Improved digital filters for evaluating

Fourier and Hankel transform integrals

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Abstract

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Introduction

Digital filtering techniques to evaluate oscillatory-type convolution integrals have become a widespread method reported in the geophysical literature. For example, Koefoed et al (1972) developed Hankel transform filters for functions encountered in electromagnetic depth soundings. Anderson (1973) extended this method to Fourier cosine transforms. Das et al (1974a, 1974b) proposed several specific filters used in transformations of resistivity sounding curves. Anderson (1974) used a standard set of Hankel and Fourier transform filters to evaluate a wide class of integral transforms required in computing electromagnetic field components about a finite electric wire source.

In all these papers, the main advantage cited for using digital filters over direct numerical integration is the increased speed of the calculation. An order of magnitude or more in speed improvement is not uncommon, but this is usually achieved with a reduction in accuracy.

This report briefly reviews the filter design process for convolution integrals. A new method is presented to improve the designed filter coefficients so that increased accuracy is possible at very little sacrifice in computational speed. A variable cutoff convolution method is described which utilizes the nature of the input kernel function and filter response to approximate the integral given a truncation tolerance. An algorithm is described for evaluating Fourier (cosine, sine) and Hankel (J_0,J_1) transform integrals for well-behaved kernels.

Additionally, a special lagged-convolution algorithm is developed to significantly decrease the overall computational time over variable convolution. This method is especially advantageous when computing a set of like-integrals where the transform parameter is equivalent to an incremented shift in the sampled filter.

Sixteen similar, but self-contained, Fortran IV computer subprograms utilizing the new filtering algorithms are listed in appendix 1. The subprogram naming convention used is given in tables 2 and 3. Each of these routines contains comments to explicitly define the calling parameters. With minor changes, the Fortran IV subprograms should be acceptable to most current-day digital computers. Some examples, using the improved filters, are tabulated in appendix 2.

Finally, the new filtering methods are discussed in light of other recently published techniques.

Filter design

The method of designing digital filters for Hankel transform integrals presented by Koefoed et al (1972) may be generalized to other type integrals expressed as a linear system convolution in the form

$$K(x) = \int_{-\infty}^{\infty} k(y)s(x-y)\,dy, \tag{1}$$

where $k(y)$ = input function (also called kernel function below)

$s(y)$ = system transfer (or filter) function

$K(x)$ = convolved output function.

We may consider (1) as either time-domain or frequency-domain convolution. By Fourier transform theory (e.g., see Papoulis, 1962) linear convolution is equivalent to multiplication in the transformed-domain; that is,

$$\hat{K}(\hat{x}) = \hat{k}(\hat{x})\hat{s}(\hat{x}), \tag{2}$$

where $\hat{K}(\hat{x}) \leftrightarrow K(x)$, $\hat{k}(\hat{x}) \leftrightarrow k(x)$, and $\hat{s}(\hat{x}) \leftrightarrow s(x)$ are Fourier transform pairs. For a known system input-output relation, the transformed system (filter) response is

$$\hat{s}(\hat{x}) = \hat{K}(\hat{x})/\hat{k}(\hat{x}), \tag{3}$$

provided $\hat{k}(\hat{x})$ is not identically zero for some $\hat{x}$, and both system input and output functions have band-limited Fourier transforms. The latter is required so that $\hat{s}(\hat{x}) \to 0$ for $\hat{x} \to \pm\infty$.

The filter impulse response is the inverse Fourier transform of $\hat{s}(\hat{x})$, but would not be useful for convolution unless truncated to a reasonable finite length. To this end, we first choose a discrete sampling of N points spaced Δx apart and obtain $\hat{s}(\hat{x}_j), j = 1,2,\ldots,N>>0$. The sampling results in a Nyquist frequency of $1/(2\Delta x)$. A suitable truncation may be obtained by cutting-off the transformed response at the Nyquist frequency. Note this is equivalent to multiplying $\hat{s}(\hat{x})$ by the Fourier transform of the function $\text{sinc}(x_j) = \sin(\pi x_j/\Delta x)/(\pi x_j/\Delta x)$.

The truncated response is inverse Fourier transformed to obtain the sampled filter impulse response (hereafter simply called filter weights or filter response). In some cases, it is advantageous to shift the resulting filter response to further minimize the magnitude of the filter tails. For example, see Koefoed (1972) for the reason of the oscillating behavior of the filter response and the suggested shift at zero crossings. If proper input-output functionals are used, then the final filter response should asymptotically approach zero in both abscissa directions.

Application of the filter weights to specific input functions in (1) is given by the non-circular convolution sum

$$K(x) = \sum_{i=1}^{NW} W_i k(x-a_i), \tag{4}$$

where W_i = filter weights; $i = 1,2,\ldots,NW$=number of weights,

x = transform parameter

a_i = filter abscissa corresponding to W_i .

Thus, the convolution integral (1) has been reduced to a straightforward summation given in (4), and the predetermined filter weights remain constant for all classes of convolution integrals having the same system transfer function.

In practice, many transform integrals encountered range over $(0,\infty)$ and therefore a transformation to the range $(-\infty,\infty)$ is required before applying (4) to a given kernel function. For example, the Hankel transform of integer order $n \geq 0$ is defined as

$$H(b) = \int_0^\infty gh(g) J_n(bg)\, dg, \tag{5}$$

where $gh(g)$ = modified kernel* for Hankel transforms,

and $J_n(bg)$ = Bessel function of the first kind of order n.

Similarly, the Fourier sine or cosine integral is given by

$$F(b) = \int_0^\infty f(g) \begin{matrix}\sin\\ \cos\end{matrix}(bg)\, dg. \tag{6}$$

If we let $x = \ln(b)$, $y = \ln(1/g)$ and after multiplying by e^x, equations (5) and (6) become respectively,

$$e^x H(e^x) = \int_{-\infty}^{\infty} \{e^{-y} h(e^{-y})\}\{ e^{x-y} J_n(e^{x-y})\}\, dy \tag{7}$$

and

$$e^x F(e^x) = \int_{-\infty}^{\infty} \{f(e^{-y})\}\{ e^{x-y} \begin{matrix}\sin\\ \cos\end{matrix}(e^{x-y})\}\, dy \tag{8}$$

---

*Many texts define $gJ_n(bg)$ as the Hankel kernel; however, we will adopt the definition given in (1) and consider $J_n(bg)$ the system transfer function with $h^*(g) = gh(g)$ the system input kernel function.

Equations (7) and (8) are expressed in the convolution form (1), which is assumed when applying equation (4). Multiplying by e^x ensures that the system response approaches zero for both large and small filter abscissas.

An important consideration in designing digital filters is the choice of known input-output function pairs. In previous work by Koefoed et al (1972) and Anderson (1973), certain known Lipschitz integrals were used to develop Hankel (J_0,J_1) and Fourier cosine filters, respectively. Both papers illustrate filter responses that are characterized as oscillatory decreasing functions in both abscissa directions. The integral forms previously used for filter design are indicated by (¢) in table 1 (The sine-transform form was previously used by the author, even though the filter weights were never published).

Table 1. Some known integral transform pairs.

F = Integral FORM; T = test TYPE; (a>0,b>0 for all F,T)

F	T	Integral	
1	1	$\int_0^\infty xe^{-ax}J_1(bx)\,dx = b/(a^2+b^2)^{3/2}$	¢ *
1	2	$\int_0^\infty x^2e^{-ax^2}J_1(bx)\,dx = b\exp(-b^2/4a)/(2a)^2$	
1	3	$\int_0^\infty J_1(a/x)J_1(bx)\,dx = J_2(2\sqrt{ab})/b$	
1	4	$\int_0^\infty e^{-ax}J_1(bx)\,dx = (\sqrt{a^2+b^2}-a)/(b\sqrt{a^2+b^2})$	
1	5	$\int_0^\infty \frac{(J_0(ax)-1)J_1(bx)}{x^2} = \begin{cases} -b(1+2\ln a/b)/4, & b<a \\ -a^2/4b, & a\le b \end{cases}$	
2	1	$\int_0^\infty e^{-ax}J_0(bx)\,dx = 1/(a^2+b^2)^{1/2}$	¢
2	2	$\int_0^\infty xe^{-ax^2}J_0(bx)\,dx = \exp(-b^2/4a)/2a$	*
2	3	$\int_0^\infty J_0(a/x)J_0(bx)\,dx = J_0(2\sqrt{ab})/b$	
2	4	$\int_0^\infty x^3/(x^4+a^4)\,J_0(bx)\,dx = \ker(ab)$	
2	5	$\int_0^\infty x/\sqrt{a^2+x^2}\,J_0(bx)\,dx = e^{-ab}/b$	
3	1	$\int_0^\infty e^{-ax}\cos(bx)\,dx = a/(a^2+b^2)$	¢
3	2	$\int_0^\infty \exp(-a^2x^2)\cos(bx)\,dx = \sqrt{\pi}\exp(-b^2/4a^2)/2a$	*
3	3	$\int_0^\infty J_0(a\sqrt{x})\cos(bx)\,dx = \sin(a^2/4b)/b$	
3	4	$\int_0^\infty 1/(a^2+x^2)\cos(bx)\,dx = \pi e^{-ab}/2a$	
3	5	$\int_0^\infty e^{-ax}(1-e^{-ax})/x\,\cos(bx)\,dx = \frac{1}{2}\ln\left\{\frac{b^2+4a^2}{b^2+a^2}\right\}$	
4	1	$\int_0^\infty e^{-ax}\sin(bx)\,dx = b/(a^2+b^2)$	¢
4	2	$\int_0^\infty x\exp(-a^2x^2)\sin(bx)\,dx = \sqrt{\pi}b\exp(-b^2/4a^2)/4a^3$	*
4	3	$\int_0^\infty \sin(a^2/x)\sin(bx)\,dx = \pi aJ_1(2a\sqrt{b})/2\sqrt{b}$	
4	4	$\int_0^\infty x/(a^2+x^2)\sin(bx)\,dx = \pi e^{-ab}/2$	
4	5	$\int_0^\infty e^{-ax}(1-e^{-ax})/x\,\sin(bx)\,dx = \tan^{-1}\{ab/(b^2+2a^2)\}$	

¢ previous design form
* new design form

If the filter response could be made to decrease more rapidly in both abscissa directions, then the filter efficiency as a convolution operator would be enhanced. Systems with rapidly decaying input-output function pairs have this desired narrow-band property. A search through tables of integrals, such as Gradshteyn and Ryzhik (1965), revealed many of the integral transform pairs given in table 1. By inspection of table 1, several promising design forms are evident. After considerable experimentation, the new filter design forms were chosen as indicated by an (*) in table 1. Note, however, the J_1 - form was not changed, because the rate of decrease of both input-output functions was deemed sufficiently rapid. The remaining filters were significantly improved by using the indicated form change.

The resulting final filter responses are illustrated in figures 1-4. In these figures, we have arbitrarily reduced the abscissa range for display purposes. The full or extended length of each filter is contained in the subprograms given in appendix 1. The reason for extending the filter length will be discussed in the next section.

Another critical factor in filter design is the choice of the sampling interval Δx. Koefoed et al (1972) discussed the fundamental sampling problem applied to a certain kernel function for a homogeneous earth model. Their analysis showed a sampling $\Delta x = (\ell n\ 10)/10$ would reproduce the kernel between sample points with an absolute error approximately less then 10^{-5}. Obviously, if we decrease the sampling a smaller error is possible, but at the sacrifice of generating many more filter weights. Thus a compromise between algorithm speed and accuracy is unavoidable.

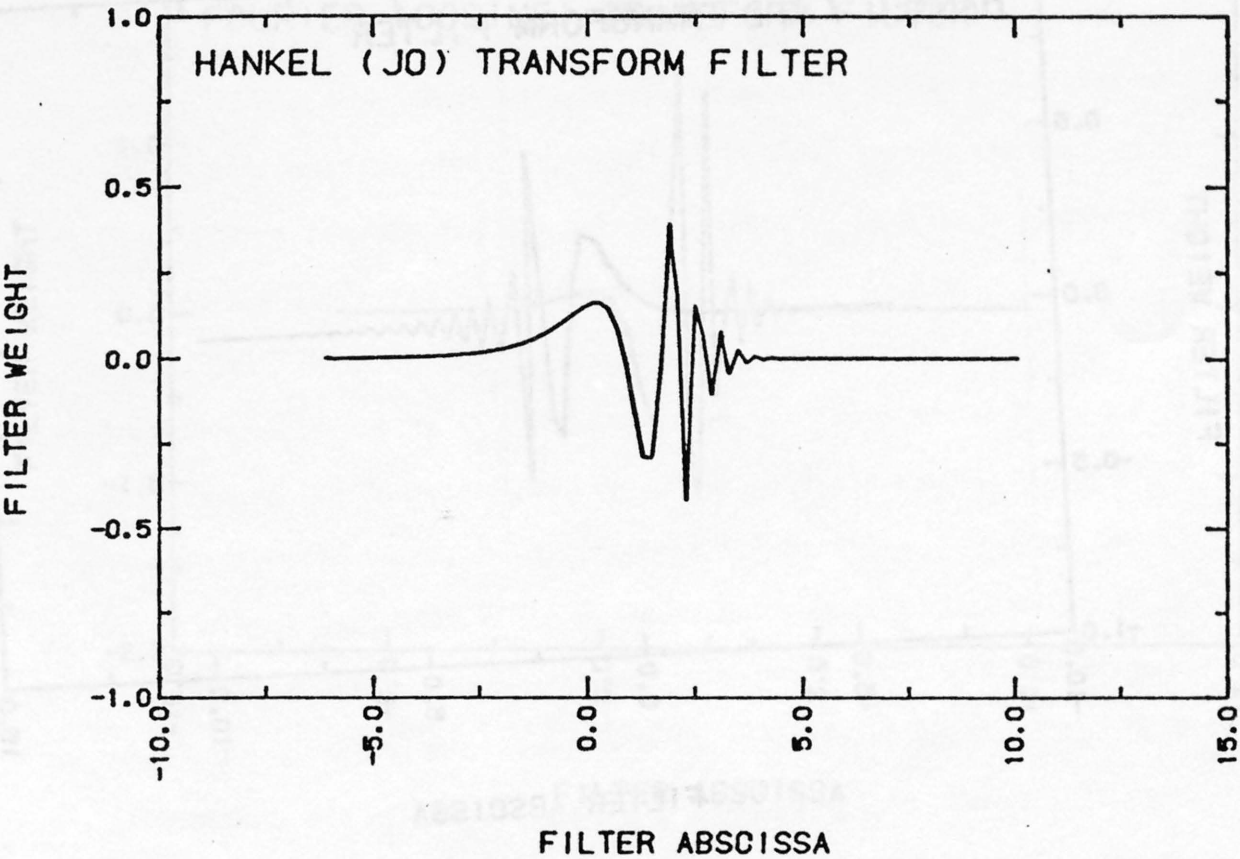

Figure 1.---Filter response for Hankel J_0-transform.

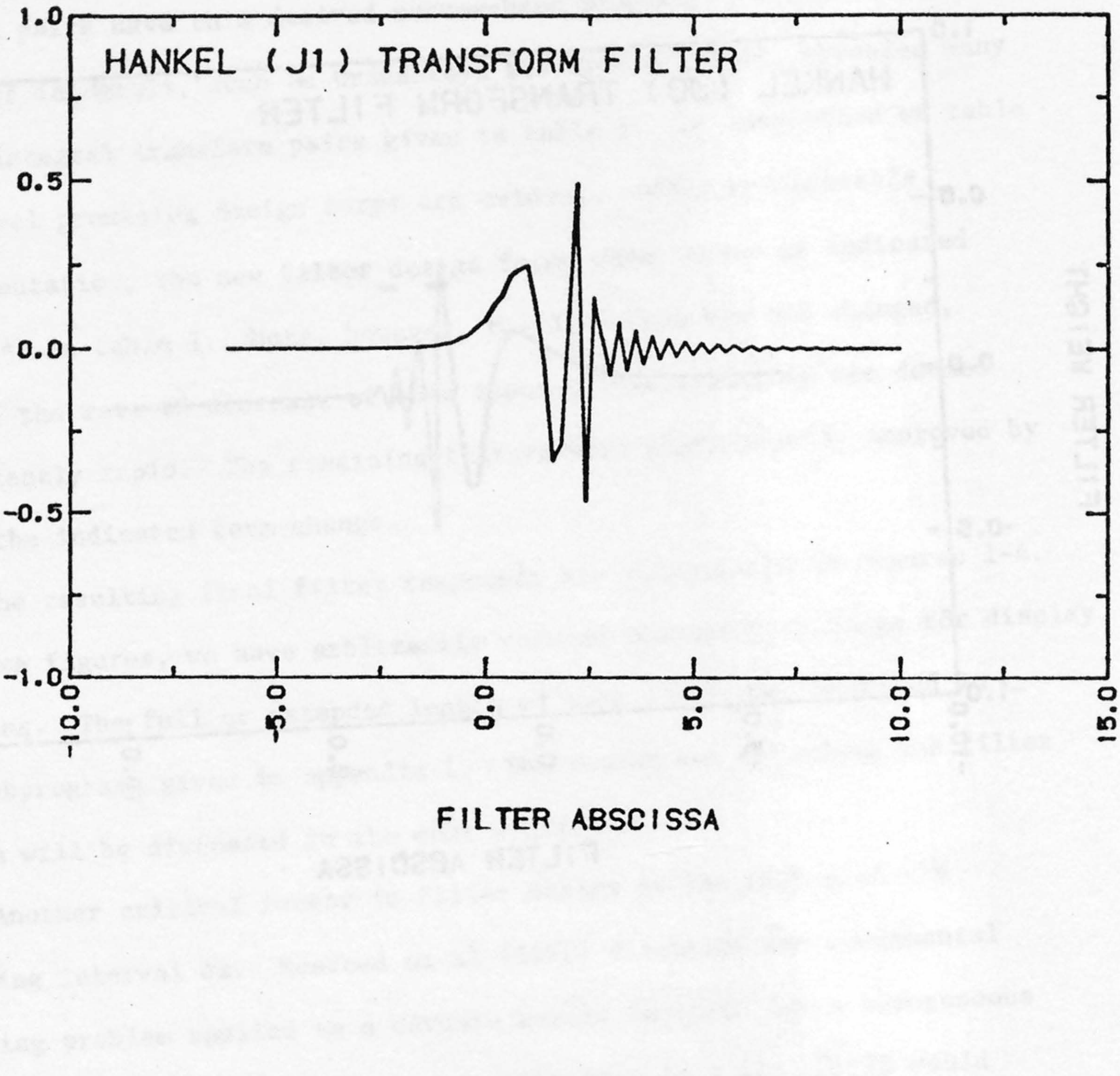

Figure 2.---Filter response for Hankel J_1-transform.

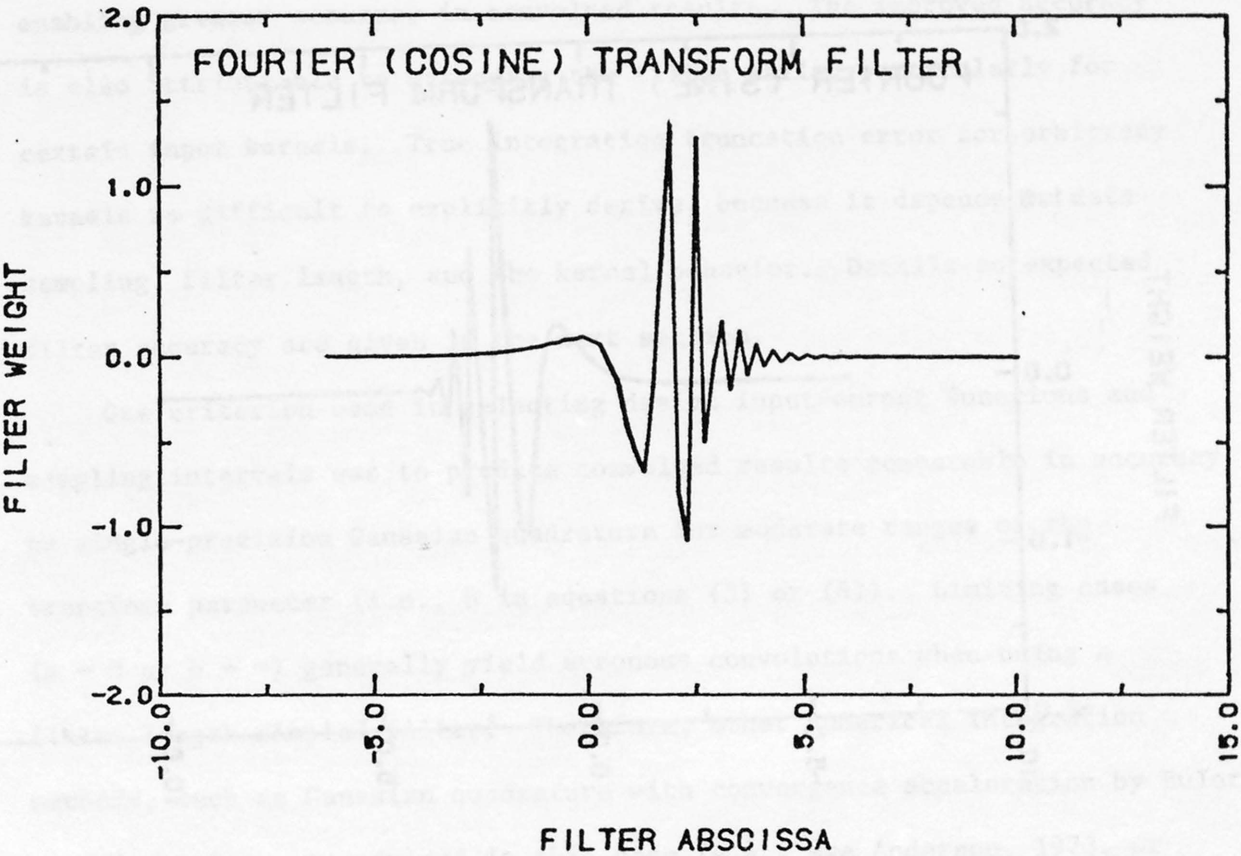

Figure 3.---Filter response for Fourier cosine-transform.

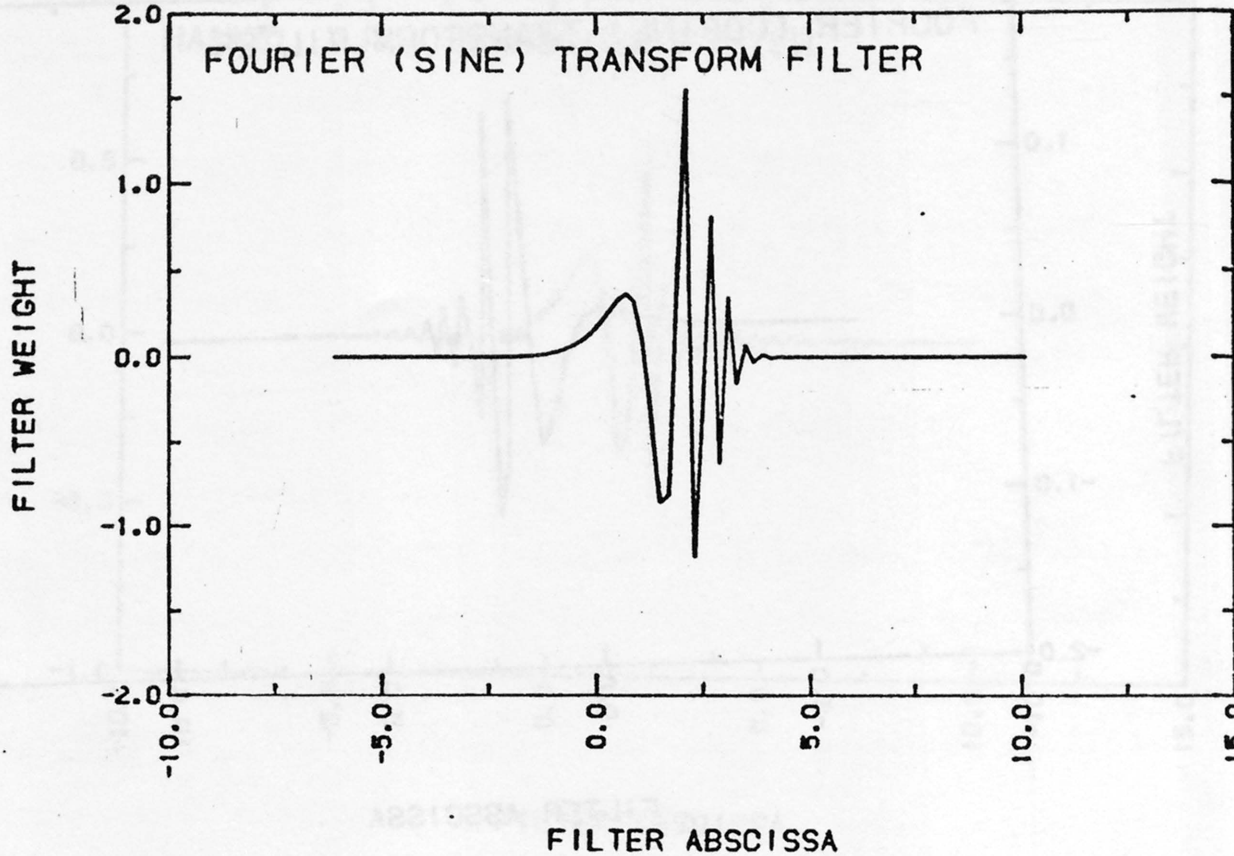

Figure 4.---Filter response for Fourier sine-transform.

Therefore, after much experimentation, the sampling was fixed at $\Delta x = .20 \sim (\ell n\ 10)/12$ for all redesigned filters. This only increases the number of weights slightly over the $(\ell n\ 10)/10$ interval, while enabling greater accuracy in convolved results. The improved accuracy is also attributable to extending the filter tails, particularly for certain input kernels. True integration truncation error for arbitrary kernels is difficult to explicitly derive, because it depends on data sampling, filter length, and the kernel behavior. Details on expected filter accuracy are given in the next section.

One criterion used in selecting design input-output functions and sampling intervals was to produce convolved results comparable in accuracy to single-precision Gaussian quadrature for moderate ranges of the transform parameter (i.e., b in equations (5) or (6)). Limiting cases ($b \to 0$ or $b \to \infty$) generally yield erronous convolutions when using a finite length sampled filter. Therefore, other numerical integration methods, such as Gaussian quadrature with convergence acceleration by Euler's transformation, are advised in this case (e.g., see Anderson, 1973, or Anderson, 1974). Methods designed exclusively for highly oscillatory integrals (e.g., see Boris and Oran, 1974) are suggested for extreme parameter problems.

Algorithms

The Fortran IV subprograms listed in appendix 1 reflect the final improved filters designed for Fourier (cosine, sine) and Hankel (J_0,J_1) transform integrals. Separate versions for both real and complex kernel functions are provided. In all cases, the kernel function must be supplied by the user as an external written function subprogram. It is mandatory that the external function (a) is monotonically decreasing as the argument increases, (b) does not have any singularities in the range of integration, and (c) is correctly coded. Violation of any of these assumptions may produce unpredictable results.

The substitution used to obtain equations (7) or (8) from (5) or (6), respectively, is implicitly assumed by all subprograms. However, the user must normalize the convolved sum after execution by the factor $e^x = b$. Sufficient comments concerning the calling parameters are given at the beginning of each routine, along with additional remarks on parameter relationships. The algorithm itself is not annotated, and consequently the following discussion outlines the general methodology and contributing factors.

The original filters published by Koefoed et al (1972), and verified by Anderson (1973), used a fixed number of weights during the convolution process. As later pointed out in a note by Verma and Koefoed (1973), many selected kernel functions damp to zero rapidly and so it is wasteful to multiply by the ends of the filter tails; that is, the contribution of the filter tails may be negligible to the total convolution sum. Consequently Verma and Koefoed proposed a truncated coefficient set approximately 15% shorter than their original filters designed for certain electromagnetic kernel functions.

This concept was further exploited and a generalization using the notion of a variable cutoff method with extended filter tails was adopted in this report. Here, a total filter length of approximately 250 weights are stored in DATA statements. For a given input kernel function obeying the above assumptions (a)-(c), and for a particular filter, we begin the convolution process in a fixed central region of the filter response containing about 20 weights (the central region is chosen such that weights decrease in absolute value beyond the region). Within the central region an absolute maximum convolved product is established. Then, depending on the nature of the kernel, the algorithm proceeds in either direction beyond the central region by accumulating products until the absolute magnitude is less than a prescribed tolerance times the central maximum. For complex kernel functions, the tolerance criterion is applied in parallel to both real and imaginary parts; both tests must be satisfied separately before the complex convolution is accepted.

Since the filter damps rapidly in both abscissa directions, the variable convolution may be terminated after using a few additional terms on either side of the central filter region, if the kernel is sufficiently decaying. Of course, this depends on the indicated tolerance controlling the truncation. In general, a smaller tolerance will result in increased accuracy, mainly because more filter weights are applied over a larger abscissa interval.

The extended filter length is particularly useful in minimizing truncation error when convolving with slowly decreasing and/or oscillatory kernel functions. Thus a large number of weights are supplied to enable increased accuracy for small tolerances, and to handle special function classes. For typically well-behaved kernels, the number of weights needed is nominal for a moderate tolerance. It should be emphasized that true integration truncation error is not directly related to the requested tolerance. The tolerance factor is simply used to terminate the variable convolution process, whereas the integration error depends on the sampling interval as well as the filter extent. However, in many cases, the toleranc may be thought of as an approximation indicator--that is, a truncation is done so that additional weights would not alter the convolution sum with respect to the specified tolerance.

In any event, it appears a larger class of integral transforms may be evaluated as a result of the variable convolution method. The uncertainty in determining exact error bounds for certain function classes may be investigated, if desired, by varying the tolerance and/or applying other numerical integration methods as control checks. More work is needed here. As a starting point, some test results are listed in appendix 2, where the transform pairs in table 1 were tested with various parameter ranges and tolerances. These results are discussed in more detail in the next section.

The basic subprograms are classified according to the notation defined in table 2.

Table 2. Basic subprogram naming convention.

General notation

```
              Hankel transform
                    ↓
   Real →    R   HANK   0  ← order 0 (or COS)
   Complex → Z   FOUR   1  ← order 1 (or SIN)
                    ↑
              Fourier transform
```

Specific names

RHANK0	RFOUR0
RHANK1	RFOUR1
ZHANK0	ZFOUR0
ZHANK1	ZFOUR1

Routines for both real and complex functions are provided, mainly to speed the operations when dealing with real functions. It should be observed, however, that it is possible to compute two real transforms in parallel using a complex routine by placing the real functions in the real and imaginary parts of an external complex function.

