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ESTIMATING STEADY-STATE EVAPORATION RATES FROM BARE SOILS UNDER CONDITIONS OF HIGH WATER TABLE

BY

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Symbols

a = a subscript, indicating a variable determined in the air, H_a cm above the soil surface.

u = a subscript, indicating a variable determined at the soil surface.

v = a subscript, indicating a variable which involves water vapor transfer.

A = sensible heat transfer into the air, cal cm^{-2} day^{-1}.

b = $-B(T_u - T_1)/L_u^*$, gm cm^{-4}.

B = (η/σ) $(\zeta/\alpha)\beta'$, gm cm^{-3} $^oK^{-1}$.

c = E/D_{hv}, cm^{-1}.

D_a = a coefficient characterizing the molecular diffusion of water vapor in free air, cm^2 day^{-1}.

D_{hv} = a coefficient characterizing the molecular diffusion of soil-water vapor caused by humidity gradients, cm^2 day^{-1}.

D_{Tv} = a coefficient characterizing the molecular diffusion of soil-water vapor caused by thermal gradients, cm^2 day^{-1} $^oK^{-1}$.

e = E/K_{sat}, rate of evaporation from the soil, dimensionless.

E = rate of evaporation from the soil, cm day^{-1}.

e_{pot} = rate of potential evaporation, dimensionless.

E_{pot} = rate of potential evaporation, cm day^{-1}.

e_∞ = ~~limiting~~ soil-limited rate of evaporation from the soil, dimensionless.

E_∞ = ~~limiting~~ soil-limited rate of evaporation from the soil, cm day^{-1}.

f(e) = a functional relation defined by equation 27.

F_m = a function which relates E and S_u, using meteorological parameters.

F_g = a function which relates E and S_u, using soil parameters.

g = acceleration of gravity, = 980 cm sec^{-2}.

$G(V_a)$ = a theoretically or empirically derived known function of wind speed, cm day^{-1} mb^{-1}.

h = relative humidity, dimensionless.

h_a = air relative humidity at height H_a above the soil surface, dimensionless.

H_a = height of meteorological measurements above the soil surface, cm.

h_u = surface soil relative humidity, dimensionless.

H_u = roughness parameter, cm (usually, for bare soils, $0.01 \leq H_u \leq 0.03$).

h_1 = soil relative humidity at depth L_u, dimensionless.

h_1^* = soil relative humidity at depth L_u^*, dimensionless.

I(y) = the integral relation defined by the right-hand side of equation 18.

k = von Karman constant = 0.41, dimensionless.

K = K_{liq} = hydraulic conductivity for liquid flow, cm day^{-1}.

K_{sat} = hydraulic conductivity of water saturated soil, cm day^{-1}.

K_{vap} = hydraulic conductivity for vapor flow, cm day^{-1}.

ℓ = $L/S_{\frac{1}{2}}$, dimensionless depth to water table.

L = total distance between the water table and soil surface, cm.

L_j = in the multilayer case, the thickness of soil layer, cm, j layers above the water table (that is, j = 1 means one layer above the water table, etc.).

L_u = thickness of the uppermost soil layer, cm.

L_u^* = thickness of the uppermost portion of the dry soil surface at which T_1 was determined, cm.

L_u' = thickness of the dry soil layer in which isothermal vapor transfer is assumed to predominate, cm.

M = molecular weight of water = 18 gm mol^{-1}.

n = an integer soil coefficient which usually ranges from 2 for clays to 5 in sands.

p = saturation vapor pressure of water, mb.

p(T) = a known relation between the saturation water vapor pressure and temperature (given in tabular or functional form), mb.

P = ambient pressure, mb (taken as P = 1000 mb in this study).

q = flux of water, cm day^{-1}.

Q_g = soil heat flux into the ground, cal cm^{-2} day^{-1}.

Q_N = net radiative flux received by the soil surface, cal cm^{-2} day^{-1}.

R = gas constant = 8.32×10^7 erg $^oK^{-1}$.

s = $S/S_{\frac{1}{2}}$, dimensionless suction.

S = soil water suction, defined as the negative of the soil water pressure head, cm of water.

S_j = in the multilayer case, the soil water suction at the upper interface of layer j, cm of water.

S_u = water suction at the soil surface, cm of water.

$S_{\frac{1}{2}}$ = a constant soil coefficient representing S at $K = \frac{1}{2} K_{sat}$, cm of water.

T = temperature, ^oK.

T_a = air temperature at H_a, ^oK.

T_u = surface soil temperature, ^oK.

T_1 = soil temperature at depth L_u^*, ^oK.

V_a = wind speed at height H_a, cm day^{-1}.

y = a variable, defined by equation 16.

$\tilde{y}$ = a variable, defined in conjunction with the right-hand side of equation 31 of the layered soil case.

z = $Z/S_{\frac{1}{2}}$, dimensionless height above water table.

Z = vertical height above the water table, cm.

α = tortuosity factor, dimensionless.

β' = $d(\log_e \rho_v)/dT$, gm cm^{-3} $^oK^{-1}$.

γ = psychrometric constant, = 0.000659 P, mb $^oK^{-1}$.

ϵ = water/air molecular ratio = 0.622 (dimensionless).

ζ = a ratio of the average temperature gradient in the air-filled soil pores to the overall soil temperature gradient, dimensionless.

η = soil porosity, dimensionless.

λ = latent heat of vaporization of water at T_a, cal gm^{-1}.

ρ_a = air density at T_a, gm cm^{-3}.

ρ_v = $\rho_v(T)$ = density of saturated water vapor, gm cm^{-3}; ρ_v is a function of temperature.

ρ_w = water density at appropriate T, gm cm^{-3}.

σ = volumetric air content of the soil, dimensionless.

$\Phi(\sigma)$ = a dimensionless function defining the effectiveness of the water-free pore space for diffusion.

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ABSTRACT

A procedure that combines meteorological and soil equations of water transfer makes it possible to estimate approximately the steady-state evaporation from bare soils under conditions of high water table. Field data required include soil-water retention curves, water table depth and a record of air temperature, air humidity and wind velocity at one elevation. The procedure takes into acount the relevant atmospheric factors and the soil's capability to conduct water in liquid and vapor forms. It neglects the effects of thermal transfer (except in the vapor case) and of salt accumulation. Homogeneous as well as layered soils can be treated. Results obtained with the method demonstrate how the soil evaporation rates depend on potential evaporation, water table depth, vapor transfer and certain soil parameters.

INTRODUCTION

It is sometimes desirable to estimate the evaporation rates from bare land surfaces and to predict approximately the variation of these rates with meteorological conditions or with man-imposed changes in the water table level. This might be rather important in certain regions during the appraisal of ground water availability. For such purposes, it is often both permissible and useful to use relatively simple estimation methods. One ~~such~~ possibility is to assume steady state of the hydraulic-gradient driven, upward flux of water and to neglect certain effects of soil temperature and of solute accumulations.

The basic approaches required for the development of this method can be found in the literature. Gardner (1958) suggested a convenient equation for describing hydraulic conductivity, the most relevant soil parameter, and from it developed methods for evaluating soil-limited evaporation in cases of high water table. Anat and others (1965) and Stallman (1967), employed Gardner's general approach, but different soil parametric equations. They demonstrated the usefulness of dimensionless curves in solving problems of the type under consideration.

The above treatments stressed the cases in which soil properties were the determining factor as far as evaporation is concerned. Cases of evaporation in which the atmospheric conditions play the decisive role can be treated by means of several, purely meteorological equations (for example, Slatyer and McIlroy, 1961).

Philip (1957a, b) showed how the effects of the soil factors on bare-soil evaporation coupled with those of the atmospheric parameters. Due to utilization of numerical methods, his approach to soil influences was more general but mathematically less convenient than the one of Gardner.

All of the studies quoted above concerned themselves with homogeneous soils and mainly with cases involving liquid transfer. Gardner indicated how to include the vapor-transfer effects, but only for selected circumstances. Philip's approach to vapor effects is more general, but again mathematically less convenient.

It is the purpose of this paper to integrate and extend the above approaches for estimating steady-state evaporation from bare soils under high water table conditions. The past approaches are unified, modified and supplemented when necessary to improve their practicability as a general (though approximate) method.

Stress has been placed on use of readily available data, simple parameter-determination techniques, dimensionless variables and simple graphical or algebraic treatments. Numerical integrations have been avoided. The older approaches are generalized so as to make them applicable to layered as well as homogeneous soils. In addition, analysis of the multilayer case is modified to allow for the treatment of evaporation affected by water vapor transfer. Examples of the results obtained with the suggested method are presented, discussed and utilized for demonstrating the role of some of the relevant factors.

THEORY

The steady-state evaporative fluxes across the boundary between any given soil-atmosphere system may be described by two functional relations. The first deals with the fluxes leaving the soil surface and entering the atmosphere. It may be represented by the meteorological equation

$$S_u = F_m(E). \qquad [1]$$

The second describes the fluxes between the water table and the soil surface and may be expressed by the soil equation

$$L = F_g\ (S_u,\ E)\ . \qquad [2]$$

In the above equations,

L = total distance between the water table and the soil surface, cm,

S_u = water suction at the soil surface, defined as the negative of the soil water pressure head, cm of water,

E = rate of evaporation from the soil, cm day $^{-1}$,

F_g = a function which relates E and S_u, using soil parameters,

F_m = a function which relates E and S_u using meteorological parameters.

Each of the above relations is an algebraic equation containing the same variables, E and S_u. Therefore, the equations can be solved simultaneously to yield values of the actual E and S_u. The determination of the actual E is the main concern of this paper.

Meteorological Equation

A relation is sought to express meteorological equation 1. However, for simplicity and ease of handling, it is best to treat the components of this relation individually.

The basic meteorological equation used is of the type generally known as the bulk aerodynamic, or Dalton equation (Slatyer and McIlroy, 1961). Its form is

$$E = G(V_a)\ [p(T_u)h_u - p(T_a)h_a]\ , \qquad [3]$$

where

$G(V_a)$ = a theoretically or empirically derived, known function of wind speed, cm day^{-1} mb^{-1},

V_a = wind speed at height H_a, cm day^{-1},

h = relative humidity, dimensionless,

T = temperature, ^oK,

p = saturation vapor pressure of water, mb,

$p(T)$ = a known relation between the saturation water vapor pressure and temperature (given in tabular or functional form), mb,

a = a subscript indicating a variable determined in the air H_a cm above the soil surface,

u = a subscript indicating a variable determined at the soil surface.

This equation, due to its simplicity, has been used extensively for estimating the loss of water by free-water surfaces, plants, and bare soils. Either ~~Both~~ its empirical form (Harbeck, 1962) or one of its modified forms (Slatyer and McIlroy, 1961, p. 3-40 to 3-44) can be ~~have been~~ employed.

The present study utilized the wind function used by van Bavel (1966)

$$G(V_a) = \left(\frac{\rho_a \epsilon k^2}{\rho_w P}\right) \frac{V_a}{(\log_e H_a/H_u)^2} , \qquad [4]$$

where

ρ_a = air density at T_a, gm cm^{-3},

ρ_w = water density at T_a gm cm^{-3},

ϵ = water/air molecular ratio = 0.622, dimensionless,

k = von Karman constant = 0.41, dimensionless,

P = ambient pressure, mb (taken as P = 1000 mb in this study),

H_a = height of meteorological measurements, above the soil surface, cm,

H_u = roughness parameter, cm (usually, for bare soils, $0.01 \le H_u \le 0.03$).

Equation 3 may be rewritten as follows in order to obtain the form required by equation 1

$$h_u = \frac{1}{p(T_u)}\left[\frac{E}{G(V_a)} + p(T_a)h_a\right] . \qquad [5]$$

The surface relative humidity, h_u , specified by equation 5 may now be substituted into the thermodynamic relation (Edelfson and Anderson, 1943)

$$S_u = -\frac{RT_u}{Mg}\log_e(h_u) \ , \tag{6}$$

where

M = molecular weight of water = 18 gm mole^{-1} ,

g = acceleration of gravity = 980 cm sec^{-2} ,

R = gas constant = 8.32×10^7 erg $^oK^{-1}$ mol^{-1} .

The above substitution would result in an equation expressing S_u in terms of atmospheric variables, the soil surface boundary temperature T_u, and E.

In order to completely attain the form of equation 1, the variables on the right-hand side of the equation sought should be, except for E, entirely meteorological. But T_u, the surface soil temperature, is present in the combination of equations 5 and 6. To replace T_u with meteorological variables and parameters, an appropriate expression for T_u may be developed as follows. First, note that T_u is related to sensible heat transfer in the air by the following equation for turbulent transfer (Slatyer and McIlroy, 1961, p. 3-53; van Bavel, 1966, p. 466)

$$A = -\lambda \gamma G(V_a)\ (T_u - T_a) \ , \tag{7}$$

where

A = sensible heat transfer into the air, cal cm^{-2} day^{-1} ,

λ = latent heat of vaporization of water at T_a, cal gm^{-1} ,

γ = psychrometric constant, = 0.000659 P, mb $^oK^{-1}$.

Second, substitute A into the following heat balance equation (Slatyer and McIlroy, 1961, p. 3-50; van Bavel, 1966, p. 456)

$$Q_N = \lambda \rho_w E + \rho_w A + Q_g \ , \qquad [8]$$

where

Q_N = net radiative flux recieved by the soil surface, cal cm^{-2} day^{-1} ,

Q_g = soil heat flux into the ground, cal cm^{-2} day^{-1} (assumed to equal zero for periods of interest in this study).

The combined equations 7 and 8, after rearrangement, yield the following for T_u

$$T_u = T_a + \frac{Q_N - Q_g - \lambda \rho_w E}{\lambda \gamma \rho_w G(V_a)} \ . \qquad [9]$$

If equation 9 were substituted into a combination of equations 5 and 6, the overall meteorological equation, equivalent to equation 1, would be obtained.

Soil Equation

The simplest system to be considered is portrayed in figure 1, Case A. A homogeneous soil is underlain by a shallow water table, with the reference height Z measured positively upward from the piezometric surface. The soil surface is at Z = L.

