

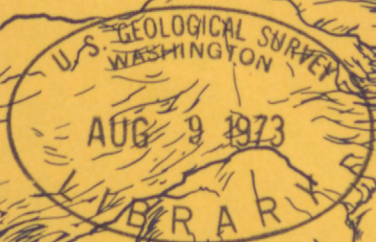
# an empirical method of estimating daily average basinwide snowmelt

U. S. Geological Survey

Water-Resources Investigations

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CALIFORNIA DEPARTMENT OF WATER RESOURCES



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<p>In this study the traditional, but relatively unreliable, degree-day method computing daily snowmelt has been modified by the addition of a daily index of the areal extent of the snowpack. The index used is actually a computed index of daily water equivalent of the pack. However, because a fairly close relation often exists between water equivalent and areal extent of the snowpack at the start of the melt season, a relation, albeit a weaker one, also often exists during the melt season between the decreasing water-equivalent index and the shrinking snowpack area. The proposed method can be readily modified to make advantageous use of any available data from a pressure-pillow snow gage in the basin.</p>				
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Rogers C. B. Murray, Secretary

GEOLOGICAL SURVEY

AN EMPIRICAL METHOD OF ESTIMATING

DAILY AVERAGE BASINWIDE SNOWMELT

By S. E. Rantz

✓ U.S. GEOLOGICAL SURVEY Water Resources Division

Water-Resources Investigations 14-73

Prepared in cooperation with the  
California Department of Water Resources

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2. Graph showing relation of water equivalent of snowpack to altitude, at start of melt season
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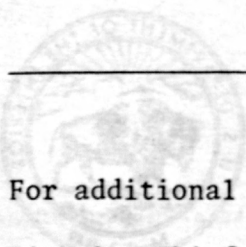


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June 1973



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## AN EMPIRICAL METHOD OF ESTIMATING DAILY AVERAGE BASINWIDE SNOWMELT

By S. E. Rantz

### ABSTRACT

This report presents an empirical method of computing daily average basinwide melt from a mountain snowpack. The term "average basinwide melt" refers to the depth at which the melted snow would stand if distributed evenly over the entire basin, rather than over the snow-covered area alone. Once obtained, the daily basinwide snowmelt values are used as input to a hydrologic routing model to compute daily values of snowmelt runoff.

The independent variables used in the empirical relation are (1) daily values of mean air temperature observed in or near the basin being studied and (2) an index of the daily water equivalent of the snowpack, based originally on snow-course data observed in or near the basin April 1 of each year. (April 1 is the approximate date when snow surveys are routinely made.) The dependent variable is daily average basinwide snowmelt, or more properly, an index of that melt. The daily index values of snowmelt needed to develop the relation are obtained by applying the hydrologic routing model, in reverse, to the streamflow records for a few snowmelt seasons, thereby computing the values of daily basinwide snowmelt required to generate the observed daily stream discharges.



The method developed here is intended for use with basins that lack either (1) the specialized meteorological data needed for applying the more sophisticated snowmelt equations that are based on rigorous physical laws of heat transfer or (2) data from a recording pressure-pillow snow gage, which gives a continuous record of water equivalent of the snowpack at a point and thereby provides an index of daily average basinwide snowmelt. Where such data deficiencies exist, daily snowmelt runoff has been commonly estimated by use of the degree-day method, in which air temperature and time-of-year (month) act as indexes of (1) the meteorological conditions that affect snowmelt and (2) the change in areal extent of the snowpack as the melt season progresses. The proposed empirical method does not provide an improved index of meteorological conditions, but, by use of a daily index of water equivalent of the snowpack, it does provide a far superior index of areal extent of the snowpack in the course of its ablation during the melt season. The proposed method can be readily modified to make advantageous use of any available data from a pressure-pillow snow gage in the basin.

The primary uses of the method would be in computing daily records of snowmelt runoff for years prior to the establishment of a stream-gaging station, and in short-term forecasting of snowmelt runoff based on temperature predictions made while the melt season is in progress. The reliability of such forecasts would be enhanced by using current streamflow records to adjust current values of the water-equivalent index of the snowpack.

## INTRODUCTION

The principles of heat transfer involved in the melting of a mountain snowpack are well understood, and an operational method, in the form of equations, is available for applying those principles to determine or predict daily snowmelt (U.S. Army Corps of Engineers, 1960). The computed snowmelt can then be routed to a stream channel. Intensive instrumentation of the watershed is required, however, to provide the meteorological data required for application of the snowmelt equations. Rarely will the hydrologist have all the necessary data, but quite often, by some improvising, he can make satisfactory estimates of the data that are lacking (Rantz, 1964). Many watersheds are equipped with one or more recording pressure-pillow snow gages, each of which gives a continuous record of water equivalent of the snowpack, thereby providing an index of daily average basinwide snowmelt. At least as often, however, the data deficiencies for a watershed are practically insurmountable, and the only information available consists of daily streamflow hydrographs for several years, a record of daily precipitation and air temperature at a climatological station in or near the basin being studied, and snow-course data for April 1 observed in or near the basin. (Snow surveys are routinely made on or about April 1 of each year, at which time the water equivalent of the snowpack is usually at its annual maximum.)

When faced with such data deficiencies, how does one study the snowmelt hydrology of a basin? In the years prior to 1955, before the development of the snowmelt equations cited above, the standard approach, and one still used by many hydrologists, was the empirical degree-day method (Wisler and Brater, 1959, p. 313-314). In that method the daily mean air temperature during a period of melt is used as an index of the integrated effect of radiation and sensible heat exchange. The degree-day method further assumes that negligible melting will occur when the air temperature is below some base value, usually 32°F. The difference, therefore, between the mean air temperature for any given date and the base temperature is the degree-days for that date. For example, if a base of 32°F is used, a day whose mean temperature is 49°F has 17 degree-days. Degree-days are then correlated with observed snowmelt runoff to provide a temperature index of melt runoff, using a separate correlation for each month of the season.

A weakness of the method lies in the fact that temperature is generally unsatisfactory as a basin index of the effect of such pertinent meteorological elements as wind, humidity, and radiation. Also, the method gives no consideration to the albedo of the snow surface, which may vary considerably with time, particularly as a result of early spring snowfalls. Because the degree-day index is correlated with basin runoff, its evaluation becomes complicated by such hydrologic factors as basin storage and lag, evapotranspiration losses, and particularly by the varying areal extent of the snowpack, which shrinks as the melt season progresses. It is not surprising, therefore, that the degree-day index for a given basin invariably shows wide variation with time, both within the snowmelt season and between snowmelt seasons, and the use of an average monthly degree-day index commonly gives highly erroneous estimates of daily snowmelt runoff.

*Purpose and scope.*--Nothing can be done about the shortcomings of the degree-day itself as an index of meteorological conditions, but the degree-day method would be improved tremendously if a satisfactory index of the areal extent of the snowpack on any given date could be improvised. The purpose of this study, therefore, was to modify the degree-day method by providing such an index.

For convenience, the basin used to test the proposed method was that of the North Yuba River (fig. 1) upstream from the U.S. Geological Survey gaging station below Goodyears Bar, Calif. In an earlier study of that basin by Rantz (1964), snowmelt had been computed, using complete meteorological data, for the melt seasons of 1956 and 1958 when the snowpack was extremely heavy, and for the melt season of 1959 when the snowpack was extremely light. Available, therefore, for those years were daily values of rainfall, air temperature, and average basinwide snowmelt, as well as the water equivalent of the snowpack on April 1. (The term "average basinwide snowmelt" refers to the depth at which the melted snow would stand if distributed evenly over the entire basin, rather than over the snow-covered area alone.) Accordingly, this study was restricted to the years 1956, 1958, and 1959. Not only were the basic data readily available for those years, but a wide range of total annual snowmelt runoff is represented by those years. Only a minimal amount of the climatological data available for the basin was utilized because the aim of the study was to demonstrate the applicability of the proposed method to meager data, and not to obtain the best possible reproduction of the North Yuba River runoff record by using all pertinent basic information.



