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USE OF HYDROLOGIC MODELS IN THE ANALYSIS OF FLOOD RUNOFF

By

JOHN SHEN

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An analog-model study of rainfall-excess versus flood-runoff relationship with special emphasis on the determination of flood frequency.

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Symbols

A	Cross-sectional area of a stream channel. Also, an admittance function.
a	Area of a sub-basin.
B	Width of the water surface in a stream channel.
C	Capacitance.
C_z	Chezy's coefficient.
D	Duration of rainfall.
E_o	A constant voltage.
E_1, E_2	Input and output voltage.
e	Voltage.
F	A factor accounting for the increase in peak due to the increase in impervious area.
g	Acceleration of gravity.
I	Rate of inflow.
I_p	Peak of inflow hydrograph.
i	Current.
K	A storage constant.
k	A storage constant describing minor storage.
L	Inductance.
L_p	Time to peak of inflow hydrograph.
L_1	First moment of a hydrograph.
L_2	Second moment of a hydrograph.
M_o	Mode.
P(f)	Power spectrum.
Q	Rate of flow. Also, rate of outflow.

Symbols - continued

q	Rate of lateral inflow per unit length of a channel.
Q_p	Peak of outflow hydrograph.
R	Resistance.
S	Quantity of storage.
S_o	Bed slope of a stream channel.
S_f	Friction slope of the flow.
S_K	Pearson's coefficient of skewness.
T	A translation lag. Also, a characteristic time for a drainage basin.
t	Time.
t_o	Time to centroid of outflow hydrograph.
t_r	Time of travel through a reservoir.
u	Mean velocity in a stream channel.
$W(f)$	A transfer admittance.
x	Distance along a stream channel in the direction of flow. Also, an exponent.
y	Depth of water in a stream channel.
λ	Lag between centroids of inflow and outflow hydrographs.
λ_p	Lag between centroid of inflow and peak of outflow.
ρ	A ratio.
σ	Standard deviation.
τ	A total lag time, between rainfall excess and flood hydrograph.

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USE OF HYDROLOGIC MODELS IN THE

ANALYSIS OF FLOOD RUNOFF

John Shen

Abstract

The analog technique is applied to the analysis of flood runoff. A quasi-linear analog model has been developed for simulating the runoff-producing characteristics of a drainage system. Where storage is linear a unique relationship correlating the inflow and outflow peaks is derived. The technique for synthesizing flood-frequency distribution is also discussed. It is found that a linear-basin system would not modify the type of probability distribution of its inflow, whereas a nonlinear-basin system would.

Introduction

With the growing interest in the development of the nation's water resources, there is an increasing need for better understanding of the complex behavior of various hydrologic systems. Current practices used in many engineering designs are largely empirical. In general, these approaches do not render consideration of the many hydrologic variables involved in a given problem, but rather attempt to treat these variables as a lump, usually by means of a coefficient. Some judgment and experience are therefore necessary for the successful application of the empirical method.

Inevitably, all empirical approaches are based on historical hydrologic records, and are thus subject to the inherent limitations. In many instances, the records were found to be insufficient to permit a complete analysis. Moreover, such information can not perspicuously reflect the effects of man-made changes that are rapidly taking place. Consequently, one must rely on the means of synthesis, through modeling technique, to reproduce the behavior of a hydrologic system, to study the interrelations of different variables for the purposes of future planning and management.

For example, in order to predict the effect of urbanization on the flood potential of a drainage system, one must synthesize its hydrologic characteristics in every important aspect on the basis of available information. And, by properly modifying these characteristics, one will be able to forecast its possible consequences.

Likewise, to achieve the optimum design of a multipurpose water-resources system, one must synthesize various combinations of system units, levels of output, and allocation of reservoir capacities, in accordance with the requirements of the design criteria.

Presently, there appear to be two general types of scheme used in system modeling--the "statistical model" and the "hydrological model":

(1) Statistical Model.--This is the technique used largely by the Harvard Group, represented by the work of Thomas and Fiering (1962). The behavior of a water-resources system is simulated by means of a synthetic sequence which is derived from the historical hydrologic events. The historical records need to be sufficiently long so as to include representative samples of dry, wet, and normal periods whereby certain statistical parameters characterizing the data are estimated. These parameters, together with some basic assumptions about the temporal and spatial distributions of the historical data, enable one to construct models for generating extended synthetic sequences. An excellent summary and discussion of the statistical techniques was presented by N. C. Matalas (1962).

(2) Hydrological Model.--This approach differs from the previous one in that it attempts to employ the physical characteristics of a hydrologic system for the modeling, such as the surface and channel roughness, the land and channel slopes, and the overland and subsurface storage, etc. The problem essentially consists of two parts: (a)determining the runoff hydrograph for a specific rainfall event; and, (b) determining the probability distribution of a classified discharge resulting from a given distribution of rainfall. Thus, by the use of a simulated drainage system, one may assess the relative importance of the many variables involved.

It is known that various investigations involving the use of hydrologic models have existed for many years. However, these studies in the past were generally of a descriptive nature. It is the intent of this study to define quantitatively the fundamental relations among the hydrologic variables with special emphasis on the rainfall-excess versus flood-runoff relationships.

One's first thought regarding the hydrological model might be to think of it as a physical laboratory model. An example of this type of approach is represented by the work of J. Amorocho and G. T. Orlob (1961). Despite its crudeness, Amorocho's and Orlob's laboratory model did succeed in illustrating some of the fundamental behavior of a simple drainage basin. The possibility of such an approach was, in fact, seriously considered during the initial planning of this study. However, in view of the large number of variable parameters that would be involved, such as the distribution of rainfall pattern; the variations of land slope and surface roughness; and, the behavior of infiltration and evapotranspiration losses, the required model would be very complex. Indeed, one could not hope to accomplish an extensive physical-model study without an enormous amount of expense. Consequently, this type of approach was not adopted at this time.

A mathematical model describing the physical behavior of a drainage system affords another possible means of hydrological modeling. In this approach, the hydrologic process would be expressed purely in terms of mathematical functions. Dooge's work (Dooge, 1959) is an outstanding example of this class. Inevitably, however, the use of mathematical models is often handicapped by one's ability to obtain their solutions, especially with the more-complex, nonlinear cases. Nevertheless the mathematical tool is an essential part for all analytical studies and is thus considered as a complementary effort to the analog-model approach described subsequently.

An analog model is necessarily based upon the many similarities which exist between the behavior of fluid flow and electric-current flow. Thus, it offers a convenient means for hydrological modeling. The advantage of such a model lies in its simplicity and flexibility. It by-passes many complex mathematical processes and allows the causes and effects to be observed readily. Hence, it is a very useful tool for research purposes. In the sections that follow, the development of various hydrologic-modeling techniques are discussed. In particular, the technique of analog simulation is largely applied.

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HYDROLOGIC MODELS FOR RAINFALL-EXCESS VERSUS FLOOD-RUNOFF SYSTEM

The process of converting rainfall excess into surface runoff is one of the oldest problems in hydrology. Yet it is also one of the most difficult endeavors. Various methods of treating such a system are known--some are based on rational analysis; others are based on empirical approaches. The methods differ from each other. Certain amounts of controversy exist. Undoubtedly, the disputes are largely due to the subjective points of view of different investigators. A more perspective outlook is indeed essential.

In a broad sense, the analyses of hydrologic systems may be classified into two schools: the "linear system" and the "nonlinear system". Briefly, a linear system relates the dependent variable to a weighted sum of the independent variables, whereas, a nonlinear system takes into account the interactions of the independent variables. The former is founded on the unit-hydrograph principles and is represented by the work of Dooge (1959), Paynter (1960), Rockwood (1956), and Kalinin (1960); the latter is based on the fluid-mechanics concept represented by the work of Amorocho (1962), Harder (1962), Liggett (1959), and Ishihara (1956). Although these individual efforts are of the same general nature, the emphases are nevertheless placed upon different applications. Whereas all real phenomena in nature are nonlinear to a greater or lesser degree, experience from various sources seems to indicate that the use of linear tools with appropriate caution and modification can lead to many valuable approximations.

Thus, before abandoning too hastily the linear concept, one should go back and examine some of the fundamental principles governing the mechanics of surface runoff. Liggett (1959) pointed out that there are two distinctly different types of problems associated with the determination of stream flow in hydrology. The first classification is that of the "upstream problem", which consists, in its elements, of a long channel into which there is a continuous inflow along the sides with little or no inflow at the upper end. There are no points with large concentrations of inflow, although the lateral inflow may vary with distance. The second classification is that of the "downstream problem", which consists of a large channel with a very small amount of lateral inflow, although there may be large concentrations of lateral inflow at the junctions with its tributaries. The main source consists of flow into the upper end.

The downstream problem is commonly referred to as the routing problem which is most satisfactorily solved by either the conventional flood-routing procedure, such as the Muskingum method, or by the more sophisticated method (Stoker, 1957) involving the unsteady-flow equations.

On the other hand, the treatment of the upstream problem, thus far, has been only cursory. Investigations rely largely on empirical procedures which were often found to be inadequate because of the greater range of variables involved. It is in this light that Liggett's analytical study represents a significant contribution.

From the hydrologic point of view, the upstream problem is perhaps more critical, especially with small drainage basins. Assuming that one has an equivalent drainage channel such as the one suggested by Liggett, then he may derive a system of flow equations as follows:

$$\frac{\partial Q}{\partial x} + B \frac{\partial y}{\partial t} = q \, , \tag{1}$$

$$\frac{\partial y}{\partial x} + \frac{1}{g} \frac{\partial u}{\partial t} + \frac{u}{g} \frac{\partial u}{\partial x} + S_f = S_o \, , \tag{2}$$

in which Q is the quantity of flow, u is the mean velocity in the channel, B is the width of the water surface in the channel, y is the depth of water in the channel, S_o is the slope of the channel, S_f is the friction slope of the flow, x is the distance along the channel, g is the acceleration of gravity, t is the time, and q is the quantity of lateral inflow per unit length of the channel. A schematic representation of the equivalent channel is shown in figure 1.

Figure 1. - Schematic representation of an equivalent drainage channel.

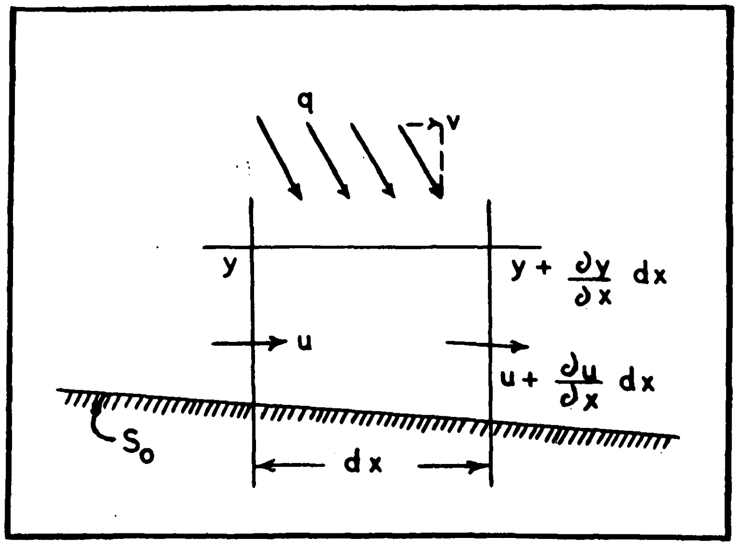

Figure 1. - Schematic representation of an equivalent drainage channel.

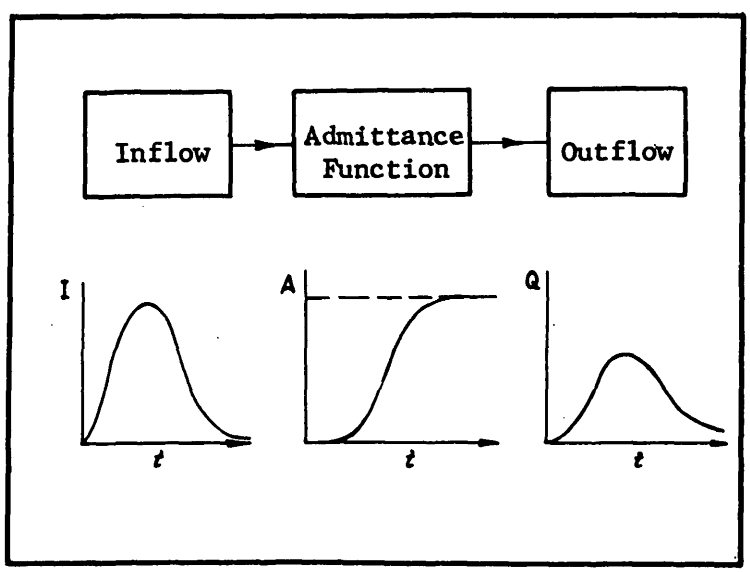

Figure 2. - A simplified routing scheme.

With the use of Chezy's relationship, $S_f = Q|Q| / C_z^2 A^2 r$, and assuming that the wave height is small in comparison with the water depth, equation 2 may be reduced to

$$\frac{\partial y}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{Q|Q|}{C_z^2 A^2 r} = S_o , \tag{3}$$

where A is the cross-sectional area of the channel, and r is the hydraulic radius.

Experience has shown that for long-period flood waves, the acceleration term $\frac{1}{gA}\frac{\partial Q}{\partial t}$ is usually of small magnitude with respect to other terms and may thus be neglected under ordinary circumstances. Hence, equation 3 may be further simplified into

$$Q = C_z A r^{\frac{1}{2}} \left(S_o - \frac{\partial y}{\partial x}\right)^{\frac{1}{2}} . \tag{4}$$

It may be readily recognized that the product of $C_z A r^{\frac{1}{2}}$ in equation 4 is a form of "conveyance function". Ordinarily it varies nonlinearly with the stage. In combination with the continuity equation (equation 1), it represents a nonlinear "admittance function". Thus, the procedure of converting rainfall excess into surface runoff may be simply stated as a process of transformation via the admittance function as illustrated in figure 2.

Figure 2. - A simplified routing scheme.

Paynter's experience (Paynter, 1960) indicates that in many cases, the admittance function of river reaches can be represented with surprising accuracy by a simple function of delay and linear storage as shown in figure 3.

Figure 3. - Linearized admittance function.

Consequently, this function can be described by two constants, i.e., a "translation lag", T, and a "storage constant", K. The translation lag is associated with the time of wave propagation down the river reach, while the storage constant is related to the overland and the valley storage.

It is interesting to note that Paynter's finding concurs with Dooge's unit-hydrograph hypothesis (Dooge, 1959) that the process of converting rainfall excess into surface runoff is a mixture of translation and reservoir action. Hence, it may be represented by a series of "linear channels" and "linear reservoirs".

Thus, the linear system may be considered as a special case of the more general nonlinear systems. Its advantage lies, of course, in the simplicity of manipulation, that is, the principles of superposition and proportionality may be readily applied.

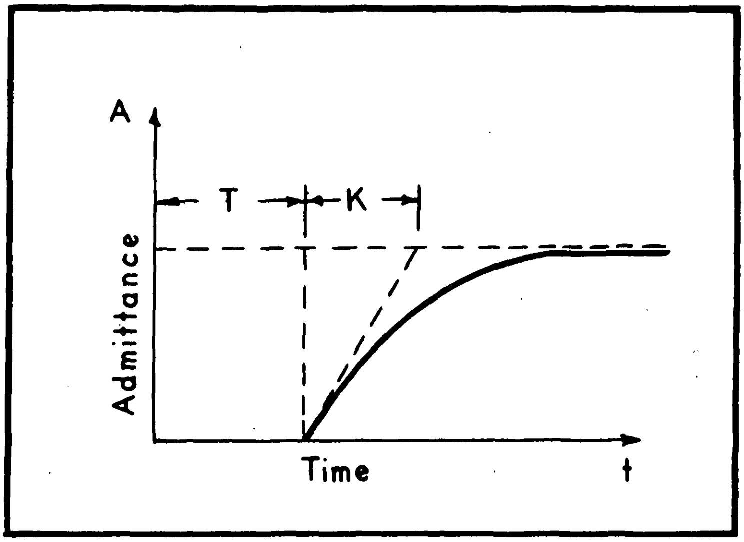

Figure 3. - Linearized admittance function.

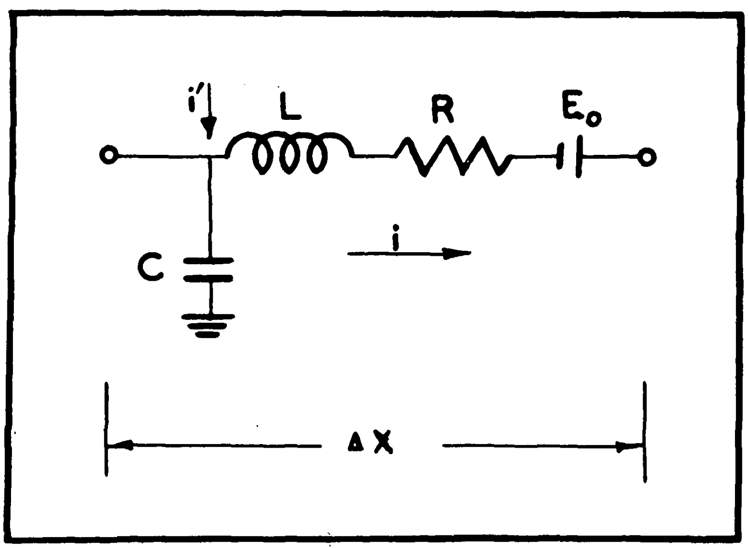

Figure 4. - A segment of transmission line.

