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PRELIMINARY DIGITAL MODEL OF GROUND-WATER FLOW IN THE MADISON GROUP, POWDER RIVER BASIN AND ADJACENT AREAS, WYOMING, MONTANA, SOUTH DAKOTA, NORTH DAKOTA, AND NEBRASKA

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7. Author(s)
Leonard F. Konikow

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Box 25046, Denver Federal Center
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For additional information write to:

U.S. Geological Survey
Water Resources Division
Box 25046, Denver Federal Center
Denver, Colo. 80225

CONTENTS

ILLUSTRATIONS

CONTENTS

PLATES

TABLES

CONVERSION FACTORS

The following report uses both the English and the metric system of units. In the text the English units are given first, and the equivalent measurement in metric units is given in parentheses. The units are frequently abbreviated, using the notations shown below. The English units can be converted to metric units by multiplying by the factors given in the following list.

English unit *To convert*	*Multiply by*	*Metric unit* *To obtain*
Feet (ft)	0.3048	Metres (m)
Feet per second (ft/s)	0.3048	Metres per second (m/s)
Feet per second per foot [(ft/s)/ft]	1.0	Metres per second per metre [(m/s)/m]
Square feet per second (ft^2/s)	0.0929	Square metres per second (m^2/s)
Cubic feet per second (ft^3/s)	2.832×10^{-2}	Cubic metres per second (m^3/s)
Miles (mi)	1.609	Kilometres (km)
Square miles (mi^2)	2.59	Square kilometres (km^2)

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ABSTRACT

A digital simulation model was used to analyze regional ground-water flow in the Madison Group aquifer in the Powder River Basin and adjacent areas. Most recharge to the aquifer originates in or near the outcrop areas of the Madison in the Bighorn Mountains and Black Hills, and most discharge occurs through springs and wells. Results from the model calculations indicate that the total flow through the aquifer in the modeled areas was approximately 200 cubic feet per second (5.7 cubic metres per second). The results also indicate that ground-water flow through the aquifer is strongly affected by (1) a zone of reduced and zero transmissivity along parts of the western margin of the Powder River Basin, (2) regional variations in aquifer transmissivity, which are partly due to changes in kinematic viscosity caused by temperature differences in the water, and (3) vertical leakage through confining beds. The aquifer can probably sustain increased ground-water withdrawals of up to several tens of cubic feet per second, but these withdrawals probably would significantly lower the potentiometric surface in the Madison aquifer in a large part of the basin. The digital model could better predict the effects of withdrawals if more accurate estimates of the storage coefficient, transmissivity, and leakance could be obtained.

INTRODUCTION

The future development of energy resources in the Powder River Basin in Wyoming and Montana (fig. 1) will be accompanied by increased demands for water, which is not abundantly available in this area. Several reports, including those by Hodson, Pearl, and Druse (1973) and Swenson (1974), indicate that large ground-water supplies might be developed from an extensive aquifer formed by the Madison Group and associated rocks. One plan has already been proposed to withdraw an average of about 20 ft^3/s (0.57 m^3/s) from a total of 40 wells drilled into the Madison aquifer in Niobrara County, Wyoming. Presently, there are insufficient data on the Madison aquifer to make a reliable and accurate prediction of the long-term effects of this proposed development or of other possible future developments.

The U.S. Geological Survey is currently investigating the hydrogeology of the Madison aquifer and developing a plan of study to (1) evaluate the quantity and quality of water available from the aquifer and (2) predict the effects of possible future developments. Achieving these objectives will require a detailed understanding of both local and regional ground-water flow through the aquifer, which, in turn, will require a comprehensive knowledge of aquifer properties, boundary conditions, hydraulic stresses, and hydrologic relationships to surface water and to other aquifers. The report on the plan of study (U.S. Geol. Survey, 1975) has set goals and priorities for intensive studies that will begin soon and attempt to accomplish the above objectives.

Purpose and scope

The overall purpose of this study was to provide preliminary quantitative descriptions of ground-water flow in the Madison aquifer that could be used as input and for guidance in developing the plan of study for future investigations of this aquifer. This objective was accomplished by developing and analyzing a digital model to simulate ground-water flow in the Madison aquifer. The model was calibrated using data that had been collected previously.

A preliminary model of this type is quite valuable in developing a plan of study. Following are the specific objectives of this model study:

1. Modify, quantify, and improve a conceptual model of ground-water flow in the aquifer system.

2. Determine deficiencies in existing data and help set priorities for data collection by identifying the types of data required and areas where the greatest need exists.

3. Make a preliminary estimate of the effects of large ground-water withdrawals on potentiometric levels, recharge, and discharge.

4. Guide the design of a monitoring network by demonstrating where the effects of pumping would most likely be readily observable.

This report presents a detailed account of the development, calibration, and analysis of the digital simulation model of ground-water flow in the Madison Group in and adjacent to the Powder River Basin. In a sense, it constitutes a supplement to the plan-of-study report (U.S. Geol. Survey, 1975).

Thus, an effort has been made here to avoid unnecessary duplication of descriptive background material that may be presented in the plan-of-study report.

This report describes the input data for the digital model in sufficient detail to allow others to duplicate the results of this study, if desired. These data can thus serve as a basis for continuing or extending this study to (1) develop a more accurate model after additional field data become available, (2) test new or revised hypotheses, or (3) evaluate the effects of new or alternative proposals for ground-water development.

Acknowledgments

This investigation was jointly sponsored by the Old West Regional Commission and the U.S. Geological Survey. Data, assistance, and valuable suggestions were provided by colleagues in the Montana, South Dakota, and Wyoming District Offices of the U.S. Geological Survey, and their help is gratefully acknowledged. The assistance and helpful suggestions of Dr. Neil Jones, Canberra College of Advanced Education, Canberra, Australia, are also appreciated.

DESCRIPTION OF STUDY AREA

Geographic setting

This study focuses on the Powder River Basin of northeastern Wyoming and southeastern Montana and adjacent areas in western South Dakota, southwestern North Dakota, and northwestern Nebraska. (See fig. 1.) Major features bounding the basin include the Bighorn Mountains on the west, the Laramie Mountains and the Hartville Uplift on the south, and the Black Hills on the east. The northeastern part of the study area is included in the Williston Basin.

The area has a semiarid climate. In general, precipitation is least near the center of the basin and greatest in the adjacent mountains and hills. The major streams draining the area include the Yellowstone, Bighorn, Tongue, Powder, Little Missouri, Belle Fourche, Cheyenne, and North Platte Rivers, and their tributaries.

Geology

Stratigraphy

Rocks of the Madison Group or Madison Limestone were deposited during the Mississippian Period. In Montana and the western part of North Dakota and South Dakota the rocks are called Madison Group, in which three formations are recognized as follows in ascending order: the Lodgepole Limestone, the Mission Canyon Limestone, and the Charles Formation. In Wyoming the three formations cannot be distinguished and the rocks are called Madison Limestone. The term "Madison Group" is generally used in this report to include rocks of Madison age in the Powder River Basin. The term "Madison Limestone" is used only to refer to occurrences of these strata in Wyoming. The strata of the Madison Group primarily consist of a thick and extensive sequence of limestone and dolomite, although some shale, evaporites, and cherty zones are present in

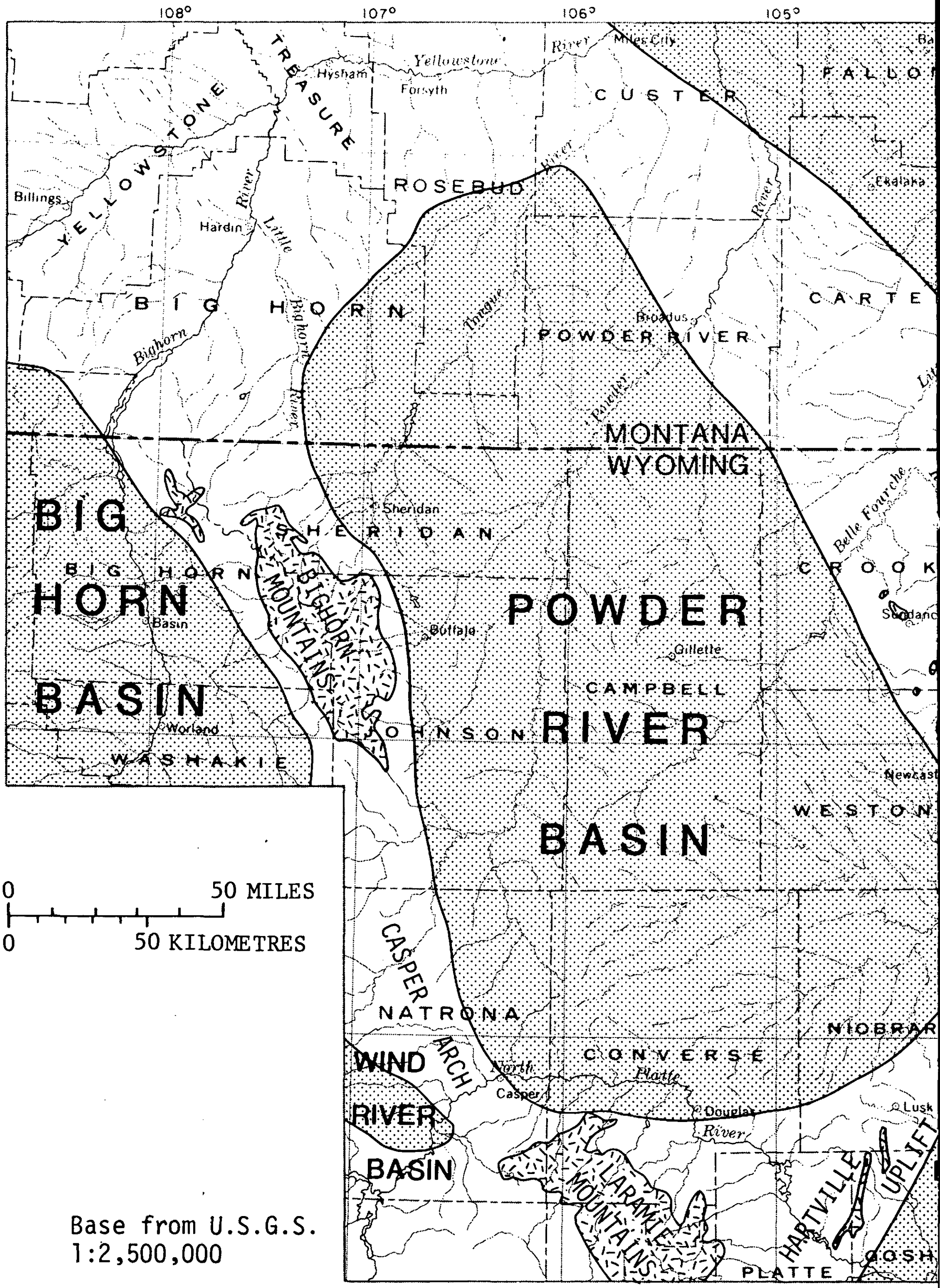

Figure 1.--Selected geographic, hydrologic, and geologic features of the Powder River Basin and adjacent areas.

EXPLANATION

Areas of exposed basement rocks

Structural basin

Geologic boundaries from "Oil and Gas Fields of the United States" by S. D. Vlissides and B. A. Quirin, 1964

Figure 1.--Continued

places. Maughan (1963) describes the occurrence of a basal unit of arkosic conglomeratic sandstone in the Laramie Range area of Wyoming. Beikman (1962) describes the Madison Limestone in the Bighorn Mountains as "a finely crystalline, thin-bedded to massive sequence of limy dolomite and limestone . . . ." Weller and others (1948) show that the Madison Limestone is approximately equivalent to the Guernsey Formation in the Hartville Uplift area and to the Pahasapa Limestone and the Englewood Formation in the Black Hills area. The thickness of the Madison Group ranges from zero, southeast of the Powder River Basin, to over 1,200 ft (366 m) in Montana and North Dakota. Detailed descriptions of the stratigraphic and sedimentary features of the Madison Group in Wyoming and southern Montana are presented by Andrichuck (1955).

Craig's (1972) map of sub-Mississippian paleogeology indicates that the Madison Group in the study area is underlain by several formations of limestone, dolomite, and sandstone, which range in age from Cambrian to Early Mississippian, and by crystalline Precambrian rocks. The Madison Group is generally overlain by Mississippian, Pennsylvanian, and Permian rocks, which include in ascending order the Amsden Formation and Tensleep Sandstone in the western part of the study area, the Hartville Formation in the Hartville Uplift area, and the Minnelusa Formation in much of the Powder River Basin and Black Hills areas. The Madison Limestone is exposed in outcrops in the Bighorn Mountains, Laramie Mountains, Hartville Uplift area, and in the Black Hills.

Because the carbonate rocks of the Madison Group are relatively soluble in water, the development of karst (solution) features is common. Sando (1974) describes ancient karst features, including enlarged joints, sink holes, caves, and solution breccias, that developed in the Madison Limestone in north-central Wyoming. He further indicates that most of the open spaces were filled by sand and residual products reworked by the early Amsden sea, which transgressed the area during the Chesterian time interval (Late Mississippian). The occurrence of large and extensive cave systems in outcrop areas of the Madison in the Bighorn Mountains and in the Black Hills is further evidence of the importance of the dissolution process in the development of secondary permeability in the Madison.

<u>Structure</u>

The Powder River Basin is a deep, elongate, asymmetrical, sedimentary basin. Beikman (1962) notes that the deepest part of the basin is on the west side, adjacent and parallel to the Bighorn Mountains, and that the surface of the Precambrian is about 21,000 ft (6,400 m) lower in the deepest part of the basin than on the nearby flank of the Bighorn Mountains.

A map showing structure contours on top of the Madison Group (Swenson, 1974) indicates that east of the Bighorn Mountains these rocks dip steeply towards the center of the basin, that north of the Laramie Mountains the dips are somewhat less steep, and that on the west side of the Black Hills the rocks dip gently westward. In the deepest parts of the basin the top of the Madison Group is more than 15,000 ft (4,570 m) below land surface.

Major faults exposed at the surface in the Powder River Basin and adjacent areas are shown on the geologic map of the Northern Great Plains (Keefer, 1974). It appears that faulting is associated with areas of steeply dipping rocks because most faults occur along the western and southern margins of the basin. The greatest displacements have been along high-angle reverse faults.

