seasons taken together provides highly conclusive evidence of a downtrend in lead.

12.3 Maumee River Trends in Total Phosphorus

12.3.1 Parametric analysis first: LOAD vs TIME

Simple linear regression: $\text{LOAD} = 444 - 0.221 \text{ TIME}$

$t = -0.42$ $p = 0.673$

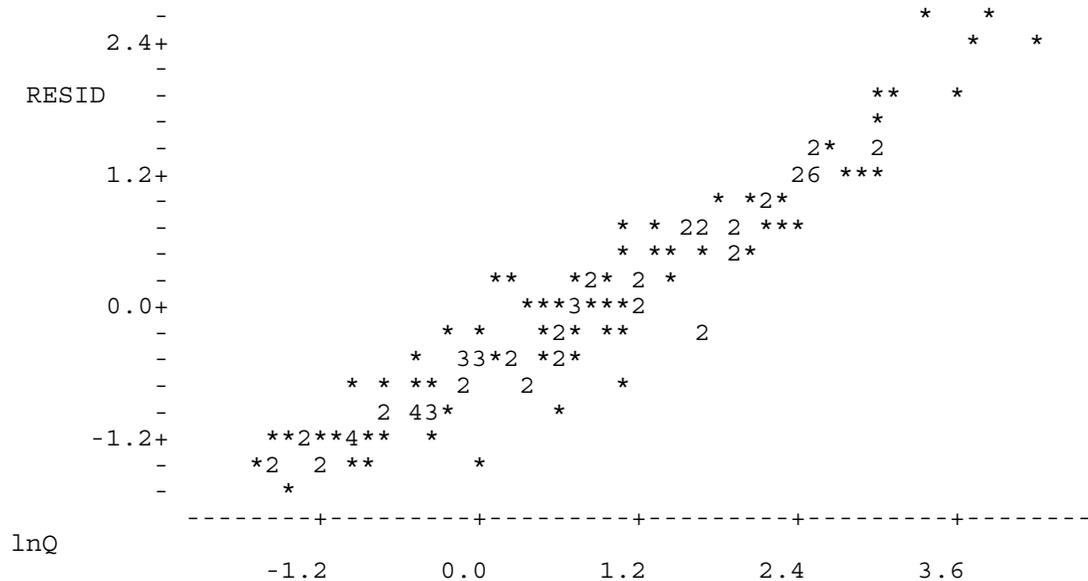
$s = 20.59$ $R\text{-sq} = 0.1\%$ $R\text{-sq(adi)} = 0.0\%$

A boxplot of the residuals shows them to be terribly skewed. A transformation is required. Try logarithms. Then the regression equation is:

$$\ln(\text{LOAD}) = 117 - 0.0592 \text{ TIME}$$

$$\begin{aligned} t &= -1.32 & p &= 0.189 \\ s &= 1.770 & \text{R-sq} &= 1.3\% & \text{R-sq(adj)} &= 0.6\% \end{aligned}$$

There is a fairly normal distribution of residuals, so a test based on regression seems legitimate. Very weak evidence of trend -- (two-sided) p-value of 0.189. But are there strong flow and/or seasonal effects? A plot of the residuals versus log of streamflow (LQ) shows a strong dependence on flow. Removing this should greatly enhance the power to detect any trend which is present.



Boxplots of residuals by month also show a strong seasonal cycle, high in the winter & spring, low in summer. The best model we could find includes time, $\ln(Q)$, $\ln(Q)^2$, and sine and cosine of $2\pi T$:

$$\text{LLOAD} = 83.3 - 0.0425 \text{ TIME} + 1.08 \ln Q + 0.0679 \ln(Q)^2 - 0.0519 \text{ SIN} + 0.141 \text{ COS}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	83.32	22.06	3.78	0.000
TIME	-0.04250	0.01115	-3.81	0.000
$\ln Q$	1.08175	0.04947	21.87	0.000
$\ln(Q)^2$	0.06789	0.01868	3.63	0.000
SIN	-0.05190	0.06252	-0.83	0.408
COS	0.14058	0.05441	2.58	0.011
s = 0.4398		R-sq = 94.1%		R-sq(adj) = 93.9%

