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A PREDICTIVE COMPUTER MODEL OF THE LOWER CRETACEOUS AQUIFER, FRANKLIN AREA, SOUTHEASTERN VIRGINIA

Oliver J. Cosner

Geological Survey
Richmond, Virginia

April 1975

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By O. J. Cosner

U. S. GEOLOGICAL SURVEY
WATER-RESOURCES INVESTIGATIONS 51-74

Prepared in cooperation with the
Virginia State Water Control Board

April 1975

UNITED STATES DEPARTMENT OF THE INTERIOR

Rogers C. B. Morton, Secretary

GEOLOGICAL SURVEY

V. E. McKelvey, Director

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U. S. Geological Survey
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Richmond, Virginia 23220

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CONVERSION TABLE

Factors for converting English units to metric units are shown to four significant figures. However, in the text the metric equivalents are shown only to the number of significant figures consistent with the values for the English units.

English	Multiply by	Metric
feet (ft)	0.3048	metres (m)
miles (mi)	1.6090	kilometres (km)
square miles (mi^2)	2.590	square kilometres (km^2)
million gallons per day (Mgal/d)	0.0438	cubic metres per second (m^3/s)
cubic feet per day per foot $(ft^3/d)/ft$	0.0929	cubic metres per day per metre $(m^3/d)/m$
feet per second (ft/s)	0.3048	metres per second (m/s)
per foot (ft^{-1})	3.280	per metre (m^{-1})

A PREDICTIVE COMPUTER MODEL OF THE LOWER CRETACEOUS AQUIFER, FRANKLIN AREA, SOUTHEASTERN VIRGINIA

By Oliver J. Cosner

ABSTRACT

The Lower Cretaceous aquifer of Southeastern Virginia is simulated in this study. The aquifer is only a few feet thick along the Fall Line, where it is near or at the surface, but it thickens and dips to the east. At Franklin where the top of the aquifer is 220 feet (67 metres) below sea level, it is about 600 feet (180 metres) thick. Thirty five miles (56 kilometres) east of Franklin, along the eastern boundary of the model area, the top is about 900 feet (270 metres) below sea level, and the thickness is estimated to be 2,000 feet (610 metres). The aquifer consists of an alternating series of permeable and semipermeable beds, which contain various mixtures of sand, gravel, silt and clay. The sediments are continental stream deposits in the western and central parts of the area, but grade to marine deposits in the eastern part. Transmissivity is zero at or near the Fall Line, and increases eastward to 19,000 cubic feet per day per foot (1,800 cubic metres per day per metre) at Franklin. Further eastward, transmissivity probably increases slightly, but then decreases as the marine phase is reached. The aquifer is overlain by a semipermeable confining layer and is underlain by relatively impermeable rocks of the pre-Cretaceous basement.

The model used is the finite-difference digital type described by Pinder (1970). Historical water levels of the aquifer were simulated from 1891 to December 1, 1972. The 1891 water-level surface was developed by running a steady-state version of the Pinder model. Simulation runs from 1891 to 1941, 1970, and 1972 are in good agreement with historical water levels. A modeling factor was used as a multiplier for varying the coefficients of transmissivity and vertical permeability of the confining layer without altering the initial (1891) surface used to start the modeling runs. This technique should be helpful in the development of other models of artesian aquifers.

Predicitive runs show that, if pumpage continues to increase as it has in the past, serious dewatering of the aquifer will occur at Franklin and possibly at other centers of increased pumpage. However, a predictive run to the year 2022 shows that if pumpage does not increase, additional drawdown would be small. Other predictive runs show the effects caused by hypothetical withdrawals at other locations.

INTRODUCTION

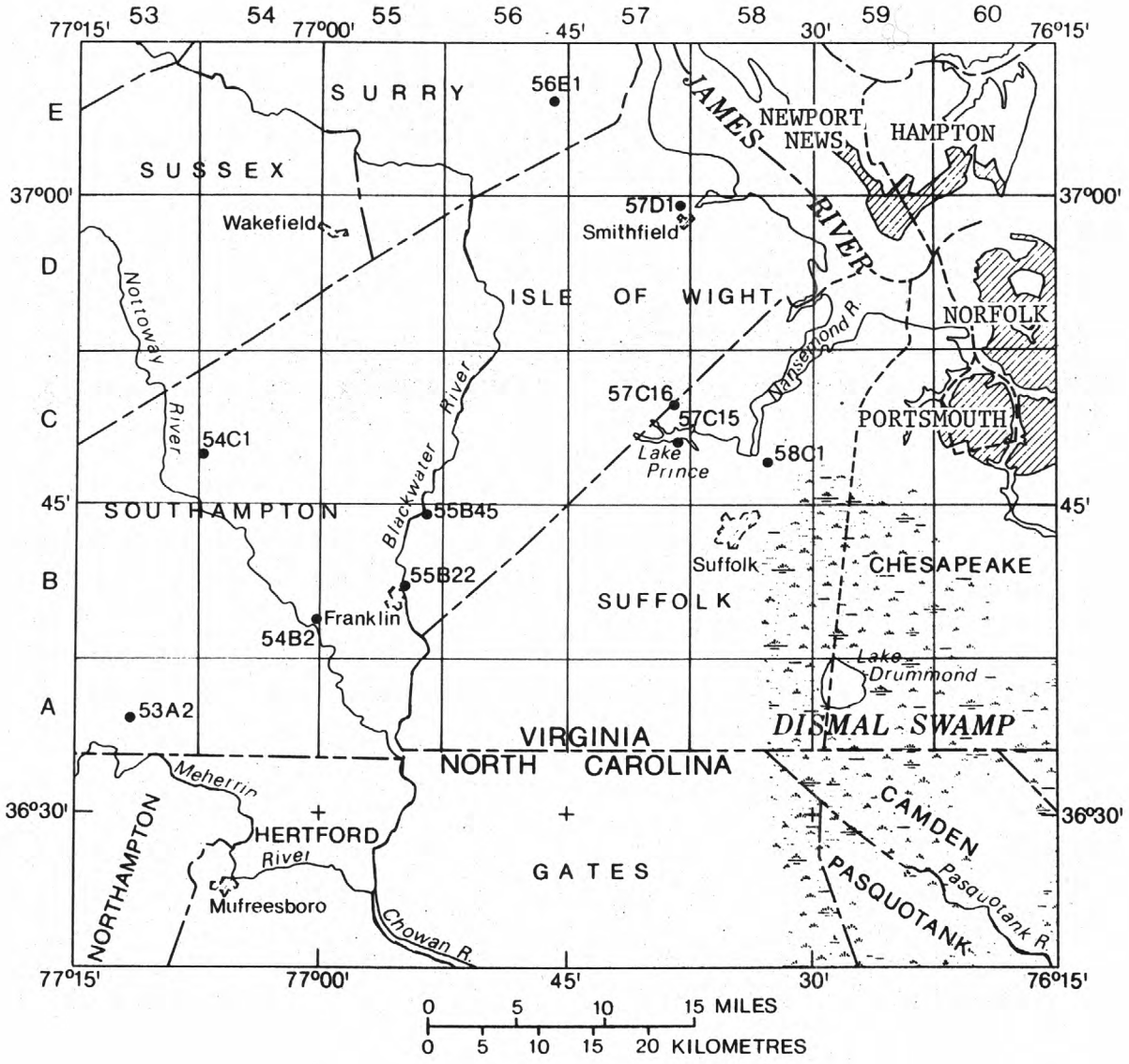
Purpose and Scope

The Lower Cretaceous aquifer is the most extensive and most productive aquifer in the Virginia Coastal Plain. The largest withdrawals of ground water from this aquifer occur in the Franklin area of southeastern Virginia. The pumpage at Franklin started about 30 years ago and has increased steadily, thereby creating a cone of depression which exceeds 5,000 mi^2 (13,000 km^2). Because there has been a significant lowering of water levels at Franklin and vicinity a tool is needed for evaluating and predicting future water levels. To fulfill this need, the creation of a calibrated digital model of the Lower Cretaceous aquifer was begun. The ultimate aim of the project is the modeling of the most important aquifers of the entire Virginia Coastal Plain. This report describes the first step of the overall plan, the digital model of the Franklin area.

The first step in building the model was to gather available historical water-level and pumpage data and analyze the hydrologic parameters of the Lower Cretaceous aquifer of the Virginia Coastal Plain. Most of the available data were for the area south of the James River, especially the Franklin area as described by Brown and Cosner (1974). This information was used to synthesize the data sets for the digital model of the Franklin area.

Well-Numbering System

The well-numbering system used by the U. S. Geological Survey in Virginia is based on the "Index to Topographic Maps of Virginia". The 7 1/2-minute quadrangles are identified by numbers and letters starting in the southwest corner of the State. The quadrangles are numbered 1 through 69 from west to east beginning at longitude 83°45' and are lettered A through Z (omitting letters I and O) from south to north, beginning at latitude 36°30'. These numbers and letters are shown in the top and left margins of figure 1. Wells are numbered serially within each 7 1/2-minute quadrangle. For example, well 54B2 is in quadrangle 54B and is the second well inventoried in that quadrangle.



EXPLANATION

55B22
●
Observation well and well number

54C
Numbers and letters in margin refer to well-numbering system

Figure 1.--Lower Cretaceous aquifer model area with well locations and well-numbering reference system.

Cooperation and Acknowledgments

This investigation was made by the U. S. Geological Survey in cooperation with the Virginia State Water Control Board, E. T. Jensen, Jr., Executive Secretary.

Acknowledgment is given to the cooperation of officials of Union Camp Corp., Hercules, Inc., and the cities of Franklin and Norfolk for furnishing information on ground-water pumpage and water levels. The author wishes to thank the following well-drilling companies for their help: Layne Atlantic Co., R. L. Magette Co., and special thanks goes to Clarence P. Pittman of Pittman Wood and Metal Products Co., for his efforts in supplying well locations and well logs.

The author expresses thanks to Charles Martin and Susan Epps of the State Water Control Board for their cooperation and indispensable help in the compilation of data and the calibration of the model. The author also wishes to thank the personnel of the College of William and Mary Computer Center and the personnel of Virginia Commonwealth University Computer Center for their cheerful and repeatedly rendered help.

Location and Extent of the Area

The Franklin model area as shown on figure 2 is somewhat larger than the Franklin study area of Brown and Cosner (1974). The boundaries of the extended model area shown in figure 3 are far beyond the area shown in figure 2 to avoid the effects of unrealistic hydrologic boundary conditions. However, there is no intent to extend credibility beyond the area shown in figure 2.

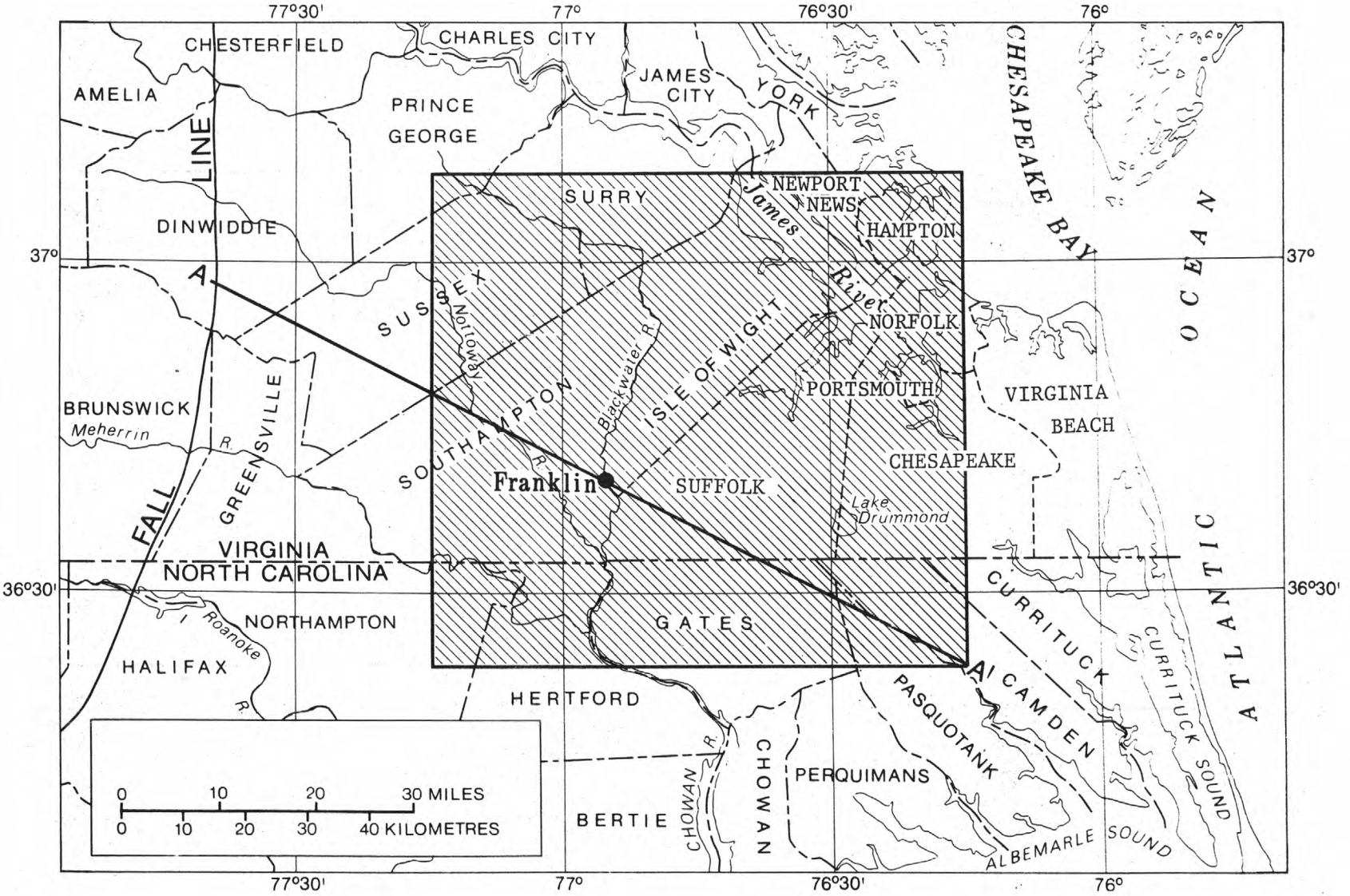


Figure 2.--Location map of the Lower Cretaceous aquifer model area indicated by shading. (The author considers this the credible area of the Franklin model.)

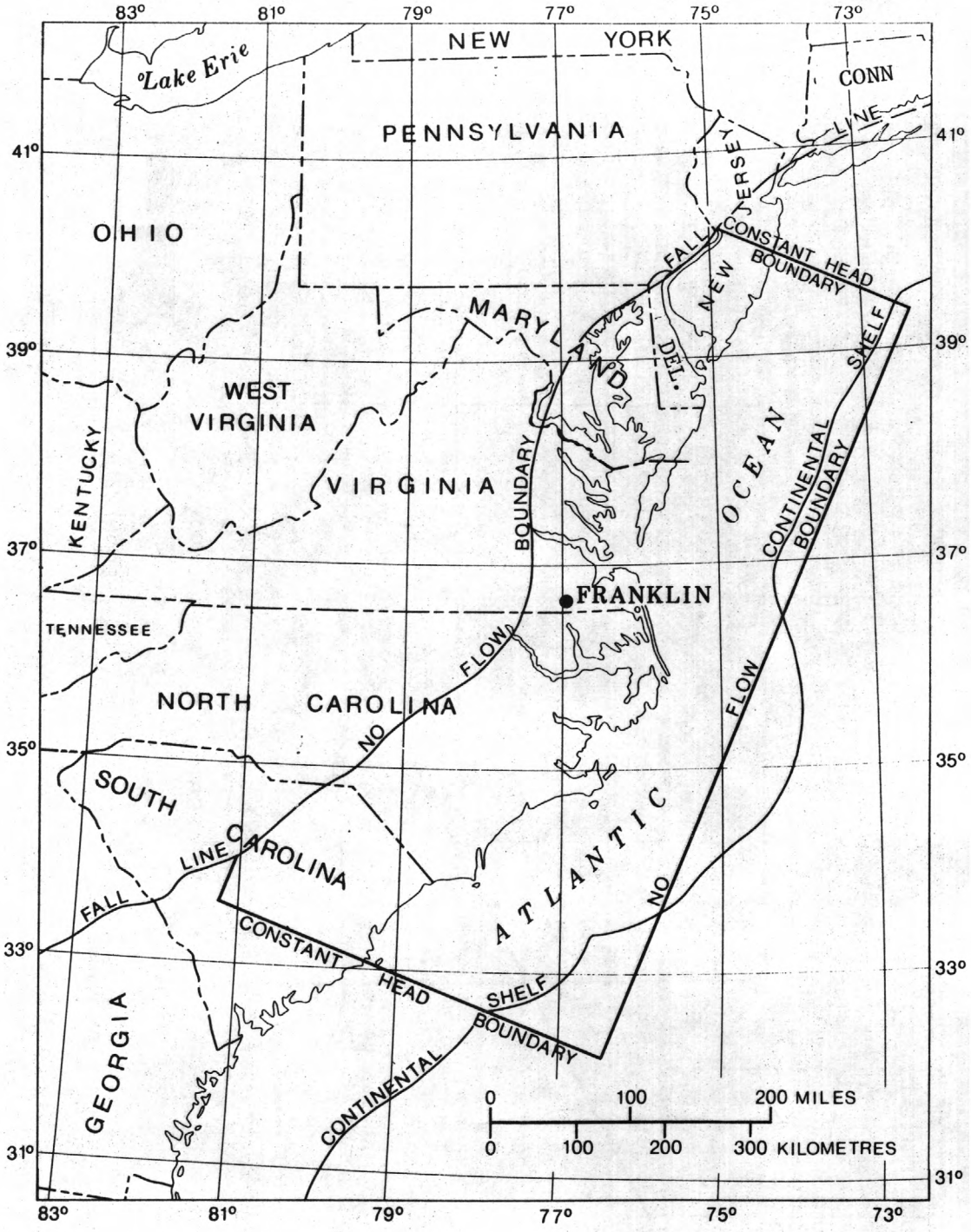


Figure 3.--Map of the east central United States showing the extended Lower Cretaceous aquifer model area.

GEOLOGIC SETTING

The Franklin model area is entirely within the Coastal Plain of Virginia and North Carolina and is underlain by unconsolidated sediments, which, in turn, rest on a massive body of hard rock called the basement complex. In this report, rocks of Triassic age or older are considered to be basement.

The names, age relationships, and general nature of the sequence of rock of the Coastal Plain are given in table 1. The diagrammatic section A-A', figure 4, (the location of section A-A' is shown on figure 2) illustrates how the Lower Cretaceous sediments, which constitute the principal aquifer, are confined below by the basement complex and above by the younger sediments, collectively called "post-Lower Cretaceous confining beds". The section also illustrates how the basement complex, at land surface in the Piedmont province extends beneath the sediments in the Coastal Plain.

POST-LOWER CRETACEOUS AQUIFERS

There are five aquifer systems in the Franklin model area (table 1). Individual aquifers are separated by beds of fine-grained sediments that act as confining layers. The water in the Pleistocene sand, the shallowest aquifer, is generally considered to be under water-table conditions, whereas that in the other aquifers is confined under artesian pressure. The deep aquifer, the Lower Cretaceous, is the major water-bearing unit and is the aquifer modeled in this study.

LOWER CRETACEOUS AQUIFER

Geology

The Lower Cretaceous deposits are composed of interbedded clay, sandy clay, and sand. Plant remains and variegated clay are common. Individual sand and clay beds are commonly not more than 40 ft (12 m) thick but may be as thick as 100 ft (30 m). The thick sand beds are the major water-bearing units.

Table 1.--Generalized lithologic and water-bearing properties of the geologic units (after Brown and Cosner, 1974).

System	Series and aquifer	Generalized lithology	Water-bearing properties
Quaternary	Holocene and Pleistocene	Surficial terrace deposits consisting of sand and clay. Alluvial deposits along streams and swamps.	Yields water for domestic and small industrial wells in Pleistocene terrace sand beds. Water is soft and low in mineral content, but may contain excessive iron (more than 0.3 mg/l).
Tertiary	Miocene	Blue and gray clay; sandy clay containing abundant shells; minor sand beds, mostly in lower part.	A minor aquifer in Surry and Isle of Wight Counties. Generally yields little water because of low permeability. Sand beds not persistent; yield poor and variable, commonly less than 10 gpm. Water is a hard calcium bicarbonate type; objectionable iron often present.
	Eocene and Paleocene	Glauconitic quartz sand and glauconitic clay to clayey sand; indurated shell and limestone layers common; subordinate blue or gray silty clay, with little glauconite; coarse sand and gravel often present at base.	Sand and limestone beds yield moderate supplies to wells, mostly in western part of area. Reported yields as much as 200 gpm. Water is generally a soft sodium bicarbonate type, generally of good quality; may have objectionable odor and color.
Cretaceous	Upper Cretaceous	Alternating beds of fine to medium quartz sand, silt, and silty or sandy clay; slightly glauconitic; a thick red to brown or gray clay locally; coarse sand and gravel beds sometimes present.	Important aquifer for domestic wells and small municipal and industrial wells. Reported yields as much as 300 gpm. Water is a soft sodium bicarbonate type and of good quality.
	Lower Cretaceous	Interbedded arkosic sand, clay, and sandy clay. Sand, white to gray, fine to coarse gravel; beds 30–40 feet thick common, locally as much as 100 feet. Clay beds are of many different colors and become more compact with depth. Individual beds vary in thickness and composition laterally and may pinch out in short distances. Plant remains common.	Major artesian aquifer for municipal and industrial supplies. Reported yields from single sand beds as much as 700 gpm. Multiscreened wells, as at Franklin, yield 2,500 gpm or more. Water is a very soft sodium bicarbonate type and excellent for most purposes; unsuitable for some industries because of high sodium and bicarbonate; high in chloride locally in southeast corner of area.
Precambrian, Paleozoic, and Triassic		Basement complex of Precambrian and Paleozoic igneous and metamorphic rocks and Triassic indurated sedimentary rocks; includes granite, andesite, gneiss, red and green shale.	Not used as aquifer in area. Water in the fractured and weathered zone. Serves as bottom confining layer for Lower Cretaceous aquifer.