For determining water transfer in liquid form, the soil's hydraulic conductivity relation is assumed to conform to an empirical function, originally suggested by Gardner (equation 11, 1958). It is presented here in a modified form (Gardner, 1964) which demonstrates more clearly the physical significance of the coefficients

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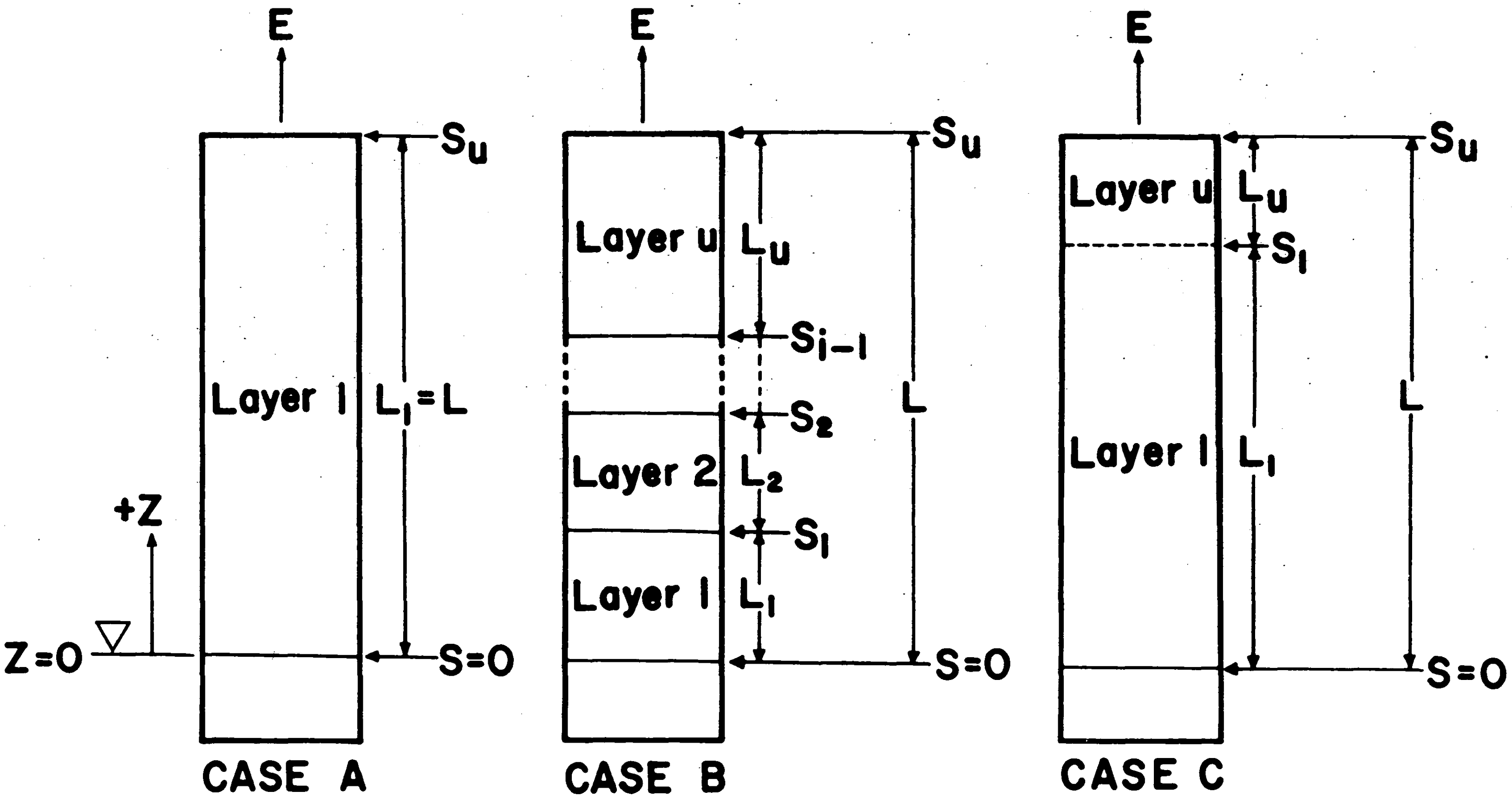

Figure 1.--The water-table-soil-atmosphere systems considered.
Case A: A homogeneous soil with water transferred exclusively in liquid form.
Case B: A layered soil with water transferred exclusively in liquid form.
Case C: A homogeneous soil with water transferred in liquid and vapor forms, the former transfer being predominant in the lower layer and the latter in the upper layer.

$$K = K(S) = \frac{K_{sat}}{\left(\frac{S}{S_{\frac{1}{2}}}\right)^n + 1} , \qquad [10]$$

where

K = hydraulic conductivity for liquid flow, cm day^{-1},

K_{sat} = hydraulic conductivity of water saturated soil, cm day^{-1},

S = soil water suction, defined as the negative of the soil water pressure head, cm of water,

$S_{\frac{1}{2}}$ = a constant coefficient representing S at $K = \frac{1}{2} K_{sat}$, cm of water,

n = an integer soil coefficient which usually ranges from 2 for clays to 5 in sands.

Assuming that Darcy's equation holds for flow in both saturated and unsaturated soils, the flux, q, which under steady state conditions must equal the evaporation rate E, may be described by

$$q = E = K\left(\frac{dS}{dZ} - 1\right) . \qquad [11]$$

On rearranging and integrating, equation 11 becomes

$$Z' = \int_o^{Z'} dZ = \int_o^{S'} \frac{dS}{\frac{E}{K(S)} + 1} , \qquad [12]$$

where $S' = S$ at $Z = Z' \leq L$.

Equation 12 with equation 10 substituted for K(S) becomes

$$Z' = \int_o^{S'} \frac{dS}{\frac{E}{K_{sat}}\left[\left(\frac{S}{S_{\frac{1}{2}}}\right)^n + 1\right] + 1} \qquad [13]$$

The above integral can be expressed in closed form (Gardner, 1958). Equation 13 expresses explicitly Z' as a function of S and E. It also defines implicitly the relation between E and S' for any given Z'. Both facts have been utilized in the past (Philip, 1957a and Gardner, 1958). However, utilization of the implicit relation is unwieldy in practice, except for n = 1 or 2. In the latter two cases the relation can easily be inverted and made explicit. In order to convert equation 13 to a more tractable form, the following transformations may be carried out. First, define the dimensionless variable,

$$e = E/K_{sat}, \qquad [14]$$

and substitute it into equation 13, obtaining

$$Z' = \frac{1}{e}\int_0^{S'} \frac{dS}{\left(\frac{S}{S_{\frac{1}{2}}}\right)^n + \left(1 + \frac{1}{e}\right)} = \frac{1}{e\left(1 + \frac{1}{e}\right)}\int_0^{S'} \frac{}{\left[\frac{S/S_{\frac{1}{2}}}{\left(1 + \frac{1}{e}\right)^{\frac{1}{n}}}\right]^n + 1} . \qquad [15]$$

Second, define a variable y by

$$y = \frac{S/S_{\frac{1}{2}}}{\left(1 + \frac{1}{e}\right)^{\frac{1}{n}}} = \frac{S}{S_{\frac{1}{2}}}\left(\frac{e}{1 + e}\right)^{\frac{1}{n}} , \qquad [16]$$

and transform the integral of equation 15 with its aid, obtaining, after rearrangement, the basic equation of this study:

$$(e + 1)\left(\frac{e}{e + 1}\right)^{\frac{1}{n}} \frac{Z'}{S_{\frac{1}{2}}} = \int_0^{y'} \frac{dy}{y^n + 1} , \qquad [17]$$

where $$y' = \frac{S'}{S_{\frac{1}{2}}}\left(\frac{e}{e+1}\right)^{\frac{1}{n}} .$$

In particular, at the soil surface, when Z' = L

$$(e+1)\left(\frac{e}{e+1}\right)^{\frac{1}{n}} \frac{L}{S_{\frac{1}{2}}} = \int_o^{y_u} \frac{dy}{y^n + 1} , \qquad [18]$$

where
$$y_u = \frac{S_u}{S_{\frac{1}{2}}}\left(\frac{e}{e+1}\right)^{\frac{1}{n}} .$$

The integral on the right-hand side of equations 17 and 18 is known in closed form for any positive n (Gradshteyn and Ryzhik, 1965, equation 2.142). The form of equations 17 or 18 makes it possible to determine the relation between e and the suction (either S' or S_u) for any n by means of simple graphs. This technique as well as the results obtained with its aid will be described presently.

For certain purposes, the use of equations 17 and 18 can be further simplified by adopting the dimensionless variables

$$s = \frac{S}{S_{\frac{1}{2}}} , \qquad [19]$$

$$z = \frac{Z}{S_{\frac{1}{2}}} , \qquad [20]$$

$$\text{and } \ell = \frac{L}{S_{\frac{1}{2}}} , \qquad [21]$$

in addition to the dimensionless $e = E/K_{sat}$ used previously. With the exception of s, these dimensionless variables are similar to those employed by Staley (cited by Anat, 1965) whose hydraulic conductivity equation is also somewhat similar to equation 10. Inspection of numerous curves indicates that $S_{\frac{1}{2}}$ of the dimensionless s matches the observed relations between K and S better than does the air entry pressure, used in this connection by Staley. The above dimensionless variables reduce the basic equation 18 to

$$(e + 1) \left(\frac{e}{e + 1}\right)^{\frac{1}{n}} \ell = \int_{o}^{y_u} \frac{dy}{y^n + 1}, \qquad [22]$$

where

$$y_u = s_u \left(\frac{e}{e + 1}\right)^{\frac{1}{n}} .$$

An analogous reduction can be carried out for equation 17.

The following reasoning leads to another useful relation which is implied by equation 18. It is clear from physical considerations, that an increase in the evaporative capacity of the atmosphere will produce an increased suction at the soil surface. This higher suction, in turn, must magnify the upward water flux through the soil. If equation 18 correctly describes reality, such a flux cannot increase without bound, because as S_u (and hence y_u) approaches infinity, the integral on the right-hand side of equation 18 approaches a finite limit, $\pi/(n \sin \frac{\pi}{n})$ (Gradshteyn and Ryzhik, 1965, equation 3.241-2 with $\mu = 1$). It follows that a limiting soil water flux and hence a soil-limited ~~limiting~~ evaporation, e_∞, exists. For any particular soil system the latter is given by

$$(e_\infty + 1)\left(\frac{e_\infty}{e_\infty + 1}\right)^{\frac{1}{n}} \frac{L}{S_{\frac{1}{2}}} = \frac{\pi}{n \sin \frac{\pi}{n}}, \qquad [23]$$

or, in completely dimensionless form,

$$(e_\infty + 1)\left(\frac{e_\infty}{e_\infty + 1}\right)^{\frac{1}{n}} \ell = \frac{\pi}{n \sin \frac{\pi}{n}}. \qquad [24]$$

The last two equations can be simplified considerably if $e_\infty \ll 1$ (that is, if $E \ll K_{sat}$). In such a case, $e_\infty + 1 \cong 1$, and equations 23 and 24 lead to

$$E_\infty \cong K_{sat}\left[\frac{S_{\frac{1}{2}}}{L}\right]^n \left[\frac{\pi}{n \sin \frac{\pi}{n}}\right]^n, \qquad [25]$$

and

$$e_\infty \cong \frac{1}{\ell^n}\left[\frac{\pi}{n \sin \frac{\pi}{n}}\right]^n. \qquad [26]$$

Equation 25 is similar to the formulas for E_{Lim} given without derivation by Gardner (1958) for $n = \frac{3}{2}$, 2, 3, 4 and yields identical numerical coefficients.

APPLICATIONS

Data Required

The equations presented above may be used to compute the estimated evaporation from bare soils under high water table conditions. The data needed for such computations are as follows.

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The meteorological data (those needed in connection with the utilization of equations 4, 5 and 9) are obtained by standard techniques or from references. These data include wind velocity, V_a, air temperature, T_a, the air relative humidity, h_a, and net radiation, Q_N. The magnitude of $p(T_a)h_a$, the water vapor pressure in the air, is determined from T_a and h_a with the aid of standard tables or formulas. Daily Q_N values may be determined either by direct measurement, or by the method outlined in Slatyer and McIlroy (Appendix II, 1961). For a given site, the latter technique can produce calculated Q_N values with the aid of standard information in the Smithsonian Meteorological Tables (List, 1951). A zero value has been assumed for Q_g in the computations of this paper. This is a reasonable assumption for daily means of Q_g, especially when these are used in conjunction with Q_N and $\lambda\rho_w E$ (see equation 9).

In addition to the above strictly meteorological data, the soil surface temperature, T_u, is needed, because it appears in equations 5 and 6. Data on this temperature are usually unavailable, hence the need for the development of equation 9 as an indirect method for T_u determination. If however, soil surface temperature data are available, it is possible to avoid the use of equation 9 and of the usually approximate Q_N data needed in connection with this equation.

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The soil equation requires knowledge of the hydraulic conductivity for a reasonable range of soil water suctions. Such data will allow evaluation of the necessary coefficients K_{sat}, n, and $S_{\frac{1}{2}}$ for a particular soil. K_{sat} can be measured directly and readily. However, the other two coefficients are more difficult to obtain. They may be computed from more routinely available data, using the technique of Marshall (1958), as modified by Millington and Quirk (1961) and by Jackson and others (1965). This technique produces, for selected magnitudes of S, a series of scaled hydraulic conductivity values $K'(S) = \xi K(S)$, where ξ is the scale factor and where K'(S) at S = 0 is designated by K'_{sat}. Note that the scale factor need not be determined to find n and $S_{\frac{1}{2}}$. If equation 10 is obeyed, a plot of $\log(\frac{K_{sat}}{K(S)} - 1)$ $[= \log(\frac{K'_{sat}}{K'(S)} - 1)]$ versus log S is linear. The slope of such a plot is equal to n and the plot's intercept with the abscissa determines $S_{\frac{1}{2}}$.

The manner of the computation of the scaled hydraulic conductivity values is adequately described in the above references. The basic information required by these computations is the characteristic relation between the soil water suction and the volumetric moisture content (that is, the water retention curve or the pore size distribution function). Such data are regularly determined in soil laboratories. Some of these ~~They~~ may also be obtained in the field by measuring the moisture content of the soil overlying a sufficiently shallow water table as a function of depth, after a prolonged period of negligible soil water fluxes.

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Homogeneous Soil

For a homogeneous soil with insignificant vapor transfer (figure 1, Case A), evaporation, E, can be computed from the meteorological and soil equations in several ways. In most cases it is convenient to compute e first. When appropriate, e may be converted to E using equation 14. In the early stages of the computation, the soil-imposed upper bounds of e (e_∞) or the bounds imposed by atmospheric factors (e_{pot}) may be needed. They can be easily computed as will be shown presently.

Equations 4, 5, 6 and 9 are combined and yield an overall meteorological equation, which expresses S_u as a function of e and which corresponds to equation 1. This equation is substituted into the soil equation 18 yielding a nonlinear algebraic equation in e. The root of this equation may be found by routine numerical methods. In this study, the method of "false position" (Hildebrand, 1956, pp. 446-447) was programmed for a digital computer, tried, and found satisfactory.

Alternately, e can be obtained by plotting the curves corresponding to the above meteorological and soil equations, the magnitude of the actual e being given by the intercept of the two curves. The meteorological equation is plotted for selected values of $e < e_{pot}$ in a straightforward manner. The soil curve is determined for selected values of $e < e_\infty$ using the following graphical procedure. Any given e value may be used with an appropriate (that is, proper n) plot of

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$$f = f(e) = (e + 1)\left(\frac{e}{e + 1}\right)^{\frac{1}{n}} \qquad [27]$$

to determine the corresponding value of f (figure 2). When f is multiplied by $\ell = L/S_{\frac{1}{2}}$, one obtains the magnitude of the left-hand side of equation 18. This magnitude is equal to the value of the integral, $I = I(y_u)$, on the right-hand side of the same equation. Then by using a plot of $I(y_u)$, given in figure 3, y_u is found. Finally, the required S_u is computed from y_u using the relevant definition, given below equation 18.

For less accurate but quicker estimates of the soil curve, dimensionless plots of the type illustrated in figures 4A and 4B may be used, the needed values of e and s being obtained for any given $z = \ell$. The limited range covered by the plot, and the necessity for a field filled with many curves are the obvious detriments of this approach. It should be noted that the curves in question also indicate the suction within the soil profile as a function of depth for any given evaporation rate.

