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SURFACE WATER TECHNIQUES

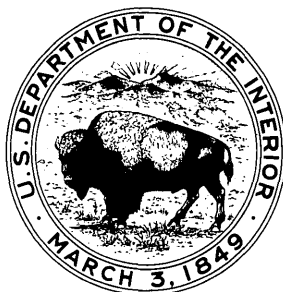
Use of Flumes in Measuring Discharge at Gaging Stations

HYDRAULIC MEASUREMENT
AND COMPUTATION

BOOK 1

CHAPTER 16

1965



UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY



SURFACE WATER TECHNIQUES

Use of Flumes in Measuring Discharge at Gaging Stations

Book 1—Hydraulic Measurement and Computation

**Chapter 16—Use of Flumes in Measuring Discharge
at Gaging Stations**

SURFACE WATER TECHNIQUES SERIES

Reports in this series that have been issued to date are listed below. An asterisk indicates the report has been published and is for sale by the Superintendent of Documents.

- Book 1, Chapter 1, Computation of Water-Surface Profiles in Open Channels, 1964
- Book 1, Chapter 1, Supplement No. 1, Backwater at Bridges, 1964
- Book 1, Chapter 2, Computation of Discharge in Tidal Reaches, 1964
- Book 1, Chapter 8, Computation of Stage-Discharge Relationships at Culverts, 1965
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- Book 1, Chapter 11, Discharge Measurements at Gaging Stations, 1965
- Book 1, Chapter 12, Discharge Ratings at Gaging Stations, 1965
- Book 1, Chapter 14, Measurement of Discharge by Dye-Dilution Methods, 1965
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- Book 1, Chapter 16, Use of Flumes in Measuring Discharge at Gaging Stations, 1965

- Book 2, Chapter 1, Storage Analyses for Water Supply, 1964

November 1965

PREFACE

The primary purpose of "Surface Water Techniques" is to provide members of the Surface Water Branch with information on technical procedures which will assist and guide them in planning and executing specialized work. The material is grouped under four main subject headings, designated as "Books," as follows:

- Book 1. Hydraulic measurement and computation
- Book 2. Hydrologic analysis
- Book 3. Instruments and equipment
- Book 4. Design of investigations

Subject matter is grouped into books and chapters with the chapter being the unit of publication and also of revision. Techniques change from time to time, thus publication by chapters permits ready revision when appropriate. Instructions contained in the chapters as initially released are considered provisional and subject to revision because of experience in use or because of advancement in knowledge, techniques or equipment.

Judgment must be used in deciding how closely to adhere to instructions. Instructions which include information on preparation of data for use in a computer must be closely followed. Instructions on methods of analysis are generally less binding and not to be utilized to the extent of inhibiting initiative or stifling progressive development. However, before using a technique which differs substantially from one which has been recommended, it should be discussed with the office of the Branch Chief.

When a technique has been sufficiently developed a new edition will be issued in similar format but in a more permanent form. Publication will be announced in "New publications of the Geological Survey" and the report will be for sale by the superintendent of Documents, but will have no automatic free distribution outside of the Survey.



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USE OF FLUMES IN MEASURING DISCHARGE AT GAGING STATIONS

By F. A. Kilpatrick

ABSTRACT

This report discusses the general principles underlying the design of various types of flumes for measuring open-channel flow. Four flumes are described in detail, and the recommended discharge ratings for each are presented.

INTRODUCTION

The use of flumes as open channel flow meters began shortly after the turn of the century. The flumes commonly utilize a contraction in channel width and a steepening of channel slope to produce critical or supercritical flow in the throat of the flume. The relation between head, or stage, and discharge is thus a function only of the characteristics of the flume and can be determined prior to installation.

Flumes have a limited but important use in gaging open-channel flow. They are used primarily as controls on small, flashy streams where a precalibration device is essential. Portable flumes are also used in making discharge measurements at sites where the use of the current meter is not feasible. Flumes cost more than any other type of control and for this reason they have not been used extensively as controls at gaging stations. The initial cost, however, is in some cases offset by the precalibrated stage-discharge relation. Equally important is the self-cleaning feature of most flumes which is why they are used principally in the Western States, where sediment is a factor in the stability of the stage-discharge relation.

The general principles which govern the design of various types of flumes are discussed in this report. Four flumes are described in detail and the discharge ratings for each are presented. A flume may be designed for a given site from the information given in the report.

PRINCIPLES GOVERNING THE DESIGN OF FLUMES

Hydraulic contractions and transitions may best be analyzed by the use of specific energy principles. The specific energy diagram of figure 1 defines for a rectangular channel the relationships between depth of flow d , and specific energy E , for various unit discharges, q . Specific energy is the energy level with reference to the streambed at a particular point. Thus a given increase in streambed elevation results in a decrease in specific energy of the same magnitude.

Specific energy is defined as

$$E = d + V^2/2g \quad (1)$$

and, if unit discharge, the discharge per foot of width is used, equation (1) becomes

$$E = d + q^2/2gd^2. \quad (2)$$

Evaluation of this equation yields the family of constant " q " curves which are asymptotic to a 45° line. The points lying on these curves and representing the minimum specific energy for a rectangular channel are uniquely defined by the equation

$$d_c = \sqrt[3]{q^2/g} \quad (3)$$

This equation defines the line of critical depths as shown in figure 1. At this critical depth, there is the unique relationship in which the velocity head is exactly half the depth of flow. Flow conditions more commonly found in rivers and streams are tranquil or subcritical and are represented by the curves above the line of critical depth. In this region depths are large, and velocities and velocity heads are relatively small. Conversely, in the supercritical flow region below the critical-depth line, depths are small, and velocities and velocity heads quite large.

Six methods employed in various flume designs, as illustrated in figures 2-7, will be discussed using the specific energy diagram of figure 1. Application of specific energy principles to abrupt contractions and short channels is not entirely correct due to accelerative and curvilinear flows. However it is the concept that is of interest here and not an exact analysis.

Type I, Tranquil Flow, Small Width Reduction

The earliest measuring or rating flumes are exemplified by figure 2 which shows subcritical flow entering a flume with side contractions. The side contractions reduce the width of the flume which results in an increase in unit discharge.

Because there is no change in bed elevation, and minor energy loss, the specific energy in the throat is about the same as in the approach. With constant specific energy, the effect of a small width contraction is a lowering of the water surface in the throat. In the example shown in figure 2, the side contraction between point A and point B causes a change in the discharge per unit width. The transition is illustrated in figure 1, as the point A on the curve $q = 1$, and the point B on the curve $q = 2$. Owing to the small degree of contraction, critical depth is not accomplished (point C on curve $q = 3$, in figure 1). It is necessary in this type of flume to measure the head in both the approach section and in the throat. For this reason, a subcritical flow meter of this type is seldom used today.

Type II, Critical Flow, Large Width Reduction

Further narrowing of the throat width, as before, results in increasing the unit discharge until a critical width is reached. (See figure 3.) This width corresponds to point "C" on figure 1 and represents the minimum specific energy which exists at the critical-flow depth. Earlier flume designs were based on measuring this depth in the throat because of the unique critical-depth: discharge relationship of equation 3.

The discharge equations for flumes conform closely to this relationship, but it can be seen that depths in the vicinity of critical flow can change radically with little change in discharge. Thus, flow close to critical is very unstable, constantly attempting to become either subcritical or supercritical.

In both type I and type II controls, the flume slope may be zero or nearly zero owing to the relatively small energy losses experienced.

In a type II control, stage may be measured at either of two locations, in the immediate approach to the flume or in the throat. Measurement in the approach will yield a more sensitive stage-discharge relationship because changes in discharge will result in greater changes in depth in subcritical flow than would like changes in discharges in critical flow. Unfortunately, the stage-discharge relationship in the approach may be unstable owing to approach conditions such as scour and fill. Consequently, stage is usually measured in the throat to alleviate influence from either upstream or downstream. Approach conditions can have some influence on flow in the throat, but it is generally insignificant. The location at which critical depth is first reached may shift further downstream into the throat as a result of excessive deposition in the approach. For this reason, and to avoid possible flow separations near the entrance, stage measurements in the throat should not be too close to the entrance.

A type II control, properly called a critical-depth meter, has the advantage of requiring measurement at only one location. It has the disadvantage that free overfall is required for best operation. Measurement upstream is not entirely satisfactory because

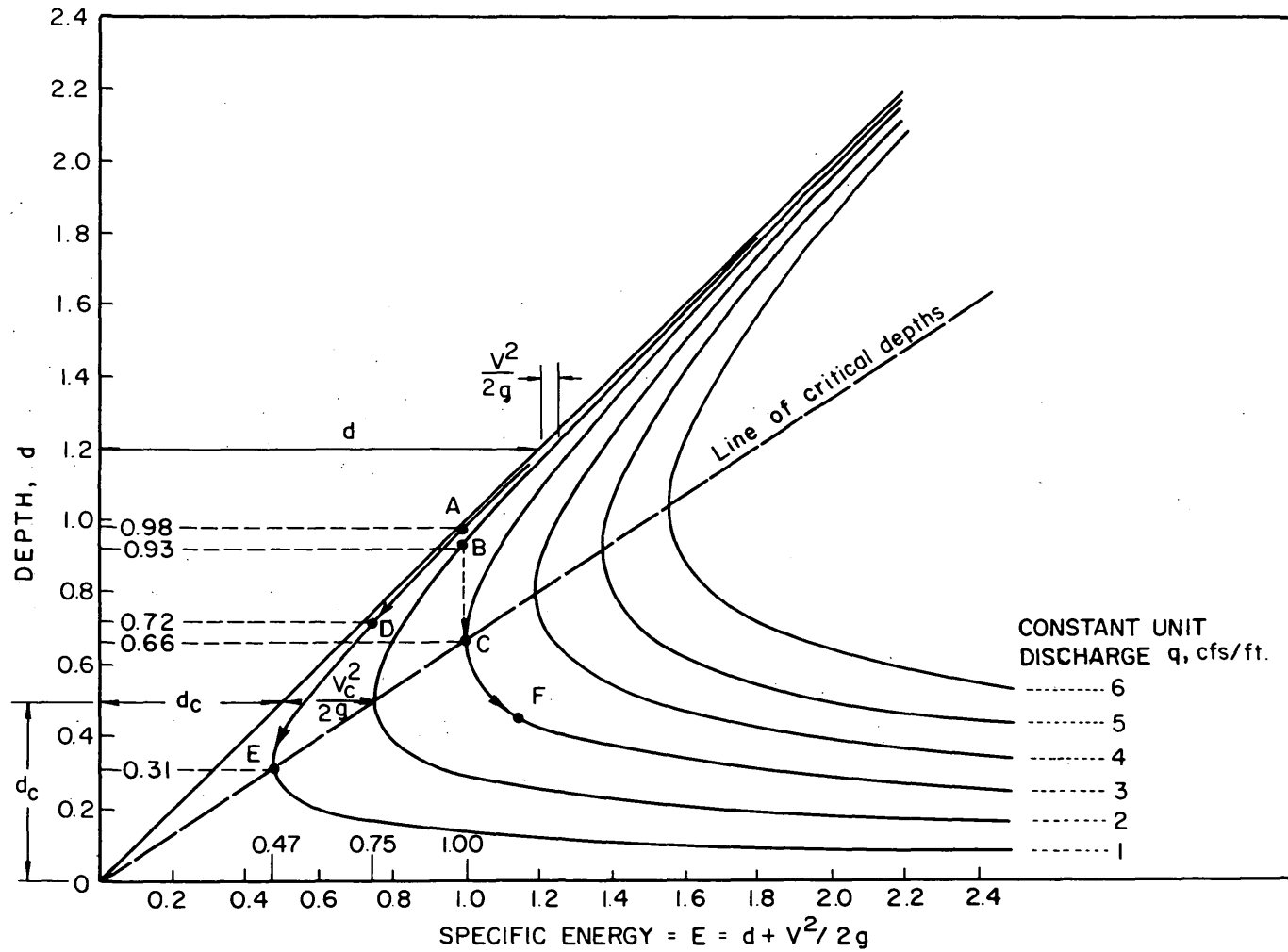


Figure 1. --Specific energy diagram for rectangular channel.