Foster, Goodwin, and Fisher (1968) used seismic methods to detect a large fault, not visible at the surface, that lies adjacent to and parallel to the eastern limit of the Bighorn Mountains. They report that the maximum throw (vertical displacement) on the fault is about 4,000 ft (1,220 m) and occurs near Buffalo, Wyo.

Ground water

The rates and directions of ground-water flow through the rocks of the Madison Group are governed primarily by the transmissivity of the aquifer, hydraulic gradients, and hydraulic stresses (recharge and discharge). Because the primary (or intergranular) porosity of the Madison appears to be low, water is stored mostly in and transmitted through secondary openings such as fractures, joints, and solution openings. The occurrence of these secondary openings is quite variable and difficult to predict, which may explain the wide range in yields of water wells drilled into the Madison. A further implication is that individual pumping tests (or drill-stem tests) are not necessarily reliable measures of the regional transmissivity of the aquifer.

Hydraulic gradients can be determined from potentiometric maps. Potentiometric data and (or) maps for the Madison Limestone are presented by Hodson (1974), Swenson (1974), Wyoming State Engineer (1974), Gries (1971), W. R. Miller (written commun., 1974), and Swenson and others (1975). These data and maps were compiled, reinterpreted, and integrated into one potentiometric map for the area included in this study (pl. 1). The potentiometric map (pl. 1) indicates that the major sources of ground water in the Madison aquifer in the study area include recharge in and near the outcrop areas in the Black Hills and Bighorn Mountains, and underflow from the Wind River Basin. Major outflows from the Madison aquifer include underflow to the east in South Dakota and to the north toward the Bighorn River in Montana, and discharge to pumping wells and flowing springs. Major cones of depression from pumping are near Midwest, Glenrock, and Newcastle, Wyo. Most springs that discharge from the Madison aquifer within the study area are located near the outcrop areas around the Black Hills.

DIGITAL SIMULATION MODEL

Background

The purpose of the simulation model is to compute the hydraulic head in an aquifer at any specified place and time. This is achieved by solving the equation of ground-water flow, which requires that the hydraulic properties, boundaries, and stresses be defined for the area modeled.

Flow equation

By following the derivation of Pinder and Bredehoeft (1968) the equation describing the transient two-dimensional flow of a homogeneous compressible fluid through a nonhomogeneous anisotropic aquifer may be written:

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) = S\frac{\partial h}{\partial t} + W(x,y,t) \qquad (1)$$

where T is the transmissivity tensor, L^2/T;
h is the hydraulic head in the aquifer, L;
S is the storage coefficient, L^0;
t is the time, T; and
W is the volume flux per unit area, L/T.

If we only consider fluxes of: (1) direct withdrawal or recharge, such as well pumpage, well injection, or evapotranspiration, and (2) steady leakage into or out of the aquifer through a confining layer or streambed, then W(x,y,t) may be expressed as:

$$W(x,y,t) = Q(x,y,t) - \frac{K_z}{m}(H_s - h) \qquad (2)$$

where Q is the rate of withdrawal (positive sign) or recharge (negative sign), L/T;
K_z is the vertical hydraulic conductivity of the confining layer or streambed, L/T;
m is the thickness of the confining layer or streambed, L; and
H_s is the hydraulic head in the source bed or stream, L.

Numerical methods

Because aquifers have variable properties and boundary conditions, exact solutions to the partial differential equation of flow (equation 1) cannot be obtained directly. Rather, a numerical solution of high accuracy is obtained using a digital computer.

Pinder and Bredehoeft (1968) showed that if the coordinate axes are aligned with the principal directions of the transmissivity tensor, equation 1 may be approximated by the following implicit finite-difference equation:

$$
\begin{aligned}
&T_{xx[i-(1/2),j]}\left[\frac{h_{i-1,j,k} - h_{i,j,k}}{(\Delta x)^2}\right] \\
&\qquad + T_{xx[i+(1/2),j]}\left[\frac{h_{i+1,j,k} - h_{i,j,k}}{(\Delta x)^2}\right] \\
&\qquad + T_{yy[i,j-(1/2)]}\left[\frac{h_{i,j-1,k} - h_{i,j,k}}{(\Delta y)^2}\right] \\
&\qquad + T_{yy[i,j+(1/2)]}\left[\frac{h_{i,j+1,k} - h_{i,j,k}}{(\Delta y)^2}\right] \\
&\quad = S\left[\frac{h_{i,j,k} - h_{i,j,k-1}}{\Delta t}\right] \\
&\qquad + \frac{q_{w(i,j)}}{\Delta x \Delta y} - \frac{K_z}{m}\left[H_{s(i,j)} - h_{i,j,k-1}\right] \qquad (3)
\end{aligned}
$$

where i,j,k are indices in the x-, y-, and time-dimensions, respectively;
$\Delta x, \Delta y, \Delta t$ are increments in the x-, y-, and time-dimensions, respectively; and
q_w is the volumetric rate of withdrawal or recharge at the (i,j) node, L^3/t.

The transmissivity terms in equation 3 are defined on boundaries between two nodes and represent the harmonic means of the transmissivities at the adjacent nodes.

The strongly implicit procedure (Stone, 1968) is used to solve equation 3 numerically. A computer program written and documented by P. C. Trescott and G. F. Pinder (written commun., 1975) was used for this analysis, which requires that the study area be subdivided into a rectangular, block-centered, finite-difference grid.

Boundary conditions

Several different types of boundary conditions can be represented in the simulation model. These include:

(1) No-flow boundary. By specifying a transmissivity equal to zero at a given node, no flow can occur across the boundary of that cell of the finite-difference grid. The numerical method used in this model also requires that the outer rows and columns of the finite-difference grid have zero transmissivities.

(2) Constant-head boundary. Where the head in the aquifer will not change over time, a constant-head condition is maintained by specifying a very high storage coefficient (10^{20}).

(3) Constant-flux. A constant rate of withdrawal or recharge may be specified for any node in the model.

(4) Vertical leakage. Vertical leakage into or out of the aquifer can occur at any node where the hydraulic head in the source bed, the vertical hydraulic conductivity of the confining layer, and the thickness of the confining layer are specified. The rate of leakage is computed implicitly by the model.

APPLICATION OF SIMULATION MODEL

Data requirements

Finite-difference grid

The limits of the modeled area were selected to include or nearly coincide with either (1) natural boundaries of the Madison Group around the Powder River Basin, (2) areas likely to be beyond the extent of major effects caused by ground-water development in the basin, or (3) the limits of the structural basin (fig. 1). Thus, the boundaries of the model were chosen on the basis of outcrop areas, the configuration of the potentiometric surface, geologic structure, and thickness variations of the Madison Group. The area included in the model exceeds 63,000 mi^2 (163,000 km^2).

The modeled area was subdivided into a rectangular finite-difference grid having 32 rows and 35 columns. A variable grid spacing was used so that the grid would be finer in areas where greater accuracy was desired. (See pl. 2.) All aquifer properties and stresses must be defined at all nodes of the grid.

By convention, nodes are located at the centers of the cells of the grid. Any specific node or cell may be referenced by citing its row (i) and column (j) location. For example, Edgemont, S. Dak., would be located in cell (27, 19).

Aquifer properties

The transmissivity of an aquifer reflects the rate at which ground water of the prevailing kinematic viscosity will flow through a unit width of the aquifer under a unit hydraulic gradient (Lohman and others, 1972). Very few data are available to describe the transmissivity of the Madison aquifer in the study area. W. R. Miller (written commun., 1974) states that data from drill-stem tests in Montana indicate that the transmissivity of the Madison ranges from 1.15×10^{-5} to 6.25×10^{-2} ft^2/s (1.07×10^{-6} to 5.81×10^{-3} m^2/s). However, the reliability of transmissivity values derived from drill-stem tests is questionable. Furthermore, the wide range in reported transmissivity values may be related to the high variability of secondary porosity and permeability development in the Madison, and may indicate that point or local tests of the Madison aquifer do not give transmissivity values that accurately reflect the ability to transmit water on a regional scale.

A flow-net analysis of the cone of depression near Midwest, Wyo., afforded an opportunity to compute a transmissivity value that is representative of a large part of the aquifer (over 200 mi^2 or 500 km^2). The method of analysis, described by Walton (1962), assumes that steady-state flow exists, with no leakage occurring to or from adjacent aquifers. The average transmissivity value was computed to be 0.013 ft^2/s ($1.2X10^{-3}$ m^2/s) using Swenson's (1974) potentiometric map and assuming a net withdrawal of about 26 ft^3/s (0.74 m^3/s). However, this transmissivity estimate may only be accurate within a factor of about two or three because of uncertainities in the long-term average rate of withdrawal, in the exact configuration of the cone of depression, and in other factors.

The storage coefficient is a measure of the volume of water released or taken into storage in an aquifer due to changes in head. Few data are available for the Madison aquifer. However, values reported in the literature for similar aquifers range between 0.00001 and 0.00025. Because steady-state flow is independent of the storage coefficient, a value for this parameter is needed only for analyses of transient (time-dependent) flow.

Boundary conditions and hydraulic stresses

Constant-head boundary cells were used where it was believed that recharge in outcrop areas or underflow into or out of the study area was sufficient to maintain the head in the aquifer at a nearly constant altitude. A summary of the areas represented as constant-head boundaries is shown in table 1. The altitudes of constant-head boundary cells were estimated either from the altitudes of the bottoms of stream valleys in or near the outcrop areas or from the potentiometric map in areas of underflow into or out of the modeled area.

Sites of known and significant well discharges from the Madison Group within the study area were represented in the model by specifying a constant flux at the corresponding nodes. These well discharges are summarized in table 2 and are believed to be accurate within a factor of less than two.

Many large springs within the study area are believed to derive some or all of their flow from the Madison aquifer. Many of these are described by Rahn and Gries (1973), Keene (1973), and Hodson (1974). The springs were simulated in the model by allowing vertical leakage at the specified nodes (also see pl. 2), which allows computed spring discharges to vary in response to temporal head changes in the aquifer. A summary of the springs represented in the simulation model is included in table 3. During the calibration of the model, described in the following section, it was necessary to allow vertical leakage in the valleys of the Tongue and Powder Rivers. The four parameters required to compute spring discharge or vertical leakage in the model using equation 2 are: (1) K_z, for which a value is assumed, (2) m, which is estimated from geologic and topographic data, (3) H_s, which is set equal to the altitude of the land surface, and (4) h, which is computed implicitly by the model. The discharge of other springs located in or near outcrop areas is not computed explicitly, but incorporated into the net recharge or discharge computed at constant-head boundaries.

Table 1.--Summary of constant-head boundaries.

Area	Number of nodes[1]	Description
1. Bighorn Mountains	8	Net effects of recharge and discharge in or near outcrop areas
2. Casper Arch	2	Underflow from Wind River Basin and recharge in outcrop areas west of model boundary.
3. Casper-Glenrock	4	Net effects of recharge from Laramie Mountains and leakage to or from North Platte River.
4. Glendo Reservoir	1	Underflow from south, leakage from Glendo Reservoir, Guernsey Reservoir, and North Platte River, and recharge in nearby outcrop areas.
5. West flank of Black Hills	11	Net effects of recharge and discharge in or near outcrop areas.
6. East flank of Black Hills	12	Net effects of recharge and discharge in or near outcrop areas.
7. East boundary of model	27	Underflow eastward to areas of lower potential.
8. Bighorn River	6	Underflow and (or) upward leakage to valleys of the Bighorn and Little Bighorn Rivers.

[1] Note that the area represented by a node varies. (See pl. 2.)

Table 2.--Summary of well discharges represented in model.

Area	Node (i,j)	Discharge ft^3/s	Discharge m^3/s
Bell Creek, Mont.	7,18	2.50	0.071
Bell Creek, Mont.	8,17	2.50	.071
Midwest, Wyo.	15,5	2.89	.082
Midwest, Wyo.	16,4	5.78	.164
Midwest, Wyo.	18,5	2.89	.082
Fiddler Creek, Wyo.	18,16	2.50	.071
Midwest, Wyo.	19,5	14.44	.409
Osage, Wyo.	19,17	2.50	.071
Newcastle, Wyo.	21,19	2.50	.071
Rapid City, S. Dak.	21,28	2.50	.071
Edgemont, S. Dak.	27,19	2.50	.071
Total		43.50	1.234

Table 3.--Summary of modeled vertical leakage.

Node (i,j)	Description	Reported discharge		Estimated thickness of confining bed		Approximate altitude of land surface	
		ft^3/s	m^3/s	ft	m	ft	m
2,14	Tongue River valley	unknown		7,100	2,164	3,250	991
2,15	Tongue River valley	unknown		6,900	2,103	3,200	975
2,23	Powder River valley	unknown		6,600	2,012	3,000	914
3,23	Powder River valley	unknown		6,600	2,012	3,050	930
13,24	Crow Creek[1]	17	0.48	500	152	3,350	1,021
14,23	Sand Creek[1]	24	.68	100	30	3,600	1,097
14,25	Spearfish Creek[1]	40	1.13	1,000	305	3,400	1,036
21,20	Stockade Beaver Creek[1]	13	.37	1,000	305	4,700	1,433
22,28	Cleghorn Spring[1]	10	.28	300	91	3,350	1,021
26,24	Beaver Creek at Buffalo Gap[1]	9	.25	500	152	3,500	1,067
27,23	Hot Springs[1]	25	.71	350	107	3,450	1,052
28,22	Cascade Spring[1]	24	.68	400	122	3,450	1,052

[1] Detailed description of springs presented by Rahn and Gries (1973).

Calibration of steady-state model

Purpose and procedure

To demonstrate that the simulation model is realistic, field observations from the aquifer must be compared with corresponding computations of the model. The best data set available for the Madison aquifer for comparative evaluation of the model is the observed potentiometric surface (pl. 1). Because most major hydraulic stresses in this aquifer have existed for many years, it was assumed that the potentiometric surface shown in plate 1 represents a steady-state (or equilibrium) flow field. For comparison with the observed potentiometric surface the model computes a steady-state potentiometric surface from the specified aquifer properties, boundaries, and hydraulic stresses. The computed steady-state potential distribution is independent of both the storage coefficient and the assumed initial conditions.

