

Improved digital filters for evaluating  
Fourier and Hankel transform integrals

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# Improved Digital Filters for

## Evaluating Fourier and Hankel Transform Integrals

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### Abstract

New algorithms are described for evaluating Fourier (cosine, sine) and Hankel ( $J_0, J_1$ ) transform integrals by means of digital filters. The filters have been designed with extended lengths so that a variable convolution operation can be applied to a large class of integral transforms having the same system transfer function. A lagged-convolution method is also presented to significantly decrease the computation time when computing a series of like-transforms over a parameter set spaced the same as the filters. Accuracy of the new filters is comparable to Gaussian integration, provided moderate parameter ranges and well-behaved kernel functions are used. A collection of Fortran IV subprograms is included for both real and complex functions for each filter type. The algorithms have been successfully used in geophysical applications containing a wide variety of integral transforms.

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## Introduction

Digital filtering techniques to evaluate oscillatory-type convolution integrals have become a widespread method reported in the geophysical literature. For example, Koefoed et al (1972) developed Hankel transform filters for functions encountered in electromagnetic depth soundings. Anderson (1973) extended this method to Fourier cosine transforms. Das et al (1974a, 1974b) proposed several specific filters used in transformations of resistivity sounding curves. Anderson (1974) used a standard set of Hankel and Fourier transform filters to evaluate a wide class of integral transforms required in computing electromagnetic field components about a finite electric wire source.

In all these papers, the main advantage cited for using digital filters over direct numerical integration is the increased speed of the calculation. An order of magnitude or more in speed improvement is not uncommon, but this is usually achieved with a reduction in accuracy.

This report briefly reviews the filter design process for convolution integrals. A new method is presented to improve the designed filter coefficients so that increased accuracy is possible at very little sacrifice in computational speed. A variable cutoff convolution method is described which utilizes the nature of the input kernel function and filter response to approximate the integral given a truncation tolerance. An algorithm is described for evaluating Fourier (cosine, sine) and Hankel ( $J_0, J_1$ ) transform integrals for well-behaved kernels.

Additionally, a special lagged-convolution algorithm is developed to significantly decrease the overall computational time over variable convolution. This method is especially advantageous when computing a set of like-integrals where the transform parameter is equivalent to an incremented shift in the sampled filter.



Sixteen similar, but self-contained, Fortran IV computer subprograms utilizing the new filtering algorithms are listed in appendix 1. The subprogram naming convention used is given in tables 2 and 3. Each of these routines contains comments to explicitly define the calling parameters. With minor changes, the Fortran IV subprograms should be acceptable to most current-day digital computers. Some examples, using the improved filters, are tabulated in appendix 2.

Finally, the new filtering methods are discussed in light of other recently published techniques.

## Filter design

The method of designing digital filters for Hankel transform integrals presented by Koefoed et al (1972) may be generalized to other type integrals expressed as a linear system convolution in the form

$$K(x) = \int_{-\infty}^{\infty} k(y)s(x-y) dy, \quad (1)$$

where  $k(y)$  = input function (also called kernel function below)

$s(y)$  = system transfer (or filter) function

$K(x)$  = convolved output function.

We may consider (1) as either time-domain or frequency-domain convolution. By Fourier transform theory (e.g., see Papoulis, 1962) linear convolution is equivalent to multiplication in the transformed-domain; that is,

$$\hat{K}(\hat{x}) = \hat{k}(\hat{x})\hat{s}(\hat{x}), \quad (2)$$

where  $\hat{K}(\hat{x}) \leftrightarrow K(x)$ ,  $\hat{k}(\hat{x}) \leftrightarrow k(x)$ , and  $\hat{s}(\hat{x}) \leftrightarrow s(x)$  are Fourier transform pairs. For a known system input-output relation, the transformed system (filter) response is

$$\hat{s}(\hat{x}) = \hat{K}(\hat{x})/\hat{k}(\hat{x}), \quad (3)$$

provided  $\hat{k}(\hat{x})$  is not identically zero for some  $\hat{x}$ , and both system input and output functions have band-limited Fourier transforms. The latter is required so that  $\hat{s}(\hat{x}) \rightarrow 0$  for  $\hat{x} \rightarrow \pm\infty$ .

The filter impulse response is the inverse Fourier transform of  $\hat{s}(\hat{x})$ , but would not be useful for convolution unless truncated to a reasonable finite length. To this end, we first choose a discrete sampling of  $N$  points spaced  $\Delta x$  apart and obtain  $\hat{s}(\hat{x}_j), j = 1, 2, \dots, N \gg 0$ . The sampling results in a Nyquist frequency of  $1/(2\Delta x)$ . A suitable truncation may be obtained by cutting-off the transformed response at the Nyquist frequency. Note this is equivalent to multiplying  $\hat{s}(\hat{x})$  by the Fourier transform of the function  $\text{sinc}(x_j) = \sin(\pi x_j / \Delta x) / (\pi x_j / \Delta x)$ .

The truncated response is inverse Fourier transformed to obtain the sampled filter impulse response (hereafter simply called filter weights or filter response). In some cases, it is advantageous to shift the resulting filter response to further minimize the magnitude of the filter tails. For example, see Koefoed (1972) for the reason of the oscillating behavior of the filter response and the suggested shift at zero crossings. If proper input-output functionals are used, then the final filter response should asymptotically approach zero in both abscissa directions.

Application of the filter weights to specific input functions in (1) is given by the non-circular convolution sum

$$K(x) = \sum_{i=1}^{NW} W_i k(x-a_i), \quad (4)$$

where  $W_i$  = filter weights;  $i = 1, 2, \dots, NW$  = number of weights,

$x$  = transform parameter

$a_i$  = filter abscissa corresponding to  $W_i$ .

Thus, the convolution integral (1) has been reduced to a straightforward summation given in (4), and the predetermined filter weights remain

constant for all classes of convolution integrals having the same system transfer function.

In practice, many transform integrals encountered range over  $(0, \infty)$  and therefore a transformation to the range  $(-\infty, \infty)$  is required before applying (4) to a given kernel function. For example, the Hankel transform of integer order  $n \geq 0$  is defined as

$$H(b) = \int_0^{\infty} gh(g) J_n(bg) dg, \quad (5)$$

where  $gh(g)$  = modified kernel\* for Hankel transforms,

and  $J_n(bg)$  = Bessel function of the first kind of order  $n$ .

Similarly, the Fourier sine or cosine integral is given by

$$F(b) = \int_0^{\infty} f(g) \frac{\sin}{\cos}(bg) dg. \quad (6)$$

If we let  $x = \ln(b)$ ,  $y = \ln(1/g)$  and after multiplying by  $e^x$ , equations (5) and (6) become respectively,

$$e^x H(e^x) = \int_{-\infty}^{\infty} \{e^{-y} h(e^{-y})\} \{e^{x-y} J_n(e^{x-y})\} dy \quad (7)$$

and

$$e^x F(e^x) = \int_{-\infty}^{\infty} \{f(e^{-y})\} \{e^{x-y} \frac{\sin}{\cos}(e^{x-y})\} dy \quad (8)$$

---

\*Many texts define  $gJ_n(bg)$  as the Hankel kernel; however, we will adopt the definition given in (1) and consider  $J_n(bg)$  the system transfer function with  $h^*(g) = gh(g)$  the system input kernel function.



Equations (7) and (8) are expressed in the convolution form (1), which is assumed when applying equation (4). Multiplying by  $e^x$  ensures that the system response approaches zero for both large and small filter abscissas.

An important consideration in designing digital filters is the choice of known input-output function pairs. In previous work by Koefoed et al (1972) and Anderson (1973), certain known Lipschitz integrals were used to develop Hankel ( $J_0, J_1$ ) and Fourier cosine filters, respectively. Both papers illustrate filter responses that are characterized as oscillatory decreasing functions in both abscissa directions. The integral forms previously used for filter design are indicated by (c) in table 1 (The sine-transform form was previously used by the author, even though the filter weights were never published).

Table 1. Some known integral transform pairs.

F = Integral FORM; T = test TYPE; (a>0,b>0 for all F,T)

1	1	$\int_0^\infty x e^{-ax} J_1(bx) dx = b/(a^2 + b^2)^{3/2}$	¢ *
1	2	$\int_0^\infty x^2 e^{-ax^2} J_1(bx) dx = b \exp(-b^2/4a)/(2a)^2$	
1	3	$\int_0^\infty J_1(a/x) J_1(bx) dx = J_2(2\sqrt{ab})/b$	
1	4	$\int_0^\infty e^{-ax} J_1(bx) dx = (\sqrt{a^2 + b^2} - a)/(b\sqrt{a^2 + b^2})$	
1	5	$\int_0^\infty \frac{(J_0(ax)-1)}{x^2} J_1(bx) dx = \begin{cases} -b(1+2\ln a/b)/4, & b < a \\ -a/4b, & a \leq b \end{cases}$	
2	1	$\int_0^\infty e^{-ax} J_0(bx) dx = 1/(a^2 + b^2)^{1/2}$	¢
2	2	$\int_0^\infty x e^{-ax^2} J_0(bx) dx = \exp(-b^2/4a)/2a$	*
2	3	$\int_0^\infty J_0(a/x) J_0(bx) dx = J_0(2\sqrt{ab})/b$	
2	4	$\int_0^\infty x^3/(x^4 + a^4) J_0(bx) dx = \ker(ab)$	
2	5	$\int_0^\infty x/\sqrt{a^2 + x^2} J_0(bx) dx = e^{-ab}/b$	
3	1	$\int_0^\infty e^{-ax} \cos(bx) dx = a/(a^2 + b^2)$	¢
3	2	$\int_0^\infty \exp(-a^2 x^2) \cos(bx) dx = \sqrt{\pi} \exp(-b^2/4a^2)/2a$	*
3	3	$\int_0^\infty J_0(a/x) \cos(bx) dx = \sin(a^2/4b)/b$	
3	4	$\int_0^\infty 1/(a^2 + x^2) \cos(bx) dx = \pi e^{-ab}/2a$	
3	5	$\int_0^\infty e^{-ax}(1-e^{-ax})/x \cos(bx) dx = \frac{1}{2} \ln \left\{ \frac{b^2 + 4a^2}{b^2 + a^2} \right\}$	
4	1	$\int_0^\infty e^{-ax} \sin(bx) dx = b/(a^2 + b^2)$	¢
4	2	$\int_0^\infty x \exp(-a^2 x^2) \sin(bx) dx = \sqrt{\pi} b \exp(-b^2/4a^2)/4a^3$	*
4	3	$\int_0^\infty \sin(a^2/x) \sin(bx) dx = \pi a J_1(2a\sqrt{b})/2\sqrt{b}$	
4	4	$\int_0^\infty x/(a^2 + x^2) \sin(bx) dx = \pi e^{-ab}/2$	
4	5	$\int_0^\infty e^{-ax}(1-e^{-ax})/x \sin(bx) dx = \tan^{-1}\{ab/(b^2 + 2a^2)\}$	

¢ previous design form  
\* new design form

If the filter response could be made to decrease more rapidly in both abscissa directions, then the filter efficiency as a convolution operator would be enhanced. Systems with rapidly decaying input-output function pairs have this desired narrow-band property. A search through tables of integrals, such as Gradshteyn and Ryzhik (1965), revealed many of the integral transform pairs given in table 1. By inspection of table 1, several promising design forms are evident. After considerable experimentation, the new filter design forms were chosen as indicated by an (\*) in table 1. Note, however, the  $J_1$  - form was not changed, because the rate of decrease of both input-output functions was deemed sufficiently rapid. The remaining filters were significantly improved by using the indicated form change.

The resulting final filter responses are illustrated in figures 1-4. In these figures, we have arbitrarily reduced the abscissa range for display purposes. The full or extended length of each filter is contained in the subprograms given in appendix 1. The reason for extending the filter length will be discussed in the next section.

Another critical factor in filter design is the choice of the sampling interval  $\Delta x$ . Koefoed et al (1972) discussed the fundamental sampling problem applied to a certain kernel function for a homogeneous earth model. Their analysis showed a sampling  $\Delta x = (\ln 10)/10$  would reproduce the kernel between sample points with an absolute error approximately less than  $10^{-5}$ . Obviously, if we decrease the sampling a smaller error is possible, but at the sacrifice of generating many more filter weights. Thus a compromise between algorithm speed and accuracy is unavoidable.

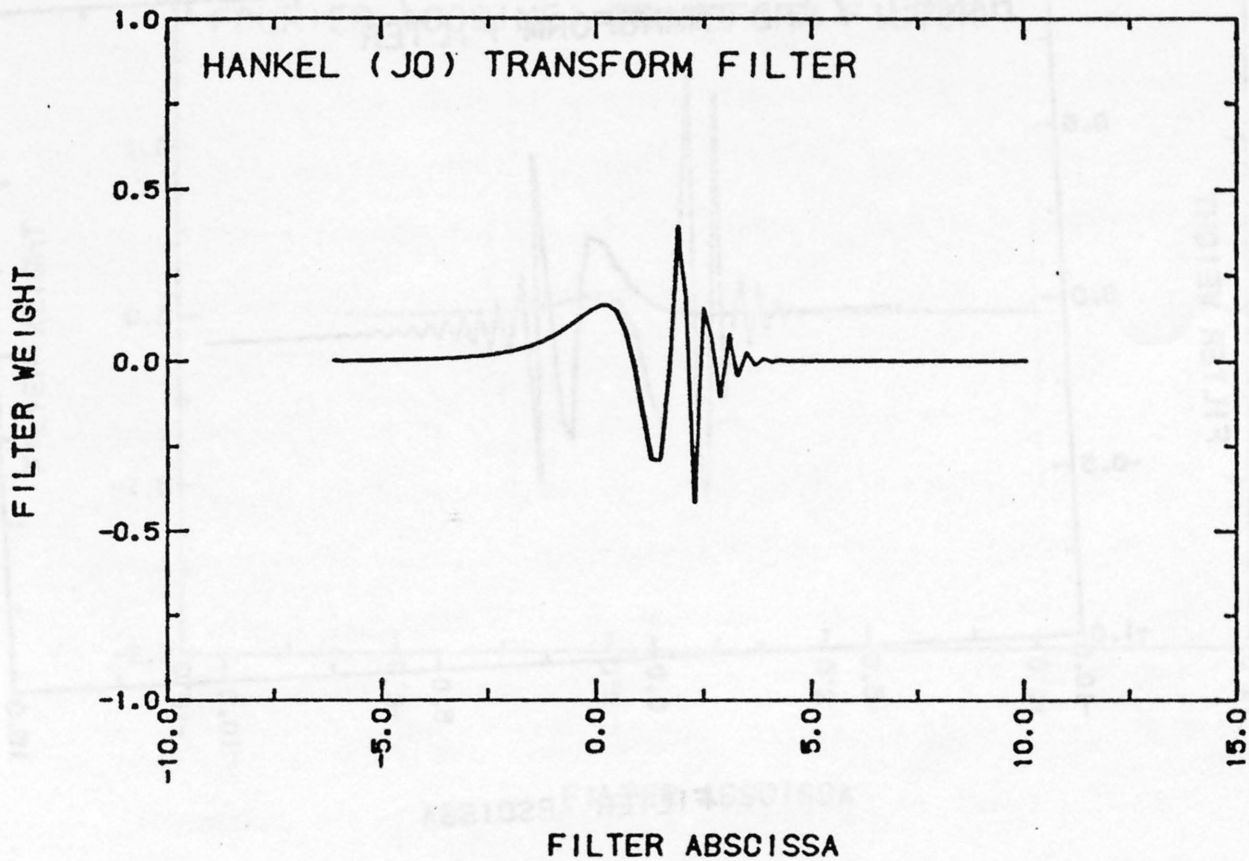


Figure 1.---Filter response for Hankel  $J_0$ -transform.



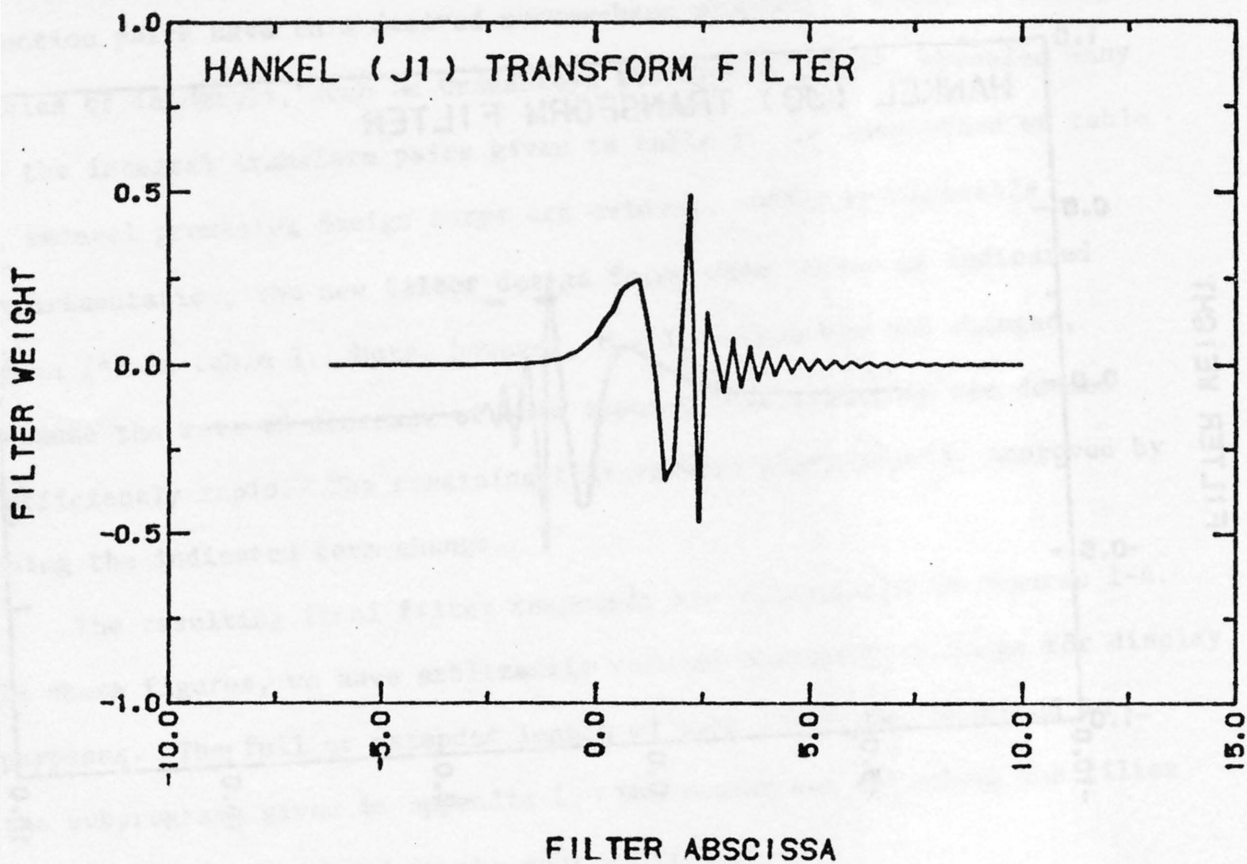


Figure 2.---Filter response for Hankel  $J_1$ -transform.

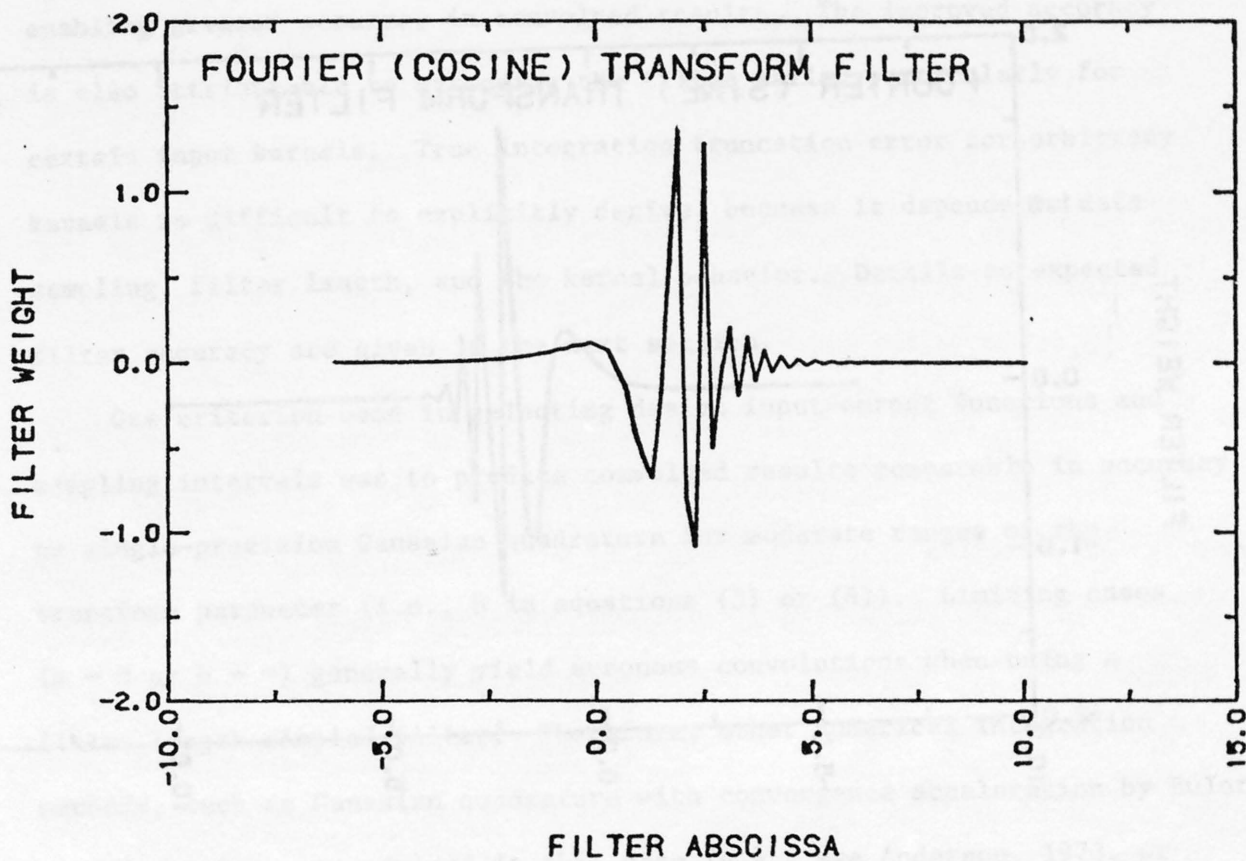


Figure 3.---Filter response for Fourier cosine-transform.

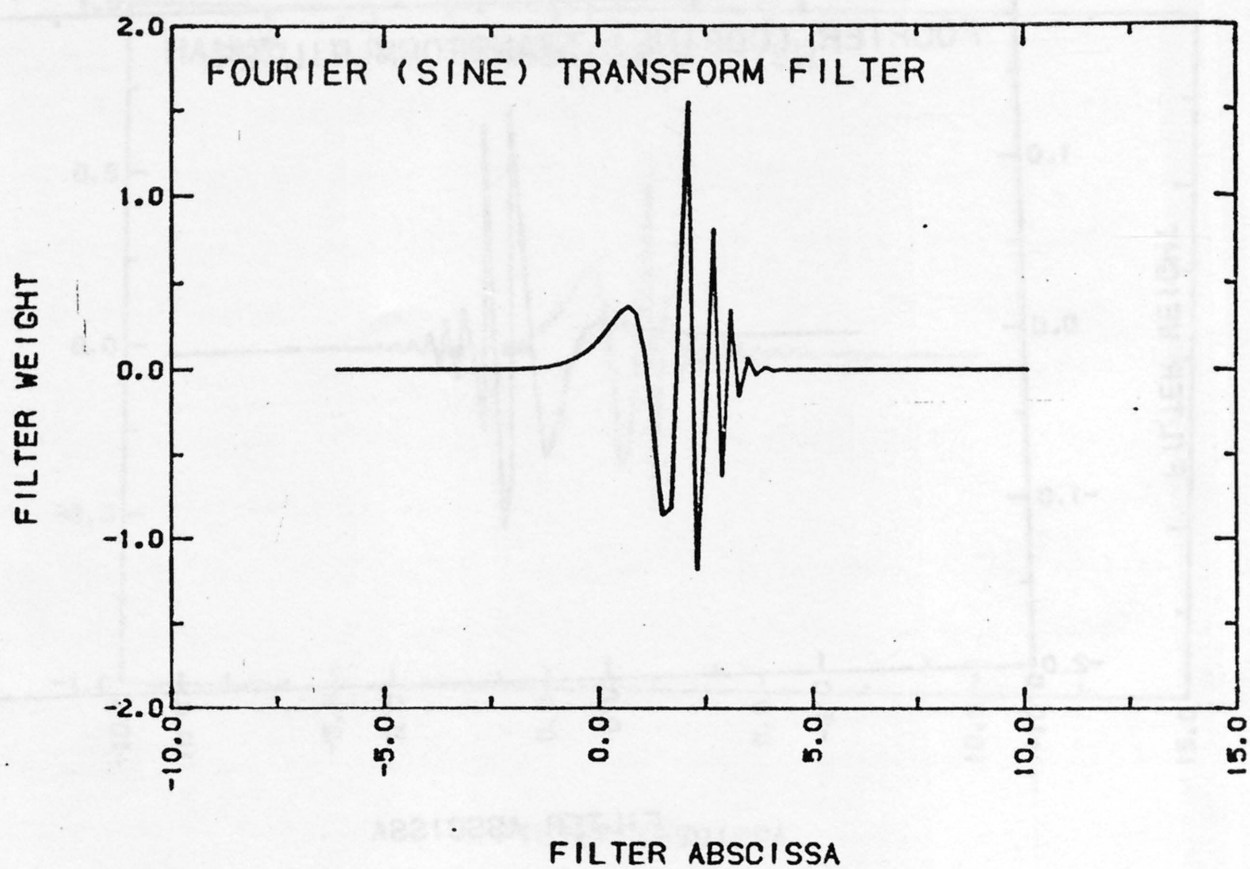


Figure 4.---Filter response for Fourier sine-transform.

Therefore, after much experimentation, the sampling was fixed at  $\Delta x = .20 \sim (\ln 10)/12$  for all redesigned filters. This only increases the number of weights slightly over the  $(\ln 10)/10$  interval, while enabling greater accuracy in convolved results. The improved accuracy is also attributable to extending the filter tails, particularly for certain input kernels. True integration truncation error for arbitrary kernels is difficult to explicitly derive, because it depends on data sampling, filter length, and the kernel behavior. Details on expected filter accuracy are given in the next section.

One criterion used in selecting design input-output functions and sampling intervals was to produce convolved results comparable in accuracy to single-precision Gaussian quadrature for moderate ranges of the transform parameter (i.e.,  $b$  in equations (5) or (6)). Limiting cases ( $b \rightarrow 0$  or  $b \rightarrow \infty$ ) generally yield erroneous convolutions when using a finite length sampled filter. Therefore, other numerical integration methods, such as Gaussian quadrature with convergence acceleration by Euler's transformation, are advised in this case (e.g., see Anderson, 1973, or Anderson, 1974). Methods designed exclusively for highly oscillatory integrals (e.g., see Boris and Oran, 1974) are suggested for extreme parameter problems.



## Algorithms

The Fortran IV subprograms listed in appendix 1 reflect the final improved filters designed for Fourier (cosine, sine) and Hankel ( $J_0, J_1$ ) transform integrals. Separate versions for both real and complex kernel functions are provided. In all cases, the kernel function must be supplied by the user as an external written function subprogram. It is mandatory that the external function (a) is monotonically decreasing as the argument increases, (b) does not have any singularities in the range of integration, and (c) is correctly coded. Violation of any of these assumptions may produce unpredictable results.

The substitution used to obtain equations (7) or (8) from (5) or (6), respectively, is implicitly assumed by all subprograms. However, the user must normalize the convolved sum after execution by the factor  $e^x = b$ . Sufficient comments concerning the calling parameters are given at the beginning of each routine, along with additional remarks on parameter relationships. The algorithm itself is not annotated, and consequently the following discussion outlines the general methodology and contributing factors.

The original filters published by Koefoed et al (1972), and verified by Anderson (1973), used a fixed number of weights during the convolution process. As later pointed out in a note by Verma and Koefoed (1973), many selected kernel functions damp to zero rapidly and so it is wasteful to multiply by the ends of the filter tails; that is, the contribution of the filter tails may be negligible to the total convolution sum. Consequently Verma and Koefoed proposed a truncated coefficient set approximately 15% shorter than their original filters designed for certain electromagnetic kernel functions.

This concept was further exploited and a generalization using the notion of a variable cutoff method with extended filter tails was adopted in this report. Here, a total filter length of approximately 250 weights are stored in DATA statements. For a given input kernel function obeying the above assumptions (a)-(c), and for a particular filter, we begin the convolution process in a fixed central region of the filter response containing about 20 weights (the central region is chosen such that weights decrease in absolute value beyond the region). Within the central region an absolute maximum convolved product is established. Then, depending on the nature of the kernel, the algorithm proceeds in either direction beyond the central region by accumulating products until the absolute magnitude is less than a prescribed tolerance times the central maximum. For complex kernel functions, the tolerance criterion is applied in parallel to both real and imaginary parts; both tests must be satisfied separately before the complex convolution is accepted.

Since the filter damps rapidly in both abscissa directions, the variable convolution may be terminated after using a few additional terms on either side of the central filter region, if the kernel is sufficiently decaying. Of course, this depends on the indicated tolerance controlling the truncation. In general, a smaller tolerance will result in increased accuracy, mainly because more filter weights are applied over a larger abscissa interval.

The extended filter length is particularly useful in minimizing truncation error when convolving with slowly decreasing and/or oscillatory kernel functions. Thus a large number of weights are supplied to enable increased accuracy for small tolerances, and to handle special function classes. For typically well-behaved kernels, the number of weights needed is nominal for a moderate tolerance. It should be emphasized that true integration truncation error is not directly related to the requested tolerance. The tolerance factor is simply used to terminate the variable convolution process, whereas the integration error depends on the sampling interval as well as the filter extent. However, in many cases, the tolerance may be thought of as an approximation indicator--that is, a truncation is done so that additional weights would not alter the convolution sum with respect to the specified tolerance.

In any event, it appears a larger class of integral transforms may be evaluated as a result of the variable convolution method. The uncertainty in determining exact error bounds for certain function classes may be investigated, if desired, by varying the tolerance and/or applying other numerical integration methods as control checks. More work is needed here. As a starting point, some test results are listed in appendix 2, where the transform pairs in table 1 were tested with various parameter ranges and tolerances. These results are discussed in more detail in the next section.

The basic subprograms are classified according to the notation defined in table 2.

Table 2. Basic subprogram naming convention.

General notation		Specific names	
<div style="text-align: center;"> Hankel transform  ↓ </div> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">Real →</div> <div style="border: 1px dashed black; padding: 5px; display: inline-block;"> R   HANK   0 </div> <div style="margin-left: 10px;">← order 0 (or COS)</div> </div> <div style="display: flex; align-items: center; margin-top: 5px;"> <div style="margin-right: 10px;">Complex →</div> <div style="border: 1px dashed black; padding: 5px; display: inline-block;"> Z   FOUR   1 </div> <div style="margin-left: 10px;">← order 1 (or SIN)</div> </div> <div style="text-align: center; margin-top: 5px;"> ↑  Fourier transform </div>		RHANK0	RFOURO
		RHANK1	RFOURI
		ZHANK0	ZFOURO
		ZHANK1	ZFOURI

Routines for both real and complex functions are provided, mainly to speed the operations when dealing with real functions. It should be observed, however, that it is possible to compute two real transforms in parallel using a complex routine by placing the real functions in the real and imaginary parts of an external complex function.

A special lagged-convolution method evolved by observing the fact that if one computes transforms over a parameter sequence  $\{b_i, i = 1, 2, \dots\}$  spaced exactly the same as the filter, then it is possible to minimize kernel function evaluations as the sequence continues. This may be accomplished by storing all kernel evaluations on the first execution. Then on subsequent calls, an incremented integer lag (equivalent to the shifted sample spacing) is used to perform lagged-products with appropriate filter weights and previously saved kernel evaluations. New kernel computations are stored only when needed by the variable convolution method. The lagged-convolution algorithm yields significant improvements in time over direct variable convolution whenever the kernel is not a simple elementary function. It should be pointed out that each lagged-convolution subprogram requires additional storage space to hold the kernel values. Also, a storage-roll feature is utilized so that the parameter sequence may be of any length.



The special lagged-convolution subprograms are classified according to the notation defined in table 3.

Table 3. Special subprogram naming convention.

General notation	Specific names
<div style="text-align: center;">Lagged Hankel transform</div> <div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: right; padding-right: 10px;">Real →</div> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <math>\begin{matrix} R &amp; \text{---} &amp; H &amp; \text{---} &amp; 0 \\ &amp; \text{LAG} &amp; &amp; &amp; \end{matrix}</math> </div> <div style="text-align: left; padding-left: 10px;">+ order 0 (or COS)</div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> <div style="text-align: right; padding-right: 10px;">Complex →</div> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <math>\begin{matrix} Z &amp; \text{---} &amp; F &amp; \text{---} &amp; 1 \\ &amp; &amp; &amp; &amp; \end{matrix}</math> </div> <div style="text-align: left; padding-left: 10px;">+ order 1 (or SIN)</div> </div> <div style="text-align: center; margin-top: 10px;">Lagged Fourier transform</div>	<div style="display: flex; justify-content: space-between; padding: 5px 0;"> <span>RLAGH0</span><span>RLAGF0</span></div> <div style="display: flex; justify-content: space-between; padding: 5px 0;"> <span>RLAGH1</span><span>RLAGF1</span></div> <div style="display: flex; justify-content: space-between; padding: 5px 0;"> <span>ZLAGH0</span><span>ZLAGF0</span></div> <div style="display: flex; justify-content: space-between; padding: 5px 0;"> <span>ZLAGH1</span><span>ZLAGF1</span></div>

Specific details on executing the lagged-convolution subprograms are given at the beginning of each routine listed in appendix 1. It is worth reading COMMENT statements: NOTES (2)-(4) for details on using the lagged-convolution method for intervals spaced differently than the filters.