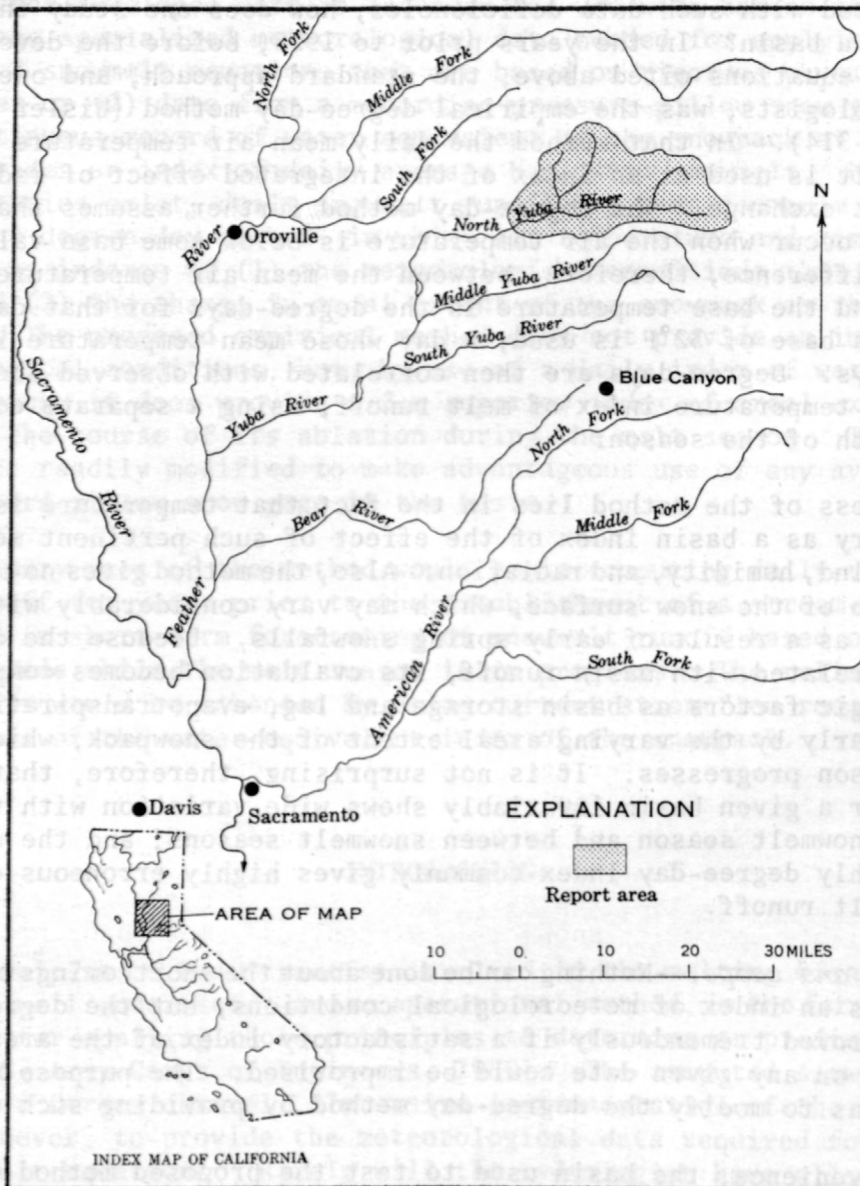


FIGURE 1.--Report area.

## DAILY AVERAGE BASINWIDE SNOWMELT

As a preliminary step in relating daily average basinwide snowmelt to air temperature and water equivalent of the snowpack, it is necessary to obtain values of daily snowmelt for use in defining the relation. In explaining how those snowmelt values are obtained, it is helpful to first discuss the procedure that would be followed for a basin having the specialized meteorological data needed to reliably compute daily snowmelt.

Where such meteorological data are available, they are used by the hydrologist in equations of heat transfer to compute the melt from each of several altitude zones in the basin. He then routes the melt, through basin storage, to the stream-gaging station he is considering. The daily melt from the various zones may be routed individually, or they may be combined before routing to provide a basinwide value of daily snowmelt. In either case, a lag factor must be introduced in recognition of the facts that (1) generally very little of the day's snowmelt reaches the ground-snow interface before noon and (2) there is generally appreciable distance between the melting snowpack and the stream-gaging station. For reasons that will become evident shortly, we assume here that the hydrologist combines the daily zonal snowmelt values for each day to obtain an average basinwide snowmelt value for that day. Rainfall that occurs on days of snowmelt augments the daily supply of water to be routed. A part of the supply infiltrates into the ground, some to be used to satisfy soil-moisture deficiencies caused by evapotranspiration, and some to reach the underlying ground-water body. The remainder of the total daily supply is assumed to be direct runoff and may be routed to the stream-gaging station by some device, such as the unit hydrograph. Ground-water outflow is likewise routed to the gaging station.

After following the above procedure to compute daily streamflow for a few years of historic record, the hydrologist compares computed discharges with those recorded. If necessary, he adjusts his basin parameters and routing coefficients to obtain closer agreement between computed and recorded discharge. He then has a satisfactory model for predicting daily streamflow from short-term predictions of meteorological conditions.

Let us assume now that the hydrologist lacks the necessary meteorological data for reliably computing snowmelt, but desires a method of predicting daily snowmelt runoff. One course of action that he might take would be as follows. He would first confine himself to the analysis of rainfall that occurred before or after the annual snowmelt period. For those rain periods that are free of snowmelt, he would compute daily basinwide precipitation from the record obtained at a precipitation gage or gages in or near the basin. In making that computation he would use an isohyetal map of mean annual precipitation to obtain the ratio of gaged precipitation to basinwide precipitation. He would next devise a routing procedure--for example, unit hydrograph--for converting basinwide precipitation to streamflow. Then, on the assumption that his routing procedure, adjusted for lag, is applicable to snowmelt routing, he would use the routing procedure in reverse. He would apply the adjusted routing procedure to observed streamflow data for several snowmelt seasons, to compute daily values of average basinwide water supply. The values obtained for rainless days would represent average basinwide snowmelt. For those days that had rain, average basinwide rainfall would be subtracted from the computed supply to obtain daily snowmelt values.



At this point the hydrologist would have no way of appraising the adequacy of his basin parameters and routing constants, because he has no idea of the true value of the daily increments of snowmelt. The daily values of snowmelt he obtained are in fact indexes of daily average basinwide snowmelt, but at this point even their adequacy as indexes, for use in an empirical snowmelt equation, cannot be evaluated. The next step to be taken is to relate the computed daily indexes of average basinwide snowmelt to air temperature and to daily indexes of water equivalent of the snowpack as determined originally from the April snow survey of each year. If that relation is satisfactory, a means becomes available for predicting daily basinwide indexes of snowmelt from air-temperature observations, after which the daily snowmelt indexes can be converted to daily runoff by use of the selected routing procedure. If the relation for predicting basinwide snowmelt indexes is unsatisfactory, basin parameters and routing constants must be adjusted by a trial-and-error procedure to produce index values of daily average basinwide snowmelt that can be related satisfactorily to air temperature and water-equivalent index.

It is with that latter relation that this paper is concerned. The inadequacy of the time-honored degree-day method has already been discussed, and the purpose of this study is to modify the method by providing an additional parameter for use--namely, daily indexes of the water equivalent of the snowpack. To test the practicality of the proposed modification, use will be made of the readily available hydrologic data from the previously mentioned study of the North Yuba River basin (Rantz, 1964). It will be assumed that the daily values of average basinwide snowmelt were obtained from a routing study such as that described above, although they were actually computed from detailed meteorological data.