The selection of a linear or a nonlinear model is a matter of refinement required of a given problem. For example, in the prediction of a flood crest as it travels down a major river channel, a certain degree of accuracy would undoubtedly be required, as human lives and large amounts of property may be involved. Moreover, in many instances, the hydrologic parameters and the channel geometry are well defined. The choice of a more complex nonlinear model is therefore justified. On the other hand, in the planning of drainage structures for small basins, the hydrologic information is generally very crude and may even be totally absent; in which case, the use of any elaborate model would be absurd. Indeed, a linear model is generally more than sufficient to give a first-order approximation for the design purposes.

In simulating a drainage system, either linear or nonlinear, one is inevitably concerned with the mathematical complexities that are involved. For instance, in studying the interrelations of the fundamental variables, one must consider a vast number of possible combinations within the specified ranges. Consequently, the simulation of such a system must be facilitated by the use of high-speed electronic computers--digital or analog.

DEVELOPMENT OF A QUASI-LINEAR ANALOG MODEL

Examples of various applications of the computer technique in a hydrologic study are known. Harder (1962) used a nonlinear analog model to simulate the flood-control system; Crawford and Linsley (1962) used a digital computer to synthesize the streamflows for three small watersheds; Baltzer and Shen (1961) utilized a power-series technique to solve the unsteady-flow equations (equations 1 and 2) on a digital computer.

In this study, the analog technique is adopted for several reasons:

1. The laws governing the behavior of fluid flow and electrical current are, in many instances, identical. Thus, an analog model may be developed on the basis of direct simulation instead of on the exact mathematical expressions which would be required by a digital computer.

2. The input and output of an analog model are expressed in continuous time-varying graphs. They are easily recognizable by an investigator. Moreover, the parameters can be readily varied. Hence, an analog model is a more flexible and convenient tool for research purposes.

3. In a cause and effect study, the data is primarily derived by means of synthesis which is often limited in scope. Hence, the operation of an analog computer is generally more economical than that of a digital computer which is ordinarily a data-processing device.

If one examines a segment of an electrical "transmission line" (figure 4), one may derive the following system of equations:

Figure 4. - A segment of transmission line.

$$\frac{\partial i}{\partial x} + C\,\frac{\partial e}{\partial t} = i' \ , \tag{5}$$

$$\frac{\partial e}{\partial x} + L\,\frac{\partial i}{\partial t} + Ri = E_o \ , \tag{6}$$

where e is the voltage, i is the current, C is the capacitance, L is the inductance, R is the resistance, x is the distance in the direction of current flow, and t is the time.

Comparing equations 5 and 6 with equations 1 and 3, it may be seen that the following variables are equivalent:

Electric variables			Hydraulic variables	
Voltage	(e)	→	Water depth	(y)
Current	(i)	→	Discharge	(Q)
Inductance	(L)	→	Inertia coefficient	$\left(\frac{1}{gA}\right)$
Capacitance	(C)	→	Surface width	(B)

Furthermore, the constant voltage (E_o) represents the fixed channel slope (S_o), and, the current (i') represents the lateral inflow (q).

Thus, the two systems of equations are analogous. The only difference lies in the friction terms. In the electrical system the resistance term is linear, whereas in the hydraulic system the resistance term is nonlinear. Thus, in order to accomplish the complete analogy, one must replace R by a variable resistance which induces a voltage drop proportional to the square of the current (Einstein and Harder, 1959). Moreover, for natural basins the storage effect may be nonlinear, that is B is a function of stage. In which case, a variable capacitance would also have to be used in the transmission-line model. The method for simulating such nonlinear reservoirs will be discussed in a later section. Figure 5 shows a variable

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Figure 5. - A variable transmission line.

---

transmission-line scheme.

In view of the previous discussion that the acceleration term $\frac{1}{gA}\frac{\partial Q}{\partial t}$ is generally of small magnitude for flood waves, the inductance element can thus be omitted from the transmission-line scheme. Accordingly, the model becomes a variable RC-type of network or an admittance function. Further simplification can be made if the system is linearized in accordance with Dooge's and Paynter's hypotheses that the admittance function is made up of two fundamental elements--the linear channel and the linear reservoir.

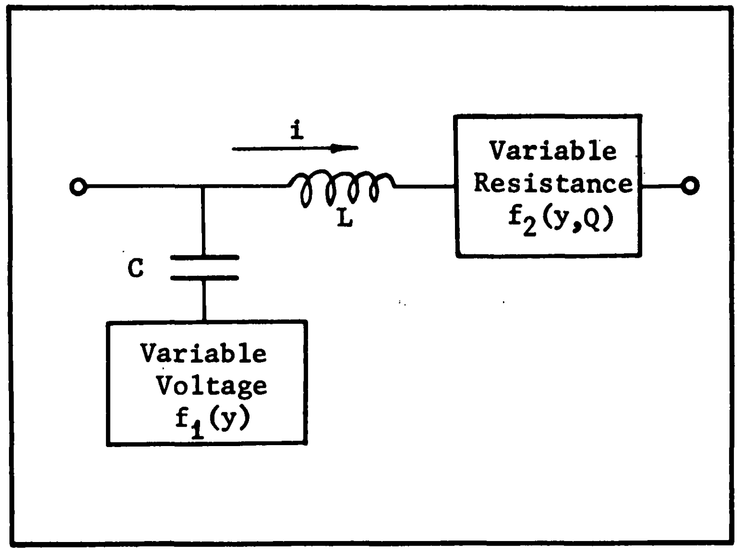

Figure 5. - A variable transmission line.

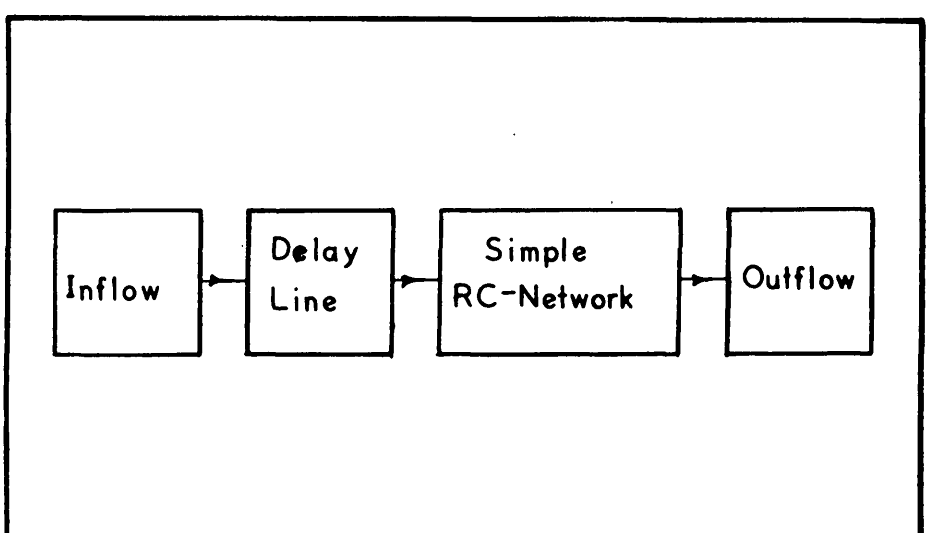

Figure 6. - Model of a linearized drainage system.

Electrically, the two elements may be simulated by a delay line and a simple RC-network. A schematic representation is shown in figure 6.

Figure 6. - Model of a linearized drainage system.

A linear reservoir is one in which the storage is linearly proportional to the outflow:

$$S = KQ \ , \qquad (7)$$

in which, S is the storage, Q is the rate of outflow, and K is a storage constant. Thus, in combination with the continuity equation,

$$I - Q = \frac{dS}{dt} \ , \qquad (8)$$

one obtains the inflow-outflow relationship for a linear reservoir,

$$I - Q = K \frac{dQ}{dt} \ , \qquad (9)$$

where I is the rate of inflow.

Equation 9 is a diffusion type of equation. It is equivalent to a simple RC circuit shown in figure 7, for which one may derive the

Figure 7. - A linear-reservoir model.

relationship

$$E_1 - E_2 = RC \frac{dE_2}{dt} \ . \qquad (10)$$

Thus, comparing equations 9 and 10, E_1 is equivalent to I; E_2 to Q; and, RC to K. The operational amplifier shown in the circuit is a voltage-transferring device which provides a means of interconnecting a series of reservoirs.

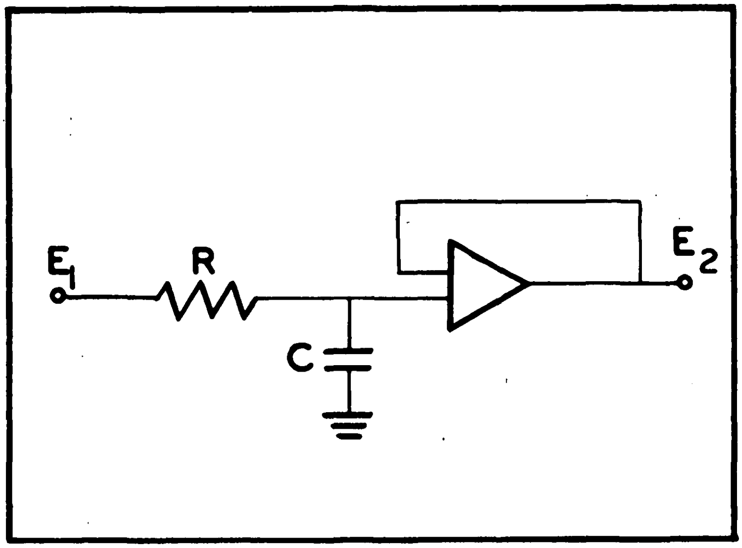

Figure 7. - A linear-reservoir model.

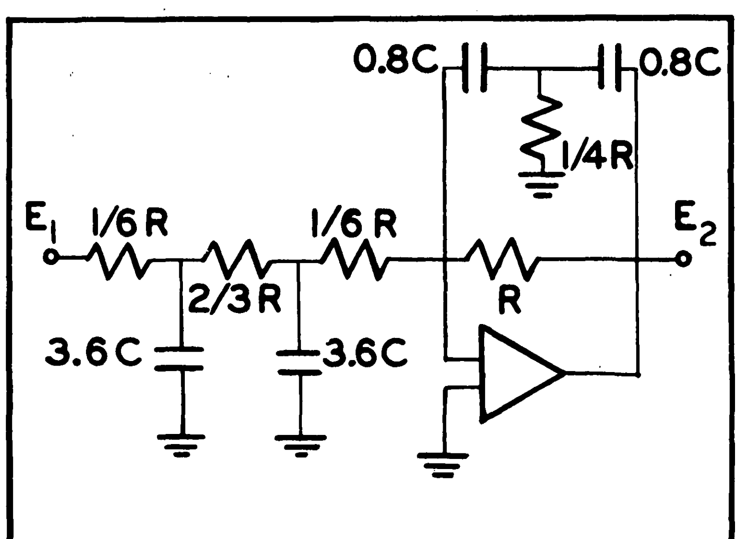

Figure 8. - A phase-shifting circuit.

A linear channel is defined as a reach in which the rating curve at every point has a linear relationship between discharge and cross-sectional area. This implies that at any point the velocity is constant for all discharges, but may vary from point to point along the reach. Thus,

$$A = T' \cdot Q \; , \tag{11}$$

where A is the channel cross-sectional area and T' is the first derivative of a translation lag, T. Combining with the continuity equation,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \; , \tag{12}$$

one obtains

$$\frac{\partial Q}{\partial x} + T' \cdot \frac{\partial Q}{\partial t} = 0 \; , \tag{13}$$

which has the solution

$$Q\ (t - T) = \text{Constant} \; . \tag{14}$$

This solution corresponds to the case of a pure translation. It indicates that a linear channel will translate any inflow hydrograph without a change of its shape.

The linear channel may be simulated electrically by a delay line. One of these devices is the phase shifter as shown in figure 8. By

Figure 8. - A phase-shifting circuit.

interconnecting a cascade of such delays, a total delay equal to the sum of the individual delays and a rise time equal to the root mean square of the individual rise time can be accomplished. This system may then be used as a time-delaying trigger mechanism to initiate the input signals at various time lags.

Whereas linear storage is found to be applicable to many natural basins, Mitchell's experience (Mitchell, 1962) indicates that the nonlinear storage is a condition which occurs with sufficient frequency to warrant careful consideration.

For a nonlinear reservoir, the relation between storage and outflow may be expressed by (Mitchell, 1962)

$$S = KQ^x , \tag{15}$$

in which, x is an exponent. Accordingly, the routing equation becomes

$$I - Q = K\, x\, Q^{x-1} \frac{dQ}{dt} . \tag{16}$$

To simulate such a nonlinear reservoir, it is necessary to have a variable capacitor so that its capacitance is a nonlinear function of Q. Figure 9 shows a nonlinear-reservoir model. It may be shown

Figure 9. - A nonlinear-reservoir model.

(Shen, 1962) that this model bears the relationship

$$E_1 - E_2 = RC\, {E_2}^{x-1} \frac{dE_2}{dt} , \tag{17}$$

if

$$E_o = E_2 - \frac{{E_2}^x}{x} . \tag{18}$$

Equation 17 is seen to be analogous to equation 16 with the condition that

$$RC = K\, x . \tag{19}$$

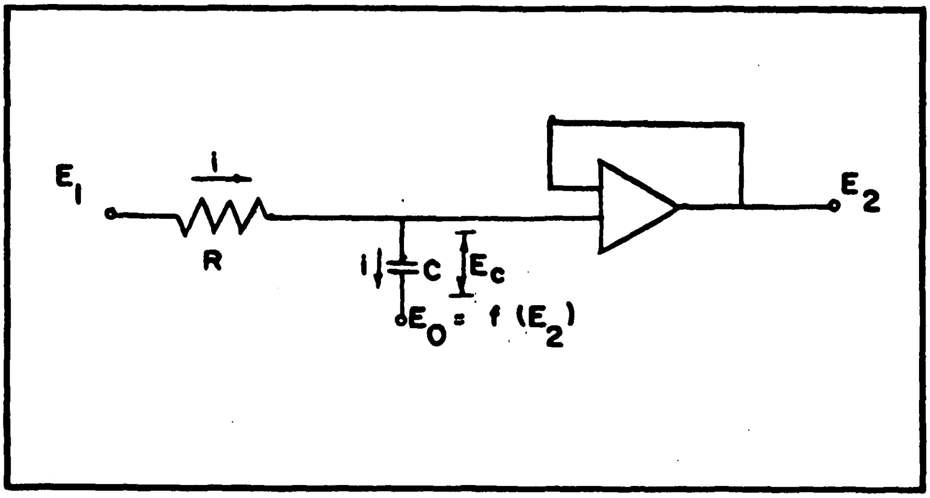

Figure 9. - A nonlinear-reservoir model.

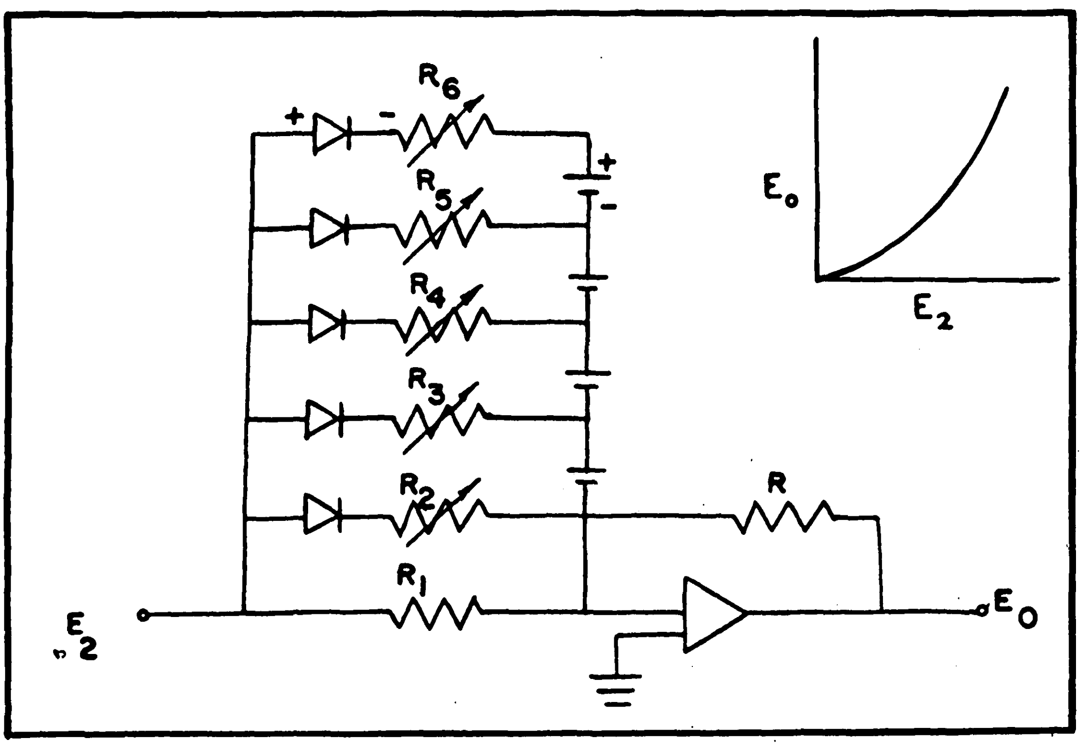

Figure 10. - A nonlinear voltage amplifier.