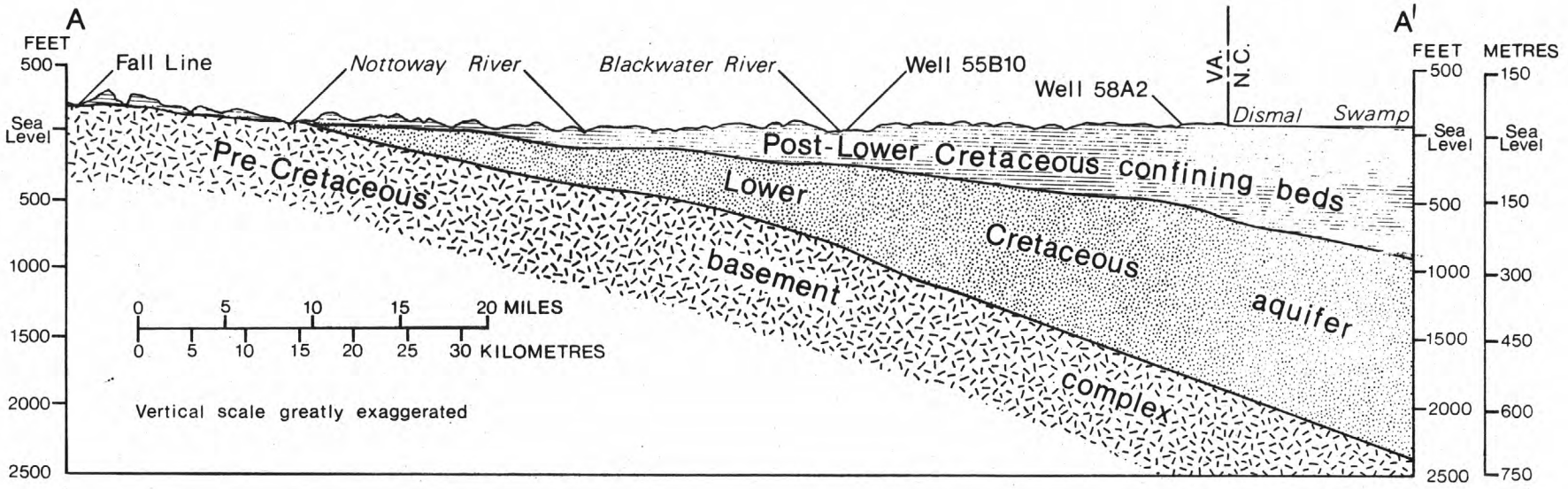


Figure 4.--Diagrammatic section A-A' of the Coastal Plain of Virginia. See figure 2 for location. Modified from Brown and Cosner (1974).

The sand is predominantly quartz, with lesser amounts of feldspar and small amounts of mica, and varies in texture from fine to coarse, with gravel occurring locally. Interstitial silt and clay are generally present. In some areas more than half the total thickness of the Lower Cretaceous sediments is water-bearing sand.

The thickness and the composition of the individual beds vary considerably over short distances. The absence of reliable horizon markers makes it difficult to correlate beds. The poor lateral continuity of these is due to their origin, which has been attributed by Cederstrom (1945, p. 21-25) to deltaic deposition. Sand was deposited in stream channels, while sand, silt and clay were deposited as natural levees and other overbank deposits in interchannel marshes, lakes, and bays. Thus, most of the sand beds are lenses, no more than a few tenths of a mile long, that grade laterally into fine-grained sediments.

The Lower Cretaceous deposits form a wedge that deepens and thickens from west to east (fig. 4). The thickness at Franklin is about 600 ft (180 m); at Norfolk about 2,000 ft (610 m).

Source of Ground Water

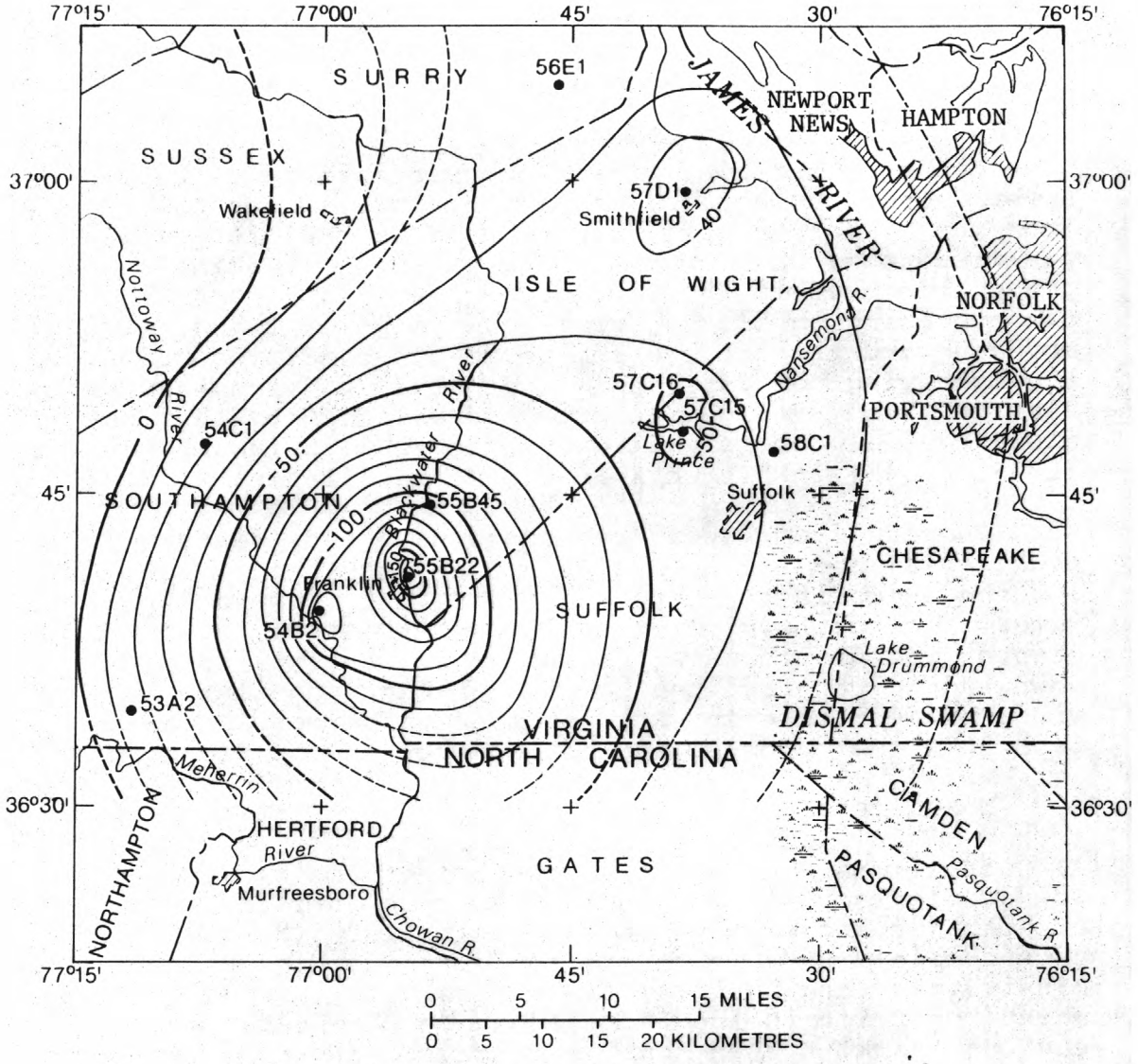
Precipitation is the source of all fresh ground water in the Coastal Plain. A part of the precipitation that soaks into the ground recharges the aquifers. A small amount of the recharge to the Lower Cretaceous aquifer occurs directly on the outcrops along the Fall Line. But, as these outcrops are of small areal extent, most of the recharge is by downward percolation from shallower ground water bodies that overlie the Lower Cretaceous aquifer. Water from upper aquifers seeping downward through confining layers into the Lower Cretaceous represents the largest source of recharge. Lateral flow through the weathered and fractured zone of the basement rocks underlying the Lower Cretaceous aquifer may contribute a small amount of recharge but is considered to be insignificant compared to leakage from the upper aquifers.

Movement of Ground Water

The prewithdrawal potentiometric surface or head of the Lower Cretaceous aquifer had a continuously decreasing gradient from the Fall Line eastward to the sea. Before ground water was withdrawn, water moved downgradient to a point at which the static head in the aquifer became higher than the head in the overlying aquifers. From this point eastward, water also moved upward through the semipermeable confining beds into the overlying aquifers and was eventually discharged at land surface. The average recharge was balanced by this natural discharge, a process called dynamic equilibrium by Hantush (1955, p. 45) in his work in the Roswell Basin, N. Mex.

Pumping from Lower Cretaceous sand beds disturbed dynamic equilibrium and in the Franklin area, the head was lowered below the heads in the upper aquifers. The upward movement of water from the Lower Cretaceous beds to overlying aquifers ceased, and ground water of the upper aquifers began moving downward to recharge the Lower Cretaceous aquifer. This downward movement of water through the semipermeable confining layers is the major source of recharge in the Franklin area.

The lateral movement of ground water within the aquifer is now (1974) toward Franklin and other pumping centers from all directions. As shown by the potentiometric map, figure 5, the gradient of the 1970 potentiometric surface is gentle east of Franklin and steeper west of Franklin. This difference is caused mainly by an eastward increase in thickness and transmissivity of the aquifer. Transmissivity increases in approximate proportion to the increase in thickness for a few miles east of Franklin then remains fairly constant for about 20 to 30 mi (32 to 48 km) to the east, to the vicinity of Suffolk, Va. From Suffolk eastward the transmissivity probably gradually decreases to the edge of the Continental Shelf because of the change in character of the Lower Cretaceous sediments. It is probable that a complete gradation from very coarse terrestial sediments to fine grained marine sediments occurs within the Lower Cretaceous from the Fall Line to the Continental Shelf.



EXPLANATION

55B22
Observation well and well number

-50
Water-level contour
Shows altitude of water level. Dashed where approximately located. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 5.--Measured water-level surface in the Lower Cretaceous aquifer for October 1970. Modified from Cosner and Brown (1972).

Transmissivity of the Aquifer

The coefficient of transmissivity is defined as the rate at which water is transmitted through a unit width of the aquifer under a unit hydraulic gradient. The variation in the character of the Lower Cretaceous sediments restricts application of a value of transmissivity determined from an aquifer test to the area of the test site. In addition, the standard methods used to determine transmissivity assume that the aquifer is of uniform thickness and uniform permeability in all directions and that wells penetrate the entire thickness of the aquifer. As none of these conditions can be wholly satisfied, any determined value of transmissivity is at best approximate.

The following is a summary of transmissivity values taken from Brown and Cosner (1974). The authors determined transmissivity by several methods. The circumference method developed by D. O. Gregg (oral commun., June 1972) was used to compute a transmissivity value from the spacing of the contours on the potentiometric map for December 1971. The value computed by this method for the area between the -50- and -70-ft (-15- and -21-m) contours around Franklin is about 19,000 (ft^3/d)/ft [1,800 (m^3/d)/m] and this is probably a good average value for the Franklin area. Estimates of transmissivity were calculated using the slope of the 1971 potentiometric surface. The lowest of these estimates is 6,000 (ft^3/d)/ft [560 (m^3/d)/m] in the southwest corner of the map area (fig. 2). Next greater is 15,000 (ft^3/d)/ft) [1,400 (m^3/d)/m] 10 mi (16 km) west of Franklin. The highest estimate calculated by the slope method is 24,000 (ft^3/d)/ft [2,200 (m^3/d)/m] for the area from 4 miles east of Franklin to the eastern boundary of the area. This estimate agrees well with Geraghty and Miller's (1967) values for the Norfolk city well field at Lake Prince in Nansemond County. The author concludes that the value of transmissivity for the Lower Cretaceous aquifer reaches a maximum a few miles east of Franklin and remains relatively constant to the vicinity of Suffolk; then it decreases eastward as the finer and finer marine facies of the aquifer occur.

Transmissivity values used in the model are in the lowest range of the values obtained at the various locations in the above discussion. Attempts at calibrating the model with the higher valued indicated these values were too high. A transmissivity at Franklin of 19,000 (ft^3/d)/ft [1,800 (m^3/d)/m] gives the best results when calibrating the model. This value also agrees with the value determined by the circumference method at Franklin by Brown and Cosner (1974). Figure 6 shows the transmissivity values included in the model along the diagrammatic section A-A'.

Storage Coefficient of the Aquifer

Few if any valid determinations of storage coefficient have been made for the Lower Cretaceous aquifer in Virginia. The values for storage coefficient used at Franklin in the model were selected from the lowest values obtianed from aquifer tests. Figure 6 shows the storage coefficient values (S) included in the model along the diagrammatic section A-A'.

Discharge of Ground Water

Withdrawal of water from the Lower Cretaceous aquifer began in the late 1800's. The artesian head was sufficient to allow many homes, farms, and industries to have flowing wells. The hydraulic ram was commonplace. The city of Franklin, which for many years depended on the Blackwater River for its water supply, switched to ground water about 1927 and had two flowing wells. Flowing wells continued to be popular, but, as more and more water was withdrawn from the Lower Cretaceous for industrial and municipal supplies, the artesian head in the aquifer declined, and so did the number of flowing wells. By the late 1940's nearly all wells tapping the Lower Cretaceous aquifer had ceased flowing.

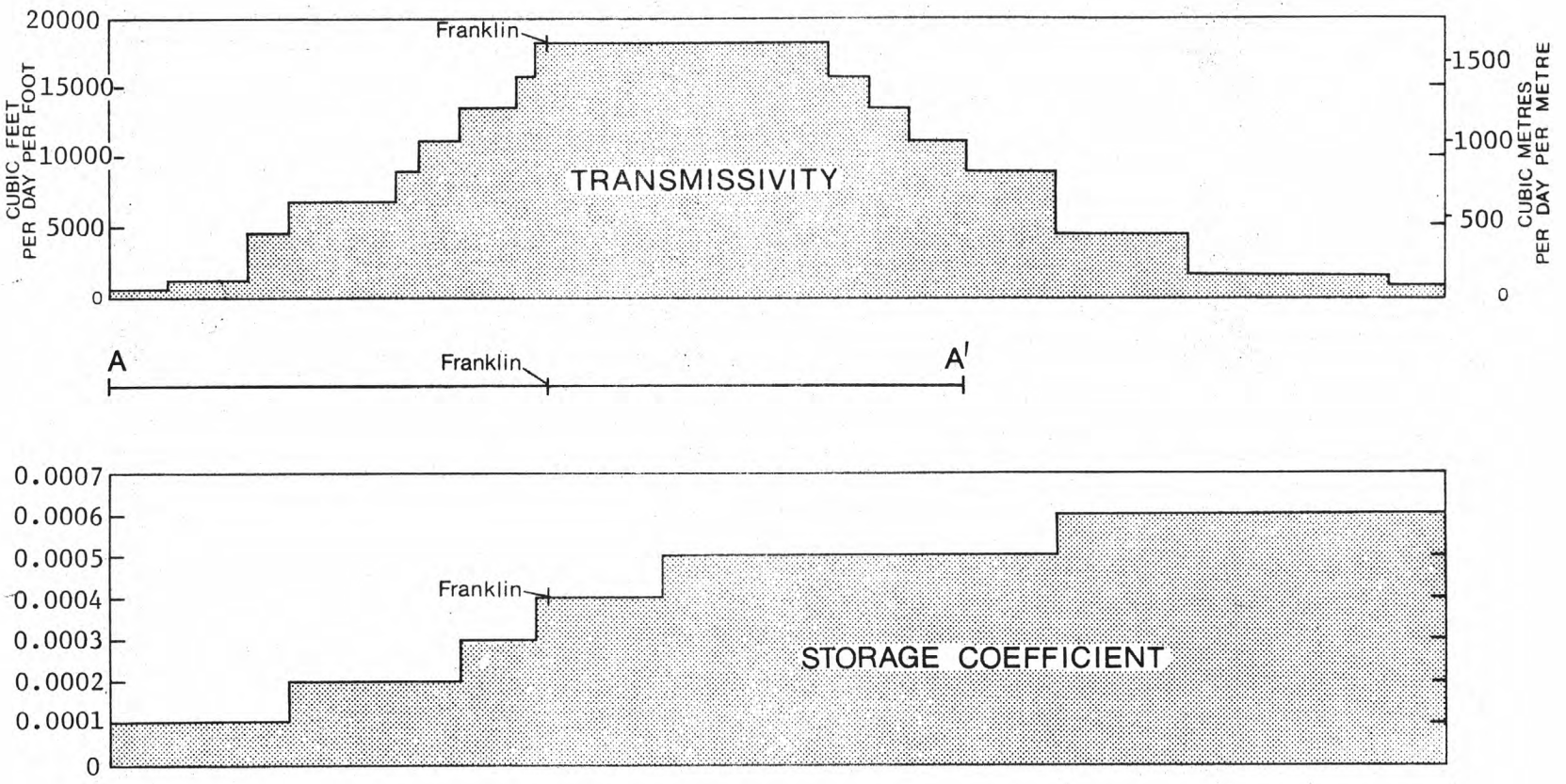


Figure 6.--Transmissivity and storage coefficient along section A-A', extended to Continental Shelf, through the City of Franklin, parallel to the northwest lines of the Lower Cretaceous model grid. See figure 2 for location.

Originally, water from the Lower Cretaceous aquifer discharged upward by leakage through the overlying confining beds; now water moves downward through the overlying confining beds and recharges the aquifer. All discharge from the Lower Cretaceous in the Franklin area is now through wells. At the end of 1971, approximately 47 Mgal/d (2.1 m^3/s) of this was pumped from wells in the Lower Cretaceous aquifer in the vicinity of Franklin About 45 Mgal/d (2.0 m /s) of this was pumped from large-yield industrial and municipal wells within a 5-mi (8 km) radius of the center of the city. Within the model area, but more remote from Franklin, an additional 23 Mgal/d (1.0 m^3/s) is pumped from the aquifer. Figures 7 and 8 show graphically the major historical pumpage from the Lower Cretaceous aquifer as entered in the model. Table 2 lists all pumpage entered in the model for historical runs.

Head in the Aquifer

Water-level measurements made during 1937-39 by Cederstrom (1945), (fig. 9), indicated that a small cone of depression had formed around Franklin. At the center of the cone, water levels were slightly lower than 20 ft (6 m) above sea level.

Figure 5 shows the configuration of the potentiometric surface in December 1970, when water levels at the center of the cone were 165 ft (50 m) below sea level. Figure 10 shows that declines of the potentiometric surface ranging from 30 to 140 ft (9 to 43 m) have occurred in the Franklin area between 1937-39 and December 1970. Decline in a small area at the center of the cone actually reached 185 ft (56 m) but is not shown on figure 10 because of scaling constraints.

During 1971, industrial withdrawals at Franklin were reduced by 15 percent. This resulted in stabilized water levels at the center of the cone of depression, although levels in wells elsewhere in the model area continue to decline. If pumpage is stabilized, outlying water levels will probably decline at decreasing rates, and the potentiometric surface will eventually reach equilibrium. Discharge through wells will be balanced by downward leakage and by recharge from the Fall Line. If pumpage increases significantly and the water level at the center of the cone declines below the top of the Lower Cretaceous aquifer (220 ft or 67 m below sea level), dewatering of the aquifer will occur.

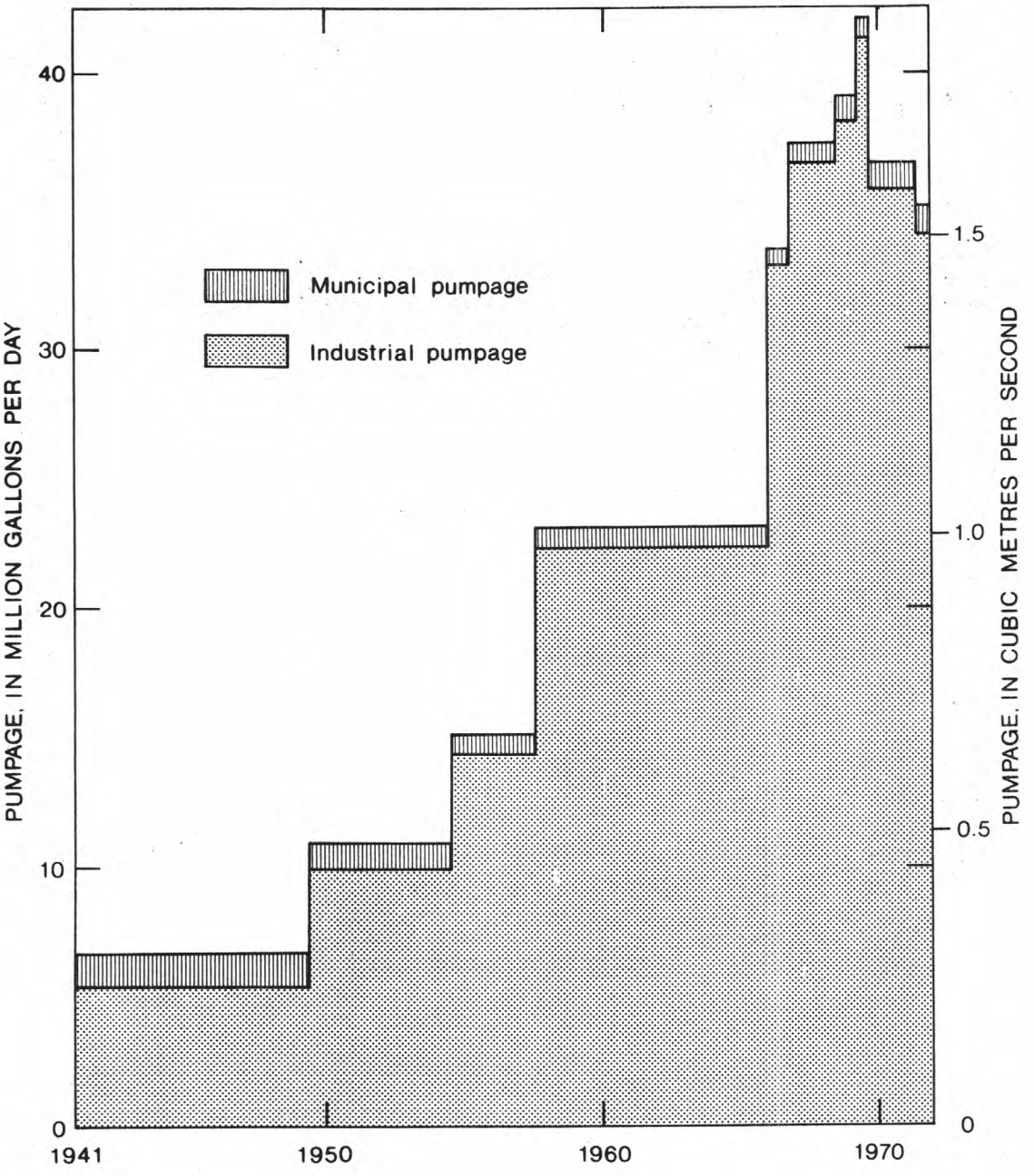


Figure 7.--Historical municipal and industrial pumpage at Franklin, Virginia.