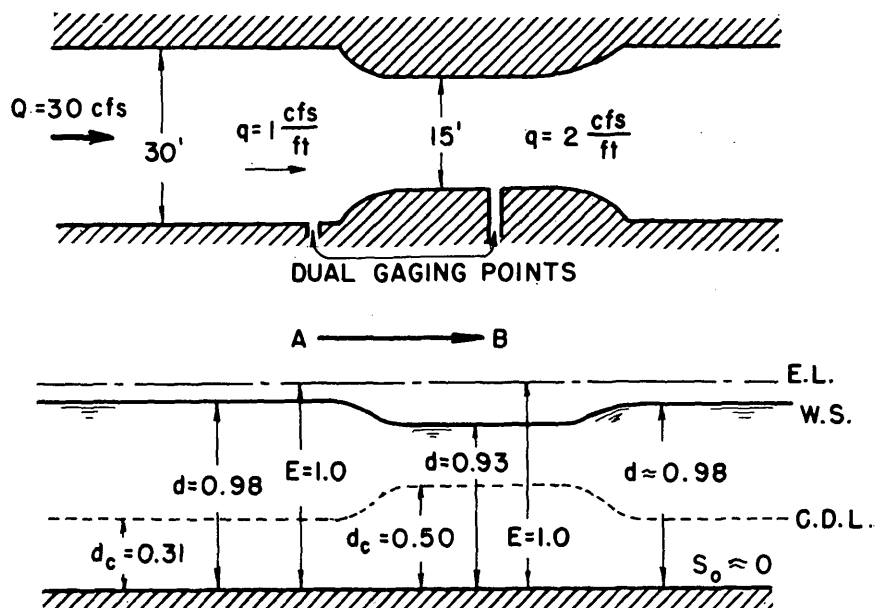


Figure 2. --Type I control, subcritical flow contraction obtained by small width reduction, horizontal bed.

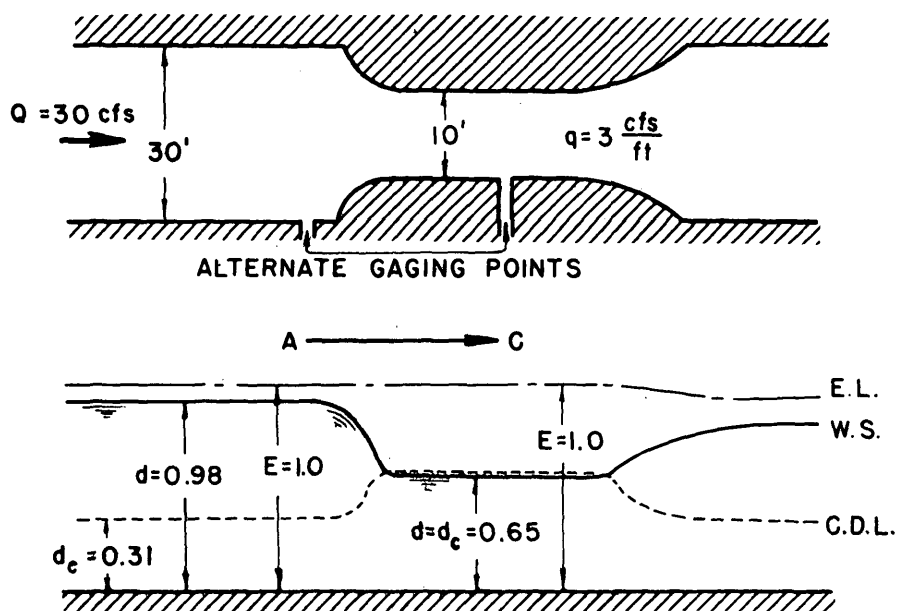


Figure 3. --Type II control, critical flow contraction obtained by large width reduction, horizontal bed.

of possible approach influences, nor is it satisfactory in the throat because of widely fluctuating water surfaces. As will be shown subsequently, much is to be gained by placing such a flume on a slope greater than critical.

Type III, Tranquil Flow,
Small Increase in Bed Elevation

Types I and II controls represented methods of obtaining measuring flumes by contracting the flow using width reductions. In these cases, as can be seen on the specific energy diagram in figure 1, the specific energy, E , is essentially constant from approach to throat. All changes in depths from approach to the throat are accomplished by going to successively larger q curves.

Flow conditions similar to those produced by the side contractions, as in types I and II, can be obtained by increasing the bed elevation. In the absence of side contractions, the unit discharges will not vary from approach to throat, but the specific energy, E , will change.

For a type III control with $E = 1.0$ in the approach and $q = 1.0$ throughout, the change in depth must be along a constant q curve. As illustrated in figure 4, this can only be effected by a reduction in specific energy. Hence, if the bed of the flume is arbitrarily raised 0.25 feet above the approach bed, the result is a direct reduction in E to 0.75 feet over the sill or to point "D" on figure 1.

This yields a depth of approximately 0.72 feet, which is still subcritical. Because q is the same in both approach and over the sill, d_c is 0.31 feet in both cases.

Raising the bed even further produces lower and lower depths across the sill until critical depth is reached at point "E" on figure 1. At this point where the specific energy is a minimum, $E = 0.47$ feet. Hence, a sill height of 0.53 feet is the critical height because a sill of greater height will produce increased stages upstream. A critical depth of 0.31 feet will exist at the sill.

Flumes which incorporate sills in their design are the least frequently used. Among the primary advantages of flumes as discharge meters are their self-cleaning characteristics. As might be expected, sills form a partial barrier to the approaching flow, which encourages deposition. Therefore, there would appear to be no advantage to flume designs incorporating sills or raised floors.

Type IV, Supercritical Flow,
Width Reduction, Steep Slope

When flumes are on approximately zero slope, as in types I, II, and III, critical depth is the minimum depth possible in the flume. When the flow in the throat reaches the critical discharge a critical contraction has been reached. Further contraction from the sides or the bottom or both will not produce supercritical flow.

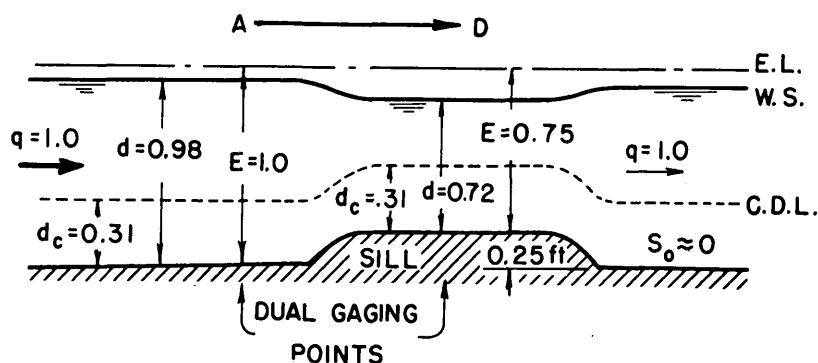


Figure 4. --Type III control, subcritical flow contraction obtained by small increase in bed elevation, horizontal bed.

The design of a flume with supercritical flow in the throat can be accomplished only by increasing the available specific energy from the approach into the throat. Whereas a rise in the flume bed decreased the specific energy, a drop in the flume bed or an increase in flume slope serves to increase the specific energy. Type IV control in figure 5, therefore, is identical with type II, but has been placed on a slope to supply the required increase in specific energy to produce supercritical flow in the throat.

Type V, Supercritical Flow, Width Reduction, Drop in Bed

Supercritical flow is obtained abruptly by dropping the bed as in type V (see figure 6). Thus, for a particular discharge the path "A-B-C-F" in figure 1 is followed; A, representing flow in the approach; A to C the effect of the side contraction or movement from one q -curve to successively higher ones; and C to F occurring because of increased specific energy provided by the slope or drop but no further contraction.

Properly, such flumes should be called supercritical-flow meters. As in the critical-flow meters, measurement of head is made either in the throat or the approach. The advantages and disadvantages of measuring in the approach have already been discussed. As previously emphasized, measurement of head in critical flow as at point "C", is undesirable since there may be large fluctuations in depth with little or no change in discharge. Therefore, head is customarily measured downstream of the point of critical depth in the region of supercritical flow. Measurement of head here may be difficult owing to the high velocities encountered under such conditions. As can be seen in figure 1, a particular disadvantage of measuring depths in this region is the lack of rating sensitivity compared with measurements in subcritical flow. The primary advantage of a supercritical-flow flume is that it has optimum self-cleaning and scouring characteristics. A discharge rating based on stage measurements in the region of supercritical flow is the least influenced by disturbances either upstream or downstream, and hence is apt to be the most stable. By the same token, such flumes are the most capable of stable operation up to high submergences.

Type VI, Supercritical Flow, Steep Slope

It should not be construed that contraction and increase in specific energy are both necessary for supercritical flow to occur. A sufficient increase in specific energy alone can produce supercritical flow. In the case of the ordinary stream-gaging control this is obtained simply by the drop created by the physical presence of the control.

As can be seen in figure 7, flow at supercritical depths can also be produced over a broad crest simply by giving it sufficient downstream slope.

A slope of 1 degree is usually sufficient to produce critical depth in the vicinity of the upstream edge of the apron, but waves and disturbances are apt to be numerous downstream. Such wave disturbances occur when flow across the apron is too close to critical and not well within the supercritical-flow range. On ordinary concrete aprons, slopes from $2\frac{1}{2}\%$ to 5% have been found to yield depths well within the supercritical-flow range.

For type VI control, if approach conditions were not subject to change, a stable discharge rating could be expected to exist by measuring stages in the subcritical region upstream. If accurate stage measurements could be made in the region of supercritical flow down on the sloping crest, a stable discharge rating would be obtained regardless of upstream or downstream disturbances. However, such a rating would be very insensitive. The addition of side contractions improves the rating sensitivity. Thus, the ideal flume is basically a broad-crested weir, flat or sloping, with side contractions.

THE PARSHALL FLUME

Development

The development of measuring flumes was instigated largely by the need for measuring irrigation flows. Prior to 1920 most flumes were similar to types I, II, III. The Venturi flume developed by V. M. Cone (1917), was the forerunner of the Parshall flume. It was similar to types I and II and required the measurement of stage both upstream and in the throat of the flume.

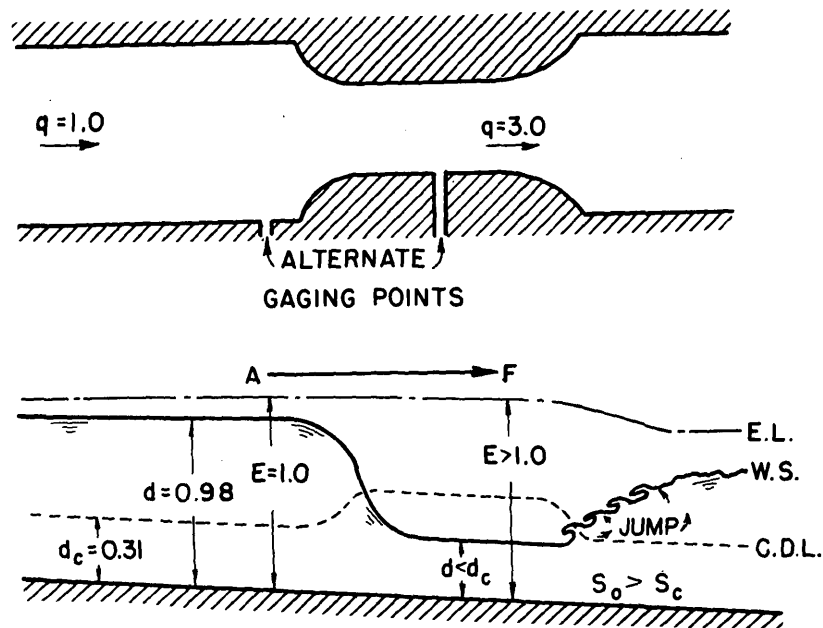


Figure 5. --Type IV control, supercritical flow contraction obtained by width reduction and sloping bed.