To produce the required nonlinear voltage, E_O, a variable voltage amplifier must be used. Figure 10 shows such a circuit

Figure 10. - A nonlinear voltage amplifier.

which consists of an operational amplifier and a group of diodes, each having a series resistance that conducts at a specific voltage level.

By interconnecting this circuit to the circuit shown in figure 9, a nonlinear reservoir is accomplished. A model such as this is entirely flexible. It may be arranged to represent any nonlinear-storage behavior of a drainage system.

In this manner, a complete drainage system may be simulated. Figure 11 illustrates such a scheme, in which a basin is subdivided

Figure 11. - Distribution of translation and storage effects of a basin.

into a number of sub-areas separated by isochrones, that is, contour lines joining all points in the basin having equal translation time to the outlet. A time-area diagram may then be constructed in accordance with the sub-areas enclosed by these isochrones. If uniform rainfall-excess occurs within the entire basin, the time-area diagram would represent the distribution of the volume of runoff. Otherwise, it must be readjusted to account for the uneven distribution of rainfall.

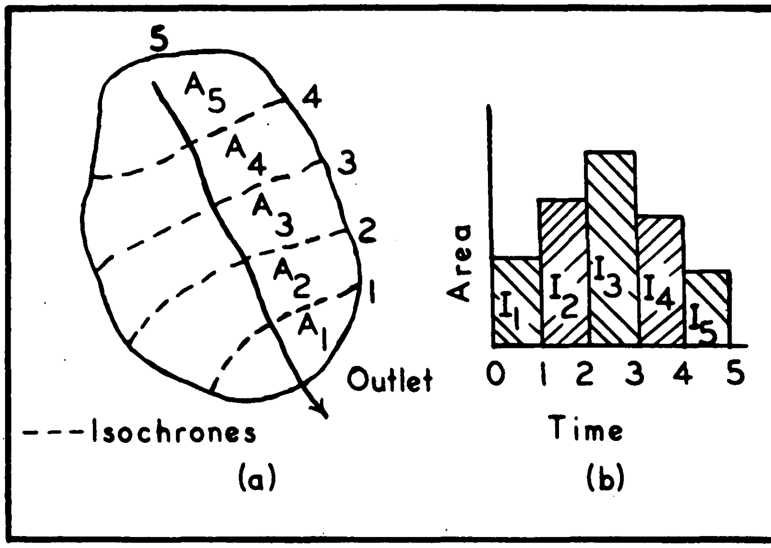

Figure 11. - Distribution of translation and storage effects of a basin.

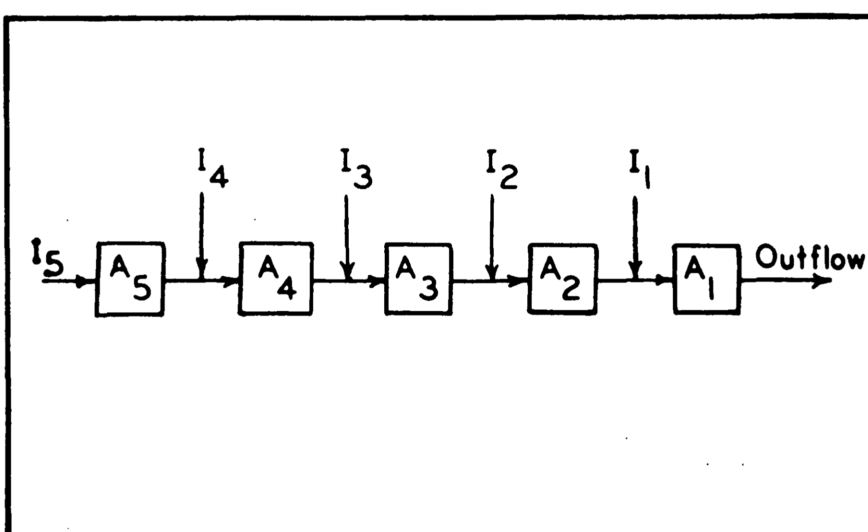

Figure 12. - A lag-and-route procedure.

Furthermore, the storage effect of each of the sub-areas may be unequal. It is thus necessary to assign different admittance functions, A_1, A_2, A_3,------, to these sub-areas. Accordingly, by routing the individual contributions through their respective admittance functions, the resulting outflow hydrograph at the outlet is determined. This lag-and-route procedure is illustrated in figure 12.

Figure 12. - A lag-and-route procedure.

Because of the manner in which the time-area diagram is constructed, such a hydrograph would be the result of an instantaneous rainfall (duration = 0 hr.). Hence, it is called the instantaneous hydrograph. To derive a hydrograph due to longer duration of rainfall, it is necessary to convert the instantaneous time-area diagram into a modified time-area diagram. This procedure firstly requires the subdivision of the drainage basin into a system of isochrones each having a time increment equal to the duration of the rain. An instantaneous time-area diagram is then constructed. Next, the rectangular inflows, I_1, I_2, I_3,------, are modified into a series of isosceles triangles (figure 13) to account for the effect of

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Figure 13. - A modified time-area diagram.

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duration. In like manner, by routing these triangular inflows through the basin system, an outflow hydrograph due to a finite-duration rainfall may be obtained.

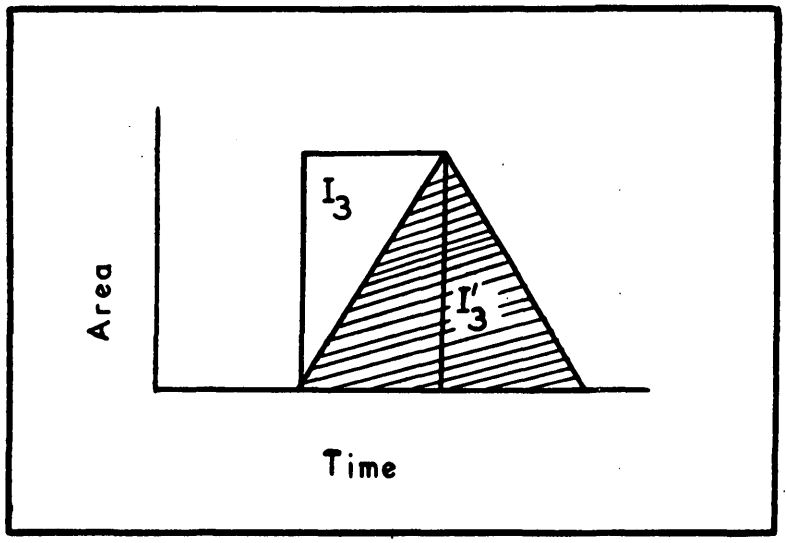

Figure 13. - A modified time-area diagram.

A remarkable simplification can be made if the admittance functions are all linear, since the conversion may be simply achieved by the principle of superposition. Letting u(o,t) be the ordinate of an instantaneous outflow hydrograph and u(D,t) be the ordinate of the corresponding outflow hydrograph having finite duration, D, due to the same amount of rainfall excess, then

$$u(D,t) = \frac{1}{D}\int_{t-D}^{t} u(o,t)dt \; . \qquad (20)$$

In practice, an instantaneous hydrograph is first integrated over a period of time. Then the same integral is delayed by D time. The difference of the two integrals divided by D is then the converted hydrograph u(D,t). This procedure, commonly referred to as the S-curve method, allows the transformation of an instantaneous hydrograph to any finite-duration hydrograph.

The analog system proposed in the foregoing discussions is necessarily a quasi-linear model, since the admittance functions may be linear as well as nonlinear. It can be carried out to varied degrees of refinement in accordance with the climatic and the physiographic features of a drainage basin. Certain simplified approaches are known to have been made. Crawford and Linsley (1962) used a linear reservoir to approximate the overall admittance effect of a small basin. Mitchell (1962) found that two linear characteristic storage functions generally suffice for the descriptions of many small basins in Illinois. In any event, one should not be over-concerned with the complexities that may be involved since the endeavor is largely facilitated through the use of electronic equipment.

SOME ANNOTATED RESULTS DERIVED FROM THE ANALOG-MODEL STUDY

It has been demonstrated that the analog model constitutes an integral part of a hydrologic investigation. Of even greater importance, perhaps, is its ability in assisting one to observe the fundamental behavior of a hydrologic system. It would be gratifying also if one could gain certain insight into some of the significant parameters that could be correlated universally with the physical characteristics of a drainage basin.

Illustrative examples

To illustrate some of the elementary models, figure 14 shows the

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Figure 14. - Outflow hydrographs through a chain of equal reservoirs.

---

routed hydrographs through a chain of 10 equal linear reservoirs from a rectangular inflow diagram.

From the results of figure 14 the storage lag between the centroids of these hydrographs or travel time, λ, is plotted against the number of reservoirs as shown in figure 15. It is interesting to

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Figure 15. - Time lags related to number of reservoirs.

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note that the lag of centroids is equal to the storage constant, K, of the reservoirs. Also shown in the same figure is the lag of peaks, λ_p, for different numbers of reservoirs in the series.

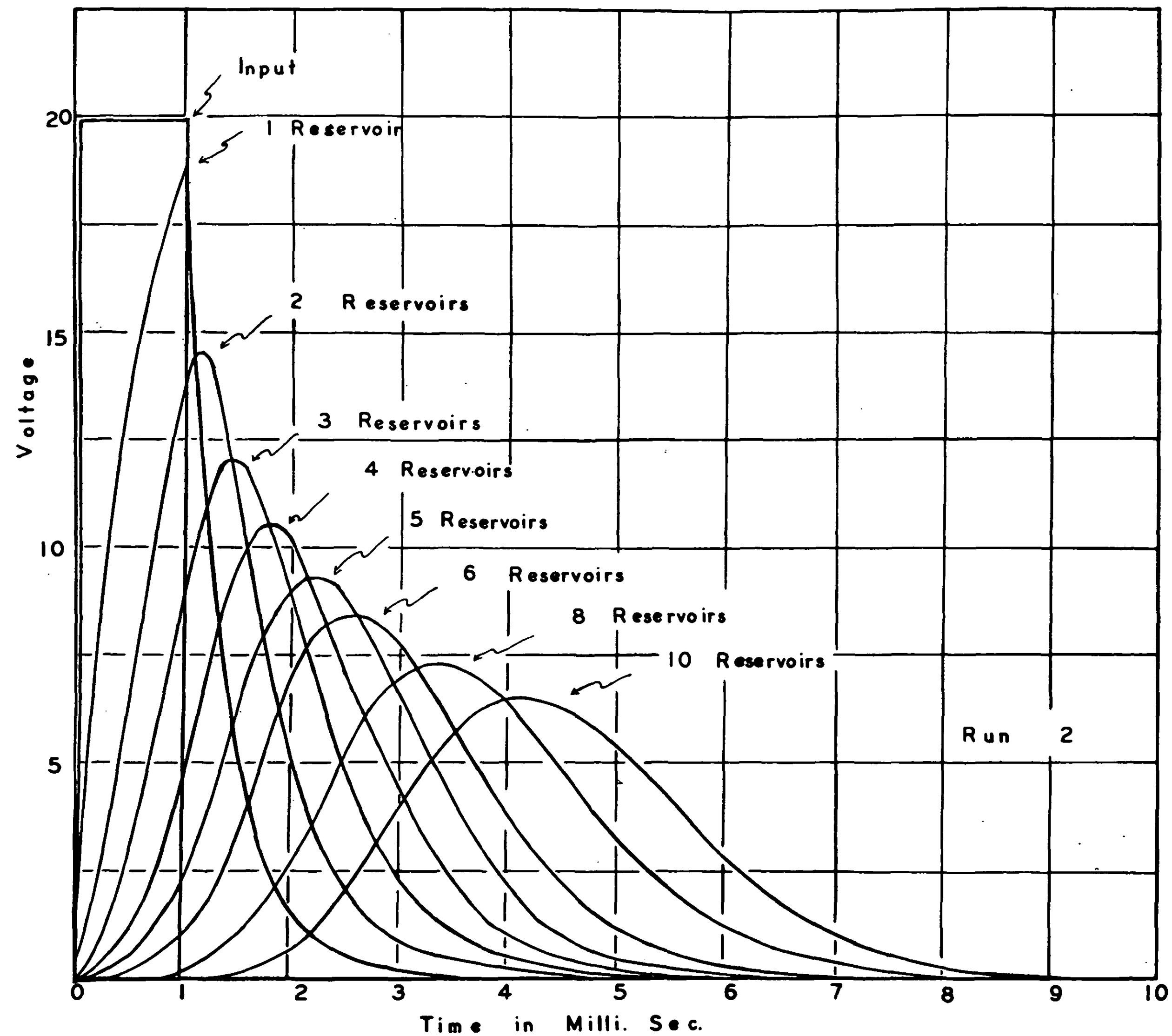

Figure 14. - Outflow hydrographs through a chain of equal reservoirs.

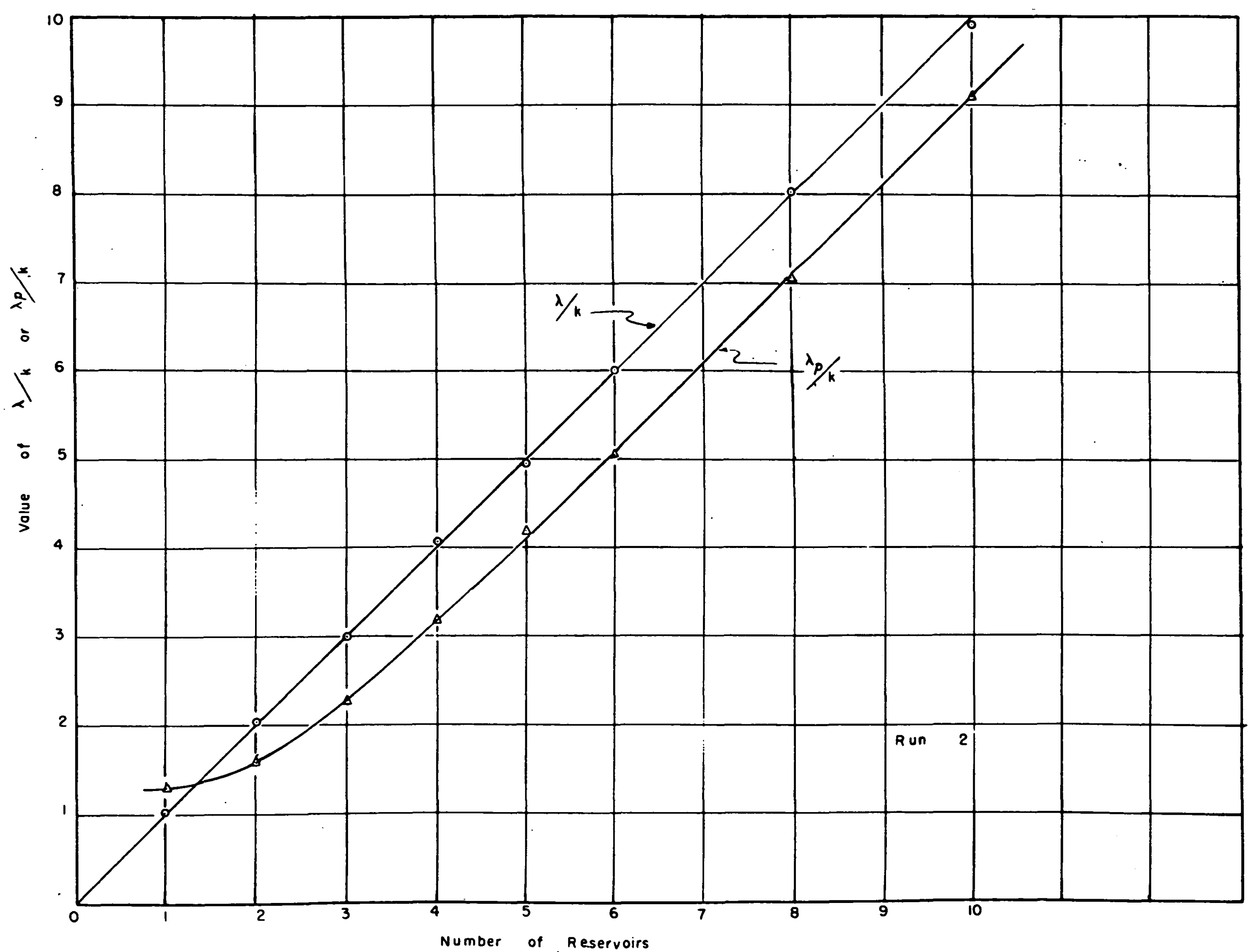

Figure 15. - Time lags related to number of reservoirs.

Accordingly, figure 16 illustrates the reduction in peak mag-

Figure 16. - Reduction in peak size due to a chain of reservoirs.

nitude as the flow is being routed through the chain of reservoirs. It may be observed that the efficiency of peak reduction declines rapidly as the number of reservoirs is increased. Also of interest is the fact that the skewness of these outflow hydrographs tends to decrease in such a systematic fashion that at the end of the tenth reservoir the hydrograph closely approximates a normal curve.

The accuracy derived from the analog models is generally within the tolerable limits for hydrologic studies if high-quality electronic components are used. For example, figure 17 illustrates an outflow

Figure 17. - A typical outflow hydrograph from a linear reservoir, K/T = 0.3.

hydrograph from a linear reservoir for a triangular inflow. In this case, the ratio of the storage constant of the reservoir, K, and the time base of the triangular-inflow diagram, T, is 0.3. For comparison, the hand-computed result is also shown by the dashed line. The difference between the two curves is less than 2 percent everywhere.

