

## THREE-DIMENSIONAL FINITE-DIFFERENCE MODEL OF GROUND-WATER SYSTEM UNDERLYING THE MUSKEGON COUNTY WASTEWATER DISPOSAL SYSTEM, MICHIGAN

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**Abstract.**—The spray irrigation system used by Muskegon County for wastewater treatment is the largest of its kind in the United States. It has 2200 hectares of irrigated farm land, 688 hectares of treatment lagoons, and 105 kilometers of drainage tile. The system has a design capacity of 1.8 cubic meters of wastewater per second. A three-dimensional finite-difference model was developed to study the effect of the disposal operation on ground-water conditions. Model calculations show that the water table at and adjacent to most of the wastewater site is lower as a result of the operation of the system to date. However, along the northwest boundary of the site, where irrigated land was not undertilled, the water table is 1 to 2 meters higher than it would be under natural conditions. Predictive simulations indicate that, even if the drainage tiles lost 75 percent of their effectiveness, the impact of disposal operations on ground-water levels would be negligible outside of the wastewater site.

Muskegon County, Mich., began construction of a 4047-ha wastewater system in 1971. By August 1974 the system was fully operative. This system, designed to use wastewater for spray irrigation of crops, is the first major regional system of its type constructed in the United States. It is located 16 km east of the city of Muskegon.

The purpose of this investigation was to develop a mathematical model of the ground-water flow system in and adjacent to the site so that this model could be used to estimate the effects of the wastewater facility on local ground-water conditions. Figure 1 shows the location and boundaries of the modeled area and the location of the wastewater facility.

The wastewater system has a design capacity of 1.8 m<sup>3</sup>/s (Bauer Engineering, Inc., 1973). Five municipalities and two industries currently pipe 1.3 m<sup>3</sup>/s to the site. Wastewater is aerated, chlorinated, and then used to irrigate corn. The treated water is sprayed on the land for 8 months; during the winter it is stored in two large lagoons, each having a surface area of 344 ha. The bottoms of the lagoons are about 3 m above the preconstruction altitude of the water table. Leak-

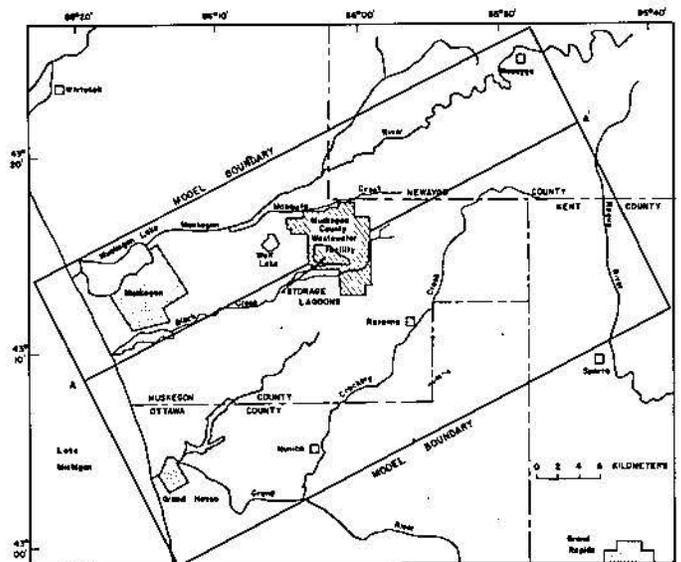


FIGURE 1.—Location of study area. Line A-A' is trace of cross section shown in figure 3.

age from the lagoons is intercepted by ditches that surround about 90 percent of the perimeter of lagoon area. Water in these seepage ditches is either pumped back to the lagoons or to adjacent streams. From the lagoons, water is pumped to center pivot rigs capable of irrigating crops at a rate of 8 cm per week. There are 54 circles irrigating an area of 2200 ha (fig. 2). Most circles are drained by corrugated polyethylene tiles having diameters of 15–25 cm. The tile is perforated with 0.2 by 3.8-cm slots and is encased in a 0.45-mm-mesh fiberglass fabric. Drainage tile lines are generally set 1.5 to 2.5 m below land surface and are spaced at 150-m intervals. They are generally just below the water table except in the northwest corner of the site, where the tile lines are few and above the water table. The total length of the tile lines is 105 km. Drainage tiles are connected to concrete collector

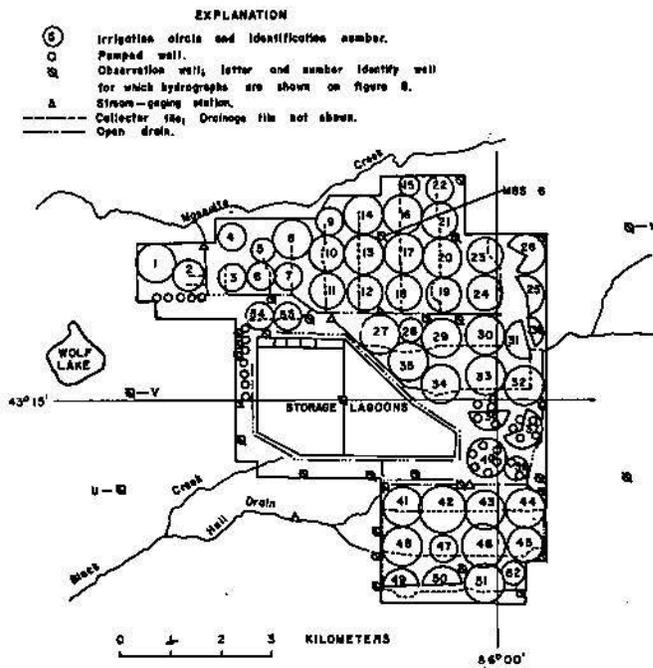


FIGURE 2.—Muskegon County wastewater system.

tiles (fig. 2) that discharge to drainage ditches. In addition to tile, there are 30 wells (designated as pumped wells on fig. 2) that are used to control ground-water levels. Seven of the wells along the northwest edge of the lagoons are pumped to reduce ground-water mounding caused by lagoon leakage. The rest of the wells are used to control mounding caused by irrigation. This system of tiles, discharge wells, seepage ditches, and drainage ditches was designed to lower ground-water levels in the project area.

A mathematical model capable of simulating the hydrologic system of the wastewater site and its vicinity was developed. The model utilizes standard numerical techniques to solve the differential equation of three-dimensional ground-water flow; it includes routines for simulating the effects of rivers, lagoons, and drainage tiles on the ground-water system. Input to the model consisted primarily of available data. The only additional field data collected were water-level measurements of about 90 wells and streamflow information obtained at five locations in the vicinity of the wastewater site.