#### RELATION OF BASINWIDE SNOWMELT TO AIR TEMPERATURE AND WATER EQUIVALENT OF SNOWPACK

##### Discussion of Parameters

The average basinwide depth of snowmelt on a given day is equal to the average melt, in inches per unit area of snow cover, multiplied by the areal extent of the snowpack on that day and divided by the total area of the basin (a constant). With the limited data to be used we have no way of determining absolute values of either daily melt or daily area of the snowpack, nor is it necessary that we have absolute values of those elements. The only absolute values we are concerned with are the final values of daily streamflow, and they will be estimated by applying index values of daily basinwide snowmelt to an appropriate routing model. The empiricism involved in that procedure differs only in degree from that used in routing precipitation on a mountain basin to a stream-gaging station. The precipitation values used as input to the precipitation routing model are usually indexes of the true basinwide precipitation, albeit much more reliable indexes than the snowmelt indexes we will be using in this study.

The use of daily mean air temperature in a relation involving snowmelt is obvious. Though imperfect as an index of climatological conditions, air temperature does reflect the integrated effect of radiation and sensible heat exchange. There is nothing to be gained by using degree-days rather than mean air temperature, because if it is desirable to subtract some constant value from mean air temperature, the correlation will so indicate and will also indicate the "best" value of that constant. To more sharply define the joint effect of progressively increasing radiation and sensible heat exchange during the snowmelt season, a separate relation is used for each half of each month of the season--April through June in westside drainage basins of the Sierra Nevada, such as that of the North Yuba River.

To obtain true daily values of the areal extent of the snowpack, it would be necessary to observe the recession of the snowline as the melt season progresses. That is rarely practical, however, and daily indexes are used in this study. The procedure used for obtaining indexes of snowpack area is based on the fact that on about April 1 of each year, when the annual snow accumulation is usually at its maximum, a fairly close relation often exists between water equivalent of the snowpack and altitude of the snowline (Potts, 1944). It was reasoned that the April water equivalent at some high altitude--one sufficiently high to remain snow-covered at the time direct snowmelt runoff ceases at the gaging station--could be used as an index of basinwide water equivalent, and, therefore, as an index of snowpack area. On the first day of direct snowmelt runoff, usually about April 1, the water equivalent at the selected altitude, as obtained from the April snow survey, would be used directly. The snowmelt, or index of snowmelt, computed for the first day, would be subtracted from the water equivalent of the first day to give the water-equivalent index to be used on the second day. The procedure would be continued throughout the season until direct snowmelt runoff ceased. The water-equivalent index used for any day would be that of the preceding day minus the computed snowmelt for that preceding day. It was obvious that the water-equivalent index, computed in the manner described above, would have to be combined with date, or at least with month, to have any meaning as an index of areal extent of the snowpack. That fact further dictated the use of separate snowmelt relations for each half-month of the season.

Direct surface runoff was assumed to start on the first day, after the snow survey of late March or early April, when the recorded hydrograph at the stream-gaging station showed the easily identifiable diurnal fluctuation that is characteristic of snowmelt runoff. Direct surface runoff was assumed to cease on the day when streamflow receded to the discharge recorded on the starting date of direct runoff, and entered the base-flow recession phase of the runoff cycle.



### Data Used in the Study

As mentioned earlier, the data used in this report were obtained from a study by Rantz (1964) of the snowmelt hydrology of the North Yuba River basin (fig. 1) upstream from the stream-gaging station below Goodyears Bar, Calif. The basin ranges in altitude from 2,450 feet to 8,590 feet, and has a drainage area of 245 square miles. Figure 2 shows the relation of water equivalent of the snowpack to altitude at the start of the snowmelt seasons of 1956, 1958, and 1959. The snowpack was extremely heavy in 1956 and 1958, and extremely light in 1959. For the purpose of this study, the water equivalent at an altitude of 7,000 feet was used as the index of areal extent of the snowpack--18 percent of the area of the total basin lies above that altitude. Table 1 shows the water equivalent of the snowpack in early April at the 7,000-foot level, as obtained from figure 2, and the duration of the period of direct snowmelt runoff, for the 3 years that are being studied.

TABLE 1.--Period of direct snowmelt runoff and water equivalent of the snowpack at altitude of 7,000 feet in 1956, 1958, and 1959

Year	Period of direct snowmelt runoff		Water equivalent, in inches, of snowpack at 7,000-foot altitude at start of melt season
	Start	End	
1956	April 6	June 30	49.1
1958	April 8	June 30	60.9
1959	April 1	May 16	20.0

All other data used are given in tables 2-4. Column 2 of the tables lists values of daily mean air temperature at Downieville Ranger Station during the periods of direct snowmelt runoff in 1956, 1958, and 1959. Downieville Ranger Station, at an altitude of 2,895 feet, is used as the index station for air temperature over the snowpack. In column 3 of the tables are values of daily average basinwide snowmelt as computed for the report by Rantz (1964). Those values are identical with the values of supply given in column 2 of table 3 of the previously cited report by Rantz, except for those days on which rain occurred. Rainfall has been subtracted from the published figures to provide values of snowmelt. Column 4 of tables 2-4 of the present report gives the accumulated values of daily snowmelt.

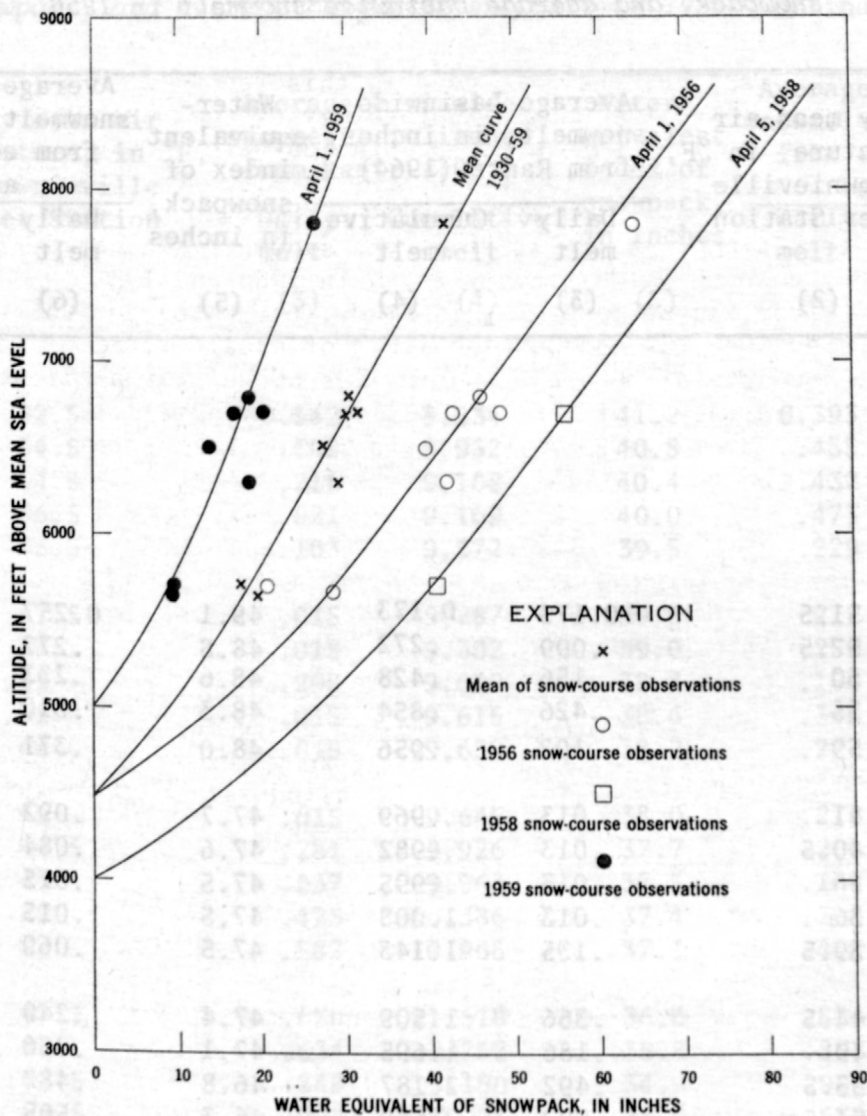


FIGURE 2.--Relation of water equivalent of snowpack to altitude, at start of melt season.