This is interpreted as a strong evidence of downtrend, with a p-value <.001 The slope (in log units) = -0.0425 per year. All coefficients are significant at $\alpha = 0.05$ except for $\sin(2\pi T)$. The sine must either be left in, or both it and the cosine taken out. To test whether together they are significant, an F test is performed. The model without these terms, with the standard error $s = 0.449$, is:

$$\text{LLOAD} = 85.1 - 0.0434 \text{ TIME} + 1.06 \text{ LQ} + 0.0748 \text{ LQSQ}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	85.09	22.51	3.78	0.000
TIME	-0.04340	0.01138	-3.81	0.000
lnQ	1.05921	0.04518	23.44	0.000
$\ln(Q)^2$	0.07481	0.01865	4.01	0.000
s = 0.4490		R-sq = 93.7%		R-sq(adj) = 93.6%

So the F test to compare these two models is:

$$F = \frac{(26.01 - 24.57) / 2.0}{0.193} = 3.73$$

Comparing to an F distribution with 2 and 127 degrees of freedom, the two-sided p-value is 0.027. Therefore reject the simpler model in favor of including the seasonal sine and cosine terms.

To predict estimates of load for the two times and two flow conditions above, natural logs of these values are input to the regression equation. The third column below reports the predicted logs of Load from the regression equation.

lnQ	Time	Predicted lnL	Bias-Corrected L
2.4	1972.5	2.3356	11.3852
0.0	1972.5	-0.6516	0.574136
2.4	1986.5	1.7406	6.27963
0.0	1986.5	-1.2467	0.316640

These predicitions must be transformed and corrected for bias. Using the Ferguson (MLE) bias correction, $0.5 \cdot s^2 = 0.5 \cdot (0.4398)^2 = 0.097$. So the bias correction equals $\exp(0.097)$, or about 10%. The four predicted total phosphorus loads are given above in the fourth column.

Therefore the percent change at high flow over the 14-year time period is:

$$(6.2763 - 11.3852) / 11.3852 = -0.448732$$

The change in percent per year is

$$-0.448732 \cdot 100 / 14 = -3.205. \text{ That is a } -3.2\% \text{ change in total P per year.}$$

The same analysis at lower flow over the 14 years is:

$$(0.31664 - 0.574136) / 0.574136 = -0.4485, \text{ the same amount as at high flow.}$$

Re-expressing the slope estimate in original units as a percent change, the average change equals -4.2% per year:

$$100 \cdot [\exp(-0.0425) - 1.0] = -4.16096$$

12.3.2 The nonparametric approach

The seasonal-Kendall test on the original observations, using 12 seasons (months): $\tau = -0.06$ with a p-value of 0.3835.

$$\log(\text{Load}) = 0.505 - 0.046 \cdot \text{time},$$

where $\text{time} = 0$ at the beginning of the first year of the record (typically a water year), and time is in units of years.