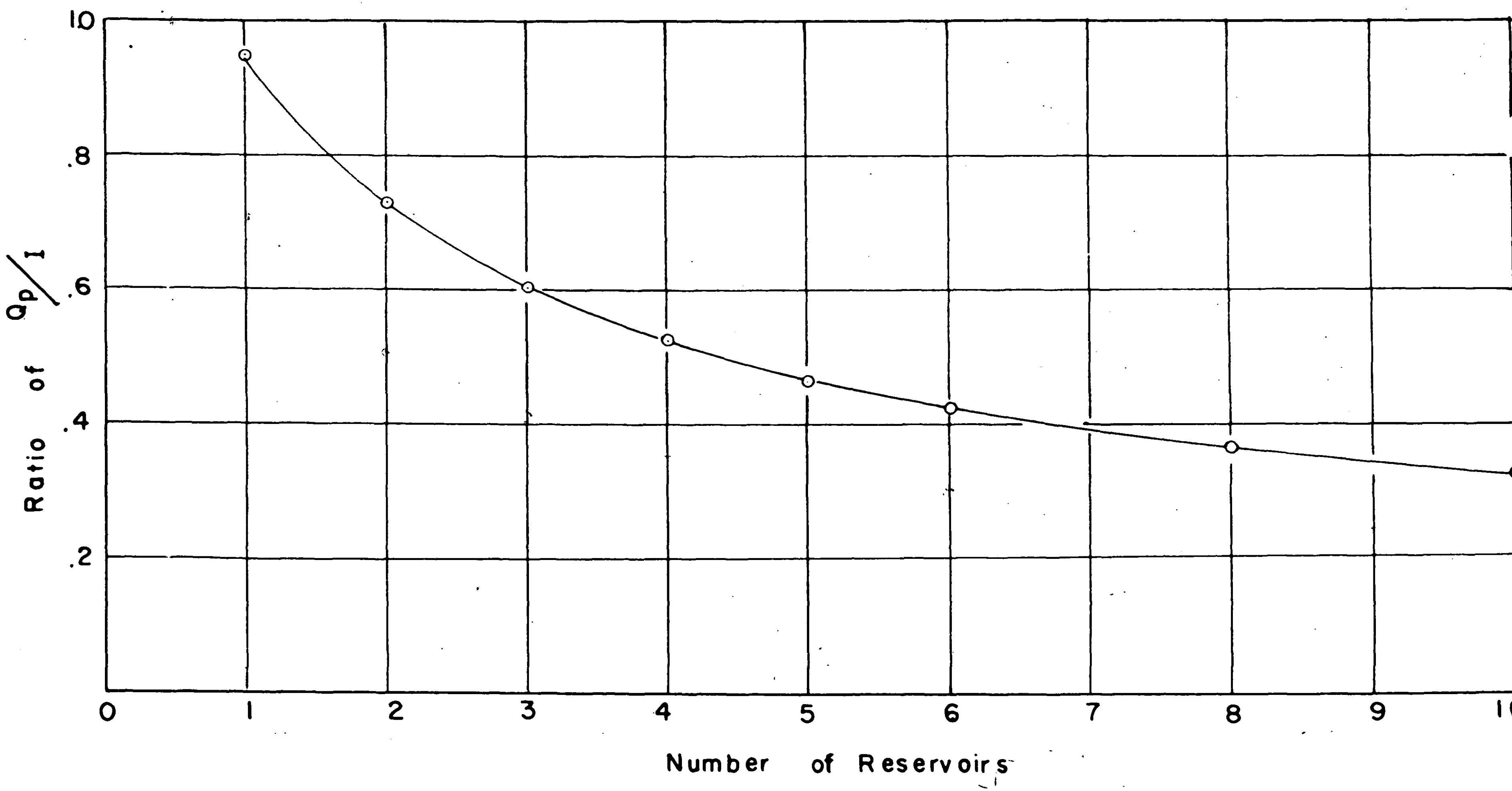

Figure 16. - Reduction in peak size due to a chain of reservoirs.

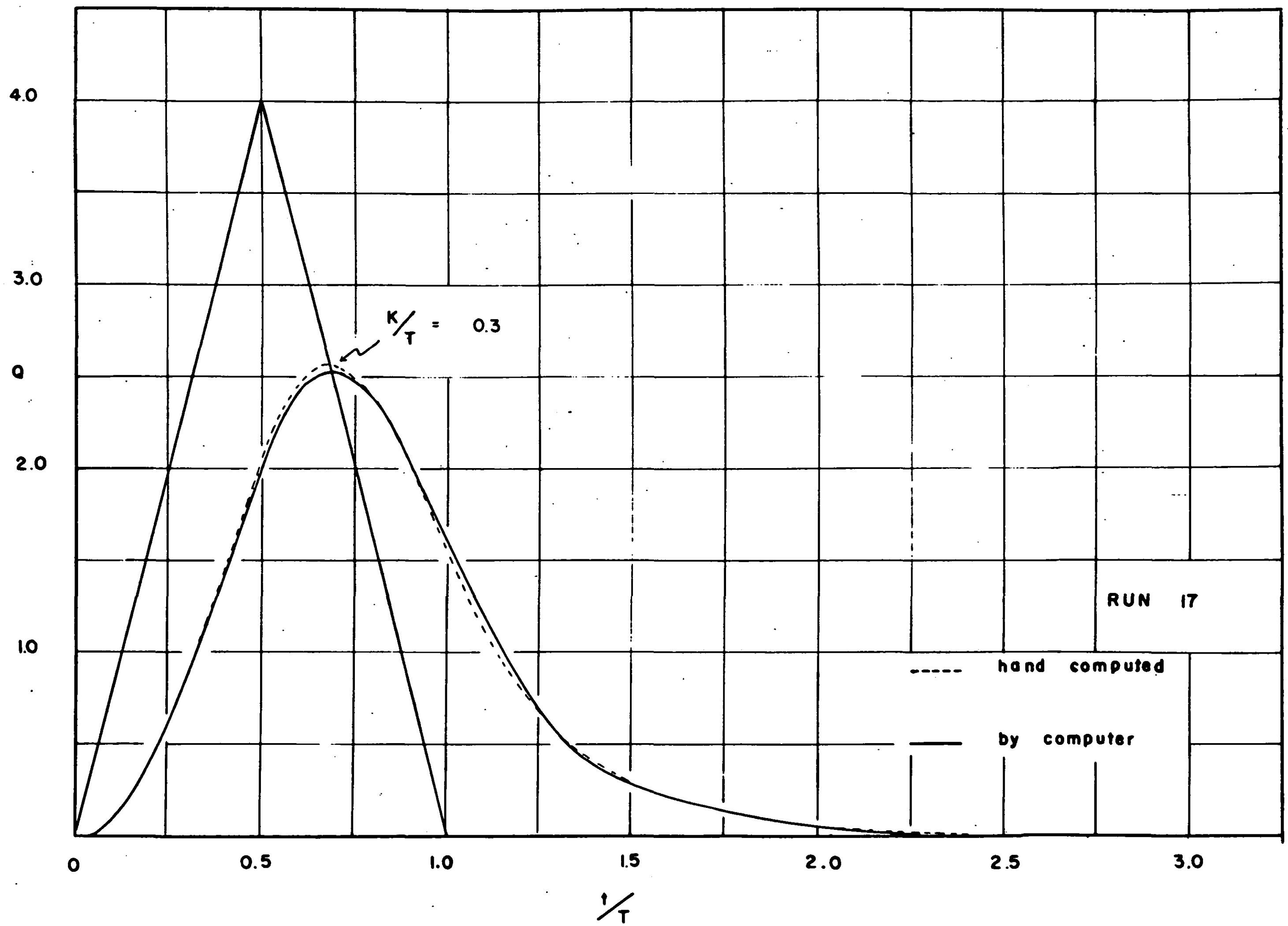

Figure 17. - A typical outflow hydrograph from a linear reservoir, K/T = 0.3.

To illustrate the effect of basin shape on flood runoff, three drainage basins of equal size and slope but of different shape are synthesized: one is rectangular in shape, one is triangular in shape with its apex facing upstream, and, the third is also triangular in shape but with its apex facing downstream. In each case, the basin is divided into 4 sub-areas separated by isochrones. Assuming that uniform instantaneous rainfall occurs over the three basins, the total volume of runoff would be equal for each basin. Furthermore, if the storage constant, K, of each of the sub-areas is proportional to its size, **a**, the ratio of K/**a** would be constant for all cases.

Figures 18 a, b and c depict the resulting hydrographs. Basin b

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Figure 18. - Effect of basin shape on flood runoff.

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appears to have manifested a higher peak when compared with the other two basins. However, the differences are not significant enough to be of any important consequence.

This result is, in fact, not too surprising if one compares notes with Richards (1955). In a hypothetical study, Richards assumed two extreme cases of linearly varying inflow hydrographs: one with a maximum concentration at the beginning and zero at the end, and one with zero concentration at the beginning and a maximum at the end. His results indicate that for ordinary small basins (time of concentration less than 6 hours) the variation in peak discharge due to these two extreme cases is from +13 percent to -20 percent approximately, as compared with the case of a uniformly distributed inflow hydrograph.

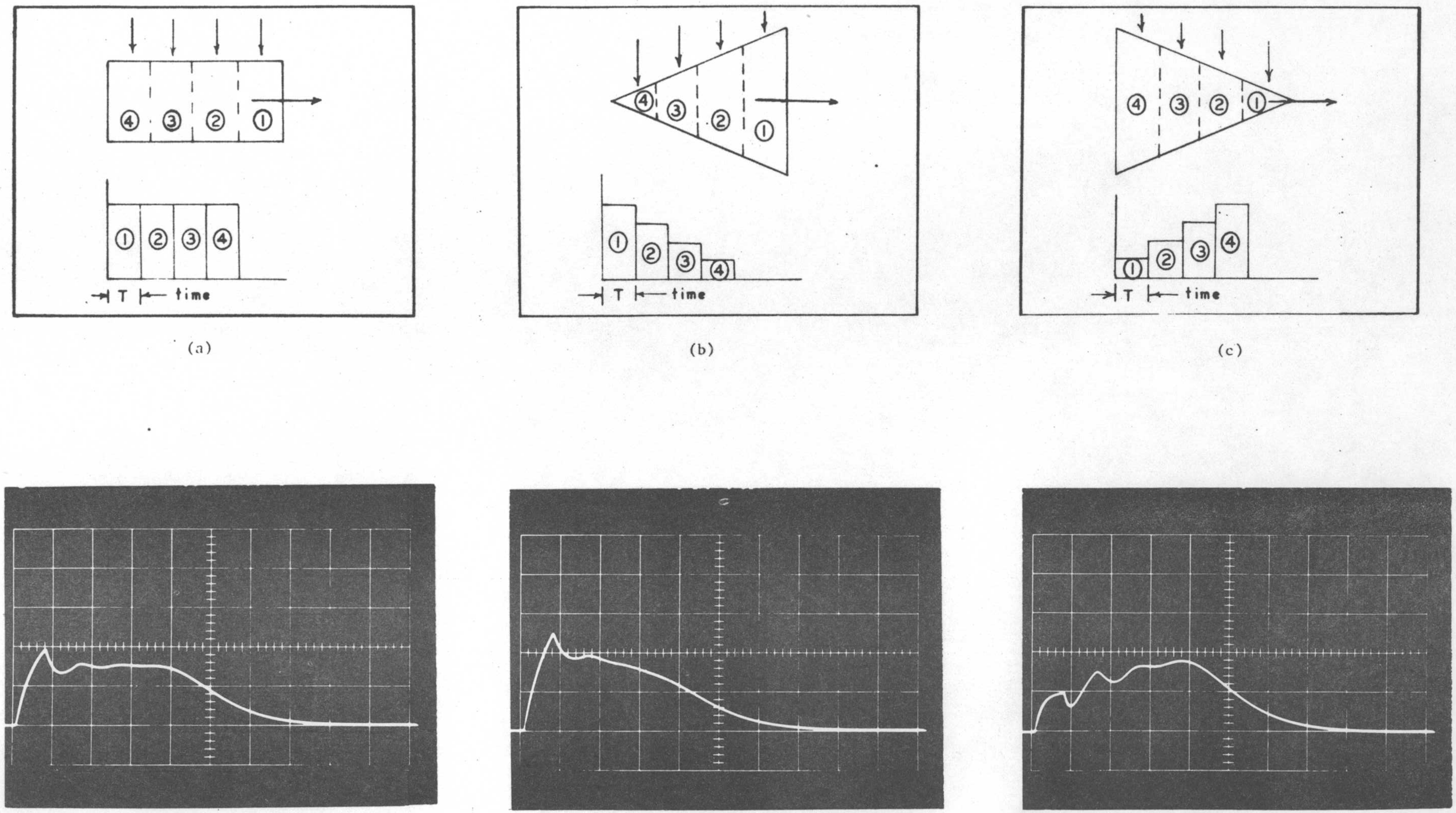


Figure 18. - Effect of basin shape on flood runoff.

A more realistic model perhaps is that shown in figure 19, for

Figure 19. - Hydrographs from a basin having its width increased geometrically in downstream direction.

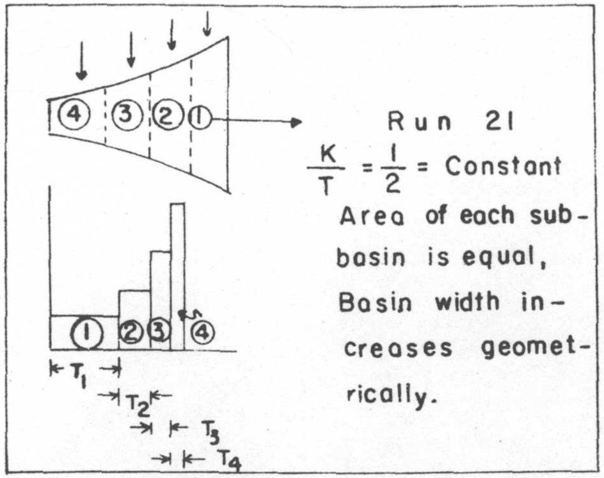
which the basin width is assumed to increase geometrically in the downstream direction. In this case, the basin is divided into 4 sub-areas that are equal in size. Moreover, the slope is assumed to decrease geometrically downstream such that K/T = constant = $\frac{1}{2}$ for each sub-area. The resulting hydrograph is shown as figure 19 a. In order to illustrate the travel of the flood wave, the corresponding hydrographs at various upstream points are also shown in figures 19 b and c.

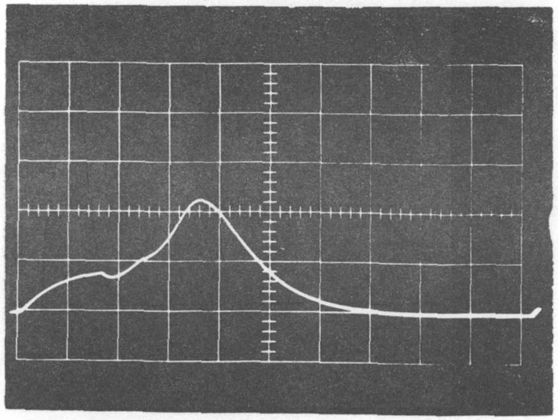
The result shown in figure 19 is intriguing in that it exhibits the case of a flashy mountain stream discharging into a flood plain where large impoundage takes place abruptly.

Storm movement also constitutes an important element in flood runoff. If a rectangular basin system such as the one shown in figure 18 a is subjected to a moving storm in either the upstream or the downstream direction, the resulting patterns of runoff would be greatly different. Figure 20 a illustrates a case in which the storm is moving

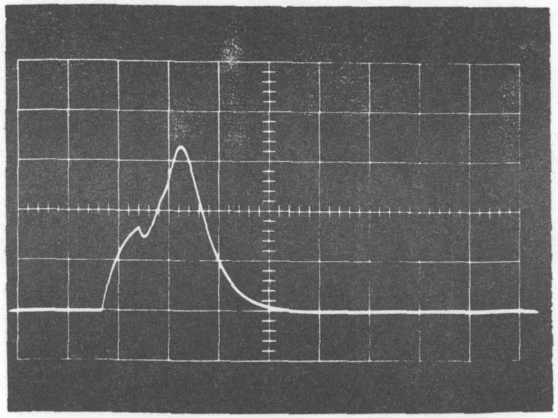
Figure 20. - Effect of storm movement.

upstream at twice the speed of travel of the flow. In like manner, figure 20 b depicts the effect of the same storm when it is moving downstream at an identical speed. It is interesting to note that the downstream movement contributed a much higher rate of flood runoff.

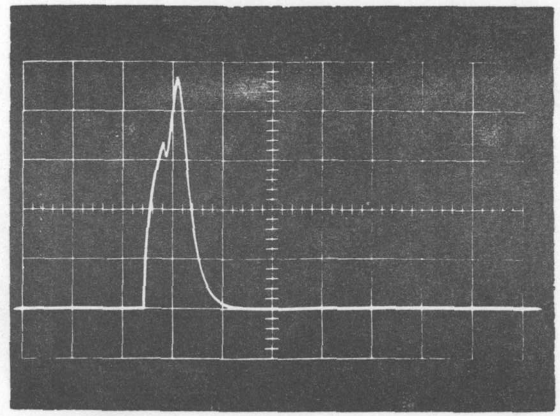




a. At end of sub-basin 1.