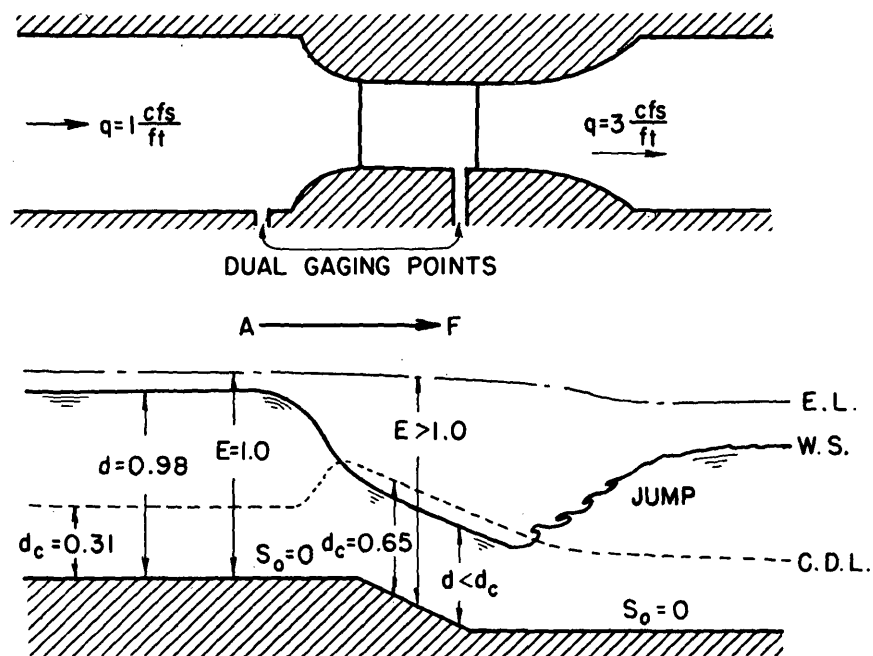


Figure 6. --Type V control, supercritical flow contraction obtained by width reduction and drop in bed.

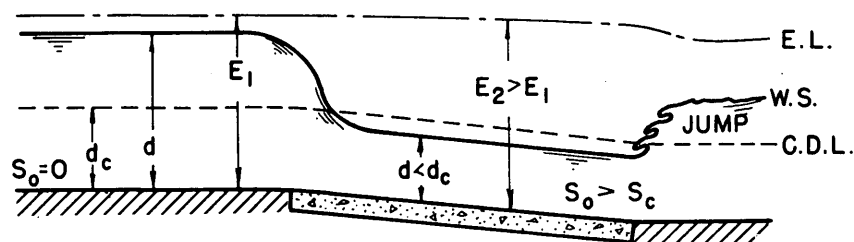


Figure 7. --Type VI control, supercritical flow contraction obtained by steepening slope.

In effect, the flume operated partially submerged much of the time.

R. L. Parshall (1926) proposed changes in the design of the Venturi flume, the most essential of which was a drop in the floor. As depicted in figure 6, this drop supplied the necessary additional specific energy to produce supercritical flow through the throat.

The throat width of the earlier flumes ranged in size from 3 inches to 8 feet. Flumes with throat widths of 10 feet to 50

feet were later constructed and field calibrated. More recently Parshall flumes of 1- and 2-inch sizes were calibrated by Robinson (1957). Head-discharge ratings are thus available for a large range in throat width.

Flume Configurations and Dimensions

Figure 8 shows the configuration of the Parshall flumes and table 1 gives the dimensions of all sizes. Flumes with throat widths from 3 inches to 8 feet contain a rounded

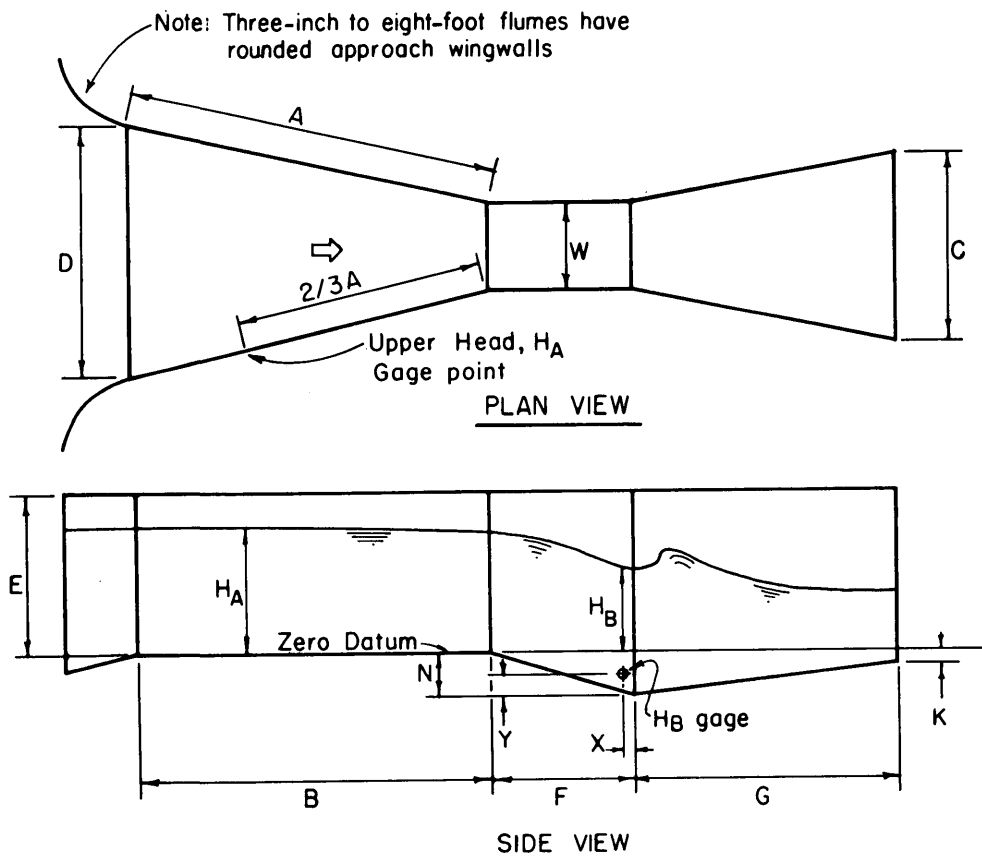


Figure 8. --Configuration and descriptive nomenclature for Parshall flumes.

entrance with the entrance floor on a slope of 25 percent.

The smaller and larger flumes do not have this feature, and it is doubtful whether any of the flumes would be materially affected by its absence as long as approach conditions are not unusual. Parshall flumes have provision for stage measurements both in the approach and in the throat, but the downstream gage is required only when submerged-flow conditions exist. The datum for both gages is the level floor in the approach. The raised floor, length G , in the downstream diverging sections is designed to reduce scour downstream and to produce more consistent discharge relationships under conditions of submergence. The percent submergence for Parshall flumes is defined as

$$\frac{H_B}{H_A} \times 100$$

with both stages referenced to common datum. Where free-fall conditions exist for all flows, the downstream gage, H_B , may be omitted and the entire diverging section left off if desired. This simplification has been used in the design of small portable Parshall measuring flumes.

Discharge Ratings

Figure 9 contains the discharge rating curves for the 2-inch through 9-inch size Parshall flumes both for free flow and for different degrees of submergence.

Table 2 gives the free-flow discharge ratings for flumes 1 to 50 feet in size. Correction factors for computing the effect of submergence for these sizes are given in figure 10. The discharge under submerged conditions is equal to the free-fall discharge minus the product of two correction factors, Q_c and k_s . Thus,

$$Q_s = Q_f - k_s Q_c$$

As can be seen in figures 9 and 10, greater submergences can be tolerated with increased flume size.

The Parshall flume was developed for use in irrigation systems and was designed to

operate with the minimum of head loss. Unfortunately, many of the flumes are placed to operate much of the time under conditions of submergence. Experience has shown the desirability of determining the discharge rating for each flume, by current-meter measurements, especially under conditions of submerged flow. Often flumes are set so low that an unstable submerged-flow discharge rating will exist because of ever-changing downstream backwater conditions. As a rule though, the Parshall flume has proven to be a satisfactory measuring device and yields entirely adequate results in measuring water for irrigation.

The original design of Parshall flumes has been maintained through the years for widths ranging from 1 inch to 50 feet. The rating results heretofore have been expressed in empirical form, either as tables or as equations. Davis (1963) has applied dimensional methods to these data to develop a semi-theoretical equation relating flow and depth for all sizes from 1 inch to 50 feet for unsubmerged flow. His equation, for flumes with side angles $\theta = \tan^{-1} 0.2$, and dropdown angles $\phi = \tan^{-1} 0.375$, is

$$Y_0 + \frac{Q_0^2}{2Y_0^2 (1 + 0.4X_0)^2} = 1.351 Q_0^{0.645},$$

in which

Y_0 = nondimensional depth, y_1/b

Q_0 = nondimensional discharge,
 $Q/g^{1/2} b^{5/2}$

X_0 = nondimensional distance, x/b

y_1 = depth at measuring section

b = channel width at throat

Q = discharge

g = acceleration of gravity

x = distance from throat crest to measuring section.

Although the equation is somewhat cumbersome, the excellent agreement between it and all published data should be useful in permitting the use of nonstandard sizes of Parshall flumes. It could also be used to

Table 1.--Dimensions and capacities of all sizes of standard Parshall measuring flumes.

Widths			Axial Lengths			Wall Depth in Con- verging Section E	Vertical distance below crest		Con- verging wall length A*	Gage Points			Free Flow Capacities	
Size; Throat width W	Upstream end D	Down- stream end C	Con- verging Section B	Throat Section F	Diverging Section G		Dip at Throat N	Lower end of flume K		H _A , dist. upstream of crest**	H _B		Min.	Max.
inches	feet	feet	feet	feet	feet	feet	feet	feet	feet	feet	x	y	cfs	cfs
1	0.549	0.305	1.17	0.250	0.67	0.5-0.75	0.094	0.062	1.19	0.79	0.026	0.042	0.005	0.15
2	.700	.443	1.33	.375	.83	0.50-0.83	.141	.073	1.36	.91	.052	.083	.01	.30
3	.849	.583	1.50	.500	1.00	1.00-2.00	.188	.083	1.53	1.02	.083	.125	.03	1.90
6	1.30	1.29	2.00	1.00	2.00	2.0	.375	.25	2.36	1.36	.167	.25	.05	3.90
9	1.88	1.25	2.83	1.00	1.50	2.5	.375	.25	2.88	1.93	.167	.25	.09	8.90
Feet														
1.0	2.77	2.00	4.41	2.0	3.0	3.0	.75	.25	4.50	3.00	.167	.25	.11	16.1
1.5	3.36	2.50	4.66	2.0	3.0	3.0	.75	.25	4.75	3.17	.167	.25	.15	24.6
2.0	3.96	3.00	4.91	2.0	3.0	3.0	.75	.25	5.00	3.33	.167	.25	.42	33.1
3.0	5.16	4.00	5.40	2.0	3.0	3.0	.75	.25	5.50	3.67	.167	.25	.61	50.4
4.0	6.35	5.00	5.88	2.0	3.0	3.0	.75	.25	6.00	4.00	.167	.25	1.30	67.9
5.0	7.55	6.00	6.38	2.0	3.0	3.0	.75	.25	6.50	4.33	.167	.25	1.60	85.6
6.0	8.75	7.00	6.86	2.0	3.0	3.0	.75	.25	7.0	4.67	.167	.25	2.60	103.5
7.0	9.95	8.00	7.35	2.0	3.0	3.0	.75	.25	7.5	5.0	.167	.25	3.00	121.4
8.0	11.15	9.00	7.84	2.0	3.0	3.0	.75	.25	8.0	5.33	.167	.25	3.50	139.5
10	15.60	12.00	14.0	3.0	6.0	4.0	1.12	.50	9.0	6.00			6	300
12	18.40	14.67	16.0	3.0	8.0	5.0	1.12	.50	10.0	6.67			8	520
15	25.0	18.33	25.0	4.0	10.0	6.0	1.50	.75	11.5	7.67			8	900
20	30.0	24.00	25.0	6.0	12.0	7.0	2.25	1.00	14.0	9.33			10	1340
25	35.0	29.33	25.0	6.0	13.0	7.0	2.25	1.00	16.5	11.00			15	1660
30	40.4	34.67	26.0	6.0	14.0	7.0	2.25	1.00	19.0	12.67			15	1990
40	50.8	45.33	27.0	6.0	16.0	7.0	2.25	1.00	24.0	16.00			20	2640
50	60.8	56.67	27.0	6.0	20.0	7.0	2.25	1.00	29.0	19.33			25	3280

