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Appendix A

Construction of Boxplots

The upper and lower limits of the central box are defined using either quartiles or hinges. These definitions are clarified below. Then the influence of each definition on the position of the whiskers is demonstrated. Definitions used by commercial software packages are listed, including one non-conventional form called a "box graph".

Quartiles
Quartiles are the 25th, 50th and 75th percentiles of a data set, as defined in chapter 1. Consider a data set $X_i, i=1,...n$. Computation of percentiles follows the equation

$$p_j = X(n+1)j$$

where $n$ is the sample size of $X_i$, $j$ is the fraction of data less than or equal to the percentile value (for the 3 quartiles, $j = .25, .50, and .75$).

Non-integer values of $(n+1)j$ imply linear interpolation between adjacent values of $X$. Computation of quartiles for two small example data sets is illustrated in Table 1.

Hinges
Tukey (1977) used values for the ends of the box which, along with the median, divided the data into four equal parts. These "fourths" or "hinges" are defined as:

Lower hinge $h_L = \text{median of all observations less than or equal to the sample median.}$
Upper hinge $h_U = \text{median of all observations equal to or greater than the overall sample median.}$

They may also be defined as:

$$\begin{align*}
\text{Lower hinge } h_L &= X_{L}, \text{ where } L = \frac{\text{integer } [(n+3)/2]}{2}, \text{ and} \\
\text{Upper hinge } h_U &= X_{U}, \text{ where } U = (n+1) - L.
\end{align*}$$

where "integer [ ]" is the integer portion of the number in brackets. For example, integer [5.7] = 5. Again, non-integer values of $L$ and $U$ imply interpolation. With hinges, however, this will always
be halfway between adjacent data points. Therefore, hinges are always either data values themselves, or averages of two data points, and so are easier to compute by hand than are percentiles. Hinges will generally be similar to quartiles for large (n> 30) sample sizes. For smaller data sets, differences will be more apparent. For example, when n=12 the lower hinge is halfway between the 3rd and 4th data points, while the lower quartile is one-quarter of the way between the two points (see Table 1). Both measures split the data into one-fourth below and three fourths above their value. Either are acceptable for use in boxplots.

Table A1
A. For the following data $X_i$ of sample size $n=11$:

\[
\begin{array}{cccccccccccc}
2 & 3 & 5 & 45 & 46 & 47 & 48 & 50 & 90 & 151 & 208 \\
\end{array}
\]

\[
\begin{align*}
p_{.25} &= \text{lower quartile} = & X_{(n+1)\cdot.25} &= X_3 &= 5. \\
p_{.75} &= \text{upper quartile} = & X_{(n+1)\cdot.75} &= X_9 &= 90. \\
p_{.50} &= \text{median} = & X_{(n+1)\cdot.50} &= X_6 &= 47. \\
h_l &= \text{lower hinge} = & \text{median} [2 \ 3 \ 5 \ 45 \ 46 \ 47] &= 25. \\
h_u &= \text{upper hinge} = & \text{median} [47 \ 48 \ 50 \ 90 \ 151 \ 208] &= 70. \\
\end{align*}
\]

B. For sample size $n=12$, and data $X_i$, $i=1,...,n$ equal to:

\[
\begin{array}{cccccccccccc}
2 & 3 & 5 & 45 & 46 & 47 & 48 & 49 & 50 & 90 & 151 & 208 \\
\end{array}
\]

\[
\begin{align*}
p_{.25} &= \text{lower quartile} = & X_{(n+1)\cdot.25} &= X_3.25 &= X_3 + 0.25\cdot(X_4 - X_3) &= 15. \\
p_{.75} &= \text{upper quartile} = & X_{(n+1)\cdot.75} &= X_9.75 &= X_9 + 0.75\cdot(X_{10} - X_9) &= 80. \\
p_{.50} &= \text{median} = & X_{(n+1)\cdot.50} &= X_6.5 &= X_6 + 0.50\cdot(X_7 - X_6) &= 47.5. \\
h_l &= \text{lower hinge} = & \text{median} [2 \ 3 \ 5 \ 45 \ 46 \ 47] &= 25. \\
h_u &= \text{upper hinge} = & \text{median} [48 \ 49 \ 50 \ 90 \ 151 \ 208] &= 70. \\
\end{align*}
\]

Figure A1. Boxplots for the Table A1 data
Figure A1 shows standard boxplots for the Table 1 data using both percentiles and hinges. Data in Table 1 were designed to maximize differences between the two measures. Real data, and larger sample sizes, will evidence much smaller differences. Note that the definitions of the box boundaries directly affect whisker lengths, and also determines which data are plotted as "outside" values.

It would be ideal if all software used the same conventions for drawing boxplots. However, that has not happened. Software written by developers who stick to the original definitions prefer hinges; those who want box boundaries to agree with tabled percentiles use quartiles. The Table 1 data can be used to determine which convention is used to produce boxplots.

Non-conventional definitions
Other statistical software use another (non-conventional) value for the box boundaries (Frigge and others, 1989). They use the next highest data value for the lower box boundary whenever n/4 is not an integer. This avoids all interpolation. Note that n, not n+1, is used.

StatView uses a percentile-type boxplot similar to the truncated boxplot, except that the upper and lower 10 percent of data are plotted as individual points. The weakness of this scheme is that 10 percent of the data will always be plotted individually at each end of the plot, and so it is less effective for defining and emphasizing unusual values. Also important is that StatView uses yet another definition for the box boundaries, X_{(n+2)\cdot j}, in calculating the quartiles. This non-conventional boxplot was called a "box graph" by Cleveland (1985).

Therefore some statistical software will produce boxes differing from conventional boxplots, particularly for small data sets.

Boxplots for Censored Data
Data sets whose values include some observations known only to be below (or above) a limit or threshold can also be effectively displayed by boxplots. First set all values below the threshold to some value less than (not equal to) the reporting limit. The actual value is not important, and could be 0, one-half the reporting limit, etc. Produce the boxplot. Then draw a line across the graph at the value of the threshold, and erase all lines below this value from the graph.

This procedure was used for data in figure A2. If less than 25 percent of the data are below the threshold, this procedure will affect at most only the lower whisker (as in the Hoover Dam through Morelos Dam boxplots). If between 25 and 75 percent are below the threshold, the box will be partially hidden below the threshold (as in the CO-UT Line and Cisco boxes). If more than 75 percent of the data are below the threshold, part of the upper whisker and outside values will be visible above the threshold, as in the Lees Ferry box. In each case, these boxplots accurately and fairly illustrate both the distribution of data above the threshold, and the percentage of data below the threshold.
A second alternative for boxplots of censored data is to estimate the percentiles falling below the threshold, and drawing dashed portions of the box below the threshold using these estimates. Helsel and Cohn (1988) have compared methods for estimating these percentiles. When multiple thresholds occur, such as thresholds which have changed over time or between laboratories, a solid line can be drawn across the plot at the highest threshold. Portions of the boxes above the highest threshold will be correct as long as each censored observation is assigned some value below its threshold. Quartiles falling below the highest threshold should be determined by using the methods recommended by Helsel and Cohn (1988). All lines below the highest threshold are only estimates, and should be drawn as dashed lines on the plot.

**Displaying confidence intervals**

As an aid for displaying whether two groups of data have different medians, confidence intervals for the median as defined in chapter 3 can be added to boxplots. When boxplots are placed side by side, their medians are significantly different if the confidence intervals do not overlap. Three methods of displaying these intervals are shown in figure A3. In the first method (A), the box is "notched" at both upper and lower limits, making the box narrower for all values within the interval. In the second (B), parentheses are drawn within the box at each limit. Shading is used in (C) to illustrate interval width. If displaying differences in medians is not of primary interest, these
methods add visual confusion to boxplots and are probably best avoided. Confusion is compounded when the interval width falls beyond the 25th or 75th percentiles. Of the three, shading seems the easiest to visualize and least confusing.