Finally, all algorithms (direct or lagged) contain a special case which may occur only when the kernel is identically zero (or underflows the machine exponent range) throughout the fixed central region of a filter. This is considered extremely rare and should not occur in typical applications. If it does occur, then a reverse scan is attempted at both filter ends and convolution proceeds toward the central region until a zero-value is again encountered.

A final comment regarding oscillatory kernels: because of the possibility of computing values near a zero of an oscillating kernel function, a very small tolerance is advised; otherwise a premature cutoff could occur during the variable convolution algorithm. The computer algorithms can be easily modified to use an alternate truncation procedure especially designed for oscillatory kernels.

## Filter tests

A large group of known integral transforms was evaluated for various parameter ranges using each subprogram. Some of these results are listed in appendix 2, where columns headed by F and T correspond to the transform pairs given in table 1. The values listed in column L represent the number of weights applied for the given tolerance. Appendix 2 has been separated into two parameter sections: (1) moderate b, and (2) small and large b. Parameter a was moderately chosen. The computed absolute error is denoted as  $|DIFF|$ , and the relative error is  $RELERR = |DIFF/EXACT|$ .

The filtered results would be expected to agree well with exact values for the design forms, especially for moderate b. Somewhat surprising is the good accuracy achieved when using slowly decreasing or oscillatory kernels. The worst case, in both sections, appears to be for  $F=T=3$ , where the kernel  $J_0(a\sqrt{x})$  is both oscillatory and slowly dampening.

It should be noted that extremely large relative errors (e.g., see line  $F=1, T=2, A=.1, B=4$ ) usually indicate the exact value is near zero. In other cases where  $RELERR > TOLERANCE$ , the magnitude of the absolute error  $|DIFF|$  is generally less than the tolerance. What exceptions do exist, are usually for extreme parameters or poorly behaved kernels.

As pointed out earlier, when the transform parameter b becomes quite small or large, the extended finite-length sampled filter may not be adequate for some kernels. In this special case, Gaussian quadrature or other numerical integration methods are often better. The results in section (2) of appendix 2 were purposely generated for very small and large b while using a small tolerance. Because of the extreme b range, many exact or filtered results underflow to identically zero (this was due to the machine word-length and floating-point exponent range  $\sim 10^{\pm 38}$ ). Nevertheless, the generality of the variable convolution method produced acceptable results in nearly all cases.

## Discussion

Other numerical methods for evaluating convolution-type integrals are noteworthy. In particular, Cornille (1972) outlines series methods and asymptotic expansions useful in numerical integration of Hankel transforms. Tsang et al (1974) present a fast Fourier transform (FFT) method in evaluating electromagnetic field integrals, and Shubert and Lin (1973) discuss computation of convolution integrals by solving an analogous differential equation numerically. The last two methods claim significant computation time improvements over direct numerical integration, namely Simpson's rule or Gaussian quadrature.

It would be interesting to compare speed and accuracy runs between these methods and the subprograms presented in this report. Seemingly the lagged-convolution method compares favorably with the differential equation approach, in that both methods take advantage of previously computed kernel values. Similarly, the FFT method and lagged-convolution apparently have comparable operations for parameter sets using the same integral form, particularly when using the same sampling interval. If various transform integral forms are required, the FFT method essentially designs a filter for each integral form; in this case, the predesigned filters of this report would have an advantage over the FFT method.



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# Appendix 1.--Source listing of subprograms

A complete listing of each subprogram named in tables 2 and 3 is given in the following order:

Subprogram	Beginning line number	Page
RHANKO	1	27
RHANK1	157	31
RFOURO	325	35
RFOUR1	504	40
ZHANKO	680	45
ZHANK1	845	49
ZFOURO	1022	54
ZFOUR1	1210	59
RLAGHO	1395	64
RLAGH1	1611	70
RLAGFO	1839	76
RLAGF1	2078	82
ZLAGHO	2314	88
ZLAGH1	2538	94
ZLAGFO	2774	100
ZLAGF1	3021	106

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1      REAL FUNCTION RHANKO(X,FUN,TOL,L)
2      C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J0(G*B)*DG" DEFINED AS THE
3      C REAL HANKEL TRANSFORM OF ORDER 0 AND ARGUMENT X(=ALOG(B))
4      C BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
5      C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
6      C
7      C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO,
8      C
9      C--PARAMETERS:
10     C
11     C      X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
12     C      FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED),
13     C              NOTE: IF PARAMS OTHER THAN G ARE REQUIRED, USE COMMON IN
14     C              CALLING PROGRAM AND IN SUBPROGRAM FUN,
15     C              THE REAL FUNCTION FUN SHOULD BE A MONOTONE
16     C              DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
17     C      TOL=    REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
18     C              IF FILTER*FUN*TOL*MAX, THEN REST OF TAIL IS TRUNCATED,
19     C              THIS IS DONE AT BOTH ENDS OF FILTER, TYPICALLY,
20     C              TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
21     C              THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
22     C              A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
23     C              BUT WITH "MORE WEIGHTS" BEING USED, TOL IS NOT DIRECTLY
24     C              RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
25     C              APPROXIMATION INDICATOR,, FOR VERY LARGE OR SMALL B,
26     C              ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
27     C      L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
28     C              CONVOLUTION (I DEPENDS ON TOL AND FUN),
29     C              MIN,L=20 AND MAX,L=193--WHICH COULD
30     C              OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
31     C              VERY FAST...
32     C
33     C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RHANKO; THE HANKEL
34     C TRANSFORM IS THEN RHANKO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
35     C THIS ROUTINE,...
36     C
37     C--USAGE-- "RHANKO" IS CALLED AS FOLLOWS;
38     C      ...
39     C      EXTERNAL RF
40     C      ...

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41      C      R=RHANKO(ALOG(B),RF,TOL,L)/B
42      C      ...
43      C      END
44      C      REAL FUNCTION RF(G)
45      C      ...USER SUPPLIED CODE...
46      C      END
47      C
48      C
49      C--NOTES:
50      C      (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
51      C      STATEMENT FUNCTION CO(II) BELOW; HOWEVER,
52      C      THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
53      C      EXP-UNDERFLOW'S TO 0.0.....
54      C      (2). SOME NON-ANSI FORTRAN STATEMENTS ARE USED
55      C      (E,G: DO 120 I=128,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
56      C      THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
57      C
58      C
59      C--JO-EXTENDED FILTER WEIGHT ARRAYS:
60      C      NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
61      C      TO SAVE STORAGE (SEE STATEMENT FUNCTION CO(II) BELOW).
62      C      DIMENSION YT(193),Y1(76),Y2(76),Y3(41)
63      C      EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1))
64      C      DATA Y1/
65      1 5.8565723E-08, 7.1143477E-11,-7.8395565E-11, 8.7489547E-11,
66      2-8.9007811E-11, 9.8790055E-11,-9.8675347E-11, 1.1118797E-10,
67      3-1.0893474E-10, 1.2543400E-10,-1.1979399E-10, 1.4200767E-10,
68      4-1.3106341E-10, 1.6153229E-10,-1.4238602E-10, 1.8486236E-10,
69      5-1.5315381E-10, 2.1319755E-10,-1.6238115E-10, 2.4824144E-10,
70      6-1.6850378E-10, 2.9243813E-10,-1.6909302E-10, 3.4934366E-10,
71      7-1.6043759E-10, 4.2417082E-10,-1.3690001E-10, 5.2458440E-10,
72      8-8.9946096E-11, 6.6188220E-10,-6.6964033E-12, 8.5276151E-10,
73      9 1.3222770E-10, 1.1219600E-09, 3.5591442E-10, 1.5061956E-09,
74      1 7.0795382E-10, 2.0600379E-09, 1.2535947E-09, 2.9646623E-09,
75      2 2.0904225E-09, 4.0409101E-09, 3.3642886E-09, 5.7687700E-09,
76      3 5.2930786E-09, 8.3164338E-09, 8.2021809E-09, 1.2083635E-08,
77      4 1.2577400E-08, 1.7666303E-08, 1.9143895E-08, 2.5953011E-08,
78      5 2.8983953E-08, 3.8268851E-08, 4.3712685E-08, 5.6590075E-08,
79      6 6.5740136E-08, 8.3864288E-08, 9.8682323E-08, 1.2448811E-07,
80      7 1.4784461E-07, 1.8501974E-07, 2.2129198E-07, 2.7524203E-07,
81      8 3.3094739E-07, 4.0974828E-07, 4.9462868E-07, 6.1030809E-07,

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82 9 7.3891802E-07, 9.0939667E-07, 1.1034727E-06, 1.3554600E-06,
83 1 1.6474556E-06, 2.0207696E-06, 2.4591294E-06, 3.0131400E-06/
84 DATA Y2/
85 1 3.6701680E-06, 4.4934101E-06, 5.4770076E-06, 6.7015208E-06,
86 2 8.1726989E-06, 9.9954201E-06, 1.2194425E-05, 1.4909101E-05,
87 3 1.8194388E-05, 2.2239184E-05, 2.7145562E-05, 3.3174088E-05,
88 4 4.0499452E-05, 4.9486730E-05, 6.0421440E-05, 7.3822001E-05,
89 5 9.0141902E-05, 1.1012552E-04, 1.3448017E-04, 1.6428337E-04,
90 6 2.0062570E-04, 2.4507680E-04, 2.9930366E-04, 3.6560582E-04,
91 7 4.4651421E-04, 5.4541300E-04, 6.6612648E-04, 8.1365181E-04,
92 8 9.9374786E-04, 1.2138120E-03, 1.4824945E-03, 1.8107657E-03,
93 9 2.2115938E-03, 2.7012675E-03, 3.2991969E-03, 4.0295817E-03,
94 1 4.9214244E-03, 6.0106700E-03, 7.3405529E-03, 8.9643708E-03,
95 2 1.0946310E-02, 1.3365017E-02, 1.6314985E-02, 1.9910907E-02,
96 3 2.4289325E-02, 2.9612896E-02, 3.6070402E-02, 4.3976936E-02,
97 4 5.3264829E-02, 6.4465091E-02, 7.7664144E-02, 9.2918324E-02,
98 5 1.1000121E-01, 1.2811102E-01, 1.4543025E-01, 1.5832248E-01,
99 6 1.6049224E-01, 1.4170064E-01, 8.8788108E-02, -1.1330934E-02,
100 7 -1.5331864E-01, -2.9094670E-01, -2.9084655E-01, -2.9708834E-02,
101 8 3.9009601E-01, 1.7999785E-01, -4.1858139E-01, 1.5317216E-01,
102 9 6.5184953E-02, -1.0751806E-01, 7.8429567E-02, -4.6019124E-02,
103 1 2.5309571E-02, -1.3904823E-02, 7.8187120E-03, -4.5190369E-03/
104 DATA Y3/
105 1 2.6729062E-03, -1.6073718E-03, 9.7715622E-04, -5.9804407E-04,
106 2 3.6749320E-04, -2.2635296E-04, 1.3960805E-04, -8.6172618E-05,
107 3 5.3212947E-05, -3.2867888E-05, 2.0304203E-05, -1.2543926E-05,
108 4 7.7499633E-06, -4.7882430E-06, 2.9584108E-06, -1.8278645E-06,
109 5 1.1293571E-06, -6.9778174E-07, 4.3113019E-07, -2.6637753E-07,
110 6 1.6458373E-07, -1.0168954E-07, 6.2429807E-08, -3.8819969E-08,
111 7 2.3985272E-08, -1.4819520E-08, 9.1563774E-09, -5.6573541E-09,
112 8 3.4954514E-09, -2.1597005E-09, 1.3343946E-09, -8.2447148E-10,
113 9 5.0941033E-10, -3.1474631E-10, 1.9447072E-10, -1.2015685E-10,
114 1 7.4241055E-11, -4.5871468E-11, 2.8343095E-11, -1.7513137E-11,
115 2 6.9049613E-12/
116 C==$$ENDATA
117 C
118 C==STATEMENT FUNCTION== INCLUDES ABSCISSA GENERATION!
119 CO(II)=FUN(EXP(-X*FLOAT(II)*.20-26.30455704))*YT(II)
120 RHANKO=0.0
121 CMAX=0.0
122 L=18

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123      DO 110 I=129,146
124      C=C0(I)
125      RHANKO=RHANKO+C
126      CMAX=AMAX1(ABS(C),CMAX)
127      110 CONTINUE
128      IF(CMAX.EQ.0.0) GO TO 150
129      CMAX=TOL*CMAX
130      DO 120 I=128,1,-1
131      C=C0(I)
132      RHANKO=RHANKO+C
133      L=L+1
134      IF(ABS(C).LE.CMAX) GO TO 130
135      120 CONTINUE
136      130 DO 140 I=147,193
137      C=C0(I)
138      RHANKO=RHANKO+C
139      L=L+1
140      IF(ABS(C).LE.CMAX) GO TO 190
141      140 CONTINUE
142      GO TO 190
143      150 DO 160 I=1,128
144      C=C0(I)
145      RHANKO=RHANKO+C
146      L=L+1
147      IF(C.EQ.0.0) GO TO 170
148      160 CONTINUE
149      170 DO 180 I=193,147,-1
150      C=C0(I)
151      RHANKO=RHANKO+C
152      L=L+1
153      IF(C.EQ.0.0) GO TO 190
154      180 CONTINUE
155      190 RETURN
156      END

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157      REAL FUNCTION RHANK1(X,FUN,TOL,L)
158      C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J1(G*B)*DG" DEFINED AS THE
159      C REAL HANKEL TRANSFORM OF ORDER 1 AND ARGUMENT X(=ALOG(B))
160      C BY CONVOLUTION FILTERING WITH PEAL FUNCTION "FUN"--AND
161      C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
162      C
163      C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO,
164      C
165      C--PARAMETERS:
166      C
167      C      X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
168      C      FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
169      C      NOTE: IF PARAMS OTHER THAN G ARE REQUIRED, USE COMMON IN
170      C      CALLING PROGRAM AND IN SUBPROGRAM FUN,
171      C      THE REAL FUNCTION FUN SHOULD BE A MONOTONE
172      C      DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
173      C      TOL=    REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
174      C      IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
175      C      THIS IS DONE AT BOTH ENDS OF FILTER, TYPICALLY,
176      C      TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
177      C      THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
178      C      A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
179      C      BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY
180      C      RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
181      C      APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
182      C      ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
183      C      L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
184      C      CONVOLUTION (L DEPENDS ON TOL AND FUN).
185      C      MIN.L=15 AND MAX.L=236--WHICH COULD
186      C      OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
187      C      VERY FAST...
188      C
189      C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RHANK1; THE HANKEL
190      C TRANSFORM IS THEN RHANK1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
191      C THIS ROUTINE....
192      C
193      C--USAGE-- "RHANK1" IS CALLED AS FOLLOWS:
194      C      ...
195      C      EXTERNAL RF
196      C      ...

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197 C      R=RHANK1(ALOG(B),RF,TOL,L)/B
198 C      ...
199 C      END
200 C      REAL FUNCTION RF(G)
201 C      ...USER SUPPLIED CODE...
202 C      END
203 C
204 C--NOTES:
205 C      (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
206 C      STATEMENT FUNCTION C1(II) BELOW; HOWEVER,
207 C      THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
208 C      EXP-UNDERFLOW'S TO 0.0,.....
209 C      (2). SOME NON-ANSI FORTRAN STATEMENTS ARE USED
210 C      (E.G; DO 20 J=85,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
211 C      THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
212 C
213 C
214 C
215 C--J1-EXTENDED FILTER WEIGHT ARRAYS:
216 C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
217 C TO SAVE STORAGE (SEE STATEMENT FUNCTION C1(II) BELOW).
218 C      DIMENSION WT(236),W1(76),W2(76),W3(76),W4(8)
219 C      EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
220 C      1 (WT(229),W4(1))
221 C      DATA W1/
222 C      1-8.8863805E-10, 1.1293811E-09,-1.2050872E-09, 1.2696232E-09,
223 C      2-1.3223909E-09, 1.3642393E-09,-1.3969439E-09, 1.4225941E-09,
224 C      3-1.4427475E-09, 1.4580582E-09,-1.4682563E-09, 1.4732179E-09,
225 C      4-1.4735606E-09, 1.4719870E-09,-1.4727091E-09, 1.4828225E-09,
226 C      5-1.5102619E-09, 1.5667752E-09,-1.6634522E-09, 1.8172900E-09,
227 C      6-2.0412753E-09, 2.3595230E-09,-2.7861077E-09, 3.3592871E-09,
228 C      7-4.0940172E-09, 5.0571015E-09,-6.2604109E-09, 7.8269461E-09,
229 C      8-9.7514701E-09, 1.2267639E-08,-1.5312389E-08, 1.9339924E-08,
230 C      9-2.4126297E-08, 3.0576829E-08,-3.8060204E-08, 4.8423732E-08,
231 C      1-6.0051116E-08, 7.6787475E-08,-9.4700993E-08, 1.2192844E-07,
232 C      2-1.4918997E-07, 1.9392737E-07,-2.3464786E-07, 3.0911127E-07,
233 C      3-3.6815394E-07, 4.9413800E-07,-5.7554168E-07, 7.9301529E-07,
234 C      4-8.9502818E-07, 1.2794292E-06,-1.3811469E-06, 2.0789668E-06,
235 C      5-2.1069398E-06, 3.4103188E-06,-3.1584463E-06, 5.6639045E-06,
236 C      6-4.6059955E-06, 9.5561672E-06,-6.4142855E-06, 1.6440205E-05,
237 C      7-8.2010619E-06, 2.8945217E-05,-8.6348466E-06, 5.2317398E-05,

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238 8-3.9915035E-06, 9.7273612E-05, 1.5220520E-05, 1.8614373E-04,
239 9 7.2023760E-05, 3.6620099E-04, 2.2062958E-04, 7.3874539E-04,
240 1 5.8623480E-04, 1.5226779E-03, 1.4538718E-03, 3.1930365E-03/
241 DATA W2/
242 1 3.4640868E-03, 6.7790882E-03, 8.0328420E-03, 1.4484339E-02,
243 2 1.8201316E-02, 3.0866143E-02, 4.0106549E-02, 6.4527872E-02,
244 3 8.4285526E-02, 1.2773175E-01, 1.6020907E-01, 2.1948043E-01,
245 4 2.3636305E-01, 2.4895051E-01, 1.2586300E-01, -5.1060445E-02,
246 5-3.4376222E-01, -2.9042175E-01, 1.1564736E-01, 4.9253231E-01,
247 6-4.6748595E-01, 1.5280945E-01, 3.3348541E-02, -8.2485252E-02,
248 7 7.9740630E-02, -6.6934498E-02, 5.5150465E-02, -4.5868721E-02,
249 8 3.8651958E-02, -3.2935834E-02, 2.8303994E-02, -2.4475127E-02,
250 9 2.1259541E-02, -1.8526278E-02, 1.6182037E-02, -1.4158101E-02,
251 1 1.2402725E-02, -1.0873526E-02, 9.5382016E-03, -8.3723743E-03,
252 2 7.3506490E-03, -6.4551136E-03, 5.6696335E-03, -4.9803353E-03,
253 3 4.3752213E-03, -3.8438703E-03, 3.3772023E-03, -2.9672872E-03,
254 4 2.6071877E-03, -2.2908274E-03, 2.0128794E-03, -1.7686708E-03,
255 5 1.5540998E-03, -1.3655866E-03, 1.1999089E-03, -1.0543497E-03,
256 6 9.2644973E-04, -8.1406593E-04, 7.1531559E-04, -6.2854459E-04,
257 7 5.5229955E-04, -4.8530352E-04, 4.2643446E-04, -3.7470650E-04,
258 8 3.2925334E-04, -2.8931382E-04, 2.5421910E-04, -2.2338147E-04,
259 9 1.9628455E-04, -1.7247455E-04, 1.5155278E-04, -1.3316089E-04,
260 1 1.1701502E-04, -1.0282066E-04, 9.0348135E-05, -7.9388568E-05/
261 DATA W3/
262 1 6.9758436E-05, -6.1296474E-05, 5.3860978E-05, -4.7327436E-05,
263 2 4.1586435E-05, -3.6541840E-05, 3.2109174E-05, -2.8214208E-05,
264 3 2.4791718E-05, -2.1784390E-05, 1.9141864E-05, -1.6819888E-05,
265 4 1.4779578E-05, -1.2986765E-05, 1.1411426E-05, -1.0027182E-05,
266 5 8.8108499E-06, -7.7420630E-06, 6.8029235E-06, -5.9777053E-06,
267 6 5.2525892E-06, -4.6154325E-06, 4.0555653E-06, -3.5636118E-06,
268 7 3.1313335E-06, -2.7514911E-06, 2.4177236E-06, -2.1244417E-06,
269 8 1.8667342E-06, -1.6402859E-06, 1.4413051E-06, -1.2664597E-06,
270 9 1.1128220E-06, -9.7781908E-07, 8.5919028E-07, -7.5494920E-07,
271 1 6.6335060E-07, -5.8286113E-07, 5.1213358E-07, -4.4998431E-07,
272 2 3.9537334E-07, -3.4738689E-07, 3.0522189E-07, -2.6817250E-07,
273 3 2.3561831E-07, -2.0701397E-07, 1.8188012E-07, -1.5979545E-07,
274 4 1.4038968E-07, -1.2333746E-07, 1.0835294E-07, -9.5185048E-08,
275 5 8.3613184E-08, -7.3443411E-08, 6.4505118E-08, -5.6648167E-08,
276 6 4.9740428E-08, -4.3665572E-08, 3.8321109E-08, -3.3616717E-08,
277 7 2.9472836E-08, -2.5819439E-08, 2.2594957E-08, -1.9745353E-08,
278 8 1.7223359E-08, -1.4987869E-08, 1.3003472E-08, -1.1240058E-08,

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279      9 9.6723739E-09,-8.2794392E-09, 7.0438407E-09,-5.9509676E-09,
280      ,1 4.9882405E-09,-4.1443813E-09, 3.4088114E-09,-2.7712762E-09/
281      DATA W4/
282      1 2.2217311E-09,-1.7504755E-09, 1.3485207E-09,-1.0080937E-09,
283      2 7.2300885E-10,-4.8860666E-10, 3.0121413E-10,-9.1649798E-11/
284 C--$$ENDATA
285 C
286 C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:
287      C1(I1)=FUN(EXP(-X+FLOAT(I1)*.20-17.0))*WT(I1)
288      RHANK1=0.0
289      CMAX=0.0
290      L=13
291      DO 10 I=86,98
292      C=C1(I)
293      RHANK1=RHANK1+C
294      CMAX=AMAX1(ABS(C),CMAX)
295      10 CONTINUE
296      IF(CMAX.EQ.0.0) GO TO 60
297      CMAX=TOL*CMAX
298      DO 20 I=85,1,-1
299      C=C1(I)
300      RHANK1=RHANK1+C
301      L=L+1
302      IF(ABS(C).LE.CMAX) GO TO 30
303      20 CONTINUE
304      30 DO 40 I=99,236
305      C=C1(I)
306      RHANK1=RHANK1+C
307      L=L+1
308      IF(ABS(C).LE.CMAX) GO TO 50
309      40 CONTINUE
310      50 RETURN
311      60 DO 70 I=1,85
312      C=C1(I)
313      RHANK1=RHANK1+C
314      L=L+1
315      IF(C.EQ.0.0) GO TO 90
316      70 CONTINUE
317      80 DO 90 I=236,99,-1
318      C=C1(I)
319      RHANK1=RHANK1+C
320      L=L+1
321      IF(C.EQ.0.0) GO TO 50
322      90 CONTINUE
323      GO TO 50
324      END

```

325 REAL FUNCTION RFOURO(X,FUN,TOL,L)  
326 C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)\*COS(G\*B)\*DG" DEFINED AS THE  
327 C REAL FOURIER COSINE TRANSFORM WITH ARGUMENT X(=ALOG(B))  
328 C BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND  
329 C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS,...  
330 C  
331 C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO,  
332 C  
333 C--PARAMETERS:  
334 C  
335 C X = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM  
336 C FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).  
337 C NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN  
338 C CALLING PROGRAM AND IN SUBPROGRAM FUN.  
339 C THE REAL FUNCTION FUN SHOULD BE A MONOTONE  
340 C DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...  
341 C TOL= REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,  
342 C IF FILTER\*FUN<TOL\*MAX, THEN REST OF TAIL IS TRUNCATED.  
343 C THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,  
344 C TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON  
345 C THE FUNCTION FUN AND PARAMETER X...IN GENERAL,  
346 C A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"  
347 C BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY  
348 C RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN  
349 C APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,  
350 C ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...  
351 C L= RESULTING NO. FILTER WTS. USED IN THE VARIABLE  
352 C CONVOLUTION (L DEPENDS ON TOL AND FUN).  
353 C MIN,L=24 AND MAX,L=281--WHICH COULD  
354 C OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING  
355 C VERY FAST...  
356 C  
357 C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RFOURO; THE FOURIER  
358 C TRANSFORM IS THEN RFOURO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM  
359 C THIS ROUTINE...  
360 C  
361 C--USAGE-- "RFOURO" IS CALLED AS FOLLOWS:  
362 C ...  
363 C EXTERNAL RF  
364 C ...



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365 C      R=RFQUR0(ALOG(B),RF,TOL,L)/B
366 C      ...
367 C      END
368 C      REAL FUNCTION RF(G)
369 C      ...USER SUPPLIED CODE...
370 C      END
371 C
372 C--NOTES:
373 C      (1). EXP=UNDERFLOW'S MAY OCCUR IN EXECUTING THE
374 C      STATEMENT FUNCTION C0(II) BELOW; HOWEVER,
375 C      THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
376 C      EXP=UNDERFLOW'S TO 0.0.....
377 C      (2). SOME NON-ANSI FORTRAN STATEMENTS ARE USED
378 C      (E.G: DO 120 I=148,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
379 C      THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
380 C
381 C
382 C--COS-EXTENDED FILTER WEIGHT ARRAYS:
383 C      NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
384 C      TO SAVE STORAGE (SEE STATEMENT FUNCTION C0(II) BELOW).
385 C      DIMENSION YT(281),Y1(76),Y2(76),Y3(76),Y4(53)
386 C      EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1)),
387 C      1 (YT(229),Y4(1))
388 C      DATA Y1/
389 C      1 5.1178101E-14, 2.9433849E-14, 2.5492522E-14, 1.9034819E-14,
390 C      2 6.4179780E-14, 1.3085746E-15, 1.1989957E-13, -1.2216234E-14,
391 C      3 1.7534103E-13, 7.9373498E-15, 2.1235658E-13, 7.9981520E-14,
392 C      4 2.3815757E-13, 1.9714260E-13, 2.8920132E-13, 3.4161340E-13,
393 C      5 4.0349917E-13, 5.2203885E-13, 5.9837223E-13, 7.8015306E-13,
394 C      6 8.8911655E-13, 1.1709731E-12, 1.3165595E-12, 1.7578463E-12,
395 C      7 1.9538564E-12, 2.6289768E-12, 2.9167697E-12, 3.9044344E-12,
396 C      8 4.3927341E-12, 5.7526904E-12, 6.6569552E-12, 8.4555678E-12,
397 C      9 1.0063229E-11, 1.2487964E-11, 1.5134682E-11, 1.8501488E-11,
398 C      1 2.2720051E-11, 2.7452598E-11, 3.4025443E-11, 4.0875985E-11,
399 C      2 5.0751668E-11, 6.1094382E-11, 7.5492982E-11, 9.1445759E-11,
400 C      3 1.1227336E-10, 1.3676464E-10, 1.6720269E-10, 2.0423244E-10,
401 C      4 2.4932743E-10, 3.0470661E-10, 3.7198526E-10, 4.5449934E-10,
402 C      5 5.5502537E-10, 6.7793669E-10, 8.2810031E-10, 1.0112626E-09,
403 C      6 1.2354800E-09, 1.5085255E-09, 1.8432253E-09, 2.2503397E-09,
404 C      7 2.7499027E-09, 3.3569525E-09, 4.1025670E-09, 5.0077487E-09,
405 C      8 6.1205950E-09, 7.4703399E-09, 9.1312760E-09, 1.1143911E-08,

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406 9 1.3622929E-08, 1.6623917E-08, 2.0324094E-08, 2.4798610E-08,  
 407 1 3.0321709E-08, 3.6992986E-08, 4.5237482E-08, 5.5183434E-08/  
 408 DATA Y2/  
 409 1 6.7491070E-08, 8.2317946E-08, 1.0069271E-07, 1.2279375E-07,  
 410 2 1.5022907E-07, 1.8316969E-07, 2.2413747E-07, 2.7322865E-07,  
 411 3 3.3441046E-07, 4.0756197E-07, 4.9894278E-07, 6.0793233E-07,  
 412 4 7.4443665E-07, 9.0679753E-07, 1.1107379E-06, 1.3525651E-06,  
 413 5 1.6573073E-06, 2.0174273E-06, 2.4728798E-06, 3.0090445E-06,  
 414 6 3.6898816E-06, 4.4879625E-06, 5.5059521E-06, 6.6935820E-06,  
 415 7 8.2160716E-06, 9.9828691E-06, 1.2260527E-05, 1.4888061E-05,  
 416 8 1.8296530E-05, 2.2202672E-05, 2.7305154E-05, 3.3109672E-05,  
 417 9 4.0751046E-05, 4.9372484E-05, 6.0820947E-05, 7.3619571E-05,  
 418 1 9.0780005E-05, 1.0976837E-04, 1.3550409E-04, 1.6365675E-04,  
 419 2 2.0227521E-04, 2.4398338E-04, 3.0197018E-04, 3.6370760E-04,  
 420 3 4.5083748E-04, 5.4213338E-04, 6.7315347E-04, 8.0800951E-04,  
 421 4 1.0051938E-03, 1.2041401E-03, 1.5011708E-03, 1.7942344E-03,  
 422 5 2.2421056E-03, 2.6730676E-03, 3.3490681E-03, 3.9815050E-03,  
 423 6 5.0028666E-03, 5.9285668E-03, 7.4730905E-03, 8.8233510E-03,  
 424 7 1.1160132E-02, 1.3119627E-02, 1.6653198E-02, 1.9472767E-02,  
 425 8 2.4800811E-02, 2.8793704E-02, 3.6762063E-02, 4.2228780E-02,  
 426 9 5.3905163E-02, 6.0804660E-02, 7.7081738E-02, 8.3874501E-02,  
 427 1 1.0377190E-01, 1.0377718E-01, 1.1892208E-01, 9.0437429E-02/  
 428 DATA Y3/  
 429 1 7.1685138E-02, -3.9473064E-02, -1.5078720E-01, -4.0489859E-01,  
 430 2 -5.6018995E-01, -6.8050388E-01, -1.5094224E-01, 6.6304064E-01,  
 431 3 1.3766748E+00, -8.0373222E-01, -1.0869629E+00, 1.2812892E+00,  
 432 4 -5.0341082E-01, -4.4274455E-02, 2.0913102E-01, -1.9999661E-01,  
 433 5 1.5207664E-01, -1.0920260E-01, 7.8169956E-02, -5.6651561E-02,  
 434 6 4.1611799E-02, -3.0880012E-02, 2.3072559E-02, -1.7311631E-02,  
 435 7 1.3021442E-02, -9.8085025E-03, 7.3943529E-03, -5.5769518E-03,  
 436 8 4.2073164E-03, -3.1745026E-03, 2.3954154E-03, -1.8076122E-03,  
 437 9 1.3640816E-03, -1.0293934E-03, 7.7682952E-04, -5.8623518E-04,  
 438 1 4.4240399E-04, -3.3386183E-04, 2.5195025E-04, -1.9013541E-04,  
 439 2 1.4348659E-04, -1.0828284E-04, 8.1716174E-05, -6.1667509E-05,  
 440 3 4.6537684E-05, -3.5119887E-05, 2.6503388E-05, -2.0000904E-05,  
 441 4 1.5093768E-05, -1.1390572E-05, 8.5559318E-06, -6.4869407E-06,  
 442 5 4.8953713E-06, -3.6942830E-06, 2.7878625E-06, -2.1038241E-06,  
 443 6 1.5875917E-06, -1.1980090E-06, 9.0398030E-07, -6.8208296E-07,  
 444 7 5.1458650E-07, -3.8817581E-07, 2.9272267E-07, -2.2067921E-07,  
 445 8 1.6623514E-07, -1.2514102E-07, 9.4034535E-08, -7.0556837E-08,  
 446 9 5.2741581E-08, -3.9298610E-08, 2.9107255E-08, -2.1413893E-08,

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447      1 1.5742032E-08,-1.1498608E-08, 8.7561571E-09,-7.2959446E-09/
448      DATA Y4/
449      1 6.8816619E-09,-8.9679825E-09, 1.4258275E-08,-1.9564299E-08,
450      2 2.0235313E-08,-1.4725545E-08, 5.4632820E-09, 3.5995580E-09,
451      3-9.5287133E-09, 1.1460041E-08,-1.0250532E-08, 7.4641748E-09,
452      4-4.4703465E-09, 2.0499053E-09,-4.4806353E-10,-4.0374336E-10,
453      5 7.0321001E-10,-6.7067960E-10, 4.9130404E-10,-2.8840747E-10,
454      6 1.2373144E-10,-1.5260443E-11,-4.2027559E-11, 6.1885474E-11,
455      7-5.9273937E-11, 4.6588766E-11,-3.2054182E-11, 1.9831637E-11,
456      8-1.1210098E-11, 5.9567021E-12,-3.2427812E-12, 2.1353868E-12,
457      9-1.8476851E-12, 1.8438474E-12,-1.8362842E-12, 1.7241847E-12,
458      1-1.5161479E-12, 1.2627657E-12,-1.0129176E-12, 7.9578625E-13,
459      2-6.2131435E-13, 4.8745900E-13,-3.8703630E-13, 3.1172547E-13,
460      3-2.5397802E-13, 2.0824130E-13,-1.7123163E-13, 1.4113344E-13,
461      4-1.1687986E-13, 9.7664016E-14,-8.2977176E-14, 7.2515267E-14,
462      5-5.6047478E-14/
463      C--$ENDATA
464      C
465      C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:
466      CO(II)=FUN(EXP(-X+FLOAT(II)*.20+30.30251236))*YT(II)
467      RFOURO=0.0
468      CMAX=0.0
469      L=22
470      DO 110 I=149,170
471      C=CO(I)
472      RFOURG=RFOURO+C
473      CMAX=AMAX1(ABS(C),CMAX)
474      110 CONTINUE
475      IF(CMAX.EQ.0.0) GO TO 150
476      CMAX=TOL*CMAX
477      DO 120 I=148,1,-1
478      C=CO(I)
479      RFOURO=RFOURO+C
480      L=L+1
481      IF(ABS(C).LE.CMAX) GO TO 130
482      120 CONTINUE
483      130 DO 140 I=171,281
484      C=CO(I)
485      RFOURO=RFOURO+C
486      L=L+1
487      IF(ABS(C).LE.CMAX) GO TO 190

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720 C--USAGE-- "ZHANKO" IS CALLED AS FOLLOWS:
721 C
722 C      ...
723 C      COMPLEX Z,ZHANKO,ZF
724 C      EXTERNAL ZF
725 C      ...
726 C      Z=ZHANKO(ALOG(B),ZF,TOL,L)/B
727 C      ...
728 C      END
729 C      COMPLEX FUNCTION ZF(G)
730 C      ...USER SUPPLIED CODE...
731 C      END
732 C--NOTES:
733 C      (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
734 C      STATEMENT FUNCTION CO(II) BELOW; HOWEVER,
735 C      THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
736 C      EXP-UNDERFLOW'S TO 0.0.....
737 C      (2). SOME NON-ANSI FORTRAN STATEMENTS ARE USED
738 C      (E.G: DO 120 I=128,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
739 C      THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
740 C
741 C
742 C
743 C--JO-EXTENDED FILTER WEIGHT ARRAYS:
744 C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
745 C TO SAVE STORAGE (SEE STATEMENT FUNCTION CO(II) BELOW).
746 C      DIMENSION YT(193),Y1(76),Y2(76),Y3(41)
747 C      EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1))
748 C      DATA Y1/
749 C      1 5.9565723E-08, 7.1143477E-11,-7.8395565E-11, 8.7489547E-11,
750 C      2-8.9007811E-11, 9.8790055E-11,-9.8675347E-11, 1.1118797E-10,
751 C      3-1.0893474E-10, 1.2543400E-10,-1.1979399E-10, 1.4200767E-10,
752 C      4-1.3106341E-10, 1.6153229E-10,-1.4238602E-10, 1.8486236E-10,
753 C      5-1.5315381E-10, 2.1319755E-10,-1.6238115E-10, 2.4824144E-10,
754 C      6-1.6850378E-10, 2.9243813E-10,-1.6909302E-10, 3.4934366E-10,
755 C      7-1.6043759E-10, 4.2417082E-10,-1.3690001E-10, 5.2458440E-10,
756 C      8-8.9946096E-11, 6.6188220E-10,-6.6964033E-12, 8.5276151E-10,
757 C      9 1.3222770E-10, 1.1219600E-09, 3.5591442E-10, 1.5061956E-09,
758 C      1 7.0795382E-10, 2.0600379E-09, 1.2535947E-09, 2.8646623E-09,
759 C      2 2.0904225E-09, 4.0409101E-09, 3.3642886E-09, 5.7687700E-09,
760 C      3 5.2930786E-09, 8.3164338E-09, 8.2021809E-09, 1.2083635E-08,

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680      COMPLEX FUNCTION ZHANKO(X,FUN,TOL,L)
681      C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J0(G*B)*DG" DEFINED AS THE
682      C COMPLEX HANKEL TRANSFORM OF ORDER 0 AND ARGUMENT X(=ALOG(B))
683      C BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
684      C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS,...
685      C
686      C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
687      C
688      C--PARAMETERS:
689      C
690      C      X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
691      C      FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
692      C              OF A REAL ARGUMENT G.
693      C      NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
694      C              CALLING PROGRAM AND IN SUBPROGRAM FUN.
695      C              THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
696      C              DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
697      C              FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RHANKO" IS ADVISED;
698      C              HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
699      C              INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
700      C      TOL=    REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
701      C              IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
702      C              THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
703      C              TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
704      C              THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
705      C              A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
706      C              BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
707      C              RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
708      C              APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
709      C              ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
710      C      L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
711      C              CONVOLUTION (L DEPENDS ON TOL AND FUN).
712      C              MIN,L=20 AND MAX,L=191--WHICH COULD
713      C              OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
714      C              VERY FAST...
715      C
716      C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZHANKO; THE HANKEL
717      C TRANSFORM IS THEN ZHANKO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
718      C THIS ROUTINE....
719      C

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488	140	CONTINUE
489		GO TO 190
490	150	DO 160 I=1,148
491		C=C0(I)
492		RFOURO=RFOURO+C
493		L=L+1
494		IF(C.EQ.0.0) GO TO 170
495	160	CONTINUE
496	170	DO 180 I=281,171,-1
497		C=C0(I)
498		RFOURO=RFOURO+C
499		L=L+1
500		IF(C.EQ.0.0) GO TO 190
501	180	CONTINUE
502	190	RETURN
503		END

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504      REAL FUNCTION RFOUR1(X,FUN,TOL,L)
505      C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*SIN(G*B)*DG" DEFINED AS THE
506      C REAL FOURIER SINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
507      C BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
508      C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
509      C
510      C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO,
511      C
512      C--PARAMETERS:
513      C
514      C      X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
515      C      FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED),
516      C      NOTE: IF PARAMS OTHER THAN G ARE REQUIRED, USE COMMON IN
517      C      CALLING PROGRAM AND IN SUBPROGRAM FUN,
518      C      THE REAL FUNCTION FUN SHOULD BE A MONOTONE
519      C      DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
520      C      TOL= REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
521      C      IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED,
522      C      THIS IS DONE AT BOTH ENDS OF FILTER, TYPICALLY,
523      C      TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
524      C      THE FUNCTION FUN AND PARAMETER X,..IN GENERAL,
525      C      A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
526      C      BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY
527      C      RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
528      C      APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
529      C      ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
530      C      L= RESULTING NO. FILTER WTS. USED IN THE VARIABLE
531      C      CONVOLUTION (L DEPENDS ON TOL AND FUN).
532      C      MIN,L=20 AND MAX,L=256--WHICH COULD
533      C      OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
534      C      VERY FAST...
535      C
536      C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RFOUR1; THE FOURIER
537      C TRANSFORM IS THEN RFOUR1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
538      C THIS ROUTINE....
539      C
540      C--USAGE-- "RFOUR1" IS CALLED AS FOLLOWS:
541      C      ...
542      C      EXTERNAL RF
543      C      ...