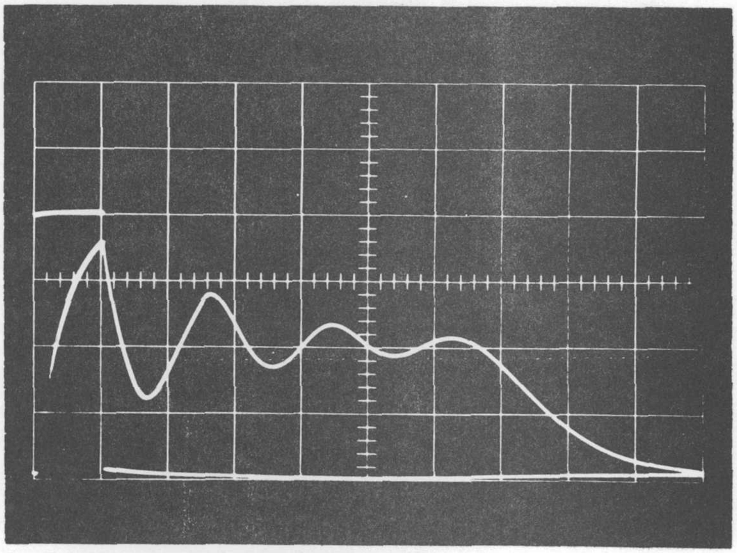


b. At end of sub-basin 2.

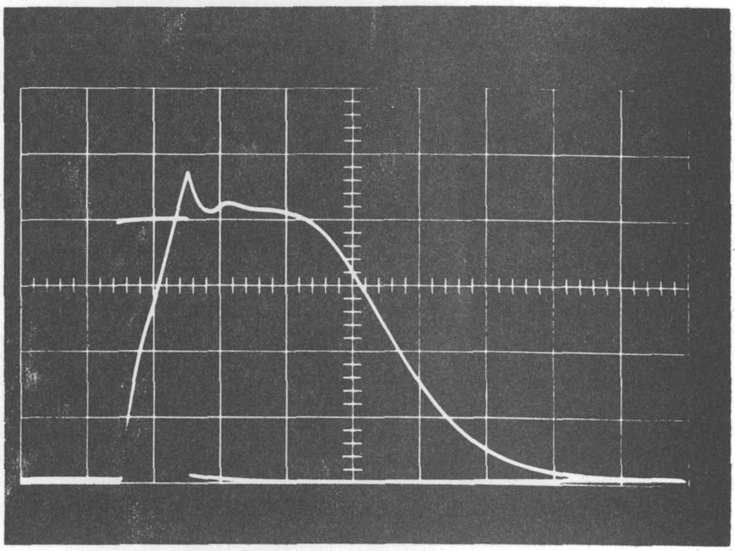


c. At end of sub-basin 3.

Figure **19.** - Hydrographs from a basin having its width increased geometrically in downstream direction.



a. Storm moving upstream.



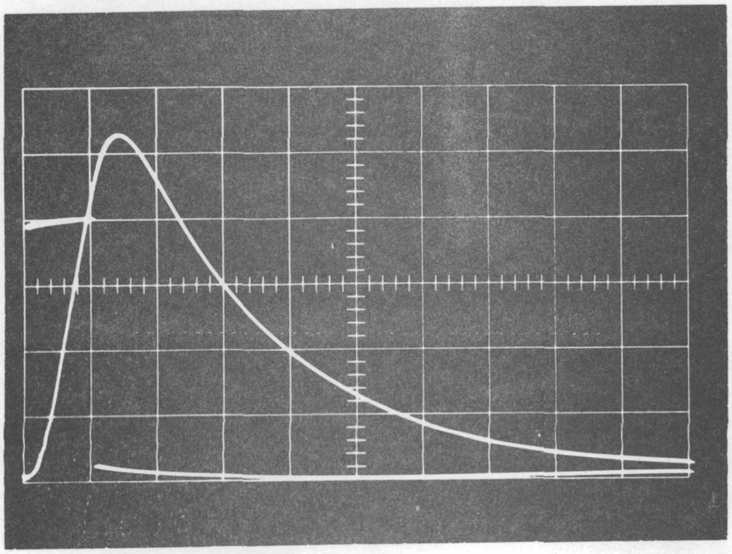
b. Storm moving downstream.

Figure 20. - Effect of storm movement.

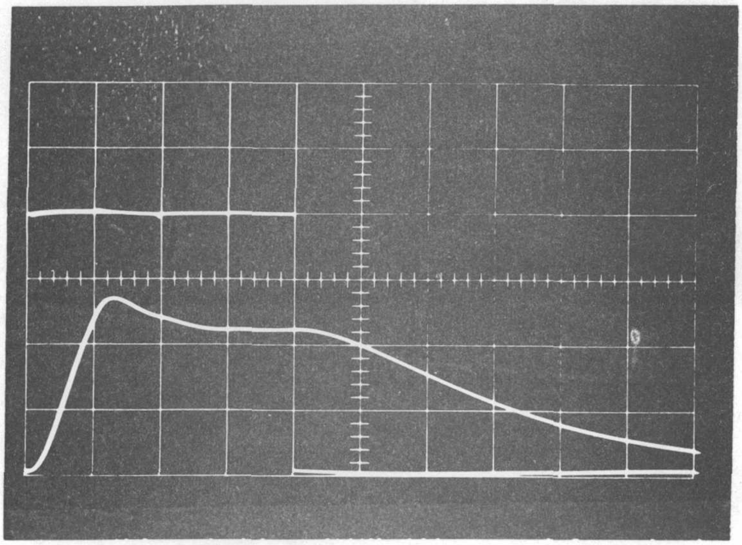
Timing of the tributary inflows is another determinative factor of the magnitude of flood runoff. Assuming that a drainage system consists of 4 tributaries which are equal in size but that their storage constants are in the ratios of 1, 2, 3 and 4, the resulting peak flow would be largely dependent upon the translation time of each of the tributaries. Figure 21-a depicts an extreme

Figure 21. - Composite hydrographs from 4 tributaries.

case for which the tributaries are assumed to have equal translation time. Conversely, figure 21-b shows another case when the tributaries have unequal translation time such that K/T ratios remain constant and equal to $\frac{1}{2}$. It may be seen that the peak flow in case a nearly doubles that in case b due to the same amount of rainfall.



a. Tributaries having equal translation time.



b. Tributaries having unequal translation time.

Figure 21. - Composite hydrograph from 4 tributaries.

Effect of nonlinear storage in a hydrologic system

The foregoing examples illustrate a few simple models which are linear in assumption. If nonlinearity exists, the degree of nonlinearity of a basin would play an important role in the magnitude of the flood runoff. As an illustration, if an isosceles triangle having a peak inflow of 4 units is routed through a linear and a nonlinear reservoir ($S = KQ^2$), the outflow hydrographs would show nearly equal flood peaks, as shown in figure 22 a. However, if the inflow is

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Figure 22. - Comparison of outflow hydrographs, linear and nonlinear reservoirs ($S = KQ^2$).

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amplified by 10 times, the resulting outflow from the nonlinear reservoir would be greatly decreased in relative magnitude, whereas the inflow-outflow ratio remains unchanged for the linear reservoir (figure 22 b). In either case, the storage constants are identical. This example shows that the relation of proportionality cannot be applied to nonlinear elements. A similar example is also shown in figure 23, in which case the reservoir has a nonlinear storage-outflow

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Figure 23. - Comparison of outflow hydrographs, linear and nonlinear reservoirs ($S = KQ^{\frac{1}{2}}$).

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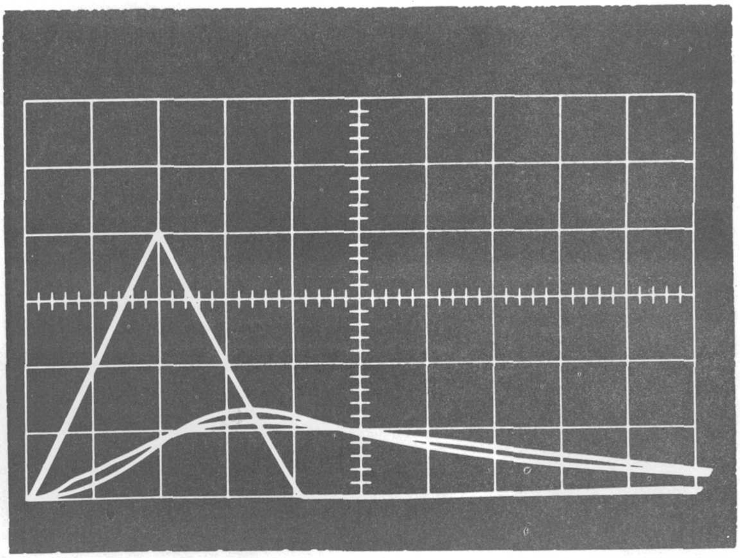
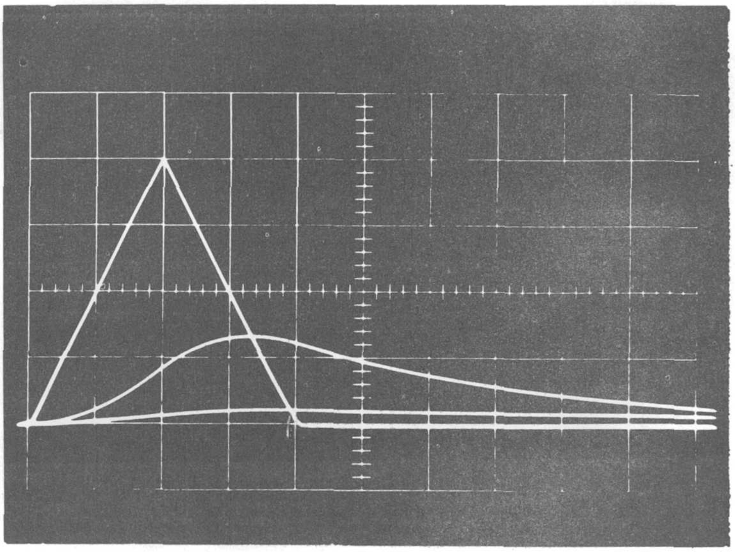
relationship of $S = KQ^{\frac{1}{2}}$, and a peak inflow of 40 units. The upper outflow curve is for the nonlinear case while the lower outflow curve is for the linear one.



a. Peak inflow = 4 units.



b. Peak inflow = 40 units.

Figure 22. - Comparison of outflow hydrographs, linear & nonlinear reservoirs ($S = KQ^2$).

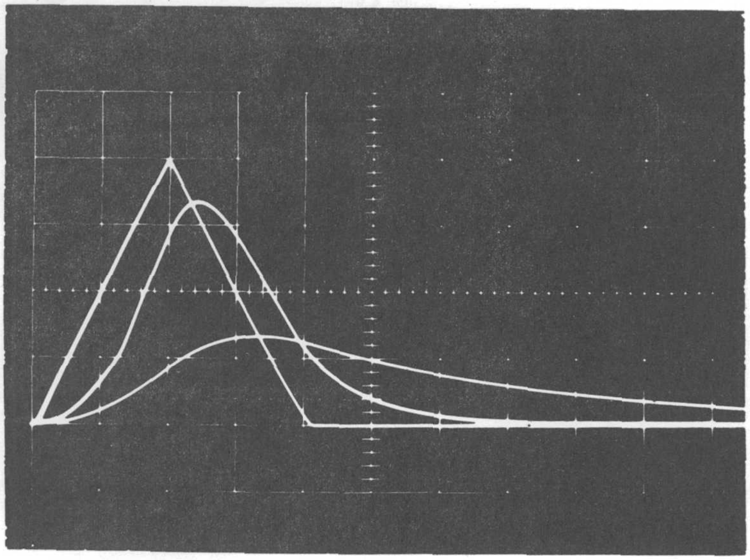


Figure 23. - Comparison of outflow hydrographs, linear and nonlinear reservoirs ($S = KQ^{\frac{1}{2}}$).

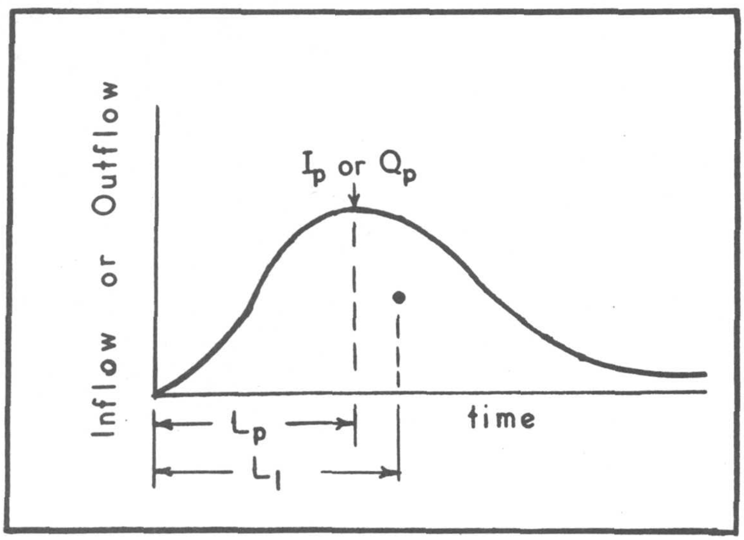


Figure 24. - Definition sketch of a hydrograph in terms of statistical parameters.

The effect of scale on nonlinear reservoirs may be realized if one examines the routing equation (equation 16)

$$I - Q = K\,x\,Q^{x-1}\frac{dQ}{dt}\ .$$

Assuming that the scales of the inflow and the outflow are increased such that $I' = \rho I$ and $Q' = \rho Q$, then the corresponding routing equation becomes

$$\rho(I - Q) = \rho^{x}\,K' x\,Q^{x-1}\frac{dQ}{dt}\ , \qquad (21)$$

in which K' is a modified storage constant. Dividing equation 21 by equation 16, one obtains

$$\rho = (\rho^{x})\,\frac{K'}{K}\ ,$$

or,

$$K' = \frac{K}{\rho^{x-1}}\ . \qquad (22)$$

Thus, the storage constant must be reduced accordingly if the same proportionality is to be maintained. The relation shown in equation 22 is very useful in the design of a nonlinear model. It provides a necessary clue in detecting the degree of nonlinearity in natural basins. A practicable method of determining the values of K and x from the recession hydrograph has also been suggested in another paper (Shen, 1962).

Description of a hydrograph in statistical parameters

A general description of the storage effect on the inflow-outflow relation is perplexed by the fact that the inflow diagram to a drainage system may have unlimited range of variation in time distribution. Thus to derive any universally applicable rule, this effect of time distribution must be adequately accounted for. From an engineering standpoint, however, it is not often of particular importance to know with great accuracy the exact shape of the outflow hydrograph as long as its peak can be determined fairly closely.

Amorocho and Orlob (1961) used a two-parameter gamma distribution to approximate the shape of a unit hydrograph. Edson (1951) and Nash (1957) also proposed similar types of schemes. Mitchell (1962) employed two dimensionless ratios, k/T and t_r/t_o, to account for the time variation of inflow and outflow, in which, T is a characteristic time for a drainage basin; k is a storage constant describing the effect of minor storage upstream from the principal reservoir; t_r is the time of travel through the reservoir; and t_o is the time to the centroid of the outflow hydrograph.

Intuitively, one should be able to describe a hydrograph by means of certain statistical parameters, that is, its mode (M_o), its first moment (L_1), its second moment (L_2), and its coefficient of skewness (S_k). Thus, the mode is equivalent to L_p, and the first moment is equivalent to the time of travel (centroid). The second moment, in conjunction with the first moment, determines the standard deviation (σ), which is a measure of dispersion of the hydrograph. Additionally, the coefficient of skewness is a measure of the skew of the hydrograph. Figure 24 shows the definition

Figure 24. - Definition sketch of a hydrograph in terms of statistical parameters.

sketch of a hydrograph in terms of these parameters. By definition, the standard deviation,

$$\sigma = \sqrt{L_2 - L_1^2}\ , \tag{23}$$

and, the coefficient of skewness (Pearson's),

$$S_K = \frac{L_1 - L_p}{\sigma}\ . \tag{24}$$

Effect of storage on peak flow

In the following analysis, an attempt is made to generalize the effect of linear storage on the peak inflow-outflow relation. Accordingly, large numbers of routings were made on the analog computer to cover a practical range of variation for each of these statistical parameters. The inflow diagrams used in these runs consisted of rectangular-shaped and triangular-shaped as well as other arbitrarily-shaped hydrographs. The corresponding results are presented in dimensionless form.

Figure 25 shows a plotting of the peak-reduction ratio,

Figure 25. - Peak-reduction ratio, Q_p/I_p , as a function of L_1/K.

---

expressed by Q_p/I_p , against the ratio of L_1/K , where K is the storage constant of the respective reservoir. It may be clearly observed that the plotted points prescribe a uniquely defined relationship. In fact, a single curve may be fitted through these points. Whereas all plotted points are within ± 15 percent of this curve, 86 percent of the data are within ± 10 percent.

A second trial is made in a plotting of Q_p/I_p versus σ/K (figure 26).

Figure 26. - Peak-reduction ratio, Q_p/I_p , as a function of σ/K.

The plotting also shows a fairly well-defined relationship, However, the correlation is less pronounced.