Figure A3. Methods for displaying confidence interval of median using a boxplot.
A. Notched boxplots  B. Parentheses  C. Shaded boxplot
## Appendix B

### Tables

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<tr>
<th>Table B1</th>
<th>Cunnane plotting positions for n = 1 to 20</th>
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<td>Table B2</td>
<td>Normal quantiles for Cunnane plotting positions of Table B1</td>
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<td>Table B3</td>
<td>Critical values for the PPCC test for normality</td>
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<td>Table B4</td>
<td>Quantiles (p-values) for the rank-sum test</td>
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<td>Quantiles (p-values) for the sign test</td>
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<td>Quantiles (p-values) for Kendall's tau (τ)</td>
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Table B2. Upper tail normal quantiles for the plotting positions of Table B1
(for lower tail quantiles, multiply all nonzero quantiles by $-1$)

| $N$ = 5 | 0.000 | 0.502 | 1.198 |
| $N$ = 6 | 0.203 | 0.649 | 1.300 |
| $N$ = 7 | 0.000 | 0.355 | 0.765 | 1.383 |
| $N$ = 8 | 0.153 | 0.475 | 0.859 | 1.453 |
| $N$ = 9 | 0.000 | 0.276 | 0.575 | 0.939 | 1.513 |
| $N$ = 10 | 0.123 | 0.377 | 0.659 | 1.007 | 1.565 |
| $N$ = 11 | 0.000 | 0.225 | 0.463 | 0.732 | 1.067 | 1.611 |
| $N$ = 12 | 0.103 | 0.313 | 0.538 | 0.796 | 1.121 | 1.653 |
| $N$ = 13 | 0.000 | 0.191 | 0.389 | 0.604 | 0.852 | 1.169 | 1.691 |
| $N$ = 14 | 0.088 | 0.267 | 0.456 | 0.663 | 0.904 | 1.212 | 1.725 |
| $N$ = 15 | 0.000 | 0.165 | 0.336 | 0.517 | 0.716 | 0.950 | 1.252 | 1.757 |
| $N$ = 16 | 0.077 | 0.234 | 0.397 | 0.571 | 0.765 | 0.992 | 1.289 | 1.787 |
| $N$ = 17 | 0.000 | 0.146 | 0.295 | 0.452 | 0.620 | 0.809 | 1.031 | 1.323 | 1.814 |
| $N$ = 18 | 0.069 | 0.208 | 0.351 | 0.502 | 0.666 | 0.849 | 1.067 | 1.354 | 1.839 |
| $N$ = 19 | 0.000 | 0.131 | 0.264 | 0.402 | 0.548 | 0.707 | 0.887 | 1.101 | 1.383 | 1.864 |
| $N$ = 20 | 0.062 | 0.187 | 0.315 | 0.449 | 0.591 | 0.746 | 0.922 | 1.133 | 1.411 | 1.886 |
Table B3. Critical $r^*$ values for the probability plot correlation coefficient test of normality (from Looney and Gulledge, 1985a)


[reject $H_0$: data are normal when PPCC $r < r^*$ ]

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Table generated by D. Helsel
Table B5. -- Quantiles (p-values) for the sign test statistic $S^+$

Quantiles for the sign test are identical to quantiles of the binomial distribution with percentile $p=0.5$. The approximation given in chapter 6 and used by most statistical software packages can be used for $n \geq 20$. Statistics textbooks that contain a table of exact quantiles for the binomial distribution for sizes below 20 include Hollander and Wolfe (1999) and Zar (1999).

An online table of exact quantiles for the binomial distribution can be found as of 5/2002 at: http://faculty.vassar.edu/lowry/binomial01.html

An example of using this online table:
Enter $n$ (the number of data pairs) and $p (=0.5)$. An exact table will be printed. P-values are cumulative probabilities, or values of the cumulative distribution function (cdf). For small values of the test statistic $S^+$ (called $k$ in the online table) – values below $n/2$, use the “Down” column to read off a one-sided p-value for the sign test. For $S^+$ larger than $n/2$, use the “Up” column. The example output below is for $n=13$. A one-sided p-value for $S^+ = 4$ (the probability of getting an $S^+ \leq 4$) is 0.133. The p-value for $S^+ = 9$ (the probability of getting an $S^+ \geq 9$) also equals 0.133. For a two-sided test, $p = 0.266$.

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Figure B1. Two-sided critical region (p-values), shaded, for the sign test. $n=13$, $S^+ = 4$ or 9.
Table B6 – Critical test statistic values for the signed-rank statistic $W^+$
(from McCornack, 1965)

The approximation given in chapter 6, used by most statistics software packages, can be used for $n > 15$ and $\alpha \geq 0.025$. For $\alpha < 0.025$, see exact tables in the McCornack paper or a textbook such as Hollander and Wolfe (1999), even for large sample sizes.

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The chi-square approximation given in chapter 7 is used by most statistics software packages. For comparing 3 to 5 groups of data with sample sizes (blocks) \( n < 10 \) in each group, an exact table should be used.

\[
\text{[reject } H_0: \text{ at } \alpha \text{ when } X_f \geq \text{ table entry]}\]

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Table B8 -- Quantiles (p-values) for Kendall's S statistic and tau correlation coefficient

For N>10 use the approximation given in section 8.2.2

One-sided p = Prob [S ≥ x] = Prob [S ≤ −x]

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Table generated by D. Helsel
## Appendix C

### Data Sets

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Data sets are available in both ASCII and MS Excel formats. See the online location from which you obtained this book for the data files HhappC.dat and HhappC.xls.
Chapter 1

1.1 For the well yield data:
   a) mean = 0.19
   b) trimmed mean = 0.05
   c) geometric mean = 0.04
   d) median = 0.04
   e) They differ because the data are skewed. The estimates which are more robust are similar, while the mean is larger.

1.2 a) standard deviation = 0.31
   b) interquartile range = 0.36
   c) MAD = 0.04
   d) skew = 2.07. quartile skew = 0.83.
   Because the data are asymmetric, the median difference is small, but the IQR and standard deviation are not.

1.3 mean = 1.64 std. dev. = 2.85
   median = 0.80 IQR = 0.61
   geometric mean = 0.04 MAD = 0.25
   skew = 3.09 quartile skew = −0.10
   The largest observation is an outlier. Though the skew appears to be strongly positive, and the standard deviation large, this is due only to the effect of that one point. The majority of the data are not skewed, as shown by the more resistant quartile skew coefficient.

   a) assuming the outlying observation is accurate, representing some high-nitrogen location which is important to have sampled, the mean must be used to compute the mass of nitrogen falling per square mile. It would probably be computed by weighting concentrations by the surface area represented by each environment. The median would under-represent this mass loading.

   b) the median would be a better "typical" concentration, and the IQR a better "typical" variability, than the mean and standard deviation. This is due to the strong effect of the one unusual point on these traditional measures.
Chapter 2

2.1 a) Either a cube root or logarithmic transform would increase symmetry.
2.2 a) The data are strongly right-skewed. A log transformation makes these data more symmetric.

2.3 Q-Q plot.
The granodiorite shows higher chloride concentrations than does the quartz monzonite. This can be seen with any of the three graphs, but most easily with the boxplot. From the Q-Q plot, the relationship does not look linear.
2.4 There appears to be no effect of the waste treatment plant.

Chapter 3

3.1 nonparametric: \( x' = 4 \) (\( \alpha /2 = .0154 \)). \( R_l = 5, \ R_u = 14. \)

\[ 0.4 \leq C_{0.5} \leq 3.0 \text{ at } \alpha = 0.031. \]

This is as close to 0.05 as the table allows.

parametric: Using the natural logs of the data,

\[ \exp(-0.045 - 2.11 \cdot \sqrt{1.63/18}) \leq GM_x \leq \exp(-0.045 + 2.11 \cdot \sqrt{1.63/18}). \]

\[ 0.51 \leq GM_x \leq 1.80. \]

Either of intervals is reasonable. The logs of the data still retain some skewness, so the nonparametric interval may be more realistic. The parametric interval might be preferred to obtain an alpha of 0.05. The choice would depend on whether the assumption of lognormality was believed.

3.2 symmetric: \[ 0.706 - 2.12 \cdot \sqrt{0.639/17} \leq \mu \leq 0.706 + 2.12 \cdot \sqrt{0.639/17} \]

\[ 0.30 \leq \mu \leq 1.12. \]

Point estimates: mean = 0.705 (assuming normal distribution).

mean = \( \exp(-0.849+0.5\cdot1.067) \)

= 0.73 (assuming a lognormal distribution).

As the logs of the data are more symmetric than the data prior to transformation, the lognormal (2nd) estimate is preferred.

3.3 Parametric 95% prediction interval:

\[ 0.19 - 2.20 \cdot \sqrt{0.0975 + (0.0975/12)} \text{ to } 0.19 + 2.20 \cdot \sqrt{0.0975 + (0.0975/12)} \]

or \(-0.53\) to \(0.91\) gallons/min/foot. Includes 0.85, so same distribution.

Nonparametric 95% prediction interval:

\( X_{[0.025\cdot13]} \) to \( X_{[0.975\cdot13]} \) \( X_{0.325} \) to \( X_{12.675} \)

The sample size is too small to produce such a wide (95%) nonparametric prediction interval. Therefore a parametric interval must be used. However, the negative lower end of the parametric prediction interval indicates that a symmetric interval is not appropriate. So an asymmetric interval resulting from taking logarithms should be used instead of the one above.

3.4 The data look relatively symmetric, so no logs taken.

mean: \( 683 \pm 126, \) or \( 557->809 \) \( \alpha = .05. \)

median: \( R_l=6, \ R_u=15 \) \( 524->894 \) \( \alpha = .041. \)
3.5 The 90th percentile = 2445 cfs. A one-sided 95% confidence interval for the 90th percentile (an upper confidence limit to insure that the intake does not go dry) is found using the large-sample approximation of equation 3.17:

\[
R_u = 365 \cdot 0.1 + z_{0.95} \cdot \sqrt{365 \cdot 0.1 \cdot (0.9)} + 0.5
\]

\[
= 36.5 + 1.645 \cdot 5.73 + 0.5 = 46.4
\]

The 46th ranked point is the upper CI, or 2700 cfs.