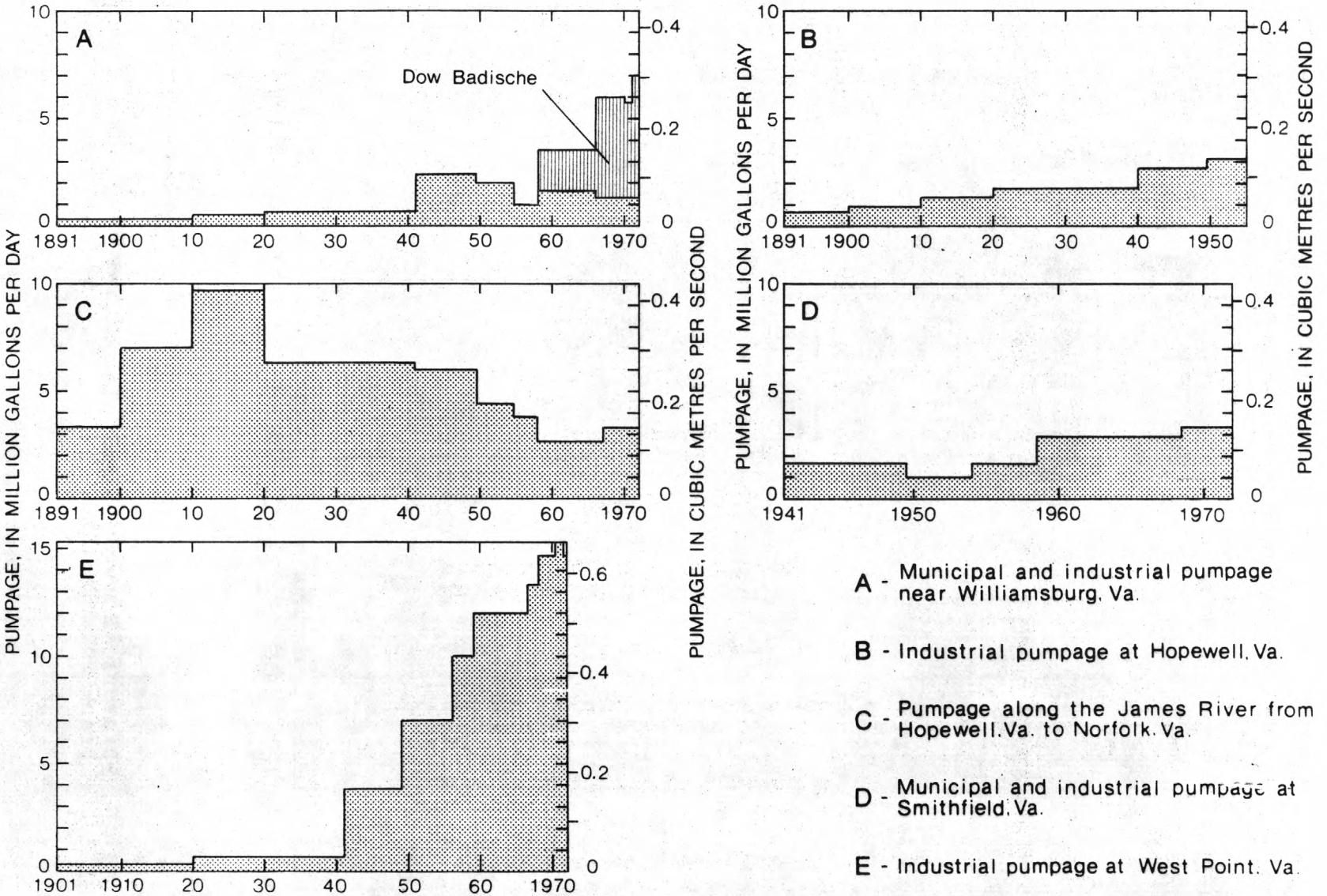
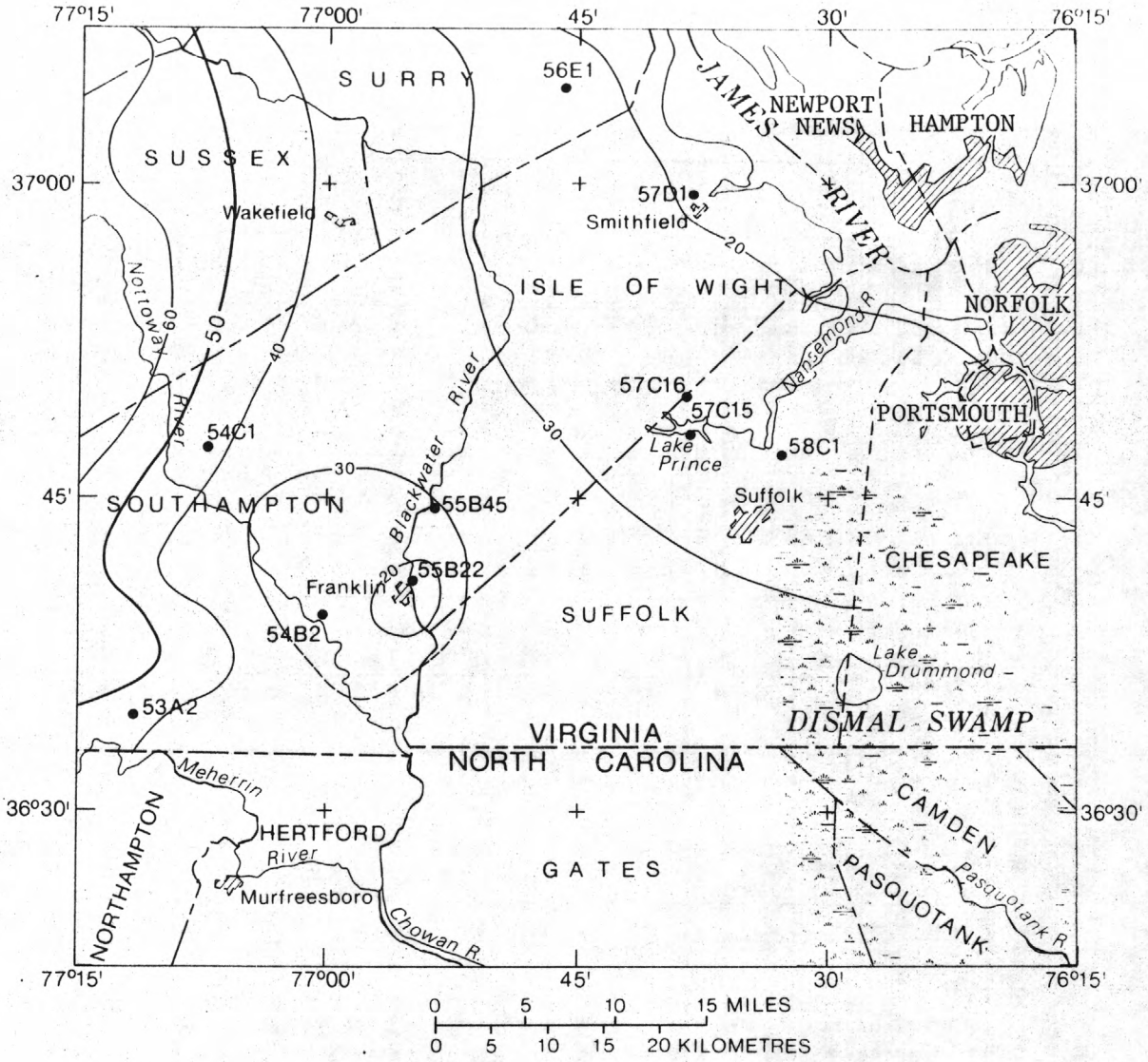


Figure 8.--Historical pumpage of the peripheral industries and municipalities of the model area.

Table 2.--Prorated withdrawal rates, in cubic feet per second, for pumping periods used in the Lower Cretaceous aquifer model.

Pumping Center		Model Run 1				Model Run 2								Model Run 3		
		January 1891 to October 1941				October 1941 to September 1970								September 1970 to December 1972		
		I	II	III	IV	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
	From	1-1 1891	12-30 1900	12-28 1910	1-3 1921	10-19 1941	3-15 1950	3-14 1955	8-10 1958	12-6 1966	4-5 1967	6-3 1969	5-14 1970	9-10 1970	1-30 1971	5-31 1972
	To	12-30 1900	12-28 1910	1-3 1921	10-19 1941	3-15 1950	3-14 1955	8-10 1958	12-6 1966	4-5 1967	6-3 1969	5-14 1970	9-10 1970	1-30 1971	5-31 1972	12-1 1972
City of Norfolk (Lake Prince wells)													8.86	21.8		
Hercules, Inc.									3.14	3.07	4.35	3.9	5.44	5.21	6.24	6.7
Wakefield						0.16	0.16	0.16	.16	.16	.16	.16	.16	.16	.15	.16
Waverly						.23	.23	.23	.23	.23	.23	.23	.23	.23	.23	.23
Smithfield						2.20	1.52	2.11	4.09	4.09	4.09	5.00	5.00	5.00	5.00	5.00
Chesapeake Corp. West Point			0.39	0.39	0.77	5.77	10.80	15.00	18.00	18.00	20.00	22.00	22.00	22.00	23.00	22.00
Dow-Badische near Williams-burg									2.50	7.00	7.00	7.00	7.00	6.84	6.68	8.59
Adjacent to James River 1/		4.97	10.38	14.96	9.55	9.38	6.77	5.06	3.95	3.95	3.95	4.87	4.87	4.87	4.87	4.87
Hopewell		1.16	1.74	2.32	4.0	4.0	4.32									
Ft. Eustis						2.0										
Branchville		.09	.14	.19	.14	.19	.19									
Boykins						.23	.23	.23	.23	.69	.69	.69	.69	.69	.69	.69
Courtland		.26	.39	.51	.75	.50	.40	.23	.23	.23		.23	.23	.23	.23	.23
Franklin		.37	.56	.74	2.60	1.95	1.50	1.00	1.05	1.05	1.17	1.30	1.64	1.64	1.64	1.64
Williamsburg		.46	.46	.62	.78	3.42	.31	1.54	2.60	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Camp Peary						3.10	2.80									
Union Camp Corp. Franklin					8.47	8.47	15.36	22.50	34.80	51.20	57.60	60.05	64.96	56.08	56.08	53.20

1/ Adjacent-to-James-River pumpage is estimated from Cederstrom (1945, 1957).

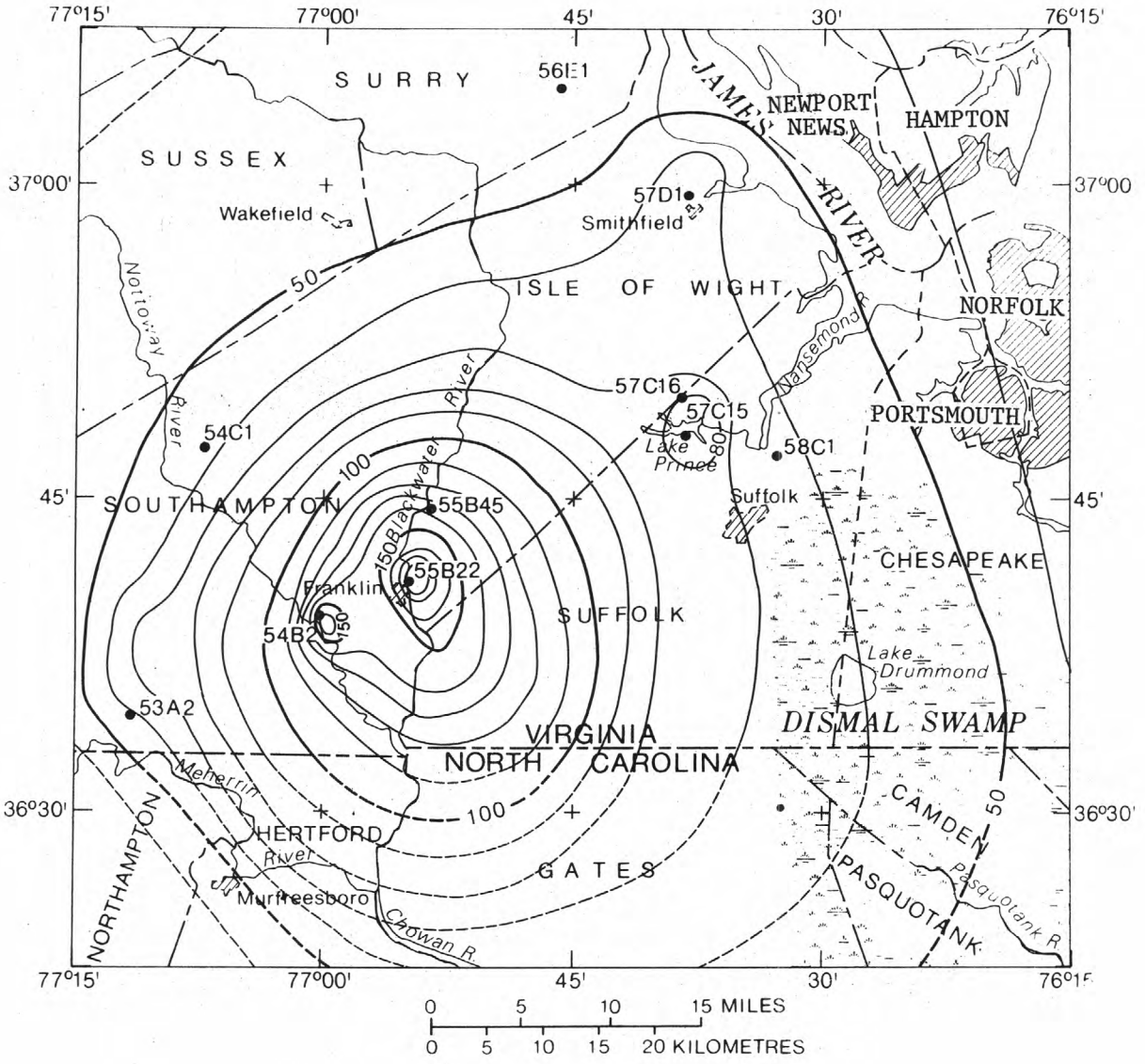


EXPLANATION

55B22
Observation well and well number

50
Water-level contour
Shows altitude of water level. Dashed where approximately located. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 9.--Measured water-level surface in the Lower Cretaceous aquifer for 1937-39. Modified from Cederstrom (1945).



EXPLANATION

55B22
Observation well and well number

50
Line of equal water-level decline
Dashed where approximately located. Interval 10 feet (3 metres).

Figure 10.--Water-level decline in the Lower Cretaceous aquifer computed from measured surfaces for period 1937-39 to October 1970.

THE AQUIFER MODEL

The iterative digital model for aquifer evaluation designed by Pinder (1970) was adopted for use in this modeling project. The model is an idealized mathematical representation of the main physical aspects of the Lower Cretaceous aquifer system in the Franklin area. The aquifer characteristics were determined by drilling and by aquifer tests. The model simulates historical conditions and predicts future hydrologic behavior of the aquifer under various hypothetical conditions.

The Pinder model as used in this study had the following features and data sets:

1. A model grid was positioned over the study area as shown in figure 11. Table 3 lists the dimensions of the entire model grid. Note that the dimensions of the rectangles are variable, being large at the extremities and smaller near the center. The center point of a grid rectangle is called a node.

2. The aquifer characteristics and hydrologic data, as listed below, were supplied to the model for each node in the grid:

   a) transmissivity

   b) storage coefficient (zero values in steady state model)

   c) pumping rate at each node where significant discharge from wells is located (zero values in steady state model)

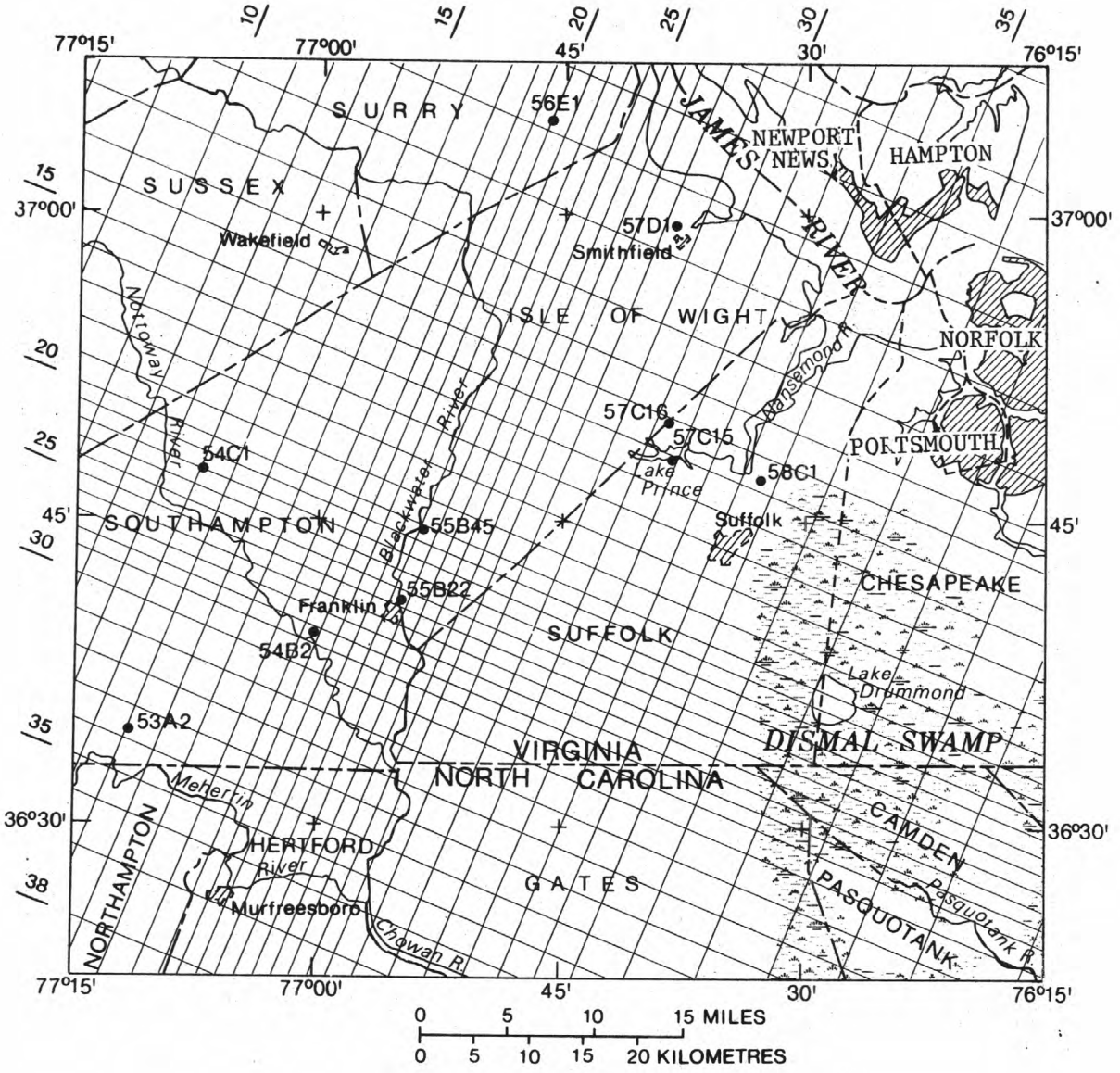
   d) thickness of the leaky confining bed

   e) head in the overlying water-table aquifer

   f) dimensions of the rectangular grid elements of the superposed finite-difference mesh

   g) initial head values in the aquifer.

Parameter cards provide multipliers for nearly all basic parameters and the parameters used in the computational scheme, such as the maximum duration of pumping and the hydraulic conductivity of the confining bed. Computer input and output are in units of feet and seconds.



EXPLANATION

55B22
Observation well and well number

Row or column number for model grid.

Figure 11.--Model grid superposed on Lower Cretaceous aquifer model base map. Note the variable grid sizes.

Table 3.--Grid Dimensions in the Lower Cretaceous Aquifer Model.