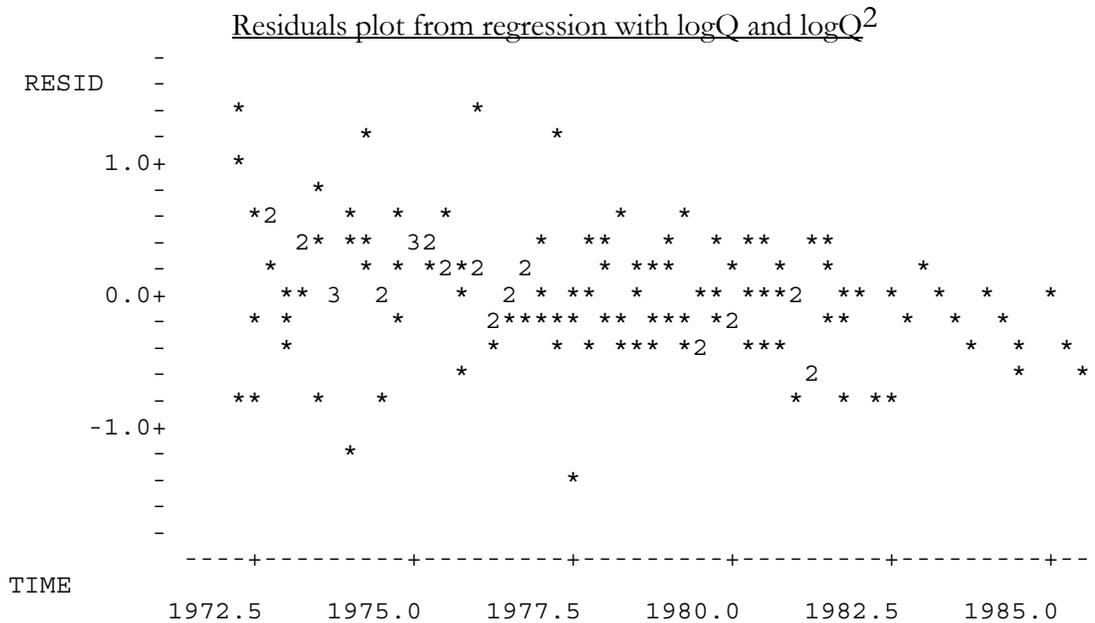
Residuals from a regression of $\log(\text{Load})$ versus $\log Q$ and $\log Q^2$ removes the effect of flow:

$$\log(\text{Load}) = -0.745 + 1.06 \log Q + 0.0758 \log Q^2$$

The S-K test on the regression residuals: $\tau = -0.25$ with $p = 0.0002$

$$\text{and } \log(\text{Load}) = 0.312 - 0.048 \cdot \text{time}$$

So, if flow is not first removed, the significant trend would be missed. Both the Seasonal Kendall on the residuals and multiple regression give a highly significant p-value. The S-K slope is 4.8% rather than 4.16% because of the effect of the low residuals during 1972-1977.



12.4 Not answered.

12.5 Not answered.

Chapter 13

13.1 Not answered

13.2 Because there is only one reporting limit, Kendall's tau can easily be computed for this data: $\tau = -0.40$ with $p = 0.023$. There is a significant decrease in TPT concentrations with depth.

13.3 Estimates of the four descriptive statistics for each of 5 multiple-threshold methods (see Helsel and Cohn, 1988) are:

<u>Method</u>	<u>MEAN</u>	<u>ST.DEV.</u>	<u>MEDIAN</u>	<u>IQR</u>
ZE (substitute zero)	12.36	75.48	0.00	1.10
HA (substitute 1/2 dl)	13.91	75.28	1.10	3.30
DL (substitute the dl)	15.45	75.19	1.30	4.10
MR (prob plot regression)	12.57	75.44	0.29	1.54
MM (lognormal MLE)	8.30	61.52	0.34	1.62

Because of the outlier at 560 $\mu\text{g/L}$ the data have more skewness than a lognormal distribution, and methods which assume a lognormal distribution for all the data (MM) would not be expected to estimate moment statistics well. It is not surprising therefore that the MLE method produces moment estimates dissimilar to the other methods. We generally select the MR moment estimates and the MM quantile estimates (those printed in bold), due to the results of Helsel and Cohn (1988).

Chapter 14

14.1 a) Contingency table

Expected values E_{ij}				
Trend in C1- (1974-81, $\alpha=0.1$)				
<u>Δ road salt appl.</u>	Down	No trend	Up	Totals
Down	5.44	23.84	16.71	46
No change	9.82	43.02	30.15	83
Up	13.73	60.13	42.14	116
Totals	29	127	89	245

Table of $\frac{(O-E)^2}{E}$ **Trend in Cl⁻ (1974-81, $\alpha=0.1$)**

Δ road salt appl.	Down	No trend	Up	Totals
Down	0.04	2.79	3.56	
No change	1.78	0.02	0.88	
Up	1.01	1.39	3.93	

$$\chi^2 = 15.39 \quad df = 4 \quad p = 0.004$$

The results indicate that the category of chloride trends is dependent on the category of salt applications, with a p-value of 0.004. Where increases in road salt occurred, there are more up trends and fewer down trends than would be expected from the marginal distributions of up trends and down trends. Where decreases in road salt occurred, there are fewer up trends than would be expected.