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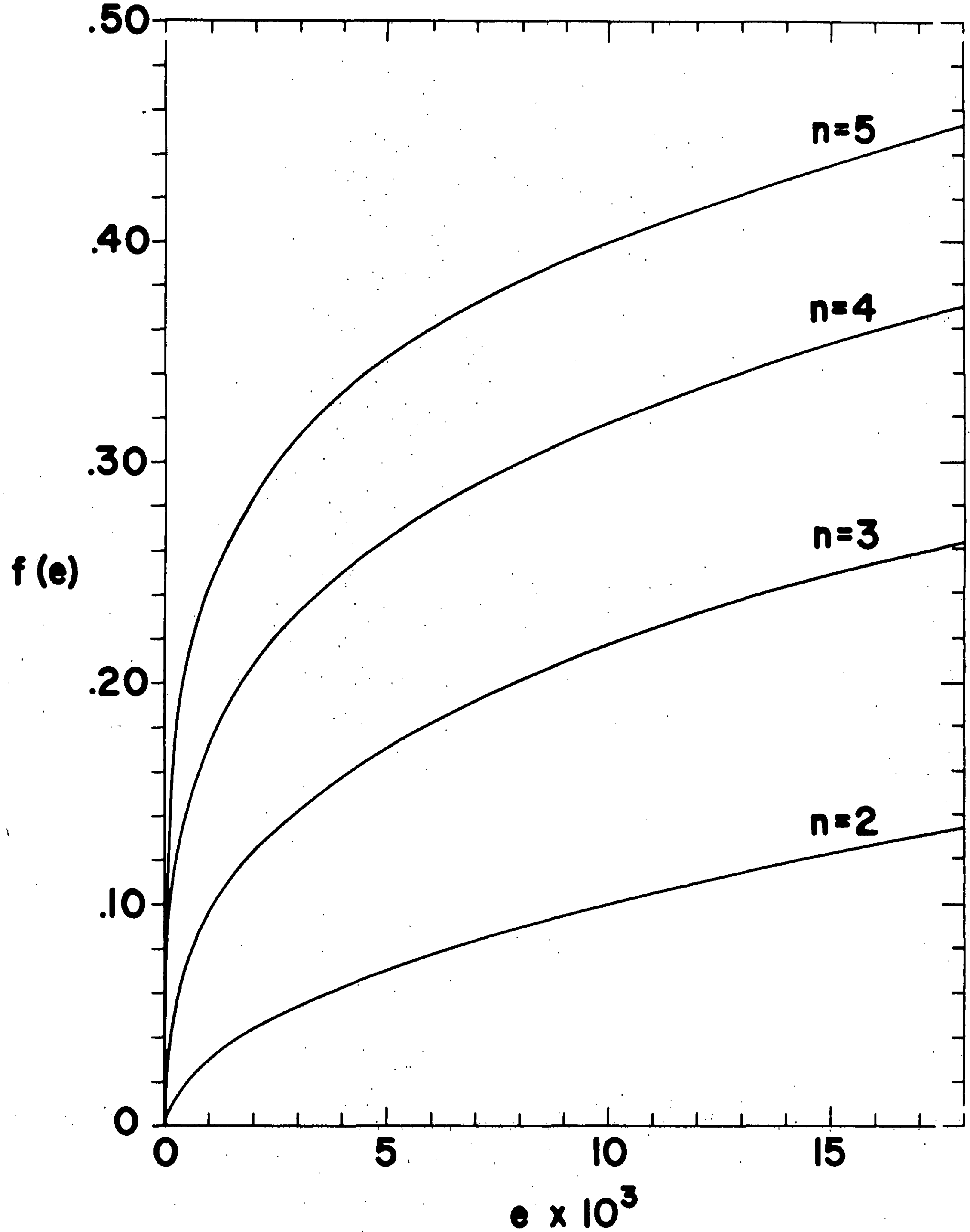

Figure 2.--Dimensionless plots of $f = f(e) = (e+1)\left(\frac{e}{e+1}\right)^{\frac{1}{n}}$ for n = 2, 3, 4, 5.

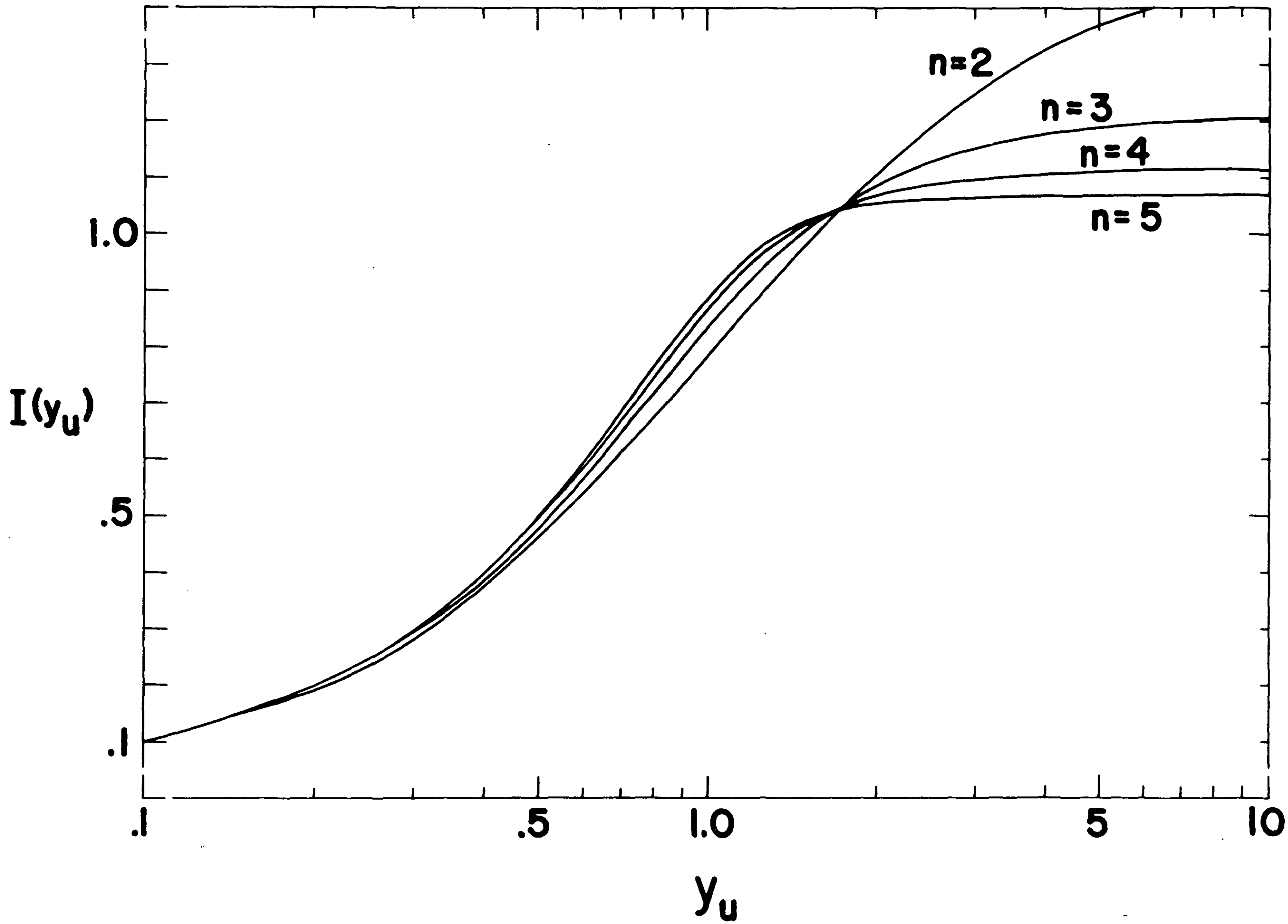

Figure 3.--Dimensionless plots of $I = I(y_u) = \int_0^{y_u} \frac{dy}{y^n+1}$ for n = 2, 3, 4, 5.

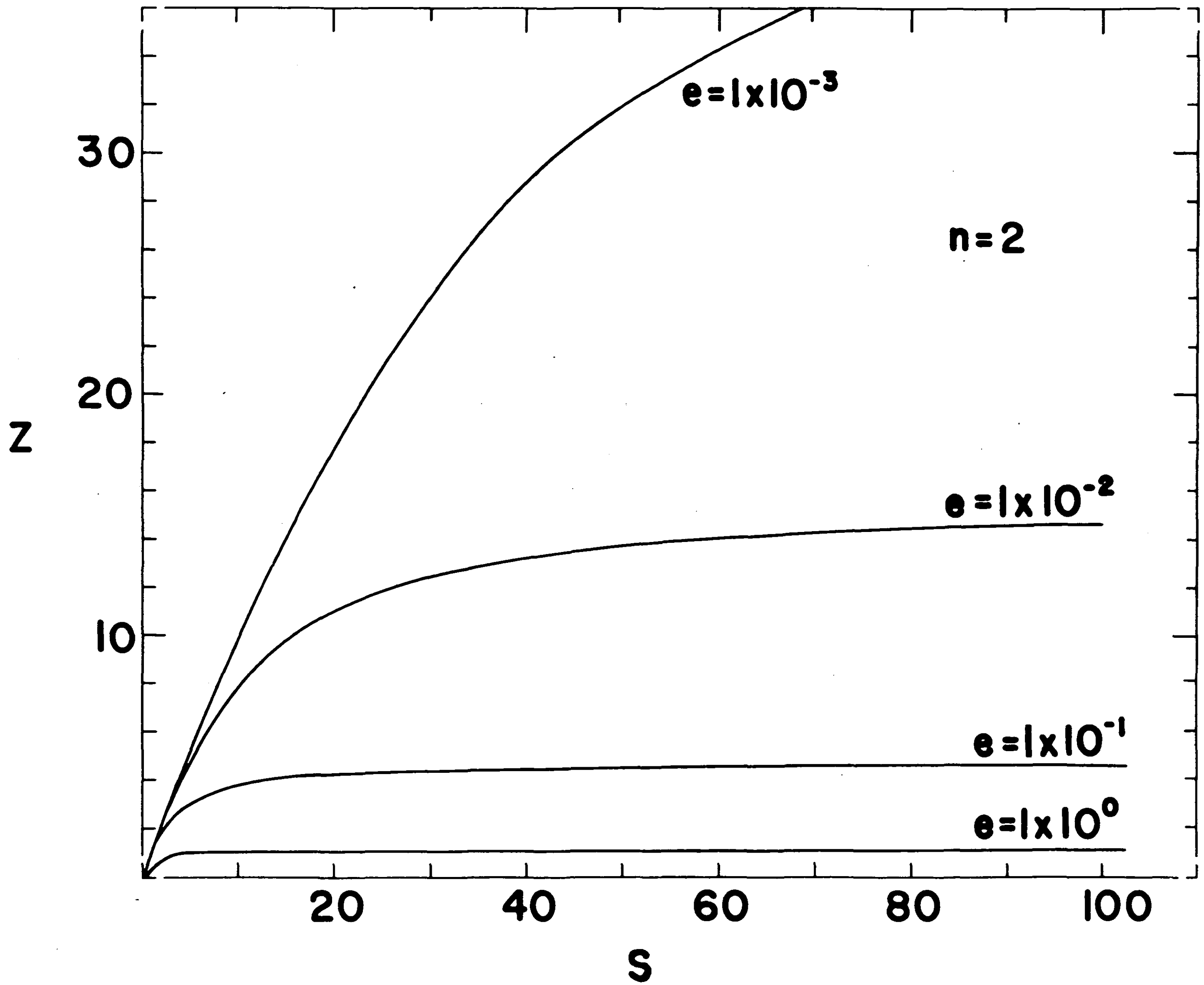

Figure 4A.--Dependence of dimensionless soil water suction, s, on dimensionless soil height z. The numbers labeling the curves indicate the magnitude of dimensionless evaporation rates, e. A. Soil parameter n = 2.

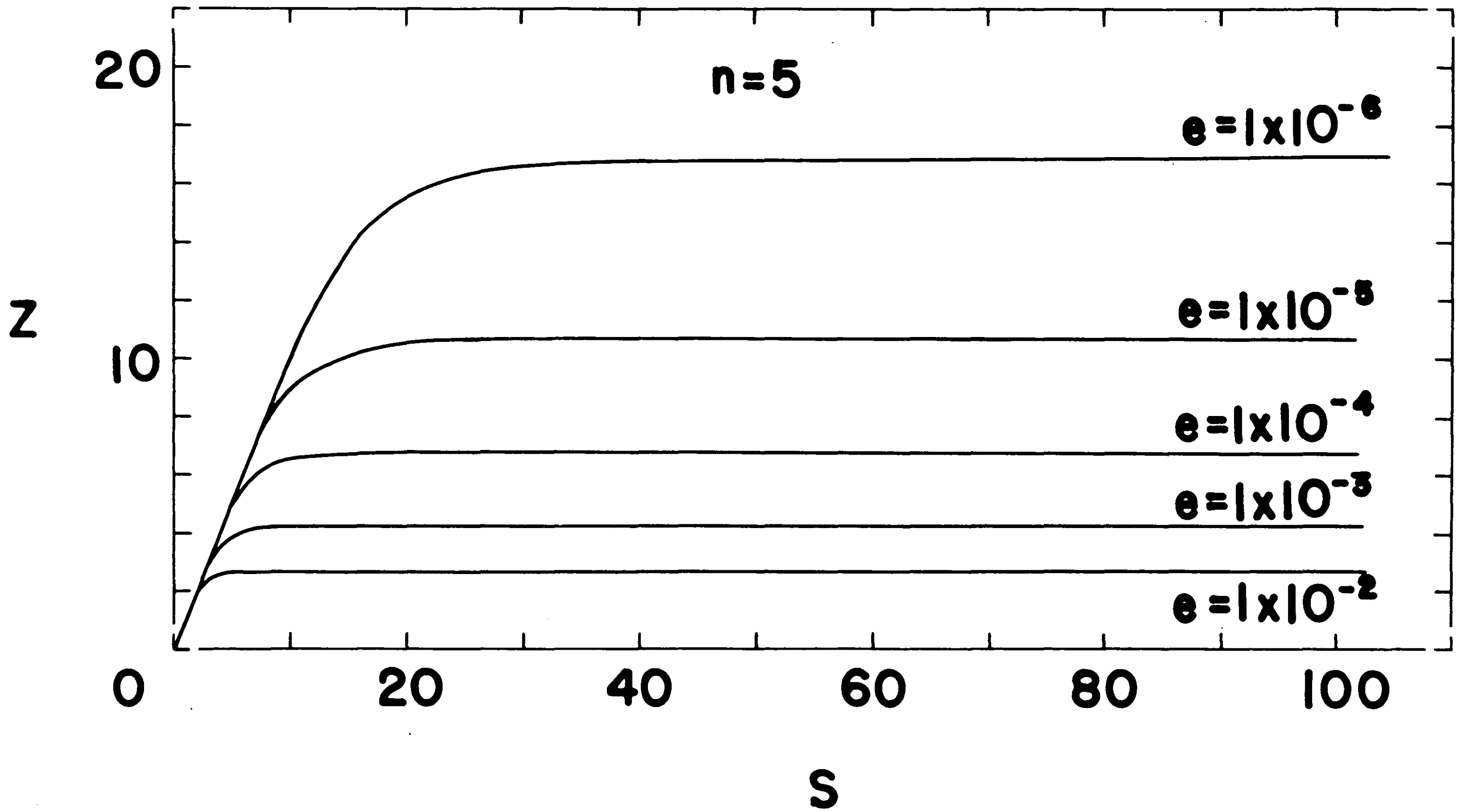

Figure 4B.--Dependence of dimensionless soil water suction, s, on dimensionless soil height, z. The numbers labeling the curves indicate the magnitude of dimensionless evaporation rates, e. B. Soil parameter n = 5.