## GEOLOGIC SETTING

### Pleistocene deposits

The western half of the modeled area (fig. 1) is underlain by sediments deposited in Pleistocene lakes

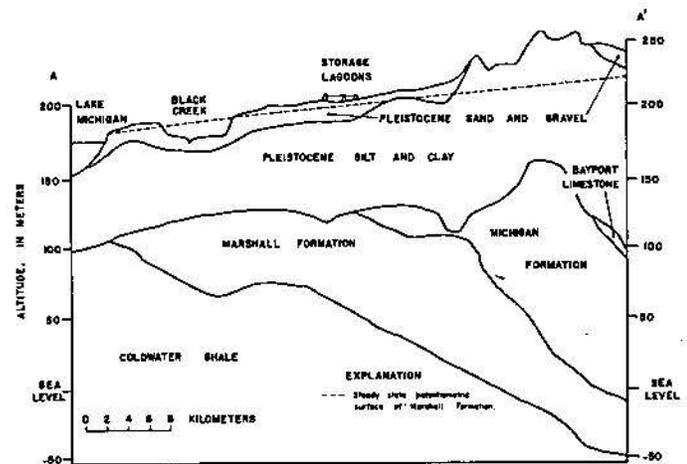


FIGURE 3.—Cross section of study area showing geologic formations. Location of section A-A' shown in figure 1.

(fig. 3) (Martin, 1955). The composition of the sediments varies from tight clay to fine gravel. Generally, the upper 6 to 24 m is well-sorted stratified sand, interlayered with fine gravel and silt. Underlying the sand and gravel are beds of relatively impermeable silt and clay. Topography in the western half is flat and poorly drained; recharge to the ground-water body is high.

Most of the eastern part of the area is underlain by morainic sediments, poorly sorted silt and clay containing lenses of sand and gravel. Thin sand and gravel beds occur along some stream valleys. The thickness of the unconsolidated deposits ranges from 30 to 165 m. Under the wastewater site these deposits are approximately 85 m thick. Undulating topography, more extensive stream development, and low recharge to the ground-water body characterize the eastern part.

### Consolidated deposits

Underlying the Pleistocene deposits are consolidated sedimentary rocks of Mississippian age (figs. 3, 4). They include the Bayport Limestone, Michigan Formation, Marshall Formation, and Coldwater Shale (Martin, 1936). The Bayport Limestone is predominantly limestone, sandstone, and shale. The Michigan Formation is composed of relatively impermeable shale, gypsum, dolomite, and limestone. These formations and the overlying silt and clay beds range in thickness from 24 m on the west to about 245 m on the east; they form a confining layer over the Marshall Formation. The Marshall Formation is composed of highly permeable gray, pink, and red sandstones (deWitt, 1960; Dorr and Eschman, 1970). This formation ranges from being absent in the western part of

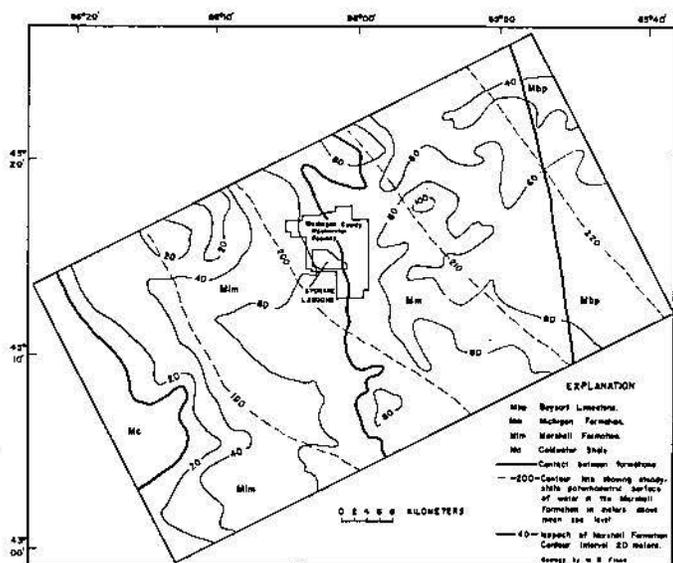


FIGURE 4.—Subcrop formations, thickness, and potentiometric surface of Marshall Formation.

the study area to as much as 100 m thick in the eastern part. The Coldwater Shale, a thick impermeable shale sequence, underlies and confines the Marshall Formation.

### HYDROLOGIC SETTING

The region studied has an area of 1620 km<sup>2</sup>. Logs of water wells drilled since 1969 and logs of exploratory oil and gas wells were used to determine the location, extent, and hydraulic properties of water-bearing materials and confining beds. Water-level measurements recorded on the well logs were used to estimate steady-state conditions.

The two principal aquifer systems are the water-table aquifer in the Pleistocene deposits and the artesian aquifer in the Marshall Formation (fig. 3). These aquifers are separated by thick confining beds of silt, clay, and shale.

#### Water-table aquifer

The water-table aquifer of the Pleistocene deposits is composed principally of sand containing some silt and gravel. The aquifer ranges from being absent where it pinches out against the morainic deposits in the eastern part to about 24 m thick in the west. Hydraulic conductivities of these materials were assigned on the basis of well-performance data within the study area and a general knowledge of the characteristics of glacial materials elsewhere. Initial estimates of trans-

missivity of the aquifer were determined from logs of wells by the relation

$$T = \sum_{i=1}^n K_i b_i, \quad (1)$$

where  $K_i$  is the hydraulic conductivity,  $b_i$  is the thickness of each lithologic unit, and  $n$  is the number of units between the water table and the base of the Pleistocene aquifer.

Initial transmissivity values were later modified during model calibration. In the model, the water-table aquifer is represented by two layers; values contoured in figure 5 were consequently defined by the equation

$$T = K_2 b_2 + K_3 b_3, \quad (2)$$

where  $K_2$  and  $K_3$  represent hydraulic conductivities and  $b_2$  and  $b_3$  represent the thickness of the two layers.

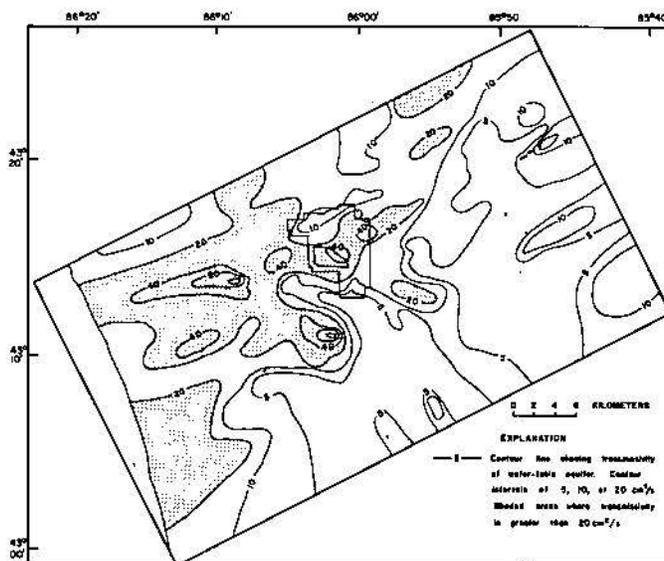


FIGURE 5.—Transmissivity of the water-table aquifer. (1 cm<sup>2</sup>/s = 8.64 m<sup>2</sup>/d.)