Chapter 4

4.1 For the before-1969 data, PPCC \( r = 0.986 \). For the after-1969 data, PPCC \( r = 0.971 \). Critical values of \( r \) are 0.948 and 0.929, respectively. Therefore normality cannot be rejected for either period at \( \alpha = 0.05 \).

4.2 For the arsenic data, PPCC \( r = 0.844 \). The critical \( r^* \) from Appendix table B3 is \( r^* = 0.959 \). Therefore reject normality. For log-transforms of the data, PPCC \( r = 0.973 \). Normality of the transformed data is not rejected.

Chapter 5

5.1 The p-value remains the same.

5.2 Given that we wish to test for a change in concentration, but the direction of the change is not specified in the problem, this should be a two-sided test. If it had stated we were looking for an increase, or a decrease, the test would have been a one-sided test.

5.3 a. Quantiles are the 12 "after" data, and 12 quantiles computed from the 19 "before" data:

<table>
<thead>
<tr>
<th>i</th>
<th>j</th>
<th>&quot;after&quot;</th>
<th>&quot;before&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.34</td>
<td>1350.00</td>
<td>1222.13</td>
</tr>
<tr>
<td>2</td>
<td>2.92</td>
<td>2260.00</td>
<td>1715.25</td>
</tr>
<tr>
<td>3</td>
<td>4.49</td>
<td>2350.00</td>
<td>1739.84</td>
</tr>
<tr>
<td>4</td>
<td>6.06</td>
<td>2870.00</td>
<td>1900.82</td>
</tr>
<tr>
<td>5</td>
<td>7.64</td>
<td>3060.00</td>
<td>2506.23</td>
</tr>
<tr>
<td>6</td>
<td>9.21</td>
<td>3140.00</td>
<td>2614.92</td>
</tr>
<tr>
<td>7</td>
<td>10.79</td>
<td>3180.00</td>
<td>2717.21</td>
</tr>
<tr>
<td>8</td>
<td>12.36</td>
<td>3430.00</td>
<td>2873.61</td>
</tr>
<tr>
<td>9</td>
<td>13.93</td>
<td>3630.00</td>
<td>3375.24</td>
</tr>
<tr>
<td>10</td>
<td>15.51</td>
<td>3780.00</td>
<td>3591.15</td>
</tr>
<tr>
<td>11</td>
<td>17.08</td>
<td>3890.00</td>
<td>3922.29</td>
</tr>
<tr>
<td>12</td>
<td>18.66</td>
<td>5290.00</td>
<td>4617.37</td>
</tr>
</tbody>
</table>
The relationship appears additive. The Hodges-Lehmann estimate (median of all possible after–before differences) = 480 cfs.

b. After regulation, the reservoir appears to be filling. Any test for change in flow should omit data during the transition period of 1969-1970. Plots of time series are always a good idea. They increase the investigator's understanding of the data. Low flows after regulation are not as low as those before. This produces the pattern seen in the Q-Q plot of the low quantiles being lower after regulation, while the upper quantiles appear the same, as shown by the drift closer to the x=y line for the higher values.

With 1969 and 70 included, $W_{RS} = 273.5$ $p=0.22$. The after flows are not significantly different. With 1969 and 70 excluded, $W_{RS} = 243.5$ $p=0.06$. The after flows are close to being significantly different -- more data after regulation is needed.
5.4 Exact test

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>R(Y)</th>
<th>R(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3.5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>4.5</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40.0</td>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.0</td>
<td>14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From table B4, Prob($W_{rs} \leq 16$) = .027. The two-sided exact p-value = 0.054

Large-sample approximation

The mean is $\mu_W = \frac{n \cdot (N+1)}{2} = \frac{4 \cdot 15}{2} = 30$

The standard deviation is given by $\sigma_W = \sqrt{\frac{n \cdot m \cdot (N+1)}{12}} = 7.0711$

$Z_{rs} = \frac{16 - \mu_W + 1/2}{\sigma_W} = -1.909$

Using linear interpolation between −1.9110 and −1.8957 in a table of the standard normal distribution gives the one-tail probability of 0.028. So the two-sided approximate p-value is 0.056.

t-test on the ranks

Replacing variable values by ranks gives

\[
\begin{align*}
\bar{x} &= 4 & S_x &= 2.582 & S^2_x &= 6.667 & n = 4 \\
\bar{y} &= 8.9 & S_y &= 3.928 & S^2_y &= 15.429 & m = 10
\end{align*}
\]

The pooled variance is:

\[
S^2 = \frac{3S^2_x + 9S^2_y}{12} = 13.2386
\]

$S = 3.639$
\[ t = \frac{\bar{x} - \bar{y}}{S \sqrt{(1/n + 1/m)}} = -2.27610 \]

Linear interpolation for a student's t with 12 degrees of freedom gives
\[
.975 + \frac{(2.27610 - 2.1788)}{(2.6810 - 2.1788)} \times .015 = .97791 \\
1.0 - .97791 = .022
\]
The two-sided rank transform p-value is .044.

### Summary

<table>
<thead>
<tr>
<th>Approach</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank-Sum Exact</td>
<td>0.054</td>
</tr>
<tr>
<td>Rank-Sum Approx.</td>
<td>0.056</td>
</tr>
<tr>
<td>t test on ranks</td>
<td>0.044</td>
</tr>
</tbody>
</table>

To compute \( \hat{\Delta} \), the \((n \times m) = 40\) differences \((X_i - Y_j = D_j)\) are:

\[
\begin{array}{c|cccccccccccc}
 (Y_1) & 0.5 & 1.5 & 2.5 & 3.5 & 4.5 & 6 & 9 & 19 & 39 & 99 \\
 (Y_2) & -0.5 & 0.5 & 1.5 & 2.5 & 3.5 & 5 & 8 & 18 & 38 & 98 \\
 (Y_3) & -1.5 & -0.5 & 0.5 & 1.5 & 2.5 & 4 & 7 & 17 & 37 & 97 \\
 (Y_4) & -2.5 & -1.5 & -0.5 & 0.5 & 1.5 & 3 & 6 & 16 & 36 & 96 \\
\end{array}
\]

\(\hat{\Delta} = \text{median of 40} D_j\text{'s (}D_{\text{rank 20} + D_{\text{rank 21}}}/2 = 3.75\]

5.5  
<table>
<thead>
<tr>
<th>Yields with fracturing</th>
<th>Yields without</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_{\text{crit}} = .932), accept normality</td>
<td>(r_{\text{crit}} = .928), reject normality</td>
</tr>
</tbody>
</table>

Because one of the groups is non-normal, the rank-sum test is performed.
\[ W_{rs} = \Sigma R_{\text{without}} = 121.5. \] The one-sided p-value from the large-sample approximation \(p = 0.032\). Reject equality. The yields from fractured rocks are higher.

5.6  
The test statistic changes very little \((W_{rs} = 123)\), indicating that most information contained in the data below detection limit is extracted using ranks. Results are the same (one-sided p-value = 0.039. Reject equality). A t-test could not be used without improperly substituting some contrived values for less-thans which might alter the conclusions.

### Chapter 6

6.1  
The sign test is computed on all data after 683 cfs is subtracted. \(S^+ = 11\). From table B5, reject if \(S^+ \geq 14\) (one-sided test). So do not reject. \(p > 0.25\).
6.2 c is not a matched pair.

6.3 a. H₀: \( \mu_{\text{South Fork}} - \mu_{\text{North Fork}} = 0 \).
    H₁: \( \mu_{\text{South Fork}} - \mu_{\text{North Fork}} \neq 0 \).

b. A boxplot of the differences shows no outliers, but the median is low. Conductance data are usually not skewed, and the PPCC \( r = 0.941 \), with normality not rejected. So a \( t \)-test on the differences is computed (parametric).

\[
\begin{array}{c}
\text{diffs} \\
-100 \quad -50 \quad 0
\end{array}
\]

c. \( t = -4.24 \quad p = 0.002 \quad \text{Reject } H₀. \)

d. Along with the boxplot above, a scatterplot shows that the South Fork is higher only once:

e. The mean difference is \(-64.7\).

6.4 Because of the data below the reporting limit, the sign test is performed on the differences Sept–June. The one-sided \( p \)-value = 0.002. Sept atrazine concentrations are significantly larger than June concs before application.

6.5 For the \( t \)-test, \( t = 1.07 \) with a one-sided \( p \)-value of 0.15. The \( t \)-test cannot reject equality of means because one large outlier in the data produces violations of the assumptions of normality and equal variance.
Chapter 7

7.1 As a log-transformed variable, pH often closely follows a normal distribution. See the following boxplots:

![Boxplots of pH for three piezometer groups](image)

pH for three piezometer groups (from Robertson et al., 1984)

The PPCC for the three groups (0.955 for BP-1, 0.971 for BP-2, and 0.946 for BP-9) cannot disprove this assumption. Therefore ANOVA will be used to test the similarity of the three groups.