Exact and approximate limiting evaporation rates, imposed by soil e_∞ or E_∞, can be obtained from equations 23 through 26, The approximate values are given directly by the appropriate equations. The exact values can be computed easily with the aid of figure 2, or if less accurate values are needed, they can be read off directly from an appropriate dimensionless plot in figure 5.

The limiting evaporation rates imposed by meteorological conditions, e_{pot}, can be computed (graphically or numerically) for any weather data by solving simultaneously equations 5 and 9, with $h_u = 1.0$ (that is, with $S_u = 0$).

Examples of results obtained with the aid of the above graphical methods are shown in figures 6, 7 and 8. The examples refer to two selected soils, Chino clay with $n = 2$, $S_{\frac{1}{2}} = 24$, and $K_{sat} = 1.95$ (Gardner and Fireman, 1958) and a coarse-textured alluvial soil taken from the 50-60 cm zone of the U.S. Geological Survey evaporation tanks near Buckeye, Arizona, with $n = 5$, $S_{\frac{1}{2}} = 44.7$ and $K_{sat} = 417$. These evaporation tanks are described by van Hylckama (1966).

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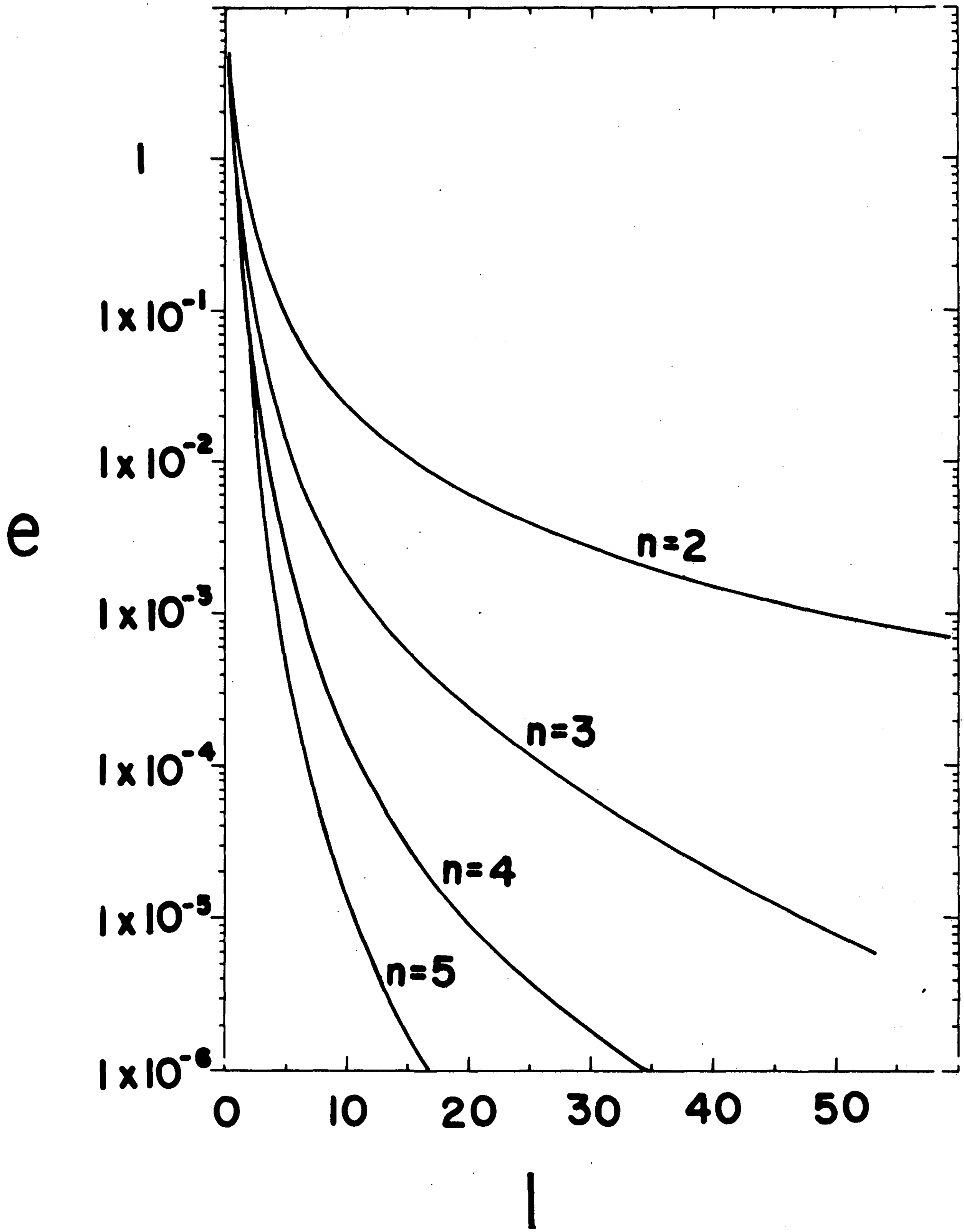

Figure 5.--Plots relating dimensionless evaporation, e, to dimensionless depth, ℓ, for n = 2, 3, 4, 5.

Application of the graphical intersection method is illustrated in figures 6A and 6B. Each figure shows meteorological curves for several arbitrarily selected atmospheric conditions and soil curves corresponding to several water table depths. Note that the soil curves approach a limiting E with increasing S_u, in agreement with the previously presented theoretical proof. The rate of approach to the actual E_∞ (or e_∞) shown by the soil curves mainly depends on the value of n characterizing the particular soil. A relatively rapid approach is exhibited by the Buckeye soil (n = 5) while in the case of Chino clay (n = 2) the approach is much more gradual. It should be noted that most of the field soils commonly found show n values which lie between 2 and 5. Hence, such soils will usually yield $E(S_u)$ plots similar to or intermediate between those shown in figures 6A and 6B. The meteorological curves also seem to approach a limiting E, but with decreasing S. The values of E, fixed by the intersection points between meteorological and soil curves of the figures in question, represent the actual evaporation rates under the particular meteorological, soil and water table conditions.

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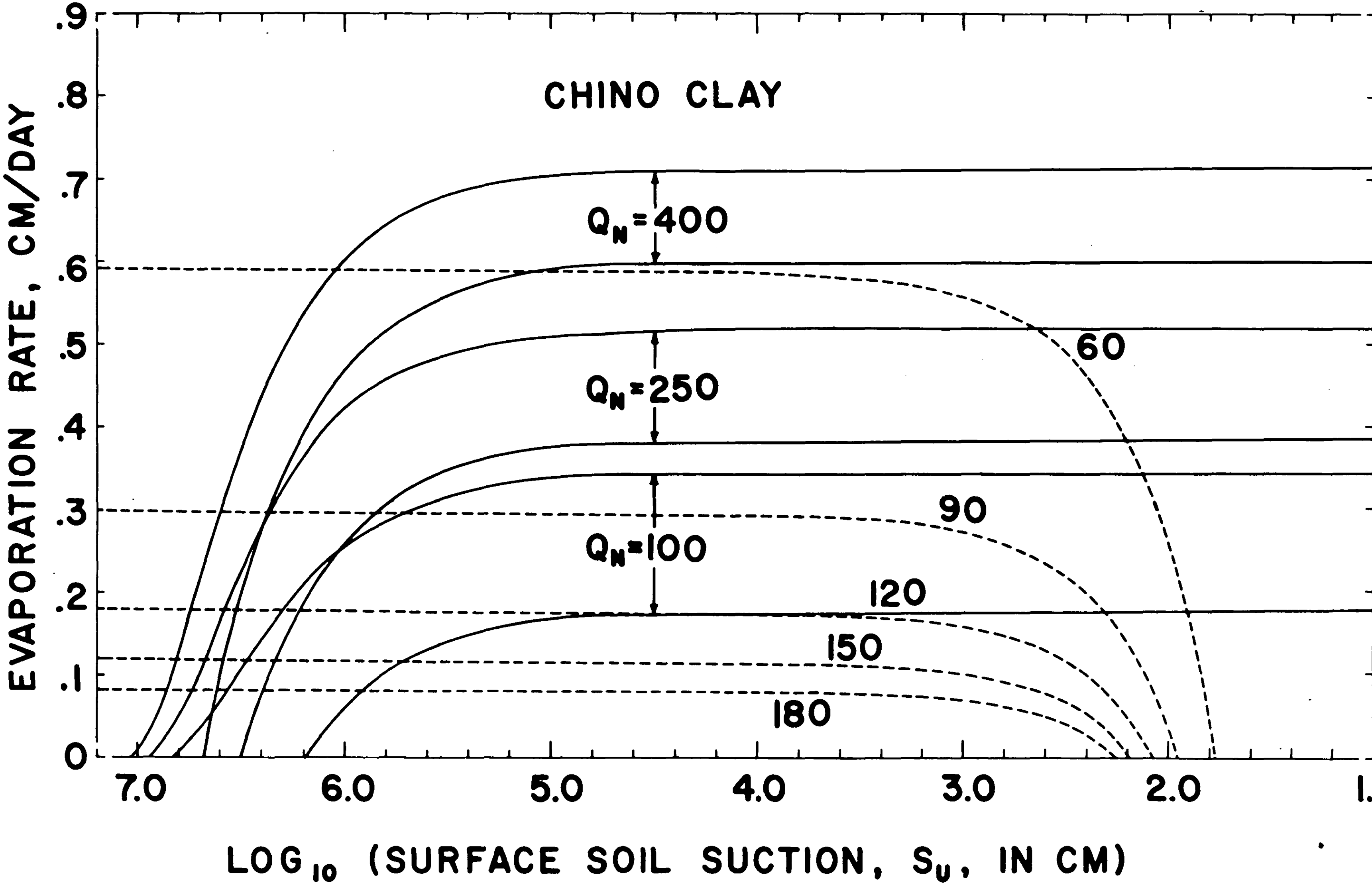

Figure 6A.--The intercept method for determining evaporation rates. The solid lines represent the meteorological curves for wind speed of 6 km/hr, air temperature of 25°C, and for the indicated Q_N values. The top and bottom curves corresponding to a given Q_N, represent air relative humidities, h_a, equal to 0.02 and 0.75, respectively. The dashed lines represent the soil curves for the indicated water table depths, L.

A. Chino clay.

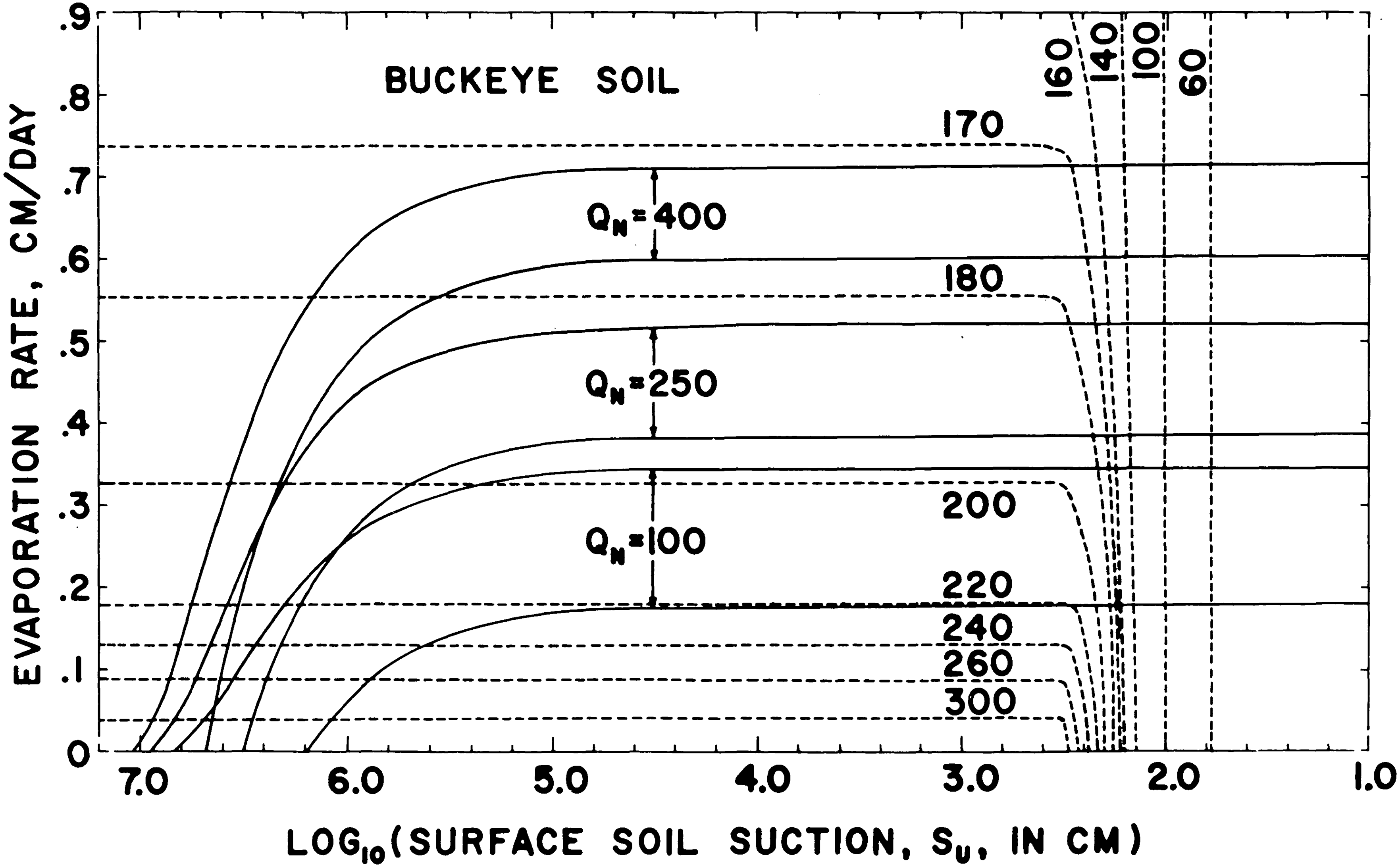

Figure 6B.--The intercept method for determining evaporation rates. The solid lines represent the meteorological curves for wind speed of 6 km/hr, air temperature of 25°C, and for the indicated Q_N values. The top and bottom curves corresponding to a given Q_N, represent air relative humidities, h_a, equal to 0.02 and 0.75, respectively. The dashed lines represent the soil curves for the indicated water table depths, L.

B. Buckeye soil.

The dependence of the actual E on weather and water table depth is demonstrated more clearly in figures 7 and 8. Figure 7A and B is concerned with the influence of the depth to water table under given meteorological conditions. This figure demonstrates that for a particular soil and meteorological condition, the evaporation rate remains essentially constant and fixed by weather, if the water table depth does not exceed a certain value. With the water table at greater depths, the evaporative flux decreases markedly because the soil becomes the limiting factor. In other words, the flux decreases because in figure 6, the pertinent meteorological curve intercepts the flat portion of the relevant soil curve. In agreement with the observations by Philip (1957b), for any given set of meteorological and soil conditions, the transition between the horizontal and descending portions of an appropriate curve in figure 7 is so sharp that it can be taken as discontinuous and its curvature can be neglected. Therefore, each curve of figure 7 consists, essentially of a horizontal part fixed by the weather, and a descending part fixed by equation 23 or 24 (that is, by figure 5).

