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UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

ANALOG-DIGITAL MODELS OF STREAM-AQUIFER SYSTEMS

By

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OPEN-FILE REPORT

WATER
RESOURCES
DIVISION

Colorado District
Denver, Colorado
March 1969

Analog-Digital Models of Stream-Aquifer Systems[a]

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[a]Report prepared in cooperation with the Colorado Water Conservation Board and the Southeastern Colorado Water Conservancy District. Publication authorized by the Director, U.S. Geological Survey.

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ABSTRACT

The best features of analog and digital computers were combined to make a management model of a stream-aquifer system. The analog model provides a means for synthesizing, verifying, and summarizing aquifer properties; the digital model permits rapid calculation of the effects of water management practices. Given specific management alternatives, a digital program can be written that will optimize operation plans of stream-aquifer systems. The techniques are demonstrated by application to a study of the Arkansas River valley in southeastern Colorado.

INTRODUCTION

Many streams are hydraulically connected to large ground-water bodies in valley-fill deposits adjacent to and underlying the stream. In the West, many alluvial valleys are irrigated both by surface and ground water. The Arkansas River valley in southeastern Colorado is typical of such systems. Figure 1 shows an idealized section of the valley.

The annual water requirement of crops in the valley is similar to that shown in Figure 2. The average yearly supply of surface water is about equal to the crop requirement but because there are few surface storage facilities, severe surface-water shortages occur during the latter part of the growing season. The ground-water reservoir offers a means of regulating the supply. Since about 1950, irrigators have installed 1,200 large-capacity wells in the valley to supply late-season deficiencies. The overall economy has benefited by this practice, but it has complicated water administration.

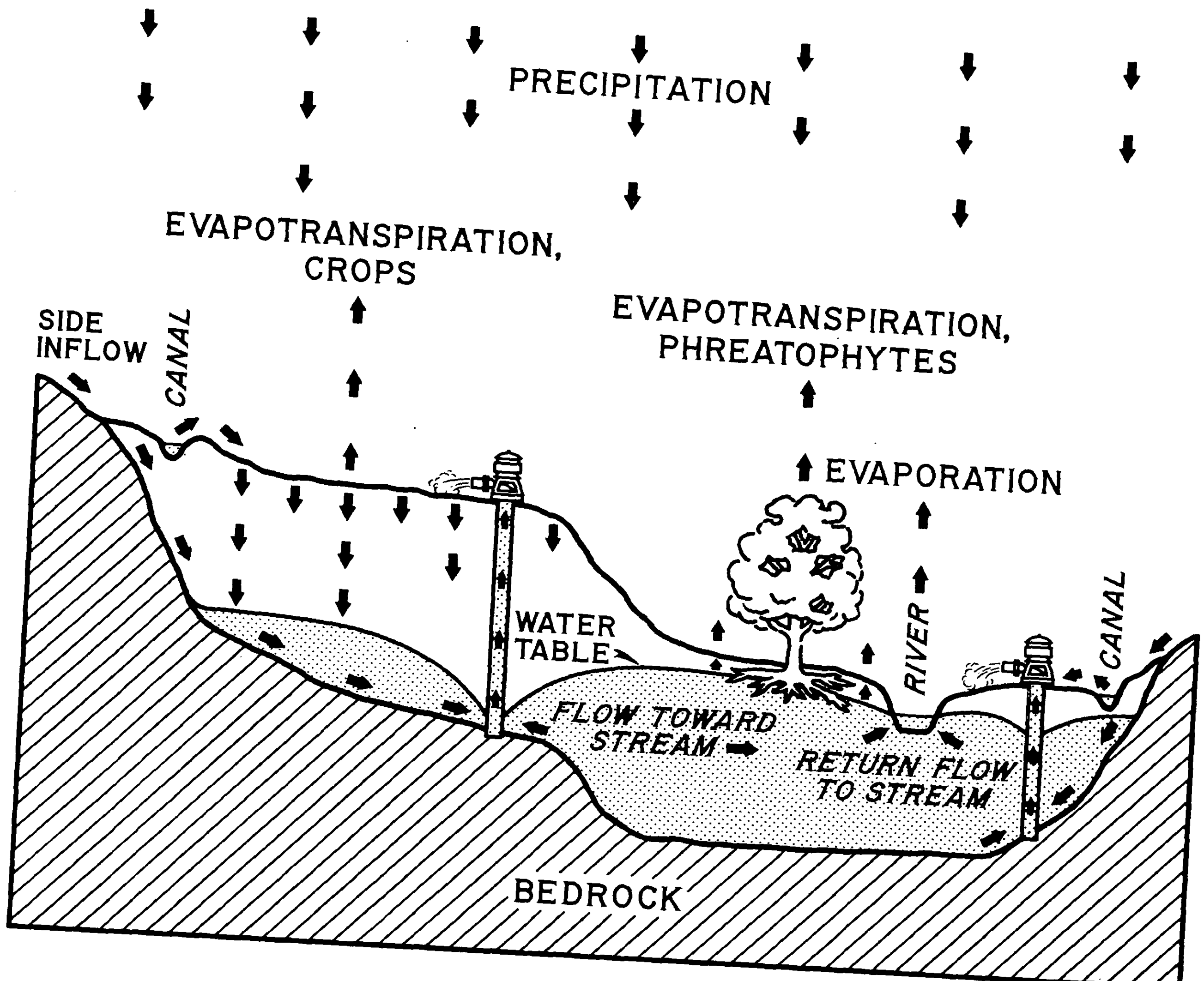


Figure 1. --Idealized section of a typical stream-aquifer system.