b) Kendall's tau

$$P = \text{no. pluses} = 5(44+25+51+55) + 32(25+55) + 14(51+55) + 44(55) = 7339$$

$$M = \text{no. minuses} = 32(14+10) + 9(14+44+10+51) + 44(10) + 25(10+51) = 3804$$

$$S = 7339 - 3804 = 3535.$$

$$\tau_b = \frac{3535}{\sqrt{\frac{(245^2 - (46^2 + 83^2 + 116^2)) (245^2 - (29^2 + 127^2 + 89^2))}{2}}}$$

$$= 3535 / 18164 = 0.19$$

To test for significance,

$$\sigma_S \cong \sqrt{\frac{1}{9} * (1 - (.19^3 + .34^3 + .47^3)) * (1 - (.12^3 + .52^3 + .36^3))}$$

$$= \sqrt{\frac{(0.85 * 0.81 * 245^3)}{9}} = 1061$$

and so $Z_S = 3534 / 1061 = 3.33$ and two-sided $p = 0.0008$.

The two variables are significantly and positively related.

c) Kendall's tau is more appropriate because

1. It includes the information that the variables are ordinal into the test. The p-value for Kendall's tau is lower than that for the contingency table, reflecting this additional information.
2. It provides a measure of the direction of association τ_b . Since τ is positive, the trends in Cl⁻ increase with increasing trends in road salt application.

$$\begin{aligned} \sigma_S &\equiv \sqrt{\frac{1}{9} \cdot (1 - (0.18^3 + 0.43^3 + 0.39^3)) \cdot (1 - (0.41^3 + 0.59^3)) \cdot 51^3} \\ &\equiv \sqrt{\frac{(0.86) \cdot (0.73) \cdot 51^3}{9}} = \sqrt{9253} = 96.2 \end{aligned}$$

$$Z_S \equiv \frac{-127+1}{96.2} = -1.31$$

and from a table of the normal distribution the one-sided p-value is $p = 0.095$. Therefore $H_0: \tau_b = 0$ is not rejected at $\alpha = 0.05$, but is for $\alpha = 0.10$. Thus there is weak evidence of a downtrend in TBT concentrations based on a split at 200 ng/L. Stronger evidence could be obtained by collecting data for subsequent years, or by obtaining better resolution of the data (the original data reported concentration values rather than a split at 200 ng/L).

Chapter 15

15.1 Logistic regression for the full model with four explanatory variables gives:

Variable Name	Parameter Estimate	Standard Error	Wald's t	p-value
Constant	-13.20539	3.55770	-3.71	0.0002
Thick	0.51527	0.15093	3.41	0.0004
Yields	0.42909	0.27484	1.56	0.0607
GW Qual	0.03035	0.32460	0.09	0.4642
Hazard	1.08952	0.29860	3.65	0.0002

with a likelihood ratio $lr_0 = 49.70$ and $p < 0.000$ as compared to the intercept-only model. However, two of the variables (Yields and GW Qual) have insignificant t-statistics. In the following model they are dropped, and lr_0 recomputed:

Variable Name	Parameter Estimate	Standard Error	Wald's t	p-value
Constant	-10.89039	2.43434	-4.47	<0.0001
Thick	0.46358	0.13575	3.41	0.0004
Hazard	1.07401	0.28301	3.80	0.0001

with a likelihood ratio $lr_0 = 52.54$ and $p < 0.000$ as compared to the intercept-only model. The partial likelihood ratio to test whether the first model is significantly better than the simpler second model is:

$$lr = lr_0(\text{simple}) - lr_0(\text{complex}) = 52.54 - 49.70 = 2.84$$

which for a chi-square distribution with 2 degrees of freedom gives:

$$p = 0.242.$$

Therefore the two additional variables (Yields and GW Qual) do not appreciably add to the explanatory power of the model.