* For sizes 1' to 8', $A = W/2 + 4$ ** H_A located 2/3 A distance from crest for all sizes; distance is wall length, not axial

Note: Flume sizes 3 inches through 8 feet have approach aprons rising at a 1:4 slope and the following entrance roundings: 3 through 9 inches, radius = 1.33 feet; 1 through 3 feet, radius = 1.67 feet; 4 through 8 feet, radius = 2.00 feet.

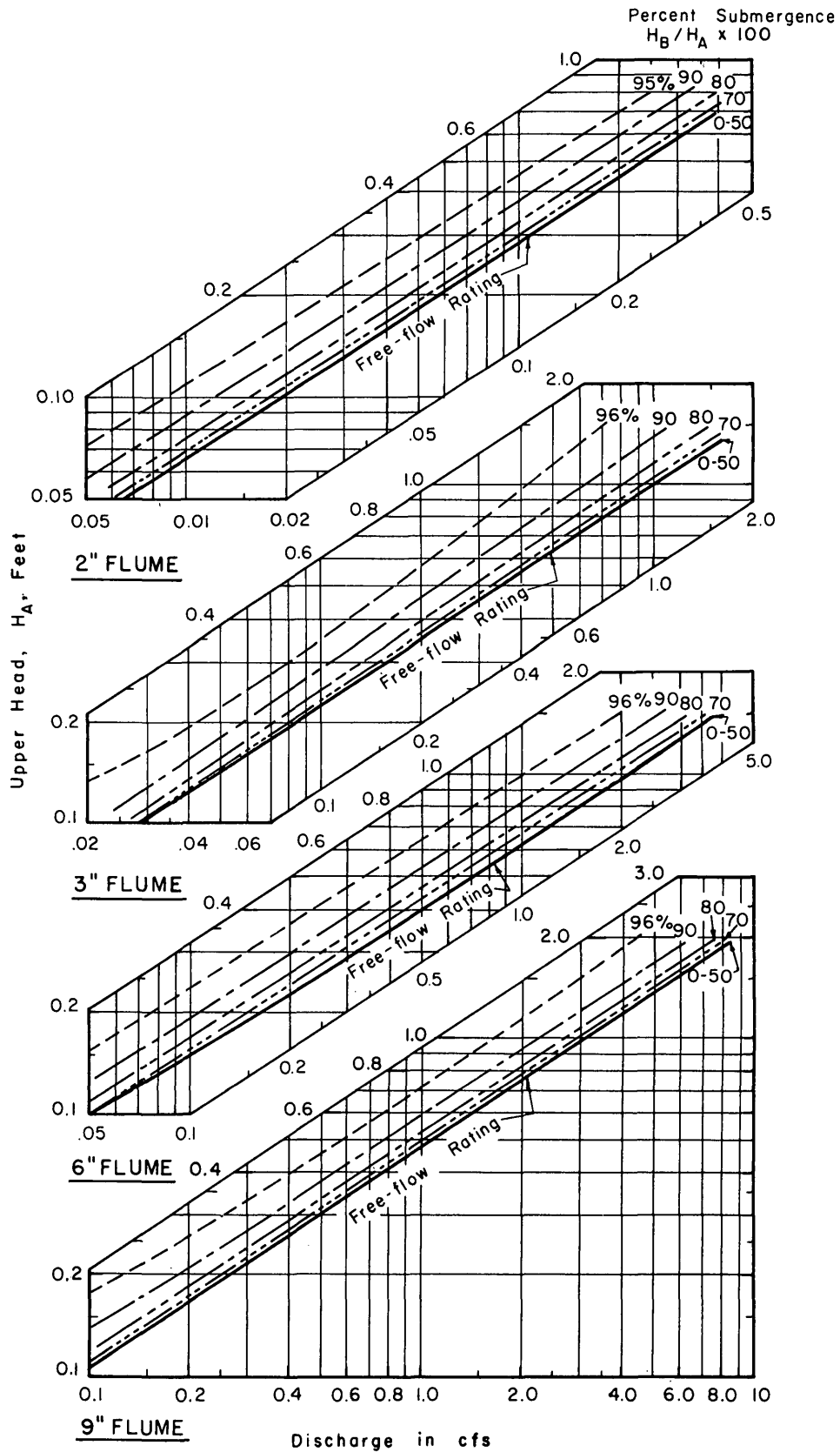


Figure 9. --Discharge rating for "inch" Parshall flumes for both free-flow and submergence conditions.

Table 2. --Discharge table for Parshall measuring flumes, sizes 1 foot to 50 feet for free-flow conditions.

H_A	1 foot	1.5 feet	2 feet	3 feet	4 feet	5 feet	6 feet	7 feet	8 feet
feet	cfs	cfs	cfs	cfs	cfs	cfs	cfs	cfs	cfs
0.10	0.11	0.15							
.15	.20	.30	0.42	0.61					
.20	.35	.51	.66	.97	1.26	1.55			
.25	.49	.71	.93	1.37	1.80	2.22	2.63	3.02	3.46
.30	.64	.94	1.24	1.82	2.39	2.96	3.52	4.08	4.62
.4	.99	1.47	1.93	2.86	3.77	4.68	5.57	6.46	7.34
.5	1.39	2.06	2.73	4.05	5.36	6.66	7.94	9.23	10.5
.6	1.84	2.73	3.62	5.39	7.15	8.89	10.6	12.4	14.1
.7	2.33	3.46	4.60	6.86	9.11	11.4	13.6	15.8	18.0
.8	2.85	4.26	5.66	8.46	11.3	14.0	16.8	19.6	22.4
.9	3.41	5.10	6.80	10.2	13.6	16.9	20.3	23.7	27.0
1.0	4.00	6.00	8.00	12.0	16.0	20.0	24.0	28.0	32.0
1.2	5.28	7.94	10.6	16.0	21.3	26.7	32.1	37.5	42.9
1.4	6.68	10.1	13.5	20.3	27.2	34.1	41.1	48.0	55.0
1.6	8.18	12.4	16.6	25.1	33.6	42.2	50.8	59.4	68.1
1.8	9.79	14.8	19.9	30.1	40.5	50.8	61.3	71.8	82.3
2.0	11.5	17.4	23.4	35.5	47.8	60.1	72.5	84.9	97.5
2.2	13.3	20.2	27.2	41.3	55.5	69.9	84.4	98.9	113.6
2.4	15.2	23.0	31.1	47.3	63.7	80.3	97.0	113.7	130.7

Table 2. --Discharge table for Parshall measuring flumes, sizes 1 foot to 50 feet for free-flow conditions. --Continued

H _A	10 feet	12 feet	15 feet	20 feet	25 feet	30 feet	40 feet	50 feet
feet	cfs	cfs	cfs	cfs	cfs	cfs	cfs	cfs
0.30	5.75	6.75	8.4	11.1	13.8	16.5	21.8	27.3
0.4	9.05	10.85	13.3	17.7	21.8	26.1	34.6	43.2
0.5	13.0	15.4	19.1	25.1	31.2	37.2	49.5	61.8
0.6	17.4	20.6	25.5	33.7	41.8	50.0	66.2	82.6
0.7	22.2	26.2	32.7	43.1	53.4	64.0	84.8	105.5
0.8	27.5	32.7	40.4	53.4	66.3	79.2	105	131
0.9	33.3	39.4	48.9	64.3	80.1	95.5	127	158
1.0	39.4	46.8	57.9	76.3	94.8	113.2	150	187
1.2	52.7	62.6	77.3	102.0	127.0	152	201	250
1.4	67.4	80.1	99.0	130.5	162	194	257	320
1.6	83.5	99.1	122.8	162	201	240	318	396
1.8	100.9	119.8	148.0	195	243	290	384	479
2.0	119.4	141.8	175.3	232	287	343	454	567
2.2	139.0	165.0	204	269	334	400	530	660
2.4	159.9	189.8	235	310	384	459	609	758
2.6	181.7	215.7	267	352	437	522	692	864
3.0	228.4	271.2	335	442	549	656	870	1084
3.5	294	347	429	566	703	840	1113	1387
4.0	363	430	531	700	870	1040	1379	1717
4.5	437	518	641	846	1051	1255	1664	2073
5.0	517	614	759	1002	1244	1486	1970	2453
5.5			885	1166	1448	1730	2295	2860
6.0			1016	1340	1664	1988	2638	3285

Note: Available data indicates that extension of the above ratings to greater heads is reliable.