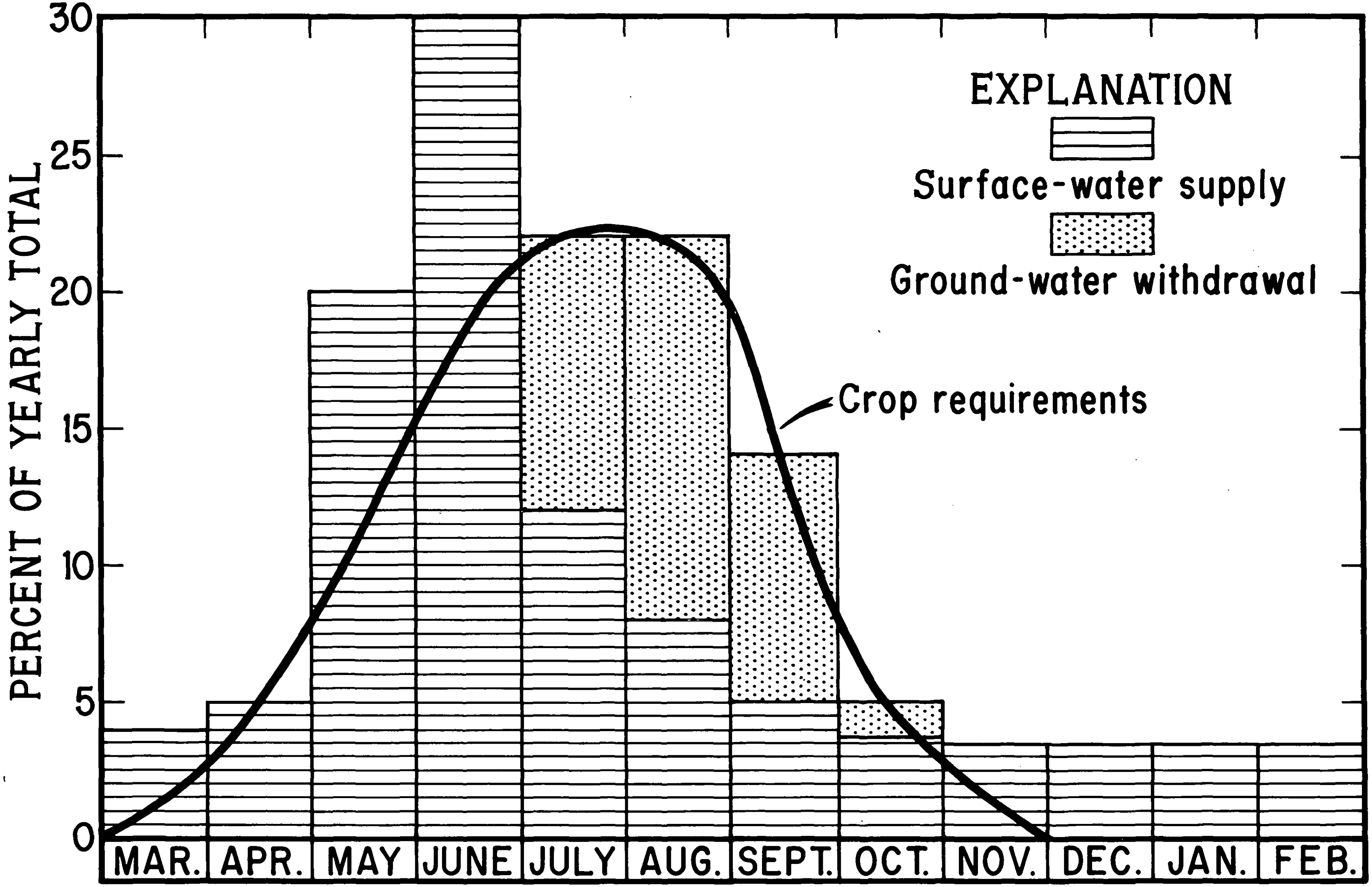


Figure 2. --Crop requirements, surface-water supply, and ground-water withdrawal in an irrigated valley.