A special lagged-convolution method evolved by observing the fact that if one computes transforms over a parameter sequence $\{\ell n\ b_i, i = 1,2,...\}$ spaced exactly the same as the filter, then it is possible to minimize kernel function evaluations as the sequence continues. This may be accomplished by storing all kernel evaluations on the first execution. Then on subsequent calls, an incremented integer lag (equivalent to the shifted sample spacing) is used to perform lagged-products with appropriate filter weights and previously saved kernel evaluations. New kernel computations are stored only when needed by the variable convolution method. The lagged-convolution algorithm yields significant improvements in time over direct variable convolution whenever the kernel is not a simple elementary function. It should be pointed out that each lagged-convolution subprogram requires additional storage space to hold the kernel values. Also, a storage-roll feature is utilized so that the parameter sequence may be of any length.

The special lagged-convolution subprograms are classified according to the notation defined in table 3.

Table 3. Special subprogram naming convention.

General notation	Specific names	
Lagged Hankel transform	RLAGH0	RLAGF0
Real → R H 0 ← order 0 (or COS)	RLAGH1	RLAGF1
LAG		
Complex → Z F 1 ← order 1 (or SIN)	ZLAGH0	ZLAGF0
Lagged Fourier transform	ZLAGH1	ZLAGF1

Specific details on executing the lagged-convolution subprograms are given at the beginning of each routine listed in appendix 1. It is worth reading COMMENT statements: NOTES (2)-(4) for details on using the lagged-convolution method for intervals spaced differently than the filters.

Finally, all algorithms (direct or lagged) contain a special case which may occur only when the kernel is identically zero (or underflows the machine exponent range) throughout the fixed central region of a filter. This is considered extremely rare and should not occur in typical applications. If it does occur, then a reverse scan is attempted at both filter ends and convolution proceeds toward the central region until a zero-value is again encountered.

A final comment regarding oscillatory kernels: because of the possibility of computing values near a zero of an oscillating kernel function, a very small tolerance is advised; otherwise a premature cutoff could occur during the variable convolution algorithm. The computer algorithms can be easily modified to use an alternate truncation procedure especially designed for oscillatory kernels.

Filter tests

A large group of known integral transforms was evaluated for various parameter ranges using each subprogram. Some of these results are listed in appendix 2, where columns headed by F and T correspond to the transform pairs given in table 1. The values listed in column L represent the number of weights applied for the given tolerance. Appendix 2 has been separated into two parameter sections: (1) moderate b, and (2) small and large b. Parameter a was moderately chosen. The computed absolute error is denoted as |DIFF|, and the relative error is RELERR=|DIFF/EXACT|.

The filtered results would be expected to agree well with exact values for the design forms, especially for moderate b. Somewhat surprising is the good accuracy achieved when using slowly decreasing or oscillatory kernels. The worst case, in both sections, appears to be for F=T=3, where the kernel $J_0(a\sqrt{x})$ is both oscillatory and slowly dampening.

It should be noted that extremely large relative errors (e.g., see line F=1, T=2, A=.1, B=4) usually indicate the exact value is near zero. In other cases where RELERR > TOLERANCE, the magnitude of the absolute error |DIFF| is generally less than the tolerance. What exceptions do exist, are usually for extreme parameters or poorly behaved kernels.

As pointed out earlier, when the transform parameter b becomes quite small or large, the extended finite-length sampled filter may not be adequate for some kernels. In this special case, Gaussian quadrature or other numerical integration methods are often better. The results in section (2) of appendix 2 were purposely generated for very small and large b while using a small tolerance. Because of the extreme b range, many exact or filtered results underflow to identically zero (this was due to the machine word-length and floating-point exponent range $\sim 10^{\pm 38}$). Nevertheless, the generality of the variable convolution method produced acceptable results in nearly all cases.

Discussion

Other numerical methods for evaluating convolution-type integrals are noteworthy. In particular, Cornille (1972) outlines series methods and asymptotic expansions useful in numerical integration of Hankel transforms. Tsang et al (1974) present a fast Fourier transform (FFT) method in evaluating electromagnetic field integrals, and Shubert and Lin (1973) discuss computation of convolution integrals by solving an analogous differential equation numerically. The last two methods claim significant computation time improvements over direct numerical integration, namely Simpson's rule or Gaussian quadrature.

It would be interesting to compare speed and accuracy runs between these methods and the subprograms presented in this report. Seemingly the lagged-convolution method compares favorably with the differential equation approach, in that both methods take advantage of previously computed kernel values. Similarly, the FFT method and lagged-convolution apparently have comparable operations for parameter sets using the same integral form, particularly when using the same sampling interval. If various transform integral forms are required, the FFT method essentially designs a filter for each integral form; in this case, the predesigned filters of this report would have an advantage over the FFT method.

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Appendix 1.--Source listing of subprograms

A complete listing of each subprogram named in tables 2 and 3 is given in the following order:

```
      REAL FUNCTION RHANKO(X,FUN,TOL,L)
C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J0(G*B)*DG" DEFINED AS THE
C  REAL HANKEL TRANSFORM OF ORDER 0 AND ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C     X       = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
C     FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
C             NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                 CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                 THE REAL FUNCTION FUN SHOULD BE A MONOTONE
C                 DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C     TOL=        REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                 IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                 THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                 TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                 THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C               A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C               BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C               RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C               APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C               ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C     L=          RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                 CONVOLUTION (L DEPENDS ON TOL AND FUN).
C               MIN.L=20 AND MAX.L=193--WHICH COULD
C                 OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                 VERY FAST...
C
C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RHANKO; THE HANKEL
C  TRANSFORM IS THEN RHANKO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE....
C
C--USAGE-- "RHANKO" IS CALLED AS FOLLOWS:
C     ...
C     EXTERNAL RF
C     ...
```

```
C         R=RHANKO(ALOG(B),RF,TOL,L)/B
C         ...
C         END
C         REAL FUNCTION RF(G)
C         ...USER SUPPLIED CODE...
C         END
C
C--NOTES;
C         (1), EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
C         STATEMENT FUNCTION CO(II) BELOW; HOWEVER,
C         THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
C         EXP-UNDERFLOW'S TO 0.0......
C         (2),  SOME NON-ANSI FORTRAN STATEMENTS ARE USED
C          (E,G: DO 120 I=128,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
C         THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
C
C
C
C--JO-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE (SEE STATEMENT FUNCTION CO(II) BELOW).
          DIMENSION YT(193),Y1(76),Y2(76),Y3(41)
          EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1))
       DATA Y1/
     1 5.8565723E-08, 7.1143477E-11,-7.8395565E-11, 8.7489547E-11,
     2-8.9007811E-11, 9.8790055E-11,-9.8675347E-11, 1.1118797E-10,
     3-1.0893474E-10, 1.2543400E-10,-1.1979399E-10, 1.4200767E-10,
     4-1.3106341E-10, 1.6153229E-10,-1.4238602E-10, 1.8486236E-10,
     5-1.5315381E-10, 2.1319755E-10,-1.6238115E-10, 2.4824144E-10,
     6-1.6850378E-10, 2.9243813E-10,-1.6909302E-10, 3.4934366E-10,
     7-1.6043759E-10, 4.2417082E-10,-1.3690001E-10, 5.2458440E-10,
     8-8.9946096E-11, 6.6188220E-10,-6.6964033E-12, 8.5276151E-10,
     9 1.3222770E-10, 1.1219600E-09, 3.5591442E-10, 1.5061956E-09,
     1 7.0795382E-10, 2.0600379E-09, 1.2535947E-09, 2.9646623E-09,
     2 2.0904225E-09, 4.0409101E-09, 3.3642886E-09, 5.7687700E-09,
     3 5.2930786E-09, 8.3164338E-09, 8.2021809E-09, 1.2083635E-08,
     4 1.2577400E-08, 1.7666303E-08, 1.9143895E-08, 2.5953011E-08,
     5 2.8983953E-08, 3.8268851E-08, 4.3712685E-08, 5.6590075E-08,
     6 6.5740136E-08, 8.3864288E-08, 9.8682323E-08, 1.2448811E-07,
     7 1.4784461E-07, 1.8501974E-07, 2.2129198E-07, 2.7524203E-07,
     8 3.3094739E-07, 4.0974828E-07, 4.9462868E-07, 6.1030809E-07,
```

```
     9 7,3891802E-07, 9,0939667E-07, 1,1034727E-06, 1,3554600E-06,
     1 1,6474556E-06, 2,0207696E-06, 2,4591294E-06, 3,0131400E-06/
      DATA Y2/
     1 3,6701680E-06, 4,4934101E-06, 5,4770076E-06, 6,7015208E-06,
     2 8,1726989E-06, 9,9954201E-06, 1,2194425E-05, 1,4909101E-05,
     3 1,8194388E-05, 2,2239184E-05, 2,7145562E-05, 3,3174088E-05,
     4 4,0499452E-05, 4,9486730E-05, 6,0421440E-05, 7,3822001E-05,
     5 9,0141902E-05, 1,1012552E-04, 1,3448017E-04, 1,6428337E-04,
     6 2,0062570E-04, 2,4507680E-04, 2,9930366E-04, 3,6560582E-04,
     7 4,4651421E-04, 5,4541300E-04, 6,6612648E-04, 8,1365181E-04,
     8 9,9374786E-04, 1,2138120E-03, 1,4824945E-03, 1,8107657E-03,
     9 2,2115938E-03, 2,7012675E-03, 3,2991969E-03, 4,0295817E-03,
     1 4,9214244E-03, 6,0106700E-03, 7,3405529E-03, 8,9643708E-03,
     2 1,0946310E-02, 1,3365017E-02, 1,6314985E-02, 1,9910907E-02,
     3 2,4289325E-02, 2,9612896E-02, 3,6070402E-02, 4,3876936E-02,
     4 5,3264829E-02, 6,4465091E-02, 7,7664144E-02, 9,2918324E-02,
     5 1,1000121E-01, 1,2811102E-01, 1,4543025E-01, 1,5832248E-01,
     6 1,6049224E-01, 1,4170064E-01, 8,8788108E-02,-1,1330934E-02,
     7-1,5331864E-01,-2,9094670E-01,-2,9084655E-01,-2,9708834E-02,
     8 3,9009601E-01, 1,7999785E-01,-4,1858139E-01, 1,5317216E-01,
     9 6,5184953E-02,-1,0751806E-01, 7,8429567E-02,-4,6019124E-02,
     1 2,5309571E-02,-1,3904823E-02, 7,8187120E-03,-4,5190369E-03/
      DATA Y3/
     1 2,6729062E-03,-1,6073718E-03, 9,7713622E-04,-5,9804407E-04,
     2 3,6749320E-04,-2,2635296E-04, 1,3960805E-04,-8,6172618E-05,
     3 5,3212947E-05,-3,2867888E-05, 2,0304203E-05,-1,2543926E-05,
     4 7,7499633E-06,-4,7882430E-06, 2,9584108E-06,-1,8278645E-06,
     5 1,1293571E-06,-6,9778174E-07, 4,3113019E-07,-2,6637753E-07,
     6 1,6458373E-07,-1,0168954E-07, 6,2829807E-08,-3,8819969E-08,
     7 2,3985272E-08,-1,4819520E-08, 9,1563774E-09,-5,6573541E-09,
     8 3,4954514E-09,-2,1597005E-09, 1,3343946E-09,-8,2447148E-10,
     9 5,0941033E-10,-3,1474631E-10, 1,9447072E-10,-1,2015685E-10,
     1 7,4241055E-11,-4,5871468E-11, 2,8343095E-11,-1,7513137E-11,
     2 6,9049613E-12/
C--$$ENDATA
C
C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:
      CO(II)=FUN(EXP(-X+FLOAT(II)*,20-26,30455704))*YT(II)
      RHANKO=0,0
      CMAX=0,0
      L=18
```

```
      DO 110 I=129,146
      C=CO(I)
      RHANKO=RHANKO+C
      CMAX=AMAX1(ABS(C),CMAX)
110   CONTINUE
      IF(CMAX.EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      DO 120 I=128,1,-1
      C=CO(I)
      RHANKO=RHANKO+C
      L=L+1
      IF(ABS(C).LE.CMAX) GO TO 130
120   CONTINUE
130   DO 140 I=147,193
      C=CO(I)
      RHANKO=RHANKO+C
      L=L+1
      IF(ABS(C).LE.CMAX) GO TO 190
140   CONTINUE
      GO TO 190
150   DO 160 I=1,128
      C=CO(I)
      RHANKO=RHANKO+C
      L=L+1
      IF(C.EQ.0.0) GO TO 170
160   CONTINUE
170   DO 180 I=193,147,-1
      C=CO(I)
      RHANKO=RHANKO+C
      L=L+1
      IF(C.EQ.0.0) GO TO 190
180   CONTINUE
190   RETURN
      END
```

```
      REAL FUNCTION RHANK1(X,FUN,TOL,L)
C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J1(G*B)*DG" DEFINED AS THE
C  REAL HANKEL TRANSFORM OF ORDER 1 AND ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C     X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
C     FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
C             NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                   CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                   THE REAL FUNCTION FUN SHOULD BE A MONOTONE
C                   DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C     TOL=          REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                   IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                   THIS IS DONE AT BOTH ENDS OF FILTER. TYPICALLY,
C                   TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                   THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C              A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C              BUT WITH "MORE WEIGHTS" BEING USED. TOL IS NOT DIRECTLY
C              RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C              APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C              ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C     L=            RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                   CONVOLUTION (L DEPENDS ON TOL AND FUN).
C              MIN.L=15 AND MAX.L=236--WHICH COULD
C                   OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                   VERY FAST...
C
C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RHANK1; THE HANKEL
C  TRANSFORM IS THEN RHANK1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE....
C
C--USAGE-- "RHANK1" IS CALLED AS FOLLOWS:
C      ...
C      EXTERNAL RF
C      ...
```

```
C       R=RHANK1(ALOG(B),RF,TOL,L)/B
C       ...
C       END
C       REAL FUNCTION RF(G)
C       ...USER SUPPLIED CODE...
C       END
C
C--NOTES;
C       (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
C       STATEMENT FUNCTION C1(II) BELOW; HOWEVER,
C       THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
C       EXP-UNDERFLOW'S TO 0.0......
C       (2).  SOME NON-ANSI FORTRAN STATEMENTS ARE USED
C        (E.G; DO 20 J=85,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
C       THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
C
C
C
C--J1-EXTENDED FILTER WEIGHT ARRAYS;
C  NOTE; ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE (SEE STATEMENT FUNCTION C1(II) BELOW).
        DIMENSION WT(236),W1(76),W2(76),W3(76),W4(8)
        EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
     1 (WT(229),W4(1))
      DATA W1/
     1-8.8863805E-10, 1.1293811E-09,-1.2050872E-09, 1.2696232E-09,
     2-1.3223909E-09, 1.3642393E-09,-1.3969439E-09, 1.4225941E-09,
     3-1.4427475E-09, 1.4580582E-09,-1.4682563E-09, 1.4732179E-09,
     4-1.4735606E-09, 1.4719870E-09,-1.4727091E-09, 1.4828225E-09,
     5-1.5102619E-09, 1.5667752E-09,-1.6634522E-09, 1.8172900E-09,
     6-2.0412753E-09, 2.3595230E-09,-2.7861077E-09, 3.3592871E-09,
     7-4.0940172E-09, 5.0571015E-09,-6.2604109E-09, 7.8269461E-09,
     8-9.7514701E-09, 1.2267639E-08,-1.5312389E-08, 1.9339924E-08,
     9-2.4126297E-08, 3.0576829E-08,-3.8060204E-08, 4.8423732E-08,
     1-6.0051116E-08, 7.6787475E-08,-9.4700993E-08, 1.2192844E-07,
     2-1.4918997E-07, 1.9392737E-07,-2.3464786E-07, 3.0911127E-07,
     3-3.6815394E-07, 4.9413800E-07,-5.7554168E-07, 7.9301529E-07,
     4-8.9502818E-07, 1.2794292E-06,-1.3811469E-06, 2.0789668E-06,
     5-2.1069398E-06, 3.4103188E-06,-3.1584463E-06, 5.6639045E-06,
     6-4.6059955E-06, 9.5561672E-06,-6.4142855E-06, 1.6440205E-05,
     7-8.2010619E-06, 2.8945217E-05,-8.6348466E-06, 5.2317398E-05,
```

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     8-3.9915035E-06, 9.7273612E-05, 1.5220520E-05, 1.8614373E-04,
     9 7.2023760E-05, 3.6620099E-04, 2.2062958E-04, 7.3874539E-04,
     1 5.8623480E-04, 1.5226779E-03, 1.4538718E-03, 3.1930365E-03/
      DATA W2/
     1 3.4640868E-03, 6.7790882E-03, 8.0328420E-03, 1.4484339E-02,
     2 1.8201316E-02, 3.0866143E-02, 4.0106549E-02, 6.4527872E-02,
     3 8.4285526E-02, 1.2773175E-01, 1.6020907E-01, 2.1948043E-01,
     4 2.3636305E-01, 2.4895051E-01, 1.2586300E-01,-5.1060445E-02,
     5-3.4376222E-01,-2.9042175E-01, 1.1564736E-01, 4.9253231E-01,
     6-4.6748595E-01, 1.5280945E-01, 3.3348541E-02,-8.2485252E-02,
     7 7.9740630E-02,-6.6934498E-02, 5.5150465E-02,-4.5868721E-02,
     8 3.8651958E-02,-3.2935834E-02, 2.8303994E-02,-2.4475127E-02,
     9 2.1259541E-02,-1.8526278E-02, 1.6182037E-02,-1.4158101E-02,
     1 1.2402225E-02,-1.0873526E-02, 9.5392016E-03,-8.3723743E-03,
     2 7.3506490E-03,-6.4551136E-03, 5.6696335E-03,-4.9803353E-03,
     3 4.3752213E-03,-3.8438703E-03, 3.3772023E-03,-2.9672872E-03,
     4 2.6071877E-03,-2.2908274E-03, 2.0128794E-03,-1.7686706E-03,
     5 1.5540998E-03,-1.3655566E-03, 1.1999089E-03,-1.0543497E-03,
     6 9.2644973E-04,-8.1406593E-04, 7.1531559E-04,-6.2854459E-04,
     7 5.5229955E-04,-4.8530352E-04, 4.2643446E-04,-3.7470650E-04,
     8 3.2925334E-04,-2.8931382E-04, 2.5421910E-04,-2.2338147E-04,
     9 1.9628455E-04,-1.7247455E-04, 1.5155278E-04,-1.3316889E-04,
     1 1.1701502E-04,-1.0282066E-04, 9.0348135E-05,-7.9388568E-05/
      DATA W3/
     1 6.9758436E-05,-6.1296474E-05, 5.3860978E-05,-4.7327436E-05,
     2 4.1586435E-05,-3.6541840E-05, 3.2109174E-05,-2.8214208E-05,
     3 2.4791718E-05,-2.1784390E-05, 1.9141864E-05,-1.6819888E-05,
     4 1.4779578E-05,-1.2986765E-05, 1.1411426E-05,-1.0027182E-05,
     5 8.8108499E-06,-7.7420630E-06, 6.8029235E-06,-5.9777053E-06,
     6 5.2525892E-06,-4.6154325E-06, 4.0555653E-06,-3.5636118E-06,
     7 3.1313335E-06,-2.7514911E-06, 2.4177236E-06,-2.1244417E-06,
     8 1.8667342E-06,-1.6402859E-06, 1.4413051E-06,-1.2664597E-06,
     9 1.1128220E-06,-9.7781908E-07, 8.5919028E-07,-7.5494920E-07,
     1 6.6335060E-07,-5.8286113E-07, 5.1213358E-07,-4.4998431E-07,
     2 3.9537334E-07,-3.4738689E-07, 3.0522189E-07,-2.6817250E-07,
     3 2.3561831E-07,-2.0701397E-07, 1.8188012E-07,-1.5979545E-07,
     4 1.4038966E-07,-1.2333746E-07, 1.0835294E-07,-9.5185048E-08,
     5 8.3613184E-08,-7.3443411E-08, 6.4505118E-08,-5.6648167E-08,
     6 4.9740428E-08,-4.3665572E-08, 3.8321109E-08,-3.3616717E-08,
     7 2.9472836E-08,-2.5819439E-08, 2.2594957E-08,-1.9745353E-08,
     8 1.7223359E-08,-1.4987869E-08, 1.3003472E-08,-1.1240058E-08,
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     9 9.6723739E-09,-8.2794392E-09, 7.0438407E-09,-5.9509676E-09,
    ,1 4.9882405E-09,-4.1443813E-09, 3.4088114E-09,-2.7712762E-09/
      DATA W4/
     1 2.2217311E-09,-1.7504755E-09, 1.3485207E-09,-1.0080937E-09,
     2 7.2300885E-10,-4.8860666E-10, 3.0121413E-10,-9.1649798E-11/
C--$$ENDATA
C
C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:
      C1(II)=FUN(EXP(-X+FLOAT(II)*.20-17.0))*WT(II)
      RHANK1=0.0
      CMAX=0.0
      L=13
      DO 10 I=86,98
      C=C1(I)
      RHANK1=RHANK1+C
      CMAX=AMAX1(ABS(C),CMAX)
   10 CONTINUE
      IF(CMAX.EQ.0.0) GO TO 60
      CMAX=TOL*CMAX
      DO 20 I=85,1,-1
      C=C1(I)
      RHANK1=RHANK1+C
      L=L+1
      IF(ABS(C).LE.CMAX) GO TO 30
   20 CONTINUE
   30 DO 40 I=99,236
      C=C1(I)
      RHANK1=RHANK1+C
      L=L+1
      IF(ABS(C).LE.CMAX) GO TO 50
   40 CONTINUE
   50 RETURN
   60 DO 70 I=1,85
      C=C1(I)
      RHANK1=RHANK1+C
      L=L+1
      IF(C.EQ.0.0) GO TO 80
   70 CONTINUE
   80 DO 90 I=236,99,-1
      C=C1(I)
      RHANK1=RHANK1+C
      L=L+1
      IF(C.EQ.0.0) GO TO 50
   90 CONTINUE
      GO TO 50
      END
```

```
      REAL FUNCTION RFOURO(X,FUN,TOL,L)
C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*COS(G*B)*DG" DEFINED AS THE
C  REAL FOURIER COSINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C     X     = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
C     FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
C             NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                   CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                   THE REAL FUNCTION FUN SHOULD BE A MONOTONE
C                   DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C     TOL=        REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                   IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                   THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                   TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                   THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C                 A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C                 BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C                 RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C                 APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C                 ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C       L=          RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                   CONVOLUTION (L DEPENDS ON TOL AND FUN).
C                 MIN.L=24 AND MAX.L=281--WHICH COULD
C                   OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                   VERY FAST...
C
C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RFOURO; THE FOURIER
C  TRANSFORM IS THEN RFOURO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE....
C
C--USAGE-- "RFOURO" IS CALLED AS FOLLOWS:
C       ...
C       EXTERNAL RF
C       ...
```

```
C        R=RFOURO(ALOG(B),RF,TOL,L)/B
C        ...
C        END
C        REAL FUNCTION RF(G)
C        ...USER SUPPLIED CODE...
C        END
C
C--NOTES:
C        (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
C        STATEMENT FUNCTION CO(II) BELOW; HOWEVER,
C        THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
C        EXP-UNDERFLOW'S TO 0.0......
C        (2). SOME NON-ANSI FORTRAN STATEMENTS ARE USED
C         (E.G: DO 120 I=148,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
C        THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
C
C
C--COS-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE (SEE STATEMENT FUNCTION CO(II) BELOW).
         DIMENSION YT(281),Y1(76),Y2(76),Y3(76),Y4(53)
         EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1)),
      1 (YT(229),Y4(1))
       DATA Y1/
      1 5.1178101E-14, 2.9433849E-14, 2.5492522E-14, 1.9034819E-14,
      2 6.4179780E-14, 1.3085746E-15, 1.1989957E-13,-1.2216234E-14,
      3 1.7534103E-13, 7.9373498E-15, 2.1235658E-13, 7.9981520E-14,
      4 2.3815757E-13, 1.9714260E-13, 2.8920132E-13, 3.4161340E-13,
      5 4.0349917E-13, 5.2203885E-13, 5.9837223E-13, 7.8015306E-13,
      6 8.8911655E-13, 1.1709731E-12, 1.3165595E-12, 1.7578463E-12,
      7 1.9538564E-12, 2.6289768E-12, 2.9167697E-12, 3.9044344E-12,
      8 4.3927341E-12, 5.7526904E-12, 6.6569552E-12, 8.4555678E-12,
      9 1.0063229E-11, 1.2487964E-11, 1.5134682E-11, 1.8501488E-11,
      1 2.2720051E-11, 2.7452598E-11, 3.4025443E-11, 4.0875985E-11,
      2 5.0751668E-11, 6.1094382E-11, 7.5492982E-11, 9.1445759E-11,
      3 1.1227336E-10, 1.3676464E-10, 1.6720269E-10, 2.0423244E-10,
      4 2.4932743E-10, 3.0470661E-10, 3.7198526E-10, 4.5449934E-10,
      5 5.5502537E-10, 6.7793669E-10, 8.2810031E-10, 1.0112626E-09,
      6 1.2354800E-09, 1.5085255E-09, 1.8432253E-09, 2.2503397E-09,
      7 2.7499027E-09, 3.3569325E-09, 4.1025670E-09, 5.0077487E-09,
      8 6.1205950E-09, 7.4703399E-09, 9.1312760E-09, 1.1143911E-08,
```

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     9 1.3622929E-08, 1.6623917E-08, 2.0324094E-08, 2.4798610E-08,
     1 3.0321709E-08, 3.6992986E-08, 4.5237482E-08, 5.5183434E-08/
      DATA Y2/
     1 6.7491070E-08, 8.2317946E-08, 1.0069271E-07, 1.2279375E-07,
     2 1.5022907E-07, 1.8316969E-07, 2.2413747E-07, 2.7322865E-07,
     3 3.3441046E-07, 4.0756197E-07, 4.9894278E-07, 6.0793233E-07,
     4 7.4443665E-07, 9.0679753E-07, 1.1107379E-06, 1.3525651E-06,
     5 1.6573073E-06, 2.0174273E-06, 2.4728798E-06, 3.0090445E-06,
     6 3.6898816E-06, 4.4879625E-06, 5.5059521E-06, 6.6935820E-06,
     7 8.2160716E-06, 9.9828691E-06, 1.2260527E-05, 1.4888061E-05,
     8 1.8296530E-05, 2.2202672E-05, 2.7305154E-05, 3.3109672E-05,
     9 4.0751046E-05, 4.9372484E-05, 6.0820947E-05, 7.3619571E-05,
     1 9.0780005E-05, 1.0976837E-04, 1.3550409E-04, 1.6365675E-04,
     2 2.0227521E-04, 2.4398338E-04, 3.0197018E-04, 3.6370760E-04,
     3 4.5083748E-04, 5.4213338E-04, 6.7315347E-04, 8.0800951E-04,
     4 1.0051938E-03, 1.2041401E-03, 1.5011708E-03, 1.7942344E-03,
     5 2.2421056E-03, 2.6730676E-03, 3.3490681E-03, 3.9815050E-03,
     6 5.0028666E-03, 5.9285668E-03, 7.4730905E-03, 8.8233510E-03,
     7 1.1160132E-02, 1.3119627E-02, 1.6653199E-02, 1.9472767E-02,
     8 2.4800811E-02, 2.8793704E-02, 3.6762063E-02, 4.2228780E-02,
     9 5.3905163E-02, 6.0804660E-02, 7.7081738E-02, 8.3874501E-02,
     1 1.0377190E-01, 1.0377718E-01, 1.1892208E-01, 9.0437429E-02/
      DATA Y3/
     1 7.1685138E-02,-3.9473064E-02,-1.5078720E-01,-4.0489859E-01,
     2-5.6018995E-01,-6.8050388E-01,-1.5094224E-01, 6.6304064E-01,
     3 1.3766748E+00,-8.0373222E-01,-1.0869629E+00, 1.2812892E+00,
     4-5.0341082E-01,-4.4274455E-02, 2.0913102E-01,-1.9999661E-01,
     5 1.5207664E-01,-1.0920260E-01, 7.8169956E-02,-5.6651561E-02,
     6 4.1611799E-02,-3.0880012E-02, 2.3072559E-02,-1.7311631E-02,
     7 1.3021442E-02,-9.8085025E-03, 7.3943529E-03,-5.5769518E-03,
     8 4.2073164E-03,-3.1745026E-03, 2.3954154E-03,-1.8076122E-03,
     9 1.3640816E-03,-1.0293934E-03, 7.7682952E-04,-5.8623518E-04,
     1 4.4240399E-04,-3.3386183E-04, 2.5195025E-04,-1.9013541E-04,
     2 1.4348659E-04,-1.0828284E-04, 8.1716174E-05,-6.1667509E-05,
     3 4.6537684E-05,-3.5119887E-05, 2.6503388E-05,-2.0000904E-05,
     4 1.5093768E-05,-1.1390572E-05, 8.5959318E-06,-6.4869407E-06,
     5 4.8953713E-06,-3.6942830E-06, 2.7878625E-06,-2.1038241E-06,
     6 1.5875917E-06,-1.1980090E-06, 9.0398030E-07,-6.8208296E-07,
     7 5.1458650E-07,-3.8817581E-07, 2.9272267E-07,-2.2067921E-07,
     8 1.6623514E-07,-1.2514102E-07, 9.4034535E-08,-7.0556837E-08,
     9 5.2741581E-08,-3.9298610E-08, 2.9107255E-08,-2.1413893E-08,
```

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      1 1.5742032E-08,-1,1498608E-08, 8,7561571E-09,-7,2959446E-09/
       DATA Y4/
      1 6,8816619E-09,-8,9679825E-09, 1,4258275E-08,-1,9564299E-08,
      2 2,0235313E-08,-1,4725545E-08, 5,4632820E-09, 3,5995580E-09,
      3-9,5287133E-09, 1,1460041E-08,-1,0250532E-08, 7,4641748E-09,
      4-4,4703465E-09, 2,0499053E-09,-4,4806353E-10,-4,0374336E-10,
      5 7,0321001E-10,-6,7067960E-10, 4,9130404E-10,-2,8840747E-10,
      6 1,2373144E-10,-1,5260443E-11,-4,2027559E-11, 6,1885474E-11,
      7-5,9273937E-11, 4,6588766E-11,-3,2054182E-11, 1,9831637E-11,
      8-1,1210098E-11, 5,9567021E-12,-3,2427812E-12, 2,1353868E-12,
      9-1,8476851E-12, 1,8438474E-12,-1,8362842E-12, 1,7241847E-12,
      1-1,5161479E-12, 1,2627657E-12,-1,0129176E-12, 7,9578625E-13,
      2-6,2131435E-13, 4,8745900E-13,-3,8703630E-13, 3,1172547E-13,
      3-2,5397802E-13, 2,0824130E-13,-1,7123163E-13, 1,4113344E-13,
      4-1,1687986E-13, 9,7664016E-14,-8,2977176E-14, 7,2515267E-14,
      5-5,6047478E-14/
C--$$ENDATA
C
C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:
       C0(II)=FUN(EXP(-X+FLOAT(II)*,20-30,30251236))*YT(II)
       RFOURO=0,0
       CMAX=0,0
       L=22
       DO 110 I=149,170
       C=C0(I)
       RFOURO=RFOURO+C
       CMAX=AMAX1(ABS(C),CMAX)
  110  CONTINUE
       IF(CMAX,EQ,0,0) GO TO 150
       CMAX=TOL*CMAX
       DO 120 I=148,1,-1
       C=C0(I)
       RFOURO=RFOURO+C
       L=L+1
       IF(ABS(C),LE,CMAX) GO TO 130
  120  CONTINUE
  130  DO 140 I=171,201
       C=C0(I)
       RFOURO=RFOURO+C
       L=L+1
       IF(ABS(C),LE,CMAX) GO TO 190
```