TABLE 2.--Daily values of mean air temperature, water-equivalent index of the snowpack, and average basinwide snowmelt in 1956

Day	Daily mean air temperature, in °F, at Downieville Ranger Station	Average basinwide snowmelt, in inches, from Rantz (1964)		Water-equivalent index of snowpack, in inches	Average basinwide snowmelt, in inches, from equations 2 and 3	
		Daily melt	Cumulative melt		Daily melt	Cumulative melt
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>April</i>						
1						
2						
3						
4						
5						
6	51.5	0.173	0.173	49.1	0.257	0.257
7	52.5	.099	.272	48.8	.272	.529
8	50	.156	.428	48.6	.233	.762
9	55	.426	.854	48.3	.310	1.072
10	59	.102	.956	48.0	.371	1.443
11	41	.013	.969	47.7	.092	1.535
12	40.5	.013	.982	47.6	.084	1.619
13	36	.013	.995	47.5	.015	1.634
14	36	.013	1.008	47.5	.015	1.649
15	39.5	.135	1.143	47.5	.069	1.718
16	44.5	.366	1.509	47.4	.249	1.967
17	49	.186	1.695	47.1	.366	2.333
18	53.5	.492	2.187	46.8	.482	2.815
19	54.5	.502	2.689	46.3	.505	3.320
20	58	.825	3.514	45.8	.592	3.912
21	58	.649	4.163	45.2	.588	4.500
22	56.5	.711	4.874	44.6	.546	5.046
23	57.5	.711	5.585	44.1	.568	5.614
24	59	.643	6.228	43.5	.601	6.215
25	50	.042	6.270	42.9	.373	6.588
26	42.5	.023	6.293	42.5	.186	6.774
27	42.5	.230	6.523	42.3	.185	6.959
28	45	.353	6.876	42.1	.246	7.205
29	46.5	.463	7.339	41.9	.283	7.488
30	51	.653	7.992	41.6	.392	7.880

TABLE 2.--Daily values of mean air temperature, water-equivalent index of the snowpack, and average basinwide snowmelt in 1956--Continued

Day	Daily mean air temperature, in °F, at Downieville Ranger Station	Average basinwide snowmelt, in inches, from Rantz (1964)		Water-equivalent index of snowpack, in inches	Average basinwide snowmelt, in inches, from equations 4 and 5	
		Daily melt	Cumulative melt		Daily melt	Cumulative melt
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>May</i>						
1	52.5	0.542	8.534	41.2	0.393	8.273
2	54.5	.398	8.932	40.8	.435	8.708
3	54.5	.216	9.148	40.4	.432	9.140
4	56.5	.021	9.169	40.0	.473	9.613
5	45.5	.103	9.272	39.5	.229	9.842
6	47.5	.015	9.287	39.3	.272	10.114
7	47	.015	9.302	39.0	.259	10.373
8	42	.298	9.600	38.7	.150	10.523
9	51	.015	9.615	38.6	.343	10.866
10	48	.015	9.630	38.2	.277	11.143
11	45	.015	9.645	38.0	.212	11.355
12	44.5	.281	9.926	37.7	.200	11.555
13	43	.037	9.963	37.5	.168	11.723
14	49	.423	10.386	37.4	.293	12.016
15	57	.582	10.968	37.1	.458	12.474
16	61.5	.650	11.618	36.6	.824	13.298
17	63	.634	12.252	35.8	.852	14.150
18	65.5	.848	13.100	34.9	.905	15.055
19	58.5	.908	14.008	34.0	.679	15.734
20	63	.990	14.998	33.4	.795	16.529
21	64	.969	15.967	32.6	.804	17.333
22	66.5	1.063	17.030	31.8	.851	18.184
23	65	.984	18.014	30.9	.788	18.972
24	60.5	.708	18.722	30.1	.652	19.624
25	61.5	.780	19.502	29.5	.664	20.288
26	63.5	.980	20.482	28.8	.698	20.986
27	58.5	.647	21.129	28.1	.561	21.547
28	55.5	.613	21.742	27.6	.481	22.028
29	61	.596	22.338	27.1	.599	22.627
30	62.5	.767	23.105	26.5	.619	23.246
31	58.5	.623	23.728	25.9	.517	23.763

TABLE 2.--Daily values of mean air temperature, water-equivalent index of the snowpack, and average basinwide snowmelt in 1956--Continued

Day	Daily mean air temperature, in °F, at Downieville Ranger Station	Average basinwide snowmelt, in inches, from Rantz (1964)		Water-equivalent index of snowpack, in inches	Average basinwide snowmelt, in inches, from equations 6 and 7	
		Daily melt	Cumulative melt		Daily melt	Cumulative melt
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>June</i>						
1	55.5	0.527	24.255	25.3	0.259	24.022
2	57.5	.493	24.748	25.1	.282	24.304
3	66	.714	25.462	24.8	.384	24.688
4	57	.304	25.766	24.4	.268	24.956
5	49	.249	26.015	24.1	.169	25.125
6	51.5	.298	26.313	24.0	.198	25.323
7	57	.331	26.644	23.8	.262	25.585
8	59.5	.312	26.956	23.5	.288	25.873
9	63	.368	27.324	23.2	.325	26.198
10	62	.377	27.701	22.9	.309	26.507
11	59	.254	27.955	22.6	.271	26.778
12	61.5	.245	28.200	22.3	.295	27.073
13	62	.220	28.420	22.0	.297	27.370
14	63.5	.091	28.511	21.7	.309	27.679
15	53	.154	28.665	21.4	.193	27.872
16	54.5	.180	28.845	21.2	.149	28.021
17	56.5	.191	29.036	21.1	.163	28.184
18	65.5	.281	29.317	20.9	.229	28.413
19	68	.272	29.589	20.7	.246	28.659
20	55.5	.190	29.779	20.4	.151	28.810
21	58	.204	29.983	20.3	.168	28.978
22	65	.238	30.221	20.1	.217	29.195
23	67	.304	30.525	19.9	.229	29.424
24	62.5	.206	30.731	19.7	.195	29.619
25	62.5	.228	30.959	19.5	.193	29.812
26	69	.234	31.193	19.3	.236	30.048
27	75	.294	31.487	19.1	.275	30.323
28	76.5	.297	31.784	18.8	.281	30.604
29	76	.258	32.042	18.5	.273	30.877
30	70	.205	32.247	18.2	.229	31.106



TABLE 3.--Daily values of mean air temperature, water-equivalent index of the snowpack, and average basinwide snowmelt in 1958