Horizontal (x) Direction

Node Number	Distance in feet	Node Number	Distance in feet	Node Number	Distance in feet
1	200,000	16	10,000	31	10,000
2	150,000	17	10,000	32	10,000
3	100,000	18	7,500	33	10,000
4	67,000	19	5,000	34	10,000
5	45,000	20	5,000	35	15,000
6	30,000	21	5,000	36	20,000
7	20,000	22	5,000	37	20,000
8	20,000	23	5,000	38	20,000
9	20,000	24	5,000	39	20,000
10	20,000	25	5,000	40	30,000
11	15,000	26	5,000	41	45,000
12	10,000	27	5,000	42	67,000
13	10,000	28	7,500	43	100,000
14	10,000	29	10,000	44	150,000
15	10,000	30	10,000	45	200,000

Table 3--Continued

Vertical (y) Direction

Node Number	Distance in feet	Node Number	Distance in feet	Node Number	Distance in feet
1	450,000	18	10,000	35	10,000
2	300,000	19	10,000	36	10,000
3	200,000	20	7,500	37	15,000
4	150,000	21	5,000	38	20,000
5	100,000	22	5,000	39	20,000
6	67,000	23	5,000	40	20,000
7	45,000	24	5,000	41	20,000
8	30,000	25	5,000	42	30,000
9	20,000	26	5,000	43	45,000
10	20,000	27	5,000	44	67,000
11	20,000	28	5,000	45	100,000
12	20,000	29	5,000	46	150,000
13	15,000	30	7,500	47	200,000
14	10,000	31	10,000	48	300,000
15	10,000	32	10,000	49	450,000
16	10,000	33	10,000		
17	10,000	34	10,000		

The Lower Cretaceous aquifer model is a representation of a simple aquifer system having the following components;

1. A water-table aquifer with a fixed water-table head for each node.

2. A confining layer whose thickness is specified at each node.

3. The artesian aquifer having a variable head.

4. Specified boundaries which are:

   a. The Fall Line; a "no flow" boundary on the west.

   b. The north and south ends of the model which are considered constant head boundaries.

   c. The continental shelf; a "no flow" boundary on the east.

A "no flow" boundary is one in which no flux (water) enters the system. A constant head boundary allows flux to enter the system in proportion to the hydraulic gradient and transmissivity values.

The runs of the model can be made for any given time period and this period is broken up into shorter periods called time steps. Each succeeding time step is 1.5 times as long as the preceeding time step. The model computes the drawdown at each node for each time step and the results are printed out at selected time steps. On the final time step the potentiometric head value is printed for each node.

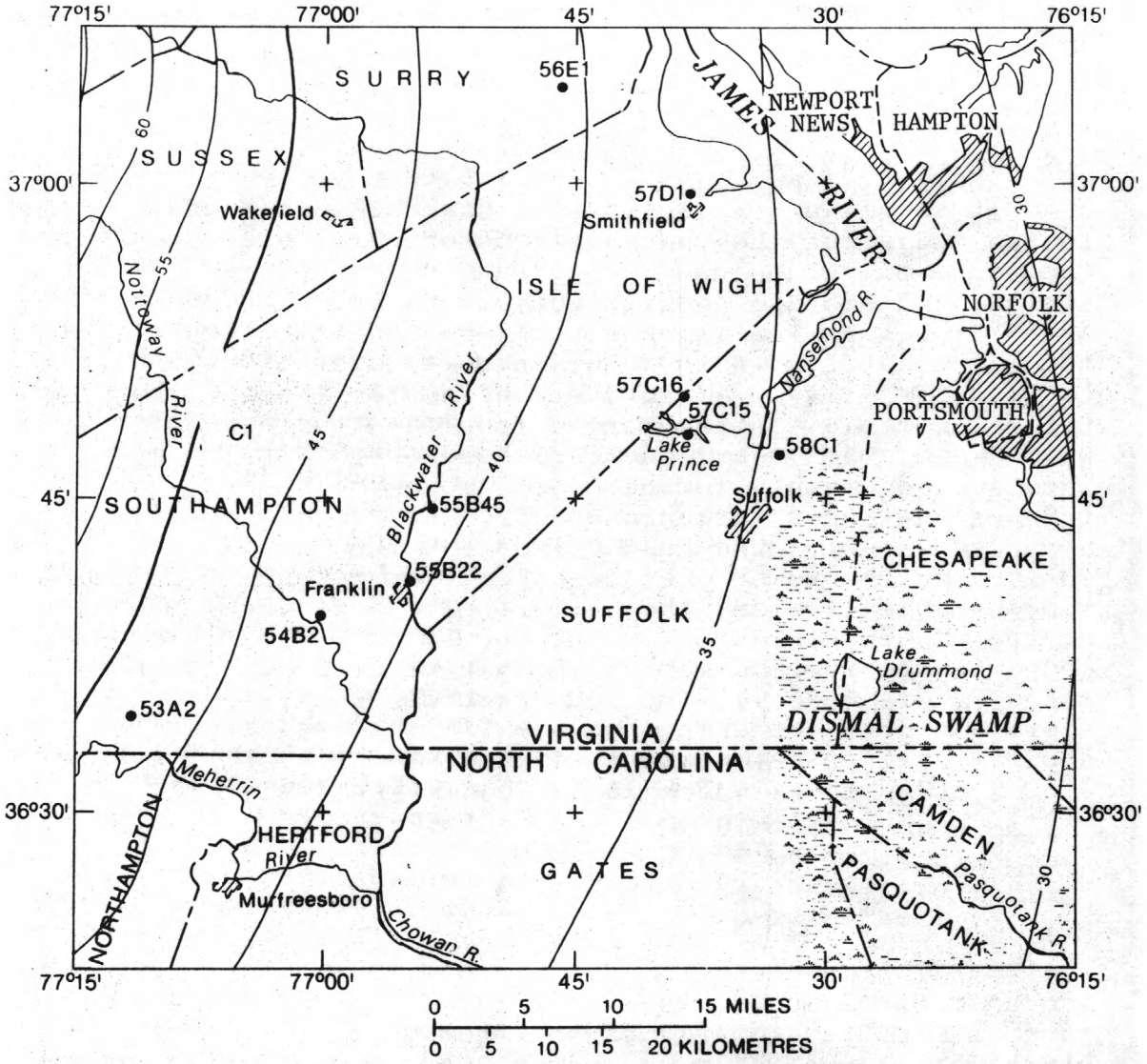
Data Arrays

It is not practical to list here all data arrays entered in the model but the values discussed below are intended to give the range of values used. Figure 6 shows values for transmissivity, and storage coefficient, along the diagrammatic section A-A' (fig. 4). The thickness of the confining layer in the model ranged from 10 ft (3 m) in some nodes along the Fall Line to 3,000 ft (914 m) along the Continental Shelf. The head for the water-table aquifer ranged from land surface in those nodes which were at or near sea level to about 50 ft (15 m) below land surface mainly in the nodes along the Fall Line. The altitude of the water table ranged from sea level to 145 ft (44 m) above sea level. The head values for the initial potentiometric surface of the Lower Cretaceous aquifer ranged from 136 ft (41 m) above sea level along the Fall Line to 6 ft (2 m) above sea level at the Continental Shelf. Another important data set in the model is the pumpage matrix (values of zero pumpage were used in steady state model). The hydraulic conductivity and the specific storage of the confining bed are included as single values which apply to all nodes, [1.05×10^{-9} ft/s (3.2×10^{-10} m/s) and 4.00×10^{-5} ft^{-1} (1.32×10^{-4} m^{-1}) respectively].

The Steady-State Surface

One can only theorize as to the exact configuration of the undisturbed steady-state surface. However, one can get a rough idea of its configuration from pressure heads in the first wells drilled in the Lower Cretaceous aquifer. Many of the first wells were flowing and no attempt was made to determine the true head of the aquifer. All that is known is that the head was above the land surface at the well. As sparse and as general as these data are, when they are coupled with the geohydrologic picture of the aquifer, a hypothetical steady state surface can be conceived. To reproduce the author's conception of this hypothetical surface was the goal of the first modeling attempts.

The geohydrologic information was compiled, placed in the model, and a run to steady state was made with no pumping from the aquifer. The simulated potentiometric surface developed is a representation of a steady-state surface for an aquifer having the characteristics of the one in the model. This surface, shown in figure 12 is a good fit with the author's conceptual steady-state surface.



EXPLANATION

55B22
Observation well and well number

50
Water-level contour
Shows altitude of water level. Contour interval 5 feet (1.5 metres). Datum is mean sea level.

Figure 12.--Map showing simulated predevelopment steady-state surface in the Lower Cretaceous aquifer.

Most undisturbed natural systems are in equilibrium from the standpoint of man's time frame. The potentiometric surface of the Lower Cretaceous aquifer was probably in equilibrium or steady state before pumpage began, and this is the optimum starting surface for modeling. Therefore, the first step in modeling the Lower Cretaceous aquifer in the Franklin area was to develop a steady-state surface.

In developing the steady-state surface it was not necessary to consider the storage coefficient since the amount of water in storage in the aquifer remains constant. Therefore, a special version of the model was used which had a storage coefficient equal to zero. A confining bed representing the entire section between the aquifer and the water table is incorporated in the model at each node. The hydraulic conductivity of this bed is read in as a single value which applies to all nodes. Values of head in the water-table aquifer were assigned for each node on the basis of topography and surface water features. The Fall Line was simulated as a zero flow boundary as was the seaward limit of the aquifer. The north and south boundaries were constant head boundaries. No pumpage was simulated and no transient effects were included in the steady-state simulation. The steady state of the model simulates the natural circulation of water from the water-table aquifer to the Lower Cretaceous aquifer in the recharge area, and from the Lower Cretaceous aquifer to the water-table aquifer in the discharge areas. Under steady-state conditions, as in the nonsteady state or transient state, the difference in head between the water-table and the potentiometric surface in the artesian aquifer is the force driving water through the confining layer.

The T/K' Modeling Factor

The concept of T/K' modeling factor was developed during the early stages of the Franklin area modeling work. The values for T (transmissivity of the aquifer) and K' (hydraulic conductivity of the confining layer) are used in the flow equation of the model in a fractional form: that is, T/K' as described in Appendix A of this report. The values of T and K' can be varied without changing the equilibrium surface as long as the value of the fraction T/K' is kept constant. The potentiometric surface obtained in the equilibrium simulation was compared with the author's conceptual prepumping surface for the Lower Cretacous aquifer and adjustments were made in the model to achieve closer agreement. At first, values for T and for K' were varied independently. Those for T were varied on a regional basis within the limits of factual data until a reasonable agreement between the model results and the conceptual surface was obtained. The results are illustrated in figure 12.

The next steps involved arranging the model for the transient phase; that is, stressing it with historical pumpage. The simulated water-level surfaces obtained in this manner were compared with the surfaces drawn from measured water levels in wells at various times during the historical period of record. To resolve major differences between the model results and the historical data, the transmissivity (T) values throughout the model were changed by a small factor using the transmissivity multiplier. At the same time, the hydraulic conductivity of the confining bed (K') was changed by the same small factor; thus the ratio of the T/K' fraction remained unchanged. The result was close agreement between the simulated transient water-level surfaces and the historical data. The changes in T and K' did not introduce any changes in the simulated steady-state surface. On subsequent trials some selective changes in transmissivity at individual nodes were introduced to further improve agreement with historical data.

After these trials, the steady-state simulation was rerun, using the new values of the revised transmissivity matrix. Only relatively minor changes in the computed steady-state surface were considered acceptable, and in all cases the new steady-state surface was used to start the transient simulation runs. In this manner steady-state and transient surfaces were kept in context with each other and the resulting simulated drawdowns were due to the pumping stress and not due to some imbalance in the initial surfaces. Ultimately both the steady-state surface and the computed drawdowns were brought into acceptable agreement with historical data.

Insensitive Modeling Parameters

In making the first runs it was found that the model was very insensitive to changes in storage coefficient and therefore the original array for storage coefficient was used throughout the runs. The values for storage coefficient along the center row of nodes is shown in figure 6. The thickness of the confining layer was also left unchanged for the period of modeling except in the very early runs.

Calibration of the Franklin Model

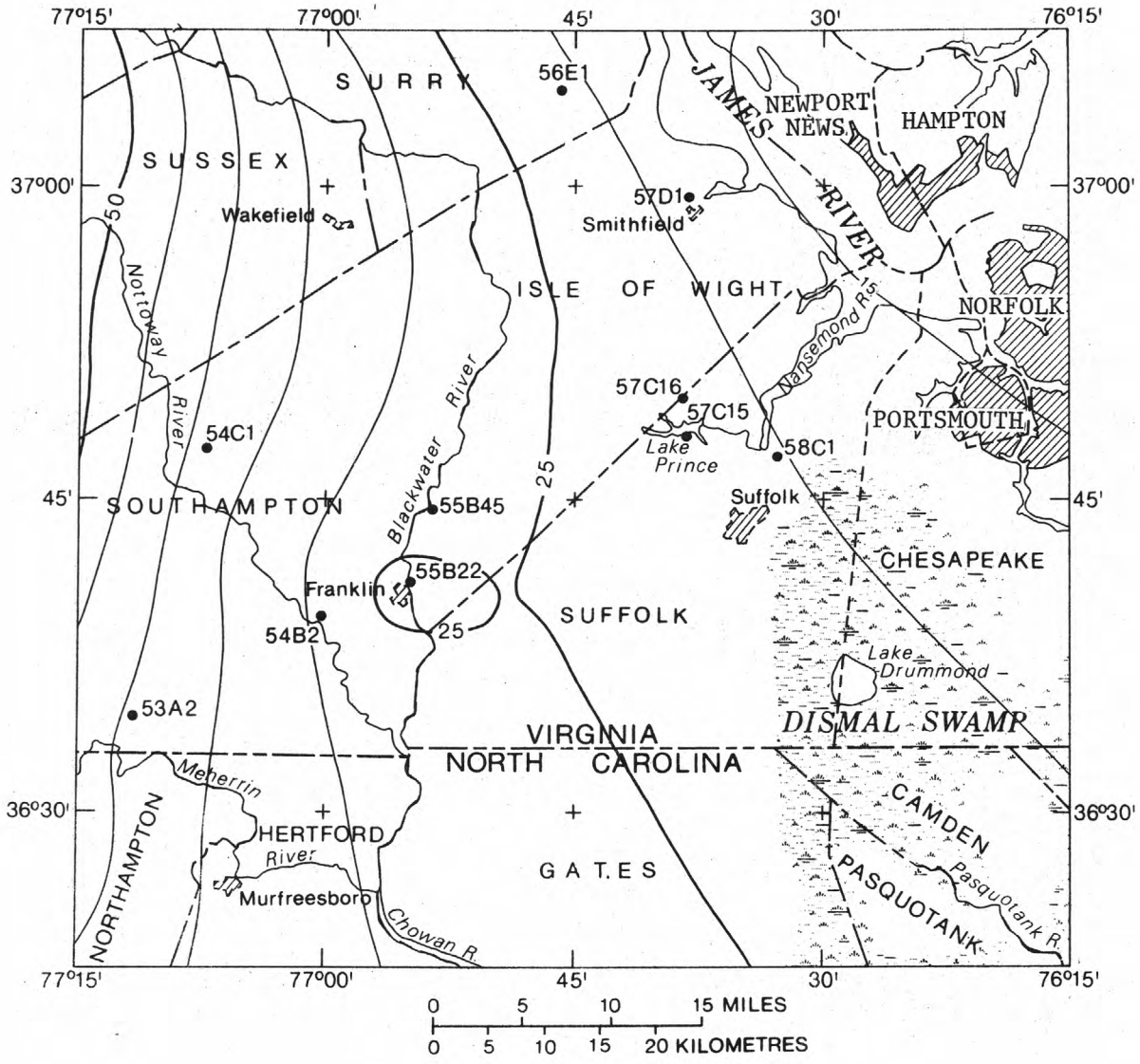
In making the model, the first order of business was to produce an equilibrium surface that met certain criteria. Having obtained this surface, the next step was to apply historical pumpage by use of the pumpage matrix in the model. This was done by making model runs that corresponded to periods of constant pumping. Each major historical increase or decrease in pumpage made it necessary to adjust the pumpage matrix and start a new pumping period. The pumping periods are grouped together in runs, each run corresponding to a significant historical period. Table 2 lists all of the historical pumpage data used in the model, assembled according to pumping periods and model runs. Although 15 pumping periods were used, they do not match exactly the historical pumpage for each site. The pumping periods were selected to correspond with the major pumping changes at the most significant pumping centers. Other pumping rates had to be averaged over these periods, so that the pumpage entered in the model for the lesser pumping centers may not correspond to actual reported pumpage but represents the quantity of water pumped during the pumping period at that site.

There are three model runs (table 2); January 1891 to October 1941, October 1941 to September 1970, and September 1970 to December 1972. At the end of each of these runs, the simulated data were compared with the measured data. This was done by plotting the altitude of the simulated surface at the node points. The model data were converted to a suitable form by a computer program (Cosner and Horwich, 1974) for input to the Geological Survey Calcomp plotter program. The contoured simulated surfaces included in this report were first drawn by the Calcomp plotter at a scale of 1 to 250,000. The measured surfaces were drawn by hand to the same scale. All maps were then reduced and placed on the page size Franklin base. In addition to the simulated and measured maps of the surfaces, decline maps were constructed. By comparing the simulated and measured maps one can judge the reliability of the model for predictive purposes.

Figure 9 is a replica of Cederstrom's 1937-39 map, slightly modified. Because pumpage at Franklin nearly doubled in 1941, the first run was stoped just prior to this increase. Annual declines at Franklin prior to 1941 were only a few tenths of a foot per year. Therefore, the simulated water-level surface for 1941 (fig. 13) is suitable for comparison with Cederstrom's 1937-39 map. The reader should note that figure 9 has a contour interval of 10 ft (3.0 m) and figure 13 has an interval of 5 ft (1.5 m). The agreement between these two surfaces is imprecise and no attempt was made for an exact fit because Cederstrom's map includes water levels from aquifers above the Lower Cretaceous aquifer. However, there is a good general agreement between the simulated water levels and water levels in the deepest wells measured by Cederstrom (1945).

Figure 14 is the simulated surface for October 1970 and figure 5 is the measured surface for that date. There is good agreement between the two maps. Comparing the simulated and measured decline maps for the periods 1941 to 1970 (fig. 15), and 1937-39 to 1970 (fig. 10), there is also close agreement. The measured water-level surface (fig. 16) and the simulated surface (fig. 14) for December 1972 and the computed and simulated decline maps for the period October 1970 to December 1972 (figs. 18 and 19) also show close agreement. The agreement between historical water levels and the simulated water levels from the model is probably best illustrated by figures 20 and 21, which show the measured and simulated hydrographs of three observation wells.

The agreement of the simulated and measured water levels is illustrated by these maps and hydrographs constitutes the verification of the Franklin model and suggests that the parameters used in the model for the various physical properties of the aquifer are good effective values. The model does not exactly duplicate the actual hydrologic system. The parameters in the model can only be considered effective values rather than duplicates of natural parameter values.

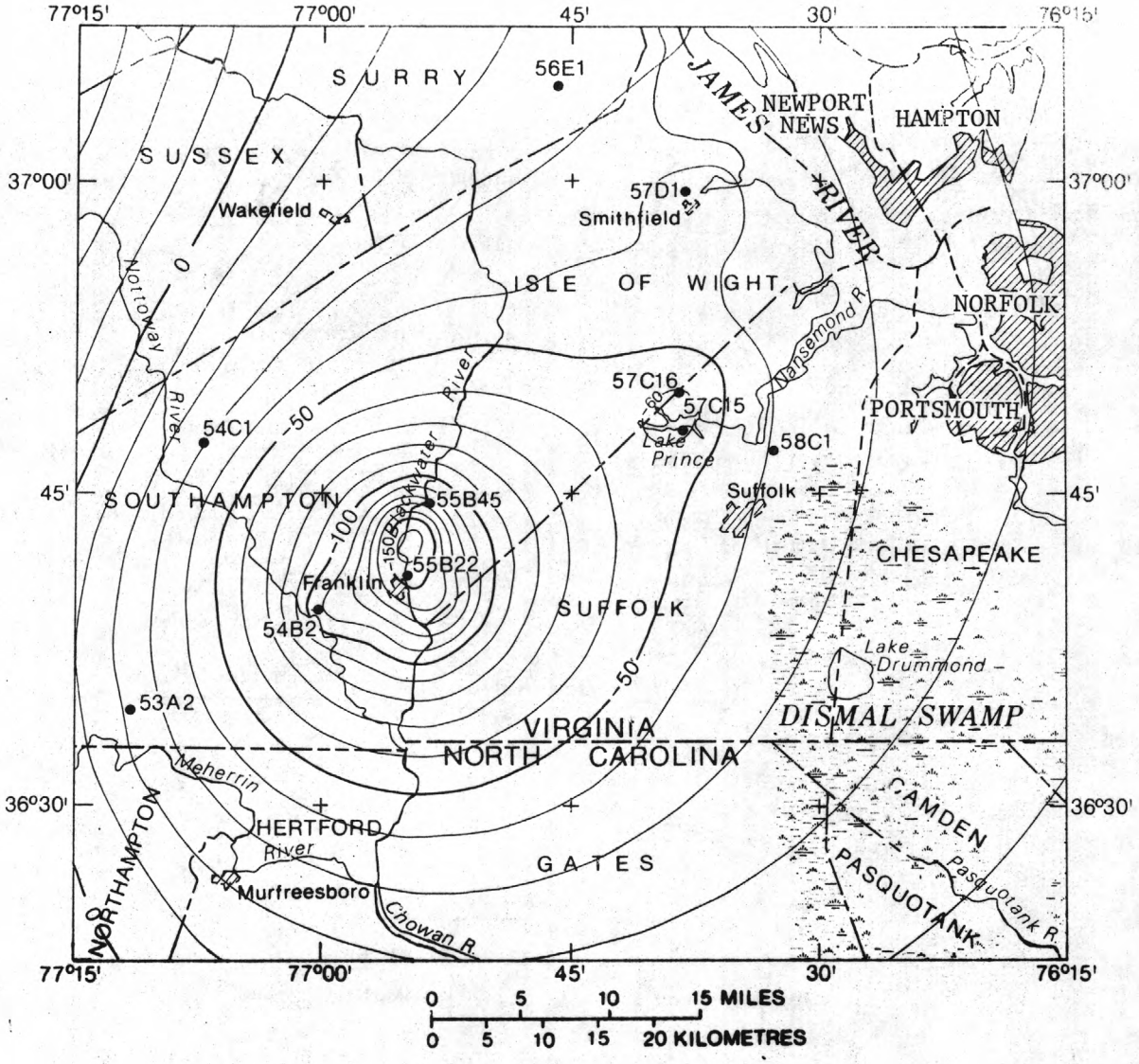


EXPLANATION

55B22
Observation well and well number

25
Water-level contour
Shows altitude of water level. Contour interval 5 feet (1.5 metres). Datum is mean sea level.

Figure 13.--Simulated water-level surface in the Lower Cretaceous aquifer for October 1941 using historical pumpage.