```

```

544      R=RFOUR1(ALOG(B),RF,TOL,L)/B
545      ...
546      END
547      C
548      REAL FUNCTION RF(G)
549      ...USER SUPPLIED CODE...
550      END
551      C
552      C--NOTES:
553      C      (1). EXP=UNDERFLOW'S MAY OCCUR IN EXECUTING THE
554      C      STATEMENT FUNCTION C1(II) BELOW; HOWEVER,
555      C      THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
556      C      EXP=UNDERFLOW'S TO 0.0,.....
557      C      (2). SOME NON-ANSI FORTRAN STATEMENTS ARE USED
558      C      (E.G: DO 20 I=190,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
559      C      THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
560      C
561      C--SIN-EXTENDED FILTER WEIGHT ARRAYS:
562      C      NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
563      C      TO SAVE STORAGE (SEE STATEMENT FUNCTION C1(II) BELOW).
564      C      DIMENSION WT(266),W1(76),W2(76),W3(76),W4(38)
565      C      EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
566      C      1 (WT(229),W4(1))
567      C      DATA W1/
568      C      1-1.1113940E-09,-1.3237246E-12, 1.3091739E-12,-1.6240954E-12,
569      C      2 1.7236636E-12,-1.8227727E-12, 1.9255992E-12,-2.0335514E-12,
570      C      3 2.1473541E-12,-2.2675549E-12, 2.3946842E-12,-2.5292661E-12,
571      C      4 2.6718110E-12,-2.8227693E-12, 2.9825171E-12,-3.1514006E-12,
572      C      5 3.3297565E-12,-3.5179095E-12, 3.7163306E-12,-3.9256378E-12,
573      C      6 4.1464798E-12,-4.3794552E-12, 4.6252131E-12,-4.8845227E-12,
574      C      7 5.1582809E-12,-5.4474462E-12, 5.7530277E-12,-6.0760464E-12,
575      C      8 6.4175083E-12,-6.7783691E-12, 7.1595239E-12,-7.5618782E-12,
576      C      9 7.9864477E-12,-8.4344110E-12, 8.9072422E-12,-9.4067705E-12,
577      C      1 9.9349439E-12,-1.0493731E-11, 1.1084900E-11,-1.1709937E-11,
578      C      2 1.2370354E-11,-1.3067414E-11, 1.3802200E-11,-1.4575980E-11,
579      C      3 1.5390685E-11,-1.6249313E-11, 1.7155934E-11,-1.8115250E-11,
580      C      4 1.9131898E-11,-2.0209795E-11, 2.1352159E-11,-2.2561735E-11,
581      C      5 2.3840976E-11,-2.5192263E-11, 2.6618319E-11,-2.8122547E-11,
582      C      6 2.9709129E-11,-3.1382670E-11, 3.3149030E-11,-3.5013168E-11,
583      C      7 3.6981050E-11,-3.9058553E-11, 4.1251694E-11,-4.3566777E-11,
584      C      8 4.6010537E-11,-4.8590396E-11, 5.1314761E-11,-5.4193353E-11,

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585 9 5.7236720E-11,-6.0455911E-11, 6.3861222E-11,-6.7461492E-11,  
586 1 7.1265224E-11,-7.5279775E-11, 7.9512249E-11,-8.3971327E-11/  
587 DATA W2/  
588 1 8.8668961E-11,-9.3621900E-11, 9.8851764E-11,-1.0438319E-10,  
589 2 1.1024087E-10,-1.1644680E-10, 1.2301979E-10,-1.2997646E-10,  
590 3 1.3733244E-10,-1.4510363E-10, 1.5330772E-10,-1.6196550E-10,  
591 4 1.7110130E-10,-1.8074257E-10, 1.9091922E-10,-2.0166306E-10,  
592 5 2.1300756E-10,-2.2498755E-10, 2.3763936E-10,-2.5100098E-10,  
593 6 2.6511250E-10,-2.8001616E-10, 2.9575691E-10,-3.1238237E-10,  
594 7 3.2994314E-10,-3.4849209E-10, 3.6808529E-10,-3.8878042E-10,  
595 8 4.1063982E-10,-4.3372666E-10, 4.5811059E-10,-4.8386049E-10,  
596 9 5.1105728E-10,-5.3977672E-10, 5.7011632E-10,-6.0215516E-10,  
597 1 6.3601273E-10,-6.7175964E-10, 7.0955028E-10,-7.4942601E-10,  
598 2 7.9161025E-10,-8.3606980E-10, 8.8317110E-10,-9.3270330E-10,  
599 3 9.8533749E-10,-1.0404508E-09, 1.0993731E-09,-1.1605442E-09,  
600 4 1.2267331E-09,-1.2942905E-09, 1.3691677E-09,-1.4429912E-09,  
601 5 1.5288164E-09,-1.6077524E-09, 1.7095998E-09,-1.7890471E-09,  
602 6 1.9129068E-09,-1.9857116E-09, 2.1491608E-09,-2.1926779E-09,  
603 7 2.4312660E-09,-2.3959044E-09, 2.7872500E-09,-2.5610596E-09,  
604 8 3.2762318E-09,-2.6082940E-09, 4.0251453E-09,-2.3550563E-09,  
605 9 5.3176554E-09,-1.3960161E-09, 7.7708747E-09, 1.1853546E-09,  
606 1 1.2760851E-08, 7.4264707E-09, 2.3342187E-08, 2.1869851E-08/  
607 DATA W3/  
608 1 4.6306744E-08, 5.4631686E-08, 9.6763087E-08, 1.2823337E-07,  
609 2 2.0832812E-07, 2.9280540E-07, 4.5580888E-07, 6.5992437E-07,  
610 3 1.0056815E-06, 1.4779183E-06, 2.2284335E-06, 3.2994604E-06,  
611 4 4.9485823E-06, 7.3545473E-06, 1.1001083E-05, 1.6380539E-05,  
612 5 2.4469550E-05, 3.6469246E-05, 5.4441527E-05, 8.1176726E-05,  
613 6 1.2113828E-04, 1.8066494E-04, 2.6954609E-04, 4.0202288E-04,  
614 7 5.996995E-04, 8.9437312E-04, 1.3338166E-03, 1.9886697E-03,  
615 8 2.9643943E-03, 4.4168923E-03, 6.5773518E-03, 9.7855105E-03,  
616 9 1.4539361E-02, 2.1558670E-02, 3.1871854E-02, 4.6903518E-02,  
617 1 6.8559512E-02, 9.9170152E-02, 1.4120770E-01, 1.9610835E-01,  
618 2 2.6192603E-01, 3.2743321E-01, 3.6407406E-01, 3.1257559E-01,  
619 3 9.0460168E-01, 3.6051039E-01,-8.6324760E-01,-8.1178720E-01,  
620 4 5.2205241E-01, 1.5449673E+00,-1.1817933E+00,-2.6759896E-01,  
621 5 8.0869203E-01,-6.2757149E-01, 3.4062630E-01,-1.5885304E-01,  
622 6 7.0472994E-02,-3.1624462E-02, 1.4894058E-02,-7.4821176E-03,  
623 7 4.0035936E-03,-2.2543784E-03, 1.3160358E-03,-7.8636604E-04,  
624 8 4.7658745E-04,-2.9125817E-04, 1.7885105E-04,-1.1012416E-04,  
625 9 6.7910334E-05,-4.1914054E-05, 2.5881544E-05,-1.5985851E-05,

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626      1 9.8751880E-06,-6.1008526E-06, 3.7692543E-06,-2.3287953E-06/
627      DATA W4/
628      1 1.4388425E-06,-8.8899353E-07, 5.4926991E-07,-3.3937048E-07,
629      2 2.0968284E-07,-1.2955437E-07, 8.0046336E-08,-4.9457371E-08,
630      3 3.0557711E-08,-1.8880390E-08, 1.1665454E-08,-7.2076428E-09,
631      4 4.4533423E-09,-2.7515696E-09, 1.7001092E-09,-1.0504494E-09,
632      5 6.4904567E-10,-4.0102999E-10, 2.4778763E-10,-1.5310321E-10,
633      6 9.4600354E-11,-5.8453314E-11, 3.6119400E-11,-2.2320056E-11,
634      7 1.3793460E-11,-8.5242656E-12, 5.2675102E-12,-3.2543076E-12,
635      8 2.0097689E-12,-1.2405412E-12, 7.6530538E-13,-4.7191929E-13,
636      9 2.9084993E-13,-1.7923661E-13, 1.1018948E-13,-6.7885902E-14,
637      1 4.2025050E-14,-2.1314731E-14/

```

C--\$ENDATA

C

C

C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:

```

642      C1(II)=FUN(EXP(-X+FLOAT(II)*.20-38.30455704))*WT(II)
643      RFOUR1=0.0
644      CMAX=0.0
645      L=18
646      DO 10 I=191,208
647      C=C1(I)
648      RFOUR1=RFOUR1+C
649      CMAX=AMAX1(ABS(C),CMAX)
650      10 CONTINUE
651      IF(CMAX.EQ.0.0) GO TO 60
652      CMAX=TOT*CMAX
653      DO 20 I=190,1,-1
654      C=C1(I)
655      RFOUR1=RFOUR1+C
656      L=L+1
657      IF(ABS(C).LE.CMAX) GO TO 30
658      20 CONTINUE
659      30 DO 40 I=209,266
660      C=C1(I)
661      RFOUR1=RFOUR1+C
662      L=L+1
663      IF(ABS(C).LE.CMAX) GO TO 50
664      40 CONTINUE
665      50 RETURN
666      60 DO 70 I=1,190

```

667		C=C1(I)
668		RFOUR1=RFOUR1+C
669		L=L+1
670		IF(C,EQ,0.0) GO TO 80
671	70	CONTINUE
672	80	DO 90 I=266,209,-1
673		C=C1(I)
674		RFOUR1=RFOUR1+C
675		L=L+1
676		IF(C,EQ,0.0) GO TO 50
677	90	CONTINUE
678		GO TO 50
679		END

```

761 4 1.2577400E-08, 1.7666303E-08, 1.9143895E-08, 2.5953011E-08,
762 5 2.8983953E-08, 3.8268851E-08, 4.3712685E-08, 5.6590075E-08,
763 6 6.5740136E-08, 8.3864288E-08, 9.8662323E-08, 1.2448811E-07,
764 7 1.4784461E-07, 1.8501974E-07, 2.2129198E-07, 2.7524203E-07,
765 8 3.3094739E-07, 4.0974828E-07, 4.9462868E-07, 6.1030809E-07,
766 9 7.3891802E-07, 9.0939667E-07, 1.1034727E-06, 1.3554600E-06,
767 1 1.6474556E-06, 2.0207695E-06, 2.4591294E-06, 3.0131400E-06/
768 DATA Y2/
769 1 3.6701680E-06, 4.4934101E-06, 5.4770076E-06, 6.7015208E-06,
770 2 8.1726989E-06, 9.9954201E-06, 1.2194425E-05, 1.4909101E-05,
771 3 1.8194388E-05, 2.2239184E-05, 2.7145562E-05, 3.3174088E-05,
772 4 4.0499452E-05, 4.9486730E-05, 6.0421440E-05, 7.3822001E-05,
773 5 9.0141902E-05, 1.1012552E-04, 1.3448017E-04, 1.6428337E-04,
774 6 2.0062570E-04, 2.4507880E-04, 2.9930366E-04, 3.6560582E-04,
775 7 4.4651421E-04, 5.4541300E-04, 6.6512648E-04, 8.1365181E-04,
776 8 9.9374786E-04, 1.2138120E-03, 1.4824945E-03, 1.8107657E-03,
777 9 2.2115938E-03, 2.7012675E-03, 3.2991969E-03, 4.0295817E-03,
778 1 4.9214244E-03, 6.0106700E-03, 7.3405529E-03, 8.9643708E-03,
779 2 1.0946310E-02, 1.3365017E-02, 1.6314985E-02, 1.9910907E-02,
780 3 2.4289325E-02, 2.9612896E-02, 3.6070402E-02, 4.3876936E-02,
781 4 5.3264829E-02, 6.4465091E-02, 7.7664144E-02, 9.2918324E-02,
782 5 1.1000121E-01, 1.2811102E-01, 1.4543025E-01, 1.5832248E-01,
783 6 1.6049224E-01, 1.4170064E-01, 8.8788108E-02, -1.1330934E-02,
784 7 -1.5331864E-01, -2.9094670E-01, -2.9084655E-01, -2.9708834E-02,
785 8 3.9009601E-01, 1.7999785E-01, -4.1858139E-01, 1.5317216E-01,
786 9 6.5184953E-02, -1.0751806E-01, 7.8429567E-02, -4.6019124E-02,
787 1 2.5309571E-02, -1.3904823E-02, 7.8187120E-03, -4.5190169E-03/
788 DATA Y3/
789 1 2.6729062E-03, -1.6073718E-03, 9.7715622E-04, -5.9804407E-04,
790 2 3.6749320E-04, -2.2635296E-04, 1.3960805E-04, -8.6172618E-05,
791 3 5.3212947E-05, -3.2867888E-05, 2.0304203E-05, -1.2543926E-05,
792 4 7.7499633E-06, -4.7882430E-06, 2.9584108E-06, -1.8278645E-06,
793 5 1.1293571E-06, -6.9778174E-07, 4.3113019E-07, -2.6637753E-07,
794 6 1.6458373E-07, -1.0168954E-07, 6.2829807E-08, -3.8819969E-08,
795 7 2.3985272E-08, -1.4819520E-08, 9.1563774E-09, -5.6573541E-09,
796 8 3.4954514E-09, -2.1397005E-09, 1.3343946E-09, -8.2447148E-10,
797 9 5.0941033E-10, -3.1474631E-10, 1.9447072E-10, -1.2015685E-10,
798 1 7.4241055E-11, -4.5871468E-11, 2.8343095E-11, -1.7513137E-11,
799 2 6.9049613E-12/
800 C---$ENDATA
801 C

```



```

802      COMPLEX FUN,C,CO,CMAX
803      DIMENSION T(2),TMAX(2)
804      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
805      C--STATEMENT FUNCTIONS-- INCLUDES ABSCISSA GENERATION:
806      CO(II)=FUN(EXP(-X+FLOAT(II)*.20+26.30455704))*YT(II)
807      ZHANKO=(0.0,0.0)
808      CMAX=(0.0,0.0)
809      L=18
810      DO 110 I=129,146
811      C=CO(I)
812      ZHANKO=ZHANKO+C
813      TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
814      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
815      110  CONTINUE
816      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
817      CMAX=TOL*CMAX
818      DO 120 I=128,1,-1
819      C=CO(I)
820      ZHANKO=ZHANKO+C
821      L=L+1
822      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
823      120  CONTINUE
824      130  DO 140 I=147,193
825      C=CO(I)
826      ZHANKO=ZHANKO+C
827      L=L+1
828      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
829      140  CONTINUE
830      GO TO 190
831      150  DO 160 I=1,128
832      C=CO(I)
833      ZHANKO=ZHANKO+C
834      L=L+1
835      IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
836      160  CONTINUE
837      170  DO 180 I=193,147,-1
838      C=CO(I)
839      ZHANKO=ZHANKO+C
840      L=L+1
841      IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
842      180  CONTINUE
843      190  RETURN
844      END

```

845                   COMPLEX FUNCTION ZHANK1(X,FUN,TOL,L)  
 846       C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)\*J1(G\*B)\*DG" DEFINED AS THE  
 847       C COMPLEX HANKEL TRANSFORM OF ORDER 1 AND ARGUMENT X(=ALOG(B))  
 848       C BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND  
 849       C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....  
 850       C  
 851       C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.  
 852       C  
 853       C--PARAMETERS:  
 854       C  
 855       C       X       = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM  
 856       C       FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)  
 857       C                   OF A REAL ARGUMENT G.  
 858       C       NOTE: IF PARAMS OTHER THAN G ARE REQUIRED, USE COMMON IN  
 859       C       CALLING PROGRAM AND IN SUBPROGRAM FUN.  
 860       C       THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE  
 861       C       DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...  
 862       C       FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RHANK1" IS ADVISED;  
 863       C       HOWEVER, TWO REAL FUNCTIONS F1(G),F2(G) MAY BE  
 864       C       INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))  
 865       C       TOL=     REAL TOLERANCE EXPECTED AT CONVOLVED TAILS--I.E.,  
 866       C                   IF FILTER\*FUN<TOL\*MAX, THEN REST OF TAIL IS TRUNCATED.  
 867       C                   THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,  
 868       C                   TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON  
 869       C                   THE FUNCTION FUN AND PARAMETER X...IN GENERAL,  
 870       C                   A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"  
 871       C                   BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY  
 872       C                   RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN  
 873       C                   APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,  
 874       C                   ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...  
 875       C       L=       RESULTING NO. FILTER WTS. USED IN THE VARIABLE  
 876       C                   CONVOLUTION (L DEPENDS ON TOL AND FUN).  
 877       C                   MIN.L=15 AND MAX.L=236--WHICH COULD  
 878       C                   OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING  
 879       C                   VERY FAST...  
 880       C  
 881       C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZHANK1; THE HANKEL  
 882       C TRANSFORM IS THEN ZHANK1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM  
 883       C THIS ROUTINE....  
 884       C

```

885 C--USAGE-- "ZHANK1" IS CALLED AS FOLLOWS:
886 C
887 C      ...
888 C      COMPLEX Z,ZHANK1,ZF
889 C      EXTERNAL ZF
890 C
891 C      ...
892 C      Z=ZHANK1(ALOG(B),ZF,TOL,L)/B
893 C      ...
894 C      END
895 C      COMPLEX FUNCTION ZF(G)
896 C      ...USER SUPPLIED CODE...
897 C      END
898 C--NOTES:
899 C      (1). EXP=UNDERFLOW'S MAY OCCUR IN EXECUTING THE
900 C      STATEMENT FUNCTION C1(II) BELOW; HOWEVER,
901 C      THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
902 C      EXP=UNDERFLOW'S TO 0.0.....
903 C      (2). SOME NON=ANSI FORTRAN STATEMENTS ARE USED
904 C      (E.G: DO 20 I=85,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
905 C      THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
906 C
907 C
908 C--J1=EXTENDED FILTER WEIGHT ARRAYS:
909 C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
910 C TO SAVE STORAGE (SEE STATEMENT FUNCTION C1(II) BELOW).
911 C      DIMENSION WT(236),W1(76),W2(76),W3(76),W4(8)
912 C      EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
913 C      1 (WT(229),W4(1))
914 C      DATA W1/
915 C      1-8.8863805E-10, 1.1293811E-09,-1.2050872E-09, 1.2696232E-09,
916 C      2-1.3223909E-09, 1.3642393E-09,-1.3969439E-09, 1.4225941E-09,
917 C      3-1.4427475E-09, 1.4580582E-09,-1.4682563E-09, 1.4732179E-09,
918 C      4-1.4735606E-09, 1.4719870E-09,-1.4727091E-09, 1.4828225E-09,
919 C      5-1.5102619E-09, 1.5667732E-09,-1.6634522E-09, 1.8172900E-09,
920 C      6-2.0412753E-09, 2.3595230E-09,-2.7861077E-09, 3.3592871E-09,
921 C      7-4.0940172E-09, 5.0571015E-09,-6.2604109E-09, 7.8269461E-09,
922 C      8-9.7514701E-09, 1.2267639E-08,-1.5312389E-08, 1.9339924E-08,
923 C      9-2.4126297E-08, 3.0576829E-08,-3.8050204E-08, 4.8423732E-08,
924 C      1-6.0051116E-08, 7.6787475E-08,-9.4700993E-08, 1.2192844E-07,
925 C      2-1.4918997E-07, 1.9392737E-07,-2.3464786E-07, 3.0911127E-07,

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926	3-3.6815394E-07,	4.9413800E-07,-5.7554169E-07,	7.9301529E-07,
927	4-8.9502818E-07,	1.2794292E-06,-1.3811469E-06,	2.0789668E-06,
928	5-2.1069398E-06,	3.4103188E-06,-3.1584463E-06,	5.6639045E-06,
929	6-4.6059955E-06,	9.5561672E-06,-6.4142855E-06,	1.6440205E-05,
930	7-8.2010619E-06,	2.8945217E-05,-8.6348466E-06,	5.2317398E-05,
931	8-3.9915035E-06,	9.7273612E-05,	1.5220520E-05,
932	9 7.2023760E-05,	3.6620099E-04,	2.2062958E-04,
933	1 5.8623480E-04,	1.5226779E-03,	1.4538718E-03,
934	DATA W2/		
935	1 3.4640868E-03,	6.7790892E-03,	8.0328420E-03,
936	2 1.8201316E-02,	3.0866143E-02,	4.0106549E-02,
937	3 8.4285526E-02,	1.2773175E-01,	1.6020907E-01,
938	4 2.3636305E-01,	2.4895051E-01,	1.2586300E-01,
939	5-3.4376222E-01,	-2.9042175E-01,	1.1564736E-01,
940	6-4.6748595E-01,	1.5280945E-01,	3.3348541E-02,
941	7 7.9740630E-02,	-6.6934498E-02,	5.5150465E-02,
942	8 3.8651958E-02,	-3.2935834E-02,	2.8303994E-02,
943	9 2.1259541E-02,	-1.8526278E-02,	1.6182037E-02,
944	1 1.2402225E-02,	-1.0873526E-02,	9.5392016E-03,
945	2 7.3506490E-03,	-6.4551136E-03,	5.6696335E-03,
946	3 4.3752213E-03,	-3.8432703E-03,	3.3772023E-03,
947	4 2.6071877E-03,	-2.2908274E-03,	2.0128794E-03,
948	5 1.5540998E-03,	-1.3655666E-03,	1.1999089E-03,
949	6 9.2644973E-04,	-8.1406593E-04,	7.1531359E-04,
950	7 5.5229955E-04,	-4.8530352E-04,	4.2643446E-04,
951	8 3.2925334E-04,	-2.8931382E-04,	2.5421910E-04,
952	9 1.9628455E-04,	-1.7247455E-04,	1.5155278E-04,
953	1 1.1701502E-04,	-1.0282066E-04,	9.0348135E-05,
954	DATA W3/		
955	1 6.9758435E-05,	-6.1296474E-05,	5.3860978E-05,
956	2 4.1586435E-05,	-3.6541840E-05,	3.2109174E-05,
957	3 2.4791718E-05,	-2.1784390E-05,	1.9141864E-05,
958	4 1.4779578E-05,	-1.2986765E-05,	1.1411426E-05,
959	5 8.8108499E-06,	-7.7420630E-06,	6.8022235E-06,
960	6 5.2525892E-06,	-4.6154325E-06,	4.0555853E-06,
961	7 3.1313335E-06,	-2.7514911E-06,	2.4177236E-06,
962	8 1.8667342E-06,	-1.6402859E-06,	1.4413051E-06,
963	9 1.1128220E-05,	-9.7781908E-07,	8.5919028E-07,
964	1 6.6335060E-07,	-5.8286113E-07,	5.1213358E-07,
965	2 3.9537334E-07,	-3.4738689E-07,	3.0522189E-07,
966	3 2.3561851E-07,	-2.0701397E-07,	1.8188012E-07,



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967      4 1.4038968E-07,-1.2333746E-07, 1.0835294E-07,-9.5185048E-08,
968      5 8.3613184E-08,-7.3443411E-08, 6.4505118E-08,-5.6648167E-08,
969      6 4.9740428E-08,-4.3665572E-08, 3.8321109E-08,-3.3616717E-08,
970      7 2.9472836E-08,-2.5819439E-08, 2.2594957E-08,-1.9745353E-08,
971      8 1.7223359E-08,-1.4987869E-08, 1.3003472E-08,-1.1240058E-08,
972      9 9.6723739E-09,-8.2794392E-09, 7.0438407E-09,-5.9509676E-09,
973      1 4.9882405E-09,-4.1443813E-09, 3.4088114E-09,-2.7712762E-09/
974      DATA W4/
975      1 2.2217311E-09,-1.7504755E-09, 1.3485207E-09,-1.0080937E-09,
976      2 7.2300885E-10,-4.8860666E-10, 3.0121413E-10,-9.1649798E-11/
977      C--$$ENDATA
978      C
979      COMPLEX FUN,C,C1,CMAX
980      DIMENSION T(2),TMAX(2)
981      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
982      C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:
983      C1(II)=FUN(EXP(-X+FLOAT(II)*.20-17.0))*WT(II)
984      ZHANK1=(0.0,0.0)
985      CMAX=(0.0,0.0)
986      L=13
987      DO 10 I=86,98
988      C=C1(I)
989      ZHANK1=ZHANK1+C
990      TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
991      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
992      10 CONTINUE
993      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 60
994      CMAX=TOL*CMAX
995      DO 20 I=85,1,-1
996      C=C1(I)
997      ZHANK1=ZHANK1+C
998      L=L+1
999      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 30
1000      20 CONTINUE
1001      30 DO 40 I=99,236
1002      C=C1(I)
1003      ZHANK1=ZHANK1+C
1004      L=L+1
1005      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 50
1006      40 CONTINUE
1007      50 RETURN

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1008      60 DO 70 I=1,85
1009          C=C1(I)
1010          ZHANK1=ZHANK1+C
1011          L=L+1
1012          IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 80
1013      70 CONTINUE
1014      80 DO 90 I=236.99,-1
1015          C=C1(I)
1016          ZHANK1=ZHANK1+C
1017          L=L+1
1018          IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 50
1019      90 CONTINUE
1020          GO TO 50
1021      END

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1022      COMPLEX FUNCTION ZFOURO(X,FUN,TOL,L)
1023      C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*COS(G*B)*DG" DEFINED AS THE
1024      C COMPLEX FOURIER COSINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
1025      C BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
1026      C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
1027      C
1028      C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
1029      C
1030      C--PARAMETERS:
1031      C
1032      C      X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
1033      C      FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
1034      C              OF A REAL ARGUMENT G.
1035      C              NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
1036      C              CALLING PROGRAM AND IN SUBPROGRAM FUN.
1037      C              THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
1038      C              DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
1039      C              FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RFOURO" IS ADVISED;
1040      C              HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
1041      C              INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
1042      C      TOL=    REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
1043      C              IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
1044      C              THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
1045      C              TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
1046      C              THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
1047      C              A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
1048      C              BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
1049      C              RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
1050      C              APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
1051      C              ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
1052      C      L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
1053      C              CONVOLUTION (L DEPENDS ON TOL AND FUN).
1054      C              MIN.L=24 AND MAX.L=261--WHICH COULD
1055      C              OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
1056      C              VERY FAST...
1057      C
1058      C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZFOURO; THE FOURIER
1059      C TRANSFORM IS THEN ZFOURO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
1060      C THIS ROUTINE....
1061      C

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1062 C--USAGE-- "ZFOURO" IS CALLED AS FOLLOWS;
1063 C
1064 C      ...
1065 C      COMPLEX Z,ZFOURO,ZF
1066 C      EXTERNAL ZF
1067 C      ...
1068 C      Z=ZFOURO(ALOG(B),ZF,TOL,L)/B
1069 C      ...
1070 C      END
1071 C      COMPLEX FUNCTION ZF(G)
1072 C      ...USER SUPPLIED CODE...
1073 C      END
1074 C
1075 C--NOTES:
1076 C      (1). EXP=UNDERFLOW'S MAY OCCUR IN EXECUTING THE
1077 C      STATEMENT FUNCTION CO(II) BELOW; HOWEVER,
1078 C      THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
1079 C      EXP=UNDERFLOW'S TO 0.0.....
1080 C      (2). SOME NON-ANSI FORTRAN STATEMENTS ARE USED
1081 C      (E.G: DO 120 I=148,1,*1), BUT IT WOULD BE SIMPLE TO CONVERT
1082 C      THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY,..
1083 C
1084 C--COS-EXTENDED FILTER WEIGHT ARRAYS:
1085 C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
1086 C TO SAVE STORAGE (SEE STATEMENT FUNCTION CO(II) BELOW).
1087 C      DIMENSION YT(281),Y1(76),Y2(76),Y3(76),Y4(53)
1088 C      EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1)),
1089 C      1 (YT(229),Y4(1))
1090 C      DATA Y1/
1091 C      1 5.1178101E-14, 2.9433849E-14, 2.5492522E-14, 1.9034819E-14,
1092 C      2 6.4179780E-14, 1.3085746E-15, 1.1989957E-13, -1.2216234E-14,
1093 C      3 1.7534103E-13, 7.9373498E-15, 2.1235658E-13, 7.9981520E-14,
1094 C      4 2.3815757E-13, 1.9714260E-13, 2.8920132E-13, 3.4161340E-13,
1095 C      5 4.0349917E-13, 5.2203885E-13, 5.9837223E-13, 7.8015306E-13,
1096 C      6 8.8911655E-13, 1.1709731E-12, 1.3165595E-12, 1.7578463E-12,
1097 C      7 1.9538564E-12, 2.6289768E-12, 2.9167697E-12, 3.9044344E-12,
1098 C      8 4.1927341E-12, 5.7526904E-12, 6.6869552E-12, 8.4555678E-12,
1099 C      9 1.0063229E-11, 1.2487964E-11, 1.5134682E-11, 1.8501488E-11,
1100 C      1 2.2720051E-11, 2.7452598E-11, 3.4025443E-11, 4.0875985E-11,
1101 C      2 5.0751668E-11, 6.1094382E-11, 7.5492982E-11, 9.1445759E-11,
1102 C      3 1.1227336E-10, 1.3576464E-10, 1.6720269E-10, 2.0423244E-10,

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1103 4 2.4932743E-10, 3.0470661E-10, 3.7198526E-10, 4.5449934E-10,
1104 5 5.5502537E-10, 6.7793669E-10, 8.2810001E-10, 1.0112626E-09,
1105 6 1.2354800E-09, 1.5085255E-09, 1.8432253E-09, 2.2503397E-09,
1106 7 2.7499027E-09, 3.3569525E-09, 4.1025670E-09, 5.0077487E-09,
1107 8 6.1205950E-09, 7.4703399E-09, 9.1312760E-09, 1.1143911E-08,
1108 9 1.3522929E-08, 1.6623917E-08, 2.0324094E-08, 2.4798610E-08,
1109 1 3.0321709E-08, 3.6992986E-08, 4.5237482E-08, 5.5183434E-08/
1110 DATA Y2/
1111 1 6.7491070E-08, 8.2317946E-08, 1.0069271E-07, 1.2279375E-07,
1112 2 1.5022907E-07, 1.8316969E-07, 2.2413747E-07, 2.7322865E-07,
1113 3 3.3441046E-07, 4.0756197E-07, 4.9894279E-07, 6.0793233E-07,
1114 4 7.4443665E-07, 9.0679753E-07, 1.1107379E-06, 1.3525651E-06,
1115 5 1.6573073E-06, 2.0174273E-06, 2.4728798E-06, 3.0090445E-06,
1116 6 3.6898816E-06, 4.4879625E-06, 5.5059521E-06, 6.6935820E-06,
1117 7 8.2160716E-06, 9.9828691E-06, 1.2260527E-05, 1.4888061E-05,
1118 8 1.8296530E-05, 2.2202672E-05, 2.7305154E-05, 3.3109672E-05,
1119 9 4.0751046E-05, 4.9372484E-05, 6.0820947E-05, 7.3619571E-05,
1120 1 9.0780005E-05, 1.0976837E-04, 1.3550409E-04, 1.6365676E-04,
1121 2 2.0227521E-04, 2.4398338E-04, 3.0197018E-04, 3.6370760E-04,
1122 3 4.5083748E-04, 5.4213238E-04, 6.7315347E-04, 8.0800951E-04,
1123 4 1.0051938E-03, 1.2041401E-03, 1.5011708E-03, 1.7942344E-03,
1124 5 2.2421056E-03, 2.6730676E-03, 3.3490681E-03, 3.9815050E-03,
1125 6 5.0028666E-03, 5.9285688E-03, 7.4730905E-03, 8.8233510E-03,
1126 7 1.1160132E-02, 1.3119527E-02, 1.6653199E-02, 1.9472767E-02,
1127 8 2.4800811E-02, 2.8793704E-02, 3.6762063E-02, 4.2228780E-02,
1128 9 5.3905163E-02, 6.0804660E-02, 7.7081738E-02, 8.3874501E-02,
1129 1 1.0377190E-01, 1.0377718E-01, 1.1892208E-01, 9.0437429E-02/
1130 DATA Y3/
1131 1 7.1685138E-02, -3.9473064E-02, -1.5078720E-01, -4.0489859E-01,
1132 2 -5.6018995E-01, -6.9050388E-01, -1.5094224E-01, 6.6304064E-01,
1133 3 1.3766748E+00, -8.0373222E-01, -1.0869629E+00, 1.2812892E+00,
1134 4 -5.0341082E-01, -4.4274455E-02, 2.0913102E-01, -1.9999661E-01,
1135 5 1.5207664E-01, -1.0920260E-01, 7.8169956E-02, -5.6651561E-02,
1136 6 4.1611799E-02, -3.0960012E-02, 2.3072559E-02, -1.7311631E-02,
1137 7 1.3021442E-02, -9.8085025E-03, 7.3943529E-03, -5.5769518E-03,
1138 8 4.2073164E-03, -3.1745026E-03, 2.3954154E-03, -1.8076122E-03,
1139 9 1.3640816E-03, -1.0293934E-03, 7.7682952E-04, -5.8623518E-04,
1140 1 4.4240399E-04, -3.3386183E-04, 2.5195025E-04, -1.9013541E-04,
1141 2 1.4348659E-04, -1.0828284E-04, 8.1716174E-05, -6.1667509E-05,
1142 3 4.6537684E-05, -3.5119887E-05, 2.6503388E-05, -2.0000904E-05,
1143 4 1.5093768E-05, -1.1390572E-05, 8.5959318E-06, -6.4869407E-06,

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1144      5 4.8953713E-06,-3.6942830E-06, 2.7878625E-06,-2.1038241E-06,
1145      6 1.5875917E-06,-1.1980090E-06, 9.0398030E-07,-6.8208296E-07,
1146      7 5.1458650E-07,-3.8817581E-07, 2.9272267E-07,-2.2067921E-07,
1147      8 1.6623514E-07,-1.2514102E-07, 9.4034535E-08,-7.0556837E-08,
1148      9 5.2741581E-08,-3.9298610E-08, 2.9107255E-08,-2.1413893E-08,
1149      1 1.5742032E-08,-1.1498608E-08, 8.7561571E-09,-7.2959446E-09/
1150      DATA Y4/
1151      1 6.8816619E-09,-8.9679825E-09, 1.4258275E-08,-1.9564299E-08,
1152      2 2.0235313E-08,-1.4725545E-08, 5.4632820E-09, 3.5995580E-09,
1153      3-9.5287133E-09, 1.1460041E-08,-1.0250532E-08, 7.4641748E-09,
1154      4-4.4703455E-09, 2.0499053E-09,-4.4806353E-10,-4.0374336E-10,
1155      5 7.0321001E-10,-5.7067960E-10, 4.9130404E-10,-2.8940747E-10,
1156      6 1.2373144E-10,-1.5260443E-11,-4.2027559E-11, 6.1885474E-11,
1157      7-5.9273917E-11, 4.6588766E-11,-3.2054182E-11, 1.9631637E-11,
1158      8-1.1210098E-11, 5.9567021E-12,-3.2427812E-12, 2.1353868E-12,
1159      9-1.8476851E-12, 1.8438474E-12,-1.8362842E-12, 1.7241847E-12,
1160      1-1.5161479E-12, 1.2627657E-12,-1.0129176E-12, 7.9578625E-13,
1161      2-6.2131435E-13, 4.8745900E-13,-3.8703630E-13, 3.1172547E-13,
1162      3-2.5397802E-13, 2.0824130E-13,-1.7123163E-13, 1.4113344E-13,
1163      4-1.1687986E-13, 9.7664016E-14,-8.2977176E-14, 7.2515267E-14,
1164      5-5.6047478E-14/
1165      C--$ENDATA
1166      C
1167      COMPLEX FUN,C,C0,CMAX
1168      DIMENSION T(2),TMAX(2)
1169      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
1170      C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION!
1171      C0(I)=FUN(EXP(-X*FLOAT(I))*20*30.30251236))*YT(I)
1172      ZFOURO=(0,0,0,0)
1173      CMAX=(0,0,0,0)
1174      L=22
1175      DO 110 I=149,170
1176      C=C0(I)
1177      ZFOURO=ZFOURO+C
1178      TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
1179      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
1180      110 CONTINUE
1181      IF(TMAX(1).EQ.0,0,AND,TMAX(2).EQ.0,0) GO TO 150
1182      CMAX=TOL*CMAX
1183      DO 120 I=148,1,-1
1184      C=C0(I)

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1185      ZFOURO=ZFOURO+C
1186      L=L+1
1187      IF (ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
1188      120  CONTINUE
1189      130  DO 140 I=171,281
1190          C=C0(I)
1191          ZFOURO=ZFOURO+C
1192          L=L+1
1193          IF (ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
1194      140  CONTINUE
1195          GO TO 190
1196      150  DO 160 I=1,148
1197          C=C0(I)
1198          ZFOURO=ZFOURO+C
1199          L=L+1
1200          IF (T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
1201      160  CONTINUE
1202      170  DO 180 I=281,171,-1
1203          C=C0(I)
1204          ZFOURO=ZFOURO+C
1205          L=L+1
1206          IF (T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
1207      180  CONTINUE
1208      190  RETURN
1209      END