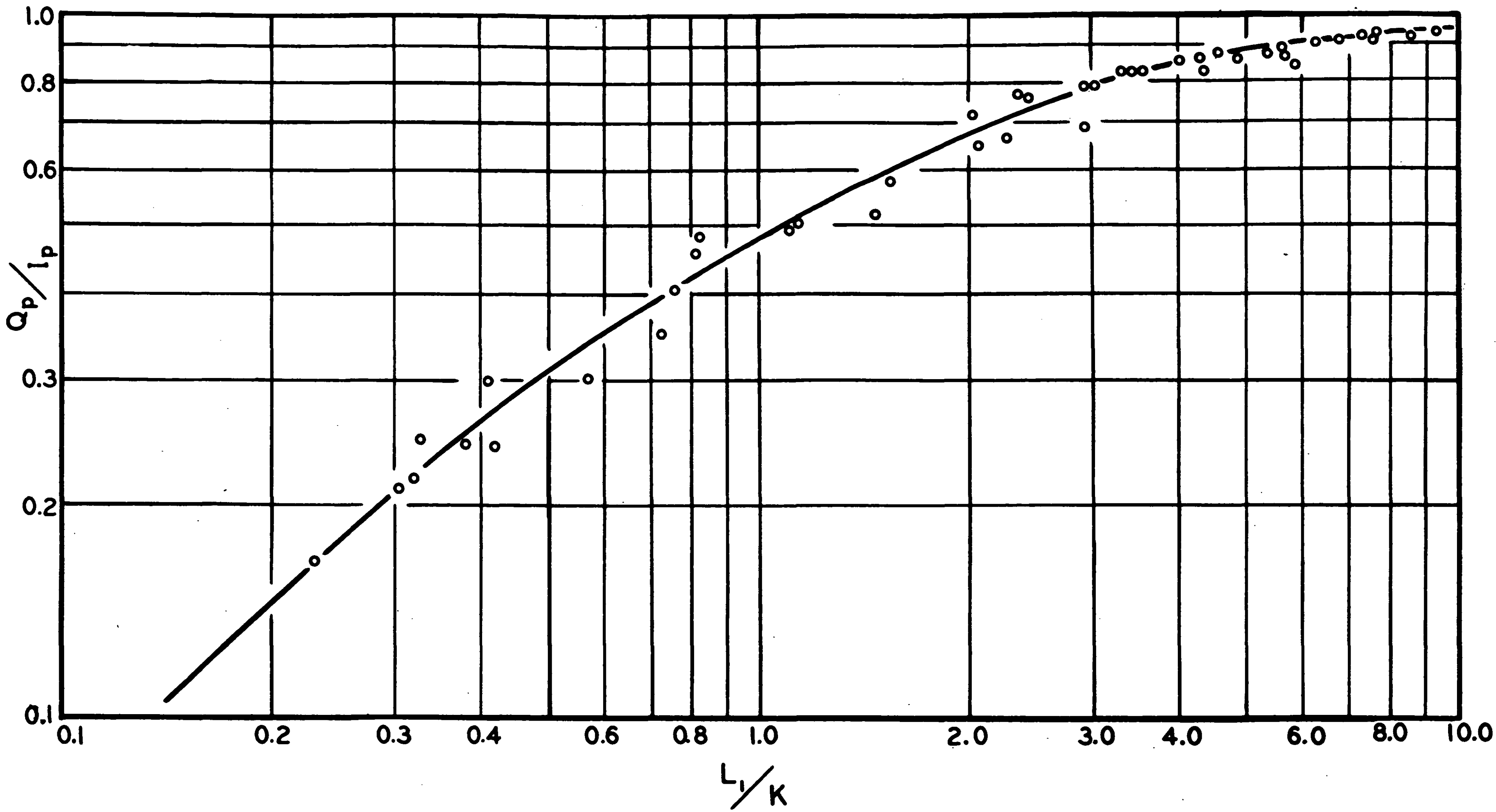


FIGURE 25

PEAK-REDUCTION RATIO, Q_p/I_p, AS A FUNCTION OF L_1/K

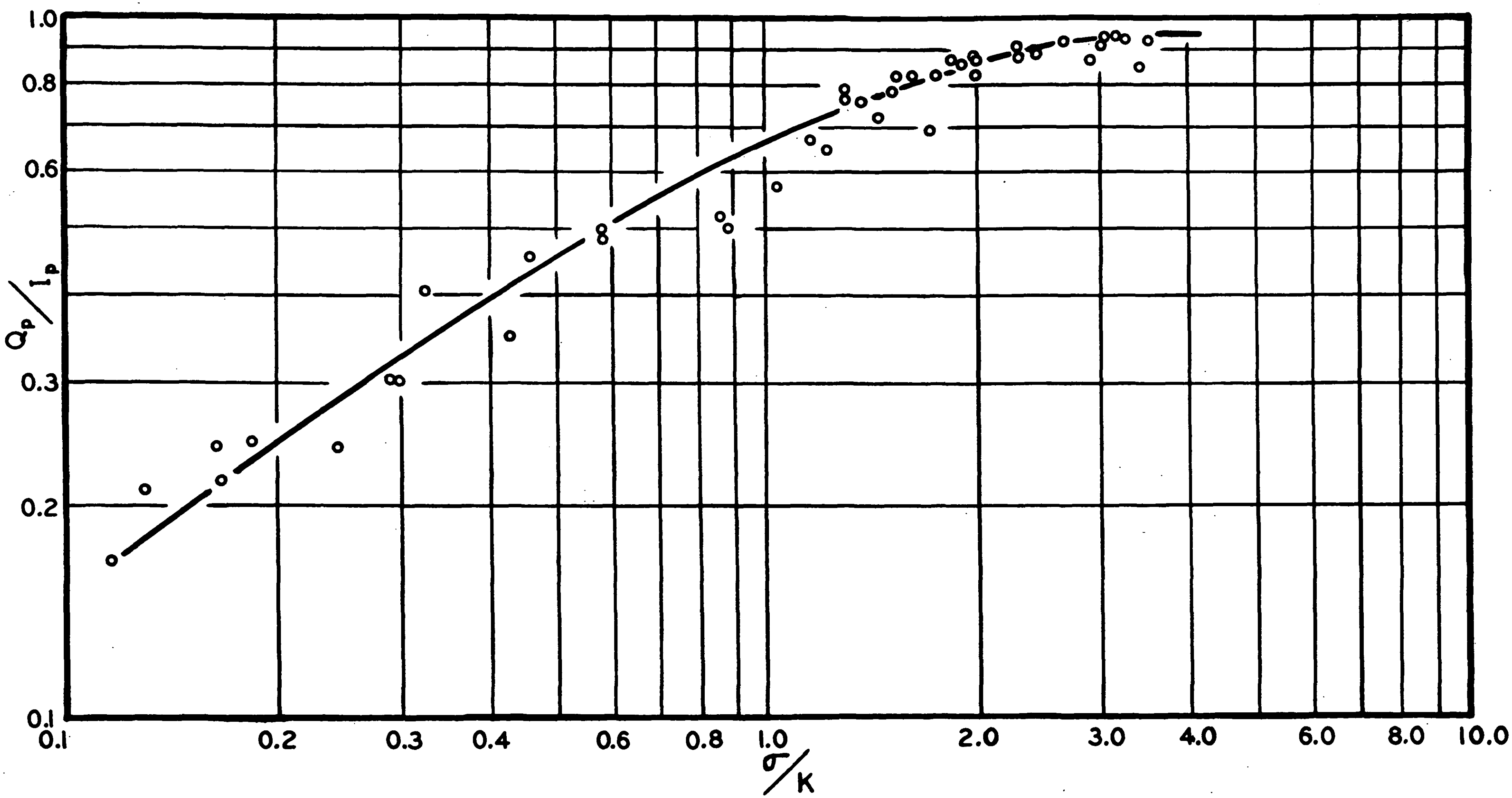


FIGURE 26

PEAK – REDUCTION RATIO, Q_p/I_p, AS A FUNCTION OF σ/K

In a similar manner, correlations with the other ratios of L_2/K and L_p/K have been tried. However, these plottings indicate a considerable amount of scatter and are thus not presented. Further attempts have also been made to improve the relation between Q_p/I_p and L_1/K by correlating its scatter with the skewness, S_k, as well as the dispersion, σ/K, of the inflow hydrographs. The ranges of S_k and σ/K covered in these trials are 0 - 0.78 and 0.065 - 3.50 respectively. The lack of correlation in either case indicates that no further improvement can be made.

Comparison with Mitchell's formula

On the basis of 128 synthetic outflow hydrographs, Mitchell derived a generalized expression (Mitchell, 1962, p. 22) for the ratio of peak inflow and outflow, I_p/Q_p, as

$$I_p/Q_p = 1 + [7.0 - 2.5(k/T)](t_r/t_o)^2, \tag{25}$$

in which, k, T, t_r and t_o are the basin parameters defined previously. The above relationship was obtained by routing consecutively an isosceles dimensionless hydrograph through a preliminary linear storage, k, and a principal linear storage, K, with consideration given to different durations of rainfall excess. Equation 25 may also be expressed as a family of curves correlating I_p/Q_p and t_r/t_o for different values of k/T.

In comparing Mitchell's formula with the general curve of Q_p/I_p versus L_1/K (figure 25), a number of routings have also been made on the analog computer. These runs were arranged in accordance with the parameters established by Mitchell. For each run, an isosceles inflow diagram was routed first through a preliminary storage and then a principal storage. Hence, the ratios of k/T, t_r/t_o, and L_1/K were determined. Accordingly, the value of I_p/Q_p, and its reciprocal, Q_p/I_p, were computed from equation 25.

Figure 27 shows a comparison of the computed values of Q_p/I_p

Figure 27. - Comparison of Mitchell's formula with figure 25.

with the curve shown in figure 25. In general the plotted points appear to have shown good agreement with this curve.

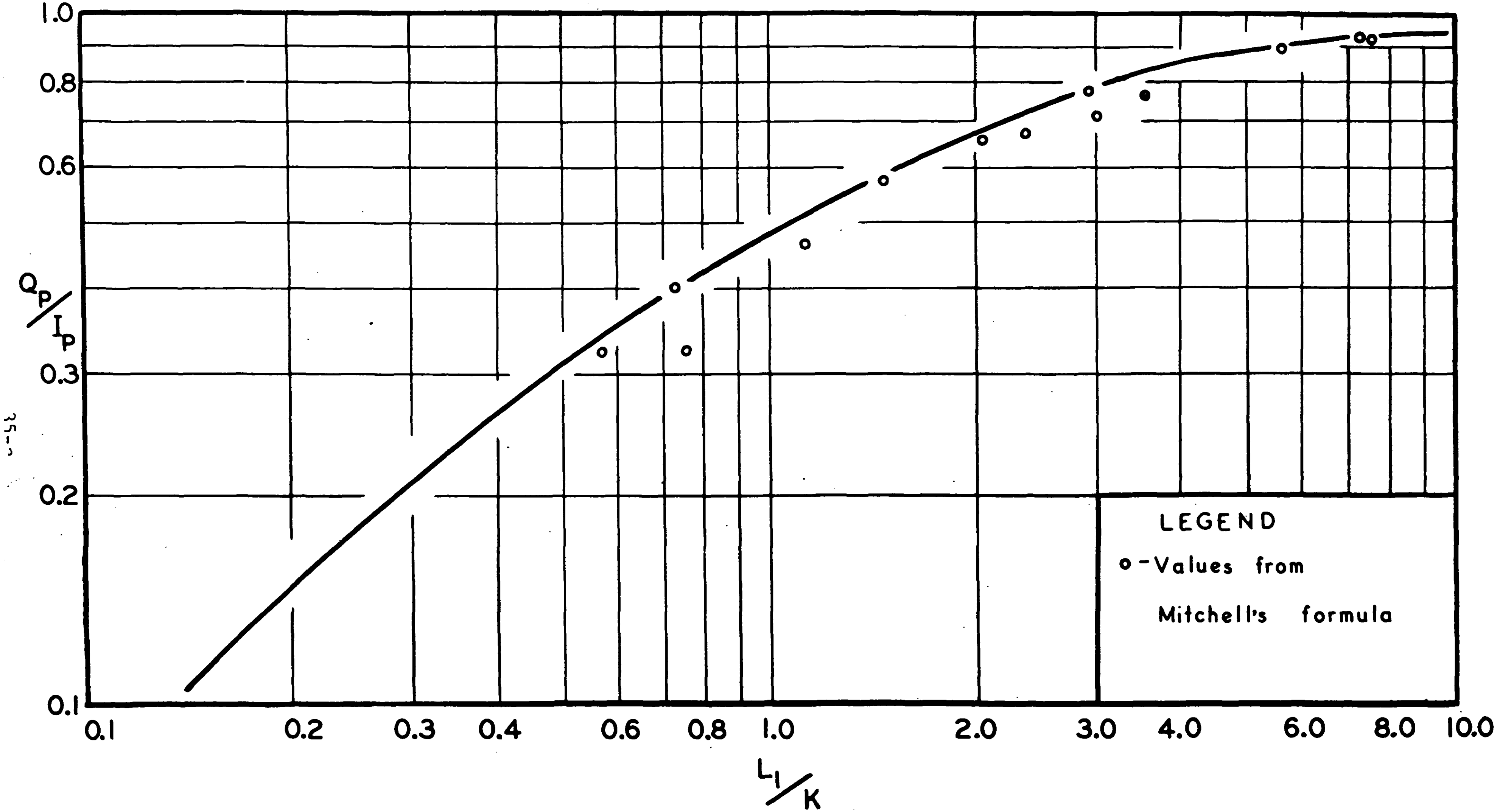


FIGURE 27

COMPARISON OF MITCHELL'S FORMULA WITH FIGURE 25

Thus, the general relationship of Q_p/I_p versus L_1/K shown in figure 25 is believed to be valid within the range of parameters tested. It provides a simple and yet useful tool for estimating the outflow peaks from the inflow hydrographs if the reservoir is linear. Moreover, it is applicable irrespective of the shape of the inflow hydrographs. It should be borne in mind, however, that this relationship can be used only for simple hydrographs, that is, single-peaked hydrographs. For multiple-peaked hydrographs, it may be necessary to undergo a separation procedure before this relationship can be applied.

From a practical point of view, the independent parameter, L_1, would be related to the physical as well as the climatical characteristics of a basin. Mitchell (1962) suggested that L_1, or t_i in his notation, is a dependent function of the rainfall duration, D, and the preliminary storage constant, k. Furthermore, he found that the variation in the ratio of k/T is generally insignificant, thus a simplified relation in dimensionless form may be expressed as

$$\frac{L_1}{T} = 1 + 0.7\left(\frac{D}{T}\right). \tag{26}$$

ANALOG-MODEL STUDY OF FLOOD-FREQUENCY DISTRIBUTION

One of the fundamental problems in hydrology is defining the probability distribution of runoff. The traditional approach to this problem has been based on a procedure involving the fitting of various probability distributions to the observed data of runoff. Frequently, this procedure met with only limited success owing to the lack of sufficient data needed by a statistical analysis. Nevertheless, these studies have set forth a basic understanding that the probability distribution of runoff is a function of the probability distribution of rainfall and the runoff-producing characteristics of a river basin. A few outstanding examples are cited as follows:

By means of multiple correlation, Benson (1962) has derived formulas for New England correlating the flood magnitude at different return periods with the drainage-area size, a main-channel slope index, a precipitation intensity-frequency factor, a winter-temperature index and an orographic factor.

On the basis of the rational method, Snyder (1958) has developed a procedure for computing the probability of flood discharge from the given rainfall-duration frequency distributions of specific drainage basins. Empirical coefficients were derived for the Washington, D. C. area to account for the runoff-producing characteristics of area, length, slope, friction and shape that are associated with the overland-flow areas as well as the sewered areas. The effects of rainfall duration and basin storage were also incorporated as a correction factor.

Paulhus and Miller (1957) employed a system of index precipitation networks in synthesizing the flood-frequency characteristics of a group of basins from the rainfall data. The procedure involves the use of a typical unit hydrograph for each basin and an empirically derived adjustment curve to account for the variations in flood magnitude and base flow.

Analyses of the influence of the basin characteristics on the rainfall-runoff probability distributions are handicapped by various sampling problems. It is seldom possible to select a sufficient number of identical drainage basins in the field. Moreover, the rainfall pattern is generally different within each basin. Thus, the principal problem is that of overcoming the sampling difficulties. An approach to this problem is offered by the use of synthetic models.

Assuming that there is a stationary, random signal generator which simulates a known frequency distribution of the inflow or rainfall excess, then by feeding the signal into a given basin model, one can examine the output from the system and analyze its frequency distribution. In order to reproduce the complete characteristics of an input distribution however, one must be able to simulate its duration distribution (flow volume) as well as its peak distribution. Thus, a complete scheme would be one such as that shown in figure 28.

Figure 28. - Schematic diagram of the frequency and duration analyzer.