Colorado water law is based on the appropriation doctrine, that is, first in time, first in right. Both surface and ground water are subject to appropriation under this doctrine where ground water is considered tributary to a stream. Ground-water withdrawals have increased consumptive use thereby reducing surface-water supplies available to holders of surface-water rights. Ground-water appropriations are junior (later in appropriation) to surface-water decrees. If the appropriation doctrine were rigidly applied, ground-water use would be severely curtailed, which would lead to inefficient use of the resource. To compromise the objectives of administering water rights and obtaining efficient use of the water, complex water-management plans must be devised. Comprehensive hydrologic studies of conjunctive use may show how laws should be modified to permit maximum utilization of the available water.

The analog model is particularly well suited for detailed quantitative descriptions of the hydrologic system. The latest digital computers also permit detailed modeling of the system, but programs become increasingly unwieldy and expensive as the number of node points increases. Both models are unwieldy and expensive to operate where many repetitive calculations are required as is the case when studying water-management plans. A technique developed for summarizing the data from a detailed hydrologic model (Jenkins, 1968b) permits construction of a simplified model that incorporates much of the accuracy of the more detailed models. By summarizing the analog results, the number of nodes used in digital computer programs can be reduced by as much as two orders of magnitude thus substantially reducing computer time. Computer time is further reduced because calculations for the simplified model use equations that are furnished routines rather than the simultaneous solution of many differential equations.

A study by the U.S. Geological Survey in cooperation with the Colorado Water Conservation Board and the Southeastern Colorado Water Conservancy District is being made to develop tools for predicting the effects of changes in water management and to optimize management objectives.

The steps in developing predictive tools are:

1. Collect data to describe the hydrologic system and construct and verify an analog model (Moore and Wood, 1967).
2. Summarize the analog model results (Jenkins, 1968b).
3. Construct a simplified digital model for testing various water-management plans.
4. Develop digital programs for optimizing certain water-management objectives.

The analog model uses a resistor-capacitor network to simulate the hydrologic system, electrical waveform and pulse generators to simulate the hydrologic input, and an oscilloscope to read the output. The model is then verified by comparing model results with field observations. The purpose of this report is to describe the transition steps between the analog and digital models, the design of the digital model, and the optimization programs.

DEFINITIONS

Definitions of Terms

erfc = the complementary error function.

i^2 erfc = the second repeated integral of the error function.

Stress = withdrawal from or recharge to the ground-water reservoir, [L^3 .]

Definitions of Symbols

Symbols for the dimensions time and length are in Roman capitals and enclosed in brackets. All other symbols are in italics. The symbols used are defined as follows:

a = the distance from the point of stress to the stream, [L].

T = the transmissivity of the aquifer, [L^2/T].

S = the specific yield of the aquifer, dimensionless.

Q = the average net rate of stress, [L^3/T].

q = the rate of the effect of the stress on the stream, [L^3/T].

v = the volume of the effect of the stress on the stream, [L^3].

t = the time since the stress began, [T].

sdf = the stream depletion factor, [T], which is the time for a steady stress to affect the stream by 28 percent of the accumulated stress (see Jenkins, 1968a, 1968b).

COMPARISON OF IDEALIZED RELATIONS WITH ANALOG-MODEL RESPONSE

The effects on streamflow of adding or subtracting water at a point within an idealized aquifer have been described mathematically by several investigators (see Jenkins, 1968a). The curves and equations are shown in Figure 3. The relations for a real system can be obtained by stressing an analog model of that system (Jenkins, 1968b). The departures between the mathematical curves and the model curves represent differences between the idealized assumptions of the mathematical model and the truer assumptions incorporated in the analog model. The added conditions accounted for in the analog model are variations in transmissivity, specific yield, sinuosity of the stream, and the irregular impermeable boundaries.

The horizontal axes of the response curves from the analog model were shifted to obtain a reasonably good fit to the idealized curve. The shifting was based on the concept of what Jenkins (1968a, 1968b) has called the stream depletion factor, or *sdf*, which is the time coordinate of the analog response curve when, for a constant pumping rate, the volume of stream depletion is 28 percent of the total volume pumped. In the idealized system, the stream depletion factor equals a^2S/T, and v/Qt is 28 percent when tT/a^2S is 1.0. The response curve from the model relates t on the abscissa to v as the ordinate. With the aid of a carefully constructed template, the point on the curve where $v/Qt = 0.28$ is determined. The time coordinate of this point is the *sdf* of the location tested. Figure 4 shows comparisons between two shifted response curves from the model of the Arkansas River valley in southeastern Colorado and the idealized relation. More than two-thirds of the response curves from the model had departures less than those shown in Figure 4.