```

1210 COMPLEX FUNCTION ZFOUR1(X,FUN,TOL,L)  
1211 C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)\*SIN(G\*B)\*DG" DEFINED AS THE  
1212 C COMPLEX FOURIER SINE TRANSFORM WITH ARGUMENT X(=ALOG(B))  
1213 C BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND  
1214 C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....  
1215 C  
1216 C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.  
1217 C  
1218 C--PARAMETERS:  
1219 C  
1220 C X = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM  
1221 C FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)  
1222 C OF A REAL ARGUMENT G.  
1223 C NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN  
1224 C CALLING PROGRAM AND IN SUBPROGRAM FUN.  
1225 C THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE  
1226 C DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...  
1227 C FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RFOUR1" IS ADVISED;  
1228 C HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE  
1229 C INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))  
1230 C TOL= REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,  
1231 C IF FILTER=FUN\*TOL\*MAX, THEN REST OF TAIL IS TRUNCATED.  
1232 C THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,  
1233 C TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON  
1234 C THE FUNCTION FUN AND PARAMETER X...IN GENERAL,  
1235 C A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"  
1236 C BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY  
1237 C RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN  
1238 C APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,  
1239 C ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...  
1240 C L= RESULTING NO. FILTER WTS. USED IN THE VARIABLE  
1241 C CONVOLUTION (L DEPENDS ON TOL AND FUN).  
1242 C MIN,L=20 AND MAX,L=256--WHICH COULD  
1243 C OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING  
1244 C VERY FAST...  
1245 C  
1246 C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZFOUR1; THE FOURIER  
1247 C TRANSFORM IS THEN ZFOUR1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM  
1248 C THIS ROUTINE....  
1249 C



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1250 C--USAGE-- "ZFOUR1" IS CALLED AS FOLLOWS:
1251 C
1252 C      ...
1253 C      COMPLEX Z,ZFOUR1,ZF
1254 C      EXTERNAL ZF
1255 C      ...
1256 C      Z=ZFOUR1(ALOG(B),ZF,TOL,L)/B
1257 C      ...
1258 C      END
1259 C      COMPLEX FUNCTION ZF(G)
1260 C      ...USER SUPPLIED CODE...
1261 C      END
1262 C
1262 C--NOTES:
1263 C      (1), EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
1264 C      STATEMENT FUNCTION C1(II) BELOW; HOWEVER,
1265 C      THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
1266 C      EXP-UNDERFLOW'S TO 0.0,.....
1267 C      (2), SOME NON-ANSI FORTRAN STATEMENTS ARE USED
1268 C      (E.G: DO 20 I=190,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
1269 C      THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
1270 C
1271 C
1272 C--SIN-EXTENDED FILTER WEIGHT ARRAYS:
1273 C      NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
1274 C      TO SAVE STORAGE (SEE STATEMENT FUNCTION C1(II) BELOW).
1275 C      DIMENSION WT(266),W1(76),W2(76),W3(76),W4(38)
1276 C      EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
1277 C      1 (WT(229),W4(1))
1278 C      DATA W1/
1279 C      1-1.1113940E+09,-1.3237246E-12, 1.3091739E-12,-1.6240954E-12,
1280 C      2 1.7236636E-12,-1.8227727E-12, 1.9255992E-12,-2.0335514E-12,
1281 C      3 2.1473541E-12,-2.2675549E-12, 2.3946842E-12,-2.5292661E-12,
1282 C      4 2.6718110E-12,-2.8227693E-12, 2.9825171E-12,-3.1514006E-12,
1283 C      5 3.3297565E-12,-3.5179095E-12, 3.7163306E-12,-3.9256378E-12,
1284 C      6 4.1464798E-12,-4.3794552E-12, 4.6252131E-12,-4.8845227E-12,
1285 C      7 5.1582809E-12,-5.4474462E-12, 5.7530277E-12,-6.0760464E-12,
1286 C      8 6.4175083E-12,-6.7783691E-12, 7.1595239E-12,-7.5618762E-12,
1287 C      9 7.9864477E-12,-8.4344110E-12, 8.9072422E-12,-9.4067705E-12,
1288 C      1 9.9349439E-12,-1.0493731E-11, 1.1084900E-11,-1.1709937E-11,
1289 C      2 1.2370354E-11,-1.3067414E-11, 1.3802200E-11,-1.4575980E-11,
1290 C      3 1.5390685E-11,-1.6249313E-11, 1.7156934E-11,-1.8115250E-11,

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1291 4 1.9131898E-11,-2.0209795E-11, 2.1352159E-11,-2.2561735E-11,
1292 5 2.3840976E-11,-2.5192263E-11, 2.6618319E-11,-2.8122547E-11,
1293 6 2.9709129E-11,-3.1382870E-11, 3.3149030E-11,-3.5013168E-11,
1294 7 3.6981050E-11,-3.9058553E-11, 4.1251694E-11,-4.3566777E-11,
1295 8 4.6010537E-11,-4.8590396E-11, 5.1314761E-11,-5.4193353E-11,
1296 9 5.7236720E-11,-6.0455911E-11, 6.3861222E-11,-6.7461492E-11,
1297 1 7.1265224E-11,-7.5279775E-11, 7.9612249E-11,-8.3971327E-11/
1298 DATA W2/
1299 1 8.8668961E-11,-9.3621900E-11, 9.8851764E-11,-1.0438319E-10,
1300 2 1.1024087E-10,-1.1644680E-10, 1.2301979E-10,-1.2997646E-10,
1301 3 1.3733244E-10,-1.4510363E-10, 1.5320772E-10,-1.6196550E-10,
1302 4 1.7110130E-10,-1.8074257E-10, 1.9091922E-10,-2.0166306E-10,
1303 5 2.1300736E-10,-2.2498755E-10, 2.3763936E-10,-2.5100098E-10,
1304 6 2.6511250E-10,-2.8001616E-10, 2.9575691E-10,-3.1238237E-10,
1305 7 3.2994314E-10,-3.4849209E-10, 3.6808529E-10,-3.8678042E-10,
1306 8 4.1063982E-10,-4.3372666E-10, 4.5811059E-10,-4.8386049E-10,
1307 9 5.1105728E-10,-5.3977672E-10, 5.7011632E-10,-6.0215516E-10,
1308 1 6.3601273E-10,-6.7175964E-10, 7.0935028E-10,-7.4942601E-10,
1309 2 7.9161025E-10,-8.3606980E-10, 8.8317110E-10,-9.3270330E-10,
1310 3 9.8533749E-10,-1.0404508E-09, 1.0993731E-09,-1.1605442E-09,
1311 4 1.2267191E-09,-1.2942905E-09, 1.3691677E-09,-1.4429912E-09,
1312 5 1.5288164E-09,-1.6077524E-09, 1.7085998E-09,-1.7890471E-09,
1313 6 1.9129066E-09,-1.9857116E-09, 2.1491608E-09,-2.1926779E-09,
1314 7 2.4312660E-09,-2.3959044E-09, 2.7872500E-09,-2.5610596E-09,
1315 8 3.2762318E-09,-2.6082940E-09, 4.0261453E-09,-2.3560563E-09,
1316 9 5.3176554E-09,-1.3960161E-09, 7.7708747E-09, 1.1853546E-09,
1317 1 1.2760851E-08, 7.4264707E-09, 2.3342187E-08, 2.1869851E-08/
1318 DATA W3/
1319 1 4.6306744E-08, 5.4631686E-08, 9.6763087E-08, 1.2823337E-07,
1320 2 2.0832812E-07, 2.9280540E-07, 4.5580888E-07, 6.5992437E-07,
1321 3 1.0056815E-06, 1.4779183E-06, 2.284335E-06, 3.2994604E-06,
1322 4 4.9485823E-06, 7.3545473E-06, 1.1001083E-05, 1.6380539E-05,
1323 5 2.4469550E-05, 3.6469246E-05, 5.4441527E-05, 8.1176726E-05,
1324 6 1.2113828E-04, 1.8066494E-04, 2.6954609E-04, 4.0202288E-04,
1325 7 5.9969995E-04, 8.9437312E-04, 1.3338166E-03, 1.9886697E-03,
1326 8 2.9643943E-03, 4.4168923E-03, 6.5773518E-03, 9.7855105E-03,
1327 9 1.4539361E-02, 2.1558670E-02, 3.1871864E-02, 4.6903518E-02,
1328 1 6.8559512E-02, 9.9170152E-02, 1.4120770E-01, 1.9610835E-01,
1329 2 2.6192603E-01, 3.2743321E-01, 3.6407406E-01, 3.1257559E-01,
1330 3 9.0460168E-02,-3.6051039E-01,-8.6324760E-01,-8.1178720E-01,
1331 4 5.2205241E-01, 1.5449873E+00,-1.1817933E+00,-2.6759896E-01,

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1332 5 8,0869203E-01,-6,2757149E-01, 3,4062630E-01,-1,5885304E-01,
1333 6 7,0472984E-02,-3,1624462E-02, 1,4894068E-02,-7,4821176E-03,
1334 7 4,0035936E-03,-2,2543784E-03, 1,3160358E-03,-7,8636604E-04,
1335 8 4,7658745E-04,-2,9125817E-04, 1,7885105E-04,-1,1012416E-04,
1336 9 6,7910334E-05,-4,1914054E-05, 2,5881544E-05,-1,5985851E-05,
1337 1 9,8751880E-06,-6,1008526E-06, 3,7692543E-06,-2,3287953E-06/
1338 DATA W4/
1339 1 1,4388425E-06,-8,8899353E-07, 5,4926991E-07,-3,3937048E-07,
1340 2 2,0968284E-07,-1,2955437E-07, 8,0046336E-08,-4,9457371E-08,
1341 3 3,0557711E-08,-1,8880390E-08, 1,1665454E-08,-7,2076428E-09,
1342 4 4,4533423E-09,-2,7515596E-09, 1,7001092E-09,-1,0504494E-09,
1343 5 6,4904567E-10,-4,0102999E-10, 2,4778763E-10,-1,5310321E-10,
1344 6 9,4600354E-11,-5,8451314E-11, 3,6119400E-11,-2,2320056E-11,
1345 7 1,3793460E-11,-8,5242656E-12, 5,2675102E-12,-3,2543076E-12,
1346 8 2,0097689E-12,-1,2405412E-12, 7,6530538E-13,-4,7191929E-13,
1347 9 2,9084993E-13,-1,7923661E-13, 1,1018948E-13,-6,7885902E-14,
1348 1 4,2025050E-14,-2,1314731E-14/

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C--88ENDATA

C  
C

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1352 COMPLEX FUN,C,C1,CMAX
1353 DIMENSION T(2),TMAX(2)
1354 EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
1355 C--8STATEMENT FUNCTION== INCLUDES ABSCISSA GENERATION:
1356 C1(I)=FUN(EXP(-X+FLOAT(I))*20-38,30455704))*WT(I)
1357 ZFOUR1=(0,0,0,0)
1358 CMAX=(0,0,0,0)
1359 L=18
1360 DO 10 I=191,208
1361 C=C1(I)
1362 ZFOUR1=ZFOUR1+C
1363 TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
1364 TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
1365 10 CONTINUE
1366 IF(TMAX(1),EQ,0,0,AND,TMAX(2),EQ,0,0) GO TO 60
1367 CMAX=TOL*CMAX
1368 DO 20 I=190,1,-1
1369 C=C1(I)
1370 ZFOUR1=ZFOUR1+C
1371 L=L+1
1372 IF(ABS(T(1)),LE,TMAX(1),AND,ABS(T(2)),LE,TMAX(2)) GO TO 30

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1373	20	CONTINUE
1374	30	DO 40 I=209,266
1375		C=C1(I)
1376		ZFOUR1=ZFOUR1+C
1377		L=L+1
1378		IF(ABS(T(1)),LE,TMAX(1),AND,ABS(T(2)),LE,TMAX(2)) GO TO 50
1379	40	CONTINUE
1380	50	RETURN
1381	60	DO 70 I=1,190
1382		C=C1(I)
1383		ZFOUR1=ZFOUR1+C
1384		L=L+1
1385		IF(T(1),EQ,0.0,AND,T(2),EQ,0.0) GO TO 80
1386	70	CONTINUE
1387	80	DO 90 I=266,209,-1
1388		C=C1(I)
1389		ZFOUR1=ZFOUR1+C
1390		L=L+1
1391		IF(T(1),EQ,0.0,AND,T(2),EQ,0.0) GO TO 50
1392	90	CONTINUE
1393		GO TO 50
1394		END



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1395      REAL FUNCTION RLAGHO(X,FUN,TOL,L,NEW)
1396 C--*** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
1397 C   INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*JO(G*B)*DG" DEFINED AS THE
1398 C   REAL HANKEL TRANSFORM OF ORDER 0 AND ARGUMENT X(=ALOG(B))
1399 C   BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
1400 C   USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS,...
1401 C
1402 C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
1403 C
1404 C--PARAMETERS:
1405 C
1406 C   * X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
1407 C             "RLAGHO" IS USEFUL ONLY WHEN X=(LAST X)=.20 *** I.E.,
1408 C             SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
1409 C             THEN SUBPROGRAM "RHANKO" IS ADVISED FOR GENERAL USE.
1410 C             (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).
1411 C   FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
1412 C             NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
1413 C             CALLING PROGRAM AND IN SUBPROGRAM FUN.
1414 C             THE REAL FUNCTION FUN SHOULD BE A MONOTONE
1415 C             DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
1416 C   TOL=     REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
1417 C             IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
1418 C             THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,
1419 C             TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
1420 C             THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
1421 C             A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
1422 C             BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY
1423 C             RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
1424 C             APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
1425 C             ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
1426 C   L=       RESULTING NO. FILTER WTS. USED IN THE VARIABLE
1427 C             CONVOLUTION (L DEPENDS ON TOL AND FUN).
1428 C             MIN.L=20 AND MAX.L=193--WHICH COULD
1429 C             OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
1430 C             VERY FAST...
1431 C   * NEW=   1 IS NECESSARY 1ST TIME OR BRAND NEW X.
1432 C             0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)=0.20
1433 C             IS ASSUMED INTERNALLY BY THIS ROUTINE.
1434 C             NOTE: IF THIS IS NOT TRUE, ROUTINE WILL

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1435 C STILL ASSUME X=(LAST X)-0.20 ANYWAY...
1436 C IT IS THE USERS RESPONSIBILITY TO NORMALIZE
1437 C BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
1438 C THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
1439 C TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
1440 C SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
1441 C ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
1442 C THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
1443 C KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
1444 C WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
1445 C WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
1446 C
1447 C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RLAGHO; THE HANKEL
1448 C TRANSFORM IS THEN RLAGHO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
1449 C THIS ROUTINE... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
1450 C
1451 C--USAGE-- "RLAGHO" IS CALLED AS FOLLOWS:
1452 C
1453 C ...
1454 C EXTERNAL RF
1455 C ...
1456 C R=RLAGHO(ALOG(B),RF,TOL,L,NEW)/B
1457 C ...
1458 C END
1459 C REAL FUNCTION RF(G)
1460 C ...USER SUPPLIED CODE...
1461 C END
1462 C
1463 C--NOTES:
1464 C (1). EXP=UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
1465 C BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
1466 C ANY & ALL EXP=UNDERFLOW'S TO 0.0...
1467 C (2). AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
1468 C METHOD, LET BMAX>BMIN>0 BE GIVEN. THEN IT CAN BE SHOWN
1469 C THAT THE ACTUAL NUMBER OF B'S IS NB=AIN(5.*ALOG(BMAX/BMIN))+1,
1470 C PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
1471 C BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING
1472 C ARGUMENTS SPACED AS X=ALOG(BMAX), X=.2, X=.2*2, ..., ALOG(BMINA),
1473 C FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
1474 C ...
1475 C NB=AIN(5.*ALOG(BMAX/BMIN))+1
1476 C NB1=NB+1

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1476 C          XO=ALOG(BMAX)+.2
1477 C          NEW=1
1478 C          DO 1 I=NB,1,-1
1479 C          X=XO-.2*(NB1-I)
1480 C          ARG(I)=EXP(X)
1481 C          ANS(I)=RLAGHO(X,RF,TOL,L,NEW)/ARG(I)
1482 C          NEW=0
1483 C          ...
1484 C          (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),ANS(I),I=1,NB FOR
1485 C          ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
1486 C          TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
1487 C          SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
1488 C          (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
1489 C          ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
1490 C          BMAX,BMIN AND BY SETTING NEW=1,...
1491 C
1492 C
1493 C--JO-EXTENDED FILTER WEIGHT ARRAYS:
1494 C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
1495 C TO SAVE STORAGE.
1496 C          DIMENSION YT(193),Y1(76),Y2(76),Y3(41)
1497 C          EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1))
1498 C          DATA Y1/
1499 C          1 5.8565723E-08, 7.1143477E-11,-7.8395565E-11, 8.7489547E-11,
1500 C          2-8.9007811E-11, 9.8790055E-11,-9.8675347E-11, 1.1118797E-10,
1501 C          3-1.0893474E-10, 1.2543400E-10,-1.1979399E-10, 1.4200767E-10,
1502 C          4-1.3106341E-10, 1.6153229E-10,-1.4238607E-10, 1.8486236E-10,
1503 C          5-1.5315381E-10, 2.1319755E-10,-1.6238115E-10, 2.4824144E-10,
1504 C          6-1.6850378E-10, 2.9243813E-10,-1.6909302E-10, 3.4934366E-10,
1505 C          7-1.6043759E-10, 4.2417082E-10,-1.3690001E-10, 5.2458440E-10,
1506 C          8-8.9946096E-11, 6.6188220E-10,-6.6964033E-12, 8.5276151E-10,
1507 C          9 1.3222770E-10, 1.1219600E-09, 3.5591442E-10, 1.5061956E-09,
1508 C          1 7.0795382E-10, 2.0600379E-09, 1.2535947E-09, 2.8646623E-09,
1509 C          2 2.0904225E-09, 4.0409101E-09, 3.3642886E-09, 5.7687700E-09,
1510 C          3 5.2930786E-09, 8.3164338E-09, 8.2021809E-09, 1.2083635E-08,
1511 C          4 1.2577400E-08, 1.7666303E-08, 1.9143895E-08, 2.5953011E-08,
1512 C          5 2.8983953E-08, 3.8268851E-08, 4.3712685E-08, 5.6590075E-08,
1513 C          6 6.5740136E-08, 8.3864288E-08, 9.8662323E-08, 1.2448811E-07,
1514 C          7 1.4784461E-07, 1.8501974E-07, 2.2129198E-07, 2.7524203E-07,
1515 C          8 3.3094739E-07, 4.0974828E-07, 4.9462868E-07, 6.1030809E-07,
1516 C          9 7.3891802E-07, 9.0939667E-07, 1.1034727E-06, 1.3554600E-06,

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1517 1 1.6474556E-06, 2.0207696E-06, 2.4591294E-06, 3.0131400E-06/
1518 DATA Y2/
1519 1 3.6701680E-06, 4.4934101E-06, 5.4770076E-06, 6.7015208E-06,
1520 2 8.1726989E-06, 9.9954201E-06, 1.2194425E-05, 1.4909101E-05,
1521 3 1.8194388E-05, 2.2239194E-05, 2.7145562E-05, 3.3174088E-05,
1522 4 4.0499453E-05, 4.9486730E-05, 6.0421440E-05, 7.3822001E-05,
1523 5 9.0141902E-05, 1.1012552E-04, 1.3448017E-04, 1.6428337E-04,
1524 6 2.0062570E-04, 2.4507680E-04, 2.9930366E-04, 3.6560582E-04,
1525 7 4.4651421E-04, 5.4541300E-04, 6.6612648E-04, 8.1365181E-04,
1526 8 9.9374786E-04, 1.2138120E-03, 1.4824945E-03, 1.8107657E-03,
1527 9 2.2115938E-03, 2.7012675E-03, 3.2991969E-03, 4.0295817E-03,
1528 1 4.9214744E-03, 6.0106700E-03, 7.3405529E-03, 8.9643708E-03,
1529 2 1.0946310E-02, 1.3365017E-02, 1.6314985E-02, 1.9910907E-02,
1530 3 2.4289325E-02, 2.9612896E-02, 3.6070402E-02, 4.3876936E-02,
1531 4 5.3264829E-02, 6.4465091E-02, 7.7664144E-02, 9.2918324E-02,
1532 5 1.1000121E-01, 1.2811102E-01, 1.4543025E-01, 1.5832248E-01,
1533 6 1.6049224E-01, 1.4170064E-01, 2.8788108E-02, -1.1330934E-02,
1534 7 -1.5331864E-01, -2.9094670E-01, -2.9084655E-01, -2.9708834E-02,
1535 8 3.9009601E-01, 1.7999785E-01, -4.1858139E-01, 1.5317216E-01,
1536 9 6.5184953E-02, -1.0751806E-01, 7.8429567E-02, -4.6019124E-02,
1537 1 2.5309571E-02, -1.3904823E-02, 7.8187120E-03, -4.5190359E-03/
1538 DATA Y3/
1539 1 2.6729062E-03, -1.6073718E-03, 9.7715622E-04, -5.9804407E-04,
1540 2 3.6749320E-04, -2.2635296E-04, 1.3960805E-04, -8.6172618E-05,
1541 3 5.3212947E-05, -3.2867888E-05, 2.0304203E-05, -1.2543926E-05,
1542 4 7.7499633E-06, -4.7882430E-06, 2.9584108E-06, -1.8278645E-06,
1543 5 1.1293571E-06, -6.9778174E-07, 4.3113019E-07, -2.6637753E-07,
1544 6 1.6458373E-07, -1.0168954E-07, 6.2829807E-08, -3.8819969E-08,
1545 7 2.3985272E-08, -1.4819520E-08, 9.1563774E-09, -5.6573541E-09,
1546 8 3.4934514E-09, -2.1597005E-09, 1.3343946E-09, -8.2447148E-10,
1547 9 5.0941033E-10, -3.1474631E-10, 1.9447072E-10, -1.2015685E-10,
1548 1 7.4241055E-11, -4.5871468E-11, 2.8343095E-11, -1.7513137E-11,
1549 2 6.9049613E-12/
1550 C--$SENDATA
1551 C
1552 DIMENSION KEY(193),SAVE(193)
1553 IF(NEW) 10,30,10
1554 10 LAG=-1
1555 X0=-X-26.30455704
1556 DO 20 IR=1,193
1557 20 KEY(IR)=0

```

```

1558      30      LAG=LAG+1
1559             RLAGHO=0.0
1560             CMAX=0.0
1561             I=0
1562             ASSIGN 110 TO M
1563             I=129
1564             GO TO 200
1565      110      CMAX=AMAX1(ABS(C),CMAX)
1566             I=I+1
1567             IF(I.LE.146) GO TO 200
1568             IF(CMAX.EQ.0.0) GO TO 150
1569             CMAX=TOL*CMAX
1570             ASSIGN 120 TO M
1571             I=128
1572             GO TO 200
1573      120      IF(ABS(C).LE.CMAX) GO TO 130
1574             I=I-1
1575             IF(I.GT.0) GO TO 200
1576      130      ASSIGN 140 TO M
1577             I=147
1578             GO TO 200
1579      140      IF(ABS(C).LE.CMAX) GO TO 190
1580             I=I+1
1581             IF(I.LE.193) GO TO 200
1582             GO TO 190
1583      150      ASSIGN 160 TO M
1584             I=1
1585             GO TO 200
1586      160      IF(C.EQ.0.0) GO TO 170
1587             I=I+1
1588             IF(I.LE.128) GO TO 200
1589      170      ASSIGN 180 TO M
1590             I=193
1591             GO TO 200
1592      180      IF(C.EQ.0.0) GO TO 190
1593             I=I-1
1594             IF(I.GE.147) GO TO 200
1595      190      RETURN
1596      C=-STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
1597      200      LOOK=I+LAG
1598             IQ=LOOK/194