The regional gradient of the water table is toward Lake Michigan. Water levels range from 250 m above mean sea level on the east side of the study area to 177 m along the shore of Lake Michigan. Water discharges from the aquifer regionally to Lake Michigan and locally to streams. Along the north edge of the wastewater facility, the water table slopes downward about 15 m/km northward toward Mosquito Creek.

Recharge to the water-table aquifer was calculated to be 20 cm/yr, based on long-term precipitation,

streamflow, and ground-water levels. This value agrees well with those of Allen, Miller, and Wood (1972) and Walton (1970) for similar geohydrologic conditions in southwestern Michigan and Illinois. Precipitation at the wastewater site and recharge for 1974-75 are shown in table 1. Recharge rates were higher than average in 1974 and 1975 because of intensive storms.

TABLE 1.—Monthly precipitation, ground-water discharge to streams, change in ground water in storage, and ground-water recharge, in centimeters, in the model area in 1974-1975.

[Negative values of change in ground water in storage indicate declining water levels. Negative values of ground-water recharge result from losses due to evapotranspiration.]

	Ground water							
	Precipitation <sup>1</sup>		Discharge to streams <sup>2</sup>		Change in storage <sup>3</sup>		Recharge <sup>4</sup>	
	1974	1975	1974	1975	1974	1975	1974	1975
January	8.9	5.1	2.3	2.0	5.1	3.0	7.4	5.0
February	3.6	5.1	2.3	1.8	-1.3	3.0	1.0	4.8
March	10.9	5.6	3.6	2.8	2.0	2.5	5.6	5.3
April	6.9	5.1	4.1	3.0	3.0	0	7.1	3.0
May	14.7	5.1	3.8	2.3	-1.3	-4.8	2.5	-2.5
June	8.6	11.9	2.5	2.0	-6.4	-0.5	-3.9	1.5
July	2.8	4.1	1.5	1.5	-6.4	-5.6	-4.9	-4.1
August	9.1	23.6	1.3	1.3	-0.5	3.6	0.8	4.9
September	1.3	2.5	1.0	3.0	-2.0	3.0	-1.0	6.0
October	3.0	2.0	1.5	2.0	1.3	-0.5	2.8	1.5
November	6.6	9.1	1.8	2.5	3.8	1.8	5.6	4.3
December	3.3	6.4	1.8	3.6	0.5	2.8	2.3	6.4
Total	79.7	85.6	27.5	27.8	-2.2	8.3	25.3	36.1

<sup>1</sup>Data from rain gage at wastewater disposal site.

<sup>2</sup>Average values calculated from streamflow hydrographs of four gaging stations.

<sup>3</sup>Average from seven observation wells assuming specific yield of 0.2.

<sup>4</sup>Total of ground-water discharge and change in ground water in storage.

### Artesian aquifer

The Marshall Formation is an artesian aquifer. Hydraulic conductivity values, determined from 7 pumping tests and 33 specific-capacity analyses (Brown and others, 1963; Walton, 1962), ranged from 8 to 800 m/d; the median value was approximately 80 m/d. The thickness of the Marshall Formation was determined from 750 oil and gas logs. In the eastern part of the study area, where the formation underlies the Michigan Formation, the thickness ranges from 18 to 100 m; the average thickness is about 73 m. In the western half, the thickness gradually decreases westward from about 70 m until the formation pinches out along the contact with the Coldwater Shale (fig. 3). Transmissivity of the Marshall Formation, determined by multiplying the formation thickness by the median hydraulic conductivity, ranges from 0 along the west border of the study area, where the Coldwater Shale is present, to 8000 m<sup>2</sup>/d just east of the wastewater site.

In most of the study area and to the east, the Marshall Formation is recharged by downward flow from the water-table aquifer. Water levels in the artesian aquifer are lower than the overlying water table; at

the east edge of the study area, this difference is as much as 30 m. Flow within the Marshall Formation is to the west. The slope of the potentiometric surface is 0.8 to 1.9 m/km (fig. 4). At the shore of Lake Michigan, the head of the Marshall Formation is about 180 m above sea level, whereas the lake level is 177 m above sea level. The vertical flow pattern here changes from downward recharge to upward discharge toward the lake.

### Confining unit

The confining beds of silt, clay, and shale that separate the water table and artesian aquifers range in thickness from 24 m in the west to about 245 m in the east. Vertical hydraulic conductivity of confining beds, 0.004 m/d, was estimated from previous investigations of similar materials in southern Michigan (Allen, Fleck, and Hanson, 1972) and in Ohio (Norris, 1963).

### MODEL DESCRIPTION

A ground-water model is a mathematical description of the movement of water in a geologic environment. When converted to a form that permits computer solution of equations containing hydrologic and hydraulic parameters, the response of the ground-water system to stress may be predicted.

#### Basic ground-water flow model

The differential equation describing flow in a porous saturated medium is

$$\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = S_s \frac{\partial h}{\partial t} + W(x, y, z, t), \quad (3)$$

where  $h$  is the head at time  $t$ ,

$K_x$ ,  $K_y$ , and  $K_z$  are principal components of the hydraulic conductivity tensor aligned with the coordinate axes,

$S_s$  is the specific storage, and

$W$  is a source term for inflow or withdrawal per unit volume of an aquifer.

The terms  $K_x$ ,  $K_y$ ,  $S_s$ ,  $W$ ,  $\frac{\partial h}{\partial x}$ ,  $\frac{\partial h}{\partial y}$ , and  $\frac{\partial h}{\partial z}$  in equation

3 are assumed to be independent of  $z$  for the thickness  $z_1$  to  $z_1 + b$ , and the equation can be integrated to give

$$\frac{\partial}{\partial x} \left( T_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( T_y \frac{\partial h}{\partial y} \right) + \left( K_z \frac{\partial h}{\partial z} \right)_{z_1+b} - \left( K_z \frac{\partial h}{\partial z} \right)_{z_1} = S \frac{\partial h}{\partial t} + bW(x, y, z, t), \quad (4)$$

where  $T_x$  and  $T_y$  are the principal components of the transmissivity tensor, and  $S$  is the storage coefficient, defined by  $S = bS_s$ .

In general, equation 4 cannot be solved analytically; however, it can be replaced with an approximating finite difference equation and thus solved numerically. The continuous entities of space and time are regarded as discrete cells, and flow is regarded as a change from one state of the system to the next in a stepwise fashion. The hydraulic properties of the material in each cell are assumed to be homogeneous, and the potentiometric head is calculated for a node at the center of each cell.