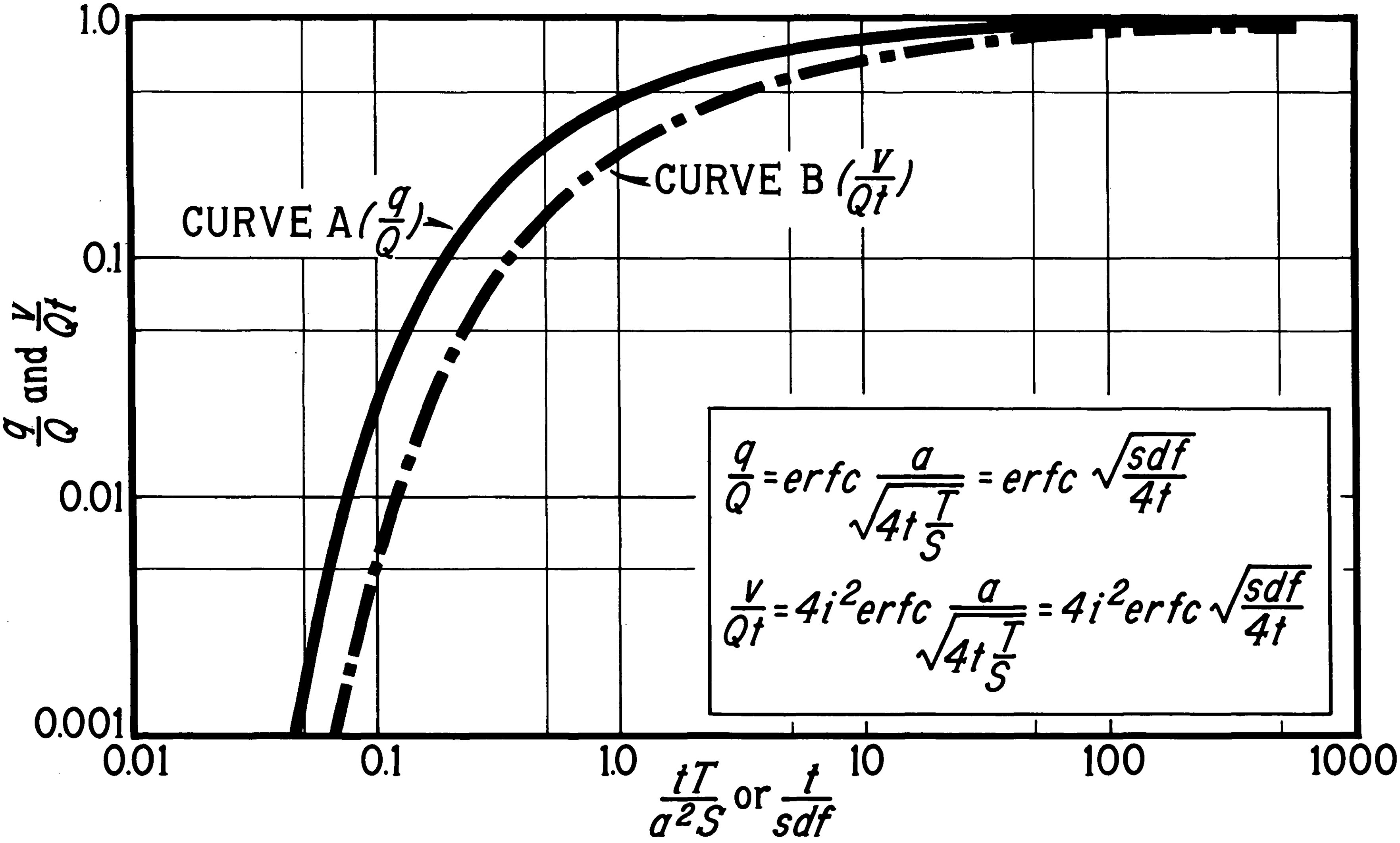
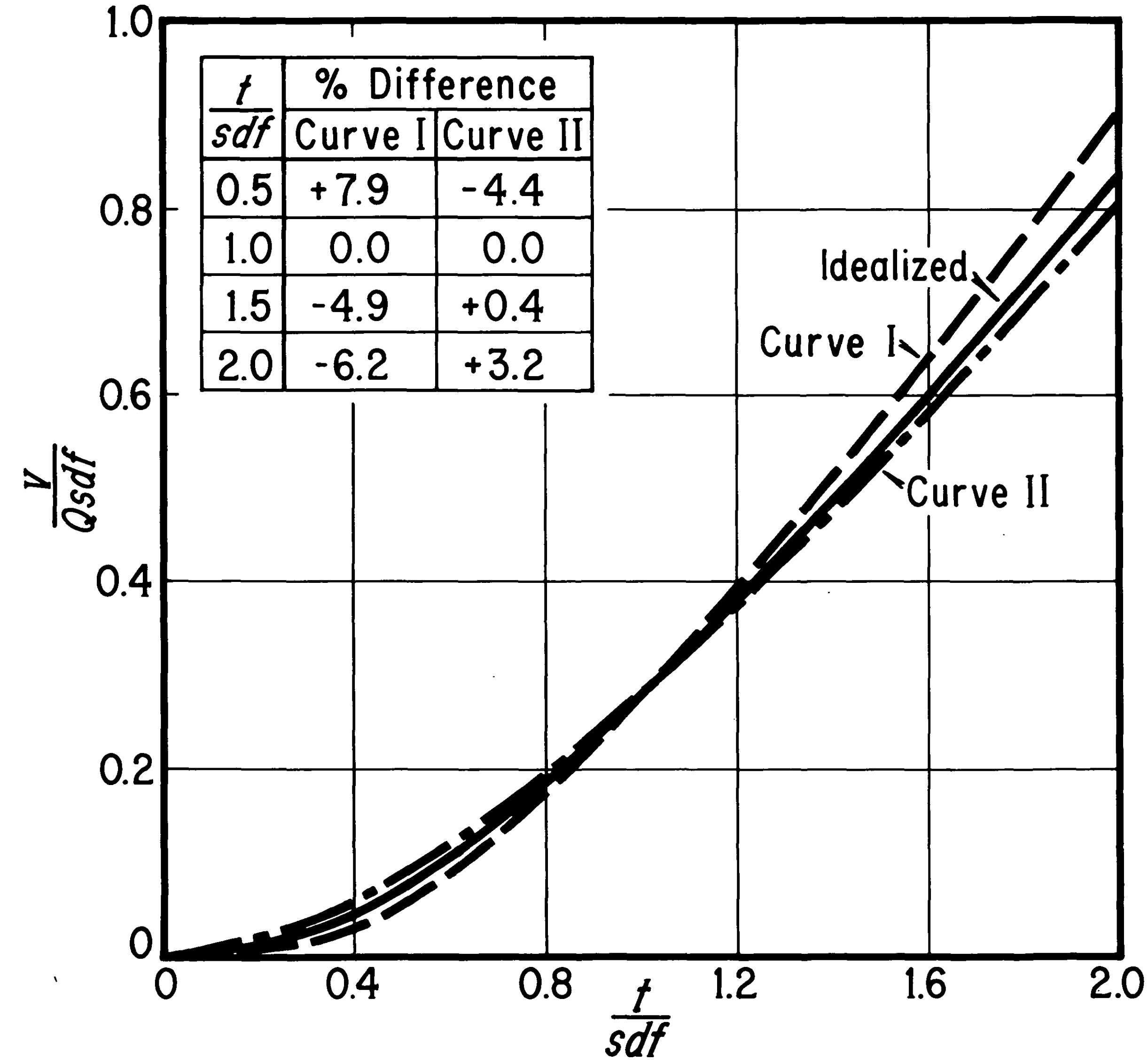


