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Chapter A5

A MODULAR FINITE-ELEMENT MODEL (MODFE) FOR AREAL AND AXISYMMETRIC GROUND-WATER-FLOW PROBLEMS, PART 3: DESIGN PHILOSOPHY AND PROGRAMMING DETAILS

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Book 6
MODELING TECHNIQUES

Steady vertical leakage

Subroutine VNBAL

Variable	Definition
CA,CB	Galerkin-weighting factors, equal to 1/3 and 2/3 respectively.
DAT	Altitude difference, $\overline{H}_{ai} - z_{ti}$ [length].
DEH	Altitude difference, $z_{ei} - \hat{h}_{i,n}$ [length].
DET	Altitude difference, $z_{ei} - z_{ti}$ [length].
DHC	Total-head change for time step; $\hat{h}_{i,n+1} - \hat{h}_{i,n}$ [length].
DHT	Altitude difference, $\hat{h}_{i,n} - z_{ti}$ [length].
DMHT	$(z_{ei} - \hat{h}_{i,n}) - (\hat{h}_{i,n} - z_{ti})$ used to compute the combination of changeover points, ϕ_{ti} and ϕ_{ei}, for leakage cases 8 and 9.
EI	Leakage coefficient, C_{ai} or C_{ei} of equations (119) and (132) respectively in Cooley (1992).
HA	Average value of controlling head, $\overline{H}_{ai}$, [length] for steady-vertical-leakage function.
HC	Aquifer head at end of time step; $\hat{h}_{i,n+1}$ [length].
HO	Aquifer head at beginning of time step; $\hat{h}_{i,n}$ [length].
PHC	Partial formulation of simplified ϕ terms, multiplies leakage coefficient for cases 2 and 6, discharge-only functions, and case 3, steady vertical-leakage function.
PHCF	Partial formulation of simplified ϕ terms, multiplies leakage coefficient for cases 8 and 9, discharge-only function.
PHE	Partial formulation of simplified ϕ terms multiplies leakage coefficient for case 5, discharge-only function.

Subroutine VNBAL (continued)

Variable	Definition
PHI	The changeover point, $-\phi_i$, of equation (115) in Cooley (1992) applied to steady vertical leakage.
PHT	Partial formulation of simplified ϕ terms, multiplies leakage coefficient for case 3, discharge-only function.
TMPA	Formulation of simplified ϕ terms and head or altitude differences, multiplies leakage coefficient to yield volumetric flow rate.
TMPB	Volumetric flow rate for nonlinear steady-leakage functions [length^3/time].
ZE	Extinction depth, z_{ei}, for discharge only function [length].
ZT	Altitude of aquifer top, z_{ti}, for nonlinear steady-leakage [length].

Subroutine VNBLSS

HI	Aquifer head from final iteration, $\hat{h}_i^{\ell+1}$ [length].
TMPA	(1) Altitude difference, $z_{ei} - z_{ti}$ for case1, or $z_{ei} - \hat{h}_i^{\ell+1}$ for case 4, that multiplies leakage coefficient for discharge-only function [length]. (2) Altitude difference, $H_{ai} - \hat{h}_i^{\ell+1}$ for case 1, or $H_{ai} - z_{ti}$, for case 4, that multiplies leakage coefficient for steady vertical-leakage function [length].
TMPB	Volumetric flow rate from nonlinear leakage functions [length^3/time].

Subroutine VNCHG

Variables that are contained in this subroutine have been defined in previous sections that give definitions of program variables in the general-storage vector G, in Fortran COMMON statements, and in subroutines VNBAL and VNBLSS.

CA,CB	Galerkin-weighting factors equal to 1/3 and 2/3 respectively.
DHP	Predicted head change over the time step.
HA	Average controlling head $\overline{H}_{ai}$ [length] for steady vertical-leakage functions.
HO	Aquifer head, $\hat{h}_{i,n}$ [length] at beginning of time step.
HP	Predicted-aquifer head at the end of time step, $\hat{h}_{i,n+1}$ [length].
NME	Index to $\underline{A}$ for the main-diagonal location of nodes that simulate nonlinear-steady leakage.
PHE	Terms involving simplified ϕ terms that multiply the leakage coefficient to form the main-diagonal term, equal to 1 - $\phi_{t\,i}$ for case 2 and 1 - $\phi_{e\,i}$ for case 6, discharge-only functions, and equal to 1 - ϕ_i for case 3, steady vertical-leakage functions.
TMPA	Terms involving head and altitude differences and/or the simplified ϕ terms that multiply the leakage coefficient EI to form the right side term for nonlinear steady leakage, by case.