If the prism being modeled is divided into "I" rows, "J" columns, and "K" layers, then equation 4 is approximated by the set of N equations (N=I×J×K) as follows:

$$\begin{aligned}
 &T_{x(i,j+1/2,k)}(h_{i,j+1,k}-h_{i,j,k})/(\Delta x_{j+1/2}\Delta x_j) \\
 &-T_{x(i,j-1/2,k)}(h_{i,j,k}-h_{i,j-1,k})/(\Delta x_{j-1/2}\Delta x_j) \\
 &+T_{y(i+1/2,j,k)}(h_{i+1,j,k}-h_{i,j,k})/(\Delta y_{i+1/2}\Delta y_i) \\
 &-T_{y(i-1/2,j,k)}(h_{i,j,k}-h_{i-1,j,k})/(\Delta y_{i-1/2}\Delta y_i) \\
 &+K_{z(i,j,k+1/2)}(h_{i,j,k+1}-h_{i,j,k})/\Delta z_{k+1/2} \\
 &-K_{z(i,j,k-1/2)}(h_{i,j,k}-h_{i,j,k-1})/\Delta z_{k-1/2} \\
 &=[S'_{i,j,k}(h_{i,j,k}-\hat{h}_{i,j,k})/\Delta t]+bW_{i,j,k} \quad (5)
 \end{aligned}$$

where  $T_{x(i,j+1/2,k)}$  is the transmissivity along the x axis between node (i,j,k) and (i,j+1k), and  $T_{x(i,j-1/2,k)}$ ,  $T_{y(i+1/2,j,k)}$ , and  $T_{y(i-1/2,j,k)}$  are similarly defined,

$K_{z(i,j,k+1/2)}$  is the hydraulic conductivity along the z axis between node (i,j,k) and node (i,j,k+1), and  $K_{z(i,j,k-1/2)}$  is similarly defined,

$\Delta x_j$ ,  $\Delta y_i$ ,  $\Delta z_k$  are the widths of column j, row i, and layer k, respectively,

$W(i,j,k)$  is the source term for cell (i,j,k),

$\Delta x_{j+1/2}$ ,  $\Delta y_{i+1/2}$ ,  $\Delta z_{k+1/2}$  is the distance between node (i,j,k) and nodes (i,j+1,k), (i+1,j,k), and (i,j,k+1), respectively,

$h_{(i,j,k)}$  is the potentiometric head at node (i,j,k) at the end of the time step,

$S'_{(i,j,k)}$  is the storage coefficient in cell(i,j,k), and

$\hat{h}_{(i,j,k)}$  is the potentiometric head at node (i,j,k) at the beginning of the time step.

Combining coefficients and bringing terms containing unknown head values to the left, equation 5 can be written in the form:

$$A_0h_{i,j,k}+A_1h_{i,j+1,k}+A_2h_{i,j-1,k}+A_3h_{i+1,j,k}+A_4h_{i-1,j,k}+A_5h_{i,j,k+1}+A_6h_{i,j,k-1}=q_0, \quad (6)$$

where  $A_n(n=0,1,\dots,6)$  are constant with respect to time, and  $q_0 = bW_{i,j,k} - [S'_{(i,j,k)}\hat{h}(i,j,k)/\Delta t]$ .

This system of N equations can be written in matrix form as  $\bar{A}h = \bar{Q}$ . (7)

Equation 7 is solved using the "Strongly Implicit Procedure" (SIP) developed by Stone (1968).

The computer program incorporating the techniques used to approximate a solution to equation 4 was developed by Trescott (1975). Modifications were

made by Larson (written commun., March 1976) to simulate tile drainage and river leakage. A typical nonequilibrium simulation with this program will consist of several recharge periods, during each of which recharge to the top layer remains constant. The recharge periods are subdivided into time steps. The length of the time step ( $\Delta t$  in eq 5) affects the accuracy of the approximation to equation 3. As the time steps get shorter, the accuracy of approximation improves, but computation time increases. Similarly, as the cell dimensions become smaller, the accuracy of approximation improves, but again the computation time increases. Thus, the length of time step and size of the cells must be selected to give acceptable results in a reasonable processing time.

The hydraulic characteristics and the initial potentiometric heads for each cell are used at the beginning of simulation to calculate those elements of matrix  $\bar{A}$  in equation 7 that remain constant throughout the simulation. The location and hydraulic characteristics of rivers and drainage tiles are used at the beginning of the simulation and stored for use throughout the simulation.

After the hydraulic constants have been calculated, recharge rates forming part of the  $W$  term in equation 4 are used with  $\Delta t$  to calculate the remaining entries of the matrices  $\bar{A}$  and  $\bar{Q}$  in equation 7. For each node, the SIP algorithm then iteratively calculates new heads that satisfy matrix equation 7. Similarly, heads are calculated for the remaining time steps and the process is repeated for the rest of the recharge periods.

### Drainage analysis

A separate analysis was made to determine the effects of the drainage tiles. In this analysis, a cross-sectional model, based on simulation techniques described by Prickett and Lonquist (1971), was utilized. The model simulated flow in a vertical plane perpendicular to a single drainage tile. The vertical depth of the cross section was assumed to be 10 m, which is representative of the thickness of the water-table aquifer; the width was taken as half the drain spacing or 75 m. To represent the drainage tile, a single node at one side of the mesh was held at a head equal to its elevation. This node was a square, 0.15 m on a side; inflow was permitted through one side and through the bottom. The simulation represented half of the flow field to a drain. The hydraulic conductivity of the drain node was reduced to a fraction of that elsewhere to represent the hydraulic resistance of the drain pipe and the surrounding fiberglass net. Also, modifications were made in the model to allow the water-table boundary to move.

During each simulation, the model was set to represent a fully saturated rectangular cross section, and recharge was applied to the uppermost node. A non-equilibrium simulation was conducted until steady-state conditions were achieved. As the nodes became dewatered during the simulation process, they were removed from the system by lowering the water-table boundary, and calculations were repeated to achieve a solution corresponding to the new boundary position. Except for the drain node and the recharge nodes, all boundaries were treated as zero-flow boundaries.

Model runs were made using values of lateral hydraulic conductivity that are in the range found at the Muskegon site. Also, several different ratios of lateral to vertical hydraulic conductivity were used. Results showed that the flow to both halves of a drainage tile could be expressed approximately as a function of hydraulic conductivity and average head above the drain by the relation

$$Q = K_L (h - V) G \text{ when } h > V, \quad (8)$$

where  $Q$  is the flow into a unit length of drainage tile,

$h$  is the average water-table elevation in the area drained by the tile,

$V$  is the elevation of the tile,

$K_L$  is the lateral hydraulic conductivity, and

$G$  is a factor that varies with the anisotropy and with the hydraulic conductivity of the drain node.

When a vertical hydraulic conductivity equal to one-tenth the lateral hydraulic conductivity was used and when the conductivity in the drain node was set at one-tenth of the lateral conductivity elsewhere,  $G$  was found to be approximately 0.1. By varying the anisotropy and the conductivity of the drain node, values of  $G$  from 0.05 to 0.40 were obtained. These values were subsequently used in checking the sensitivity of the three-dimensional model under various drainage conditions.