Day	Daily mean air temperature, in °F, at Downieville Ranger Station	Average basinwide snowmelt, in inches, from Rantz (1964)		Water-equivalent index of snowpack, in inches	Average basinwide snowmelt, in inches, from equations 2 and 3	
		Daily melt	Cumulative melt		Daily melt	Cumulative melt
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>April</i>						
1	8.7	.520	.520	60.9	.122	.122
2	9.2	.521	1.041	60.9	.164	.286
3	9.2	.521	1.562	60.8	.234	.520
4	10.1	.521	2.083	60.6	.277	.797
5	11.0	.521	2.604	60.4	.250	1.047
6	11.0	.521	3.125	60.1	.318	1.365
7	11.0	.521	3.646	59.9	.292	1.657
8	42.0	.098	3.744	59.5	.274	1.931
9	44.5	.161	3.905	59.2	.535	2.466
10	48.5	.186	4.091	58.4	.546	3.012
11	51.0	.128	4.219	57.9	.470	3.482
12	49.5	.168	4.387	57.4	.453	3.935
13	53.5	.419	4.806	57.0	.538	4.473
14	52.0	.438	5.244	56.4	.607	5.080
15	51.0	.328	5.572	55.8	.604	5.684
16	53.0	.356	5.928	55.2	.143	5.827
17	53.5	.324	6.252	55.1	.157	5.984
18	51.0	.489	6.741	54.9	.270	6.254
19	50.5	.301	7.042	54.6	.355	6.609
20	53.5	.413	7.455	54.3	.311	6.920
21	56.0	.749	8.204	54.0	.437	7.357
22	56.0	.203	8.407	53.5	.463	7.820
23	40.0	.071	8.478	53.1	.405	8.225
24	40.5	.034	8.512			
25	44.5	.262	8.774			
26	47.5	.524	9.298			
27	46.0	.122	9.420			
28	50.5	.138	9.558			
29	51.5	.655	10.213			
30	49.5	.361	10.574			

TABLE 3.--Daily values of mean air temperature, water-equivalent index of the snowpack, and average basinwide snowmelt in 1958--Continued

Day	Daily mean air temperature, in °F, at Downieville Ranger Station	Average basinwide snowmelt, in inches, from Rantz (1964)		Water-equivalent index of snowpack, in inches	Average basinwide snowmelt, in inches, from equations 4 and 5	
		Daily melt	Cumulative melt		Daily melt	Cumulative melt
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>May</i>						
1	54.5	0.641	7.569	52.7	0.528	8.753
2	56	.724	8.293	52.1	.564	9.317
3	54	.746	9.039	51.6	.506	9.823
4	57.5	.915	9.954	51.1	.595	10.418
5	60	1.021	10.975	50.5	.655	11.073
6	55.5	.663	11.638	49.8	.531	11.604
7	53.5	.636	12.274	49.3	.476	12.080
8	60	.826	13.100	48.8	.638	12.718
9	62	.964	14.064	48.2	.683	13.401
10	64	.365	14.429	47.5	.725	14.126
11	52	.268	14.697	46.8	.420	14.546
12	46.5	.206	14.903	46.4	.282	14.828
13	44	.508	15.411	46.1	.220	15.048
14	53	.705	16.116	45.9	.438	15.486
15	57.5	.723	16.839	45.4	.544	16.030
16	64	.995	17.834	44.9	1.107	17.137
17	66	1.053	18.887	43.8	1.154	18.291
18	65.5	1.091	19.978	42.6	1.104	19.395
19	60	.960	20.938	41.5	.882	20.277
20	63.5	.798	21.736	40.6	.983	21.260
21	63	.700	22.436	39.6	.942	22.202
22	65	.609	23.045	38.7	.987	23.189
23	54.5	.595	23.640	37.7	.625	23.814
24	54	.427	24.067	37.1	.599	24.413
25	60	.923	24.990	36.5	.776	25.189
26	59	.832	25.822	35.7	.728	25.917
27	58	.757	26.579	35.0	.684	26.601
28	55	.648	27.227	34.3	.583	27.184
29	57	.520	27.747	33.7	.630	27.814
30	58.5	.656	28.403	33.1	.661	28.475
31	59.5	.578	28.981	32.4	.675	29.150

TABLE 3.--Daily values of mean air temperature, water-equivalent index of the snowpack, and average basinwide snowmelt in 1958--Continued

Day	Daily mean air temperature, in °F, at Downieville Ranger Station	Average basinwide snowmelt, in inches, from Rantz (1964)		Water-equivalent index of snowpack, in inches	Average basinwide snowmelt, in inches, from equations 6 and 7	
		Daily melt	Cumulative melt		Daily melt	Cumulative melt
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>June</i>						
1	55.5	0.483	29.464	31.7	0.325	29.475
2	59	.201	29.665	31.4	.377	29.852
3	52.5	.115	29.780	31.0	.271	30.123
4	51	.410	30.190	30.8	.246	30.369
5	61	.416	30.606	30.5	.396	30.765
6	59.5	.434	31.040	30.1	.369	31.134
7	54.5	.193	31.233	29.8	.291	31.425
8	56	.153	31.386	29.5	.310	31.735
9	52.5	.140	31.526	29.2	.256	31.991
10	49.5	.294	31.820	28.9	.210	32.201
11	56.5	.187	32.007	28.7	.309	32.510
12	56.5	.168	32.175	28.4	.305	32.815
13	50	.135	32.310	28.1	.211	33.026
14	61	.399	32.709	27.9	.363	33.389
15	65.5	.383	33.092	27.5	.419	33.808
16	68.5	.298	33.390	27.1	.327	34.135
17	69	.327	33.717	26.8	.328	34.463
18	72	.290	34.007	26.4	.352	34.815
19	66	.264	34.271	26.1	.291	35.106
20	63	.221	34.492	25.8	.260	35.366
21	68	.273	34.765	25.5	.303	35.669
22	67.5	.250	35.015	25.2	.295	35.964
23	73	.307	35.322	24.9	.341	36.305
24	68	.249	35.571	24.6	.292	36.597
25	65	.261	35.832	24.3	.262	36.859
26	70	.307	36.139	24.0	.302	37.161
27	65.5	.162	36.301	23.7	.260	37.421
28	61.5	.200	36.501	23.5	.224	37.645
29	62	.200	36.701	23.3	.226	37.871
30	59	.187	36.888	23.0	.199	38.070



TABLE 4.--Daily values of mean air temperature, water-equivalent index of the snowpack, and average basinwide snowmelt in 1959

Day	Daily mean air temperature, in °F, at Downieville Ranger Station	Average basinwide snowmelt, in inches, from Rantz (1964)		Water-equivalent index of snowpack, in inches	Average basinwide snowmelt, in inches, from equations 2 and 3	
		Daily melt	Cumulative melt		Daily melt	Cumulative melt
(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>April</i>						
1	55.0	0.193	0.193	20.0	0.225	0.225
2	58.5	.340	.533	19.8	.263	.488
3	55.0	.373	.906	19.5	.224	.712
4	57.0	.647	1.553	19.3	.245	.957
5	57.0	.481	2.034	19.0	.244	1.201
6	56.0	.399	2.433	18.8	.232	1.433
7	57.0	.013	2.446	18.6	.243	1.676
8	53.5	.013	2.459	18.3	.203	1.879
9	51.5	.162	2.621	18.1	.181	2.060
10	54.0	.260	2.881	17.9	.208	2.268
11	54.0	.462	3.343	17.7	.207	2.475
12	53.5	.401	3.744	17.5	.201	2.676
13	53.0	.139	3.883	17.3	.195	2.871
14	50.5	.215	4.098	17.1	.167	3.038
15	49.5	.359	4.457	17.0	.156	3.194
16	49.0	.306	4.763	16.8	.239	3.433
17	52.0	.200	4.963	16.6	.289	3.722
18	46.5	.137	5.100	16.3	.194	3.916
19	48.5	.266	5.366	16.1	.227	4.143
20	51.5	.279	5.645	15.9	.277	4.420
21	54.0	.360	6.005	15.6	.317	4.737
22	54.5	.435	6.440	15.3	.324	5.061
23	58.5	.274	6.714	14.9	.387	5.448
24	60.5	.591	7.305	14.6	.418	5.866
25	58.5	.010	7.315	14.1	.381	6.247
26	48.0	.010	7.325	13.8	.210	6.457
27	43.0	.165	7.490	13.5	.128	6.585
28	48.5	.251	7.741	13.4	.216	6.801
29	59.0	.345	8.086	13.2	.383	7.184
30	62.0	.376	8.462	12.8	.428	7.612

TABLE 4.--Daily values of mean air temperature, water-equivalent index of the snowpack, and average basinwide snowmelt in 1959--Continued