Subroutine VNFMCO

EC	Leakage coefficient (1/3) $R_a^e\Delta^e$ or(1/3)$R_e^e\Delta^e$ of equations (119) and (132) respectively in Cooley (1992).
L	Zone number where nonlinear-steady leakage is simulated.
NA,NB,NC	Node numbers in an element.
NBE	Beginning (lowest) element number in zone L.

Subroutine VNFMCO (continued)

Variable	Definition
NDC	Index to ND for locating nodes in element.
NE	Counter used in computing leakage coefficient E from combined-element input.
NEND	Ending (highest) element number in zone L.
NO	Number of elements in zone L.
NTE	Index variable for element areas.
VNCF	Conductance terms, R_a^e or R_e^e for steady vertical leakage [time $^{-1}$].

Subroutine VNINIT

Variable	Definition
NVNZ	Number of zones for nonlinear, steady vertical leakage.

Subroutine VNPRED

Variables that are contained in this subroutine have been defined in previous sections that give definitions of program variables in the general-storage vector G, in Fortran COMMON statements, and in other subroutines.

Table 23.—Variable names by subroutine for nonlinear steady vertical leakage

Main program variable	Subroutine						
	VNBAL	VNBLSS	VNCHG	VNCORR	VNFMCO	VNINIT	VNPRED
DT	DT	DT					
G						G	
G(IAA)				A			A
G(IARA)					AR		
G(IBA)				B			B
G(IDHA)	DH			DH			
G(IECA)	E	E		E	E		E
G(IHA)	H	H		H	H		H
G(IHBA)	DHB						
G(IHSA)	HS	HS	HS	HS	HS		HS
G(INA)	IN	IN		IN			IN
G(INDA)					ND		
G(ITPA)	TOP	TOP		TOP			TOP
G(IYGA)	R	R					
IECA						IECA	
IHSA						IHSA	
ISTP			ISTP				
NVNZ					NBNC	NBNC	
TIME			TIME				

Subroutine SWBDMP

Variable	Definition
B	(1) Computed displacement, $\underline{\delta}_{\ell}$, from subroutines BAND and MICCG.
	(2) Damped displacements, $\rho_{\ell}\,\underline{\delta}_{\ell}$, of equation (239) in Cooley (1992).
DSPA	Absolute value of maximum displacement $\lvert e_{\ell} \rvert$ of equation (243) in Cooley (1992).
DSP	Maximum displacement from the current iteration e_{ℓ}.
DSPO	Initial value of maximum displacement e_{ℓ} or the maximum value from the previous iteration.
RP	Estimate of the damping parameter ρ^*.
SPR	Ratio of damped to undamped displacements, given by equation (241) in Cooley (1992).

Subroutine SWFMCO

IW	Number of storage locations assigned to each node for storing components of reduced matrix A.
IPTK	Indicator variable to supress printout of aquifer thickness at each node.
IPTP	Indicator variable to suppress printout of altitude of aquifer top.
MBM1	Number of storage locations allocated to each node for storing elements of JPT.
ND	Index to $\underline{A}$ for locating transmissivity terms.
NP	Index to $\underline{JPT}$.

Subroutine SWINIT

Variable	Definition
DSMX	Maximum allowable displacement [length].
DSP	Maximum computed displacement [length].
NITSW	Maximum number of water-table iterations.
TOLSW	Closure tolerance for steady state, ε_s, of equation (240) in Cooley (1992).

Subroutine SWTHK

HO	Aquifer head from previous iteration, $\hat{h}^{\ell}$ [length].
THKI	Temporary-storage variable for aquifer thickness at node I.
THKK	Temporary-storage variable for aquifer thickness at node K.

Subroutine TKOUT

Variables that are contained in this subroutine have been defined in previous sections that give definitions of program variables in the general-storage vector G, in Fortran COMMON statements, and in other subroutines.