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1599      IR=MOD(LOOK,194)
1600      IF(IR,EQ,0) IR=1
1601      IROLL=10*193
1602      IF(KEY(IR).LE,IROLL) GO TO 220
1603 210    C=SAVE(IR)*YT(I)
1604      PLAGH0=RLAGH0+C
1605      L=L+1
1606      GO TO M,(110,120,140,160,180)
1607 220    KEY(IR)=IROLL+IR
1608      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
1609      GO TO 210
1610      END

```

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1511      REAL FUNCTION RLACHI(X,FUN,TOL,L,NEW)
1512      C--*** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
1513      C INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J1(G*B)*DG" DEFINED AS THE
1514      C REAL HANKEL TRANSFORM OF ORDER 1 AND ARGUMENT X(=ALOG(B))
1515      C BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
1516      C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
1517      C
1518      C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
1519      C
1520      C--PARAMETERS:
1521      C
1522      C      * X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
1523      C                "RLACHI" IS USEFUL ONLY WHEN X=(LAST X)=.20 *** I.E.,
1524      C                SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
1525      C                THEN SUBPROGRAM "RHANK1" IS ADVISED FOR GENERAL USE.
1526      C                (ALSO SEE PARM 'NEW' & NOTES (2)-(3) BELOW).
1527      C      FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
1528      C                NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
1529      C                CALLING PROGRAM AND IN SUBPROGRAM FUN.
1530      C                THE REAL FUNCTION FUN SHOULD BE A MONOTONE
1531      C                DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
1532      C      TOL=      REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
1533      C                IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
1534      C                THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,
1535      C                TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
1536      C                THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
1537      C                A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
1538      C                BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY
1539      C                RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
1540      C                APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
1541      C                ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
1542      C      L=        RESULTING NO. FILTER WTS. USED IN THE VARIABLE
1543      C                CONVOLUTION (L DEPENDS ON TOL AND FUN).
1544      C                MIN.L=15 AND MAX.L=236--WHICH COULD
1545      C                OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
1546      C                VERY FAST...
1547      C      * NEW=     1 IS NECESSARY 1ST TIME OR BRAND NEW X.
1548      C      *          0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)=0.20
1549      C                IS ASSUMED INTERNALLY BY THIS ROUTINE.
1550      C                NOTE: IF THIS IS NOT TRUE, ROUTINE WILL

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1651 C STILL ASSUME X=(LAST X)=0.20 ANYWAY...
1652 C IT IS THE USERS RESPONSIBILITY TO NORMALIZE
1653 C BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
1654 C THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
1655 C TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
1656 C SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
1657 C ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
1658 C THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
1659 C KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
1660 C WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
1661 C WHEN NEEDED (DEPENDS ON PARS TOL AND FUN)
1662 C
1663 C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RLAGH1; THE HANKEL
1664 C TRANSFORM IS THEN RLAGH1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
1665 C THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
1666 C
1667 C--USAGE-- "RLAGH1" IS CALLED AS FOLLOWS:
1668 C
1669 C     ...
1670 C     EXTERNAL RF
1671 C     ...
1672 C     R=RLAGH1(ALOG(B),RF,TOL,L,NEW)/B
1673 C     ...
1674 C     END
1675 C     REAL FUNCTION RF(G)
1676 C     ...USER SUPPLIED CODE...
1677 C     END
1678 C--NOTES;
1679 C (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
1680 C BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
1681 C ANY & ALL EXP-UNDERFLOW'S TO 0.0....
1682 C (2). AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
1683 C METHOD, LET BMAX>=BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
1684 C THAT THE ACTUAL NUMBER OF B'S IS NB=AIN(5.*ALOG(BMAX/BMIN))+1,
1685 C PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
1686 C BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING
1687 C ARGUMENTS SPACED AS X=ALOG(BMAX),X=.,2,X=.,2*2,...,ALOG(BMINA).
1688 C FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
1689 C
1690 C     ...
1691 C     NB=AIN(5.*ALOG(BMAX/BMIN))+1
1692 C     NB1=NB+1

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1692      C      X0=ALOG(BMAX)+.2
1693      C      NEW=1
1694      C      DO 1 I=NB,1,-1
1695      C      X=X0-.2*(NB1-I)
1696      C      ARG(I)=EXP(X)
1697      C      ANS(I)=RLAGH1(X,RF,TOL,L,NEW)/ARG(I)
1698      C      1      NEW=0
1699      C      ...
1700      C      (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),ANS(I),I=1,NB FOR
1701      C      ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
1702      C      TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
1703      C      SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD,
1704      C      (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
1705      C      ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
1706      C      BMAX,BMIN AND BY SETTING NEW=1....
1707      C
1708      C
1709      C--J1-EXTENDED FILTER WEIGHT ARRAYS:
1710      C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
1711      C TO SAVE STORAGE.
1712      C      DIMENSION WT(236),W1(76),W2(76),W3(76),W4(8)
1713      C      EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
1714      C      1 (WT(229),W4(1))
1715      C      DATA W1/
1716      C      1-8.8863805E-10, 1.1293811E-09,-1.2050872E-09, 1.2696232E-09,
1717      C      2-1.3223909E-09, 1.3642393E-09,-1.3969439E-09, 1.4225941E-09,
1718      C      3-1.4427475E-09, 1.4580582E-09,-1.4682563E-09, 1.4732179E-09,
1719      C      4-1.4735606E-09, 1.4719870E-09,-1.4727091E-09, 1.4828225E-09,
1720      C      5-1.5102619E-09, 1.5667752E-09,-1.6634522E-09, 1.8172900E-09,
1721      C      6-2.0412753E-09, 2.3595230E-09,-2.7864077E-09, 3.3592871E-09,
1722      C      7-4.0940172E-09, 5.0571015E-09,-6.2604109E-09, 7.8269461E-09,
1723      C      8-9.7514701E-09, 1.2267639E-08,-1.5312389E-08, 1.9339924E-08,
1724      C      9-2.4126297E-08, 3.0576829E-08,-3.8060204E-08, 4.8423732E-08,
1725      C      1-6.0051116E-08, 7.6787475E-08,-9.4700993E-08, 1.2192844E-07,
1726      C      2-1.4518997E-07, 1.9392737E-07,-2.3464786E-07, 3.0911127E-07,
1727      C      3-3.6815394E-07, 4.9413800E-07,-5.7554168E-07, 7.9301529E-07,
1728      C      4-8.9502818E-07, 1.2794292E-06,-1.3811469E-06, 2.0789668E-06,
1729      C      5-2.1069398E-06, 3.4103188E-06,-3.1584463E-06, 5.6639045E-06,
1730      C      6-4.6059955E-06, 9.5561672E-06,-6.4142855E-06, 1.6440205E-05,
1731      C      7-8.2010619E-06, 2.8945217E-05,-8.6348466E-06, 5.2317398E-05,
1732      C      8-3.9915035E-06, 9.7273612E-05, 1.5220520E-05, 1.8614373E-04,

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1733	9	7.2023760E-05,	3.6620099E-04,	2.2062958E-04,	7.3874539E-04,
1734	1	5.8623480E-04,	1.5226779E-03,	1.4538718E-03,	3.1930365E-03/
1735		DATA W2/			
1736	1	3.4640868E-03,	6.7790882E-03,	8.0328420E-03,	1.4484339E-02,
1737	2	1.8201316E-02,	3.0866143E-02,	4.0106549E-02,	6.4527872E-02,
1738	3	8.4285526E-02,	1.2773175E-01,	1.6020907E-01,	2.1948043E-01,
1739	4	2.3636305E-01,	2.4895051E-01,	1.2586300E-01,	-5.1060445E-02,
1740	5	-3.4376222E-01,	-2.9042175E-01,	1.1564736E-01,	4.9253231E-01,
1741	6	-4.6748595E-01,	1.5280945E-01,	3.3348541E-02,	-8.2485252E-02,
1742	7	7.9740630E-02,	-6.6934498E-02,	5.5150465E-02,	-4.5868721E-02,
1743	8	3.8651959E-02,	-3.2935834E-02,	2.8303994E-02,	-2.4475127E-02,
1744	9	2.1259541E-02,	-1.8526278E-02,	1.6182037E-02,	-1.4158101E-02,
1745	1	1.2402225E-02,	-1.0873526E-02,	9.5392016E-03,	-8.3723743E-03,
1746	2	7.3506490E-03,	-6.4551136E-03,	5.6626335E-03,	-4.9803353E-03,
1747	3	4.3752213E-03,	-3.8438703E-03,	3.3772023E-03,	-2.9672872E-03,
1748	4	2.4071877E-03,	-2.2908274E-03,	2.0128794E-03,	-1.7686706E-03,
1749	5	1.5540992E-03,	-1.3655666E-03,	1.1999089E-03,	-1.0543497E-03,
1750	6	9.2644973E-04,	-8.1406593E-04,	7.1571557E-04,	-6.2854459E-04,
1751	7	5.5229955E-04,	-4.8530352E-04,	4.2643446E-04,	-3.7470650E-04,
1752	8	3.2925334E-04,	-2.8931382E-04,	2.5421910E-04,	-2.2338147E-04,
1753	9	1.9628455E-04,	-1.7247455E-04,	1.5155278E-04,	-1.3316889E-04,
1754	1	1.1701502E-04,	-1.0282066E-04,	9.0348135E-05,	-7.9388568E-05/
1755		DATA W3/			
1756	1	6.9758436E-05,	-6.1296474E-05,	5.3860978E-05,	-4.7327436E-05,
1757	2	4.1586435E-05,	-3.6541840E-05,	3.2109174E-05,	-2.8214208E-05,
1758	3	2.4791718E-05,	-2.1784390E-05,	1.9141864E-05,	-1.6819888E-05,
1759	4	1.4779578E-05,	-1.2986765E-05,	1.1411426E-05,	-1.0027182E-05,
1760	5	8.8108499E-06,	-7.7420630E-06,	6.8029235E-06,	-5.9777053E-06,
1761	6	5.2525892E-06,	-4.6154325E-06,	4.0555653E-06,	-3.5636118E-06,
1762	7	3.1313335E-06,	-2.7514911E-06,	2.4177236E-06,	-2.1244417E-06,
1763	8	1.8667342E-06,	-1.6402859E-06,	1.4413051E-06,	-1.2664597E-06,
1764	9	1.1128220E-06,	-9.7781908E-07,	8.5919028E-07,	-7.5494920E-07,
1765	1	6.6335060E-07,	-5.8286113E-07,	5.1213358E-07,	-4.4998431E-07,
1766	2	3.9537334E-07,	-3.4738689E-07,	3.0522189E-07,	-2.6817250E-07,
1767	3	2.3561831E-07,	-2.0701397E-07,	1.8188012E-07,	-1.5979545E-07,
1768	4	1.4038968E-07,	-1.2333746E-07,	1.0835294E-07,	-9.5185048E-08,
1769	5	8.3613184E-08,	-7.3443411E-08,	6.4505118E-08,	-5.6648167E-08,
1770	6	4.9740428E-08,	-4.3665572E-08,	3.8321109E-08,	-3.3616717E-08,
1771	7	2.9472836E-08,	-2.5819439E-08,	2.2594957E-08,	-1.9745353E-08,
1772	8	1.7223359E-08,	-1.4987869E-08,	1.3003472E-08,	-1.1240058E-08,
1773	9	9.6723739E-09,	-8.2794392E-09,	7.0438407E-09,	-5.9509676E-09,



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1774      1 4.9882405E-09,-4.1443813E-09, 3.4088114E-09,-2.7712762E-09/
1775      DATA W4/
1776      1 2.2217311E-09,-1.7504753E-09, 1.3485207E-09,-1.0080937E-09,
1777      2 7.2300885E-10,-4.8860666E-10, 3.0121413E-10,-9.1649798E-11/
1778      C--$ENDATA
1779      C
1780      DIMENSION KEY(236),SAVE(236)
1781      IF(NEW) 10,30,10
1782      10      LAG=-1
1783      X0=-X-17.0
1784      DO 20 IR=1,236
1785      20      KEY(IR)=0
1786      30      LAG=LAG+1
1787      RLAGH1=0.0
1788      CMAX=0.0
1789      L=0
1790      ASSIGN 110 TO M
1791      I=R6
1792      GO TO 200
1793      110      CMAX=AMAX1(ABS(C),CMAX)
1794      I=I+1
1795      IF(I,LE,98) GO TO 200
1796      IF(CMAX,EQ,0.0) GO TO 150
1797      CMAX=Y01*CMAX
1798      ASSIGN 120 TO M
1799      I=R5
1800      GO TO 200
1801      120      IF(ABS(C).LE,CMAX) GO TO 130
1802      I=I+1
1803      IF(I,GT,0) GO TO 200
1804      130      ASSIGN 140 TO M
1805      I=99
1806      GO TO 200
1807      140      IF(ABS(C).LE,CMAX) GO TO 190
1808      I=I+1
1809      IF(I,LE,236) GO TO 200
1810      GO TO 190
1811      150      ASSIGN 160 TO M
1812      I=1
1813      GO TO 200
1814      160      IF(C,EQ,0.0) GO TO 170

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1815      I=I+1
1816      IF(I,LE,85) GO TO 200
1817 170    ASSIGN 180 TO M
1818      I=236
1819      GO TO 200
1820 180    IF(C,EQ,0,0) GO TO 190
1821      I=I-1
1822      IF(I,GE,99) GO TO 200
1823 190    RETURN
1824 C=STORE/PRETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
1825 200    LOOK=I+LAG
1826      IQ=LOOK/237
1827      IR=MOD(LOOK,237)
1828      IF(IR,EQ,0) IR=1
1829      IROLL=IQ*236
1830      IF(KEY(IR),LE,IROLL) GO TO 220
1831 210    C=SAVE(IR)*WT(I)
1832      RLAGH1=RLAGH1+C
1833      L=L+1
1834      GO TO M,(110,120,140,160,180)
1835 220    KEY(IR)=IROLL+IR
1836      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
1837      GO TO 210
1838      END

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1839      REAL FUNCTION PLAGFO(X,FUN,TOL,L,NEW)
1840      C--*** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
1841      C INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*COS(G*B)*DG" DEFINED AS THE
1842      C REAL FOURIER COSINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
1843      C BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
1844      C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
1845      C
1846      C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO,
1847      C
1848      C--PARAMETERS:
1849      C
1850      C      * X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
1851      C      "PLAGFO" IS USEFUL ONLY WHEN X=(LAST X)=.20 *** I.E.,
1852      C      SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
1853      C      THEN SUBPROGRAM "RFOURO" IS ADVISED FOR GENERAL USE,
1854      C      (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).
1855      C      FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
1856      C      NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
1857      C      CALLING PROGRAM AND IN SUBPROGRAM FUN,
1858      C      THE REAL FUNCTION FUN SHOULD BE A MONOTONE
1859      C      DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
1860      C      TOL=      REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
1861      C      IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
1862      C      THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,
1863      C      TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
1864      C      THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
1865      C      A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
1866      C      BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY
1867      C      RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
1868      C      APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
1869      C      ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
1870      C      L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
1871      C      CONVOLUTION (L DEPENDS ON TOL AND FUN).
1872      C      MIN.L=24 AND MAX.L=281--WHICH COULD
1873      C      OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
1874      C      VERY FAST...
1875      C      * NEW= 1 IS NECESSARY 1ST TIME OR BRAND NEW X.
1876      C      0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)=0.20
1877      C      IS ASSUMED INTERNALLY BY THIS ROUTINE.
1878      C      NOTE: IF THIS IS NOT TRUE, ROUTINE WILL

```

```

1879 C          STILL ASSUME X=(LAST X)-0.20 ANYWAY...
1880 C          IT IS THE USERS RESPONSIBILITY TO NORMALIZE
1881 C          BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
1882 C          THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
1883 C          TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
1884 C          SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
1885 C          ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
1886 C          THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
1887 C          KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
1888 C          WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
1889 C          WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
1890 C
1891 C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RLAGFO; THE FOURIER
1892 C TRANSFORM IS THEN RLAGFO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
1893 C THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
1894 C
1895 C--USAGE-- "RLAGFO" IS CALLED AS FOLLOWS:
1896 C
1897 C      ...
1898 C      EXTERNAL RF
1899 C      ...
1900 C      R=RLAGFO(ALOG(B),RF,TOL,L,NEW)/B
1901 C      ...
1902 C      END
1903 C      REAL FUNCTION RF(G)
1904 C      ...USER SUPPLIED CODE...
1905 C      END
1906 C
1907 C--NOTES:
1908 C      (1). EXP=UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
1909 C      BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
1910 C      ANY & ALL EXP=UNDERFLOW'S TO 0.0....
1911 C      (2). AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
1912 C      METHOD, LET BMAX>BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
1913 C      THAT THE ACTUAL NUMBER OF B'S IS NB=AIN(5,*ALOG(BMAX/BMIN))+1,
1914 C      PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
1915 C      BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING
1916 C      ARGUMENTS SPACED AS X=ALOG(BMAX),X=.2,X=.2*2,...,ALOG(BMINA).
1917 C      FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
1918 C      ...
1919 C      NB=AIN(5,*ALOG(BMAX/BMIN))+1
1920 C      NB1=NB+1

```

```

1920      C      X0=ALOG(BMAX)+.2
1921      C      NEW=1
1922      C      DO 1 I=NB,1,-1
1923      C      X=X0-.2*(NB1-I)
1924      C      ARG(I)=EXP(X)
1925      C      ANS(I)=RLAGFO(X,RF,TOL,L,NEW)/ARG(I)
1926      C      1      NEW=0
1927      C      ...
1928      C      (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),ANS(I),I=1,NB FOR
1929      C      ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
1930      C      TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
1931      C      SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
1932      C      (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
1933      C      ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
1934      C      BMAX,BMIN AND BY SETTING NEW=1....
1935      C
1936      C
1937      C--COS-EXTENDED FILTER WEIGHT ARRAYS:
1938      C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
1939      C TO SAVE STORAGE,
1940      C      DIMENSION YT(281),Y1(76),Y2(76),Y3(76),Y4(53)
1941      C      EQUIVALENCE (YT(1),Y1(1)),(Y1(77),Y2(1)),(YT(153),Y3(1)),
1942      C      1 (YT(229),Y4(1))
1943      C      DATA Y1/
1944      C      1 5.1178101E-14, 2.9433849E-14, 2.5492522E-14, 1.9034819E-14,
1945      C      2 6.4179780E-14, 1.3085746E-15, 1.1989957E-13, -1.2216234E-14,
1946      C      3 1.7534103E-13, 7.9373498E-15, 2.1235658E-13, 7.9981520E-14,
1947      C      4 2.3815757E-13, 1.9714260E-13, 2.8920132E-13, 3.4161340E-13,
1948      C      5 4.0349917E-13, 5.2203885E-13, 5.9837223E-13, 7.8015306E-13,
1949      C      6 8.8911653E-13, 1.1709731E-12, 1.3165595E-12, 1.7578463E-12,
1950      C      7 1.9538564E-12, 2.6289768E-12, 2.9167697E-12, 3.9044344E-12,
1951      C      8 4.3927341E-12, 5.7526904E-12, 6.6569352E-12, 8.4555678E-12,
1952      C      9 1.0063229E-11, 1.2487964E-11, 1.5134682E-11, 1.8501488E-11,
1953      C      1 2.2720051E-11, 2.7452598E-11, 3.4025443E-11, 4.0875985E-11,
1954      C      2 5.0751668E-11, 6.1094382E-11, 7.5492982E-11, 9.1445759E-11,
1955      C      3 1.1227336E-10, 1.3676464E-10, 1.6720269E-10, 2.0423244E-10,
1956      C      4 2.4932743E-10, 3.0470661E-10, 3.7198526E-10, 4.5449934E-10,
1957      C      5 5.5502537E-10, 6.7793669E-10, 8.2810001E-10, 1.0112626E-09,
1958      C      6 1.2354800E-09, 1.5085245E-09, 1.8432753E-09, 2.2503397E-09,
1959      C      7 2.7499027E-09, 3.3569525E-09, 4.1025670E-09, 5.0077487E-09,
1960      C      8 6.1205950E-09, 7.4703399E-09, 9.1312760E-09, 1.1143911E-08,

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1961	9	1.3622929E-08,	1.6623917E-08,	2.0324094E-08,	2.4798610E-08,
1962	1	3.0321709E-08,	3.6992986E-08,	4.5237482E-08,	5.5183434E-08/
1963		DATA Y2/			
1964	1	6.7491070E-08,	8.2317946E-08,	1.0069271E-07,	1.2279375E-07,
1965	2	1.5022907E-07,	1.8316969E-07,	2.2413747E-07,	2.7322865E-07,
1966	3	3.3441046E-07,	4.0756197E-07,	4.9894278E-07,	6.0793233E-07,
1967	4	7.4443665E-07,	9.0679753E-07,	1.1107379E-06,	1.3525651E-06,
1968	5	1.6573073E-06,	2.0174273E-06,	2.4728798E-06,	3.0090445E-06,
1969	6	3.6898816E-06,	4.4879625E-06,	5.5059521E-06,	6.6935820E-06,
1970	7	8.2160716E-06,	9.9828691E-06,	1.2260527E-05,	1.4888061E-05,
1971	8	1.8296530E-05,	2.2202672E-05,	2.7305154E-05,	3.3109672E-05,
1972	9	4.0751046E-05,	4.9372484E-05,	6.0020947E-05,	7.3619571E-05,
1973	1	9.0780005E-05,	1.0976837E-04,	1.3550409E-04,	1.6365676E-04,
1974	2	2.0227521E-04,	2.4398338E-04,	3.0197018E-04,	3.6370760E-04,
1975	3	4.5083748E-04,	5.4213338E-04,	6.7315347E-04,	8.0800951E-04,
1976	4	1.0051938E-03,	1.2041401E-03,	1.5011708E-03,	1.7942344E-03,
1977	5	2.2421056E-03,	2.6730676E-03,	3.3490681E-03,	3.9815050E-03,
1978	6	5.0028666E-03,	5.9285668E-03,	7.4730905E-03,	8.8233510E-03,
1979	7	1.1160132E-02,	1.3119627E-02,	1.6653199E-02,	1.9472767E-02,
1980	8	2.4800811E-02,	2.8793704E-02,	3.6762063E-02,	4.2228780E-02,
1981	9	5.3905163E-02,	6.0804660E-02,	7.7081738E-02,	8.3874501E-02,
1982	1	1.0377190E-01,	1.0377718E-01,	1.1892208E-01,	9.0437429E-02/
1983		DATA Y3/			
1984	1	7.1685138E-02,	-3.9473064E-02,	-1.5078720E-01,	-4.0489859E-01,
1985	2	-5.6018995E-01,	-6.8050388E-01,	-1.5094224E-01,	6.6304064E-01,
1986	3	1.3766748E+00,	-8.0373222E-01,	-1.0869629E+00,	1.2812892E+00,
1987	4	-5.0341082E-01,	-4.4274455E-02,	2.0913102E-01,	-1.9999661E-01,
1988	5	1.5207664E-01,	-1.0920260E-01,	7.8169956E-02,	-5.6651561E-02,
1989	6	4.1611799E-02,	3.0880012E-02,	2.3072559E-02,	-1.7311631E-02,
1990	7	1.3021442E-02,	-9.8085025E-03,	7.3943529E-03,	-5.5769518E-03,
1991	8	4.2073164E-03,	-3.1745026E-03,	2.3954154E-03,	-1.8076122E-03,
1992	9	1.3640816E-03,	-1.0293934E-03,	7.7682952E-04,	-5.8623518E-04,
1993	1	4.4240399E-04,	-3.3386183E-04,	2.5195025E-04,	-1.9013541E-04,
1994	2	1.4348659E-04,	-1.0928284E-04,	8.1716174E-05,	-6.1667509E-05,
1995	3	4.6537684E-05,	-3.5119897E-05,	2.6503388E-05,	-2.0000904E-05,
1996	4	1.5093768E-05,	-1.1390572E-05,	8.5959318E-06,	-6.4869407E-06,
1997	5	4.8953713E-06,	-3.6942930E-06,	2.7878625E-06,	-2.1038241E-06,
1998	6	1.5875917E-06,	-1.1980090E-06,	9.0398030E-07,	-6.8208296E-07,
1999	7	5.1458650E-07,	-3.8817581E-07,	2.9272267E-07,	-2.2067921E-07,
2000	8	1.6623514E-07,	-1.2514102E-07,	9.4034535E-08,	-7.0556837E-08,
2001	9	5.2741581E-08,	-3.9298610E-08,	2.9107255E-08,	-2.1413893E-08,

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2002      1 1.5742032E-08,-1.1498608E-08, 8.7561571E-09,-7.2959446E-09/
2003      DATA Y4/
2004      1 6.8816619E-09,-8.9679825E-09, 1.4258275E-08,-1.9564299E-08,
2005      2 2.0235313E-08,-1.4725545E-08, 5.4632820E-09, 3.5995580E-09,
2006      3-9.5287133E-09, 1.1460041E-08,-1.0250532E-08, 7.4641748E-09,
2007      4-4.4703465E-09, 2.0499053E-09,-4.4006353E-10,-4.0374336E-10,
2008      5 7.0321001E-10,-6.7067960E-10, 4.9130404E-10,-2.8840747E-10,
2009      6 1.2373144E-10,-1.5260443E-11,-4.2027559E-11, 6.1885474E-11,
2010      7-5.9273937E-11, 4.6588766E-11,-3.2054182E-11, 1.9831637E-11,
2011      8-1.1210098E-11, 5.9567021E-12,-3.2427812E-12, 2.1353868E-12,
2012      9-1.8476651E-12, 1.8438474E-12,-1.8362842E-12, 1.7241847E-12,
2013      1-1.5161479E-12, 1.2627657E-12,-1.0129176E-12, 7.9578625E-13,
2014      2-6.2131435E-13, 4.8743900E-13,-3.8701630E-13, 3.1172547E-13,
2015      3-2.5397802E-13, 2.0824130E-13,-1.7123163E-13, 1.4113344E-13,
2016      4-1.1687986E-13, 9.7664016E-14,-8.2977176E-14, 7.2515267E-14,
2017      5-5.6047478E-14/
2018      C=--$SENDATA
2019      DIMENSION KEY(281),SAVE(281)
2020      IF(NEW) 10,30,10
2021      10      LAG=-1
2022      X0=X-30.30251236
2023      DO 20 IR=1,281
2024      20      KEY(IR)=0
2025      30      LAG=LAG+1
2026      RLAGFO=0.0
2027      CMAX=0.0
2028      L=0
2029      ASSIGN 110 TO M
2030      I=149
2031      GO TO 200
2032      110      CMAX=AMAX1(ABS(C),CMAX)
2033      I=I+1
2034      IF(I.LE.170) GO TO 200
2035      IF(CMAX.EQ.0.0) GO TO 150
2036      CMAX=TOL*CMAX
2037      ASSIGN 120 TO M
2038      I=148
2039      GO TO 200
2040      120      IF(ABS(C).LE.CMAX) GO TO 130
2041      I=I-1
2042      IF(I.GT.0) GO TO 200

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2043      130      ASSIGN 140 TO M
2044      I=171
2045      GO TO 200
2046      140      IF(ABS(C).LE.CMAX) GO TO 190
2047      I=I+1
2048      IF(I.LE.281) GO TO 200
2049      GO TO 190
2050      150      ASSIGN 160 TO M
2051      I=1
2052      GO TO 200
2053      160      IF(C.EQ.0.0) GO TO 170
2054      I=I+1
2055      IF(I.LE.148) GO TO 200
2056      170      ASSIGN 180 TO M
2057      I=281
2058      GO TO 200
2059      180      IF(C.EQ.0.0) GO TO 190
2060      I=I-1
2061      IF(I.GE.171) GO TO 200
2062      190      RETURN
2063      C==STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
2064      200      LOOK=I+LAG
2065      IQ=LOOK/282
2066      IR=MOD(LOOK,282)
2067      IF(IR.EQ.0) IR=1
2068      IROLL=IQ*281
2069      IF(KEY(IR).LE.IROLL) GO TO 220
2070      210      C=SAVE(IR)*YT(I)
2071      RLAGF0=RLAGF0+C
2072      L=L+1
2073      GO TO M,(110,120,140,160,180)
2074      220      KEY(IR)=IROLL+IR
2075      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
2076      GO TO 210
2077      END