Anova Table:

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piez Gp</td>
<td>2</td>
<td>7.07</td>
<td>3.54</td>
<td>9.57</td>
<td>0.002</td>
</tr>
<tr>
<td>Error</td>
<td>15</td>
<td>5.54</td>
<td>0.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>17</td>
<td>12.61</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The groups are declared different. Statistics for each are:

<table>
<thead>
<tr>
<th>GP</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Pooled Std. Dev = 0.608</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP-1</td>
<td>6</td>
<td>7.65</td>
<td>0.596</td>
<td></td>
</tr>
<tr>
<td>BP-2</td>
<td>6</td>
<td>6.68</td>
<td>0.279</td>
<td></td>
</tr>
<tr>
<td>BP-9</td>
<td>6</td>
<td>8.20</td>
<td>0.822</td>
<td></td>
</tr>
</tbody>
</table>

A Tukey's test on the data is then computed to determine which groups are different. The least significant range for Tukey's test is

\[ LSR = q(0.95, 2, 15) \sqrt{0.37/6} = 3.01 \times 0.248 = 0.75 \]

Any group mean pH which differs by more than 0.75 is significantly different by the Tukey's multiple comparison test. Therefore two piezometer groups are contaminated, significantly higher than the uncontaminated BP-2 group:

BP-9 ≅ BP-1 > BP-2
Since the sample sizes are small (n=6 for each group) one might prefer a Kruskal-Wallis test to protect against any hidden non-normality:

\[
\begin{array}{llll}
\text{GP} & N & \text{MEDIAN} & R_j \\
\text{BP-1} & 6 & 7.60 & 11.3 \\
\text{BP-2} & 6 & 6.75 & 3.6 \\
\text{BP-9} & 6 & 8.00 & 13.6 \\
\end{array}
\]

Overall Median = 9.5

\[
K = 11.59 \quad \chi^2_{0.95,2} = 5.99. \quad \text{Reject } H_0, \text{ with } p = 0.003.
\]

ANOVA and Kruskal-Wallis tests give identical results.

7.2 Boxplots of the data indicate skewness. Therefore the Kruskal-Wallis test is computed:

\[
K = 7.24 \quad \text{Corrected for ties, } K = 7.31. \quad p = 0.027
\]

Reject that all groups have the same median chloride concentration.

The medians are ranked as granodiorite > qtz monzonite > ephemeral. Individual K-W tests are computed for adjacent pairs at \( \alpha = 0.05 \):

granodiorite \( \cong \) qtz monzonite (p = 0.086)

qtz monzonite \( \cong \) ephemeral (p = 0.27). So:

7.3 Median polish for the data of strata 1:

<table>
<thead>
<tr>
<th></th>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
<th>Year median</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969</td>
<td>25.25</td>
<td>11.25</td>
<td>10.25</td>
<td>10.75</td>
<td>8.75</td>
</tr>
<tr>
<td>1970</td>
<td>16.5</td>
<td>2.5</td>
<td>1.5</td>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td>1971</td>
<td>15</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>-1.50</td>
</tr>
<tr>
<td>Season median</td>
<td>14.25</td>
<td>0.25</td>
<td>-0.75</td>
<td>-0.25</td>
<td>2.25</td>
</tr>
</tbody>
</table>
Corbicula densities were 14 units higher in winter than in other seasons, and 9 to 10 units higher in 1969 than 1970 or 1971. Those effects dominated all others. This is shown by a plot of the two-way polished medians:

\[ \text{Corbicula densities} = 14 \text{ units higher in winter than in other seasons, and 9 to 10 units higher in 1969 than 1970 or 1971.} \]

The residuals are skewed, as shown in a boxplot:

\[ \text{The residuals are skewed, as shown in a boxplot:} \]

However, a residuals plot of cell residuals versus the comparison value shows outliers, but an overall slope of zero, stating that no power transformation will improve the situation very much.

\[ \text{However, a residuals plot of cell residuals versus the comparison value shows outliers, but an overall slope of zero, stating that no power transformation will improve the situation very much.} \]
7.4 Due to the outliers noted above, ranks of the Corbicula data were used to test the effects of season and year on the number of organisms.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>2</td>
<td>1064.7</td>
<td>532.33</td>
<td>13.78</td>
<td>0.000</td>
</tr>
<tr>
<td>Season</td>
<td>3</td>
<td>1300.7</td>
<td>433.57</td>
<td>11.23</td>
<td>0.000</td>
</tr>
<tr>
<td>Year*Season</td>
<td>6</td>
<td>560.8</td>
<td>93.46</td>
<td>2.42</td>
<td>0.057</td>
</tr>
<tr>
<td>Error</td>
<td>24</td>
<td>926.8</td>
<td>38.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>35</td>
<td>3853.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A two-way ANOVA on the ranks indicates that both season and year are significant influences on the density of Corbicula, and that there is no interaction. This is illustrated well by the plot of polished medians above.

7.5 Not answered.

Chapter 8

8.1 The plot of uranium versus total dissolved solids looks like it could be nonlinear near the 0 TDS boundary. So Spearman's rho was computed, and rho = 0.72 with t_r = 4.75 and p<0.001.

8.2 Pearson's r = 0.637 with t_r = 3.79 and p<0.001. Kendall's tau = 0.53 with p<0.001. The suggestion of nonlinearity would favor either rho or tau, though the nonlinearity is not serious in this case.

8.3 Not answered.
Chapter 9

9.1 A residuals plot for the untransformed variables shows strong curvature. A log-log regression gives an acceptable plot, with one outlier not influencing the line:

\[ \log(\text{Yield}) = 6.74 + 1.39 \log(\text{Grain Size}) \]

\[ t = 8.14 \quad p < 0.001 \]

9.2 The overall mean yield will be the average of estimates of mean yield for the four wells from the regression equation. Applying the \( \frac{1}{2} s^2 \) correction factor to obtain the mean yield rather than the median, the estimated mean yields are:

46.104 120.830 316.669 556.380  with overall mean = 260 gal/day/ft².

9.3 Here are some possible transformations, including the log. Can logQ be improved on?

<table>
<thead>
<tr>
<th>explanatory variable</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q 0.5</td>
<td>51.1%</td>
</tr>
<tr>
<td>log Q</td>
<td>57.3%</td>
</tr>
<tr>
<td>Q -0.25</td>
<td>57.4%</td>
</tr>
<tr>
<td>Q -0.5</td>
<td>55.4%</td>
</tr>
<tr>
<td>Q -1</td>
<td>47.9%</td>
</tr>
<tr>
<td>( 1/(1+0.00001Q) )</td>
<td>41.8%</td>
</tr>
<tr>
<td>( 1/(1+0.0001Q) )</td>
<td>47.6%</td>
</tr>
<tr>
<td>( 1/(1+0.001Q) )</td>
<td>58.5%</td>
</tr>
<tr>
<td>( 1/(1+0.01Q) )</td>
<td>52.4%</td>
</tr>
<tr>
<td>( 1/(1+0.1Q) )</td>
<td>48.5%</td>
</tr>
</tbody>
</table>

There are perhaps two other good candidate explanatory variables on this list, \( Q^{0.25} \), and \( 1/(1+0.001Q) \). Neither improve significantly over logQ, based on \( R^2 \) or on residuals plots. A residuals plot and probability plot of residuals for the hyperbolic transformation having \( b=0.001 \) are below.

When \( b=0.00001 \) or smaller, the model is virtually identical to the linear model \( \text{TDS} = b_0+b_1Q \) [a power transformation with \( \theta =1 \)]. When \( b=0.1 \) or larger, the model is virtually identical to the inverse model \( \text{TDS} = b_0+b_1(1/Q) \) [a power transformation with \( \theta =-1 \)]. Values of \( b \) in between these provide functions similar to moving down the ladder of powers from \( \theta =1 \) to \( \theta =-1 \). The advantage of using the hyperbolic function is its interpretability as a mixing of ground and surface waters (Johnson et al., 1969).

9.4 If the objective is to predict LOAD, then that (or its transform) should be the dependent variable. The regression statistics (especially PRESS) will then tell how well the predictions will do. If \( \ln(C) \) is used as the dependent variable, the standard error \( s = 0.3394 \), exactly
the same as in the equation for ln(LOAD), but $R^2=17.3\%$ rather than 67.9\% for ln(LOAD). The $t$ statistic on $\beta_1$ is $-4.43$, also significant but not as much as when $y = \ln(LOAD)$. In other words, the error of the $\ln(C)$ values is exactly the same magnitude as the errors of $\ln(LOAD)$. The percent variation explained drops from 67.9\% to 17.3\%, the difference being the strong effect of $Q$ on variation in LOAD. Note the changes in regression coefficients. The previous model was $\ln(LOAD) = 0.789 + 0.761 \ln(Q)$. This one is $\ln(C) = -0.194 - 0.239 \ln(Q)$. The intercept decreased by an amount equal to $\ln(2.7)$ (the log of the unit conversion coefficient) and the slope decreased by exactly 1 because $Q$ is removed from both sides. The standard errors of the coefficients are both unchanged.