Index

- 3-dimensional rotation, 60
- 7-day 10-year low flow, 83
- additive relation, 42, 142
 - checking for, 186
- adjacent values, 26
- adjusted R^2 , 399
- adjusted variable Kendall test, 336
- adjusted variable plots, 301
- AIC, 399
- Akaike's information criteria, 399
- aligned-ranks, 192
- alpha level, 107
- alpha level, 68
- alternate hypothesis, 104
- analysis of variance, 158
 - ANOVA table, 168
 - assumptions of, 166
 - multi-factor, 170
 - on the ranks, 163
 - on within-block ranks, 191
 - one-factor, 164
 - two-factor ANOVA table, 174
 - two-way rank tests, 170
 - unbalanced design, 179
- angles, judgment of on graphs, 417
- ANOVA. see analysis of variance
- area, judgment of on graphs, 416
- assumption of independence, 252
- asymmetric confidence intervals, 69
 - for the mean, 76
- attained significance level, 108
- autocorrelation. see serial correlation
- bar chart, 62, 206
 - grouped, 422
 - stacked, 421
- bias correction, 258
- bisquare weight function, 284
- block effect, 196
- blocking, 181, 197
- Blom plotting position, 24
- boxplots, 24, 39, 423
 - side-by-side, 129
- boxplots, 128, 207
- bulging rule, 230
- business graphics, 412
- categorical variables
 - Kruskal-Wallis test for, 382
- categories, 19
- censored data, 3, 124, 128
 - guidelines for use, 373
 - in trend tests, 353
 - nonparametric tests, 366
 - parametric tests, 366
 - regression, 370
- characteristics of data, 2
- chemometrics, 372
- coefficient of determination, 228
- coefficient of skewness, 11
- Cohen's method, 353, 360
- color, use of in graphs, 412
- comparing among distributions, 35
- comparison value, 186
- compliance with water quality standards, 83
- confidence intervals, 66
 - for percentiles, 82
 - for percentiles (nonparametric), 84

- confidence intervals, **cont.**
 - for percentiles (parametric), 90
 - for regression line, 242
 - for skewed data, 69
 - for the mean, 75
 - for the median, 70
- confidence level, 68
- constant variance assumption, 255
- contingency tables, 378
 - for censored data, 372
- continuity correction, 122, 141, 147
- control chart, 93
- Cook's D, 250
- correction for ties
 - Kendall's tau, 215
 - rank-sum test, 123
- correlation, 209
 - monotonic, 210
- correlation coefficient, 210
 - linear, 218
 - nonparametric, 212
- critical values
 - lower, 113
 - upper, 113
- cumulative distribution function, 23
- cumulative line graph, 419
- cumulative logits, 406
- Cunnane plotting position, 22, 24, 114
- degrees of freedom, 160, 167, 297, 310, 380
- detection limit, 3, 128
 - guidelines for use of data, 373
 - more than one, 354
 - trend tests for data below, 353
- DFFITs, 250
- difference between group means, 135
 - confidence interval for, 135
- differences between groups
 - estimators, 132
- discriminant function analysis, 402
- dot and line plot, 36, 38
- dot charts, 423
- Duncan's multiple range test, 199, 200
- Durbin-Watson statistic, 253
- efficiency
 - asymptotic relative, 102
 - Kendall vs. OLS, 268
- equal variance, 124
- error rate, 107, 110
 - overall, 199
 - pairwise, 199
- error sum of squares, 167, 196
- exogenous variables in trend tests, 330
- expected value, 224
- explanatory variable, 99, 158, 222
- extension of records, 278
- factor, 158
- factorial ANOVA, 170
 - assumptions of, 173
- far-out values, 26
- fixed effects, 180
- flood-frequency, 24
- flow-duration, 24
- framed rectangle, 421
- Friedman test, 170, 187, 192
 - large sample approximation, 188
- F-test, 168, 174
- geometric mean, 7, 73
 - estimation for censored data, 361
- geometric mean functional regression, 276
- graphical analysis, 19
- graphical comparisons, 35
- graphical methods, 128
- graphics, 205
- graphs
 - pie chart, 416
 - angle and slope, 417
 - boxplots, 423
 - cumulative line graphs, 419
 - dot charts, 423
 - framed rectangles, 421