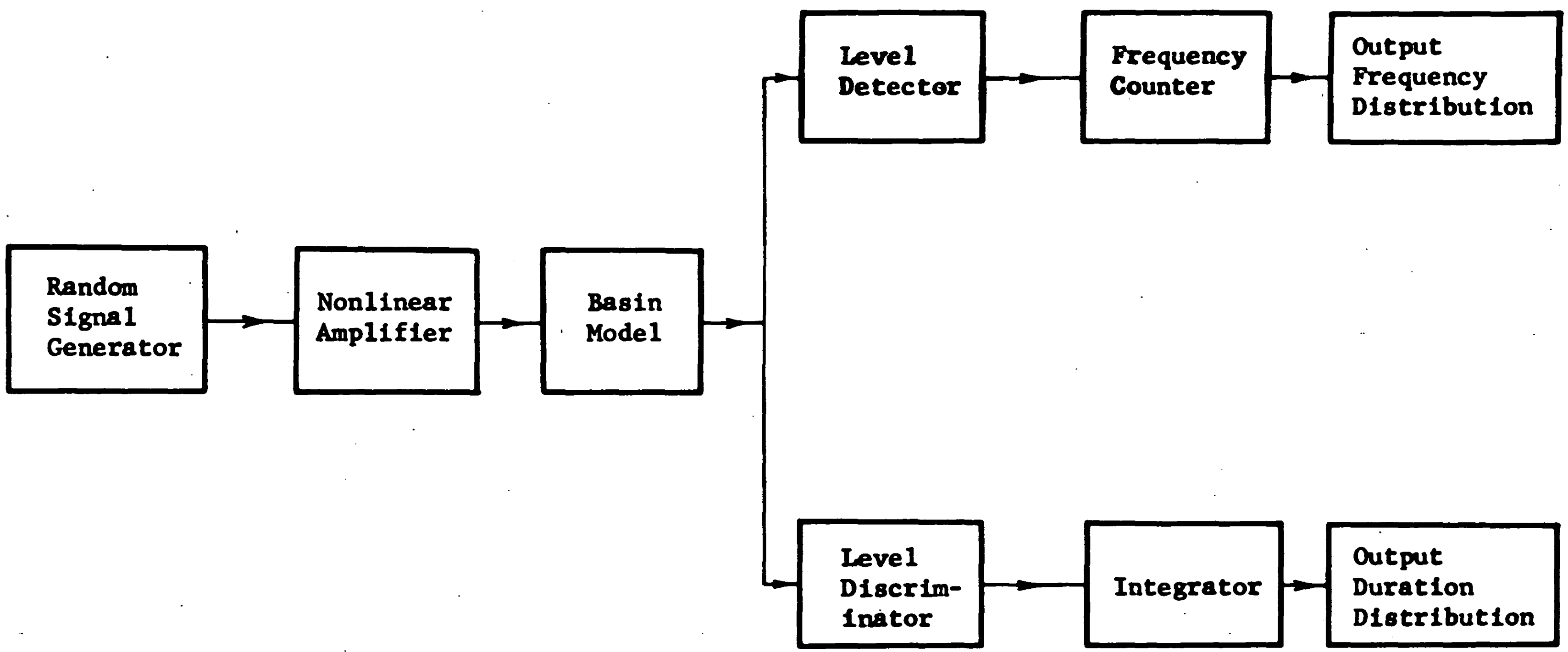


Figure 28. - Schematic diagram of the frequency and duration analyzer.

Accordingly, three basic electronic components have been constructed consisting of: a frequency synthesizer which produces random signals having various types of peak distribution; a frequency analyzer which determines the frequency of occurrence of the peaks above a certain level of magnitude; and, a duration analyzer which determines the percentage in time during which the signal equals or exceeds a certain level.

As a simple illustration, figure 29 shows the appearance of the

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Figure 29. - Random input and output through a linear reservoir.

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input and output signals through a linear reservoir, for which the input has a normal distribution. Accordingly, figure 30 shows the

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Figure 30. - Probability distribution of inflow and outflow, linear reservoir.

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probability distributions of the simulated peak inflow and outflow as plotted on a piece of normal-probability paper. It is interesting to note that both distributions show the appearance of straight lines which indicate that the output is again normally distributed.

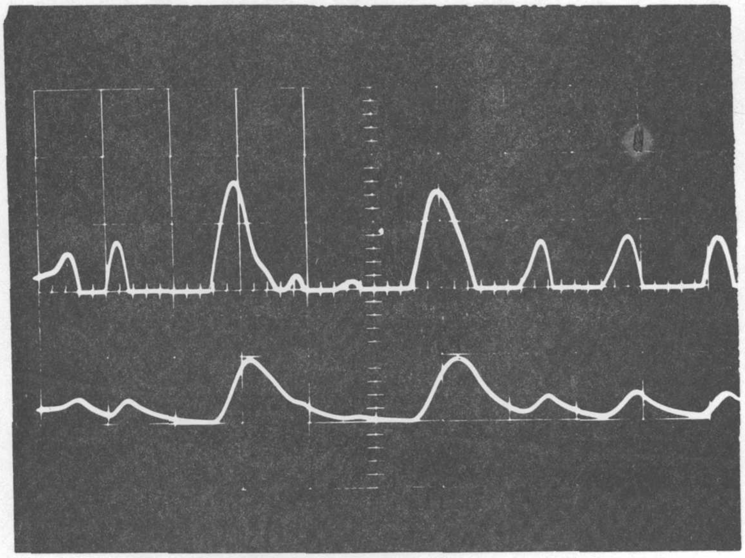

Figure 29. - Random input and output through a linear reservoir.

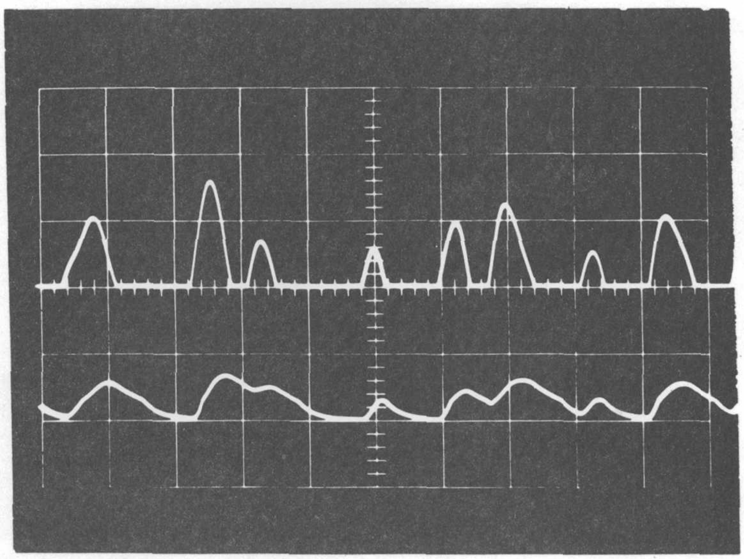

Figure 31. - Random input and output through a nonlinear reservoir, $S = KQ^2$.

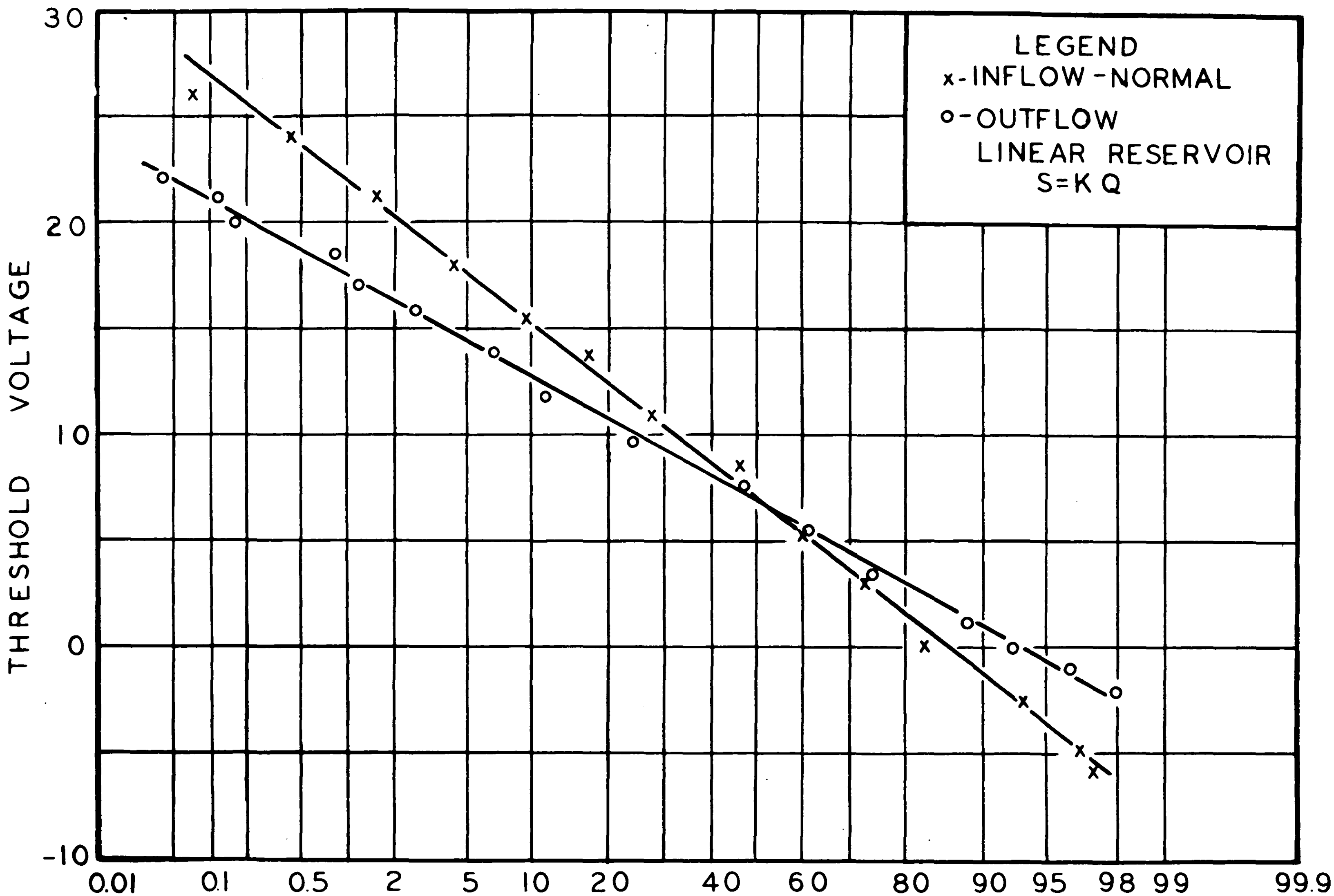

FIGURE 30-PROBABILITY DISTRIBUTIONS OF INFLOW AND OUTFLOW, LINEAR RESERVOIR

On the other hand, if the same input signal is fed into a nonlinear reservoir (figure 31), for which $S = KQ^2$, one would observe

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Figure 31. - Random input and output through a nonlinear reservoir, $S = KQ^2$.

---

that the frequency distribution of the peak outflow becomes skewed (figure 32).

Figure 32. - Probability distribution of inflow and outflow, nonlinear reservoir.

In a similar manner, figure 33 depicts the distribution of an

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Figure 33. - Probability distribution of inflow and outflow, linear reservoir, log-normal input.

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input signal that is log-normally distributed. Again, the output from a linear system shows a likely log-normal distribution.

The foregoing illustrations indicate that a linear-basin system would not alter the type of probability distribution of its inflow whereas a nonlinear-basin system would. The levels of magnitude are, of course, modified in either case.

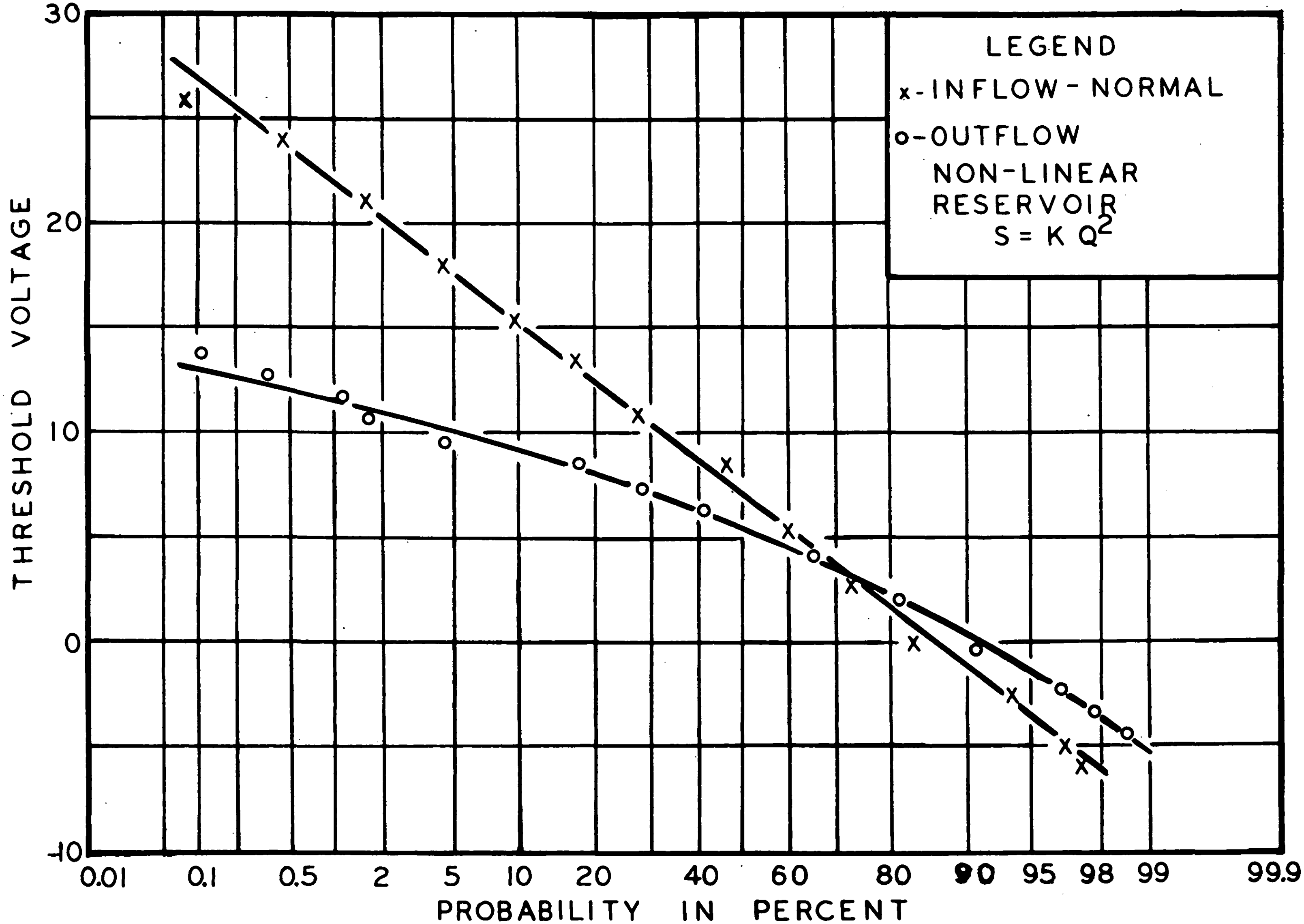

FIGURE 32-PROBABILITY DISTRIBUTION OF INFLOW AND OUTFLOW, NON-LINEAR RESERVOIR

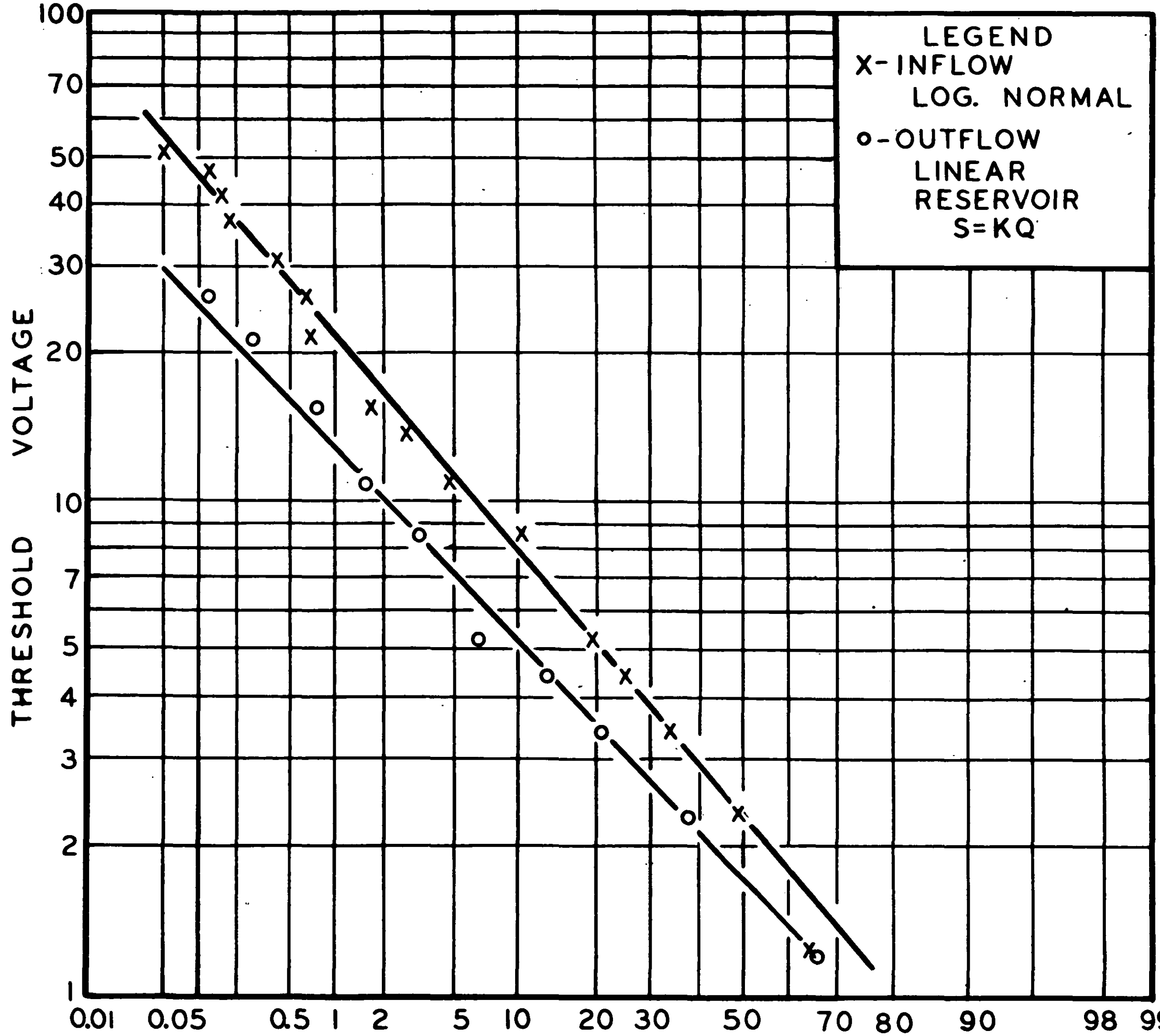

FIGURE 33-PROBABILITY DISTRIBUTION OF INFLOW AND OUTFLOW LINEAR RESERVOIR, LOG.-NORMAL INFLOW

The invariance of probability distribution with respect to a linear process is expected to be applicable to the distribution of flow volume or the duration distribution. To illustrate, figure 34

Figure 34. - Duration distributions of inflow and outflow, linear reservoir.