The calibration procedure aims to minimize differences between the observed and computed potentiometric surfaces by adjusting the input data (aquifer properties, boundary conditions, and hydraulic stresses) to modify the model's output. Although the large number of interrelated factors affecting ground-water flow makes this a highly subjective procedure, the degree of allowable adjustment of any parameter generally is directly proportional to the uncertainty of its value or specification. For example, because pumping rates are relatively well known, their values were not adjusted. But because the transmissivity is poorly known, various values were assumed over a range of several orders of magnitude.

Assuming various values for given parameters also helps to achieve one objective of the calibration procedure, namely to determine the sensitivity of the model to factors that affect ground-water flow. From this we may infer which factors greatly affect flow in the Madison aquifer. Evaluating the importance of each factor helps determine which data must be defined most accurately and which data are already adequate or require only minimal definition.

Another objective of calibrating the steady-state model is to improve the conceptual model of the aquifer. The conceptual model consists of our understanding of the physical and functional nature of the aquifer, including sources of recharge and discharge, rates and directions of flow, variations in aquifer properties and hydraulic potential, and its relation to surface water and other aquifers. Because the simulation model numerically integrates the effects of these several factors that affect ground-water flow, the computed results are internally consistent with all input data, and we can determine if any element of our conceptual model must be revised. In fact, the map depicting the observed potentiometric surface may be reinterpreted as a result of feedback from the model's output. In a sense, any adjustment of input data constitutes a modification of the conceptual model.

Because the observed potentiometric surface is the basis of the calibration procedure, the accuracy of the calibrated model is restricted by the accuracy of the observed potentiometric surface. In general, the observed potentiometric surface is based on relatively limited and imprecise data. Thus, the model has to be recalibrated after any major corrections or revisions are made in the observed potentiometric surface.

Adjustments of input data

Development of the present model was an evolutionary process in which successive adjustments and modifications to the model were based on the results of previous simulations. The first few models assumed uniform transmissivity, zero pumpage, and zero leakage. Adjustments of the transmissivity value over a range of several orders of magnitude produced a corresponding change in the computed total flow through the aquifer. (See pl. 2.) Available data on present recharge to and discharge from the Madison Group in this study area indicate that the total ground-water flow through the aquifer might range between 100 and 400 ft^3/s (2.83 and 11.3 m^3/s). Figure 2 indicates that a uniform transmissivity between about 0.01 and 0.04 ft^2/s (0.0009 and 0.004 m^2/s) would produce the expected flow. The average transmissivity computed for the area of the cone of depression at Midwest, Wyo., is near the lower end of this range. However, the head distribution computed by the model for these first few simulations did not agree very well with the observed potentiometric surface (pl. 1).

The next major adjustment to the model was to account for known well withdrawals from the Madison aquifer (listed in table 2). Most of the well withdrawals occur near Midwest, Wyo. The computed potentiometric surface for the Midwest area, where the large cone of depression is located, agreed best with the observed potentiometric surface for transmissivity values between 0.010 and 0.025 ft^2/s (0.0009 and 0.0023 m^2/s). However, computed heads were still hundreds of feet too high throughout most of the Powder River Basin area, even after well withdrawals were simulated.

Next a hypothesis was developed that stated that the zone of steep hydraulic gradient along the western margin of the Powder River Basin (see pl. 1) represented a zone of low transmissivity. Because this steep gradient zone coincides closely with the area where structure maps indicate that the Madison Group is most steeply dipping, and because faulting is associated with areas of steeply dipping rocks along the western and southern margins of the basin, it is possible that a major fault has offset the Madison in the subsurface and produced a barrier to flow through the aquifer. The concept (or hypothesis) that the Madison aquifer is discontinuous across a narrow zone along the western and southern margins of the basin is consistent with the observed potentiometric data, although it requires a reinterpretation of the data's significance. Total offset would form a barrier and flow would tend to be parallel to the boundary, rather than perpendicular, as indicated by the interpretation which is implicit in plate 1. Several simulation trials tested different combinations of reduced and zero transmissivities in this zone. The final selection for this preliminary model is shown in plate 2. The best results were obtained when the transmissivity was set equal to zero in the middle reach of the zone of steep hydraulic gradient, and the regional transmissivity was reduced by a factor of 100 in the northern and southern extentions of this zone. The effect of these modifications was to lower heads in the center of the basin by reducing recharge and underflow from the west. However, computed heads were still higher than observed heads in much of the area.

The next modifications allowed vertical leakage at nodes corresponding with major springs (listed in table 3). Computed spring discharges should agree with observed spring discharges. Because computed heads were still too high in the northern part of the modeled area, vertical leakage was also allowed in the northernmost parts of the Tongue River and Powder River valleys.

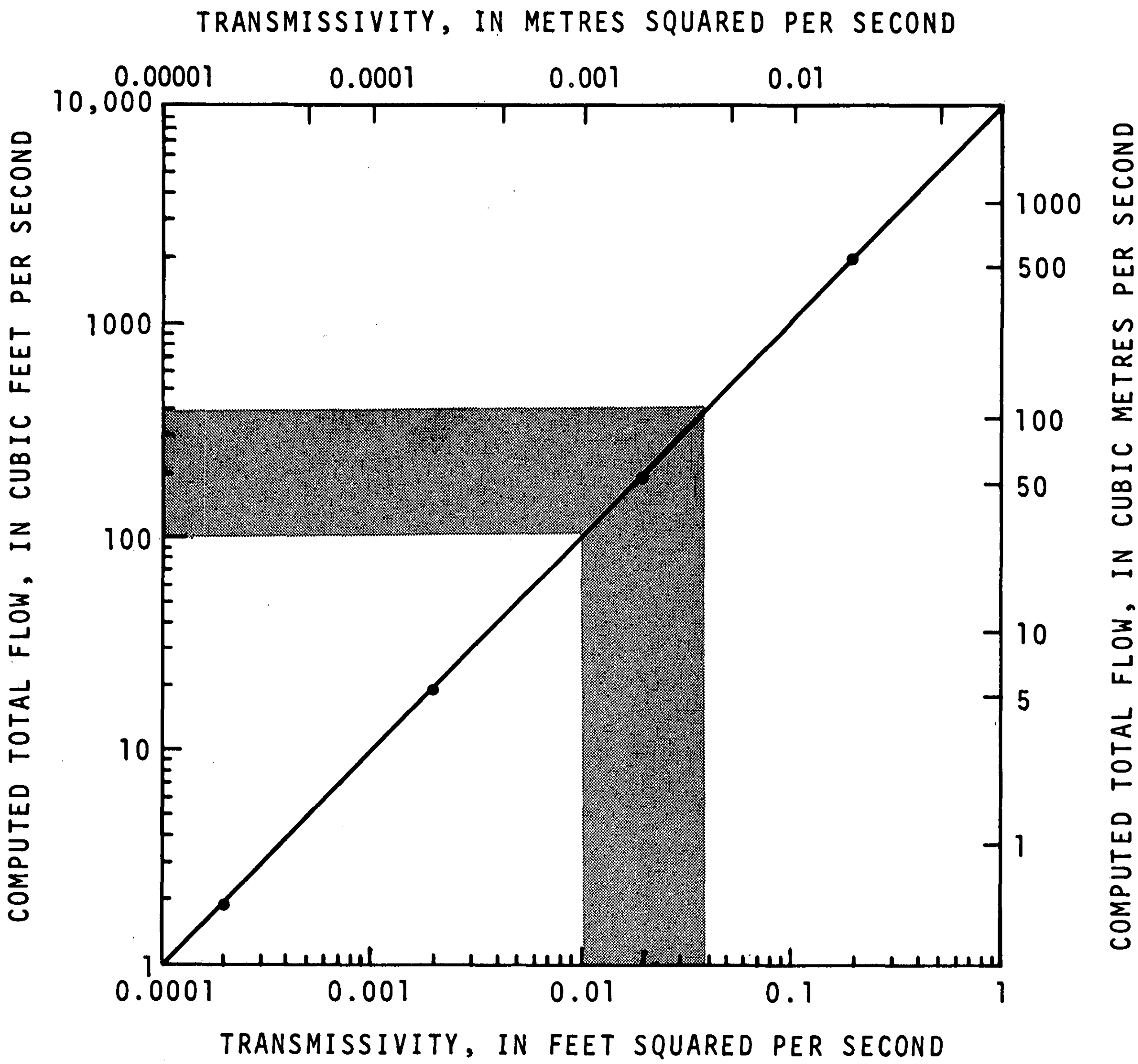

Figure 2.--Relation between assumed uniform transmissivity and total flow computed by simulation model. Pattern defines band of reasonable values.

Although no measurements indicate whether or not significant ground-water discharge occurs in these areas, the potentiometric surface contains troughs which indicate that flow converges at or near these areas. Also, the altitude of the land surface is less than the altitude of the potentiometric surface, indicating that water from the Madison could flow to the land surface. Computed spring discharges were highly sensitive to changes in the assumed altitudes of nearby constant-head cells, and moderately sensitive to adjustments of aquifer transmissivity and vertical hydraulic conductivity. Simulation tests were made for vertical hydraulic conductivity values that ranged over two orders of magnitude, and the final value selected was 2.0×10^{-7} ft/s (6.1×10^{-8} m/s). Following these modifications, computed heads were still too high in the central and southern parts of the Powder River Basin.

At this point it was concluded that the observed head distribution cannot be explained on the basis of a constant and uniform transmissivity. Although little or no information is available to describe the spatial variations in the hydraulic properties of the aquifer within the modeled area, it is known that the temperature of the ground water in the Madison varies from about 5°C (Celsius) in or near some outcrop areas to over 100°C in some deeper parts of the basin. Figure 3 shows that the kinematic viscosity of water decreases as its temperature increases. The curve in figure 3 is based on data presented by Lohman and others (1972) and Lange (1969). Thus, warm water can be transmitted through a given rock at a lower hydraulic gradient than can equal quantities of cold water. Because the kinematic viscosity of water is dependent on its temperature and transmissivity is inversely proportional to kinematic viscosity, the effective transmissivity of the aquifer will vary as a function of the ground-water temperature.

Observed temperatures in the Madison were plotted in plate 3 and contoured at a 20°C interval. The temperature data were reported by Hodson (1974), B. B. Hanshaw (written commun., 1975), W. R. Miller (written commun., 1975), and J. E. Powell (written commun., 1975). The contours in plate 3 delineate temperature zones, which in turn were used to define transmissivity zones.

Comparing the temperature data to the structure contour map indicates that the temperature is related to the altitude and depth of burial of the Madison. Temperature contours in the central and deepest part of the basin were estimated by (1) extrapolating the gradient of the temperature from margins of the basin toward the center of the basin, and (2) visual correlation with structure contours. These lines of evidence suggest that the temperature of water in the Madison may exceed 120°C in the deepest parts of the basin. These high temperatures indicate that large parts of this study area may warrant further study as a possible geothermal resource.

The previous trial and error simulations indicated that a mean transmissivity of 0.023 ft^2/s (2.1×10^{-3} m^2/s) near Midwest, Wyo., would provide an adequate but preliminary standard of reference for adjusting the transmissivity in other areas. The average transmissivity within a given temperature zone was computed by multiplying the transmissivity at Midwest by the ratio of the kinematic viscosity at the prevailing water temperature at Midwest to the kinematic viscosity in the given temperature zone. The transmissivity adjustments are summarized in table 4. The standard temperature of 83°C is the average reported temperature in eight wells in the Midwest cone of depression.

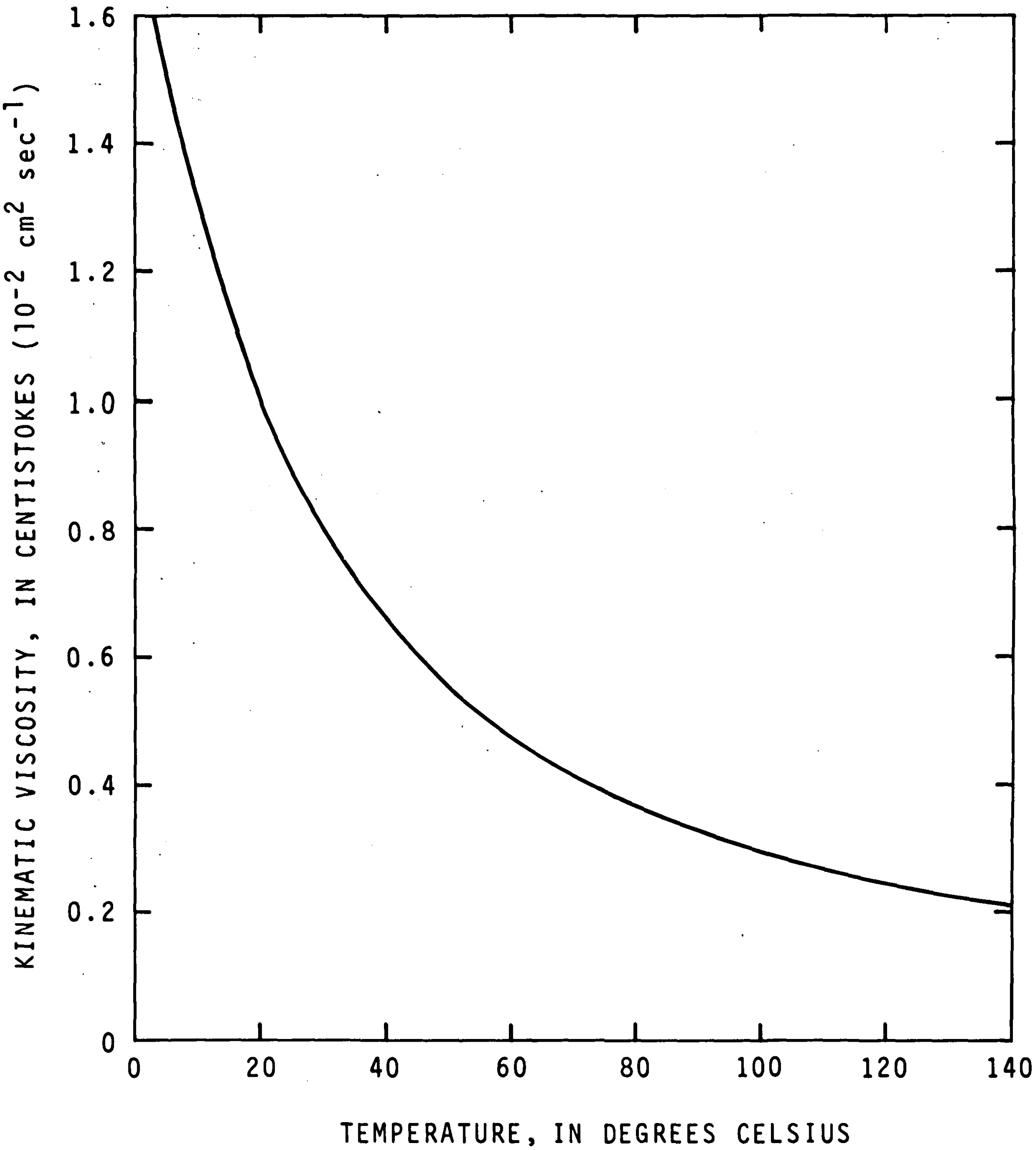

Figure 3.--Relation between kinematic viscosity and temperature of water.