```
C--USAGE-- "ZHANKO" IS CALLED AS FOLLOWS:
C          ...
C          COMPLEX Z,ZHANKO,ZF
C          EXTERNAL ZF
C          ...
C          Z=ZHANKO(ALOG(B),ZF,TOL,L)/B
C          ...
C          END
C          COMPLEX FUNCTION ZF(G)
C          ...USER SUPPLIED CODE...
C          END
C
C--NOTES:
C          (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
C          STATEMENT FUNCTION CO(II) BELOW; HOWEVER,
C          THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
C          EXP-UNDERFLOW'S TO 0.0.......
C          (2).  SOME NON-ANSI FORTRAN STATEMENTS ARE USED
C           (E.G: DO 120 I=128,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
C          THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
C
C
C
C--JO-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE (SEE STATEMENT FUNCTION CO(II) BELOW).
           DIMENSION YT(193),Y1(76),Y2(76),Y3(41)
           EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1))
        DATA Y1/
      1 5.8565723E-08, 7.1143477E-11,-7.8395565E-11, 8.7489547E-11,
      2-8.9007811E-11, 9.8790055E-11,-9.8675347E-11, 1.1118797E-10,
      3-1.0893474E-10, 1.2543400E-10,-1.1979399E-10, 1.4200767E-10,
      4-1.3106341E-10, 1.6153229E-10,-1.4238602E-10, 1.8486236E-10,
      5-1.5315381E-10, 2.1319755E-10,-1.6238115E-10, 2.4824144E-10,
      6-1.6850378E-10, 2.9243813E-10,-1.6909302E-10, 3.4934366E-10,
      7-1.6043759E-10, 4.2417082E-10,-1.3690001E-10, 5.2458440E-10,
      8-8.9946096E-11, 6.6188220E-10,-6.6964033E-12, 8.5276151E-10,
      9 1.3222770E-10, 1.1219600E-09, 3.5591442E-10, 1.5061956E-09,
      1 7.0795382E-10, 2.0600379E-09, 1.2535947E-09, 2.8646623E-09,
      2 2.0904225E-09, 4.0409101E-09, 3.3642886E-09, 5.7687700E-09,
      3 5.2930786E-09, 8.3164338E-09, 8.2021809E-09, 1.2083635E-08,
```

```
      COMPLEX FUNCTION ZHANKO(X,FUN,TOL,L)
C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J0(G*B)*DG" DEFINED AS THE
C  COMPLEX HANKEL TRANSFORM OF ORDER 0 AND ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C     X     = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
C     FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
C             OF A REAL ARGUMENT G.
C           NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C             CALLING PROGRAM AND IN SUBPROGRAM FUN.
C             THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
C             DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C           FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RHANKO" IS ADVISED;
C           HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
C           INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
C     TOL=    REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C             IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C             THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C             TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C             THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C           A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C           BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C           RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C           APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C           ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C     L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C             CONVOLUTION (L DEPENDS ON TOL AND FUN).
C           MIN.L=20 AND MAX.L=193--WHICH COULD
C             OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C             VERY FAST...
C
C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZHANKO; THE HANKEL
C  TRANSFORM IS THEN ZHANKO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE....
C
```

```
  140   CONTINUE
        GO TO 190
  150   DO 160 I=1,148
        C=CO(I)
        RFOURO=RFOURO+C
        L=L+1
        IF(C.EQ.0.0) GO TO 170
  160   CONTINUE
  170   DO 180 I=281,171,-1
        C=CO(I)
        RFOURO=RFOURO+C
        L=L+1
        IF(C.EQ.0.0) GO TO 190
  180   CONTINUE
  190   RETURN
        END
```

```
      REAL FUNCTION RFOUR1(X,FUN,TOL,L)
C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*SIN(G*B)*DG" DEFINED AS THE
C  REAL FOURIER SINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C     X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
C     FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
C             NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                THE REAL FUNCTION FUN SHOULD BE A MONOTONE
C                DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C     TOL=         REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C             A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C             BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C             RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C             APPPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C             ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C      L=        RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                CONVOLUTION (L DEPENDS ON TOL AND FUN).
C             MIN.L=20 AND MAX.L=266--WHICH COULD
C                OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                VERY FAST...
C
C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RFOUR1; THE FOURIER
C  TRANSFORM IS THEN RFOUR1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE....
C
C--USAGE-- "RFOUR1" IS CALLED AS FOLLOWS:
C     ...
C     EXTERNAL RF
C     ...
```

```
C       R=RFOUR1(ALOG(B),RF,TOL,L)/B
C       ...
C       END
C       REAL FUNCTION RF(G)
C       ...USER SUPPLIED CODE...
C       END
C
C--NOTES:
C       (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
C       STATEMENT FUNCTION C1(II) BELOW; HOWEVER,
C       THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
C       EXP-UNDERFLOW'S TO 0.0......
C       (2). SOME NON-ANSI FORTRAN STATEMENTS ARE USED
C        (E.G: DO 20 I=190,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
C       THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
C
C
C--SIN-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE (SEE STATEMENT FUNCTION C1(II) BELOW).
        DIMENSION WT(266),W1(76),W2(76),W3(76),W4(38)
        EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
     1 (WT(229),W4(1))
      DATA W1/
     1-1.1113940E-09,-1.3237246E-12, 1.5091739E-12,-1.6240954E-12,
     2 1.7236636E-12,-1.8227727E-12, 1.9255992E-12,-2.0335514E-12,
     3 2.1473541E-12,-2.2675549E-12, 2.3946842E-12,-2.5292661E-12,
     4 2.6718110E-12,-2.8227693E-12, 2.9825171E-12,-3.1514006E-12,
     5 3.3297565E-12,-3.5179095E-12, 3.7163306E-12,-3.9256378E-12,
     6 4.1464798E-12,-4.3794552E-12, 4.6252131E-12,-4.8845227E-12,
     7 5.1582809E-12,-5.4474462E-12, 5.7530277E-12,-6.0760464E-12,
     8 6.4175083E-12,-6.7783691E-12, 7.1595239E-12,-7.5618782E-12,
     9 7.9864477E-12,-8.4344110E-12, 8.9072422E-12,-9.4067705E-12,
     1 9.9349439E-12,-1.0493731E-11, 1.1084900E-11,-1.1709937E-11,
     2 1.2370354E-11,-1.3067414E-11, 1.3802200E-11,-1.4575980E-11,
     3 1.5390685E-11,-1.6249313E-11, 1.7155934E-11,-1.8115250E-11,
     4 1.9131898E-11,-2.0209795E-11, 2.1352159E-11,-2.2561735E-11,
     5 2.3840976E-11,-2.5192263E-11, 2.6618319E-11,-2.8122547E-11,
     6 2.9709129E-11,-3.1382670E-11, 3.3149030E-11,-3.5013168E-11,
     7 3.6981050E-11,-3.9058553E-11, 4.1251694E-11,-4.3566777E-11,
     8 4.6010537E-11,-4.8590396E-11, 5.1314761E-11,-5.4193353E-11,
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     9 5.7236720E-11,-6.0455911E-11, 6.3861222E-11,-6.7461492E-11,
     1 7.1265224E-11,-7.5279775E-11, 7.9512249E-11,-8.3971327E-11/
      DATA W2/
     1 8.8668961E-11,-9.3621900E-11, 9.8851764E-11,-1.0438319E-10,
     2 1.1024087E-10,-1.1644680E-10, 1.2301979E-10,-1.2997646E-10,
     3 1.3733244E-10,-1.4510363E-10, 1.5330772E-10,-1.6196550E-10,
     4 1.7110130E-10,-1.8074257E-10, 1.9091922E-10,-2.0166306E-10,
     5 2.1300756E-10,-2.2498755E-10, 2.3763936E-10,-2.5100098E-10,
     6 2.6511250E-10,-2.8001616E-10, 2.9575691E-10,-3.1238237E-10,
     7 3.2994314E-10,-3.4849209E-10, 3.6808529E-10,-3.8878042E-10,
     8 4.1063982E-10,-4.3372666E-10, 4.5811059E-10,-4.8386049E-10,
     9 5.1105728E-10,-5.3977672E-10, 5.7011632E-10,-6.0215516E-10,
     1 6.3601273E-10,-6.7175964E-10, 7.0955028E-10,-7.4942601E-10,
     2 7.9161025E-10,-8.3606980E-10, 8.8317110E-10,-9.3270330E-10,
     3 9.8533749E-10,-1.0404508E-09, 1.0993731E-09,-1.1605442E-09,
     4 1.2267391E-09,-1.2942905E-09, 1.3691677E-09,-1.4429912E-09,
     5 1.5288164E-09,-1.6077524E-09, 1.7095998E-09,-1.7890471E-09,
     6 1.9129068E-09,-1.9857116E-09, 2.1491608E-09,-2.1926779E-09,
     7 2.4312660E-09,-2.3959044E-09, 2.7872500E-09,-2.5610596E-09,
     8 3.2762318E-09,-2.6082940E-09, 4.0251453E-09,-2.3560563E-09,
     9 5.3176554E-09,-1.3960161E-09, 7.7708747E-09, 1.1853546E-09,
     1 1.2760851E-08, 7.4264707E-09, 2.3342187E-08, 2.1869851E-08/
      DATA W3/
     1 4.6306744E-08, 5.4631686E-08, 9.6763087E-08, 1.2823337E-07,
     2 2.0832812E-07, 2.9280540E-07, 4.5580888E-07, 6.5992437E-07,
     3 1.0056815E-06, 1.4779183E-06, 2.2284335E-06, 3.2994604E-06,
     4 4.9485823E-06, 7.3545473E-06, 1.1001083E-05, 1.6380539E-05,
     5 2.4469550E-05, 3.6469246E-05, 5.4441527E-05, 8.1176726E-05,
     6 1.2113828E-04, 1.8066494E-04, 2.6954609E-04, 4.0202288E-04,
     7 5.9969995E-04, 8.9437312E-04, 1.3338166E-03, 1.9886697E-03,
     8 2.9643943E-03, 4.4168923E-03, 6.5773518E-03, 9.7855105E-03,
     9 1.4539361E-02, 2.1558670E-02, 3.1871854E-02, 4.6903518E-02,
     1 6.8559512E-02, 9.9170152E-02, 1.4120770E-01, 1.9610835E-01,
     2 2.6192603E-01, 3.2743321E-01, 3.6407406E-01, 3.1257559E-01,
     3 9.0460168E-02,-3.6051039E-01,-8.6324760E-01,-8.1178720E-01,
     4 5.2205241E-01, 1.5449873E+00,-1.1817933E+00,-2.6759896E-01,
     5 8.0869203E-01,-6.2757149E-01, 3.4062630E-01,-1.5885304E-01,
     6 7.0472994E-02,-3.1624462E-02, 1.4894068E-02,-7.4821176E-03,
     7 4.0035936E-03,-2.2543784E-03, 1.3160358E-03,-7.8636604E-04,
     8 4.7658745E-04,-2.9125817E-04, 1.7885105E-04,-1.1012416E-04,
     9 6.7910334E-05,-4.1914054E-05, 2.5881544E-05,-1.5985851E-05,
```

```
     1 9.8751880E-06,-6.1008526E-06, 3.7692543E-06,-2.3287953E-06/
      DATA W4/
     1 1.4388425E-06,-8.8899353E-07, 5.4926991E-07,-3.3937048E-07,
     2 2.0968284E-07,-1.2955437E-07, 8.0046336E-08,-4.9457371E-08,
     3 3.0557711E-08,-1.8880390E-08, 1.1665454E-08,-7.2076428E-09,
     4 4.4533423E-09,-2.7515696E-09, 1.7001092E-09,-1.0504494E-09,
     5 6.4904567E-10,-4.0102999E-10, 2.4778763E-10,-1.5310321E-10,
     6 9.4600354E-11,-5.8453314E-11, 3.6119400E-11,-2.2320056E-11,
     7 1.3793460E-11,-8.5242656E-12, 5.2676102E-12,-3.2543076E-12,
     8 2.0097689E-12,-1.2405412E-12, 7.6530538E-13,-4.7191929E-13,
     9 2.9084993E-13,-1.7923661E-13, 1.1018948E-13,-6.7885902E-14,
     1 4.2025050E-14,-2.1314731E-14/
C--$$ENDATA
C
C
C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:
      C1(II)=FUN(EXP(-X+FLOAT(II)*.20-38.30455704))*WT(II)
      RFOUR1=0.0
      CMAX=0.0
      L=18
      DO 10 I=191,208
      C=C1(I)
      RFOUR1=RFOUR1+C
      CMAX=AMAX1(ABS(C),CMAX)
   10 CONTINUE
      IF(CMAX.EQ.0.0) GO TO 60
      CMAX=TOL*CMAX
      DO 20 I=190,1,-1
      C=C1(I)
      RFOUR1=RFOUR1+C
      L=L+1
      IF(ABS(C).LE.CMAX) GO TO 30
   20 CONTINUE
   30 DO 40 I=209,266
      C=C1(I)
      RFOUR1=RFOUR1+C
      L=L+1
      IF(ABS(C).LE.CMAX) GO TO 50
   40 CONTINUE
   50 RETURN
   60 DO 70 I=1,190
```

```
      C=C1(I)
      RFOUR1=RFOUR1+C
      L=L+1
      IF(C.EQ.0.0) GO TO 80
   70 CONTINUE
   80 DO 90 I=266,209,-1
      C=C1(I)
      RFOUR1=RFOUR1+C
      L=L+1
      IF(C.EQ.0.0) GO TO 50
   90 CONTINUE
      GO TO 50
      END
```

```
     4 1,2577400E-08, 1.7666303E-08, 1.9143895E-08, 2.5953011E-08,
     5 2.8983953E-08, 3.8268851E-08, 4.3712685E-08, 5.6590075E-08,
     6 6.5740136E-08, 8.3864288E-08, 9.8662323E-08, 1.2448811E-07,
     7 1.4784461E-07, 1.8501974E-07, 2.2129198E-07, 2.7524203E-07,
     8 3.3094739E-07, 4.0974828E-07, 4.9462868E-07, 6.1030809E-07,
     9 7.3891802E-07, 9.0939667E-07, 1,1034727E-06, 1,3554600E-06,
     1 1,6474556E-06, 2.0207696E-06, 2,4591294E-06, 3.0131400E-06/
      DATA Y2/
     1 3.6701680E-06, 4.4934101E-06, 5.4770076E-06, 6.7015208E-06,
     2 8.1726989E-06, 9.9954201E-06, 1,2194425E-05, 1.4909101E-05,
     3 1.8194388E-05, 2.2239184E-05, 2.7145562E-05, 3.3174088E-05,
     4 4.0499452E-05, 4.9486730E-05, 6.0421440E-05, 7.3822001E-05,
     5 9.0141902E-05, 1.1012552E-04, 1.3448017E-04, 1.6428337E-04,
     6 2.0062570E-04, 2.4507680E-04, 2.9930366E-04, 3.6560582E-04,
     7 4.4651421E-04, 5.4541300E-04, 6.6512648E-04, 8.1365181E-04,
     8 9.9374786E-04, 1.2138120E-03, 1.4824945E-03, 1.8107657E-03,
     9 2.2115938E-03, 2.7012678E-03, 3.2991969E-03, 4.0295817E-03,
     1 4.9214244E-03, 6.0106700E-03, 7.3405529E-03, 8.9643708E-03,
     2 1.0946310E-02, 1.3365017E-02, 1.6314985E-02, 1.9910907E-02,
     3 2.4289325E-02, 2.9612896E-02, 3.6070402E-02, 4.3876936E-02,
     4 5.3264829E-02, 6.4465091E-02, 7.7664144E-02, 9.2918324E-02,
     5 1.1000121E-01, 1.2811102E-01, 1.4543025E-01, 1.5832248E-01,
     6 1.6049224E-01, 1.4170064E-01, 8.8788108E-02,-1.1330934E-02,
     7-1.5331864E-01,-2.9094670E-01,-2.9004655E-01,-2.9708834E-02,
     8 3.9009601E-01, 1.7939785E-01,-4.1858139E-01, 1.5317216E-01,
     9 6.5184953E-02,-1.0751806E-01, 7.8429567E-02,-4.6019124E-02,
     1 2.5309571E-02,-1.3904823E-02, 7.8187120E-03,-4.5190369E-03/
      DATA Y3/
     1 2.6729062E-03,-1.6073718E-03, 9.7715622E-04,-5.9804407E-04,
     2 3.6749320E-04,-2.2635296E-04, 1.3960805E-04,-8.6172618E-05,
     3 5.3212947E-05,-3.2867888E-05, 2.0304203E-05,-1.2543926E-05,
     4 7.7499633E-06,-4.7882430E-06, 2.9584108E-06,-1.8278645E-06,
     5 1.1293571E-06,-6.9778174E-07, 4.3113019E-07,-2.6637753E-07,
     6 1.6458373E-07,-1.0168954E-07, 6.2829807E-08,-3.8819969E-08,
     7 2.3985272E-08,-1.4819520E-08, 9.1563774E-09,-5.6573541E-09,
     8 3.4954514E-09,-2.1597005E-09, 1.3343946E-09,-8.2447148E-10,
     9 5.0941033E-10,-3.1474631E-10, 1.9447072E-10,-1.2015685E-10,
     1 7.4241055E-11,-4.5871468E-11, 2.8343095E-11,-1.7513137E-11,
     2 6.9049613E-12/
C--$$ENDATA
C
```

```
      COMPLEX FUN,C,CO,CMAX
      DIMENSION T(2),TMAX(2)
      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
C--STATEMENT FUNCTIONS-- INCLUDES ABSCISSA GENERATION:
      CO(II)=FUN(EXP(-X+FLOAT(II)*.20-26.30455704))*YT(II)
      ZHANKO=(0.0,0.0)
      CMAX=(0.0,0.0)
      L=18
      DO 110 I=129,146
      C=CO(I)
      ZHANKO=ZHANKO+C
      TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
  110 CONTINUE
      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      DO 120 I=128,1,-1
      C=CO(I)
      ZHANKO=ZHANKO+C
      L=L+1
      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
  120 CONTINUE
  130 DO 140 I=147,193
      C=CO(I)
      ZHANKO=ZHANKO+C
      L=L+1
      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
  140 CONTINUE
      GO TO 190
  150 DO 160 I=1,128
      C=CO(I)
      ZHANKO=ZHANKO+C
      L=L+1
      IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
  160 CONTINUE
  170 DO 180 I=193,147,-1
      C=CO(I)
      ZHANKO=ZHANKO+C
      L=L+1
      IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
  180 CONTINUE
  190 RETURN
      END
```

```
      COMPLEX FUNCTION ZHANK1(X,FUN,TOL,L)
C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J1(G*B)*DG" DEFINED AS THE
C  COMPLEX HANKEL TRANSFORM OF ORDER 1 AND ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C      X     = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
C      FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
C              OF A REAL ARGUMENT G.
C           NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                 CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                 THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
C                 DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C              FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RHANK1" IS ADVISED;
C              HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
C              INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
C      TOL=       REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                 IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                 THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                 TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                 THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C              A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C              BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C              RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C              APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C              ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C      L=         RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                 CONVOLUTION (L DEPENDS ON TOL AND FUN).
C              MIN.L=15 AND MAX.L=236--WHICH COULD
C                 OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                 VERY FAST...
C
C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZHANK1; THE HANKEL
C  TRANSFORM IS THEN ZHANK1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE....
C
```

```
C--USAGE-- "ZHANK1" IS CALLED AS FOLLOWS;
C       ...
C       COMPLEX Z,ZHANK1,ZF
C       EXTERNAL ZF
C       ...
C       Z=ZHANK1(ALOG(B),ZF,TOL,L)/B
C       ...
C       END
C       COMPLEX FUNCTION ZF(G)
C       ...USER SUPPLIED CODE...
C       END
C
C--NOTES;
C       (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
C       STATEMENT FUNCTION C1(II) BELOW; HOWEVER,
C       THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
C       EXP-UNDERFLOW'S TO 0.0......
C       (2).  SOME NON-ANSI FORTRAN STATEMENTS ARE USED
C        (E.G; DO 20 I=85,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
C       THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
C
C
C
C--J1-EXTENDED FILTER WEIGHT ARRAYS;
C  NOTE; ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE (SEE STATEMENT FUNCTION C1(II) BELOW).
        DIMENSION WT(236),W1(76),W2(76),W3(76),W4(8)
        EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
     1 (WT(229),W4(1))
       DATA W1/
     1-8.8863805E-10, 1.1293811E-09,-1.2050872E-09, 1.2696232E-09,
     2-1.3223909E-09, 1.3642393E-09,-1.3969439E-09, 1.4225941E-09,
     3-1.4427475E-09, 1.4580582E-09,-1.4682563E-09, 1.4732179E-09,
     4-1.4735606E-09, 1.4719870E-09,-1.4727091E-09, 1.4828225E-09,
     5-1.5102619E-09, 1.5667732E-09,-1.6634522E-09, 1.8172900E-09,
     6-2.0412753E-09, 2.3595230E-09,-2.7861077E-09, 3.3592871E-09,
     7-4.0940172E-09, 5.0571015E-09,-6.2604109E-09, 7.8269461E-09,
     8-9.7514701E-09, 1.2267639E-08,-1.5312389E-08, 1.9339924E-08,
     9-2.4126297E-08, 3.0576829E-08,-3.8050204E-08, 4.8423732E-08,
     1-6.0051116E-08, 7.6787475E-08,-9.4700993E-08, 1.2192844E-07,
     2-1.4918997E-07, 1.9392737E-07,-2.3464786E-07, 3.0911127E-07,
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     3-3.6815394E-07, 4.9413800E-07,-5.7554168E-07, 7.9301529E-07,
     4-8.9502818E-07, 1.2794292E-06,-1.3811469E-06, 2.0789668E-06,
     5-2.1069398E-06, 3.4103188E-06,-3.1584463E-06, 5.6639045E-06,
     6-4.6059955E-06, 9.5561672E-06,-6.4142855E-06, 1.6440205E-05,
     7-8.2010619E-06, 2.8945217E-05,-8.6348466E-06, 5.2317398E-05,
     8-3.9915035E-06, 9.7273612E-05, 1.5220520E-05, 1.8614373E-04,
     9 7.2023760E-05, 3.6620099E-04, 2.2062958E-04, 7.3874539E-04,
     1 5.8623480E-04, 1.5226779E-03, 1.4538718E-03, 3.1930365E-03/
      DATA W2/
     1 3.4640868E-03, 6.7790892E-03, 8.0328420E-03, 1.4484339E-02,
     2 1.8201316E-02, 3.0866143E-02, 4.0106549E-02, 6.4527872E-02,
     3 8.4285526E-02, 1.2773175E-01, 1.6020907E-01, 2.1948043E-01,
     4 2.3636305E-01, 2.4895051E-01, 1.2586300E-01,-5.1060445E-02,
     5-3.4376222E-01,-2.9042175E-01, 1.1564736E-01, 4.9253231E-01,
     6-4.6748595E-01, 1.5280945E-01, 3.3348541E-02,-8.2485252E-02,
     7 7.9740630E-02,-6.6934498E-02, 5.5150465E-02,-4.5868721E-02,
     8 3.8651958E-02,-3.2935834E-02, 2.8303994E-02,-2.4475127E-02,
     9 2.1259541E-02,-1.8526278E-02, 1.6182037E-02,-1.4158101E-02,
     1 1.2402225E-02,-1.0873526E-02, 9.5392016E-03,-8.3723743E-03,
     2 7.3506490E-03,-6.4551136E-03, 5.6696335E-03,-4.9803353E-03,
     3 4.3752213E-03,-3.8438703E-03, 3.3772023E-03,-2.9672872E-03,
     4 2.6071877E-03,-2.2908274E-03, 2.0128794E-03,-1.7686706E-03,
     5 1.5540998E-03,-1.3655666E-03, 1.1999089E-03,-1.0543497E-03,
     6 9.2644973E-04,-8.1406693E-04, 7.1531559E-04,-6.2854459E-04,
     7 5.5229955E-04,-4.8530352E-04, 4.2643446E-04,-3.7470650E-04,
     8 3.2925334E-04,-2.8931382E-04, 2.5421910E-04,-2.2338147E-04,
     9 1.9628455E-04,-1.7247455E-04, 1.5155278E-04,-1.3316889E-04,
     1 1.1701502E-04,-1.0282066E-04, 9.0348135E-05,-7.9388568E-05/
      DATA W3/
     1 6.9758436E-05,-6.1296474E-05, 5.3860978E-05,-4.7327436E-05,
     2 4.1586435E-05,-3.6541840E-05, 3.2109174E-05,-2.8214208E-05,
     3 2.4791718E-05,-2.1784390E-05, 1.9141864E-05,-1.6819888E-05,
     4 1.4779578E-05,-1.2986765E-05, 1.1411426E-05,-1.0027182E-05,
     5 8.8108499E-06,-7.7420630E-06, 6.8029235E-06,-5.9777053E-06,
     6 5.2525892E-06,-4.6154325E-06, 4.0555653E-06,-3.5636118E-06,
     7 3.1313335E-06,-2.7514911E-06, 2.4177236E-06,-2.1244417E-06,
     8 1.8667342E-06,-1.6402859E-06, 1.4413051E-06,-1.2664597E-06,
     9 1.1128220E-06,-9.7781908E-07, 8.5919028E-07,-7.5494920E-07,
     1 6.6335060E-07,-5.8286113E-07, 5.1213358E-07,-4.4998431E-07,
     2 3.9537334E-07,-3.4738689E-07, 3.0522189E-07,-2.6817250E-07,
     3 2.3561831E-07,-2.0701397E-07, 1.8188012E-07,-1.5979545E-07,
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     4 1.4038968E-07,-1.2333746E-07, 1.0835294E-07,-9.5185048E-08,
     5 8.3613184E-08,-7.3443411E-08, 6.4505118E-08,-5.6648167E-08,
     6 4.9740428E-08,-4.3665572E-08, 3.8321109E-08,-3.3616717E-08,
     7 2.9472836E-08,-2.5819439E-08, 2.2594957E-08,-1.9745353E-08,
     8 1.7223359E-08,-1.4987869E-08, 1.3003472E-08,-1.1240058E-08,
     9 9.6723739E-09,-8.2794392E-09, 7.0438407E-09,-5.9509676E-09,
     1 4.9882405E-09,-4.1443813E-09, 3.4088114E-09,-2.7712762E-09/
      DATA W4/
     1 2.2217311E-09,-1.7504755E-09, 1.3485207E-09,-1.0080937E-09,
     2 7.2300885E-10,-4.8860666E-10, 3.0121413E-10,-9.1649798E-11/
C--$$ENDATA
C
      COMPLEX FUN,C,C1,CMAX
      DIMENSION T(2),TMAX(2)
      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:
      C1(II)=FUN(EXP(-X+FLOAT(II)*.20-17.0))*WT(II)
      ZHANK1=(0.0,0.0)
      CMAX=(0.0,0.0)
      L=13
      DO 10 I=86,98
      C=C1(I)
      ZHANK1=ZHANK1+C
      TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
   10 CONTINUE
      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 60
      CMAX=TOL*CMAX
      DO 20 I=85,1,-1
      C=C1(I)
      ZHANK1=ZHANK1+C
      L=L+1
      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 30
   20 CONTINUE
   30 DO 40 I=99,236
      C=C1(I)
      ZHANK1=ZHANK1+C
      L=L+1
      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 50
   40 CONTINUE
   50 RETURN
```

```
60   DO 70 I=1,85
     C=C1(I)
     ZHANK1=ZHANK1+C
     L=L+1
     IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 80
70   CONTINUE
80   DO 90 I=236,99,-1
     C=C1(I)
     ZHANK1=ZHANK1+C
     L=L+1
     IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 50
90   CONTINUE
     GO TO 50
     END
```

```
      COMPLEX FUNCTION ZFOURO(X,FUN,TOL,L)
C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*COS(G*B)*DG" DEFINED AS THE
C  COMPLEX FOURIER COSINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C        X     = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
C        FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
C                OF A REAL ARGUMENT G.
C               NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                  CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                  THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
C                  DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C                FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RFOURO" IS ADVISED;
C                HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
C                INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
C.       TOL=    REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                  IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                  THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                  TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                  THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C                A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C                BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C                RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C                APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C                ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C          L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                  CONVOLUTION (L DEPENDS ON TOL AND FUN).
C                MIN.L=24 AND MAX.L=281--WHICH COULD
C                  OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                  VERY FAST...
C
C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZFOURO; THE FOURIER
C  TRANSFORM IS THEN ZFOURO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE....
C
```