- graphs, **cont.**
- grouped bar charts, 422
 - hidden scale breaks, 428
 - judgments of length, 420
 - misleading, 424
 - overlapping histograms, 429
 - precision of, 411
 - stacked bar charts, 422
 - use of color, 412
 - use of numbers on, 427
 - use of perspective, 424
 - use of shading, 413
- Gringorten plotting position, 24
- Gumbel distribution, 90
- harmonic mean, 199
- Hazen plotting position, 24
- heavy tails, 2
- heteroscedasticity, 124, 230, 255, 280
- hinges, 25
- histograms, 206, 429
- histograms, 19, 36
- Hodges-Lehmann estimator, 132, 155
- confidence interval for, 133, 155
 - for step trend, 349
- homoscedasticity, 51
- homoscedasticity, 13
- hypothesis tests, 97, 109
- choice of, 101
 - classification, 99
 - classification of, 106
 - exact, 103
 - with censored data, 365
- independent groups, 117
- inequality of variance, 124
- influence, 4, 248, 250
- interaction, 172
- intercept, 226
- confidence interval for, 240
 - deletion of, 240
 - nonparametric, 268
- interquartile range, 9, 24
- interval estimates, 66
- invariance to rotation, 280
- IQR, 9
- iteratively weighted least squares, 283
- joint probability, 379
- Kendall's nonparametric line;, 370
- Kendall's nonparametric line; .i.Sen slope estimate; .regression
- nonparametric;, 266
- Kendall's S statistic, 272
- Kendall's tau, 326
- for categorical variables., 386
 - for censored data, 372
 - large sample approximation;, 213
 - tie correction;, 215
- Kendall's tau; .i.tau;, 212
- Kendall's tau-b;), 386
- kite diagram, 54
- Kruskal-Wallis test, 158, 159
- for categorical variables, 382
 - large sample approximation, 160
 - rank transform approximation, 163
- ladder of powers, 14, 15, 31, 119, 230
- large sample approximation, 103, 121
- least normal squares, 279
- least significant range, 198
- least squares, 228
- length, judgment of on graphs, 420
- leverage, 248
- likelihood ratio, 397
- likelihood- R^2 , 398
- line of organic correlation, 276
- linear regression. see regression
- linearity, 13
- linearity, 46
- LOC, 276
- log likelihood, 397

- logistic regression, 395
 - for censored data, 371
 - for multiple responses, 403
- logistic transformation, 396
- logit, 396
- loglinear models, 390
- lognormal, 2
- lognormal distribution, 73
- LOWESS, 47, 288
 - use in trend tests, 335
- LOWESS, 325
- MAD, 10, 284
- Mann-Kendall trend test, 326
 - with censored data, 354
- Mann-Whitney test, 118
- MARA, 192
- marginal probability, 379
- mass transport, 258
- matched-pair tests
 - graphical presentation of, 152
- maximum-likelihood, 360
 - tobit regression, 370
- mean
 - asymmetric confidence interval for, 76
 - confidence interval for, 75
- mean difference, 157
- mean square error, 228
- mean squares, 167, 174
- measures of location, 3
- median, 6
 - confidence interval for, 70
 - test for differences in, 118, 159
- Median Aligned-Ranks ANOVA, 192
- median difference, 154
- median polish, 182
- mixed effects, 181
- MLE, 360
 - tobit regression, 370
- mode, 7
- monotonic correlation, 210
- monotonic trend, 327
- MOVE, 276
- moving average, 286
- moving medians, 286
- multiple comparison test
 - graphical display of, 208
- multiple comparison tests, 197
 - for categorical variables, 385
 - nonparametric, 198, 203
- multiple regression, 237
 - use as a trend test, 337
- multiplicative relation, 43
- multiply-censored data, 353
 - hypothesis tests for, 368
 - nonparametric tests for, 368
- multivariate graphical methods, 52
- nominal response variables, 406
- non-normality, 92, 124
- nonparametric interval estimate, 70
- nonparametric prediction intervals, 77
- nonparametric test for percentiles, 86
- nonparametric tests, 101
 - comparison to parametric, 102
 - for censored data, 366
- normal distribution, 2, 26, 31, 113
- normal probability plot, 24, 114
- normal quantiles, 28, 114
- normal scores, 114
- normality
 - of test statistics, 121
 - test of, 102, 113, 166
- normality assumption, 150
- null hypothesis, 104, 107, 108
 - not rejecting, 109
 - rejecting, 109
- odds ratio, 396
- OLS, 222
- one-sided p-value, 112
- one-sided tests, 105, 109
- ordinal variables, 386