Day	Daily mean air temperature, in °F, at Downieville Ranger Station	Average basinwide snowmelt, in inches, from Rantz (1964)		Water-equivalent index of snowpack, in inches	Average basinwide snowmelt, in inches, from equations 4 and 5	
		Daily melt	Cumulative melt		Daily melt	Cumulative melt
(1)	(2)	(3)	(4)	(5)	(6)	(7)
May						
1	53.5	0.013	8.475	12.4	0.203	7.815
2	39	.013	8.488	12.2	.044	7.859
3	40	.056	8.544	12.1	.054	7.913
4	41.5	.090	8.634	12.1	.070	7.983
5	47	.067	8.701	12.0	.130	8.113
6	48	.148	8.849	11.9	.140	8.253
7	54	.199	9.048	11.7	.203	8.456
8	59.5	.255	9.303	11.5	.260	8.716
9	55.5	.234	9.537	11.3	.216	8.932
10	55.5	.192	9.729	11.1	.214	9.146
11		.243	9.972	10.9	.300	9.446
12	64	.220	10.192	10.6	.297	9.743
13	68.5	.305	10.497	10.3	.339	10.082
14	57.5	.150	10.647	9.9	.224	10.306
15	44	.043	10.690	9.7	.089	10.395
16	44	.041	10.731	9.6	.073	10.468

### Derivation of the Relations

The earlier discussion of parameters (p. 6-7) indicates the rationale used in the derivation of the snowmelt relations. Date within the melt season is a factor in the effectiveness of: (1) Air temperature as an index of the heat transfer to the snowpack, which in turn is an index of average snowmelt; and (2) water-equivalent index at a single altitude level as an index of basinwide water equivalent, which in turn is an index of areal extent of the snowpack. Consequently, the snowmelt season was divided into six periods and a separate relation was used for each. Those periods were April 1-15 and 16-30, May 1-15 and 16-31, and June 1-15 and 16-30. Because daily average basinwide snowmelt is the daily product of average melt and areal extent of the snowpack, divided by total basin area (a constant), we expect the form of the daily equation for each of the six periods to be:

$$Sm = a(WE + b) (T + c), \quad (1)$$

where  $Sm$  = average basinwide snowmelt, in inches,

$WE$  = water-equivalent index of the snowpack, in inches, at 7,000 feet,

$T$  = mean air temperature, in degrees Fahrenheit, at Downieville  
Ranger Station,

and  $a$ ,  $b$ , and  $c$  are constants. Furthermore, we expect  $a$  to be positive,  $b$  to be either positive or negative, and  $c$  to be negative.

The data from tables 2-4 were plotted with  $Sm$  (col. 3) as ordinate,  $T$  (col. 2) as abscissa, and  $WE$  as the parameter for a family of curves. The value of  $WE$  used for any day was equal to the difference between accumulated snowmelt (col. 4) on the preceding day and the value of  $WE$  on the first day of snowmelt. The general position of the plotted points verified the predicted model, as expressed by equation 1. The scatter of those points, however, was quite chaotic; the scatter diagrams are not shown here. A fact immediately apparent was that minus 35 was a suitable value of  $c$ , because all trial curves tended to converge to a snowmelt value of zero when  $T = 35^\circ\text{F}$ . That simplified the ensuing trial-and-error procedure to obtain values of  $a$  and  $b$  that would not only fit the plotted points as satisfactorily as possible, but would also provide a fairly uniform transition between relations for each of the six 2-week periods. The equations of daily snowmelt that were finally derived follow.

April 1-15	$Sm = 0.00015 (WE + 55) (T - 35)$	(2)
April 16-30	$Sm = 0.00030 (WE + 40) (T - 35)$	(3)
May 1-15	$Sm = 0.00040 (WE + 15) (T - 35)$	(4)
May 16-31	$Sm = 0.00085 (WE) (T - 35)$	(5)
June 1-15	$Sm = 0.00050 (WE) (T - 35)$	(6)
June 16-30	$Sm = 0.00036 (WE) (T - 35)$	(7)



## DISCUSSION OF RESULTS

Tables 2-4 show the melt computed by applying equations 2-7 to the index values listed in column 2 (air temperature) and column 5 (water equivalent) of the tables. In column 5, the water-equivalent index at the 7,000-foot altitude is reduced on each successive day by the computed snowmelt for the previous day. For example, on the first page of table 2, the melt season starts April 6, 1956, with a water equivalent of 49.1 inches, obtained from the snow surveys of early April of 1956. The computed melt April 6, shown in column 6, is 0.26 inch. The water equivalent, therefore, April 7 is 48.8 inches (49.1 minus 0.26). The computed snowmelt values in column 6, when compared with those in column 3, show only fair agreement. That degree of agreement is probably as high as can be expected, because the index values used as independent variables in equations 2-7 are not completely satisfactory. They are, however, the only ones available for use in most basins. Incidentally, it should not be inferred that the values of daily average basinwide snowmelt in column 3, which are used as a standard of comparison, are without error. The values in column 3 were obtained from the 1964 study of the North Yuba River basin. In that study the computed daily values of point snowmelt are considered to be satisfactory, but the daily values of areal extent of the snowpack were derived from a continuous inventory of snowmelt and are subject to considerable error.

Column 7 of tables 2-4 gives accumulated values of computed daily snowmelt. Those values are plotted as dotted-line curves in figures 3-5. For visual comparison, the accumulated values of melt in column 4 of tables 2-4, obtained from the 1964 study of the North Yuba River basin, are plotted as solid-line curves in figures 3-5. The two curves have fair agreement in 1956 and 1958, and what seems to be poor agreement in 1959 (fig. 5). However, most of the discrepancy in 1959 is attributable to the poor agreement in the first 5 days in April, during which equation 2 underestimated the total 5-day melt by 0.8 inch. That error is carried through the entire 1959 snowmelt period in the dotted-line curve of cumulative daily snowmelt.

Where the snowpack covers a large range of altitude, it is not unusual for large errors in computed daily snowmelt to occur during the first few days of the melt season. At that time of the year a "heat deficit" of varying magnitude often exists in the high-altitude part of the snowpack, and the heat received there during those days is not available to cause melt, but instead primes the snowpack, or readies it, for later melt (Rantz, 1964, p. 20). Furthermore, the albedo of the snowpack at that time tends to vary from year to year because the occurrence of snowfall is highly variable in early spring. Referring again to figure 5, if, during April 1959, the hydrologist had kept an additional cumulative record of daily average basinwide snowmelt, computed by applying his routing model in reverse to currently observed streamflow, he would have noticed April 5 that equation 2 had badly underestimated the melt for the first 5 days of the month. He could then have adjusted his values of accumulated snowmelt and water equivalent for use in equation 2 for the days following April 5. On resuming his computations, using equations 2-5, he would have obtained daily values of average basinwide snowmelt, which when accumulated would give the dashed-line curve in figure 5. That curve agrees satisfactorily with the solid-line curve in figure 5.