If LOAD were computed by using the regression for $\ln(C)$ and then multiplying that result by 2.7 $Q$, exactly the same estimates would result as when using the equation for ln(LOAD). This is true regardless of which estimation method is employed (median, MLE, or Smearing), and will always be true for log-log regression estimation. The moral of the story is: if your boss thinks that you shouldn't use ln(LOAD) as the dependent variable and you can't convince him or her otherwise, go ahead and predict ln(C), and from that ln(LOAD), and you will still get the results you got doing it the simple way.

### Chapter 10

**10.1 Slopes**

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>30</th>
<th>10</th>
<th>15</th>
<th>13</th>
<th>9.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>-10</td>
<td>7.5</td>
<td>7.33</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>25</td>
<td>16</td>
<td>8.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>55</td>
<td>7</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>62</td>
<td>-6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ranked slopes: $-10, -6, 0.5, 4, 7.33, 7.55, 8.67, 9.2, 10, 13, 15, 16, 25, 30$

a) Median slope $= 8.67 = \text{Theil slope estimator } \hat{\beta}_1$

b) $\tau = \frac{S}{n(n-1)/2} = \frac{11}{6 \cdot 5/2} = 0.73$

c) \begin{align*}
\text{intercept } \hat{\beta}_0 & = Y_{\text{med}} - \hat{\beta}_1 \cdot X_{\text{med}} = 47.5 - 8.67 \cdot 3.5 = 17.17 \\
Y & = 17.17 + 8.67 \cdot X \quad \text{is the Kendall-Theil equation} \\
(Y & = 10.07 + 9.17 \cdot X \quad \text{is the OLS equation for the same data})
\end{align*}
Appendix D  Answers

10.2

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Slopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>30 10 15 47.5 9.2</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>-10 7.5 53.33 4</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>25 85 8.67</td>
</tr>
<tr>
<td>4</td>
<td>55</td>
<td>145 0.5</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>-144</td>
</tr>
<tr>
<td>6</td>
<td>56</td>
<td></td>
</tr>
</tbody>
</table>

Ranked slopes: -144, -10, 0.5, 4, 7.5, 8.67, 9.2, 10, 15, 25, 30, 47.5, 53.33, 85, 145

a) Median slope = 10 = Theil slope estimator \( \hat{b}_1 \)
Median X = 3.5
Median Y = 47.5

b) S and \( \tau \) are unchanged

c) \( \hat{b}_0 = 47.5 - 10 \cdot 3.5 = 12.5 \)
\( Y = 12.5 + 10 \cdot X \) the Kendall-Theil equation is similar to ex. 10.1.
\( Y = -8.33 + 21 \cdot X \) the OLS slope has changed a lot from ex. 10.1.

d) the p-value is unchanged.

e) for a 95% confidence interval, the closest entry in table B8 to \( \alpha/2 = 0.025 \) is
p = 0.028 for \( X_U = 11 \). From eqs. 10.3 and 10.4,
\( R_U = \frac{(15 + 11)}{2} = 13 \) for \( N = 15 \) and \( X_U = 11 \).

The rank \( R_l \) of the pairwise slope corresponding to the lower confidence limit is
\( R_l = \frac{(15 - 11)}{2} + 1 = 3 \).

So an \( \alpha = 0.054 \) confidence limit for \( \hat{\beta}_1 \) is the interval between the 3rd and 13th ranked pairwise slope (the 3rd slope in from either end), or
\( 0.5 \leq \hat{\beta}_1 \leq 53.3 \).

10.3  The unweighted OLS regression equation is
\( Y = -8.3 + 21.0 \cdot X \)  \( t = 1.41 \)  \( p = 0.23 \)
The residuals are then divided by 6(MAD), where the MAD is the median of the absolute values of the residuals. Bisquare weights are computed for each data point:

<table>
<thead>
<tr>
<th>pt #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight</td>
<td>0.999</td>
<td>0.996</td>
<td>0.935</td>
<td>0.954</td>
<td>0.179</td>
<td>0.631</td>
</tr>
</tbody>
</table>

A first weighted least squares is then computed:

\[ Y = 3.1 + 13.1 X \quad t = 1.49 \quad p = 0.21 \]

Bisquare weights are again computed for each data point, using residuals from the first WLS:

<table>
<thead>
<tr>
<th>pt #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight</td>
<td>0.984</td>
<td>0.952</td>
<td>0.938</td>
<td>1.000</td>
<td>0.000</td>
<td>0.746</td>
</tr>
</tbody>
</table>

A second WLS is then computed:

\[ Y = 10.4 + 8.80 X \quad t = 2.73 \quad p = 0.07 \]

Though the slope has diminished from the OLS line, the significance has greatly increased due to the lower weight of the outlier. Note the similarity between this WLS and the Kendall's robust line. A residuals plot shows that the WLS line fits most of the data much better than with OLS. The outlier's influence on the slope has diminished, and its residual remains large.
Some object: "Isn't this WLS line the same as throwing away the outlier -- it has a weight of zero?" The difference is that the outlier was determined to be downweighted to zero by the data itself, not an arbitrary decision by the data analyst. Weighted least squares also allows outliers to have partial weights, not simply a zero or one weight as with discarding the outlier. So WLS is far less arbitrary and far more consistent in its assignment of weights to all data points than is throwing away outliers.

10.4 Lowering = \(-2.07 - 0.167 \times \text{Years}\)  
\[ r^2 = 0.76 \]

OLS does not follow the data as well as the smooth because the data are nonlinear.

10.5 Plotting the 20 years of joint data shows that curvature and heteroscedasticity exist, and transformation is required before regression. Thus the natural logs of both are taken. A linear relation results, as shown in the following plot.
Regression between the 20 year joint record at the two stations is:

\[
\ln(\text{Short}) = 1.095 + 0.507 \times \ln(\text{Base}) \quad t = 6.00 \quad p < 0.001 \quad R^2 = 0.67
\]

Using this equation and the 30 additional years of record at Base, 30 years of simulated flows at Short are generated. Now the LOC is used to generate estimates of the "Short" 30-year record. Summary statistics for the 20 years of joint \(\ln(\text{Base})\) and \(\ln(\text{Short})\) records are as follows:

<table>
<thead>
<tr>
<th></th>
<th>(n)</th>
<th>Mean</th>
<th>Stdev</th>
<th>Median</th>
<th>P25</th>
<th>P75</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln(\text{Short}))</td>
<td>20</td>
<td>2.319</td>
<td>0.524</td>
<td>2.160</td>
<td>1.862</td>
<td>2.850</td>
</tr>
<tr>
<td>(\ln(\text{Base}))</td>
<td>20</td>
<td>2.414</td>
<td>0.844</td>
<td>2.190</td>
<td>1.802</td>
<td>3.200</td>
</tr>
</tbody>
</table>

From equation 10.10,

\[
Y_i = \bar{Y} + \text{sign}[r] \times \frac{s_y}{s_x} \times (X_i - \bar{X}), \quad \text{or} \quad \ln(\text{Short}) = 2.319 + (0.524/0.844) \times (\ln(\text{Base}) - 2.414)
\]

\[
= 0.820 + 0.621 \times \ln(\text{Base})
\]

Note how the slope and intercept for LOC differ from the regression coefficients.

Summary statistics for the estimated flows at "Short" by the two methods are compared to the true 30-year record from Appendix C13 in the following table.

<table>
<thead>
<tr>
<th></th>
<th>(n)</th>
<th>Mean</th>
<th>Stdev</th>
<th>Median</th>
<th>P25</th>
<th>P75</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS est.</td>
<td>30</td>
<td>2.2087</td>
<td>0.4975</td>
<td>2.0228</td>
<td>1.7731</td>
<td>2.6249</td>
</tr>
<tr>
<td>LOC est.</td>
<td>30</td>
<td>2.184</td>
<td>0.609</td>
<td>1.956</td>
<td>1.651</td>
<td>2.694</td>
</tr>
<tr>
<td>true values</td>
<td>30</td>
<td>2.079</td>
<td>0.613</td>
<td>1.930</td>
<td>1.630</td>
<td>2.290</td>
</tr>
</tbody>
</table>

The standard deviation for the regression estimate is too small, as expected.
Boxplots are presented below for three groups: the 30-year estimates using regression and LOC combined with the 20-year record at Short, and the actual 50-year record. LOC comes closer to correctly estimating the lowest and highest flows. The regression estimates are too low for high flows, and too high for low flows. They "regress" toward the mean more than the actual data because the standard deviation of the estimates is too small, as $R^2 < 1$.