- ordinary least squares, 222
- outliers, 12, 92
 - tests for, 92
- outliers, 2, 31, 38, 127, 250
- outside values, 26
- overall error rate, 199
- paired observations, 138
- paired t-test, 148
 - assumptions of, 148
 - computation, 149
- pairwise comparisons, 199
- pairwise error rate, 199
- parametric c. i. for the median, 73
- parametric prediction intervals, 80
- parametric tests, 100
 - comparison with nonparametric, 102
 - with censored data, 365, 366
- partial likelihood ratio, 398
- Pearson Type III distribution,, 90
- Pearson's r , 218
- percent exceedance, 30
- percentiles
 - confidence interval for, 83
 - parametric tests for, 91
 - water quality, 83
- percentiles, 9
- perceptual tasks for interpreting graphs, 412
- perspective, use of in graphs, 424
- pie chart, 62
- pie charts, 416
- Piper diagram, 58, 59
- Piper smooth, 59
- plotting position, 22, 23, 30, 114
- point estimates, 66
- polar smooth, 48, 292
- population**, 2
- positive skewness, 10
- power, 95, 100, 107
 - lack of, 124, 127
 - loss of, 102, 150
- power, 102
- power transformation
 - effect on paired t-test, 151
 - effect on rank-sum test, 118
 - effect on signed-rank test, 144
- power transformations, 177
 - avoiding, 265
 - effect on t-test, 128
 - invariance to, 327
 - WLS as an alternative, 282
- PPCC test, 113
- precision of graphs, 411
- prediction interval, 66, 243
 - asymmetric, 81
 - nonparametric, 77, 79, 81, 244
 - one-sided, 79
 - parametric, 80, 81
 - symmetric, 81
 - two-sided, 78
- prediction residual, 249
- PRESS statistic, 249
- principal components analysis, 59
- probability paper, 29
- probability plot, 27, 31, 35, 41
- probability plot correlation coefficient, 35
- probability plot correlation coefficient, 113
- profile plot, 53
- p-value, 108, 111
 - one-sided, 113
 - two-sided, 112, 113
- Q-Q plot, 42, 43, 128, 129
 - construction, 45
- quality control, 93
- quantile plot, 22, 206
- quantile-quantile plot, 129
- quantiles, 22, 83
- quantiles, 42
- quartile, 9
- quartile skew coefficient, 11
- r squared, 228