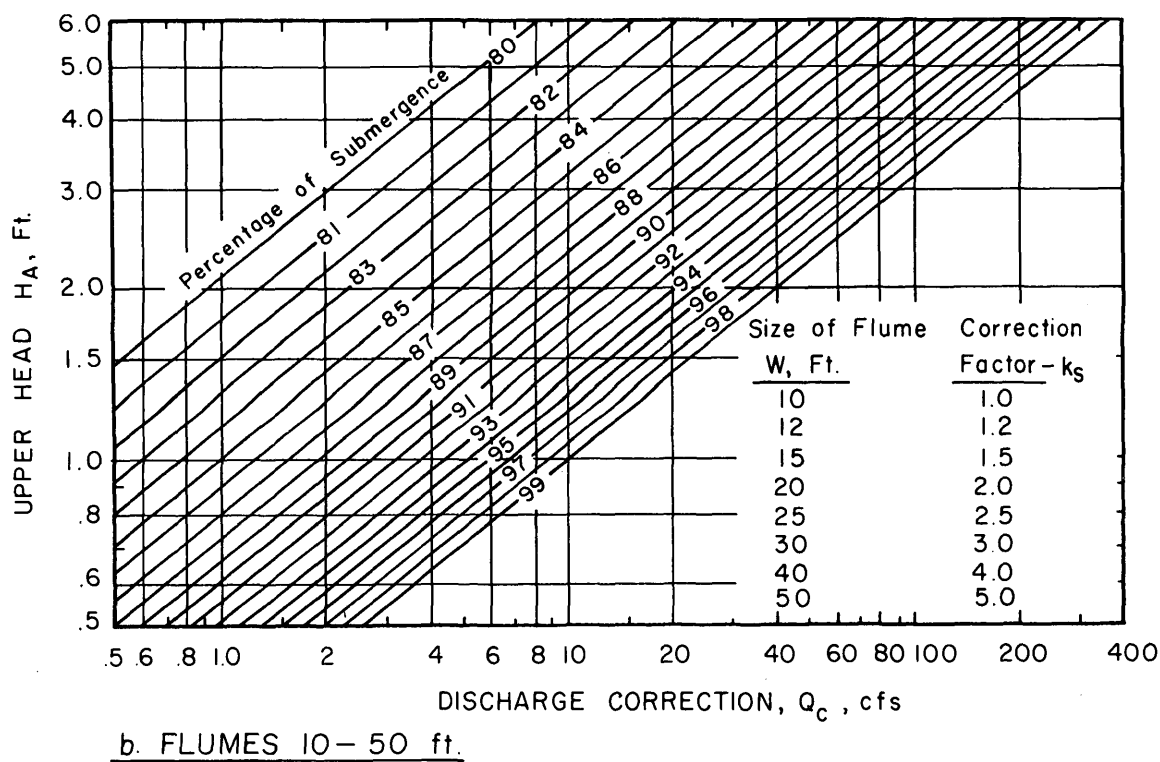
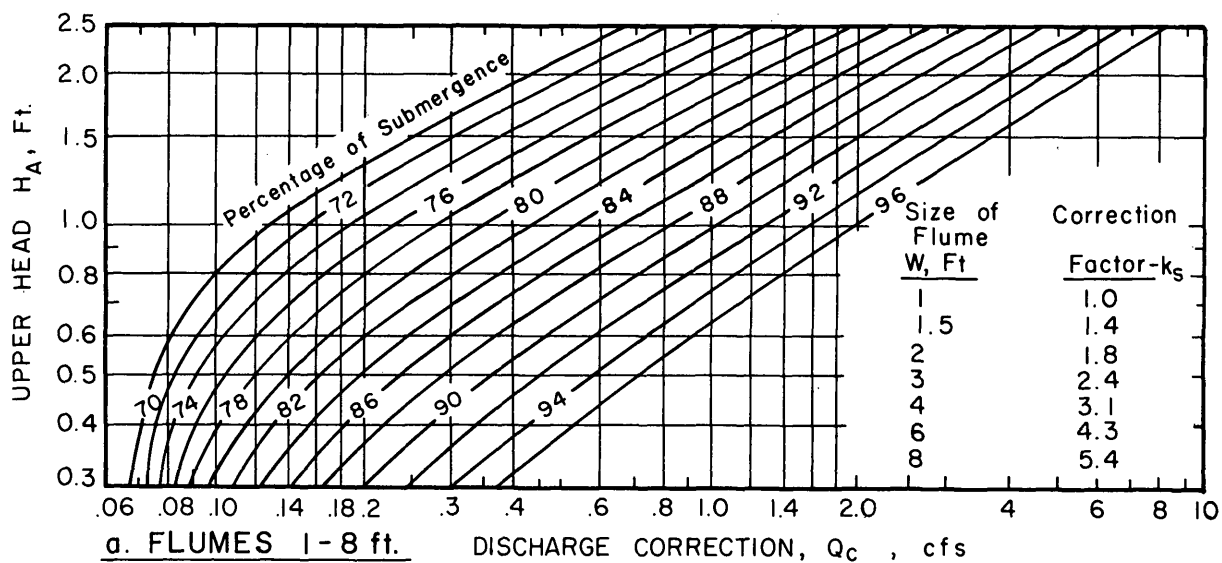


Figure 10. --Correction factors for submerged flow through 1- to 50-ft Parshall flumes.

correct calibration curves of standard-size flumes that do not conform with the specified dimensions of throat width or upstream measuring distance.

Using the same data on which Davis' equation above is based, Dodge (1963) proposed the following equation:

$$Y_0 = 1.19 Q_0^{0.645} X_0^{0.0494},$$

which gives reliable results for flumes with throat widths up to 6 feet.

Continued studies such as these should someday help to unify all Parshall flume data for submerged flows, and for other side angles and dropdown angles.

Use of Parshall Flumes in Natural Channels

The Parshall flume has been used in natural stream channels with varying degrees of success. Generally it will operate satisfactorily in channels where the sediment discharge is not too great or does not contain large rocks and other debris. Its use on flashy, cobble-strewn streams has been relatively unsuccessful. Because of the vertical-wall design, a given flume has a relatively small discharge capacity. While this is no handicap in irrigation systems, it may be on natural streams. Its lack of sensitivity at low flows is frequently overcome by placing temporary weirs in the throat. At some gaging stations small Parshall flumes have been combined with other types of weirs or controls. Installations of this type are generally unsatisfactory because the upstream stage is measured upstream from the flume and thus the rating is influenced by sediment deposits in the approach. A simple depressed "V" section in the weir control will prove just as adequate and less expensive.

Portable Parshall Flumes

Figure 11 shows a portable modified Parshall flume, designed by C. A. Taylor and H. C. Troxell (1931), which has the diverging section removed. The flume is used only under free-flow conditions. The rating shown represents a slightly greater flow for a given head than that for the standard 3-inch Parshall flume.

THE SAN DIMAS FLUME

Design and Development

As previously noted the Parshall flume while satisfactory for measuring irrigation flows, is generally not suited for measurement of debris-laden flows. A flume for measuring such flows was developed in southern California at the San Dimas Experimental Forest (1938). This flume is a supercritical-flow flume rather than a critical-depth flume, as titled by the designers, because head measurements are made in the throat below the point where critical depth occurs. To produce supercritical flow, the floor of the flume was set on a 3° slope. The San Dimas flume as originally designed was rectangular in section with the configuration and dimensional relationships shown in figure 12. The stage gaging point was placed 3 feet below the entrance regardless of the size of the flume. This was in recognition of the fact that the flume amounted to a broad-crested weir with side contractions.

Discharge Ratings for Original Design

Figure 12 shows the discharge ratings for the different size flumes. The ratings for the 1-, 2- and 3-foot flumes were determined from tests on structures of this size. From this data a general equation

$$Q = 6.35 W^{1.04} H^{1.5-n}$$

was developed where W is the throat width in feet, H the pressure head in the throat in feet; and $n = 0.179W^{0.32}$. This general expression was found to apply to flume sizes 1 through 4 foot, but discharge measurements on a 10-foot flume differed from the general expression as shown in figure 12. This was also true of a ½-foot flume tested later. The differences probably result from the fixed location of the gaging point but variable total length of the flume. For the 10-foot flume it is quite possible that for higher flows, the head was measured in the region of curvilinear flow near the free over fall or that parallel flow in the throat was not realized.

The San Dimas flume is capable of measuring flows containing considerable sediment, rock and other debris. Because head measurements are made in supercritical flow in

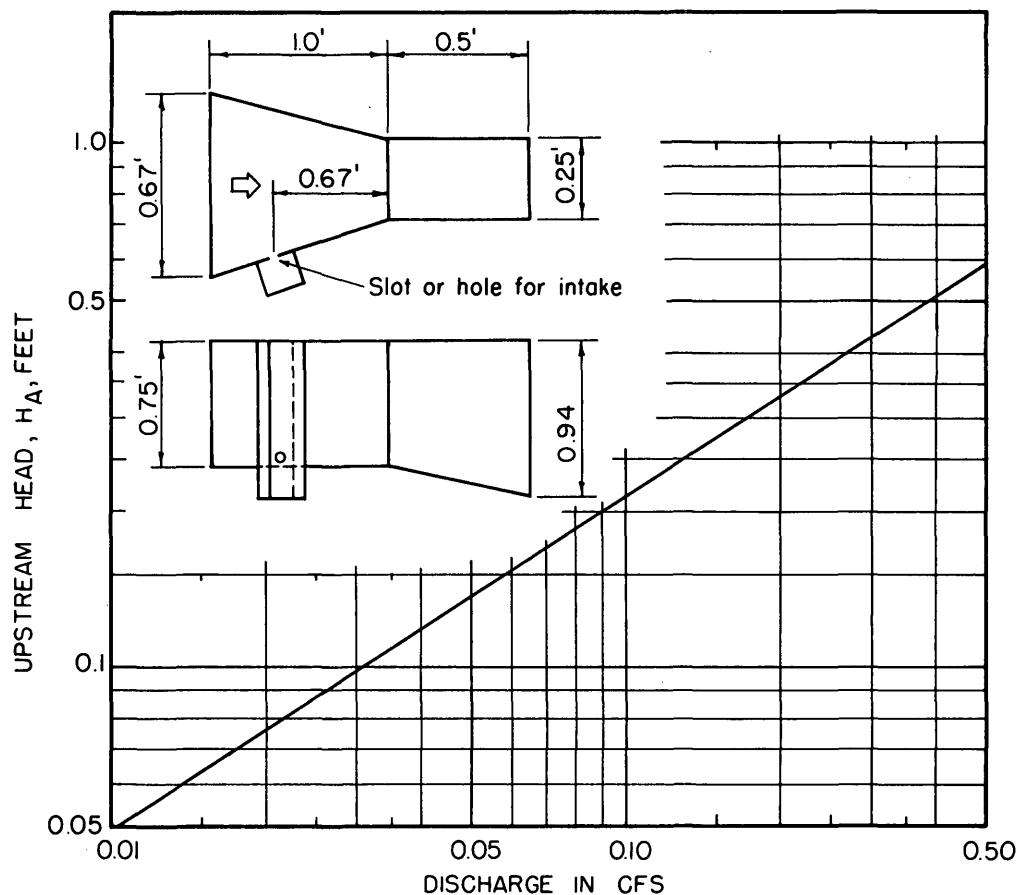


Figure 11. --Modified 3-inch Parshall flume with rating for free-flow conditions.

the throat, and critical depth occurs upstream, the discharge ratings should be independent of upstream and downstream disturbances. Considerable variation in approach conditions as a result of deposition of rocks and debris should have no material effect on the ratings.

As in the case of the Parshall flume, the San Dimas flume is rectangular in section and is not sensitive or accurate at low flows. For this reason the flumes are ordinarily operated in conjunction with various sharp-crested weirs.

Modified San Dimas Flumes

In 1947, additional tests were performed by Bermel (1950) on San Dimas flumes with various modifications. Ratings for 3 different modified flumes are shown in figure 13. The essential changes can be seen by comparison of figure 13 with figure 12. The modifications consisted of reducing the degree of contraction of the flume relative to

the natural channel, a less abrupt entrance, and the measurement of head at the midpoint of the flume, regardless of length.

A design criterion was comparability of depth of flow in the flume with pressure-head measurements. A wide range of approach and entrance conditions as well as several intake designs were tested to obtain the best comparability. The more gradual entrance and the slot-type intake were found to yield the most comparable results. Tests do indicate a slight difference between depth and pressure head.