It is this characteristic form of the curve that leads to the simplicity of the following procedure for determining the actual E. The appropriate soil-limited evaporation, E_{∞}, may be determined with equation 23 or 24, and plotted against depth to the water table. The appropriate meteorologically controlled potential evaporation, E_{pot},

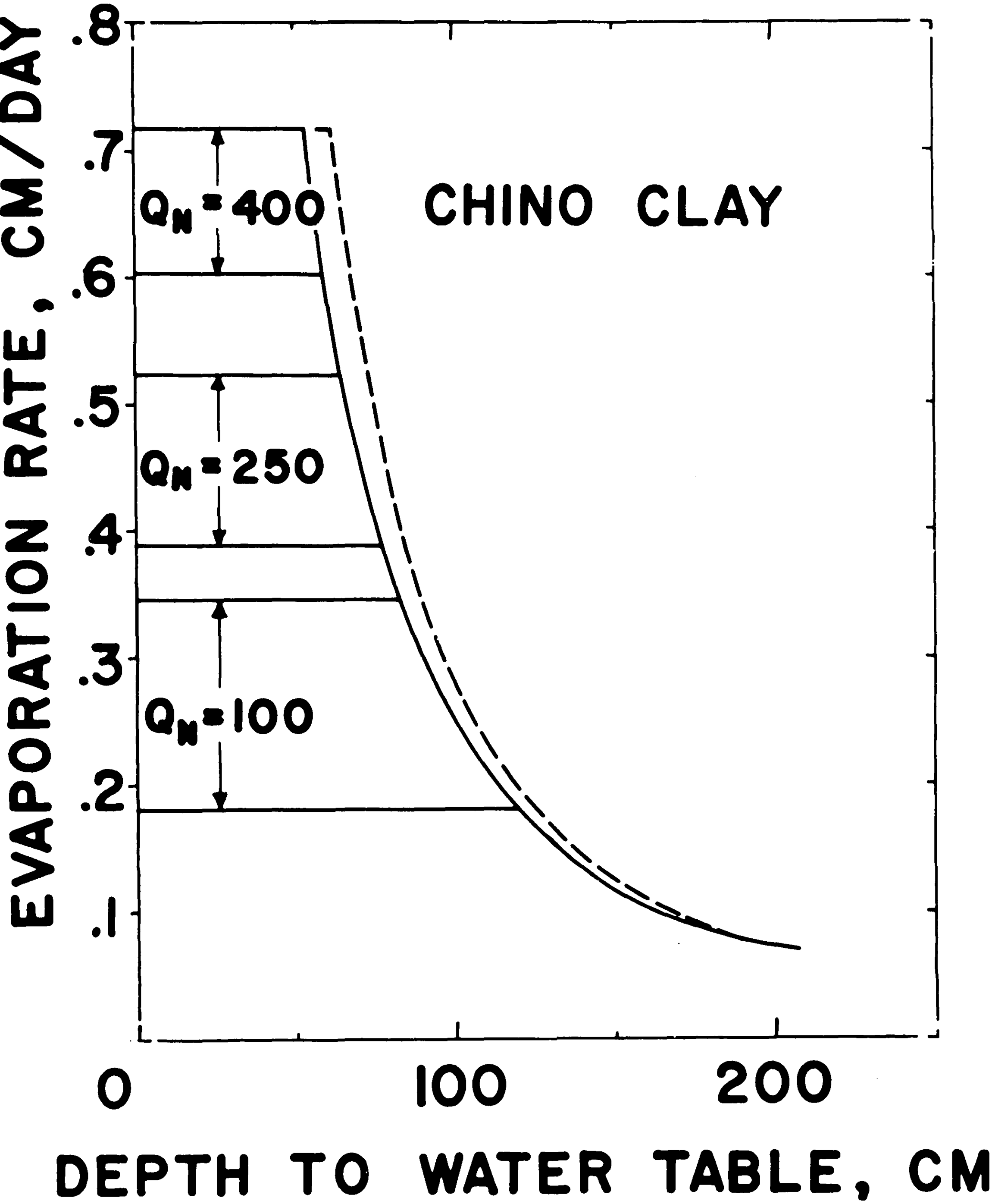

Figure 7A.--Relation between evaporation rates and water table depths, calculated by the intersection method (solid lines). The indicated meteorological conditions are identical with those of figure 6. The descending solid line also represents the exact soil-limited rates of evaporation obtained from equation 23. The dashed line represents the approximate soil limited rates of evaporation and is obtained from equation 25. The exact and approximate curves coincide in B.

A. Chino clay.

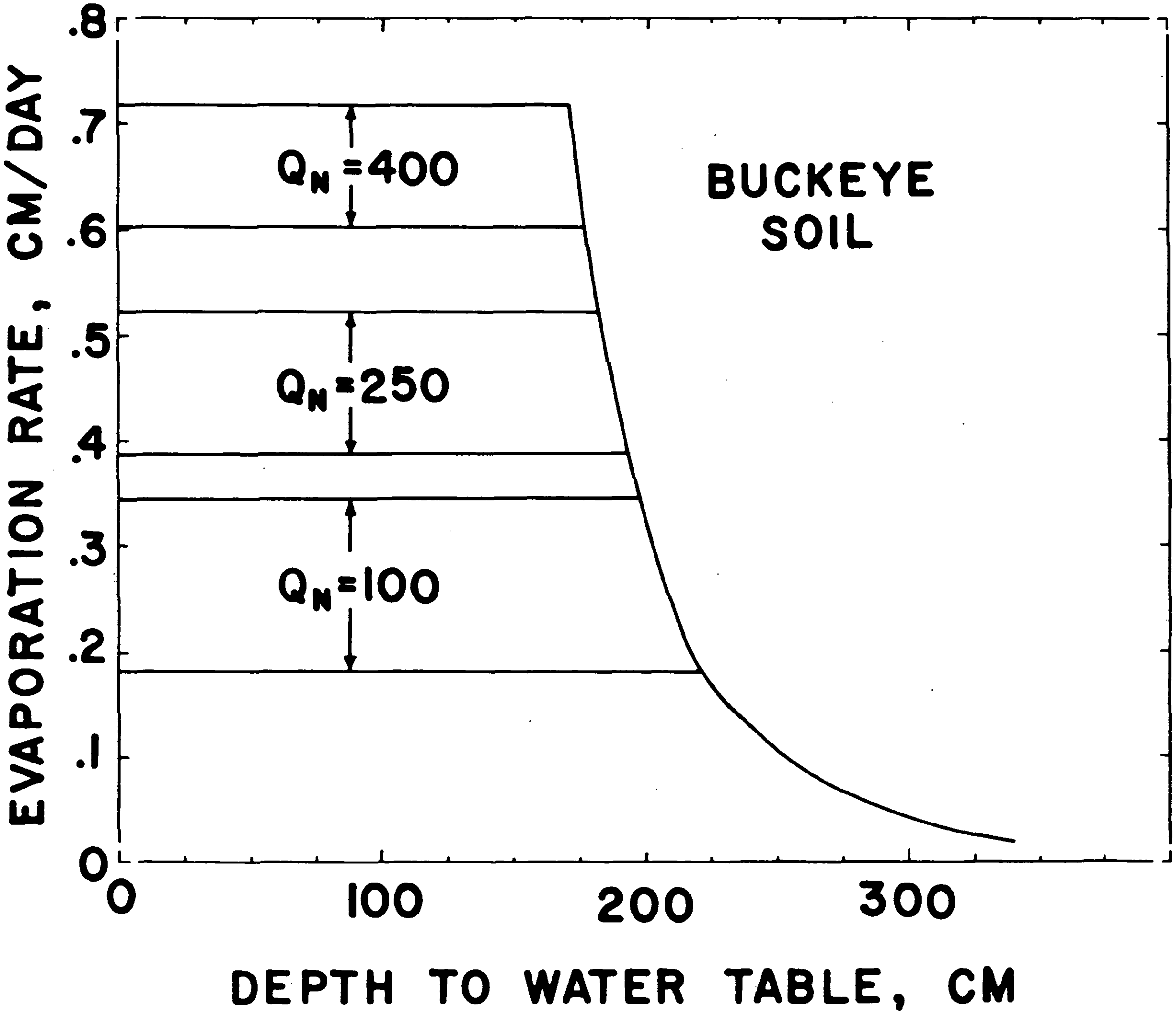

Figure 7B.--Relation between evaporation rates and water table depths, calculated by the intersection method (solid lines). The indicated meteorological conditions are identical with those of figure 6. The descending solid line also represents the exact soil-limited rates of evaporation obtained from equation 23. The dashed line represents the approximate soil limited rates of evaporation and is obtained from equation 25. The exact and approximate curves coincide in B.

B. Buckeye soil.

may then be entered as a straight, horizontal line. The actual ~~maximum~~ evaporation for any given water-table depth may be taken as the lowermost portions of the two intersecting curves.

Note that if $E \ll K_{sat}$, as in the case illustrated in figure 7B, the exact and approximate E_{∞} curves essentially coincide. Hence, equations 25 or 26 may be used for estimating E under such circumstances. On the other hand, figure 7A illustrates a case in which such a coincidence does not occur. As a result, the approximate E_{∞} curve overestimates the actual E in the descending portion of the E curve.

Figure 8 illustrates, for several water table depths in Chino soil how efficiently the atmosphere can remove soil water under various meteorological conditions. The index of the meteorological conditions is the potential (that is, $S_u = 0$) evaporation, E_{pot}. The efficiency of removal is measured by the ration E/E_{pot}. For a given water table depth, the figure demonstrates that the maximum efficiency of water removal (= 1.0) occurs at small values of E_{pot}. For any given water table depth, as E_{pot} increases, the efficiency remains at a maximum until a certain limiting E_{pot} is reached. Thereupon the efficiency declines rapidly. This transition point is fixed by the water table depth and occurs when the evaporation rate becomes limited by the soil's inability to conduct water rapidly enough.

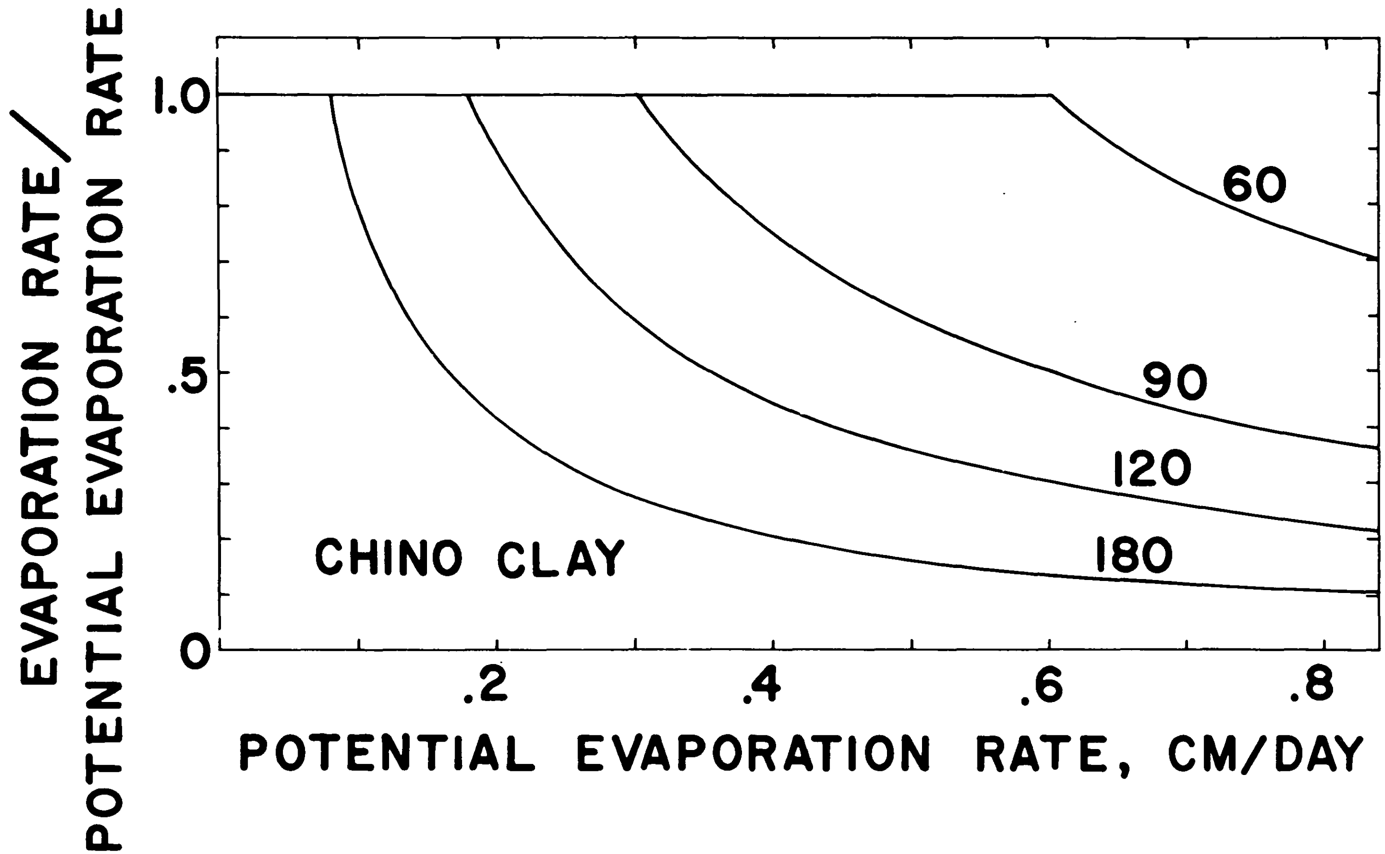

Figure 8.--The dependence of relative evaporation rates, E/E_{pot}, upon the potential evaporation rates, E_{pot}, for Chino clay. Numbers labeling the curves indicate the depths to water table.

Layered Soil

In a manner analogous to the homogeneous case, steady state evaporation in a layered system unaffected by vapor transfer may be described by the functional relations appropriate to each layer. For a soil with i layers above the water table, (figure 1, Case B), these relations may be symbolized by

Soil layer 1 (lowermost): $L_1 = F_{g_1} (S_1 , E)$, [28-1]

Soil layer 2 : $L_2 = F_{g_2} (S_1 , S_2 , E)$, [28-2]

Soil layer 3 : $L_3 = F_{g_3} (S_2 , S_3 , E)$, [28-3]

. . . .

Soil layer i (uppermost): $L_u = F_{g_i} (S_{i-1} , S_u , E)$, [28-i]

The atmosphere : $E = F_m (S_u)$. [29]

In any one of the equations 28-j above ($j = 1,2,...i$), S_{j-1} and S_j are, respectively, the suctions at the lower and upper interface of layer j. Note that S_o is known ($S_o = 0$). Therefore, it does not appear in equation 28-1. In addition, in conformance with the earlier symbolism, S_i ~~it~~ is designated as S_u (see equation 28-i). Presently, the subscript j will also be used for subscripting the coefficients n, $S_{\frac{1}{2}}$ and K_{sat} of the layer j.

The above set of equations may be solved simultaneously since it contains as many equations as unknowns. Such a solution may be achieved using either a numerical or graphical (intercept) method. The latter method will be described presently. Note that, as in the homogeneous case, the present approach is possible due to the fact that two adjacent layers exhibit identical suctions at their common interface.