Table 4.--Transmissivity adjustments based on temperature variations.

Temperature range (°C)	Mean temperature of zone (°C)	Kinematic viscosity, in centistokes (10^{-2} cm^2 sec^{-1})	Viscosity ratio[1]	Adjusted transmissivity	
				ft^2/s	m^2/s
0-20	10	1.31	.271	0.006	0.0006
20-40	30	.804	.441	.010	.0009
40-60	50	.556	.639	.015	.0013
60-80	70	.416	.853	.020	.0018
standard[2]	83	.355	1.00	.023	.0021
80-100	90	.328	1.08	.025	.0023
100-120	110	.269	1.32	.030	.0028
120-140	130	.227	1.56	.036	.0033

[1] This multiplication factor equals the ratio of (1) the kinematic viscosity at the temperature of the standard of reference, to (2) the kinematic viscosity at the mean temperature of a zone.

[2] All transmissivity adjustments are referenced to the standard transmissivity at 83°C.

These adjustments increased the transmissivity in the center of the basin and decreased it near the outcrop areas. This in turn resulted in lower computed heads in the center of the basin and achieved better agreement between the observed and computed potentiometric surfaces. However, lower transmissivities in and near the outcrop areas produced spring discharges that were too low. Hence, transmissivities in the outcrop areas only were readjusted upward by a factor of 5 to offset the transmissivity adjustment for temperature variations. The intrinsic permeability of the rocks in the Madison Group may be greater in or near the outcrop areas because of increased weathering, fracturing, and solution.

Results

The steady-state potentiometric surface computed by the calibrated model for this preliminary study is presented in plate 4. The major features of the observed potentiometric surface are generally well reproduced. The computed surface shows: (1) the major cone of depression near Midwest, Wyo.; (2) a zone of steep hydraulic gradient close to and parallel to the Bighorn Mountains; (3) relatively steep hydraulic gradients surrounding the Black Hills; (4) a discontinuity, which acts as a no-flow boundary, between the Midwest area and the center of the Powder River Basin; (5) small cones of depression developed around major spring discharge areas; and (6) underflow out of the area across parts of the model's northern and eastern boundaries. Because the hypothesis proposing a hydraulic discontinuity along the western margin of the Powder River Basin represents a significant revision of the conceptual model, and has a major effect on the flow field in the aquifer, the hypothesis should be further tested to verify the existence, origin, extent, and impact of the discontinuity. Other discontinuities may also exist elsewhere in the basin. If a discontinuity is caused by faulting or by an abrupt facies change, it should be identifiable using geophysical methods.

Computed heads were generally too low in the northeastern quarter of the modeled area and too high in the rest of the modeled area. The difference between the observed and computed heads is less than 300 ft (91 m) in about 80 percent of the modeled area. However, computed heads were over 400 ft (122 m) too high at some places in the southern part of the Powder River Basin. Although some of these differences can be attributed to errors in the interpretation of observed data, most of the differences are undoubtedly caused by errors in the input data to the simulation model. For example, unknown local and regional variations of either transmissivity or vertical leakage might be the cause of these differences. These errors will not be easily resolved without the collection of additional field data.

Observed and computed spring discharges were also compared as part of the calibration procedure. Table 5 itemizes the vertical leakage rates computed by the model, and shows that computed total spring discharge was only about one-half of the observed total. The computed discharge at a specific node was within 25 percent of the observed only for Sand Creek, Stockade Beaver Creek, and Beaver Creek at Buffalo Gap. One possible explanation for the discrepancy between the observed and computed data is that the observed flows of some of the springs may not be derived entirely from the Madison Group. For example, Keene (1973) suggests that Cascade Spring derives its water from the Minnelusa Formation, which overlies the Madison. This hypothesis may also apply to a degree to some of the other springs. It is also possible that springs are

Table 5.--Comparison of computed and observed vertical leakage.

Location	Node	Computed leakage		Observed leakage	
		ft^3/s	m^3/s	ft^3/s	m^3/s
Tongue River	2,14-15	9.1	0.26	unknown	
Powder River	2-3,23	7.0	.20	unknown	
Crow Creek[1]	13,24	5.7	.16	17	0.48
Sand Creek[1]	14,23	21.	.59	24	.68
Spearfish Creek[1]	14,25	5.9	.17	40	1.13
Stockade Beaver Creek[1]	21,20	11.6	.33	13	.37
Cleghorn Spring[1]	22,28	6.6	.19	10	.28
Beaver Creek at Buffalo Gap[1]	26,24	11.2	.32	9	.25
Hot Springs[1]	27,23	9.9	.28	25	.71
Cascade Spring[1]	28,22	7.0	.20	24	.68

[1] Spring discharge.

localized in zones of higher transmissivity, due to either fractures or solution openings. Such variations in aquifer properties cannot yet be described.

A mass balance for each simulation run is calculated to check the numerical accuracy of the solution. As part of these calculations, the net flux contributed by each hydrologic component of the model is computed and is tabulated as part of the hydrologic budget for the aquifer. The hydrologic budget provides a measure of the relative importance of each element of the budget. The hydrologic budget for the final calibration of the steady-state model is presented in table 6. As expected, it shows that the Bighorn Mountains and Black Hills supply most of the recharge to the Madison aquifer, and that springs are the largest single source of discharge. The total flux through the aquifer is slightly over 200 ft^3/s (5.7 m^3/s). However, because much of the spring discharge is simply derived from recharge in nearby outcrop areas, significantly less than 200 ft^3/s (5.7 m^3/s) is actually flowing through the major part of the area of the aquifer.

The hydrologic budget also indicates that there is very little flow either to or from the southern margins of the basin under existing steady-state conditions. The extent and hydraulic continuity of the Madison aquifer in the area south of the Powder River Basin and east of the Laramie Mountains are poorly defined. Additional study in this area is therefore needed because an understanding of the flow system in the Madison aquifer requires a knowledge of its boundaries as well as its hydraulic properties.

Steady-state vertical leakage is another element of the hydrologic budget that cannot presently be verified. Improved model results obtained when vertical leakage was allowed in the Tongue and Powder River valleys do not prove it is actually occurring. However, it may be more widespread than has been projected in the model. Frickel and Shown (1974) presented a summary of streamflow data available for the Powder River Basin. The computed rates of ground-water discharge due to vertical leakage represent less than one percent of the reported average annual flows of the Tongue and Powder Rivers. Thus, even if leakage occurs at the computed rates, it probably could not be detected from streamflow records. Verification may require considerable study and test drilling.

Of course, vertical leakage need not flow directly to the land surface. Ground water may flow between the Madison aquifer and underlying or overlying aquifers, depending upon the relative hydraulic potentials. Because data were not yet available to map the potentiometric surfaces of either underlying or overlying aquifers, it was assumed that no steady-state leakage occurred to or from either underlying or overlying aquifers. But this factor must be more carefully evaluated in future studies. Sufficient potentiometric data may be available for aquifers that overlie the Madison.

Regional transmissivity variations appear to have an important effect on heads and flow in the Madison aquifer. Although part of this effect is caused by changes in the viscosity of the ground water, additional transmissivity variations are caused by variations in the properties of rocks. Both are important factors, but the latter is presently unknown. Because of the difficulty of directly measuring regional variations in properties of the rocks that can be quantitatively related to transmissivity, geophysical methods would seem to offer an alternative means for solving this problem for the Madison.

Table 6.--Elements of hydrologic budget computed by simulation model.

	Computed flux[1]	
	ft^3/s	m^3/s
Total well discharge[2]	- 43.5	-1.23
Constant-head boundaries:		
Bighorn Mountains	49.9	1.41
Casper Arch	22.0	.62
Casper-Glenrock	1.1	.03
Glendo Reservoir	0.4	.01
West flank of Black Hills	75.1	2.13
East flank of Black Hills	53.6	1.52
East boundary of model	- 32.0	- .92
Bighorn River area	- 31.6	- .89
Vertical leakage:		
Tongue River valley	- 9.1	- .26
Powder River valley	- 7.0	- .20
Total spring discharge[3]	- 78.9	-2.23
Total flux:		
Recharge	202.1	5.72
Discharge	-202.1	-5.72

[1] A positive value in this table indicates recharge to the aquifer; a negative value denotes discharge.

[2] See table 2.

[3] See table 5.

Because of the uncertainty in defining many of the factors that affect ground-water flow in the Madison aquifer, the final calibrated model described in this preliminary report contains some errors. Although additional adjustments of the model's parameters may remove some of the remaining differences between the observed and computed data, the quantity and quality of presently available data do not justify the expense of making further adjustments to the input data of this preliminary model. Even though many factors that affect ground-water flow in the Madison are by no means clearly defined or understood, it seemed advisable to use the present model to make preliminary estimates of the range of possible effects that developments of the Madison aquifer might impose on water levels, recharge, discharge, and pattern of flow.

Evaluation of transient model

Purpose and procedure

Any large-scale, sustained, withdrawals of water from the Madison aquifer will necessarily affect the potentiometric levels, recharge, and discharge of the aquifer. If the extent, magnitude, and timing of these effects could be accurately predicted, then proposed developments could be evaluated objectively on the basis of physical evidence. The main purpose of modifying the steady-state model into a transient ground-water flow model in this study is to demonstrate the capability of this modeling technique to predict the effects of major ground-water withdrawals. This predictive capability is illustrated by an example in which the transient model is used to analyze the effects of proposed withdrawals in Niobrara County, Wyo. The example helps to determine which physical parameters must be included in a transient model, which data are adequately defined, and which must be better understood. A secondary objective of the transient analysis is the estimation of the range of effects to be expected from this proposed development. This, in turn, can be used as a guide to design a monitoring network for the Madison aquifer.

The transient model requires a value for the storage coefficient. Because this parameter is poorly defined for the Madison aquifer, simulation tests were made using three values (0.00001, 0.00005, and 0.00025) that differ by a factor of five and cover the range of values reported in the literature for similar aquifers. If ground-water withdrawals produce significant drawdown in the Madison, leakage might be induced from or leakage reduced to overlying or underlying formations. Thus, the model also tests a variety of leakage conditions, ranging from zero leakage to unlimited leakage, by assuming a range of values of leakance (the ratio of vertical hydraulic conductivity to thickness of the confining bed). Using different combinations of values for the storage and leakance coefficients produces a family of computed aquifer responses. The response of the real aquifer should then be included in the set of responses computed by the model.

The transient model was run for a total simulation period of 100 years after the start of pumping. Although it is not anticipated that pumping would occur at one location for 100 years, this duration allows results to be analyzed and compared for any shorter anticipated project life. It also permits an estimate of the maximum possible effects of continuous withdrawal, because in most cases analyzed, the aquifer has reestablished steady-state flow in less than 100 years. The size of the initial time step used in the model was approximately 10 hours. Subsequent time steps were increased by a factor of

4.0, so that nine time steps were required to simulate 100 years. The model computes heads, drawdowns, and a mass balance for each time step.

The potentiometric levels computed by the calibrated steady-state model were used as initial conditions for each of the tests of the transient model. All computed changes are thus relative to one internally consistent set of base data. The only change to the aquifer properties, boundaries, and hydraulic stresses used in the steady-state model, other than storage and leakance, was the addition of a pumping stress of 20 ft^3/s (0.57 m^3/s) in Niobrara County, Wyo. In the model the total pumpage was distributed over three nodes, as follows: 50 percent at (27,16), 25 percent at (27,17), and 25 percent at (28,15).

Nonleaky confined aquifer

The assumption of nonleaky conditions would produce the greatest possible drawdown for a given pumping rate. The computed decline (or drawdown) of the potentiometric surface over time at selected points is illustrated in figures 4 to 8. Each figure shows three curves, representing the simulation results for three different assumed storage coefficients. It should be emphasized that these curves represent a numerical solution to a mathematical problem that is defined by the specified aquifer properties, boundary conditions, and hydraulic stresses. The computed curves will represent the true aquifer responses only to the degree that the modeled aquifer specifications actually approximate the true aquifer properties, boundary conditions, and hydraulic stresses.

Figure 4 shows that after 100 years of pumping the potentiometric surface in Niobrara County, Wyo., near the pumping wells would decline by nearly 700 ft (213 m), regardless of the storage coefficient. The drawdowns shown in figure 4 were computed at node (27,16), which is also a pumping node. The computed drawdown does not represent the drawdown in the pumping well, but rather represents an average drawdown for the area of the cell represented by that node. The relative positions of the three curves in figure 4 indicate that the cone of depression would spread most rapidly for the smallest storage coefficient. All three curves show that major effects would occur here even after only 1 year of pumping, when the drawdown would range from 220 to 550 ft (67 to 168 m), depending on the storage coefficient.