Table 24.—Variable names by subroutine for nonlinear steady-state conditions

Main program variable	Subroutine				
	SWBDMP	SWFMCO	SWINIT	SWTHK	TKOUT
G(IAA)		A		A	
G(IBA)	B			B	
G(IHA)				H	
G(IJPA)		JPT		JPT	
G(INA)		IN		IN	
G(ITKA		THK		THK	THK
G(ITPA)		TOP		TOP	
G(IQA)				Q	
DSMX	DSMX				
DSP	DSP		DSP		
DSPA	DSPA			DSPA	
DSPO	DSPO				
G			G		
RP	RP		RP		
TOLSW			TOLSW	TOLSW	
ITER	ITER			ITER	
ITKA			ITKA		
ITPA			ITPA		
NITSW			NITSW		

Direct-Solution Method

Subroutine BAND

Variable	Definition
A	Element of upper triangular matrix A.
B	The intermediate vector y when it appears in the DO 150 and DO 160 loops; the computed values of head difference when used in the DO 170 and DO 180 loops.
C	A factor u_{ij}/α_{ii} used in upper triangularization of matrix A when it appears in the DO 140 loop. A coefficient used to compute $\underline{y}$ when it appears in the DO 160 loop.
C1	The main diagonal in $\underline{D}$, $1/\alpha_{ii}$ where α_{ii} is the main diagonal in matrix $\underline{\underline{u}}$.
IB	Index to the off diagonal of the expanded A matrix.
ID	Index for the main diagonal of the expanded A matrix.
IRBW	Number of elements in semi-bandwidth of $\underline{\underline{A}}$ that are factored, including the main diagonal.
MR	Amount of storage required for composing upper-triangular matrix A.
NP	Number of terms in <u>JPT</u>.
NR	Amount of storage required for composing upper-triangular matrix A.

Subroutine INITB

Variable	Definition
NA	Length of vector storage associated with program vector A for storing terms used to form coefficients of matrix equations, computed as (MBWC+1) × NNDS.
NB	Length of vector storage needed to store element areas in program vector AR, computed as 2 × NELS.

Subroutine INITB (continued)

Variable	Definition
NBCZ	Number of head-dependent (Cauchy-type) boundary zones.
NC	Amount of computer storage needed for reduced-matrix A, factored into an upper-triangular matrix computed as MBW× (NNDS-NHDS-MBW) + (MBW×(MBW+1))/2.
ND	Amount of storage needed for the pointer vector JPT computed as (MBWC-1)×NNDS.
NE	Amount of storage needed for the ND vector that stores node numbers of each element, computed as 4×NELS.
NF	Amount of computer storage needed for right side of matrix equations (1) through (5) computed as the greater value of NNDS-NHDS or NELS.
TIME	Total simulation time.
TITLE	Title of simulation and description of scale change for length terms.

Subroutine SETB

Variable	Definition
IB, IE, IB1, IE1	Starting (IB and IB1) and ending (IE and IE1) locations in general storage vector G for writing terms to storage files.
IBND	Bandwidth of reduced-matrix A.
JB, JE	Indexes for combining incidences of two elements.
MAXND	Largest node number in element for computing reduced-matrix bandwidth.
MINND	Smallest node number in element for computing reduced-matrix bandwidth.
NDC	Index for node numbers in vector ND.

Subroutine SETB (continued)

Variable	Definition
NE	Counter for elements.
NTMP	Current value of reduced-matrix bandwidth.

Table 25.—Variable names by subroutine for direct-solution method

Main program variable	Subroutine BAND	INITB	SETB
G		G	
G(IAA)	A		A
G(IATA)	AT		
G(IBA)	B		
G(IJPA)	JPT		
G(INA)	IN		IN
G(INDA)			ND
IBND	IBND		IBND
NBCZ		NBCZ	
TIME		TIME	
TITLE		TITLE	