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depicts a case in which the simulated inflow to a linear reservoir has a log-normal duration distribution. For the curve shown, the inflow at different levels is plotted against the percent in time during which the flow is less than that indicated. It is seen that the outflow exhibits another log-normal distribution. Furthermore, the two distributions have equal mean values of discharge at 20.4. The standard deviations are 6.38 and 2.95 for the inflow and outflow respectively. Thus, the effect of a linear reservoir upon inflow is to reduce its variance of distribution.

The foregoing examples illustrate, in principle, that the relation of inflow and outflow can be described by an analysis of duration distribution and an analysis of peak distribution. Thus, for a particular problem, it would be necessary to simulate these two properties individually such that the inflow characteristics may be completely represented. The techniques described may, of course, be applied to more complex drainage systems as well.

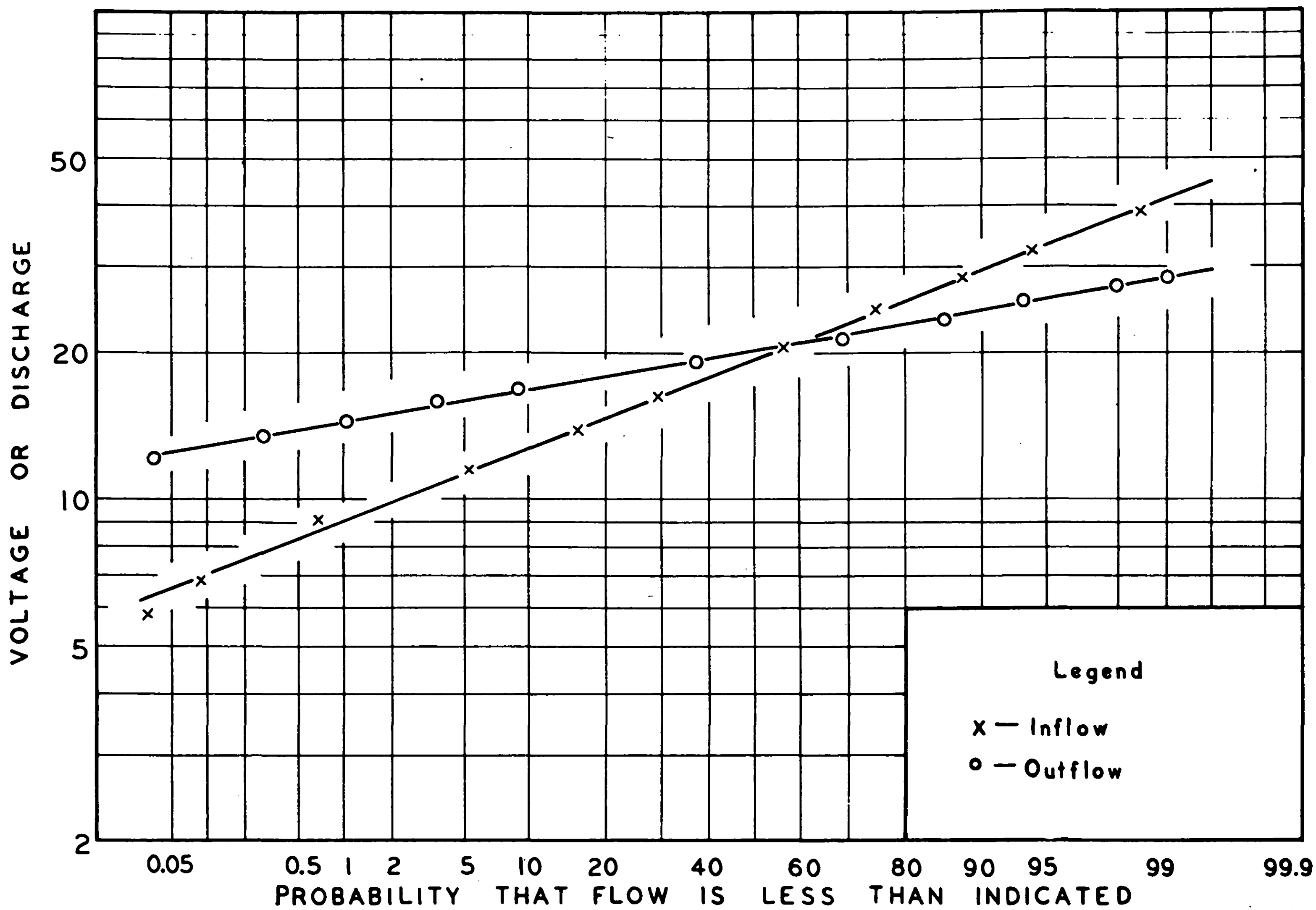

FIGURE 34.- DURATION DISTRIBUTIONS OF INFLOW AND OUTFLOW, LINEAR RESERVOIR.

Mathematical treatment, in particular the power-spectrum analysis, of the linear transformation on a stationary random process is well known in the field of communication. In which case, the power spectrum of the output, $P_o(f)$, can be simply related to the power spectrum of the input, $P_i(f)$, by

$$P_o(f) = P_i(f)|W(f)|^2, \tag{27a}$$

where W(f) is the transfer admittance of the linear system. Accordingly, the variance of the output may be determined as

$$\sigma_o^2 = \int P_i(f)|W(f)|^2 df. \tag{27b}$$

For nonlinear systems, however, the process becomes rather involved--for example, the work of Wiener (1942) and George (1959). A more rigorous treatment of this type of problem was offered by Langbein (1958). Using the queuing theory, Langbein demonstrated a technique of probability routing pertinent to reservoir storage analyses. The method is nonparametric in that it is unaffected by the kind of frequency distribution back of the probabilities. It is valid for nonlinear as well as linear reservoirs.

Thus the principal advantage of using the analog technique in flood-frequency analysis lies in its simplicity in dealing with the more complex drainage systems when the mathematical process becomes too complicated. Once a system model is constructed, the inflow-outflow frequency relationship can be readily determined. Its ability is virtually unlimited.

SUMMARY AND OUTLOOK

One of the fundamental questions that often arises in hydrology today is: "What are the effects of man-made changes on the frequency distribution of flood flows?" This is the kind of question that has not been satisfactorily answered by field investigations. It is also in this area that the analog technique is believed to be able to play a very important role.

The primary objective of this study is to bridge some of the missing links encountered in field studies. One specific example is that of determining the effect of urbanization on the frequency and magnitude of peak flow. Carter (1961), in a study on the magnitude and frequency of floods in suburban Washington, D. C. areas, found that the lag time, τ, between rainfall excess and the flood hydrograph is decreased in the ratio of 1.20/3.10 because of storm sewers and improvements to the principal stream channels. Formulas were developed to express τ in terms of the total length of the channel and a weighted slope for the developed and underveloped areas. Furthermore, using 18 streams in the area, he has derived a regression equation correlating the annual flood, $\bar{Q}$, with τ and the drainage area, A, expressed in square miles. It is

$$\frac{\bar{Q}}{F} = 223\, A^{0.85}\, \tau^{-0.45}, \tag{28}$$

where F is a factor accounting for the increase in peak due to the percentage increase in the impervious area. Accordingly, if the effect of imperviousness is first accounted for, the variation in peak magnitude would be inversely proportional to the 0.45th power of the change in τ due to suburban development:

$$\frac{\bar{Q}'}{\bar{Q}} \propto \left(\frac{\tau'}{\tau}\right)^{-0.45} \tag{29}$$

Carter's conclusion is extremely interesting in that it appears to concur with some of the results derived from this study. Assuming that uniform rainfall of a short duration occurs over a drainage basin before and after the change, then the inflow time-area diagrams could be approximately represented by isosceles triangles (Mitchell, 1962; Snyder, 1958). Accordingly, for an equal volume of rainfall excess, the peak inflows, I_p, would be inversely proportional to the first moments, L_1, of the triangular diagrams. Hence,

$$\frac{I_p'}{I_p} = \frac{L_1}{L_1'} \; . \tag{30}$$

Furthermore, from the general curve in figure 25, an average slope of 0.56 may be obtained; that is

$$\frac{Q_p'/I_p'}{Q_p/I_p} \propto \left(\frac{L_1'/K}{L_1/K}\right)^{0.56} \; . \tag{31}$$

Assuming that the storage effect, K, of the basin was not altered by the suburban development, then by combining equations 30 and 31, one obtains the relation:

$$\frac{Q_p'}{Q_p} \propto \left(\frac{L_1'}{L_1}\right)^{-0.44} \; . \tag{32}$$

The resemblance between equation 32 and Carter's equation 29 is indeed remarkable. In essence, τ is equivalent to $L_1 + K - D/2$ if the storage is linear, where D is the duration of rainfall. Thus for rainfalls having durations approximately equal to 2K, $\tau \approx L_1$. Consequently, equations 29 and 32 are nearly identical.

A more rigorous approach to this problem can be made by using the analog technique of frequency analysis described previously. If it is possible to estimate the change in τ or L_1 due to urbanization (such as the formulas suggested by Carter and Snyder), models may then be built to account for these effects. Thus, by knowing the frequency distribution of runoff before the change and the frequency distribution of rainfall, the expected frequency distribution of runoff subsequent to the change may be synthesized. The advantage of this type of analysis lies in the fact that it not only accounts for the distribution of peak flows but also renders consideration to the effects of duration distribution and sequential correlation. Work along this line is in progress.

The foregoing discussion thus far involves only surface runoff. It begins from rainfall excess with the assumption that the losses due to evaporation and infiltration, etc., are already taken into account. Admittedly, these losses can be of a very significant nature especially in the arid lands. The most complete efforts known to the author are those due to Crawford and Linsley (1962) and Chow (1962). However, these suggested methods are still largely empirical at this time. It appears that further fundamental researches in the physical processes are necessary.

During a private discussion, Dr. Jacob Rubin indicated that he is currently undertaking a project on the mechanics of infiltration in the Menlo Park office of the Geological Survey. His approach, which incorporates the dynamic behavior of soil-moisture profile, appears to have shown considerable promise. It is believed that an analog-model study of this infiltration aspect would be mutually beneficial.

The technique of frequency synthesis described previously also carries certain statistical inferences. For example, it enables one to analyze the expected errors due to short-term samples. By synthesizing various types of probability distributions, such as normal, log-normal, Gumbel, Pearson, etc., it is also possible to determine the most suitable sampling scheme to be used for each type of distribution.

Other proposed endeavors consist of applying the suggested modeling techniques to actual field basins. As an initial effort, three existing experimental basins might be selected: one in the humid region, one in the arid region and one in the urbanized region. Accordingly, linear and nonlinear models would be constructed for each basin so as to evaluate the degree of nonlinearity that might be encountered in these differed environments.

It appears that the high-speed computers, in particular the analog computers, offer a convenient means for analyzing complex hydrologic systems. The analysis may be accomplished either by the direct simulation or by the indirect solution of the mathematical relations describing such systems. With the benefit of the rapid advancement of modern electronic techniques, a hydrologist may greatly broaden his realm of hydrologic know-how.

References

Amorocho, Jaime, and Orlob, G. T., 1961, Nonlinear analysis of hydrologic systems: Water Resources Center, Contribution no. 40, Univ. of California, Berkeley.

Baltzer, R. A., and Shen, John, 1961, Computation of flows in tidal reaches by finite-difference technique: National Coastal and Shallow Water Research Conf. Proc., 1st, Los Angeles, California, 1961, p. 258-264.

Benson, M. A., 1962, Factors influencing the occurrence of floods in a humid region of diverse terrain: U. S. Geol. Survey Water-Supply Paper 1580-B.

Carter, R. W., 1961, Magnitude and frequency of floods in suburban areas: U. S. Geol. Survey Prof. Paper 424-B.

Chow, V. T., 1962, Hydrologic determination of waterway areas for the design of drainage structures in small drainage basins: Univ. of Illinois, Eng. Expt. Sta. Bull. 462.

Crawford, N. H., and Linsley, R. K., 1962, Synthesis of continuous streamflow hydrographs on a digital computer: Stanford Univ., Dept. Civil Eng. Tech. Rept. 12.

Dooge, J. C. I., 1959, A general theory of the unit hydrograph: Jour. Geophys. Research, v. 64, no. 2, Feb., p. 241-256.

Edson, C. G., 1951, Parameters for relating unit hydrographs to watershed characteristics: Am. Geophys. Union Trans., v. 32, p. 951-956.

Einstein, H. A., and Harder, J. A., 1959, An electric analog model of a tidal estuary: Jour. Waterways and Harbors Div., Am. Soc. Civil Engineers Proc., v. 85, no. WW3, Paper 2173, Sept., p. 153-165.

George, D. A., 1959, Continuous nonlinear systems: Research Lab. Electronics, Massachusetts Inst. Technology, Tech. Rept. 355.

Harder, J. A., 1962, Analog models for flood control systems: Jour. Hydraulics Div., Am. Soc. Civil Engineers Proc., v. 88, no. HY 2, Paper 3074, March, p. 63-74.

Ishihara, Tojiro, and others, 1956, Electronic analog computer for flood flows: Proc. Regional Tech. Conf. on Water Resources Development in Asia and the Far East, U. N. Economic Comm., Flood Control Series no. 9, Bangkok, p. 170-174.

Kalinin, G. P., and Levin, A. G., 1960, Use of an electronic analog computer in the forecasting of rain floods: Meterologiya i Gidrologiya, no. 12, Moscow, Dec., p. 14-18. Translation by U. S. Joint Publications Research Service, Washington, D. C.

Langbein, W. G., 1958, Queuing theory and water storage: Am. Soc. Civil Engineers Proc. Paper 1811, v. 84, no. HY 3, 24 p.

Liggett, J. A., 1959, Unsteady open channel flow with lateral inflow: Tech. Rept. 2, July, Dept. Civil Eng., Stanford Univ.

Matalas, N. C., 1962, Some comments on synthetic hydrology: unpublished paper presented at the Symposium on Synthetic Hydrology and Simulation Techniques, July, Phoenix, Arizona.

Mitchell, W. D., 1962, Effect of reservoir storage on peak flow: U. S. Geol. Survey Water-Supply Paper 1580-C, p. 24.

Nash, J. E., 1959, Form of the instantaneous unit hydrograph: General Assembly Toronto Proc., Internat. Assoc. Sci. Hydrology, v. 3, p. 114-121.

Paulhus, Joseph L. H., and Miller, J. F., 1957, Flood frequencies derived from rainfall data: Am. Soc. Civil Engineers Proc. Paper 1451, v. 83, no. HY 6, 18 p.

Paynter, H. M., 1960, Flood routing by admittance methods: A Palimpsest on the Electronic Analog Art, G. A. Philbrick Researches, Inc., Boston, Massachusetts, p. 240-245.

Richards, B. D., 1955, Flood estimating and control: London, Chapman and Hall.

Rockwood, D. M., and Hildebrand, C. E., 1956, An electronic analog for multiple-stage reservoir-type storage routing: Corps of Engineers, U. S. Army, Tech. Bull. 18, March, Portland, Oregon.

Shen, John, 1962 a, Simulation technique for hydrologic models: unpublished paper presented at the Symposium on Synthetic Hydrology and Simulation Techniques, July, Phoenix, Arizona.

- - , 1962 b, A method for determining the storage-outflow characteristics for nonlinear reservoirs: U. S. Geol. Survey Prof. Paper 450-E, p. 167.

Snyder, Franklin F., 1958, Synthetic flood frequency: Am. Soc. Civil Engineers Proc. Paper 1808, v. 84, no. HY 5, 22 p.

Stoker, J. J., 1957, Water waves: New York, Interscience Publishers, Inc., p. 291-308, 451-505.

Thomas, H. A., and Fiering, M. B., 1962, Mathematical synthesis of streamflow sequences for the analysis of river basins by simulation: Design of Water-Resources Systems, Harvard Univ. Press, Cambridge, Massachusetts, p. 459-493.

Wiener, N., 1942, Response of a nonlinear device to noise: Radiation Lab., Massachusetts Inst. Technology, Rept. 129.