Similarly, figure 5 shows that the maximum drawdown predicted for node (27,19), near Edgemont, S. Dak., is about 360 ft (110 m). Again, the time-rate of drawdown is sensitive to the assumed value of the storage coefficient. For example, 250 ft (76 m) of drawdown would occur after 1 year for S = 0.00001, after 5 years for S = 0.00005, and after 25 years for S = 0.00025. For this and other areas away from the pumping nodes in Niobrara County, the model predicts the decline in the potentiometric surface that would be measured in an observation well. Figure 6 shows the time-drawdown curves computed for node (21,19), near Newcastle, Wyo. Here the maximum drawdown after 100 years would be about 100 ft (30 m). Figure 7 presents the time-drawdown curves computed for node (11,12), near Gillette, Wyo. The curves show that the maximum drawdown at this location would be approximately 125 ft (38 m). Figure 8 illustrates the time-drawdown curves for node (25,30), in northeastern Custer County, S. Dak. In this area the drawdowns would always be small, not exceeding 10 ft (3 m).

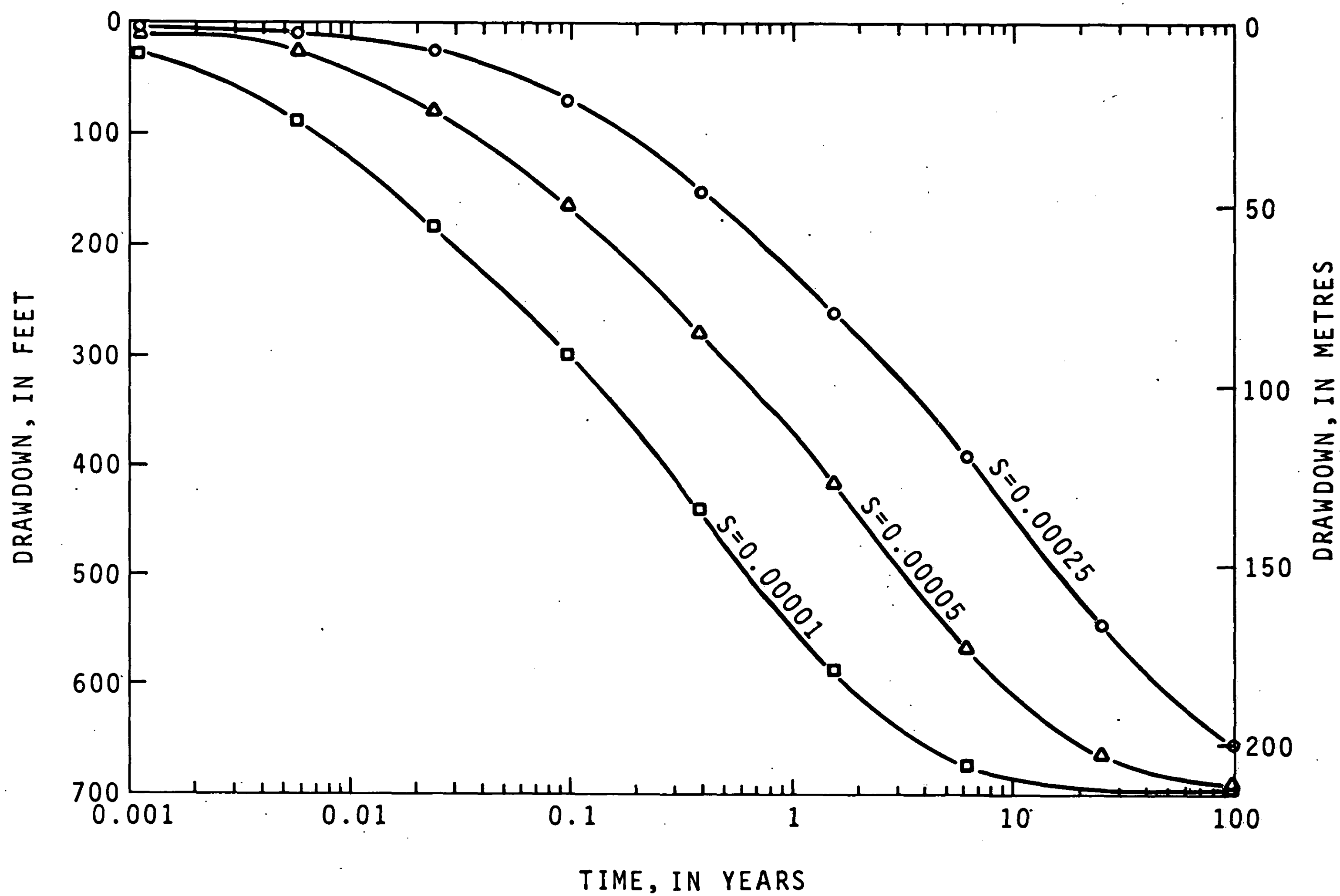

Figure 4.--Nonleaky time-drawdown curves for node (27,16), near hypothetical pumping wells in Niobrara County, Wyo.

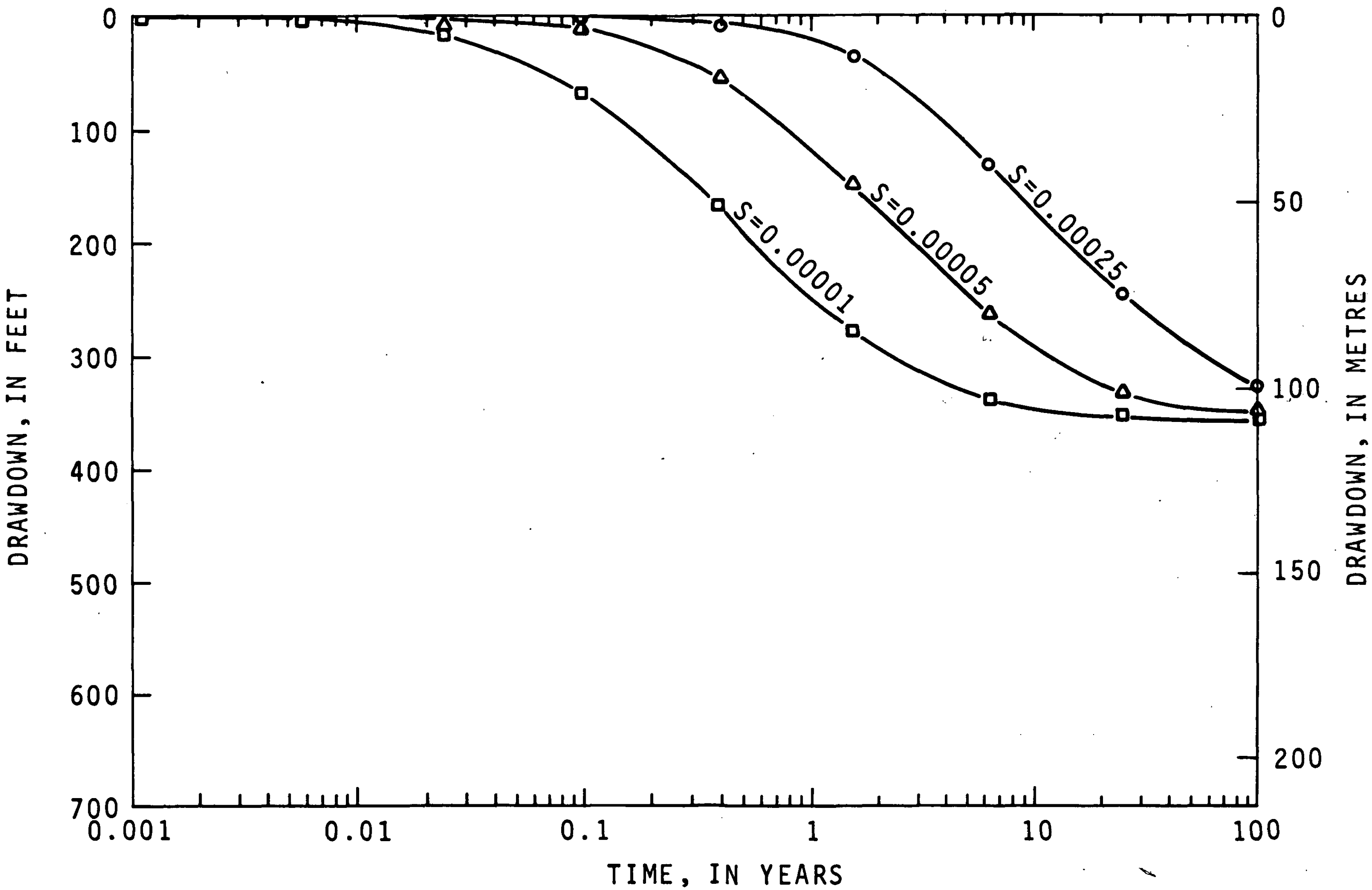

Figure 5.--Nonleaky time-drawdown curves for node (27,19), near Edgemont, S. Dak.

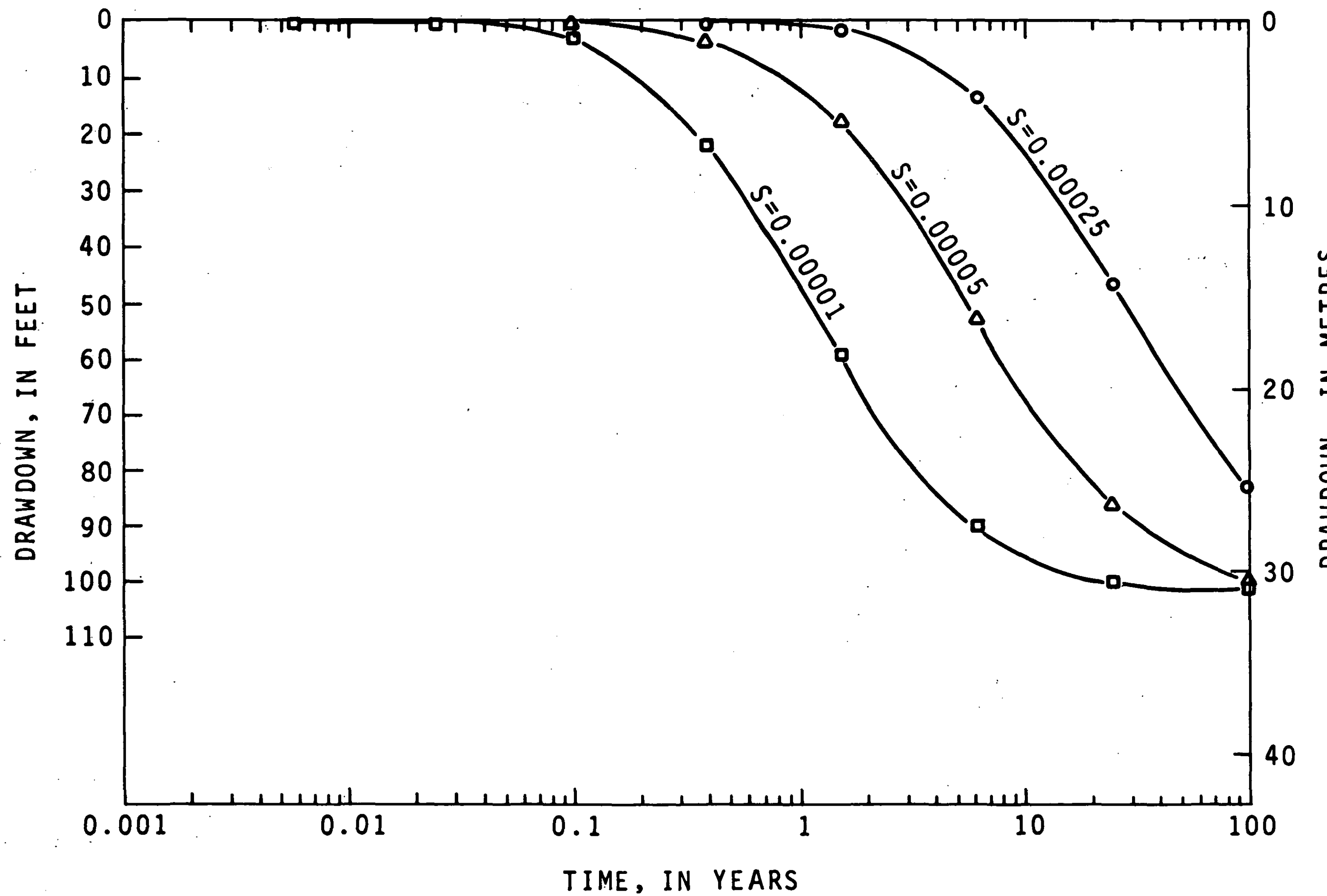

Figure 6.--Nonleaky time-drawdown curves for node (21,19) near Newcastle, Wyo.

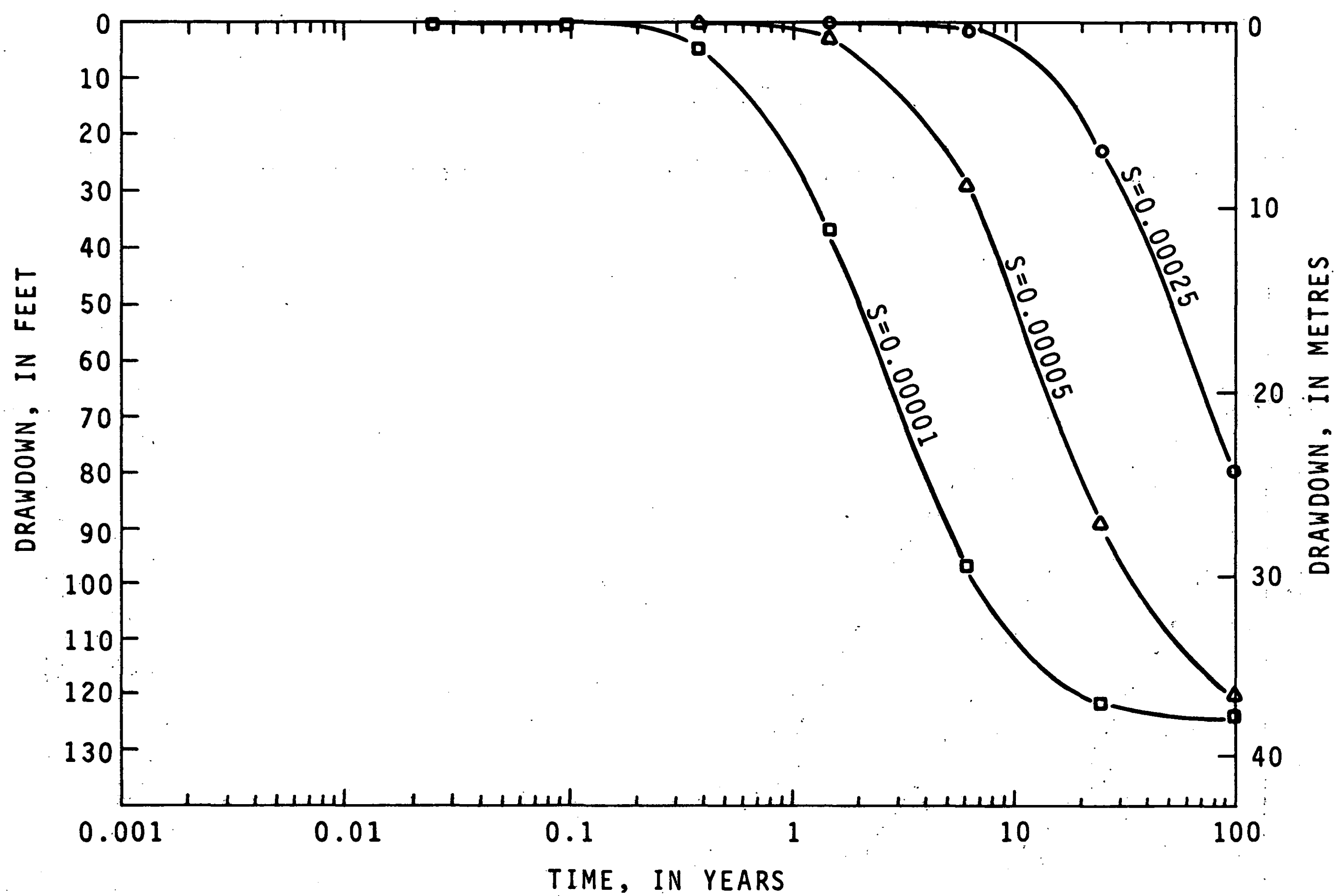

Figure 7.--Nonleaky time-drawdown curves for node (11,12), near Gillette, Wyo.