It will be recalled that the intercept method discussed previously involves finding the intersection between plots representing the meteorological and soil equations. In applying the intercept method to the layered case, one must deal with ~~different~~ sets of parameters which differ from layer to layer. Hence, E and S should be plotted rather than their dimensionless counterparts, although e may be employed in certain computations involving single layers.

The meteorological curve needed is plotted with the aid of ~~from~~ equations 4, 5, 6 and 9, as it was in the homogeneous soil case. The graph of the soil equation involves S_u (in addition to E), that is ~~i.e.~~, the surface suction of the uppermost layer. To plot such a graph for a layered soil system, a procedure for obtaining S_u from any given E must be used. This procedure involves the determination, for a given E value, of the suction, S_j, at the upper surface of each successive soil layer, starting with j = 1 and ending with the appropriate value of S_u for j = i.

The equation for computing such a suction at the lowermost layer 1, (figure 1 Case B), is:

$$(e_1 + 1)\left(\frac{e_1}{e_1 + 1}\right)^{\frac{1}{n_1}} \frac{L_1}{(S_{\frac{1}{2}})_1} = \int_0^{y_1} \frac{dy}{y^{n_1} + 1}, \qquad [30]$$

where $e_1 = E/(K_{sat})_1$,

$$y_1 = \frac{S_1}{(S_{\frac{1}{2}})_1}\left(\frac{e_1}{e_1 + 1}\right)^{\frac{1}{n_1}} .$$

Note that equation 30 is identical with ~~to~~ equation 18, because of the physical similarity of the respective situations. The graphical procedure for obtaining S_1 (utilizing figures 2 and 3) described for equation 18 is applicable here.

In the second step, the following equation is used for the relations in layer 2:

$$(e_2 + 1)\left(\frac{e_2}{e_2 + 1}\right)^{\frac{1}{n_2}} \frac{L_2}{(S_{\frac{1}{2}})_2} = \int_{\tilde{y}_1}^{y_2} \frac{dy}{y^{n_2} + 1} , \qquad [31]$$

where $$e_2 = E/(K_{sat})_2 ,$$

$$y_2 = \frac{S_2}{(S_{\frac{1}{2}})_2}\left(\frac{e_2}{e_2 + 1}\right)^{\frac{1}{n_2}} ,$$

$$\tilde{y}_1 = \frac{S_1}{(S_{\frac{1}{2}})_2}\left(\frac{e_2}{e_2 + 1}\right)^{\frac{1}{n_2}} ,$$

$$(\tilde{y}_1 \neq y_1) \; !$$

The derivation of the above equation is identical in principle to that of equation 17. However, the lower boundary condition here is $S = S_1$ and not zero, as it was in equation 17.

Equation 31, for ease in handling, is rearranged:

$$(e_2 + 1)\left(\frac{e_2}{e_2 + 1}\right)^{\frac{1}{n_2}} \frac{L_2}{(S_{\frac{1}{2}})_2} + \int_o^{\tilde{y}_1} \frac{dy}{y^{n_2} + 1} = \int_o^{y_2} \frac{dy}{y^{n_2} + 1} \quad . \qquad [32]$$

With the aid of equation 32 one can find S_2 for the given S_1 and E values. To accomplish this, first compute $\tilde{y}_1$ and e_2, using the relevant definitions given in connection with equation 31. The integral ~~term~~ on the left-hand side of equation 32, $I(\tilde{y}_1)$ is then evaluated employing the appropriate curve of figure 3. Next, a technique identical with that of the homogeneous case (and involving figure 2) is used to determine the magnitude of the first term of equation 32, $f(e_2)\ L_2/(S_{\frac{1}{2}})_2$. Addition of the latter term to the previously computed $I(\tilde{y}_1)$ yields the value of $I(y_2)$ from which S_2 is computed using figure 3.

Equations such as ~~like~~ equation 32, with subscript 2 replaced by $j = 3,4,\ldots i$, may be written for each additional soil layer. Thus the calculation procedure may be carried stepwise up the soil profile. The equation for the uppermost layer, leading to the S_u values sought is:

$$(e_i + 1)\left(\frac{e_i}{e_i + 1}\right)^{\frac{1}{n_i}} \frac{L_i}{(S_{\frac{1}{2}})_i} + \int_o^{\tilde{y}_{i-1}} \frac{dy}{y^{n_i} + 1} = \int_o^{y_u} \frac{dy}{y^{n_i} + 1} \ , \qquad [33]$$

with the definitions of $\tilde{y}_{i-1}$ and y_u similar to those of analogous terms in equation 31.

Often, the only information sought is ~~only~~ the dependence of the soil-limited evaporation, E_∞, upon the water table depth. Such information may be obtained for multilayered systems without determining the individual soil curve and without using graphical or numerical means. Most of the required procedure consists of computing, for various E values of interest, the suctions at the lower surfaces of successive soil layers, starting with the uppermost layer, i, and finishing with the layer just above the one in which the water table can be found. These computations are followed by calculation of the water table position in the lowest soil layer, 1.

The ~~relevant~~ equation for the uppermost layer required by such a procedure is derived from equation 33, by noting that e_∞ is associated with an infinite S_u and hence with an infinite y_u. This in turn implies that the integral on the right of equation 33 is equal to $\pi/[n_i \sin(\pi/n_i)]$ (see the derivation of equation 23). Using this fact, one obtains, after rearrangement, the following equation for the uppermost layer:

$$\frac{\pi}{n_u \sin\left(\frac{\pi}{n_u}\right)} - (e_u + 1)\left(\frac{e_u}{e_u + 1}\right)^{\frac{1}{n_u}} \frac{L_u}{(S_{\frac{1}{2}})_u} = \int_0^{\tilde{y}_{u-1}} \frac{dy}{y^{n_u} + 1} \quad [34]$$

The value of the left-hand side of equation 34 can be computed for the known parameters involved. From this value, $\tilde{y}_{u-1}$ is determined with the aid of figure 3. The definition of $\tilde{y}_{u-1}$ provides the means of calculating the corresponding S_{i-1}.

The underlying layers, j = i-1, i-2,...,2, are described by equations identical in form to equation 32, but with index 2 replaced by indices appropriate to the particular layer. These equations may be successively solved for S_{j-1}, progressing downwards, in the manner closely resembling the one described in the preceding paragraph. In each step, the suction previously determined at the lower interface provides the suction value for the upper interface of the analyzed layer. This procedure may be carried out stepwise, down the soil profile, for any number of discrete soil layers, until the lowermost layer is reached. At this point, equation 30 is used with y_1 known from the solution of the equation appropriate to the layer just above. This equation is applicable because the suction at the lower surface of layer 1 (the water table surface) is equal to zero for all cases. Equation 30 may be used for determining the value of L_1 which corresponds to the value of E employed. The final result of such a computation for a given value of E is the relevant depth to the water table expressed as the sum total of soil layer thicknesses. Note that as computations for various E values progress, the water table position may be found to shift from one soil layer to an adjacent one. Such cases would necessitate an appropriate adjustment in the computation procedure outlined above.

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Figure 9 demonstrates the application of either of the above two computation methods to Buckeye soil with: i. no crust; ii. the same soil overlain by a slightly salt-cemented upper crust ($n = 4$, $S_{\frac{1}{2}} = 28.1$, $K_{sat} = 47$) of either 3 or 10 cm thickness; and iii. the same soil overlain by the 10 cm crust of the previous layer plus an uppermost 10 cm layer of a hypothetical soil ($n = 3$, $S_{\frac{1}{2}} = 20$, $K_{sat} = 20$). The figure shows clearly that a relatively thin less permeable layer may markedly decrease evaporation rates.

Effects of Vapor Transfer

If a homogeneous soil in contact with a water table is sufficiently dry near the surface, water transfer in the dessicated, upper region involves primarily vapor rather than liquid flow. Vapor flux in this layer may depend significantly on soil-temperature gradients. The probable existence of such a transfer can be detected by noting that, in general, appreciable vapor-transfer influences in soils tend to occur when $h \leq 0.8$ (Philip and de Vries, 1957; Rose, 1963b; Jackson, 1964). To utilize this fact, derive h_u, using the previously described procedures for cases unaffected by vapor-transfer (for example, after computing E, employ equation 5 to evaluate h_u). If the derived h_u value is smaller than 0.8, it might be expected that E can be significantly affected by vapor-transfer.

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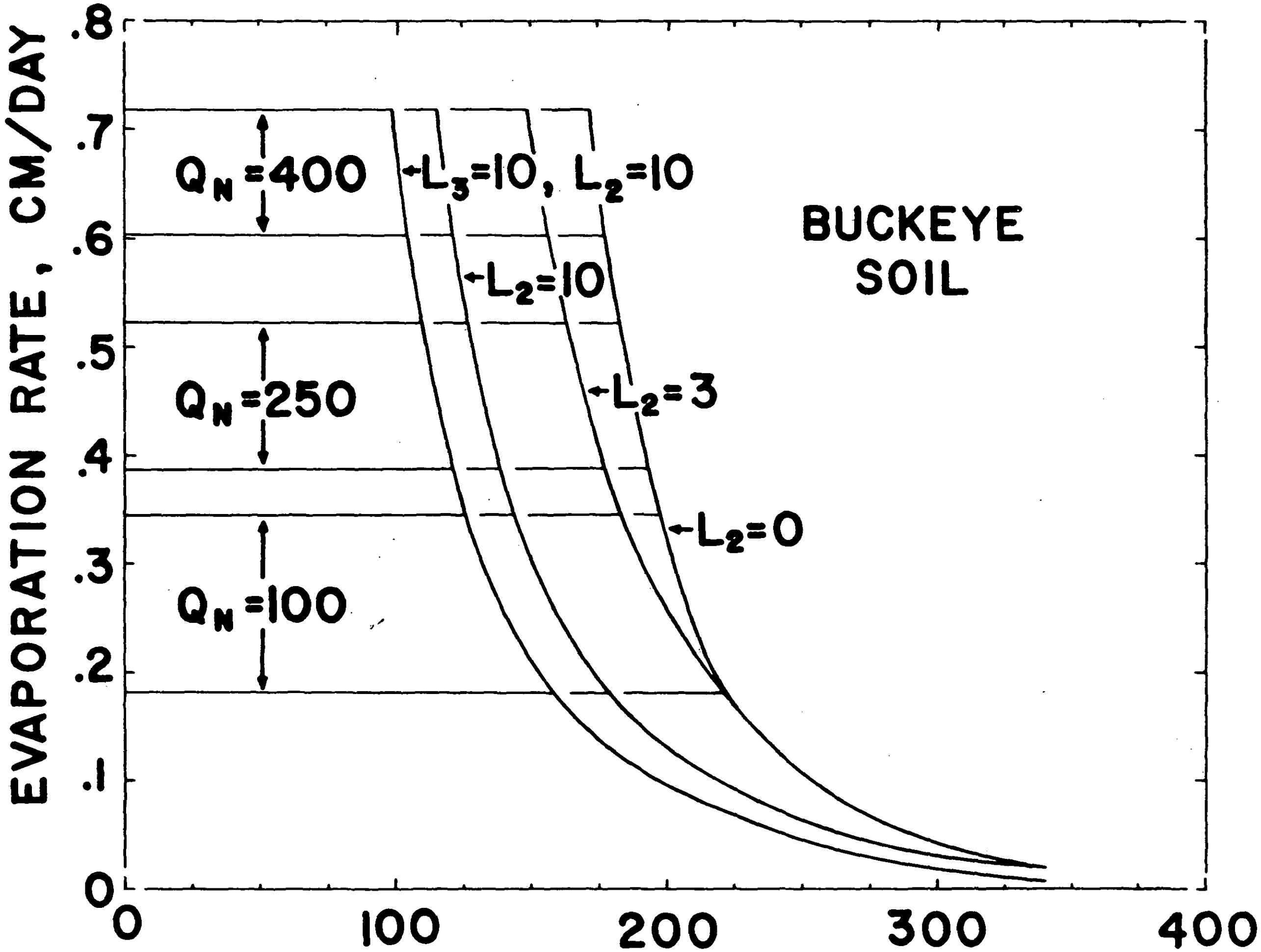

Figure 9.--The influence of layering on the relation between evaporation rate and depth to water table. Soil evaporation limiting curves are shown for i. the homogeneous case (L_2 = 0); ii. a two-layered soil, with the upper layer thickness, L_2, of either 3 or 10 cm; and iii. a three-layered soil with the thicknesses of intermediate and uppermost layers equal to L_2 = 10 cm and L_3 = 10 cm, respectively.

When the dessicated, upper layer in question is present, a more or less exact evaluation of E involves numerical integrations and is based on heat-transfer as well as water-transfer equations. The approach outlined below avoids this relatively complex procedure, but it is clearly approximate. This approach utilizes a suggestion originally made by Gardner (1958) and a theory of vapor-transfer in soils developed by Philip and de Vries (1957; see also de Vries, 1958).

Gardner suggested that the homogeneous soil-water system in question may be represented approximately by a two-layered column (figure 1, Case C) in which water is being transported exclusively in vapor form within the upper layer u, while in the lower layer 1 only liquid flow take place. The theory of Philip and de Vries (1957), applied to the dry, upper soil layer of such a system may be formulated in terms of humidity and temperature gradients. Such a formulation results in the following equation of vapor flow

$$E = - D_{hv} \frac{dh}{dz} - D_{Tv} \frac{dT}{dz} \qquad [35]$$

where

D_{hv} = a coefficient characterizing the molecular diffusion of soil-water vapor caused by humidity gradients, $cm^2\ day^{-1}$,

D_{Tv} = a coefficient characterizing the molecular diffusion of soil-water vapor caused by thermal gradients, $cm^2\ day^{-1}\ {}^oK^{-1}$.

It can be shown (Penman, 1940, Philip and de Vries, 1957) that the coefficient D_{hv} is described by

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$$D_{hv} = D_a\,\Phi(\sigma)\,\{P/[P - hp(T)]\}\,\rho_v(T)/\rho_w\ , \qquad [36]$$

where

D_a = a coefficient characterizing the molecular diffusion of water vapor in free air, $cm^2\ day^{-1}$

$= 50.91\ T^{2.3}/P$, (de Vries, 1958),

P = ambient pressure, mb,

σ = volumetric air content of the soil, dimensionless,

$\Phi(\sigma)$ = a dimensionless function defining the effectiveness of the water-free pore space for diffusion

$\cong \alpha\,\sigma$,

α = tortuosity factor, dimensionless $\cong 0.66$,

$\rho_v = \rho_v(T)$ = density of saturated water vapor, gm cm^{-3}; ρ_v is a function of temperature.

According to the Philip and de Vries theory, the coefficient D_{Tv} is given by

$$D_{Tv} = D_a \eta\,\{P/[P - hp(T)]\}\,(d\rho_v/dT)\,\zeta\,h/\rho_w\ , \qquad [37]$$

where

η = soil porosity, dimensionless,

ζ = a ratio of the average temperature gradient in the air-filled soil pores to the overall soil temperature gradient; this ratio depends upon soil porosity, water content, temperature and quartz content; it usually varies between 1.3 and 2.3, except in extremely dry, compact soils in which it may reach the value of 3.2, especially if the soil contains much quartz (see Philip and de Vries, 1957, and Rose, 1968),

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Note that

$$D_{Tv} = Bh\, D_{hv} \,, \qquad [38]$$

where

$B = (\eta/\sigma)\ (\zeta/\alpha)\ [d(\log_e \rho_v)/dT] = (\eta/\sigma)\ (\zeta/\alpha)\ \beta'$, gm cm^{-3} $^{o}K^{-1}$

$\beta' = d(\log_e \rho_v)/dT \cong 0.1516 - 3.22 \times 10^{-4} T$; the latter empirical equation has been fitted for $290^{o}K < T < 360^{o}K$ using data from List (1951).