Chapter 11

11.1 The full multiple regression model contains strong multi-collinearity. The VIFs among the four percentage variables are huge:

10.6 Not answered.
\[ \text{LOGTN} = -1.3 + 0.596 \text{LOGDA} + 0.346 \text{LOGIMP} + 0.0314 \text{MMJTEMP} \\
- 0.0494 \text{MSRAIN} + 0.040 \text{PRES} + 0.035 \text{PNON} + 0.037 \text{PCOMM} + 0.024 \text{PIND} \]

\[ n = 42 \quad s = 0.61 \quad R^2 = 0.59 \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Err(β)</th>
<th>t-ratio</th>
<th>p</th>
<th>VIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept ( \beta_0 )</td>
<td>-1.28</td>
<td>24.60</td>
<td>-0.05</td>
<td>0.959</td>
<td></td>
</tr>
<tr>
<td>Slopes ( \beta_k )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOGDA</td>
<td>0.596</td>
<td>0.121</td>
<td>4.94</td>
<td>0.000</td>
<td>1.8</td>
</tr>
<tr>
<td>LOGIMP</td>
<td>0.346</td>
<td>0.228</td>
<td>1.52</td>
<td>0.138</td>
<td>3.8</td>
</tr>
<tr>
<td>MMJTEMP</td>
<td>0.031</td>
<td>0.019</td>
<td>1.65</td>
<td>0.107</td>
<td>10.1</td>
</tr>
<tr>
<td>MSRAIN</td>
<td>-0.049</td>
<td>0.021</td>
<td>-2.32</td>
<td>0.026</td>
<td>9.1</td>
</tr>
<tr>
<td>PRES</td>
<td>0.040</td>
<td>0.245</td>
<td>0.16</td>
<td>0.873</td>
<td>9227.2</td>
</tr>
<tr>
<td>PNON</td>
<td>0.035</td>
<td>0.246</td>
<td>0.14</td>
<td>0.888</td>
<td>3062.2</td>
</tr>
<tr>
<td>PCOMM</td>
<td>0.037</td>
<td>0.245</td>
<td>0.15</td>
<td>0.882</td>
<td>8311.4</td>
</tr>
<tr>
<td>PIND</td>
<td>0.024</td>
<td>0.246</td>
<td>0.10</td>
<td>0.922</td>
<td>2026.2</td>
</tr>
</tbody>
</table>

Table 11.9  Regression statistics and VIF's for Exercise 11.1

To determine why the multi-collinearity is so strong, the correlation matrix is computed.

<table>
<thead>
<tr>
<th>LOG</th>
<th>MMJ</th>
<th>MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOGT</td>
<td>LOGDA</td>
<td>IMP</td>
</tr>
<tr>
<td>LOGDA</td>
<td>0.565</td>
<td></td>
</tr>
<tr>
<td>LOGIMP</td>
<td>0.058</td>
<td>-0.382</td>
</tr>
<tr>
<td>MMJTEMP</td>
<td>-0.205</td>
<td>-0.188</td>
</tr>
<tr>
<td>MSRAIN</td>
<td>-0.259</td>
<td>-0.083</td>
</tr>
<tr>
<td>PRES</td>
<td>0.294</td>
<td>0.210</td>
</tr>
<tr>
<td>PNON</td>
<td>-0.042</td>
<td>0.319</td>
</tr>
<tr>
<td>PCOMM</td>
<td>-0.218</td>
<td>-0.441</td>
</tr>
<tr>
<td>PIND</td>
<td>-0.131</td>
<td>0.060</td>
</tr>
</tbody>
</table>

Surprisingly, the percentage terms do not have large pair-wise correlation coefficients. Instead, they are strongly related in that the four of them add to 100%, except for rounding error. This is why the VIFs are so large. Therefore at least one of them should be dropped. The variable with the smallest partial F (PIND) could be chosen. This brings the VIF down from over 9000 to 10, still large. In order to save much time the Cp and PRESS statistics can be computed for all possible models. The results below show that the best 5-variable model, containing LOGDA, LOGIMP, MMJTEMP,
MSRAIN, and PIND, is the best in terms of prediction errors (PRESS) and model bias/standard error (Cp). VIFs are below 10 ($R^2 < 0.9$) and so are acceptable.

A residuals plot from the minimum PRESS and Cp model shows no hint of curvature or increasing variance. Therefore this model is preferred.
11.2 First compare those models which have equal numbers of parameters and eliminate the ones with higher SSE.

- Compare 4 to 7, eliminate 4
- Compare 3 to 6, eliminate 3
- Compare 2 to 5, eliminate 2
- Compare 6 to 8, eliminate 6

Now, for the remaining models (1, 5, 7, 8, 9, 10) perform F tests between pairs of nested models. The order in which to proceed is arbitrary.

<table>
<thead>
<tr>
<th>Compare</th>
<th>F</th>
<th>dfnum</th>
<th>dfdenom</th>
<th>Fcrit</th>
<th>conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Models 1 to 5</td>
<td>11.18</td>
<td>1</td>
<td>123</td>
<td>3.9</td>
<td>reject H0, eliminate model 1</td>
</tr>
<tr>
<td>Models 5 to 7</td>
<td>0.28</td>
<td>2</td>
<td>121</td>
<td>3.1</td>
<td>do not reject, eliminate model 7</td>
</tr>
<tr>
<td>Models 5 to 8</td>
<td>1.39</td>
<td>1</td>
<td>122</td>
<td>3.9</td>
<td>do not reject, eliminate model 8</td>
</tr>
<tr>
<td>Models 5 to 9</td>
<td>0.77</td>
<td>3</td>
<td>120</td>
<td>2.68</td>
<td>do not reject, eliminate model 9</td>
</tr>
<tr>
<td>Models 5 to 10</td>
<td>0.88</td>
<td>5</td>
<td>118</td>
<td>2.29</td>
<td>do not reject, eliminate model 10</td>
</tr>
</tbody>
</table>

**Model 5 is the preferred model.**
Another possible approach is to use either PRESS or Mallows Cp.

<table>
<thead>
<tr>
<th>Model</th>
<th>p</th>
<th>$s^2$</th>
<th>Cp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0.5636</td>
<td>14.29</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.5350</td>
<td>8.37</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.5343</td>
<td>9.17</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>0.5359</td>
<td>10.51</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.5183</td>
<td>4.41</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>0.5207</td>
<td>5.98</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>0.5245</td>
<td>7.84</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>0.5166</td>
<td>5.00</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>0.5212</td>
<td>8.06</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>0.5208</td>
<td>9.95</td>
</tr>
</tbody>
</table>

The results are interpreted as: the transport curve is quadratic with a shift in intercept for the winter months. Only two seasons (not three) can be distinguished. The slope of the curve does not change with season.

11.3 Not answered.

11.4 Not answered.

Chapter 12

12.1 Regression

\[ \text{load} = 25,250 - 12.6 \text{ year} \quad r^2 = 10.6\% \]
\[ (t) \quad (1.53) \quad (-1.50) \quad \text{two-sided p value} = 0.134 \]

Multiple regression

\[ \text{load} = 28,152 - 14.4 \text{ year} + 0.696 q \quad r^2 = 88.3\% \]
\[ (t) \quad (4.69) \quad (-4.60) \quad (10.91) \quad \text{two-sided p value} = 0.0001 \]

Mann-Kendall

\[ \text{load} = 11,800 - 5.8 \text{ year} \quad \text{two-sided p value} = 0.415 \]

Mann-Kendall on Residuals

Regression model is \[ \text{load} = -110 + 0.681q \quad r^2 = 74.5\% \]
\[ (t) \quad (-1.24) \quad (7.44) \]

Kendall fit: \[ \text{residual} = 28,250 - 14.4 \text{ year} \quad \text{two-sided p value} = 0.0001 \]
therefore \[ \text{load} = -110 + 0.681q + \text{residual} \]
\[ = -110 + 0.681q + 28,250 - 14.4 \text{ year} \]
\[ = 28,140 - 14.4 \text{ year} + 0.681q \]
12.2 Winter: $P = 16, M = 34, S = -18$
1 tie of 3, 2 ties of 2
$\text{Var}[S] = 159.33$
$Z = -1.347$
$p = 0.18$ very little evidence of downtrend in winter lead

Spring: $P = 27, M = 38, S = -11$
3 ties of 2, 1 tie of 5
$\text{Var}[S] = 249$
$Z = 0.633$
$p = 0.53$ no evidence of downtrend in spring lead

Summer: $P = 16, M = 33, S = -17$
1 tie of 4
$\text{Var}[S] = 156.33$
$Z = -1.28$
$p = 0.20$ very little evidence of downtrend in summer lead

Fall: $P = 11, M = 37, S = -26$
1 tie of 4, 1 tie of 2
$\text{Var}[S] = 155.33$
$Z = 2.005$
$p = 0.045$ fairly strong evidence of downtrend in fall lead

Seasonal Kendall: $S = -72$
$\text{VAR}[S] = 720$
$Z = -2.646$
$p (2\text{-sided}) = 0.008$

Thus, even though the evidence from no individual season was highly conclusive, the data from all seasons taken together provides highly conclusive evidence of a downtrend in lead.