Figure 3. --Curves showing effects of aquifer stress on streamflow.



$\frac{t}{sdf}$	% Difference Curve I	% Difference Curve II
0.5	+7.9	-4.4
1.0	0.0	0.0
1.5	-4.9	+0.4
2.0	-6.2	+3.2

Figure 4. --Departures of two analog-model curves from the idealized curve.

SUMMARIZING ANALOG MODEL RESULTS

If one accepts the departures from the idealized relation as tolerable, a new description of the hydrologic system can be shown on a map. Such a map was prepared for the Arkansas River valley in southeastern Colorado by stressing the analog model at 266 points and obtaining the value of the *sdf*. Lines were constructed connecting points of equal *sdf*.

Figure 5 shows a map of a segment of a hypothetical valley similar to the Arkansas River valley. At the scale of the Arkansas River valley analog model, the grids would have the spacing shown. Figure 6 shows the same segment of the system with lines of equal *sdf*. The lines connect points where hydrologic stresses on the aquifer have identical proportionate effects on the stream. For example, a constant ground-water withdrawal would reduce the stream volume by 28 percent of the volume withdrawn after being imposed for 30 days anywhere along the line nearest the river, after 120 days anywhere along the second line from the river, and so on. The reach is divided into four *sdf* bands. If stresses are uniform over each band, they can be represented as a single stress at a single point, insofar as effect on the stream is concerned. The mean *sdf* value of each band is shown in Figure 6.