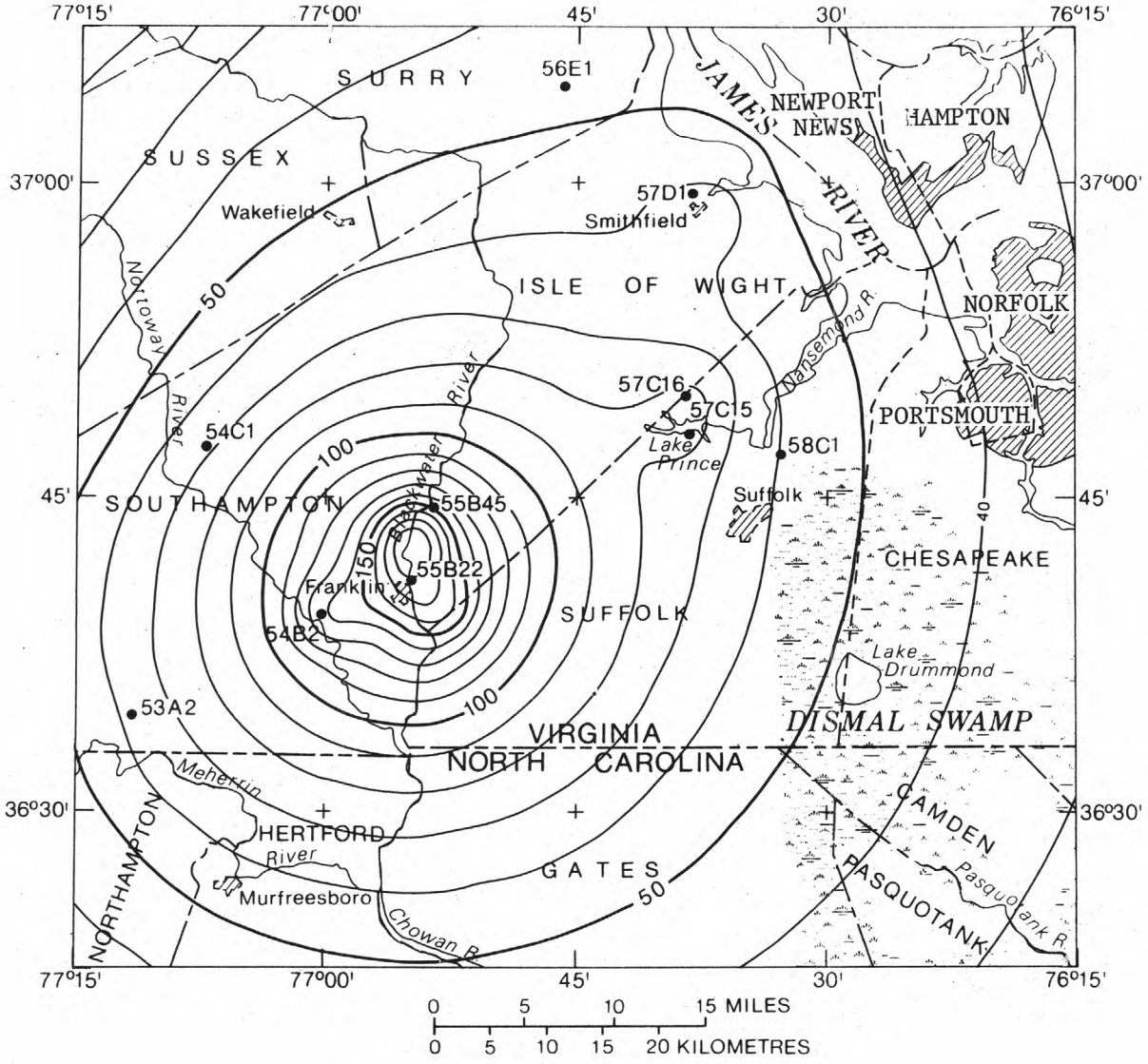


EXPLANATION

55B22
Observation well and well number

-50
Water-level contour
Shows altitude of water level. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 14.--Simulated water-level surface in the Lower Cretaceous aquifer for October 1970 using historical pumpage.

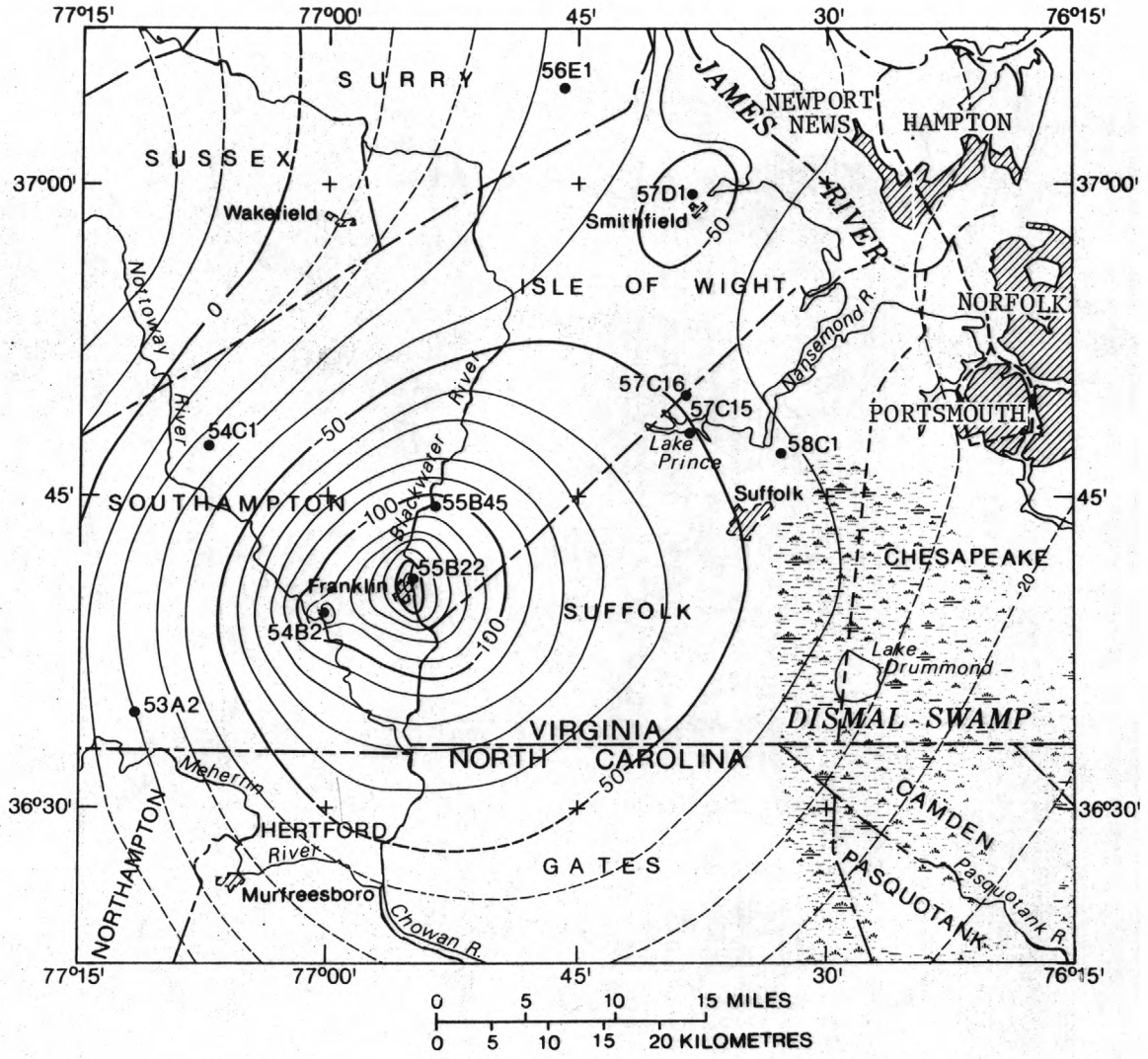


EXPLANATION

55B22
Observation well and well number

100
Line of equal water-level decline
Interval 10 feet (3 metres).

Figure 15.--Simulated water-level decline for period October 1941 to October 1970 using historical pumpage.

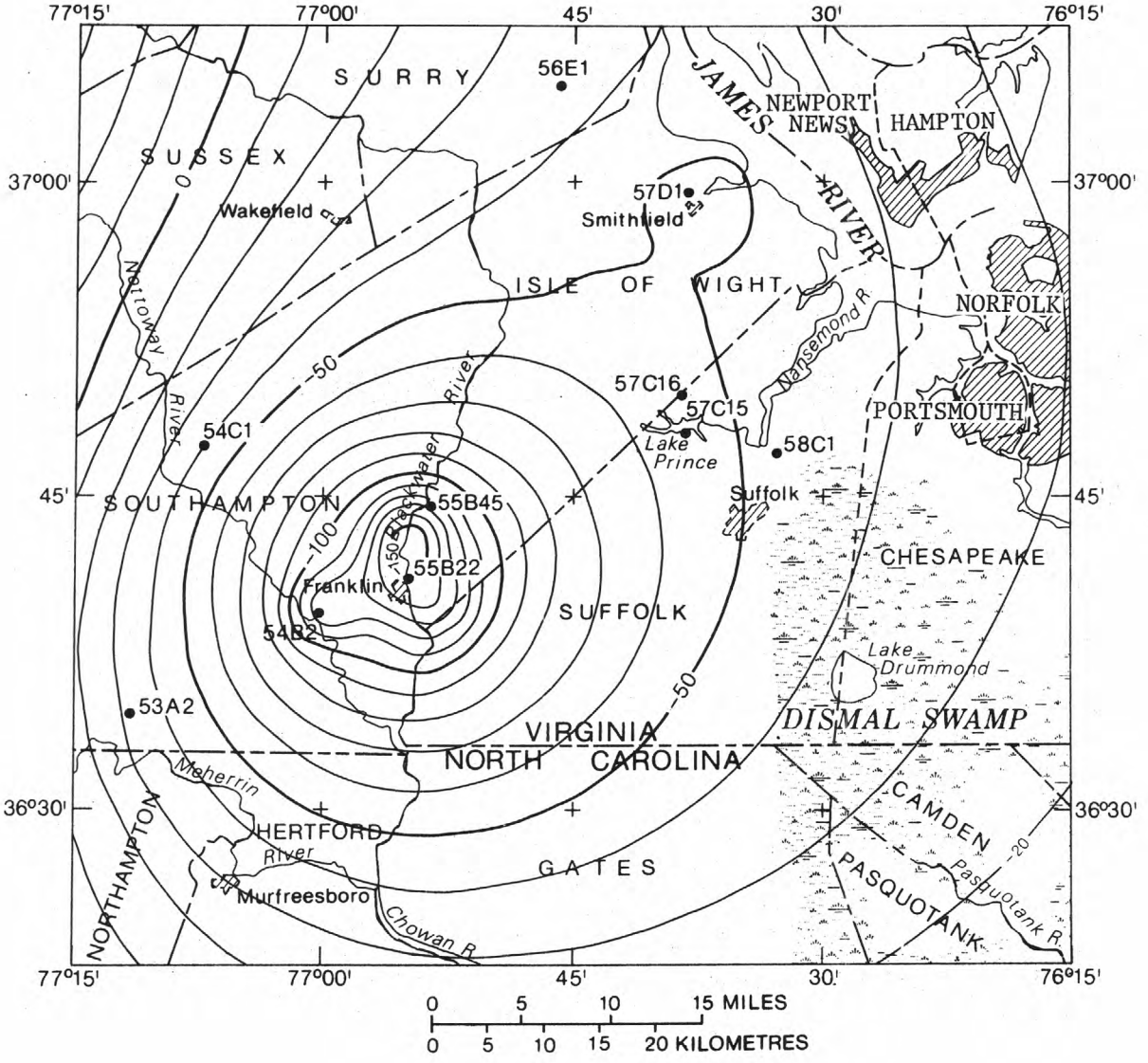


EXPLANATION

55B22
Observation well and well number

—50—
Water-level contour
Shows altitude of water level. Dashed where approximately located. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 16.--Measured water-level surface in the Lower Cretaceous aquifer for December 1972.

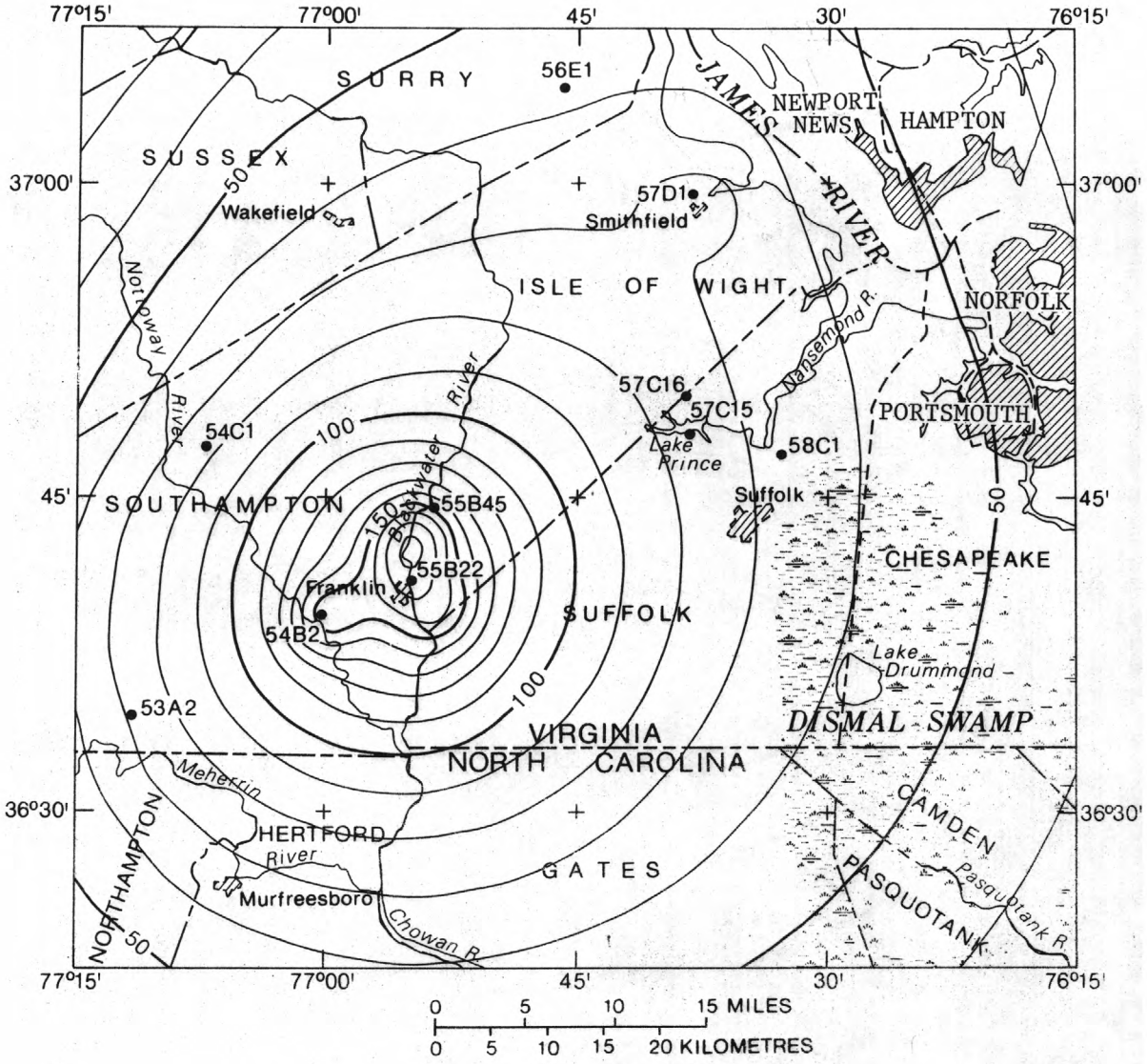


EXPLANATION

55B22
Observation well and well number

—100—
Water-level contour
Shows altitude of water level. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 17.--Simulated water-level surface in the Lower Cretaceous aquifer for December 1972 using historical pumpage.

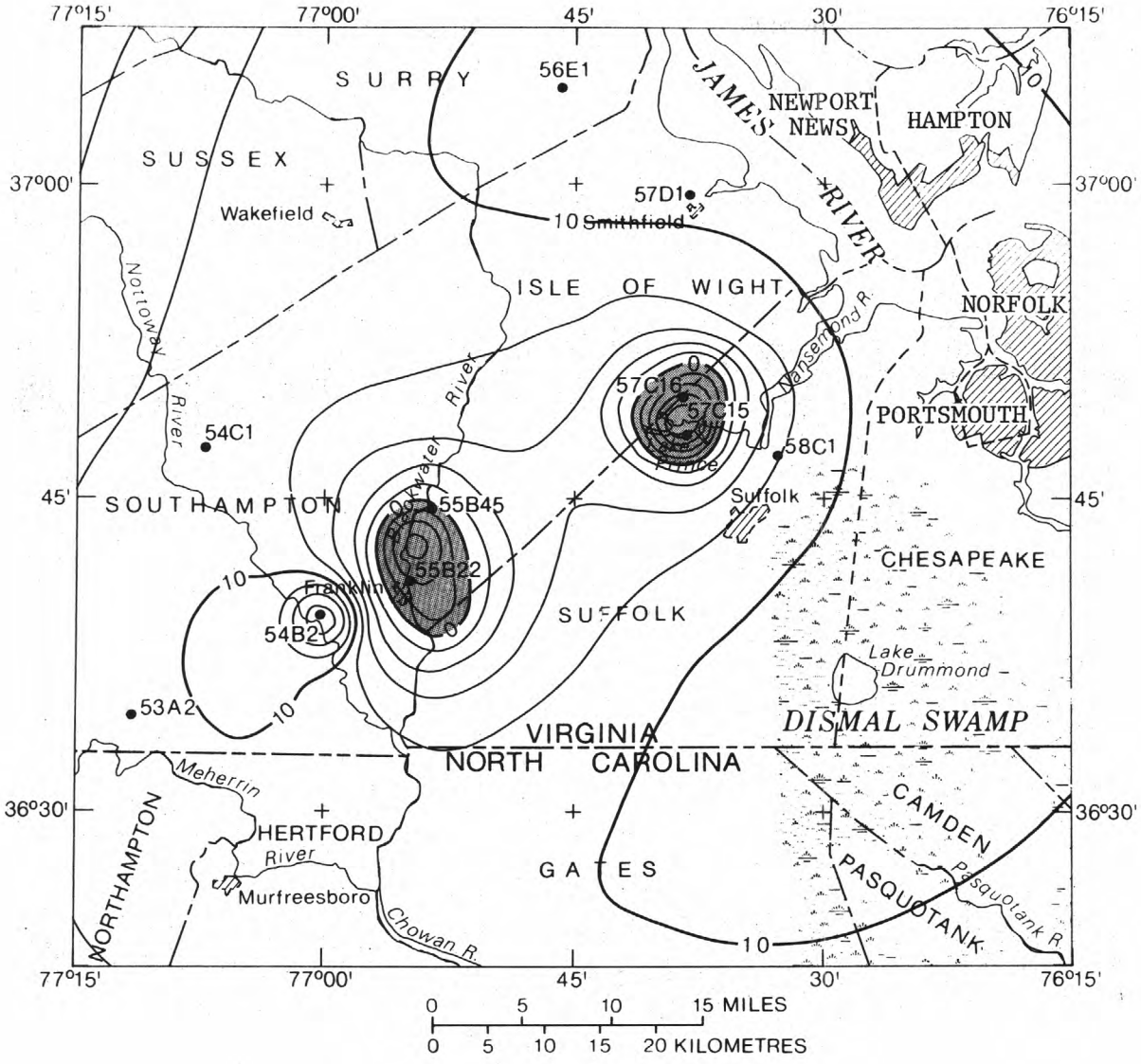


EXPLANATION

55B22
Observation well and well number

100
Line of equal water-level decline
Interval 10 feet (3 metres).

Figure 18.--Simulated water-level decline in the Lower Cretaceous aquifer for period 1941 to December 1972 using historical pumpage.



EXPLANATION

55B22
Observation well and well number

10
Line of equal water-level decline
Interval 2 feet (0.6 metres).

Area where water levels rose

Figure 19.--Simulated water-level change in the Lower Cretaceous aquifer for period October 1970 to December 1972 using historical pumpage. Areas shaded where water levels rose.