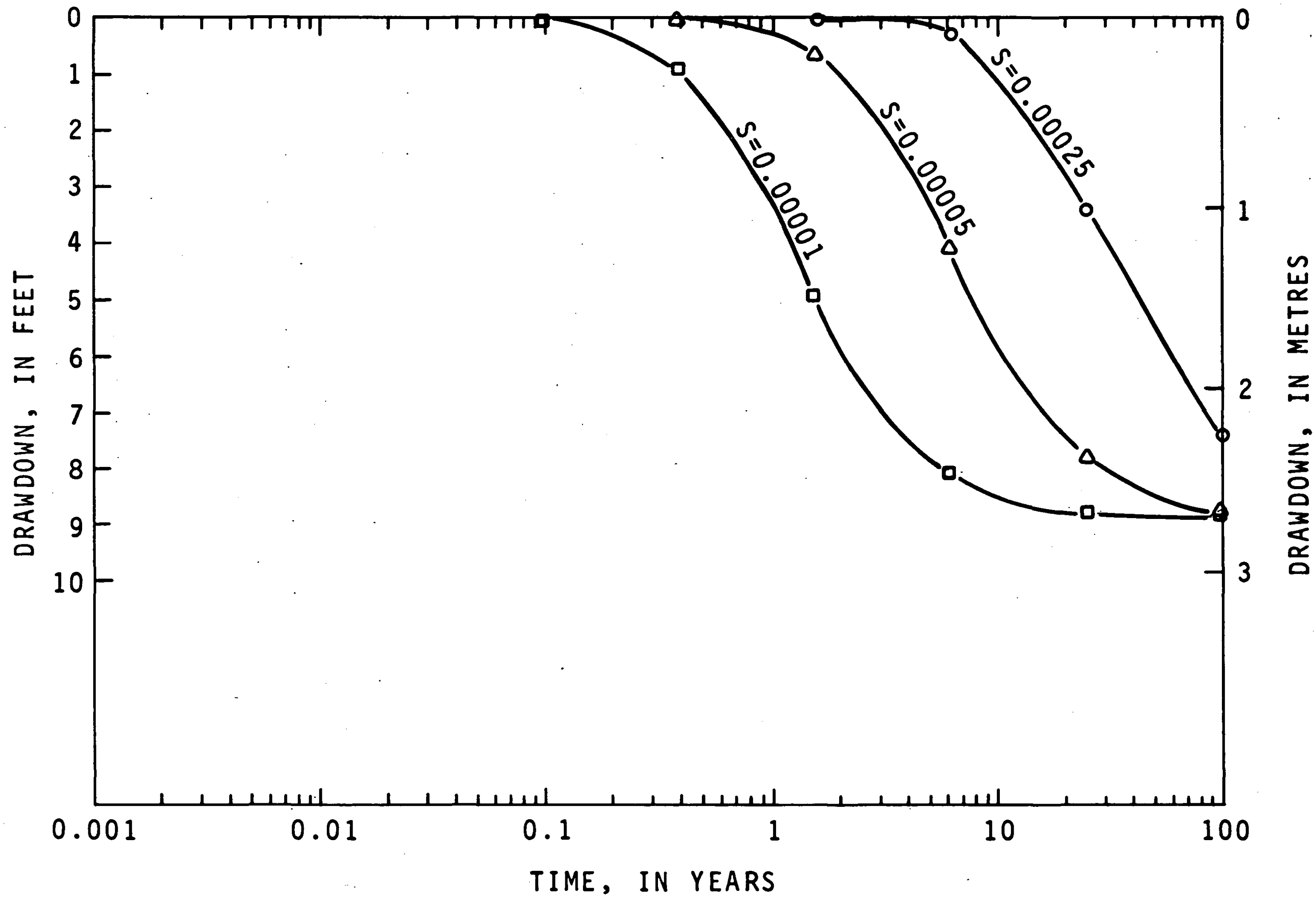

Figure 8.--Nonleaky time-drawdown curves for node (25,30), northeastern Custer County, S. Dak.

The areal variations in drawdown for S = 0.00005 in a nonleaky confined aquifer after 100 years of hypothetical pumping are contoured in plate 5. This map clearly indicates that the greatest effects would occur closest to the center of pumping in Niobrara County, Wyo., but that drawdowns over 50 ft (15 m) would occur up to 175 mi (282 km) from the center of pumping. Significant drawdowns would occur throughout most of the Powder River Basin, but little drawdown would occur east of the Black Hills. If the center of pumping were located elsewhere in the Powder River Basin, the center of the cone of depression would shift accordingly. If the center of pumping were located in an area of higher transmissivity, and further from no-flow boundaries, the magnitude of the drawdown would probably be less than shown in plate 5, although the area of influence would probably be as extensive.

An analysis of the mass balance and hydrologic budget indicates that the well withdrawals of 20 ft^3/s (0.57 m^3/s), which were added to the steady-state model, are partly derived from increased recharge, partly from decreased discharge, and partly from a reduction in the amount of water stored in the aquifer. The last factor is most important soon after pumping begins and least significant as the aquifer approaches a new steady-state flow system after many years. The most significant decrease in discharge was computed for vertical leakage at node (28,22), which represents discharge from Cascade Spring. The computed decrease in discharge of Cascade Spring with time is shown in figure 9. The computed maximum reduction in flow of this spring is 4.0 ft^3/s (0.11 m^3/s). A maximum reduction in discharge of about 1.2 ft^3/s (0.034 m^3/s) was also computed for Hot Springs at node (27,23) and for Stockade Beaver Creek at node (21,20).

The rate of drawdown over time depends in part on the storage coefficient of the aquifer. Without field measurements the storage coefficient cannot be accurately defined. Aquifer tests using one or more observation wells can be used to determine the storage coefficient. Ideally aquifer tests should be performed in several different parts of the basin to determine if this parameter has a significant spatial variation.

Leaky confined aquifer

The nonleaky analyses produce estimates of the maximum possible drawdowns for the given aquifer properties and withdrawal rates. However, it is probable that the significant drawdowns created by large ground-water withdrawals would cause sufficient head differences between the Madison aquifer and adjacent aquifers to induce vertical leakage into the Madison. This vertical leakage is a form of recharge that would offset some of the effects of well discharge, and reduce the extent and magnitude of the cone of depression.

In the transient leaky analysis, the heads in both the aquifer and the source bed were initially set equal to the computed steady-state heads. Thus, vertical leakage would occur only after a change in head occurs in the aquifer. This model analysis is not based on the geological identity of the possible source beds or confining beds, and it assumes that the head in the source bed will remain constant over time. A range in values for the leakance factor (K_z/m) of the confining bed was tested by the model. To illustrate the difference between aquifer responses under leaky conditions compared with nonleaky conditions, all leaky tests were made using the intermediate value of the storage coefficient (S = 0.00005).

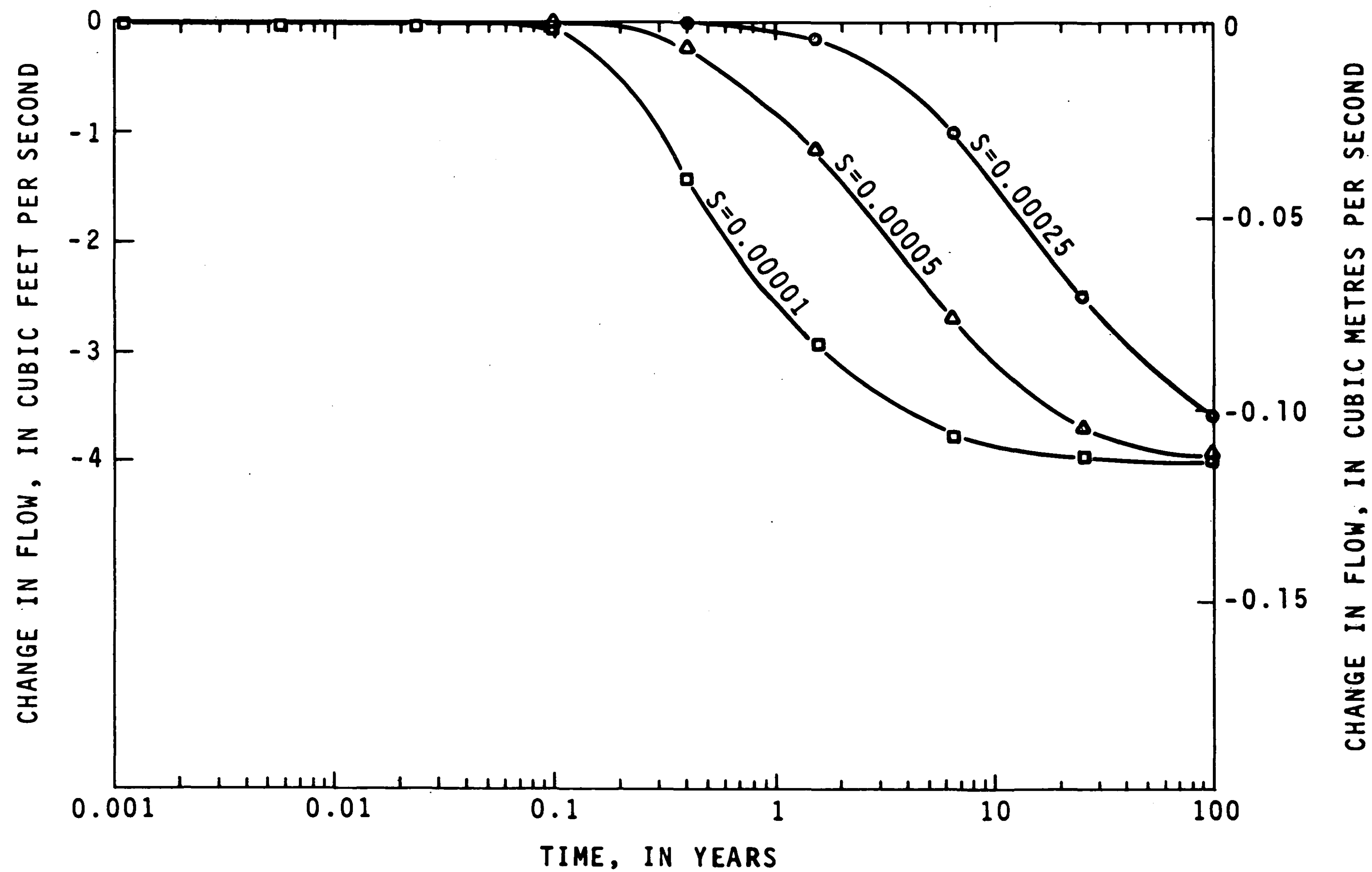

Figure 9.--Computed decrease over time in vertical leakage at node (28,22), which represents the discharge of Cascade Spring, S. Dak., for nonleaky aquifer conditions.

A value of $K_z/m = 0$ represents the lower limit of leakance, or nonleaky conditions, and aquifer responses for this value were presented in figures 4 through 9. A value of $K_z/m = 10^{-9}$ represented an upper limit of leakance, because it allowed unlimited leakage that immediately replaced any well withdrawals and precluded any significant drawdown from occurring. Figures 10 through 13 present time-drawdown for several values of leakance, and indicate that a value of K_z/m between 10^{-13} and 10^{-11} (ft/s)/ft [(m/s)/m] would produce reasonable aquifer responses. In general there is not much difference between the curves for $K_z/m = 0$ and $K_z/m = 10^{-13}$. Hence the latter value may be too conservative. On the other hand, a value of 10^{-11} appears to allow excessive leakage, and may be too high.

Comparison of the leaky and nonleaky curves shown in figures 10 through 13 indicates that the major effects of leakage are to reduce the magnitude of drawdown and to stabilize the cone of depression at an earlier time. For example, figure 10 shows a significant difference between the nonleaky curve ($K_z/m = 0$) and the leaky curve for $K_z/m = 10^{-12}$. At this node near the pumping wells the nonleaky curve stabilizes at a maximum drawdown of about 790 ft (241 m) after 100 years, while the leaky curve ($K_z/m = 10^{-12}$) stabilizes at a maximum drawdown of about 390 ft (119 m) after only 20 years. Similar effects are shown in figures 11 through 13.