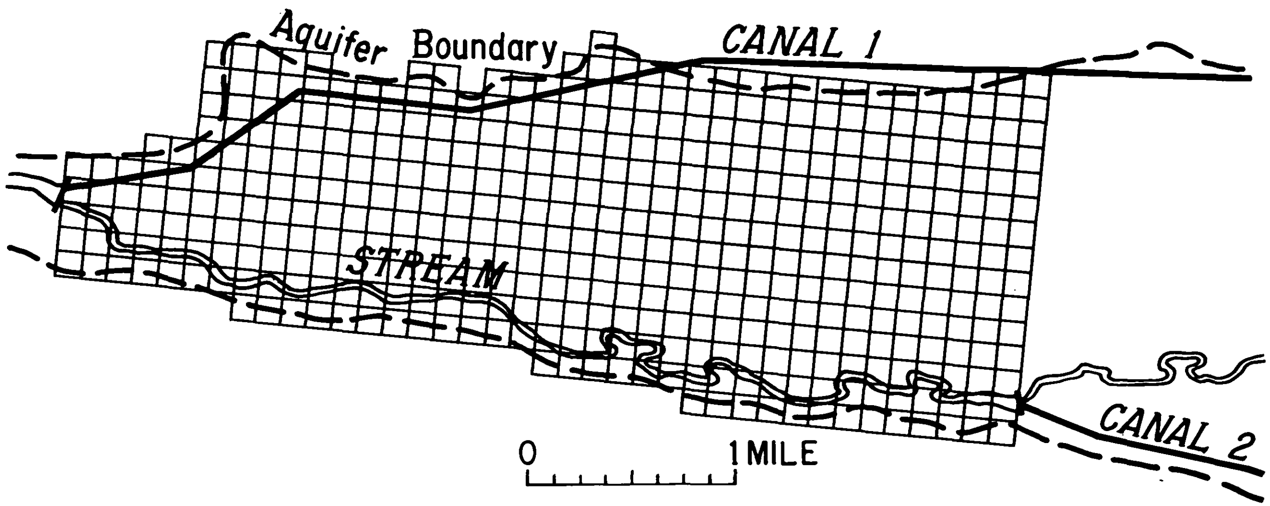


Figure 5. --Hypothetical stream-aquifer system showing analog grid network.

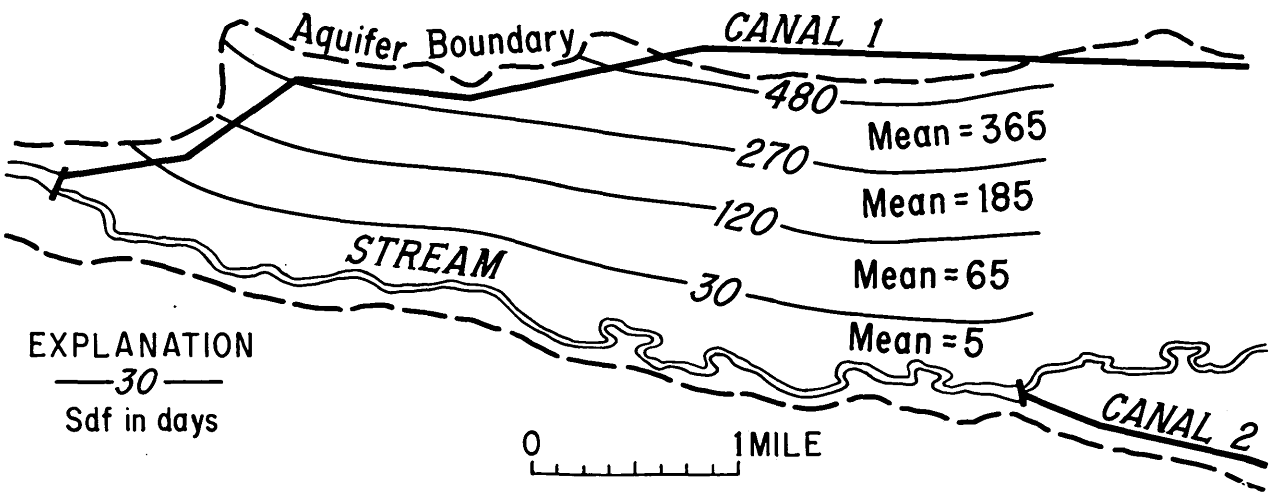


Figure 6. --Hypothetical stream-aquifer system showing lines of equal stream depletion factors.

This map provides a basis for constructing a simplified model of the system. Instead of the 400-node grid used for the analog model, the new model uses a grid consisting of only four bands along the stream for one reach of valley. Figure 6 shows only the single reach, but the model of the entire system consists of many reaches, which generally correspond to reaches between surface diversion points.

ANALYSIS OF A WATER-MANAGEMENT PLAN

The simplified model is designed to predict the availability of surface water at successive diversion points downstream and to show the changes in ground-water storage in each reach. The stress on each cell shown on the map is assumed to be uniformly distributed, thus mean values of the *sdf* can be used. Changes in streamflow caused by the stresses on the various cells are calculated from a simple program on the digital computer using the classical equations shown in Figure 3 (see Jenkins, 1968a). Changes in storage are calculated as the difference between the accumulated stress and the effect on the stream. After calibrating this digital model with historical records of streamflow, diversions, pumpage, change in ground-water storage, and rainfall, the model is ready to predict the results of any proposed changes in water-management practices.