TYPE HS, H, AND HL FLUMES

The U.S. Soil Conservation Service (1962) has HS, H, and HL type flumes in use on many small watersheds. The configuration and proportions of the 3 types of H flumes are shown in figure 14. It should be noted that all dimensions are proportional to the total depth or height of a given flume. The design

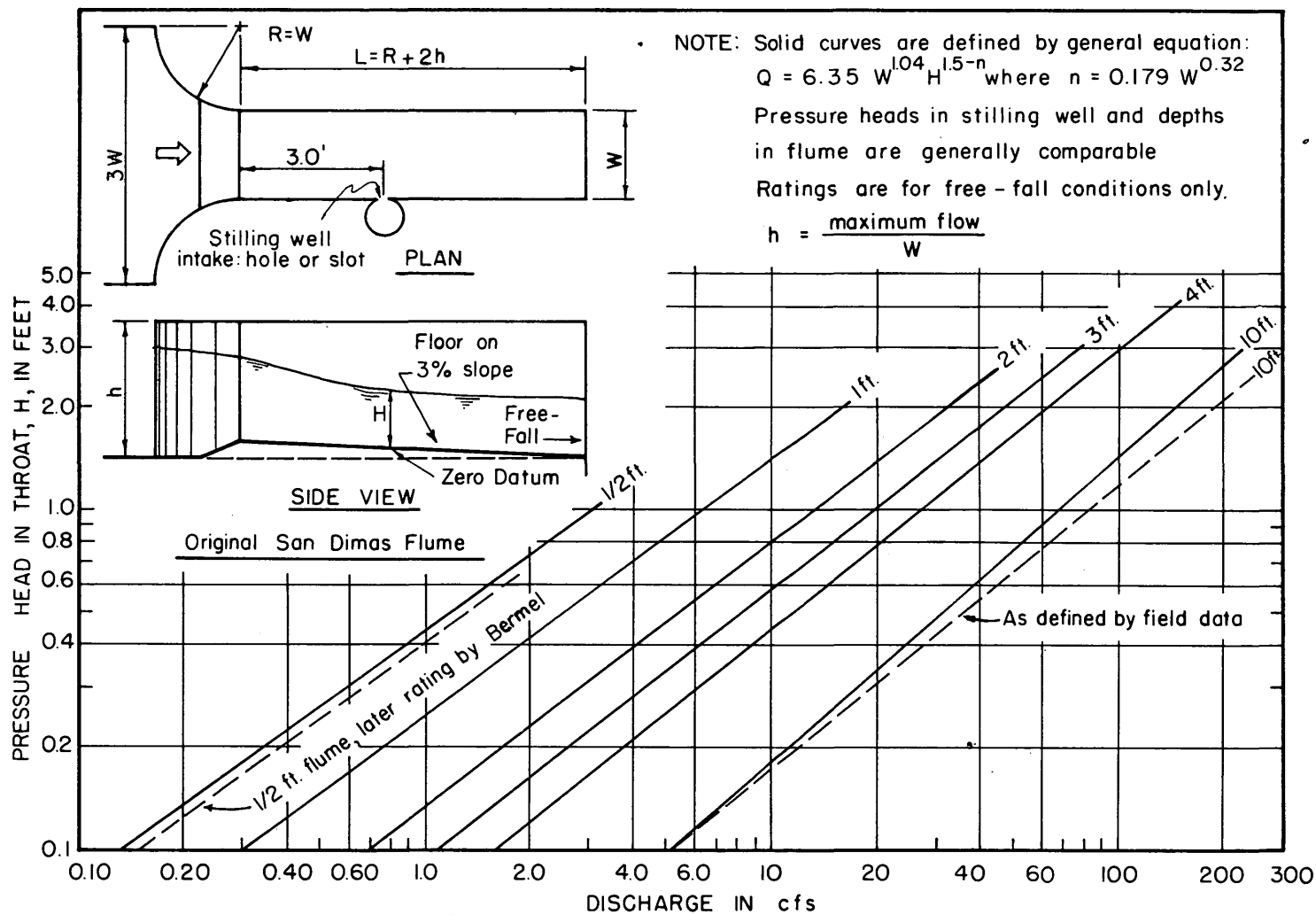


Figure 12. --Discharge ratings for different sizes of San Dimas flumes as originally designed.

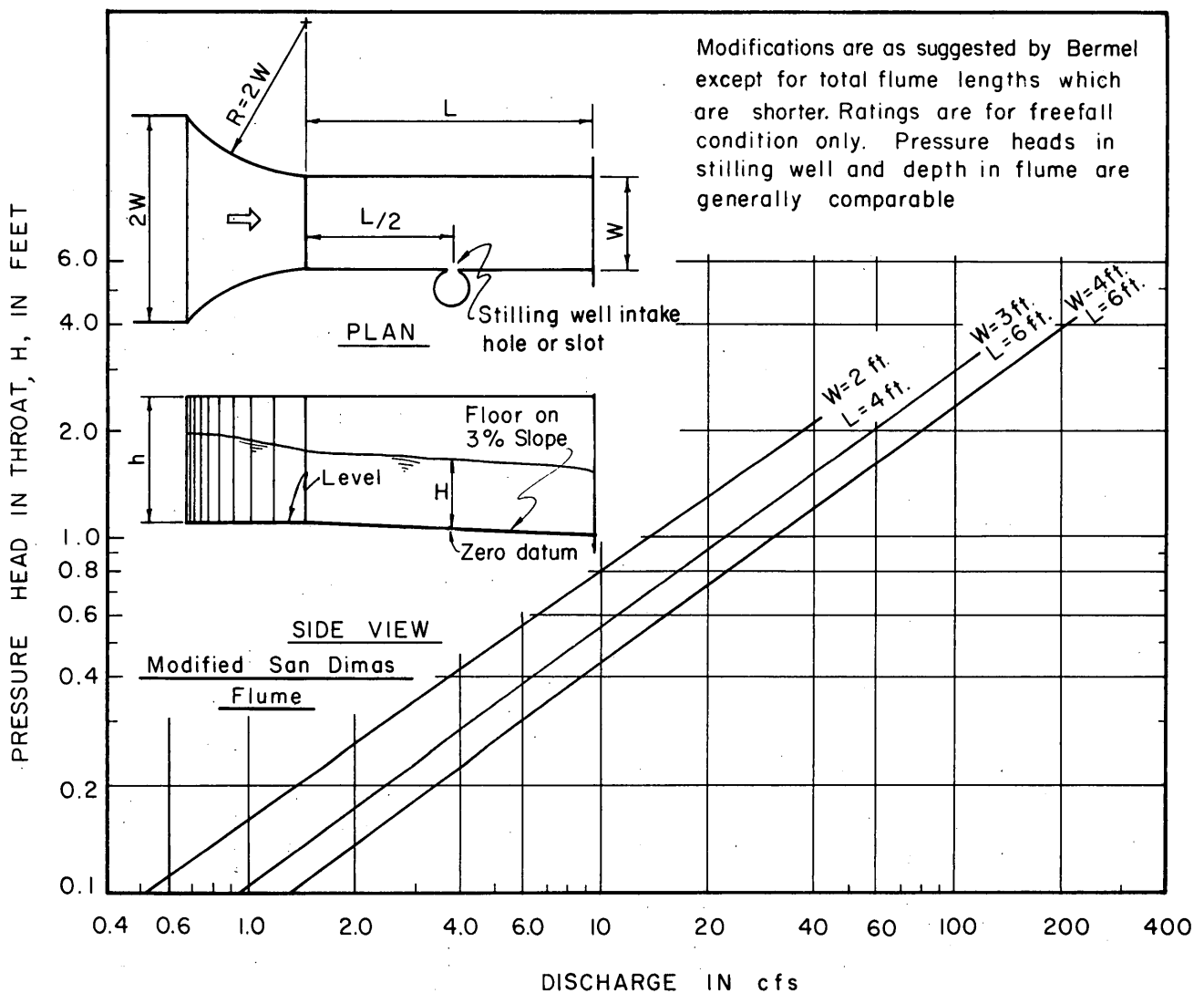


Figure 13. --Discharge ratings for different sizes of modified San Dimas flumes.

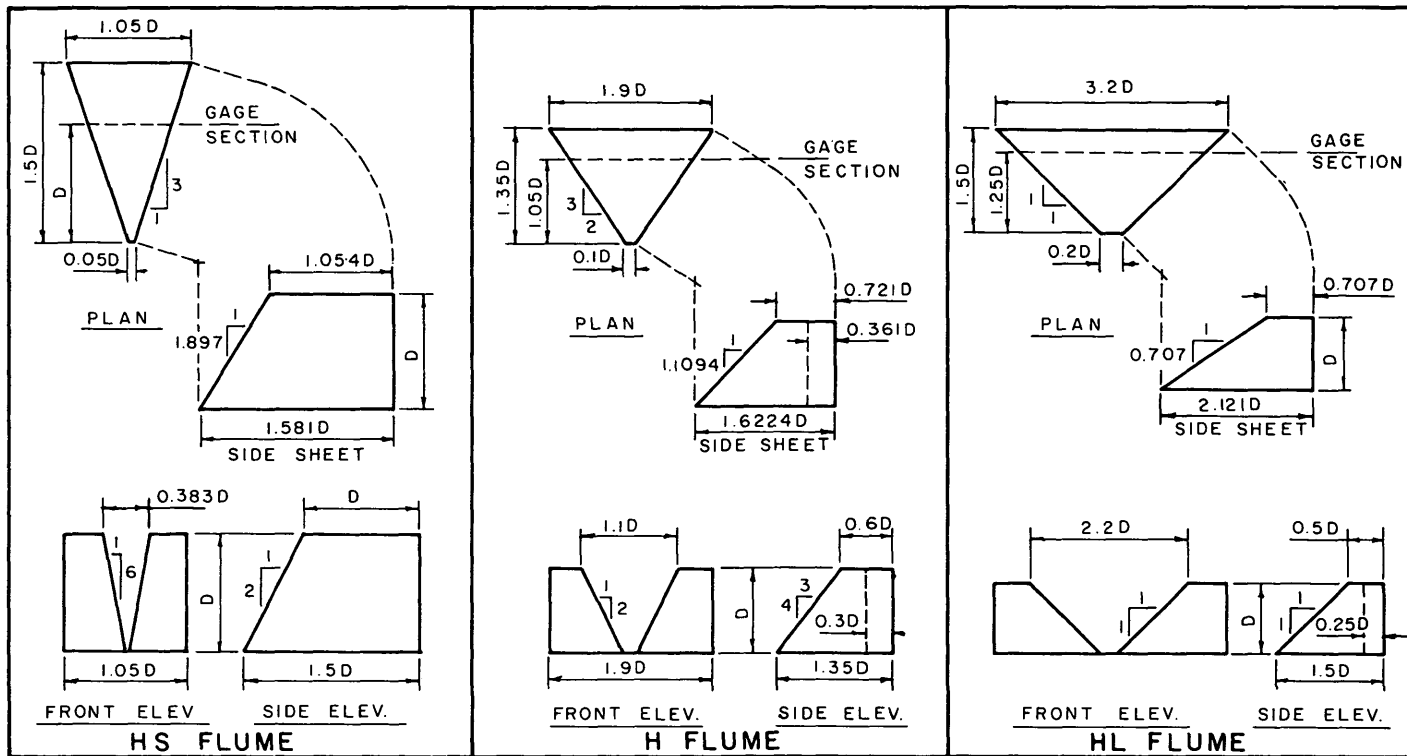


Figure 14. --Configuration and proportions of types HS, H, and HL flumes.

attempts to combine the sensitivity and accuracy of the sharp-crested weir and the self-cleaning features of the flume. The result is a compromise of both. This flume is recommended for small streams where sediment would be a problem if the conventional sharp-crested weir were used, but not where movement of coarse sediment and debris occurs on a large scale.

The H flumes do not exactly fit any of the flume examples shown in figures 2-7 and are in fact more weir than flume. Stage is measured upstream in subcritical flow and critical depth occurs over the crest as with any weir. If sediment accumulates in the approach, the discharge rating will be unstable. The design of the H flumes does decrease to some extent the accumulation of sediment.

Head is measured on the converging approach wall in a zone of accelerated flow. While this design feature is perhaps undesirable, it is probably better than measuring further upstream where approach conditions more seriously influence the rating. The converging walls are also conducive to self cleaning.

Table 3 gives the discharge ratings for the various types and size of flumes. These flumes are intended to operate under free-flow conditions although submergences up to 50% have no significant effect on the ratings.

Where the H flumes can be used, they probably have an advantage over the other flumes discussed in simplicity of design and construction. The 3 plain surfaces composing the flume are ordinarily made of metal plate and can be prefabricated for assembly in the field. The flumes are usually mounted or cast into a concrete headwall. In many instances lightweight sheet piling could be quickly driven to form both headwall and cut-off for these flumes.

THE TRAPEZOIDAL FLUME

General Design

Both the Parshall and San Dimas type flumes have narrow ranges of discharge because of their vertical walls and are unsuited to the accurate measurement of low discharges. The obvious solution to this problem was the sloping of the flume walls to produce a flume of trapezoidal shape.