```

2078 REAL FUNCTION RLAGF1(X,FUN,TOL,L,NEW)  
2079 C--\*\*\* A SPECIAL LAGGED\* CONVOLUTION METHOD TO COMPUTE THE  
2080 C INTEGRAL FROM 0 TO INFINITY OF "FUN(G)\*SIN(G\*B)\*DG" DEFINED AS THE  
2081 C REAL FOURIER SINE TRANSFORM WITH ARGUMENT X(=ALOG(B))  
2082 C BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND  
2083 C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....  
2084 C  
2085 C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO,  
2086 C  
2087 C--PARAMETERS:  
2088 C  
2089 C \* X \* REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM  
2090 C "RLAGF1" IS USEFUL ONLY WHEN X=(LAST X)=.20 \*\*\* I.E.,  
2091 C SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,  
2092 C THEN SUBPROGRAM "RFOUR1" IS ADVISED FOR GENERAL USE,  
2093 C (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).  
2094 C FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).  
2095 C NOTE: IF PARAMS OTHER THAN G ARE REQUIRED, USE COMMON IN  
2096 C CALLING PROGRAM AND IN SUBPROGRAM FUN.  
2097 C THE REAL FUNCTION FUN SHOULD BE A MONOTONE  
2098 C DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...  
2099 C TOL= REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,  
2100 C IF FILTER\*FUN<TOL\*MAX, THEN REST OF TAIL IS TRUNCATED.  
2101 C THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,  
2102 C TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON  
2103 C THE FUNCTION FUN AND PARAMETER X...IN GENERAL,  
2104 C A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"  
2105 C BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY  
2106 C RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN  
2107 C APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,  
2108 C ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...  
2109 C L= RESULTING NO. FILTER PTS. USED IN THE VARIABLE  
2110 C CONVOLUTION (L DEPENDS ON TOL AND FUN).  
2111 C MIN. L=20 AND MAX. L=256--WHICH COULD  
2112 C OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING  
2113 C VERY FAST...  
2114 C \* NEW= 1 IS NECESSARY 1ST TIME OR BRAND NEW X.  
2115 C 0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)=0.20  
2116 C IS ASSUMED INTERNALLY BY THIS ROUTINE.  
2117 C NOTE: IF THIS IS NOT TRUE, ROUTINE WILL

```

2118 C          STILL ASSUME X=(LAST X)=0.20 ANYWAY...
2119 C          IT IS THE USERS RESPONSIBILITY TO NORMALIZE
2120 C          BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
2121 C          THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
2122 C          TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
2123 C          SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
2124 C          ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
2125 C          THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
2126 C          KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
2127 C          WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
2128 C          WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
2129 C
2130 C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RLAGF1; THE FOURIER
2131 C   TRANSFORM IS THEN RLAGF1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
2132 C   THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
2133 C
2134 C--USAGE-- "RLAGF1" IS CALLED AS FOLLOWS:
2135 C
2136 C          ...
2137 C          EXTERNAL RF
2138 C          ...
2139 C          R=RLAGF1(ALOG(B),RF,TOL,L,NEW)/B
2140 C          ...
2141 C          END
2142 C          REAL FUNCTION RF(G)
2143 C          ...USER SUPPLIED CODE...
2144 C          END
2145 C
2146 C--NOTES:
2147 C   (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
2148 C   BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
2149 C   ANY & ALL EXP-UNDERFLOW'S TO 0.0....
2150 C   (2). AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
2151 C   METHOD, LET BMAX>BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
2152 C   THAT THE ACTUAL NUMBER OF B'S IS NB=AIN(5.*ALOG(BMAX/BMIN))+1,
2153 C   PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
2154 C   BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING
2155 C   ARGUMENTS SPACED AS X=ALOG(BMAX), X=.2, X=.2*2, ..., ALG(BMINA).
2156 C   FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
2157 C
2158 C          ...
2159 C          NB=AIN(5.*ALOG(BMAX/BMIN))+1
2160 C          NB1=NB+1

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```

2159      C      X0=ALOG(BMAX)+.2
2160      C      NEW=1
2161      C      DO 1 I=NB,1,-1
2162      C      X=X0-.2*(NB1-I)
2163      C      ARG(I)=EXP(X)
2164      C      ANS(I)=RLAGF1(X,RF,TOL,L,NEW)/ARG(I)
2165      C      1      NEW=0
2166      C      ...
2167      C      (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),ANS(I),I=1,NB FOR
2168      C      ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
2169      C      TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
2170      C      SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
2171      C      (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
2172      C      ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
2173      C      BMAX,BMIN AND BY SETTING NEW=1,...
2174      C
2175      C
2176      C--SIN-EXTENDED FILTER WEIGHT ARRAYS:
2177      C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
2178      C TO SAVE STORAGE.
2179      C      DIMENSION WT(266),W1(76),W2(76),W3(76),W4(38)
2180      C      EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
2181      C      1 (WT(229),W4(1))
2182      C      DATA W1/
2183      C      1-1.1113940E-09,-1.3237246E-12, 1.5091739E-12,-1.6240934E-12,
2184      C      2 1.7236636E-12,-1.8227727E-12, 1.9255992E-12,-2.0335514E-12,
2185      C      3 2.1473541E-12,-2.2675549E-12, 2.3946842E-12,-2.5292661E-12,
2186      C      4 2.6718110E-12,-2.8227693E-12, 2.9825171E-12,-3.1514006E-12,
2187      C      5 3.1297565E-12,-3.5179095E-12, 3.7161306E-12,-3.9256378E-12,
2188      C      6 4.1464798E-12,-4.3794552E-12, 4.6252131E-12,-4.8845227E-12,
2189      C      7 5.1582809E-12,-5.4474462E-12, 5.7510277E-12,-6.0760464E-12,
2190      C      8 6.4175083E-12,-6.7783691E-12, 7.1325239E-12,-7.5618782E-12,
2191      C      9 7.9864477E-12,-8.4344110E-12, 8.9072422E-12,-9.4067705E-12,
2192      C      1 9.9349439E-12,-1.0493731E-11, 1.1084900E-11,-1.1709937E-11,
2193      C      2 1.2370354E-11,-1.3067414E-11, 1.3802200E-11,-1.4575980E-11,
2194      C      3 1.5390688E-11,-1.6249313E-11, 1.7155934E-11,-1.8115250E-11,
2195      C      4 1.9131898E-11,-2.0209795E-11, 2.1352159E-11,-2.2561735E-11,
2196      C      5 2.3840976E-11,-2.5192263E-11, 2.6618319E-11,-2.8122547E-11,
2197      C      6 2.9709129E-11,-3.1382870E-11, 3.3149030E-11,-3.5013168E-11,
2198      C      7 3.6981050E-11,-3.9058553E-11, 4.1251694E-11,-4.3566777E-11,
2199      C      8 4.6010537E-11,-4.8590396E-11, 5.1314761E-11,-5.4193353E-11,

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2200 9 5.7236720E-11,-6.0455911E-11, 6.3861222E-11,-6.7461492E-11,  
 2201 1 7.1265224E-11,-7.5279775E-11, 7.9512249E-11,-8.3971327E-11/  
 2202 DATA W2/  
 2203 1 8.8668961E-11,-9.3621900E-11, 9.8851764E-11,-1.0438319E-10,  
 2204 2 1.1024087E-10,-1.1644680E-10, 1.2301979E-10,-1.2997646E-10,  
 2205 3 1.3733244E-10,-1.4510363E-10, 1.5330772E-10,-1.6196550E-10,  
 2206 4 1.7110130E-10,-1.8074257E-10, 1.9091922E-10,-2.0166306E-10,  
 2207 5 2.1300756E-10,-2.2498755E-10, 2.3761936E-10,-2.5100098E-10,  
 2208 6 2.6511250E-10,-2.8001616E-10, 2.9575691E-10,-3.1238237E-10,  
 2209 7 3.2994314E-10,-3.4849209E-10, 3.6808529E-10,-3.8878042E-10,  
 2210 8 4.1063982E-10,-4.3372666E-10, 4.5811059E-10,-4.8386049E-10,  
 2211 9 5.1105728E-10,-5.3277672E-10, 5.7011632E-10,-6.0215516E-10,  
 2212 1 6.3601273E-10,-6.7175964E-10, 7.0956028E-10,-7.4942601E-10,  
 2213 2 7.9161025E-10,-8.3606280E-10, 8.8317110E-10,-9.3270330E-10,  
 2214 3 9.8533749E-10,-1.0404508E-09, 1.0893731E-09,-1.1605442E-09,  
 2215 4 1.2267391E-09,-1.2942905E-09, 1.3691577E-09,-1.4479912E-09,  
 2216 5 1.5288164E-09,-1.6077524E-09, 1.7085998E-09,-1.7890471E-09,  
 2217 6 1.9129068E-09,-1.9857116E-09, 2.1491608E-09,-2.1926779E-09,  
 2218 7 2.4312660E-09,-2.3959044E-09, 2.7872500E-09,-2.5610596E-09,  
 2219 8 3.2762318E-09,-2.6082940E-09, 4.0261453E-09,-2.3560563E-09,  
 2220 9 5.3176554E-09,-1.3960161E-08, 7.7708747E-09, 1.1853546E-09,  
 2221 1 1.2760851E-08, 7.4264707E-09, 2.3342187E-08, 2.1869851E-08/  
 2222 DATA W3/  
 2223 1 4.6306744E-08, 5.4631636E-08, 9.6763087E-08, 1.2823337E-07,  
 2224 2 2.0832812E-07, 2.9280540E-07, 4.5580888E-07, 6.9992437E-07,  
 2225 3 1.0056815E-06, 1.4779183E-06, 2.2284335E-06, 3.2994604E-06,  
 2226 4 4.9485823E-06, 7.3545473E-06, 1.1001083E-05, 1.6380539E-05,  
 2227 5 2.4469550E-05, 3.6469246E-05, 5.4441527E-05, 8.1176726E-05,  
 2228 6 1.2113828E-04, 1.8066494E-04, 2.6954609E-04, 4.0202288E-04,  
 2229 7 5.9969995E-04, 8.9437312E-04, 1.3338166E-03, 1.9886697E-03,  
 2230 8 2.9643943E-03, 4.4168923E-03, 6.5773518E-03, 9.7855105E-03,  
 2231 9 1.4639316E-02, 2.1558670E-02, 3.1871864E-02, 4.6903518E-02,  
 2232 1 6.8559512E-02, 9.9170152E-02, 1.4120770E-01, 1.9610835E-01,  
 2233 2 2.6192603E-01, 3.2743321E-01, 3.6407406E-01, 3.1257559E-01,  
 2234 3 9.0460168E-02,-3.6051039E-01,-8.6324760E-01,-8.1178720E-01,  
 2235 4 5.2205241E-01, 1.5449873E+00,-1.1817933E+00,-2.6759896E-01,  
 2236 5 8.0869203E-01,-6.2757149E-01, 3.4062630E-01,-1.5883304E-01,  
 2237 6 7.0472984E-02,-3.1624462E-02, 1.4894068E-02,-7.4821176E-03,  
 2238 7 4.0035936E-03,-2.2543784E-03, 1.3160358E-03,-7.8636604E-04,  
 2239 8 4.7658745E-04,-2.9125817E-04, 1.7885105E-04,-1.1012416E-04,  
 2240 9 6.7910334E-05,-4.1914084E-05, 2.5881544E-05,-1.5985851E-05,

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2241      1 9.8751880E-06,-6.1008526E-06, 3.7692543E-06,-2.3287953E-06/
2242      DATA W4/
2243      1 1.4388425E-06,-8.8899353E-07, 5.4926991E-07,-3.3937048E-07,
2244      2 2.0968284E-07,-1.2955437E-07, 8.0046336E-08,-4.9457371E-08,
2245      3 3.0557711E-08,-1.8880390E-08, 1.1665454E-08,-7.2076428E-09,
2246      4 4.4533423E-09,-2.7515696E-09, 1.7001092E-09,-1.0504494E-09,
2247      5 6.4904567E-10,-4.0102999E-10, 2.4778763E-10,-1.5310321E-10,
2248      6 9.4600354E-11,-5.8453314E-11, 3.6119400E-11,-2.2320056E-11,
2249      7 1.3793460E-11,-8.5242636E-12, 5.2675102E-12,-3.2543076E-12,
2250      8 2.0097689E-12,-1.2405412E-12, 7.6530538E-13,-4.7191929E-13,
2251      9 2.9084993E-13,-1.7923661E-13, 1.1018948E-13,-6.7885902E-14,
2252      1 4.2025050E-14,-2.1314731E-14/
2253      C--ssENDATA
2254      C
2255      DIMENSION KEY(266),SAVE(266)
2256      IF(NEW) 10,30,10
2257      LAG=1
2258      X0=-X-38.30455704
2259      DO 20 IR=1,266
2260      20      KEY(IR)=0
2261      30      LAG=LAG+1
2262      RLAGF1=0.0
2263      CMAX=0.0
2264      L=0
2265      ASSIGN 110 TO M
2266      I=191
2267      GO TO 200
2268      110      CMAX=AMAX1(ABS(C),CMAX)
2269      I=I+1
2270      IF(I.LE.208) GO TO 200
2271      IF(CMAX.EQ.0.0) GO TO 130
2272      CMAX=TOL*CMAX
2273      ASSIGN 120 TO M
2274      I=190
2275      GO TO 200
2276      120      IF(ABS(C).LE.CMAX) GO TO 130
2277      I=I-1
2278      IF(I.GT.0) GO TO 200
2279      130      ASSIGN 140 TO M
2280      I=209
2281      GO TO 200

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2282      140      IF(ABS(C).LE.CMAX) GO TO 190
2283              I=I+1
2284              IF(I.LE.266) GO TO 200
2285              GO TO 190
2286      150      ASSIGN 160 TO M
2287              I=1
2288              GO TO 200
2289      160      IF(C.EQ.0.0) GO TO 170
2290              I=I+1
2291              IF(I.LE.190) GO TO 200
2292      170      ASSIGN 180 TO M
2293              I=266
2294              GO TO 200
2295      180      IF(C.EQ.0.0) GO TO 190
2296              I=I-1
2297              IF(I.GE.209) GO TO 200
2298      190      RETURN
2299      C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
2300      200      LOOK=I+LAG
2301              IQ=LOOK/267
2302              IR=MOD(LOOK,267)
2303              IF(IR.EQ.0) IR=1
2304              IROLL=IQ*266
2305              IF(KEY(IR).LE.IROLL) GO TO 220
2306      210      C=SAVE(IR)*WT(I)
2307              PLAGF1=RLAGF1+C
2308              I=L+1
2309              GO TO M,(110,120,140,160,180)
2310      220      KEY(IR)=IROLL+IR
2311              SAVE(IR)=FUN(EXP(XO+FLOAT(LOOK)*.20))
2312              GO TO 210
2313      END

```

2314 COMPLEX FUNCTION ZLAGHO(X,FUN,TOL,L,NEW)  
2315 C----- A SPECIAL LAGGED\* CONVOLUTION METHOD TO COMPUTE THE  
2316 C INTEGRAL FROM 0 TO INFINITY OF "FUN(G)\*J0(G\*B)\*DG" DEFINED AS THE  
2317 C COMPLEX HANKEL TRANSFORM OF ORDER 0 AND ARGUMENT X(=ALOG(B))  
2318 C BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND  
2319 C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....  
2320 C  
2321 C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO,  
2322 C  
2323 C--PARAMETERS:  
2324 C  
2325 C \* X = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM  
2326 C "ZLAGHO" IS USEFUL ONLY WHEN X=(LAST X)=.20 \*\*\* I.E.,  
2327 C SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,  
2328 C THEN SUBPROGRAM "ZHANKO" IS ADVISED FOR GENERAL USE.  
2329 C (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).  
2330 C FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)  
2331 C OF A REAL ARGUMENT G,  
2332 C NOTE: IF PARAMS OTHER THAN G ARE REQUIRED, USE COMMON IN  
2333 C CALLING PROGRAM AND IN SUBPROGRAM FUN.  
2334 C THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE  
2335 C DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...  
2336 C FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RLAGHO" IS ADVISED;  
2337 C HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE  
2338 C INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))  
2339 C TOL= REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,  
2340 C IF FILTER\*FUN<TOL\*MAX, THEN REST OF TAIL IS TRUNCATED.  
2341 C THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,  
2342 C TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON  
2343 C THE FUNCTION FUN AND PARAMETER X...IN GENERAL,  
2344 C A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"  
2345 C BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY  
2346 C RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN  
2347 C APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,  
2348 C ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...  
2349 C L= RESULTING NO. FILTER WTS. USED IN THE VARIABLE  
2350 C CONVOLUTION (L DEPENDS ON TOL AND FUN).  
2351 C MIN.L=20 AND MAX.L=193--WHICH COULD  
2352 C OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING  
2353 C VERY FAST...



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2354 C      * NEW= 1 IS NECESSARY 1ST TIME OR BRAND NEW X.
2355 C      0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)=0.20
2356 C      IS ASSUMED INTERNALLY BY THIS ROUTINE.
2357 C      NOTE: IF THIS IS NOT TRUE, ROUTINE WILL
2358 C      STILL ASSUME X=(LAST X)=0.20 ANYWAY...
2359 C      IT IS THE USERS RESPONSIBILITY TO NORMALIZE
2360 C      BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
2361 C      THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
2362 C      TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
2363 C      SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
2364 C      ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
2365 C      THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
2366 C      KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
2367 C      WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
2368 C      WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
2369 C
2370 C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZLAGHO; THE HANKEL
2371 C TRANSFORM IS THEN ZLAGHO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
2372 C THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
2373 C
2374 C--USAGE-- "ZLAGHO" IS CALLED AS FOLLOWS:
2375 C
2376 C      ...
2377 C      COMPLEX Z,ZLAGHO,ZF
2378 C      EXTERNAL ZF
2379 C
2380 C      ...
2381 C      Z=ZLAGHO(ALOG(B),ZF,TOL,L,NEW)/B
2382 C
2383 C      ...
2384 C      COMPLEX FUNCTION ZF(G)
2385 C      ...USER SUPPLIED CODE...
2386 C      END
2387 C
2388 C--NOTES:
2389 C      (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
2390 C      BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
2391 C      ANY & ALL EXP-UNDERFLOW'S TO 0.0....
2392 C      (2). AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
2393 C      METHOD, LET BMAX>BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
2394 C      THAT THE ACTUAL NUMBER OF B'S IS NB=AIN(5.*ALOG(BMAX/BMIN))+1,
2395 C      PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
2396 C      BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING

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2395 C ARGUMENTS SPACED AS X=ALOG(BMAX),X=.,2,X=.,2*2,...,ALOG(BMINA).
2396 C FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
2397 C
2398 C     ...
2399 C     NB=AIN(5.*ALOG(BMAX/BMIN))+1
2400 C     NB1=NB+1
2401 C     X0=ALOG(BMAX)+.2
2402 C     NEW=1
2403 C     DO 1 I=NB,1,-1
2404 C     X=X0-.2*(NB1-I)
2405 C     ARG(I)=EXP(X)
2406 C     Z(I)=ZLAGH0(X,ZF,TOL,L,NEW)/ARG(I)
2407 C     1 NEW=0
2408 C     ...
2409 C (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),Z(I),I=1,NB FOR
2410 C ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
2411 C TO SPLINE=INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
2412 C SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
2413 C (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
2414 C ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
2415 C BMAX,BMIN AND BY SETTING NEW=1,...
2416 C
2417 C --JO--EXTENDED FILTER WEIGHT ARRAYS:
2418 C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
2419 C TO SAVE STORAGE.
2420 C     DIMENSION YT(193),Y1(76),Y2(76),Y3(41)
2421 C     EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1))
2422 C     DATA Y1/
2423 C     1 5.8565723E-08, 7.1143477E-11,-7.8395565E-11, 8.7489547E-11,
2424 C     2-8.9007811E-11, 9.8790055E-11,-9.8675347E-11, 1.1118797E-10,
2425 C     3-1.0893474E-10, 1.2543400E-10,-1.1979399E-10, 1.4200767E-10,
2426 C     4-1.3106341E-10, 1.6153279E-10,-1.4238807E-10, 1.8486236E-10,
2427 C     5-1.5315381E-10, 2.1319759E-10,-1.6238115E-10, 2.4824144E-10,
2428 C     6-1.6850378E-10, 2.9243813E-10,-1.6909302E-10, 3.4934366E-10,
2429 C     7-1.6043759E-10, 4.2417082E-10,-1.3690001E-10, 5.2458440E-10,
2430 C     8-8.9946096E-11, 6.6188220E-10,-6.6964033E-10, 8.5276151E-10,
2431 C     9 1.3222770E-10, 1.1219600E-09, 3.5591442E-10, 1.5061956E-09,
2432 C     1 7.0795382E-10, 2.0600379E-09, 1.2535947E-09, 2.8646623E-09,
2433 C     2 2.0904225E-09, 4.0409101E-09, 3.3642886E-09, 5.7687700E-09,
2434 C     3 5.2930786E-09, 8.3164338E-09, 8.2021809E-09, 1.2083635E-08,
2435 C     4 1.2577400E-08, 1.7666303E-08, 1.9143895E-08, 2.5953011E-08,

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2518      GO TO 200
2519      IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
2520      I=I-1
2521      IF(I.GE.147) GO TO 200
2522      190      RETURN
2523      C=-STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
2524      200      LOOK=I+LAG
2525      IQ=LOOK/194
2526      IR=MOD(LOOK,194)
2527      IF(IR.EQ.0) IR=1
2528      IROLL=IQ*193
2529      IF(KEY(IR).LE.IROLL) GO TO 220
2530      210      C=SAVE(IR)*YT(I)
2531      ZLAGH0=ZLAGH0+C
2532      L=L+1
2533      GO TO M,(110,120,140,160,180)
2534      220      KEY(IR)=IROLL+IR
2535      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
2536      GO TO 210
2537      END

```

2538 COMPLEX FUNCTION ZLAGH1(X,FUN,TOL,L,NEW)  
2539 C--\*\*\* A SPECIAL LAGGED\* CONVOLUTION METHOD TO COMPUTE THE  
2540 C INTEGRAL FROM 0 TO INFINITY OF "FUN(G)\*J1(G\*B)\*DG" DEFINED AS THE  
2541 C COMPLEX HANKEL TRANSFORM OF ORDER 1 AND ARGUMENT X(=ALOG(B))  
2542 C BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN" AND  
2543 C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....  
2544 C  
2545 C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO,  
2546 C  
2547 C--PARAMETERS:  
2548 C  
2549 C \* X = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM  
2550 C "ZLAGH1" IS USEFUL ONLY WHEN X=(LAST X)\*.20 \*\*\* I.E.,  
2551 C SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,  
2552 C THEN SUBPROGRAM "ZHANK1" IS ADVISED FOR GENERAL USE.  
2553 C (ALSO SEE PARM 'NEW' & NOTES (2)-(3) BELOW).  
2554 C FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)  
2555 C OF A REAL ARGUMENT G.  
2556 C NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN  
2557 C CALLING PROGRAM AND IN SUBPROGRAM FUN,  
2558 C THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE  
2559 C DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...  
2560 C FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RLAGH1" IS ADVISED;  
2561 C HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE  
2562 C INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))  
2563 C TOL= REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,  
2564 C IF FILTER\*FUN<TOL\*MAX, THEN REST OF TAIL IS TRUNCATED.  
2565 C THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,  
2566 C TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON  
2567 C THE FUNCTION FUN AND PARAMETER X...IN GENERAL,  
2568 C A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"  
2569 C BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY  
2570 C RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN  
2571 C APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,  
2572 C ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...  
2573 C L= RESULTING NO. FILTER WTS. USED IN THE VARIABLE  
2574 C CONVOLUTION (L DEPENDS ON TOL AND FUN).  
2575 C MIN,L=15 AND MAX,L=236--WHICH COULD  
2576 C OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING  
2577 C VERY FAST...

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2578 C      * NEW= 1 IS NECESSARY 1ST TIME OR BRAND NEW X,
2579 C      0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)=0.20
2580 C      IS ASSUMED INTERNALLY BY THIS ROUTINE.
2581 C      NOTE: IF THIS IS NOT TRUE, ROUTINE WILL
2582 C      STILL ASSUME X=(LAST X)=0.20 ANYWAY...
2583 C      IT IS THE USERS RESPONSIBILITY TO NORMALIZE
2584 C      BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
2585 C      THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
2586 C      TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
2587 C      SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
2588 C      ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
2589 C      THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
2590 C      KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
2591 C      WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
2592 C      WHEN NEEDED (DEPENDS ON PARAMS TOL AND FUN)
2593 C
2594 C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZLAGH1; THE HANKEL
2595 C TRANSFORM IS THEN ZLAGH1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
2596 C THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
2597 C
2598 C--USAGE-- "ZLAGH1" IS CALLED AS FOLLOWS:
2599 C
2600 C      ...
2601 C      COMPLEX Z,ZLAGH1,ZF
2602 C      EXTERNAL ZF
2603 C      ...
2604 C      Z=ZLAGH1(ALOG(B),ZF,TOL,L,NEW)/B
2605 C      ...
2606 C      END
2607 C      COMPLEX FUNCTION ZF(C)
2608 C      ...USER SUPPLIED CODE...
2609 C      END
2610 C
2611 C--NOTES:
2612 C      (1), EXP=UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
2613 C      BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
2614 C      ANY & ALL EXP=UNDERFLOW'S TO 0.0....
2615 C      (2), AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
2616 C      METHOD, LET BMAX>=BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
2617 C      THAT THE ACTUAL NUMBER OF S'S IS NB=AIN(5,*ALOG(BMAX/BMIN))+1,
2618 C      PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
2619 C      BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING

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2619 C ARGUMENTS SPACED AS X=ALOG(BMAX),X=.2,X=.2*2,...,ALOG(BMINA).
2620 C FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
2621 C
2622 C ***
2623 C NB=AIN(5,ALOG(BMAX/BMIN))+1
2624 C NB1=NB+1
2625 C X0=ALOG(BMAX)+.2
2626 C NEW=1
2627 C DO 1 I=NB,1,-1
2628 C X=X0-.2*(NB1-I)
2629 C ARG(I)=EXP(X)
2630 C Z(I)=ZLAGH1(X,ZF,TOL,L,NEW)/ARG(I)
2631 C 1 NEW=0
2632 C ***
2633 C (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),Z(I),I=1,NB FOR
2634 C ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
2635 C TO SPLINE=INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
2636 C SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
2637 C (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
2638 C ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
2639 C BMAX,BMIN AND BY SETTING NEW=1....
2640 C
2641 C --J1--EXTENDED FILTER WEIGHT ARRAYS:
2642 C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
2643 C TO SAVE STORAGE,
2644 C DIMENSION WT(236),W1(76),W2(76),W3(76),W4(8)
2645 C EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
2646 C 1 (WT(229),W4(1))
2647 C DATA W1/
2648 C 1=.8,8863805E-10, 1.1293811E-09,-1.2050872E-09, 1.2696232E-09,
2649 C 2=-1.3223909E-09, 1.3642393E-09,-1.3969439E-09, 1.4225941E-09,
2650 C 3=-1.4427475E-09, 1.4580562E-09,-1.4682563E-09, 1.4732179E-09,
2651 C 4=-1.4735606E-09, 1.4719870E-09,-1.4727091E-09, 1.4828225E-09,
2652 C 5=-1.5102619E-09, 1.5667752E-09,-1.6634522E-09, 1.8172900E-09,
2653 C 6=-2.0412751E-09, 2.3595230E-09,-2.7861077E-09, 3.3592871E-09,
2654 C 7=-4.0940172E-09, 5.0571015E-09,-6.2604109E-09, 7.8269461E-09,
2655 C 8=-9.7514701E-09, 1.2267639E-08,-1.5312389E-08, 1.9339924E-08,
2656 C 9=-2.4126297E-08, 3.0576829E-08,-3.8060204E-08, 4.8423732E-08,
2657 C 1=-6.0051116E-08, 7.6787475E-08,-9.4700993E-08, 1.2192844E-07,
2658 C 2=-1.4918997E-07, 1.9392737E-07,-2.3464786E-07, 3.0911127E-07,
2659 C 3=-3.6815394E-07, 4.9413800E-07,-5.7554168E-07, 7.9301529E-07,

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2660 4=8.9502818E-07, 1.2794292E-06, -1.3811469E-06, 2.0789668E-06,
2661 5=2.1069398E-06, 3.4103188E-06, -3.1584463E-06, 5.6639045E-06,
2662 6=4.6059955E-06, 9.5561672E-06, -6.4142855E-06, 1.6440205E-05,
2663 7=8.2010619E-06, 2.8945217E-05, -8.6348466E-06, 5.2317398E-05,
2664 8=3.9915035E-06, 9.7273612E-05, 1.5220520E-05, 1.8614373E-04,
2665 9 7.2023760E-05, 3.6620099E-04, 2.2062958E-04, 7.3874539E-04,
2666 1 5.8623480E-04, 1.5226779E-03, 1.4538718E-03, 3.1930365E-03/
2667 DATA W2/
2668 1 3.4640868E-03, 6.7790882E-03, 8.0328420E-03, 1.4484339E-02,
2669 2 1.8201316E-02, 3.0866143E-02, 4.0106549E-02, 6.4527872E-02,
2670 3 8.4285526E-02, 1.2773175E-01, 1.6026907E-01, 2.1948043E-01,
2671 4 2.3636305E-01, 2.4895051E-01, 1.2586300E-01, -5.1060445E-02,
2672 5=3.4376222E-01, -2.9042175E-01, 1.1564736E-01, 4.9253231E-01,
2673 6=4.6748595E-01, 1.5280945E-01, 3.3348541E-02, -8.2485252E-02,
2674 7 7.9740630E-02, -6.6934498E-02, 5.5150465E-02, -4.5868721E-02,
2675 8 3.8651958E-02, -3.2935834E-02, 2.8303994E-02, -2.4475127E-02,
2676 9 2.1259541E-02, -1.8526278E-02, 1.6182037E-02, -1.4158101E-02,
2677 1 1.2402225E-02, -1.0873526E-02, 9.5392016E-03, -8.3723743E-03,
2678 2 7.3506490E-03, -6.4551136E-03, 5.4696335E-03, -4.9803353E-03,
2679 3 4.3752213E-03, -3.8438703E-03, 3.3772023E-03, -2.9672872E-03,
2680 4 2.6071877E-03, -2.2908274E-03, 2.0128794E-03, -1.7686706E-03,
2681 5 1.5540998E-03, -1.3655666E-03, 1.1999089E-03, -1.0543497E-03,
2682 6 9.2644973E-04, -8.1406593E-04, 7.1531559E-04, -6.2854459E-04,
2683 7 5.5229955E-04, -4.8530352E-04, 4.2643446E-04, -3.7470650E-04,
2684 8 3.2925334E-04, -2.8931382E-04, 2.5421910E-04, -2.2338147E-04,
2685 9 1.9628455E-04, -1.7247455E-04, 1.5155278E-04, -1.3316889E-04,
2686 1 1.1701502E-04, -1.0282066E-04, 9.0343135E-05, -7.9388568E-05/
2687 DATA W3/
2688 1 6.9758436E-05, -6.1296474E-05, 5.3860978E-05, -4.7327436E-05,
2689 2 4.1586435E-05, -3.6541840E-05, 3.2109174E-05, -2.8214208E-05,
2690 3 2.4791718E-05, -2.1784390E-05, 1.9141864E-05, -1.6819888E-05,
2691 4 1.4779578E-05, -1.2988765E-05, 1.1411426E-05, -1.0027182E-05,
2692 5 8.8108499E-06, -7.7420630E-06, 6.8029235E-06, -5.9777053E-06,
2693 6 5.2525892E-06, -4.6154325E-06, 4.0555553E-06, -3.5636118E-06,
2694 7 3.1313335E-06, -2.7514911E-06, 2.4177236E-06, -2.1244417E-06,
2695 8 1.8667342E-06, -1.6402859E-06, 1.4413051E-06, -1.2664597E-06,
2696 9 1.1128220E-06, -9.7781908E-07, 8.5919028E-07, -7.5494920E-07,
2697 1 6.6335060E-07, -5.8286113E-07, 5.1213358E-07, -4.4998431E-07,
2698 2 3.9537334E-07, -3.4738689E-07, 3.0522189E-07, -2.6817250E-07,
2699 3 2.3561831E-07, -2.0701397E-07, 1.8198012E-07, -1.5979545E-07,
2700 4 1.4038968E-07, -1.2333746E-07, 1.0835294E-07, -9.5185048E-08,

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2701      5 8.3613184E-08,-7.3443411E-08, 6.4505118E-09,-5.6648167E-08,
2702      6 4.9740428E-08,-4.3665572E-08, 3.8321109E-08,-3.3616717E-08,
2703      7 2.9472836E-08,-2.5819439E-08, 2.2594957E-08,-1.9745353E-08,
2704      8 1.7223359E-08,-1.4987869E-08, 1.3003472E-08,-1.1240058E-08,
2705      9 9.6723739E-09,-8.2794392E-09, 7.0438407E-09,-5.9509676E-09,
2706      1 4.9882405E-09,-4.1443813E-09, 3.4088114E-09,-2.7712762E-09/
2707      DATA W4/
2708      1 2.2217311E-09,-1.7504755E-09, 1.3485207E-09,-1.0080937E-09,
2709      2 7.2300885E-10,-4.8860666E-10, 3.0121413E-10,-9.1649798E-11/
2710      C==ssENDATA
2711      C
2712      COMPLEX FUN,C,CMAX,SAVE
2713      DIMENSION KEY(236),SAVE(236),T(2),TMAX(2)
2714      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
2715      IF(NEW) 10,30,10
2716      10      LAG=-1
2717      X0=-X-17.0
2718      DO 20 IR=1,236
2719      20      KEY(IR)=0
2720      30      LAG=LAG+1
2721      ZLAGH1=(0.0,0.0)
2722      CMAX=(0.0,0.0)
2723      L=0
2724      ASSIGN 110 TO M
2725      I=86
2726      GO TO 200
2727      110      TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
2728      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
2729      I=I+1
2730      IF(I.LE.98) GO TO 200
2731      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
2732      CMAX=TOL*CMAX
2733      ASSIGN 120 TO M
2734      I=85
2735      GO TO 200
2736      120      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
2737      I=I-1
2738      IF(I.GT.0) GO TO 200
2739      130      ASSIGN 140 TO M
2740      I=99
2741      GO TO 200

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2436 5 2.8983953E-08, 3.8268851E-08, 4.3712685E-08, 5.6590075E-08,
2437 6 6.5740136E-08, 8.3864288E-08, 9.8662323E-08, 1.2448811E-07,
2438 7 1.4784461E-07, 1.8501974E-07, 2.2129198E-07, 2.7524203E-07,
2439 8 3.3094739E-07, 4.0974828E-07, 4.9462968E-07, 6.1030809E-07,
2440 9 7.3891802E-07, 9.0039667E-07, 1.1034727E-06, 1.3554600E-06,
2441 1 1.6474556E-06, 2.0207696E-06, 2.4591294E-06, 3.0131400E-06/
2442 DATA Y2/
2443 1 3.6701680E-06, 4.4934101E-06, 5.4770076E-06, 6.7015208E-06,
2444 2 8.1726989E-06, 9.9954201E-06, 1.2194425E-05, 1.4909101E-05,
2445 3 1.8194388E-05, 2.2239184E-05, 2.7145562E-05, 3.3174088E-05,
2446 4 4.0499452E-05, 4.9486730E-05, 6.0421440E-05, 7.3822001E-05,
2447 5 9.0141902E-05, 1.1012552E-04, 1.3448017E-04, 1.6428337E-04,
2448 6 2.0062570E-04, 2.4507680E-04, 2.9930366E-04, 3.6560582E-04,
2449 7 4.4651421E-04, 5.4541300E-04, 6.6512648E-04, 8.1365181E-04,
2450 8 9.9374786E-04, 1.2138120E-03, 1.4824945E-03, 1.8107657E-03,
2451 9 2.2115938E-03, 2.7012675E-03, 3.2991969E-03, 4.0295817E-03,
2452 1 4.9214244E-03, 6.0106700E-03, 7.3405529E-03, 8.9643708E-03,
2453 2 1.0946310E-02, 1.3365017E-02, 1.6314985E-02, 1.9910907E-02,
2454 3 2.4289325E-02, 2.9612896E-02, 3.6070402E-02, 4.3876936E-02,
2455 4 5.3264829E-02, 6.4465091E-02, 7.7664144E-02, 9.2918324E-02,
2456 5 1.1000121E-01, 1.2811102E-01, 1.4543025E-01, 1.5832248E-01,
2457 6 1.6049224E-01, 1.4170064E-01, 8.8788108E-02, -1.1330934E-02,
2458 7 -1.5331864E-01, -2.9094670E-01, -2.9084655E-01, -2.9708834E-02,
2459 8 3.9009601E-01, 1.7999785E-01, -4.1868139E-01, 1.5317216E-01,
2460 9 6.5184953E-02, -1.0751806E-01, 7.8429567E-02, -4.6019124E-02,
2461 1 2.5309571E-02, -1.3904823E-02, 7.8187120E-03, -4.5190369E-03/
2462 DATA Y3/
2463 1 2.6729062E-03, -1.6073718E-03, 9.7715622E-04, -5.9804407E-04,
2464 2 3.6749320E-04, -2.2635296E-04, 1.3960805E-04, -8.6172618E-05,
2465 3 5.3212947E-05, -3.2867888E-05, 2.0304203E-05, -1.2543926E-05,
2466 4 7.7499633E-06, -4.7882430E-06, 2.9564108E-06, -1.8278645E-06,
2467 5 1.1293571E-06, -6.9778174E-07, 4.3113019E-07, -2.6637753E-07,
2468 6 1.6458373E-07, -1.0168954E-07, 6.2829807E-08, -3.8819969E-08,
2469 7 2.3985272E-08, -1.4819520E-08, 9.1563774E-09, -5.6573541E-09,
2470 8 3.4954514E-09, -2.1597005E-09, 1.3343946E-09, -8.2447148E-10,
2471 9 5.0941033E-10, -3.1474631E-10, 1.9447072E-10, -1.2015685E-10,
2472 1 7.4241055E-11, -4.5871468E-11, 2.8343095E-11, -1.7513137E-11,
2473 2 6.9049613E-12/
2474 C--$SENDATA
2475 C
2476 COMPLEX FUN,C,CMAX,SAVE

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2477      DIMENSION KEY(193),SAVE(193),T(2),TMAX(2)
2478      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
2479      IF(NEW) 10,30,10
2480      10    LAG=-1
2481           X0=-X-26.30455704
2482           DO 20 IR=1,193
2483      20    KEY(IR)=0
2484      30    LAG=LAG+1
2485           ZLAGH0=(0.0,0.0)
2486           CMAX=(0.0,0.0)
2487           L=0
2488           ASSIGN 110 TO M
2489           I=129
2490           GO TO 200
2491      110    TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
2492           TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
2493           I=I+1
2494           IF(I.LE.146) GO TO 200
2495           IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
2496           CMAX=TOL*CMAX
2497           ASSIGN 120 TO M
2498           I=128
2499           GO TO 200
2500      120    IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
2501           I=I+1
2502           IF(I.GT.0) GO TO 200
2503      130    ASSIGN 140 TO M
2504           I=147
2505           GO TO 200
2506      140    IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
2507           I=I+1
2508           IF(I.LE.193) GO TO 200
2509           GO TO 190
2510      150    ASSIGN 160 TO M
2511           I=1
2512           GO TO 200
2513      160    IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
2514           I=I+1
2515           IF(I.LE.128) GO TO 200
2516      170    ASSIGN 180 TO M
2517           I=193

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2742      140      IF(ABS(T(1)),LE,TMAX(1),AND,ABS(T(2)),LE,TMAX(2)) GO TO 190
2743      I=I+1
2744      IF(I,LE,236) GO TO 200
2745      GO TO 190
2746      150      ASSIGN 160 TO M
2747      I=1
2748      GO TO 200
2749      160      IF(T(1),EQ,0,0,AND,T(2),EQ,0,0) GO TO 170
2750      I=I+1
2751      IF(I,LE,65) GO TO 200
2752      170      ASSIGN 180 TO M
2753      I=236
2754      GO TO 200
2755      180      IF(T(1),EQ,0,0,AND,T(2),EQ,0,0) GO TO 190
2756      I=I-1
2757      IF(I,GE,99) GO TO 200
2758      190      RETURN
2759      C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
2760      200      LOOK=I+LAG
2761      IQ=LOOK/237
2762      IR=MOD(LOOK,237)
2763      IF(IR,EQ,0) IR=1
2764      IROLL=IQ*236
2765      IF(KEY(IR),LE,IROLL) GO TO 220
2766      210      C=SAVE(IR)*WT(I)
2767      ZLAGH1=ZLAGH1+C
2768      L=L+1
2769      GO TO M,(110,120,140,160,180)
2770      220      KEY(IR)=IROLL+IR
2771      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
2772      GO TO 210
2773      END

```

2774                   COMPLEX FUNCTION ZLAGFO(X,FUN,TOL,L,NEW)  
2775 C--\*\*\* A SPECIAL LAGGED\* CONVOLUTION METHOD TO COMPUTE THE  
2776 C INTEGRAL FROM 0 TO INFINITY OF "FUN(G)\*COS(G\*B)\*DG" DEFINED AS THE  
2777 C COMPLEX FOURIER COSINE TRANSFORM WITH ARGUMENT X(=ALOG(B))  
2778 C BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND  
2779 C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....  
2780 C  
2781 C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.  
2782 C  
2783 C--PARAMETERS:  
2784 C  
2785 C     \* X       = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM  
2786 C               "ZLAGFO" IS USEFUL ONLY WHEN X=(LAST X)=.20 \*\*\* I.E.,  
2787 C               SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,  
2788 C               THEN SUBPROGRAM "ZFOUR0" IS ADVISED FOR GENERAL USE.  
2789 C               (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).  
2790 C     FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)  
2791 C               OF A REAL ARGUMENT G.  
2792 C               NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN  
2793 C               CALLING PROGRAM AND IN SUBPROGRAM FUN.  
2794 C               THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE  
2795 C               DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...  
2796 C               FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RLAGFO" IS ADVISED;  
2797 C               HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE  
2798 C               INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))  
2799 C     TOL=      REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,  
2800 C               IF FILTER=FUN\*TOL\*MAX, THEN REST OF TAIL IS TRUNCATED.  
2801 C               THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,  
2802 C               TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON  
2803 C               THE FUNCTION FUN AND PARAMETER X...IN GENERAL,  
2804 C               A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"  
2805 C               BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY  
2806 C               RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN  
2807 C               APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,  
2808 C               ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...  
2809 C     L=        RESULTING NO. FILTER WTS. USED IN THE VARIABLE  
2810 C               CONVOLUTION (L DEPENDS ON TOL AND FUN).  
2811 C               MIN,L=24 AND MAX,L=281--WHICH COULD  
2812 C               OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING  
2813 C               VERY FAST...