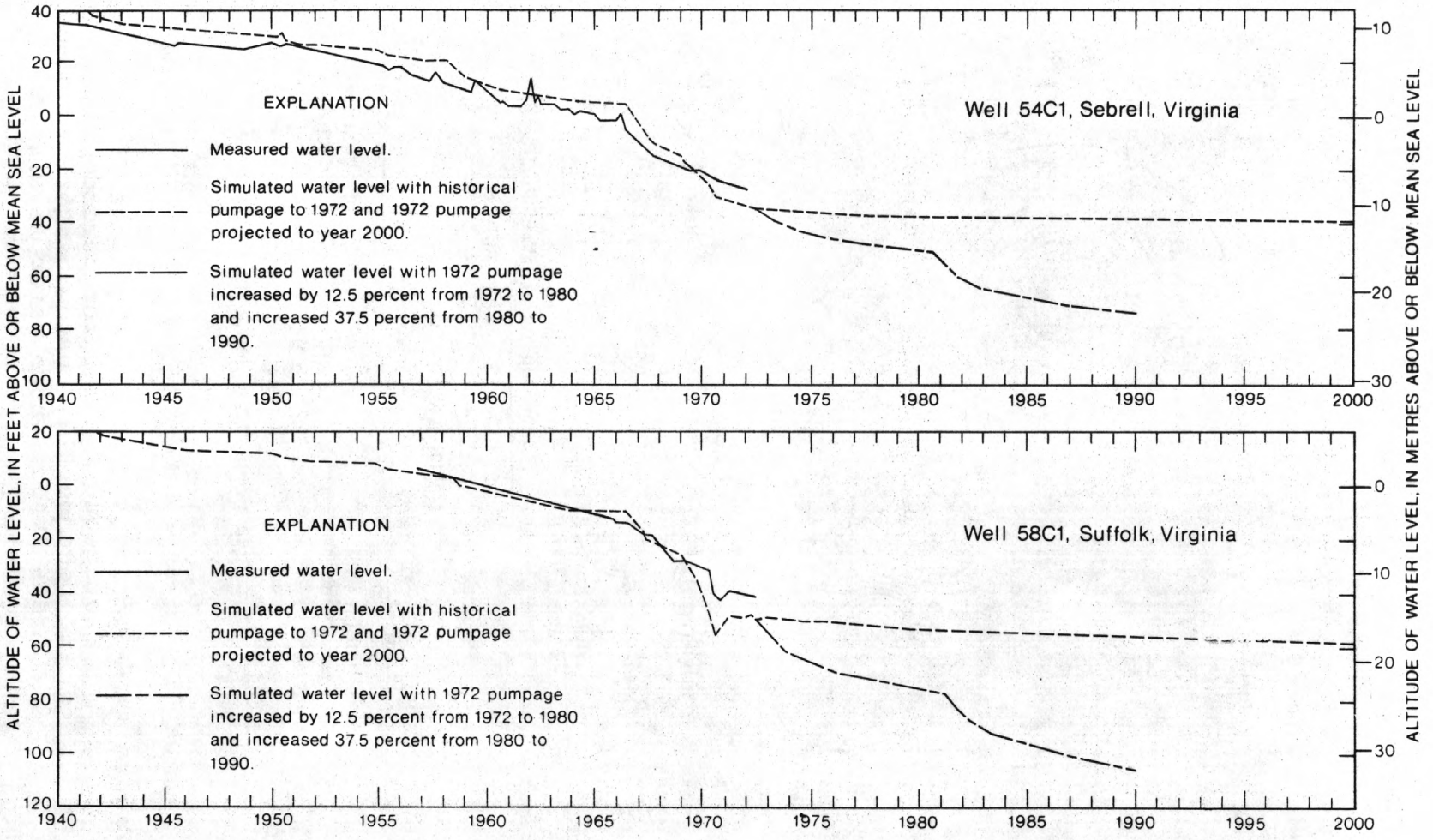


Figure 20.--Simulated and measured hydrographs for wells 54C1 at Sebrell, Virginia and 58C1, near Suffolk, Virginia.

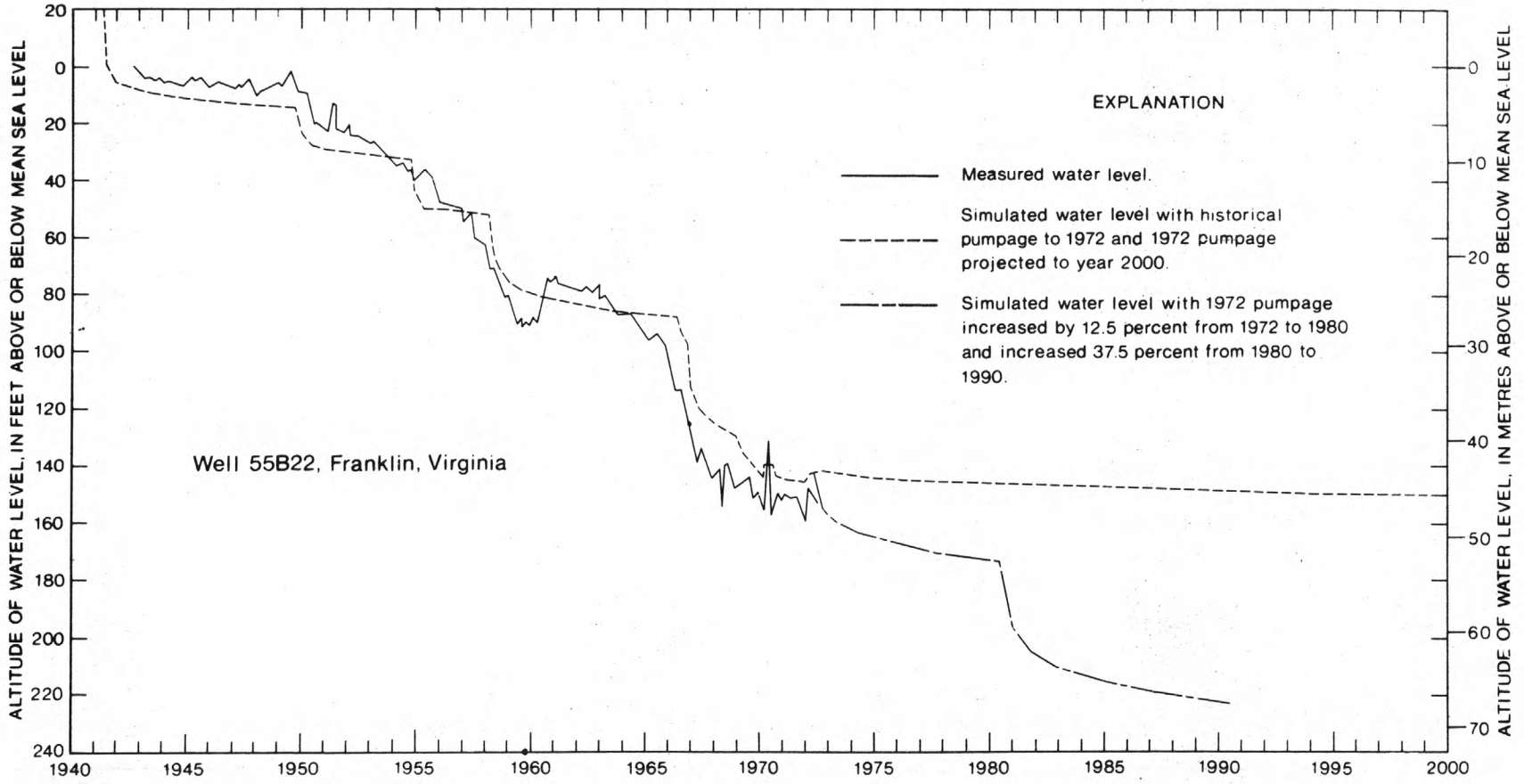


Figure 21.--Hydrographs of simulated and measured water levels for well 55B22, the Franklin city observation well.

Discrepancies in the Model

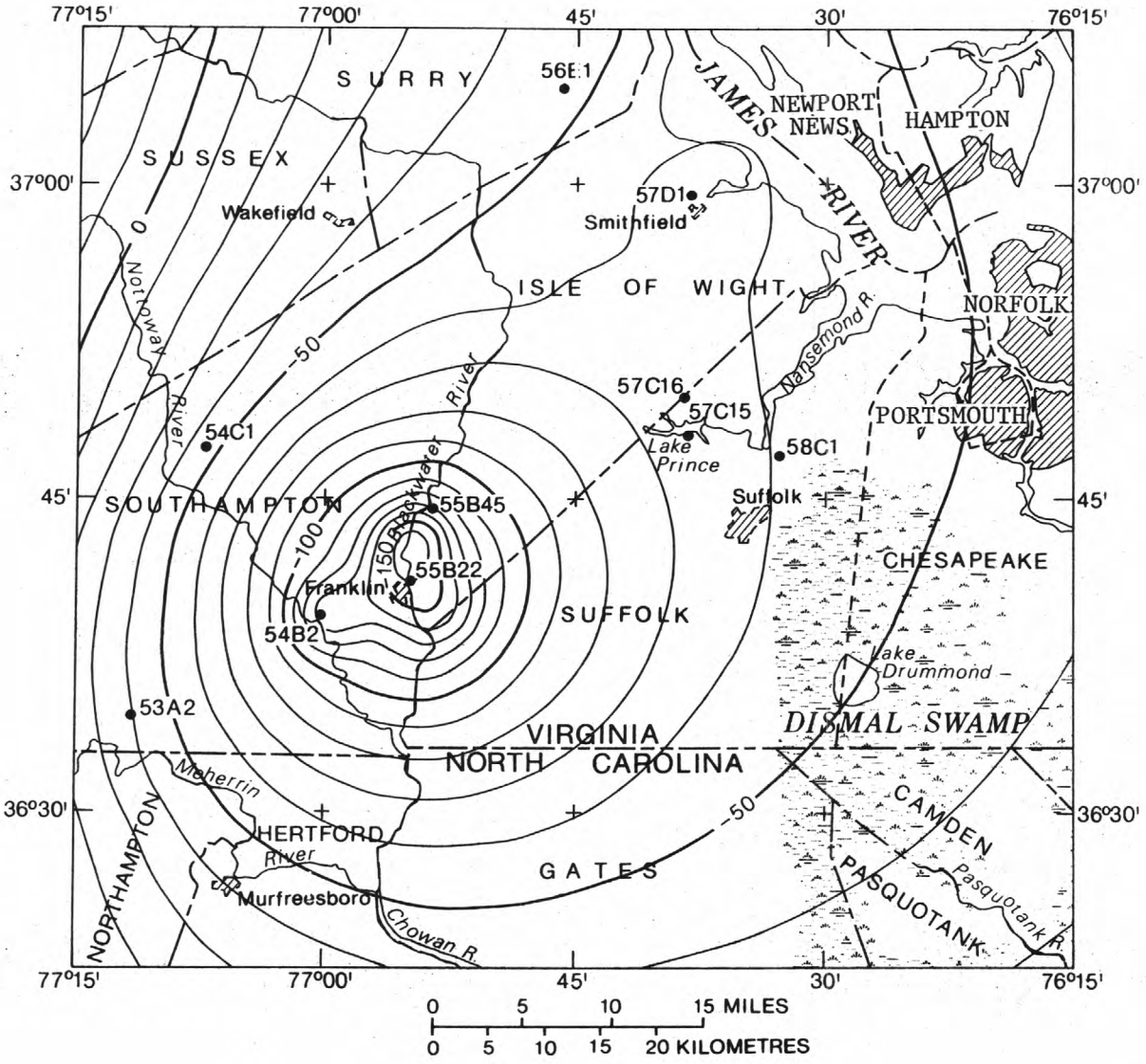
Minor discrepancies between simulated surfaces and actual measured surfaces are caused by the nature of the simulated data and the measured data and the manner in which they are plotted. One reason is that the data points for the simulated water surface represent the average value over a rectangular area. The size and shape of these areas are specified by the model grid dimensions (table 3). Thus each nodal value in the model represents an average value for a rather large area; while the data used on a measured surface map are point data; that is, each data point represents a water-level measurement in a well. Contours on the measured contour map are drawn by interpolation between the well data.

Secondly, the simulated contour maps were drawn by an automatic plotter using data from a computer contouring program. The computer program accepts the model output as random data and generates contour-data points based on an equal-sized rectangular grid. Near the center, the model-data points are close together and some of them must be ignored for input to the computer program.

Thirdly, it was not possible to enter the historical pumpage from 7 mi (11 km) west southwest of Franklin near well 54B2 and from the Lake Prince wells, 57C15 and 57C16, into the model at the geographical center of these well fields. Pumpage must be entered in the model at nodes which are slightly offset from these pumpage centers. In addition, at West Point, Virginia, many miles from the center of the model there is a large withdrawal entered at a long narrow node that covers more area than the well field.

Use of the Model

The model will be used to predict future water levels. The two main factors in accurately predicting future conditions are the validity of the model in representing the physical parameters of the aquifer system and the accuracy of the predictions of ground-water withdrawal. The hydrographs in figures 20 and 21 show predicted water levels under two different sets of conditions. They show part of a predictive run to the year 2022 using December 1972 pumpage, and a predictive run to 1990 with pumpage increased by 12.5 percent from 1972 to 1980 and increased by an additional 37.5 percent from 1980 to 1990. Contour maps for these two predicted surfaces are shown in figures 22 and 23.

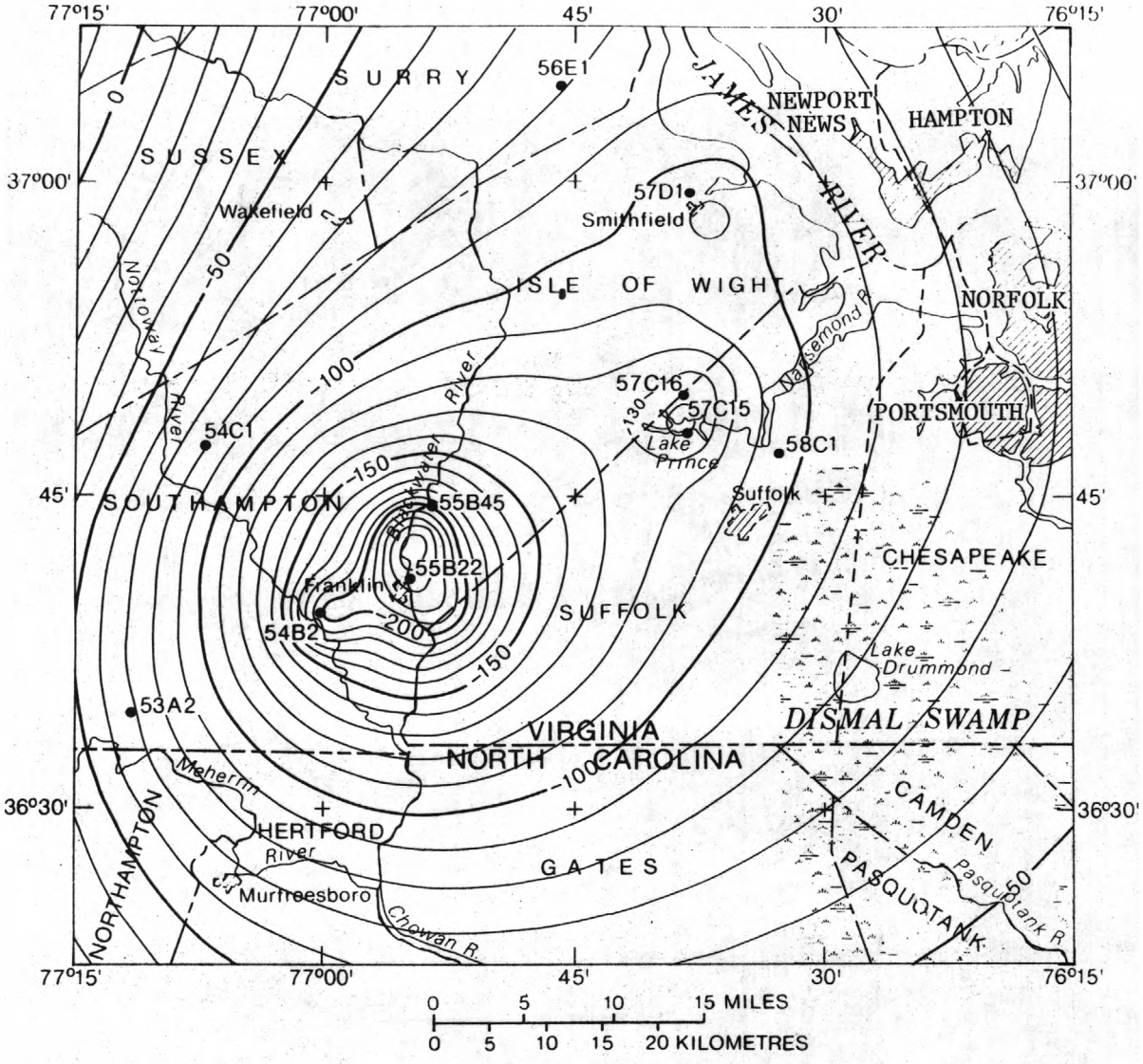


EXPLANATION

55B22
Observation well and well number

—50—
Water-level contour
Shows altitude of water level. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 22.--Simulated water-level surface in the Lower Cretaceous aquifer for January 2022 using 1972 pumpage.

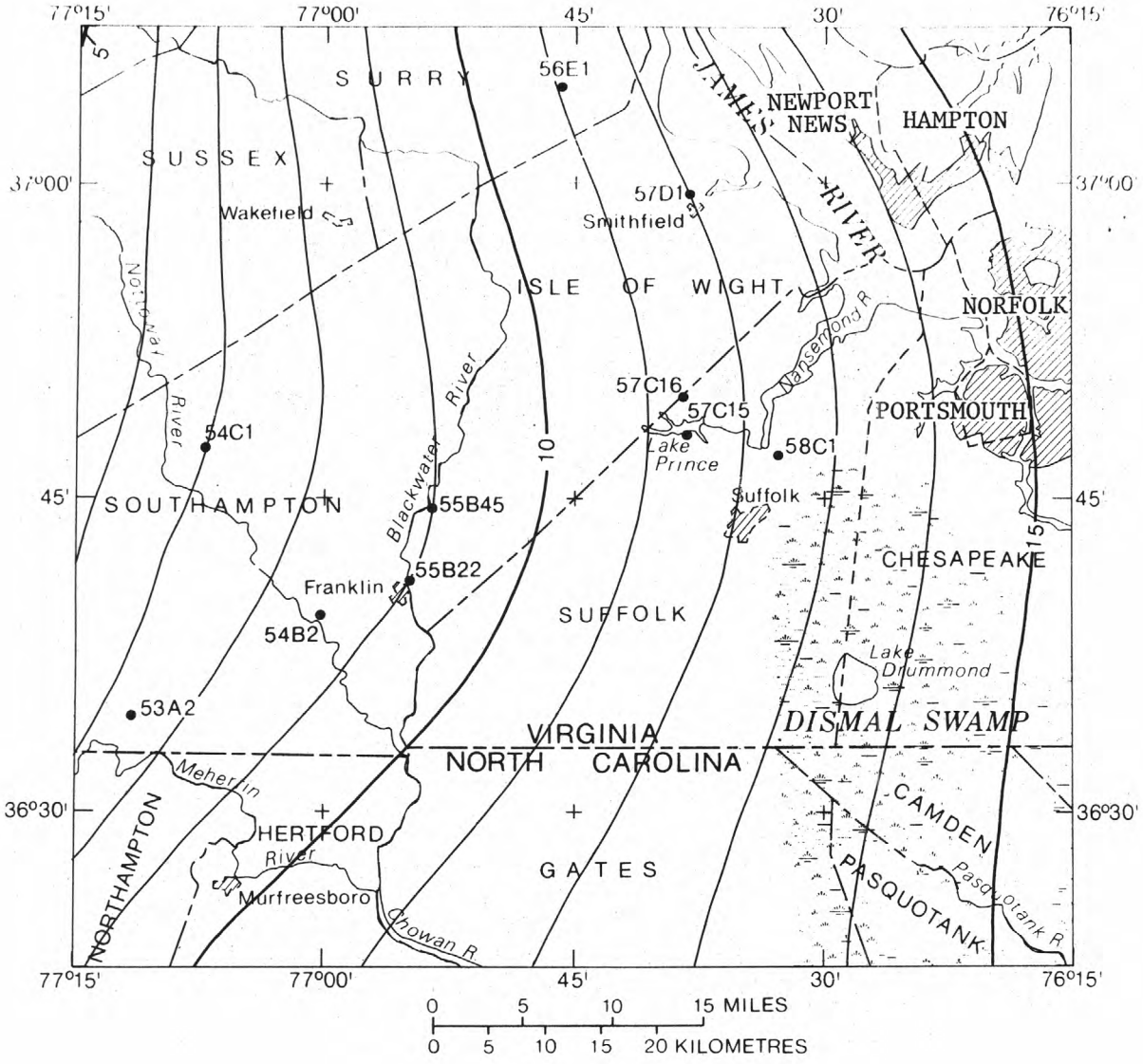


EXPLANATION

55B22
Observation well and well number

-50
Water-level contour
Shows altitude of water level. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 23.--Simulated water-level surface in the Lower Cretaceous aquifer for December 1990 using 1.125 x [1972 pumpage plus 7.2 Mgal/d (0.32 m^3/s) at Lake Prince well field] from 1972 to 1980 and 1.375 x [1972 pumpage plus 7.2 Mgal/d (0.32 m^3/s) at Lake Prince well field] from 1980 to 1990.



EXPLANATION

55B22
Observation well and well number

10
Line of equal water-level decline
Interval 1 foot (0.3 metres)

Figure 24.--Simulated water-level decline in the Lower Cretaceous aquifer for period December 1972 to January 2022 using 1972 pumpage.

The predictive run to 2022 using 1972 pumpage (figures 20, 21 and 22) shows a pronounced leveling trend of the potentiometric surface with the most drawdown occurring east of Franklin (figure 24). This leveling out is a striking development and, although it was not entirely unexpected, it is a trend that has been only very slightly indicated by measured ground-water levels. A close inspection of the hydrographs of measured water levels in figures 20 and 21 does reveal that the leveling starts but is obscured by periodic increases of withdrawals.