It follows from the expressions for D_{hv} and D_{Tv} given above, that equation 35 can be written

$$-\frac{E}{D_{hv}} = \frac{dh}{dz} + Bh\,\frac{dT}{dz} \,. \qquad [39]$$

Utilization of equation 39 is facilitated by the following approximations, which are made possible by the low water content of the upper soil layer in question.

First, in dry soils, the volumetric air content, σ, is approximately constant and equal either to porosity, η, less the water content of air-dry soil, or to η alone, if the latter water content is negligible. Hence the ratio η/σ of B is approximately constant and often equal to unity. It may be noted that of the other factors determining B, only ζ depends on variables other than temperature. In this study ζ, which in all soils is of the same general order of magnitude, will be taken as a constant and all the computations needed for determining E will be carried out twice: once for the probable minimum value of ζ, ($\zeta \cong 1.3$) and another time for the corresponding maximum value ($\zeta \cong 2.3$). It can be shown that such calculations lead to the estimation of the probable upper and lower bounds of the E value sought. It follows from the above considerations that in this study it will be possible to regard B as determined by temperature alone.

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Second, in dry soils, the ratio $P/[P - hp(T)]$ is approximately equal to unity. Hence, the coefficient D_{hv} of equation 39 can be regarded as a function of temperature alone.

In addition to the above approximations and to soil and meteorological data, needed in the previously discussed cases, the approach under consideration requires two new assumptions, as well as acquisition of information about soil temperature, T_1, at one small depth, L_u^*. The latter depth is defined here as one which may exhibit a significant gradient of the mean daily temperature. In the computations of this study, the L_u^* value was taken as 2 cm.

The first new assumption required is that the temperature gradient in the dessicated, L_u cm deep, upper layer does not vary with depth and is approximately equal to $(T_u - T_1)/L_u^*$.

The second new premise is based on the fact that D_{hv} and B, though temperature-dependent, do not vary greatly with T. Due to this, the following can be assumed for temperature ranges commonly met near the soil surface: D_{hv} amd B are independent of temperature, if they are evaluated at the mean temperature of the upper soil layer defined as $\frac{1}{2}(T_u + T_1)$.

It will be noted that the above two premises tend to imply that the depth of the upper layer in question, L_u, is not very different from L_u^*. If the procedure to be derived presently yields results which are strongly at variance with this implied assumption, a satisfactory assessment of E may require certain special measures. These will be described in due course.

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With the aid of the above approximations and premises, equation 39 can be easily integrated. First, this equation is rewritten in a slightly different form,

$$\frac{dh}{dz} = b(h - \frac{c}{b}) , \qquad [40]$$

where

$$b = - B (T_u - T_l)/L_u^* ,$$

$$c = E/D_{hv} .$$

Second, h of equation 40 is replaced by a new variable, φ, defined by φ = h - (c/b). The resulting equation in φ is readily solved by separation of variables, using boundary conditions, which state that the variable h assumes the values of h_l and h_u at $Z = (L - L_u)$ and $Z = L$, respectively. This solution yields, after rearrangement, the working equation of the procedure under consideration

$$L_u = \frac{2.3}{b} \log_{10} \frac{h_u - (c/b)}{h_l - (c/b)} . \qquad [41]$$

Equation 41 makes it possible to compute L_u for a given E, if the relevant soil properties are known, and if the given value of E is plausible under the assumptions made. Note that because of the latter limitation, when b is positive (that is, when $T_u < T_l$), there will exist certain arbitrarily chosen E values for which equation 41 cannot yield a meaningful answer.

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The value of h_1 in equation 41 may be taken as corresponding to the soil moisture suction at which the vapor transfer influences become sufficiently important. According to theoretical considerations of Philip and de Vries (1957) and measurements by Rose (1963b), liquid flow commences at $h \cong 0.6$. Jackson's experiments (1964) suggest for desorption that this value may lie between 0.5 and 0.8. The commencement of appreciable vapor-transfer influences probably occurs at somewhat higher values of h than those associated with the commencement of liquid flow. Hence, perhaps h_1 could be taken as at least equal to 0.8. For a given soil, the soundness of this choice can be checked and possibly improved by comparing the value of $K = K_{liq}$ at $h = 0.8$ (computed with the aid of equations 6 and 10) and the value of the corresponding coefficient of isothermal vapor transfer (Rose, 1963a), $K_{vap} = (Mgh\ D_{hv})/(RT)$. If $K_{vap} \cong K_{liq}$, it is very probable that the value of h_1 chosen was suitable.

Whichever reasonable value of h_1 is used, it is found that the interface suction S_1, which corresponds to h_1, is relatively high and usually exceeds 10,000 cm. For suctions of this magnitude the rate of water flow in the moist soil below the interface in question is essentially soil limited. Hence, the rate of water transfer in the lower, moist soil layer can be evaluated with the aid of equation 23. If the thickness of the moist layer is taken as $L_1 = L - L_u$, then for steady state conditions and any given L, equation 23 (with its L replaced by $L - L_u$) in effect expresses E as an increasing function of the dry layer depth, L_u.

Another relation, giving E as a decreasing function of L_u is expressed by the just derived equation 41. The two equations linking E and L_u, (equations 23 and 41), can be solved simultaneously, either graphically or numerically, yielding the actual E and L_u.

If the value of L_u thus obtained is of a different order of magnitude than L_u^*, the actual E which corresponds to L_u should be reassessed. If $L_u \ll L_u^*$, it might be desirable to acquire new T_l data for an appropriately small L_u^* and to repeat the original procedure, using the new T_l. On the other hand, if $L_u \gg L_u^*$, it is advisable to consider the upper layer, u, as consisting of two sublayers. In the upper sublayer, nonisothermal vapor transfer can be taken as the predominant manner of water transfer. Equation 41 describes the relevant relations for such a region. If the depth of this sublayer is assumed to be L_u^* and if E is given, humidity, h_l^*, at the bottom of the sublayer in question can be computed, since it follows from equation 41 that

$$h_l^* = (c/b) + [h_u - (c/b)]/\exp(-bL_u^*) \quad [42]$$

In the lower part of layer u, isothermal vapor transfer can be assumed to be the dominant mode of water flow. Such a flow is described by equation 35, with $dT/dz = 0$. Integration of this equation, leads to the relation

$$L_u' = (h_l - h_l^*)/(E/D_{hv}) \quad [43]$$

in which L_u' is the depth of the lower sublayer.

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For a given E, the reassessment procedure just outlined can produce a corresponding value of L_u (= $L'_u + L^*_u$). Thus, a relation between E and L_u can be obtained for a set of arbitrarily selected E values. As in the first vapor-case procedure described above, such a relation can be used in conjunction with equation 23 in order to determine, either graphically or numerically, the desired values of the actual E and L_u.

The above procedures for including vapor-transfer in the evaporation computations were tried out with the data of the Buckeye soil on hand, and several estimated T_l values. The results obtained showed that under the conditions tested ($T_u > T_l$), the E value was somewhat increased by the vapor-transfer influences. However, this increase did not exceed 0.01 cm/day (less than five percent of E) and hence could be neglected for most practical purposes (compare with the results of Hanks and Gardner, 1965). The reason for so slight an increase probably was two-fold. Firstly, the values of D_{hv} and D_{Tv} are rather small. Secondly, when $T_u > T_l$, thermal transfer is counteracting the influence of the humidity gradients. If, contrary to the above experience, conditions are such that significant vapor-transfer effects are suspected, the methods given in this section can be used to estimate such influences.

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DISCUSSION, EXPERIMENTAL TEST AND CONCLUSIONS

Of the relations which can be computed with the aid of the approach presented in the preceding pages, the one which might be most useful in hydrologic practice is described by the plots of E versus L, as those in figures 7 and 9. A summary of the procedure based on using these plots is given in the Appendix. The results obtained in this study confirm Philip's (1957b) contention that for all practical purposes, plots of this kind can be prepared by assuming that, for any given L, the actual E(L) is the smaller of E_{pot} and $E_{\infty}(L)$. In such cases, the latter two quantities may be calculated, respectively, with the aid of the appropriate meteorological and soil equations. It follows that the actual E is either atmosphere-limited or soil-limited. This implies that the region on the E(L) plots in which both atmospheric and soil factors are influential is so small that it can be neglected. For a Yolo light clay, Philip noted that the impreciseness due to such a neglect as compared with the exact solution was smaller than could be exhibited on a graph of the scale he used. The experience of this study, in which two very different soils were used, was similar.

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The reason for the narrowness of this region of imprecision is suggested by the shapes of the curves shown in figures 6A and 6B. An inspection of these curves reveals that the S_u axis may be divided into the following three regions: (1) a low suction region (roughly $S_u < 6 \times 10^3$) in which the soil curves may show relatively steep slopes, but the meteorological curves are nearly horizontal and are fixed by $E \cong E_{pot}$; (2) an intermediate suction region (approximately, $6 \times 10^3 < S_u < 6 \times 10^4$) in which soil and meteorological curves are nearly horizontal and approach their respective, limiting E values; (3) a high suction region ($S_u > 6 \times 10^4$) in which the meteorological curves exhibit appreciable slopes, whereas the soil curves are practically horizontal and are fixed by $E \cong E_\infty$. From the above it is clear that an intersection between meteorological and soil curves which occurs in the low-suction range results in $E \cong E_{pot}$. On the other hand, when such an intersection occurs on the high-suction range, one obtains $E \cong E_\infty$. The intersections which occur in the intermediate range involve plots with nearly horizontal slopes of opposite sign. Hence, for any given set of weather and soil parameters only a limited range of water table depths will produce intersection values confined to the intermediate S_u range. In this range the values of E may vary somewhat. However, the almost horizontal character of the curves specifies that the actual value of E lies between the very nearly equal values of E_{pot} and E_∞. Therefore, for all practical purposes, $E \cong E_{pot} \cong E_\infty$ for the intermediate range. It follows from the above considerations that in all three suction ranges in question, the actual E must be almost equal either to E_∞ or to E_{pot} or to both of these quantities.

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It should be noted that the conclusion just stated very probably is not restricted to the soil and weather conditions treated in the examples of this study and that it can be expected to be applicable rather generally. The reasons for this are as follows. Equation 3 demonstrates that the meteorologically determined E is a linear function of h_u. However, it follows from equation 6 that $h_u = \exp\left[(-MgS_u)/(RT_u)\right] = \exp\left[\mu^2 \times 10^{-7} S_u\right]$ where $\mu^2 < 8$ for the commonly found surface soil temperatures, T_u. Hence, h_u deviates appreciably from 1.0 (that is, from full saturation) only if the S_u value is very high. This accounts for the fact that the meteorological curve deviates from the horizontal only in the high S_u range. On the other hand, it follows from equation 22 and figure 3 ~~demonstrate~~ that for the usual soil parameters and evaporation rates, the soil curves almost reach their limiting level when S_u is still relatively low. This accounts for the soil curve contribution to the peculiarities of the E(L) relation under consideration.

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An inspection of figures 6A and 6B indicates that in the intermediate suction range, the $E(S_u)$ curves drawn for soils with low valued n parameters (for example, n = 2) show the largest slopes, whereas the soils with large n values (for example, n = 5) exhibit the smallest slopes. Further confirmation of this conclusion is implied by the curves of figure 3. It follows from such a conclusion ~~implies~~ that in the intermediate suction zone, the lower the value of n, the larger the limited range of transitional E values which depend on both soil and meteorological factors. This might suggest that the E(L) estimation procedure under consideration is least precise in cases of soils with low values of n. However, even in these cases the procedure was found sufficiently accurate for most practical purposes. Note also that the appreciably sloping parts of the soil and meteorological curves shift towards the intermediate suction region as the limiting E values decrease. Hence, the relative importance of the transitional E range increases as the magnitude of the limiting E decreases. But for these small values of E, the absolute importance of any imprecision of E in the intermediate suction range is insignificant.

In order to test in a preliminary way the applicability of the E(L) estimation procedure considered above, its results were compared with actual field observations. The field data were obtained from two large, bare-soil evaporation tanks located at Buckeye, Arizona and described by van Hylckama (1966). The local soil contained in these tanks had a slightly salt-cemented upper layer (layer 2 discussed above in the layered soil case) that appeared to be somewhat thicker than 10 cm.

The tanks were provided with apparatus for automatically maintaining a pre-selected water table depth, and for recording the quantity of water required to do this. Consequently, data on the actual daily rate of evporation for a selected depth to water table was readily obtained.

Meteorological data, which included air temperatures, air relative humidities, and wind velocities, were also collected hourly at the site. These variables, converted to a daily average basis, along with tabular radiation data appropriate to the site (List, 1951), were used to calculate E_{pot} by the methods described earlier in this paper. In addition to the above, soil temperatures at the depths of 5 and 10 cm were recorded.

Data sets of various periods were chosen for analysis, primarily on the basis of completeness of both actual E and E_{pot} information, and achievement of a steady state. All these sets were selected from March to October data of three consecutive years.

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To provide information needed for calculation of E_∞, undisturbed soil cores were obtained from the evaporation tanks at 0-10 cm and 50-60 cm depths. The cores, 3 cm thick and 5 cm in diameter, were taken in successive, spaced pairs from each depth zone with a soil sampler provided with retainer rings. Using the local ground water of the area, saturated hydraulic conductivities and moisture retention curves were determined on cores selected from the two layers. Each moisture retention curve was determined in two parts. A ceramic plate with a hanging water column was utilized for the 0 to 0.1 bar suction range, whereas a pressure plate apparatus (Richards, 1954, Method 32) was used for suctions between 0.1 and 1.0 bar. The moisture retention curves for the two soil zones were used to compute the appropriate n and $S_{\frac{1}{2}}$ parameters by the methods described earlier in this report. These parameters were then used, along with the K_{sat} data, to calculate E_∞ by the procedure outlined in the section of this paper concerned with layered soil.

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A comparison of the observed evaporation rates with calculated E_∞ and E_{pot} is illustrated in figure 10. The circles on the plot indicate average observed evaporation rates for three depths to water table. Due to the insufficiency of the available data, these averages do not carry the same weight. The circles, indicating the 120, 146, and 156 cm depths, represent the averages from 55, 13, and 6 days, respectively, The bars connected in figure 10 to the appropriate circles by dotted lines represent the average E_{pot} values calculated from the meteorological data obtained for identical time periods. Also plotted in figure 10 are two E_∞ curves calculated, as mentioned above, for a layered soil case. The upper and lower E_∞ curves correspond, respectively, to assumed 10 cm and 15 cm depths of the cemented, upper soil layers.

Figure 10 shows that under the conditions studied, evaporation was soil limited. Also, this figure demonstrates that the observed values correlate reasonably well with those predicted by the layered soil estimating technique. Since layered soils are quite prevalent in the field, the method's suitability for handling such situations is an attractive feature.

In spite of these encouraging results it must be stressed that due to the various premises involved in the derivation of the theoretical meteorological and soil relations of this study, the procedure suggested here is subject to obvious limitations.