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2814      C      * NEW= 1 IS NECESSARY 1ST TIME OR BRAND NEW X,
2815      C      0 FOR ALL SURSEQUENT CALLS WHERE X=(LAST X)-0.20
2816      C      IS ASSUMED INTERNALLY BY THIS ROUTINE.
2817      C      NOTE: IF THIS IS NOT TRUE, ROUTINE WILL
2818      C      STILL ASSUME X=(LAST X)-0.20 ANYWAY,..
2819      C      IT IS THE USERS RESPONSIBILITY TO NORMALIZE
2820      C      BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
2821      C      THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
2822      C      TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
2823      C      SIMPLE ELEMENTARY FUNCTION,..DUE TO INTERNALLY SAVING
2824      C      ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1,..
2825      C      THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
2826      C      KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
2827      C      WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
2828      C      WHEN NEEDED (DEPENDS ON PARAMS TOL AND FUN)
2829      C
2830      C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZLAGFO; THE FOURIER
2831      C      TRANSFORM IS THEN ZLAGFO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
2832      C      THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL,..
2833      C
2834      C--USAGE-- "ZLAGFO" IS CALLED AS FOLLOWS:
2835      C      ...
2836      C      COMPLEX Z ZLAGFO,ZF
2837      C      EXTERNAL ZF
2838      C      ...
2839      C      Z=ZLAGFO(ALOG(B),ZF,TOL,L,NEW)/B
2840      C      ...
2841      C      END
2842      C      COMPLEX FUNCTION ZF(G)
2843      C      ...USER SUPPLIED CODE...
2844      C      END
2845      C
2846      C--NOTES:
2847      C      (1). EXP=UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
2848      C      BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
2849      C      ANY & ALL EXP=UNDERFLOW'S TO 0.0....
2850      C      (2). AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
2851      C      METHOD, LET BMAX>=BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
2852      C      THAT THE ACTUAL NUMBER OF B'S IS NB=AIN(5,*ALOG(BMAX/BMIN))+1,
2853      C      PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
2854      C      BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING

```

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2855 C ARGUMENTS SPACED AS X=ALOG(BMAX),X=.,2,X=.,2*2,...,ALOG(BMINA),
2856 C FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
2857 C
2858 C      ***
2859 C      NB=AIN(5.*ALOG(BMAX/BMIN))+1
2860 C      NB1=NB+1
2861 C      X0=ALOG(BMAX)+.2
2862 C      NEW=1
2863 C      DO 1 I=NB,1,-1
2864 C      X=X0-.2*(NB1-I)
2865 C      ARG(I)=EXP(X)
2866 C      Z(I)=ZLAGFO(X,ZF,TOL,L,NEW)/ARG(I)
2867 C      1 NEW=0
2868 C      ***
2869 C      (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),Z(I),I=1,NB FOR
2870 C      ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
2871 C      TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
2872 C      SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
2873 C      (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
2874 C      ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
2875 C      BMAX,BMIN AND BY SETTING NEW=1,...
2876 C
2877 C --COS-EXTENDED FILTER WEIGHT ARRAYS:
2878 C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
2879 C TO SAVE STORAGE.
2880 C      DIMENSION YT(281),Y1(76),Y2(76),Y3(76),Y4(53)
2881 C      EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1)),
2882 C      1 (YT(229),Y4(1))
2883 C      DATA Y1/
2884 C      1 5.1178101E-14, 2.9433849E-14, 2.5492522E-14, 1.9034819E-14,
2885 C      2 6.4179780E-14, 1.3085746E-15, 1.1989957E-13,-1.2216234E-14,
2886 C      3 1.7534103E-13, 7.9373498E-15, 2.1235658E-13, 7.9981520E-14,
2887 C      4 2.3815757E-13, 1.9714260E-13, 2.8920132E-13, 3.4161340E-13,
2888 C      5 4.0349917E-13, 5.2203885E-13, 5.9837223E-13, 7.8015306E-13,
2889 C      6 8.8911655E-13, 1.1709731E-12, 1.3165595E-12, 1.7578463E-12,
2890 C      7 1.9538564E-12, 2.6289768E-12, 2.9167697E-12, 3.9044344E-12,
2891 C      8 4.3927341E-12, 5.7526904E-12, 6.6569552E-12, 8.4555678E-12,
2892 C      9 1.0063229E-11, 1.2487964E-11, 1.5134582E-11, 1.8501488E-11,
2893 C      1 2.2720051E-11, 2.7452598E-11, 3.4025443E-11, 4.0875985E-11,
2894 C      2 5.0751668E-11, 6.1094382E-11, 7.5492982E-11, 9.1445759E-11,
2895 C      3 1.1227336E-10, 1.3676464E-10, 1.6720269E-10, 2.0423244E-10,

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2896 4 2.4932743E-10, 3.0470661E-10, 3.7198526E-10, 4.5449934E-10,
2897 5 5.5502537E-10, 6.7793669E-10, 8.2810001E-10, 1.0112626E-09,
2898 6 1.2354800E-09, 1.5085255E-09, 1.8432253E-09, 2.2503397E-09,
2899 7 2.7499027E-09, 3.3569525E-09, 4.1025670E-09, 5.0077487E-09,
2900 8 6.1205950E-09, 7.4703399E-09, 9.1312760E-09, 1.1143911E-08,
2901 9 1.3622929E-08, 1.6623917E-08, 2.0324094E-08, 2.4798610E-08,
2902 1 3.0321709E-08, 3.6992986E-08, 4.5237482E-08, 5.3183434E-08/
2903 DATA Y2/
2904 1 6.7491070E-08, 8.2317946E-08, 1.0060271E-07, 1.2279375E-07,
2905 2 1.5022907E-07, 1.8316969E-07, 2.2413747E-07, 2.7322865E-07,
2906 3 3.3441045E-07, 4.0756197E-07, 4.9894278E-07, 6.0793233E-07,
2907 4 7.4443665E-07, 9.0679753E-07, 1.1107379E-06, 1.3525651E-06,
2908 5 1.6573073E-06, 2.0174273E-06, 2.4722798E-06, 3.0090445E-06,
2909 6 3.6898816E-06, 4.4879625E-06, 5.5056521E-06, 6.6935820E-06,
2910 7 8.2160716E-06, 9.9828691E-06, 1.2280527E-05, 1.4888061E-05,
2911 8 1.8296530E-05, 2.2202672E-05, 2.7305154E-05, 3.3109672E-05,
2912 9 4.0751046E-05, 4.9372484E-05, 6.0820947E-05, 7.3619571E-05,
2913 1 9.0780005E-05, 1.0976837E-04, 1.3550409E-04, 1.6365676E-04,
2914 2 2.0227521E-04, 2.4398338E-04, 3.0197018E-04, 3.6370760E-04,
2915 3 4.5083748E-04, 5.4213338E-04, 6.7315347E-04, 8.0800951E-04,
2916 4 1.0051938E-03, 1.2041401E-03, 1.5011708E-03, 1.7942344E-03,
2917 5 2.2421056E-03, 2.6730676E-03, 3.3490681E-03, 3.9815050E-03,
2918 6 5.0028666E-03, 5.9285668E-03, 7.4730905E-03, 8.8233510E-03,
2919 7 1.1160132E-02, 1.3119627E-02, 1.6663199E-02, 1.9472767E-02,
2920 8 2.4800811E-02, 2.8793704E-02, 3.5762063E-02, 4.2228780E-02,
2921 9 5.3905163E-02, 6.0804660E-02, 7.7081738E-02, 8.3874501E-02,
2922 1 1.0377190E-01, 1.0377718E-01, 1.1892208E-01, 9.0437429E-02/
2923 DATA Y3/
2924 1 7.1685138E-02, -3.9473064E-02, -1.5078720E-01, -4.0489859E-01,
2925 2 -5.6018995E-01, -6.8050388E-01, -1.5094224E-01, 6.6304064E-01,
2926 3 1.3766748E+00, -8.0373222E-01, -1.0869629E+00, 1.2812892E+00,
2927 4 -5.0341082E-01, -4.4274455E-02, 2.0913102E-01, -1.9999661E-01,
2928 5 1.5207664E-01, -1.0920260E-01, 7.8169956E-02, -5.6651561E-02,
2929 6 4.1611799E-02, -3.0880012E-02, 2.3072559E-02, -1.7311631E-02,
2930 7 1.3021442E-02, -9.8085025E-03, 7.3943529E-03, -5.5769518E-03,
2931 8 4.2073164E-03, -3.1745026E-03, 2.3954154E-03, -1.8076122E-03,
2932 9 1.3640816E-03, -1.0293934E-03, 7.7682952E-04, -5.8623518E-04,
2933 1 4.4240399E-04, -3.3386183E-04, 2.5195025E-04, -1.9013541E-04,
2934 2 1.4348659E-04, -1.0828234E-04, 8.1716174E-05, -6.1667509E-05,
2935 3 4.6537684E-05, -3.5119887E-05, 2.6503388E-05, -2.0000904E-05,
2936 4 1.5093768E-05, -1.1390572E-05, 8.5959318E-06, -6.4869407E-06,

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2937      5 4.8953713E-06,-3.6942830E-06, 2.7878625E-06,-2.1038241E-06,
2938      6 1.5875917E-06,-1.1980090E-06, 9.0398030E-07,-6.8208296E-07,
2939      7 5.1458650E-07,-3.8817581E-07, 2.9272267E-07,-2.2067921E-07,
2940      8 1.6623514E-07,-1.2514102E-07, 9.4034535E-08,-7.0556837E-08,
2941      9 5.2741581E-08,-3.9298610E-08, 2.9107255E-08,-2.1413893E-08,
2942      1 1.5742032E-08,-1.1498608E-08, 8.7561571E-09,-7.2959446E-09/
2943      DATA Y4/
2944      1 6.8816619E-09,-8.9679825E-09, 1.4258275E-08,-1.9564299E-08,
2945      2 2.0235313E-08,-1.4725545E-08, 5.4632820E-09, 3.5995580E-09,
2946      3-9.5287133E-09, 1.1460041E-08,-1.0280532E-08, 7.4641748E-09,
2947      4-4.4703465E-09, 2.0499053E-09,-4.4806353E-10,-4.0374336E-10,
2948      5 7.0321001E-10,-6.7067960E-10, 4.9130404E-10,-2.8840747E-10,
2949      6 1.2373144E-10,-1.5260443E-11,-4.2027559E-11, 6.1885474E-11,
2950      7-5.9273937E-11, 4.6588766E-11,-3.2054182E-11, 1.9831637E-11,
2951      8-1.1210098E-11, 5.9567021E-12,-3.2427912E-12, 2.1353868E-12,
2952      9-1.8476851E-12, 1.8438474E-12,-1.8362842E-12, 1.7241847E-12,
2953      1-1.5161479E-12, 1.2627657E-12,-1.0129176E-12, 7.9578625E-13,
2954      2-6.2131435E-13, 4.8745900E-13,-3.8703630E-13, 3.1172547E-13,
2955      3-2.5397802E-13, 2.0824130E-13,-1.7123163E-13, 1.4113344E-13,
2956      4-1.1687986E-13, 9.7604016E-14,-8.2977176E-14, 7.2515267E-14,
2957      5-5.6047478E-14/
2958      C--98ENDATA
2959      COMPLEX FUN,C,CMAX,SAVE
2960      DIMENSION KEY(281),SAVE(281),T(2),TMAX(2)
2961      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
2962      IF(NEW) 10,30,10
2963      10      LAG=-1
2964      X0=-X-30.30251236
2965      DO 20 IR=1,281
2966      20      KEY(IR)=0
2967      30      LAG=LAG+1
2968      ZLAGFO=(0.0,0.0)
2969      CMAX=(0.0,0.0)
2970      L=0
2971      ASSIGN 110 TO M
2972      I=149
2973      GO TO 200
2974      110      TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
2975      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
2976      I=I+1
2977      IF(I,LE,170) GO TO 200

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2978      IF(TMAX(1),EQ.0.0.AND,TMAX(2),EQ.0.0) GO TO 150
2979      CMAX=TOL*CMAX
2980      ASSIGN 120 TO M
2981      I=148
2982      GO TO 200
120      IF(ABS(T(1)),LE,TMAX(1),AND,ABS(T(2)),LE,TMAX(2)) GO TO 130
2984      I=I-1
2985      IF(I,GT,0) GO TO 200
2986      130      ASSIGN 140 TO M
2987      I=171
2988      GO TO 200
2989      140      IF(ABS(T(1)),LE,TMAX(1),AND,ABS(T(2)),LE,TMAX(2)) GO TO 190
2990      I=I+1
2991      IF(I,LE,281) GO TO 200
2992      GO TO 190
2993      150      ASSIGN 160 TO M
2994      I=1
2995      GO TO 200
2996      160      IF(T(1),EQ.0.0.AND,T(2),EQ.0.0) GO TO 170
2997      I=I+1
2998      IF(I,LE,148) GO TO 200
2999      170      ASSIGN 180 TO M
3000      I=281
3001      GO TO 200
3002      180      IF(T(1),EQ.0.0.AND,T(2),EQ.0.0) GO TO 190
3003      I=I-1
3004      IF(I,GE,171) GO TO 200
3005      190      RETURN
3006      C=-STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
3007      200      LOOK=Y+LAG
3008      IQ=LOOK/282
3009      IR=MOD(LOOK,282)
3010      IF(IR,EQ,0) IR=1
3011      IROLL=IQ*281
3012      IF(KEY(IR),LE,IROLL) GO TO 220
3013      210      C=SAVE(IR)*YT(I)
3014      ZLAGFO=ZLAGFO+C
3015      I=L+1
3016      GO TO M,(110,120,140,160,180)
3017      220      KEY(IP)=IROLL+IR
3018      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
3019      GO TO 210
3020      END

```

3021 COMPLEX FUNCTION ZLAGF1(X,FUN,TOL,L,NEW)  
 3022 C--\*\*\* A SPECIAL LAGGED\* CONVOLUTION METHOD TO COMPUTE THE  
 3023 C INTEGRAL FROM 0 TO INFINITY OF "FUN(G)\*SIN(G\*B)\*DG" DEFINED AS THE  
 3024 C COMPLEX FOURIER SINE TRANSFORM WITH ARGUMENT X(=ALOG(B))  
 3025 C BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND  
 3026 C USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....  
 3027 C  
 3028 C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO,  
 3029 C  
 3030 C--PARAMETERS:  
 3031 C  
 3032 C \* X = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM  
 3033 C "ZLAGF1" IS USEFUL ONLY WHEN X=(LAST X)=.20 \*\*\* I.E.,  
 3034 C SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,  
 3035 C THEN SUBPROGRAM "ZFOUR1" IS ADVISED FOR GENERAL USE.  
 3036 C (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).  
 3037 C FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)  
 3038 C OF A REAL ARGUMENT G.  
 3039 C NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN  
 3040 C CALLING PROGRAM AND IN SUBPROGRAM FUN.  
 3041 C THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE  
 3042 C DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...  
 3043 C FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RLAGF1" IS ADVISED;  
 3044 C HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE  
 3045 C INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))  
 3046 C TOL= REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,  
 3047 C IF FILTER\*FUN<TOL\*MAX, THEN REST OF TAIL IS TRUNCATED.  
 3048 C THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,  
 3049 C TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON  
 3050 C THE FUNCTION FUN AND PARAMETER X...IN GENERAL,  
 3051 C A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"  
 3052 C BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY  
 3053 C RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN  
 3054 C APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,  
 3055 C ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...  
 3056 C L= RESULTING NO. FILTER WTS. USED IN THE VARIABLE  
 3057 C CONVOLUTION (L DEPENDS ON TOL AND FUN).  
 3058 C MIN.L=20 AND MAX.L=266--WHICH COULD  
 3059 C OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING  
 3060 C VERY FAST...

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3061 C      * NEW= 1 IS NECESSARY 1ST TIME OR BRAND NEW X.
3062 C      0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)=0.20
3063 C      IS ASSUMED INTERNALLY BY THIS ROUTINE.
3064 C      NOTE: IF THIS IS NOT TRUE, ROUTINE WILL
3065 C      STILL ASSUME X=(LAST X)=0.20 ANYWAY...
3066 C      IT IS THE USERS RESPONSIBILITY TO NORMALIZE
3067 C      BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
3068 C      THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
3069 C      TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
3070 C      SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
3071 C      ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
3072 C      THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
3073 C      KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
3074 C      WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
3075 C      WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
3076 C
3077 C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZLAGF1; THE FOURIER
3078 C TRANSFORM IS THEN ZLAGF1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
3079 C THIS ROUTINE..., WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
3080 C
3081 C--USAGE-- "ZLAGF1" IS CALLED AS FOLLOWS:
3082 C      ...
3083 C      COMPLEX Z,ZLAGF1,ZF
3084 C      EXTERNAL ZF
3085 C      ...
3086 C      Z=ZLAGF1(ALOG(B),ZF,TOL,L,NEW)/B
3087 C      ...
3088 C      END
3089 C      COMPLEX FUNCTION ZF(G)
3090 C      ...USER SUPPLIED CODE...
3091 C      END
3092 C
3093 C--NOTES:
3094 C      (1). EXP=UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
3095 C      BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
3096 C      ANY & ALL EXP=UNDERFLOW'S TO 0.0....
3097 C      (2). AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
3098 C      METHOD, LET BMAX>=BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
3099 C      THAT THE ACTUAL NUMBER OF B'S IS NB=AIN(5.*ALOG(BMAX/BMIN))+1,
3100 C      PROVIDED BMAX/BMIN>=1, THE USER MAY THEN ASSUME AN "ADJUSTED"
3101 C      BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING

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3102 C ARGUMENTS SPACED AS X=ALOG(BMAX),X=.2,X=.2*2,...,ALOG(BMINA).
3103 C FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
3104 C
3105 C     ...
3106 C     NB=AIN(TS,ALOG(BMAX/BMIN))+1
3107 C     NB1=NB+1
3108 C     X0=ALOG(BMAX)+.2
3109 C     NEW=1
3110 C     DO 1 I=NB,1,-1
3111 C     X=X0-.2*(NB1-I)
3112 C     ARG(I)=EXP(X)
3113 C     Z(I)=ZLAGF1(X,ZF,TOL,L,NEW)/ARG(I)
3114 C     1 NEW=0
3115 C     ...
3116 C (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),Z(I),I=1,NB FOR
3117 C ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
3118 C TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
3119 C SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
3120 C (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
3121 C ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
3122 C BMAX,BMIN AND BY SETTING NEW=1,...
3123 C
3124 C --BIN-EXTENDED FILTER WEIGHT ARRAYS:
3125 C NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
3126 C TO SAVE STORAGE,
3127 C     DIMENSION WT(266),W1(76),W2(76),W3(76),W4(38)
3128 C     EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
3129 C     1 (WT(229),W4(1))
3130 C     DATA W1/
3131 C     1-1.1113940E-09,-1.3237246E-12, 1.5091739E-12,-1.6240954E-12,
3132 C     2 1.7236636E-12,-1.8227727E-12, 1.9255992E-12,-2.0335514E-12,
3133 C     3 2.1473541E-12,-2.2678549E-12, 2.3946842E-12,-2.5292661E-12,
3134 C     4 2.6718110E-12,-2.8227693E-12, 2.9825171E-12,-3.1514006E-12,
3135 C     5 3.3297565E-12,-3.5179095E-12, 3.7163306E-12,-3.9256378E-12,
3136 C     6 4.1464798E-12,-4.3794852E-12, 4.6252131E-12,-4.8848227E-12,
3137 C     7 5.1582809E-12,-5.4474462E-12, 5.7530277E-12,-6.0760464E-12,
3138 C     8 6.4175083E-12,-6.7783691E-12, 7.1595239E-12,-7.5618782E-12,
3139 C     9 7.9864477E-12,-8.4344110E-12, 8.9072422E-12,-9.4067705E-12,
3140 C     1 9.9349439E-12,-1.0493731E-11, 1.1084900E-11,-1.1709937E-11,
3141 C     2 1.2370354E-11,-1.3067414E-11, 1.3602200E-11,-1.4575980E-11,
3142 C     3 1.5390685E-11,-1.6249313E-11, 1.7195934E-11,-1.8115250E-11,

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3143 4 1.9131898E-11,-2.0209795E-11, 2.1352159E-11,-2.2561735E-11,
3144 5 2.3840976E-11,-2.5192263E-11, 2.6618319E-11,-2.8122547E-11,
3145 6 2.9709129E-11,-3.1382870E-11, 3.3149030E-11,-3.5013168E-11,
3146 7 3.6981050E-11,-3.9058553E-11, 4.1251694E-11,-4.3566777E-11,
3147 8 4.6010537E-11,-4.9590396E-11, 5.1214761E-11,-5.4193353E-11,
3148 9 5.7236720E-11,-6.0455911E-11, 6.3861222E-11,-6.7461492E-11,
3149 1 7.1265224E-11,-7.5279775E-11, 7.9512249E-11,-8.3971327E-11/
3150 DATA W2/
3151 1 8.8668961E-11,-9.3621900E-11, 9.8851764E-11,-1.0438319E-10,
3152 2 1.1024087E-10,-1.1644680E-10, 1.2301979E-10,-1.2997646E-10,
3153 3 1.3703244E-10,-1.4510363E-10, 1.5330772E-10,-1.6196550E-10,
3154 4 1.7110130E-10,-1.8074257E-10, 1.9091922E-10,-2.0166306E-10,
3155 5 2.1300756E-10,-2.2498755E-10, 2.3763936E-10,-2.5100098E-10,
3156 6 2.6511250E-10,-2.8001616E-10, 2.9575691E-10,-3.1238237E-10,
3157 7 3.2994314E-10,-3.4849209E-10, 3.6808529E-10,-3.8878042E-10,
3158 8 4.1063982E-10,-4.3372666E-10, 4.5811059E-10,-4.8386049E-10,
3159 9 5.1105728E-10,-5.3977672E-10, 5.7011632E-10,-6.0215516E-10,
3160 1 6.3601273E-10,-6.7175964E-10, 7.0955028E-10,-7.4942601E-10,
3161 2 7.9161025E-10,-8.3606980E-10, 8.8317110E-10,-9.3270330E-10,
3162 3 9.8533749E-10,-1.0404508E-09, 1.0923731E-09,-1.1605442E-09,
3163 4 1.2267391E-09,-1.2942905E-09, 1.3691677E-09,-1.4429912E-09,
3164 5 1.5288164E-09,-1.6077524E-09, 1.7085998E-09,-1.7890471E-09,
3165 6 1.9129068E-09,-1.9857116E-09, 2.1491608E-09,-2.1926779E-09,
3166 7 2.4312660E-09,-2.3959044E-09, 2.7872500E-09,-2.5610596E-09,
3167 8 3.2762318E-09,-2.6082940E-09, 4.0261453E-09,-2.3560563E-09,
3168 9 5.3176554E-09,-1.3960161E-09, 7.7708747E-09, 1.1853546E-09,
3169 1 1.2760851E-08, 7.4264707E-09, 2.3342187E-08, 2.1869851E-08/
3170 DATA W3/
3171 1 4.6306744E-08, 5.4631686E-08, 9.8763087E-08, 1.2823337E-07,
3172 2 2.0832612E-07, 2.9280540E-07, 4.5580888E-07, 6.5992437E-07,
3173 3 1.0056815E-06, 1.4779183E-06, 2.2284335E-06, 3.2994604E-06,
3174 4 4.9485823E-06, 7.3543473E-06, 1.1001083E-05, 1.6380539E-05,
3175 5 2.4469550E-05, 3.6462246E-05, 5.4441527E-05, 8.1176726E-05,
3176 6 1.2113829E-04, 1.8066491E-04, 2.6954609E-04, 4.0202288E-04,
3177 7 5.9969995E-04, 8.9437312E-04, 1.3338166E-03, 1.9886697E-03,
3178 8 2.9643943E-03, 4.4168923E-03, 6.5773518E-03, 9.7855105E-03,
3179 9 1.4539361E-02, 2.1558670E-02, 3.1871864E-02, 4.6903518E-02,
3180 1 6.8559512E-02, 9.9170152E-02, 1.4120770E-01, 1.9610835E-01,
3181 2 2.6192603E-01, 3.2743321E-01, 3.6407406E-01, 3.1257559E-01,
3182 3 9.0460168E-02, -3.6051039E-01, -8.6324760E-01, -8.1178720E-01,
3183 4 5.2205241E-01, 1.5449873E+00, -1.1817933E+00, -2.6759896E-01,

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3184      5 8.0869203E-01,-6.2757149E-01, 3.4062630E-01,-1.5885304E-01,
3185      6 7.0472984E-02,-3.1624462E-02, 1.4894063E-02,-7.4821176E-03,
3186      7 4.0035936E-03,-2.2543784E-03, 1.3160358E-03,-7.8636604E-04,
3187      8 4.7658745E-04,-2.9125817E-04, 1.7885105E-04,-1.1012416E-04,
3188      9 6.7910334E-05,-4.1914054E-05, 2.5881544E-05,-1.5985851E-05,
3189      1 9.8751880E-06,-6.1008526E-06, 3.7692543E-06,-2.3287953E-06/
3190      DATA W4/
3191      1 1.4388425E-06,-8.8899353E-07, 5.4926991E-07,-3.3937048E-07,
3192      2 2.0966284E-07,-1.2955437E-07, 8.0046336E-08,-4.9457371E-08,
3193      3 3.0557711E-08,-1.6880390E-08, 1.1665454E-08,-7.2076428E-09,
3194      4 4.4533423E-09,-2.7515696E-09, 1.7001092E-09,-1.0504494E-09,
3195      5 6.4904567E-10,-4.0102999E-10, 2.4776763E-10,-1.5310321E-10,
3196      6 9.4600354E-11,-5.8483314E-11, 3.6119400E-11,-2.2320056E-11,
3197      7 1.3793460E-11,-8.5242656E-12, 5.2675102E-12,-3.2543078E-12,
3198      8 2.0097689E-12,-1.2405412E-12, 7.6530538E-13,-4.7191929E-13,
3199      9 2.9084993E-13,-1.7923661E-13, 1.1018948E-13,-6.7885902E-14,
3200      1 4.2025050E-14,-2.1314731E-14/
3201      C--$$ENDATA
3202      C
3203      COMPLEX FUN,C,CMAX,SAVE
3204      DIMENSION KEY(266),SAVE(266),T(2),TMAX(2)
3205      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
3206      IF(NEW) 10,30,10
3207      LAG=-1
3208      X0=-X-3R.30455704
3209      DO 20 IR=1,266
3210      20 KEY(IR)=0
3211      30 LAG=LAG+1
3212      ZLAGF1=(0.0,0.0)
3213      CMAX=(0.0,0.0)
3214      L=0
3215      ASSIGN 110 TO M
3216      I=191
3217      GO TO 200
3218      110 TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
3219      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
3220      I=I+1
3221      IF(I.LE.203) GO TO 200
3222      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
3223      CMAX=TOL*CMAX
3224      ASSIGN 120 TO M

```