```
C--USAGE-- "ZFOURO" IS CALLED AS FOLLOWS:
C        ...
C        COMPLEX Z,ZFOURO,ZF
C        EXTERNAL ZF
C        ...
C        Z=ZFOURO(ALOG(B),ZF,TOL,L)/B
C        ...
C        END
C        COMPLEX FUNCTION ZF(G)
C        ...USER SUPPLIED CODE...
C        END
C
C--NOTES:
C        (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
C        STATEMENT FUNCTION CO(II) BELOW; HOWEVER,
C        THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
C        EXP-UNDERFLOW'S TO 0.0......
C        (2). SOME NON-ANSI FORTRAN STATEMENTS ARE USED
C         (E.G: DO 120 I=148,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
C        THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
C
C
C--COS-EXTENDED FILTER WEIGHT ARRAYS:
C   NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C   TO SAVE STORAGE (SEE STATEMENT FUNCTION CO(II) BELOW).
        DIMENSION YT(281),Y1(76),Y2(76),Y3(76),Y4(53)
        EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1)),
     1 (YT(229),Y4(1))
      DATA Y1/
     1 5.1178101E-14, 2.9433849E-14, 2.5492522E-14, 1.9034819E-14,
     2 6.4179780E-14, 1.3085746E-15, 1.1989957E-13,-1.2216234E-14,
     3 1.7534103E-13, 7.9373498E-15, 2.1235658E-13, 7.9981520E-14,
     4 2.3815757E-13, 1.9714260E-13, 2.8920132E-13, 3.4161340E-13,
     5 4.0349917E-13, 5.2203885E-13, 5.9837223E-13, 7.8015306E-13,
     6 8.8911655E-13, 1.1709731E-12, 1.3165595E-12, 1.7578463E-12,
     7 1.9538564E-12, 2.6289768E-12, 2.9167697E-12, 3.9044344E-12,
     8 4.3927341E-12, 5.7526904E-12, 6.6569552E-12, 8.4555678E-12,
     9 1.0063279E-11, 1.2487964E-11, 1.5134682E-11, 1.8501488E-11,
     1 2.2720051E-11, 2.7452598E-11, 3.4025443E-11, 4.0875985E-11,
     2 5.0751668E-11, 6.1094382E-11, 7.5492982E-11, 9.1445759E-11,
     3 1.1227336E-10, 1.3676464E-10, 1.6720269E-10, 2.0423244E-10,
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     4 2.4932743E-10, 3.0470661E-10, 3.7198526E-10, 4.5449934E-10,
     5 5.5502537E-10, 6.7793669E-10, 8.2810001E-10, 1.0112626E-09,
     6 1.2354800E-09, 1.5085255E-09, 1.8432253E-09, 2.2503397E-09,
     7 2.7499027E-09, 3.3569525E-09, 4.1025670E-09, 5.0077487E-09,
     8 6.1205950E-09, 7.4703399E-09, 9.1312760E-09, 1.1143911E-08,
     9 1.3622929E-08, 1.6623917E-08, 2.0324094E-08, 2.4798610E-08,
     1 3.0321709E-08, 3.6992986E-08, 4.5237482E-08, 5.5183434E-08/
      DATA Y2/
     1 6.7491070E-08, 8.2317946E-08, 1.0069271E-07, 1.2279375E-07,
     2 1.5022907E-07, 1.8316969E-07, 2.2413747E-07, 2.7322865E-07,
     3 3.3441046E-07, 4.0756197E-07, 4.9894278E-07, 6.0793233E-07,
     4 7.4443665E-07, 9.0679753E-07, 1.1107379E-06, 1.3525651E-06,
     5 1.6573073E-06, 2.0174273E-06, 2.4728798E-06, 3.0090445E-06,
     6 3.6898816E-06, 4.4879625E-06, 5.5059521E-06, 6.6935820E-06,
     7 8.2160716E-06, 9.9828691E-06, 1.2260527E-05, 1.4888061E-05,
     8 1.8296530E-05, 2.2202672E-05, 2.7305154E-05, 3.3109672E-05,
     9 4.0751046E-05, 4.9372484E-05, 6.0820947E-05, 7.3619571E-05,
     1 9.0780005E-05, 1.0976837E-04, 1.3550409E-04, 1.6365676E-04,
     2 2.0227521E-04, 2.4398338E-04, 3.0197018E-04, 3.6370760E-04,
     3 4.5083748E-04, 5.4213238E-04, 6.7315347E-04, 8.0800951E-04,
     4 1.0051938E-03, 1.2041401E-03, 1.5011708E-03, 1.7942344E-03,
     5 2.2421056E-03, 2.6730676E-03, 3.3490681E-03, 3.9815050E-03,
     6 5.0028666E-03, 5.9285688E-03, 7.4730905E-03, 8.8233510E-03,
     7 1.1160132E-02, 1.3119527E-02, 1.6653199E-02, 1.9472767E-02,
     8 2.4800811E-02, 2.8793704E-02, 3.6762063E-02, 4.2228780E-02,
     9 5.3905163E-02, 6.0804660E-02, 7.7081738E-02, 8.3874501E-02,
     1 1.0377190E-01, 1.0377718E-01, 1.1892208E-01, 9.0437429E-02/
      DATA Y3/
     1 7.1685138E-02,-3.9473064E-02,-1.5078720E-01,-4.0489859E-01,
     2-5.6018995E-01,-6.8050388E-01,-1.5094224E-01, 6.6304064E-01,
     3 1.3766748E+00,-8.0373222E-01,-1.0869629E+00, 1.2812892E+00,
     4-5.0341082E-01,-4.4274455E-02, 2.0913102E-01,-1.9999661E-01,
     5 1.5207664E-01,-1.0920260E-01, 7.8169956E-02,-5.6651561E-02,
     6 4.1611799E-02,-3.0860012E-02, 2.3072559E-02,-1.7311631E-02,
     7 1.3021442E-02,-9.8085025E-03, 7.3943529E-03,-5.5769518E-03,
     8 4.2073164E-03,-3.1745026E-03, 2.3954154E-03,-1.8076122E-03,
     9 1.3640816E-03,-1.0293934E-03, 7.7682952E-04,-5.8623518E-04,
     1 4.4240399E-04,-3.3386183E-04, 2.5195025E-04,-1.9013541E-04,
     2 1.4348659E-04,-1.0828284E-04, 8.1716174E-05,-6.1667509E-05,
     3 4.6537684E-05,-3.5119887E-05, 2.6503388E-05,-2.0000904E-05,
     4 1.5093768E-05,-1.1390572E-05, 8.5959318E-06,-6.4869407E-06,
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     5 4.8953713E-06,-3.6942830E-06, 2.7878625E-06,-2.1038241E-06,
     6 1.5875917E-06,-1.1980090E-06, 9.0398030E-07,-6.8208296E-07,
     7 5.1458650E-07,-3.8817581E-07, 2.9272267E-07,-2.2067921E-07,
     8 1.6623514E-07,-1.2514102E-07, 9.4034535E-08,-7.0556837E-08,
     9 5.2741581E-08,-3.9298610E-08, 2.9107255E-08,-2.1413893E-08,
     1 1.5742032E-08,-1.1498608E-08, 8.7561571E-09,-7.2959446E-09/
      DATA Y4/
     1 6.8816619E-09,-8.9679825E-09, 1.4258275E-08,-1.9564299E-08,
     2 2.0235313E-08,-1.4725545E-08, 5.4632820E-09, 3.5995580E-09,
     3-9.5287133E-09, 1.1460041E-08,-1.0250532E-08, 7.4641748E-09,
     4-4.4703465E-09, 2.0499053E-09,-4.4806353E-10,-4.0374336E-10,
     5 7.0321001E-10,-5.7067960E-10, 4.9130404E-10,-2.8840747E-10,
     6 1.2373144E-10,-1.5260443E-11,-4.2027559E-11, 6.1885474E-11,
     7-5.9273937E-11, 4.6588766E-11,-3.2054182E-11, 1.9631637E-11,
     8-1.1210098E-11, 5.9567021E-12,-3.2427812E-12, 2.1353868E-12,
     9-1.8476851E-12, 1.8438474E-12,-1.8362842E-12, 1.7241847E-12,
     1-1.5161479E-12, 1.2627657E-12,-1.0129176E-12, 7.9578625E-13,
     2-6.2131435E-13, 4.8745900E-13,-3.8703630E-13, 3.1172547E-13,
     3-2.5397802E-13, 2.0824130E-13,-1.7123163E-13, 1.4113344E-13,
     4-1.1687986E-13, 9.7664016E-14,-8.2977176E-14, 7.2515267E-14,
     5-5.6047478E-14/
C--$$ENDATA
C
      COMPLEX FUN,C,CO,CMAX
      DIMENSION T(2),TMAX(2)
      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION!
      CO(II)=FUN(EXP(-X+FLOAT(II)*.20-30.30251236))*YT(II)
      ZFOURO=(0.0,0.0)
      CMAX=(0.0,0.0)
      L=22
      DO 110 I=149,170
      C=CO(I)
      ZFOURO=ZFOURO+C
      TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
  110 CONTINUE
      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      DO 120 I=148,1,-1
      C=CO(I)
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      ZFOURO=ZFOURO+C
      L=L+1
      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
  120 CONTINUE
  130 DO 140 I=171,281
      C=CO(I)
      ZFOURO=ZFOURO+C
      L=L+1
      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
  140 CONTINUE
      GO TO 190
  150 DO 160 I=1,148
      C=CO(I)
      ZFOURO=ZFOURO+C
      L=L+1
      IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
  160 CONTINUE
  170 DO 180 I=281,171,-1
      C=CO(I)
      ZFOURO=ZFOURO+C
      L=L+1
      IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
  180 CONTINUE
  190 RETURN
      END
```

```
      COMPLEX FUNCTION ZFOUR1(X,FUN,TOL,L)
C--INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*SIN(G*B)*DG" DEFINED AS THE
C  COMPLEX FOURIER SINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C     X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
C     FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
C             OF A REAL ARGUMENT G.
C            NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C               CALLING PROGRAM AND IN SUBPROGRAM FUN.
C               THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
C               DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C             FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RFOUR1" IS ADVISED;
C             HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
C             INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
C     TOL=    REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C               IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C               THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C               TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C               THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C             A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C             BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C             RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C             APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C             ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C       L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C               CONVOLUTION (L DEPENDS ON TOL AND FUN).
C             MIN.L=20 AND MAX.L=266--WHICH COULD
C               OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C               VERY FAST...
C
C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZFOUR1; THE FOURIER
C  TRANSFORM IS THEN ZFOUR1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE....
C
```

```
C--USAGE-- "ZFOUR1" IS CALLED AS FOLLOWS:
C        ...
C        COMPLEX Z,ZFOUR1,ZF
C        EXTERNAL ZF
C        ...
C        Z=ZFOUR1(ALOG(B),ZF,TOL,L)/B
C        ...
C        END
C        COMPLEX FUNCTION ZF(G)
C        ...USER SUPPLIED CODE...
C        END
C
C--NOTES:
C        (1), EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE
C        STATEMENT FUNCTION C1(II) BELOW; HOWEVER,
C        THIS IS OK PROVIDED THE MACHINE SYSTEM SETS ANY & ALL
C        EXP-UNDERFLOW'S TO 0.0.....
C        (2), SOME NON-ANSI FORTRAN STATEMENTS ARE USED
C         (E.G: DO 20 I=190,1,-1), BUT IT WOULD BE SIMPLE TO CONVERT
C        THESE STATEMENTS TO ANSI FORTRAN, IF NECESSARY...
C
C
C--SIN-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE (SEE STATEMENT FUNCTION C1(II) BELOW).
         DIMENSION WT(266),W1(76),W2(76),W3(76),W4(38)
         EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
      1 (WT(229),W4(1))
       DATA W1/
      1-1.1113940E-09,-1.3237246E-12, 1.5091739E-12,-1.6240954E-12,
      2 1.7236636E-12,-1.8227727E-12, 1.9255992E-12,-2.0335514E-12,
      3 2.1473541E-12,-2.2675549E-12, 2.3946842E-12,-2.5292661E-12,
      4 2.6718110E-12,-2.8227693E-12, 2.9825171E-12,-3.1514006E-12,
      5 3.3297565E-12,-3.5179095E-12, 3.7163306E-12,-3.9256378E-12,
      6 4.1464798E-12,-4.3794552E-12, 4.6252131E-12,-4.8845227E-12,
      7 5.1582809E-12,-5.4474462E-12, 5.7530277E-12,-6.0760464E-12,
      8 6.4175083E-12,-6.7783691E-12, 7.1595239E-12,-7.5618782E-12,
      9 7.9864477E-12,-8.4344110E-12, 8.9072422E-12,-9.4067705E-12,
      1 9.9349439E-12,-1.0493731E-11, 1.1084900E-11,-1.1709937E-11,
      2 1.2370354E-11,-1.3067414E-11, 1.3802200E-11,-1.4575980E-11,
      3 1.5390685E-11,-1.6249313E-11, 1.7156934E-11,-1.8115250E-11,
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     4 1.9131898E-11,-2.0209795E-11, 2.1352159E-11,-2.2561735E-11,
     5 2.3840976E-11,-2.5192263E-11, 2.6618319E-11,-2.8122547E-11,
     6 2.9709129E-11,-3.1382870E-11, 3.3149030E-11,-3.5013168E-11,
     7 3.6981050E-11,-3.9058553E-11, 4.1251694E-11,-4.3566777E-11,
     8 4.6010537E-11,-4.8590396E-11, 5.1314761E-11,-5.4193353E-11,
     9 5.7236720E-11,-6.0455911E-11, 6.3861222E-11,-6.7461492E-11,
     1 7.1265224E-11,-7.5279775E-11, 7.9512249E-11,-8.3971327E-11/
      DATA W2/
     1 8.8668961E-11,-9.3621900E-11, 9.8851764E-11,-1.0438319E-10,
     2 1.1024087E-10,-1.1644680E-10, 1.2301979E-10,-1.2997646E-10,
     3 1.3733244E-10,-1.4510363E-10, 1.5330772E-10,-1.6196550E-10,
     4 1.7110130E-10,-1.8074257E-10, 1.9091922E-10,-2.0166306E-10,
     5 2.1300756E-10,-2.2498755E-10, 2.3763936E-10,-2.5100098E-10,
     6 2.6511250E-10,-2.8001616E-10, 2.9575691E-10,-3.1238237E-10,
     7 3.2994314E-10,-3.4849209E-10, 3.6808529E-10,-3.8878042E-10,
     8 4.1063982E-10,-4.3372666E-10, 4.5811059E-10,-4.8386049E-10,
     9 5.1105728E-10,-5.3977672E-10, 5.7011632E-10,-6.0215516E-10,
     1 6.3601273E-10,-6.7175964E-10, 7.0955028E-10,-7.4942601E-10,
     2 7.9161025E-10,-8.3606980E-10, 8.8317110E-10,-9.3270330E-10,
     3 9.8533749E-10,-1.0404508E-09, 1.0993731E-09,-1.1605442E-09,
     4 1.2267391E-09,-1.2942905E-09, 1.3691677E-09,-1.4429912E-09,
     5 1.5288164E-09,-1.6077524E-09, 1.7085998E-09,-1.7890471E-09,
     6 1.9129068E-09,-1.9857116E-09, 2.1491608E-09,-2.1926779E-09,
     7 2.4312660E-09,-2.3959044E-09, 2.7872500E-09,-2.5610596E-09,
     8 3.2762318E-09,-2.6082940E-09, 4.0261453E-09,-2.3560563E-09,
     9 5.3176554E-09,-1.3960161E-09, 7.7708747E-09, 1.1853546E-09,
     1 1.2760851E-08, 7.4264707E-09, 2.3342187E-08, 2.1869851E-08/
      DATA W3/
     1 4.6306744E-08, 5.4631686E-08, 9.6763087E-08, 1.2823337E-07,
     2 2.0832812E-07, 2.9280540E-07, 4.5580888E-07, 6.5992437E-07,
     3 1.0056815E-06, 1.4779183E-06, 2.2284335E-06, 3.2994604E-06,
     4 4.9485823E-06, 7.3545473E-06, 1.1001083E-05, 1.6380539E-05,
     5 2.4469550E-05, 3.6469246E-05, 5.4441527E-05, 8.1176726E-05,
     6 1.2113828E-04, 1.8066494E-04, 2.6954609E-04, 4.0202288E-04,
     7 5.9969995E-04, 8.9437312E-04, 1.3338166E-03, 1.9886697E-03,
     8 2.9643943E-03, 4.4168923E-03, 6.5773518E-03, 9.7855105E-03,
     9 1.4539361E-02, 2.1558670E-02, 3.1871864E-02, 4.6903518E-02,
     1 6.8559512E-02, 9.9170152E-02, 1.4120770E-01, 1.9610835E-01,
     2 2.6192603E-01, 3.2743321E-01, 3.6407406E-01, 3.1257559E-01,
     3 9.0460168E-02,-3.6051039E-01,-8.6324760E-01,-8.1178720E-01,
     4 5.2205241E-01, 1.5449873E+00,-1.1817933E+00,-2.6759896E-01,
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     5 8,0869203E-01,-6,2757149E-01, 3,4062630E-01,-1,5885304E-01,
     6 7,0472984E-02,-3,1624462E-02, 1,4894068E-02,-7,4821176E-03,
     7 4,0035936E-03,-2,2543784E-03, 1,3160358E-03,-7,8636604E-04,
     8 4,7658745E-04,-2,9125817E-04, 1,7885105E-04,-1,1012416E-04,
     9 6,7910334E-05,-4,1914054E-05, 2,5881544E-05,-1,5985851E-05,
     1 9,8751880E-06,-6,1008526E-06, 3,7692543E-06,-2,3287953E-06/
      DATA W4/
     1 1,4388425E-06,-8,8899353E-07, 5,4926991E-07,-3,3937048E-07,
     2 2,0968284E-07,-1,2955437E-07, 8,0046336E-08,-4,9457371E-08,
     3 3,0557711E-08,-1,8880390E-08, 1,1665454E-08,-7,2076428E-09,
     4 4,4533423E-09,-2,7515596E-09, 1,7001092E-09,-1,0504494E-09,
     5 6,4904567E-10,-4,0102999E-10, 2,4778763E-10,-1,5310321E-10,
     6 9,4600354E-11,-5,8453314E-11, 3,6119400E-11,-2,2320056E-11,
     7 1,3793460E-11,-8,5242656E-12, 5,2675102E-12,-3,2543076E-12,
     8 2,0097689E-12,-1,2405412E-12, 7,6530538E-13,-4,7191929E-13,
     9 2,9084993E-13,-1,7923661E-13, 1,1018948E-13,-6,7885902E-14,
     1 4,2025050E-14,-2,1314731E-14/
C--$$ENDATA
C
C
      COMPLEX FUN,C,C1,CMAX
      DIMENSION T(2),TMAX(2)
      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
C--STATEMENT FUNCTION-- INCLUDES ABSCISSA GENERATION:
      C1(II)=FUN(EXP(-X+FLOAT(II)*,20-38,30455704))*WT(II)
      ZFOUR1=(0,0,0,0)
      CMAX=(0,0,0,0)
      L=18
      DO 10 I=191,208
      C=C1(I)
      ZFOUR1=ZFOUR1+C
      TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
   10 CONTINUE
      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 60
      CMAX=TOL*CMAX
      DO 20 I=190,1,-1
      C=C1(I)
      ZFOUR1=ZFOUR1+C
      L=L+1
      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 30
```

```
20    CONTINUE
30    DO 40 I=209,266
      C=C1(I)
      ZFOUR1=ZFOUR1+C
      L=L+1
      IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 50
40    CONTINUE
50    RETURN
60    DO 70 I=1,190
      C=C1(I)
      ZFOUR1=ZFOUR1+C
      L=L+1
      IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 80
70    CONTINUE
80    DO 90 I=266,209,-1
      C=C1(I)
      ZFOUR1=ZFOUR1+C
      L=L+1
      IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 50
90    CONTINUE
      GO TO 50
      END
```

```
      REAL FUNCTION RLAGHO(X,FUN,TOL,L,NEW)
C-**** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
C  INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J0(G*B)*DG" DEFINED AS THE
C  REAL HANKEL TRANSFORM OF ORDER 0 AND ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C    * X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
C               "RLAGHO" IS USEFUL ONLY WHEN X=(LAST X)-.20 *** I.E.,
C               SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
C               THEN SUBPROGRAM "RHANKO" IS ADVISED FOR GENERAL USE.
C               (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).
C     FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
C             NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                   CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                   THE REAL FUNCTION FUN SHOULD BE A MONOTONE
C                   DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C       TOL=        REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                   IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                   THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                   TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                   THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C               A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C               BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C               RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C               APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C               ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C       L=          RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                   CONVOLUTION (L DEPENDS ON TOL AND FUN).
C               MIN.L=20 AND MAX.L=193--WHICH COULD
C                   OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                   VERY FAST...
C     * NEW=    1 IS NECESSARY 1ST TIME OR BRAND NEW X.
C               0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)-0.20
C                 IS ASSUMED INTERNALLY BY THIS ROUTINE.
C                 NOTE: IF THIS IS NOT TRUE, ROUTINE WILL
```

```
C              STILL ASSUME X=(LAST X)-0.20 ANYWAY...
C              IT IS THE USERS RESPONSIBILITY TO NORMALIZE
C              BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
C            THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
C              TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
C              SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
C              ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
C              THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
C              KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
C              WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
C              WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
C
C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RLAGHO; THE HANKEL
C  TRANSFORM IS THEN RLAGHO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
C
C--USAGE-- "RLAGHO" IS CALLED AS FOLLOWS:
C        ...
C        EXTERNAL RF
C        ...
C        R=RLAGHO(ALOG(B),RF,TOL,L,NEW)/B
C        ...
C        END
C        REAL FUNCTION RF(G)
C        ...USER SUPPLIED CODE...
C        END
C
C--NOTES:
C        (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
C        BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
C        ANY & ALL EXP-UNDERFLOW'S TO 0.0....
C        (2).  AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
C        METHOD, LET BMAX>=BMIN>0 BE GIVEN. THEN IT CAN BE SHOWN
C        THAT THE ACTUAL NUMBER OF B'S IS NB=AINT(5.*ALOG(BMAX/BMIN))+1,
C        PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
C        BMINA=BMAX*EXP(-.2*(NB-1)).  THE METHOD GENERATES THE DECREASING
C        ARGUMENTS SPACED AS X=ALOG(BMAX),X-.2,X-.2*2,...,ALOG(BMINA).
C        FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
C            ...
C            NB=AINT(5.*ALOG(BMAX/BMIN))+1
C            NB1=NB+1
```

```
C            X0=ALOG(BMAX)+.2
C            NEW=1
C            DO 1 I=NB,1,-1
C            X=X0-.2*(NB1-I)
C            ARG(I)=EXP(X)
C            ANS(I)=RLAGH0(X,RF,TOL,L,NEW)/ARG(I)
C       1    NEW=0
C            ...
C       (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),ANS(I),I=1,NB FOR
C       ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
C       TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
C       SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
C       (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
C       ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
C       BMAX,BMIN AND BY SETTING NEW=1,...
C
C
C--J0-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE.
      DIMENSION YT(193),Y1(76),Y2(76),Y3(41)
      EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1))
     DATA Y1/
    1 5.8565723E-08, 7.1143477E-11,-7.8395565E-11, 8.7489547E-11,
    2-8.9007811E-11, 9.8790055E-11,-9.8675347E-11, 1.1118797E-10,
    3-1.0893474E-10, 1.2543400E-10,-1.1979399E-10, 1.4200767E-10,
    4-1.3106341E-10, 1.6153229E-10,-1.4238607E-10, 1.8486236E-10,
    5-1.5315381E-10, 2.1319755E-10,-1.6238115E-10, 2.4824144E-10,
    6-1.6850378E-10, 2.9243813E-10,-1.6909302E-10, 3.4934366E-10,
    7-1.6043759E-10, 4.2417082E-10,-1.3690001E-10, 5.2458440E-10,
    8-8.9946096E-11, 6.6188220E-10,-6.6964033E-12, 8.5276151E-10,
    9 1.3222770E-10, 1.1219600E-09, 3.5521442E-10, 1.5061956E-09,
    1 7.0795382E-10, 2.0600379E-09, 1.2535947E-09, 2.8646623E-09,
    2 2.0904225E-09, 4.0409101E-09, 3.3642886E-09, 5.7687700E-09,
    3 5.2930786E-09, 8.3164338E-09, 8.2021809E-09, 1.2083635E-08,
    4 1.2577400E-08, 1.7666303E-08, 1.9143895E-08, 2.5953011E-08,
    5 2.8983953E-08, 3.8268851E-08, 4.3712685E-08, 5.6590075E-08,
    6 6.5740136E-08, 8.3864288E-08, 9.8662323E-08, 1.2448811E-07,
    7 1.4784461E-07, 1.8501974E-07, 2.2129198E-07, 2.7524203E-07,
    8 3.3094739E-07, 4.0974828E-07, 4.9462868E-07, 6.1030809E-07,
    9 7.3891802E-07, 9.0939667E-07, 1.1034727E-06, 1.3554600E-06,
```

```
     1 1.6474556E-06, 2.0207696E-06, 2.4591294E-06, 3.0131400E-06/
      DATA Y2/
     1 3.6701680E-06, 4.4934101E-06, 5.4770076E-06, 6.7015208E-06,
     2 8.1726989E-06, 9.9954201E-06, 1.2194425E-05, 1.4909101E-05,
     3 1.8194368E-05, 2.2239184E-05, 2.7145562E-05, 3.3174088E-05,
     4 4.0499457E-05, 4.9486730E-05, 6.0421440E-05, 7.3822001E-05,
     5 9.0141902E-05, 1.1012552E-04, 1.3448017E-04, 1.6428337E-04,
     6 2.0062570E-04, 2.4507680E-04, 2.9930366E-04, 3.6560582E-04,
     7 4.4651421E-04, 5.4541300E-04, 6.6612648E-04, 8.1365181E-04,
     8 9.9374786E-04, 1.2138120E-03, 1.4824945E-03, 1.8107657E-03,
     9 2.2115938E-03, 2.7012675E-03, 3.2991969E-03, 4.0295817E-03,
     1 4.9214744E-03, 6.0106700E-03, 7.3405529E-03, 8.9643708E-03,
     2 1.0946310E-02, 1.3365017E-02, 1.6314985E-02, 1.9910907E-02,
     3 2.4289325E-02, 2.9612896E-02, 3.6070402E-02, 4.3876936E-02,
     4 5.3264829E-02, 6.4465091E-02, 7.7664144E-02, 9.2918324E-02,
     5 1.1000121E-01, 1.2811102E-01, 1.4543025E-01, 1.5832248E-01,
     6 1.6049224E-01, 1.4170064E-01, 8.8788108E-02,-1.1330934E-02,
     7-1.5331864E-01,-2.9094670E-01,-2.9084655E-01,-2.9708834E-02,
     8 3.9009601E-01, 1.7999785E-01,-4.1858139E-01, 1.5317216E-01,
     9 6.5184953E-02,-1.0751806E-01, 7.8429567E-02,-4.6019124E-02,
     1 2.5309571E-02,-1.3904823E-02, 7.8187120E-03,-4.5190369E-03/
      DATA Y3/
     1 2.6729062E-03,-1.6073718E-03, 9.7715622E-04,-5.9804407E-04,
     2 3.6749320E-04,-2.2635296E-04, 1.3960805E-04,-8.6172618E-05,
     3 5.3212947E-05,-3.2867888E-05, 2.0304203E-05,-1.2543926E-05,
     4 7.7499633E-06,-4.7882430E-06, 2.9584108E-06,-1.8278645E-06,
     5 1.1293571E-06,-6.9778174E-07, 4.3113019E-07,-2.6637753E-07,
     6 1.6458373E-07,-1.0168954E-07, 6.2829807E-08,-3.8819969E-08,
     7 2.3985272E-08,-1.4819520E-08, 9.1563774E-09,-5.6573541E-09,
     8 3.4954514E-09,-2.1597005E-09, 1.3343946E-09,-8.2447148E-10,
     9 5.0941033E-10,-3.1474631E-10, 1.9447072E-10,-1.2015685E-10,
     1 7.4241055E-11,-4.5871468E-11, 2.8343095E-11,-1.7513137E-11,
     2 6.9049613E-12/
C--$$ENDATA
C
      DIMENSION KEY(193),SAVE(193)
      IF(NEW) 10,30,10
10    LAG=-1
      X0=-X-26.30455704
      DO 20 IR=1,193
20    KEY(IR)=0
```

```
30    LAG=LAG+1
      RLAGHO=0.0
      CMAX=0.0
      L=0
      ASSIGN 110 TO M
      I=129
      GO TO 200
110   CMAX=AMAX1(ABS(C),CMAX)
      I=I+1
      IF(I.LE.146) GO TO 200
      IF(CMAX.EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      ASSIGN 120 TO M
      I=128
      GO TO 200
120   IF(ABS(C).LE.CMAX) GO TO 130
      I=I-1
      IF(I.GT.0) GO TO 200
130   ASSIGN 140 TO M
      I=147
      GO TO 200
140   IF(ABS(C).LE.CMAX) GO TO 190
      I=I+1
      IF(I.LE.193) GO TO 200
      GO TO 190
150   ASSIGN 160 TO M
      I=1
      GO TO 200
160   IF(C.EQ.0.0) GO TO 170
      I=I+1
      IF(I.LE.128) GO TO 200
170   ASSIGN 180 TO M
      I=193
      GO TO 200
180   IF(C.EQ.0.0) GO TO 190
      I=I-1
      IF(I.GE.147) GO TO 200
190   RETURN
C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
200   LOOK=I+LAG
      IQ=LOOK/194
```

```
      IR=MOD(LOOK,194)
      IF(IR.EQ.0) IR=1
      IROLL=IO*193
      IF(KEY(IR).LE.IROLL) GO TO 220
210   C=SAVE(IR)*YT(I)
      RLAGHO=RLAGHO+C
      L=L+1
      GO TO M,(110,120,140,160,180)
220   KEY(IR)=IROLL+IR
      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
      GO TO 210
      END
```

```
      REAL FUNCTION RLAGH1(X,FUN,TOL,L,NEW)
C-**** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
C  INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J1(G*B)*DG" DEFINED AS THE
C  REAL HANKEL TRANSFORM OF ORDER 1 AND ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C         * X     = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
C                   "RLAGH1" IS USEFUL ONLY WHEN X=(LAST X)-.20 *** I.E.,
C                   SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
C                   THEN SUBPROGRAM "RHANK1" IS ADVISED FOR GENERAL USE.
C                   (ALSO SEE PARM 'NEW' & NOTES (2)-(3) BELOW).
C         FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
C                 NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                       CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                       THE REAL FUNCTION FUN SHOULD BE A MONOTONE
C                       DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C             TOL=      REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                       IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                       THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                       TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                       THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C                   A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C                   BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C                   RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C                   APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C                   ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C             L=        RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                       CONVOLUTION (L DEPENDS ON TOL AND FUN).
C                   MIN.L=15 AND MAX.L=236--WHICH COULD
C                       OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                       VERY FAST...
C           * NEW=  1 IS NECESSARY 1ST TIME OR BRAND NEW X.
C           *       0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)-0.20
C                   IS ASSUMED INTERNALLY BY THIS ROUTINE.
C                       NOTE:  IF THIS IS NOT TRUE, ROUTINE WILL
```

```
C              STILL ASSUME X=(LAST X)-0.20 ANYWAY...
C              IT IS THE USERS RESPONSIBILITY TO NORMALIZE
C              BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
C            THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
C              TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
C              SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
C              ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
C              THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
C              KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
C              WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
C              WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
C
C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RLAGH1; THE HANKEL
C  TRANSFORM IS THEN RLAGH1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
C
C--USAGE-- "RLAGH1" IS CALLED AS FOLLOWS:
C     ...
C     EXTERNAL RF
C     ...
C     R=RLAGH1(ALOG(B),RF,TOL,L,NEW)/B
C     ...
C     END
C     REAL FUNCTION RF(G)
C     ...USER SUPPLIED CODE...
C     END
C
C--NOTES:
C     (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
C     BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
C     ANY & ALL EXP-UNDERFLOW'S TO 0.0...
C     (2).  AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
C     METHOD, LET BMAX>=BMIN>0 BE GIVEN. THEN IT CAN BE SHOWN
C     THAT THE ACTUAL NUMBER OF B'S IS NB=AINT(5.*ALOG(BMAX/BMIN))+1,
C     PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
C     BMINA=BMAX*EXP(-.2*(NB-1)).  THE METHOD GENERATES THE DECREASING
C     ARGUMENTS SPACED AS X=ALOG(BMAX),X-.2,X-.2*2,...,ALOG(BMINA).
C     FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
C              ...
C              NB=AINT(5.*ALOG(BMAX/BMIN))+1
C              NB1=NB+1
```