A very simplified analysis is shown in Figure 7. The only stresses on the system are pumping and recharge. In the first analysis, (Case 1, Figure 7), pumping is uniformly distributed within the aquifer area as indicated by the well locations. The streamflow passing the headgate of Canal 1 is steady at 50 cfs (cubic feet per second). The resultant streamflow at the headgate of Canal 2 and the change in ground-water storage are shown by the solid lines (Figure 7). Note that the stream is practically dry twice during the year. If Canal 2 has a senior right of 20 cfs, some alteration in the pattern of use is necessary to supply this right.

If manipulation of the pattern of withdrawal is possible, the same amount of water could be withdrawn from the outer *sdf* band. The resulting hydrographs are shown by the dashed lines (Case 2, Figure 7). Now the minimum streamflow at the headgate of Canal 2 is about 20 cfs, which meets the water-right requirement. The hydrographs can be altered also by changing the timing of stresses. Changes in either time or space, or both in combination, can be analyzed to achieve specific objectives.

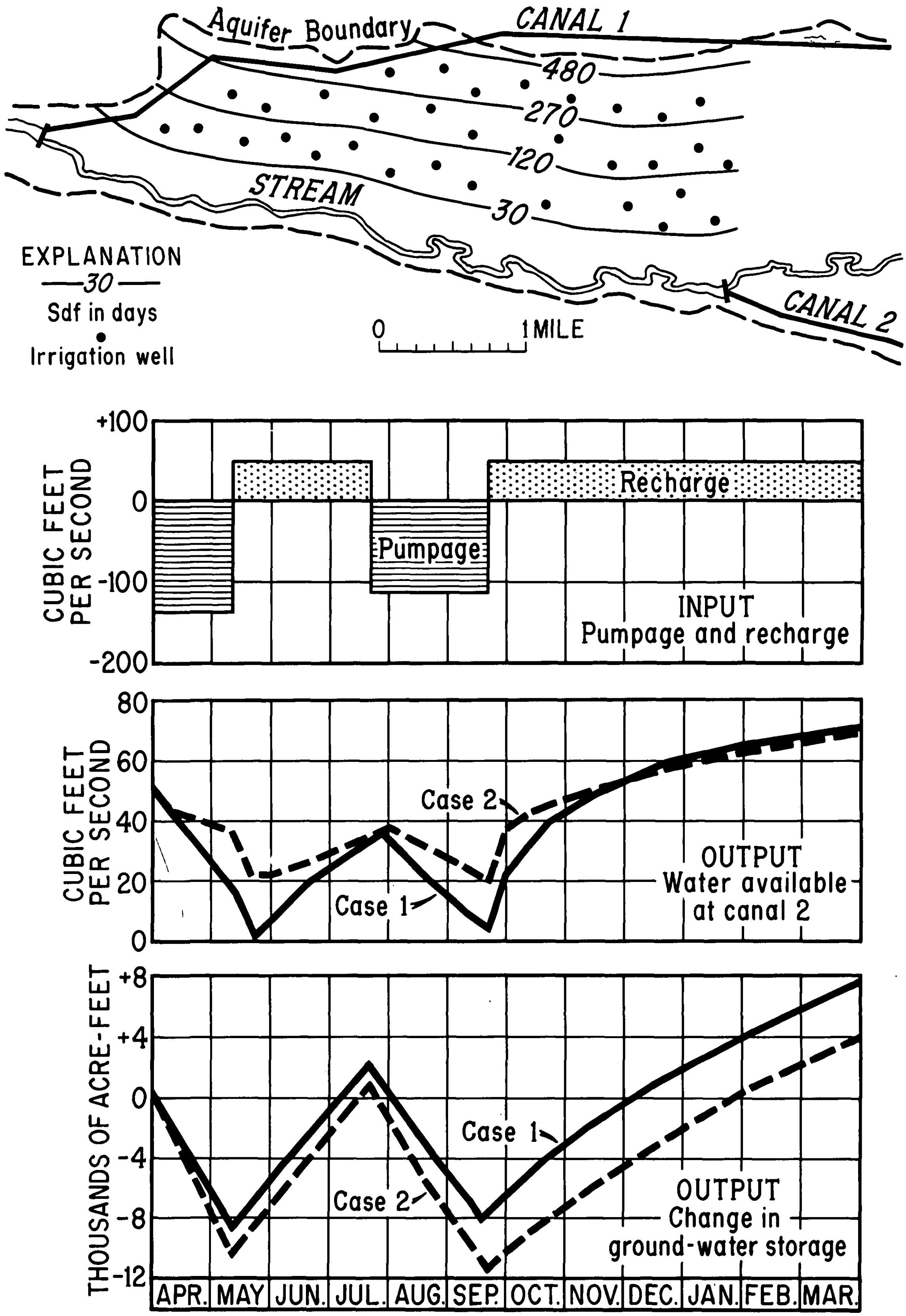


Figure 7. --Diagrams showing the results of a digital computer analysis of a hypothetical stream-aquifer system.