12.3 Maumee River Trends in Total Phosphorus

12.3.1 Parametric analysis first: LOAD vs TIME
Simple linear regression: $LOAD = 444 - 0.221 \text{ TIME}$
$t = -0.42$ \( p = 0.673 \)
$s = 20.59$ \( \text{R-sq} = 0.1\% \) \( \text{R-sq(adj)} = 0.0\% \)
A boxplot of the residuals shows them to be terribly skewed. A transformation is required. Try logarithms. Then the regression equation is:
\[ \ln(LOAD) = 117 - 0.0592 \times \text{TIME} \]
\[ t = -1.32 \quad \text{p} = 0.189 \]
\[ s = 1.770 \quad \text{R-sq} = 1.3\% \quad \text{R-sq(adj)} = 0.6\% \]

There is a fairly normal distribution of residuals, so a test based on regression seems legitimate. Very weak evidence of trend -- (two-sided) p-value of 0.189. But are there strong flow and/or seasonal effects? A plot of the residuals versus log of streamflow (LQ) shows a strong dependence on flow. Removing this should greatly enhance the power to detect any trend which is present.

Boxplots of residuals by month also show a strong seasonal cycle, high in the winter & spring, low in summer. The best model we could find includes time, ln(Q), ln(Q)^2, and sine and cosine of 2\piT:
\[ LLOAD = 83.3 - 0.0425 \times \text{TIME} + 1.08 \ln(Q) + 0.0679 \ln(Q)^2 - 0.0519 \text{SIN} + 0.141 \text{COS} \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>Stdev</th>
<th>t-ratio</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>83.32</td>
<td>22.06</td>
<td>3.78</td>
<td>0.000</td>
</tr>
<tr>
<td>TIME</td>
<td>-0.0425</td>
<td>0.0115</td>
<td>-3.81</td>
<td>0.000</td>
</tr>
<tr>
<td>lnQ</td>
<td>1.08175</td>
<td>0.04947</td>
<td>21.87</td>
<td>0.000</td>
</tr>
<tr>
<td>ln(Q)^2</td>
<td>0.06789</td>
<td>0.01868</td>
<td>3.63</td>
<td>0.000</td>
</tr>
<tr>
<td>SIN</td>
<td>-0.05190</td>
<td>0.06252</td>
<td>-0.83</td>
<td>0.408</td>
</tr>
<tr>
<td>COS</td>
<td>0.14058</td>
<td>0.05441</td>
<td>2.58</td>
<td>0.011</td>
</tr>
</tbody>
</table>

\[ s = 0.4398 \quad \text{R-sq} = 94.1\% \quad \text{R-sq(adj)} = 93.9\% \]
This is interpreted as a strong evidence of downtrend, with a p-value < .001. The slope (in log units) = −0.0425 per year. All coefficients are significant at \( \alpha = 0.05 \) except for \( \text{sine}(2\pi T) \). The sine must either be left in, or both it and the cosine taken out. To test whether together they are significant, an F test is performed. The model without these terms, with the standard error \( s = 0.449 \), is:

\[
L\text{LOAD} = 85.1 - 0.0434 \text{ TIME} + 1.06 \ln Q + 0.0748 \ln (Q)^2
\]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>Stdev</th>
<th>t-ratio</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>85.09</td>
<td>22.51</td>
<td>3.78</td>
<td>0.000</td>
</tr>
<tr>
<td>\text{TIME}</td>
<td>-0.04340</td>
<td>0.01138</td>
<td>-3.81</td>
<td>0.000</td>
</tr>
<tr>
<td>\ln Q</td>
<td>1.05921</td>
<td>0.04518</td>
<td>23.44</td>
<td>0.000</td>
</tr>
<tr>
<td>\ln (Q)^2</td>
<td>0.07481</td>
<td>0.01865</td>
<td>4.01</td>
<td>0.000</td>
</tr>
<tr>
<td>( s = 0.4490 )</td>
<td>( R\text{-sq} = 93.7% )</td>
<td>( R\text{-sq(adj)} = 93.6% )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

So the F test to compare these two models is:

\[
F = \frac{(26.01 - 24.57) / 2.0}{0.193} = 3.73
\]

Comparing to an F distribution with 2 and 127 degrees of freedom, the two-sided p-value is 0.027. Therefore reject the simpler model in favor of including the seasonal sine and cosine terms.

To predict estimates of load for the two times and two flow conditions above, natural logs of these values are input to the regression equation. The third column below reports the predicted logs of Load from the regression equation.

<table>
<thead>
<tr>
<th>\ln Q</th>
<th>Time</th>
<th>Predicted \ln L</th>
<th>Bias-Corrected L</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>1972.5</td>
<td>2.3356</td>
<td>11.3852</td>
</tr>
<tr>
<td>0.0</td>
<td>1972.5</td>
<td>-0.6516</td>
<td>0.574136</td>
</tr>
<tr>
<td>2.4</td>
<td>1986.5</td>
<td>1.7406</td>
<td>6.27963</td>
</tr>
<tr>
<td>0.0</td>
<td>1986.5</td>
<td>-1.2467</td>
<td>0.316640</td>
</tr>
</tbody>
</table>

These predictions must be transformed and corrected for bias. Using the Ferguson (MLE) bias correction, \( 0.5 \cdot s^2 = 0.5 \cdot (0.4398)^2 = 0.097 \). So the bias correction equals \( \exp(0.097) \), or about 10%. The four predicted total phosphorus loads are given above in the fourth column.

Therefore the percent change at high flow over the 14-year time period is:

\[
\frac{(6.2763 - 11.3852)}{11.3852} = -0.448732
\]

The change in percent per year is

\[
-0.448732 \cdot 100 / 14 = -3.205 \text{. That is a} -3.2\% \text{ change in total P per year.} \]
The same analysis at lower flow over the 14 years is:
\[
(0.31664 - 0.574136) / 0.574136 = -0.4485, \text{ the same amount as at high flow.}
\]
Re-expressing the slope estimate in original units as a percent change, the average change equals \(-4.2\%\) per year:
\[
100 \times \exp(-0.0425) - 1.0 = -4.16096
\]

### 12.3.2 The nonparametric approach

The seasonal-Kendall test on the original observations, using 12 seasons (months): \(\text{Tau} = -0.06\) with a \(p\)-value of 0.3835.

\[
\log(\text{Load}) = 0.505 - 0.046 \times \text{time},
\]
where time = 0 at the beginning of the first year of the record (typically a water year), and time is in units of years.

Residuals from a regression of \(\log(\text{Load})\) versus \(\log Q\) and \(\log Q^2\) removes the effect of flow:
\[
\log(\text{Load}) = -0.745 + 1.06 \log Q + 0.0758 \log Q^2
\]
The S-K test on the regression residuals: \(\text{tau} = -0.25\) with \(p = 0.0002\)
and \(\log(\text{Load}) = 0.312 - 0.048 \times \text{time}\)

So, if flow is not first removed, the significant trend would be missed. Both the Seasonal Kendall on the residuals and multiple regression give a highly significant \(p\)-value. The S-K slope is 4.8\% rather than 4.16\% because of the effect of the low residuals during 1972-1977.
12.4 Not answered.

12.5 Nat answered.

Chapter 13

13.1 Not answered

13.2 Because there is only one reporting limit, Kendall's tau can easily be computed for this data: $\tau = -0.40$ with $p = 0.023$. There is a significant decrease in TPT concentrations with depth.

13.3 Estimates of the four descriptive statistics for each of 5 multiple-threshold methods (see Helsel and Cohn, 1988) are:

<table>
<thead>
<tr>
<th>Method</th>
<th>MEAN</th>
<th>ST.DEV.</th>
<th>MEDIAN</th>
<th>IQR</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZE (substitute zero)</td>
<td>12.36</td>
<td>75.48</td>
<td>0.00</td>
<td>1.10</td>
</tr>
<tr>
<td>HA (substitute 1/2 dl)</td>
<td>13.91</td>
<td>75.28</td>
<td>1.10</td>
<td>3.30</td>
</tr>
<tr>
<td>DL (substitute the dl)</td>
<td>15.45</td>
<td>75.19</td>
<td>1.30</td>
<td>4.10</td>
</tr>
<tr>
<td>MR (prob plot regression)</td>
<td>12.57</td>
<td>75.44</td>
<td>0.29</td>
<td>1.54</td>
</tr>
<tr>
<td>MM (lognormal MLE)</td>
<td>8.30</td>
<td>61.52</td>
<td>0.34</td>
<td>1.62</td>
</tr>
</tbody>
</table>

Because of the outlier at 560 µg/L the data have more skewness than a lognormal distribution, and methods which assume a lognormal distribution for all the data (MM) would not be expected to estimate moment statistics well. It is not surprising therefore that the MLE method produces moment estimates dissimilar to the other methods. We generally select the MR moment estimates and the MM quantile estimates (those printed in bold), due to the results of Helsel and Cohn (1988).