### Muskegon model

Applying the principles of modeling ground-water flow to the Muskegon wastewater disposal site entailed developing a grid and devising a method for simulating effects of lagoons, rivers, and drainage tiles. The area modeled was divided with a rectilinear grid into three water-bearing layers as shown in figure 6. Each layer has 44 rows and 48 columns. Layer 1, the lowermost layer, represents the Marshall Formation. Layer 2 represents all the water-table aquifer except the upper 6 m; layer 3 is the upper 6 m. The water-table aquifer was divided into two

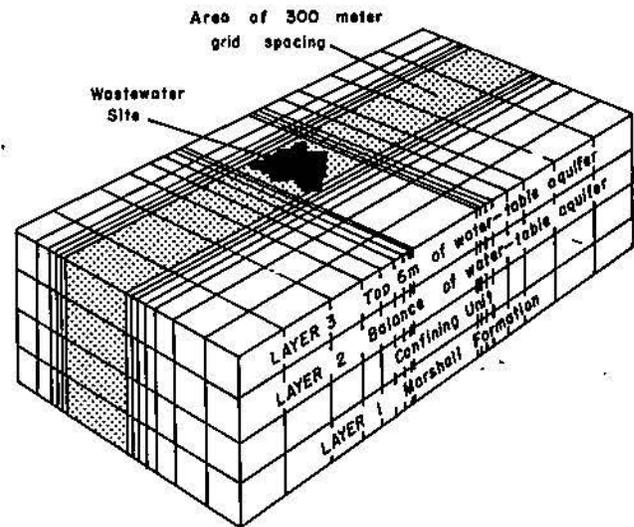


FIGURE 6.—Grid spacing used in finite-difference model. Horizontal spacing ranges from 300 m at wastewater site to 5000 m at edge of model.

layers, so that the drains and streams could be simulated in the uppermost layer as shallow features, rather than having their effect distributed through the full thickness of the aquifer. The confining unit between the Marshall Formation and the water-table aquifer was not treated as a separate layer; its effect was incorporated into the vertical hydraulic conductivities of layers 1 and 2.

The horizontal grid was designed so that the smallest cells (300 m on a side) were at the wastewater site (shaded area in figure 6). The grid spacing increased by a factor of 1.5 to the boundaries of the model area. Moving the boundaries far beyond the wastewater site greatly diminishes the sensitivity of the model at the site to the boundary conditions. All boundaries of the model area were treated as constant flow boundaries except the western boundary of the top layer, which was assumed to have a constant head equal to the elevation of Lake Michigan.

Storage effects were simulated only in the uppermost layer of the model. Compressive storage in the unconfined aquifer below the water table was not simulated, nor was artesian storage in the Marshall Sandstone. Compressive storage in the confining unit above the Marshall was also neglected. It was assumed that these compressive effects would be negligible in comparison to the water-table storage in the uppermost layer. Thus, the model grid below the uppermost layer was purely transmissive in character.

Flow to the drainage tiles was simulated using equation 8, with  $G$  equal to 0.1. In the model, this was done by adding the terms  $(GL_{i,j} K_{i,j} h_{i,j,k})$  and  $(GL_{i,j}$

$K_{i,j}$ ,  $V_{i,j}$ ) to the left and right of equation 6, respectively, for each cell in layer 3 that contained drainage tiles, in each time step during which the head exceeded the tile elevation. In these expressions

$l_{i,j}$  is the length of tile in the cell,

$K_{i,j}$  is the hydraulic conductivity of the cell, and

$V_{i,j}$  is the average tile elevation in the cell.

During model calibration, values of  $G$  in the range 0.05 to 0.40 were used in equation 8. The response of the model was not particularly sensitive to variation in  $G$  within this range, and 0.1 was retained as giving the best results.

The effects of streams on the ground-water system were simulated using the method described by Prickett and Lonquist (1971, p. 33). The method is based on assumptions that a streambed layer separates the stream from the aquifer and that seepage from the stream to the aquifer becomes constant when the water level in the aquifer falls below the bottom of the streambed. Under these assumptions, the rate of flow through the streambed is expressed by the equation

$$Q_r = K' A (M - y)/b \text{ when } h < y \quad (9)$$

$$\text{and } Q_r = K' A (M - h)/b \text{ when } h \geq y, \quad (10)$$

where  $Q_r$  is the flow between the stream and the aquifer (it is positive when the flow is from the stream to the aquifer),

$K'$  is the hydraulic conductivity,

$b$  is the thickness of the streambed layer,

$A$  is the area of streambed within the model cell,

$M$  is the elevation of the stream surface,

$y$  is the elevation of the bottom of the streambed layer, and

$h$  is the head of a cell in layer 3 of the model.

In the Muskegon model,  $K'$  was taken as one-tenth of the local lateral hydraulic conductivity in layer 3 of the model, and  $b$  was taken as 1.5 m. Other values of these parameters were also tried during model calibration.

Recharge to the water-table aquifer from the lagoons was treated as an evenly distributed, constant seepage. In the initial model calibrations, the seepage rate through the bottom of the lagoons was assumed to be 0.58 m<sup>3</sup>/s. This estimate was based upon pumpage records from the seepage ditches around the lagoons. Approximately 0.66 m<sup>3</sup>/s is pumped from these ditches, and it was assumed that 80 percent of this, or 0.53 m<sup>3</sup>/s, represents seepage from the lagoons. The balance represents drainage from other sources—that is, from surrounding irrigation circles or from regional ground-water flow. Also, because the seepage

ditches do not completely enclose the lagoons, it was assumed that only 90 percent of the seepage from the lagoons was intercepted by the ditches. A total seepage of 0.58 m<sup>3</sup>/s was, therefore, used.

If this estimate of lagoon seepage is accurate, it implies that the vertical hydraulic conductivity beneath the lagoons is considerably lower than that of the streambed or that of the water-table zone. This may reflect the effect of deposition of organic matter in the lagoons, even though the system has been operated only since 1974.

The seepage ditches around the lagoons were simulated as streams, by using equations 9 and 10, the streambed conductivity was assumed to be one-tenth of the local lateral hydraulic conductivity, and the streambed layer thickness was assumed to be 1.5 m. Seepage into the ditches in computer simulations was found to be in close agreement with field data.

#### Data base

Data required for model development included hydraulic properties of the hydrogeologic units, hydraulic properties of rivers and drainage tiles, and constant recharge and discharge sources. Heads and base-flow calculated by the model were compared with measured water levels and streamflow for purposes of calibration.

Initial estimates of the transmissivity of each cell in layers 1 and 2 and of horizontal hydraulic conductivity of each cell in layer 3 were made from lithologic data. Specific yield of layer 3 was set to 0.2. The storage coefficient for layers 1 and 2 was set to 0. The vertical hydraulic conductivity between the bottom two layers at each horizontal grid location was assumed to be equal to the vertical hydraulic conductivity of the confining unit divided by the thickness of the confining unit at the grid location. The vertical hydraulic conductivity between the top two layers at each grid location was initially assumed to be equal to one-tenth of the horizontal hydraulic conductivity of the cell in the top layer at that grid location.

Engineering blueprints were used to estimate length and average elevation of drainage tiles. For each cell, the length and average elevations of all tiles in the cell were used.

The width, length, and elevation of rivers in each cell of the upper layer were determined from topographic maps. Thickness of the streambed layer, as noted previously, was assumed to be 1.5 m. The vertical hydraulic conductivity of the streambed in each cell was initially assumed to be one-tenth of the horizontal hydraulic conductivity of the cell. A stream leakage factor was calculated independently of the

model program from width, length, hydraulic conductivity, and streambed thickness. The leakage factor and the stream elevation were used in model computations.