The most promising design of a trapezoidal flume and the one recommended here, was developed by A. R. Chamberlain (1957) and A. R. Robinson (1959). The configuration of the flume and dimensions for 1-foot and 3-foot flumes are shown in figure 15. This flume is of type IV shown in figure 5 but with sloping walls and measurement of stage (piezometric head) only in the supercritical-flow section. This flume features walls at 30° slope with the horizontal; a flume-floor slope of 5% to produce supercritical flow; and 3 sections, an approach section, a converging section and a throat or supercritical-flow section. With a 1-foot throat the flume has a range in flow of from 1 to 350 cfs. The addition of a removable 120° V-notch weir to the throat exit is suggested for low flows. The discharge capacity of the flume can be increased by extending the walls where channel conditions permit, or by increasing the throat width.

Discharge Relationships for One-Foot Flume

This flume has been extensively tested in the laboratory and the field under a wide range of conditions. The rating curve for the 1-foot flume shown in figure 16 is based on model data and field discharge measurements of numerous prototype installations. Above 50 cfs the rating is extended using model relationships. Except with extremely adverse approach conditions, this rating may be expected to remain stable up to submergences of nearly 80%.

The original design of this flume specified a 5% bed slope for the entire structure. Under ordinary conditions flow will pass from subcritical in the approach section to supercritical in the throat. However, at very low flows it is possible for supercritical flow to exist in the approach section due to the steep slope. This is the case illustrated in figure 7. To overcome this, it is recommended that the approach section be flat and the converging and throat section remain at 5% slope.

The Three-Foot Trapezoidal Flume

To extend the range and hence the potential of this design, a flume of approximately the same configuration but with a large throat width of 3 feet was recently installed. The dimensions for this larger flume are shown in the table of figure 15.

Table 3. -- Discharge rating table for various sizes of HS , H , and HL flumes.

Flume Type and Size, D	Head in feet																		
	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.80	1.00	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0	4.5
HS																			
0.4	0.0010	0.004	0.018	0.044	0.085														
.6	.0014	.005	.021	.049	.092	0.15	0.23												
.8	.0017	.006	.024	.054	.10	.16	.24	0.47											
1.0	.0021	.007	.027	.060	.11	.18	.26	.50	0.82										
H																			
0.5	0.002	0.010	0.04	0.11	0.20	0.35													
.75	.003	.013	.05	.12	.22	.37	0.57												
1.0	.004	.015	.06	.13	.24	.40	.60	1.16	1.96										
1.5	.006	.020	.07	.16	.28	.45	.67	1.27	2.09	3.20	4.60								
2.0	.007	.025	.08	.18	.32	.51	.74	1.38	2.25	3.38	4.82	6.58	8.67	11.1					
2.5	.009	.030	.10	.21	.36	.56	.82	1.49	2.41	3.59	5.06	6.84	8.98	11.5	19.4				
3.0	.010	.035	.11	.23	.40	.62	.89	1.60	2.57	3.80	5.33	7.16	9.33	11.9	19.9	31.0			
4.5	.015	.050	.16	.31	.52	.78	1.11	1.94	3.04	4.42	6.11	8.12	10.50	13.2	21.6	32.7	46.8	63.9	84.5
HL																			
4.0	0.03	0.09	0.28	0.56	0.94	1.42	2.01	3.53	5.56	8.06	11.2	14.9	19.2	24.3	39.9	60.3	85.9	117	

Note: Ratings derived from tests made by the Soil Conservation Service at Washington, D.C., and Minneapolis, Minn.

DIMENSIONS OF TRAPEZOIDAL SUPERCRITICAL FLOW FLUME												
Flume Size, W_T ft.	Width of Approach Section, W_A ft.	ANGLES		LENGTHS			Flume Height h , ft.	Sloping Wall Length D , ft.	CAPACITIES		FLOOR SLOPES	
		Sloping Walls ϕ	Converging Walls ϕ	Approach Section L_A , ft.	Converging Section L_C , ft.	Throat Section L_T , ft.			Min. cfs	Max. cfs	Approach Section	Converging & Throat Sect.
1	5.0	30°	21.8°	5.0	5.0	5.0	4.0	8.0	0*	350	5 %**	5 %
3	9.0	30°	21.8°	Omitted	7.5	6.5	5.0	10.0	1.0	600	0 %	5 %

* With "V"-notch weir

** Optional, may be level; see text

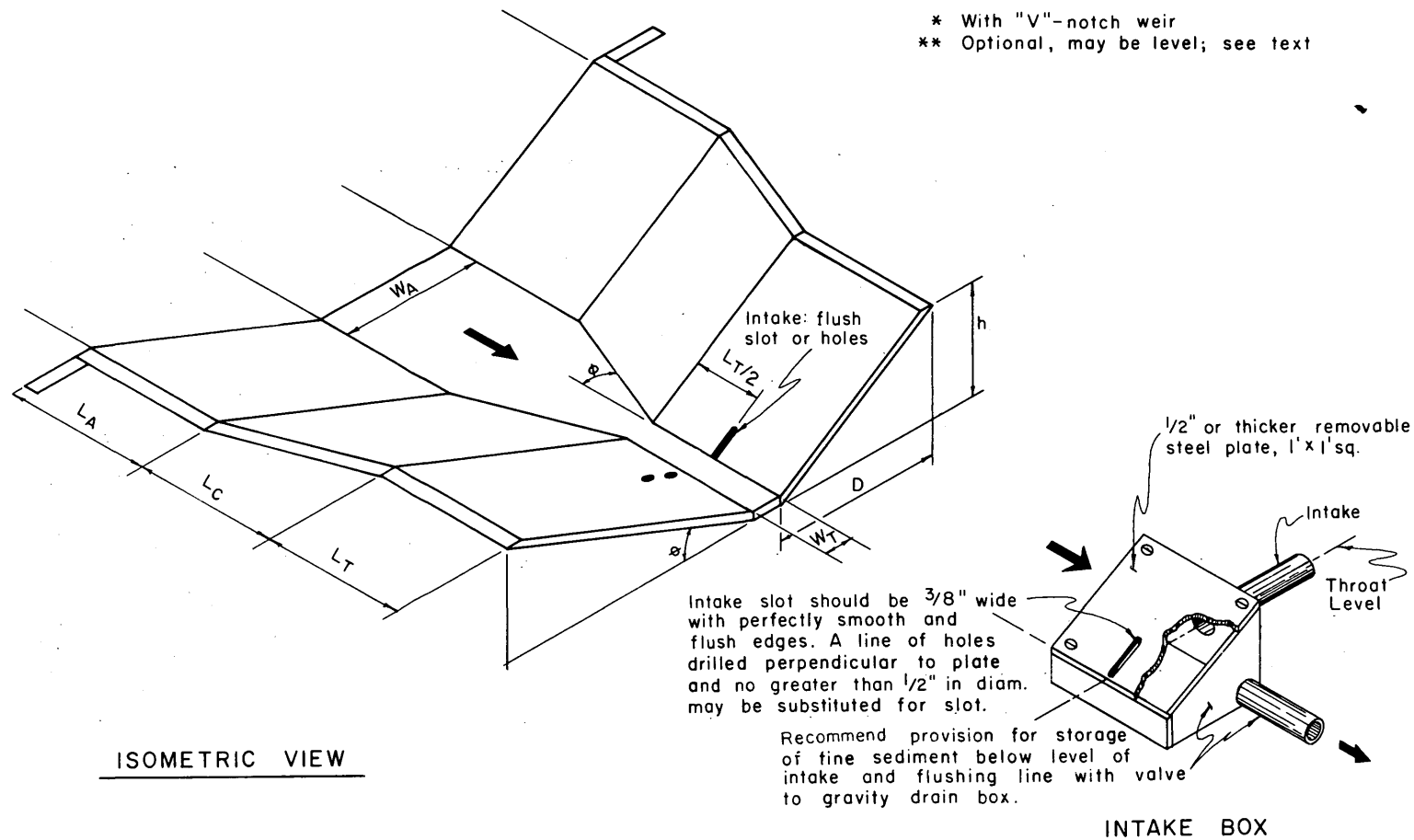


Figure 15. --Configuration of the trapezoidal supercritical flow flume.

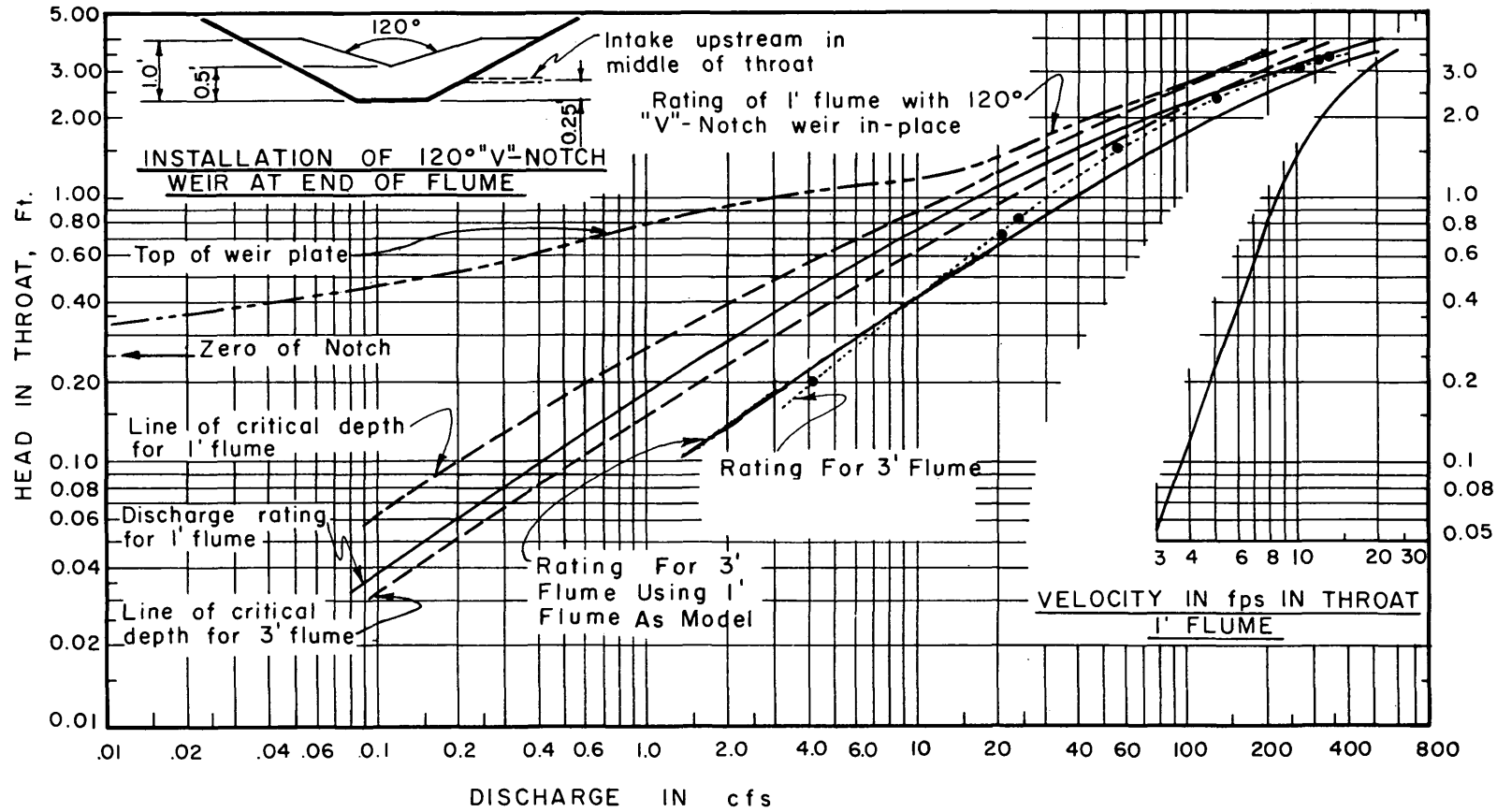


Figure 16. --Discharge rating curves and related data for one-and three-foot trapezoidal supercritical flow flumes.