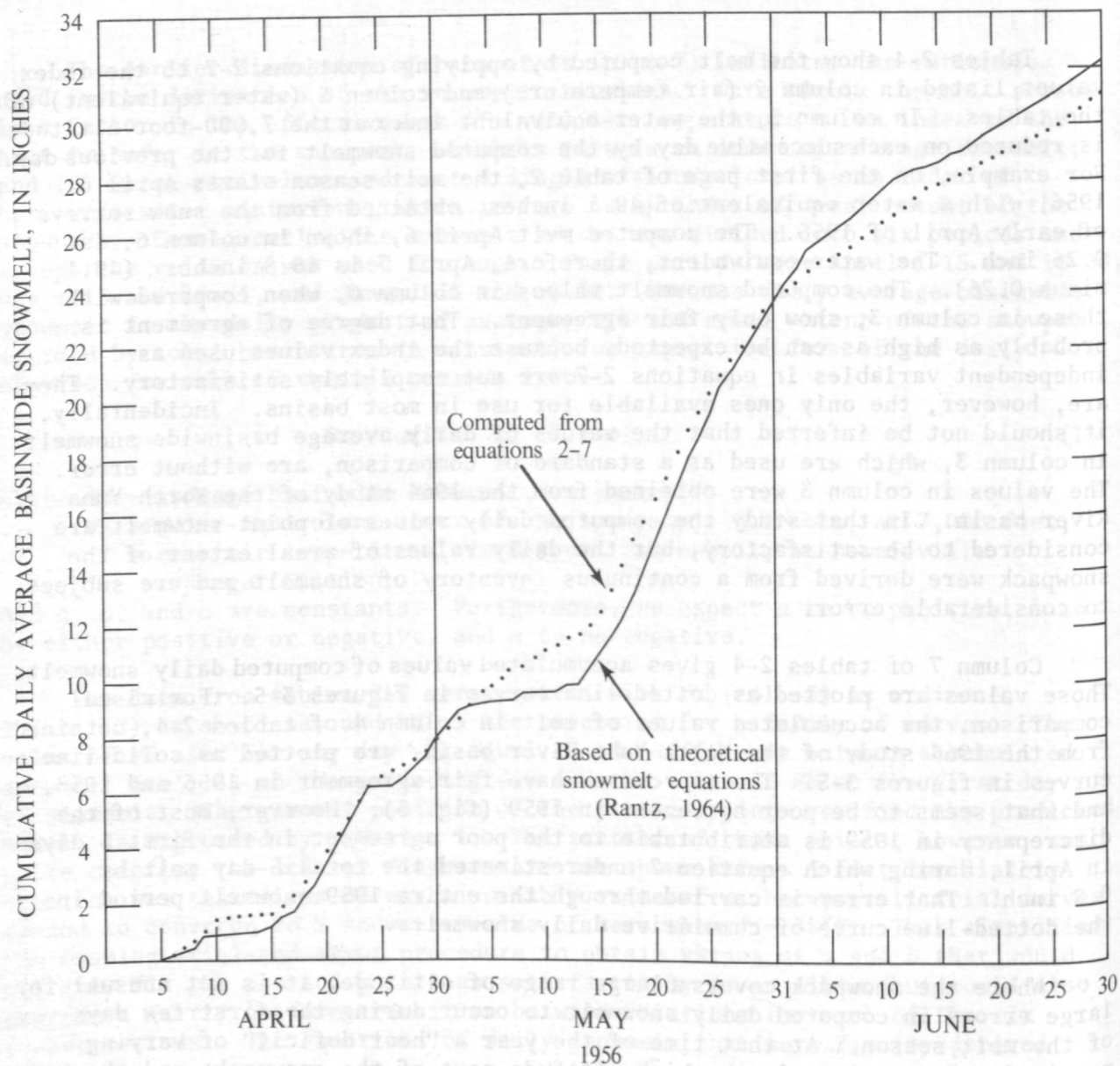


FIGURE 3.--Comparison of graphs of cumulative daily average basinwide snowmelt for 1956, based on theoretical and empirical snowmelt equations.

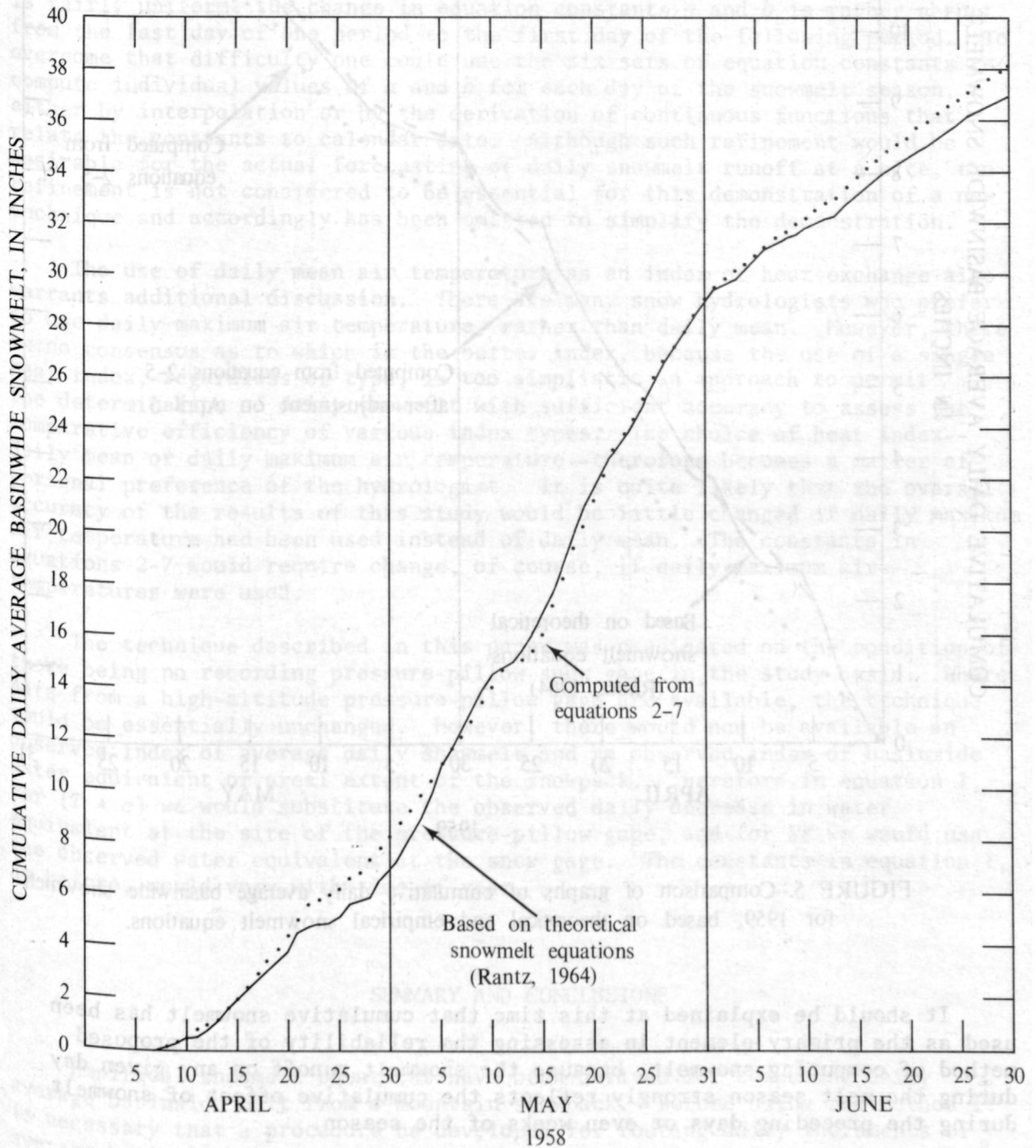


FIGURE 4.--Comparison of graphs of cumulative daily average basinwide snowmelt for 1958, based on theoretical and empirical snowmelt equations.