In calculating irrigation rates, the design capacities and number of hours of operating time of the irrigation rigs were considered. Total recharge for each cell in the model area was calculated from lagoon leakage, amount of irrigation, and natural recharge.

Water levels from two sources were available. A long-term average water-table map was derived from levels reported in driller's well records. Well hydrographs were drawn using water levels measured twice a month since 1970 in 90 observation wells.

### Calibration

A model is calibrated by repetitively running the computer program using available hydraulic data. The results of each simulation are then used to refine estimates of hydraulic parameters for subsequent simulations. Development of the Muskegon model involved a steady-state calibration and a transient calibration. In each of these calibrations, it was observed that head differences between the upper and middle layers of the grid were negligible—that is, that heads in the upper 6 m of the water-table aquifer were essentially equal to those in the lower part of the aquifer. In presenting each simulation result, therefore, a single map of water-table aquifer head or changes in head has been utilized, rather than presenting separate maps for the two model layers representing the unconfined aquifer. Results are not shown for the artesian aquifer, inasmuch as changes in the potentiometric surface were small in all simulations.

During the steady-state calibration a specific yield of 0 was set in all cells of the uppermost layer, thereby eliminating the time dependence in equation 3. The purpose of steady-state calibration was to match the long-term average water table before construction of the wastewater facility with the water table calculated by the model. A uniform recharge of 20 cm/yr was used in computations.

A series of steady-state simulations was made to determine the sensitivity of the model to variations in the hydraulic parameters. The simulations indicated that the model was insensitive to changes in vertical hydraulic conductivity or stream leakage. The model, however, was very sensitive to changes in horizontal hydraulic conductivity and transmissivity. Best results were obtained when these parameters were reduced to half their initial values. The comparison between computed and measured water-table positions after steady-state calibration is shown in figure 7.

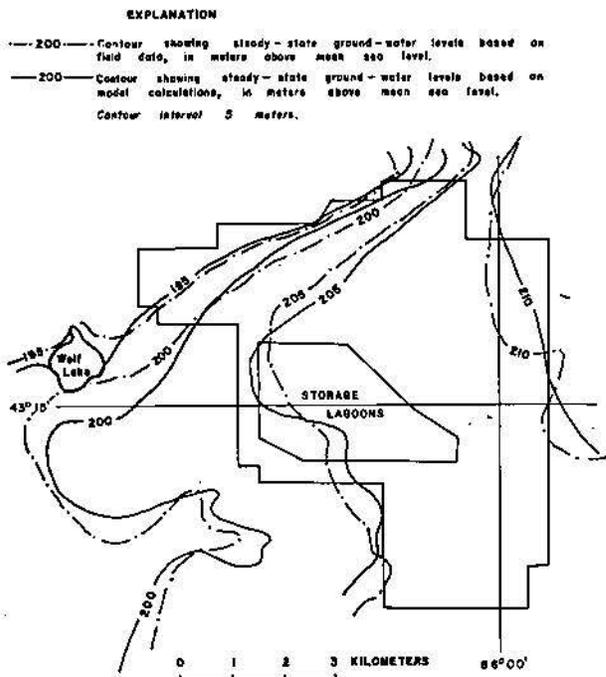


FIGURE 7.—Steady-state water table from field measurements and from model calculations.

Along the north edge of the area, step ground-water gradients caused difficulty in matching water levels.

Transient calibration was used to test assumptions regarding the specific yield and the hydraulic characteristics of the wastewater facility and to further refine other parameters. The period January 1974 to December 1975 was used for transient calibration. This period included the first growing season during which the facility operated at full capacity, as well as some earlier periods of operation at part capacity. Twenty-four monthly recharge periods were simulated. Initial heads used in the transient calibration were those calculated in the steady-state model.

Only minor changes were made in hydraulic conductivity and transmissivity during transient calibration. The model was not particularly sensitive to changes in specific yield, and the original value of 0.2 was ultimately retained because it gave the best results. Stream location and stream surface elevation proved to be important factors. For this reason an effort was made to include all streams regardless of size and to establish stream surface elevations as accurately as possible.

Figure 8 shows four representative hydrographs after transient calibration and illustrates the final match between computed and observed water-level trends. Figure 9 compares measured water levels in the water-table aquifer during July 1975 with those calculated by the model during transient calibration.

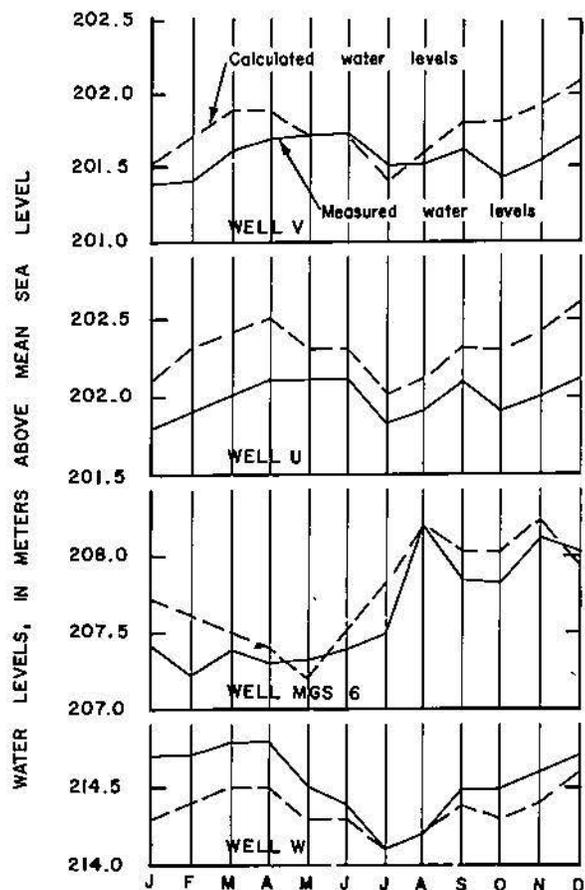


FIGURE 8.—Hydrographs of selected wells during 1975 from field measurements and from model calculations. Location of wells shown in figure 2.

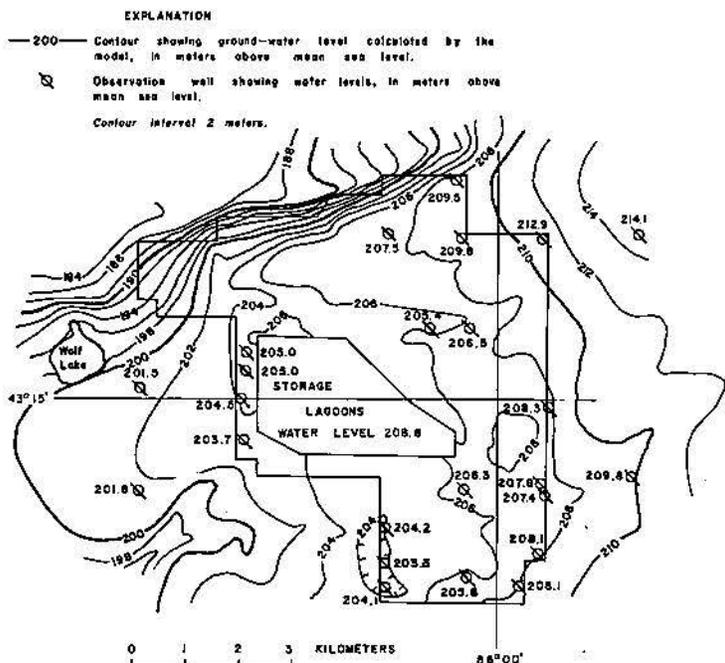


FIGURE 9.—Altitude of water table during July 1975 from field measurements and from model calculations.