Chapter 14

14.1 a) Contingency table

<table>
<thead>
<tr>
<th>Δ road salt appl.</th>
<th>Down</th>
<th>No trend</th>
<th>Up</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Down</td>
<td>5.44</td>
<td>23.84</td>
<td>16.71</td>
<td>46</td>
</tr>
<tr>
<td>No change</td>
<td>9.82</td>
<td>43.02</td>
<td>30.15</td>
<td>83</td>
</tr>
<tr>
<td>Up</td>
<td>13.73</td>
<td>60.13</td>
<td>42.14</td>
<td>116</td>
</tr>
<tr>
<td>Totals</td>
<td>29</td>
<td>127</td>
<td>89</td>
<td>245</td>
</tr>
</tbody>
</table>

**Trend in Cl⁻ (1974-81, α=0.1)**

<table>
<thead>
<tr>
<th>Δ road salt appl.</th>
<th>Expected values Eij</th>
</tr>
</thead>
<tbody>
<tr>
<td>Down</td>
<td>5.44</td>
</tr>
<tr>
<td>No change</td>
<td>9.82</td>
</tr>
<tr>
<td>Up</td>
<td>13.73</td>
</tr>
</tbody>
</table>
The results indicate that the category of chloride trends is dependent on the category of salt applications, with a \( p \)-value of 0.004. Where increases in road salt occurred, there are more up trends and fewer down trends than would be expected from the marginal distributions of up trends and down trends. Where decreases in road salt occurred, there are fewer up trends than would be expected.

### Table of \( \frac{(0-E)^2}{E} \)

**Trend in Cl\(^{-}\) (1974-81, \( \alpha=0.1 \))**

<table>
<thead>
<tr>
<th>( \Delta ) road salt appl.</th>
<th>Down</th>
<th>No trend</th>
<th>Up</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Down</td>
<td>0.04</td>
<td>2.79</td>
<td></td>
<td>3.56</td>
</tr>
<tr>
<td>No change</td>
<td>1.78</td>
<td>0.02</td>
<td></td>
<td>0.88</td>
</tr>
<tr>
<td>Up</td>
<td>1.01</td>
<td>1.39</td>
<td></td>
<td>3.93</td>
</tr>
</tbody>
</table>

\( \chi^2 = 15.39 \quad \text{df} = 4 \quad p = 0.004 \)

b) Kendall's tau

\[
P = \text{no. pluses} = 5(44+25+51+55) + 32(25+55) + 14(51+55) + 44(55) = 7339
\]

\[
M = \text{no. minuses} = 32(14+10) + 9(14+44+10+51) + 44(10) + 25(10+51) = 3804
\]

\[
S = 7339 - 3804 = 3535.
\]

\[
\tau_b = \frac{3535}{\sqrt{(245^2 - (46^2 + 83^2 + 116^2))(245^2 - (29^2 + 127^2 + 89^2))}} = 0.19
\]

To test for significance,

\[
\sigma_S = \sqrt{\frac{1}{9} \left( \left( -1 + (1.19^3 + 0.34^3 + 0.47^3) \right) \left( -0.13^3 + 0.52^3 + 0.36^3 \right) \right) = 1061}
\]

and so \( Z_S = 3534/1061 = 3.33 \) and two-sided \( p = 0.0008 \).

The two variables are significantly and positively related.

c) Kendall's tau is more appropriate because

1. It includes the information that the variables are ordinal into the test. The \( p \)-value for Kendall's tau is lower than that for the contingency table, reflecting this additional information.

2. It provides a measure of the direction of association \( \tau_b \). Since \( \tau \) is positive, the trends in Cl\(^{-}\) increase with increasing trends in road salt application.
14.2 Based on the tables below, there is a significant association between location of the well and the probability of detecting volatiles. The more protected downdip wells indeed have less chance of being contaminated than do the outcrop wells.

<table>
<thead>
<tr>
<th>Location</th>
<th>Non-detects</th>
<th>Detect VOC</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downdip</td>
<td>91.92</td>
<td>23.08</td>
<td>115</td>
</tr>
<tr>
<td>Outcrop</td>
<td>143.08</td>
<td>35.92</td>
<td>179</td>
</tr>
<tr>
<td>Totals</td>
<td>235</td>
<td>59</td>
<td>294</td>
</tr>
</tbody>
</table>

Table of \( \frac{(0-E)^2}{E} \)

<table>
<thead>
<tr>
<th>Location</th>
<th>Non-detects</th>
<th>Detect VOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downdip</td>
<td>2.16</td>
<td>8.59</td>
</tr>
<tr>
<td>Outcrop</td>
<td>1.39</td>
<td>5.52</td>
</tr>
</tbody>
</table>

\[ \chi^2 = 17.647 \quad df = 1 \quad p = <0.0001 \]

14.3 To test for association, Kendall's tau-b is used because both time and concentration variables are ordinal. Computations are shown in the boxes below.

<table>
<thead>
<tr>
<th>Year</th>
<th>( \leq 200 )</th>
<th>&gt; 200</th>
<th>( A_i )</th>
<th>( a_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>0.18</td>
</tr>
<tr>
<td>1989</td>
<td>9</td>
<td>13</td>
<td>22</td>
<td>0.43</td>
</tr>
<tr>
<td>1990</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>0.39</td>
</tr>
</tbody>
</table>

\[ C_j = 21 \quad C_j = 30 \quad 51 \]

\[ c_j = 0.41 \quad c_j = 0.59 \quad 1.0 \]

The number of pluses \( P = 2(13+10) + 9(10) = 136 \)
The number of minuses \( M = 7(9+10) + 13(10) = 263 \)
So \( S = 136 - 263 = -127 \).

To compute the denominator of \( \tau_b \),

\[ SS_a = 9^2 + 22^2 + 20^2 = 965. \]
\[ SS_c = 21^2 + 30^2 = 1341. \]

and \( \tau_b = \frac{-127}{\sqrt{(51^2 - 965)(51^2 - 1341)}} = -0.18. \)

From equation 14.9 the approximate value of \( \sigma_S \) is
\[ \sigma_S \equiv \sqrt{\frac{1}{9} \cdot (1-(0.183^3+0.433^3+0.393^3)) \cdot (1-(0.413^3+0.593^3)) \cdot 51^3} \]

\[ \equiv \sqrt{\frac{(0.86)(0.73) \cdot 51^3}{9}} = \sqrt{9253} = 96.2 \]

\[ Z_S \equiv \frac{-127+1}{96.2} = -1.31 \]

and from a table of the normal distribution the one-sided p-value is \( p = 0.095 \). Therefore \( H_0: \tau_b = 0 \) is not rejected at \( \alpha = 0.05 \), but is for \( \alpha = 0.10 \). Thus there is weak evidence of a downtrend in TBT concentrations based on a split at 200 ng/L. Stronger evidence could be obtained by collecting data for subsequent years, or by obtaining better resolution of the data (the original data reported concentration values rather than a split at 200 ng/L).

### Chapter 15

15.1 Logistic regression for the full model with four explanatory variables gives:

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>Wald's t</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-13.20539</td>
<td>3.55770</td>
<td>-3.71</td>
<td>0.0002</td>
</tr>
<tr>
<td>Thick</td>
<td>0.51527</td>
<td>0.15093</td>
<td>3.41</td>
<td>0.0004</td>
</tr>
<tr>
<td>Yields</td>
<td>0.42909</td>
<td>0.27484</td>
<td>1.56</td>
<td>0.0607</td>
</tr>
<tr>
<td>GW Qual</td>
<td>0.03035</td>
<td>0.32460</td>
<td>0.09</td>
<td>0.4642</td>
</tr>
<tr>
<td>Hazard</td>
<td>1.08952</td>
<td>0.29860</td>
<td>3.65</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

with a likelihood ratio \( l_{ro} = 49.70 \) and \( p<0.000 \) as compared to the intercept-only model. However, two of the variables (Yields and GW Qual) have insignificant t-statistics. In the following model they are dropped, and \( l_{ro} \) recomputed:

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>Wald's t</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-10.89039</td>
<td>2.43434</td>
<td>-4.47</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Thick</td>
<td>0.46358</td>
<td>0.13575</td>
<td>3.41</td>
<td>0.0004</td>
</tr>
<tr>
<td>Hazard</td>
<td>1.07401</td>
<td>0.28301</td>
<td>3.80</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

with a likelihood ratio \( l_{ro} = 52.54 \) and \( p<0.000 \) as compared to the intercept-only model. The partial likelihood ratio to test whether the first model is significantly better than the simpler second model is:

\[ lr = l_{ro}(simple) - l_{ro}(complex) = 52.54 - 49.70 = 2.84 \]
which for a chi-square distribution with 2 degrees of freedom gives:

\[ p = 0.242. \]

Therefore the two additional variables (Yields and GW Qual) do not appreciably add to the explanatory power of the model.
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