Data are being compiled to use in a program to analyze the Arkansas River valley in southeastern Colorado, shown schematically in Figure 8. The valley has been divided into 26 reaches with as many as 20 *sdf* bands per reach. The amount of computer time necessary for an analysis of such a complex system is not known. However, the calculations for Figure 7 took less than a minute on an IBM 360/65 computer, and an analysis of 10 years of record for a 4-reach segment of the Arkansas River valley required less than 2 minutes.

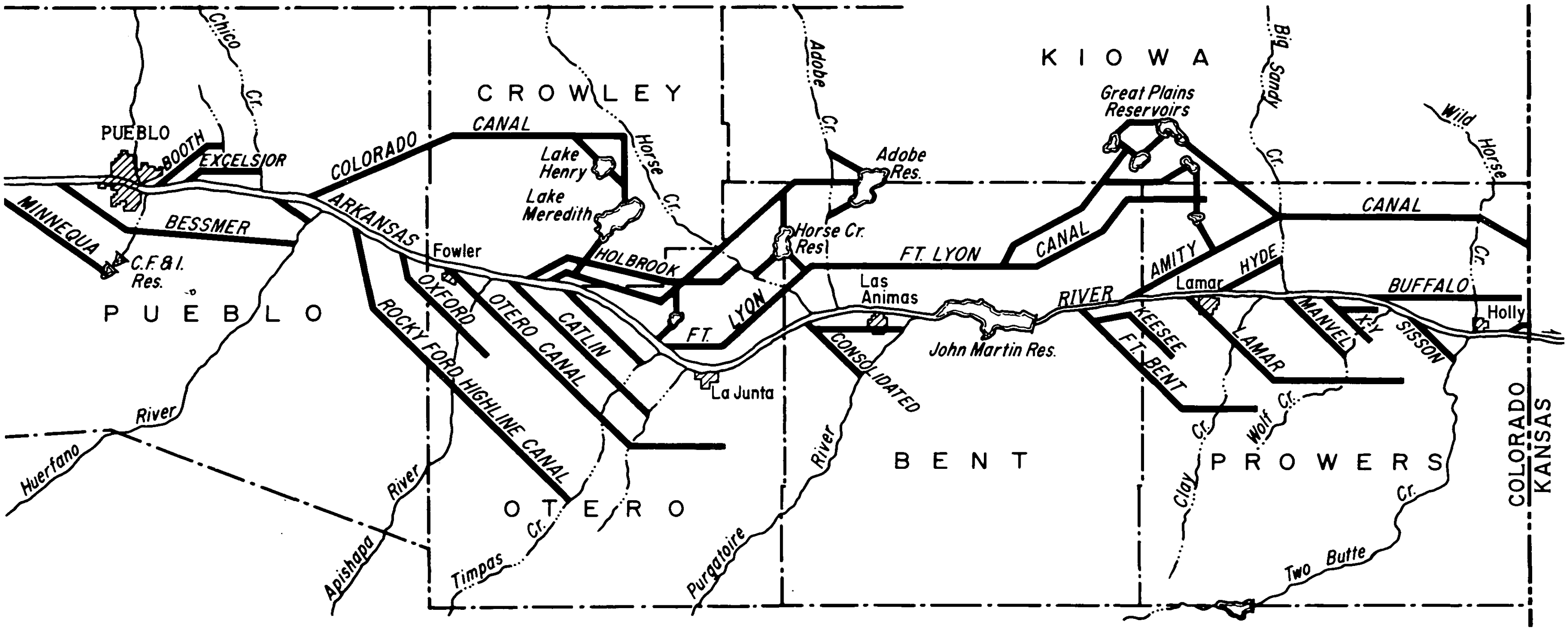


Figure 8. --Schematic map showing the surface-water distribution system in the Arkansas River valley in Colorado

OPTIMIZING MANAGEMENT PLANS

The very simple example shown in Figure 7 illustrates how withdrawals from the ground-water reservoir can be manipulated to regulate the supply. The administrator might choose as his objective function maintaining total diversions (surface water plus ground water) as nearly constant as possible from year to year, allowing irrigators to plan operations on a firm basis. His decision variables would be when, where, and how much to pump or recharge. Ideally one would use the part of the ground-water reservoir most distant from the stream to achieve the objective. Ground-water storage would be reduced during dry years and replenished during wet years. However, in a real system, many constraints modify ideal operation. Aside from legal questions concerning water rights, the system itself presents constraints. The storage capacity of the aquifer near the edges of the valley may be too small for complete regulation; pumping in the vicinity of the stream may be necessary to prevent waterlogging; deterioration of water quality may limit use; there are many others. U.S. Geological Survey specialists in the fields of Operations Research and Systems Analysis are developing a digital program that will provide plans of optimum water use within the legal and physical constraints. One program being tested is to furnish water to ditches in the order of priority of water rights, maximizing the number of water rights served. Optimum deliveries depend on achieving maximum use of ground water and surface water during the irrigation season.

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