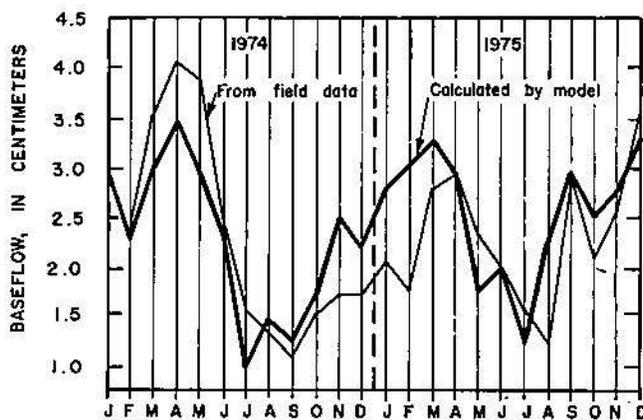


FIGURE 10.—Hydrographs of monthly baseflow from field measurements and from model calculations.

Figure 10 shows the comparison between the amount of observed and computed baseflow from the vicinity of the wastewater site during the period of transient calibration.

### INITIAL MODEL APPLICATIONS

#### Effects of the system through 1975

The model was used to study the past effect of the operation of the wastewater system on regional ground-water levels. Water levels for 1975 were calculated as if the system had not been constructed. The impact the wastewater system has had on the regional water table was determined by subtracting the calculated water levels from the actual levels. Figure 11

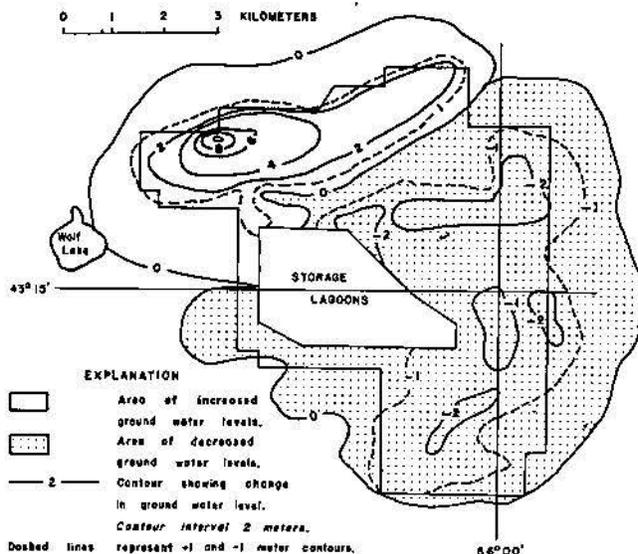


FIGURE 11.—Changes in ground-water levels, in the water-table aquifer, caused by operation of the wastewater system, July 1975.

shows this impact. In most of the site the effect of the facility has been to lower ground-water levels. However, water levels have risen in the northwestern part of the area where there are relatively few drains.

### Predictive simulations

Several predictive simulations were made assuming a variety of operational conditions. In one simulation, it was assumed that irrigation would be maintained at a uniform rate of 4 cm per week (the average rate for 1975) in all irrigation circles, the lagoons would leak steadily at a rate of 0.58 m<sup>3</sup>/s, and that drainage tile performance would again be described by equation 8, with  $G$  taken as 0.1. The December 1975 water-level configuration was used as the starting surface, and the simulation was carried to steady state. In this and in all subsequent predictive simulations, steady state was attained after 3 or 4 years of operation. The simulations were nevertheless carried for several additional years. The results for 10 years of operation are presented here for each simulation as the steady-state condition.

The results of the initial predictive simulation are

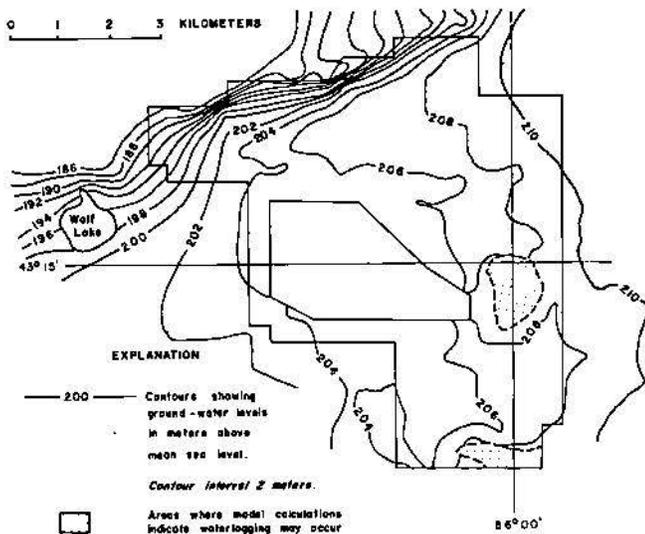


FIGURE 12.—Steady-state water table from model calculations with irrigation of 4 cm per week, normal lagoon leakage, and tile seepage.

shown in figure 12. The effects are primarily at the wastewater site. For a few areas within the site, the computed water levels are above land surface. These areas are shown on figure 12 as being waterlogged—that is, the water table in these areas would be approximately at land surface.

The results shown in figure 12 do not imply that waterlogging is a necessary consequence of the waste

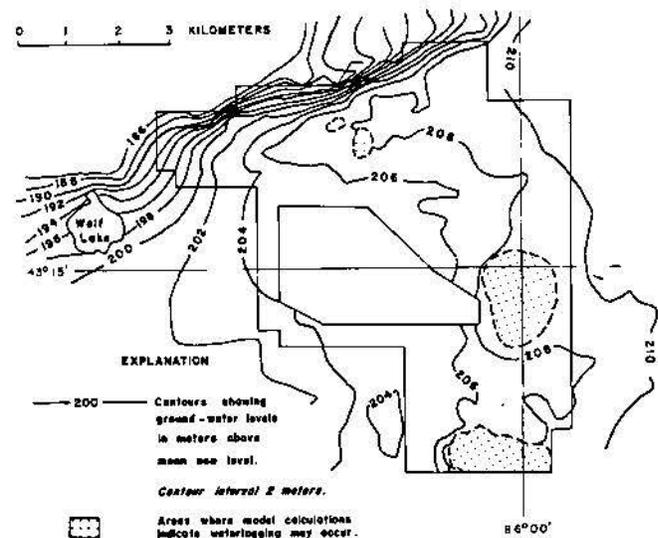


FIGURE 13.—Steady-state water table from model calculations with irrigation of 8 cm per week, normal lagoon leakage, and tile seepage.

disposal operation, but, rather, that irrigation at a rate of 4 cm per week over the entire area would probably cause such problems. In practice, irrigation rates will vary from one part of the system to another and will be managed so as to avoid waterlogging, or, alternatively, the number of drainage tiles could be increased to avoid waterlogging.