```
C             X0=ALOG(BMAX)+.2
C             NEW=1
C             DO 1 I=NB,1,-1
C             X=X0-.2*(NB1-I)
C             ARG(I)=EXP(X)
C             ANS(I)=RLAGH1(X,RF,TOL,L,NEW)/ARG(I)
C         1   NEW=0
C             ...
C        (3).  IF RESULTS ARE STORED IN ARRAYS ARG(I),ANS(I),I=1,NB FOR
C        ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
C        TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
C        SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
C        (4).  IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
C        ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
C        BMAX,BMIN AND BY SETTING NEW=1....
C
C
C--J1-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE.
      DIMENSION WT(236),W1(76),W2(76),W3(76),W4(8)
      EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
     1 (WT(229),W4(1))
      DATA W1/
     1-8.8863805E-10, 1.1293811E-09,-1.2050872E-09, 1.2696232E-09,
     2-1.3223909E-09, 1.3642393E-09,-1.3969439E-09, 1.4225941E-09,
     3-1.4427475E-09, 1.4580582E-09,-1.4682563E-09, 1.4732179E-09,
     4-1.4735606E-09, 1.4719870E-09,-1.4727091E-09, 1.4828225E-09,
     5-1.5102619E-09, 1.5667752E-09,-1.6634522E-09, 1.8172900E-09,
     6-2.0412753E-09, 2.3595230E-09,-2.7861077E-09, 3.3592871E-09,
     7-4.0940172E-09, 5.0571015E-09,-6.2604109E-09, 7.8269461E-09,
     8-9.7514701E-09, 1.2267639E-08,-1.5312389E-08, 1.9339924E-08,
     9-2.4126297E-08, 3.0576829E-08,-3.8060204E-08, 4.8423732E-08,
     1-6.0051116E-08, 7.6787475E-08,-9.4700993E-08, 1.2192844E-07,
     2-1.4918997E-07, 1.9392737E-07,-2.3464786E-07, 3.0911127E-07,
     3-3.6815394E-07, 4.9413800E-07,-5.7554168E-07, 7.9301529E-07,
     4-8.9502818E-07, 1.2794292E-06,-1.3811469E-06, 2.0789668E-06,
     5-2.1069398E-06, 3.4103188E-06,-3.1584463E-06, 5.6639045E-06,
     6-4.6059955E-06, 9.5561672E-06,-6.4142855E-06, 1.6440205E-05,
     7-8.2010619E-06, 2.8945217E-05,-8.6348466E-06, 5.2317398E-05,
     8-3.9915035E-06, 9.7273612E-05, 1.5220520E-05, 1.8614373E-04,
```

```
     9 7.2023760E-05, 3.6620099E-04, 2.2062958E-04, 7.3874539E-04,
     1 5.8623480E-04, 1.5226779E-03, 1.4538718E-03, 3.1930365E-03/
      DATA W2/
     1 3.4640868E-03, 6.7790882E-03, 8.0328420E-03, 1.4484339E-02,
     2 1.8201316E-02, 3.0866143E-02, 4.0106549E-02, 6.4527872E-02,
     3 8.4285526E-02, 1.2773175E-01, 1.6020907E-01, 2.1948043E-01,
     4 2.3636305E-01, 2.4895051E-01, 1.2586300E-01,-5.1060445E-02,
     5-3.4376222E-01,-2.9042176E-01, 1.1564736E-01, 4.9253231E-01,
     6-4.6748595E-01, 1.5280945E-01, 3.3348541E-02,-8.2485252E-02,
     7 7.9740630E-02,-6.6934498E-02, 5.5150465E-02,-4.5868721E-02,
     8 3.8651958E-02,-3.2935834E-02, 2.8303994E-02,-2.4475127E-02,
     9 2.1259541E-02,-1.8526278E-02, 1.6182037E-02,-1.4158101E-02,
     1 1.2402225E-02,-1.0873526E-02, 9.5392016E-03,-8.3723743E-03,
     2 7.3506490E-03,-6.4551136E-03, 5.6696335E-03,-4.9803353E-03,
     3 4.3752213E-03,-3.8438703E-03, 3.3772023E-03,-2.9672872E-03,
     4 2.6071877E-03,-2.2908274E-03, 2.0128794E-03,-1.7686706E-03,
     5 1.5540998E-03,-1.3655666E-03, 1.1999089E-03,-1.0543497E-03,
     6 9.2644973E-04,-8.1406593E-04, 7.1531557E-04,-6.2854459E-04,
     7 5.5229955E-04,-4.8530352E-04, 4.2643446E-04,-3.7470650E-04,
     8 3.2925334E-04,-2.8931382E-04, 2.5421910E-04,-2.2338147E-04,
     9 1.9628455E-04,-1.7247455E-04, 1.5155278E-04,-1.3316889E-04,
     1 1.1701502E-04,-1.0282066E-04, 9.0348135E-05,-7.9388568E-05/
      DATA W3/
     1 6.9758436E-05,-6.1296474E-05, 5.3860978E-05,-4.7327436E-05,
     2 4.1586435E-05,-3.6541840E-05, 3.2109174E-05,-2.8214208E-05,
     3 2.4791718E-05,-2.1784390E-05, 1.9141864E-05,-1.6819888E-05,
     4 1.4779578E-05,-1.2986765E-05, 1.1411426E-05,-1.0027182E-05,
     5 8.8108499E-06,-7.7420630E-06, 6.8029235E-06,-5.9777053E-06,
     6 5.2525892E-06,-4.6154325E-06, 4.0555653E-06,-3.5636118E-06,
     7 3.1313335E-06,-2.7514911E-06, 2.4177236E-06,-2.1244417E-06,
     8 1.8667342E-06,-1.6402859E-06, 1.4413051E-06,-1.2664597E-06,
     9 1.1128220E-06,-9.7781908E-07, 8.5919028E-07,-7.5494920E-07,
     1 6.6335060E-07,-5.8286113E-07, 5.1213358E-07,-4.4998431E-07,
     2 3.9537334E-07,-3.4738689E-07, 3.0522189E-07,-2.6817250E-07,
     3 2.3561831E-07,-2.0701397E-07, 1.8188012E-07,-1.5979545E-07,
     4 1.4038968E-07,-1.2333746E-07, 1.0835294E-07,-9.5185048E-08,
     5 8.3613184E-08,-7.3443411E-08, 6.4505118E-08,-5.6648167E-08,
     6 4.9740428E-08,-4.3665572E-08, 3.8321109E-08,-3.3616717E-08,
     7 2.9472836E-08,-2.5819439E-08, 2.2594957E-08,-1.9745353E-08,
     8 1.7223359E-08,-1.4987869E-08, 1.3003472E-08,-1.1240058E-08,
     9 9.6723739E-09,-8.2794392E-09, 7.0438407E-09,-5.9509676E-09,
```

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      1 4.9882405E-09,-4.1443813E-09, 3.4088114E-09,-2.7712762E-09/
       DATA W4/
      1 2.2217311E-09,-1.7504753E-09, 1.3485207E-09,-1.0080937E-09,
      2 7.2300885E-10,-4.8860666E-10, 3.0121413E-10,-9.1649798E-11/
C--$$ENDDATA
C
      DIMENSION KEY(236),SAVE(236)
      IF(NEW) 10,30,10
   10 LAG=-1
      X0=-X-17.0
      DO 20 IR=1,236
   20 KEY(IR)=0
   30 LAG=LAG+1
      RLAGH1=0.0
      CMAX=0.0
      L=0
      ASSIGN 110 TO M
      I=86
      GO TO 200
  110 CMAX=AMAX1(ABS(C),CMAX)
      I=I+1
      IF(I.LE.98) GO TO 200
      IF(CMAX.EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      ASSIGN 120 TO M
      I=85
      GO TO 200
  120 IF(ABS(C).LE.CMAX) GO TO 130
      I=I-1
      IF(I.GT.0) GO TO 200
  130 ASSIGN 140 TO M
      I=99
      GO TO 200
  140 IF(ABS(C).LE.CMAX) GO TO 190
      I=I+1
      IF(I.LE.236) GO TO 200
      GO TO 190
  150 ASSIGN 160 TO M
      I=1
      GO TO 200
  160 IF(C.EQ.0.0) GO TO 170
```

```
      I=I+1
      IF(I.LE.85) GO TO 200
170   ASSIGN 180 TO M
      I=236
      GO TO 200
180   IF(C.EQ.0.0) GO TO 190
      I=I-1
      IF(I.GE.99) GO TO 200
190   RETURN
C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
200   LOOK=I+LAG
      IO=LOOK/237
      IR=MOD(LOOK,237)
      IF(IR.EQ.0) IR=1
      IROLL=IO*236
      IF(KEY(IR).LE.IROLL) GO TO 220
210   C=SAVE(IR)*WT(I)
      RLAGH1=RLAGH1+C
      L=L+1
      GO TO M,(110,120,140,160,180)
220   KEY(IR)=IROLL+IR
      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
      GO TO 210
      END
```

```
      REAL FUNCTION RLAGFO(X,FUN,TOL,L,NEW)
C--*** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
C  INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*COS(G*B)*DG" DEFINED AS THE
C  REAL FOURIER COSINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C     * X    = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
C              "RLAGFO" IS USEFUL ONLY WHEN X=(LAST X)-.20 *** I.E.,
C              SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
C              THEN SUBPROGRAM "RFOURO" IS ADVISED FOR GENERAL USE.
C              (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).
C      FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
C            NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                THE REAL FUNCTION FUN SHOULD BE A MONOTONE
C                DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C        TOL=    REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C              A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C              BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C              RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C              APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C              ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C        L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                CONVOLUTION (L DEPENDS ON TOL AND FUN).
C              MIN.L=24 AND MAX.L=281--WHICH COULD
C                OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                VERY FAST...
C     * NEW=   1 IS NECESSARY 1ST TIME OR BRAND NEW X.
C              0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)-0.20
C                IS ASSUMED INTERNALLY BY THIS ROUTINE.
C                NOTE:  IF THIS IS NOT TRUE, ROUTINE WILL
```

```
C              STILL ASSUME X=(LAST X)-0.20 ANYWAY...
C              IT IS THE USERS RESPONSIBILITY TO NORMALIZE
C              BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
C             THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
C              TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
C              SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
C              ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
C              THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
C              KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
C              WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
C              WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
C
C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RLAGFO; THE FOURIER
C  TRANSFORM IS THEN RLAGFO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
C
C--USAGE-- "RLAGFO" IS CALLED AS FOLLOWS;
C       ...
C       EXTERNAL RF
C       ...
C       R=RLAGFO(ALOG(B),RF,TOL,L,NEW)/B
C       ...
C       END
C       REAL FUNCTION RF(G)
C       ...USER SUPPLIED CODE...
C       END
C
C--NOTES;
C       (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
C       BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
C       ANY & ALL EXP-UNDERFLOW'S TO 0.0....
C       (2).  AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
C       METHOD, LET BMAX>=BMIN>0 BE GIVEN. THEN IT CAN BE SHOWN
C       THAT THE ACTUAL NUMBER OF B'S IS NB=AINT(5.*ALOG(BMAX/BMIN))+1,
C       PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
C       BMINA=BMAX*EXP(-.2*(NB-1)).  THE METHOD GENERATES THE DECREASING
C       ARGUMENTS SPACED AS X=ALOG(BMAX),X-.2,X-.2*2,...,ALOG(BMINA).
C       FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE;
C       ...
C             NB=AINT(5.*ALOG(BMAX/BMIN))+1
C             NB1=NB+1
```

```
C             X0=ALOG(BMAX)+.2
C             NEW=1
C             DO 1 I=NB,1,-1
C             X=X0-.2*(NB1-I)
C             ARG(I)=EXP(X)
C             ANS(I)=RLAGF0(X,RF,TOL,L,NEW)/ARG(I)
C         1   NEW=0
C             ...
C       (3).  IF RESULTS ARE STORED IN ARRAYS ARG(I),ANS(I),I=1,NB FOR
C       ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
C       TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
C       SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
C       (4).  IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
C       ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
C       BMAX,BMIN AND BY SETTING NEW=1....
C
C
C--COS-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE.
      DIMENSION YT(281),Y1(76),Y2(76),Y3(76),Y4(53)
      EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1)),
     1 (YT(229),Y4(1))
      DATA Y1/
     1 5.1178101E-14, 2.9433849E-14, 2.5492522E-14, 1.9034819E-14,
     2 6.4179780E-14, 1.3085746E-15, 1.1989957E-13,-1.2216234E-14,
     3 1.7534103E-13, 7.9373498E-15, 2.1235658E-13, 7.9981520E-14,
     4 2.3815757E-13, 1.9714260E-13, 2.8920132E-13, 3.4161340E-13,
     5 4.0349917E-13, 5.2203885E-13, 5.9837223E-13, 7.8015306E-13,
     6 8.8911655E-13, 1.1709731E-12, 1.3165595E-12, 1.7578463E-12,
     7 1.9538564E-12, 2.6289768E-12, 2.9167697E-12, 3.9044344E-12,
     8 4.3927341E-12, 5.7526904E-12, 6.6569552E-12, 8.4555678E-12,
     9 1.0063229E-11, 1.2487964E-11, 1.5134682E-11, 1.8501488E-11,
     1 2.2720051E-11, 2.7452598E-11, 3.4025443E-11, 4.0875985E-11,
     2 5.0751668E-11, 6.1094382E-11, 7.5492982E-11, 9.1445759E-11,
     3 1.1227336E-10, 1.3676464E-10, 1.6720259E-10, 2.0423244E-10,
     4 2.4932743E-10, 3.0470661E-10, 3.7192526E-10, 4.5449934E-10,
     5 5.5502537E-10, 6.7793669E-10, 8.2810001E-10, 1.0112626E-09,
     6 1.2354800E-09, 1.5085255E-09, 1.8432753E-09, 2.2503397E-09,
     7 2.7499027E-09, 3.3569525E-09, 4.1025670E-09, 5.0077487E-09,
     8 6.1205950E-09, 7.4703399E-09, 9.1312760E-09, 1.1143911E-08,
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     9 1.3622929E-08, 1.6623917E-08, 2.0324094E-08, 2.4798610E-08,
     1 3.0321709E-08, 3.6992986E-08, 4.5237482E-08, 5.5183434E-08/
      DATA Y2/
     1 6.7491070E-08, 8.2317946E-08, 1.0069271E-07, 1.2279375E-07,
     2 1.5022907E-07, 1.8316969E-07, 2.2413747E-07, 2.7322865E-07,
     3 3.3441046E-07, 4.0756197E-07, 4.9894278E-07, 6.0793233E-07,
     4 7.4443665E-07, 9.0679753E-07, 1.1107379E-06, 1.3525651E-06,
     5 1.6573073E-06, 2.0174273E-06, 2.4728798E-06, 3.0090445E-06,
     6 3.6898816E-06, 4.4879625E-06, 5.5059521E-06, 6.6935820E-06,
     7 8.2160716E-06, 9.9828691E-06, 1.2260527E-05, 1.4888061E-05,
     8 1.8296530E-05, 2.2202672E-05, 2.7305154E-05, 3.3109672E-05,
     9 4.0751046E-05, 4.9372484E-05, 6.0820947E-05, 7.3619571E-05,
     1 9.0780005E-05, 1.0976837E-04, 1.3550409E-04, 1.6365676E-04,
     2 2.0227521E-04, 2.4398338E-04, 3.0197018E-04, 3.6370760E-04,
     3 4.5083748E-04, 5.4213338E-04, 6.7315347E-04, 8.0800951E-04,
     4 1.0051938E-03, 1.2041401E-03, 1.5011708E-03, 1.7942344E-03,
     5 2.2421056E-03, 2.6730676E-03, 3.3490681E-03, 3.9815050E-03,
     6 5.0028666E-03, 5.9285668E-03, 7.4730905E-03, 8.8233510E-03,
     7 1.1160132E-02, 1.3119627E-02, 1.6653199E-02, 1.9472767E-02,
     8 2.4800811E-02, 2.8793704E-02, 3.6762063E-02, 4.2228780E-02,
     9 5.3905163E-02, 6.0804660E-02, 7.7081738E-02, 8.3874501E-02,
     1 1.0377190E-01, 1.0377718E-01, 1.1892208E-01, 9.0437429E-02/
      DATA Y3/
     1 7.1685138E-02,-3.9473064E-02,-1.5078720E-01,-4.0489859E-01,
     2-5.6018995E-01,-6.8050388E-01,-1.5094224E-01, 6.6304064E-01,
     3 1.3766748E+00,-8.0373222E-01,-1.0869629E+00, 1.2812892E+00,
     4-5.0341082E-01,-4.4274455E-02, 2.0913107E-01,-1.9999661E-01,
     5 1.5207664E-01,-1.0920260E-01, 7.8169956E-02,-5.6651561E-02,
     6 4.1611799E-02,-3.0880012E-02, 2.3077559E-02,-1.7311631E-02,
     7 1.3021442E-02,-9.8085025E-03, 7.3943529E-03,-5.5769518E-03,
     8 4.2073164E-03,-3.1745026E-03, 2.3954154E-03,-1.8076122E-03,
     9 1.3640816E-03,-1.0293934E-03, 7.7682952E-04,-5.8623518E-04,
     1 4.4240399E-04,-3.3386183E-04, 2.5195025E-04,-1.9013541E-04,
     2 1.4348659E-04,-1.0828284E-04, 8.1716174E-05,-6.1667509E-05,
     3 4.6537684E-05,-3.5119887E-05, 2.6503388E-05,-2.0000904E-05,
     4 1.5093768E-05,-1.1390572E-05, 8.5959318E-06,-6.4869407E-06,
     5 4.8953713E-06,-3.6942930E-06, 2.7878625E-06,-2.1038241E-06,
     6 1.5875917E-06,-1.1980090E-06, 9.0398030E-07,-6.8208296E-07,
     7 5.1458650E-07,-3.8817581E-07, 2.9272267E-07,-2.2067921E-07,
     8 1.6623514E-07,-1.2514102E-07, 9.4034535E-08,-7.0556837E-08,
     9 5.2741581E-08,-3.9298610E-08, 2.9107255E-08,-2.1413893E-08,
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      1 1,5742032E-08,-1,1498608E-08, 8,7561571E-09,-7,2959446E-09/
       DATA Y4/
      1 6,8816619E-09,-8,9679825E-09, 1,4258275E-08,-1,9564299E-08,
      2 2,0235313E-08,-1,4725545E-08, 5,4632820E-09, 3,5995580E-09,
      3-9,5287133E-09, 1,1460041E-08,-1,0250532E-08, 7,4641748E-09,
      4-4,4703465E-09, 2,0499053E-09,-4,4006353E-10,-4,0374336E-10,
      5 7,0321001E-10,-6,7067960E-10, 4,9130404E-10,-2,8840747E-10,
      6 1,2373144E-10,-1,5260443E-11,-4,2027559E-11, 6,1885474E-11,
      7-5,9273937E-11, 4,6588766E-11,-3,2054182E-11, 1,9831637E-11,
      8-1,1210098E-11, 5,9567021E-12,-3,2427812E-12, 2,1353868E-12,
      9-1,8476651E-12, 1,8438474E-12,-1,8362842E-12, 1,7241847E-12,
      1-1,5161479E-12, 1,2627657E-12,-1,0129176E-12, 7,9578625E-13,
      2-6,2131435E-13, 4,8743900E-13,-3,8703630E-13, 3,1172547E-13,
      3-2,5397802E-13, 2,0824130E-13,-1,7123163E-13, 1,4113344E-13,
      4-1,1687986E-13, 9,7664016E-14,-8,2977176E-14, 7,2515267E-14,
      5-5,6047478E-14/
C--$$ENDATA
       DIMENSION KEY(281),SAVE(281)
       IF(NEW) 10,30,10
10     LAG=-1
       X0=-X-30,30251236
       DO 20 IR=1,281
20     KEY(IR)=0
30     LAG=LAG+1
       RLAGF0=0,0
       CMAX=0,0
       L=0
       ASSIGN 110 TO M
       I=149
       GO TO 200
110    CMAX=AMAX1(ABS(C),CMAX)
       I=I+1
       IF(I,LE,170) GO TO 200
       IF(CMAX,EQ,0,0) GO TO 150
       CMAX=TOL*CMAX
       ASSIGN 120 TO M
       I=148
       GO TO 200
120    IF(ABS(C),LE,CMAX) GO TO 130
       I=I-1
       IF(I,GT,0) GO TO 200
```

```
130     ASSIGN 140 TO M
        I=171
        GO TO 200
140     IF(ABS(C).LE.CMAX) GO TO 190
        I=I+1
        IF(I.LE.281) GO TO 200
        GO TO 190
150     ASSIGN 160 TO M
        I=1
        GO TO 200
160     IF(C.EQ.0.0) GO TO 170
        I=I+1
        IF(I.LE.148) GO TO 200
170     ASSIGN 180 TO M
        I=281
        GO TO 200
180     IF(C.EQ.0.0) GO TO 190
        I=I-1
        IF(I.GE.171) GO TO 200
190     RETURN
C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
200     LOOK=I+LAG
        IQ=LOOK/282
        IR=MOD(LOOK,282)
        IF(IR.EQ.0) IR=1
        IROLL=IQ*281
        IF(KEY(IR).LE.IROLL) GO TO 220
210     C=SAVE(IR)*YT(I)
        RLAGF0=RLAGF0+C
        L=L+1
        GO TO M,(110,120,140,160,180)
220     KEY(IR)=IROLL+IR
        SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
        GO TO 210
        END
```

```
      REAL FUNCTION RLAGF1(X,FUN,TOL,L,NEW)
C----- A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
C  INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*SIN(G*B)*DG" DEFINED AS THE
C  REAL FOURIER SINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH REAL FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C  * X     * REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
C            "RLAGF1" IS USEFUL ONLY WHEN X=(LAST X)-.20 *** I.E.,
C            SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
C            THEN SUBPROGRAM "RFOUR1" IS ADVISED FOR GENERAL USE.
C            (ALSO SEE PARM !NEW! & NOTES (2)-(4) BELOW).
C    FUN(G)= EXTERNAL DECLARED REAL FUNCTION NAME (USER SUPPLIED).
C          NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C              CALLING PROGRAM AND IN SUBPROGRAM FUN.
C              THE REAL FUNCTION FUN SHOULD BE A MONOTONE
C              DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C      TOL=    REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C              IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C              THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C              TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C              THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C            A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C            BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C            RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C            APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C            ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C      L=      RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C              CONVOLUTION (L DEPENDS ON TOL AND FUN).
C            MIN.L=20 AND MAX.L=266--WHICH COULD
C              OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C              VERY FAST...
C    * NEW=  1 IS NECESSARY 1ST TIME OR BRAND NEW X.
C            0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)-0.20
C            IS ASSUMED INTERNALLY BY THIS ROUTINE.
C              NOTE:  IF THIS IS NOT TRUE, ROUTINE WILL
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```
C               STILL ASSUME X=(LAST X)-0.20 ANYWAY...
C               IT IS THE USERS RESPONSIBILITY TO NORMALIZE
C               BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
C            THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
C               TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
C               SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
C               ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
C               THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
C               KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
C               WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
C               WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
C
C--THE RESULTING REAL CONVOLUTION SUM IS GIVEN IN RLAGF1; THE FOURIER
C  TRANSFORM IS THEN RLAGF1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
C
C--USAGE-- "RLAGF1" IS CALLED AS FOLLOWS;
C        ...
C        EXTERNAL RF
C        ...
C        R=RLAGF1(ALOG(B),RF,TOL,L,NEW)/B
C        ...
C        END
C        REAL FUNCTION RF(G)
C        ...USER SUPPLIED CODE...
C        END
C
C--NOTES;
C        (1). EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
C        BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
C        ANY & ALL EXP-UNDERFLOW'S TO 0.0....
C        (2). AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
C        METHOD, LET BMAX>=BMIN>0 BE GIVEN. THEN IT CAN BE SHOWN
C        THAT THE ACTUAL NUMBER OF B'S IS NB=AINT(5.*ALOG(BMAX/BMIN))+1,
C        PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
C        BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING
C        ARGUMENTS SPACED AS X=ALOG(BMAX),X-.2,X-.2*2,...,ALOG(BMINA).
C        FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE;
C        ...
C            NB=AINT(5.*ALOG(BMAX/BMIN))+1
C            NB1=NB+1
```

```
C           X0=ALOG(BMAX)+.2
C           NEW=1
C           DO 1 I=NB,1,-1
C           X=X0-.2*(NB1-I)
C           ARG(I)=EXP(X)
C           ANS(I)=RLAGF1(X,RF,TOL,L,NEW)/ARG(I)
C         1 NEW=0
C           ...
C       (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),ANS(I),I=1,NB FOR
C       ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
C       TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
C       SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
C       (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
C       ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
C       BMAX,BMIN AND BY SETTING NEW=1....
C
C
C--SIN-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE.
      DIMENSION WT(266),W1(76),W2(76),W3(76),W4(38)
      EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
     1 (WT(229),W4(1))
      DATA W1/
     1-1.1113940E-09,-1.3237246E-12, 1.5091739E-12,-1.6240954E-12,
     2 1.7236636E-12,-1.8227727E-12, 1.9255992E-12,-2.0335514E-12,
     3 2.1473541E-12,-2.2675549E-12, 2.3946842E-12,-2.5292661E-12,
     4 2.6718110E-12,-2.8227693E-12, 2.9825171E-12,-3.1514006E-12,
     5 3.3297565E-12,-3.5179095E-12, 3.7163306E-12,-3.9256378E-12,
     6 4.1464798E-12,-4.3794552E-12, 4.6252131E-12,-4.8845227E-12,
     7 5.1582809E-12,-5.4474462E-12, 5.7530277E-12,-6.0760464E-12,
     8 6.4175083E-12,-6.7783691E-12, 7.1595239E-12,-7.5618782E-12,
     9 7.9864477E-12,-8.4344110E-12, 8.9072422E-12,-9.4067705E-12,
     1 9.9349439E-12,-1.0493731E-11, 1.1084900E-11,-1.1709937E-11,
     2 1.2370354E-11,-1.3067414E-11, 1.3802200E-11,-1.4575980E-11,
     3 1.5390685E-11,-1.6249313E-11, 1.7155934E-11,-1.8115250E-11,
     4 1.9131898E-11,-2.0209795E-11, 2.1352159E-11,-2.2561735E-11,
     5 2.3840976E-11,-2.5192263E-11, 2.6618319E-11,-2.8122547E-11,
     6 2.9709129E-11,-3.1382870E-11, 3.3149030E-11,-3.5013168E-11,
     7 3.6981050E-11,-3.9058553E-11, 4.1251694E-11,-4.3566777E-11,
     8 4.6010537E-11,-4.8590396E-11, 5.1314761E-11,-5.4193353E-11,
```

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     9 5.7236720E-11,-6.0455911E-11, 6.3861222E-11,-6.7461492E-11,
     1 7.1265224E-11,-7.5279775E-11, 7.9512249E-11,-8.3971327E-11/
      DATA W2/
     1 8.8668961E-11,-9.3621900E-11, 9.8851764E-11,-1.0438319E-10,
     2 1.1024087E-10,-1,1644680E-10, 1,2301979E-10,-1,2997646E-10,
     3 1,3733244E-10,-1,4510363E-10, 1,5330772E-10,-1,6196550E-10,
     4 1,7110130E-10,-1,8074257E-10, 1,9091922E-10,-2,0166306E-10,
     5 2,1300756E-10,-2,2498755E-10, 2,3763936E-10,-2,5100098E-10,
     6 2,6511250E-10,-2,8001616E-10, 2,9575691E-10,-3,1238237E-10,
     7 3,2994314E-10,-3,4849209E-10, 3,6808529E-10,-3,8878042E-10,
     8 4,1063982E-10,-4,3372666E-10, 4,5811059E-10,-4,8386049E-10,
     9 5,1105728E-10,-5,3977672E-10, 5,7011632E-10,-6,0215516E-10,
     1 6,3601273E-10,-6,7175964E-10, 7,0955028E-10,-7,4942601E-10,
     2 7,9161025E-10,-8,3606980E-10, 8,8317110E-10,-9,3270330E-10,
     3 9,8533749E-10,-1,0404508E-09, 1,0993731E-09,-1,1605442E-09,
     4 1,2267391E-09,-1,2942905E-09, 1,3691677E-09,-1,4479912E-09,
     5 1,5288164E-09,-1,6077524E-09, 1,7085998E-09,-1,7890471E-09,
     6 1,9129068E-09,-1,9857116E-09, 2,1491608E-09,-2,1926779E-09,
     7 2,4312660E-09,-2,3959044E-09, 2,7872500E-09,-2,5610596E-09,
     8 3,2762318E-09,-2,6082940E-09, 4,0261453E-09,-2,3560563E-09,
     9 5,3176554E-09,-1,3960161E-09, 7,7708747E-09, 1,1853546E-09,
     1 1,2760851E-08, 7,4264707E-09, 2,3342187E-08, 2,1869851E-08/
      DATA W3/
     1 4,6306744E-08, 5,4631686E-08, 9,6763087E-08, 1,2823337E-07,
     2 2,0832812E-07, 2,9280540E-07, 4,5580888E-07, 6,5992437E-07,
     3 1,0056815E-06, 1,4779183E-06, 2,2284335E-06, 3,2994604E-06,
     4 4,9485823E-06, 7,3545473E-06, 1,1001083E-05, 1,6380539E-05,
     5 2,4469550E-05, 3,6469246E-05, 5,4441527E-05, 8,1176726E-05,
     6 1,2113828E-04, 1,8066494E-04, 2,6954609E-04, 4,0202288E-04,
     7 5,9969995E-04, 8,9437312E-04, 1,3338166E-03, 1,9886697E-03,
     8 2,9643943E-03, 4,4168923E-03, 6,5773518E-03, 9,7855105E-03,
     9 1,4539361E-02, 2,1558670E-02, 3,1871864E-02, 4,6903518E-02,
     1 6,8559512E-02, 9,9170152E-02, 1,4120770E-01, 1,9610835E-01,
     2 2,6192603E-01, 3,2743321E-01, 3,6407406E-01, 3,1257559E-01,
     3 9,0460168E-02,-3,6051039E-01,-8,6324760E-01,-8,1178720E-01,
     4 5,2205241E-01, 1,5449873E+00,-1,1817933E+00,-2,6759896E-01,
     5 8,0869203E-01,-6,2757149E-01, 3,4062630E-01,-1,5885304E-01,
     6 7,0472984E-02,-3,1624462E-02, 1,4894068E-02,-7,4821176E-03,
     7 4,0035936E-03,-2,2543784E-03, 1,3160358E-03,-7,8636604E-04,
     8 4,7658745E-04,-2,9125817E-04, 1,7885105E-04,-1,1012416E-04,
     9 6,7910334E-05,-4,1914054E-05, 2,5881544E-05,-1,5985851E-05,
```