This flume does not strictly conform in scale to that of the 1-foot flume owing to certain practical limitations. The principal difference is in the throat length which is shorter than the 15 feet dictated by the three-fold increase in scale. However the flume is essentially a broad-crested weir and strict adherence to scale is not only impractical, but it is doubtful that exact hydraulic similarity would be obtained. Experience with the 1-foot flume made the presence of the approach section of questionable value. For this reason the approach section L_A (figure 15), as originally designed for the 1-foot flume, was omitted in the case of the 3-foot flume, and the approximate configuration of the converging section, L_C , was extended by the use of rock fill upstream to meet the natural channel banks. A level concrete floor was placed in the approach to the flume for use as a discharge measuring section.

This 3-foot flume has a range in discharge from 3 to approximately 600 cfs. The rating shown in figure 16 by the solid line was developed from that of the 1-foot flume using the Froude number criterion.

Recent discharge measurements made on this 3-foot flume do not check this rating but define the dotted curve shown on figure 16. As shown in the accompanying table of figure 15, the throat length for the 3-foot flume is only 6.5 feet compared with 15 feet for the scaled-up 1 foot flume. The difference in ratings, as well as visual observation of the water surface at high flows, indicates that the throat length is too short and that the intake is too far upstream. The effect has been to measure head at high flows near the point where critical depth exists and perhaps even within subcritical flow. The surging water surface and the widely fluctuating stage trace observed on the recorder at the higher flows would seem to verify this conclusion.

If the flume throat is lengthened to about 10 feet and the intake is placed at a distance of 6.5 feet from the upstream edge of the throat section, the resulting rating should be close to that defined by the solid line rating for the 3-foot flume.

THE CONSTRUCTION AND OPERATION OF FLUMES

Stage Measurements

Velocities in the throat of the trapezoidal flume range from about 3 fps to in excess of 20 fps. Such high velocities are characteristic of all critical- or supercritical-flow flumes and are necessary if the structures are to be kept clear of sediment, rock, and other debris. Such high velocities also make it impractical to place staff gages in the throat section although an overhead type wire-weight gage has been used successfully.

As a rule, regardless of how much large rocks and stones a stream carries, it also has fine sand and sediment which may be expected to enter the intake system. The bubbler gage improves response, lessens freezing problems, but will not remedy entirely the sediment problem. All flumes, regardless of whether or not conventional intakes or bubbler gages are used should have provision for periodically flushing the intake system of fine sediment. The high elevation of most flumes makes it feasible to use a simple gravity drain flushing system.

If flumes are installed on streams where rapid changes in stage may be expected, intake lag may be a crucial factor. The bubbler gage will solve this problem and has been used successfully under such conditions by mounting the orifice within the intake box and not in the high-velocity flow.

Where conventional intakes are employed, lag time may be reduced by using large intakes, placing the gage stilling well as close to the flume as possible and making the diameter of the stilling well as small as practical.

Regardless of whether conventional intakes or bubbler gages are used, considerable surge may be expected.

Discharge Measurements in Flumes

Most flumes have converging approach sections or other prepared sections where current-meter discharge measurements will

commonly be made. Experience has indicated that contrary to what might be expected, such measurements may be poorer than if they had been made at a point in the natural channel. Some of the errors in these measurements have occurred from failure to observe the following precautions:

1. The measuring section should preferably be in a region of parallel flow away from the flume throat. In the case of the Parshall flume, the measuring section would have to be in the converging section, but should be located $2/3$ of the length of this section upstream from the throat, or opposite the upstream intakes.
2. Because of high velocities and interference from the presence of the engineer, measurements should be made from a bridge or plank across the flume with the meter suspended on a rod.
3. The floor in the approach section of most flumes is supposed to be uniform. There is therefore no compensation of depth readings due to rounding and all depths should be read to hundredths instead of tenths. Because of the high velocities, the most satisfactory method of measuring depth is to let the base plate of the wading rod touch the water surface and then read on an auxiliary scale the distance through which the rod is lowered to the floor of the flume.
4. Widths should be measured accurately to the nearest tenth of a foot with a graduated tape instead of a tag line. If the approach section is converging, the width should be measured through the center of the meter cups, not at the rod.
5. Velocity measurements should be made as close to each wall as possible, and again, half a foot from the walls, as the greatest variation may be expected in this region.

Winter Operation

Relatively small installations such as weirs and flumes have been successfully operated under severe winter conditions by the use of removable roof covers and liquid-gas fueled infra-red heater systems.

The 3-foot trapezoidal flume as previously described has yielded ice-free records by means of such an installation. The limited experience to date indicates that the roof and

heater system should conform to the configuration shown in figure 17. Extension of the roof a short distance over the approach section provides an ice-free measuring section. The size of infra-red heater and gas tank depend on local climatic conditions and exposure. Cost of operation will generally be from \$0.25 to \$0.50 per day.

Construction Features

Flumes have customarily been constructed of reinforced concrete, but concrete block, steel, and wood, have also been tried. Very high velocities and heavy sediment and debris loads demand that solid construction be employed. These high velocities also produce considerable uplift forces on the structures. Good concrete and good concreting techniques should be employed or erosion of the flume throat may be expected. Flume dimensions, especially those of the throat sections must be followed if precalibrated ratings are to be used.

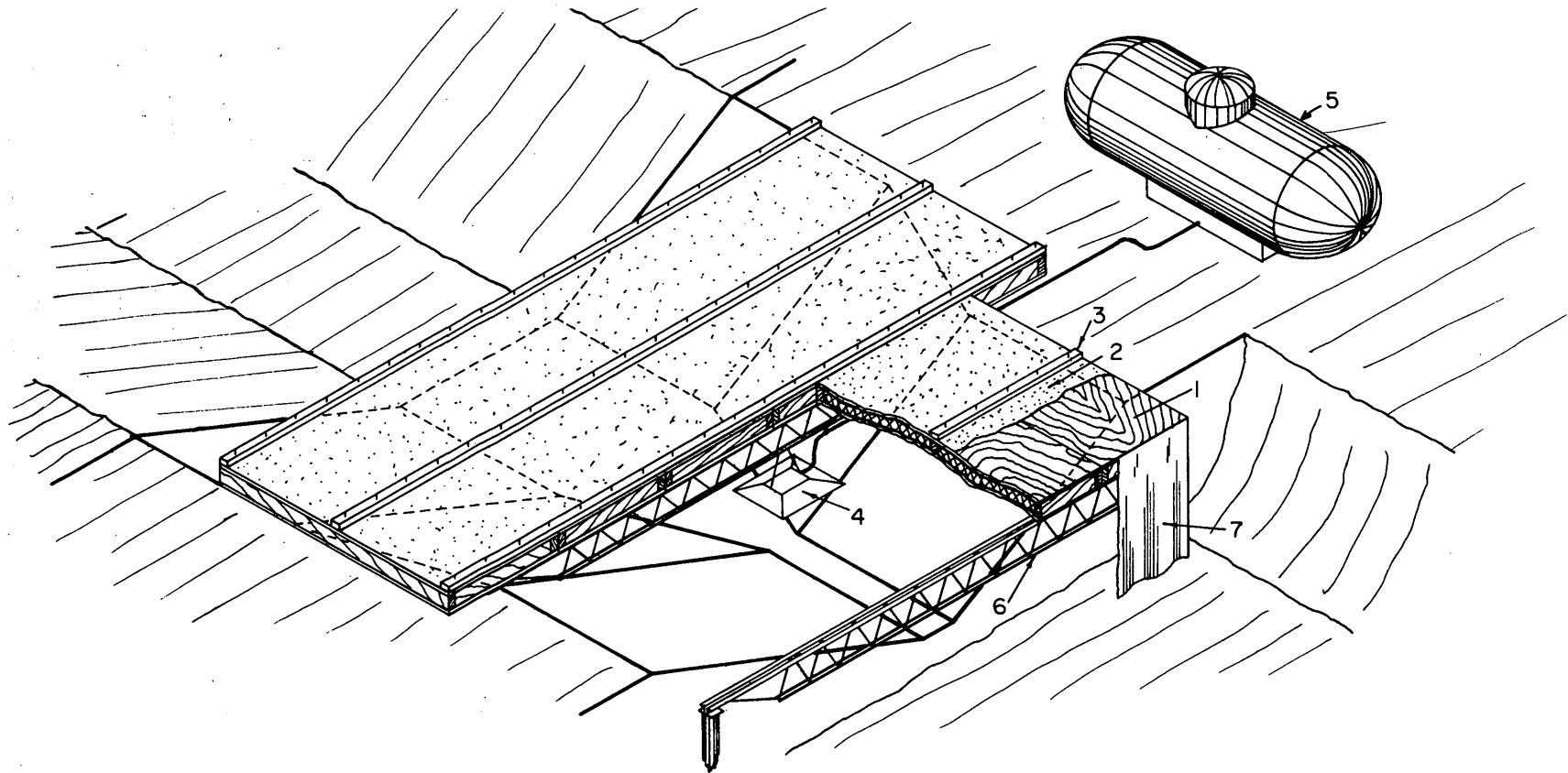
SUMMARY AND CONCLUSIONS

The 4 flume designs covered in this report are intended as precalibrated devices to be used where field ratings cannot be readily obtained. If precalibrated ratings are used, the structures must be carefully built to the exact dimensions specified. These factors necessarily increase the cost of such structures.

Evidence seems to indicate that the most critical dimensions for any of the flumes discussed are those of the throat. For critical- and supercritical-flow flumes, if a smooth, gradual transition from the natural channel to the flume throat is accomplished, the stage-discharge relation for the flume will be essentially a function of the geometry of the throat.

In many instances, nondescript flume-type structures could be built where the self-cleaning features of flumes are needed if field rating is possible. The principles outlined early in the report should aid the engineer in designing the structures to suit each specific need. The following pertinent design features should be kept in mind in the design of supercritical-flow flumes.

1. To obtain supercritical flow in the throat a floor slope of between 3 and 5% should be used.



1. Roof sections composed of a series insulated plywood boxes.
2. Roll-type roofing.
3. Roofing joints capped with wood battens held by scaffolding nails for easy removal. Where rain rather than just snow may be expected, battens should be omitted and entire roof given greater slope.
4. Infra-red heater suspended over throat section of flume.
5. Propane gas tank for fueling heater; tank should not be enclosed.
6. Light, prefab steel roof member for roof supports; wooden members may be substituted. Roof should project 3 ft beyond flume exit and at least 3 ft upstream over approach section.
7. Canvas flap or "drape" both upstream and downstream. Enclosure should be vented if natural ventilation inadequate.

Figure 17. --Trapezoidal flume with infra-red heater and roof installation for winter operation.

2. Side walls at less than 30° are not recommended as adverse wave disturbances will result.

3. The approach floor or apron should be level or nearly so.

4. Smooth and gradually converging side wingwalls forming a gradual transition from the natural channel to the flume throat should be used. The flow must enter the flume smoothly. The flume should be carefully aligned in the channel.

5. The flume length should not be excessive as a series of waves will be produced which will travel the length of the flume. The flume length should be between 2 and 3 times the maximum head at the measuring section.

6. Head should be measured at about the midpoint of the throat. Intakes should be absolutely flush with the throat walls.

7. The flume should not contract the natural channel by more than a factor of two.

8. All flumes must be strongly constructed, usually of reinforced concrete. Unless they are constructed on bedrock, considerable scour protection downstream is essential.

9. Discharge measurement should not be made at sections immediately downstream of a flume or in the flume throat.

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