A second predictive simulation was made in which the irrigation rate was maintained at a uniform rate

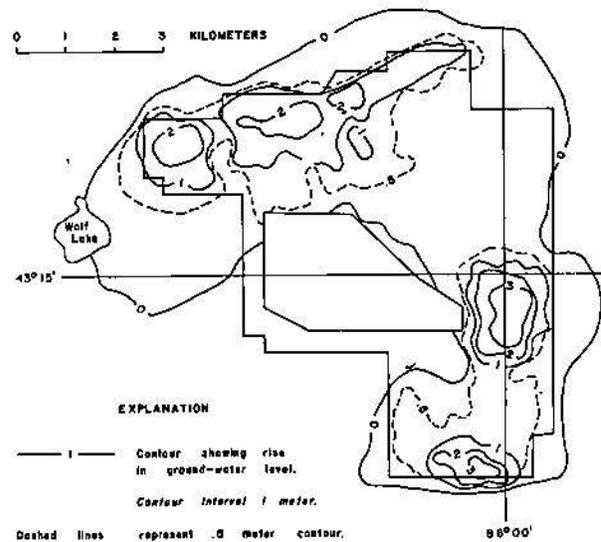


FIGURE 14.—Predicted rise in ground-water levels in the water-table aquifer, if irrigation is increased from 4 cm per week to 8 cm per week. In the southern part of the site, the increase of 4 cm per week in irrigation rate causes a rise of as much as 3 m.

of 8 cm per week (the design irrigation rate). Performance of drainage tiles and the lagoon seepage were the same as in the first predictive simulation. The 1975 water-level surface was again taken as the initial condition, and the simulation was continued to steady state. Figure 13 shows contours of the water-table elevation after steady state was reached. The increase in the irrigation rate from 4 to 8 cm per week caused the waterlogged area to increase from 150 ha to 400 ha. Figure 14 shows the rise in water level that may be expected with an increase in irrigation rate from 4 cm per week to 8 cm per week. The figure shows the difference between water-level elevations in figures 12 and 13.

A third predictive simulation assumed that the irrigation rate would be maintained at 8 cm per week, that lagoon leakage would continue at  $0.58 \text{ m}^3/\text{s}$ , but that the efficiency of the drainage tiles would be severely reduced by clogging. In the model, the factor  $G$  in equation 8 was reduced from 0.1 to 0.025, and thus the flow to the drains, for a given head differential, would be only 25 percent of that in earlier simulations. The 1975 water-table surface was again taken as the starting condition, and calculations were continued until steady state was achieved. Figure 15 shows contours

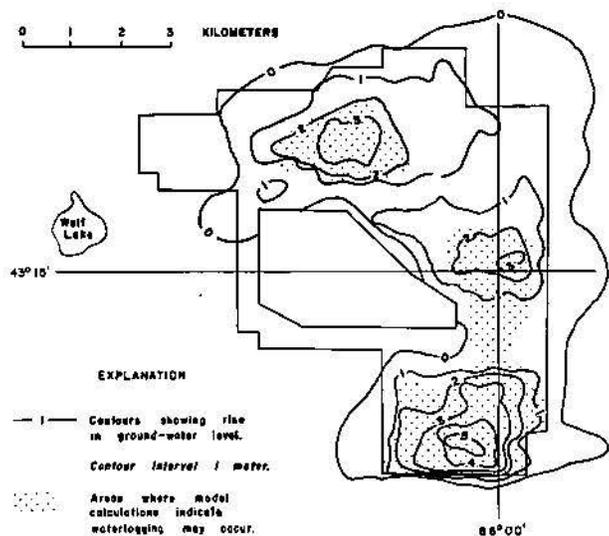


FIGURE 15.—Predicted rise in ground-water levels, in the water-table aquifer, when seepage into drainage tile is reduced by 75 percent.

of the change in water level that would result from clogging of the drains. The contours represent the rise in water level, above the levels shown in figure 13, that would result if drainage efficiency were reduced but other factors remained as in the previous simulation. Nearly half of the irrigated area becomes waterlogged. However, long before waterlogging became that exten-

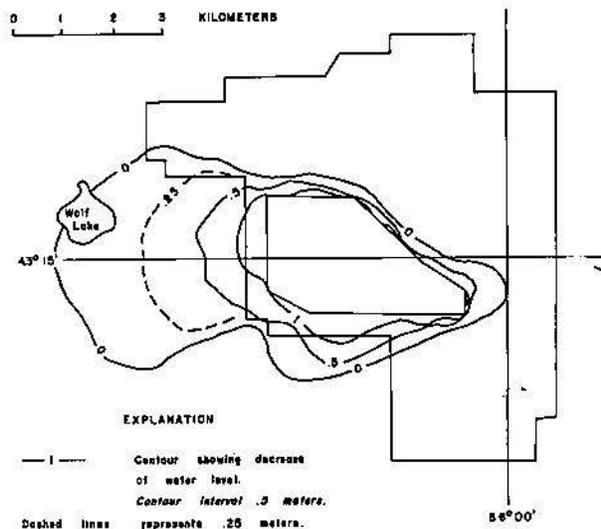


FIGURE 16.—Predicted decline in ground-water levels, in the water-table aquifer, if the bottoms of the storage lagoons become sealed.

sive, irrigation practices or drainage tiles would be modified.

In the final predictive simulation, seepage from the lagoons was reduced to zero while the other factors remained unchanged. The purpose of this simulation was to determine the effect on water levels if the bottoms of the lagoons became effectively sealed with organic matter. The 1975 water table was again taken as the starting condition, and the simulation was continued to steady state. Figure 16 shows contours of the changes in water level that would result from sealing the lagoons; that is, the changes caused by elimination of lagoon seepage while all other factors were maintained as in the second simulation. The changes in water level are relatively minor and are restricted to the immediate vicinity of the lagoons and a small area to the west that extends almost to Wolf Lake. There is virtually no reduction in the waterlogged areas from those in figure 13.

## SUMMARY

The ground-water system in the vicinity of the Muskegon wastewater site was simulated using a three-dimensional finite difference model. During calibration of the model for steady-state conditions a constant recharge rate of 20 cm/year was used for simulated conditions before the wastewater disposal system was constructed. Irrigation systems, lagoons, and drainage tiles were incorporated in the model for calibration under transient flow conditions.

The calibrated model showed that except in the

northwest corner, where tile lines are few and are above the water table, ground-water levels are lower than they would be under natural conditions. Predictive simulations indicate that, even if the drainage tiles lose 75 percent of their effectiveness, the impact of disposal operations on ground-water levels outside of the wastewater site will be small.

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