The areal variations in drawdown computed with $S = 0.00005$ and $K_z/m = 10^{-12}$ after 100 years of pumping are illustrated in plate 6, which shows that drawdowns in excess of 50 ft (15 m) are essentially within about 30 mi (48 km) of the center of pumping. A comparison of plate 6 with plate 5 clearly illustrates that vertical leakage significantly reduces the extent and magnitude of the cone of depression. Although plate 6 shows drawdowns after 100 years, there was essentially no change in the computed head distribution after 20 years of pumping. Leakance values smaller than 10^{-12} would produce a more extensive cone of depression, while values greater than 10^{-12} would result in a less extensive cone of depression.

Because some of the pumped water is derived from vertical leakage, less water must be derived from an increase in recharge, from a decrease in discharge, or from water stored in the aquifer. Figure 14 shows that the computed decrease in vertical leakage at node (28,22), which represents discharge from Cascade Spring, would be about 1.1 ft^3/s (0.031 m^3/s) if $K_z/m = 10^{-12}$ (ft/s)/ft [(m/s)/m], whereas the computed decrease was almost 4.0 ft^3/s (0.11 m^3) for equivalent nonleaky conditions.

It is apparent that vertical leakage, if it occurs, could have a significant impact on the response of the Madison aquifer to large ground-water withdrawals. If the confining beds are extremely thick and (or) highly compressible, then the specific storage of the confining bed should be considered to account for transient releases of water from storage in the confining bed. Also, if vertical leakage is significant, it is likely that withdrawals from the Madison will cause the head in the source bed to decrease. The effects of transient flow in both the confining bed and source bed would probably produce less vertical leakage into the Madison than the model computed by assuming a constant head in the source bed. These effects would produce more complex response curves, which may initially follow the curves for a given leakance value, but then diverge in the direction of greater drawdown. These transient effects might be analyzed by treating the Madison either as part of a two-layer aquifer system or as part of a three-dimensional flow system. Existing digital

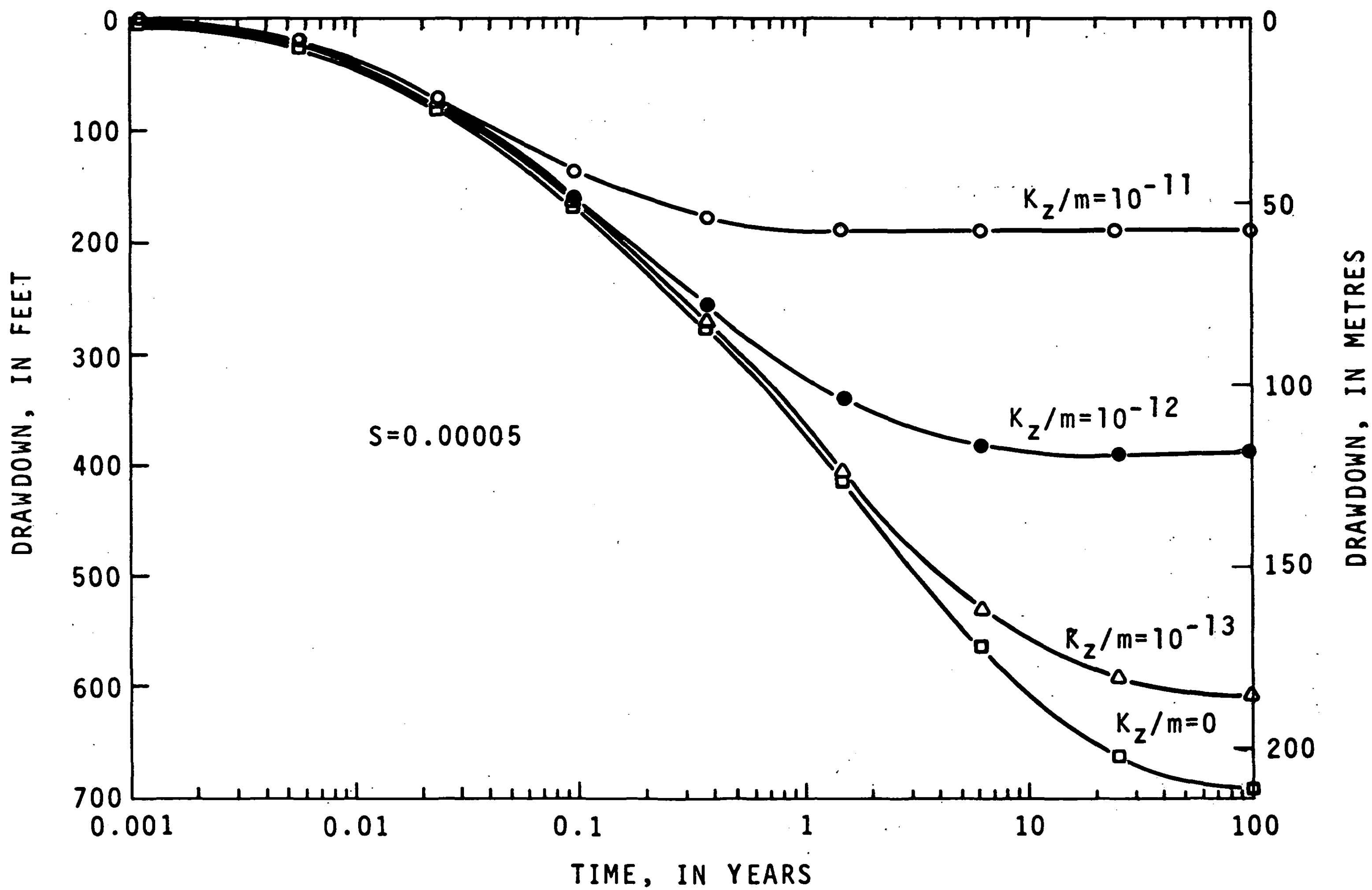

Figure 10.--Leaky time-drawdown curves for node (27,16), near hypothetical pumping wells in Niobrara County, Wyo.

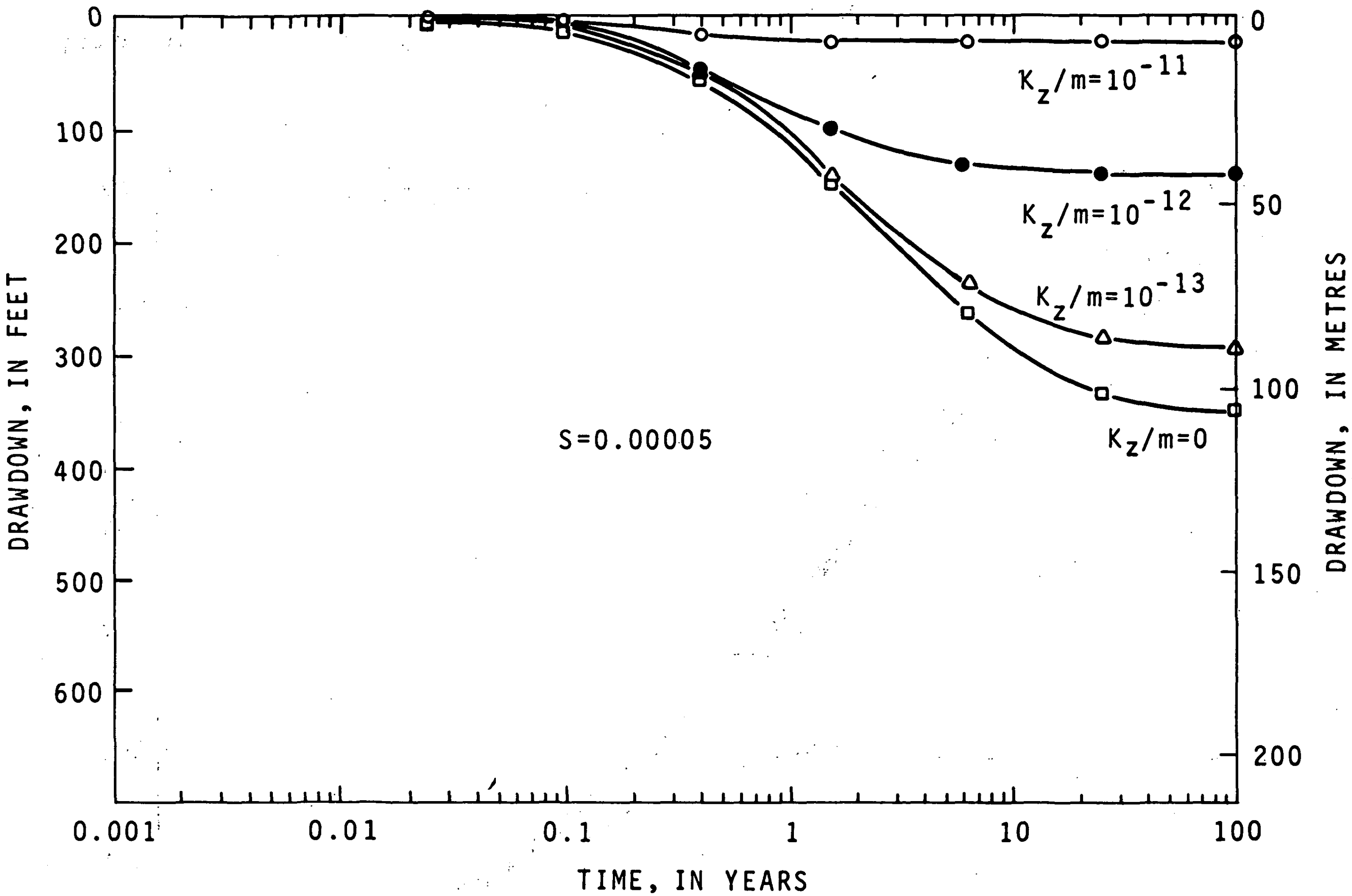

Figure 11.--Leaky time-drawdown curves for node (27,19), near Edgemont, S. Dak.

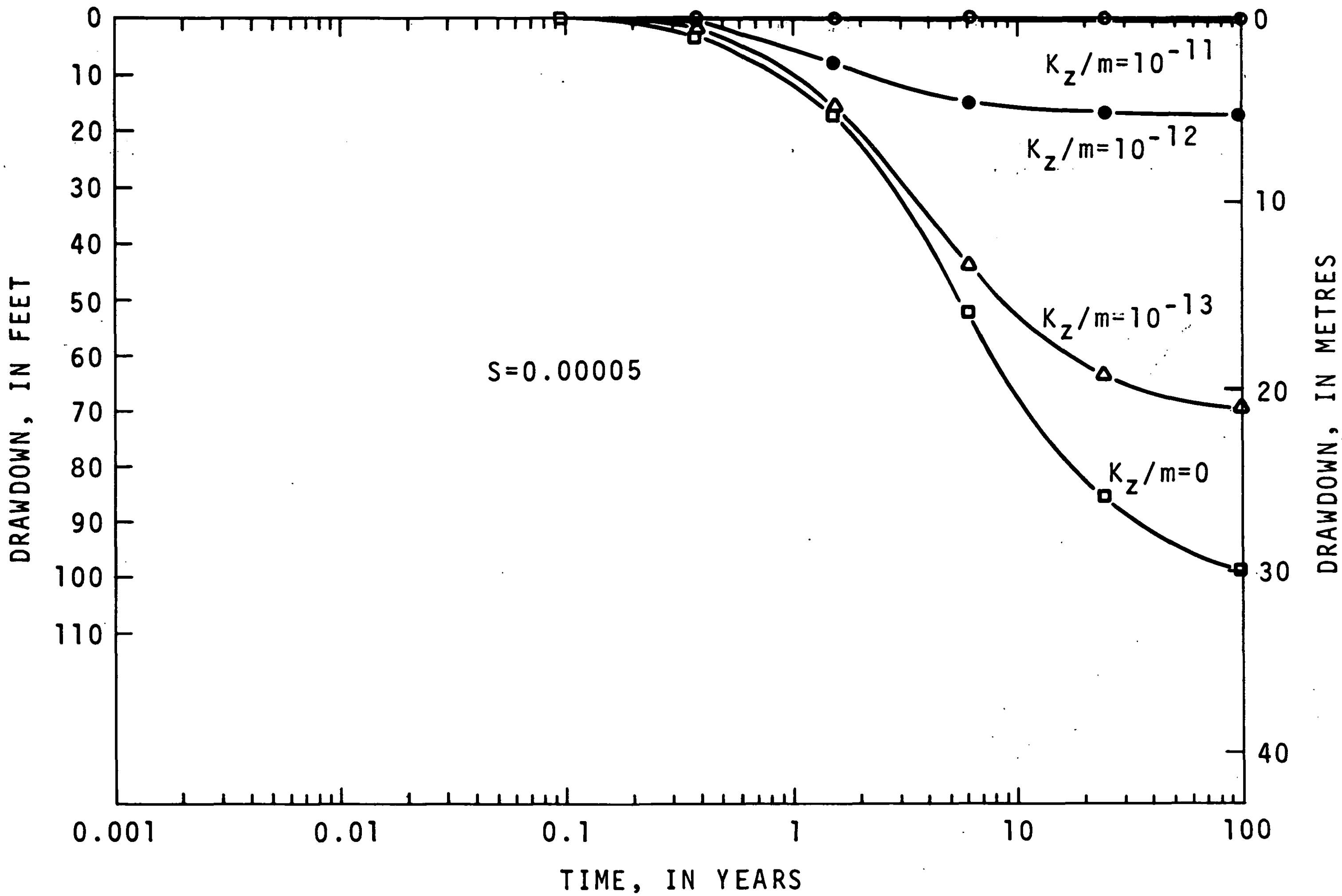

Figure 12.--Leaky time-drawdown curves for node (21,19), near Newcastle, Wyo.