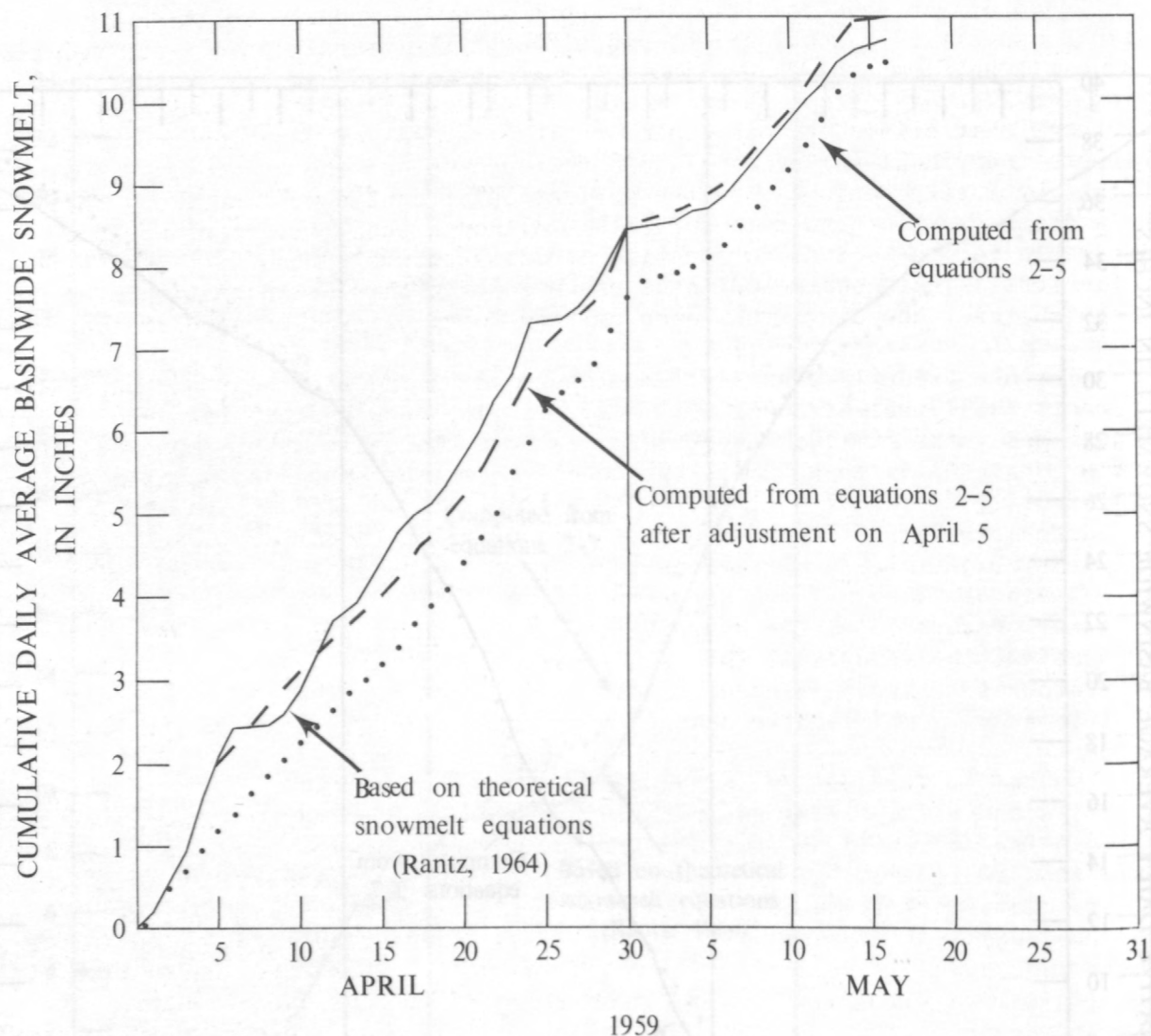


FIGURE 5.--Comparison of graphs of cumulative daily average basinwide snowmelt for 1959, based on theoretical and empirical snowmelt equations.

It should be explained at this time that cumulative snowmelt has been used as the primary element in assessing the reliability of the proposed method of computing snowmelt, because the snowmelt runoff on any given day during the melt season strongly reflects the cumulative effect of snowmelt during the preceding days or even weeks of the season.

Each of the equations 2-7 represents an average daily relation for use within a specified 2-week period, and although the transition between relations is fairly uniform, the change in equation constants  $a$  and  $b$  is rather abrupt from the last day of one period to the first day of the following period. To overcome that difficulty one could use the six sets of equation constants to compute individual values of  $a$  and  $b$  for each day of the snowmelt season, either by interpolation or by the derivation of continuous functions that relate the constants to calendar date. Although such refinement would be desirable for the actual forecasting of daily snowmelt runoff at a site, the refinement is not considered to be essential for this demonstration of a new technique and accordingly has been omitted to simplify the demonstration.

The use of daily mean air temperature as an index of heat exchange also warrants additional discussion. There are many snow hydrologists who prefer to use daily maximum air temperature, rather than daily mean. However, there is no consensus as to which is the better index, because the use of a single heat index, regardless of type, is too simplistic an approach to permit the determination of daily snowmelt with sufficient accuracy to assess the comparative efficiency of various index types. The choice of heat index--daily mean or daily maximum air temperature--therefore becomes a matter of personal preference of the hydrologist. It is quite likely that the overall accuracy of the results of this study would be little changed if daily maximum air temperature had been used instead of daily mean. The constants in equations 2-7 would require change, of course, if daily maximum air temperatures were used.

The technique described in this paper was predicated on the condition of there being no recording pressure-pillow snow gage in the study basin. Where data from a high-altitude pressure-pillow gage are available, the technique would be essentially unchanged. However, there would now be available an observed index of average daily snowmelt and an observed index of basinwide water equivalent or areal extent of the snowpack. Therefore in equation 1, for  $(T + c)$  we would substitute the observed daily decrease in water equivalent at the site of the pressure-pillow gage, and for  $WE$  we would use the observed water equivalent at the snow gage. The constants in equation 1, as before, would vary with time of year.

## SUMMARY AND CONCLUSIONS

Empirical snowmelt equations have been derived for computing daily average basinwide melt from a mountain snowpack. Before using the method it is necessary that a procedure be developed for routing daily increments of average basinwide snowmelt to the gaging station under consideration. That routing procedure, when applied in reverse to the streamflow records for a few snowmelt seasons, provides the snowmelt values to be used as the dependent variable in evaluating the constants in the proposed empirical snowmelt equations. Individual values of the constants are derived for each 2-week period of the snowmelt season.

The daily snowmelt equations that were derived are of the form

$$Sm = a(WE + b) (T + c),$$

where  $Sm$  = average basinwide snowmelt, in inches,

$WE$  = water-equivalent index of the snowpack, in inches, at some selected high altitude,

$T$  = mean air temperature, in degrees Fahrenheit, at a climatological station in or near the study basin,

and  $a$ ,  $b$ , and  $c$  are constants.

The water-equivalent index at the start of the melt season is determined from the routine snow surveys made in or near the basin on or about April 1. The index for each subsequent day is obtained by subtracting the computed snowmelt for the preceding day from the water-equivalent index of the preceding day. The water-equivalent index is, in effect, an index of the areal extent of the snowpack on each day.

The proposed snowmelt equations seem to be a great improvement over those used in the traditional degree-day method. It is intended that the method described above be used only in the absence of the specialized meteorological data needed for applying the more sophisticated snowmelt equations that are based on rigorous physical laws of heat transfer and in the absence of data from a high-altitude pressure-pillow snow gage in the study basin. However, it would seem that the snowmelt equations can be readily modified to make advantageous use of any available data from a pressure-pillow gage in the basin, as explained on page 23.

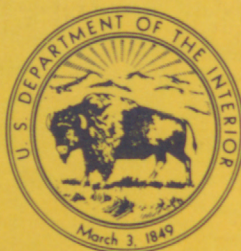
#### REFERENCES CITED

- Potts, H. L., 1944, A photographic snow-survey method of forecasting runoff: Am. Geophys. Union Trans., pt. 1, p. 149-152.
- Rantz, S. E., 1964, Snowmelt hydrology of a Sierra Nevada stream: U.S. Geol. Survey Water-Supply Paper 1779-R, 36 p.
- U.S. Army Corps of Engineers, 1960, Runoff from snowmelt: Eng. Manual 1110-2-1406, 59 p.
- Wisler, C. O., and Brater, E. F., 1959, Hydrology: New York, John Wiley and Sons, Inc., 408 p.



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