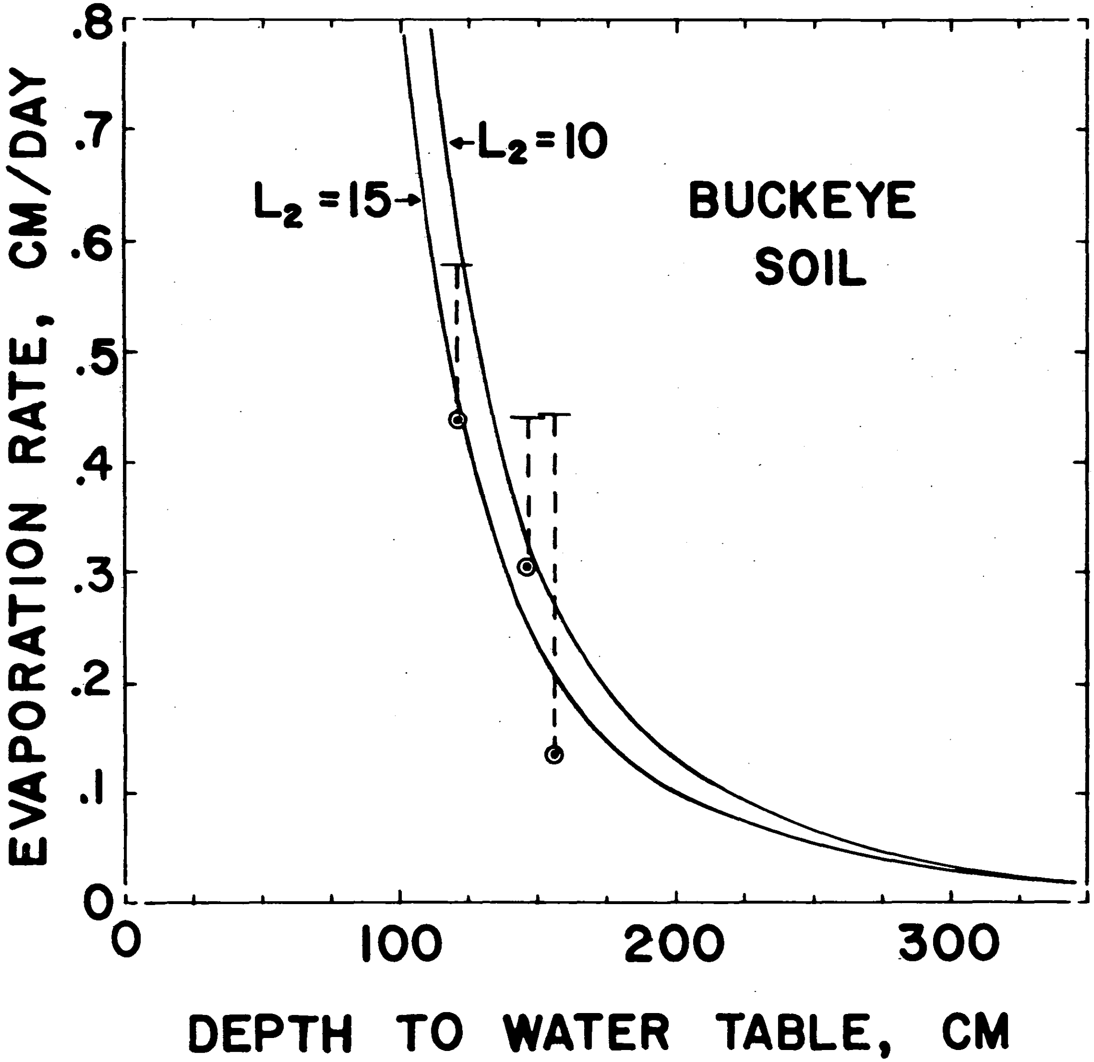

Figure 10.--Influence of water-table depth on actual and estimated rates of evaporation from the Buckeye tanks. Computed, soil-limited evaporation levels are indicated by the solid lines. The soils involved are two-layered, with the upper layer thickness, L_2 cm. Each circle and each bar connected with it represent, respectively, an observed mean evaporation rate and the corresponding, calculated mean potential evaporation.

Steady state conditions were assumed throughout this paper. However, in nature the systems considered are seldom in such a state, principally because of the variations in meteorological conditions, in soil salt content and in water-table depth.

Owing to the periodicity of the meteorological and water-table changes it might be hoped that use of daily averages for the input data will decrease the errors inherent in a steady state model applied to transient situations. Gardner and Hillel (1962) have suggested that the circadian variation in evaporation rate is effectively damped in the upper few centimeters, and that the overall evaporation rate is subject to little error. However, it is doubtful that such errors are diminished to negligible proportions.

The changes in soil salt-content and water-table depth are relatively slow and therefore their short period effects might be negligible. However, their long-range influences could be of very considerable importance and should be taken into account, perhaps by assuming a series of steady states, with different experimentally determined soil parameters and measured or predicted water-table depths. The effect of salts accumulating and often precipitating in the surface layers might be particularly significant, especially when leaching rains are infrequent and ground water solute-content is relatively high.

Under various conditions, the thermal transfer of water might significantly change the evaporation rate. In this study, such a transfer was taken into account only in the last case treated (that is, the case affected by vapor flowing within the upper, relatively dry soil layer). Thus, the thermal transfer of liquid water was entirely neglected. This approximation seems to be justified by the following: (1) in moist soils such a transfer usually is negligibly small in comparison with the coexisting liquid flow due to pressure gradients; (2) in dry soils such a transfer is usually insignificant in comparison with the coexisting, thermal flow of vapor (see Philip, 1957a, and de Vries, 1958, for some typical relative magnitudes of the relevant transfer coefficients).

The analysis of the vapor-affected case presented in this study attempts to treat the most important of the thermal influences taken into account by the Philip--de Vries theory. However, it must be stressed, that the simple analysis under consideration is based on several assumptions, which are extraneous to the above theory and which can be only approximately valid. These assumptions may influence the vapor-effect correction, though it is very doubtful that they could change it sufficiently to be of practical significance.

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Some of the temperature effects which all the suggested procedures do take into account involve soil surface temperature, T_u. In practice inaccuracies will exist in data needed to compute the effects of T_u (especially those involved in determining Q_N). This may contribute to the imprecision of the approach in question. Also, if a zero value is assumed for Q_g, as was done in this study, the surface temperature, T_u, may be over- or underestimated. Information on the thermal conductivity of the soil and soil temperature at a shallow soil depth could make it possible to account for a non-zero Q_g, but this would necessitate the gathering of additional data with possibly negligible improvement of the overall estimate.

The soil data employed might be less precise than desirable. This could be at least partly due to the inapplicability of the empirical equation 10, or to the inaccuracy of the methods suggested for deriving hydraulic conductivity information from the soil water retention curve. In addition, it might be impossible to take into account adequately the variability of field soils.

Finally, inherent in the method are all the limitations of the basic meteorological and soil equations (equations 3, 4, 9, 10, Darcy's law and the Philip--de Vries theory).

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With the surface-temperature equation 9 included in the computation scheme, the procedure described in this paper might be called quadri-combinational, because it is an algorithm which combines a soil equation of water flow and meteorological equations of heat balance, vapor transfer and sensible heat transfer. Alternate forms of some of these equations could be employed and, possibly, other ways of including them in the algorithm could be devised. The relative merits of such variants of the proposed approach will have to be determined experimentally.

It follows from the above considerations that the technique described in this paper is only approximate and that therefore it should be used with appropriate care. It is possible to devise changes in this technique which would considerably improve its precision. However, these changes would impair the method's relative simplicity and its dependence on generally available data. The preliminary experimental results cited above as well as theoretical considerations seem to indicate that in spite of its limitations, the procedure for evaluating evaporation presented in this paper can yield useful, approximate estimates. It should be used primarily when simplicity is needed and precision of estimates is not crucial.

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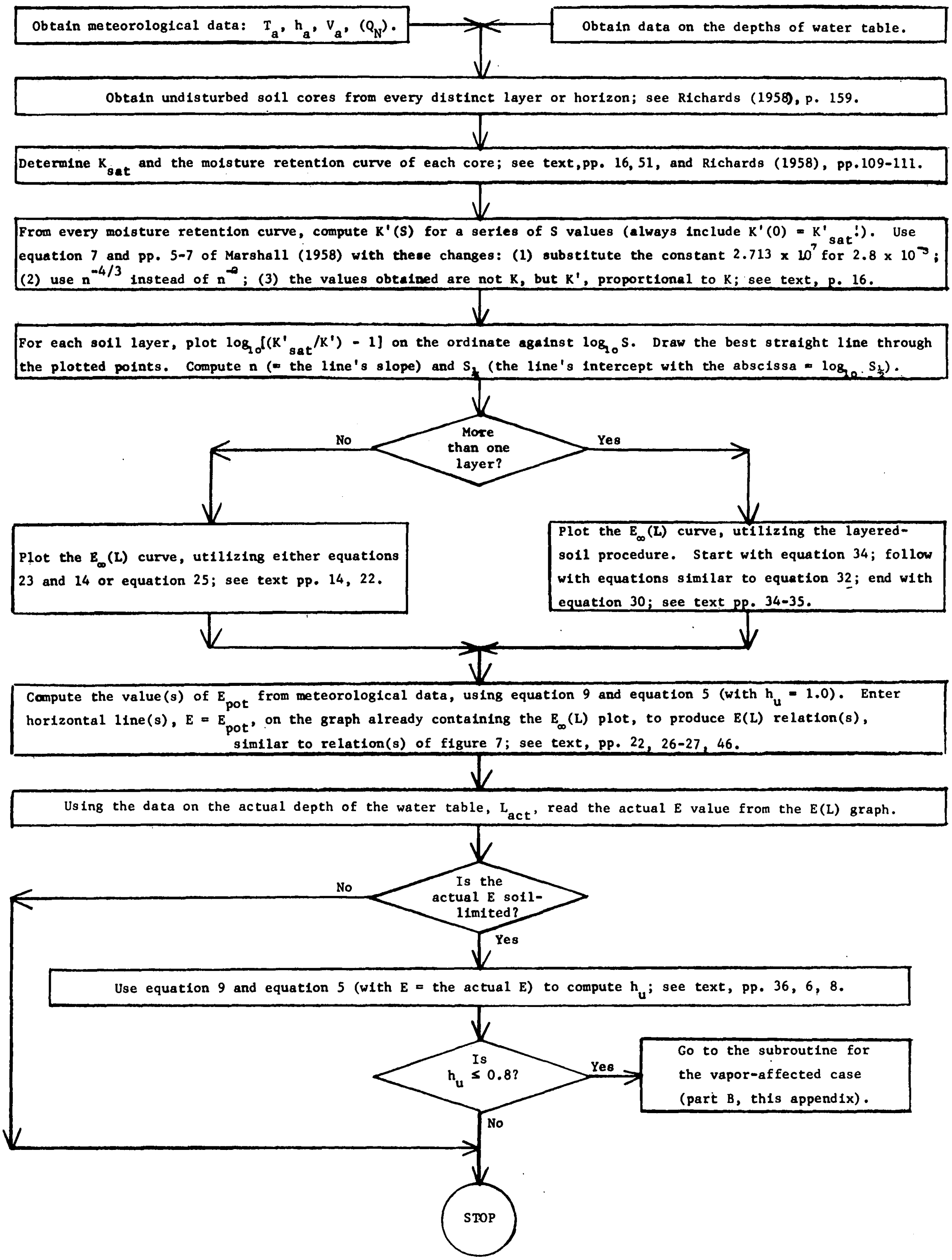
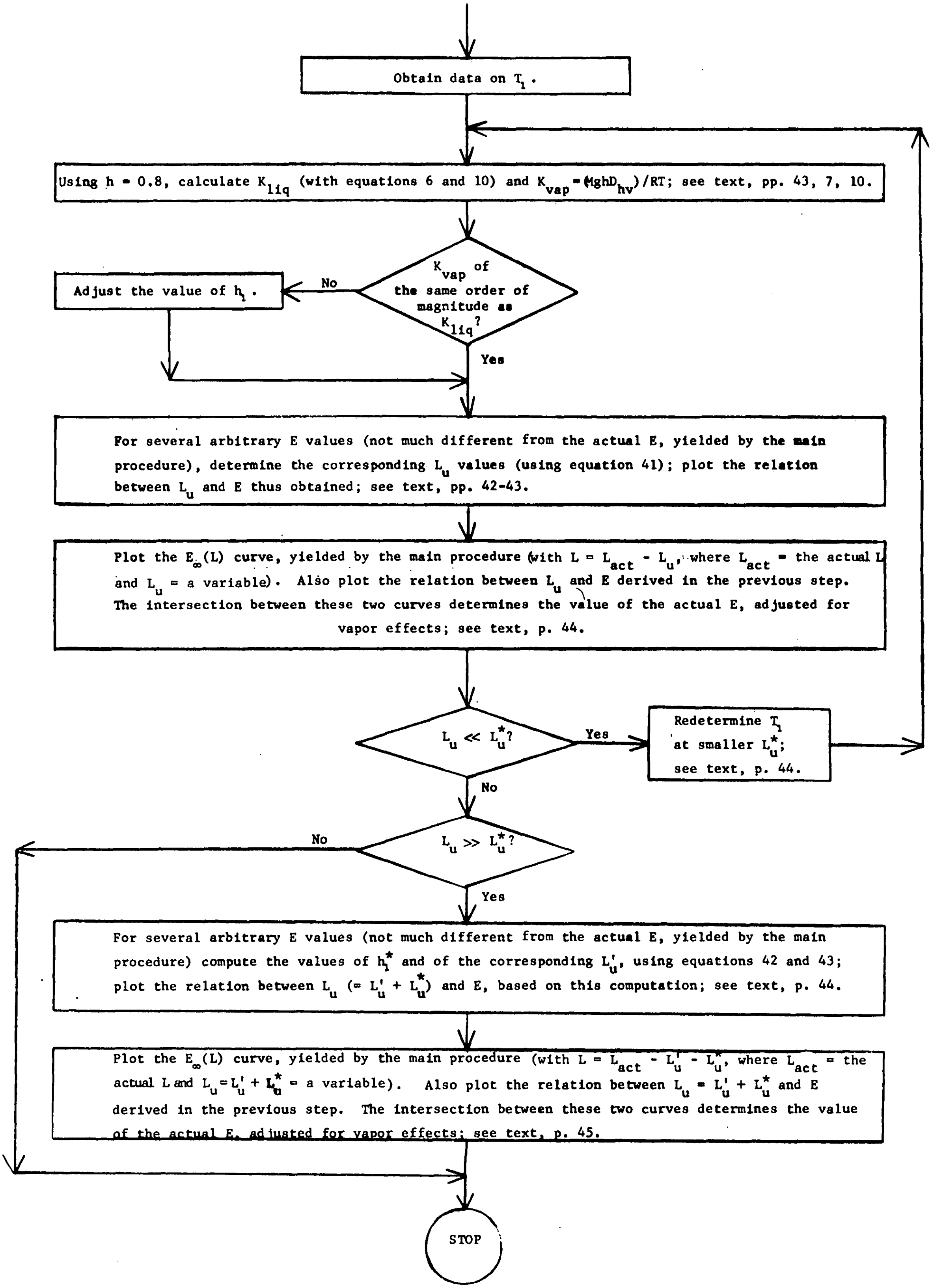
APPENDIX

Flow charts for estimating steady-state evaporation rates from bare soils.

A. The main procedure.



B. The subroutine for the vapor-affected case.



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