```
     1 9.8751880E-06,-6.1008526E-06, 3.7692543E-06,-2.3287953E-06/
      DATA W4/
     1 1.4388425E-06,-8.8899353E-07, 5.4926991E-07,-3.3937048E-07,
     2 2.0968284E-07,-1.2955437E-07, 8.0046336E-08,-4.9457371E-08,
     3 3.0557711E-08,-1.8880390E-08, 1.1665454E-08,-7.2076428E-09,
     4 4.4533423E-09,-2.7515696E-09, 1.7001092E-09,-1.0504494E-09,
     5 6.4904567E-10,-4.0102999E-10, 2.4778763E-10,-1.5310321E-10,
     6 9.4600354E-11,-5.8453314E-11, 3.6119400E-11,-2.2320056E-11,
     7 1.3793460E-11,-8.5242636E-12, 5.2675102E-12,-3.2543076E-12,
     8 2.0097689E-12,-1.2405412E-12, 7.6530538E-13,-4.7191929E-13,
     9 2.9084993E-13,-1.7923661E-13, 1.1018948E-13,-6.7885902E-14,
     1 4.2025050E-14,-2.1314731E-14/
C--$$ENDATA
C
      DIMENSION KEY(266),SAVE(266)
      IF(NEW) 10,30,10
10    LAG=-1
      X0=-X-38.30455704
      DO 20 IR=1,266
20    KEY(IR)=0
30    LAG=LAG+1
      RLAGF1=0.0
      CMAX=0.0
      L=0
      ASSIGN 110 TO M
      I=191
      GO TO 200
110   CMAX=AMAX1(ABS(C),CMAX)
      I=I+1
      IF(I.LE.208) GO TO 200
      IF(CMAX.EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      ASSIGN 120 TO M
      I=190
      GO TO 200
120   IF(ABS(C).LE.CMAX) GO TO 130
      I=I-1
      IF(I.GT.0) GO TO 200
130   ASSIGN 140 TO M
      I=209
      GO TO 200
```

```
140      IF(ABS(C).LE.CMAX) GO TO 190
         I=I+1
         IF(I.LE.266) GO TO 200
         GO TO 190
150      ASSIGN 160 TO M
         I=1
         GO TO 200
160      IF(C.EQ.0.0) GO TO 170
         I=I+1
         IF(I.LE.190) GO TO 200
170      ASSIGN 180 TO M
         I=266
         GO TO 200
180      IF(C.EQ.0.0) GO TO 190
         I=I-1
         IF(I.GE.209) GO TO 200
190      RETURN
C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
200      LOOK=I+LAG
         IQ=LOOK/267
         IR=MOD(LOOK,267)
         IF(IR.EQ.0) IR=1
         IROLL=IQ*266
         IF(KEY(IR).LE.IROLL) GO TO 220
210      C=SAVE(IR)*WT(I)
         RLAGF1=RLAGF1+C
         L=L+1
         GO TO M,(110,120,140,160,180)
220      KEY(IR)=IROLL+IR
         SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
         GO TO 210
         END
```

```
      COMPLEX FUNCTION ZLAGHO(X,FUN,TOL,L,NEW)
C--*** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
C  INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J0(G*B)*DG" DEFINED AS THE
C  COMPLEX HANKEL TRANSFORM OF ORDER 0 AND ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C      * X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
C                 "ZLAGHO" IS USEFUL ONLY WHEN X=(LAST X)-.20 *** I.E.,
C                 SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
C                 THEN SUBPROGRAM "ZHANKO" IS ADVISED FOR GENERAL USE.
C                 (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).
C     FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
C                 OF A REAL ARGUMENT G.
C             NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                   CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                   THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
C                   DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C                 FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RLAGHO" IS ADVISED;
C                 HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
C                 INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
C       TOL=        REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                   IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                   THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                   TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                   THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C                 A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C                 BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C                 RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C                 APPROXIMATION INDICATOR.. FOR VERY LARGE OR SMALL B,
C                 ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C       L=          RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                   CONVOLUTION (L DEPENDS ON TOL AND FUN).
C                 MIN.L=20 AND MAX.L=193--WHICH COULD
C                   OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                   VERY FAST...
```

```
C        * NEW*  1 IS NECESSARY 1ST TIME OR BRAND NEW X,
C                0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)-0.20
C                  IS ASSUMED INTERNALLY BY THIS ROUTINE.
C                  NOTE: IF THIS IS NOT TRUE, ROUTINE WILL
C                  STILL ASSUME X=(LAST X)-0.20 ANYWAY...
C                  IT IS THE USERS RESPONSIBILITY TO NORMALIZE
C                  BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
C                THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
C                  TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
C                  SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
C                  ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
C                  THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
C                  KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
C                  WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
C                  WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
C
C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZLAGHO; THE HANKEL
C  TRANSFORM IS THEN ZLAGHO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
C
C--USAGE-- "ZLAGHO" IS CALLED AS FOLLOWS:
C
C        ...
C        COMPLEX Z,ZLAGHO,ZF
C        EXTERNAL ZF
C        ...
C        Z=ZLAGHO(ALOG(B),ZF,TOL,L,NEW)/B
C        ...
C        END
C        COMPLEX FUNCTION ZF(G)
C        ...USER SUPPLIED CODE...
C        END
C
C--NOTES:
C        (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
C        BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
C        ANY & ALL EXP-UNDERFLOW'S TO 0.0...
C        (2).  AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
C        METHOD, LET BMAX>=BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
C        THAT THE ACTUAL NUMBER OF B'S IS NB=AINT(5.*ALOG(BMAX/BMIN))+1,
C        PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
C        BMINA=BMAX*EXP(-.2*(NB-1)).  THE METHOD GENERATES THE DECREASING
```

```
C        ARGUMENTS SPACED AS X=ALOG(BMAX),X-.2,X-.2*2,...,ALOG(BMINA).
C        FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
C             ...
C             NB=AINT(5.*ALOG(BMAX/BMIN))+1
C             NB1=NB+1
C             X0=ALOG(BMAX)+.2
C             NEW=1
C             DO 1 I=NB,1,-1
C             X=X0-.2*(NB1-I)
C             ARG(I)=EXP(X)
C             Z(I)=ZLAGH0(X,ZF,TOL,L,NEW)/ARG(I)
C        1    NEW=0
C             ...
C        (3). IF RESULTS ARE STORED IN ARRAYS ARG(I),Z(I),I=1,NB FOR
C        ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
C        TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
C        SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
C        (4). IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
C        ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
C        BMAX,BMIN AND BY SETTING NEW=1....
C
C
C--J0-EXTENDED FILTER WEIGHT ARRAYS:
C   NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C   TO SAVE STORAGE.
      DIMENSION YT(193),Y1(76),Y2(76),Y3(41)
      EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1))
      DATA Y1/
     1 5.8565723E-08, 7.1143477E-11,-7.8395565E-11, 8.7489547E-11,
     2-8.9007811E-11, 9.8790055E-11,-9.8675347E-11, 1.1118797E-10,
     3-1.0893474E-10, 1.2543400E-10,-1.1979399E-10, 1.4200767E-10,
     4-1.3106341E-10, 1.6153279E-10,-1.4238602E-10, 1.8486236E-10,
     5-1.5315381E-10, 2.1319755E-10,-1.6238115E-10, 2.4824144E-10,
     6-1.6850378E-10, 2.9243813E-10,-1.6909302E-10, 3.4934366E-10,
     7-1.6043759E-10, 4.2417082E-10,-1.3690001E-10, 5.2458440E-10,
     8-8.9946096E-11, 6.6188220E-10,-6.6964033E-12, 8.5276151E-10,
     9 1.3222770E-10, 1.1219600E-09, 3.5591442E-10, 1.5061956E-09,
     1 7.0795382E-10, 2.0600379E-09, 1.2535947E-09, 2.8646623E-09,
     2 2.0904225E-09, 4.0409101E-09, 3.3642886E-09, 5.7687700E-09,
     3 5.2930786E-09, 8.3164338E-09, 8.2021809E-09, 1.2083635E-08,
     4 1.2577400E-08, 1.7666303E-08, 1.9143895E-08, 2.5953011E-08,
```

```
      GO TO 200
180   IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
      I=I-1
      IF(I.GE.147) GO TO 200
190   RETURN
C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
200   LOOK=I+LAG
      IQ=LOOK/194
      IR=MOD(LOOK,194)
      IF(IR.EQ.0) IR=1
      IROLL=IQ*193
      IF(KEY(IR).LE.IROLL) GO TO 220
210   C=SAVE(IR)*YT(I)
      ZLAGHO=ZLAGHO+C
      L=L+1
      GO TO M,(110,120,140,160,180)
220   KEY(IR)=IROLL+IR
      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
      GO TO 210
      END
```

```
      COMPLEX FUNCTION ZLAGH1(X,FUN,TOL,L,NEW)
C--*** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
C  INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*J1(G*B)*DG" DEFINED AS THE
C  COMPLEX HANKEL TRANSFORM OF ORDER 1 AND ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C      * X     = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE HANKEL TRANSFORM
C                "ZLAGH1" IS USEFUL ONLY WHEN X=(LAST X)-.20 *** I.E.,
C                SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
C                THEN SUBPROGRAM "ZHANK1" IS ADVISED FOR GENERAL USE.
C                (ALSO SEE PARM 'NEW' & NOTES (2)-(3) BELOW).
C      FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
C                OF A REAL ARGUMENT G.
C              NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                   CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                   THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
C                   DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C                FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RLAGH1" IS ADVISED;
C                HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
C                INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
C          TOL=  REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                   THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                   TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                   THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C                A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C                BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C                RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C                APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C                ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C          L=       RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                   CONVOLUTION (L DEPENDS ON TOL AND FUN).
C                MIN.L=15 AND MAX.L=236--WHICH COULD
C                   OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                   VERY FAST...
```

```
C          * NEW=  1 IS NECESSARY 1ST TIME OR BRAND NEW X,
C                  0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)-0,20
C                    IS ASSUMED INTERNALLY BY THIS ROUTINE,
C                    NOTE:  IF THIS IS NOT TRUE, ROUTINE WILL
C                    STILL ASSUME X=(LAST X)-0,20 ANYWAY,..
C                    IT IS THE USERS RESPONSIBILITY TO NORMALIZE
C                    BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
C                 THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
C                    TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
C                    SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
C                    ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
C                    THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
C                    KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
C                    WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
C                    WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
C
C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZLAGH1; THE HANKEL
C   TRANSFORM IS THEN ZLAGH1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C   THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
C
C--USAGE-- "ZLAGH1" IS CALLED AS FOLLOWS:
C        ...
C        COMPLEX Z,ZLAGH1,ZF
C        EXTERNAL ZF
C        ...
C        Z=ZLAGH1(ALOG(B),ZF,TOL,L,NEW)/B
C        ...
C        END
C        COMPLEX FUNCTION ZF(C)
C        ...USER SUPPLIED CODE...
C        END
C
C--NOTES:
C        (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
C        BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
C        ANY & ALL EXP-UNDERFLOW'S TO 0.0....
C        (2).  AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
C        METHOD, LET BMAX>=BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
C        THAT THE ACTUAL NUMBER OF B'S IS NB=AINT(5.*ALOG(BMAX/BMIN))+1,
C        PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
C        BMINA=BMAX*EXP(-.2*(NB-1)). THE METHOD GENERATES THE DECREASING
```

```
C           ARGUMENTS SPACED AS X=ALOG(BMAX),X-.2,X-.2*2,...,ALOG(BMINA).
C           FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
C           ...
C                NB=AINT(5.*ALOG(BMAX/BMIN))+1
C                NB1=NB+1
C                X0=ALOG(BMAX)+.2
C                NEW=1
C                DO 1 I=NB,1,-1
C                X=X0-.2*(NB1-I)
C                ARG(I)=EXP(X)
C                Z(I)=ZLAGH1(X,ZF,TOL,L,NEW)/ARG(I)
C           1    NEW=0
C           ...
C           (3).  IF RESULTS ARE STORED IN ARRAYS ARG(I),Z(I),I=1,NB FOR
C           ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
C           TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
C           SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
C           (4).  IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
C           ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
C           BMAX,BMIN AND BY SETTING NEW=1....
C
C
C--J1-EXTENDED FILTER WEIGHT ARRAYS:
C  NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C  TO SAVE STORAGE.
      DIMENSION WT(236),W1(76),W2(76),W3(76),W4(8)
      EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
     1 (WT(229),W4(1))
      DATA W1/
     1-8.8863805E-10, 1.1293811E-09,-1.2050872E-09, 1.2696232E-09,
     2-1.3223909E-09, 1.3642393E-09,-1.3969439E-09, 1.4225941E-09,
     3-1.4427475E-09, 1.4580582E-09,-1.4682563E-09, 1.4732179E-09,
     4-1.4735606E-09, 1.4719870E-09,-1.4727091E-09, 1.4828225E-09,
     5-1.5102619E-09, 1.5667752E-09,-1.6634522E-09, 1.8172900E-09,
     6-2.0412753E-09, 2.3595230E-09,-2.7861077E-09, 3.3592871E-09,
     7-4.0940172E-09, 5.0571015E-09,-6.2604109E-09, 7.8269461E-09,
     8-9.7514701E-09, 1.2267639E-08,-1.5312389E-08, 1.9339924E-08,
     9-2.4126297E-08, 3.0576829E-08,-3.8060204E-08, 4.8423732E-08,
     1-6.0051116E-08, 7.6787475E-08,-9.4700993E-08, 1.2192844E-07,
     2-1.4918997E-07, 1.9392737E-07,-2.3464786E-07, 3.0911127E-07,
     3-3.6815394E-07, 4.9413800E-07,-5.7554168E-07, 7.9301529E-07,
```