- random effects, 180
- randomized complete block design, 182, 187
- rank transform test, 194
- rank transformation test, 123, 170, 203
- ranks, 7
- ranks, 104
- rank-sum test, 110, 118
 - an alternative to logistic regression, 402
 - as a test for trend, 349
- rating curve, 258
- record extension, 278
- regression, 221
 - for censored data, 369
- regression
 - as a test for trend, 328
 - assumptions, 224
 - confidence interval on mean response, 242
 - diagnosing problems, 232
 - guide to model selection, 263, 316
 - hypothesis testing, 238
 - non-normal residuals, 268
 - nonparametric, 266
 - normality assumption, 236
 - robust, 269, 283
 - validation of equation, 249
- regression diagnostics, 238, 246
- rejection region, 113
- reliability, 66
- replicates, 194
- replication
 - ANOVA without, 194
- residuals, 31, 226
 - prediction, 249
 - standardized, 249
 - studentized, 249
 - testing for normality, 236
 - use in trend tests, 332
- residuals plot, 187, 232
 - trend, 234
- resistant, 6
- response variable, 99, 222
- rho, 217
- risk tolerance, 108
- RMSE, 358
- robust, 11
- robust regression, 269
- root mean squared error, 358
- sample**, 2
- sample size, estimating, 95
- sampling design, 95
- Satterthwaite's approximation, 126
- scale breaks, 428
- scatterplot matrix, 61
- scatterplots, 46, 423
- schematic plot, 26
- seasonal Kendall test, 339
- seasonal rank-sum test, 350
- seasonal variation, 234
 - differences among seasons, 345
 - graphics for display of, 344
 - modeling, 338
 - use of periodic functions, 342
- Sen slope estimate, 266
- serial correlation, 252
 - remedies, 254
- shading on graphs, 413
- Shapiro-Wilk test, 115
- sign test, 138, 187
 - computation, 138
 - large sample approximation, 141
- signed-rank test, 142, 192
 - large sample approximation, 146
 - rank transform approximation, 148
- significance level, 107, 110
- simple boxplot, 25
- skew, 10
- skewed data, 124
- skewness, 2, 31, 69
- skewness, 38, 40, 127

- slope, 226
 - confidence interval for, 240
 - judgment of on graphs, 417
 - test of significance, 238
- smearing estimator, 259
- smooth, 46, 47, 48
 - lower, 292
 - LOWESS, 286
 - middle, 286
 - outer, 293
 - upper, 292
- smooth, 47
- smoothing, 286
- smoothness factor, 289
- spatial trend, 326
- Spearman's rho, 217
- spread, 8, 51
- stacked bar chart, 62
- standard deviation, 9, 36
- standard error, 36, 228
- standardized residual, 249
- star diagram, 54, 58
- statistical maps, 413
- statistical tables, 113
- stem and leaf diagram, 20
- step trend, 349
 - when to test for, 351
- Stiff diagram, 53, 58
- studentized range, 198
- student's t statistic, 75
- sum of squares, 196
- summary statistics, 10
 - for censored data, 358
 - with multiple reporting limits, 364
- sums of squares, 173
- symmetric confidence intervals, 68, 75
- symmetry, 13
 - assumption of, 151
- table of test statistic quantiles, 110
- tables, deficiencies of, 410
- tails of the distribution, 10, 31
- target population**, 2
- tau, 212
- t-distribution
 - noncentral, 90
- test statistic, 108
- Theil slope estimate
 - computation, 266
 - confidence interval for, 273
 - efficiency, 268
 - for trends, 330
 - with censored data, 354
- tie correction
 - for tests with censored data, 354
- tobit regression, 366, 370
- tolerance intervals, 83
- tolerance probability, 96
- transformation bias
 - in regression, 258
 - of MLE, 360
- transformations, 31, 103, 166, 177, 255
 - consequences in regression, 256
- transformations, 13, 230
- t-ratio, 232
- trend
 - exponential change, 347
- trend analysis
 - including exogenous variables, 330
 - nonparametric, 335
 - step trends, 349
 - use of transformations, 347
 - with censored data, 353
- trend slope, 330
 - seasonal, 341
- trilinear diagram, 57
- trimmed mean, 7
- truncated boxplot, 26

- t-test, 103, 124
 - as a test for trend, 349
 - assumptions of, 124
 - computation of, 125
 - for multiple comparisons, 199
 - on ranks, 127
 - problems with, 124
 - violation of assumptions, 127
- Tukey's multiple comparison test, 199, 201
- two-factor ANOVA, 193, 194
- two-sided tests, 105, 111
- Type I error, 107
- Type II error, 107
- unequal sample sizes, 179
- unequal variances, 126
- variance, 9
 - confidence interval for, 240
- violation of test assumptions, 150
- Wald's t-statistic, 398
- Weibull plotting position, 24
- weight function, 288
- weighted least squares, 281
- whisker, 25
- WLS, 281