```

3225      I=190
3226      GO TO 200
3227 120    IF (ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
3228      I=I+1
3229      IF (I.GT.0) GO TO 200
3230 130    ASSIGN 140 TO M
3231      I=209
3232      GO TO 200
3233 140    IF (ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
3234      I=I+1
3235      IF (I.LE.266) GO TO 200
3236      GO TO 190
3237 150    ASSIGN 160 TO M
3238      I=1
3239      GO TO 200
3240 160    IF (T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
3241      I=I+1
3242      IF (I.LE.190) GO TO 200
3243 170    ASSIGN 180 TO M
3244      I=266
3245      GO TO 200
3246 180    IF (T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
3247      I=I+1
3248      IF (I.GE.209) GO TO 200
3249 190    RETURN
3250  C=-STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
3251 200    LOOK=I+LAG
3252      IQ=LOOK/267
3253      IR=MOD(LOOK,267)
3254      IF (IR.EQ.0) IR=1
3255      IROLL=IQ*266
3256      IF (KEY(IR).LE.IROLL) GO TO 220
3257 210    C=SAVE(IR)*WT(I)
3258      ZLAGF1=ZLAGF1+C
3259      L=L+1
3260      GO TO M, (110,120,140,160,180)
3261 220    KEY(IR)=IROLL+IR
3262      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
3263      GO TO 210
3264      END

```

## Appendix 2.---Test results.

## FILTER TESTS:

--MODERATE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
1	1	31	0,1000E+00	0,2000E+00	0,17888544E+02	0,17887801E+02	0,7429E-03	0,4153E-04	0,1000E-03
1	1	35	0,1000E+00	0,1000E+01	0,98518533E+00	0,98502134E+00	0,1640E-03	0,1665E-03	0,1000E-03
1	1	41	0,1000E+00	0,4000E+01	0,62441452E+01	0,62422531E+01	0,1892E-04	0,3030E-03	0,1000E-03
1	1	41	0,1000E+01	0,2000E+00	0,18857320E+00	0,18857222E+00	0,9816E-06	0,5205E-05	0,1000E-03
1	1	31	0,1000E+01	0,1000E+01	0,35355338E+00	0,35354393E+00	0,9455E-05	0,2674E-04	0,1000E-03
1	1	34	0,1000E+01	0,4000E+01	0,57067205E+01	0,57065136E+01	0,2068E-05	0,3625E-04	0,1000E-03
1	2	31	0,1000E+00	0,2000E+00	0,45241871E+01	0,45242038E+01	-0,1675E-04	0,3702E-05	0,1000E-03
1	2	24	0,1000E+00	0,1000E+01	0,20521249E+01	0,20521447E+01	-0,1976E-04	0,9629E-05	0,1000E-03
1	2	25	0,1000E+00	0,4000E+01	0,42483543E+15	-0,30106629E+04	0,3011E-04	0,7087E+11	0,1000E-03
1	2	85	0,1000E+01	0,2000E+00	0,49502491E+01	0,49503862E+01	-0,1371E-05	0,2769E-04	0,1000E-03
1	2	28	0,1000E+01	0,1000E+01	0,19470019E+00	0,19469971E+00	0,4824E-06	0,2478E-05	0,1000E-03
1	2	23	0,1000E+01	0,4000E+01	0,18315639E+01	0,18311789E+01	0,3851E-05	0,2102E-03	0,1000E-03
1	3	70	0,1000E+00	0,2000E+00	0,49667307E+01	0,49671476E+01	-0,3968E-05	0,7990E-04	0,1000E-03
1	3	63	0,1000E+00	0,1000E+01	0,48354037E+01	0,48359028E+01	-0,4990E-05	0,1032E-03	0,1000E-03
1	3	54	0,1000E+00	0,4000E+01	0,43657927E+01	0,43650769E+01	0,7659E-05	0,1754E-03	0,1000E-03
1	3	54	0,1000E+01	0,2000E+00	0,46748903E+00	0,46752756E+00	-0,3853E-04	0,8243E-04	0,1000E-03
1	3	48	0,1000E+01	0,1000E+01	0,35283407E+00	0,35288725E+00	-0,5318E-04	0,1507E-03	0,1000E-03
1	3	51	0,1000E+01	0,4000E+01	0,91032035E+01	0,90828397E+01	0,2036E-03	0,2237E-02	0,1000E-03
1	4	36	0,1000E+00	0,2000E+00	0,27639320E+01	0,27635086E+01	0,4234E-03	0,1532E-03	0,1000E-03
1	4	41	0,1000E+00	0,1000E+01	0,90049628E+00	0,90031951E+00	0,1768E-03	0,1963E-03	0,1000E-03
1	4	47	0,1000E+00	0,4000E+01	0,24375195E+00	0,24370805E+00	0,4390E-04	0,1801E-03	0,1000E-03
1	4	67	0,1000E+01	0,2000E+00	0,97096664E+01	0,97096572E+01	0,9220E-07	0,9496E-06	0,1000E-03
1	4	45	0,1000E+01	0,1000E+01	0,29289122E+00	0,29289033E+00	0,2898E-05	0,9895E-05	0,1000E-03
1	4	39	0,1000E+01	0,4000E+01	0,18936609E+00	0,18934510E+00	0,2099E-04	0,1109E-03	0,1000E-03
1	5	44	0,1000E+00	0,2000E+00	-0,12500000E-01	-0,12490173E-01	-0,9827E-05	0,7862E-03	0,1000E-03
1	5	48	0,1000E+00	0,1000E+01	-0,25000000E-02	-0,24991255E-02	-0,8744E-06	0,3498E-03	0,1000E-03
1	5	54	0,1000E+00	0,4000E+01	-0,62499999E-03	-0,62492175E-03	-0,7825E-07	0,1252E-03	0,1000E-03
1	5	58	0,1000E+01	0,2000E+00	-0,21094379E+00	-0,21091424E+00	-0,2554E-04	0,1211E-03	0,1000E-03
1	5	41	0,1000E+01	0,1000E+01	-0,25000000E+00	-0,24967052E+00	-0,3295E-03	0,1318E-02	0,1000E-03
1	5	45	0,1000E+01	0,4000E+01	-0,62500000E-01	-0,62485539E-01	-0,1446E-04	0,2314E-03	0,1000E-03
2	1	67	0,1000E+00	0,2000E+00	0,44721360E+01	0,44719433E+01	0,1927E-03	0,4309E-04	0,1000E-03
2	1	68	0,1000E+00	0,1000E+01	0,99503718E+00	0,99495371E+00	0,8347E-04	0,8389E-04	0,1000E-03
2	1	70	0,1000E+00	0,4000E+01	0,24992191E+00	0,24989002E+00	0,3189E-04	0,1276E-03	0,1000E-03
2	1	81	0,1000E+01	0,2000E+00	0,98058067E+00	0,98057036E+00	0,1033E-04	0,1053E-04	0,1000E-03
2	1	69	0,1000E+01	0,1000E+01	0,70710678E+00	0,70707841E+00	0,2836E-04	0,4011E-04	0,1000E-03
2	1	68	0,1000E+01	0,4000E+01	0,24253562E+00	0,24252472E+00	0,1090E-04	0,4496E-04	0,1000E-03



FILTER TESTS:

--MODERATE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
2	2	45	0.1000E+00	0.2000E+00	0.45241871E+01	0.45241634E+01	0.2366E-04	0.5230E-05	0.1000E-03
2	2	39	0.1000E+00	0.1000E+01	0.41042499E+00	0.41064876E+00	-0.2238E-03	0.5452E-03	0.1000E-03
2	2	37	0.1000E+00	0.4000E+01	0.21241771E-16	-0.23456432E-04	0.2346E-04	0.1104E+13	0.1000E-03
2	2	66	0.1000E+01	0.2000E+00	0.49502492E+00	0.49502430E+00	0.6184E-06	0.1249E-05	0.1000E-03
2	2	43	0.1000E+01	0.1000E+01	0.38940039E+00	0.38940220E+00	-0.1814E-05	0.4659E-05	0.1000E-03
2	2	38	0.1000E+01	0.4000E+01	0.91578196E-02	0.91939307E-02	-0.3611E-04	0.3943E-02	0.1000E-03
2	3	62	0.1000E+00	0.2000E+00	0.49004989E+01	0.49005751E+01	-0.7617E-04	0.1554E-04	0.1000E-03
2	3	59	0.1000E+00	0.1000E+01	0.90247240E+00	0.90254273E+00	-0.7033E-04	0.7793E-04	0.1000E-03
2	3	56	0.1000E+00	0.4000E+01	0.15956649E+00	0.15946533E+00	0.1012E-03	0.6340E-03	0.1000E-03
2	3	54	0.1000E+01	0.2000E+00	0.40489027E+01	0.40491113E+01	-0.2086E-03	0.5152E-04	0.1000E-03
2	3	54	0.1000E+01	0.1000E+01	0.22389075E+00	0.22289707E+00	0.9937E-03	0.4438E-02	0.1000E-03
2	3	50	0.1000E+01	0.4000E+01	-0.99287451E-01	-0.99289170E-01	0.1719E-05	0.1732E-04	0.1000E-03
2	4	55	0.1000E+00	0.2000E+00	0.40280331E+01	0.40280157E+01	0.1788E-04	0.4439E-05	0.1000E-03
2	4	47	0.1000E+00	0.1000E+01	0.24204740E+01	0.24204629E+01	0.1112E-04	0.4593E-05	0.1000E-03
2	4	41	0.1000E+00	0.4000E+01	0.10626239E+01	0.10626192E+01	0.4739E-05	0.4459E-05	0.1000E-03
2	4	44	0.1000E+01	0.2000E+00	0.17331428E+01	0.17331362E+01	0.6586E-05	0.3800E-05	0.1000E-03
2	4	37	0.1000E+01	0.1000E+01	0.28670621E+00	0.28668803E+00	0.1818E-04	0.6342E-04	0.1000E-03
2	4	32	0.1000E+01	0.4000E+01	-0.36178848E-01	-0.36165248E-01	-0.1360E-04	0.3759E-03	0.1000E-03
2	5	63	0.1000E+00	0.2000E+00	0.49009933E+01	0.49005916E+01	0.4018E-03	0.8198E-04	0.1000E-03
2	5	59	0.1000E+00	0.1000E+01	0.90481741E+00	0.90475772E+00	0.7970E-04	0.8808E-04	0.1000E-03
2	5	55	0.1000E+00	0.4000E+01	0.16758001E+00	0.16755579E+00	0.2422E-04	0.1445E-03	0.1000E-03
2	5	57	0.1000E+01	0.2000E+00	0.40936537E+01	0.40932181E+01	0.4377E-03	0.1069E-03	0.1000E-03
2	5	53	0.1000E+01	0.1000E+01	0.36787944E+00	0.36779106E+00	0.8838E-04	0.2402E-03	0.1000E-03
2	5	50	0.1000E+01	0.4000E+01	0.45789098E-02	0.45607461E-02	0.1816E-04	0.3967E-02	0.1000E-03
3	1	69	0.1000E+00	0.2000E+00	0.20000000E+01	0.19927443E+01	0.2557E-03	0.1279E-03	0.1000E-03
3	1	63	0.1000E+00	0.1000E+01	0.99009900E-01	0.98740095E-01	0.2698E-03	0.2725E-02	0.1000E-03
3	1	65	0.1000E+00	0.4000E+01	0.62460962E-02	0.61180166E-02	0.1281E-03	0.2051E-01	0.1000E-03
3	1	85	0.1000E+01	0.2000E+00	0.96153846E+00	0.96153253E+00	0.5923E-05	0.6160E-05	0.1000E-03
3	1	73	0.1000E+01	0.1000E+01	0.50000000E+00	0.49996341E+00	0.3659E-04	0.7317E-04	0.1000E-03
3	1	66	0.1000E+01	0.4000E+01	0.58823530E-01	0.58798929E-01	0.2460E-04	0.4182E-03	0.1000E-03
3	2	71	0.1000E+00	0.2000E+00	0.32602466E+01	0.32594248E+01	0.8218E-03	0.2521E-03	0.1000E-03
3	2	60	0.1000E+00	0.1000E+01	0.12307667E+09	-0.33243167E-03	0.3324E-03	0.2701E+07	0.1000E-03
3	2	62	0.1000E+00	0.4000E+01	0.00000000E+00	-0.12467560E-03	0.1247E-03	0.0000E+00	0.1000E-03
3	2	116	0.1000E+01	0.2000E+00	0.87740982E+00	0.87740730E+00	0.1423E-05	0.1622E-05	0.1000E-03
3	2	72	0.1000E+01	0.1000E+01	0.69019422E+00	0.69018453E+00	0.9686E-05	0.1403E-04	0.1000E-03
3	2	65	0.1000E+01	0.4000E+01	0.16231813E-01	0.16274730E-01	-0.4292E-04	0.2644E-02	0.1000E-03



FILTER TESTS:

--MODERATE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
3	3	75	0.1000E+00	0.2000E+00	0.52498371E-01	0.50164499E-01	0.2334E-02	0.3734E-01	0.1000E-03
3	3	70	0.1000E+00	0.1000E+01	0.24999974E-02	0.19681623E-02	0.5318E-03	0.2127E+00	0.1000E-03
3	3	75	0.1000E+00	0.4000E+01	0.15625000E-03	0.81810369E-05	0.1481E-03	0.9476E+00	0.1000E-03
3	3	81	0.1000E+01	0.2000E+00	0.47449231E+01	0.47157100E+01	0.2921E-01	0.6157E-02	0.1000E-03
3	3	81	0.1000E+01	0.1000E+01	0.24740396E+00	0.24656339E+00	0.8406E-03	0.3398E-02	0.1000E-03
3	3	69	0.1000E+01	0.4000E+01	0.15614829E-01	0.15613548E-01	0.1282E-05	0.8208E-04	0.1000E-03
3	4	105	0.1000E+00	0.2000E+00	0.15396825E+02	0.15396825E+02	0.7355E-04	0.4777E-05	0.1000E-03
3	4	89	0.1000E+00	0.1000E+01	0.14213153E+02	0.14213038E+02	0.1153E-03	0.8110E-05	0.1000E-03
3	4	77	0.1000E+00	0.4000E+01	0.10529363E+02	0.10529074E+02	0.2885E-03	0.2740E-04	0.1000E-03
3	4	83	0.1000E+01	0.2000E+00	0.12860592E+01	0.12860433E+01	0.1596E-04	0.1241E-04	0.1000E-03
3	4	75	0.1000E+01	0.1000E+01	0.57786367E+00	0.57782742E+00	0.3625E-04	0.6274E-04	0.1000E-03
3	4	68	0.1000E+01	0.4000E+01	0.28770133E-01	0.28741126E-01	0.2901E-04	0.1008E-02	0.1000E-03
3	5	72	0.1000E+00	0.2000E+00	0.23500182E+00	0.23198409E+00	0.1773E-04	0.7544E-04	0.1000E-03
3	5	63	0.1000E+00	0.1000E+01	0.14635181E-01	0.14612547E-01	0.2263E-04	0.1547E-02	0.1000E-03
3	5	65	0.1000E+00	0.4000E+01	0.93604082E-03	0.92614403E-03	0.9897E-05	0.1057E-01	0.1000E-03
3	5	91	0.1000E+01	0.2000E+00	0.67851199E+00	0.67850966E+00	0.2332E-05	0.3437E-05	0.1000E-03
3	5	75	0.1000E+01	0.1000E+01	0.45814536E+00	0.45812513E+00	0.1992E-04	0.4349E-04	0.1000E-03
3	5	67	0.1000E+01	0.4000E+01	0.81259472E-01	0.81239938E-01	0.1954E-04	0.2404E-03	0.1000E-03
4	1	43	0.1000E+00	0.2000E+00	0.40000001E+01	0.39998593E+01	0.1408E-03	0.3519E-04	0.1000E-03
4	1	42	0.1000E+00	0.1000E+01	0.99009901E+00	0.98998680E+00	0.1122E-03	0.1133E-03	0.1000E-03
4	1	43	0.1000E+00	0.4000E+01	0.24984385E+00	0.24978788E+00	0.5617E-04	0.2248E-03	0.1000E-03
4	1	54	0.1000E+01	0.2000E+00	0.19230769E+00	0.19229963E+00	0.8058E-05	0.4190E-04	0.1000E-03
4	1	45	0.1000E+01	0.1000E+01	0.50000000E+00	0.49998964E+00	0.1036E-04	0.2072E-04	0.1000E-03
4	1	41	0.1000E+01	0.4000E+01	0.23529412E+00	0.23528237E+00	0.1175E-04	0.4994E-04	0.1000E-03
4	2	34	0.1000E+00	0.2000E+00	0.32602467E+02	0.32598331E+02	0.4136E-02	0.1268E-03	0.1000E-03
4	2	29	0.1000E+00	0.1000E+01	0.61539334E-08	-0.47976662E-03	0.4798E-03	0.7796E+05	0.1000E-03
4	2	33	0.1000E+00	0.4000E+01	0.00000000E+00	-0.91403347E-04	0.9140E-04	0.0000E+00	0.1000E-03
4	2	77	0.1000E+01	0.2000E+00	0.87740881E+01	0.87735921E+01	0.4960E-05	0.5653E-04	0.1000E-03
4	2	36	0.1000E+01	0.1000E+01	0.34509711E+00	0.34512091E+00	-0.2380E-04	0.6898E-04	0.1000E-03
4	2	31	0.1000E+01	0.4000E+01	0.32463625E-01	0.32615788E-01	-0.1522E-03	0.4687E-02	0.1000E-03
4	3	61	0.1000E+00	0.2000E+00	0.15692261E-01	0.15692609E-01	-0.3485E-06	0.2221E-04	0.1000E-03
4	3	57	0.1000E+00	0.1000E+01	0.15629554E-01	0.15629367E-01	0.1870E-06	0.1196E-04	0.1000E-03
4	3	56	0.1000E+00	0.4000E+01	0.15395891E-01	0.15395833E-01	0.5832E-07	0.3788E-05	0.1000E-03
4	3	48	0.1000E+01	0.2000E+00	0.14188663E+01	0.14188398E+01	0.2646E-04	0.1865E-04	0.1000E-03
4	3	47	0.1000E+01	0.1000E+01	0.90591722E+00	0.90595682E+00	-0.1396E-03	0.1541E-03	0.1000E-03
4	3	33	0.1000E+01	0.4000E+01	-0.51870313E-01	-0.52478133E-01	0.6078E-03	0.1172E-01	0.1000E-03

# FILTER TESTS:

--MODERATE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
4	4	53	0.1000E+00	0.2000E+00	0.15396925E+01	0.15396696E+01	0.2287E-04	0.1486E-04	0.1000E-03
4	4	48	0.1000E+00	0.1000E+01	0.14213153E+01	0.14213026E+01	0.1267E-04	0.8911E-05	0.1000E-03
4	4	43	0.1000E+00	0.4000E+01	0.10529363E+01	0.10529249E+01	0.1135E-04	0.1078E-04	0.1000E-03
4	4	46	0.1000E+01	0.2000E+00	0.12860592E+01	0.12860532E+01	0.5335E-05	0.4148E-05	0.1000E-03
4	4	40	0.1000E+01	0.1000E+01	0.57786367E+00	0.57784101E+00	0.2266E-04	0.3921E-04	0.1000E-03
4	4	36	0.1000E+01	0.4000E+01	0.28770139E-01	0.28764924E-01	0.5214E-05	0.1812E-03	0.1000E-03
4	5	44	0.1000E+00	0.2000E+00	0.32175055E+00	0.32173968E+00	0.1086E-04	0.3376E-04	0.1000E-03
4	5	42	0.1000E+00	0.1000E+01	0.97726907E-01	0.97719884E-01	0.7022E-05	0.7186E-04	0.1000E-03
4	5	43	0.1000E+00	0.4000E+01	0.24963601E-01	0.24958918E-01	0.4683E-05	0.1876E-03	0.1000E-03
4	5	58	0.1000E+01	0.2000E+00	0.97726907E-01	0.97722064E-01	0.4843E-05	0.4956E-04	0.1000E-03
4	5	46	0.1000E+01	0.1000E+01	0.32175055E+00	0.32175108E+00	-0.5327E-06	0.1656E-05	0.1000E-03
4	5	42	0.1000E+01	0.4000E+01	0.21866895E+00	0.21866028E+00	0.8669E-05	0.3964E-04	0.1000E-03

# FILTER TESTS:

--SMALL AND LARGE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
1	1	90	0.6000E-01	0.1000E-03	0.46296103E+00	0.46296097E+00	0.5960E-07	0.1287E-06	0.1000E-03
1	1	99	0.6000E-01	0.1000E-02	0.46277013E+01	0.46277012E+01	0.1788E-06	0.3864E-07	0.1000E-05
1	1	56	0.6000E-01	0.1000E-01	0.44432158E+02	0.44432158E+02	=0.4768E-06	0.1073E-07	0.1000E-05
1	1	56	0.6000E-01	0.1000E+02	0.99994601E-02	0.99993678E-02	0.9232E-07	0.9232E-05	0.1000E-05
1	1	68	0.6000E-01	0.1000E+03	0.99999946E-04	0.99999028E-04	0.9177E-09	0.9177E-05	0.1000E-05
1	1	80	0.6000E-01	0.1000E+04	0.10000000E-05	0.99999063E-06	0.9365E-11	0.9365E-05	0.1000E-05
1	2	72	0.6000E-01	0.1000E-03	0.69444444E-02	0.69088186E-02	0.3563E-04	0.5130E-02	0.1000E-05
1	2	83	0.6000E-01	0.1000E-02	0.69444154E-01	0.69416584E-01	0.2757E-04	0.3970E-03	0.1000E-05
1	2	95	0.6000E-01	0.1000E-01	0.69415516E+00	0.69422265E+00	=0.6749E-04	0.9723E-04	0.1000E-05
1	2	37	0.6000E-01	0.1000E+02	0.00000000E+00	=0.35976074E-05	0.3588E-05	0.0000E+00	0.1000E-05
1	2	49	0.6000E-01	0.1000E+03	0.00000000E+00	=0.10092124E-06	0.1009E-06	0.0000E+00	0.1000E-05
1	2	61	0.6000E-01	0.1000E+04	0.00000000E+00	0.15157228E-08	=0.1546E-08	0.0000E+00	0.1000E-05
1	3	132	0.6000E-01	0.1000E-03	0.30025840E-01	0.30004624E-01	0.2122E-04	0.7066E-03	0.1000E-05
1	3	124	0.6000E-01	0.1000E-02	0.30003488E-01	0.29998075E-01	0.5413E-05	0.1804E-03	0.1000E-05
1	3	112	0.6000E-01	0.1000E-01	0.29994547E-01	0.29992835E-01	0.1713E-05	0.5710E-04	0.1000E-05
1	3	81	0.6000E-01	0.1000E+02	0.24432444E-01	0.24420041E-01	0.2403E-05	0.9835E-04	0.1000E-05
1	3	70	0.6000E-01	0.1000E+03	0.81646506E-03	0.79878296E-03	0.1768E-04	0.2166E-01	0.1000E-05
1	3	77	0.6000E-01	0.1000E+04	0.12958937E-03	0.20875710E-03	=0.7917E-04	0.6109E+00	0.1000E-05
1	4	90	0.6000E-01	0.1000E-03	0.13892209E-01	0.13885417E-01	0.6792E-05	0.4889E-03	0.1000E-05
1	4	99	0.6000E-01	0.1000E-02	0.13885643E+00	0.13885759E+00	=0.3161E-05	0.2276E-04	0.1000E-05
1	4	99	0.6000E-01	0.1000E-01	0.13606078E+01	0.13606075E+01	0.2086E-06	0.1533E-06	0.1000E-05
1	4	78	0.6000E-01	0.1000E+02	0.99400011E-01	0.99409989E-01	0.2142E-07	0.2155E-06	0.1000E-05
1	4	89	0.6000E-01	0.1000E+03	0.99940000E-02	0.99939980E-02	0.1979E-08	0.1980E-06	0.1000E-05
1	4	99	0.6000E-01	0.1000E+04	0.99994001E-03	0.99993982E-03	0.1692E-09	0.1892E-06	0.1000E-05
1	5	169	0.6000E-01	0.1000E-03	=0.34484648E-03	=0.34247055E-03	=0.2376E-05	0.6890E-02	0.1000E-05
1	5	97	0.6000E-01	0.1000E-02	=0.27971723E-02	=0.23093421E-02	0.1217E-04	0.5298E-02	0.1000E-05
1	5	86	0.6000E-01	0.1000E-01	=0.11458798E-01	=0.11446143E-01	=0.1266E-04	0.1104E-02	0.1000E-05
1	5	69	0.6000E-01	0.1000E+02	=0.90000000E-04	=0.89994577E-04	=0.5422E-08	0.6025E-04	0.1000E-05
1	5	66	0.6000E-01	0.1000E+03	=0.90000000E-05	=0.89967643E-05	=0.1032E-06	0.1147E-01	0.1000E-05
1	5	67	0.6000E-01	0.1000E+04	=0.90000000E-06	=0.35992661E-06	=0.5401E-06	0.6001E+00	0.1000E-05
2	1	141	0.6000E-01	0.1000E-03	0.16666643E+02	0.16667156E+02	=0.5131E-03	0.3078E-04	0.1000E-05
2	1	147	0.6000E-01	0.1000E-02	0.16664352E+02	0.16664339E+02	0.1335E-04	0.8012E-06	0.1000E-05
2	1	107	0.6000E-01	0.1000E-01	0.16439899E+02	0.16439890E+02	0.8583E-05	0.5221E-06	0.1000E-05
2	1	100	0.6000E-01	0.1000E+02	0.99998200E-01	0.99998040E-01	0.1602E-06	0.1602E-05	0.1000E-05
2	1	104	0.6000E-01	0.1000E+03	0.99999981E-02	0.99999813E-02	0.1688E-07	0.1688E-05	0.1000E-05
2	1	106	0.6000E-01	0.1000E+04	0.10000000E-02	0.99999826E-03	0.1746E-08	0.1746E-05	0.1000E-05

FILTER TESTS:

--SMALL AND LARGE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
2	2	123	0.6000E-01	0.1000E-03	0.83333329E+01	0.83332961E+01	0.3684E-04	0.4420E-05	0.1000E-05
2	2	135	0.6000E-01	0.1000E-02	0.83332986E+01	0.83332628E+01	0.3576E-04	0.4292E-05	0.1000E-05
2	2	146	0.6000E-01	0.1000E-01	0.83298618E+01	0.83298272E+01	0.3457E-04	0.4150E-05	0.1000E-05
2	2	53	0.6000E-01	0.1000E+02	0.00000000E+00	-0.87690539E-07	0.8769E-07	0.0000E+00	0.1000E-05
2	2	64	0.6000E-01	0.1000E+03	0.00000000E+00	-0.69847859E-09	0.6985E-09	0.0000E+00	0.1000E-05
2	2	74	0.6000E-01	0.1000E+04	0.00000000E+00	-0.68299573E-11	0.6830E-11	0.0000E+00	0.1000E-05
2	3	104	0.6000E-01	0.1000E-03	0.99999401E+04	0.99999401E+04	0.0000E+00	0.0000E+00	0.1000E-05
2	3	99	0.6000E-01	0.1000E-02	0.99993999E+03	0.99993958E+03	0.4196E-03	0.4196E-06	0.1000E-05
2	3	93	0.6000E-01	0.1000E-01	0.99980008E+02	0.99970007E+02	0.4768E-05	0.4771E-07	0.1000E-05
2	3	80	0.6000E-01	0.1000E+02	0.48421967E-01	0.48381934E-01	0.4003E-04	0.8268E-03	0.1000E-05
2	3	76	0.6000E-01	0.1000E+03	-0.21005938E-02	-0.21312196E-02	0.3063E-04	0.1458E-01	0.1000E-05
2	3	81	0.6000E-01	0.1000E+04	-0.10787793E-03	-0.13489555E-03	0.2702E-04	0.2504E+00	0.1000E-05
2	4	109	0.6000E-01	0.1000E-03	0.12139683E+02	0.12139647E+02	0.3612E-04	0.2975E-05	0.1000E-05
2	4	97	0.6000E-01	0.1000E-02	0.98370975E+01	0.98370711E+01	0.2646E-04	0.2690E-05	0.1000E-05
2	4	85	0.6000E-01	0.1000E-01	0.75345125E+01	0.75344868E+01	0.1568E-04	0.2081E-05	0.1000E-05
2	4	52	0.6000E-01	0.1000E+02	0.69312070E+00	0.69311239E+00	0.2310E-05	0.3332E-05	0.1000E-05
2	4	44	0.6000E-01	0.1000E+03	-0.65303751E-03	-0.58497731E-03	0.3194E-04	0.4891E-01	0.1000E-05
2	4	50	0.6000E-01	0.1000E+04	0.23959041E-19	-0.14292987E-08	0.1429E-08	0.5966E+11	0.1000E-05
2	5	104	0.6000E-01	0.1000E-03	0.99999399E+04	0.99999401E+04	0.9888E-02	0.9888E-06	0.1000E-05
2	5	99	0.6000E-01	0.1000E-02	0.99993999E+03	0.99993917E+03	0.8240E-03	0.8240E-06	0.1000E-05
2	5	93	0.6000E-01	0.1000E-01	0.99940018E+02	0.99939934E+02	0.8392E-04	0.8397E-06	0.1000E-05
2	5	76	0.6000E-01	0.1000E+02	0.54881163E-01	0.54880907E-01	0.2561E-06	0.4667E-05	0.1000E-05
2	5	71	0.6000E-01	0.1000E+03	0.24787522E-04	0.24751129E-04	0.3639E-07	0.1468E-02	0.1000E-05
2	5	72	0.6000E-01	0.1000E+04	0.87565108E-29	-0.94142595E-10	0.9414E-10	0.1075E+20	0.1000E-05
3	1	165	0.6000E-01	0.1000E-03	0.16666620E+02	0.16666548E+02	0.7224E-04	0.4334E-05	0.1000E-05
3	1	171	0.6000E-01	0.1000E-02	0.16662038E+02	0.16661968E+02	0.7057E-04	0.4235E-05	0.1000E-05
3	1	111	0.6000E-01	0.1000E-01	0.16216216E+02	0.16216264E+02	-0.4745E-04	0.2926E-05	0.1000E-05
3	1	97	0.6000E-01	0.1000E+02	0.59997840E-03	0.59948624E-03	0.4922E-06	0.8203E-03	0.1000E-05
3	1	105	0.6000E-01	0.1000E+03	0.59999979E-05	0.59371834E-05	0.6281E-07	0.1047E-01	0.1000E-05
3	1	113	0.6000E-01	0.1000E+04	0.60000000E-07	0.53624399E-07	0.6376E-08	0.1063E+00	0.1000E-05
3	2	154	0.6000E-01	0.1000E-03	0.14770439E+02	0.14770373E+02	0.6545E-04	0.4431E-05	0.1000E-05
3	2	166	0.6000E-01	0.1000E-02	0.14769423E+02	0.14769357E+02	0.6568E-04	0.4447E-05	0.1000E-05
3	2	160	0.6000E-01	0.1000E-01	0.14668231E+02	0.14668203E+02	0.2849E-04	0.1942E-05	0.1000E-05
3	2	92	0.6000E-01	0.1000E+02	0.00000000E+00	-0.60348027E-06	0.6035E-06	0.0000E+00	0.1000E-05
3	2	102	0.6000E-01	0.1000E+03	0.00000000E+00	-0.60924473E-07	0.6092E-07	0.0000E+00	0.1000E-05
3	2	111	0.6000E-01	0.1000E+04	0.00000000E+00	-0.64220242E-08	0.6422E-08	0.0000E+00	0.1000E-05



# FILTER TESTS:

--SMALL AND LARGE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
3	3	122	0.6000E-01	0.1000E-03	0.41211856E+04	0.18869898E+04	0.2234E+04	0.5421E+00	0.1000E-05
3	3	111	0.6000E-01	0.1000E-02	0.78332690E+03	0.78320348E+03	0.1234E+00	0.1576E-03	0.1000E-05
3	3	115	0.6000E-01	0.1000E-01	0.89878548E+01	0.90028914E+01	-0.1504E-01	0.1673E-02	0.1000E-05
3	3	109	0.6000E-01	0.1000E-02	0.90000000E-05	0.84589171E-05	0.5411E-06	0.6012E-01	0.1000E-05
3	3	116	0.6000E-01	0.1000E-03	0.89999999E-07	0.34126007E-07	0.5557E-07	0.6175E+00	0.1000E-05
3	3	119	0.6000E-01	0.1000E-04	0.90000000E-09	-0.56650029E-08	0.6565E-08	0.7294E+01	0.1000E-05
3	4	178	0.6000E-01	0.1000E-03	0.26179781E+02	0.26179666E+02	0.1154E-03	0.4408E-05	0.1000E-05
3	4	178	0.6000E-01	0.1000E-02	0.26178368E+02	0.26178254E+02	0.1142E-03	0.4362E-05	0.1000E-05
3	4	167	0.6000E-01	0.1000E-01	0.26164235E+02	0.26164123E+02	0.1123E-03	0.4292E-05	0.1000E-05
3	4	105	0.6000E-01	0.1000E-02	0.14367855E+02	0.14367918E+02	-0.6354E-04	0.4422E-05	0.1000E-05
3	4	95	0.6000E-01	0.1000E-03	0.64893580E-01	0.64898063E-01	-0.4482E-05	0.6907E-04	0.1000E-05
3	4	97	0.6000E-01	0.1000E-04	0.22924472E-24	-0.17707694E-05	0.1771E-05	0.7724E+19	0.1000E-05
3	5	24	0.6000E-01	0.1000E-03	0.69314612E+00	0.00000000E+00	0.6931E+00	0.1000E+01	0.1000E-05
3	5	140	0.6000E-01	0.1000E-02	0.69304302E+00	0.69304010E+00	0.2921E-05	0.4214E-05	0.1000E-05
3	5	121	0.6000E-01	0.1000E-01	0.68290791E+00	0.68290789E+00	0.2235E-07	0.3273E-07	0.1000E-05
3	5	96	0.6000E-01	0.1000E-02	0.53999724E-04	0.54005471E-04	-0.5747E-08	0.1064E-03	0.1000E-05
3	5	107	0.6000E-01	0.1000E-03	0.55129425E-06	0.59000857E-06	-0.3871E-07	0.7022E-01	0.1000E-05
3	5	103	0.6000E-01	0.1000E-04	0.15493047E-07	0.39944348E-07	-0.2445E-07	0.1578E+01	0.1000E-05
4	1	201	0.6000E-01	0.1000E-03	0.277777701E+01	0.27764766E+01	0.1293E-04	0.4657E-03	0.1000E-05
4	1	209	0.6000E-01	0.1000E-02	0.27770064E+00	0.27768146E+00	0.1918E-04	0.6906E-04	0.1000E-05
4	1	98	0.6000E-01	0.1000E-01	0.27027027E+01	0.27025859E+01	0.1168E-03	0.4323E-04	0.1000E-05
4	1	63	0.6000E-01	0.1000E-02	0.99996400E-01	0.99996226E-01	0.1742E-06	0.1742E-05	0.1000E-05
4	1	67	0.6000E-01	0.1000E-03	0.99999965E-02	0.99999630E-02	0.3353E-07	0.3353E-05	0.1000E-05
4	1	68	0.6000E-01	0.1000E-04	0.10000000E-02	0.99999744E-03	0.2561E-08	0.2561E-05	0.1000E-05
4	2	190	0.6000E-01	0.1000E-03	0.20514498E+00	0.20513150E+00	0.1348E-04	0.6570E-04	0.1000E-05
4	2	202	0.6000E-01	0.1000E-02	0.20513088E+01	0.20511751E+01	0.1337E-03	0.6516E-04	0.1000E-05
4	2	130	0.6000E-01	0.1000E-01	0.20372544E+02	0.20371338E+02	0.1205E-02	0.5916E-04	0.1000E-05
4	2	49	0.6000E-01	0.1000E-02	0.00000000E+00	-0.15159947E-06	0.1516E-06	0.0000E+00	0.1000E-05
4	2	59	0.6000E-01	0.1000E-03	0.00000000E+00	-0.15342038E-08	0.1534E-08	0.0000E+00	0.1000E-05
4	2	68	0.6000E-01	0.1000E-04	0.00000000E+00	-0.17274365E-10	0.1727E-10	0.0000E+00	0.1000E-05
4	3	199	0.6000E-01	0.1000E-03	0.56548658E-02	0.56476873E-02	0.7179E-05	0.1269E-02	0.1000E-05
4	3	185	0.6000E-01	0.1000E-02	0.56548556E-02	0.56507584E-02	0.4100E-05	0.7251E-03	0.1000E-05
4	3	148	0.6000E-01	0.1000E-01	0.56547650E-02	0.56546729E-02	0.9208E-07	0.1628E-04	0.1000E-05
4	3	70	0.6000E-01	0.1000E-02	0.55536882E-02	0.55536082E-02	0.8004E-07	0.1441E-04	0.1000E-05
4	3	68	0.6000E-01	0.1000E-03	0.46962638E-02	0.46968285E-02	-0.5647E-06	0.1202E-03	0.1000E-05
4	3	60	0.6000E-01	0.1000E-04	0.44587228E-04	0.38695227E-04	0.5892E-05	0.1321E+00	0.1000E-05



FILTER TESTS:

--SMALL AND LARGE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
4	4	120	0.6000E-01	0.1000E-03	0.15707869E+01	0.15707801E+01	0.6795E-05	0.4326E-05	0.1000E-05
4	4	102	0.6000E-01	0.1000E-02	0.15707021E+01	0.15706954E+01	0.6661E-05	0.4241E-05	0.1000E-05
4	4	84	0.6000E-01	0.1000E-01	0.15698541E+01	0.15698475E+01	0.6646E-05	0.4233E-05	0.1000E-05
4	4	56	0.6000E-01	0.1000E+02	0.86207129E+00	0.86207535E+00	-0.4061E-05	0.4710E-05	0.1000E-05
4	4	50	0.6000E-01	0.1000E+03	0.38936148E-02	0.38941748E-02	-0.5599E-06	0.1438E-03	0.1000E-05
4	4	53	0.6000E-01	0.1000E+04	0.13754695E-25	-0.31456986E-08	0.3146E-08	0.2287E+18	0.1000E-05
4	5	20	0.6000E-01	0.1000E-03	0.83333198E-03	0.00000000E+00	0.8333E-03	0.1000E+01	0.1000E-05
4	5	138	0.6000E-01	0.1000E-02	0.83319832E-02	0.83314434E-02	0.5398E-06	0.6479E-04	0.1000E-05
4	5	127	0.6000E-01	0.1000E-01	0.82007444E-01	0.82003179E-01	0.4265E-05	0.5201E-04	0.1000E-05
4	5	62	0.6000E-01	0.1000E+02	0.59994960E-02	0.59994866E-02	0.7334E-08	0.1272E-05	0.1000E-05
4	5	56	0.6000E-01	0.1000E+03	0.59999948E-03	0.60000441E-03	-0.4926E-08	0.8210E-05	0.1000E-05
4	5	59	0.6000E-01	0.1000E+04	0.59999999E-04	0.60005852E-04	-0.6853E-08	0.1142E-03	0.1000E-05