The leveling trend is very important because it reaffirms the correctness of the concept of the way a leaky aquifer system behaves. If withdrawals from an aquifer continue at a constant rate over a long period of time, they will cause a cone of depression to develop that will grow until sufficient water is induced to enter the aquifer by the gradient across the confining bed to balance the amount of water being pumped, giving a new steady-state condition. Thus as time continues, the shape and depth of the cone of depression become more and more stable and if pumpage is kept constant, the cone will eventually stabilize. The model clearly shows that this is happening in the Franklin area now.

Additional model runs were made assuming several time periods and combinations of pumpage. The potentiometric maps and decline maps (figs. 25-33) are in Appendix B.

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APPENDIX A

The T/K' Modeling Factor

The basic flow equation, in finite difference form, is as follows:

$$T'_{xx(i-\frac{1}{2},j)} \frac{h_{i-1,j,k}-h_{i,j,k}}{\Delta x_i} + T'_{xx(i+\frac{1}{2},j)} \frac{h_{i+1,j,k}-h_{i,j,k}}{\Delta x_i}$$

$$+ T'_{yy(i,j-\frac{1}{2})} \frac{h_{i,j-1,k}-h_{i,j,k}}{\Delta y_j} + T'_{yy(i,j+\frac{1}{2})} \frac{h_{i,j+1,k}-h_{i,j,k}}{\Delta y_j}$$

$$= S_{i,j} \frac{h_{i,j,k}-h_{i,j,k-1}}{\Delta t} + q_{w(i,j,k)} - G_{(i,j,k)} \qquad (1)$$

This is a modified form of the equation given by Pinder (1970) based on work by Pinder and Bredehoeft (1968). In equation (1) the principal directions of the transmissivity tensor are assumed to be aligned with the coordinate axes, and

$$T'_{xx(i+\frac{1}{2},j)} = \frac{2T_{xx(i,j)}T_{xx(i+1,j)}}{T_{xx(i,j)}\Delta x_{i+1}+T_{xx(i+1,j)}\Delta x_i}$$

is the harmonic mean of

$$\frac{T_{xx(i,j)}}{\Delta x_i}, \quad \frac{T_{xx(i+1,j)}}{\Delta x_{i+1}}$$

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and each of the other T' terms is similarly defined as a harmonic mean of transmissivity divided by grid spacing at successive nodes along the x or y axis. $T_{xx(i,j)}$ and $T_{yy(i,j)}$ are the principal components of the transmissivity tensor at node i,j. Δx_i and Δy_j are the dimensions, along the x and y axes, of the rectangular grid element centered at node i,j. $S_{i,j}$ is the storage coefficient of the aquifer at node i,j; Δt is the length of the time step, $t_k - t_{k-1}$; $h_{i,j,k}$ is the head at node i,j at time t_k; $q_{w(i,j,k)}$ is the total rate of pumpage from wells at node i,j during time step k; and $G_{(i,j,k)}$ is the flux from a confining bed at node i,j during time step k. $G_{(i,j,k)}$ includes leakage through the confining bed from an overlying water table aquifer or source, and may also include transient leakage of water released from storage within the confining bed, evaluated using techniques described by Bredehoeft and Pinder (1970). If steady-state conditions are assumed, the flux from the confining bed reduces to

$$G_{i,j} = \frac{K'}{L_{i,j}} (\hat{h}_{i,j} - h_{i,j}) \qquad (2)$$

where K' is the vertical hydraulic conductivity of the confining layer; $L_{i,j}$ is the thickness of the confining layer at node i,j; $\hat{h}_{i,j}$ is the head in the overlying water table aquifer at node i,j; and the time subscript, k, has been dropped. Under steady-stage conditions, moreover, the term

$$S_{i,j} \frac{h_{i,j,k} - h_{i,j,k-1}}{\Delta t}$$

of equation (1)

is zero. Finally, we wish to consider the steady state which prevailed in the Lower Cretaceous aquifer prior to any pumping, so that the term $q_{w(i,j,k)}$ of equation (1) may also be considered zero. Thus for this case equation (1) reduces to

$$T'_{xx(i-\frac{1}{2},j)} \frac{(h_{i-1,j}-h_{i,j})}{\Delta x_i} + T'_{xx(i+\frac{1}{2},j)} \frac{(h_{i+1,j}-h_{i,j})}{\Delta x_i}$$

$$+ T'_{yy(i,j-\frac{1}{2})} \frac{(h_{i,j-1}-h_{i,j})}{\Delta y_j} + T'_{yy(i,j+\frac{1}{2})} \frac{(h_{i,j+1}-h_{i,j})}{\Delta y_j}$$

$$+ \frac{K'}{L_{i,j}} (\hat{h}_{i,j}-h_{i,j}) = 0 \qquad (3)$$

where the time subscript (k) has been dropped. Dividing through by $\frac{K'}{L_{i,j}}$ gives

$$\frac{T'_{xx(i-\frac{1}{2},j)} L_{i,j}}{K' \Delta x_i} (h_{i-1,j}-h_{i,j}) + \frac{T'_{xx(i+\frac{1}{2},j)} L_{i,j}}{K' \Delta x_i} (h_{i+1,j}-h_{i,j})$$

$$+ \frac{T'_{yy(i,j-\frac{1}{2})} L_{i,j}}{K' \Delta y_j} (h_{i,j-1}-h_{i,j})$$

$$+ \frac{T'_{yy(i,j+\frac{1}{2})} L_{i,j}}{K' \Delta y_j} (h_{i,j+1}-h_{i,j}) + (\hat{h}_{i,j}-h_{i,j}) = 0 \qquad (4)$$

The solution to equation (4) is determined by four coefficients:

$$\frac{T'_{xx(i-\frac{1}{2},j)}}{K'}\,\frac{L_{i,j}}{\Delta x_i}\ ;\quad \frac{T'_{xx(i+\frac{1}{2},j)}}{K'}\,\frac{L_{i,j}}{\Delta x_i}\ ;$$

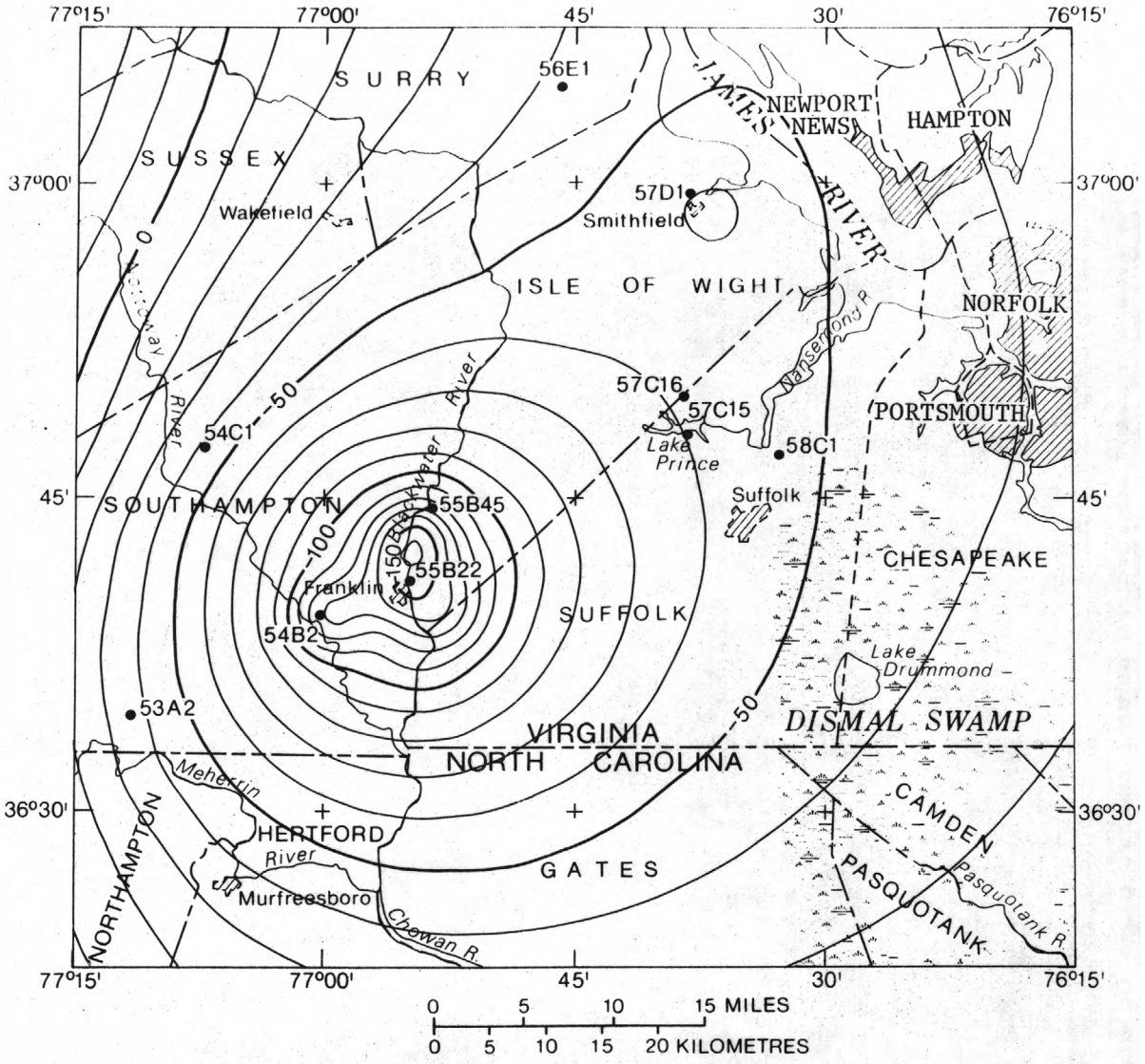
$$\frac{T'_{yy(i,j-\frac{1}{2})}}{K'}\,\frac{L_{i,j}}{\Delta y_j}\ ;\quad \frac{T'_{yy(i,j+1)}}{K'}\,\frac{L_{i,j}}{\Delta y_j}$$

and by the boundary conditions. The individual values of T'_{xx}, T'_{yy}, and K' may be altered arbitrarily without affecting the solution so long as the ratios,

$$\frac{T'_{xx(i-\frac{1}{2},j)}}{K'}\ ,\quad \frac{T'_{xx(i+\frac{1}{2},j)}}{K'}\ ,\quad \frac{T'_{yy(i,j-\frac{1}{2})}}{K'}\ \text{and}\ \frac{T'_{yy(i,j+\frac{1}{2})}}{K'}$$

remain unchanged.

Appendix B

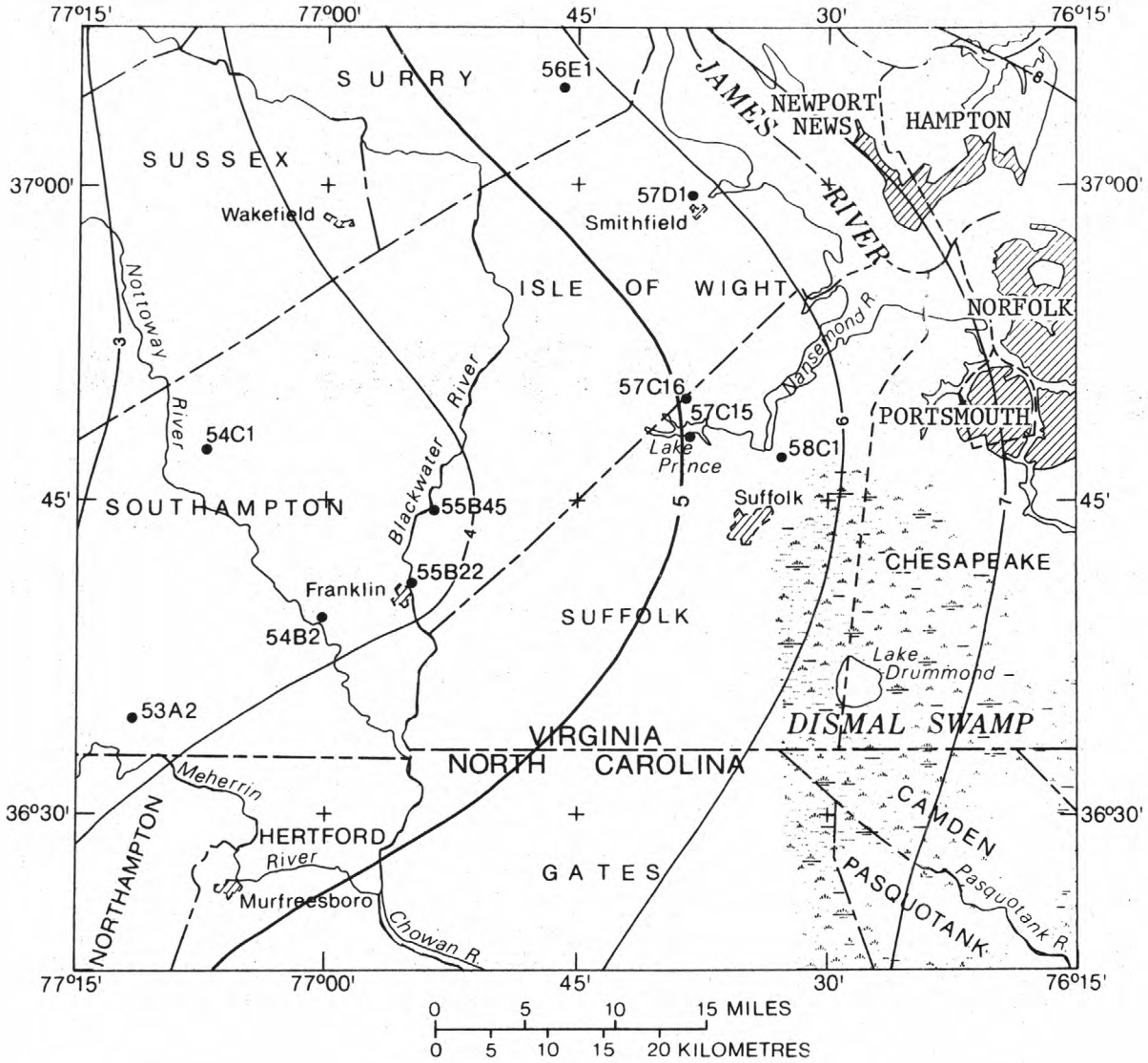


EXPLANATION

55B22
Observation well and well number

—50—
Water-level contour
Shows altitude of water level. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 25.--Simulated water-level surface in the Lower Cretaceous aquifer for December 1980 with pumpage at 1972 level.



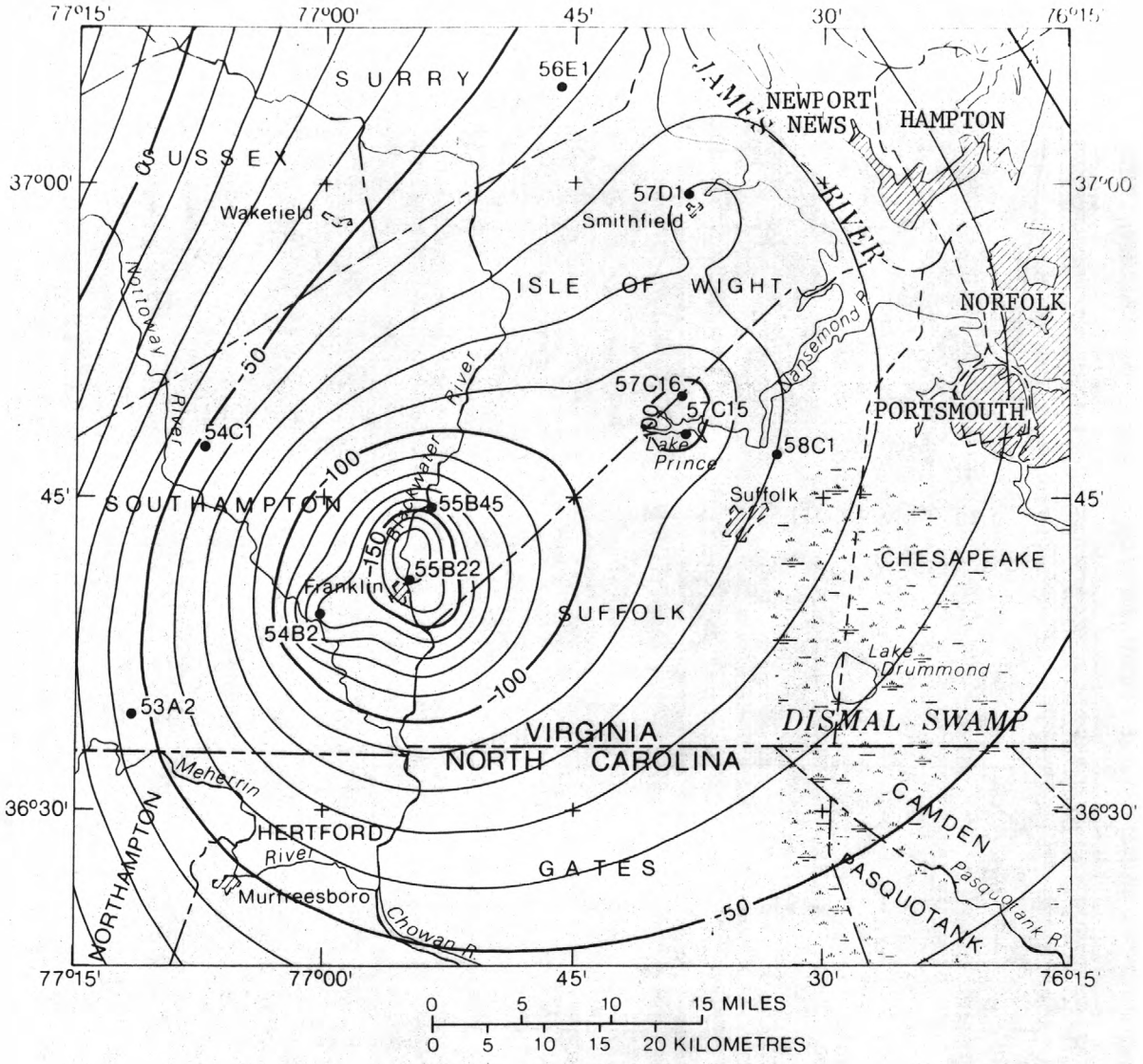
EXPLANATION

55B22
Observation well and well number

5
Line of equal water-level decline

Interval 1 foot (0.3 metres)

Figure 26.--Simulated water-level decline in the Lower Cretaceous aquifer for period December 1972 December 1980 with pumpage at 1972 level.

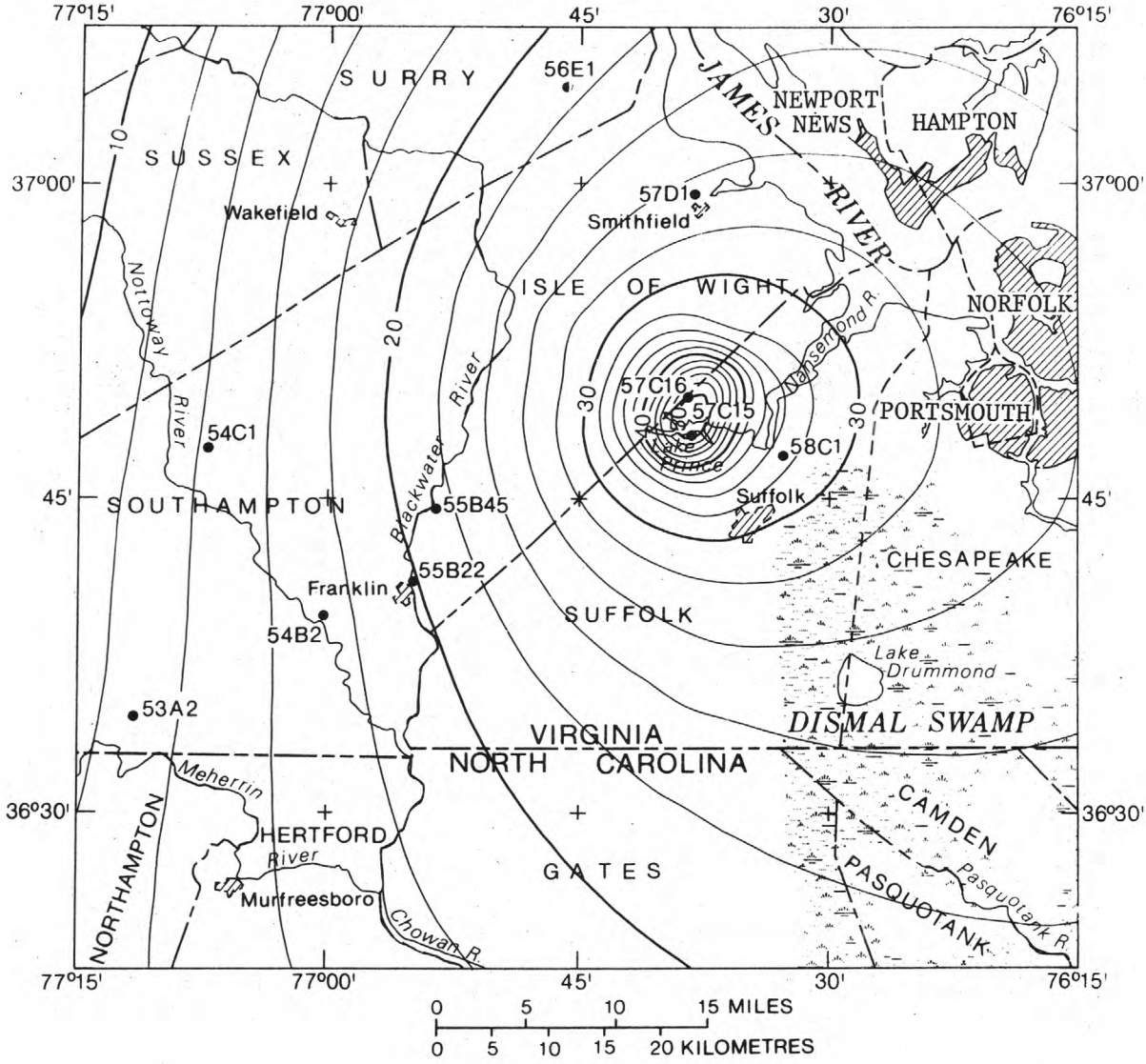


EXPLANATION

55B22
Observation well and well number

-50
Water-level contour
Shows altitude of water level. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 27.--Simulated water-level surface in the Lower Cretaceous for January 2022 using 1972 pumpage plus 7.2 Mgal/d (0.32 m^3/s) at Lake Prince well field.