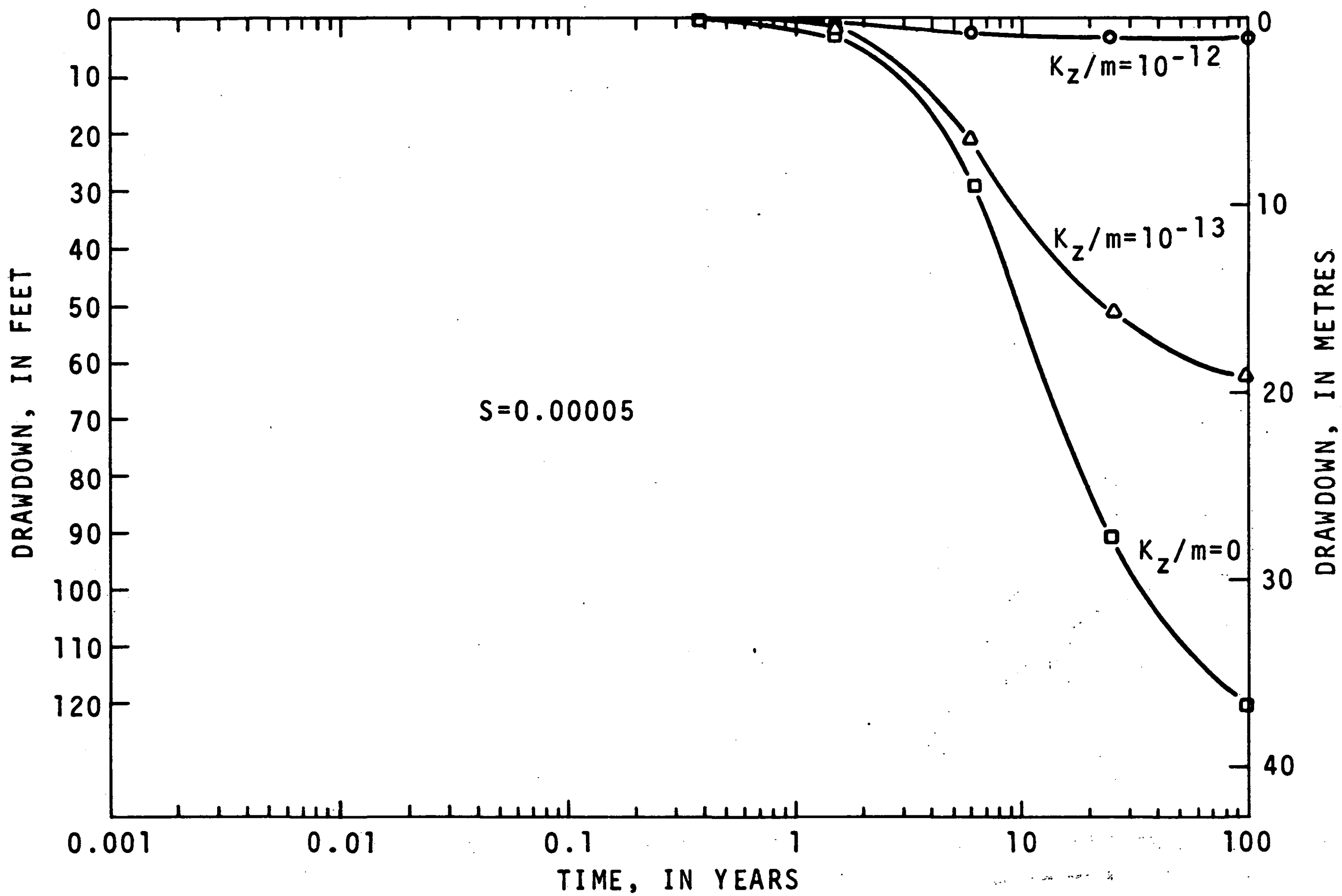

Figure 13.--Leaky time-drawdown curves for node (11,12), near Gillette, Wyo.

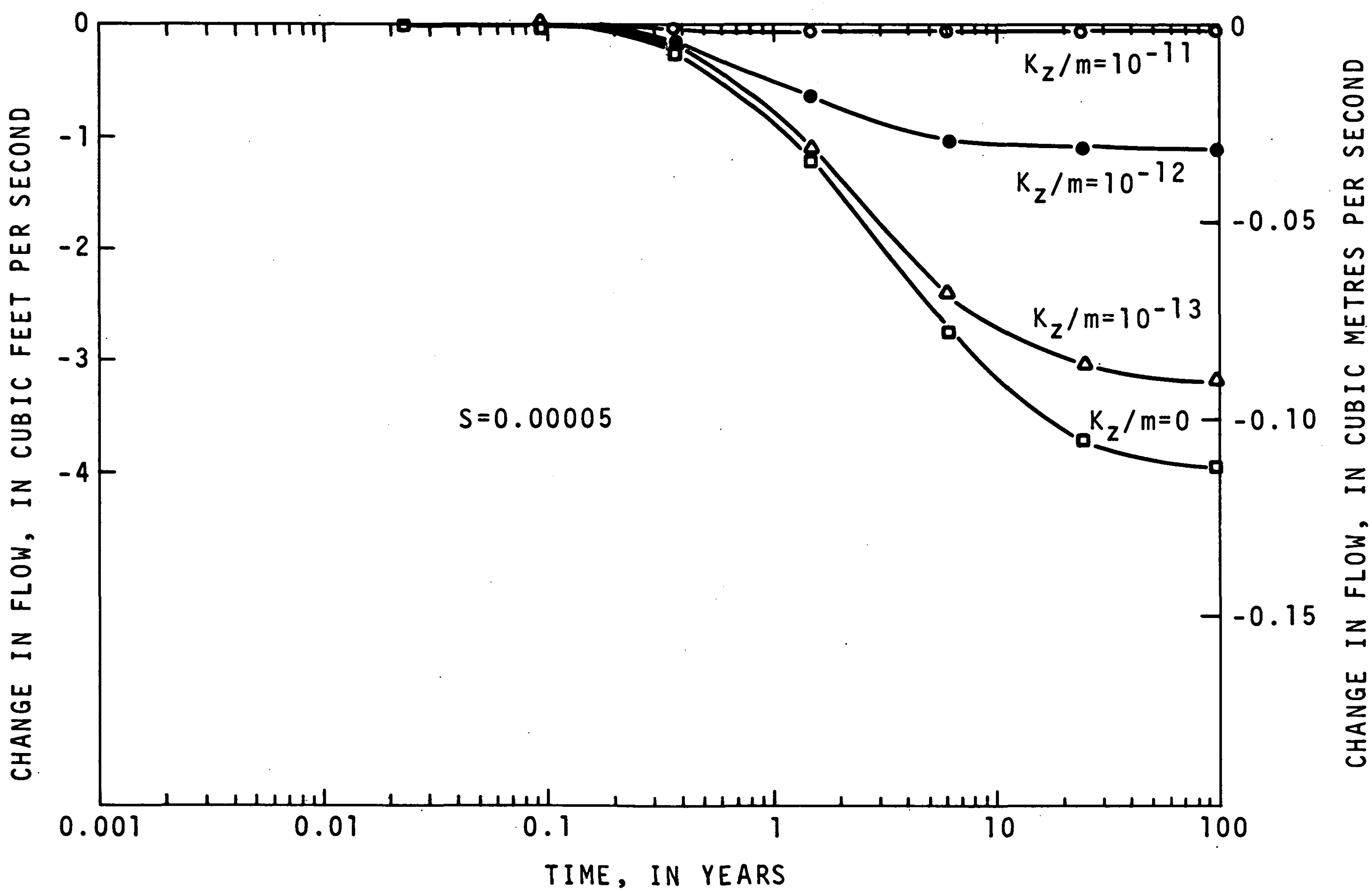

Figure 14.--Computed decrease over time in vertical leakage at node (28,22), which represents the discharge of Cascade Spring, S. Dak., for leaky aquifer conditions.

models are available to simulate and analyze either situation, but a better understanding of possible source beds, confining beds, and their relation to the Madison and other aquifers is needed. The leakance (K_z/m) is a critical parameter that must be defined on the basis of observed data. A long-term pumping test of the Madison that uses observation wells in both the Madison and in adjacent aquifers could help to define the nature of vertical leakage and allow more accurate predictions to be made of aquifer responses.

SUMMARY AND CONCLUSIONS

1. A preliminary digital model was calibrated to simulate existing steady-state ground-water flow in the Madison aquifer in the Powder River Basin and adjacent areas. The model integrated numerous factors that affect ground-water flow in the Madison, and calculated the total flow through the aquifer in the modeled area to be approximately 200 ft^3/s (5.7 m^3/s). Most recharge to the aquifer originates in or near the outcrop areas of the Madison Limestone in the Bighorn Mountains and Black Hills. Most discharge from the aquifer occurs through springs and wells and as underflow past the eastern boundary of the study area. As additional data become available, the model should be adjusted, modified, and recalibrated. The model can be used to predict the hydraulic effects of proposed ground-water development, but the accuracy of the predictions is limited by the accuracy of the input data that describe the aquifer properties and boundary conditions.

2. Many of the questions raised during this model study can only be answered by additional hydrologic, geologic, geophysical, and geochemical investigations. For example, water-table altitudes in or near the outcrop areas should be mapped to refine the altitudes selected to represent the corresponding constant-head boundary cells. Also, the south-central boundary of the modeled area is poorly understood, and the possibility of flow across this boundary cannot yet be precisely evaluated. The hydraulic continuity and importance of the Madison aquifer south of this boundary is also uncertain. The approximation of a no-flow boundary seems reasonable, except for minor flow from the area between Glendo Reservoir and Guernsey Reservoir in Wyoming. As another example, the observed potentiometric surface exhibits anomalies in several areas, such as near the northern boundary of the modeled area, that could represent the influence either of steady-state vertical leakage in zones that may coincide with parts of some river valleys, or of major transmissivity changes in the aquifer. The difference is significant. Although some simplifying assumptions and approximations are necessary to understand and model the Madison aquifer, it must ultimately be considered not as an independent entity in itself, but rather as just one major element of a complex, three-dimensional, ground-water flow system.

3. The hypothesis that a hydraulic discontinuity exists in the Madison aquifer along the western margin of the Powder River Basin is tentatively accepted, mainly on the basis of the flow-model analysis. This zone of reduced and zero transmissivity appears to be associated with the area of steepest structural dip of the Madison Group, and therefore may be caused by faulting. This zone has a major effect on the computed flow of ground water in the aquifer, and further investigations should attempt to verify its existence, origin, extent, and impact. The possibility that discontinuities exist elsewhere in the Madison should also be considered.

4. The regional average transmissivity probably lies between 0.010 and 0.025 ft^2/s (0.0009 and 0.0023 m^2/s). However, regional variations in aquifer transmissivity have an important effect on heads and flow in the Madison aquifer. Part of the transmissivity variation is due to changes in the kinematic viscosity of the ground water, which are related to temperature changes of the water within the aquifer. Additional transmissivity variations are due to variations in the properties of the rocks, and a means to evaluate this factor is needed.

5. Additional temperature data are needed to define changes in the kinematic viscosity of the ground water more accurately. Such data are most needed near the axis of the Powder River Basin, where a better estimate of the maximum temperatures could also help to evaluate the feasibility of developing geothermal energy from the Madison aquifer.

6. An analysis of the range of possible transient responses of the aquifer to large scale well withdrawals indicates that the Madison aquifer could probably sustain increased development in the Powder River Basin area involving pumping of up to several tens of cubic feet per second from wells tapping the Madison aquifer. The greatest effect would be the lowering of the potentiometric surface, but a decrease in the flow of certain springs and streams also would occur.

7. The rate at which ground-water pumping effects spread through the aquifer is strongly related to the storage coefficient. The storage coefficient for the Madison aquifer probably ranges between 0.00001 and 0.00025, but values of greater accuracy and precision are needed and should be based on aquifer tests.

8. The extent and magnitude of the effects of ground-water pumping are strongly affected by vertical leakage, which may be induced or changed by drawdown in the Madison aquifer. A value of leakance between 10^{-13} and 10^{-11} (ft/s)/ft [(m/s)/m] appears reasonable, but cannot yet be substantiated. Also, additional work is needed to identify and describe the source beds, the confining beds, the potentiometric surface of the source beds, the thickness of the confining beds, and the vertical hydraulic conductivity of the confining beds. The latter may require aquifer tests with observation wells in both the pumped aquifer and the source beds. Accurate long-term predictions may also require estimates of the specific storage of the confining beds and the transmissivity and storage coefficient of the source beds.

9. Steady-state leakage between the Madison aquifer and underlying or overlying formations cannot yet be evaluated. Leakage may account for some of the unexplained variations or anomalies in the potentiometric surface, and should be more carefully evaluated in future studies. If leakage can occur under the stresses of transient conditions, it can likewise occur under steady-state flow.

10. A monitoring network should be designed and in operation as soon as possible so that background conditions of flow, heads, and water quality can be established prior to any future large-scale development. The great depth to the Madison Group will make the cost of drilling wells very expensive. So before observation wells are drilled, the possibility of utilizing existing wells to obtain data should be considered thoroughly. However, new wells would offer the opportunity to collect relevant hydraulic, geologic, geophysical,

and geochemical data for the Madison aquifer under carefully controlled conditions within some of the very large areas where data are not presently available.

Observation wells should be located throughout the Powder River Basin. There seems to be little need for permanent observation wells to be located either east of the Black Hills or west of Midwest, Wyo., because the effects of pumping probably would not spread into these areas. Some observation wells should be located in or near outcrop areas to help evaluate the effects of seasonal or long-term climatic cycles on recharge to the Madison aquifer. Although the accuracy of measuring the future effects of development will be approximately proportional to the number of observation wells that are maintained, regional effects probably can be determined with a network in which observation wells are not located closer than about 50 mi (80 km) apart. Water levels in the primary observation wells initially should be monitored continuously to determine the short-term range of fluctuations and evaluate the optimal frequency for future, routine, noncontinuous measurements. Perhaps at a minimal cost, secondary or supplementary observation wells could be selected from existing wells not included in the basic network. These could then be measured infrequently, such as once every 6 months to 2 years. Several primary and (or) secondary observation wells might be selected to monitor water-level changes in aquifers overlying the Madison. These would help measure the effects of vertical leakage.

11. Major ground-water development will result in a reduction of flow from some springs that discharge from the Madison aquifer; therefore a long-term monitoring network should include gages to measure their discharge. Future studies should also aim to determine what part, if any, of the discharge of Cascade Spring is derived from the Madison. If a significant part of the flow of Cascase Spring is from the Madison, it should be considered as a prime candidate for inclusion in the monitoring network. If it is not, then this preliminary digital model must be modified accordingly.

12. As the results of monitoring become available in the future, the efficiency and effectiveness of the monitoring network should be reevaluated frequently, and revised if necessary. Any specific proposal for large-scale development should be considered individually to ascertain the need for additional hydraulic data and observation wells.

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