```
     4-8,9502818E-07, 1,2794292E-06,-1,3811469E-06, 2,0789668E-06,
     5-2,1069398E-06, 3,4103188E-06,-3,1584463E-06, 5,6639045E-06,
     6-4,6059955E-06, 9,5561672E-06,-6,4142855E-06, 1,6440205E-05,
     7-8,2010619E-06, 2,8945217E-05,-8,6348466E-06, 5,2317398E-05,
     8-3,9915035E-06, 9,7273612E-05, 1,5220520E-05, 1,8614373E-04,
     9 7,2023760E-05, 3,6620099E-04, 2,2062958E-04, 7,3874539E-04,
     1 5,8623480E-04, 1,5276779E-03, 1,4539718E-03, 3,1930365E-03/
      DATA W2/
     1 3,4640868E-03, 6,7790882E-03, 8,0328420E-03, 1,4484339E-02,
     2 1,8201316E-02, 3,0866143E-02, 4,0106549E-02, 6,4527872E-02,
     3 8,4285526E-02, 1,2773175E-01, 1,6020907E-01, 2,1948043E-01,
     4 2,3636305E-01, 2,4895051E-01, 1,2586300E-01,-5,1060445E-02,
     5-3,4376222E-01,-2,9042175E-01, 1,1564736E-01, 4,9253231E-01,
     6-4,6748595E-01, 1,5280945E-01, 3,3348541E-02,-8,2485252E-02,
     7 7,9740630E-02,-6,6934498E-02, 5,5150465E-02,-4,5868721E-02,
     8 3,8651958E-02,-3,2935834E-02, 2,8303994E-02,-2,4475127E-02,
     9 2,1259541E-02,-1,8526278E-02, 1,6182037E-02,-1,4158101E-02,
     1 1,2402225E-02,-1,0873526E-02, 9,5392016E-03,-8,3723743E-03,
     2 7,3506490E-03,-6,4551136E-03, 5,6696335E-03,-4,9803353E-03,
     3 4,3752213E-03,-3,8438703E-03, 3,3772023E-03,-2,9672872E-03,
     4 2,6071877E-03,-2,2908274E-03, 2,0128794E-03,-1,7686706E-03,
     5 1,5540998E-03,-1,3655666E-03, 1,1999099E-03,-1,0543497E-03,
     6 9,2644973E-04,-8,1406593E-04, 7,1531559E-04,-6,2854459E-04,
     7 5,5229955E-04,-4,8530352E-04, 4,2643446E-04,-3,7470650E-04,
     8 3,2925334E-04,-2,8931382E-04, 2,5421910E-04,-2,2338147E-04,
     9 1,9628455E-04,-1,7247455E-04, 1,5155278E-04,-1,3316889E-04,
     1 1,1701502E-04,-1,0282066E-04, 9,0343135E-05,-7,9388568E-05/
      DATA W3/
     1 6,9758436E-05,-6,1296474E-05, 5,3860978E-05,-4,7327436E-05,
     2 4,1586435E-05,-3,6541840E-05, 3,2109174E-05,-2,8214208E-05,
     3 2,4791718E-05,-2,1784390E-05, 1,9141864E-05,-1,6819888E-05,
     4 1,4779578E-05,-1,2986765E-05, 1,1411426E-05,-1,0027182E-05,
     5 8,8108499E-06,-7,7420630E-06, 6,8029235E-06,-5,9777053E-06,
     6 5,2525892E-06,-4,6154325E-06, 4,0555653E-06,-3,5636118E-06,
     7 3,1313335E-06,-2,7514911E-06, 2,4177236E-06,-2,1244417E-06,
     8 1,8667342E-06,-1,6402859E-06, 1,4413051E-06,-1,2664597E-06,
     9 1,1128220E-06,-9,7781908E-07, 8,5919028E-07,-7,5494920E-07,
     1 6,6335060E-07,-5,8286113E-07, 5,1213358E-07,-4,4998431E-07,
     2 3,9537334E-07,-3,4738689E-07, 3,0522189E-07,-2,6817250E-07,
     3 2,3561831E-07,-2,0701397E-07, 1,8188012E-07,-1,5979545E-07,
     4 1,4038968E-07,-1,2333746E-07, 1,0835294E-07,-9,5185048E-08,
```

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     5 8.3613184E-08,-7.3443411E-08, 6.4505118E-08,-5.6648167E-08,
     6 4.9740428E-08,-4.3665572E-08, 3.8321109E-08,-3.3616717E-08,
     7 2.9472836E-08,-2.5819439E-08, 2.2594957E-08,-1.9745353E-08,
     8 1.7223359E-08,-1.4987869E-08, 1.3003472E-08,-1.1240058E-08,
     9 9.6723739E-09,-8.2794392E-09, 7.0438407E-09,-5.9509676E-09,
     1 4.9882405E-09,-4.1443813E-09, 3.4088114E-09,-2.7712762E-09/
      DATA W4/
     1 2.2217311E-09,-1.7504755E-09, 1.3485207E-09,-1.0080937E-09,
     2 7.2300885E-10,-4.8860666E-10, 3.0121413E-10,-9.1649798E-11/
C--$$ENDDATA
C
      COMPLEX FUN,C,CMAX,SAVE
      DIMENSION KEY(236),SAVE(236),T(2),TMAX(2)
      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
      IF(NEW) 10,30,10
10    LAG=-1
      X0=-X-17.0
      DO 20 IR=1,236
20    KEY(IR)=0
30    LAG=LAG+1
      ZLAGH1=(0.0,0.0)
      CMAX=(0.0,0.0)
      L=0
      ASSIGN 110 TO M
      I=86
      GO TO 200
110   TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
      I=I+1
      IF(I.LE.98) GO TO 200
      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      ASSIGN 120 TO M
      I=85
      GO TO 200
120   IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
      I=I-1
      IF(I.GT.0) GO TO 200
130   ASSIGN 140 TO M
      I=99
      GO TO 200
```

```
     5 2.8983953E-08, 3.8268851E-08, 4.3712685E-08, 5.6590075E-08,
     6 6.5740136E-08, 8.3864288E-08, 9.8662323E-08, 1.2448811E-07,
     7 1.4784461E-07, 1.8501974E-07, 2.2129198E-07, 2.7524203E-07,
     8 3.3094738E-07, 4.0974828E-07, 4.9462868E-07, 6.1030809E-07,
     9 7.3891802E-07, 9.0939667E-07, 1.1034727E-06, 1.3554600E-06,
     1 1.6474556E-06, 2.0207696E-06, 2.4591294E-06, 3.0131400E-06/
      DATA Y2/
     1 3.6701680E-06, 4.4934101E-06, 5.4770076E-06, 6.7015208E-06,
     2 8.1726989E-06, 9.9954201E-06, 1.2194425E-05, 1.4909101E-05,
     3 1.8194388E-05, 2.2239184E-05, 2.7145562E-05, 3.3174088E-05,
     4 4.0499452E-05, 4.9486730E-05, 6.0421440E-05, 7.3822001E-05,
     5 9.0141902E-05, 1.1012552E-04, 1.3448017E-04, 1.6428337E-04,
     6 2.0062570E-04, 2.4507680E-04, 2.9930366E-04, 3.6560582E-04,
     7 4.4651421E-04, 5.4541300E-04, 6.6612648E-04, 8.1365181E-04,
     8 9.9374786E-04, 1.2138120E-03, 1.4824945E-03, 1.8107657E-03,
     9 2.2115938E-03, 2.7012675E-03, 3.2991969E-03, 4.0295817E-03,
     1 4.9214244E-03, 6.0106700E-03, 7.3405529E-03, 8.9643708E-03,
     2 1.0946310E-02, 1.3365017E-02, 1.6314985E-02, 1.9910907E-02,
     3 2.4289325E-02, 2.9612896E-02, 3.6070402E-02, 4.3876936E-02,
     4 5.3264829E-02, 6.4465091E-02, 7.7664144E-02, 9.2918324E-02,
     5 1.1000121E-01, 1.2811102E-01, 1.4543025E-01, 1.5832248E-01,
     6 1.6049224E-01, 1.4170064E-01, 8.8788108E-02,-1.1330934E-02,
     7-1.5331864E-01,-2.9094670E-01,-2.9084655E-01,-2.9708834E-02,
     8 3.9009601E-01, 1.7999785E-01,-4.1858139E-01, 1.5317216E-01,
     9 6.5184953E-02,-1.0751806E-01, 7.8429567E-02,-4.6019124E-02,
     1 2.5309571E-02,-1.3904823E-02, 7.8187120E-03,-4.5190369E-03/
      DATA Y3/
     1 2.6729062E-03,-1.6073718E-03, 9.7715622E-04,-5.9804407E-04,
     2 3.6749320E-04,-2.2635296E-04, 1.3960805E-04,-8.6172618E-05,
     3 5.3212947E-05,-3.2867888E-05, 2.0304203E-05,-1.2543926E-05,
     4 7.7499633E-06,-4.7882430E-06, 2.9584108E-06,-1.8278645E-06,
     5 1.1293571E-06,-6.9778174E-07, 4.3113019E-07,-2.6637753E-07,
     6 1.6458373E-07,-1.0168954E-07, 6.2829807E-08,-3.8819969E-08,
     7 2.3985272E-08,-1.4819520E-08, 9.1563774E-09,-5.6573541E-09,
     8 3.4954514E-09,-2.1597005E-09, 1.3343946E-09,-8.2447148E-10,
     9 5.0941033E-10,-3.1474631E-10, 1.9447072E-10,-1.2015685E-10,
     1 7.4241055E-11,-4.5871468E-11, 2.8343095E-11,-1.7513137E-11,
     2 6.9049613E-12/
C--$$ENDATA
C
      COMPLEX FUN,C,CMAX,SAVE
```

```
      DIMENSION KEY(193),SAVE(193),T(2),TMAX(2)
      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
      IF(NEW) 10,30,10
10    LAG=-1
      X0=-X-26.30455704
      DO 20 IR=1,193
20    KEY(IR)=0
30    LAG=LAG+1
      ZLAGHO=(0.0,0.0)
      CMAX=(0.0,0.0)
      L=0
      ASSIGN 110 TO M
      I=129
      GO TO 200
110   TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
      I=I+1
      IF(I.LE.146) GO TO 200
      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      ASSIGN 120 TO M
      I=128
      GO TO 200
120   IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
      I=I-1
      IF(I.GT.0) GO TO 200
130   ASSIGN 140 TO M
      I=147
      GO TO 200
140   IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
      I=I+1
      IF(I.LE.193) GO TO 200
      GO TO 190
150   ASSIGN 160 TO M
      I=1
      GO TO 200
160   IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
      I=I+1
      IF(I.LE.128) GO TO 200
170   ASSIGN 180 TO M
      I=193
```

```
140   IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
      I=I+1
      IF(I.LE.236) GO TO 200
      GO TO 190
150   ASSIGN 160 TO M
      I=1
      GO TO 200
160   IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
      I=I+1
      IF(I.LE.85) GO TO 200
170   ASSIGN 180 TO M
      I=236
      GO TO 200
180   IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
      I=I-1
      IF(I.GE.99) GO TO 200
190   RETURN
C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
200   LOOK=I+LAG
      IQ=LOOK/237
      IR=MOD(LOOK,237)
      IF(IR.EQ.0) IR=1
      IROLL=IQ*236
      IF(KEY(IR).LE.IROLL) GO TO 220
210   C=SAVE(IR)*WT(I)
      ZLAGH1=ZLAGH1+C
      L=L+1
      GO TO M,(110,120,140,160,180)
220   KEY(IR)=IROLL+IR
      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
      GO TO 210
      END
```

```
      COMPLEX FUNCTION ZLAGFO(X,FUN,TOL,L,NEW)
C-=*** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
C  INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*COS(G*B)*DG" DEFINED AS THE
C  COMPLEX FOURIER COSINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C       * X     = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
C                 "ZLAGFO" IS USEFUL ONLY WHEN X=(LAST X)-.20 *** I.E.,
C                 SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
C                 THEN SUBPROGRAM "ZFOURO" IS ADVISED FOR GENERAL USE.
C                 (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).
C       FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
C                 OF A REAL ARGUMENT G.
C               NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                     CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                     THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
C                     DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C                 FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RLAGFO" IS ADVISED;
C                 HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
C                 INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
C         TOL=      REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                   IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                   THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                   TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                   THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C                 A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C                 BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C                 RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C                 APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C                 ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C         L=        RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                   CONVOLUTION (L DEPENDS ON TOL AND FUN).
C                 MIN.L=24 AND MAX.L=281--WHICH COULD
C                   OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                   VERY FAST...
```

```
C         * NEW=  1 IS NECESSARY 1ST TIME OR BRAND NEW X.
C                 0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)-0.20
C                   IS ASSUMED INTERNALLY BY THIS ROUTINE.
C                   NOTE: IF THIS IS NOT TRUE, ROUTINE WILL
C                   STILL ASSUME X=(LAST X)-0.20 ANYWAY...
C                   IT IS THE USERS RESPONSIBILITY TO NORMALIZE
C                   BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
C                THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
C                   TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
C                   SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
C                   ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
C                   THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
C                   KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
C                   WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
C                   WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
C
C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZLAGFO; THE FOURIER
C  TRANSFORM IS THEN ZLAGFO/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
C
C--USAGE-- "ZLAGFO" IS CALLED AS FOLLOWS:
C
C         ...
C         COMPLEX Z ZLAGFO,ZF
C         EXTERNAL ZF
C         ...
C         Z=ZLAGFO(ALOG(B),ZF,TOL,L,NEW)/B
C         ...
C         END
C         COMPLEX FUNCTION ZF(G)
C         ...USER SUPPLIED CODE...
C         END
C
C--NOTES:
C         (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
C         BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
C         ANY & ALL EXP-UNDERFLOW'S TO 0.0...
C         (2).  AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
C         METHOD, LET BMAX>=BMIN>0 BE GIVEN. THEN IT CAN BE SHOWN
C         THAT THE ACTUAL NUMBER OF B'S IS NB=AINT(5.*ALOG(BMAX/BMIN))+1,
C         PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
C         BMINA=BMAX*EXP(-.2*(NB-1)).  THE METHOD GENERATES THE DECREASING
```

```
C          ARGUMENTS SPACED AS X=ALOG(BMAX),X-.2,X-.2*2,...,ALOG(BMINA).
C          FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
C                ...
C                NB=AINT(5.*ALOG(BMAX/BMIN))+1
C                NB1=NB+1
C                X0=ALOG(BMAX)+.2
C                NEW=1
C                DO 1 I=NB,1,-1
C                X=X0-.2*(NB1-I)
C                ARG(I)=EXP(X)
C                Z(I)=ZLAGF0(X,ZF,TOL,L,NEW)/ARG(I)
C          1     NEW=0
C                ...
C          (3).  IF RESULTS ARE STORED IN ARRAYS ARG(I),Z(I),I=1,NB FOR
C          ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
C          TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
C          SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
C          (4).  IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
C          ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
C          BMAX,BMIN AND BY SETTING NEW=1....
C
C
C--COS-EXTENDED FILTER WEIGHT ARRAYS:
C   NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C   TO SAVE STORAGE.
       DIMENSION YT(281),Y1(76),Y2(76),Y3(76),Y4(53)
       EQUIVALENCE (YT(1),Y1(1)),(YT(77),Y2(1)),(YT(153),Y3(1)),
     1 (YT(229),Y4(1))
      DATA Y1/
     1 5.1178101E-14, 2.9433849E-14, 2.5492522E-14, 1.9034819E-14,
     2 6.4179780E-14, 1.3085746E-15, 1.1989957E-13,-1.2216234E-14,
     3 1.7534103E-13, 7.9373498E-15, 2.1235650E-13, 7.9981520E-14,
     4 2.3815757E-13, 1.9714260E-13, 2.8920132E-13, 3.4161340E-13,
     5 4.0349917E-13, 5.2203885E-13, 5.9837223E-13, 7.8015306E-13,
     6 8.8911655E-13, 1.1709731E-12, 1.3165595E-12, 1.7578463E-12,
     7 1.9538564E-12, 2.6289768E-12, 2.9167697E-12, 3.9044344E-12,
     8 4.3927341E-12, 5.7526904E-12, 6.6569552E-12, 8.4555678E-12,
     9 1.0063229E-11, 1.2487964E-11, 1.5134582E-11, 1.8501488E-11,
     1 2.2720051E-11, 2.7452598E-11, 3.4025443E-11, 4.0875985E-11,
     2 5.0751668E-11, 6.1094382E-11. 7.5492982E-11, 9.1445759E-11,
     3 1.1227336E-10, 1.3676464E-10, 1.6720269E-10, 2.0423244E-10,
```

```
     4 2.4932743E-10, 3.0470661E-10, 3.7198526E-10, 4.5449934E-10,
     5 5.5502537E-10, 6.7793669E-10, 8.2810001E-10, 1.0112626E-09,
     6 1.2354800E-09, 1.5085255E-09, 1.8432253E-09, 2.2503397E-09,
     7 2.7499027E-09, 3.3569525E-09, 4.1025670E-09, 5.0077487E-09,
     8 6.1205950E-09, 7.4703399E-09, 9.1312760E-09, 1.1143911E-08,
     9 1.3622929E-08, 1.6623917E-08, 2.0324094E-08, 2.4798610E-08,
     1 3.0321709E-08, 3.6992986E-08, 4.5237482E-08, 5.5183434E-08/
      DATA Y2/
     1 6.7491070E-08, 8.2317946E-08, 1.0069271E-07, 1.2279375E-07,
     2 1.5022907E-07, 1.8316969E-07, 2.2413747E-07, 2.7322865E-07,
     3 3.3441046E-07, 4.0756197E-07, 4.9094278E-07, 6.0793233E-07,
     4 7.4443665E-07, 9.0679753E-07, 1.1107379E-06, 1.3525651E-06,
     5 1.6573073E-06, 2.0174273E-06, 2.4728798E-06, 3.0090445E-06,
     6 3.6898816E-06, 4.4879625E-06, 5.5059521E-06, 6.6935820E-06,
     7 8.2160716E-06, 9.9828691E-06, 1.2260527E-05, 1.4888061E-05,
     8 1.8296530E-05, 2.2202672E-05, 2.7305154E-05, 3.3109672E-05,
     9 4.0751046E-05, 4.9372484E-05, 6.0820947E-05, 7.3619571E-05,
     1 9.0780005E-05, 1.0976837E-04, 1.3550409E-04, 1.6365676E-04,
     2 2.0227521E-04, 2.4398338E-04, 3.0197018E-04, 3.6370760E-04,
     3 4.5083748E-04, 5.4213338E-04, 6.7315347E-04, 8.0800951E-04,
     4 1.0051938E-03, 1.2041401E-03, 1.5011708E-03, 1.7942344E-03,
     5 2.2421056E-03, 2.6730676E-03, 3.3490681E-03, 3.9815050E-03,
     6 5.0028666E-03, 5.9285668E-03, 7.4730905E-03, 8.8233510E-03,
     7 1.1160132E-02, 1.3119627E-02, 1.6653199E-02, 1.9472767E-02,
     8 2.4800811E-02, 2.8793704E-02, 3.6752063E-02, 4.2228780E-02,
     9 5.3905163E-02, 6.0804660E-02, 7.7081738E-02, 8.3874501E-02,
     1 1.0377190E-01, 1.0377718E-01, 1.1892208E-01, 9.0437429E-02/
      DATA Y3/
     1 7.1685138E-02,-3.9473064E-02,-1.5078720E-01,-4.0489859E-01,
     2-5.6018995E-01,-6.8050388E-01,-1.5094224E-01, 6.6304064E-01,
     3 1.3766748E+00,-8.0373222E-01,-1.0869629E+00, 1.2812892E+00,
     4-5.0341082E-01,-4.4274455E-02, 2.0913102E-01,-1.9999661E-01,
     5 1.5207664E-01,-1.0920260E-01, 7.8169956E-02,-5.6651561E-02,
     6 4.1611799E-02,-3.0880012E-02, 2.3072559E-02,-1.7311631E-02,
     7 1.3021442E-02,-9.8085025E-03, 7.3943529E-03,-5.5769518E-03,
     8 4.2073164E-03,-3.1745026E-03, 2.3954154E-03,-1.8076122E-03,
     9 1.3640816E-03,-1.0293934E-03, 7.7682952E-04,-5.8623518E-04,
     1 4.4240399E-04,-3.3386183E-04, 2.5195025E-04,-1.9013541E-04,
     2 1.4348659E-04,-1.0828284E-04, 8.1716174E-05,-6.1667509E-05,
     3 4.6537684E-05,-3.5119887E-05, 2.6503388E-05,-2.0000904E-05,
     4 1.5093768E-05,-1.1390572E-05, 8.5959318E-06,-6.4869407E-06,
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     5 4.8953713E-06,-3.6942830E-06, 2.7878625E-06,-2.1038241E-06,
     6 1.5875917E-06,-1.1980090E-06, 9.0398030E-07,-6.8208296E-07,
     7 5.1458650E-07,-3.8817581E-07, 2.9272267E-07,-2.2067921E-07,
     8 1.6623514E-07,-1.2514102E-07, 9.4034535E-08,-7.0556837E-08,
     9 5.2741581E-08,-3.9298610E-08, 2.9107255E-08,-2.1413893E-08,
     1 1.5742032E-08,-1.1498608E-08, 8.7561571E-09,-7.2959446E-09/
      DATA Y4/
     1 6.8816619E-09,-8.9679825E-09, 1.4258275E-08,-1.9564299E-08,
     2 2.0235313E-08,-1.4725545E-08, 5.4632820E-09, 3.5995580E-09,
     3-9.5287133E-09, 1.1460041E-08,-1.0250532E-08, 7.4641748E-09,
     4-4.4703465E-09, 2.0499053E-09,-4.4806353E-10,-4.0374336E-10,
     5 7.0321001E-10,-6.7067960E-10, 4.9130404E-10,-2.8840747E-10,
     6 1.2373144E-10,-1.5260443E-11,-4.2027559E-11, 6.1885474E-11,
     7-5.9273937E-11, 4.6588766E-11,-3.2054182E-11, 1.9831637E-11,
     8-1.1210098E-11, 5.9567021E-12,-3.2427812E-12, 2.1353868E-12,
     9-1.8476851E-12, 1.8438474E-12,-1.8362842E-12, 1.7241847E-12,
     1-1.5161479E-12, 1.2627657E-12,-1.0129176E-12, 7.9578625E-13,
     2-6.2131435E-13, 4.8745900E-13,-3.8703630E-13, 3.1172547E-13,
     3-2.5397802E-13, 2.0824130E-13,-1.7123163E-13, 1.4113344E-13,
     4-1.1687986E-13, 9.7664016E-14,-8.2977176E-14, 7.2515267E-14,
     5-5.6047478E-14/
C--$$ENDATA
      COMPLEX FUN,C,CMAX,SAVE
      DIMENSION KEY(281),SAVE(281),T(2),TMAX(2)
      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
      IF(NEW) 10,30,10
10    LAG=-1
      X0=-X-30.30251236
      DO 20 IR=1,281
20    KEY(IR)=0
30    LAG=LAG+1
      ZLAGF0=(0.0,0.0)
      CMAX=(0.0,0.0)
      L=0
      ASSIGN 110 TO M
      I=149
      GO TO 200
110   TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
      I=I+1
      IF(I.LE.170) GO TO 200
```

```
      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      ASSIGN 120 TO M
      I=148
      GO TO 200
120   IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
      I=I-1
      IF(I.GT.0) GO TO 200
130   ASSIGN 140 TO M
      I=171
      GO TO 200
140   IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
      I=I+1
      IF(I.LE.281) GO TO 200
      GO TO 190
150   ASSIGN 160 TO M
      I=1
      GO TO 200
160   IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
      I=I+1
      IF(I.LE.148) GO TO 200
170   ASSIGN 180 TO M
      I=281
      GO TO 200
180   IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
      I=I-1
      IF(I.GE.171) GO TO 200
190   RETURN
C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
200   LOOK=I+LAG
      IQ=LOOK/282
      IR=MOD(LOOK,282)
      IF(IR.EQ.0) IR=1
      IROLL=IQ*281
      IF(KEY(IR).LE.IROLL) GO TO 220
210   C=SAVE(IR)*YT(I)
      ZLAGF0=ZLAGF0+C
      L=L+1
      GO TO M,(110,120,140,160,180)
220   KEY(IR)=IROLL+IR
      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
      GO TO 210
      END
```

```
      COMPLEX FUNCTION ZLAGF1(X,FUN,TOL,L,NEW)
C--*** A SPECIAL LAGGED* CONVOLUTION METHOD TO COMPUTE THE
C  INTEGRAL FROM 0 TO INFINITY OF "FUN(G)*SIN(G*B)*DG" DEFINED AS THE
C  COMPLEX FOURIER SINE TRANSFORM WITH ARGUMENT X(=ALOG(B))
C  BY CONVOLUTION FILTERING WITH COMPLEX FUNCTION "FUN"--AND
C  USING A VARIABLE CUT-OFF METHOD WITH EXTENDED FILTER TAILS....
C
C--BY W.L.ANDERSON, U.S.GEOLOGICAL SURVEY, DENVER, COLORADO.
C
C--PARAMETERS:
C
C      * X      = REAL ARGUMENT(=ALOG(B) AT CALL) OF THE FOURIER TRANSFORM
C                 "ZLAGF1" IS USEFUL ONLY WHEN X=(LAST X)-.20 *** I.E.,
C                 SPACED SAME AS FILTER USED--IF THIS IS NOT CONVENIENT,
C                 THEN SUBPROGRAM "ZFOUR1" IS ADVISED FOR GENERAL USE.
C                 (ALSO SEE PARM 'NEW' & NOTES (2)-(4) BELOW).
C      FUN(G)= EXTERNAL DECLARED COMPLEX FUNCTION NAME (USER SUPPLIED)
C                 OF A REAL ARGUMENT G.
C             NOTE: IF PARMS OTHER THAN G ARE REQUIRED, USE COMMON IN
C                   CALLING PROGRAM AND IN SUBPROGRAM FUN.
C                   THE COMPLEX FUNCTION FUN SHOULD BE A MONOTONE
C                   DECREASING FUNCTION AS THE ARGUMENT G BECOMES LARGE...
C                 FOR REAL-ONLY FUNCTIONS, SUBPROGRAM "RLAGF1" IS ADVISED;
C                 HOWEVER, TWO REAL-FUNCTIONS F1(G),F2(G) MAY BE
C                 INTEGRATED IN PARALLEL BY WRITING FUN=CMPLX(F1(G),F2(G))
C         TOL=      REAL TOLERANCE EXCEPTED AT CONVOLVED TAILS--I.E.,
C                   IF FILTER*FUN<TOL*MAX, THEN REST OF TAIL IS TRUNCATED.
C                   THIS IS DONE AT BOTH ENDS OF FILTER.  TYPICALLY,
C                   TOL <= .0001 IS USUALLY OK--BUT THIS DEPENDS ON
C                   THE FUNCTION FUN AND PARAMETER X...IN GENERAL,
C                 A "SMALLER TOL" WILL USUALLY RESULT IN "MORE ACCURACY"
C                 BUT WITH "MORE WEIGHTS" BEING USED.  TOL IS NOT DIRECTLY
C                 RELATED TO TRUNCATION ERROR, BUT GENERALLY SERVES AS AN
C                 APPROXIMATION INDICATOR... FOR VERY LARGE OR SMALL B,
C                 ONE SHOULD USE A SMALLER TOL THAN RECOMMENDED ABOVE...
C         L=        RESULTING NO. FILTER WTS. USED IN THE VARIABLE
C                   CONVOLUTION (L DEPENDS ON TOL AND FUN).
C                 MIN.L=20 AND MAX.L=266--WHICH COULD
C                   OCCUR IF TOL IS VERY SMALL AND/OR FUN NOT DECREASING
C                   VERY FAST...
```

```
C        * NEW= 1 IS NECESSARY 1ST TIME OR BRAND NEW X,
C               0 FOR ALL SUBSEQUENT CALLS WHERE X=(LAST X)-0.20
C                 IS ASSUMED INTERNALLY BY THIS ROUTINE.
C                 NOTE; IF THIS IS NOT TRUE, ROUTINE WILL
C                 STILL ASSUME X=(LAST X)-0.20 ANYWAY...
C                 IT IS THE USERS RESPONSIBILITY TO NORMALIZE
C                 BY CORRECT B=EXP(X) OUTSIDE OF CALL (SEE USAGE BELOW).
C              THE LAGGED CONVOLUTION METHOD PICKS UP SIGNIFICANT
C                 TIME IMPROVEMENTS WHEN THE KERNEL IS NOT A
C                 SIMPLE ELEMENTARY FUNCTION...DUE TO INTERNALLY SAVING
C                 ALL KERNEL FUNCTION EVALUATIONS WHEN NEW=1...
C                 THEN WHEN NEW=0, ALL PREVIOUSLY CALCULATED
C                 KERNELS WILL BE USED IN THE LAGGED CONVOLUTION
C                 WHERE POSSIBLE, ONLY ADDING NEW KERNEL EVALUATIONS
C                 WHEN NEEDED (DEPENDS ON PARMS TOL AND FUN)
C
C--THE RESULTING COMPLEX CONVOLUTION SUM IS GIVEN IN ZLAGF1; THE FOURIER
C  TRANSFORM IS THEN ZLAGF1/B WHICH IS TO BE COMPUTED AFTER EXIT FROM
C  THIS ROUTINE.... WHERE B=EXP(X), X=ARGUMENT USED IN CALL...
C
C--USAGE-- "ZLAGF1" IS CALLED AS FOLLOWS:
C
C        ...
C        COMPLEX Z,ZLAGF1,ZF
C        EXTERNAL ZF
C        ...
C        Z=ZLAGF1(ALOG(B),ZF,TOL,L,NEW)/B
C        ...
C        END
C        COMPLEX FUNCTION ZF(G)
C        ...USER SUPPLIED CODE...
C        END
C
C--NOTES;
C        (1).  EXP-UNDERFLOW'S MAY OCCUR IN EXECUTING THE SUBPROGRAM
C        BELOW; HOWEVER, THIS IS OK PROVIDED THE MACHINE SYSTEM SETS
C        ANY & ALL EXP-UNDERFLOW'S TO 0.0....
C        (2).  AS AN AID TO UNDERSTANDING & USING THE LAGGED CONVOLUTION
C        METHOD, LET BMAX>=BMIN>0 BE GIVEN, THEN IT CAN BE SHOWN
C        THAT THE ACTUAL NUMBER OF B'S IS NB=AINT(5.*ALOG(BMAX/BMIN))+1,
C        PROVIDED BMAX/BMIN>=1. THE USER MAY THEN ASSUME AN "ADJUSTED"
C        BMINA=BMAX*EXP(-.2*(NB-1)).  THE METHOD GENERATES THE DECREASING
```

```
C          ARGUMENTS SPACED AS X=ALOG(BMAX),X-.2,X-.2*2,...,ALOG(BMINA).
C          FOR EXAMPLE, ONE MAY CONTROL THIS WITH THE CODE:
C                ...
C                NB=AINT(5.*ALOG(BMAX/BMIN))+1
C                NB1=NB+1
C                X0=ALOG(BMAX)+.2
C                NEW=1
C                DO 1 I=NB,1,-1
C                X=X0-.2*(NB1-I)
C                ARG(I)=EXP(X)
C                Z(I)=ZLAGF1(X,ZF,TOL,L,NEW)/ARG(I)
C             1  NEW=0
C                ...
C          (3).  IF RESULTS ARE STORED IN ARRAYS ARG(I),Z(I),I=1,NB FOR
C          ARG IN (BMINA,BMAX), THEN THESE ARRAYS MAY BE USED, FOR EXAMPLE,
C          TO SPLINE-INTERPOLATE AT A DIFFERENT (LARGER OR SMALLER)
C          SPACING THAN USED IN THE LAGGED CONVOLUTION METHOD.
C          (4).  IF A DIFFERENT RANGE OF B IS DESIRED, THEN ONE MAY
C          ALWAYS RESTART THE ABOVE PROCEDURE IN (2) WITH A NEW
C          BMAX,BMIN AND BY SETTING NEW=1,...
C
C
C--SIN-EXTENDED FILTER WEIGHT ARRAYS:
C   NOTE: ABSCISSA CORRESPONDING TO WEIGHT IS GENERATED
C   TO SAVE STORAGE.
           DIMENSION WT(266),W1(76),W2(76),W3(76),W4(38)
           EQUIVALENCE (WT(1),W1(1)),(WT(77),W2(1)),(WT(153),W3(1)),
     1 (WT(229),W4(1))
      DATA W1/
     1-1.1113940E-09,-1.3237246E-12, 1.5091739E-12,-1.6240954E-12,
     2 1.7236636E-12,-1.8227727E-12, 1.9255992E-12,-2.0335514E-12,
     3 2.1473541E-12,-2.2675549E-12, 2.3946842E-12,-2.5292661E-12,
     4 2.6718110E-12,-2.8227693E-12, 2.9825171E-12,-3.1514006E-12,
     5 3.3297565E-12,-3.5179095E-12, 3.7163306E-12,-3.9256378E-12,
     6 4.1464798E-12,-4.3794552E-12, 4.6252131E-12,-4.8845227E-12,
     7 5.1582809E-12,-5.4474462E-12, 5.7530277E-12,-6.0760464E-12,
     8 6.4175083E-12,-6.7783691E-12, 7.1595239E-12,-7.5618782E-12,
     9 7.9864477E-12,-8.4344110E-12, 8.9072422E-12,-9.4067705E-12,
     1 9.9349439E-12,-1.0493731E-11, 1.1084900E-11,-1.1709937E-11,
     2 1.2370354E-11,-1.3067414E-11, 1.3802200E-11,-1.4575980E-11,
     3 1.5390685E-11,-1.6249313E-11, 1.7155934E-11,-1.8115250E-11,
```

```
     4 1.9131898E-11,-2.0209795E-11, 2.1352159E-11,-2.2561735E-11,
     5 2.3840976E-11,-2.5192263E-11, 2.6618319E-11,-2.8122547E-11,
     6 2.9709129E-11,-3.1382870E-11, 3.3149030E-11,-3.5013168E-11,
     7 3.6981050E-11,-3.9058553E-11, 4.1251694E-11,-4.3566777E-11,
     8 4.6010537E-11,-4.9590396E-11, 5.1314761E-11,-5.4193353E-11,
     9 5.7236720E-11,-6.0455911E-11, 6.3861222E-11,-6.7461492E-11,
     1 7.1265224E-11,-7.5279775E-11, 7.9512249E-11,-8.3971327E-11/
      DATA W2/
     1 8.8668961E-11,-9.3621900E-11, 9.8851764E-11,-1.0438319E-10,
     2 1.1024087E-10,-1.1644680E-10, 1.2301979E-10,-1.2997646E-10,
     3 1.3733244E-10,-1.4510363E-10, 1.5330772E-10,-1.6196550E-10,
     4 1.7110130E-10,-1.8074257E-10, 1.9091922E-10,-2.0166306E-10,
     5 2.1300756E-10,-2.2498755E-10, 2.3763936E-10,-2.5100098E-10,
     6 2.6511250E-10,-2.8001616E-10, 2.9575691E-10,-3.1238237E-10,
     7 3.2994314E-10,-3.4849209E-10, 3.6808529E-10,-3.8878042E-10,
     8 4.1063982E-10,-4.3372566E-10, 4.5811059E-10,-4.8386049E-10,
     9 5.1105728E-10,-5.3977672E-10, 5.7011632E-10,-6.0215516E-10,
     1 6.3601273E-10,-6.7175964E-10, 7.0955020E-10,-7.4942601E-10,
     2 7.9161025E-10,-8.3606980E-10, 8.8317110E-10,-9.3270330E-10,
     3 9.8533749E-10,-1.0404508E-09, 1.0993731E-09,-1.1605442E-09,
     4 1.2267391E-09,-1.2942905E-09, 1.3691677E-09,-1.4429912E-09,
     5 1.5288164E-09,-1.6077524E-09, 1.7085998E-09,-1.7890471E-09,
     6 1.9129068E-09,-1.9857116E-09, 2.1491608E-09,-2.1926779E-09,
     7 2.4312660E-09,-2.3959044E-09, 2.7872500E-09,-2.5610596E-09,
     8 3.2762318E-09,-2.6082940E-09, 4.0251453E-09,-2.3560563E-09,
     9 5.3176554E-09,-1.3960161E-09, 7.7708747E-09, 1.1853546E-09,
     1 1.2760851E-08, 7.4264707E-09, 2.3342187E-08, 2.1869851E-08/
      DATA W3/
     1 4.6306744E-08, 5.4631686E-08, 9.6763087E-08, 1.2823337E-07,
     2 2.0832812E-07, 2.9280540E-07, 4.5580888E-07, 6.5992437E-07,
     3 1.0056815E-06, 1.4779183E-06, 2.2294335E-06, 3.2994604E-06,
     4 4.9485823E-06, 7.3545473E-06, 1.1001083E-05, 1.6380539E-05,
     5 2.4469550E-05, 3.6469246E-05, 5.4441527E-05, 8.1176726E-05,
     6 1.2113828E-04, 1.8066494E-04, 2.6954609E-04, 4.0202288E-04,
     7 5.9969995E-04, 8.9437312E-04, 1.3338166E-03, 1.9886697E-03,
     8 2.9643943E-03, 4.4168923E-03, 6.5773518E-03, 9.7855105E-03,
     9 1.4539361E-02, 2.1558670E-02, 3.1871864E-02, 4.6903518E-02,
     1 6.8559512E-02, 9.9170152E-02, 1.4120770E-01, 1.9610835E-01,
     2 2.6192603E-01, 3.2743321E-01, 3.6407406E-01, 3.1257559E-01,
     3 9.0460168E-02,-3.6051039E-01,-8.6324760E-01,-8.1178720E-01,
     4 5.2205241E-01, 1.5449873E+00,-1.1817933E+00,-2.6759896E-01,
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```
     5 8.0869203E-01,-6.2757149E-01, 3.4062630E-01,-1.5885304E-01,
     6 7.0472984E-02,-3.1624462E-02, 1.4894068E-02,-7.4821176E-03,
     7 4.0035936E-03,-2.2543784E-03, 1.3160358E-03,-7.8636604E-04,
     8 4.7658745E-04,-2.9125817E-04, 1.7885105E-04,-1.1012416E-04,
     9 6.7910334E-05,-4.1914054E-05, 2.5881544E-05,-1.5985851E-05,
     1 9.8751880E-06,-6.1008526E-06, 3.7692543E-06,-2.3287953E-06/
      DATA W4/
     1 1.4388425E-06,-8.8899353E-07, 5.4926991E-07,-3.3937048E-07,
     2 2.0968284E-07,-1.2955437E-07, 8.0046336E-08,-4.9457371E-08,
     3 3.0557711E-08,-1.8880390E-08, 1.1665454E-08,-7.2076428E-09,
     4 4.4533423E-09,-2.7515696E-09, 1.7001092E-09,-1.0504494E-09,
     5 6.4904567E-10,-4.0102999E-10, 2.4778763E-10,-1.5310321E-10,
     6 9.4600354E-11,-5.8453314E-11, 3.6119400E-11,-2.2320056E-11,
     7 1.3793460E-11,-8.5242656E-12, 5.2675102E-12,-3.2543076E-12,
     8 2.0097689E-12,-1.2405412E-12, 7.6530538E-13,-4.7191929E-13,
     9 2.9084993E-13,-1.7923661E-13, 1.1016948E-13,-6.7885902E-14,
     1 4.2025050E-14,-2.1314731E-14/
C--$$ENDATA
C
      COMPLEX FUN,C,CMAX,SAVE
      DIMENSION KEY(266),SAVE(266),T(2),TMAX(2)
      EQUIVALENCE (C,T(1)),(CMAX,TMAX(1))
      IF(NEW) 10,30,10
10    LAG=-1
      X0=-X-38.30455704
      DO 20 IR=1,266
20    KEY(IR)=0
30    LAG=LAG+1
      ZLAGF1=(0.0,0.0)
      CMAX=(0.0,0.0)
      L=0
      ASSIGN 110 TO M
      I=191
      GO TO 200
110   TMAX(1)=AMAX1(ABS(T(1)),TMAX(1))
      TMAX(2)=AMAX1(ABS(T(2)),TMAX(2))
      I=I+1
      IF(I.LE.208) GO TO 200
      IF(TMAX(1).EQ.0.0.AND.TMAX(2).EQ.0.0) GO TO 150
      CMAX=TOL*CMAX
      ASSIGN 120 TO M
```

```
      I=190
      GO TO 200
  120 IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 130
      I=I-1
      IF(I.GT.0) GO TO 200
  130 ASSIGN 140 TO M
      I=209
      GO TO 200
  140 IF(ABS(T(1)).LE.TMAX(1).AND.ABS(T(2)).LE.TMAX(2)) GO TO 190
      I=I+1
      IF(I.LE.266) GO TO 200
      GO TO 190
  150 ASSIGN 160 TO M
      I=1
      GO TO 200
  160 IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 170
      I=I+1
      IF(I.LE.190) GO TO 200
  170 ASSIGN 180 TO M
      I=266
      GO TO 200
  180 IF(T(1).EQ.0.0.AND.T(2).EQ.0.0) GO TO 190
      I=I-1
      IF(I.GE.209) GO TO 200
  190 RETURN
C--STORE/RETRIEVE ROUTINE (DONE INTERNALLY TO SAVE CALL'S)
  200 LOOK=I+LAG
      IO=LOOK/267
      IR=MOD(LOOK,267)
      IF(IR.EQ.0) IR=1
      IROLL=IO*266
      IF(KEY(IR).LE.IROLL) GO TO 220
  210 C=SAVE(IR)*WT(I)
      ZLAGF1=ZLAGF1+C
      L=L+1
      GO TO M,(110,120,140,160,180)
  220 KEY(IR)=IROLL+IR
      SAVE(IR)=FUN(EXP(X0+FLOAT(LOOK)*.20))
      GO TO 210
      END
```

Appendix 2.---Test results.

FILTER TESTS: --MODERATE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
1	1	31	0,1000E+00	0,2000E+00	0,17888544E+02	0,17887801E+02	0,7429E-03	0,4153E-04	0,1000E-03
1	1	35	0,1000E+00	0,1000E+01	0,98518533E+00	0,98502134E+00	0,1640E-03	0,1665E-03	0,1000E-03
1	1	41	0,1000E+00	0,4000E+01	0,62441452E-01	0,62422531E-01	0,1892E-04	0,3030E-03	0,1000E-03
1	1	41	0,1000E+01	0,2000E+00	0,18857320E+00	0,18857222E+00	0,9816E-06	0,5205E-05	0,1000E-03
1	1	31	0,1000E+01	0,1000E+01	0,35355338E+00	0,35354393E+00	0,9455E-05	0,2674E-04	0,1000E-03
1	1	34	0,1000E+01	0,4000E+01	0,57067205E-01	0,57065136E-01	0,2068E-05	0,3625E-04	0,1000E-03
1	2	31	0,1000E+00	0,2000E+00	0,45241871E+01	0,45242038E+01	-0,1675E-04	0,3702E-05	0,1000E-03
1	2	24	0,1000E+00	0,1000E+01	0,20521249E+01	0,20521447E+01	-0,1976E-04	0,9629E-05	0,1000E-03
1	2	25	0,1000E+00	0,4000E+01	0,42483543E-15	-0,30106629E-04	0,3011E-04	0,7087E+11	0,1000E-03
1	2	85	0,1000E+01	0,2000E+00	0,49502491E-01	0,49503862E-01	-0,1371E-05	0,2769E-04	0,1000E-03
1	2	28	0,1000E+01	0,1000E+01	0,19470019E+00	0,19469971E+00	0,4824E-06	0,2478E-05	0,1000E-03
1	2	23	0,1000E+01	0,4000E+01	0,18315639E-01	0,18311789E-01	0,3851E-05	0,2102E-03	0,1000E-03
1	3	70	0,1000E+00	0,2000E+00	0,49667507E-01	0,49671476E-01	-0,3968E-05	0,7990E-04	0,1000E-03
1	3	63	0,1000E+00	0,1000E+01	0,48354037E-01	0,48359028E-01	-0,4990E-05	0,1032E-03	0,1000E-03
1	3	54	0,1000E+00	0,4000E+01	0,43657927E-01	0,43650268E-01	0,7659E-05	0,1754E-03	0,1000E-03
1	3	54	0,1000E+01	0,2000E+00	0,46748903E+00	0,46752756E+00	-0,3853E-04	0,8243E-04	0,1000E-03
1	3	48	0,1000E+01	0,1000E+01	0,35283407E+00	0,35288725E+00	-0,5318E-04	0,1507E-03	0,1000E-03
1	3	51	0,1000E+01	0,4000E+01	0,91032035E-01	0,90828397E-01	0,2036E-03	0,2237E-02	0,1000E-03
1	4	36	0,1000E+00	0,2000E+00	0,27639320E+01	0,27635086E+01	0,4234E-03	0,1532E-03	0,1000E-03
1	4	41	0,1000E+00	0,1000E+01	0,90049628E+00	0,90031951E+00	0,1768E-03	0,1963E-03	0,1000E-03
1	4	47	0,1000E+00	0,4000E+01	0,24375195E+00	0,24370805E+00	0,4390E-04	0,1801E-03	0,1000E-03
1	4	67	0,1000E+01	0,2000E+00	0,97096664E-01	0,97096572E-01	0,9220E-07	0,9496E-06	0,1000E-03
1	4	45	0,1000E+01	0,1000E+01	0,29289322E+00	0,29289033E+00	0,2898E-05	0,9895E-05	0,1000E-03
1	4	39	0,1000E+01	0,4000E+01	0,18936609E+00	0,18934510E+00	0,2099E-04	0,1109E-03	0,1000E-03
1	5	44	0,1000E+00	0,2000E+00	-0,12500000E-01	-0,12490173E-01	-0,9827E-05	0,7862E-03	0,1000E-03
1	5	48	0,1000E+00	0,1000E+01	-0,25000000E-02	-0,24991255E-02	-0,8744E-06	0,3498E-03	0,1000E-03
1	5	54	0,1000E+00	0,4000E+01	-0,62499999E-03	-0,62492175E-03	-0,7825E-07	0,1252E-03	0,1000E-03
1	5	58	0,1000E+01	0,2000E+00	-0,21094379E+00	-0,21091824E+00	-0,2554E-04	0,1211E-03	0,1000E-03
1	5	41	0,1000E+01	0,1000E+01	-0,25000000E+00	-0,24967052E+00	-0,3295E-03	0,1318E-02	0,1000E-03
1	5	45	0,1000E+01	0,4000E+01	-0,62500000E-01	-0,62485539E-01	-0,1446E-04	0,2314E-03	0,1000E-03
2	1	67	0,1000E+00	0,2000E+00	0,44721360E+01	0,44719433E+01	0,1927E-03	0,4309E-04	0,1000E-03
2	1	68	0,1000E+00	0,1000E+01	0,99503718E+00	0,99495371E+00	0,8347E-04	0,8389E-04	0,1000E-03
2	1	70	0,1000E+00	0,4000E+01	0,24992191E+00	0,24989002E+00	0,3189E-04	0,1276E-03	0,1000E-03
2	1	81	0,1000E+01	0,2000E+00	0,98058067E+00	0,98057034E+00	0,1033E-04	0,1053E-04	0,1000E-03
2	1	69	0,1000E+01	0,1000E+01	0,70710678E+00	0,70707841E+00	0,2836E-04	0,4011E-04	0,1000E-03
2	1	68	0,1000E+01	0,4000E+01	0,24253562E+00	0,24252472E+00	0,1090E-04	0,4496E-04	0,1000E-03