Iterative, Conjugate-Gradient Method

Subroutine INITCG

Variable	Definition
NA	Length of vector storage associated with program vector A for storing terms that form coefficients of matrix equations, computed as (MBWC+1) × NNDS.
NB	Length of vector storage associated with vector AR for storing element areas, computed as 2 × NELS.
NBCZ	Number of head-dependent (Cauchy-type) boundary zones.
NC	Length of vector storage associated with upper-triangular matrix U resulting from incomplete factorization, computed as NA+(MBWC+3) × (NNDS-NHDS).
ND	Length of vector storage associated with pointer vector JPT, computed as (MBWC-1) × NNDS.
NE	Length of vector storage associated with vector ND for storing node numbers of each element, computed as 4 × NELS.
NF	Length of vector storage for B vector which stores computed head changes, computed as NNDS-NHDS.
NG	Amount of storage (= NF) used to allocate space for $\underline{X}$, $\underline{P}$, and $\underline{R}$ in subroutine MICCG.
TIME	Total simulation time.
TITLE	Title of simulation and description of scale change for length terms.
TOL	Closure tolerance for MICCG solution.

Subroutine MICCG

AF(ID+J)	Term $\tilde{U}_{ij} = \tilde{U}_{ij}/\tilde{\alpha}_{ii}$, of equation (282) in Cooley (1992).

Subroutine MICCG (continued)

Variable	Definition
AF(KD)	Main-diagonal term $\tilde{\alpha}_{ii}$ of $\underline{\underline{\tilde{U}}}$ computed by equation (278) in Cooley (1992).
AF(KD+L)	Off-diagonal term $\tilde{u}_{ij}$, of $\underline{\underline{\tilde{U}}}$ computed by equation (274) in Cooley (1992).
AF(LD)	Main-diagonal term $\tilde{\alpha}_{ii}$, of $\underline{\underline{\tilde{U}}}$ computed by equation (278) in Cooley (1992).
B(I)	Scaled-head change for iteration [length].
C	Factor $\tilde{u}_{ki}/\tilde{a}_{kk}$ for formulating coefficients of $\underline{\underline{\tilde{U}}}$.
ID	Index to main-diagonal element of $\underline{\underline{\tilde{U}}}$, α_{ii}.
IT	Iteration number.
KNT	Counter for iteration number.
NP	Index for pointer vector JPT.
PIV	Factor $1/\tilde{\alpha}^{kk}$ for formulating coefficients of $\underline{\underline{\tilde{U}}}$.
PMAX	Largest scaled displacement for iteration.
PX	$\underline{p}_k^T \underline{\underline{A}}\, \underline{p}_k$ of equation (271) in Cooley (1992).
P(I)	Vectors $\underline{p}_k$ of equation (271) in Cooley (1992).
RMAX	Largest scaled residual, $\lvert r^k \rvert / a_{ii}$, of equation (289) in Cooley (1992).
RX	$\underline{s}_k^T \underline{r}_k$ of equation (271) in Cooley (1992).
S	β_k of equation (271) in Cooley (1992).

subroutine MICCG (continued)

Variable	Definition
TMPA	Temporary variable used to compute f_{ij} of equation (277) in Cooley (1992) for row-sum agreement, scaled displacement W P(I), and scaled residual $\lvert r_i^k \rvert / a_{ii}$ for row-sum agreement.
W	Scaling factor relating flow-balance residuals from incomplete factorization of $\underline{A}$ to those computed with $\underline{A}$.
X(K)	y_i of equation (283) in Cooley (1992).
X(M)	y_i of equation (283) and s_i of equation (284) in Cooley (1992).
X(N)	y_i of equation (283) in Cooley (1992).

Subroutine SETG

Variable	Definition
IB	Starting location in general-storage vector G for element area.
IB1	Starting location in general-storage vector G for element incidences.

Table 26.—Variable names by subroutine for iterative, conjugate-gradient method

Main program variable	Subroutine INITCG	Subroutine MICCG	Subroutine SETCG
G	G		
G(IAA)		A	A
G(IAFA)		AF	
G(IBA)		B	
G(IJPA)		JPT	
G(INA)		IN	
G(IPA)		P	
G(IRA)		R	
G(IXA)		X	
IAFA	IAFA		
IPA	IPA		
IRA	IRA		
IXA	IXA		
NBCZ	NBCZ		
TIME	TIME		
TITLE	TITLE		
TOL	TOL	TOL	