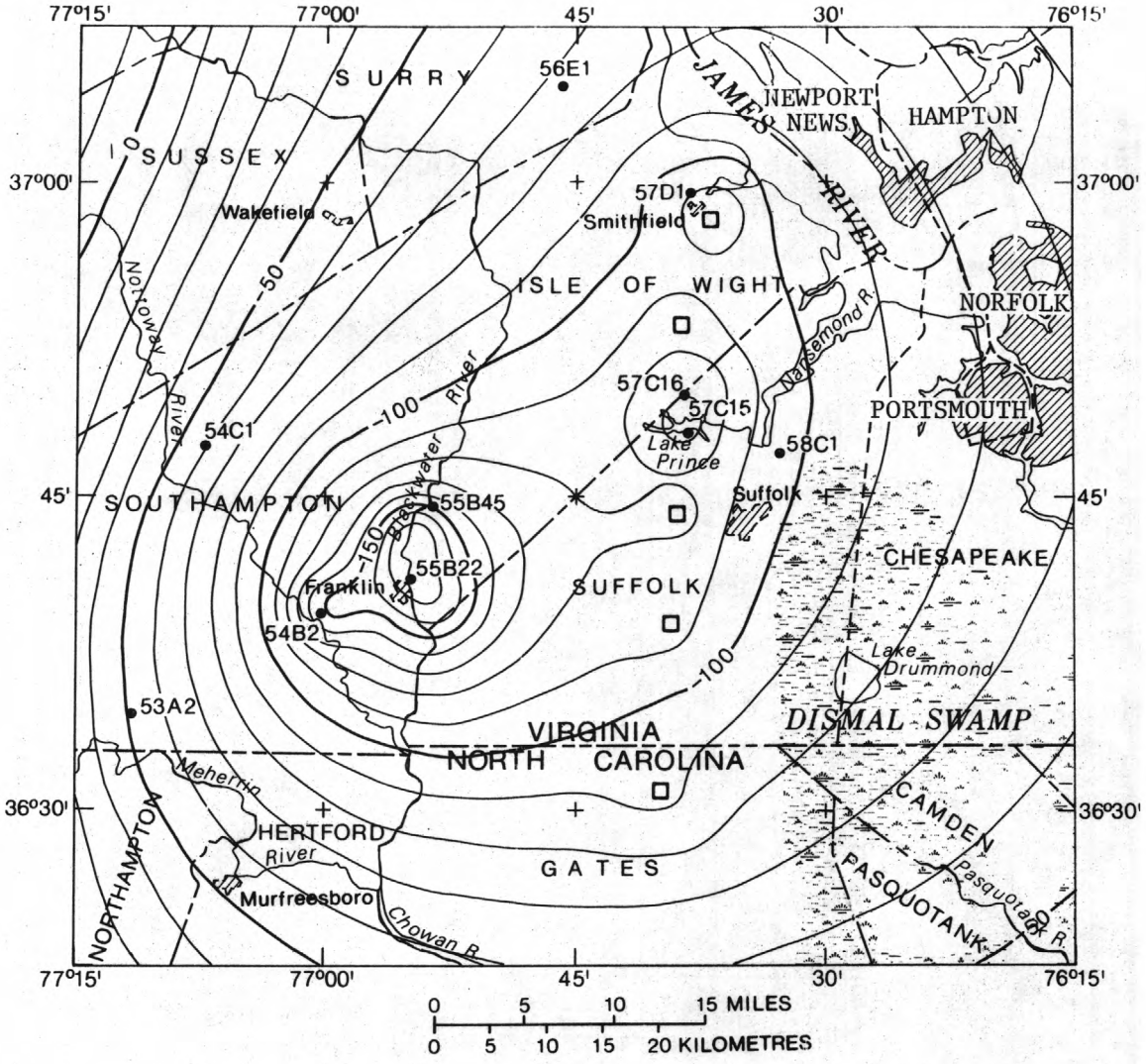


EXPLANATION

55B22
Observation well and well number

20
Line of equal water-level decline
Interval 2 feet (0.6 metres)

Figure 28.--Simulated water-level decline in the Lower Cretaceous aquifer for period December 1972 to January 2022 using 1972 pumpage plus 7.2 Mgal/d (0.32 m^3/s) at Lake Prince well field.



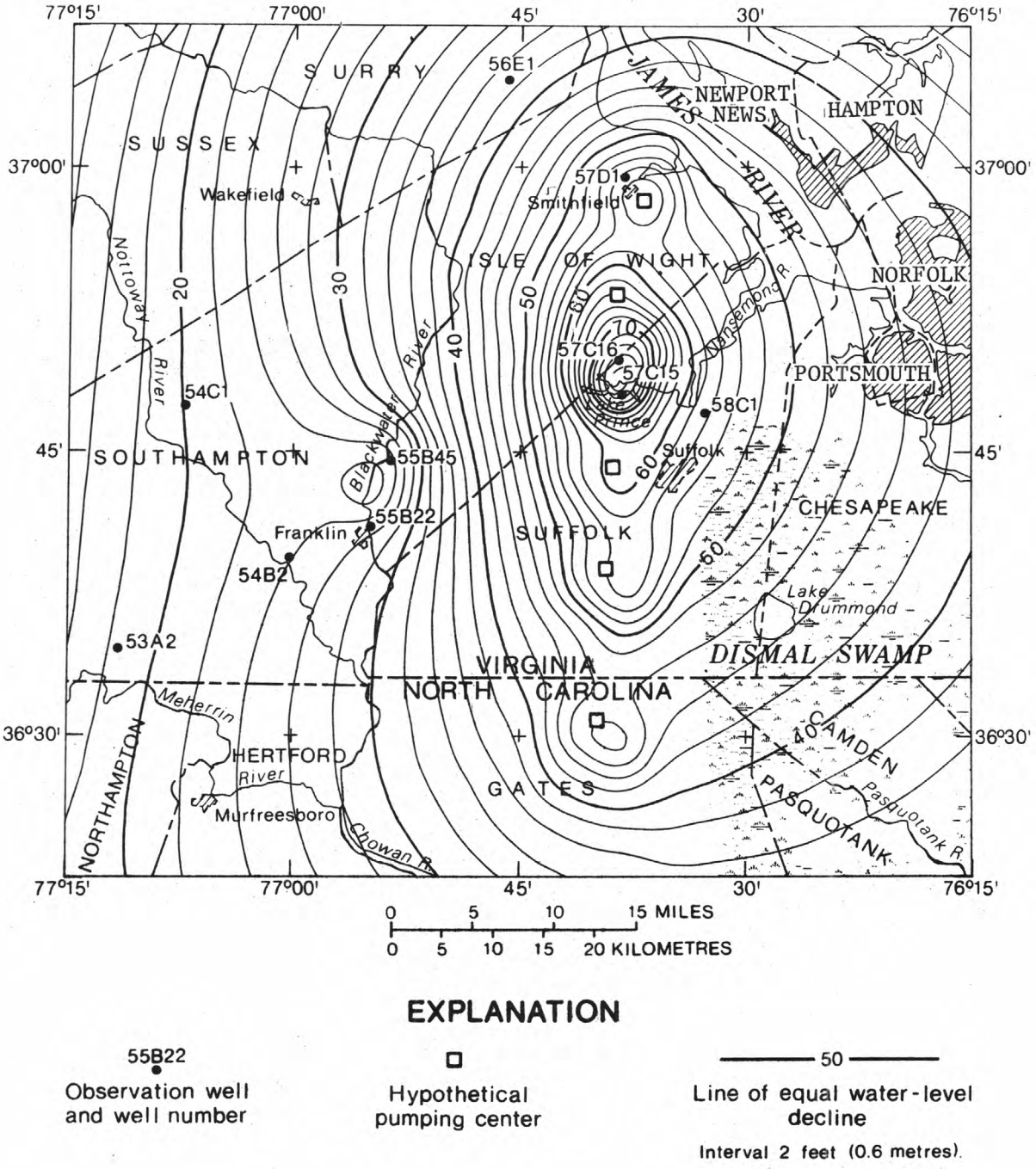
EXPLANATION

55B22
Observation well and well number

Hypothetical pumping center

-50
Water-level contour
Shows altitude of water level. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 29.--Simulated water-level surface in the Lower Cretaceous aquifer for 50 year projection (January 2022) using 1972 pumpage plus 7.2 Mgal/d (0.32 m^3/s) at Lake Prince and 14 Mgal/d (0.61 m^3/s) divided evenly between five hypothetical pumping centers.



EXPLANATION

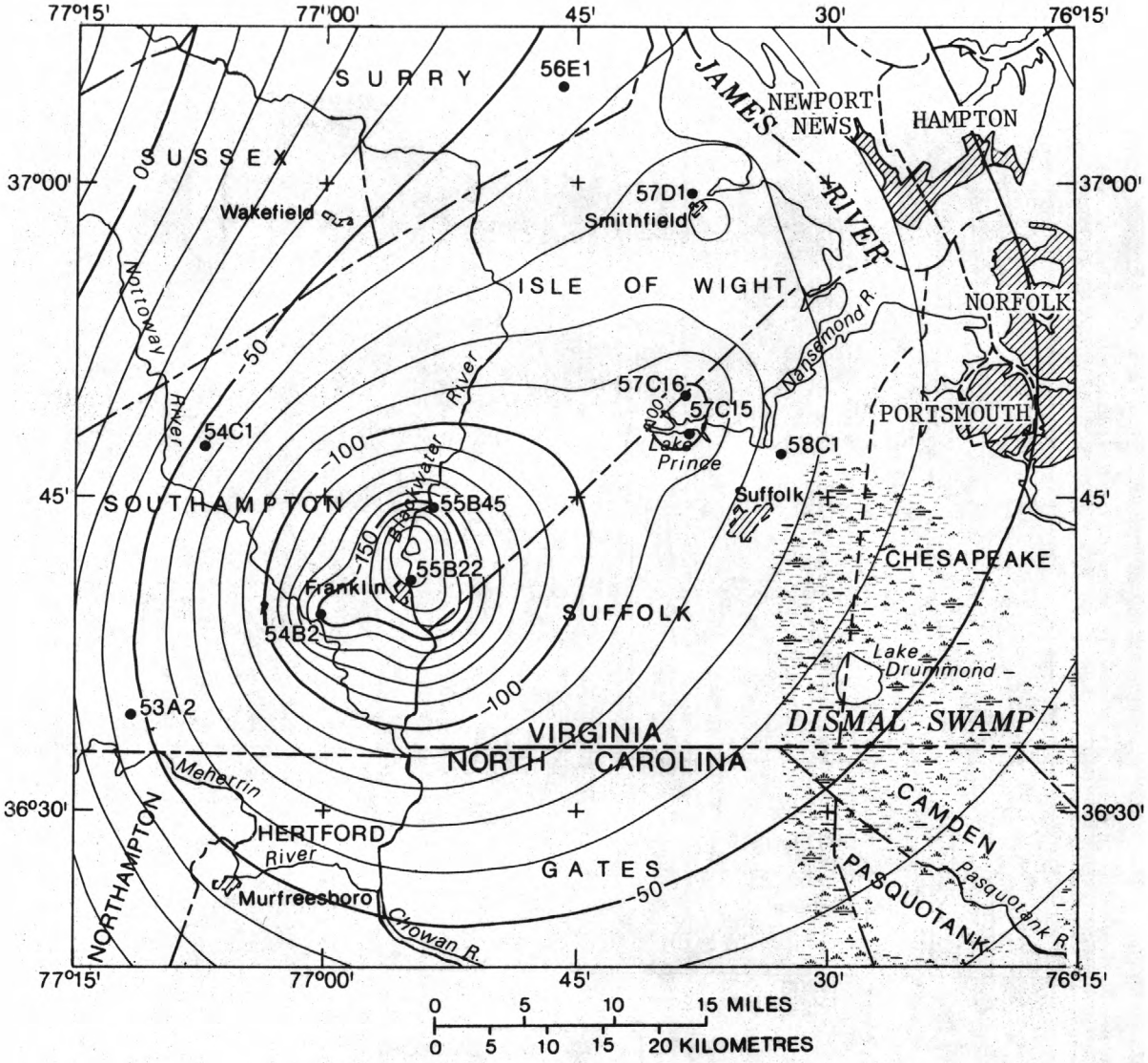
55B22
Observation well and well number

Hypothetical pumping center

50
Line of equal water-level decline

Interval 2 feet (0.6 metres).

Figure 30.--Simulated water-level decline in the Lower Cretaceous aquifer for period December 1972 to January 2022 using 1972 pumpage plus 7.2 Mgal/d (0.32 m^3/s) at Lake Prince well field and 14 Mgal/d (0.61 m^3/s) divided evenly between five hypothetical pumping centers.

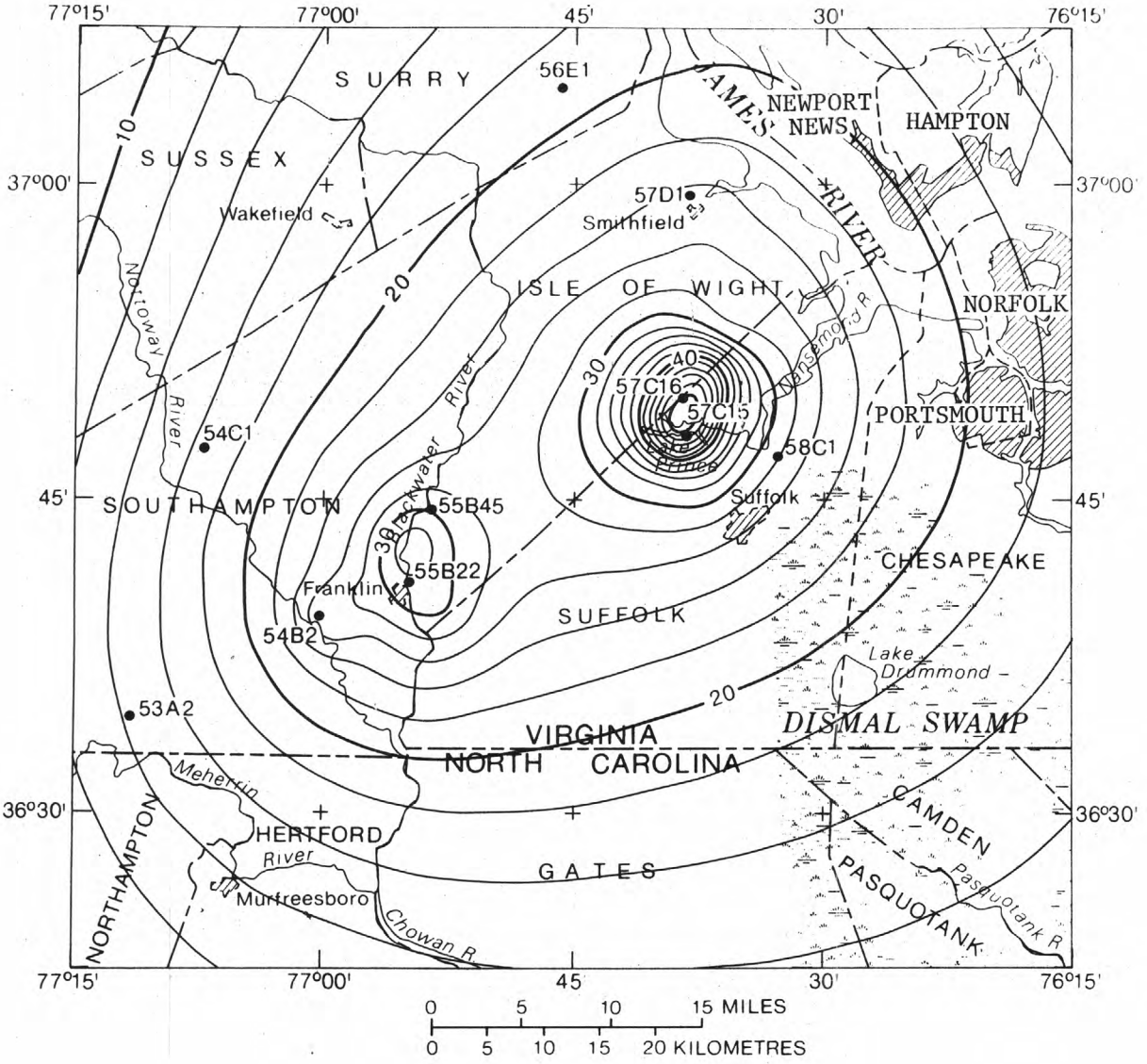


EXPLANATION

55B22
Observation well and well number

-50
Water-level contour
Shows altitude of water level. Contour interval 10 feet (3 metres). Datum is mean sea level.

Figure 31.--Simulated water-level surface in the Lower Cretaceous aquifer for December 1980 using 1.125 x [1972 pumpage + 7.2 Mgal/d (0.32 m^3/s) at Lake Prince well field].

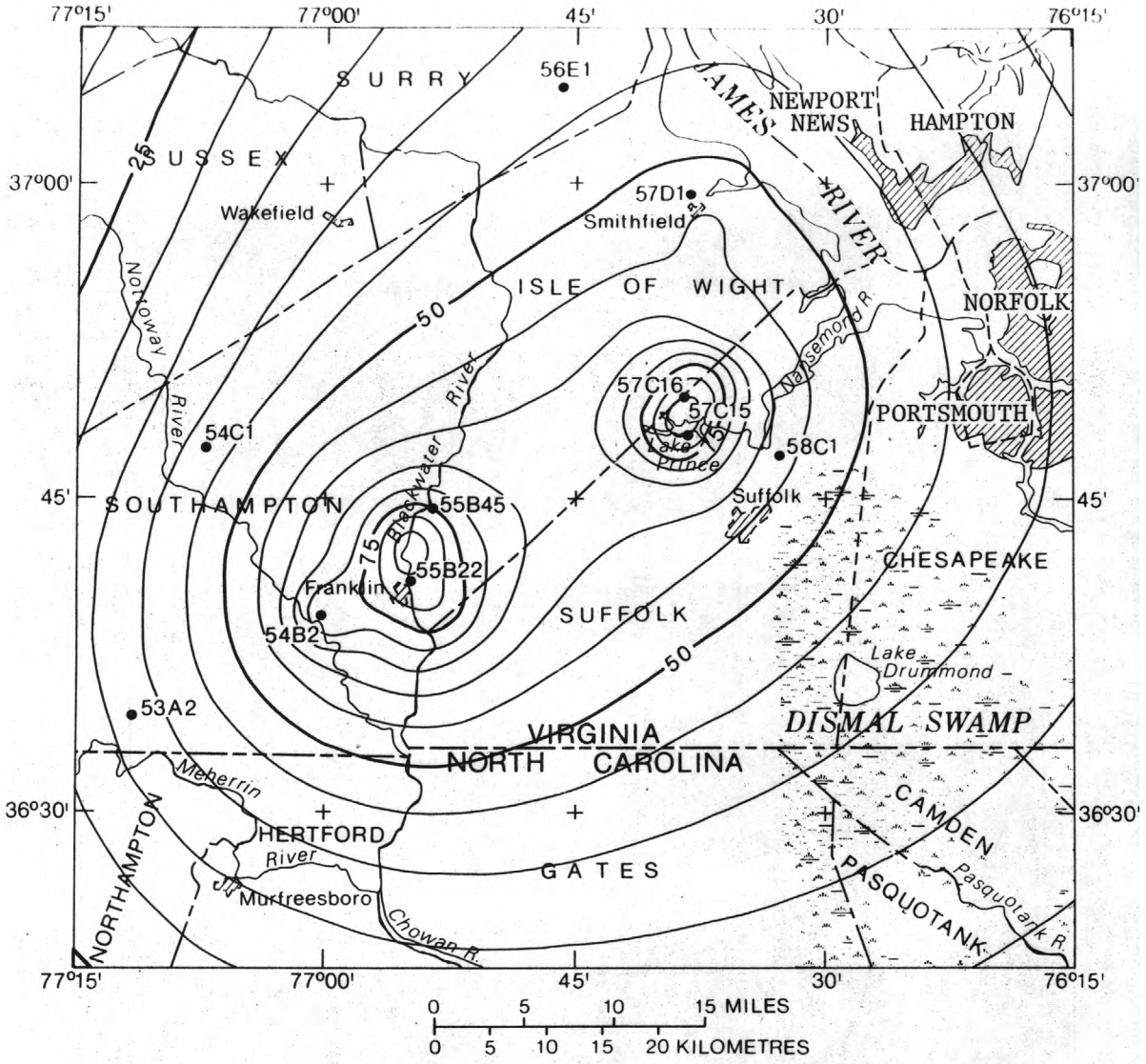


EXPLANATION

55B22
Observation well and well number

20
Line of equal water-level decline
Interval 2 feet (0.6 metres)

Figure 32.--Simulated water level decline in the Lower Cretaceous aquifer for period 1972 to 1980 using 1.125 x [1972 pumpage + 7.2 Mgal/d (0.32 m^3/s) at Lake Prince well field].



EXPLANATION

55B22
Observation well and well number

50
Line of equal water-level decline

Interval 5 feet (1.5 metres).

Figure 33.--Simulated water-level decline in the Lower Cretaceous aquifer for period December 1972 to January 1990 with 1.125 x [1972 pumpage + 7.2 Mgal/d (0.32 m /s) at Lake Prince well field] from December 1972 to December 1980, and 1.375 x [1972 pumpage + 7.2 Mgal/d (0.32 m /s) at Lake Prince well field] from December 1980 to December 1990.

[redacted]