FILTER TESTS: --MODERATE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
2	2	45	0,1000E+00	0,2000E+00	0,45241871E+01	0,45241634E+01	0,2366E-04	0,5230E-05	0,1000E-03
2	2	39	0,1000E+00	0,1000E+01	0,41042499E+00	0,41064876E+00	-0,2238E-03	0,5452E-03	0,1000E-03
2	2	37	0,1000E+00	0,4000E+01	0,21241771E-16	-0,23456432E-04	0,2346E-04	0,1104E+13	0,1000E-03
2	2	66	0,1000E+01	0,2000E+00	0,49502492E+00	0,49502430E+00	0,6184E-06	0,1249E-05	0,1000E-03
2	2	43	0,1000E+01	0,1000E+01	0,38940039E+00	0,38940220E+00	-0,1814E-05	0,4659E-05	0,1000E-03
2	2	38	0,1000E+01	0,4000E+01	0,91578196E-02	0,91939307E-02	-0,3611E-04	0,3943E-02	0,1000E-03
2	3	62	0,1000E+00	0,2000E+00	0,49004989E+01	0,49005751E+01	-0,7617E-04	0,1554E-04	0,1000E-03
2	3	59	0,1000E+00	0,1000E+01	0,90247240E+00	0,90254273E+00	-0,7033E-04	0,7793E-04	0,1000E-03
2	3	56	0,1000E+00	0,4000E+01	0,15956649E+00	0,15946533E+00	0,1012E-03	0,6340E-03	0,1000E-03
2	3	54	0,1000E+01	0,2000E+00	0,40489027E+01	0,40491113E+01	-0,2086E-03	0,5152E-04	0,1000E-03
2	3	54	0,1000E+01	0,1000E+01	0,22389075E+00	0,22289707E+00	0,9937E-03	0,4438E-02	0,1000E-03
2	3	50	0,1000E+01	0,4000E+01	-0,99287451E-01	-0,99289170E-01	0,1719E-05	0,1732E-04	0,1000E-03
2	4	55	0,1000E+00	0,2000E+00	0,40280331E+01	0,40280157E+01	0,1788E-04	0,4439E-05	0,1000E-03
2	4	47	0,1000E+00	0,1000E+01	0,24204740E+01	0,24204629E+01	0,1112E-04	0,4593E-05	0,1000E-03
2	4	41	0,1000E+00	0,4000E+01	0,10626239E+01	0,10626192E+01	0,4739E-05	0,4459E-05	0,1000E-03
2	4	44	0,1000E+01	0,2000E+00	0,17331428E+01	0,17331362E+01	0,6586E-05	0,3800E-05	0,1000E-03
2	4	37	0,1000E+01	0,1000E+01	0,28670621E+00	0,28668803E+00	0,1818E-04	0,6342E-04	0,1000E-03
2	4	32	0,1000E+01	0,4000E+01	-0,36178848E-01	-0,36165248E-01	-0,1360E-04	0,3759E-03	0,1000E-03
2	5	63	0,1000E+00	0,2000E+00	0,49009933E+01	0,49005916E+01	0,4018E-03	0,8198E-04	0,1000E-03
2	5	59	0,1000E+00	0,1000E+01	0,90483741E+00	0,90475772E+00	0,7970E-04	0,8808E-04	0,1000E-03
2	5	55	0,1000E+00	0,4000E+01	0,16758001E+00	0,16755579E+00	0,2422E-04	0,1445E-03	0,1000E-03
2	5	57	0,1000E+01	0,2000E+00	0,40936537E+01	0,40932161E+01	0,4377E-03	0,1069E-03	0,1000E-03
2	5	53	0,1000E+01	0,1000E+01	0,36787944E+00	0,36779106E+00	0,8838E-04	0,2402E-03	0,1000E-03
2	5	50	0,1000E+01	0,4000E+01	0,45789098E-02	0,45607461E-02	0,1816E-04	0,3967E-02	0,1000E-03
3	1	69	0,1000E+00	0,2000E+00	0,20000000E+01	0,19997443E+01	0,2557E-03	0,1279E-03	0,1000E-03
3	1	63	0,1000E+00	0,1000E+01	0,99009900E-01	0,98740095E-01	0,2698E-03	0,2725E-02	0,1000E-03
3	1	65	0,1000E+00	0,4000E+01	0,62460962E-02	0,61180166E-02	0,1281E-03	0,2051E-01	0,1000E-03
3	1	85	0,1000E+01	0,2000E+00	0,96153846E+00	0,96153253E+00	0,5923E-05	0,6160E-05	0,1000E-03
3	1	73	0,1000E+01	0,1000E+01	0,50000000E+00	0,49996341E+00	0,3659E-04	0,7317E-04	0,1000E-03
3	1	66	0,1000E+01	0,4000E+01	0,58823530E-01	0,58798929E-01	0,2460E-04	0,4182E-03	0,1000E-03
3	2	71	0,1000E+00	0,2000E+00	0,32602466E+01	0,32594248E+01	0,8218E-03	0,2521E-03	0,1000E-03
3	2	60	0,1000E+00	0,1000E+01	0,12307867E-09	-0,33241167E-03	0,3324E-03	0,2701E+07	0,1000E-03
3	2	62	0,1000E+00	0,4000E+01	0,00000000E+00	-0,12467560E-03	0,1247E-03	0,0000E+00	0,1000E-03
3	2	116	0,1000E+01	0,2000E+00	0,87740882E+00	0,87740739E+00	0,1423E-05	0,1622E-05	0,1000E-03
3	2	72	0,1000E+01	0,1000E+01	0,69019422E+00	0,69018453E+00	0,9686E-05	0,1403E-04	0,1000E-03
3	2	65	0,1000E+01	0,4000E+01	0,16231813E-01	0,16274730E-01	-0,4292E-04	0,2644E-02	0,1000E-03

FILTER TESTS: --MODERATE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
3	3	75	0.1000E+00	0.2000E+00	0.62498371E-01	0.60164499E-01	0.2334E-02	0.3734E-01	0.1000E-03
3	3	70	0.1000E+00	0.1000E+01	0.24999974E-02	0.19681623E-02	0.5318E-03	0.2127E+00	0.1000E-03
3	3	75	0.1000E+00	0.4000E+01	0.15625000E-03	0.81810369E-05	0.1481E-03	0.9476E+00	0.1000E-03
3	3	81	0.1000E+01	0.2000E+00	0.47449231E+01	0.47157100E+01	0.2921E-01	0.6157E-02	0.1000E-03
3	3	81	0.1000E+01	0.1000E+01	0.24740396E+00	0.24656339E+00	0.8406E-03	0.3398E-02	0.1000E-03
3	3	69	0.1000E+01	0.4000E+01	0.15614829E-01	0.15613548E-01	0.1282E-05	0.8208E-04	0.1000E-03
3	4	105	0.1000E+00	0.2000E+00	0.15396925E+02	0.15396851E+02	0.7355E-04	0.4777E-05	0.1000E-03
3	4	89	0.1000E+00	0.1000E+01	0.14213153E+02	0.14213038E+02	0.1153E-03	0.8110E-05	0.1000E-03
3	4	77	0.1000E+00	0.4000E+01	0.10529363E+02	0.10529074E+02	0.2885E-03	0.2740E-04	0.1000E-03
3	4	83	0.1000E+01	0.2000E+00	0.12860592E+01	0.12860433E+01	0.1596E-04	0.1241E-04	0.1000E-03
3	4	75	0.1000E+01	0.1000E+01	0.57786367E+00	0.57782742E+00	0.3625E-04	0.6274E-04	0.1000E-03
3	4	68	0.1000E+01	0.4000E+01	0.28770139E-01	0.28741126E-01	0.2901E-04	0.1008E-02	0.1000E-03
3	5	72	0.1000E+00	0.2000E+00	0.23500182E+00	0.23498409E+00	0.1773E-04	0.7544E-04	0.1000E-03
3	5	63	0.1000E+00	0.1000E+01	0.14635181E-01	0.14612547E-01	0.2263E-04	0.1547E-02	0.1000E-03
3	5	65	0.1000E+00	0.4000E+01	0.93604082E-03	0.92614403E-03	0.9897E-05	0.1057E-01	0.1000E-03
3	5	91	0.1000E+01	0.2000E+00	0.67851199E+00	0.67850966E+00	0.2332E-05	0.3437E-05	0.1000E-03
3	5	75	0.1000E+01	0.1000E+01	0.45814536E+00	0.45812543E+00	0.1992E-04	0.4349E-04	0.1000E-03
3	5	67	0.1000E+01	0.4000E+01	0.81259472E-01	0.81239935E-01	0.1954E-04	0.2404E-03	0.1000E-03
4	1	43	0.1000E+00	0.2000E+00	0.40000001E+01	0.39998593E+01	0.1408E-03	0.3519E-04	0.1000E-03
4	1	42	0.1000E+00	0.1000E+01	0.99009901E+00	0.98998680E+00	0.1122E-03	0.1133E-03	0.1000E-03
4	1	43	0.1000E+00	0.4000E+01	0.24984385E+00	0.24978768E+00	0.5617E-04	0.2248E-03	0.1000E-03
4	1	54	0.1000E+01	0.2000E+00	0.19230769E+00	0.19229963E+00	0.8058E-05	0.4190E-04	0.1000E-03
4	1	45	0.1000E+01	0.1000E+01	0.50000000E+00	0.49998964E+00	0.1036E-04	0.2072E-04	0.1000E-03
4	1	41	0.1000E+01	0.4000E+01	0.23529412E+00	0.23528237E+00	0.1175E-04	0.4994E-04	0.1000E-03
4	2	34	0.1000E+00	0.2000E+00	0.32602467E+02	0.32598331E+02	0.4136E-02	0.1268E-03	0.1000E-03
4	2	29	0.1000E+00	0.1000E+01	0.61539334E-08	-0.47976862E-03	0.4798E-03	0.7796E+05	0.1000E-03
4	2	33	0.1000E+00	0.4000E+01	0.00000000E+00	-0.91403347E-04	0.9140E-04	0.0000E+00	0.1000E-03
4	2	77	0.1000E+01	0.2000E+00	0.87740881E-01	0.87735921E-01	0.4960E-05	0.5653E-04	0.1000E-03
4	2	36	0.1000E+01	0.1000E+01	0.34509711E+00	0.34512091E+00	-0.2380E-04	0.6898E-04	0.1000E-03
4	2	31	0.1000E+01	0.4000E+01	0.32463625E-01	0.32615788E-01	-0.1522E-03	0.4687E-02	0.1000E-03
4	3	61	0.1000E+00	0.2000E+00	0.15692261E-01	0.15692609E-01	-0.3485E-06	0.2221E-04	0.1000E-03
4	3	57	0.1000E+00	0.1000E+01	0.15629554E-01	0.15629367E-01	0.1870E-06	0.1196E-04	0.1000E-03
4	3	56	0.1000E+00	0.4000E+01	0.15395891E-01	0.15395833E-01	0.5832E-07	0.3788E-05	0.1000E-03
4	3	48	0.1000E+01	0.2000E+00	0.14188663E+01	0.14188398E+01	0.2646E-04	0.1865E-04	0.1000E-03
4	3	47	0.1000E+01	0.1000E+01	0.90591722E+00	0.90605692E+00	-0.1396E-03	0.1541E-03	0.1000E-03
4	3	33	0.1000E+01	0.4000E+01	-0.51870313E-01	-0.52478133E-01	0.6078E-03	0.1172E-01	0.1000E-03

FILTER TESTS: **MODERATE B**

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
4	4	53	0.1000E+00	0.2000E+00	0.15396925E+01	0.15396696E+01	0.2287E-04	0.1486E-04	0.1000E-03
4	4	48	0.1000E+00	0.1000E+01	0.14213153E+01	0.14213026E+01	0.1267E-04	0.8911E-05	0.1000E-03
4	4	43	0.1000E+00	0.4000E+01	0.10529363E+01	0.10529249E+01	0.1135E-04	0.1078E-04	0.1000E-03
4	4	46	0.1000E+01	0.2000E+00	0.12860592E+01	0.12860539E+01	0.5335E-05	0.4148E-05	0.1000E-03
4	4	40	0.1000E+01	0.1000E+01	0.57786367E+00	0.57784101E+00	0.2266E-04	0.3921E-04	0.1000E-03
4	4	36	0.1000E+01	0.4000E+01	0.28770139E-01	0.28764924E-01	0.5214E-05	0.1812E-03	0.1000E-03
4	5	44	0.1000E+00	0.2000E+00	0.32175055E+00	0.32173968E+00	0.1086E-04	0.3376E-04	0.1000E-03
4	5	42	0.1000E+00	0.1000E+01	0.97726907E-01	0.97719884E-01	0.7022E-05	0.7186E-04	0.1000E-03
4	5	43	0.1000E+00	0.4000E+01	0.24963601E-01	0.24958918E-01	0.4683E-05	0.1876E-03	0.1000E-03
4	5	58	0.1000E+01	0.2000E+00	0.97726907E-01	0.97722064E-01	0.4843E-05	0.4956E-04	0.1000E-03
4	5	46	0.1000E+01	0.1000E+01	0.32175055E+00	0.32175108E+00	-0.5327E-06	0.1656E-05	0.1000E-03
4	5	42	0.1000E+01	0.4000E+01	0.21866895E+00	0.21866028E+00	0.8669E-05	0.3964E-04	0.1000E-03

FILTER TESTS: --SMALL AND LARGE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
1	1	90	0,6000E-01	0.1000E-03	0,46296103E+00	0,46296097E+00	0,5960E-07	0,1287E-06	0,1000E-05
1	1	99	0,6000E-01	0.1000E-02	0,46277013E+01	0,46277012E+01	0,1788E-06	0,3864E-07	0,1000E-05
1	1	56	0,6000E-01	0.1000E-01	0,44432158E+02	0,44432158E+02	-0,4768E-06	0,1073E-07	0,1000E-05
1	1	56	0,6000E-01	0.1000E+02	0,99994601E-02	0,99993678E-02	0,9232E-07	0,9232E-05	0,1000E-05
1	1	68	0,6000E-01	0.1000E+03	0,99999946E-04	0,99999028E-04	0,9177E-09	0,9177E-05	0,1000E-05
1	1	80	0,6000E-01	0.1000E+04	0,10000000E-05	0,99999063E-06	0,9365E-11	0,9365E-05	0,1000E-05
1	2	72	0,6000E-01	0.1000E-03	0,69444441E-02	0,69088186E-02	0,3563E-04	0,5130E-02	0,1000E-05
1	2	83	0,6000E-01	0.1000E-02	0,69444154E-01	0,69416584E-01	0,2757E-04	0,3970E-03	0,1000E-05
1	2	95	0,6000E-01	0.1000E-01	0,69415516E+00	0,69422265E+00	-0,6749E-04	0,9723E-04	0,1000E-05
1	2	37	0,6000E-01	0.1000E+02	0,00000000E+00	-0,35876074E-05	0,3588E-05	0,0000E+00	0,1000E-05
1	2	49	0,6000E-01	0.1000E+03	0,00000000E+00	-0,10092424E-06	0,1009E-06	0,0000E+00	0,1000E-05
1	2	61	0,6000E-01	0.1000E+04	0,00000000E+00	0,15457228E-08	-0,1546E-08	0,0000E+00	0,1000E-05
1	3	132	0,6000E-01	0.1000E-03	0,30025840E-01	0,30004624E-01	0,2122E-04	0,7066E-03	0,1000E-05
1	3	124	0,6000E-01	0.1000E-02	0,30003488E-01	0,29998075E-01	0,5413E-05	0,1804E-03	0,1000E-05
1	3	112	0,6000E-01	0.1000E-01	0,29994547E-01	0,29992835E-01	0,1713E-05	0,5710E-04	0,1000E-05
1	3	81	0,6000E-01	0.1000E+02	0,24432444E-01	0,24430041E-01	0,2403E-05	0,9835E-04	0,1000E-05
1	3	70	0,6000E-01	0.1000E+03	0,81646506E-03	0,79878296E-03	0,1768E-04	0,2166E-01	0,1000E-05
1	3	77	0,6000E-01	0.1000E+04	0,12958937E-03	0,20875710E-03	-0,7917E-04	0,6109E+00	0,1000E-05
1	4	90	0,6000E-01	0.1000E-03	0,13892209E-01	0,13885417E-01	0,6792E-05	0,4889E-03	0,1000E-05
1	4	99	0,6000E-01	0.1000E-02	0,13885643E+00	0,13885959E+00	-0,3161E-05	0,2276E-04	0,1000E-05
1	4	99	0,6000E-01	0.1000E-01	0,13606078E+01	0,13606075E+01	0,2086E-06	0,1533E-06	0,1000E-05
1	4	78	0,6000E-01	0.1000E+02	0,99400011E-01	0,99399989E-01	0,2142E-07	0,2155E-06	0,1000E-05
1	4	89	0,6000E-01	0.1000E+03	0,99940000E-02	0,99939980E-02	0,1979E-08	0,1980E-06	0,1000E-05
1	4	99	0,6000E-01	0.1000E+04	0,99994001E-03	0,99993982E-03	0,1692E-09	0,1892E-06	0,1000E-05
1	5	109	0,6000E-01	0.1000E-03	-0,34484648E-03	-0,34247055E-03	-0,2376E-05	0,6890E-02	0,1000E-05
1	5	97	0,6000E-01	0.1000E-02	-0,22971723E-02	-0,23093421E-02	0,1217E-04	0,5298E-02	0,1000E-05
1	5	86	0,6000E-01	0.1000E-01	-0,11458798E-01	-0,11446142E-01	-0,1266E-04	0,1104E-02	0,1000E-05
1	5	69	0,6000E-01	0.1000E+02	-0,90000000E-04	-0,89994577E-04	-0,5422E-08	0,6025E-04	0,1000E-05
1	5	66	0,6000E-01	0.1000E+03	-0,90000000E-05	-0,98967645E-05	-0,1032E-06	0,1147E-01	0,1000E-05
1	5	67	0,6000E-01	0.1000E+04	-0,90000000E-06	-0,35992661E-06	-0,5401E-06	0,6001E+00	0,1000E-05
2	1	141	0,6000E-01	0.1000E-03	0,16666643E+02	0,16667156E+02	-0,5131E-03	0,3078E-04	0,1000E-05
2	1	147	0,6000E-01	0.1000E-02	0,16664352E+02	0,16664339E+02	0,1335E-04	0,8012E-06	0,1000E-05
2	1	107	0,6000E-01	0.1000E-01	0,16439899E+02	0,16439890E+02	0,8583E-05	0,5221E-06	0,1000E-05
2	1	100	0,6000E-01	0.1000E+02	0,99998200E-01	0,99998040E-01	0,1602E-06	0,1602E-05	0,1000E-05
2	1	104	0,6000E-01	0.1000E+03	0,99999981E-02	0,99999813E-02	0,1688E-07	0,1688E-05	0,1000E-05
2	1	106	0,6000E-01	0.1000E+04	0,10000000E-02	0,99999826E-03	0,1746E-08	0,1746E-05	0,1000E-05

FILTER TESTS!   --SMALL AND LARGE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
2	2	123	0.6000E-01	0.1000E-03	0.83333329E+01	0.83332961E+01	0.3684E-04	0.4420E-05	0.1000E-05
2	2	135	0.6000E-01	0.1000E-02	0.83332986E+01	0.83332628E+01	0.3576E-04	0.4292E-05	0.1000E-05
2	2	146	0.6000E-01	0.1000E-01	0.83298618E+01	0.83298272E+01	0.3457E-04	0.4150E-05	0.1000E-05
2	2	53	0.6000E-01	0.1000E+02	0.00000000E+00	-0.87690539E-07	0.8769E-07	0.0000E+00	0.1000E-05
2	2	64	0.6000E-01	0.1000E+03	0.00000000E+00	-0.69847859E-09	0.6985E-09	0.0000E+00	0.1000E-05
2	2	74	0.6000E-01	0.1000E+04	0.00000000E+00	-0.68299573E-11	0.6830E-11	0.0000E+00	0.1000E-05
2	3	104	0.6000E-01	0.1000E-03	0.99999401E+04	0.99999401E+04	0.0000E+00	0.0000E+00	0.1000E-05
2	3	99	0.6000E-01	0.1000E-02	0.99993999E+03	0.99993958E+03	0.4196E-03	0.4196E-06	0.1000E-05
2	3	93	0.6000E-01	0.1000E-01	0.99940008E+02	0.99940003E+02	0.4768E-05	0.4771E-07	0.1000E-05
2	3	80	0.6000E-01	0.1000E+02	0.48421967E-01	0.48381934E-01	0.4003E-04	0.8268E-03	0.1000E-05
2	3	76	0.6000E-01	0.1000E+03	-0.21005938E-02	-0.21312196E-02	0.3063E-04	0.1458E-01	0.1000E-05
2	3	81	0.6000E-01	0.1000E+04	-0.10787793E-03	-0.13489555E-03	0.2702E-04	0.2504E+00	0.1000E-05
2	4	109	0.6000E-01	0.1000E-03	0.12139683E+02	0.12139647E+02	0.3612E-04	0.2975E-05	0.1000E-05
2	4	97	0.6000E-01	0.1000E-02	0.98370975E+01	0.98370711E+01	0.2646E-04	0.2690E-05	0.1000E-05
2	4	85	0.6000E-01	0.1000E-01	0.75345125E+01	0.75344968E+01	0.1568E-04	0.2081E-05	0.1000E-05
2	4	52	0.6000E-01	0.1000E+02	0.69312070E+00	0.69311839E+00	0.2310E-05	0.3332E-05	0.1000E-05
2	4	44	0.6000E-01	0.1000E+03	-0.65303751E-03	-0.68497731E-03	0.3194E-04	0.4891E-01	0.1000E-05
2	4	50	0.6000E-01	0.1000E+04	0.23959041E-19	-0.14292987E-08	0.1429E-08	0.5966E+11	0.1000E-05
2	5	104	0.6000E-01	0.1000E-03	0.99999399E+04	0.99999301E+04	0.9888E-02	0.9888E-06	0.1000E-05
2	5	99	0.6000E-01	0.1000E-02	0.99993999E+03	0.99993917E+03	0.8240E-03	0.8240E-06	0.1000E-05
2	5	93	0.6000E-01	0.1000E-01	0.99940018E+02	0.99939934E+02	0.8392E-04	0.8397E-06	0.1000E-05
2	5	76	0.6000E-01	0.1000E+02	0.54881163E-01	0.54880907E-01	0.2561E-06	0.4667E-05	0.1000E-05
2	5	71	0.6000E-01	0.1000E+03	0.24787522E-04	0.24751129E-04	0.3639E-07	0.1468E-02	0.1000E-05
2	5	72	0.6000E-01	0.1000E+04	0.87565108E-29	-0.94142596E-10	0.9414E-10	0.1075E+20	0.1000E-05
3	1	165	0.6000E-01	0.1000E-03	0.16666620E+02	0.16666548E+02	0.7224E-04	0.4334E-05	0.1000E-05
3	1	171	0.6000E-01	0.1000E-02	0.16662038E+02	0.16661968E+02	0.7057E-04	0.4235E-05	0.1000E-05
3	1	111	0.6000E-01	0.1000E-01	0.16216216E+02	0.16216264E+02	-0.4745E-04	0.2926E-05	0.1000E-05
3	1	97	0.6000E-01	0.1000E+02	0.59997840E-03	0.59948624E-03	0.4922E-06	0.8203E-03	0.1000E-05
3	1	105	0.6000E-01	0.1000E+03	0.59999979E-05	0.59371834E-05	0.6281E-07	0.1047E-01	0.1000E-05
3	1	113	0.6000E-01	0.1000E+04	0.60000000E-07	0.53624399E-07	0.6376E-08	0.1063E+00	0.1000E-05
3	2	154	0.6000E-01	0.1000E-03	0.14770439E+02	0.14770373E+02	0.6545E-04	0.4431E-05	0.1000E-05
3	2	166	0.6000E-01	0.1000E-02	0.14769423E+02	0.14769357E+02	0.6568E-04	0.4447E-05	0.1000E-05
3	2	160	0.6000E-01	0.1000E-01	0.14668231E+02	0.14668203E+02	0.2849E-04	0.1942E-05	0.1000E-05
3	2	92	0.6000E-01	0.1000E+02	0.00000000E+00	-0.60348027E-06	0.6035E-06	0.0000E+00	0.1000E-05
3	2	102	0.6000E-01	0.1000E+03	0.00000000E+00	-0.60924473E-07	0.6092E-07	0.0000E+00	0.1000E-05
3	2	111	0.6000E-01	0.1000E+04	0.00000000E+00	-0.64220242E-08	0.6422E-08	0.0000E+00	0.1000E-05

FILTER TESTS: --SMALL AND LARGE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
3	3	122	0.6000E-01	0.1000E-03	0.41211856E+04	0.18869898E+04	0.2234E+04	0.5421E+00	0.1000E-05
3	3	111	0.6000E-01	0.1000E-02	0.78332690E+03	0.78320348E+03	0.1234E+00	0.1576E-03	0.1000E-05
3	3	115	0.6000E-01	0.1000E-01	0.89878548E+01	0.90028914E+01	-0.1504E-01	0.1673E-02	0.1000E-05
3	3	109	0.6000E-01	0.1000E+02	0.90000000E-05	0.84589171E-05	0.5411E-06	0.6012E-01	0.1000E-05
3	3	116	0.6000E-01	0.1000E+03	0.89999999E-07	0.34426007E-07	0.5557E-07	0.6175E+00	0.1000E-05
3	3	119	0.6000E-01	0.1000E+04	0.90000000E-09	-0.56650029E-08	0.6565E-08	0.7294E+01	0.1000E-05
3	4	178	0.6000E-01	0.1000E-03	0.26179781E+02	0.26179666E+02	0.1154E-03	0.4408E-05	0.1000E-05
3	4	178	0.6000E-01	0.1000E-02	0.26178368E+02	0.26178254E+02	0.1142E-03	0.4362E-05	0.1000E-05
3	4	167	0.6000E-01	0.1000E-01	0.26164235E+02	0.26164123E+02	0.1123E-03	0.4292E-05	0.1000E-05
3	4	105	0.6000E-01	0.1000E+02	0.14367855E+02	0.14367918E+02	-0.6354E-04	0.4422E-05	0.1000E-05
3	4	95	0.6000E-01	0.1000E+03	0.64893580E-01	0.64898063E-01	-0.4482E-05	0.6907E-04	0.1000E-05
3	4	97	0.6000E-01	0.1000E+04	0.22924492E-24	-0.17707694E-05	0.1771E-05	0.7724E+19	0.1000E-05
3	5	24	0.6000E-01	0.1000E-03	0.69314612E+00	0.00000000E+00	0.6931E+00	0.1000E+01	0.1000E-05
3	5	140	0.6000E-01	0.1000E-02	0.69304302E+00	0.69304010E+00	0.2921E-05	0.4214E-05	0.1000E-05
3	5	121	0.6000E-01	0.1000E-01	0.68290791E+00	0.68290789E+00	0.2235E-07	0.3273E-07	0.1000E-05
3	5	96	0.6000E-01	0.1000E+02	0.53999724E-04	0.54005471E-04	-0.5747E-08	0.1064E-03	0.1000E-05
3	5	107	0.6000E-01	0.1000E+03	0.55129425E-06	0.59000578E-06	-0.3871E-07	0.7022E-01	0.1000E-05
3	5	103	0.6000E-01	0.1000E+04	0.15493047E-07	0.39944348E-07	-0.2445E-07	0.1578E+01	0.1000E-05
4	1	201	0.6000E-01	0.1000E-03	0.27777701E-01	0.27764766E-01	0.1293E-04	0.4657E-03	0.1000E-05
4	1	209	0.6000E-01	0.1000E-02	0.27770064E+00	0.27768146E+00	0.1918E-04	0.6906E-04	0.1000E-05
4	1	98	0.6000E-01	0.1000E-01	0.27027027E+01	0.27025859E+01	0.1168E-03	0.4323E-04	0.1000E-05
4	1	63	0.6000E-01	0.1000E+02	0.99996400E-01	0.99996226E-01	0.1742E-06	0.1742E-05	0.1000E-05
4	1	67	0.6000E-01	0.1000E+03	0.99999965E-02	0.99999630E-02	0.3353E-07	0.3353E-05	0.1000E-05
4	1	68	0.6000E-01	0.1000E+04	0.10000000E-02	0.99999744E-03	0.2561E-08	0.2561E-05	0.1000E-05
4	2	190	0.6000E-01	0.1000E-03	0.20514498E+00	0.20513150E+00	0.1348E-04	0.6570E-04	0.1000E-05
4	2	202	0.6000E-01	0.1000E-02	0.20513088E+01	0.20511751E+01	0.1337E-03	0.6516E-04	0.1000E-05
4	2	130	0.6000E-01	0.1000E-01	0.20372544E+02	0.20371338E+02	0.1205E-02	0.5916E-04	0.1000E-05
4	2	49	0.6000E-01	0.1000E+02	0.00000000E+00	-0.15159947E-06	0.1516E-06	0.0000E+00	0.1000E-05
4	2	59	0.6000E-01	0.1000E+03	0.00000000E+00	-0.15342038E-08	0.1534E-08	0.0000E+00	0.1000E-05
4	2	68	0.6000E-01	0.1000E+04	0.00000000E+00	-0.17274365E-10	0.1727E-10	0.0000E+00	0.1000E-05
4	3	199	0.6000E-01	0.1000E-03	0.56548658E-02	0.56476873E-02	0.7179E-05	0.1269E-02	0.1000E-05
4	3	185	0.6000E-01	0.1000E-02	0.56548566E-02	0.56507584E-02	0.4100E-05	0.7251E-03	0.1000E-05
4	3	148	0.6000E-01	0.1000E-01	0.56547650E-02	0.56546729E-02	0.9208E-07	0.1628E-04	0.1000E-05
4	3	70	0.6000E-01	0.1000E+02	0.55536882E-02	0.55536082E-02	0.8004E-07	0.1441E-04	0.1000E-05
4	3	68	0.6000E-01	0.1000E+03	0.46962638E-02	0.46968285E-02	-0.5647E-06	0.1202E-03	0.1000E-05
4	3	60	0.6000E-01	0.1000E+04	0.44587226E-04	0.38695227E-04	0.5892E-05	0.1321E+00	0.1000E-05

FILTER TESTS: --SMALL AND LARGE B--

F	T	L	A	B	EXACT	FILTERED	DIFF	RELERR	TOLERANCE
4	4	120	0.6000E-01	0.1000E-03	0.15707869E+01	0.15707801E+01	0.6795E-05	0.4326E-05	0.1000E-05
4	4	102	0.6000E-01	0.1000E-02	0.15707021E+01	0.15706954E+01	0.6661E-05	0.4241E-05	0.1000E-05
4	4	84	0.6000E-01	0.1000E-01	0.15698541E+01	0.15698475E+01	0.6646E-05	0.4233E-05	0.1000E-05
4	4	56	0.6000E-01	0.1000E+02	0.86207129E+00	0.86207535E+00	-0.4061E-05	0.4710E-05	0.1000E-05
4	4	50	0.6000E-01	0.1000E+03	0.38936148E-02	0.38941748E-02	-0.5599E-06	0.1438E-03	0.1000E-05
4	4	53	0.6000E-01	0.1000E+04	0.13754695E-25	-0.31456986E-08	0.3146E-08	0.2287E+18	0.1000E-05
4	5	20	0.6000E-01	0.1000E-03	0.83333198E-03	0.00000000E+00	0.8333E-03	0.1000E+01	0.1000E-05
4	5	138	0.6000E-01	0.1000E-02	0.83319832E-02	0.83314434E-02	0.5398E-06	0.6479E-04	0.1000E-05
4	5	127	0.6000E-01	0.1000E-01	0.82007444E-01	0.82003179E-01	0.4265E-05	0.5201E-04	0.1000E-05
4	5	62	0.6000E-01	0.1000E+02	0.59994960E-02	0.59994886E-02	0.7334E-08	0.1222E-05	0.1000E-05
4	5	66	0.6000E-01	0.1000E+03	0.59999948E-03	0.60000441E-03	-0.4926E-08	0.8210E-05	0.1000E-05
4	5	69	0.6000E-01	0.1000E+04	0.59999999E-04	0.60006852E-04	-0.6853E-08	0.1142E-03	0.1000E-05